NTNU – Trondheim
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A Multi-Stage Stochastic Facility Routing Model for Humanitarian Logistics Planning

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A Multi-Stage Stochastic Facility Routing Model for Humanitarian Logistics Planning

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June 8, 2012

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Preface

This master thesis is conducted as the final part leading to achievement of a Master of Science degree in Industrial Economics and Technology Management at the Norwegian University of Science and Technology (NTNU), spring of 2012. The work presented integrates the fields of humanitarian logistics and operations research. It is written as a continuation and extension of the specialization project within the field of Managerial Economics and Operations Research the preceding fall of 2011. We call attention to the fact that the main title of the thesis has been altered from its original version as stated in the master contract.

We wish to express our sincere gratitude to our academic supervisor Associate Professor Lars Magnus Hvattum from the Department of Industrial Economics and Technology Management at NTNU, for his immeasurable patience, his good humour and invaluable counsel throughout the bygone year. We would also like to give special thanks Tørris Jæger, Head of Disaster Management at the Norwegian Red Cross, for providing first-hand field knowledge of the area of disaster management.

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Abstract

This thesis presents a humanitarian logistics decision model to be used in the event of a disaster. The operations under consideration span from opening of local distribution facilities and initial allocation of supplies, to last mile distribution of aid. An introduction of the field of disaster management is given, which forms the basis for the following description of the disaster response problem faced by the decision maker.

Two mathematical models are developed aiming to enable efficient decision making. The mathematical models solve the disaster response problem and seek to maximize the utility of distribution of aid amongst beneficiaries. Utility is expressed in terms of amount of satisfied demand and cost-effectiveness.

The main mathematical model is formulated as a multi-stage mixed-integer stochastic model to account for the difficulty in predicting the outcome of a disaster. The model will be applied to earthquakes in particular for reasons of concreteness. Accessibility of new information implicates initiation of distinct operations in the humanitarian supply chain, be it facility location and supply allocation, or last mile distribution planning and execution. The realized level of demand, in addition to the transportation resources available to the decision maker for execution of last mile aid distribution, are parameters treated as random due to uncertainty. Complete information regarding these variables is revealed in stage two. As a direct consequence of treating demand as an uncertain parameter, marginal utility will also be subject to stochasticity. Also, the state of the distribution network is treated as a random parameter due to uncertainty arising from the vulnerability of the local infrastructure. Reception of complete information concerning the state of the infrastructure indicates transition from stage 2 to stage 3.

The mathematical models are applied to an illustrative example to demonstrate their application as decision-making tools in practice. An assessment of the applicability and validity of the stochastic program is made, based on several test instances generated by the authors. Results show that instances of considerable size are challenging to solve due to the complexity of the stochastic programming model. Still, optimal solutions may be found within a reasonable time frame. Moreover, findings prove the value of the stochastic programming model to be significant as compared with an deterministic expected value approach.

Sammendrag

Denne masteroppgaven presenterer en beslutningsmodell innen fagfeltet humanitær logistikk, til bruk i en katastrofesituasjon. Beslutningene omfatter åpning av lokale distribusjonsfasiliteter, initiell allokering av forsyninger og distribusjon av disse. Som relevant bakgrunn for problemstillingen vil en introduksjon av fagfeltet katastrofeledelse bli presentert, før problemet beslutningstageren står overfor blir beskrevet.

To matematiske modeller er utviklet. De har til hensikt å tilrettelegge for effektiv beslutningstagning. Modellene løser problemet knyttet til katastrofehåndtering ved å maksimere nytten av effektiv distribusjon av nødhjelp. Nytte er definert som tilfredsstillelse av etterspørsel og kostnadseffektivitet.

Den mest omfattende modellen er formulert som en fler-stegs stokastisk modell med enkelte heltallsrestriksjoner. Denne tar høyde for utfordringene knyttet til uforutsigbarhet i omfanget av katastrofer generelt, og er i oppgaven applikert på jordskjelv spesielt. Tilgjengeliggjøring av ny informasjon markerer initialiseringen av nye handlinger i den humanitære verdikjeden, i form av plassering av fasiliteter, nødhjepsallokering, og planlegging og gjennomføring av distribusjonen. Reell etterspørsel og beslutningstakerens tilgjengelige transportressurser er to av faktorene som er gjenstand for usikkerhet. Fullstendig informasjon om disse, blir realisert i steg to. En direkte konsekvens av å betrakte etterspørsel som en usikker parameter, er at marginalnytten også er stokastisk. Tilstanden til distribusjonsnettverket er den siste usikre parameteren som betraktes, på grunn av sårbarhet i den lokale infrastrukturen. Tilgjengeliggjøring av informasjon angående nettverkets tilstand markerer begynnelsen til det tredje steget.

De matematiske modellene er anvendt på et illustrativt eksempel for å demonstrere deres gyldighet som beslutningsstøtteverktøy i praksis. En vurdering av gyldigheten til den stokastiske modellen blir deretter gjennomført basert på instanser generert av forfatterne. Resultatet viser at omfattende instanser er utfordrende på grunn av kompleksiteten til den stokastiske modellen, men at optimale løsninger oppnås innen rimelig løsningstid. Funn angir dessuten verdien til den stokastiske løsningen til å være signifikant bedre enn en tilsvarende deterministis løsning basert på forventningsverdier.

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List of Abbreviations

CTP	Covering Tour Problem
DFU	Demand Fulfillment Utility
DRP	Disaster Response Problem
EEV	Expected result of using the deterministic expected value solution
EV	Expected Value Problem
EVPI	Expected Value of Perfect Information
FEMA	Federal Emergency Management Agency
FLP	Facility Location Problem
FRP	Facility Routing Problem
ICD	International Central Depot
IFRC	International Federation of Red Cross and Red Crescent Societies
LDC	Local Distribution Center
LP	Linear Programming
MIP	Mixed Integer Problem
MSP	Muliti-Stage Stochastic Programming
MU	Monetary Utility
NFP	Network Flow Problem
POD	Point of Distribution
RAP	Resource Allocation Problem
RP	Recourse Problem
SCAP	Single Commodity Allocation Problem
SMIP	Stochastic Mixed Integer Program
SP	Stochastic Programming
VRP	Vehicle Routing Problem
VSS	Value of the Stochastic Solution
WS	Wait-and-See Solution

1 Introduction

Natural disasters such as droughts, earthquakes, hurricanes and floods have proven a global challenge in their unpredictable nature and potential scale of impact represented by fatalities and social, environmental and economic costs. The Haiti earthquake of 2010 efficiently demonstrated the potential severity of events following a natural disaster. It killed 222 570 people and affected a total of 3.9 million others. The disaster caused an estimated US$ 8.0 billion worth of damages, and led to the collapse of around 70 per cent of buildings and homes [Guha-Sapir et al., 2011]. In 2011, 302 natural disasters at large claimed over 29 780 lives worldwide, affected nearly 206 million others and caused record economic damages of US$ 366 billion [Guha-Sapir, 2012]. These figures substantiate the vitality of providing aid of the appropriate kind and amount to those affected in the most efficient and effective way possible, in order to prevent loss and suffering [Christopher and Tatham, 2011].

The Center for Research on the Epidemiology of Disasters (CRED) defines a disaster as:

> [A] situation or event which overwhelms local capacity, necessitating a request to a national or international level for external assistance; an unforeseen and often sudden event that causes great damage, destruction and human suffering. [Guha-Sapir et al., 2011, page 7]

The number, magnitude and economical impact of disasters are on the increase along with the overall size of the global population. Hence, scholars agree that advance in the management of disaster operations is imperative, and will contribute to an improvement in readiness, increase response speed, ease recovery and provide institutional learning over time [Altay and Green III, 2006, Christopher and Tatham, 2011, de la Torre et al., 2011, Thomas and Kopczak, 2005].

A term commonly applied to describe the process of distributing required aid and supplies in disaster relief situations, is humanitarian logistics. Thomas and Kopczak [2005] introduced the following widely adopted definition of the term:

> [T]he process of planning, implementing and controlling the efficient, cost-effective flow and storage of goods and materials, as well as related information, from the point of origin to the point of consumption for the purpose of alleviating the suffering of vulnerable people. [p.2]

A range of aspects common to the variety of disasters, impedes the execution of efficient humanitarian logistics. Uncertainty and unpredictability characterizes the surroundings of disasters, and the logistical activities are to be performed in rapidly changing environments. Knowledge of timing and location of events is substantially difficult, if not impossible, to predict with any significant degree of certainty [Christopher and Tatham, 2011]. The same applies to the total magnitude of events immediately following a disaster. This translates into a number of different aspects: Uncertainty regarding the nature of demand, capacity of facilities to be used in the distribution process, the transportation capacity, and amount of supply available for the decision maker, along with several other factors. Adding to this is the implicit urgency of need. Not only is the decision maker in charge of the distribution process required to make decisions based on limited and unreliable information, he must also make these at the earliest possible point in time following a disaster in order to prevent lives from being lost [Altay and Green III, 2006, de la Torre et al., 2011].

Accompanying decision making based on insufficient data, is the risk of making irreversible critical decisions [Altay and Green III, 2006]. Yet another complicating factor is the potentially severe damages done to the infrastructure both in relation to communication and transportation network in the area surrounding the point of impact [Christopher and Tatham, 2011]. This efficiently impedes the generation of reliable distribution routes from source of supply to location of need. As a result of this, estimates suggest that logistics in terms of movement of goods and people accounts for the largest amount of expenditures in any disaster relief operation [Clark and Culkin, 2007, Christopher and Tatham, 2011, Thomas and Kopczak, 2005].

Against a backdrop of uncertainty, the scope of this thesis is to present a decision making tool for use in catastrophic events which incorporates stochastic aspects. The purpose of considering elements of randomness is to enable viable decisions when information is limited, in order to reduce the risk of generating infeasible plans. A multi-stage stochastic model consisting of three stages is proposed in order to capture uncertainty in demand, capacity of the vehicle fleet and the state of the infrastructure. Maximizing the utility provided to the affected society constitutes the objective of the model. It is to be applied on a multi-commodity network flow problem with multiple modes of transportation. The initial stage concerns facility location decisions, whereas the last two stages involve last mile distribution decisions.

Decisions regarding location of local distribution centers in the affected area, along with the amount of supplies to provide these with, are made in the initial stage of the distribution process. A tentative last mile distribution plan is generated in the second stage determining the number of vehicles to use, and the load to assign to these vehicles. The decision are made according to realized demand and capacity, whereas information regarding the state of the local infrastructure is yet to be realized. The actual routes of the vehicles, are drawn up in the third and final stage based on realization of the infrastructure and the predetermined load of the selected vehicles. These decisions are made while seeking adherence to the initially developed plan to the extent possible. Although readily applicable to a wider range of disasters owing the the generality of the model, this thesis will primarily deal with humanitarian response in relation to earthquakes for reasons of concreteness.

The thesis is organized as follows. Section 2 gives a general presentation of the field of humanitarian logistics as an activity contained by disaster management. This entails an introduction of the humanitarian supply chain and its unique features as compared to its commercial counterpart. Also the distribution partners involved in the distribution process and the initial assessments necessary to map out the scope of the situation are described.

Section 3 states the disaster response problem at hand in terms of important attributes to the problem. Aspects of uncertainty, the criticality of considering fairness in distribution and the objective of distribution are discussed in greater detail. Following is a review of literature relevant to the disaster response problem in order to illustrate the differentiating value of the model proposed in this thesis, given in Section 4. Section 5 presents the underlying deterministic model, which forms the basis for the stochastic framework representing the center of interest in the remainder of this thesis. Underlying assumptions and limitations of the model is stated, along with the complete mathematical formulation and an accompanying exposition of its constituent elements.

Motivation for deployment of multi-stage stochastic programming and a description of the implications of its use is provided in Section 6, leading up to the core of the thesis: a formal introduction of the multi-stage stochastic model in Section 7. As for the deterministic model, preliminary assumptions and limitations are listed followed by the mathematical formulation of the model and consecutive interpretation. Issues related to the implementation of both mathematical models in commercial software are given in Section 8, with emphasis on the process of instance generation aiming to develop realistic test data. Section 9 provides the reader with a synthetic, yet comprehensible, numerical example in order to illustrate the characteristics, application and output of the models. Finally, an evaluation and validation of the stochastic programming model is given in Section 10 in terms of computational efficiency and applicative value as compared to its deterministic counterpart. The thesis is rounded off with conclusions of the findings and directions for future work in Section 11.

2 Disaster Management

This section provides a description of disaster management in general and the field of humanitarian logistics. A review of the main phases and associated activities involved in disaster management will be given, in addition to a clarification of central terminology to be used hereinafter.

In advance of, during, and in the aftermath of a disaster, efforts are exerted to minimize potential impact on the community in terms of harm to life, property and the environment at large. Disaster management is a generic and universal term often used to capture a large set of actions taken in handling catastrophes. Key activities performed include the attempt to decrease exposure to the consequences of disasters, developing measures to address initial impact as well as post-disaster response and recovery needs [Coppola, 2007].

2.1 The Four Phases of Disaster Management

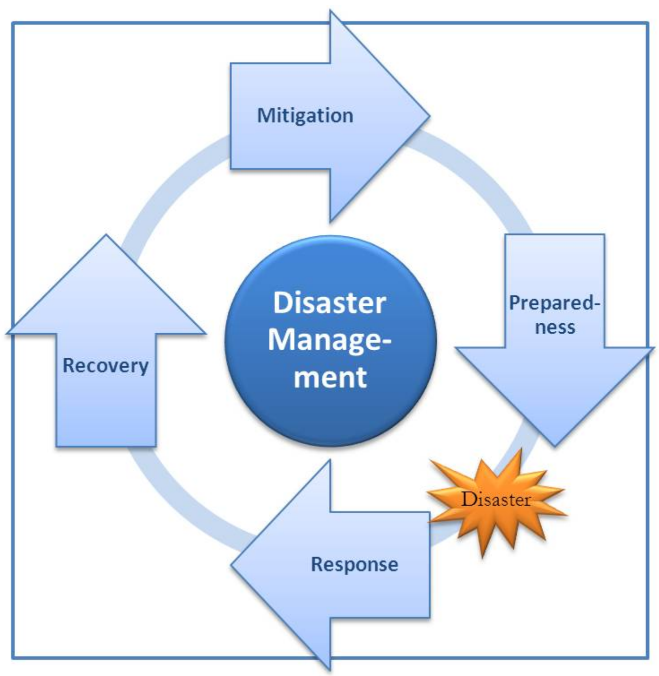

Figure 1: The four phases of disaster management

Disaster management is widely recognized to consist of four distinct phases; mitigation, preparedness, response and recovery [FEMA, 2012], [Sangiamkul and van Hillegersberg, 2011, Rawls and Turnquist, 2012, Altay and Green III, 2006]. Their occurrence depends on the point in time when activities are performed relative to the point of impact of the disaster in question, as illustrated in Figure 1.

During mitigation, measures are taken to prevent the onset of a disaster, or reduce its impact should one occur. The objective is to protect people and structures, and reduce the costs of response and recovery. Mitigation involves identification of potential events and associated likelihood of occurrence and consequences, combined with development of a strategy aiming to eliminate risks or reduce losses. Building of wind shelters and tsunami resistant shelters are examples of mitigation measures taken in the US.

Preparedness seeks to build an emergency management function to enable rapid response for disasters that cannot be eliminated. Development of emergency operation plans to identified hazards, recruitment and training of staff, identification of resources and supplies and designation of facilities, are amongst the activities carried out in this phase.

As opposed to mitigation and preparedness, response is executed in the time period subsequent to disaster impact. The central function during this phase is execution of preparedness plans by conducting operations seeking to save lives and reduce damage. Among the actions taken are emergency assistance and restoration of critical infrastructure. Different assessments of the situation at hand are also an important aspect of the response.

During the recovery phase, actions are taken to stabilize and restore the community in order to enable return to self-reliance. In addition, protection against future hazards is considered. Also this phase takes place in the aftermath of the immediate impact of the disaster, and can have both short and long term perspectives. Recovery includes activities such as restoration of infrastructure, provision of loans and grants to individuals and businesses, crisis counseling and provision of legal services.

The scope of this thesis is confined to immediate post-disaster response, where uncertainty concerning the accuracy of information upon which a distribution regime is built influences the outcome. The remainder of this thesis thus focuses solely on the humanitarian response phase.

2.2 Relevant Features of Humanitarian Logistics

Movement of goods and people accounts for up to 80 % of the costs in any disaster relief operation, making it the most expensive part of the operation [Clark and Culkin, 2007, Sangiamkul and van Hillegersberg, 2011]. Creation of an effective disaster supply chain is in consequence an essential function of disaster management in order to enable delivery of necessary goods to disaster relief organizations in a timely manner. Sheu [2007] states that the first three days following any disaster are the most critical, an argument emphasizing the vitality of effectiveness in distribution of aid.

The function performing humanitarian response is often referred to as humanitarian logistics. This term is widely used, and covers operations ranging from supply chain strategies and processes to technologies which will maintain the flow of goods and materials required by the humanitarian agencies [Baldini et al., 2011]. At its simplest, the operations involve procurement, dispatch of aid for shipment to the beneficiary region, storage in either national or regional warehouses, and eventually transport to the extended and final distribution points where the aid is handed to the beneficiaries [Maspero and Ittmann, 2008]. Sangiamkul and van Hillegersberg [2011] argue that logistic activities are executed in each of the four phases in disaster management, however with differing volume, urgency and variety of supplies.

2.2.1 The Humanitarian Supply Chain

To better understand the decisions and priorities made in operations related to humanitarian logistics, we need to assess central characteristics of the humanitarian supply chain as compared to its commercial counterpart. On a strategic level, Beamon [2004] differentiates between the two supply chains in terms of their goals and performance measures. When the humanitarian supply chain seeks to minimize the loss of lives and alleviate suffering, the commercial supply chain mainly wishes to maximize profit. Apte [2010] states that money is not a panacea in humanitarian logistics. Rather, lack of preparation in terms of disrupted infrastructure and prepositioning of goods, amplified with poor last mile distribution, prove to result in huge quantities of available aid never reaching the disaster victims. With regards to performance measures, it is recognized that the traditional cost minimizing measures are not central in humanitarian supply chains. There exists other measures such as time required to respond to a disaster, or the ability to meet the needs of the disaster which describe the performance more appropriately. Vitoriano et al. [2010] add to these fairness of the distribution, reliability and security of the operation routes.

When considering tactical and operational facets of humanitarian supply chains, Blecken [2010] emphasizes four important aspects: 1) uncertainty in critical factors; 2) underinvestment in research and infrastructure; 3) poor local transportation and communication infrastructure; and 4) lack of professional staff and training of logistics personnel. These aspects will be discussed in further detail in the following.

In situations of humanitarian response, crucial attributes to the problem are uncertain. This uncertainty is reflected in the number of people affected and corresponding demand, the amount of available information, and the lack of information related to personnel, equipment and the state of the infrastructure. The first two aspects may stem from inconsistent communication between the receiver of relief and the corresponding providers of information, i.e. between the affected population and rescuers, locals or reporters. As opposed to business logistics in which the demand information is provided actively and directly by customer themselves, the sources of on-the-spot relief demand information can be limited and almost unidentifiable in the immediate aftermath of a disaster [Balcik and Beamon, 2008]. Knowledge of aggregated demand is in addition a prerequisite in emergency logistics, as opposed to the disaggregated demand information which is conventionally treated in business logistics. To a certain extent, such relief demand information is rather fuzzy and hard to predict. This is due to lack of referable historical time series data, which further substantiates the need for real-time relief demand forecasting [Sheu, 2007].

The way humanitarian supply chains are funded with consistent under-investment in research and infrastructure, constitutes yet another differentiating aspect of the humanitarian supply chain [Blecken, 2010]. This in turn limits the potential to improve their efficiency and responsiveness. Stocks pre-positioned in preparedness for emergency relief response are typically not sufficient to cover the totality of demand. In an emergency situation, unmet demand can result in loss of life. The overriding objective in the event of a disaster is thus to mitigate the effect of the emergency by keeping the level of unmet demand at a minimum [Shen et al., 2009b].

Blecken [2010] gives special mention to the poor local transportation and communication infrastructure that tend to characterize the humanitarian supply chain, even in advance of a disaster. After a disaster has struck, severe damage to roads and infrastructure at large frequently occurs. Such an outcome inhibits efficient distribution of required aid and triggers the need for alternative modes of transportation. In terms of issues related to import of humanitarian aid, the Logistics Cluster [2012a] emphasizes the obstacles caused by national customs regulations. All goods entering or exiting a country have to undergo certain government control procedures and formalities, and complicated customs procedures can potentially cause delays resulting in congestion at port of entry. This in turn affects the flow of goods by 1) increasing the turn-around time for feeder vessels and railway wagons; 2) causing complex and non-transparent administrative requirements, often pertaining to documentation and 3) entailing high costs for processing trade information.

In terms of human resources, a lack of professional staff, training of logistics personnel and standardized work processes impose challenges both in planning and execution of distribution. These challenges are reinforced by high staff turnover among field logistics personnel, with an annual rate of up to 80% [Thomas and Kopczak, 2005]. Maspero and Ittmann [2008] suggest that the cause is due to an absence of clear career paths, adequate training and transfer of experience and knowledge, which is aggravated by a high pressure work environment.

2.2.2 Central Distributors

There are several contributors across the different tiers in the humanitarian supply chain. The Federal Emergency Management Agency (FEMA) [2008] and the International Federation of Red Cross and Red Crescent Societies (IFRC) [2012a] give descriptions of central actors, out of which mainly three are relevant for the remaining parts of this thesis. They comprise the International Central Depots (ICDs), the Local Distribution Centers (LDCs) and the different Points of Distribution (PODs). The relationship between these actors is depicted in Figure 2. An individual presentation of the three will be given in the following.

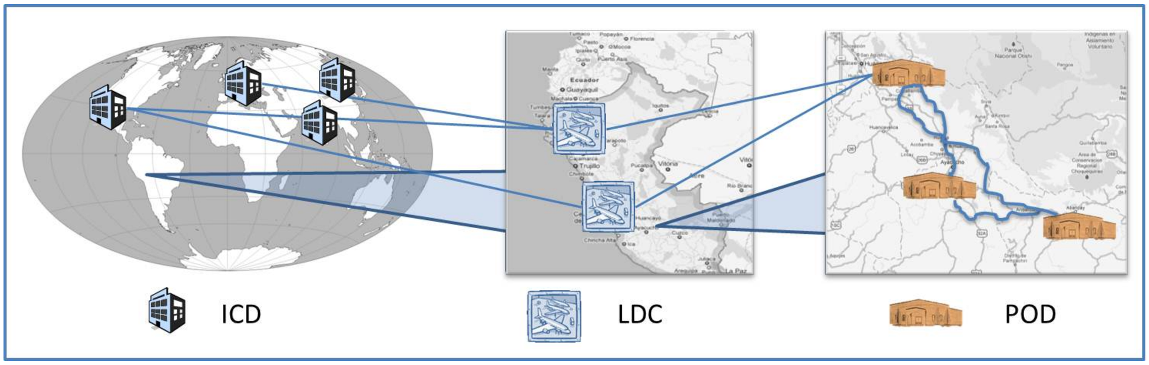

Figure 2: The relationship between relevant distributors at different levels in the humanitarian supply chain

The ICDs serve as the initial source of supply. They deliver stock, warehousing and fleet services, as well as general logistics support to operations. The IFRC has established regional logistics units in Dubai, Kuala Lumpur and Panama which keep pre-positioned commodities in stock [IFRC, 2012a]. Standard non-food and shelter items such as blankets, hygiene parcels, kitchen sets, jerry cans, buckets, mosquito nets, plastic sheeting, shelter kits and tents are among the supplies held at the ICDs. The key objective of each unit is to be able to deliver specified relief items globally to 5,000 families within 48 hours of a request, and to a further 15,000 families within two weeks. The three units contain stockpiles sufficient to meet the needs of 300,000 people [IFRC, 2012b]. The structure and mandate of the regional units are identical. However, there are differences in the regions in terms of exposure to disasters and scale of needs. Each regional unit is responsible for a designated area in accordance with Figure 3. The Panama ICD covers America, the Dubai ICD serves Europe, Africa and the Middle East, whereas the Kuala Lumpur ICD supplies Asia and the Pacific [IFRC, 2012b].

The LDCs are classified as one of two types; temporary or permanent. The permanently established LDCs serve as year-round emergency supply storages, and are prepared to handle minor seasonal natural disasters such as floods and hurricanes on their own. LDCs of this type also serve as distribution centers should major catastrophes occur, but will under these circumstances require additional supply from an ICD. The fact that they hold a certain level of stock at all times, enables them to provide initial help faster. The stationary

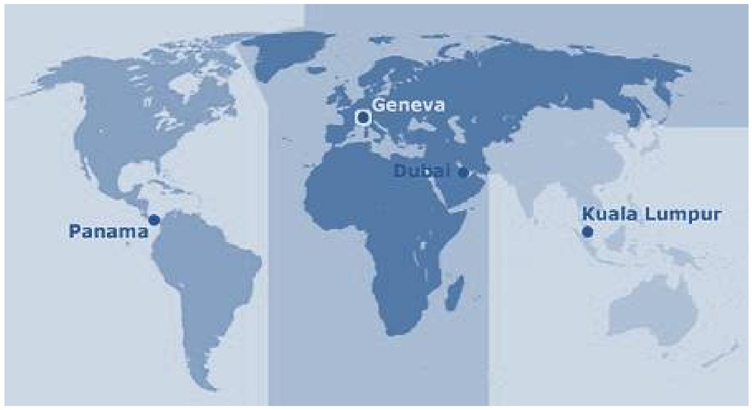

Figure 3: Location of International Central Depots providing initial supplies

LDCs are thus subject to accompanying holding and operating costs [Zhu et al., 2008]. The temporary LDCs on the other hand, are non-stationary and non-operational in times of no crisis. They are commonly located at airports, train stations, harbors or other sites adequate for handling large inflows and outflows of goods and personnel, and can serve as drop points. LDCs of this type are provisional and do not have an explicit holding cost due to the temporary nature of the supply [Zhu et al., 2008].

There is an impending risk that possible sites for LDCs are destroyed during the disaster. Ascertaining feasible destinations for LDCs prior to the crisis is thus a crucial element in ensuring efficient allocation of aid [FEMA, 2008].

The Federal Emergency Management Agency (FEMA) [2008] describes a POD as a centralized location at which the public can collect life sustaining commodities. Shelf stable food, bottled water and limited amounts of ice, tarps, and blankets exemplify commodities of this nature. The actual amount and type of commodities sent to the different PODs are determined at the LDCs, as is the type and location of PODs to activate. The LDCs are also in charge of operation and eventually demobilization of the PODs [FEMA, 2008].

2.2.3 Initial Post-Disaster Assessments

In the event of large-scale disasters, international aid agencies, national authorities and occasionally the United Nations have to coordinate their emergency response to ensure efficient help. There exists a range of institutions seeking to assist in this process by providing scientific information detailing the extent of the disaster. The European Mediterranean Seismological Centre (EMSC), which supplies rapid earthquake information services, represents an organisation of this sort. When an earthquake hits a certain area, seismological networks of seismic stations around the globe collect real time parametric data. These data are automatically associated, merged and processed at the EMSC in order to publish an up-to-date catalogue of seismicity in terms of location and magnitude of earthquakes. The information is published online at EMSC's website. For potentially destructive earthquakes, this information is also disseminated via SMS, email or fax to registered end-users within 20-30 minutes after the earthquake's occurrence [Mazet-Roux et al., 2010]. If an aid agency decides to act, secondary information will be gathered. As opposed to first-hand information, secondary information refers to sporadic information which has been collected from affected people or other organisations. It can either relate to an earlier relevant situation or to the current one. Gathering information from affected areas has become easier due to organisations such as Ushahidi. This agency provides an emergency platform that handles SMS, MMS and emails sent to a specific known number from locals on-site. The platform both translates messages into English and sorts information regarding type and location of need. Besides websites such as Ushahidi; Twitter, Facebook, Google Maps and other technology features make it easier to visualize the consequences of the earthquake [Bulkley, 2010, Jæger, 2012a].

Generally, a local agent on-site will send in a team of experts to complete initial assessments regarding the extent of the disaster and people affected. The aim of the initial assessments is to better understand the situation in order to identify potential problems, and corresponding sources and consequences of these problems, which can complicate the response process. Upon major disaster, a rapid assessment should be conducted [IFRC, 2008]. Its purpose is to gather information concerning the needs of the affected population, existing capacity, possible areas of intervention and resource requirements. A rapid assessment normally takes a week or less. High-priority information can however be gathered between 2 and 72 hours after the disaster has struck. Several assumptions need to be made in order to be able to conduct the assessment owing to lack of information, limited time, security and safety limits. A rapid assessment should be followed by a detailed assessment to enable recommendations to be made; to support decisions concerning start-up of operations in new areas; or to monitor the situation in case of gradual change in state of affairs. Detailed assessments generally take about one month, depending on the complexity of issues and available resources [IFRC, 2008]. There are situations in which additional assessments are unnecessary due to inaccessibility of the affected area, adequacy of existing information or because assessments have already been carried out by other agencies. Cooperate between agencies involved in disaster operations is fortunately common.

3 Problem Description

This section will focus on the response phase of humanitarian logistics, as described in the preceding sections. The problem to be addressed will be stated in a general manner, and the timing and extent of decisions relevant to the decision maker will be described. Aspects concerning facility location and last mile distribution of aid to beneficiaries will be presented, and relevant elements of uncertainty with the prospect of complicating the distribution process will be accounted for. As will the vitality of considering fairness in distribution. Finally, the primary goal of disaster response operations will be explicated.

3.1 The Disaster Response Problem

This thesis considers the planning problem for a humanitarian supply chain in the event of an earthquake, the Disaster Response Problem (DRP). The main task is to establish at which drop-points supplies should arrive and be managed for further distribution, and the international depots from which these supplies should originate. The planning problem includes creation of a distribution plan for the available vehicle types and commodities, from point of supply to point of consumption via local distribution centers, in order to meet the immediate needs of the affected population. The problem is complicated by limited information and uncertain systems, resulting in distribution planning activities of high complexity. Figure 4 seeks to illustrate the course of events in disaster response as presented in this thesis, both in terms of the actions taken and the information received throughout the operation.

Shortly after an earthquake has occurred, aid agencies and/or the government will decide whether or not it is necessary to initiate emergency response. If so, local agents will start gathering information about the consequences of the disaster. The only accurate and reliable available information is the magnitude and the location of the earthquake. To manage the emergency response and distribution of goods efficiently and effectively, new or updated information has to be gathered continuously.

The local agent sends a team of experts into the area of relevance to complete an initial assessment of the extent of the disaster and the needs of the people affected. The assessment will serve as a basis for an appeal that lists specific items and quantities needed to provide immediate relief to the affected populations [IFRC, 2008].

Based on identification of the location of the demand points, the team of experts informs the local agent where to inaugurate PODs. The location of the PODs, together with knowledge of the state of the infrastructure prior to the earthquake, serve as the basis for determination of the LDCs to open and operate.

Based on historical data and the estimated impact caused by earthquakes of different magnitudes, the decision maker will develop a series of possible outcomes of the earthquake [Rawls and Turnquist, 2010, Barbarosoğlu and Arda, 2004]. Estimated demand for equipment, medical supplies/teams, food and non-food items at each PODs are listed based on the initial assessment and previous experience. In addition to the location of the LDCs, this list is information communicated to the ICDs. If conduction of the assessment is too time consuming due to inability of inspection or other obstacles impeding obtainment of information, a needs analysis based on similar cases will form the basis for the initial distribution from the ICDs. It should however be noted, that communicated information to the ICDs is only an estimate, and demand may be higher or lower for each commodity type at each POD. Goods are expected to be delivered to the LDCs as soon as the list of items are communicated, and less than 72 hours after the earthquake has hit [FEMA, 2008]. During these hours of waiting, calls are made to traditional government donors and the public to secure commitments of cash and in-kind donations.

If the local agent managing the LDCs stands in need of supplies from the ICDs, an account of the distribution plan and the associated costs will be required. Generally, aid agencies will not dispatch supplies if disposable funds are insufficient to cover the costs of final distribution to demand points [Jæger, 2012a]. As goods are sent from the ICDs, medical teams, vehicles and volunteers are engaged to the operating LDCs. Seeing as how the number of vehicles and volunteers to arrive on time is uncertain, the knowledge of exact transportation capacity at each LDC is not realized until packing of the vehicles that has actually arrived, is started.

Approximately between 48 and 72 hours after the disaster has struck, supplies enabling the local agent to provide drivers and others managing the LDC with an initial distribution plan, would normally have arrived at the LDCs. The vehicles are packed and dispatched according to this plan. In some cases, the vehicles will be prevented from completing their initial routes due to infrastructure damage. For those to which this applies, the local agent will be consulted and an alternative route will be generated based on updated information about the network. As time passes, accrued amounts of information about the state of the infrastructure will become available via satellite pictures and first-hand experience provided by drivers and local reports. Some vehicles will reach their planned destination PODs, whilst others may have to change destination along the way because of obstacles. Either way, their duties are considered completed when the commodities they carry are delivered.

3.2 Sources of Uncertainty Relating to Disaster Response

In this section, elements of uncertainty relevant in the planning of distribution of aid in event of disasters caused by earthquakes will be discussed. We will focus on the random elements of greatest importance when planning distribution of humanitarian aid.

Delay in supply to the LDCs is a major problem caused by restricting toll barriers, absence of emergency exception laws and local governments' unwillingness to receive help [de la Torre et al., 2011]. However, these aspects go beyond the scope of this thesis, and these political and legal issues will not be treated further because of their unpredictable and nation dependent nature making them difficult to generalize. Also the amount of donations made by the public, aid agencies and governments are hard to prognosticate in the immediate aftermath of the catastrophe.

There are several other factors however, which influence the development of the local agent's strategies and restrict the number of alternatives. Firstly, demand is uncertain. The districts in need may be situated in remote areas, and the disaster site might be in a state of chaos making an complete overview impossible to achieve [Thomas and Kopczak, 2005]. Secondly, the size of the vehicle fleet and the available medical personnel are highly uncertain factors in humanitarian logistics. The consequences of not reaching a POD are severe, and uncertainty in the state of the infrastructure is the third main element of uncertainty considered in this study. To the extent that this study concerns uncertainty, its focus lays mainly on these three aspects and the consequences that they entail.

Unpredictable demand patterns increase the complexity of the distribution plan [Balcik et al., 2008]. Demand can fluctuate unexpectedly due to a number of reasons. These reasons include after-shock damages, people returning to greater self-sufficiency, beneficiaries moving between different areas in hopes of find greater relief, or unexpected challenges such as outbreak of disease epidemics [de la Torre et al., 2011].

Volunteer organizations state that they generally do not possess their own vehicle fleet. The implication of this fact is that multiple independent local drivers and vehicles need to be hired and managed internally [de la Torre et al., 2011]. This in turn, complicates prediction of the size of the vehicle fleet, its total capacity, the experience and knowledge held by local drivers and the employable technology in the vehicles. In case of insufficiency of available vehicles, agencies may be required to import the amount needed. The ease of importing vehicles for a short time period and the ability to transport these to the requiring LDCs, influence the final vehicle capacity. An additional limitation arise from regular cost-benefit analysis. Engaging an excessive number of vehicles, especially high technology vehicles, will be very expensive and a waste of resources. The engaged amount should thus, to the extent possible, correlate with the amount needed in order to perform distribution [Jæger, 2012a].

Within the first 72 hours after impact, the vehicle fleet and the number of drivers available at the LDCs will be more or less confirmed, and an initial distribution plan will be determined. At this point in time we assume that the team of experts have more precise information concerning the level and nature of demand at each POD, which will naturally affect the distribution plan.

The initial distribution plan is subject to alternation because of the potentially severe damage typically caused to the local distribution network by an earthquake. Roads, bridges and airports are often destroyed. The accessibility of the different recipients may be reduced accordingly depending on their location relative to the earthquake's epicenter, and the quality of the infrastructure connecting the LDCs and the PODs. The attributes of a vehicle may in addition restrict it from using certain paths in the infrastructure. The cause of failures in parts of the infrastructure may be due to factors such as natural gas explosions, consequent fires, building, bridge or road collapse or road blockage [Günneç and Salman, 2007]. As a result, operable infrastructure needs to be estimated based on the decision maker's judgment and experience, and the distribution plan and emergency strategy based on this input may in effect not be applicable.

