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Bidding Revealed

An Empirical Analysis of Selling Hydropower through Elspot

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Industrial Economics and Technology Management
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Sammendrag

I det deregulerte nordiske kraftmarkedet byr kraftprodusenter for å selge morgendagens kraftproduksjon i elektrisistetsauksjonen Elspot. Denne avhandlingen presenterer en empirisk analyse av bud avgitt av tre norske vannkraftprodusenter over fire to-ukers perioder i 2011. Som pristakere maksimerer produsentene sin profitt ved å by på sin marginalkostnad, som er avhengig av både kjente og ukjente variabler. I tillegg må produsentene følge eksterne markedsrestriksjoner og interne tekniske og hydrologiske restriksjoner når de byr. Mønstre i de avgitte budene blir funnet og forklaringer på budgivningen blir gitt. Videre blir suboptimaliteter og potensielle irrasjonaliteter i budgivningen avdekket. En to-stegs stokastisk blandet heltalls lineær modell som genererer optimale bud utvikles for å bistå i analysen. Vannkraftprodusentene optimaliserer ikke i sin budgivning, men kommer ofte nær det optimale resultatet.

An empirical analysis of selling hydropower through Elspot

BIDDING REVEALED

June 7, 2012

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Abstract

In the deregulated Nordic electricity market, power producers bid to sell tomorrow's power in the day-ahead auction Elspot. This thesis presents an empirical analysis of bids submitted by three medium to large sized Norwegian reservoir hydropower producers over four two-week periods in 2011. Being price takers, the producers maximize their profits by bidding their marginal cost, which is dependent on both known and unknown variables. Additionally, producers must abide by both external market restrictions and internal technical and hydrological restrictions when bidding. Patterns in the submitted bids are found and explanations for the bidding behavior are given. Furthermore, suboptimalities and potential irrationalities in the bidding are revealed. A two-stage stochastic mixed-integer linear program generating optimal bids is developed to assist in the analysis. The hydropower producers do not optimize their bidding, yet often come close to the optimal result.

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Preface

This thesis has been written for the degree of Master of Technology at the Norwegian University of Science and Technology (NTNU), Department of Industrial Economics and Technology Management within the field of Applied Economics and Operations Research. We would specially like to thank our teaching supervisor, Professor Stein-Erik Fleten for helpful assistance and valuable discussions. In addition, we owe Associate Professor Trine Boomsma from University of Copenhagen thanks for constructive feedback. Last but not least, we thank the three anonymous Norwegian hydropower producers and SKM Market Predictor who have provided us with invaluable data. Without their contribution this thesis would not have been possible.

It should be noted that from the problem description in our thesis contracts, this paper was meant to include a theoretical and simplified model for solving the bidding problem. Learned in early stages of the work, such a model gave a very poor representation of the real world, and would be of little assistance when analysing actual bids. Instead we have developed an optimal bidding model seeking to replicate the reality hydropower producers face more accurately.

Trondheim, June 7, 2012

Erik Nicholas Alnæs Roger Grøndahl

Contents

1 Introduction

Hydropower producers in the Nordic countries are free to choose what to do with their water and where to sell their power. However, besides bilateral agreements with large industrial power consumers, the only marketplace to trade true volumes of physical power is the day-ahead auction Elspot organized by Nord Pool Spot ASA (NPS). In total 92 TWh, or 74 %, of the electricity produced in Norway was in 2010 traded through Elspot (Nord Pool ASA, 2011). Througout this thesis we will be referring to Elspot and Nord Pool Spot as if they are the same. At 12:00 both distributors and producers submit bids to Nord Pool Spot for buying or selling electricity for the coming day, that is the next 12–36 hours. The participants can use several combinations of prices and volumes for each hour, thus creating a piece-wise linear bid function, in addition to other types of bids.

After receiving all bids, NPS sets the uniform system and area spot prices. At around 13:00 the prices become public and a producer will learn how big a volume he is committed to produce for every hour the next day. All power producers and suppliers have balancing responsibility, overseen through the transmission system operator, Statnett. At 19:00 detailed production plans needs to be submitted to Statnett. The commitment from Elspot is included in the balance, so if a producer for some reason fails to hit the committed volume he needs to make up for this through other means. For this purpose Nord Pool Spot also organizes an intraday trading market, Elbas, where participants can buy and sell electricity directly from 14:00 to 2 hours prior delivery. However, in Norway the liquidity of Elbas is lacking and the volumes are less than 1 % of Elspot. That is mainly because of late Norwegian Elbas entry and an already well functioning Regulating Power Market (RKM), organized by Statnett (Statnett, 2009). After the production plans are submitted, Statnett accept bids for upwards or downwards regulation of production balance. The participants choose both the volume, hour and price of their bid, however the RKM price will be set as a uniform price for all accepted participants within the same spot area. The information on whether the bid is accepted can be given by Statnett until 15 minutes prior delivery, thus such a reaction time is a requirement for bidding in RKM. The prices for a potential upwards regulation will per definition be at least 5 NOK/MWh higher than the corresponding Elspot price, though a producer can never be certain there will be need of any regulation and thus can not rely solely on bidding RKM power. On the other hand, if you end up short on your balance as a producer you need to buy the gap at a RKM price of minimum 5 NOK above Elspot.