3.3 Considering Fairness in Distribution of Aid

Clark and Culkin [2007] propose three principles said to define humanitarianism: humanity, impartiality and neutrality. In short, these state that suffering should be alleviated wherever it is found, giving priority to the most urgent needs without discrimination.

The concept of fairness is vital to consider when distributing emergency supplies. The Sphere Handbook [The Sphere Project, 2011] is part of an initiative to determine and promote standards by which the global community responds to the plight of helping people affected by disasters. The Sphere Handbook argue that agencies should provide aid impartially and in accordance with need:

> Access to health services should be based on the principles of equity and impartiality, ensuring equal access according to need without any discrimination. Equity should be ensured so that similar food rations are provided to similarly affected populations and population sub-groups. [The Sphere Project, 2011, p. 182]

Generally, the needs of the most vulnerable sub-groups of the population, such as wounded, children, pregnants and women, is prioritized [Jæger, 2012a]. Consequently, the marginal utility of the delivered items reduces in line with reception of aid by the most indigent at each POD. Even though need for relief may still exist in an area, helping people of higher levels of distress in other regions before continuing to deliver to the initial region might be of greater utility. The requirement of prioritizing is caused by shortage in supply or capacity, or damages in the distribution network.

3.4 The Problem in a Nutshell

After an earthquake has struck, rapid response is of pronounced importance. As soon as the local agent receives any information, decisions regarding which LDCs to open, type and quantity of resources to order, and way of procurement and storage of the supplies have to be made. Acquirement of the appropriate means of transportation and personnel in addition to generation of efficient distribution plans, are crucial in order to keep fatalities to a minimum. The decision support tool presented in this thesis treats these challenges and serves as a support platform for decision making. When utilizing such a support platform, it is highly relevant that the decisions subject to implementation as suggested by the decision support tool, comply with the local agent's intuition and judgment. This will serve to ensure both efficient and logical decision making. The decision support tool has to provide sound solutions for the entirety of decisions made, both during the planning process as well as during implementation.

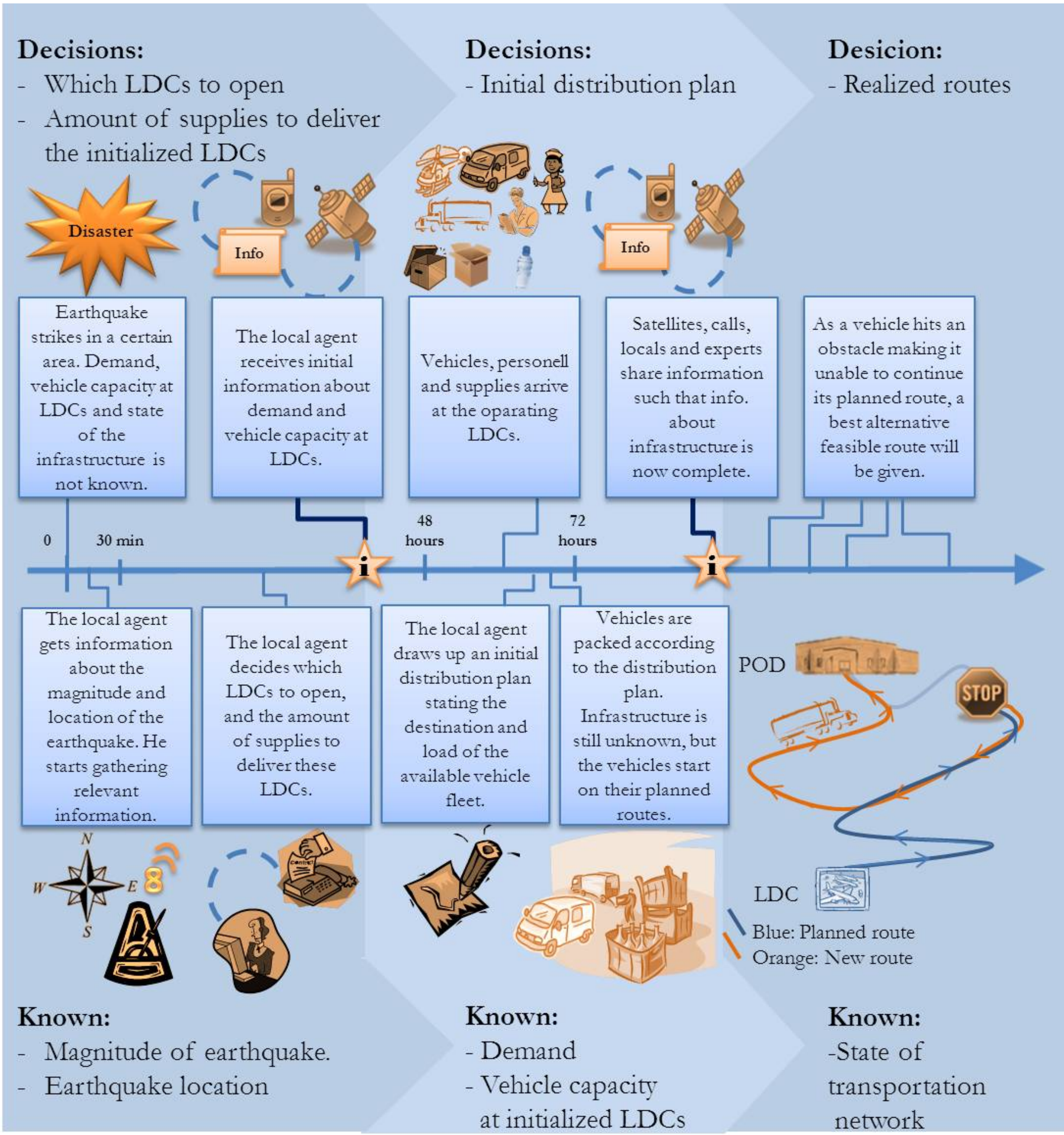

Figure 4: Timeline indicating flow of information and content of actions taken during emergency response

4 Literature Review

The number of studies conducted in the area of humanitarian logistics since its onset in the 1990s, has seen an increase in line with the number and impact of disasters [Hentenryck et al., 2010]. Different scholars have covered a variety of different classic models such as vehicle routing, network flow, facility location, location routing and supply chain management, in order to best describe the mechanisms of the humanitarian supply chain. The different aspects related to the humanitarian supply chain are reflected in the level of complexity of the models.

In the following we will present approaches taken by different researchers while considering characteristics relevant to this study. The features that distinguish our model from these will firstly be specified. This section serves as a foundation for comparison with the different models of other scholars presented in the following sections. An overview of characteristics in the models proposed by the researchers mentioned, is given in Table 1, in comparison with the model to be presented.

4.1 Characteristics of the Proposed Model

This report considers a three-stage stochastic programming model for distribution of humanitarian aid in the context of the disaster response problem. The first stage seeks to choose the most expedient allocation of commodities between a set of different local distribution centers sourcing from an international central depot, enabling efficient distribution in the stages to follow. Demand, vehicle capacity at the LDCs, and the state of the distribution network are subject to uncertainty in the initial stage. Last mile distribution of goods from the initialized local distribution centers to the final recipients is handled in the second stage, with infrastructure treated as the component of uncertainty. The third and final stage seeks to take corrective measures as to the routes of the vehicles after they have departed from their respective LDCs, based on realization of the state of the infrastructure. This model assumes pre-positioned international distribution centers, and is able to handle multiple commodity types and several modes of transportation. Considering the objective, the model seeks to maximize the beneficiaries' total utility in terms of the amount of demand satisfied at the expense of the remaining beneficiaries, thus achieving fairness of distribution. To the best of our knowledge, this is the first study to investigate such a problem.

4.2 Segmentation of the Humanitarian Supply Chain

Various approaches are taken by different scholars in terms of their choice of focal point within the humanitarian supply chain. The relationship between central depots (ICDs) providing initial supplies to local distribution centers (LDCs), is among the most common choices of tier [Balcik and Beamon, 2008, Campbell and Jones, 2011, Yi and Özdamar, 2007]. These models are mainly restricted to determining the number and location of LDCs to initialize, in addition to the amount of supply to be stocked at the distribution centers chosen. Last mile distribution considering the final stage of humanitarian logistics is another widely applied practice [Balcik et al., 2008, Barbarosoğlu and Arda, 2004, Hsueh et al., 2008, Nolz et al., 2010, Özdamar et al., 2004, Shen et al., 2009b,a, Vitoriano et al., 2009, 2010]. These models aim to develop vehicle schedules in order to allocate supplies between the affected population from the available pre-established distribution centers. Several scholars have also chosen to consider a wider part of the supply chain, integrating decisions concerning both pre-positioning of emergency supplies and last mile distribution of these supplies [Clark and Culkin, 2007, Günneç and Salman, 2007, Hentenryck et al., 2010, Mete and Zabinsky, 2010, Rawls and Turnquist, 2010, 2012, Salmerón and Apte, 2009, Tzeng et al., 2007, Yi and Özdamar, 2007, Zhu et al., 2008].

In addition to different selections of tiers to consider within the humanitarian supply chain, researchers also differs in terms of their choice of phase in disaster management, as described in Section 2.1. However, they show a distinct tendency to favor the phases covering the period just prior to or in the immediate wake of a disaster, or a combination of the two. Last mile distribution decision are obviously taken solely in the response phase, whereas facility location and pre-positioning decisions can be made both in preparation for, or in response to disaster events.

Campbell and Jones [2011] adopt a classic news vendor approach in order to consider prepositioning of supplies in preparation for a disaster. Rawls and Turnquist [2010] on the other hand, have developed an emergency response planning tool that determines the location and quantities of various types of emergency supplies to be pre-positioned, under uncertainty about if, or where, a natural disaster will occur. Their proposed model combines facility location, decisions concerning stocking levels of supplies, and distribution of those supplies to multiple demand locations after an event. They thus cover both the preparedness and response phase. As a continuation of their model, Rawls and Turnquist [2012] focus on the same two phases in constructing a dynamic allocation model to optimize pre-event planning for meeting short-term demands for supplies under uncertainty about location and quantity of demands which have to be met.

4.3 Choices of Fundamental Model Formulations

Last mile distribution of humanitarian aid is mainly formulated as vehicle routing problems (VRP) [Balcik et al., 2008, Günneç and Salman, 2007] or different variants of the network flow problem (NFP) [Barbarosoğlu and Arda, 2004, Vitoriano et al., 2009, 2010]. Özdamar et al. [2004] however, present a hybrid problem integrating the multi-period multi-commodity NFP with the VRP. Single commodity allocation problems (SCAP), resource allocation problems (denoted by RAP in this study), covering tour problems (CTP), facility location problems (FLP) and facility routing problems (FRP) have also been the subject of interest for a number of scholars. The latter problem integrates the discrete FLP and the VRP, and is addressed by Günneç and Salman [2007], Rawls and Turnquist [2010, 2012], Salmerón and Apte [2009] and Yi and Özdamar [2007] by means of network flow models.

Yi and Özdamar [2007] adopt a mixed integer multi-commodity network flow model which optimizes the locations and capacities of facilities, as well as vehicle routes and schedules. In addition to detailed vehicle routes and load instructions, the model provides optimal allocation of medical personnel. Balcik and Beamon [2008] handle facility location decisions exclusively, by employing a variant of the maximal covering location model. The model sets out to determine the number and locations of distribution centers, and the amount of relief supplies to be stocked at each distribution center in order to meet the needs of the people affected by a disaster. Balcik et al. [2008] consider a vehicle-based last mile distribution system, in which a local distribution center stores and distributes emergency relief supplies to a number of demand locations. The main decisions are allocation of relief supplies among the demand locations, and determination of delivery schedules by means of a VRP throughout the planning horizon. They propose a mixed integer programming model that determines delivery schedules for vehicles, and equitably allocates resources based on supply, vehicle capacity and delivery time restrictions. Transportation cost is minimized while the benefit to aid recipients is maximized.

Hentenryck et al. [2010] propose models for the SCAP that divide the distribution process into two phases. First, initial supply amounts are allocated in the network, followed by delivery to final recipients. They address the difficulty of treating both the storage problem and the routing problem simultaneously by proposing a three-stage algorithm. They decompose the decisions related to storage, customer allocation and routing, and apply mixed integer programming, constraint programming and large neighborhood search respectively. Zhu et al. [2008] attack a similar problem, the RAP, by means of a multi-commodity and multi-modal transportation network flow model. The model sets out to decide the type and amount of commodities to be maintained in local reserve depots in order to cope with slight disasters, while cooperating with local government for serious disasters. As opposed to the two latter researchers mentioned, Nolz et al. [2010] dwell on a CTP arising in a post-disaster situation. As a solution to this problem, a hybrid method based on generic algorithms, variable neighborhood search and path relinking aiming to distribute supply of aliment, shelter and medicine among the population affected by the disaster is developed. The outcome is a set of vehicle routes which satisfies criteria set by the authors.

4.4 Treatment of Uncertain Information

Several researchers recognize the need to consider the impact of random events accompanying natural disasters. Balcik et al. [2008] use a rolling-horizon framework to capture the inherent supply and demand uncertainties. To reflect vehicle-road compatibility, they associate the travel costs on arcs with vehicle types. Using this approach, if a road is damaged or cannot be used by a specific vehicle, the cost of traveling along that arc is assigned a large number. This preprocessing phase enables decision makers to consider the transportation infrastructure, and eliminates infeasible or undesirable routes. When making inventory decisions, Campbell and Jones [2011] account for risk of deterioration or inaccessibility of pre-allocated supplies and supply points after a disaster has struck by introducing a probability value associated with each potential supply point.

Vitoriano et al. [2009] and Vitoriano et al. [2010] on the other hand, have chosen to model uncertainty concerning the extent of detrimental effects caused to infrastructure via reliability analysis. Reliability is defined as the probability of executing an activity with success. This interpretation of reliability is translated to the proposed goal programming models by means of a measure stating the probability of being able to cross all arcs included in the applied solution. The aspect of security is considered in a similar manner by assigning each arc with a probability of vehicle ransack at crossing. This probability is used to calculate a global measure stating the probability of not being ransacked throughout the entire distribution process. These two attributes are included in separate goal constraints in which deviation from a given target is calculated, deviations which are to be minimized in the objective function. The humanitarian aid distribution system suggested by Vitoriano et al. [2009] additionally allows for choice of a second approach in which maximum ransack probability of arcs is minimized, and minimum reliability of arcs is maximized.

Another approach taken to cope with aspects of uncertainty caused by disasters, is multistage stochastic programming (MSP). The authors included in this literature review solely employ two-stage stochastic programming models. Barbarosoğlu and Arda [2004] consider uncertainty in demand when planning the transportation of humanitarian aid during emergency response. In the first stage, goods are pre-positioned by allowing movement of goods between existing supply depots. In the second stage, a transportation plan is drawn up based on existing supply and the realization of uncertain demand and arc capacities. Hentenryck et al. [2010] and Rawls and Turnquist [2010] model immediate post-disaster response under uncertainty in physical damage caused by the disaster. They both include pre-disaster first stage decisions of locating and stocking warehouses which can be damaged by the disaster. In the second stage, routes are constructed after obtaining information about demand and remaining supply. Demand, transportation network and surviving stock of various commodities after an event are all subject to uncertainty. In a similar manner, Rawls and Turnquist [2012] treat demand and infrastructure as stochastic elements, yet omitting potential deterioration of pre-positioned supply. Günneç and Salman

[2007] also propose a two-stage multi-criteria stochastic programming model. The facilities to be opened are chosen in the first stage, together with their capacities and the storage quantities of each commodity. In the second stage, the distribution of the commodities is optimized. This model accounts for failure in parts of the infrastructure in the event of an earthquake. The infrastructure is represented by links that may be non-operational after the disaster, with an estimated probability of failure. The demand nodes are also subject to deterioration, which in effect produces stochastic demand.

4.5 Objective Function Terms

Günneç and Salman [2007], Vitoriano et al. [2009], Hentenryck et al. [2010], Salmerón and Apte [2009] and Vitoriano et al. [2010] have all chosen multi-criteria optimization models to capture and balance the often conflicting objectives faced by the decision maker, as mentioned in Section 2.2.1. Vitoriano et al. [2009, 2010] propose goal programming models for designing vehicle routes in aid distribution problems. Main criteria involved in a disaster response operation such as time, cost, reliability, security and equity, are taken into account. The models consist of two network flow models, one for vehicle flow and another for load flow, as well as the relation between the two. Clark and Culkin [2007] wish to give greater priority to minimization of unsatisfied demand than to minimization of costs, in accordance with the performance measures essential to the humanitarian supply chain as stated in Section 2.2.1. In order to honor this priority policy, they introduce weightings of the objectives which should reflect the importance of the component in question. Single-objective models have also been utilized. Minimization of costs [Barbarosoğlu and Arda, 2004, Rawls and Turnquist, 2010, Balcik et al., 2008, Campbell and Jones, 2011] and unsatisfied demand [Balcik and Beamon, 2008, Özdamar et al., 2004] are the general approaches.

4.6 Consideration of Fairness

The notion of fairness within humanitarian logistics is subject to several different descriptions. In their review of literature presenting operations research models in transportation of relief goods, de la Torre et al. [2011] describes egalitarian policies as maximization of equality measures such as delivery quantity and speed. By adopting this definition of fairness, the models developed by Campbell and Jones [2011], Nolz et al. [2010], Hentenryck et al. [2010] and Mete and Zabinsky [2010] can be considered egalitarian in terms of delivery speed, in their minimization of time to deliver goods to beneficiaries. Similarly, we can argue that the model presented by Tzeng et al. [2007] is egalitarian in delivery quantity by their introduction of an objective function maximizing satisfaction of fairness and minimizing unfair distribution. Clark and Culkin [2007] address the aspect of fairness by enforcing a distribution policy whereby a minimum amount of a certain item has to be delivered to each demand point. Vitoriano et al. [2009] and Vitoriano et al. [2010] seek to obtain equity of distribution by adding a goal constraint which minimizes the maximum deviation of the load supplied proportional to the demand. The desired effect is to achieve delivery of aid to the less propitious areas in terms of their location, which otherwise would be surpassed. Balcik et al. [2008] also recognizes the need for incorporation of equity in relief aid distribution among affected areas while minimizing suffering. Obtainment of equity is achieved by requiring that their model serve an "equal allocation principle". This principle states that supplies should be allocated proportionally among the demand locations based on demand amounts and population vulnerabilities, and balance the unsatisfied and late-satisfied demand among demand locations over time.

4.7 The Use of Algorithms and Heuristics

Of the scholars treated in this review, several utilize algorithms in the problem solving process due to the computational complexity of problems describing disaster situations. By developing heuristic algorithms, they are able to solve large-scale instances of their defined problems. As large, complex real-world problems may prevent an optimizer from finding an optimal, or even feasible, solution using reasonable effort, the utilization of heuristics ensure that at least a feasible solution is produced within minimal time and storage requirements. Zhu et al. [2008] design an LP-relaxation algorithm by introducing an LP-rounding technique as a means to handle the potential large amount of integer variables which their resource allocation model (RAM) produces. Firstly, the LP-relaxation of the RAM is obtained by relaxing all variables' integer constraints. Secondly, the LP-relaxation is solved. Finally, an integer solution is obtained according to the fractional optimal solution of the LP-relaxation.

Hsueh et al. [2008] have developed a two-phase heuristic comprised of route construction and improvement of routes to tackle the dynamic vehicle routing problem for relief logistics which they present. Hentenryck et al. [2010], on the other hand, propose a multi-stage hybrid-optimization decomposition for SCAPs. The procedure involves a combination of a MIP model for stochastic commodity storage, a hybrid constraint programming/MIP model for multi-trip vehicle routing, and a large neighborhood search model for minimizing the latest delivery time in multiple vehicle routing. The use of these different technologies enables an exploitation of the structure of each individual optimization subproblem, and high quality solutions to real-world benchmarks. Özdamar et al. [2004] introduce an iterative solution approach based on Lagrangian relaxation, to be able deal with significantly large scale emergencies. Their proposed model is decomposed into two multi-commodity network flow models for commodity- and vehicle-flow respectively. These sub-models are coupled by means of relaxed arc capacity constraints using Lagrangian relaxation. Also Yi and Özdamar [2007] seek to make their location-distribution model for coordinating logistics support and evacuation operations in disaster response activities applicable to larger problem instances, which they achieve by post processing. The post processing is enabled through a routing algorithm that is pseudo-polynomial in the number of vehicles utilized, followed by the solution of a linear system of equations defined in a very restricted domain.

4.8 Distribution Systems in the Supply Chain

Of the various attributes ascribed to the distribution system of the humanitarian supply chain, the number of depots and characteristics of the vehicle fleet used in the distribution process have proven to be of the most relevance when comparing the literature considered in this review to the model presented in this study. Nolz et al. [2010] have chosen to present a multi-objective covering tour problem arising in a post-natural disaster situation, in which a single depot is available for utilization for the decision maker. The vehicle fleet considered is homogeneous in the sense that it consists of vehicles of equal capacity. Also Günneç and Salman [2007] fail to include multiple modes of transportation. In an attempt of achieving pre-positioning of emergency supplies, Rawls and Turnquist [2010, 2012] consider several potential storage facilities, but omit considering characteristics of a vehicle fleet. In the two-stage stochastic model specified for biological terrorism emergency scenarios proposed by Shen et al. [2009a], the problem is modeled by a single depot in which the decision maker utilizes a heterogeneous vehicle fleet. The most widely applied approach considering the literature treated in this review however, is modeling with multiple depots and a heterogeneous vehicle fleet. This is demonstrated by the model given by Mete and Zabinsky [2010] for the storage and distribution problem of medical supplies for disasters in general.

Table 1: An overview of relevant characteristics in models addressing the disaster response problem

	Problem type						*Uncertainty treatment*					
	VRP	*NFP*	*FRP*	*RAP/ SCAP*	*CTP*	*FLP*	*Stoch. (sup/)dem*	*Stoch. network*	*Stoch. cap.*	*Two-stage*	*Multi-stage*	*No stoch. elements*
This report		x	x				x	x	x		x	
Balcik and Beamon [2008]					x	x	x			x		
Balcik et al. [2008]	x											x
Barbarosoğlu and Arda [2004]		x					x	x		x		
Campbell and Jones [2011]						x						x
Clark and Culkin [2007]		x	x									x
Günneç and Salman [2007]		x	x				x	x		x		
Hentenryck et al. [2010]	x			x			x	x		x		
Hsueh et al. [2008]	x											x
Mete and Zabinsky [2010]		x	x				x	x		x		
Nolz et al. [2010]					x							x
Özdamar et al. [2004]	x	x										x
Rawls and Turnquist [2010]		x	x				x	x		x		
Rawls and Turnquist [2012]		x	x				x	x		x		
Salmerón and Apte [2009]		x	x				x	x		x		
Shen et al. [2009b]	x						x	x		x		
Shen et al. [2009a]	x						x	x		x		
Tzeng et al. [2007]			x									x
Vitoriano et al. [2009]		x										x
Vitoriano et al. [2010]		x										x
Yi and Özdamar [2007]		x	x									x
Zhu et al. [2008]				x			x	x		x		

	Objective function				*Depot*		*Vehicle fleet*		*Uses algorithms*
	Min cost	*Max sat. demand*	*Single obj.*	*Multiple obj.*	*Single depot*	*Multiple depots*	*Hetero. vehicles*	*Homo. vehicles*	
This report	x*	x	x			x	x		
Balcik and Beamon [2008]		x	x			x		+	
Balcik et al. [2008]	x	x		x	x		x		
Barbarosoğlu and Arda [2004]	x					x	x		
Campbell and Jones [2011]	x		x		x	x			
Clark and Culkin [2007]	x	x		x*		x	x		
Günneç and Salman [2007]	x	x		x**		x		x	
Hentenryck et al. [2010]	x	x		x**		x		x	Y
Hsueh et al. [2008]				x**	x		x		Y
Mete and Zabinsky [2010]	x	x		x**		x	x		
Nolz et al. [2010]				x**	x			x	Y
Özdamar et al. [2004]		x	x			x	x		Y
Rawls and Turnquist [2010]	x	x		x		x			Y
Rawls and Turnquist [2012]	x	x		x		x			
Salmerón and Apte [2009]		x		x**		x	x		
Shen et al. [2009b]		x		x**	x		x		Y
Shen et al. [2009a]		x		x**	x		x		Y
Tzeng et al. [2007]	x	x		x**		x	x		
Vitoriano et al. [2009]	x			x**		x	x		
Vitoriano et al. [2010]	x			x**		x	x		
Yi and Özdamar [2007]		x	x			x	x		Y
Zhu et al. [2008]	x		x			x	x		Y

* Minimization of cost is not the main goal of the model and is in effect given a near negligible weight
** Not all of the attributes taken into consideration by the model are listed

5 Presentation of the Deterministic Model

When omitting aspects of uncertainty, the problem described in Section 3 translates into a deterministic programming problem. We start by explaining the handling of nonlinear elements, followed by statement of the underlying assumptions and limitation used in the construction of the deterministic model. Finally, a formal presentation of the complete deterministic model will be given.

5.1 Handling Elements of Nonlinearity

Nonlinear problems represent an important group of optimization problems [Lundgren et al., 2010]. The different model formulations applied, vary according to the way the nonlinear relations are expressed. Mathematical nonlinear problems with integrality constraints on given variables are especially difficult to solve, and Lundgren et al. [2010] argues that few commercial softwares have solution processes which is efficient in solving these type of problems.

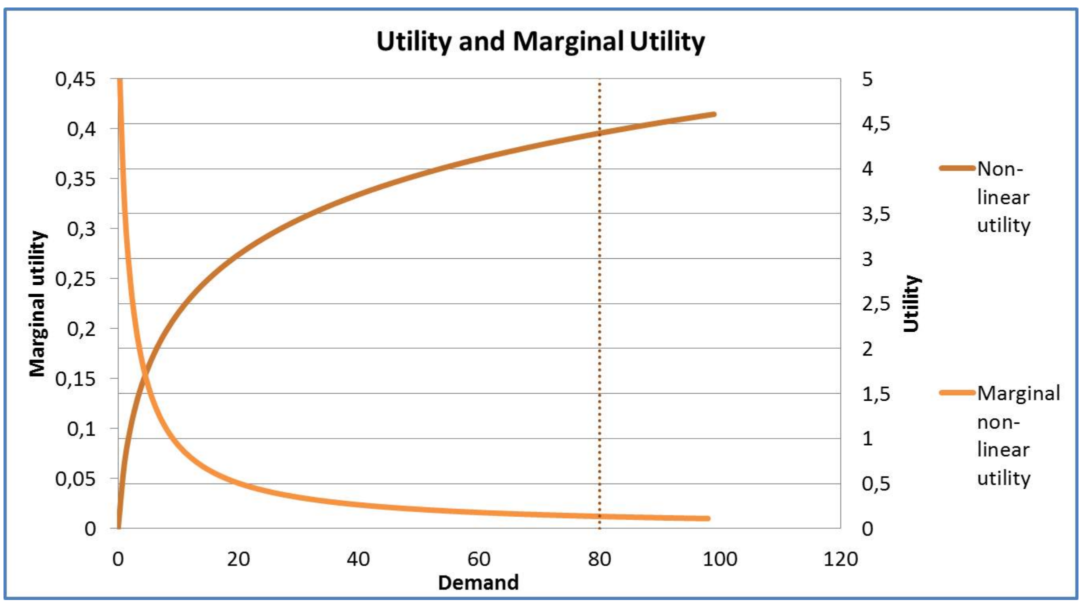

Figure 5: The nonlinear utility function and its marginal utility function

Taylor series expansion and quadratic, as well as linear, approximations are common methods used to avoid nonlinear functions. In order to tackle the challenging aspect of fairness described in Section 3.3, this problem will be subject to nonlinear maximization of utility, as depicted in Figure 5.

With reference to this illustration, the most obvious approximation approach to utilize for the mathematical models developed in this thesis, is that of linear approximations.

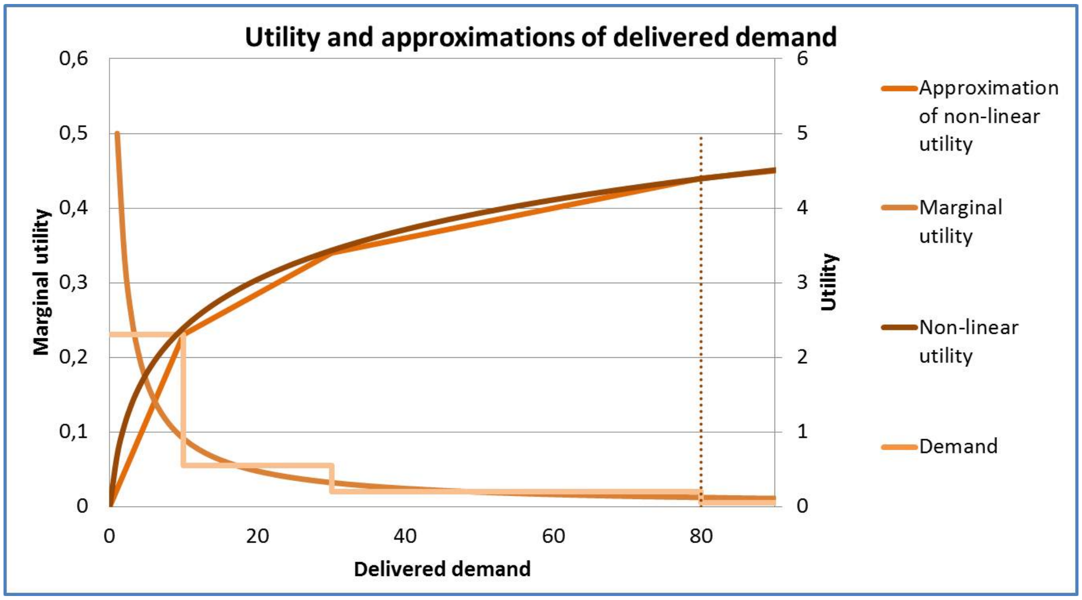

Figure 6: An illustration of the objective function and appurtenant approximations

The two lines substituting for the original utility function are indicated in Figure 6. This specific example considers four utility intervals, where the utility is close to zero as the total demand of 80 is fulfilled. There is no more demand to be met exceeding this upper limit. The effect of introducing nonlinear utility is that the marginal utility of delivering unit number 1, is higher than that of delivering item number 20.

5.2 Limitations and Assumptions under Perfect Information

The Achilles' heel of a mathematical model is represented by its input data. A realistic model will not be able to provide the user with useful information unless the input data are precise and realistic. Deterministic models suffer from the assumption of perfect information. According to Wallace [2003], a decision maker who works under this assumption, acts as *an economic man*. The economic man has knowledge of the significance of an optimal solution, which he aims to achieve, but limited information obstructs him from being sure that he is taking the right decisions.

The overall goal of the problem raised in this thesis formed by the society in general and by the decision makers, is to maximize total utility. Formulated as a facility routing problem, this model includes multi-commodity network flow with capacitated arcs, nonlinear utilities and multi-modal vehicle fleet. The model presented in this section considers a single possible earthquake scenario exclusively, and returns the optimal solution values of the decision variables describing the facility routing problem. These values are generated based on perfect information about the effects of the earthquake, and are applied immediately after impact. This mathematical model is formulated as a deterministic linear program.

In the following, the limitations and assumptions applicable to the deterministic model are listed:

1. Emergency supplies including rescue teams, medical personnel, food, water, machinery, medicines and necessity items, are modeled as commodities. Restricted amounts of initial supply is assumed to be located at the ICDs. ICDs are modeled as source nodes experiencing outflow exclusively, whereas the LDCs are modeled as transshipment nodes. Neither of these are exposed to demand. Flow of commodities between LDCs is allowed. All listed LDCs are able to operate and are subject to initialization. Commodities are assumed to be held in stock at the LDCs for a limited time period only, and holding costs are thus ignored. The LDCs portray the locations at which supplies are received for further distribution to final demand destinations. Also planning of the distribution process and loading of the vehicles assigned to perform last mile distribution, is executed at the LDCs. The PODs are able to serve as both transshipment nodes and termination nodes, and experience a given level of demand.

2. The ICDs are assumed to be situated far away from the affected area, making aircrafts the most appropriate means of transportation for initial supply. Ship freight is considered too time-consuming to be used for providing immediate help [Jæger, 2012b]. The paths connecting the ICDs and LDCs thus constitute of incapacitated arcs which will handle a fleet of vehicles in a single mode. Roads, walking paths and air routes connecting the LDCs and the PODs on the other hand, are represented by capacitated arcs able to handle a multi-modal vehicle fleet in order to conform with real life network properties. The arc capacities are assumed known immediately after the earthquake has hit. Time and cost spent to traverse an arc using a certain vehicle type, depend on the length and quality of the arc. All commodities must pass through an LDC, and direct shipment from ICDs to PODs is disallowed.

3. The different modes of transportation at the decision maker's disposal are modeled by means of a corresponding range of vehicle types. In order to prevent vehicles from traversing certain arcs with which they are incompatible, a set is defined for each vehicle type containing the arcs it is allowed to traverse. Restricting arc capacities can also prevent vehicles from traversing certain arcs.

4. Efficient distribution is to be ensured by the objective function which by implication minimizes costs. However, satisfying demand is of much higher importance in the

event of a disaster, and the utility related to each monetary unit saved is less than that of demand satisfaction, regardless of the distance from the earthquake and the number of items delivered thus far.

5. According to Tzeng et al. [2007], fairness in humanitarian logistics entails exerting effort to ensure that the required relief materials are distributed to all demand points. This model assumes different utility factors to reward fulfillment of demand depending on the number of commodities delivered. This implies that the marginal utility factor for each node and each commodity will vary in line with the number of commodities received by the POD in question. Ability to prioritize the most vulnerable part of the affected population as discussed in Section 3.3, is thus enabled. The model uses utility intervals to create such an effect. To avoid nonlinearities in the model formulation, a method which approximates the utility of demand fulfillment is adopted.

6. The budget can be restrictive or non-restrictive depending on the input value provided by the user of the model. To ensure efficiency of flow, an economical term will be added to the objective function.

7. Cultural and political issues are not considered when formulating the model. Neither are tax issues. As a result of this, delay in supply will not be necessary to take into account.

8. Variables representing commodities and satisfied demand are modeled as continuous variables in order to reduce the complexity of the model.

9. All parameters are known to the decision maker, and all decisions are made simultaneously.

5.3 Deterministic Model Formulation

This section provides a formal presentation of the deterministic model. The sets, indices, parameters and variables that form the foundation upon which the mathematical formulation is built, will be stated. Thereafter follows a detailed description of the model in terms of its constituting constraints. All sets are denoted by calligraphic upper-case letters, indices and variables by standard lower-case letters, and constants by standard upper-case letters.

Sets

$\mathcal{B}$	- set of commodity types
$\mathcal{K}$	- set of utility intervals
$\mathcal{N}$	- set of nodes
$\mathcal{V}$	- set of vehicle types

Indices of Main Sets

b	- commodity type	$b \in \mathcal{B}$
i, j, j'	- node	$i, j, j' \in \mathcal{N}$
k	- utility interval	$k \in \mathcal{K}$
v	- vehicle type	$v \in \mathcal{V}$

Derived Sets

$\mathcal{A}$	- subset of arcs which vehicles traveling between ICDs and LDCs are allowed to traverse	
$\mathcal{A}_v$	- subset of arcs which vehicle type v traveling between LDCs and PODs are allowed to traverse	
$\mathcal{I}$	- set of ICDs	$\mathcal{I} \subset \mathcal{N}$
$\mathcal{L}$	- set of LDCs	$\mathcal{L} \subset \mathcal{N}$
$\mathcal{P}$	- set of PODs	$\mathcal{P} \subset \mathcal{N}$
$\mathcal{V}_j$	- subset of vehicle types allowed to travel into node j	$V_j \subseteq \mathcal{V}$

Indices of Derived Sets

(i, j)	- arc	$(i, j) \in \mathcal{A} \cup \mathcal{A}_v$

Parameters

C^I	- unit capacity of vehicles traveling between ICDs and LDCs
C^L_v	- unit capacity of vehicle type v
C^A_{ij}	- unit capacity of arc (i, j)
C^C_j	- capacity at LDC j
D_{jb}	- demand of commodity type b at POD j
E^C_b	- cost associated with shipping one unit of commodity type b
E^I_{ij}	- cost associated with traveling from ICD i to LDC j
E^L_{ijv}	- cost associated with traveling from node i to node j for vehicle type v
E^O_i	- cost associated with opening LDC i
F^I_i	- total available number of vehicles at ICD i
F^L_{iv}	- total available number of vehicles of vehicle type v at LDC i
H^B	- available budget
H^T	- upper convoy time limit

M^D_{jbk} - utility factor for satisfied demand at POD j of commodity type b in utility interval k
M^B - utility factor for residual budget
Q^C_b - unit size of commodity type b
Q^V_v - unit size of vehicle type v
S_{ib} - supply of commodity type b at ICD i
T^I_{ij} - time spent traveling from ICD i to LDC j
T^L_{ijv} - time spent traveling from node i to node j for vehicle type v
U_{jbk} - size of utility interval k for commodity type b at POD j

Variables

$l_i = \begin{cases} 1, \text{ if LDC } i \text{ is opened} \\ 0, \text{ otherwise} \end{cases}$
x^I_{ij} - number of vehicles to travel from ICD i to LDC j
y^I_{ijb} - amount of commodity type b sent from ICD i to LDC j
$z^I_{ij} = \begin{cases} 1, \text{ if a vehicle traverses arc } (i, j) \\ 0, \text{ otherwise} \end{cases}$
d_{jbk} - amount of satisfied demand of commodity type b at POD j in utility interval k
x^L_{ijv} - number of vehicles of type v to travel from node i to node j
y^L_{ijbv} - amount of commodity type b sent from node i to node j with vehicle type v
$z^L_{ijv} = \begin{cases} 1, \text{ if vehicle type } v \text{ traverses arc } (i, j) \\ 0, \text{ otherwise} \end{cases}$
w - level of residual budget

Objective Function

$$\max \sum_{j\in\mathcal{P}} \sum_{b\in\mathcal{B}} \sum_{k\in\mathcal{K}} M^D_{jbk} d_{jbk} + M^B w \tag{5.1}$$

The aim of the model is to maximize the utility provided by means of humanitarian aid distribution to the population affected by an disaster. Utility is measured in terms of weighted level of demand fulfillment and economic efficiency. The first term of the utility function represents the total number of units of commodities delivered across all demand points, adjusted by a factor reflecting the urgency of need. The last term gives the level of residual budget, scaled in order to correspond to the first term.

Facility Location Constraints

Supply Constraints:

$$\sum_{j\in\mathcal{L}} y^I_{ijb} \leq S_{ib} \qquad\qquad i \in \mathcal{I}, b \in \mathcal{B} \tag{5.2}$$

In the initial step of the distribution process, we wish to ensure that the amount of commodities dispatched from the origin of supply, the ICDs, does not exceed the disposable level of stock at the given ICD. This condition is confirmed by adding the foregoing Supply Constraints (5.2).

Vehicle Flow Constraints:

$$\sum_{j\in\mathcal{L}} x^I_{ij} \leq F^I_i \qquad\qquad i \in \mathcal{I} \tag{5.3}$$

$$x^I_{ij} - F^I_i z^I_{ij} \leq 0 \qquad\qquad (i,j) \in \mathcal{A} \tag{5.4}$$

$$\sum_{b\in\mathcal{B}} Q_b y^I_{ijb} - C^I x^I_{ij} \leq 0 \qquad\qquad (i,j) \in \mathcal{A} \tag{5.5}$$

$$z^I_{ij} - x^I_{ij} \leq 0 \qquad\qquad (i,j) \in \mathcal{A} \tag{5.6}$$

The Vehicle Flow Constraints (5.3) - (5.6) apply to the single mode of transportation providing distribution between ICDs and LDCs. Constraints (5.3) limit the outbound number of vehicles at the ICDs according to the size of the available vehicle fleet. Constraints (5.4) equivalently limit the number of vehicles traversing an arc between an ICD and an LDC. Constraints (5.5) ensure vehicle capacity compliance, whereas Constraints (5.6) assure coherence between the integer variable defining quantity of vehicles traversing an arc and the corresponding binary variable.

Transshipment Constraints

LDC Capacity Constraints:

$$\sum_{i\in\mathcal{I}}\sum_{b\in\mathcal{B}} Q_b y^I_{ijb} - C^C_j l_j \leq 0 \qquad\qquad j \in \mathcal{L} \tag{5.7}$$

The transshipment centers, LDCs, are assumed to operate under capacity limitations, and the purpose of the LDC Capacity Constraints (5.7) are to keep total inbound flow from ICDs to the LDC in question within its capacity range. As the model enables a choice of which LDCs to open depending on the initialization cost, these constraints also provide prevention of inflow to inoperative LDCs.

Transshipment Commodity Flow Balance Constraints:

$$\sum_{j\in\mathcal{N}\setminus\mathcal{I}}\sum_{v\in\mathcal{V}_j} y^L_{ijbv} - \sum_{j\in\mathcal{I}} y^I_{jib} - \sum_{j\in\mathcal{N}\setminus\mathcal{I}}\sum_{v\in\mathcal{V}_i} y^L_{jibv} \leq 0 \qquad i\in\mathcal{L}, b\in\mathcal{B} \qquad (5.8)$$

The Commodity Flow Balance Constraints (5.8) seek to balance inbound versus outbound flow of commodities at the LDCs, in order to ensure conservation of the available quantity of commodities in the network. We consequently avoid appearance of exaggerated commodity flow at transshipment, whilst allowing for transshipment through the LDCs.

Last Mile Distribution Constraints

Demand Satisfaction Constraints:

$$\sum_{v\in\mathcal{V}_j}\sum_{(i,j)\in\mathcal{A}_v} y^L_{ijbv} - \sum_{v\in\mathcal{V}}\sum_{(j,i)\in\mathcal{A}_v} y^L_{jibv} - \sum_{k\in\mathcal{K}} d_{jbk} = 0 \qquad j\in\mathcal{P}, b\in\mathcal{B} \qquad (5.9)$$

$$\sum_{k\in\mathcal{K}} d_{jbk} \leq D_{jb} \qquad j\in\mathcal{P}, b\in\mathcal{B} \qquad (5.10)$$

$$d_{jbk} \leq U_{jbk} \qquad j\in\mathcal{P}, b\in\mathcal{B}, k\in\mathcal{K} \qquad (5.11)$$

The set of Demand Satisfaction Constraints (5.9) define the level of demand fulfillment for each demand point, POD, as the difference between ingoing and outgoing quantity of commodities. The second set of constraints, Constraints (5.10), restrains demand satisfaction from exceeding the actual demand, thus eliminating wasteful use of resources. The last set of constraints, Constraints (5.11), register the level of demand satisfaction achieved within each utility interval. Each interval corresponds to a specific level of urgency of reception of a given commodity. Combined with the first term of the Objective Function (5.1), Constraints (5.11) ensure that distribution between the accessible demand points is performed according to urgency of need.

Vehicle Flow Constraints:

$$\sum_{(i,j)\in\mathcal{A}_v} x^L_{ijv} - \sum_{(j,i)\in\mathcal{A}_v} x^L_{jiv} - F^L_{iv} l_i \leq 0 \qquad i\in\mathcal{L}, v\in\mathcal{V} \qquad (5.12)$$

$$x^L_{ijv} - (\sum_{j'\in\mathcal{L}} F^L_{j'v}) z^L_{ijv} \leq 0 \qquad v\in\mathcal{V}, (i,j)\in\mathcal{A}_v \qquad (5.13)$$

$$\sum_{b\in\mathcal{B}} Q_b y^L_{ijbv} - C^L_v x^L_{ijv} \leq 0 \qquad v\in\mathcal{V}, (i,j)\in\mathcal{A}_v \qquad (5.14)$$

$$z^L_{ijv} - x^L_{ijv} \leq 0 \qquad v\in\mathcal{V}, (i,j)\in\mathcal{A}_v \qquad (5.15)$$

$$\sum_{(j,i)\in\mathcal{A}_v} x^L_{jiv} - \sum_{(i,j)\in\mathcal{A}_v} x^L_{ijv} \leq 0 \qquad j\in\mathcal{P}, v\in\mathcal{V}_j \qquad (5.16)$$

The vehicle fleet executing last mile distribution is assumed to consist of vehicles of diffent types in terms of capacity, size and arcs valid for access in the distribution network. The Vehicle Flow Constraints (5.12) - (5.16) consequently apply to the available range of vehicle types. Constraints (5.12) keep the number of vehicles departing from an LDC within the available amount, adjusted by the number of vehicles entering the given LDC from other LDCs and PODs. In an equivalent manner, Constraints (5.13) ensure that the number of vehicles traversing an LDC's outgoing arcs is within the total vehicle quantity limit. The capacity limitations of the different vehicle types are met via Constraints (5.14), whilst Constraints (5.15) assure coherence between the integer variable defining quantity of vehicles traversing an arc and the corresponding binary variable. The final set of vehicle flow constraints, Constraints (5.16), balances in and outflow of vehicles at the PODs. Their purpose is to impede an increase in the number of vehicles leaving a POD as compared to the number entering the POD, in order to prevent artificial enlargement of the vehicle flow.

Arc Capacity Constraints:

$$\sum_{v \in \mathcal{V}_j} Q_v^V x_{ijv}^L \leq C_{ij}^A \qquad i,j \in \mathcal{N} \backslash \mathcal{I} \tag{5.17}$$

As stated in Assumption 2, the arcs connecting the LDCs and PODs are subject to capacity limitations. These limits apply to the grand total of vehicles, across all types, traversing an arc during the full length of the distribution process. The Arc Capacity Constraints (5.17) will hinder vehicle flow beyond the capacity of an arc with regard to this fact.

Commodity Flow Constraints:

$$\sum_{v \in \mathcal{V}} \sum_{(i,j) \in \mathcal{A}_v : i \in \mathcal{L}} y_{ijbv}^L - \sum_{j \in \mathcal{P}} \sum_{k \in \mathcal{K}} d_{jbk} = 0 \qquad b \in \mathcal{B} \tag{5.18}$$

$$\sum_{(j,i) \in \mathcal{A}_v} y_{jibv}^L - \sum_{(i,j) \in \mathcal{A}_v} y_{ijbv}^L \leq 0 \qquad j \in \mathcal{P}, b \in \mathcal{B}, v \in \mathcal{V}_j \tag{5.19}$$

The complete amount of commodities initially planned for dispatch from the LDCs should contribute in satisfying final demand, as ensured by Constraints (5.18). The Commodity Flow Balance Constraints (5.19) correspond to Vehicle Flow Constraints (5.16), now with the intention of balancing in- and outflow of commodities at the PODs.

Efficiency Constraints

Budget Constraints:

$$\sum_{(i,j) \in \mathcal{A}} \sum_{b \in \mathcal{B}} E_b^C y_{ijb}^I + \sum_{(i,j) \in \mathcal{A}} E_{ij}^I x_{ij}^I + \sum_{v \in \mathcal{V}} \sum_{(i,j) \in \mathcal{A}_v} E_{ijv}^L x_{ijv}^L + \sum_{i \in \mathcal{L}} E_i^O l_i + w = H^B \tag{5.20}$$

The Budget Constraints ensure that total cost of distribution does not surpass available budget, and calculate residual budget, if any. Total distribution cost include 1) cost per unit of commodities sent from ICDs to LDCs; 2) cost per vehicle traveling between ICDs and LDCs, and LDCs and PODs, with arc-specific cost values; and 3) cost of opening the LDCs chosen.

Convoy Travel Time Constraints:

$$\sum_{(i,j)\in\mathcal{A}} T^I_{ij} z^I_{ij} + \sum_{v\in\mathcal{V}} \sum_{(i,j)\in\mathcal{A}_v} T^L_{ijv} z^L_{ijv} \leq H^T \tag{5.21}$$

Constraints (5.21) account for the time spent serving the different PODs. If this constraint is binding, it has the effect of making the vehicles move in convoy.

Non-negativity Constraints

$$l_i \in \{0,1\} \quad i \in \mathcal{L} \tag{5.22}$$
$$x^I_{ij} \geq 0 \quad \text{integer}, (i,j) \in \mathcal{A} \tag{5.23}$$
$$y^I_{ijb} \geq 0 \quad (i,j) \in \mathcal{A}, b \in \mathcal{B} \tag{5.24}$$
$$z^I_{ij} \in \{0,1\} \quad (i,j) \in \mathcal{A} \tag{5.25}$$
$$d_{jbk} \geq 0 \quad j \in \mathcal{P}, b \in \mathcal{B}, k \in \mathcal{K} \tag{5.26}$$
$$x^L_{ijv} \geq 0 \quad \text{integer}, v \in \mathcal{V}, (i,j) \in \mathcal{A}_v \tag{5.27}$$
$$y^L_{ijbv} \geq 0 \quad b \in \mathcal{B}, v \in \mathcal{V}, (i,j) \in \mathcal{A}_v \tag{5.28}$$
$$z^L_{ijv} \in \{0,1\} \quad v \in \mathcal{V}, (i,j) \in \mathcal{A}_v \tag{5.29}$$
$$w \geq 0 \tag{5.30}$$

Constraints (5.22) - (5.30) enforce non-negative values for the variables used to represent the range of decisions made in the distribution process.

6 An Introduction to Multistage Stochastic Programming

This section deals with aspects of multi-stage stochastic programming (MSP) relevant for the SP model to be proposed. The aim of MSP is to determine optimal decisions in problems involving uncertain data. Stochastic, as opposed to deterministic, programming entails inclusion of data subject to random variation [Birge and Louveaux, 2011]. In the context of SP, Wallace [2003] replaces the economic man described in Section 5.2, with the *the administrative man*. The administrative man makes decisions under limited knowledge, and seeks solutions according to the knowledge he holds at the time of decision which are only just sufficient. The nature of these solutions however, might change over time as relevant information becomes available. The local agent described in Section 3.2 is restrained by limited information, but seeks to produce optimal distribution plans. He thus fits the given description of the administrative man.

Following is a description and comparison of approaches used to handle elements of uncertainty in different mathematical problems. A brief introduction of multi-stage recourse problems in general is given in Section 6.2, followed by a preliminary account of criteria applied in valuation and validation of stochastic solutions in Section 6.3.

For a thorough introduction to stochastic programming in general, we refere to Birge and Louveaux [2011] and Higle [2005]. The reader is assumed to be familiar with basic concepts of optimization and mathematical programming.

6.1 Handling Uncertainties Inherent in Real-Life Events

Most problems describing real-life decision-making processes to a greater or lesser extent consist of uncertain parameters. Prices, demand, costs, weather, technology or, as in this case, extent of damages exemplify aspects of this kind. Consideration of uncertainty is essential in development of models which are able to produce reliable and sound decisions applicable to the total range of possible real-life outcomes.

By applying a deterministic problem formulation, elements of uncertainty are neglected as perfect information is assumed. Methods such as sensitivity analysis or scenario analysis are often used to ensure validity of solutions [Midthun, 2011]. Sensitivity analysis seeks to evaluate the effect variation of parameters of interest has on the optimal solution. Scenario analysis is another approach often used in situations where we are able to identify a finite number of possible realizations of the uncertain parameters. Based on this range of realizations, we are able to define a number of scenarios. In turn, for each of these different scenarios separately, a deterministic problem will be formulated and solved. An optimal solution will be located based on analysis of the resulting array of scenario solutions.

Midthun [2011] argues that stochastic programming (SP) is the appropriate tool for making decisions under uncertainty. Wallace [2003] states that SP can in some cases prove a by far superior approach, when compared with its deterministic counterpart. Shapiro and Philpott [2007] claim that SP models will take advantage of the fact that probability distributions governing the data are known or can be estimated. The models aim to establish decisions that will perform well for the totality of future outcomes under consideration. Hence, the decision maker's goal is to determine a policy which is feasible for all or almost all the possible realizations of the uncertain parameter, and to optimize the expectation of some function of the decisions and the random variables [Shapiro and Philpott, 2007].

With these arguments at hand, uncertain parameters relating to the DRP described in Section 3, will be dealt with by means of stochastic programming. The remainder of this section is therefore devoted to aspects relating to SP.

6.2 Characteristics of Multi-Stage Recourse Problems

Real-life problems normally include parameters which are of unknown values at a given point in time, but which will be revealed as the situation progresses. Recourse models utilize the flexibility of being able to postpone certain decisions until information regarding relevant uncertain parameters is realized [Sen and Higle, 1999]. Each decision is defined in terms of the point in time at which it is to be made, and in accordance with elimination of uncertainty regarding decisive parameters [Wallace, 2003]. Decisions which can be delayed, so-called recourse actions, provide an opportunity to adjust to the realized information. This will compensate for some bad effects that might have been experienced because of earlier decisions. Such adaption is referred to as recourse. Recourse models are therefore always presented as models constituting of two or more stages, allowing for exploitation of the continuous revelation of relevant information throughout the length of the planning process [Higle, 2005].

In two-stage recourse problems, all parameters are revealed prior to the second stage decisions being determined. However, if obtainment of new information takes place at several future points in time, a multi-stage recourse model will serve to describe reality more accurately. Multi-stage models provides the advantage of being able to consider an extended planning horizon whilst avoiding poor initial decisions. An important byproduct in such a planning process is the generation of recourse plans for each of the different scenarios describing possible future outcomes.

A key feature of multi-stage problems is the evolution of the random phenomena over time [Sen and Higle, 1999]. As an implication, the decision problem faced at a certain stage can vary dramatically depending on the outcomes realized in the previous period. In addition, decisions made at one stage will have an impact on the range of options available in future periods. The variables generally depend on the values of random parameters of previous stages. For ease of understanding and convenience, this dependency is depicted in Figure 7 by means of two interchangeable scenario trees.

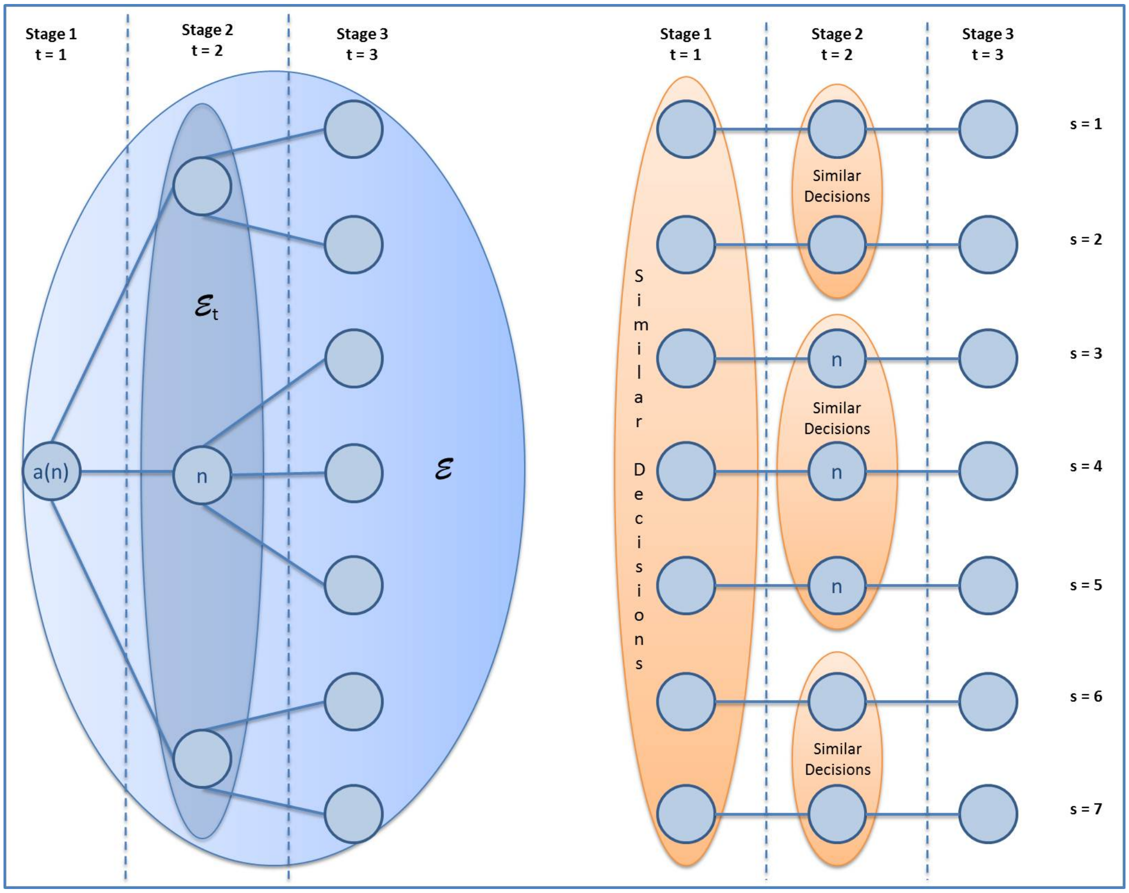

Figure 7: Scenario trees illustrating the totality of future scenarios, in addition to the course of decisions and the interdependencies between decision made at different stage

The figure shows a structured distributional representation of the stochastic elements in a problem. In general, the nodes of the scenario tree represent possible stages at which decisions can be made and uncertainty is resolved, throughout the course of events. New information is revealed where at least two branches lead out from a single node. Scenarios are defined as specific paths leading from the root node to a leaf node. The total number of scenarios thus equals the number of leaf nodes. Transition from one stage to the next takes place as new information is obtained. Both trees shown in Figure 7 consist of seven distinct scenarios, s, and three separate stages, t.

The left part of Figure 7 is a simple scenario tree. The root node corresponds to the initial stage where limited information exists regarding the true nature of random variables. The dashed lines separate the stages, whereas the ellipsoids encircle nodes belonging to the same stage, $\mathcal{E}_t$. The notation $a(n)$ is used in succeeding chapters to denote the predecessor node, also denoted by parent node, of node n.

The right part of Figure 7 illustrates each possible scenario separately. The nodes of the right tree have the same functions as in the left part, and the ellipsoids represent nodes that share the same stochastic elements. The same decisions are thus to be made across all nodes contained within an ellipsoid. The ellipsoids represent the non-anticipativity constraints.

In addition to illustrating the course of events, the two different trees represent two different approaches taken when formulating SP models. The first scenario tree translates into a node formulation, whereas the second tree denotes a scenario formulation. The increased number of nodes of the latter tree implies that a more complicated computational process will be needed in order to solve a model based on scenario formulation, as opposed to the node formulation of the left tree. This fact serves as grounds for choosing node formulation in the MSP model proposed.

6.3 Valuation of the Stochastic Solution in Multi-Stage Problems

When making decisions, aspects of randomness and uncertainty may be vital to consider in order to produce resilient decisions applicable to the range of scenarios which might occur. Omitting elements of uncertainty in stochastic environments might result in suboptimal, or even infeasible, solutions. A commonly applied approach is substituting ucertain parameters by approximations or expected values in deterministic models. However, Kall and Wallace [1994] argue that turning to stochastic programming when working with decisions affected by uncertainty is crucial.

Stochastic programming models generally involve an increase in the level of complexity as compared with their deterministic counterparts, making them both harder and more time-consuming to solve for realistically sized problems [Maggioni et al., 2012]. Being able to determine values stating the benefit of a more realistic SP model over a more simplistic deterministic model can thus produce evident indications as to whether or not introducing a SP model will be worthwhile, or if a deterministic model will suffice [Maggioni et al., 2012]. In order to assess the value of stochastic solutions, several scholars tend to employ the concept of measures calculated based on different levels of available information [Birge and Louveaux, 2011, Escudero et al., 2007, Maggioni et al., 2012]. Whereas an extensive range of different measures for valuation have been adopted in literature, the commonly applied concepts of the Expected Value of Perfect Information (EVPI) and the Value of the Stochastic Solution (VSS) for multi-stage stochastic models will form the basis for evaluation of the model proposed in this thesis.

The bounds given by the EVPI and VSS yield a quantitative standard of comparison for determining the importance of considering uncertainties. These provide valuable insight as to the potential benefit from solving the stochastic program over a deterministic program in which random parameters have replaced expected values. The Value of the Stochastic Solution indicates the expected gain from solving a stochastic model rather than its deterministic counterpart. The Expected Value of Perfect Information states how much a decision maker should be willing to pay in order to obtain perfect information about the future. The two measures are generally defined in terms of the 'wait-and-see' (WS) solution, the solution of the Recourse Problem (RP) and the expected result of using an expected value solution (EEV). An introduction of the terms, followed by a mathematical deduction of the evaluation measures will be given in the in the consecutive part of this section.

6.3.1 Calculation of the Expected Value of Perfect Information

The term Recourse Problem (RP) is commonly used in literature to denote the optimal solution given by the stochastic programming model [Birge and Louveaux, 2011, Escudero et al., 2007]. This notation will for ease of reference be adopted in this thesis in conjunction with valuation of the proposed model. The RP value is also referred to as the 'here-and-now'-solution, as opposed to the 'wait-and-see'-solution (WS). Assuming that perfect information about the future realization of events could be obtained, and each specific scenario independently solved to optimality, the WS value represents the expected optimal solution value. This value is found by separately solving each specific scenario to optimality [Escudero et al., 2007]. With these two definitions at hand, the EVPI can be formulated as the difference between the wait-and-see and the here-and-now solution:

$$EVPI = WS - RP \tag{6.1}$$

An occurence of high EVPI values serve as justification for investing in improved forecasting techniques, whereas low EVPI values should discourage such investment [Birge and Louveaux, 2011].

6.3.2 Calculation of the Value of the Stochastic Solution

When faced with elements of uncertainty, an approach frequently resorted to is solving the Expected Value Problem (EV). The EV is obtained by replacing all random variables by their expected values, and solving a deterministic program. As opposed to the stochastic programming model producing the RP value, this approach involves a problem of considerable less size and complexity. By fixing all decision variables in the multi-stage stochastic program at the optimal values obtained by using the expected value solution until the final stage, the Expected result of using the Expected Value solution (EEV) is produced [Birge, 1982, Escudero et al., 2007].

Escudero et al. [2007] also describe a second method for computation of EEV, termed the expected result of using dynamic solutions of the average scenario. The deterministic average scenario problem is solved in the first step of the procedure. The stochastic values are replaced by their expected values at all subsequent stages. The resulting optimal first stage variables are saved. At the following stage in the scenario tree, an average scenario problem is solved for each scenario tree node leading up to the final stage. For each of the nodes, the random parameters of subsequent stages are estimated by their expected values, and all the variables of the preceding stages are fixed at the optimal solution values obtained in the chain leading up to the node in question. The optimal values of the variables of the current stage are still to be saved. The same procedure applies to the final stage, except that use of expected values is no longer relevant. The dynamic EEV value is calculated based on the optimal values obtained for each scenario tree node of the final stage. In this way, we achieve a more realistic use of the expected value solution, as more precise information is added to the model. In order to be able to distinguish the two procedures described, they will be referred to as the static approach and dynamic approach in what follows.

By employing the values described so far, the VSS value is found as the difference between the solution to the stochastic programming model, RP, and the expected result of using a deterministic expected value solution, EEV:

$$VSS = RP - EEV \tag{6.2}$$

This value enables us to calculate the maximum amount that we would be prepared to pay to ignore uncertainty over the total time horizon the model applies to.

Large VSS values indicate that uncertainty is of importance to the optimal solution, and that the deterministic solution is of less value. Stochastic programming is deemed appropriate when VSS is high [Barbarosoğlu and Arda, 2004]. Comparing the EVPI and the VSS, the latter is of greatest pertinence to the decision maker in situations where gathering more information about the future can prove cumbersome, such as ours.

For maximization problems, Birge [1982] argues that the following relationship between the values defining the evaluation measures applies:

$$EEV \leq RP \leq WS \tag{6.3}$$

The first relation, $EEV \leq RP$, implies that a solution generated by a stochastic program will always perform better than the expected result of using the EV solution. The last relation, $RP \leq WS$, on the other hand, suggests that obtaining perfect information is always to be preferred.

Due to certain attributes of the proposed MSP model, some requisite assumptions will need to be made when estimating the different measures for valuation. The values presented will be adjusted accordingly. Hence, calculations of the measures will not coincide strictly with the theory presented in the preceding. This is to be explained in greater detail in Section 10.2.1.

7 Presentation of the Multi-Stage Stochastic Programming Model

This section presents the complete representation of the SP model, including the assumptions and limitations used in the construction of the model as well as all necessary sets, definitions, functions and restrictions.

7.1 Assumptions and Limitations

Deterministic models implicitly assume perfect information. The assumption of perfect information might lead to inflexible and extreme decisions [Midthun et al., 2009]. In the context of this disaster response problem (DRP), decisions are adapted to the expected demand, number of available vehicles and the state of the infrastructure. If deviations from the expected distribution network occur, these can potentially result in bad outcomes. Hence, it seems reasonable to develop a SP model which considers these unknown conditions. Due to the characteristics of the DRP described in Section 3 we have chosen to formulate the model as a three-stage stochastic mixed integer program (SMIP). This enhances reality representation, which is an important issue in efficient operations research-modeling processes. However, it should be noted that there is a trade-off between reality representation and solution speed [Nygreen et al., 1998]. For discussion of this aspect, we refer to Section 10.1.

Even though the assumption of perfect information is relaxed in the three-stage SP approach proposed, budget and supply among others are still treated as deterministic parameters. In real life, budget will be stochastic in the sense that it varies with the effect of the earthquake. Normally the budget is restrictive, but large enough to at least satisfy the demand of highest importance. Supply at the ICDs is normally certain, but due to potential delays, supply at the LDC might be stochastic. Still, both supply and budget are treated as deterministic elements in this model, based on the following two reasons. Firstly, the model does not consider explicit time periods, resulting in unrealistic implementation of the late arrival of supplies as stochastic variables. Secondly, we seek to restrict the number of stochastic parameters in order to reduce computational time. In a sector characterized by extreme time sensitivity, a computational efficient model with minor errors will be preferred to an model that takes days to run.

A general challenge in SP is the potentially immense number of possible realization of the stochastic elements. The number of realizations, translated into an equivalent number of future scenarios, is the product of possible outcomes for each distinct random value of a parameter. Demand, size of the available vehicle fleet, utilities and state of the last-mile transportation network comprise the parameters modeled stochastically. In a model spanning three stages, this implies generation of an unlimited number of possible scenarios. We assume that a discrete approximation of the possible outcomes in stage 2 and stage 3, along with respective probability distributions, can be constructed, as illustrated in Figure 7, Section 6.2. It is recognized that such an approach depends highly upon the specific sample drawn, and is thus subject to error. Still, careful choice of input parameters is assumed to reduce the risk of critical errors.

The most significant assumptions concern the choice of elements to be treated stochastically, which will be based upon the aspects discussed in Section 3.2. Immediately following an earthquake, the decision maker wishes to provide aid at the earliest possible point in time. However, as described in Section 3.2, some beneficiaries may return to greater self-sufficiency as the situation progresses, whilst others may relocate to different areas in hopes of finding greater relief. The arise of unexpected challenges such as eruption of disease epidemics can also bring about change in the level of demand at different PODs. This argument serves as the grounds for considering demand as a stochastic parameter. The same reasoning applies to the utility factor, as well as the utility interval size, which is dependent on identical factors. As discussed in Section 2.2.3, allocation of vehicles can often prove troublesome due to limited availability of vehicles and difficulty in predicting the actual level of availability. The range of different types of vehicles accessible to the decision maker will be considered known in advance, whereas the number of vehicles per type at disposal at each LDC will be subject to uncertainty. Vehicle capacity is hence modeled as a stochastic parameter in terms of the number of disposable vehicles, exclusively.

Allocation of drivers and other personnel, as well as allocation of vehicles, are time consuming processes, as mentioned in Section 3.2. However, time consuming to the extent that it is carried out within the time taken for supplies to arrive at the operational LDCs. At this point in time, further assessments providing updated information concerning the level of demand across PODs has been conducted, and demand information is now assumed certain. The same applies to utilities and utility interval sizes. Also the disposable number of vehicles of each type is treated as a matter of certainty at this stage, as it is assumed that the vehicles have arrived at the LDCs. The uncertain parameter of stage 2 yet to be realized, is that of the transportation network. We expect that the network will be reduced in consequence of an earthquake, which implies that a node could potentially be totally disconnected from the rest of the distribution network.

Based on assessments of the extent of the earthquake, the decision maker is able to submit initial estimates of the amount of supplies required in the affected area and the LDCs which are most expedient to initialize. It should however be noted, that neither ICDs nor LDCs will be subject to destruction by the earthquake. The LDCs that are actually destroyed during the earthquake will not be eligible and simply excluded from the selection of available LDC locations, as this information is assumed to be known when the initial decisions are made.

In order to fully comprehend the SP model, the conception of the three stages needs be explained in further detail. Allocation of commodities and other necessary items, as well as selection of LDCs to initialize, are modeled as decision made in stage 1. The first reception of updated information marks the transition from stage 1 to stage 2. As information becomes available in stage 2, the decision maker will be able to generate initial routes, stating the load assigned to these vehicles and their destination PODs based on knowledge of demand, resources which is now at his disposal and anticipation of the state of the network. In accordance with this plan, vehicles are packed, and dispatched from the LDCs as soon as they are loaded.

Mathematically, as information about the state of the distribution network is realized, the 3rd stage takes effect. In reality, if a planned route proves fully operative and able of being carried out according to plan, this information will not be needed. Destinations are in effect potentially able of being reached prior to reception of information, and corrective measures regarding the vehicle and respective load serving this POD, decisions relating to stage 3, will not be made. As a result, the value of the 3rd stage variables will be determined subsequent to the completion of deliverance. Should however, a vehicle hit an obstacle in advance of the relevant information being provided, it will be forced to wait until complete information regarding the state of the distribution network is gained and a modified route is generated. Hence, the transition from stage 2 to stage 3 is diffuse, and will only take place for the set of vehicles faced with an obstacle when carrying out their initially planned route. The 3rd stage variables describing the actual routes of vehicles which never reaches the final stage, will coincide with the corresponding 2nd stage variables. Potential waiting time will not be considered in the proposed model, and does not affect the total convoy time subject to restriction. The variables indicating the full range of decision are depicted in Figure 8 and corresponds to Figure 4 in Section 3.

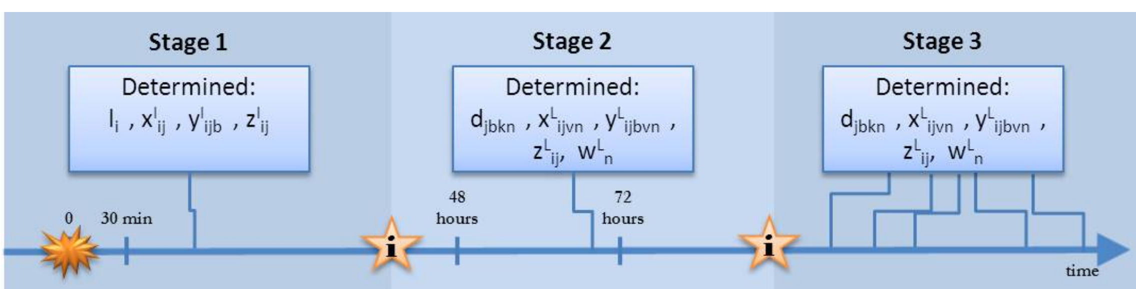

Figure 8: An illustration of the decision variables determining point

Only the decisions made in the final stage of the distribution process indicate the actual implementation of flow of commodities and vehicles from point of supply to point of consumption. Notwithstanding, the decisions of the 2nd stage are regarded equally important. This is due the fact that the planned route generated in stage 2 may prove able of completion, which is to be preferred due to ease of management and increased predictability throughout the distribution process. In real life, being able to predict and prepare for the tasks to be performed will be of importance to the personnel involved in humanitarian logistics, to ensure stable and comfortable working conditions.

Further assumptions concern repacking of vehicles at PODs during last mile distribution in order to combine loads. This will allow for further shipment if arc capacities or arc costs restrict the vehicles of the same type to which this applies, to proceed. Vehicles are permitted to travel empty to be able to relieve other vehicles, when necessary. All vehicles end their routes at the final destination POD, as defined by their designated distribution routes generated in stage 2, given that they are able to reach the POD in question according to plan and delivering to another POD does not provide an increase in the level of utility. If the given destination POD is inaccessible to the vehicle type in question, this vehicle is free to serve other PODs. As a direct consequence of this, the assigned load will not go to waste and contribute to satisfaction of demand.