In addition to the RKM, Statnett also coordinates a market for primary real time frequency regulation (Statnett, 2011). All power producers are obliged to support the system with a share of so-called rotating reserves. These reserves will be activated automatically when the frequency deviates above 50.1 Hz or below 49.9 Hz. Through FNR the producers bid in additional rotating reserves which in the end will be paid off at uniform marginal cost based FNR prices, that is if they are used at all.

Since producers can not be guaranteed any production in the closer to real-time markets, Nordic producers looking to sell in an efficient market must necessarily bid much of their power into day-ahead Elspot. The problem faced by the flexible hydropower producers, hereby referred to as the bidding problem, thus consists of how much power to offer in Elspot for tomorrow, at what prices and for which hours through what type of bids. This problem is further complicated through regulations on water flows and reservoir levels, variable feed-in fees to the grid owner, as well as start-up costs and variable efficiency curves for the turbines.

This thesis analyzes a set of bids submitted to Elspot by three Norwegian reservoir hydro producers in four two-week periods in 2011. Through this empirical analysis we show how the bids can be related to the complications just mentioned, and show whether they are dealt with rationally in the bidding to NPS. To the authors' knowledge this is something that has not been done before.

As a price taker in a competitive market you achieve your optimal outcome by offering your good to marginal cost. The same goes for power producers. However, where nuclear or thermal power plants can relate their marginal costs to the cost of fuel, hydropower producers get their water for free. For flexible hydro producers, producers with adjustable reservoirs, the marginal costs translates into the opportunity cost of not being able to sell power from this water at a later stage (Pereira et al., 1998). And determining the latter part is far from easy, as value of an additional unit of water in the reservoirs, the marginal water value, depends on more than just future price expectations. It is also dependent on the current reservoir level, local inflow expectations and the size of reservoir compared to it's average inflow and production capacity (Tipping, 2007).

Some of the aforementioned aspects can be observed and controlled, some can be easily modeled, while others are more difficult to predict or estimate. Fleten and Kristoffersen (2007) develop a multistage stochastic MIP model for short term production planning given uncertain inflow and prices. They look at a cascaded two-reservoir system and estimate the marginal water values as functions of reservoir levels through the use of averaging forward and futures contracts, while establishing operating rules for the relation between the two reservoir levels. The concave water value functions are then approximated by piecewise linear functions to be consistent with a mixed-integer linear formulation. Start-up costs are modeled through the use of binary variables describing the running state of the turbines. The horizon modeled is 7 stages of 24 hours, where the first stage includes the bidding decisions and the coming decisions the unit commitments.

Fleten et al. (2011) build on a lot from the previous mentioned model, however work only with one reservoir and one turbine. They extend the model with a long-term formulation without hourly resolution or binary variables. This is partly done to make the model less dependent on the water values, as water values in this model is included as a constant end-of-horizon value. This is also in line with the logic presented in Wolfgang et al. (2009), where using backward dynamic programming implies that this week's water value will be independent of the end point value if the end of the planning period is far enough ahead. Fleten et al. (2011) still model the prices as stochastic, whereas the inflow is modeled as deterministic, justified by being a model solely for winter seasons with low inflow. Löhndorf et al. (2011) similarly formulate the bidding problem as an intraday problem considering bidding into the day-ahead market, whereas the longer-term interday problem is modeled as a Markov decision process managing storage operations over time. The intraday problem is formulated as a stochastic program followed by the interday problem integrating stochastic dual dynamic programming with approximate dynamic programming. They show that the difference between the approximate and the optimal solution is negligible and that their method is applicable to real world settings. De Ladurantaye et al. (2007) optimize the day-ahead bidding for a generic set of power plants in the same hydrological system. They use a stochastic programming model to generate optimal bids given a pre-generated 24 hour production plan. We refer to Faria and Fleten (2011) for a wider review on bidding strategies, including thermal generation.

To further analyze the bids we also develop a model for optimal bidding and short term production planning. The model draws a lot from Fleten et al. (2011), however differs in a few aspects. The goal of our model is not to work as direct decision support, but rather to assist in the analyses of the aforementioned historical bids. Since the bids come from several producers and different hydrological systems the model needs to be generic with respect to the number of reservoirs and turbines. We do not have exact historical information about the marginal water values the producers used for bidding, neither do we believe ourselves able to recreate them. On that logic we have instead formulated a model where the reservoir levels at the start and end of each two-week period are fixed. This would also imply that the inflow over the two-week period needs to be known, and we model it as deterministic seeing that inflow forecasts for such a short period are usually quite good (Doorman, 2009). We have also developed a an alternative constraint that binds the total water usage per power station instead of fixing end reservoir levels. The day-ahead prices are modeled as stochastic, however the model is limited to a two-stage formulation where all post-day-ahead prices in one scenario follow deterministically. This is due to computational issues learned from Fleten et al. (2011), as well as the fact that actual bidding does not take place until the day before production. The bidding model assists in the bid analysis through comparisons between actual bids from a hydrological cascade, and model generated bids. We also use the model for testing the value of block bids. Additionally, a full two-week runthrough with iterative actual spot price realizations have been carried out, both for a 300 scenario stochastic model and a 1-scenario deterministic model. The resulting difference in surprisingly small. The same model have also bee run through with the actual spot prices as the sole price scenario to reveal the true value of perfect information.

The outline of the thesis is as follows. In Section 2 we present the premises that flexible hydro producers have to deal with when bidding day-ahead, particularly in Norway. Section 3 presents the bids and gives the main empirical analysis, structured around the producers' decisions of deciding the bid volumes, setting the price points and deciding what type of bids to use, respectively. In Section 4 we introduce and formulate the optimal bidding model and present some results from running it. Finally, we conclude the thesis in Section 5.

2 A flexible hydro producer's bidding premises

Compared to the assumptions made in simplified academic models, the bidding problem the hydropower producers face is often far more complex. This section will elaborate on the factors that complicate the bidding process and how the producers address these issues in actual bidding. We have divided the section into the premises that are more or less exogenous, while the second part concerns the factors the producers to some extent can control themselves before bidding.

2.1 External premises for bidding

Options to selling in Elspot Like mentioned in the introduction, there exists alternatives for producers besides selling all power in Elspot, as well as options for deviating from the realized commitment. Setting aside regulation capacity for fixing other market participants' imbalances in RKM or the FNR might give extra income. However, unintentionally deviating from the balance commitment will always yield producers a negative difference in expected profit. If you as a producer are unable to reach your commitment, you must pay a higher than spot price for buying the up-regulation. On the other side, if you end up producing more than commitment you get paid an expectation on less than spot price. So the producers might as well do their bidding right the first time around in Elspot.

Power balancing entities also have the option of participating in the weekly Regulating Capacity Options Market (RKOM), where they will be paid a premium to set aside a certain volume of for RKM bidding in the coming week. The participants can still choose their own bid prices, but have to bid the agreed volume from 05:00 to 23:00, Monday through Sunday.

Elspot market rules We wish to familiarize the reader with the hourly bid matrix and the other types of bids used in Elspot. In Table 2.1 we present an hourly bid matrix in the form it is submitted to NPS. Note however that it only displays the first and last four hours of the day. The bid matrix consists strictly of hourly bids, other bids are submitted in separate forms. In the matrix, the hours of the day are to the left on the vertical axis, while the price points are at the top of the horizontal axis. The bid volumes connected to the price points and hours of the day make up the rest. We observe the strictly non-decreasing bid volumes as the price points go up within each hour, a requirement from NPS. Positive bid volumes are bids to purchase electricity and negative are bids to produce and sell. The technical minimum and maximum price points are respectively -2100 and 21000 NOK/MWh, in euros -200 and 2000 EUR/MWh. Participants must always include these price points in the bid matrix and are allowed to use an additional 62 price points. Only using 8 as in Table 2.1 is not much. Furthermore, the smallest allowed bid ticks are 1 NOK/MWh and 0.1 EUR/MWh for the prices, and 0.1 MWh for the volumes. Generating a close to optimal bid matrix for a large hydro power producer is a very complex task.

To split up the problem, hourly bid matrices are normally generated for each separate power station in their portfolio. These are then aggregated to a single matrix submitted to Nord Pool Spot.

Table 2.1: Bid matrix consisting of hourly bids for a hydro producer in the form it is submitted to Nord Pool Spot. The top row shows price points in NOK/MWh, the left column represents the hours of the day, and the remaining table body comprises the bidded volumes in MWhs where bids to sell are denoted by a negative sign.