In order to enable control of inflow versus outflow of commodities and vehicles at the LDCs, a dummy node system in which each LDC is provided with a corresponding dummy LDC has been introduced. Figure 9 illustrates this system. By prohibiting inflow to the original LDC from all nodes apart from the ICDs, but allowing both inflow and outflow to the dummy LDCs, we allow for transshipment at the LDCs. In this way, the chance of successfully reaching a POD is increased as the number of possible paths is extended. The LDCs and their respective dummy LDCs are connected to the identical set of nodes, but not to each other. The arcs connected to the original LDC are only outgoing, whereas the arcs connected to the respective dummy LDC are symmetric. This prevents an artificial increase in commodity and/or vehicle flow beyond the amounts really existing in the network. The dummy LDCs, as opposed to their respective original LDCs, are empty nodes. They are neither provided with any supplies or vehicles, nor do they experience any demand. The dummy LDCs will be utilized should a vehicle need to return to the node of origin, or use an LDC for transshipment purposes. Even though the LDC and corresponding dummy LDC are represented by two different nodes in the transportation network, they are in reality situated at the exact same location. If an LDC is to be used for transshipment only, opening costs should not be induced. Transshipment will thus take place at the dummy LDC, which will not be subject to opening costs.

A decisive and significant attribute of the proposed model is the enablement of initialization of routes prior to reception of information regarding the final state of the distribution network. The dispatched vehicles will thus need to be forced to follow the planned routes to the extent possible. They will either be able to complete the given route, or need to be given an alternative plan of attack should an obstacle hinder this route from being carried through. In order to tackle the mathematical challenge of modifying a route, two almost identical networks has been created. The original network utilized in stage 2 includes

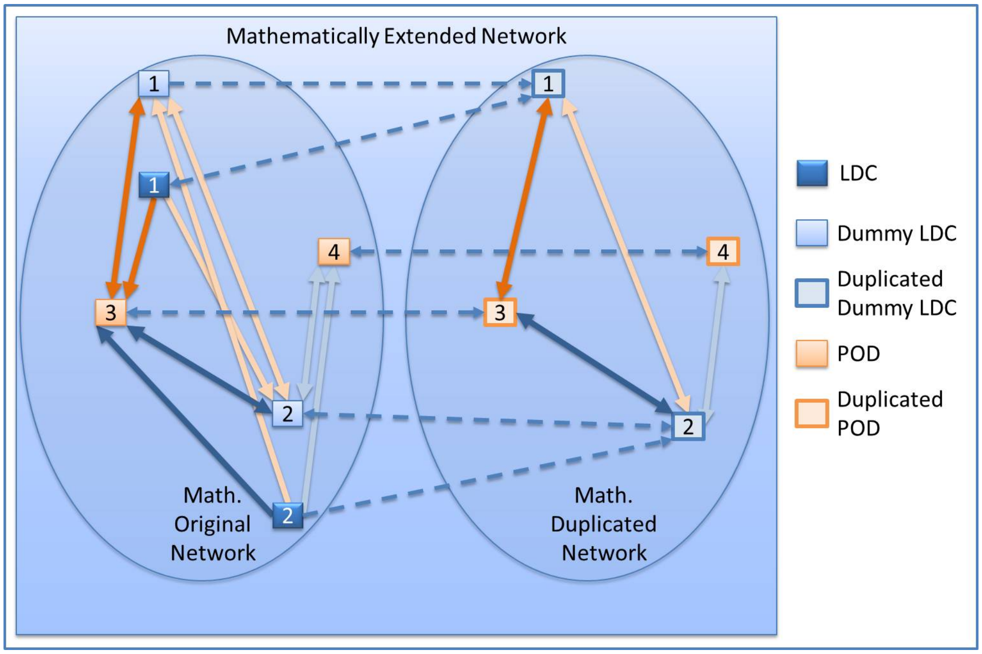

Figure 9: An illustration of the utilized dummy node system and the duplicated network - Dotted arcs depict links connecting the original network and the duplicated network, whereas unbroken arcs of the same color represent corresponding arcs

the nodes representing LDCs, dummy LDCs and PODs. The mathematically extended network builds upon this network, but also includes a duplicated network consisting of the the dummy LDCs and PODs only. The relationship between the different networks are illustrated in Figure 9. As long as the arcs to be traversed exist in stage 3, flow will occur in the original network according to the initially generated distribution plan. If an obstacle impedes this route from being followed, the original network will be abandoned and the vehicle in question will proceed in the duplicated network according to a modified route. Flow on links connecting corresponding nodes of the original and duplicated network represent artificial flow created for modeling purposes only, as to indicate a change of network and transition to stage 3. It will thus not induce costs as it does not actually take place in real life situations. For the same reason, the connecting links are incapacitated. All other arcs can only handle a limited number of vehicles, and the arc capacities are generated such that flow in both directions are accounted for. When a vehicle traverses an arc, the remaining capacity of that arc is reduced accordingly. The implication of introducing a duplicated network and a dummy node system is that arcs which in reality represent the same distance, will be modeled as separate arcs between different nodes. This will need to be considered when formulating the model.

The modeling approach taken is designed to illustrate the actual course of events in a realistic manner. The assumptions presented in this section joint with Item (1) - (8) from the deterministic model in Section 5.2 form the complete set of assumptions applied to the MSP model developed in the following section.

7.2 Formal Definitions

The following section give a formal definition of the sets, indices, constants and variables used in the mathematical formulation of the stochastic programming model developed in this study. As for the deterministic model, all sets are denoted by calligraphic upper-case letters, indices and variables by standard lower-case letters and constants by standard upper-case letters.

Network Quantities

Q_I - total number of ICDs in the original network
Q_L - total number of LDCs in the original network
Q_P - total number of PODs in the original network
Q_N - total number of nodes in the original network

Main Sets

$\mathcal{B}$ - set of commodity types
$\mathcal{E}$ - set of scenario tree nodes in the scenario tree
$\mathcal{K}$ - set of utility intervals
$\mathcal{N}$ - set of nodes

$$\mathcal{N} = \{1, \ldots, Q_I, \\ Q_I + 1, \ldots, Q_I + Q_L, \\ Q_I + Q_L + 1, \ldots, Q_I + 2Q_L, \\ Q_I + 2Q_L + 1, \ldots, Q_I + 2Q_L + Q_P = Q_N\}$$

$\mathcal{T}$ - set of stages

$$\mathcal{T} = \{1, 2, 3\}$$

$\mathcal{V}$ - set of vehicle types

Indices of Main Sets

b	- commodity type	$b \in \mathcal{B}$
i, j, j'	- node	$i, j, j' \in \mathcal{N}$
k	- utility interval	$k \in \mathcal{K}$
n	- scenario tree node	$n \in \mathcal{E}$
t	- stage	$t \in \mathcal{T}$
v	- vehicle type	$v \in \mathcal{V}$

Derived Sets

$\mathcal{A}$ - subset of arcs which vehicles traveling between ICDs and LDCs are allowed to traverse

$\mathcal{A}_{vn}$ - subset of arcs which vehicle type v traveling between LDCs, dummy LDCs and PODs are allowed to traverse in scenario tree node n

$\mathcal{A}'_{vn}$ - subset of arcs and duplicated arcs which vehicle type v traveling dummy LDCs and PODs, and the duplicated dummy LDC and the duplicated PODs are allowed to traverse in scenario tree node n,
$\mathcal{A}'_{vn} = \mathcal{A}_{vn} \cup \{(i, (i + Q_L + Q_P)) : i \in \mathcal{L}^D \cup \mathcal{P}\} \cup \{((i + Q_L + Q_P), i) : i \in \mathcal{L}^D \cup \mathcal{P}\}$
$\cup \{((i + Q_L + Q_P), (j + Q_L + Q_P)) : (i, j) \in \mathcal{A}_{vn}, i, j \in \mathcal{L}^D \cup \mathcal{P}\}$
$\cup \{(i, (i + 2Q_L + Q_P)) : i \in \mathcal{L}\}$

$\mathcal{E}_t$ - set of scenario tree nodes at stage t,
$\mathcal{E}_t \subset \mathcal{E}$

$\mathcal{I}$ - set of ICDs,
$\mathcal{I} = \{1, \dots, Q_I\}, \mathcal{I} \subset \mathcal{N}$

$\mathcal{L}$ - set of LDCs,
$\mathcal{L} = \{1, \dots, Q_L\}, \mathcal{L} \subset \mathcal{N}$

$\mathcal{L}^{D'}$ - duplicated set of dummy LDCs,
$\mathcal{L}^{D'} = \{1, \dots, 2Q_L\}$

$\mathcal{L}^D$ - set of dummy LDCs,
$\mathcal{L}^D = \{1, \dots, Q_L\}, \mathcal{L}^{\mathcal{D}} \subset \mathcal{L}^{D'} \wedge \mathcal{L}^{\mathcal{D}} \subset \mathcal{N}$

$\mathcal{P}'$ - duplicated set of PODs,
$\mathcal{P}' = \{1, \dots, 2Q_P\}$

$\mathcal{P}$ - set of PODs,
$\mathcal{P} = \{1, \dots, Q_P\}, \mathcal{P} \subset \mathcal{P}' \wedge \mathcal{P} \subset \mathcal{N}$

$\mathcal{V}_{jn}$ - subset of vehicle types allowed to travel into node j in scenario tree node n,
$V_{jn} \subseteq \mathcal{V}$

Indices of Subsets

(i, j)	- arc	$(i, j) \in \mathcal{A} \cup \mathcal{A}_{vn} \cup \mathcal{A}'_{vn}$
$a(n)$	- predecessor scenario tree node of scenario tree node n	$a(n) \in \mathcal{E} \backslash \mathcal{E}_3$

Deterministic Parameters

C^I - unit capacity of vehicles traveling between ICDs and LDCs
C_v^L - unit capacity of vehicle type v
C_j^C - unit capacity at an LDC j
E_b^C - unit cost of commodity type b
E_{ij}^I - cost associated with traveling from ICD i to LDC j
E_i^O - cost associated with opening an LDC i
F_i^I - total available number of vehicles at ICD i
H^B - available budget
H^T - upper convoy time limit
M^B - utility factor for residual budget
P_n - probability of scenario tree node n occurring
Q_b^C - unit size of commodity type b
Q_v^V - unit size of vehicle type v
S_{ib} - supply of commodity type b at ICD i
T_{ij}^I - time spent traveling from ICD i to LDC j

Stochastic Parameters

C_{ijn}^A - unit capacity of arc (i, j) in scenario tree node n
D_{jbn} - demand of commodity type b at POD j in scenario tree node n
E_{ijvn}^L - cost associated with traveling from node i to node j for vehicle type v in scenario tree node n
F_{ivn}^L - total expected number of vehicles of vehicle type v at LDC i in scenario tree node n
M_{jbkn}^D - utility factor for satisfied demand at POD j of commodity type b in utility interval k in scenario tree node n
T_{ijvn}^L - time spent traveling from node i to node j for vehicle type v in scenario tree node n
U_{jbkn} - size of utility interval k for commodity type b at POD j in scenario tree node n

First Stage Variables

$$l_i = \begin{cases} 1, \text{ if LDC } i \text{ is opened} \\ 0, \text{ otherwise} \end{cases}$$

x_{ij}^I - number of vehicles to travel from ICD i to LDC j

y^I_{ijb} - amount of commodity type b sent from ICD i to LDC j

$z^I_{ij} = \begin{cases} 1, \text{ if a vehicle traverses arc } (i,j) \\ 0, \text{ otherwise} \end{cases}$

Second and Third Stage Variables

d_{jbkn} - amount of satisfied demand of commodity type b at POD j in utility interval k and scenario tree node n

x^L_{ijvn} - number of vehicles of type v to travel from node i to node j in scenario tree node n

y^L_{ijbvn} - amount of commodity type b sent from node i to node j with vehicle type v in scenario tree node n

$z^L_{ijvn} = \begin{cases} 1, \text{ if vehicle type } v \text{ traverses arc } (i,j) \text{ in scenario tree node } n \\ 0, \text{ otherwise} \end{cases}$

w_n - level of residual budget in scenario tree node n

7.3 Model Formulation

This section will explicate the stochastic programming model in its entirety. Each constraint, or group of connected constraints, will be presented separately and chronologically according to the stage it applies to, accompanied by a brief explanation. The complete model excluding explanations is in addition given in Appendix A.

Objective Function

$$\max \sum_{j\in\mathcal{P}}\sum_{b\in\mathcal{B}}\sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{E}\setminus\mathcal{E}_1} P_n M_{jbkn} d_{jbkn} + \sum_{n\in\mathcal{E}\setminus\mathcal{E}_1} P_n M^B w_n \tag{7.1}$$

As defined in Section 3.4, the primary aim of the proposed model is to provide the local agent with a sensible decision scheme explicitly stating the course of distribution of necessary emergency supplies to the affected population in the event of an earthquake. The model seeks to achieve this goal by considering effectiveness and fairness of distribution. Effectiveness is measured in terms of cost, which by implication expresses time and distance involved in executing the distribution plan generated by the model. The vitality of considering effectiveness is reflected in the urgency of the situation accompanying natural disasters. Pushed to extremes, time of delivery translates into number of lives saved. The aspect of fairness also relates to level of urgency in reception of aid, as discussed in Section 3.3.

In light of these facts, the chosen objective function maximizes utility accumulated over the scenario tree nodes, n, of stage 2 and 3. The two terms constituting the utility function express utility in terms of level of demand fulfillment and residual monetary budget respectively, in the same manner as for the deterministic model stated in Section 5. What separates the stochastic programming model from its deterministic counterpart however, is the inclusion of different disaster scenarios reflecting the uncertainty in outcome following an earthquake. The objective function in itself does not constitute a valid basis for evaluation of the model's performance. This is due to the duplication of utility values caused by summing both 2nd and 3rd stage utility. Yet, the reason for including both stages is the desire to produce efficient decisions for both the planned and the realized distribution plan. These decisions, planned as well as realized, should be made according to the level of urgency in need at the PODs for every scenario tree node.

Constraints Stage t = {1}

The initial decisions concern which LDCs to open, and the amount of supply to provide these LDCs with. The 1st stage decisions are made under uncertainty regarding the capacity of the vehicle fleet performing last mile distribution, the level of final demandand the distribution network connecting the nodes in the network, in accordance with the description of the Disaster Response Problem given in Section 3.2 .

Supply Constraints:

$$\sum_{j\in\mathcal{L}} y^{I}_{ijb} \leq S_{ib} \qquad i \in \mathcal{I}, b \in \mathcal{B} \tag{7.2}$$

The Supply Constraints (7.2) ensure that the amount of commodities dispatched from the ICDs is kept within the amount available at the ICDs.

LDC Capacity Constraints:

$$\sum_{i\in\mathcal{I}}\sum_{b\in\mathcal{B}} Q_b y^{I}_{ijb} - C^{C}_{j} l_j \leq 0 \qquad j \in \mathcal{L} \tag{7.3}$$

The different LDCs are assumed to be subject to capacity limitations in terms of size of the facility. Constraints (7.3) prohibit violation of these capacity restrictions for the initialized LDCs, in addition to preventing inflow of commodities to the LDCs which are excluded from the distribution process. These constraints correspond to LDC Capacity Constraints (5.7) of the deterministic model given i Section 5.3.

Vehicle Flow Constraints:

$$\sum_{j \in \mathcal{L}} x_{ij}^I \leq F_i^I \qquad i \in \mathcal{I} \tag{7.4}$$

$$x_{ij}^I - F_i^I z_{ij}^I \leq 0 \qquad (i,j) \in \mathcal{A} \tag{7.5}$$

$$\sum_{b \in \mathcal{B}} Q_b y_{ijb}^I - C^I x_{ij}^I \leq 0 \qquad (i,j) \in \mathcal{A} \tag{7.6}$$

$$z_{ij}^I - x_{ij}^I \leq 0 \qquad (i,j) \in \mathcal{A} \tag{7.7}$$

Vehicle Flow Constraints (7.4) - (7.7) are exact replications of the Vehicle Flow Constraints (5.3) - (5.6) of Section 5.3. The reader is thus referred to this section for an explanation of the group of constraints.

Constraints Linking Stage t={1} and t={2}

The transition from stage 1 to stage 2 of the distribution process takes place as the LDCs have received their designated amounts of supply from the ICDs, in addition to information about vehicle fleet capacity and demand. The 2nd stage decisions indicate the *expected* last mile distribution routes, and represent the planned routes. They are generated based on uncertain information regarding the state of the infrastructure connecting the LDCs and PODs, as described in Section 3.2.

Transshipment Commodity Flow Balance Constraints:

$$\sum_{v \in \mathcal{V}} \sum_{(i,j) \in \mathcal{A}_{vn}} y_{ijbvn}^L - \sum_{(j,i) \in \mathcal{A}} y_{jib}^I \leq 0 \qquad i \in \mathcal{L}, b \in \mathcal{B}, n \in \mathcal{E}_2 \tag{7.8}$$

The linking constraints connecting the 1st and 2nd stage, control the planned number of commodities to be used to serve final demand, and are intended to balance in- and outgoing flow of supplies at the LDCs chosen for operation. The constraints ensure that the amount of commodities dispatched from each LDC, does not exceed the number of commodities received from the ICDs. These constraints correspond to the Transshipment Commodity Flow Balance Constraints (5.8) of the deterministic model given i Section 5. However, inbound flow of commodities from PODs and other LDCs is omitted from the stochastic formulation owing to the dummy node system introduced to the stochastic programming model. The purpose of this system is to enable control of inflow versus outflow of commodities at the LDCs across the three stages.

Constraints Stage t = {2}

Planned Demand Satisfaction Constraints:

$$\sum_{v\in\mathcal{V}_{jn}}\sum_{(i,j)\in\mathcal{A}_{vn}} y^L_{ijbvn} - \sum_{v\in\mathcal{V}_{jn}}\sum_{(j,i)\in\mathcal{A}_{vn}} y^L_{jibvn} - \sum_{k\in\mathcal{K}} d_{jbkn} = 0 \qquad j\in\mathcal{P}, b\in\mathcal{B}, n\in\mathcal{E}_2 \tag{7.9}$$

$$\sum_{k\in\mathcal{K}} d_{jbkn} \leq D_{jbn} \qquad j\in\mathcal{P}, b\in\mathcal{B}, n\in\mathcal{E}_2 \tag{7.10}$$

$$d_{jbkn} \leq U_{jbkn} \qquad j\in\mathcal{P}, b\in\mathcal{B}, k\in\mathcal{K}, n\in\mathcal{E}_2 \tag{7.11}$$

Constraints (7.9) - (7.11) are equivalent to Constraints (5.9) - (5.11) of the deterministic model, and apply to the 2nd stage scenario tree nodes. The reader is referred to Section 5.3 for an explanation of the constraints.

Planned Vehicle Flow Constraints:

$$\sum_{(i,j)\in\mathcal{A}_{vn}} x^L_{ijvn} - F^L_{ivn} l_i \leq 0 \qquad i\in\mathcal{L}, v\in\mathcal{V}, n\in\mathcal{E}_2 \tag{7.12}$$

$$x^L_{ijvn} - (\sum_{j'\in\mathcal{L}} F^L_{j'vn}) z^L_{ijvn} \leq 0 \qquad v\in\mathcal{V}, n\in\mathcal{E}_2, (i,j)\in\mathcal{A}_{vn} \tag{7.13}$$

$$\sum_{b\in\mathcal{B}} Q^C_b y^L_{ijbvn} - C^L_v x^L_{ijvn} \leq 0 \qquad v\in\mathcal{V}, n\in\mathcal{E}_2, (i,j)\in\mathcal{A}_{vn} \tag{7.14}$$

$$\sum_{(j,i)\in\mathcal{A}_{vn}} x^L_{jivn} - \sum_{(i,j)\in\mathcal{A}_{vn}} x^L_{ijvn} \leq 0 \qquad j\in\mathcal{L}^D\cup\mathcal{P}, n\in\mathcal{E}_2, v\in\mathcal{V}_{jn} \tag{7.15}$$

The capacity of the vehicle fleet is amongst the realized information of stage 2. Constraints (7.12) ensure that the number of vehicles departing from an LDC does not exceed the total available quantity of vehicles across the variety of types assigned to that LDC. Constraints (7.13) consider the number of vehicles traversing an existing arc, making sure that this quantity does not exceed the total number of available vehicles across all LDCs. Their purpose is to force the binary flow variables indicating whether or not an arc is subject to passage, to one if a vehicle type traverses a given arc. Constraints (7.14) keep total load assigned to the different vehicles within the vehicle's capacity across all arcs which they are allowed to pass. Constraints (7.15) represent the vehicle flow balance constraints balancing in- and outgoing flow of vehicles at the dummy LDCs and PODs. They prohibit an increase in the number of vehicles passing through a relevant node between point of arrival and point of departure.

Arc Capacity Constraints:

$$\sum_{v \in \mathcal{V}} Q_v^V (x_{ijvn}^L + x_{(i+Q_L)jvn}^L) \leq C_{ijn}^A \qquad n \in \mathcal{E}_2, (i,j) \in \mathcal{A}_{vn} : i \in \mathcal{L}, j \in \mathcal{N} \backslash \mathcal{I} \tag{7.16}$$

$$\sum_{v \in \mathcal{V}} Q_v^V x_{ijvn}^L \leq C_{ijn}^A \qquad n \in \mathcal{E}_2, (i,j) \in \mathcal{A}_{vn} : i \in \mathcal{P}, j \in \mathcal{N} \backslash \mathcal{I} \tag{7.17}$$

Constraints (7.16) and Constraints (7.17) enforce arc capacity compliance by considering the size and number of vehicles seeking to traverse an arc. They apply to the existing outgoing arcs of LDCs and PODs, respectively. Due to the dummy node system, the outgoing arcs of an LDC equal those of the respective dummy LDC, consequently rendering it necessary to consider them concurrently.

Planned Commodity Flow Constraints:

$$\sum_{v \in \mathcal{V}} \sum_{(i,j) \in \mathcal{A}_{vn} : i \in \mathcal{L}} y_{ijbvn}^L - \sum_{j \in \mathcal{P}} \sum_{k \in \mathcal{K}} d_{jbkn} = 0 \qquad b \in \mathcal{B}, n \in \mathcal{E}_2 \tag{7.18}$$

$$\sum_{(j,i) \in \mathcal{A}_{vn}} y_{jibvn}^L - \sum_{(i,j) \in \mathcal{A}_{vn}} y_{ijbvn}^L \leq 0 \qquad j \in \mathcal{L}^D \cup \mathcal{P}, b \in \mathcal{B}, v \in \mathcal{V}, n \in \mathcal{E}_2 \tag{7.19}$$

We wish the complete amount of commodities initially planned for dispatch from the LDCs to contribute in satisfying final demand. This circumstance is guaranteed by means of Constraints (7.18). The Realized Commodity Flow Balance Constraints (7.19) balances inflow versus outflow of commodities at the dummy LDCs and PODs. They prohibit an increase in the number of commodities shipped through a relevant node between point of arrival and point of departure.

Constraints Linking Stage t={2} and t={3}

Demand Satisfaction Linking Constraints:

$$\sum_{j \in \mathcal{P}} \sum_{k \in \mathcal{K}} d_{jbkn} - \sum_{j \in \mathcal{P}} \sum_{k \in \mathcal{K}} d_{jbka(n)} \leq 0 \qquad b \in \mathcal{B}, n \in \mathcal{E}_3 \tag{7.20}$$

$$\begin{aligned} &\sum_{k \in \mathcal{K}} d_{jbka(n)} - \sum_{k \in \mathcal{K}} d_{jbkn} \\ &+ \Big(\sum_{(i,j) \in \mathcal{A}'_{vn}} \sum_{v \in \mathcal{V}_{jn}} y_{ijbvn}^L - \sum_{(i,j) \in \mathcal{A}_{va(n)}} \sum_{v \in \mathcal{V}_{ja(n)}} y_{ijbva(n)}^L \Big) \leq 0 \qquad j \in \mathcal{P}, b \in \mathcal{B}, n \in \mathcal{E}_3 \end{aligned} \tag{7.21}$$

Constraints (7.20) and (7.21) comprise the links coordinating decision regarding the amount of demand to be satisfied in the second and the third stage. Constraints (7.20) prevent total planned demand satisfaction in stage 3 from exceeding the total actual demand satisfaction of the planned stage across all demand points. In stage 2 the vehicles are packed according to realized capacity and demand, and tentative vehicle routes are generated. As the different vehicles start their designated routes generated in stage 2 before stage 3 is initialized, we are prevented from repacking the vehicles, and are in effect unable to add additional cargo to increase total demand satisfaction in stage 3. The decision maker should however be allowed to vary the amount to be delivered at the different destinations between stage 2 and 3. This is provided for by Constraints (7.21), which enable the model to change the level of demand fulfillment at a demand point should it prove costly to reach or inaccessible, or provide less utility than expected. These constraints also imply that PODs can receive more than planned such that available commodities are not wasted.

Vehicle Flow Linking Constraints:

$$\sum_{(i,j)\in\mathcal{A}'_{vn}} x^L_{ijvn} - \sum_{(i,j)\in\mathcal{A}_{va(n)}} x^L_{ijva(n)} \leq 0 \qquad i \in \mathcal{L}, v \in \mathcal{V}, n \in \mathcal{E}_3 \tag{7.22}$$

$$x^L_{ijva(n)} - x^L_{ijvn} + \Big(\sum_{(j',i)\in\mathcal{A}_{vn}} x^L_{j'ivn} - \sum_{(j',i)\in\mathcal{A}_{va(n)}} x^L_{j'iva(n)}\Big) \leq 0 \quad v \in \mathcal{V}, n \in \mathcal{E}_3, (i,j) \in \mathcal{A}_{vn} \tag{7.23}$$

$$x^L_{ijvn} - x^L_{ijva(n)} \leq 0 \quad v \in \mathcal{V}, n \in \mathcal{E}_3, (i,j) \in \mathcal{A}_{vn} \tag{7.24}$$

The planned and realized last mile distribution decisions made in the 2nd and 3rd stage respectively, require coordination. We thus need to enforce 3rd stage adherence to the initial plan developed and initialized in the 2nd stage, and ensure that it is followed to the extent feasible. The local agent is prohibited from repacking or loading additional vehicles between stage 2 and 3. In that regard, Constraints (7.22) make sure that the number of vehicles upon which the realized distribution plan of stage 3 is based, does not surpass the loaded and potentially dispatched number of vehicles of stage 2.

Transition between the 2nd and 3rd stage takes place as a vehicle meets an infrastructural obstacle which calls for diversion of planned flow. The network of stage 3 will at best equal that of stage 2, and thus constitutes a subset of the operable arcs of stage 2. An enforcement of the initial distribution plan is hence exclusively viable for their mutual arcs. Constraints (7.23) - (7.24) coordinating vehicle flow in the last two stages, need thus only apply to their common set of arcs. Constraints (7.23) require the realized vehicle quantity traversing an arc in the final stage to equal that of stage 2, given that the flow of vehicles into the start node of that arc does not diverge between stages. If however, the number of vehicles able to reach the start node is less in the final stage than in stage 2, the constraints allow for a reduction in the number of vehicles traversing that arc accordingly. The vehicles are forced over in the duplicated network should a change of route prove necessary, and Constraints (7.24) are included to prohibit increase in vehicle flow in the mathematical original network. Flow from the original into the duplicated network thus indicates transition from stage 2 to stage 3. Both increase and decrease in vehicle quantity to cross an arc is permitted in the duplicated network, to allow for unrestrained drawing of final routes. Hence, all route modifications causing an increase in flow in the final stage must be made in the duplicated network.

Commodity Flow Linking Constraints:

$$\sum_{(i,j)\in\mathcal{A}'_{vn}} y^L_{ijbvn} - \sum_{(i,j)\in\mathcal{A}_{va(n)}} y^L_{ijbva(n)} \leq 0 \quad i\in\mathcal{L}, b\in\mathcal{B}, v\in\mathcal{V}, n\in\mathcal{E}_3 \tag{7.25}$$

$$y^L_{ijbva(n)} - y^L_{ijbvn} + \Big(\sum_{(j',i)\in\mathcal{A}_{vn}} y^L_{j'ibvn} - \sum_{(j',i)\in\mathcal{A}_{va(n)}} y^L_{j'ibva(n)}\Big) \leq 0 \quad b\in\mathcal{B}, v\in\mathcal{V}, n\in\mathcal{E}_3, (i,j)\in\mathcal{A}_{vn} \tag{7.26}$$

$$y^L_{ijbvn} - y^L_{ijbva(n)} \leq 0 \quad b\in\mathcal{B}, v\in\mathcal{V}, n\in\mathcal{E}_3, (i,j)\in\mathcal{A}_{vn} \tag{7.27}$$

In the exact same fashion as the Vehicle Flow Linking Constraints, the Commodity Flow Linking Constraints (7.25) - (7.27) merge the decisions made in the 2nd and 3rd stage concerning the amount of a commodity type to traverse an arc in a specific vehicle type.

Constraints Stage t = {3}

The majority of constraints describing the 2nd stage also relate to stage 3. The 3rd stage constraints for which this applies are hence modified versions of constraints also stated for stage 2. The most intuitive modification is that of the different networks they consider. The constraints will not be explained in greater detail, but rather be accompanied by a referral back to their 2nd stage equivalent.

Realized Demand Satisfaction Constraints:

$$\sum_{v \in \mathcal{V}_{jn}} \sum_{(i,j) \in \mathcal{A}'_{vn}} y^L_{ijbvn} - \sum_{v \in \mathcal{V}_{jn}} \sum_{(j,i) \in \mathcal{A}'_{vn}} y^L_{jibvn} - \sum_{k \in \mathcal{K}} d_{jbkn} = 0 \qquad j \in \mathcal{P}, b \in \mathcal{B}, n \in \mathcal{E}_3 \tag{7.28}$$

$$\sum_{k \in \mathcal{K}} d_{jbkn} \leq D_{jbn} \qquad j \in \mathcal{P}, b \in \mathcal{B}, n \in \mathcal{E}_3 \tag{7.29}$$

$$d_{jbkn} \leq U_{jbkn} \qquad j \in \mathcal{P}, b \in \mathcal{B}, k \in \mathcal{K}, n \in \mathcal{E}_3 \tag{7.30}$$

Constraints (7.28) - (7.30) correspond to Constraints (7.9) - (7.11) of stage 2.

Realized Vehicle Flow Constraints:

$$\sum_{(i,j) \in \mathcal{A}'_{vn}} x^L_{ijvn} - F^L_{iva(n)} l_i \leq 0 \qquad i \in \mathcal{L}, v \in \mathcal{V}, n \in \mathcal{E}_3 \tag{7.31}$$

$$x^L_{ijvn} - (\sum_{j' \in \mathcal{L}} F^L_{j'va(n)}) z^L_{ijvn} \leq 0 \qquad v \in \mathcal{V}, n \in \mathcal{E}_3, (i,j) \in \mathcal{A}'_{vn} \tag{7.32}$$

$$\sum_{b \in \mathcal{B}} Q_b y^L_{ijbvn} - C^L_v x^L_{ijvn} \leq 0 \qquad v \in \mathcal{V}, n \in \mathcal{E}_3, (i,j) \in \mathcal{A}'_{vn} \tag{7.33}$$

$$z^L_{ijvn} - x^L_{ijvn} \leq 0 \qquad v \in \mathcal{V}, n \in \mathcal{E}_3, (i,j) \in \mathcal{A}'_{vn} \tag{7.34}$$

$$\sum_{(j,i) \in \mathcal{A}'_{vn}} x^L_{jivn} - \sum_{(i,j) \in \mathcal{A}'_{vn}} x^L_{ijvn} \leq 0 \qquad j \in \mathcal{L}^{D'} \cup \mathcal{P}', n \in \mathcal{E}_3, v \in \mathcal{V}_{jn} \tag{7.35}$$

Constraints (7.31) - (7.33) and (7.35) correspond to Constraints (7.12) - (7.15) of stage 2. As information regarding the capacity of the vehicle fleet is acquired at stage 2, the parameter stating the 3rd stage vehicle availability is that of the preceding stage. Constraints (7.34) are in addition added to the vehicle flow constraints of stage 3. Their purpose is to force the binary flow variables indicating whether or not an arc is subject to passage to zero if a vehicle type does not traverse a given arc.

Arc Capacity Constraints:

$$\sum_{v\in\mathcal{V}} Q_v^V(x_{ijvn}^L + x_{(i+Q_L)jvn}^L + x_{(i+2Q_L+Q_P)(j+Q_L+Q_P)vn}^L) \leq C_{ijn}^A \qquad n\in\mathcal{E}_3, \; (i,j)\in\mathcal{A}_{vn} : i\in\mathcal{L}, j\in\mathcal{N}\backslash\mathcal{I} \tag{7.36}$$

$$\sum_{v\in\mathcal{V}} Q_v^V(x_{ijvn}^L + x_{(i+Q_L+Q_P)(j+Q_L+Q_P)vn}^L) \leq C_{ijn}^A \qquad n\in\mathcal{E}_3, \; (i,j)\in\mathcal{A}_{vn} : i\in\mathcal{P}, j\in\mathcal{N}\backslash\mathcal{I} \tag{7.37}$$

Constraints (7.36) - (7.37) correspond to Constraints (7.16) - (7.17) of stage 2. However, a duplication of the distribution network enabling in transit modification of the initially planned route, is introduced to the 3rd stage. The arcs signifying connection between an LDC and the remaining nodes in the mathematical original network will in consequence be subject to a threefold duplication. The arcs connecting a POD and the remaining nodes in the original network are similarly subject to a twofold duplication. While considering this, Constraints (7.36) and (7.37) ensure arc capacity compliance for arcs having LDCs and PODs as points of departure, respectively.

Realized Commodity Flow Balance Constraints:

$$\sum_{(j,i)\in\mathcal{A}'_{vn}} y_{jibvn}^L - \sum_{(i,j)\in\mathcal{A}'_{vn}} y_{ijbvn}^L \leq 0 \qquad j\in\mathcal{L}^{D'}\cup\mathcal{P}', b\in\mathcal{B}, v\in\mathcal{V}, n\in\mathcal{E}_3 \tag{7.38}$$

Constraints (7.38) correspond to Constraints (7.19) of stage 2, with the addition of also applying to the LDCs and PODs constituting the mathematical duplicated network.

Efficiency Constraints Applied to All Stages

Budget Constraints:

$$\sum_{i\in\mathcal{L}} E_i^O l_i + \sum_{(i,j)\in\mathcal{A}}\sum_{b\in\mathcal{B}} E_b^C y_{ijb}^I + \sum_{(i,j)\in\mathcal{A}} E_{ij}^I x_{ij}^I + \sum_{v\in\mathcal{V}}\sum_{(i,j)\in\mathcal{A}_{vn}} E_{ijvn}^L x_{ijvn}^L + w_n = H^B \qquad n\in\mathcal{E}\backslash\mathcal{E}_1 \tag{7.39}$$

Constraints (7.39) ensure that the budget is honored, and calculate the residual budget. They apply to the last two stages separately, but both take 1st stage costs into account.

Convoy Travel Time Constraints:

$$\sum_{(i,j)\in\mathcal{A}} T^I_{ij} z^I_{ij} + \sum_{v\in\mathcal{V}} \sum_{(i,j)\in\mathcal{A}'_{vn}} T^L_{ijvn} z^L_{ijvn} \leq H^T \qquad n \in \mathcal{E}_3 \tag{7.40}$$

Constraints (7.40) account for the time spent serving the different PODs by keeping convoy time within a given bound. However, seeing how the binary vehicle flow variables merely indicate whether or not one or more vehicles traverse an arc, the calculated time value should not be taken as the total delivery time.

Non-Negativity Constraints for All Variables

$$l_i \in \{0,1\} \qquad i \in \mathcal{L} \tag{7.41}$$

$$x^I_{ij} \geq 0 \qquad \text{integer}, (i,j) \in \mathcal{A} \tag{7.42}$$

$$y^I_{ijb} \geq 0 \qquad (i,j) \in \mathcal{A}, b \in \mathcal{B} \tag{7.43}$$

$$z^I_{ij} \in \{0,1\} \qquad (i,j) \in \mathcal{A} \tag{7.44}$$

$$d_{jbkn} \geq 0 \qquad j \in \mathcal{P}, b \in \mathcal{B}, k \in \mathcal{K}, n \in \mathcal{E}_2 \cup \mathcal{E}_3 \tag{7.45}$$

$$x^L_{ijvn} \geq 0 \qquad \text{integer}, v \in \mathcal{V}, n \in \mathcal{E}_2, (i,j) \in \mathcal{A}_{vn} \tag{7.46}$$

$$y^L_{ijbvn} \geq 0 \qquad v \in \mathcal{V}, n \in \mathcal{E}_2, (i,j) \in \mathcal{A}_{vn}, b \in \mathcal{B} \tag{7.47}$$

$$z^L_{ijvn} \in \{0,1\} \qquad v \in \mathcal{V}, n \in \mathcal{E}_2, (i,j) \in \mathcal{A}_{vn} \tag{7.48}$$

$$x^L_{ijvn} \geq 0 \qquad \text{integer}, v \in \mathcal{V}, n \in \mathcal{E}_3, (i,j) \in \mathcal{A}'_{vn} \tag{7.49}$$

$$y^L_{ijbvn} \geq 0 \qquad v \in \mathcal{V}, n \in \mathcal{E}_3, (i,j) \in \mathcal{A}'_{vn}, b \in \mathcal{B} \tag{7.50}$$

$$z^L_{ijvn} \in \{0,1\} \qquad v \in \mathcal{V}, n \in \mathcal{E}_3, (i,j) \in \mathcal{A}'_{vn} \tag{7.51}$$

$$w_n \geq 0 \qquad n \in \mathcal{E}\backslash\mathcal{E}_1 \tag{7.52}$$

Constraints (7.41) - (7.44) represent the non-negativity constraints describing the decisions made in the initial stage of the distribution process. The non-negativity constraints associated with the decision variables of the second and final stage are given by Constraints (7.46) - (7.48) and (7.49) - (7.52), respectively, whereas Constraints (7.45) relate to both last two stages.

8 Implementation of the Models

Three commercial softwares are utilized to obtain and illustrate the solutions produced by the models presented in Section 5.3 and Section 7.3 to fictive problems. Generation of instances in Xcode or Microsoft Visual Studio form the first step of the implementation process, followed by optimization of the problem in Xpress in the 2nd step. The third and final step is executed in MATLAB and is intended to simplify interpretation of the output produced by Xpress. This section describes the use of the three commercial softwares, in addition to the process of generating different instances to be used for testing of the models.

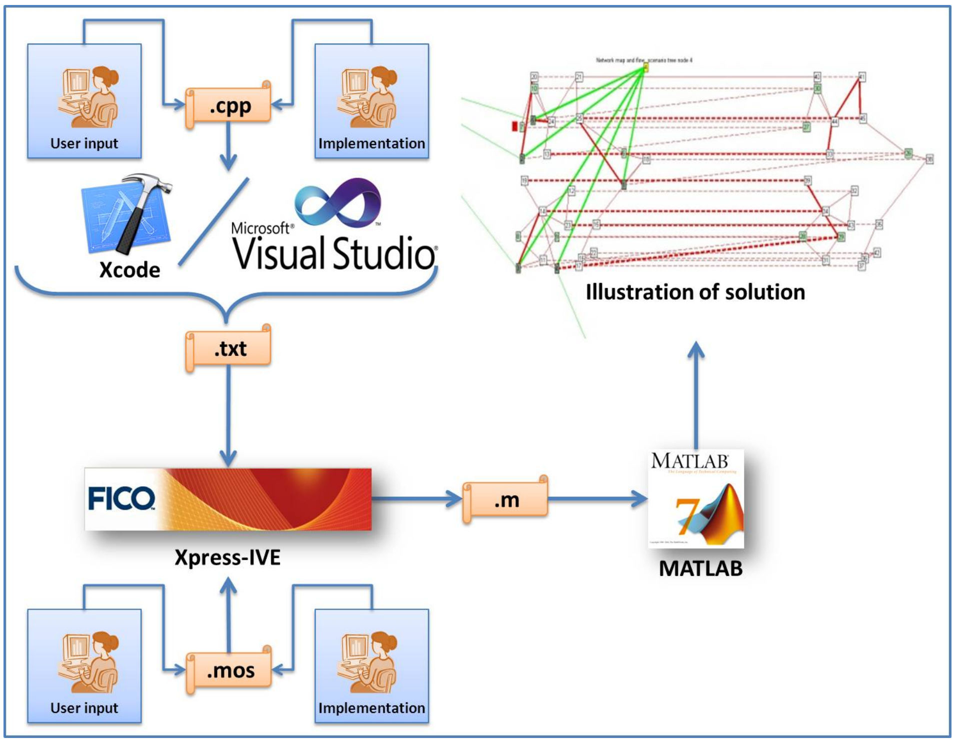

Figure 10: An outline of the steps involved in the implementation process

All programming code is written by the authors. A $c++$ code enables the user to feed input into the console to generate data of interest. The data file generated consists of sets, subsets, parameters and other required figures to be read by the optimizer. The output produced by the optimizer is in turn used as input into a software which plots a map to visualize the location of the totality of nodes constituting the mathematically extended network, in addition to the flow of vehicles in this network. Figure 10 serves to illustrate the total implementation process.

8.1 Xcode Version 3.0 and Microsoft Visual Studio 2010 Version 10.0.40219.1 SP1Rel

The two softwares are both suites of tools including c++ source code. We have used the programs to build and compile the written c++ code which generates instances based on input data from the user. The written code can be run by both programs. The number of PODs, $|\mathcal{P}|$, LDCs, $|\mathcal{L}|$, commodity types, $|\mathcal{B}|$, and possible outcomes in stage $t = 2$ and $t = 3$, represent a few of the basic input data which the user will be inquired to feed to the console. Other parameters required to be able to solve the mathematical models will be generated automatically based on the characteristics of the problem. The generation of instances will be described in greater detail in Section 8.4. The output of the executed code is a text file. This file contains all parameters necessary for successful compilation of the mathematical model. Two separate codes have been produced, applicable to the deterministic- and the stochastic programming problem respectively. The latter will be provided in digital files to be enclosed with the thesis.

8.2 Xpress-IVE 64bit Version 1.22.04

Both the deterministic and the stochastic programming models are implemented in mosel and run by the Xpress optimization suite. Xpress is a commercial software which enables optimization of mathematical problems. We use Xpress Mosel Version 3.2.3 and Xpress Optimizer Version 22.01.09. Mosel code is written such that data is read directly from the generated text file where the model builder can access it. The software uses LP relaxation, branch-and-bound algorithms and other presolve procedures to reduce the size of the problem prior to preforming a global search to find an optimal solution. Some elements and restrictions are added to the mosel code in order to reduce the number of variables created. For large problems, the size of the computer memory is critical. If the computer memory is sufficient, the global search will by default terminate when the optimality gap is less than 0.001%. When the optimizations are carried out, output will be generated if an optimal solution is found or the process is terminated by the user. The output gives a clear solution to the mathematical problem, in addition to code to be read by Matlab. The mosel code produced for the proposed MSP model, the deterministic model and the calculators of EEV' and WS' values will be provided in digital files to be enclosed with the thesis.

8.3 Matlab R2011b Version 7.13.0.564

Matlab is a numerical computing environment and fourth-generation programming language. We use this software to visualize the entire array of nodes and existing arcs of the mathematically extended network, in addition to the arcs that will be traversed in the distribution process, i.e. the final vehicle routes. By implementing the results into Matlab, we are able to track the randomness of the arcs related to each scenario, and illustrate the use of the duplicated network. The arcs that experience flow are represented by thicker lines, whereas and the connecting links are given by dashed lines. The ICDs are indicated by orange boxes in each corner, the LDCs and the PODs by blue and white boxes respectively, and the epicenter by a red box.

8.4 Instance Generation

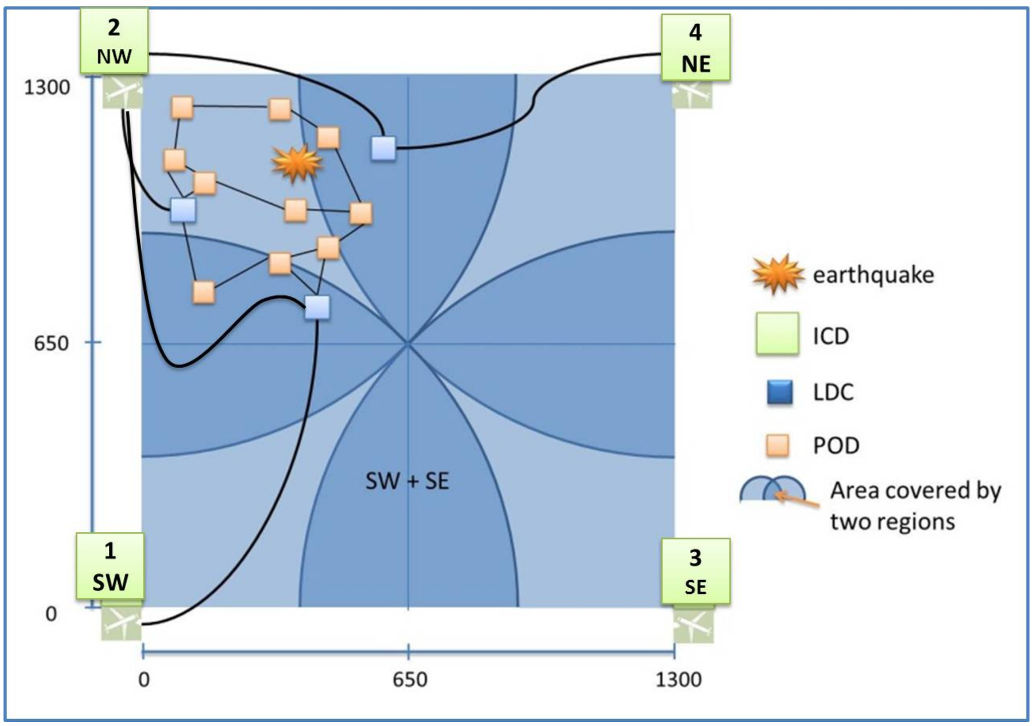

Figure 11: Illustration of the world as assumed in generation of instances - The distances of the figure does not agree with reality

The totality of instances used to demonstrate the applicability of the models proposed in this thesis are generated in accordance with the descriptions given in Section 8.1− 8.3. Due to differences in certain attributes of the deterministic and the MSP model, some of the parameters required by the two models will be of different nature. This section describes the generation of stochastic data files exclusively, as this generation process is the most comprehensive.

Generation of instances is semi-automatic, in the respect that the user will be required to feed the following data to the console: The number of commodity types, ($|\mathcal{B}|$), the number of LDCs, ($|\mathcal{L}|$), the number of PODs, ($|\mathcal{P}|$), vehicle types, ($|\mathcal{V}|$), budget, (H^B), utility factor for residual budget (M^B), upper convoy time limit, (H^T), unit size of the commodity types, (Q_b^C), unit specific costs (E_b^C, E_i^O), the number of possibilities in stage 2 and 3, ($|\mathcal{E}_2|$ and $\frac{|\mathcal{E}_3|}{|\mathcal{E}_2|}$), available vehicles, ($F_i^I$ and F_{iv1}^L), supply, (S_{ib}), capacities, (C^I, C_v^L and C_j^C) and assumed demand (D_{jb1}).

Based on this input data, the program assigns values to the remaining sets and parameters. Vehicle size is generated based on the given vehicle capacity. The number of vehicles of each of the given types disposable at the LDCs will increase or decrease randomly with up to 30% in the second stage. The utility factor for residual budget is given a value high enough to ensure efficient vehicle flow, yet low enough to ensure higher priority of the utility of demand fulfillment, M_{jbkn}. For every instance generated, the number of ICDs will be equal to four and located as shown in Figure 11, one in each corner. The ICDs' range of distribution cover a quarter of a circle with a radius of 920 length units. LDCs located in certain areas might in effect be covered by two ICDs. The darker blue fields in Figure 11 indicates the areas of dual coverage. The arcs connecting ICDs and LDCs have unlimited capacity in terms of the total number of vehicles that they are able to carry. The epicenter of the earthquake is given random coordinates, ranging from 0 and 1300 length units in each direction, within the reference frame depicted in the figure. The epicenter will thus be placed within one of the four quadrants in the system of coordinates, and the LDCs and PODs will be given random locations within this same quadrant.

The expected demand at each POD is defined by the user. This value is however subject to uncertainty in the first stage of the SP model, and demand will vary with up to 30% from the assumed value for the 2nd stage scenario tree nodes. There is greatest likelihood of increase in demand moving from stage 1 to stage 2. The successor nodes of each 2nd stage scenario tree node will experience the same level of demand as their predecessor. Four utility intervals, $\mathcal{K}$, will be applied to all instances generated. The size of these utility intervals, U_{jbk}, correlates with the level of demand at the three different stages. As the number of delivered items increases, the utility interval size increases accordingly, as shown in Figure 6 in Section 5.1. The utility factor for demand fulfillment across the set of PODs, M_{jbkn}, depends on a given POD's distance from the epicenter, and will change in line with the expected and realized demand in stage 1 and stage 2 respectively.

The anticipated network of stage 1 is constructed such that all LDCs and PODs are connected to at least two other nodes; the two closest to the node in question. A minimum of one arc within a set of linked nodes, will be linked to an LDC. All arcs of the network will be assigned a random quality, $1, 2$ or 3. The higher the number, the higher the quality and capacity. High quality values also imply less travel time and cost associated with traversing a given arc. Travel time and cost are functions of the distance between the nodes. After an earthquake has hit, nodes closer to the epicenter are statistically connected to less arcs than those situated further away, based on an assumption of correlation between distance from the epicenter and extent of damages. Arcs of lower quality are most likely to prove inoperable, and total capacity is in effect subject to reduction.

Each LDC is assigned a corresponding dummy LDC. The dummy LDCs are placed "north" of their respective original LDC. The real network consists of ICDs, LDCs and PODs. The mathematical original network on the other hand, comprises LDCs, dummy LDCs and PODs, whereas the duplicated network consists of duplicated dummy LDCs and duplicated PODs only. The mathematically extended network includes both the duplicated and the original network. For ease of visualization, the nodes of the duplicated network are all located "far east" of their original nodes.

The leaf tree nodes of stage 3 are given an equal probability of occurrence of $P_3 = \frac{1}{|\mathcal{E}_3|}$, as are the scenario tree nodes in stage 2, $P_2 = \frac{1}{|\mathcal{E}_2|}$.

9 An Illustrative Example

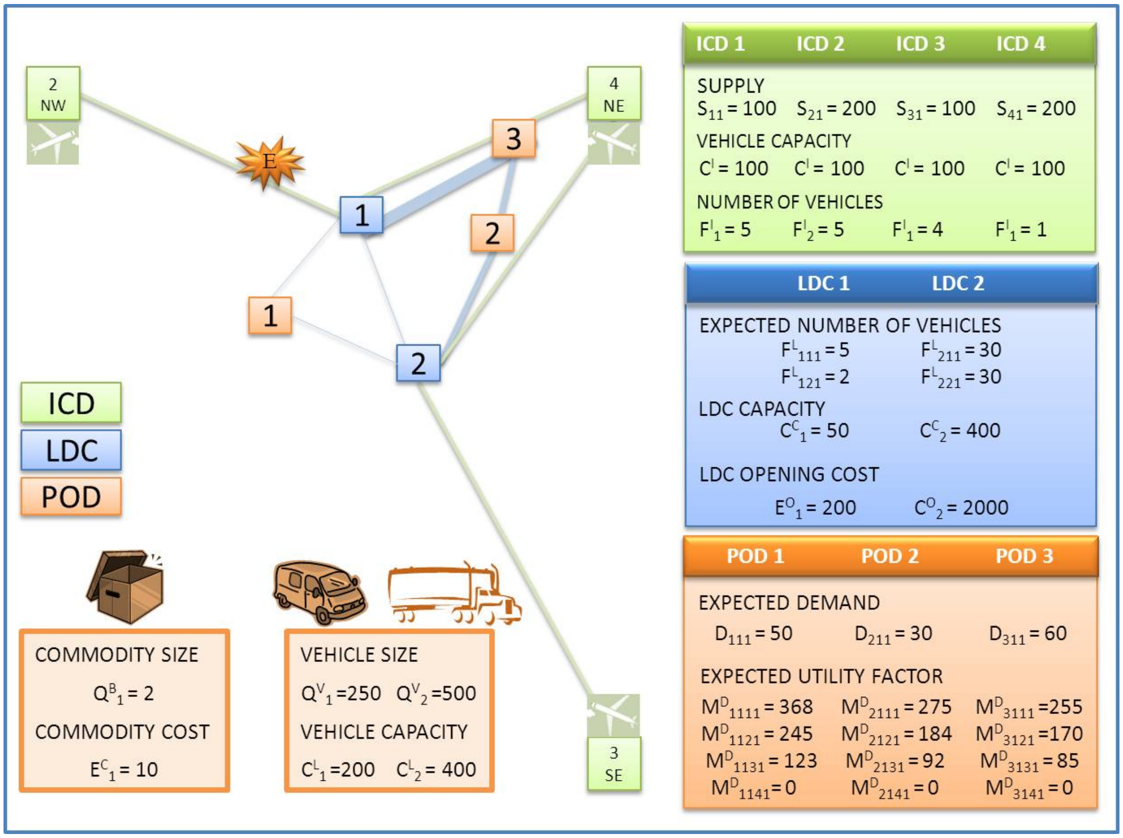

Figure 12: An outline of the characteristics of the illustrative example

A numerical example will be given in the following sections to illustrate how the two proposed models can be used to optimize the planning and distribution of humanitarian aid. The MSP model will however be the main focal point. How important features of the models such as utility factors, utility size, distribution network, different costs and capacities, together with commodity and vehicle sizes, may influence the allocation of commodities will be subject to analysis. The major aim of this section is to provide a comprehensive interpretation of the mathematical models, and illustrate their applicability. Numerical solutions will be presented and compared in Section 9.3 The results obtained in this illustrative example were found using Microsoft Visual Studio, Xpress-IVE and MATLAB, as explained in Section 8.

A problem consisting of four ICDs, $|\mathcal{I}| = 4$, two LDCs, $|\mathcal{L}| = 2$, and three PODs, $|\mathcal{P}| = 3$ is considered. Two vehicle types, $|\mathcal{V}| = 2$, are available to distribute a single type of commodities. Initial supply is set at sufficient level, whilst vehicle capacity and budget might prove insufficient in some scenarios. An overview of this distribution problem is given in Figure 12.

The arcs between the ICDs and the LDCs are incapacitated, because of the assumption of standard vehicles as stated in Section 5.2. Flow from ICDs to LDCs will however be restricted by the capacity of the LDC. The green lines in Figure 12 symbolize possible routes leading from ICDs to LDCs. Arcs connecting LDCs and PODs are divided into three categories according to their quality. In Figure 12 thicker arcs indicate higher quality, and thus higher capacity. Travel time and cost depend on the length and quality of the respective arc. The thinner the arc, the longer the travel times, and the higher the costs induced. Traversing a poor-quality arc exposes the vehicles to a higher level of stress than the good-quality arcs, and is consequently more expensive. Therefore, thicker arcs are preferred when possible.

The available supply at the different ICDs is sufficient to serve the totality demand, but due to restrictive aircraft capacity at least two vehicles will need to be utilized. This will imply an increase in total costs, a matter subject to minimization. Only ICDs covering the affected area will be able to provide initial supply, as described in Section 8.4. ICD 1 is located too far away from the affected area, whereas ICD 4 only has one vehicle at disposal making it unable to singly provide all aid necessary. As ICD 3 is situated even further away from the affected area than ICD 4 and ICD 2, this option will be the most expensive.

By observing the relation between LDC capacities and LDC costs, we would assume the only LDC chosen for operation to be LDC 2. Only 25 units of the commodity type can be handled at LDC 1, which is less than 18% of expected demand, necessitating initialization of both LDCs. Total opening costs and transportation costs will consequently be substantially higher for this option. If LDC 1 is to be opened, the increase in utility of demand fulfillment induced by being able to deliver an additional 25 units must be greater than the reduction in monetary utility implied by the increase in associated costs. This is due to the nature of Objective Function (5.1).

Considering the PODs exclusively, we would expect POD 1 to receive the total quantity of commodities demanded in the first utility interval prior to satisfaction of demand at the remaining POD, seeing as how this will provide highest expected utility. POD 3 should receive at least some items, prior to the level of demand satisfied at POD 1 reaching the second utility interval. This effect is produced as a result of considering the aspect of fairness, as described in Sections 3.3 and 4.6.

In terms of commodity type and vehicle types, it is apparent that vehicle type 1 will be able to deliver 100 units of the commodity, whereas vehicle type 2 is able to transport 200 units. The size of the two different vehicle types will restrict the number of vehicles of type 2 traversing poor quality arcs with a capacity of 1000, to be less than or equal to two.

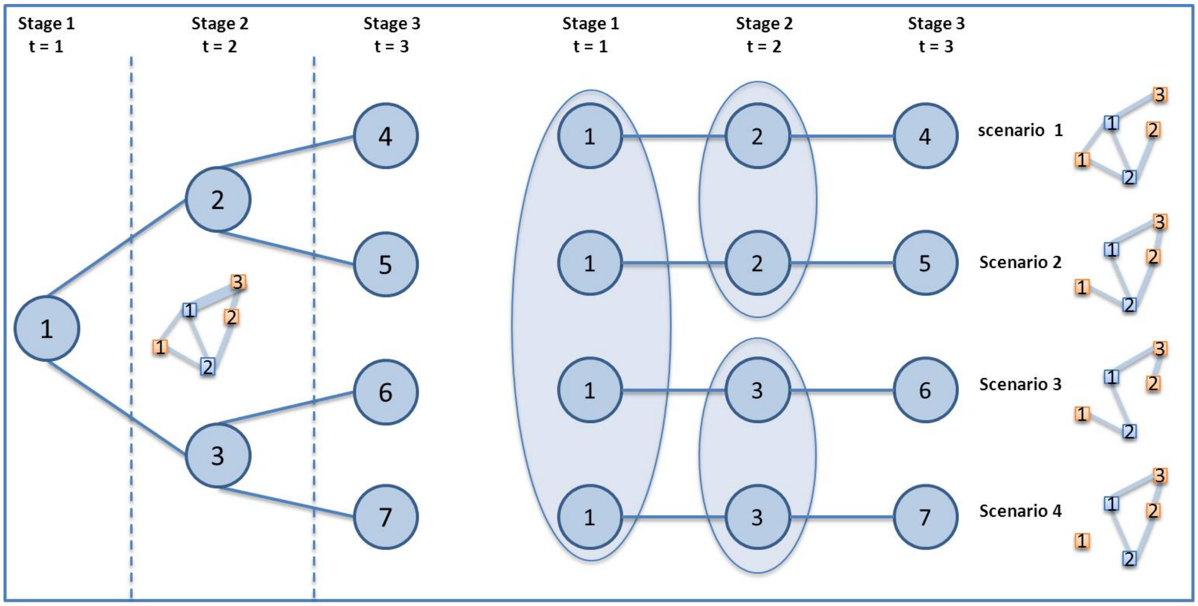

Figure 13: The scenario tree applicable to the illustrative example, in addition to the networks employed at stage 1 and 2 and for each of the different scenarios

Two possible outcomes can unfold from a parent node in both stage 1 and 2, resulting in a total of seven scenario tree nodes and four distinct scenarios as illustrated in Figure 13. The different realized networks are shown for the respective scenarios, as well as the coupling of decisions for each of the scenarios. Existence of an arc in the networks depicted in Figure 13 indicates that at least one, but not necessarily all, of the available vehicle types will be allowed to traverse the arc in question.

To enable comparison of the stochastic and the deterministic model, the latter model is applied to each of the four scenarios in addition to the average scenario problem described in Section 6.3.2.

The remaining utilized data not mentioned in the preceding discussion, will be given in Appendix C.

9.1 The Deterministic Approach

The initial problem takes place in the immediate aftermath of an earthquake. The solution to the expected value problem considering 1st stage parameters only is given in Figure 14.

By solving the deterministic model for each of the four possible scenarios separately, we

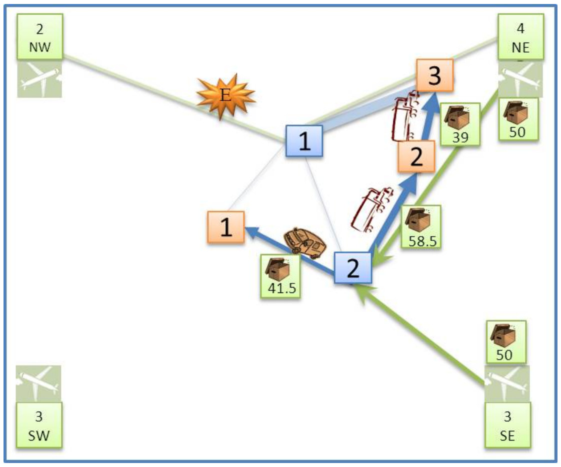

Figure 14: Optimal distribution strategy for the 1st stage average scenario problem

are able to calculate the expected optimal solution. The decision maker will determine an optimal strategy for distribution based on the characteristics of each scenario. These solutions will be better than (or equally good as) the one of the average scenario problem. The optimal strategy in terms of choice of LDCs to open and flow of goods from ICDs to LDCs is identical for all of the scenarios. The optimal last mile distribution strategy however, differs between scenarios. Comparing these decision schemes with that of the average scenario problem, deviations in distribution plans are recognized for the majority of scenarios. The different optimal strategies are depicted in Figure 15 and 14.

9.2 The Stochastic Approach

Solving the illustrative problem by means of the MSP model naturally results in generation of solutions different from that of the deterministic approach. The optimal value of objective function will receive little attention in the following discussion, due to the nature of the objective function. Both planned as well as realized DFU and MU, are included for maximization, resulting in an objective function value which is difficult to interpret.

Presentation of the results for each of the stages will be given in the following sections. For each of the stages, two illustrations showing the vehicle flow are provided for ease of visualization. The left picture illustrates the flow as defined by the mathematical decision variables based on the mathematically expanded network, as described in Section 7.1. As this network includes fictive nodes and arcs, a second illustration to the right of the Matlab-map is provided in order to show the flow of goods and vehicles as it would appear in real-life.

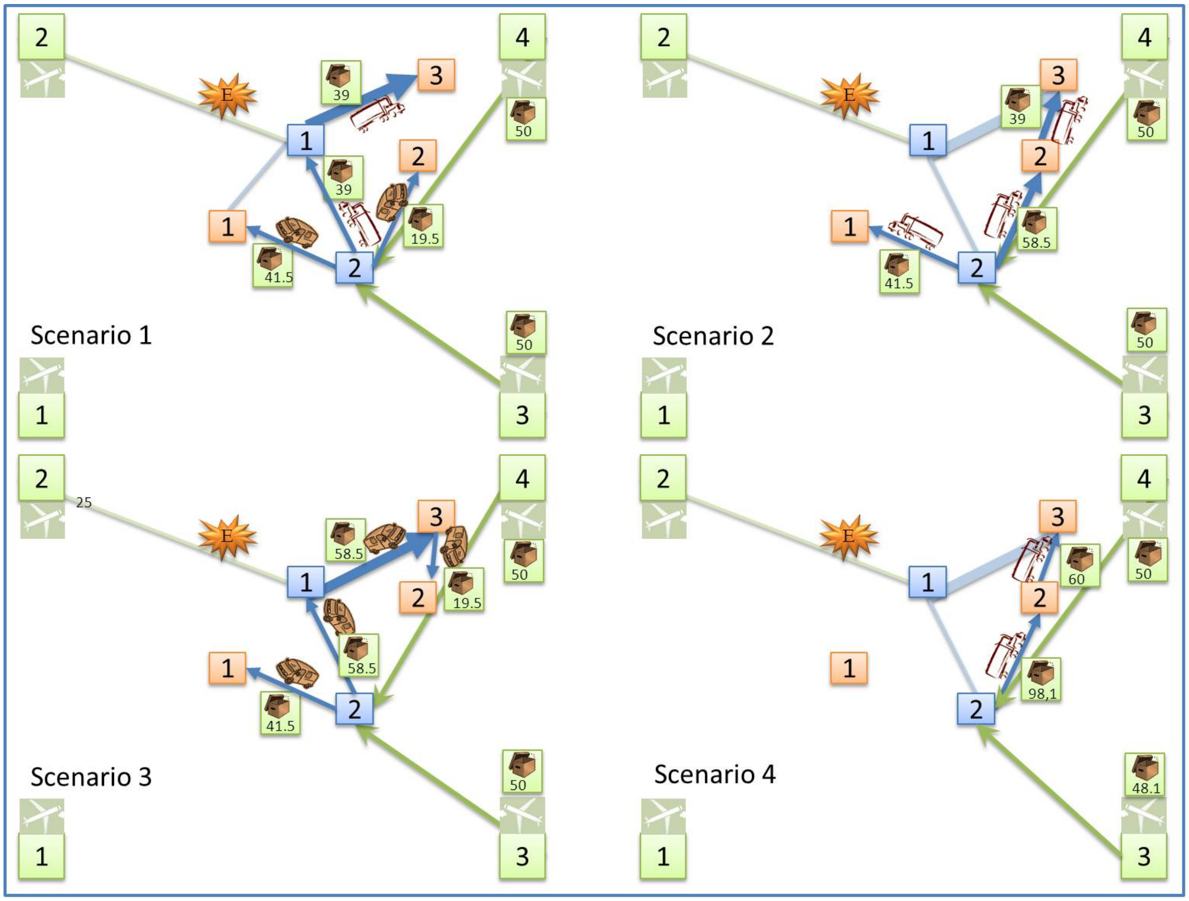

Figure 15: Optimal distribution strategies for the different scenarios when solved separately

9.2.1 Decision Scheme for Stage 1

The decisions made in the first stage are not directly represented in the objective function, but still have a large impact on the objective function value. Initialization and operation of an LDC imply both opening costs and transportation costs upon distribution. The initial decisions also affect the range of possible last mile distribution routes which can be applied at the following stages. Providing supply to a single LDC only, will entail greater risk of being disconnected from certain nodes as compared with operation of several LDCs. Such a situation can potentially prove catastrophic for those in need. The decision concerning the amount of supplies to deliver to the LDCs is decisive for the achievable level of demand fulfillment at stage 3, and is made based on rough estimates. LDC capacity, available supply and available aircrafts restrict the quantities which can be allocated to the different LDCs, but even within these limits dispatching excessive amounts of supply can result in shortage in budget, and is thus unfavorable.

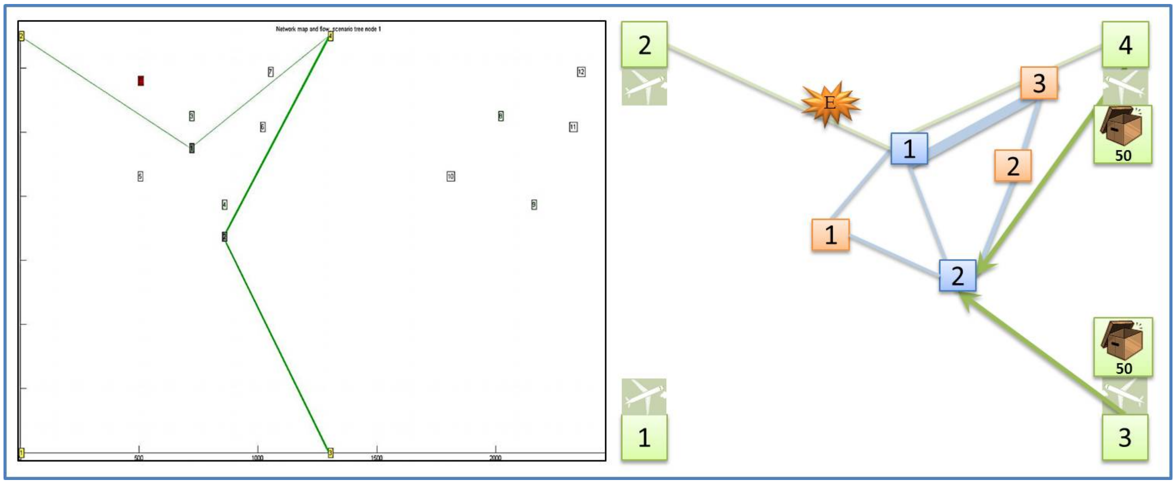

Figure 16: The flow from ICDs to initialized LDCs at stage 1
The duplicated network shown in the left map, is not subject to use at this stage

Figure 16 presents the range of 1st stage decisions for this example. Due to insufficient funds, total expected demand exceeds the 100 units delivered to LDC 2. The two aircrafts utilized are fully loaded, and dispatch of additional units would require use of yet another aircraft. This would induce costs beyond the residual budget. The restrictive nature of the budget also prevents initialization of LDC 1, which is less attractive to operate due to inferior capacity.

9.2.2 Decision Scheme for Stage 2

Firstly, we wish to describe the nature of the MSP model's Objective Function (7.1), and emphasize the necessity of considering the terms it contains. Special mention will be given to the inclusion of variables of the 2nd stage, as this is not intuitively perceivable.

The demand satisfaction term is included to ensure achievement of fair and sensible distribution for the initial distribution plan as planned in stage 2. Upon loading and dispatch of the vehicles chosen to perform last mile distribution, it will be of practicality due to both managing and operating purposes that their initially assigned destinations correspond to the actual final destination. If the 2nd stage decisions are omitted from the objective function, production of initial plans in which the vehicles are assigned to suboptimal destinations can occur, as the model only focuses on the final stage decisions. Suboptimal planning in the 2nd stage arises from the desire to spend as little as possible of the budget, due to the maximization of residual budget. The demand satisfaction term will be bounded by the amount of supplies received at the LDCs at stage 1. The entire amount of supplies planned to be dispatched from the LDCs is required to contribute to fulfillment of demand at the PODs. Also, delivery of items exceeding the quantity demanded is disallowed.

The economical term for stage 2 is also a part of the objective function in order to ensure that efficient decisions are made. Should this term be omitted, the model would produce plans in which a superfluous number of vehicles is dispatched from the LDCs in order to increase the opportunity set for the following stage. The initial distribution plan would in addition implement purposeless traversal of arcs by this set of vehicles until the budget is spent or the upper limit for convoy time is reached, based on the same line of reasoning. We seek to avoid this effect as no local agent would wish to implement an initial plan produced on these grounds. The initial decision scheme generated in stage 2, should be logical and readily apparent to the local agent and in accordance with reality.

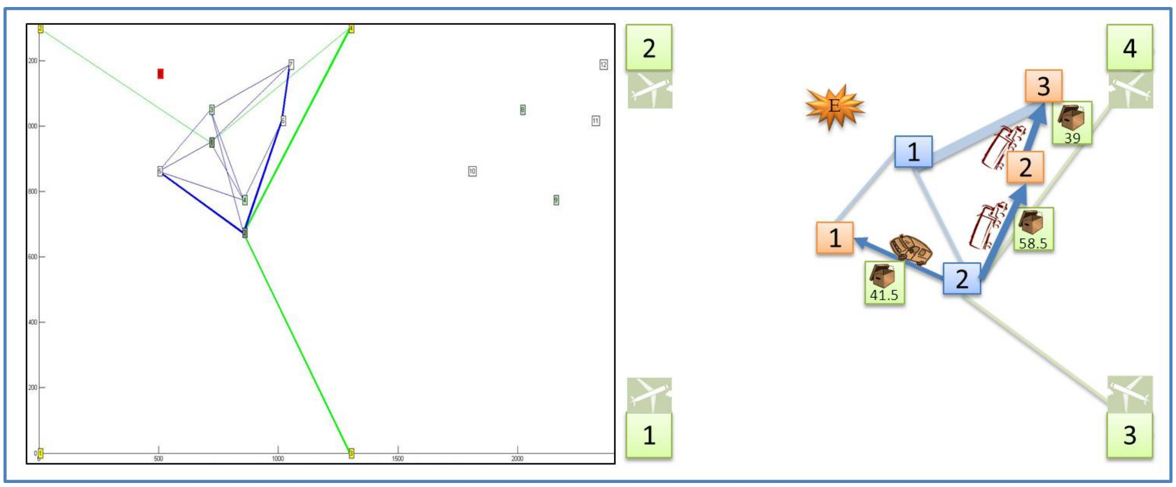

Figure 17: Planned vehicle and commodity flow in scenario tree node 2, Stage 2
The duplicated network shown in the left map, is not subject to use at this stage

Complete information concerning the realized number of vehicles per available type as well as actual level of demand and accompanying utility factors, will be attained at stage 2. This knowledge, in addition to that of the realized distribution networks in stage 3, will affect the choice of routes, and can cause differences in the routes planned for each of the scenario tree nodes in stage 2. This is illustrated by Figure 17 and Figure 18.

Two vehicles are packed and planned to travel two different distances in scenario tree node 2, as shown in Figure 17. The mathematical flow of this scenario tree node is identical to one of the realized routes to be presented in the following section. The flow related to scenario tree node 3 however, differs by its use of dummy LDC 1, as given in Figure 18 (left). LDC 1 is used for transshipment, but due to attributes of the mathematical model, flow must go

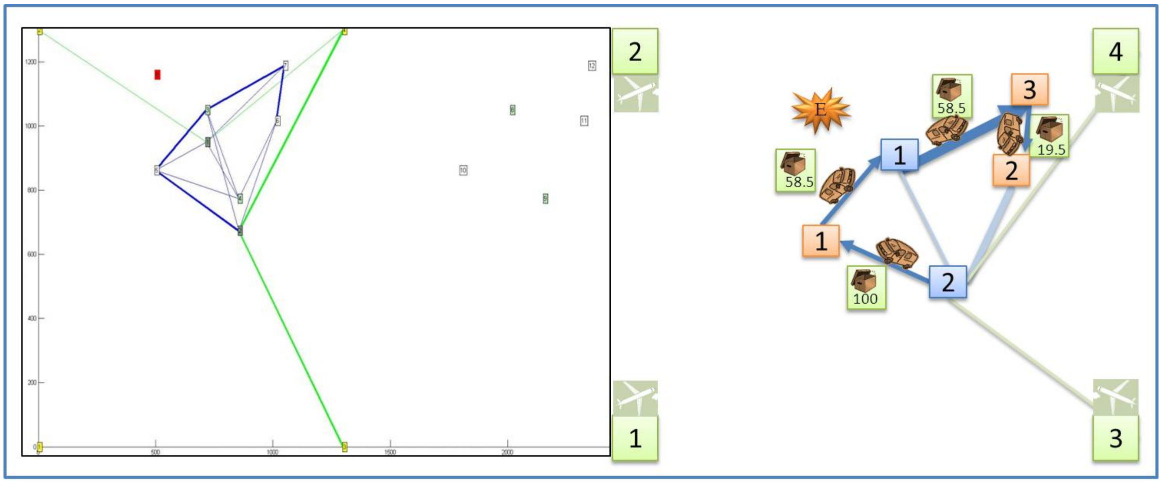

Figure 18: Planned vehicle- and commodity flow in scenario tree node 3, Stage 2
The duplicated network shown in the left map, is not subject to use at this stage

through the respective dummy LDC as it is prevented from passing through the real LDC. The flow as it would appear in the real-life network is illustrated in the right part of the figure. A single vehicle is packed at LDC 2, and will serve some PODs via LDC 1.

9.2.3 Decision Scheme for Stage 3

The networks realized in the 3rd stage upon which the final distribution plans are built, are depicted in Figure 13. The four different scenarios will be described individually in the following.

The planned route for scenario 1, scenario tree node 4, is given by scenario tree node 2 as shown in Figure 17. Both vehicles are packed and dispatched from the LDCs as soon as loading is complete according to this plan. Vehicle type 1 set to serve POD 1, follows this plan from point of supply to point of consumption. Hence, it never reaches stage 3 as it does not require additional information about the network to complete its task. This is illustrated in the left map in Figure 19 by means of a red line indicating the route followed by vehicle type 1. This line stays within the mathematical original network throughout the entire period of distribution.

The vehicle planned to served POD 2 and POD 3 however, hits an obstacle after reaching and providing delivery to POD 2. The road between POD 2 and POD 3 proves impassable, and the plan needs to be changed accordingly. To avoid more dead-ends, the driver makes a call to the local agent and receives information about the complete state of the network. The reception of this information indicates transition from stage 2 to stage 3. The vehicle

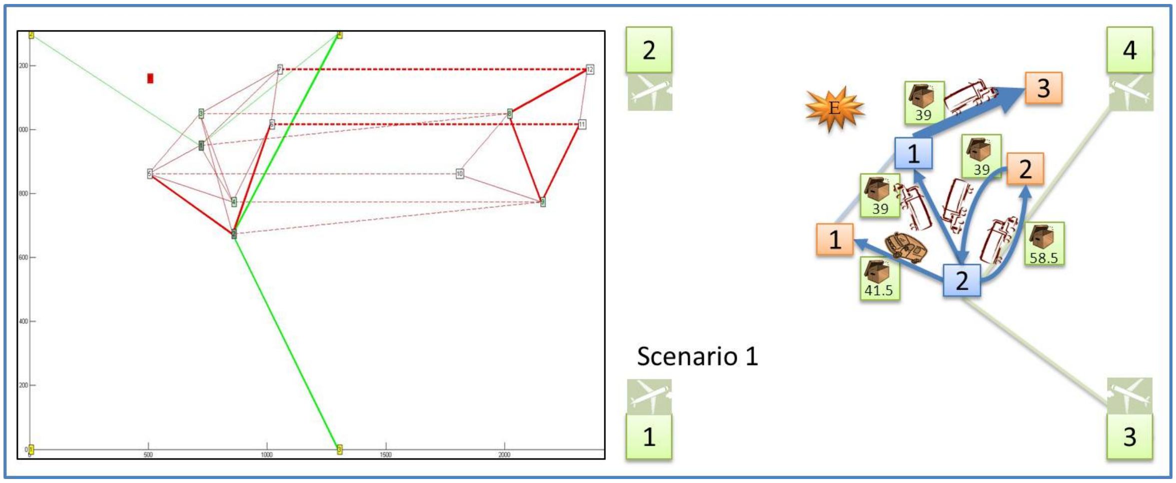

Figure 19: Realized vehicle and commodity flow in the scenario tree node 4, Scenario 1

is no longer required to follow the original plan, and further distribution is carried through in the duplicated network (left part of Figure 19) symbolizing transition from stage 2 to 3. The realized route as followed in the real network is illustrated in the right part of Figure 19. The modified route leading to vehicle type 2's final destination comprises the optimal route considering the realized network of this specific scenario.

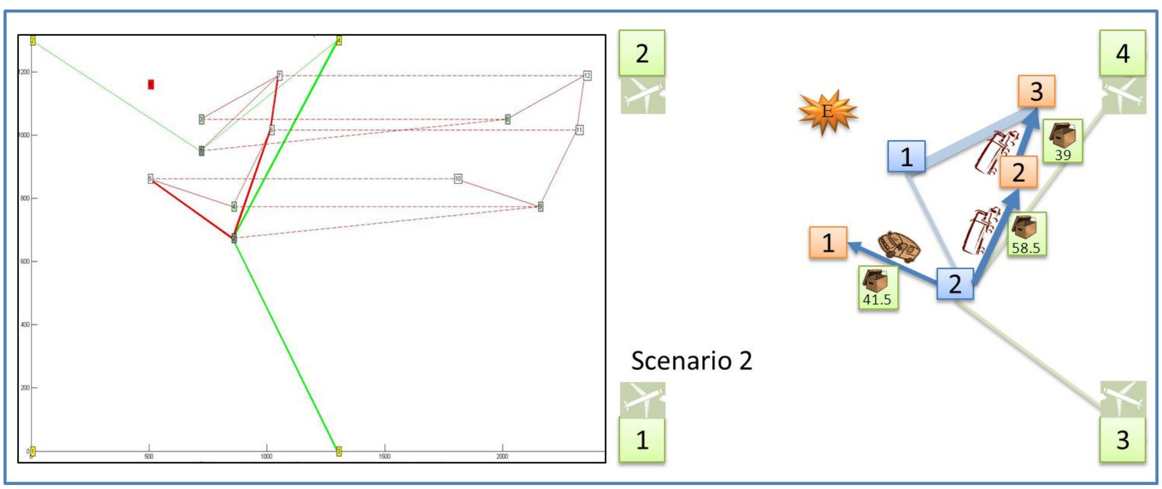

Figure 20: Realized vehicle and commodity flow in the scenario tree node 5, Scenario 2

The initial distribution plan in Scenario 2 equals that of scenario 1. As the totality of arcs constituting the path to be followed are intact, complete network information will never be required and the 3rd stage is in consequence never reached. The 2nd and 3rd stage variables are in effect identical. This is illustrated in Figure 20.

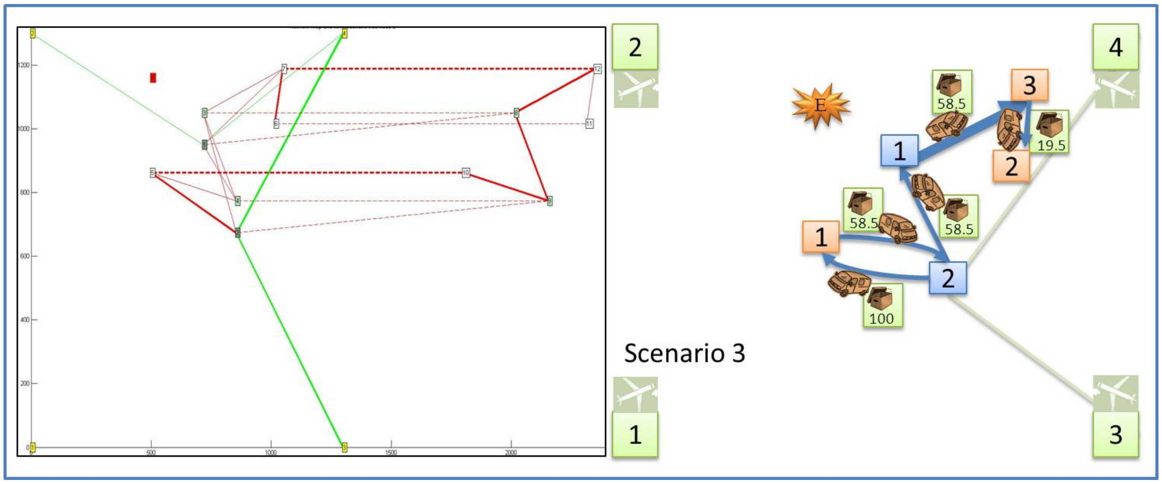

Figure 21: Realized vehicle and commodity flow in the scenario tree node 6, Scenario 3

The initial plan for scenario 3 is impossible to enforce owing to reduction of arcs in the realized network. The first distance however, from LDC 2 to POD 1, will be traveled by the vehicle according to the 2nd stage plan. As visualized in the left map in Figure 21, the duplicated network is used by the vehicle to return to LDC 2. From this node it travels via the dummy LDC 1, thus using this as a transshipment node. This does not induce opening cost, as the node is not used to process and manage supplies. When the vehicle finally reaches POD 3, the decision variables in the mathematical model can assume one of two values, both resulting in the exact same route. The optimizer may choose to make the vehicle return to the original network, deliver the units demanded and travel directly to POD 2, as shown in the left map. Or, the decision variables may be set such that the vehicle delivers the items in POD 3, returns to the duplicated network, proceeds to the duplicated POD 2 and finally returns to the original network to deliver to POD 2. The objective function value does not differ between these two choices of decisions, and the algorithm used by the solver is thus free to decide between the two. The real-life result provided will be the same, as showed in the right map of Figure 21. Decision of this kind constitute symmetric solutions.

In the final scenario, the initial distribution plan proves inapplicable even before dispatch. In practice, this would most likely imply waiting time at the LDC in order for information about the network to be realized. As soon as this information becomes available, an optimal actual route based on the realization of the network will be generated. Because POD 1 proves impossible to reach, the driver will be provided with a new distribution plan stating the destinations to serve and the amount of supplies to delivery based on the load carried by the vehicle. Constraint (7.28) and (7.29) prevent the vehicle from transporting more

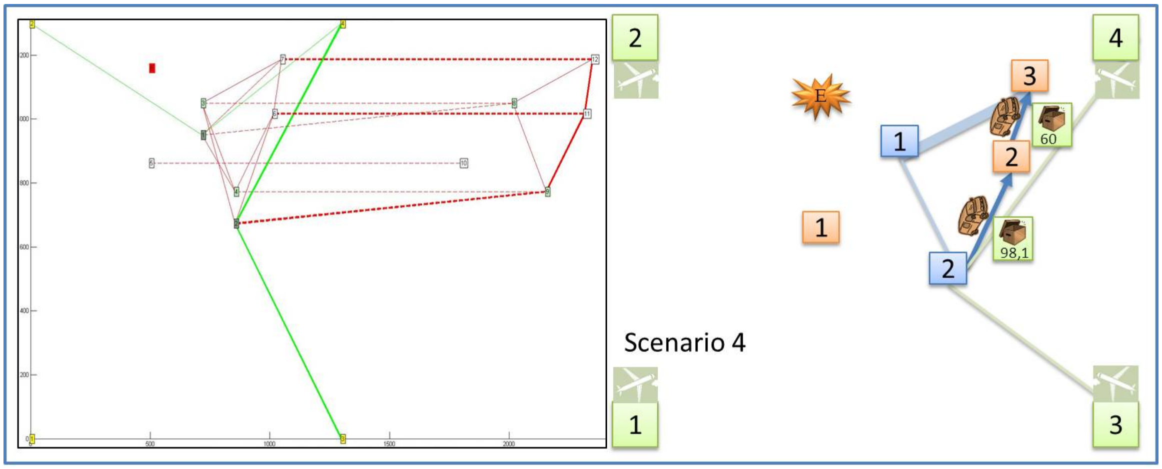

Figure 22: Realized vehicle and commodity flow in the scenario tree node 7, Scenario 4

than what is delivered and demanded at demand points, forcing the vehicle to unload 1.9 items at the LDC before dispatch. The left map in Figure 22 shows the path taken by the vehicle in stage 3 leading from the LDC into the duplicated network and further to the duplicated POD 2. The vehicle is to satisfy demand at POD 2, and thus traverses the dotted arc connecting the duplicated POD 2 and the real POD 2 to deliver the planned amount of supplies. As opposed to in scenario 3, the vehicle now returns to the duplicated network to complete the route, before ending his route at POD 3 in the original network. As can be seen from the right map, POD 2 receives 38.1 units and not the planned amount of 19.9, whereas POD 3 receives an additional 21 units relative to the planned amount. The model permits change in final destinations to minimize waste of commodities. This will be the case when demand points prove inaccessible or provide less utility than expected as compared to other demand points.

9.3 Comparison of the Two Approaches

The two measures for valuation, EVPI and VSS, described in Section 6.3 form a sensible basis for comparison of the two approaches presented in the preceding sections. The calculation procedure for these measures will be described in Section 10.2.1. Even though the objective function of the MSP model includes both planned and realized utility, only 3rd stage Demand Fulfillment Utility (DFU) and Monetary Utility (MU) will be considered to enable comparison with the deterministic approach. DFU and MU will be presented separately as they have more obvious interpretations when analyzed individually. Also the WS, EEV and RP solutions presented are given in terms of 3rd stage DFU and MU in addition to total value for reasons of interpretation. The resulting values are denoted by

WS', EEV', RP', EVPI' and VSS' to indicate that they represent adjusted values.

MU depends on the remaining budget. Increasing the budget may in effect provoke a situation in which the solutions generated are identical with that of a lower budget case with all other parameters fixed, but an improvement in objective function value is experienced due to the increased contribution to residual budget. The primary goal in our model is to find a solution which satisfies demand to the greatest extent possible. We do however, seek cost-efficient solutions. Including MU for both the 2nd and 3rd stage in the objective function is essential in order to achieve efficiency of distribution .

Evaluation of the value of perfect information for this illustrative example is based on the EVPI' given in Table 3. The total EVPI' shows that the value of perfect information is negligible, as an improvement of only 0.19% in the solution will be provided by obtainment of better forecasts, as compared to the MSP solution. Receiving perfect information will only increase total monetary utility, as the same level of utility of demand fulfillment is provided by both the solution of the MSP model and that of the deterministic model using perfect information. The EVPI'$_{MU}$ is positive because the local agent will allocate less supplies to the LDCs due to his knowledge of inaccessibility of POD 1 in scenario 4 .

Table 3: Valuation measures for the illustrative example

	Scenarios				*Calculated*			*EVPI'*		*VSS'*	
	1	*2*	*3*	*4*	*WS'*	*EEV'*	*RP'*	*Value*	*%*	*Value*	*%*
DFU	20780	20780	22780	16723	20265	12921	20265	0	0.00	7344	56.84
MU	1536	1842	1414	2041	1709	1865	1668	41	2.46	-197	-10.56
Total	22316	22622	24194	18764	21974	14786	21933	41	0.19	7147	48.34

The VSS' values presented in Table 3 clearly show that the proposed MSP model serves as a more robust decision support tool than an expected value approach. In this example, the load assigned to the vehicles will prevent efficient delivery of goods to beneficiaries in an average scenario problem. The reason for this is that basing allocation of supplies amongst vehicles on expected values will result in all units being assigned to vehicle type 2. This vehicle type is less likely to reach all of the PODs. The MSP model considers all possible scenarios simultaneously, and thus provides a solution more likely to be applicable across the total range of future outcomes. The DFU produced by use of the MSP model is thus larger than that of the EEV model. A negative VSS'$_{MU}$, indicates that less funds are spent when the EEV approach is applied, as compared to the solution of the MSP model. This however is not an intentional saving, but occurs because the vehicles are unable to follow the routes generated by the EEV model. In summary, the VSS' values clearly show that the MSP model is to be preferred over the deterministic expected value model for this example. They also indicate that uncertainty is crucial to consider in order to obtain sound solutions.

10 Computational Study

In this section, the applicability, quality and value of the proposed MSP model will be evaluated by conducting a series of numerical tests. The most important findings will be presented in two separate sequences; the first consisting of tests regarding computational efficiency of the MSP model, whereas the second seeks to validate the proposed model and demonstrate its value as compared to deterministic approaches. All instances are either stopped as run time exceeds 43200s (12hrs), or when a MIP solution within 0.10 % of the optimal solution has been found, unless otherwise stated.

Computational efficiency will be assessed by changing 1) the configurations of the network in terms of number of PODs; 2) the number of LDCs; 3) the number of possible outcomes in stage 2; 4) the number of possible outcomes per parent node in stage 3; and 5) the size of the budget. At least 30 instances are considered for each of the aspects subject to testing. All tests are conducted for at least 15 different values for the parameter of interest, to enable identification of possible trends. In addition, two separate sets of corresponding values are generated to be able to present average values. Using an average provides the advantage of being able to hedge against the effect of non-representative extremities. All results relating to computational efficiency were obtained using Xpress-IVE on PCs connected to a computational cluster, equipped with 2x AMD Opteron 2431 2,4 GHz processors and 24 GB RAM.

The last sequence of tests makes use of a set of different test cases to illustrate how different input parameters may affect the optimal solution and the corresponding routing decisions produced by the MSP model, as compared that of deterministic approaches. Four corresponding instances are generated for each of the test cases considered in order to be able to state the results in terms of average values, based on the same reasoning as given above. The results are given in terms of the Expected Value of Perfect Information (EVPI) and the Value of the Stochastic Solution (VSS). The results concerning the value of the MSP model provided below were obtained using Xpress IVE on a PC equipped with Intel(R) Core(TM) i7-2600 3.40GHz processor and 16.0 GB RAM.

A complete outline of the average solution times and the average problem sizes related to the different tests performed to demonstrate the computational efficiency of the proposed model is given in Appendix E - G. Output files containing the results produced by these different tests, as well as the input data files used in validation of the model, will be provided in digital files to be enclosed with the thesis.

10.1 Computational Efficiency of the Stochastic Program

Limitations of the proposed model in relation to problem size will be thoroughly discussed in this section. Questions regarding the applicability of the model as well as its function as a decision support tool, will be addressed.

Apart from for the tests which evaluate the effect of altering the number of scenario tree nodes at different stages in the model, the total number of scenario tree nodes will be set to 13. Stage 2 will include three scenario tree nodes, and stage 3 a total of nine with three successor nodes per parent node, resulting in nine different future scenarios. The number of possible scenarios considered when evaluating disaster response problems generally ranges from 8 to 100, which is the case for scholars such as Vitoriano et al. [2010] and Rawls and Turnquist [2010]. Contemplation of nine scenarios is thus deemed a realistic figure for the tests to be conducted. Scholars tend to consider only a limited number of demand points for distribution. Vitoriano et al. [2010] treat 9 demand points (PODs), while 6 and 30 PODs are treated by Barbarosoğlu and Arda [2004] and Rawls and Turnquist [2010] respectively. Static characteristics of the problem which serve as a basis for the following tests are given in Table 4.

Table 4: Static problem characteristics - common for all instances

# of	*# of*	*# of*	*Successor nodes*	
LDCs	*PODs*	*vehicle types*	*stage 2*	*stage 3*
5	15	2	3	3

10.1.1 Varying the Number of LDCs

The computational results in this part of the study are based on instances in which the only parameter subject to change is the number of LDCs. Two different testings regarding two different numbers of PODs have been performed for a range of varying LDC-quantities. As stated introductorily, two corresponding sets of instances have been generated in order to create an average. Solutions for $2 * 16$ instances with 15 PODs, and $2 * 8$ instances with 40 PODs have been calculated for the first and second testing respectively. The maximum number of LDCs in the instances are 90. This will serve to illustrate computational time and change in optimal solutions in a comprehensive and reliable manner.

A large number of LDCs will imply several decisions in all 3 stages, and hence presumably higher computational time as compared to a case consisting of fewer LDCs. This presumption is verified by all instances generated, and is reflected by the average values depicted in Figure 23.

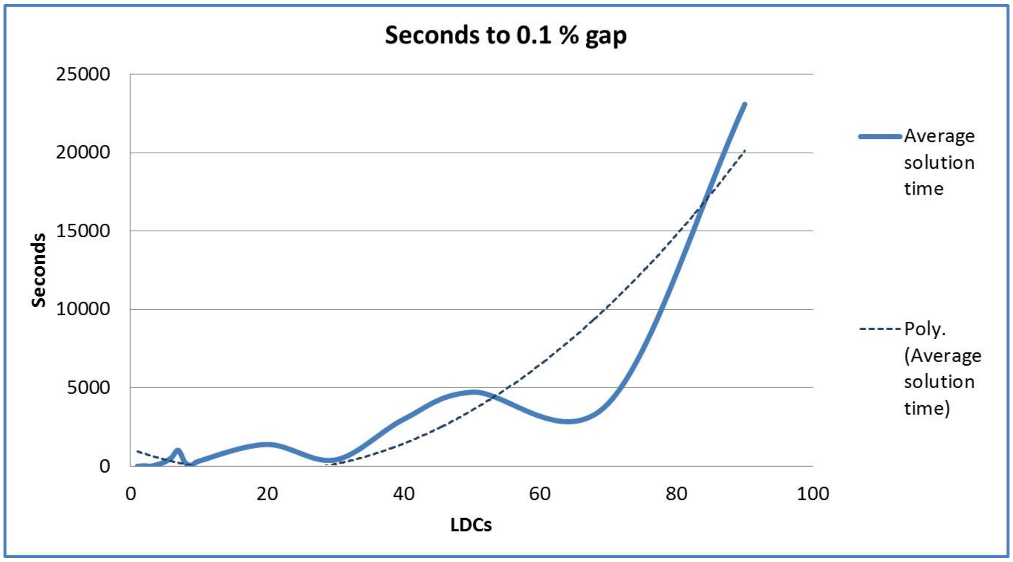

Figure 23: Average solution time when varying the number of LDCs - All instances are solved to a gap of 0.1 %

Only one of the instances considered was unable to find an optimal solution within 12 hours; that of 90 LDCs. In Figure 23, this value has been equaled out by a similar instance of less computational time. A network consisting of a quantity of LDCs six times that of the number of PODs, is considered unlikely. Thus, this time overrun is not regarded as critical. All remaining instances were solved far within the acceptable time frame, indicating computational efficiency for varying levels of LDCs. By interpretation of the trend line, we expect the solution time to further increase for higher numbers of LDCs. This result is assumed to be applicable also to cases with input parameters different from those considered, and we conclude that number of LDCs correlates with computational time.

All other parameters held constant, increase in the number of LDCs implies higher levels of available capacity both in terms of total vehicle capacity and total handling capacity. Higher objective function values would be a natural expectation in situations of wider ranges of LDCs for operation. However, results show that the marginal value of having extra LDCs to choose from experiences a steep decline as the the number of LDCs exceeds approximately 25 % the number of PODs. These results are given in Figure 24 and Figure 25. Across all cases tested, the overall trend is initialization of averagely 27 % of the available LDCs. The implication of these results is that expanding the number of LDCs increases computational time without necessarily adding any extra value to the solution. Moreover, the intangible cost of having several possible LDCs to manage, maintain, keep updated and prepared for operation may cause greater inconvenience to the decision maker and his team, than the value of having these extra LDCs available.

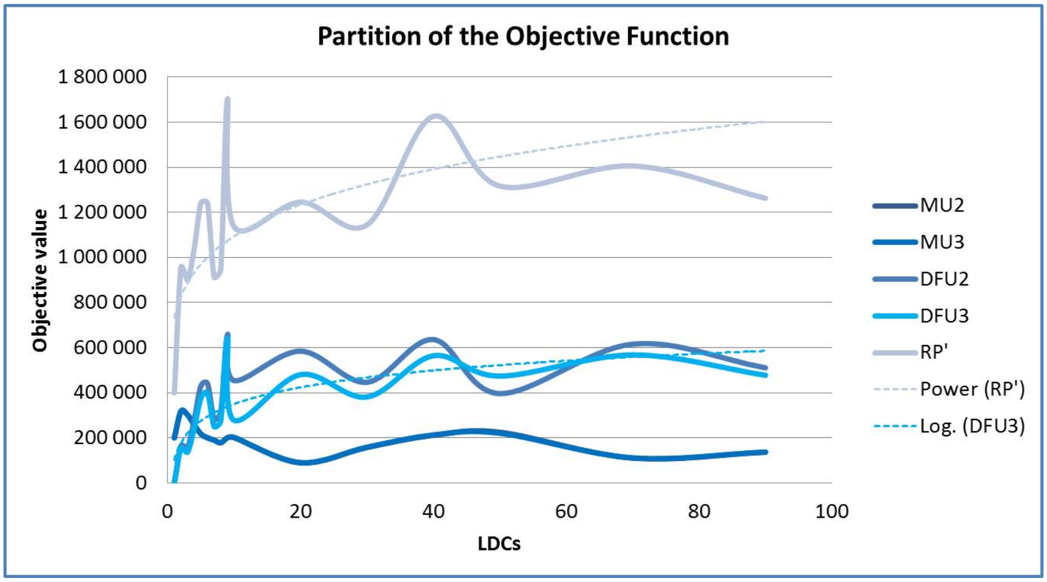

Figure 24: Average objective function value when varying the number of LDCs - Instances are based on a network consisting of 15 PODs and solved to a gap of 0.1 %

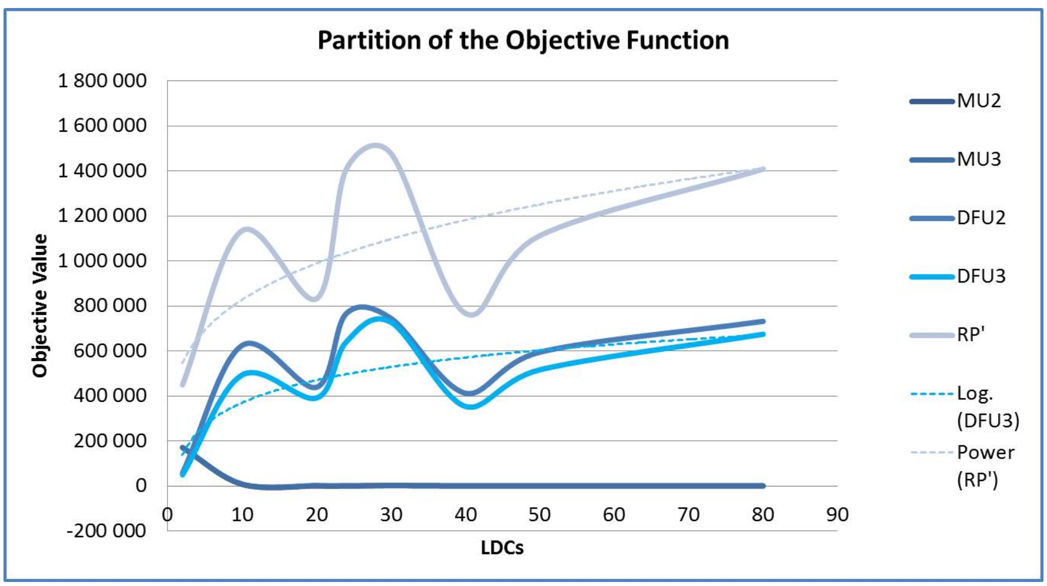

Figure 25: Average objective function value when varying the number of LDCs - Instances are based on a network consisting of 40 PODs and solved to a gap of 0.1 %

10.1.2 Varying the Number of PODs

The computational results in this part of the study are based on instances in which the only parameter subject to change is the number of PODs. Increasing the number of PODs will expand the number of feasible solutions in addition to the number of symmetric solutions. This will entail an increase in computational time as reflected by a larger number of constraints and variables.

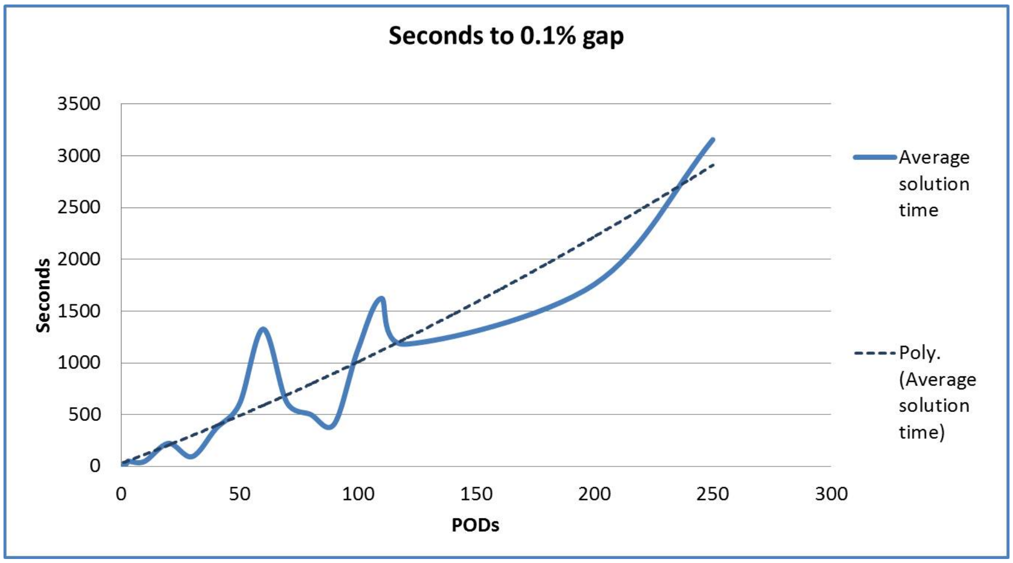

Figure 26: Average solution time when varying the number of PODs - All instances are solved to a gap of 0.1 %

Figure 26 shows that a decision maker will still be able to obtain an optimal solution within an acceptable time frame. Solutions are found within 45 minutes when considering up to 250 PODs. It is however reasonable to assume that a larger number of arcs would be connected to each node in the network in real-life situations. Despite this fact, we argue that the instances reflect a realistic network based on comparison with the work of other researchers, such as Barbarosoğlu and Arda [2004], Hsueh et al. [2008], Mete and Zabinsky [2010], Özdamar et al. [2004]. These articles provide examples with significantly lower numbers of PODs as compared with the 250 considered in this study.

As the number of PODs increases, so does the chance of achieving higher objective function values, given that budget, capacity and supply is sufficient in meeting demand. Comparison of optimal objective function values across instances is thus deemed difficult (and futile) due to differences in level of attainable utility. However, Figure 27 shows indication of growth in optimal objective value in line with an increasing number of PODs. In addition, substantial variation in the objective function value is observed. The reason for such vari-

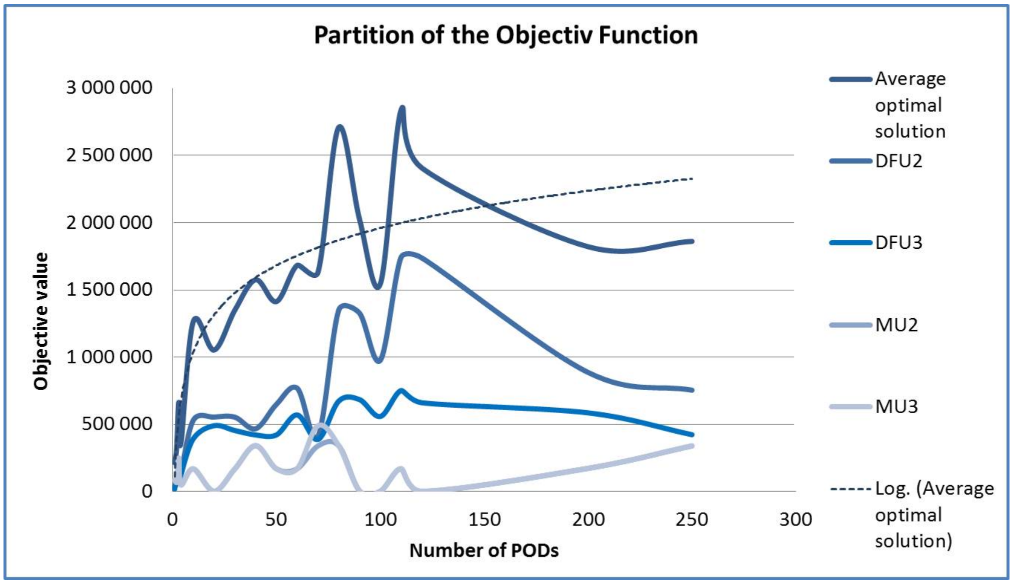

Figure 27: Average objective function value when varying the number of PODs - All instances are solved to a gap of 0.1 %

ance is ambiguous. Even if additional utility is attainable, an increase in this value may still not occur. PODs in the immediate vicinity of the earthquake generate higher utility of demand fulfillment than remote PODs. The additional utility of fulfilling demand at remote PODs may provide less additional utility as compared to utility lost by the cost incurred in providing this aid. Furthermore, PODs close to the earthquake are more likely to prove inaccessible owing to larger damage to this part of the distribution network, and objective function might in fact drop. Restrictive capacity, supply and budget can also obviously impede further improvement of solutions, and entail decreasing marginal utility. By studying Figure 27, it can be observed that even if the average optimal solution increases, this is mainly caused by additional utility of demand fulfillment at stage 2, DFU2. The realized demand fulfillment utility, DFU3, is fairly constant. Based on this, we conclude that if the number of demand points increases as the event progresses, with all other parameters held constant, growth in the realized utility of providing aid will not necessarily be experienced. Occurrence of after-shocks as an example can bring about such a situation of sudden adding of PODs.

The test results also show that the number of constraints and variables in the initial matrix as well as in the presolved matrix, correlates with computational time. This is illustrated in Table 5 and demonstrates the importance of presolve. According to Baricelli et al. [1998], change of certain coefficient values as to obtain tighter constraints may be critical in order to find solutions to a problem within reasonable time. The sets of instances tested do

Table 5: Correlation between constraints/variables and solution time when varying the number of PODs

Matrix		*Presolved*	
Constraints	*Variables*	*Constraints*	*Variables*
0.9208	0.9324	0.9192	0.8987

not require unreasonable computational time, but should the size of the problems grow, presolving may still prove to be of vital importance .

10.1.3 Varying Budget and Number of Successor Nodes per Parent Node in the Scenario Tree

The effects of altering the number of successor nodes per parent node in both stage 2 and stage 3, as well as budget, represent the final aspect subject to analysis. All other parameters are held constant. By varying the number of successors individually, inspection of the complexity of the model is enabled by study of the effect of varying the number of scenario tree nodes in stage 2, or stage 3, whilst maintaining the number of scenarios. Figure 28 illustrates two possible instances of this type. The scenario tree at the left has three successor nodes per parent node in stage 2 and two in stage 3, whereas the right instance has two successor nodes per parent node in stage 2 and three in stage 3.

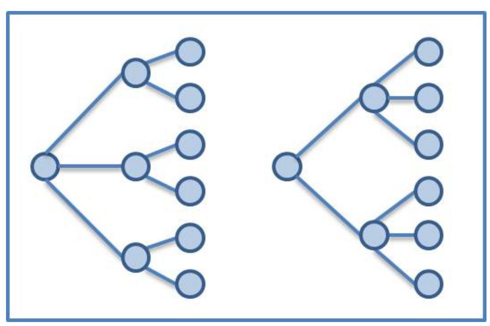

Figure 28: Illustration of two instances subject to varying numbers of successor nodes per parent node - Both consist of six scenarios

The tests executed demonstrate an effect which contradicts the argument made by Baricelli et al. [1998], stating that tighter constraints will lead to decline in solution time. Two sets of instances were tested for varying numbers of successor nodes per parent node for both stage 2 and stage 3. The first instance considers a situation of limited budget amounting to 100 000 monetary units, as opposed to excessive budget of 500 000 monetary units in the second instance. The effect the sufficiency versus insufficiency of funds has on solution time is unambiguous. Average computational time is higher for instances of limited budget, regardless of the number of scenarios, and regardless of whether the highest number of successor nodes per parent node is contained by stage 2 or 3.

A possible explanation is that an instance of this kind could prove hard to solve, as compared with an instance with a larger feasible area, if the optimizer finds several nearly feasible solutions which could have produced high objective function values. The optimizer uses branch and bound techniques when solving the problem, and would need to perform a high number of branchings before realizing that the solution is actually infeasible. This would require extensive computational time, a likely explanation for the results found here.

The average solution times when varying the number of successor nodes in stage 2 and 3 are plotted in Figure 29 and Figure 30 respectively. The instances have been solved to a gap of 1 % due to high run times, some even exceeding 43 200 sec. A solution within 1 % of the best bound, is tolerable in a real life situation.

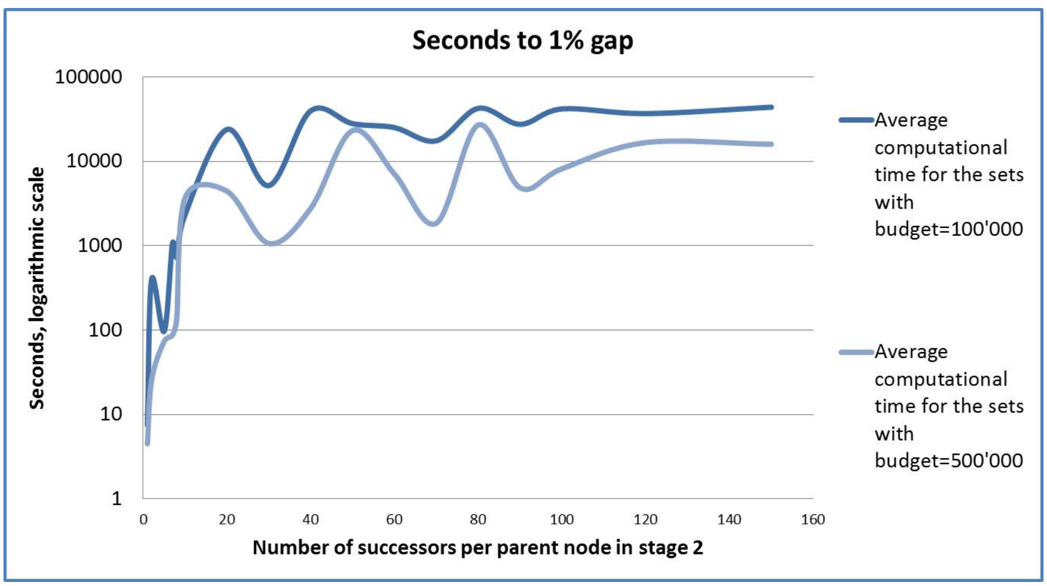

Figure 29: Average solution time when varying the number of successor nodes per parent node in stage 2 - All instances are solved to a gap of 1 %

Another clear tendency made visible by the two figures is the increase in solution time as the number of scenarios increases. The ability to solve to optimality depends on both the level of budget and the number of successor nodes for each stage, as demonstrated in Table 6. It is worth noticing however, that the computational times required to locate an

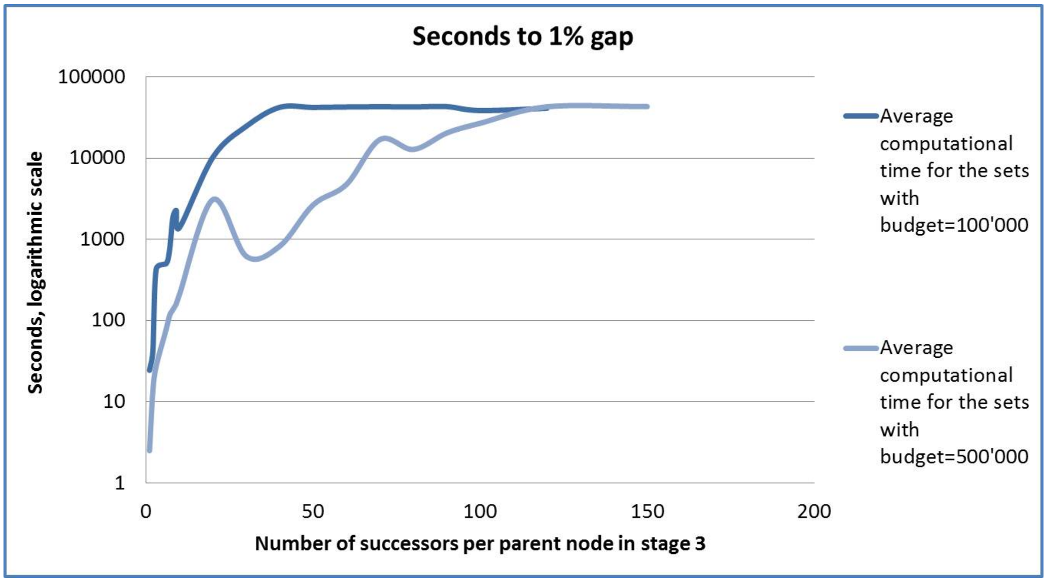

Figure 30: Average solution time when varying the number of successor nodes per parent node in stage 3 - All instances are solved to a gap of 1 %

optimal solution are in general considerably lower than 43 200 seconds, but the verification of this solution to ensure its optimality has proved time consuming due to slow reduction of the best bound. Figure 31 serves to illustrate this fact. The green boxes represents a feasible solution, and the yellow line the best bound. The solution proved to be optimal is found long before the optimizer reach a gap of 0.1%.

Table 6: Ability to find optimal solutions when varying the number of successor nodes per parent node in stage 2 and 3

Budget	*# of nodes per parent node*		*# of scenarios*	*Solved to optimality?*
	stage 2	*stage 3*		
100	10	3	30	yes
	20	3	60	no
	3	6	18	yes
	3	7	21	no
500	40	3	120	yes
	50	3	150	no
	3	20	60	yes
	3	30	90	no

Figure 32 visualizes the problem size in terms of the number of variables and constraints. For instances of identical scenario quantities, the problem size is largest for instances with

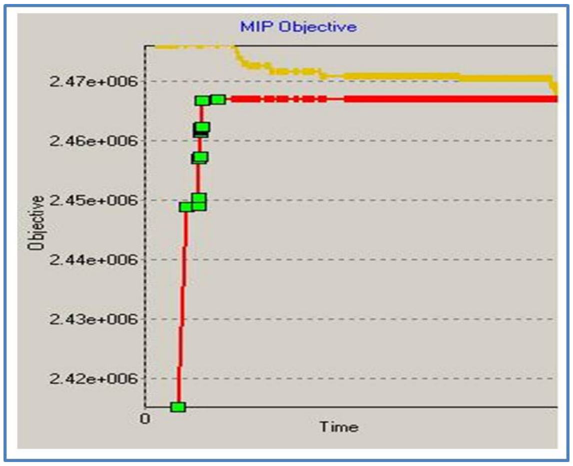

Figure 31: Illustration of reduction in best bound to find optimal solution as depicted in Xpress-IVE

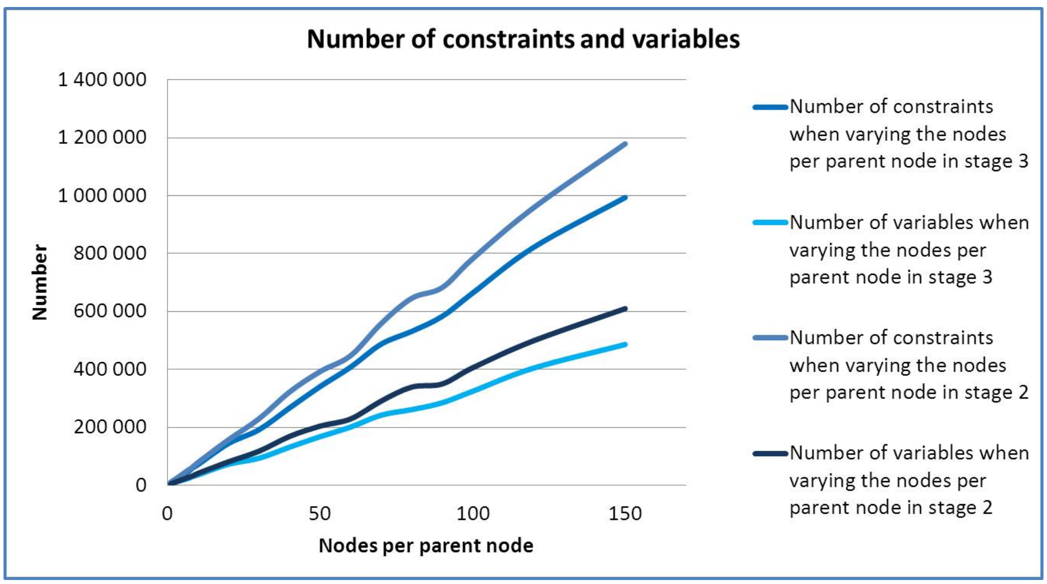

Figure 32: Total number of constraints and variables when varying the number of successor nodes per parent node in stage 2 and 3

the highest number of successor nodes per parent node in stage 2, as opposed to in stage 3. We would thus expect that these instances would be more time consuming to solve. Despite of this, Figure 33 shows that the average computational time is higher when the number of successor nodes per parent node is greater in stage 3 than in stage 2. A possible explanation

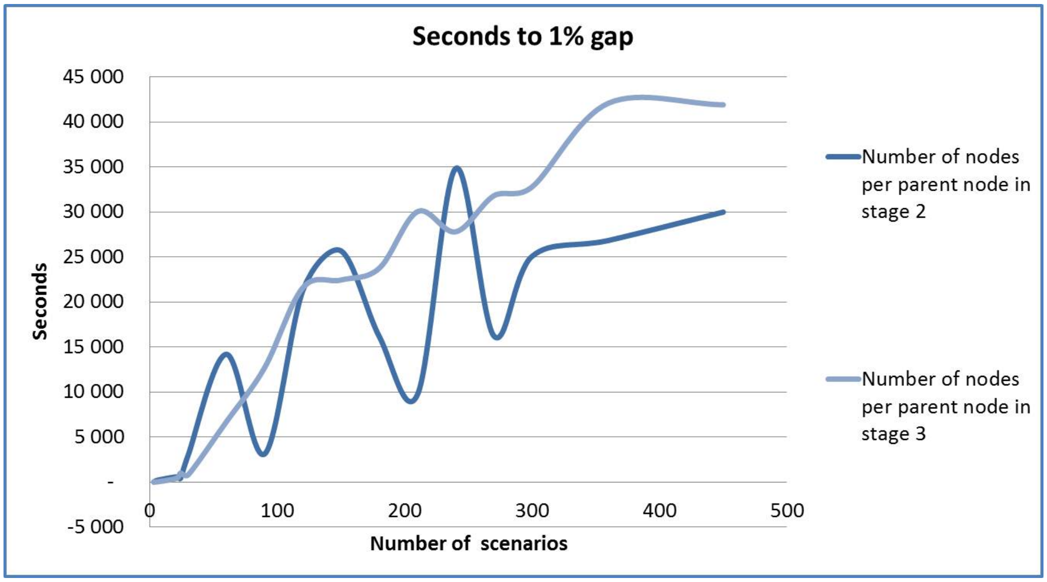

Figure 33: Average problem size when varying the number of successor nodes per parent node in stage 2 and 3 - All instances are solved to a gap of 1 %

is that even though the real size of the problem is bigger in terms of variables and constraints, the complexity, in addition to the amount of symmetric solutions, of a problem in which the highest number of successor nodes is contained by stage 3 might be greater. Hence, computational time does not only depend on the problem size, but also on which of the constraints and variables an increase in the problem size effects. The implication of this is that information regarding the state of the distribution network is of greater significance for solution time, than knowledge of actual demand and vehicle capacity.

A further finding demonstrated by the tests performed concerning the configuration of the scenario tree instances, regards the objective function value. Figure 34 depicts the average values for each of the four terms constituting the objective function, as well as the total value. Both in case of sufficient and insufficient budget, the values produced when the highest number of successor nodes are held by the 2nd stage exceed that of highest number of successor nodes in stage 3. The reason for this can be explained by the possibility of not being able to reach a certain POD. Inaccessibility of a demand point will yield zero demand fulfillment, whereas a reduction in demand or available vehicle capacity will still enable fulfillment of demand, only not to the extent planned.

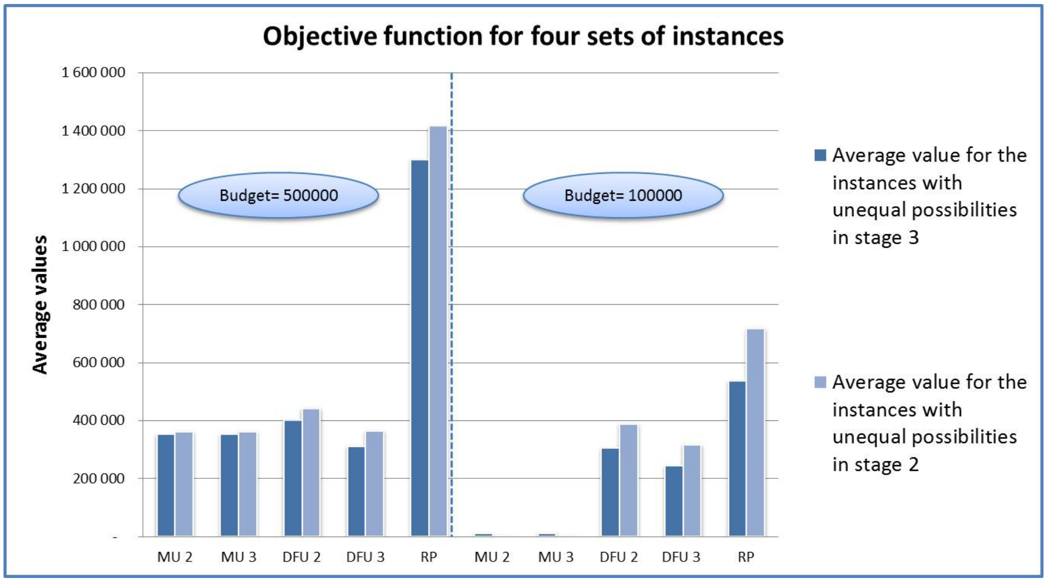

Figure 34: Average objective function values for high and low budget instances, when varying the number of successor nodes per parent node in stage 2 and 3

10.2 Validation of the Multi-Stage Stochastic Program

This section will provide a quantitative validation of the proposed multi-stage SP model. The validation will be based on the two measures for valuation presented in Section 6.3; the Expected Value of Perfect Information (EVPI) and the Value of the Stochastic Solution (VSS). These measures will be applied to a range of different test cases of interest, which serves to illustrate how the model performs under different conditions.

The section will be introduced by a preliminary explanation of the chosen method for calculation of the the Expected Value solution (EEV), a prerequisite for finding the VSS. The base case example, upon which the different test cases are built, will be presented, and extended into a description of its test versions. An overview of the full range of EVPI and VSS measures obtained will be given, followed by an evaluation and analysis of the results. Finally, based on the analysis, the robustness of the proposed model and its ability to produce resilient outcomes in situations characterized by substantial uncertainty, will be discussed.

10.2.1 Explanatory Remarks for Calculation of the Valuation Measures

In order to achieve a comprehensive and elucidative assessment of the value of the MSP model, the dynamic approach for calculation of the EEV, based on the description given in Section 6.3, will be adopted. To further improve the validation process, several instances will be generated for all cases, such that the valuation measures to be presented in the following are represented by their respective average values. In this way we are able to hedge against extreme outcomes and present representative values for the measures. A more veracious account of the value of the model can thus be given.

All instances representing the same test case are identical in terms of input data for all parameters apart from those representing the characteristics of the network. The characteristics of the different networks are given in terms of the parameters stating operability or inoperativeness across the array of their constituting arcs, in addition to the cost of and time needed to traverse these. All other factors held constant, these stochastic characteristics of the network can potentially have a tremendous impact on the optimal solution.

We have chosen to utilize the dynamic approach for calculation of the EEV value exclusively, as it is expected to produce a more realistic result for the EEV solution than the static approach. This is because we are able to take corrective measures in the 2nd stage before the 3rd stage is initialized. By adjusting the EV solution to the outcome of the 2nd stage, improved conformity with the realized situation of the 2nd stage is achieved. Such an approach would be in greater accordance with the decision making process taking place in real life disaster situations. Also, in order to be able to apply the static approach, Linking

Constraints (7.21), (7.23) and (7.26) would occasionally need to be relaxed due to their confining disposition. This would enable avoidance of the generation of infeasible solutions in estimation of EEV values. Constraint relaxation increases the dimensions of the problem, and hence the feasible region, which enables us to find feasible solutions in cases where the constraints are too restrictive. This EEV value would in effect be closer (or equally close) to the RP value than the true optimal EEV value, thus not giving an accurate reflection of the actual value of the MSP model.

Owing to certain attributes of the proposed MSP model, some requisite assumptions regarding the expected network have been made when estimating the Expected result of using the Expected Value solution (EEV). This information is required as input in the EV calculations, which form the basis for the consecutive EEV calculation. The expected network will be represented by the initially expected network of stage 1. This might not be entirely mathematically correct, but we argue that the 1st stage input data still represents valid values for expectation. Mathematically, finding the expected network would be a cumbersome process as the network is represented by a 0/1 matrix. Construction of a more advanced EEV calculator is beyond the scope of this thesis, and a calculator which suffice for the purpose of validating the developed model has rather been created.

All EEV and RP values are given in terms of optimal 3rd stage values exclusively, as opposed to the objective function value produced by the original MSP model which include both 2nd stage and 3rd stage optimal variable values. This enables comparison of these values with the WS value produced by the deterministic model. Whereas the MSP model generates both an anticipated distribution plan and a realized distribution plan, both subject to maximization, the deterministic model only performs a single last mile distribution optimization. It should also be noted that the values produced, both in terms of EVPI and VSS, are only to be used for comparison of different instances, and are not to be interpreted as an indication of the true economical value of the proposed MSP model in a literal sense. This is due to the nature of the objective function of the model, as explained in Section 7.3.

10.2.2 Introduction of the Base Case

As an initial action in the validation process seeking to validate the MSP model proposed, an example denoted the base case has been generated and applied. This base case will serve as a platform, upon which a range of test cases seeking to reflect the performance of the model are built. It will also act as a point of reference in evaluation of the valuation measures produced for the different test cases. By consciously and singly changing input parameters of interest, pertinent test cases will be generated.

The characteristics of the base case is inspired by real earthquake data from the earthquake which hit Haiti on January 12, 2010. Some modifications and assumptions regarding the data set are made when adequate information is lacking, or the available information is incompatible with the attributes of the model. The adopted data is provided via Logistics Cluster [2012b].

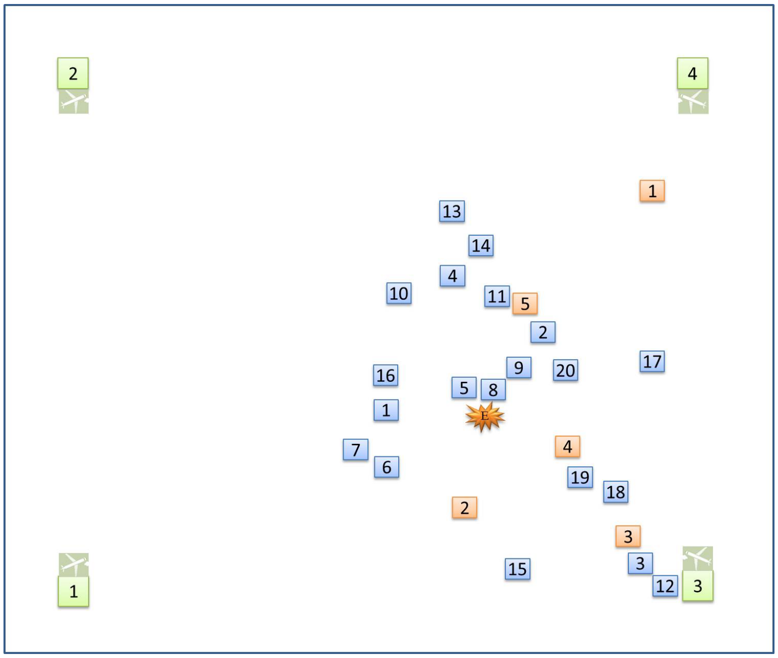

Figure 35: Configuration of the distribution network applied during validation of the model

The size of the network chosen as a basis for the entirety of base cases, and hence all descendant test cases, is in immediate agreement with the distribution network utilized during the disaster in Haiti. It consists of $|\mathcal{P}| = 20$ PODs and $|\mathcal{L}| = 5$ LDCs, as depicted by red and blue boxes respectively in Figure 35. The reciprocal location of the nodes will also remain unaltered throughout the range of instances. The operativeness of the arcs connecting these nodes will vary between base cases in order to ensure representativeness of the valuation measures presented. They will however be identical for all test cases relating to the same base case. Arc capacities and the cost and time needed to traverse an arc, will in effect be identical for appurtenant cases. In addition to the parameters listed in Table 7, those omitted from Table 8 will be treated as static values, and are thus common for the entire array of cases irrespective of the originating base case.

In order to keep computational time within a reasonable frame, $|\mathcal{N}| = 9$ scenarios has been considered. As argued in Section 10.1, this is deemed a realistic number of future outcomes. $|\mathcal{V}| = 2$ vehicle types (v-types) of different capacities have been placed at the decision maker's disposal in order for him to distribute $|\mathcal{B}| = 3$ different commodity types (c-types) in varying quantities to the PODs. Complete demand for each of the c-types across all demand

Table 7: Static problem characteristics - common for all cases

# of	*# of*	*# of*	*Successor Nodes*	
LDCs	*PODs*	*vehicle types*	*stage 2*	*stage 3*
5	20	2	3	3

nodes is given in Table 8, as is the capacity of the LDCs and the available distribution budget. The vehicle capacity of each v-type is also given, along with the accumulated capacity across the total number of available vehicles. Seeing how the commodity types are of different sizes; total demand, total LDC capacity and total vehicle capacity are converted into an equivalent unit of measurement (st. units). By stating these quantities in terms of standard units, as opposed to number of items, we allow for comparison. We do not wish to limit the time available to serve the PODs, as our main priority is to satisfy demand as extensively as possible. The total convoy time is thus set at a level high enough to guarantee that it will not affect the solution, and will not be a parameter of interest. We also assume an abundance of supplies at the ICDs, based on the description of the problem given in Section 3.1.

Table 8: Characteristics of the base case - subject to change in the descendant test cases

	Demand per c-type				*LDC cap*						*V-cap per v-type*		
Bud-get	*1*	*2*	*3*	*Tot dem*	*1*	*2*	*3*	*4*	*5*	*Tot cap*	*1*	*2*	*Tot cap*
5E+5	1020	2360	4380	30000	674	3014	2095	9796	9021	30000	88	45	30000

10.2.3 Presentation of the Test Cases

In evaluation of the proposed SP model serving as a decision support tool for support in development of distribution plans in real-life disasters, we seek to challenge the model on aspects expected to influence its performance. We wish to register and illustrate the impact of changing input parameters expected to entail a reduction or increase in feasible region. In order to do so, we expose the decision maker to a situation of both restrictive and non-restrictive funding streams, reflected in the available amount of budget. The total level of demand is also made subject to change relative to the available level of LDC capacity, vehicle capacity and budget. The intention is to demonstrate efficiency in distribution when demand exceeds the level of resources needed, and also when the level of resources is sufficient relative to the level of demand.

The total level of available vehicle capacity across the different vehicle types may or may not affect the solution by dictating the outbound amount of supplies to be distributed. As argued in Section 3.1, a scarcity of transportation capacity is often the case in distribution of relief. By changing the vehicle capacity of each vehicle type whilst leaving total number of available vehicles unaltered, this aspect will be examined. The same reasoning applies to total level of LDC capacity.

The preceding argumentation serves as the motivation behind the choice of considering the effect of changing budget, total LDC capacity, total demand and total vehicle capacity. An overview of the cases we have chosen to consider is given in Table 9. It should be noted that the levels of demand and capacity subject to change are stated in terms of *anticipated* quantities as given in the initial phase of distribution. This is due to the fact that realized levels of demand and capacity are randomly generated, thus exempt from manual manipulation. Four different versions of the base case and descendant test cases are created, and an average value to represent the valuation measures are calculated based on corresponding cases, as explained in Section 10.2.1.

Table 9: Overview of the test cases

Case	*Variable parameter*	*Value* (st. units)
1	Total demand	40 000
2	Total demand	1 000
3	Total LDC capacity	40 000
4	Total LDC capacity	1 000
5	Total vehicle capacity	40 000
6	Total vehicle capacity	1 000
7	Budget	1E+06
8	Budget	3E+05

10.2.4 Results and Discussion

The values of the WS, EEV and RP solutions are given in terms of average third stage utility on the grounds stated in Section 10.2.1, and denoted by WS', EEV' and RP' to indicate that they represent adjusted values. The overall results from the test cases are given in Table 10, where WS', EEV' and RP' are given along with EVPI' and VSS'. The EVPI' and VSS' are given both in terms of average third stage utility and in percent. The majority of instances have been solved to a gap of 0.1 %. If a solution has not been found within 3600 sec, the current solution has been accepted. The average run time to optimality for the SP solution is 1132.24 sec. The run times to optimality for the EEV and WS models are negligible for the cases under consideration.

Table 10: Resulting valuation measures in terms of average values

				EVPI'		*VSS'*	
Case	*WS'*	*EEV'*	*RP'*	*Value*	*%*	*Value*	*%*
Base case	802 028.0	564 314.5	743 843.8	58 184.3	8.1	179 529.3	31.4
1	770 623.0	552 018.0	712 042.8	58 580.3	8.6	160 024.8	28.5
2	217 182.5	106 212.4	201 895.5	15 287.0	7.8	95 683.1	92.6
3	802 176.8	566 538.5	743 765.5	58 411.3	8.2	177 227.0	30.8
4	371 568.0	242 460.8	339 980.0	31 588.0	9.1	97 519.3	39.7
5	1 023 402.5	653 499.5	919 238.3	104 164.3	11.7	265 738.8	40.8
6	144 814.5	118 339.2	132 709.3	12 105.2	9.6	14 370.1	12.4
7	852 053.3	619 224.0	795 126.7	56 926.5	7.4	175 902.7	27.8
8	707 662.3	445 005.3	595 680.0	111 982.3	18.7	150 674.8	33.8

Case 1 and 2 test the effect of increasing and decreasing total demand respectively, relative to the base case. Comparing the VSS' of Case 1 and 2 with the base case show that the value of the stochastic solution decreases when there is a shortage in resources. As the SP model does not have the resources needed to satisfy demand, the feasible area is reduced resulting in a decrease in the quality of the SP solution. Moreover, when shortage in supply resources are experienced, stochasticity is not the critical restraining factor which depreciate the solution, but the fact that supply is scarce. When, on the contrary, resources are in excess, the value of the VSS' experiences a substantial increase of 2 times the VSS' of the base case. This implies that evaluating the distribution policy and making decisions using a SP model is essential when resources exceed demand.

Considering the VSS' for Case 5 and 6, similar results are obtained by altering total vehicle capacity. The same reasoning as for Case 1 and 2 is applied. The adjusted value of the stochastic solution declines as vehicle capacity becomes insufficient. Consequently, toleration of the additional effort and complexity that the SP model entails is less convincingly justified. Under limited vehicle capacity, solving the deterministic EV problem provides a substitutable solution, but at a lower complexity level. As vehicle capacity exceeds demand and total LDC capacity, VSS' increases. The SP model gives a solution with higher levels of utility accumulated over every scenario than the expected result of using an EV solution, and is preferred.

Varying total LDC capacity in Case 3 and 4 indicates that further increase in capacity as compared to the base case has no effect on neither the EVPI' nor the VSS' value. Increasing LDC capacity beyond this level will not contribute to more efficient distribution, as it will not be utilized in order to fulfill demand to a higher extent. For the same reasons, an increase in LDC capacity will not increase the value of the stochastic solution. Still, the VSS' is high, and can justify the use of the SP model.

Restrictive parameters of the cases chosen will prevent the utilization of higher levels of capacity, as several factors influence the solution of the model. However, insufficiency in total LDC capacity yields an increase in the value of perfect information. Total LDC capacity is dispersed across several LDCs. Hence, selecting the LDCs which will enable the local agent in charge of final distribution, to provide the highest possible level of demand fulfillment, is of the utmost significance. The level of supplies received by an LDC is confined by the capacity of the LDC in question. When LDC capacity is restricted, the value of the stochastic solution shows an upward trend as expected value solutions are more likely to suggest suboptimal choices of LDCs to initialize. We would also be willing to pay to receive more accurate information regarding the elements of uncertainty which influence the decisions to be made.

An increase or decrease in budget will only have a slight impact on the VSS' and EVPI' as compared to the base case example. The budget is sufficient for supplying the demand nodes in the base case. A further increase will not necessarily improve the solution in terms of an increase in the level of demand fulfillment. A reduction in budget such that it is not sufficient to supply all demand nodes and limits the allowed arcs of the v-types, increases EVPI' and VSS' as compared with when budget is in excess. This is in accordance with the previous results. The effect however, is smaller than in the previous cases, as the differences in VSS' and EVPI' are far less than in the other cases. The reason for this is that the budget has a small impact on the optimal solution as compared to the level of demand fulfillment. This is because of the diminutive value given the utility factor for residual budget, in agreement with the main objective of humanitarian logistics as given in Section 2.2.1. Scarcity of vehicle and LDC capacity will influence the amount of satisfied demand, which is the main priority of our proposed model when maximizing the objective function.

The average VSS' of the base cases demonstrates that the solution brought forth by the SP model yields a considerable improvement over the deterministic model, when total LDC and vehicle capacity and demand have equal values. Average VSS' values of the test cases ranging from 12.4% to 92.6% indicate that this result, to a greater or lesser extent, applies to all the test cases. Furthermore, based on the level of the values, we argue that the stochastic programming approach taken is to be deemed appropriate, as large VSS values indicate that uncertainty is of importance to the optimal solution. An average EVPI' ranging from 7.4% to 18.7% demonstrates that obtaining higher quality forecasts of the state of the infrastructure, the level of demand and level of resources in the events of a disaster can prove somewhat beneficial for the cases under consideration.

Comparing the percentual EVPI' of the different test cases with that of the base case, the overall trend is an increase in its value in events of shortage of resources, and decline in situations of excess. We expect this effect to become even more evident as the level of excess or deficiency is further increased. There is less value of perfect information when the decision maker has adequate amounts of resources available, and an investment in improved forecasting techniques should not be carried through. On the other hand, when demand exceeds available resources, we would be interested in investing in techniques to achieve better forecasts. Excessive amounts of available transportation resources will increase the feasible area and make it easier for the SP model to produce sound solutions. Constraint (7.12) and (7.31) will in these cases be non-binding, and are in effect redundant. Because of the excessive amount of transportation resources, the MSP model potentially has a larger number of alternative solutions, and can be more certain of being able to satisfy demand. In consequence, when supply and capacity exceed demand perfect information is not a necessary prerequisite for producing a good solution and of less worth.

For all test cases, WS'≤SP'≤EEV', which is consistent with the findings of Birge [1982] as presented in Section 6.3.2. When considering all the test cases simultaneously, it is readily apparent that the potential benefit from solving the stochastic program over solving the deterministic expected value program is highly convincing in situations of resource excess, and more so than in situations of resource deficiency. The EVPI' values similarly indicate that the potential worth of more accurate forecasts of stochastic elements in the immediate aftermath of a disaster is generally of some value. However, acquiring forecasts of higher quality can more often than not prove too arduous in the event of an earthquake, causing the VSS to be of greater pertinency to the decision maker than the EVPI, as argued in Section 6.3.2.

11 Conclusive Remarks

This thesis proposes a comprehensive approach to handling the disaster response problem, which comprises of facility location and last mile distribution of humanitarian aid in the event of an earthquake. Both a deterministic and a stochastic programming model have been developed as decision support tools for the underlying problem. The deterministic model serves as an important foundation for the extension into the stochastic model, and is a valid base for comparison of the two.

Crucial assumptions and limitations of the deterministic model confine its applicability in terms of generation of reliable solutions. Neither the information flow associated with the real-life problem, nor its appurtenant sequence of decisions are reflected in this model, implying poor representation of the reality. It does however perform well in terms of solution speed. Hence, the deterministic model has served as a platform for the advanced stochastic programming model, where an improvement of reality representation has been of importance in the modeling process.

The stochastic programming model resembles the deterministic model in terms of its structure. However, an obvious improvement from its deterministic counterpart is the introduction of uncertainty in a number of essential parameters. Enabling postponement of decisions and ability to coordinate the point of decision and the point of realization of relevant information constitute further enhancements. By doing so, both information flow and the related time are considered. We believe that despite an increase in solution time, inclusion of the elements of uncertainty form a major improvement and the implications of including stochasticity substantiate the model's applicability as a decision tool. The reasons for this, are the impact randomness has on the underlying problem, and the ability to approach a realistic approximation. In what follows, three main contributions of the proposed stochastic programming model will be emphasized.

Firstly, the stochastic programming model consists of three distinct stages, each relating to a different set of decisions influencing the final outcome of the distribution process. Introduction of multiple stages enables consideration of an extended planning horizon. In accordance with the situation faced by organizations providing distribution of humanitarian aid, this accentuates the actual course of events as they appear in reality. The use of recourse actions has proved appropriate due to the nature of the information flow related to disaster events. A range of different factors will be subject to uncertainty due to the unpredictability of events, and this initially unknown information will be realized at different points in time throughout the distribution process. As the consequences of a disaster are both random and next to impossible to predict, including several stages has enabled consideration of multiple stochastic elements, providing greater compliance with reality. In addition, this stage-wise structure has allowed for a combination of both facility location and last mile distribution in a single model, optimizing both planning problems simultaneously.

Secondly, the stochastic model explicitly considers fairness of distribution, an aspect deemed vital to consider in relation to humanitarian logistics. Being able to plan immediate response according to the most urgent needs as reflected in the utility induced by providing aid, is crucial in order to keeping the number of fatalities to a minimum.

Thirdly, the model created allows for dispatch of aid prior to reception of complete information regarding the state of the infrastructure, entailing a substantial improvement in responsiveness. The initial distribution plan is generated based on the expected outcome of the earthquake and the corresponding impact on the infrastructure. Modification of the initially generated routes is executed when an obstacle is hit which prevents a route from being carried through according to plan. As a result, postponement of the requirement for complete and reliable information regarding the realized state of the distribution network is achieved, and aid will be provided at the earliest possible point in time. In order to being able to handle early dispatch and modification of routes, a duplicated network has been introduced to the model proposed. Furthermore, the model is formulated such that it is able to cope with change of final destinations should demand points included in the initial plan prove inaccessible. This reduces wasteful use of resources and enables the decision maker to make use of the totality of commodities dispatched from the LDCs in order to obtain the highest possible level of utility.

These contributions are substantiated by the measures presented for a range of instances of interest, which is indicated by the value of the stochastic solution. All test results prove the stochastic model to provide the decision maker with better solutions than corresponding expected value solutions. Despite the occurrence of some symmetric solutions arising from the use of the duplicated network, the solution times are acceptable even in cases of overwhelming problem sizes. Findings of the computational study provide possible users of the model with relevant contemplations. These are reflected by the decline in marginal utility when the number of LDCs exceeds 27 % that of the demand points. A further major finding is the increase in solution time entailed by occurrence of a larger number of realizations regarding uncertain parameters in stage 3 than in stage 2. Hence, complete information concerning the state of the infrastructure will be of greater importance to the decision maker, as compared with knowledge of actual demand and vehicle capacity.

A possible improvement of the model is represented by enabling return to LDC for reloading in order to provide further delivery of aid. This would possibly compensate for shortages in vehicle capacity, but would require introduction of time indices to symbolize the sequence of vehicles flow. Adding time indices increases the complexity of the model, and has therefore been omitted from consideration in the model proposed to ensure computational efficiency. Improvement could also be achieved by use of algorithms or dynamic constraint generation to eliminate consideration of 2nd stage decisions in the objective function proposed for the stochastic model. This would produce a logical and readily comprehensible objective function of greater interpretive value.

Based on the arguments given in this report and outlined above, we conclude that we have successfully developed a multi-stage stochastic programming model that provides a decision maker aiming to optimize humanitarian aid distribution with a sufficient decision support tool.

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A The Multi-Stage Stochastic Programming Model

Network Quantities

Q_I	- total number of ICDs in the original network
Q_L	- total number of LDCs in the original network
Q_P	- total number of PODs in the original network
Q_N	- total number of nodes in the original network

Main Sets

$\mathcal{B}$ - set of commodity types

$\mathcal{E}$ - set of scenario tree nodes in the scenario tree

$\mathcal{K}$ - set of utility intervals

$\mathcal{N}$ - set of nodes

$$\begin{aligned}\mathcal{N} = \{&1, \ldots, Q_I,\\ &Q_I + 1, \ldots, Q_I + Q_L,\\ &Q_I + Q_L + 1, \ldots, Q_I + 2Q_L,\\ &Q_I + 2Q_L + 1, \ldots, Q_I + 2Q_L + Q_P = Q_N\}\end{aligned}$$

$\mathcal{T}$ - set of stages

$$\mathcal{T} = \{1, 2, 3\}$$

$\mathcal{V}$ - set of vehicle types

Indices of Main Sets

b	- commodity type	$b \in \mathcal{B}$
i, j, j'	- node	$i, j, j' \in \mathcal{N}$
k	- utility interval	$k \in \mathcal{K}$
n	- scenario tree node	$n \in \mathcal{E}$
t	- stage	$t \in \mathcal{T}$
v	- vehicle type	$v \in \mathcal{V}$

Derived Sets

$\mathcal{A}$ - subset of arcs, (i, j), which vehicles traveling between ICDs and LDCs are allowed to traverse

$\mathcal{A}_{vn}$ - subset of arcs, (i, j), which vehicle type v traveling between LDCs, dummy LDCs and PODs is allowed to traverse in scenario tree node n

$\mathcal{A}'_{vn}$ - subset of arcs and duplicated arcs, (i, j), which vehicle type v traveling between LDCs, dummy LDCs and PODs, and the duplicated dummy LDC and the duplicated PODs is allowed to traverse in scenario tree node n,
$\mathcal{A}'_{vn} = \mathcal{A}_{vn} \cup \{(i, (i + Q_L + Q_P)) : i \in \mathcal{L}^D \cup \mathcal{P}\} \cup \{((i + Q_L + Q_P), i) : i \in \mathcal{L}^D \cup \mathcal{P}\}$
$\cup \{((i + Q_L + Q_P), (j + Q_L + Q_P)) : (i, j) \in \mathcal{A}_{vn}, i, j \in \mathcal{L}^D \cup \mathcal{P}\}$
$\cup \{(i, (i + 2Q_L + Q_P)) : i \in \mathcal{L}\}$

$\mathcal{E}_t$ - set of scenario tree nodes at stage t,
$\mathcal{E}_t \subset \mathcal{E}$

$\mathcal{I}$ - set of ICDs,
$\mathcal{I} = \{1, \ldots, Q_I\}, \mathcal{I} \subset \mathcal{N}$

$\mathcal{L}$ - set of LDCs,
$\mathcal{L} = \{1, \ldots, Q_L\}, \mathcal{L} \subset \mathcal{N}$

$\mathcal{L}^{D'}$ - duplicated set of dummy LDCs,
$\mathcal{L}^{D'} = \{1, \ldots, 2Q_L\}$

$\mathcal{L}^D$ - set of dummy LDCs,
$\mathcal{L}^D = \{1, \ldots, Q_L\}, \mathcal{L}^{\mathcal{D}} \subset \mathcal{L}^{D'} \wedge \mathcal{L}^{\mathcal{D}} \subset \mathcal{N}$

$\mathcal{P}'$ - duplicated set of PODs,
$\mathcal{P}' = \{1, \ldots, 2Q_P\}$

$\mathcal{P}$ - set of PODs,
$\mathcal{P} = \{1, \ldots, Q_P\}, \mathcal{P} \subset \mathcal{P}' \wedge \mathcal{P} \subset \mathcal{N}$

$\mathcal{V}_{jn}$ - subset of vehicle types allowed to travel into node j in scenario tree node n,
$V_{jn} \subset \mathcal{V}$

Indices of Subsets

(i, j)	- arc	$(i, j) \in \mathcal{A} \cup \mathcal{A}_{vn} \cup \mathcal{A}'_{vn}$
$a(n)$	- predecessor scenario tree node of scenario tree node n	$a(n) \in \mathcal{E}$

Deterministic Parameters

C^I - unit capacity of vehicles traveling between ICDs and LDCs
C_v^L - unit capacity of vehicle type v
C_j^C - unit capacity at an LDC j
E_b^C - unit cost of commodity type b
E_{ij}^I - cost associated with traveling from ICD i to LDC j
E_i^O - cost associated with opening an LDC i
F_i^I - total available number of vehicles at ICD i

H^B	- available budget
H^T	- upper convoy time limit
M^B	- utility factor for residual budget
P_n	- probability of scenario tree node n occurring
Q_b^C	- unit size of commodity type b
Q_v^V	- unit size of vehicle type v
a_{ib}	- supply of commodity type b at ICD i
T_{ij}^I	- time spent traveling from ICD i to LDC j

Stochastic Parameters

C_{ijn}^A	- unit capacity of arc (i,j) in scenario tree node n
D_{jbn}	- demand of commodity type b at POD j in scenario tree node n
E_{ijvn}^L	- cost associated with traveling from node i to node j for vehicle type v in scenario tree node n
F_{ivn}^L	- total available number of vehicles of vehicle type v at LDC i in scenario tree node n
M_{jbkn}^D	- utility factor for satisfied demand at POD j of commodity type b in utility interval k in scenario tree node n
T_{ijvn}^L	- time spent traveling from node i to node j for vehicle type v in scenario tree node n
U_{jbkn}	- size of utility interval k for commodity type b at POD j in scenario tree node n

First Stage Variables

$$l_i = \begin{cases} 1, \text{ if LDC } i \text{ is opened} \\ 0, \text{ otherwise} \end{cases}$$

x_{ij}^I	- number of vehicles to travel from ICD i to LDC j
y_{ijb}^I	- amount of commodity type b sent from ICD i to LDC j

$$z_{ij}^I = \begin{cases} 1, \text{ if a vehicle traverses arc } (i,j) \\ 0, \text{ otherwise} \end{cases}$$

Second and Third Stage Variables

d_{jbkn}	- amount of satisfied demand of commodity type b at POD j in utility interval k and scenario tree node n
x_{ijvn}^L	- number of vehicles of type v to travel from node i to node j in scenario tree node n

y^L_{ijbvn} - amount of commodity type b sent from node i to node j with vehicle type v in scenario tree node n

$z^L_{ijvn} = \begin{cases} 1, \text{ if vehicle type } v \text{ traverses arc } (i,j) \text{ in scenario tree node } n \\ 0, \text{ otherwise} \end{cases}$

w_n - level of residual budget in scenario tree node n

Objective Function

$$\max \sum_{j \in \mathcal{P}} \sum_{b \in \mathcal{B}} \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{E} \setminus \mathcal{E}_1} P_n M_{jbkn} d_{jbkn} + \sum_{n \in \mathcal{E} \setminus \mathcal{E}_1} P_n M^B w_n \tag{A.1}$$

Constraints Stage t = {1}

$$\sum_{j \in \mathcal{L}} y^I_{ijb} \leq a_{ib} \qquad i \in \mathcal{I}, b \in \mathcal{B} \tag{A.2}$$

$$\sum_{i \in \mathcal{I}} \sum_{b \in \mathcal{B}} Q_b y^I_{ijb} - C^C_j l_j \leq 0 \qquad j \in \mathcal{L} \tag{A.3}$$

$$\sum_{j \in \mathcal{L}} x^I_{ij} \leq F^I_i \qquad i \in \mathcal{I} \tag{A.4}$$

$$x^I_{ij} - F^I_i z^I_{ij} \leq 0 \qquad (i,j) \in \mathcal{A} : i \in \mathcal{I} \tag{A.5}$$

$$\sum_{b \in \mathcal{B}} Q_b y^I_{ijb} - C^I x^I_{ij} \leq 0 \qquad (i,j) \in \mathcal{A} \tag{A.6}$$

$$z^I_{ij} - x^I_{ij} \leq 0 \qquad (i,j) \in \mathcal{A} \tag{A.7}$$

Constraints Linking Stage t={1} and t={2}

$$\sum_{v \in \mathcal{V}} \sum_{(i,j) \in \mathcal{A}_{vn}} y^L_{ijbvn} - \sum_{(j,i) \in \mathcal{A}} y^I_{jib} \leq 0 \qquad i \in \mathcal{L}, b \in \mathcal{B}, n \in \mathcal{E}_2 \tag{A.8}$$

Constraints Stage t = {2}

$$\sum_{v\in\mathcal{V}_{jn}}\sum_{(i,j)\in\mathcal{A}_{vn}} y^L_{ijbvn} - \sum_{v\in\mathcal{V}_{jn}}\sum_{(j,i)\in\mathcal{A}_{vn}} y^L_{jibvn} - \sum_{k\in\mathcal{K}} d_{jbkn} = 0 \qquad j\in\mathcal{P}, b\in\mathcal{B}, n\in\mathcal{E}_2 \tag{A.9}$$

$$\sum_{k\in\mathcal{K}} d_{jbkn} \le D_{jbn} \qquad j\in\mathcal{P}, b\in\mathcal{B}, n\in\mathcal{E}_2 \tag{A.10}$$

$$d_{jbkn} \le U_{jbkn} \qquad j\in\mathcal{P}, b\in\mathcal{B}, k\in\mathcal{K}, n\in\mathcal{E}_2 \tag{A.11}$$

$$\sum_{(i,j)\in\mathcal{A}_{vn}} x^L_{ijvn} - F^L_{ivn} l_i \le 0 \qquad i\in\mathcal{L}, v\in\mathcal{V}, n\in\mathcal{E}_2 \tag{A.12}$$

$$x^L_{ijvn} - (\sum_{j'\in\mathcal{L}} F^L_{j'vn}) z^L_{ijvn} \le 0 \qquad v\in\mathcal{V}, n\in\mathcal{E}_2, (i,j)\in\mathcal{A}_{vn} \tag{A.13}$$

$$\sum_{b\in\mathcal{B}} Q^C_b y^L_{ijbvn} - C^L_v x^L_{ijvn} \le 0 \qquad v\in\mathcal{V}, n\in\mathcal{E}_2, (i,j)\in\mathcal{A}_{vn} \tag{A.14}$$

$$\sum_{(j,i)\in\mathcal{A}_{vn}} x^L_{jivn} - \sum_{(i,j)\in\mathcal{A}_{vn}} x^L_{ijvn} \le 0 \qquad j\in\mathcal{L}^D\cup\mathcal{P}, n\in\mathcal{E}_2, v\in\mathcal{V}_{jn} \tag{A.15}$$

$$\sum_{v\in\mathcal{V}} Q^V_v (x^L_{ijvn} + x^L_{(i+Q_L)jvn}) \le C^A_{ijn} \qquad n\in\mathcal{E}_2, (i,j)\in\mathcal{A}_{vn} : i\in\mathcal{L}, j\in\mathcal{N}\backslash\mathcal{I} \tag{A.16}$$

$$\sum_{v\in\mathcal{V}} Q^V_v x^L_{ijvn} \le C^A_{ijn} \qquad n\in\mathcal{E}_2, (i,j)\in\mathcal{A}_{vn} : i\in\mathcal{P}, j\in\mathcal{N}\backslash\mathcal{I} \tag{A.17}$$

$$\sum_{v\in\mathcal{V}}\sum_{(i,j)\in\mathcal{A}_{vn}:i\in\mathcal{L}} y^L_{ijbvn} - \sum_{j\in\mathcal{P}}\sum_{k\in\mathcal{K}} d_{jbkn} = 0 \qquad b\in\mathcal{B}, n\in\mathcal{E}_2 \tag{A.18}$$

$$\sum_{(j,i)\in\mathcal{A}_{vn}} y^L_{jibvn} - \sum_{(i,j)\in\mathcal{A}_{vn}} y^L_{ijbvn} \le 0 \qquad j\in\mathcal{L}^D\cup\mathcal{P}, b\in\mathcal{B}, v\in\mathcal{V}, n\in\mathcal{E}_2 \tag{A.19}$$

Constraints Linking Stage t={2} and t={3}

$$\sum_{j\in\mathcal{P}}\sum_{k\in\mathcal{K}} d_{jbkn} - \sum_{j\in\mathcal{P}}\sum_{k\in\mathcal{K}} d_{jbka(n)} \leq 0 \qquad b\in\mathcal{B}, n\in\mathcal{E}_3 \qquad \text{(A.20)}$$

$$\sum_{k\in\mathcal{K}} d_{jbka(n)} - \sum_{k\in\mathcal{K}} d_{jbkn} + \Big(\sum_{(i,j)\in\mathcal{A}'_{vn}}\sum_{v\in\mathcal{V}_{jn}} y^L_{ijbvn} - \sum_{(i,j)\in\mathcal{A}_{va(n)}}\sum_{v\in\mathcal{V}_{ja(n)}} y^L_{ijbva(n)}\Big) \leq 0 \qquad j\in\mathcal{P}, b\in\mathcal{B}, n\in\mathcal{E}_3 \qquad \text{(A.21)}$$

$$\sum_{(i,j)\in\mathcal{A}'_{vn}} x^L_{ijvn} - \sum_{(i,j)\in\mathcal{A}_{va(n)}} x^L_{ijva(n)} \leq 0 \qquad i\in\mathcal{L}, v\in\mathcal{V}, n\in\mathcal{E}_3 \qquad \text{(A.22)}$$

$$x^L_{ijva(n)} - x^L_{ijvn} + \Big(\sum_{(j',i)\in\mathcal{A}_{vn}} x^L_{j'ivn} - \sum_{(j',i)\in\mathcal{A}_{va(n)}} x^L_{j'iva(n)}\Big) \leq 0 \qquad v\in\mathcal{V}, n\in\mathcal{E}_3, (i,j)\in\mathcal{A}_{vn} \qquad \text{(A.23)}$$

$$x^L_{ijvn} - x^L_{ijva(n)} \leq 0 \qquad v\in\mathcal{V}, n\in\mathcal{E}_3, (i,j)\in\mathcal{A}_{vn} \qquad \text{(A.24)}$$

$$\sum_{(i,j)\in\mathcal{A}'_{vn}} y^L_{ijbvn} - \sum_{(i,j)\in\mathcal{A}_{va(n)}} y^L_{ijbva(n)} \leq 0 \qquad i\in\mathcal{L}, b\in\mathcal{B}, v\in\mathcal{V}, n\in\mathcal{E}_3 \qquad \text{(A.25)}$$

$$y^L_{ijbva(n)} - y^L_{ijbvn} + \Big(\sum_{(j',i)\in\mathcal{A}_{vn}} y^L_{j'ibvn} - \sum_{(j',i)\in\mathcal{A}_{va(n)}} y^L_{j'ibva(n)}\Big) \leq 0 \qquad b\in\mathcal{B}, v\in\mathcal{V}, n\in\mathcal{E}_3, (i,j)\in\mathcal{A}_{vn} \qquad \text{(A.26)}$$

$$y^L_{ijbvn} - y^L_{ijbva(n)} \leq 0 \qquad b\in\mathcal{B}, v\in\mathcal{V}, n\in\mathcal{E}_3, (i,j)\in\mathcal{A}_{vn} \qquad \text{(A.27)}$$

Constraints Stage t = {3}

$$\sum_{v\in\mathcal{V}_{jn}}\sum_{(i,j)\in\mathcal{A}'_{vn}} y^L_{ijbvn} - \sum_{v\in\mathcal{V}_{jn}}\sum_{(j,i)\in\mathcal{A}'_{vn}} y^L_{jibvn} - \sum_{k\in\mathcal{K}} d_{jbkn} = 0 \qquad j\in\mathcal{P}, b\in\mathcal{B}, n\in\mathcal{E}_3 \tag{A.28}$$

$$\sum_{k\in\mathcal{K}} d_{jbkn} \le D_{jbn} \qquad j\in\mathcal{P}, b\in\mathcal{B}, n\in\mathcal{E}_3 \tag{A.29}$$

$$d_{jbkn} \le U_{jbkn} \qquad j\in\mathcal{P}, b\in\mathcal{B}, k\in\mathcal{K}, n\in\mathcal{E}_3 \tag{A.30}$$

$$\sum_{(i,j)\in\mathcal{A}'_{vn}} x^L_{ijvn} - F^L_{iva(n)} l_i \le 0 \qquad i\in\mathcal{L}, v\in\mathcal{V}, n\in\mathcal{E}_3 \tag{A.31}$$

$$x^L_{ijvn} - (\sum_{j'\in\mathcal{L}} F^L_{j'va(n)}) z^L_{ijvn} \le 0 \qquad v\in\mathcal{V}, n\in\mathcal{E}_3, (i,j)\in\mathcal{A}'_{vn} \tag{A.32}$$

$$\sum_{b\in\mathcal{B}} Q_b y^L_{ijbvn} - C^L_v x^L_{ijvn} \le 0 \qquad v\in\mathcal{V}, n\in\mathcal{E}_3, (i,j)\in\mathcal{A}'_{vn} \tag{A.33}$$

$$z^L_{ijvn} - x^L_{ijvn} \le 0 \qquad v\in\mathcal{V}, n\in\mathcal{E}_3, (i,j)\in\mathcal{A}'_{vn} \tag{A.34}$$

$$\sum_{(j,i)\in\mathcal{A}'_{vn}} x^L_{jivn} - \sum_{(i,j)\in\mathcal{A}'_{vn}} x^L_{ijvn} \le 0 \qquad j\in\mathcal{L}^{D'}\cup\mathcal{P}', n\in\mathcal{E}_3, v\in\mathcal{V}_{jn} \tag{A.35}$$

$$\sum_{v\in\mathcal{V}} Q^V_v (x^L_{ijvn} + x^L_{(i+Q_L)jvn} + x^L_{(i+2Q_L+Q_P)(j+Q_L+Q_P)vn}) \le C^A_{ijn} \qquad n\in\mathcal{E}_3, (i,j)\in\mathcal{A}_{vn}: i\in\mathcal{L}, j\in\mathcal{N}\backslash\mathcal{I} \tag{A.36}$$

$$\sum_{v\in\mathcal{V}} Q^V_v (x^L_{ijvn} + x^L_{(i+Q_L+Q_P)(j+Q_L+Q_P)vn}) \le C^A_{ijn} \qquad n\in\mathcal{E}_3, (i,j)\in\mathcal{A}_{vn}: i\in\mathcal{P}, j\in\mathcal{N}\backslash\mathcal{I} \tag{A.37}$$

$$\sum_{(j,i)\in\mathcal{A}'_{vn}} y^L_{jibvn} - \sum_{(i,j)\in\mathcal{A}'_{vn}} y^L_{ijbvn} \le 0 \qquad j\in\mathcal{L}^{D'}\cup\mathcal{P}', b\in\mathcal{B}, v\in\mathcal{V}, n\in\mathcal{E}_3 \tag{A.38}$$

Efficiency Constraints Applied to All Stages

$$\sum_{i\in\mathcal{L}} E_i^O l_i + \sum_{(i,j)\in\mathcal{A}} \sum_{b\in\mathcal{B}} E_b^C y_{ijb}^I + \sum_{(i,j)\in\mathcal{A}} E_{ij}^I x_{ij}^I + \sum_{v\in\mathcal{V}} \sum_{(i,j)\in\mathcal{A}_{vn}} E_{ijvn}^L x_{ijvn}^L + w_n = H^B \qquad n \in \mathcal{E}\backslash\mathcal{E}_1 \tag{A.39}$$

$$\sum_{(i,j)\in\mathcal{A}} T_{ij}^I z_{ij}^I + \sum_{v\in\mathcal{V}} \sum_{(i,j)\in\mathcal{A}'_{vn}} T_{ijvn}^L z_{ijvn}^L \le H^T \qquad n \in \mathcal{E}_3 \tag{A.40}$$

Non-Negativity Constraints for All Variables

$$l_i \in \{0,1\} \qquad i \in \mathcal{L} \tag{A.41}$$
$$x_{ij}^I \ge 0 \qquad \text{integer}, (i,j) \in \mathcal{A} \tag{A.42}$$
$$y_{ijb}^I \ge 0 \qquad (i,j) \in \mathcal{A}, b \in \mathcal{B} \tag{A.43}$$
$$z_{ij}^I \in \{0,1\} \qquad (i,j) \in \mathcal{A} \tag{A.44}$$
$$d_{jbkn} \ge 0 \qquad j \in \mathcal{P}, b \in \mathcal{B}, k \in \mathcal{K}, n \in \mathcal{E}_2 \cup \mathcal{E}_3 \tag{A.45}$$
$$x_{ijvn}^L \ge 0 \qquad \text{integer}, v \in \mathcal{V}, n \in \mathcal{E}_2, (i,j) \in \mathcal{A}_{vn} \tag{A.46}$$
$$y_{ijbvn}^L \ge 0 \qquad v \in \mathcal{V}, n \in \mathcal{E}_2, (i,j) \in \mathcal{A}_{vn}, b \in \mathcal{B} \tag{A.47}$$
$$z_{ijvn}^L \in \{0,1\} \qquad v \in \mathcal{V}, n \in \mathcal{E}_2, (i,j) \in \mathcal{A}_{vn} \tag{A.48}$$
$$x_{ijvn}^L \ge 0 \qquad \text{integer}, v \in \mathcal{V}, n \in \mathcal{E}_3, (i,j) \in \mathcal{A}'_{vn} \tag{A.49}$$
$$y_{ijbvn}^L \ge 0 \qquad v \in \mathcal{V}, n \in \mathcal{E}_3, (i,j) \in \mathcal{A}'_{vn}, b \in \mathcal{B} \tag{A.50}$$
$$z_{ijvn}^L \in \{0,1\} \qquad v \in \mathcal{V}, n \in \mathcal{E}_3, (i,j) \in \mathcal{A}'_{vn} \tag{A.51}$$
$$w_n \ge 0 \qquad n \in \mathcal{E}\backslash\mathcal{E}_1 \tag{A.52}$$

B Data for the Illustrative Example

General Parameters

Number of ICDs	4
Number of LDCs	2
Number of PODs	3
Number of scenario tree nodes	7
Budget	100,000
Utility factor for residual budget	1
Upper convoy time limit	200,000
Aircraft capacity from the ICDs	100
Commodity types	1
Commodity cost	10
Commodity size	2

ICD Specific Parameters

	ICD			
	1	*2*	*3*	*4*
Supply	100	200	100	200
Number of vehicles	5	5	4	1

LDC Specific Parameters

	LDC	
	1	*2*
Capacity	200	400
Opening cost	200	2000
Expected number of v-type1	5	6
Expected number of v-type2	2	3

POD Specific Parameters

	POD		
	1	*2*	*3*
Expected demand	50	30	60

Vehicle Specific Parameters

	Vehicle type	
	1	*2*
Capacity	200	400
Size	250	500

Arcs Connecting the ICDs and the LDCs

	Travel cost		*Travel time*	
ICD	*LDC 1*	*LDC 2*	*LDC 1*	*LDC 2*
1	3688	3458	3688	3458
2	2331	3014	2331	3014
3	3472	2676	3472	2676
4	1972	2070	1972	2070

Arcs Connecting the LDCs and PODs

	Arc cost				
Node	*LDC 1*	*LDC 2*	*POD1*	*POD 2*	*POD 3*
LDC 1	-	266	139	0	127
LDC 2	266	-	180	145	0
POD 1	139	180	-	0	0
POD 2	0	145	0	-	87
POD 3	127	0	0	87	-

	Arc capacity				
Node	*LDC 1*	*LDC 2*	*POD1*	*POD 2*	*POD 3*
LDC 1	-	1000	1000	0	3000
LDC 2	1000	-	1000	2000	0
POD 1	1000	1000	-	0	0
POD 2	0	2000	0	-	2000
POD 3	3000	0	0	2000	-

	Arc capacity				
Node	*LDC 1*	*LDC 2*	*POD1*	*POD 2*	*POD 3*
LDC 1	-	532	278	0	254
LDC 2	532	-	360	291	0
POD 1	278	360	-	0	0
POD 2	0	291	0	-	175
POD 3	254	0	0	175	-

C Results from Varying the Number of LDCs

	Time relation			*Objective value*				
# of LDCs	*Seconds to 10%*	*Seconds to 1%*	*Seconds to 0.1%*	*MU stage 2*	*MU stage 3*	*DFU stage 2*	*DFU stage 3*	*SP value*
1	-		-	200 000	200 000	-	-	400 000
2	1	15	38	318 020	318 139	164 642	153 422	954 222
3	1	9	14	302 050	302 167	157 169	137 271	898 656
4	1	30	125	257 388	257 693	281 533	243 489	1 040 102
5	1	10	275	217 287	217 600	430 768	375 877	1 241 532
6	1	30	556	200 303	200 940	441 373	398 632	1 241 247
7	4	135	1 019	189 581	190 291	281 914	250 287	912 072
8	8	55	234	178 316	178 976	314 865	276 440	948 597
9	10	45	70	200 987	201 351	657 987	642 031	1 702 355
10	6	50	333	201 286	202 582	454 028	278 071	1 135 967
20	22	146	1 399	89 101	90 715	584 873	480 905	1 245 594
30	18	228	411	157 457	158 901	446 594	381 426	1 144 377
40	14	1 014	2 998	211 669	213 150	636 296	564 365	1 625 479
70	20	504	4 015	110 542	111 402	615 768	568 095	1 405 806
90	133	2 142	23 097	136 476	137 506	510 618	477 748	1 262 346

|POD|=15

	Time relation			*Objective value*				
# of LDCs	*Seconds to 10%*	*Seconds to 1%*	*Seconds to 0.1%*	*MU stage 2*	*MU stage 3*	*DFU stage 2*	*DFU stage 3*	*SP value*
2	1	1	3	171 324	171 343	57 544	49 796	450 006
10	17	106	412	7 648	8 571	624 853	493 564	1 134 635
20	22	484	3 812	382	1 859	438 973	391 174	832 388
24	7	1 092	2 734	100	668	766 828	640 728	1 408 323
30	27	95	1 479	1 912	2 508	745 489	728 194	1 478 103
40	93	10 189	25 199	74	1 187	412 564	353 839	767 664
50	65	556	1 749	2	1 224	595 022	516 648	1 112 896
80	91	1 002	4 097	308	1 245	732 052	674 853	1 408 457

|POD|=40

D Results from Varying the Number of PODs

Instances					*Problem size*			
					Matrix		*Presolved*	
Number of PODs	*Seconds to 10%*	*Seconds to 1%*	*Seconds to 0.1%*	*Optimal solution*	*Constraints*	*Variables*	*Constraints*	*Variables*
1	0	8	9	215 865	7 1654	277	3 252	2 327
2	0	13	20	325 475	8 3864	971	3 756	2 766
3	0	13	53	664 778	8 9605	283	4 022	3 019
4	0	7	48	358 084	10 5266	131	4 810	3 592
10	8	13	50	1 265 132	15 5338	297	6 656	5 154
20	2	56	223	1 053 211	30 1361	4 817	12 376	9 740
30	13	55	95	1 349 945	46 6812	1 007	18 159	14 296
40	5	76	364	1 575 943	63 7912	5 637	17 469	13 712
50	9	71	603	1 413 668	87 3773	3 639	58 429	28 442
60	8	168	1327	1 683 645	107 632	37 275	32 354	25 697
70	75	169	620	1 631 337	136 025	45 191	40 801	32 234
80	4	52	503	2 708 796	160 352	48 391	41 649	33 192
90	25	85	408	2 025 363	191 177	54 477	47 767	37 981
100	72	404	1125	1 544 751	226 316	61 723	54 930	43 531
110	53	1139	1624	2 838 265	259 657	66 217	58 394	46 389
120	50	100	1181	2 406 784	297 028	73 581	63 428	50 508
200	204	211	1759	11 821 461	684 953	119 083	105 305	83 964
250	118	210	3156	1 861 296	1 005 664	148 419	132 192	255 282

E Results from Varying the Number of Successor Nodes per Parent Node in Stage 2

Budget =100,000

	Time relation			*Objective value*				
# of succ. s=2	*Seconds to 10%*	*Seconds to 1%*	*Seconds to 0.1%*	*MU stage 2*	*MU stage 3*	*DFU stage 2*	*DFU stage 3*	*SP value*
1	2	8	331	194	124	243 700	235 032	479 050
2	1	398	613	109	308	400 779	364 208	765 404
5	4	97	2 530	36 925	36 925	496 956	440 042	1 010 848
7	243	1 071	3 398	4 143	4 609	386 160	279 868	674 780
8	51	730	4 885	13 317	13 568	152 719	150 689	330 293
10	267	2 342	21 994	181	547	226 274	192 622	419 624
20	935	23 943	43 000	890	1 203	337 564	228 750	568 407
30	1 017	5 178	24 060	11 685	11 949	161 022	156 234	340 890
40	9 042	39 812	43 188	217	471	265 829	251 188	517 705
50	3 225	28 072	43 289	670	1 176	342 350	270 942	615 138
60	340	25 239	43 202	285	1 356	940 611	483 770	1 426 022
70	300	17 590	25 337	3 685	4 674	196 327	170 464	375 150
80	12 875	42 641	43 238	2 600	2 928	259 992	242 950	508 470
90	25 858	27 576	43 200	8 893	9 329	400 816	287 551	706 589
100	12 671	41 974	43 218	318	352	273 114	264 029	537 813
120	36 918	36 920	43 226	234	627	1 166 220	1 132 040	2 299 121
150	33 116	43 998	44 922	298	768	388 296	249 749	639 111

Budget =500,000

	Time relation			*Objective value*				
# of succ. s=2	*Seconds to 10%*	*Seconds to 1%*	*Seconds to 0.1%*	*MU stage 2*	*MU stage 3*	*DFU stage 2*	*DFU stage 3*	*SP value*
1	-	5	31	377 332	377 680	269 389	262 814	1 287 214
2	8	26	118	344 749	344 537	513 444	467 801	1 670 530
5	4	74	750	419 952	420 636	393 380	332 222	1 566 190
7	7	88	3 739	341 014	341 828	449 200	367 900	1 499 941
8	8	134	783	379 677	380 285	367 893	288 358	1 416 212
10	5 451	3 701	2 651	352 728	353 206	295 820	260 354	631 054
20	2 865	4 448	24 113	372 870	373 478	305 867	235 911	1 288 124
30	660	1 067	9 236	365 449	365 761	298 538	278 147	1 307 895
40	2 778	2 778	29 290	340 235	340 573	278 746	260 810	610 182
50	4 354	23 301	43 169	364 098	364 725	386 328	336 222	1 451 372
60	4 933	7 090	19 898	380 915	381 781	689 796	396 605	1 849 096
70	1 855	1 855	26 687	343 259	344 296	635 764	494 500	1 817 819
80	11 061	27 065	43 138	343 312	344 008	487 940	420 515	1 595 774
90	3 156	4 873	43 102	375 151	375 862	431 239	323 762	1 506 014
100	5 624	8 201	43 172	351 093	351 874	405 072	343 138	1 451 176
120	16 762	16 762	43 223	348 246	348 826	862 678	827 465	2 387 214
150	7 958	15 986	43 229	351 360	351 894	460 769	310 744	737 384

F Results from Varying Budget and Number of Successor Nodes per Parent Node in the Scenario Tree

Budget = 100,000

	Time relation			*Objective value*				
# of succ. s=3	*Seconds to 10%*	*Seconds to 1%*	*Seconds to 0.1%*	*MU stage 2*	*MU stage 3*	*DFU stage 2*	*DFU stage 3*	*SP value*
1	7	25	323	1 810	2 004	297 012	260 786	561 611
2	1	42	5 304	2 748	2 798	285 490	263 348	554 383
5	75	503	22 128	1 507	2 320	313 276	249 122	566 223
7	51	741	24 777	84	270	300 685	281 815	582 853
8	35	1 868	43 201	10 954	12 128	251 927	140 005	207 507
10	83	1 403	21 947	2 127	2 611	394 552	339 284	738 572
20	460	10 321	42 051	1 657	2 252	280 708	225 359	509 975
30	557	24 815	43 203	1 132	1 857	411 531	382 290	796 809
40	7 902	42 367	43 201	1 789	2 593	355 300	306 963	666 644
50	860	42 236	43 203	92	569	253 086	196 897	450 643
60	3 310	42 747	43 215	978	2 721	264 016	193 707	230 711
70	1 806	43 103	43 204	1 826	2 860	328 276	230 326	563 288
80	2 577	42 784	43 238	1 812	3 136	253 818	195 094	453 859
90	54 012	43 200	43 200	202	1 568	296 006	215 717	513 492
100	5 002	38 620	43 209	83	1 299	307 156	208 956	517 494
120	61 026	41 199	43 378	757	2 257	257 130	190 270	225 207
150	13 535	40 615	43 391	179 065	180 238	362 949	285 016	1 007 267

Budget = 500,000

	Time relation			*Objective value*				
# of succ. s=3	*Seconds to 10%*	*Seconds to 1%*	*Seconds to 0.1%*	*MU stage 2*	*MU stage 3*	*DFU stage 2*	*DFU stage 3*	*SP value*
1	-	3	43	361 677	362 497	373 215	304 503	1 401 891
2	-	14	43	365 008	365 126	346 992	307 889	1 385 014
5	12	79	723	363 319	364 567	388 270	290 352	1 406 507
7	4	115	3 421	347 018	347 566	398 984	359 967	1 453 534
8	11	137	5 111	363 868	365 428	265 028	165 535	1 159 858
10	10	212	1 342	346 293	347 020	481 181	401 882	1 576 375
20	3 092	3 095	23 500	338 622	340 581	391 368	286 408	678 490
30	389	618	22 982	350 488	351 412	517 410	462 550	1 681 859
40	136	828	43 201	345 222	346 305	462 619	384 305	1 538 450
50	375	2 665	43 203	367 835	369 046	307 228	234 602	1 278 711
60	489	4 764	43 200	361 228	362 734	373 091	257 382	1 354 435
70	4 657	16 987	43 205	339 439	340 654	488 162	341 544	754 900
80	1 134	12 825	43 202	338 947	340 681	350 525	286 482	658 318
90	8 545	20 433	43 240	364 650	366 185	374 459	269 124	1 374 417
100	23 677	26 968	43 267	344 117	346 072	419 389	277 776	1 387 353
120	21 427	43 004	43 208	362 859	363 881	299 081	194 093	1 219 913
150	69 409	43 205	43 205	355 413	356 995	610 088	489 194	1 811 689