Hour	-2100	39	40	299	300	350	400	21000
1	0	0	-25.7	-25.7	-82.8	-107.7	-127.7	-127.7
2	0	0	-25.7	-25.7	-82.8	-107.7	-127.7	-127.7
3	0	0	-25.7	-25.7	-82.8	-107.7	-127.7	-127.7
4	0	0	-25.7	-25.7	-82.7	-107.6	-127.6	-127.6
.	.	.	.	.	.	.	.	.
21	0	0	-25.7	-25.7	-76.8	-101.7	-121.7	-121.7
22	0	0	-25.7	-25.7	-76.8	-101.7	-121.7	-121.7
23	0	0	-25.7	-25.7	-82.7	-107.6	-127.6	-127.6
24	0	0	-25.7	-25.7	-82.7	-107.6	-127.6	-127.6

The second most common type of bid is the block bid. Block bids are bids that span at least three consecutive hours at a fixed volume and price. It comes with an all-or-nothing condition and an intertemporal condition. This implies that the bid is either fully accepted or rejected for all hours it is submitted for, and thus cannot be interpolated. A sales block is accepted only if the average spot price over its hours is below the bid price. Table 2.2 shows a typical set of block bids, successive in time. Additionally, the second block is linked to the first. Linked block bids are block bids that come with an only-if condition. A linked block is only considered for appectance if the block it is linked to, the mother block, is accepted. When settling the market, the market operator will therefore disregard the linked block until the mother block is potentially accepted, and first then decide whether to accept the linked block or not.

Table 2.2: A typical set of two successive block bids, the evaluation of the second being dependent on the acceptance of the first.

Start	Stop	Volume	Price	Block	Linked to
00:00	07:00	-40	200	B0007	
07:00	22:00	-40	200	B0724	B0007

After receiving all bids, NPS sets the uniform system and area spot prices. The system price is the price for every traded MWh that would equal supply and demand in Norway, Sweden, Denmark, Finland, and Estonia combined, while at the same time generating the largest possible socio-economic surplus. The area spot prices are set the same way as the system price, except that the balance between production, consumption, import, and export needs to be maintained within a geographically defined spot area. The algorithm for setting area prices therefore includes capacities for trade across spot area borders. If all area border crossing transmission lines have spare capacity, the spot prices will equal the system price. The price most relevant for the market participants is the area spot price as it is the price they must pay or receives for each MW of electricity. The area spot price is from here referred to as the spot price, or simply price. The system price is calculated regardless by the market operator as it used for several purposes, e.g. in the financial markets and for calculating the variable feed-in fee.

Most often, the spot price set by Nord Pool Spot will not equal any of the price points chosen by the producer. Thus the exact hourly bid commitment will be an interpolated value between volumes of the two neighboring price points.

Feed-in fees Power producers in Norway pay a fee when delivering power to the electricity grid, from here on referred to as the feed-in fee. The feed-in fee consists of a fixed part of 8 NOK/MWh paid to the TSO, Statnett, and a variable part paid to the grid owner. The fixed part of the fee is set for several years at a time to cover costs for Statnett and is equal for all power stations and all hours of the week. The variable part equals the marginal loss rate multiplied with the system price for every respective hour. The marginal loss rate is set by the TSO on a weekly basis to account for the changes in grid losses, caused by the marginal power production from each respective power station. If your production is closer to the power drain, you might in fact improve the grid situation by supplying the grid. Thus the marginal loss rate can be both positive and negative. The marginal loss rate is given as two different values over the span of a week, one for weekdays (07:00 – 22:00) and one for weekends and nights (22:00 – 06:00). With regards to bidding, the fixed part should not have any effect other than shifting the curve up by its amount, whereas the variable part should be added or subtracted in determining the price points for every hour. Note that even though the rates are given in advance, the system price for the next day is still unknown and system price variations will add uncertainty to the fee.

The waterways Hydropower stations are usually found in cascades with other power stations along a waterway. Thus, the decisions made for one reservoir and power station might affect decisions that must be made further downstream. Unless very near, or at full capacity, the reservoir levels and capacities in this study are large enough to cope with inflow from reservoirs above on a day to day basis without being forced to release its own water. It is therefore difficult to recognize if and how a bid for reservoir power station affects the bids in a reservoir station below. Thus we see the cascade problem concerning more longer-term planning than hour to hour dependence in the bidding. In the case where a run-of-river power plant lies below a reservoir hydropower plant one might be able to observe subsequent increased bidding in the run-of-river plant if the producer bids to release water from the reservoir above. However, we have no bids for the described set-up of power stations. We will therefore give little more attention to how producers interconnect their bidding within cascades.

When scheduling the water release, and consequently bidding to sell power, power producers must abide by local regulations regarding the water systems. There can be requirements on the minimum or maximum levels of water in a reservoir or flow in the rivers. The reasons for regulating the water systems can be many, but it often concerns preservation of wildlife, ensuring access to water for drinking and irrigation, as well as proactive flood prevention.

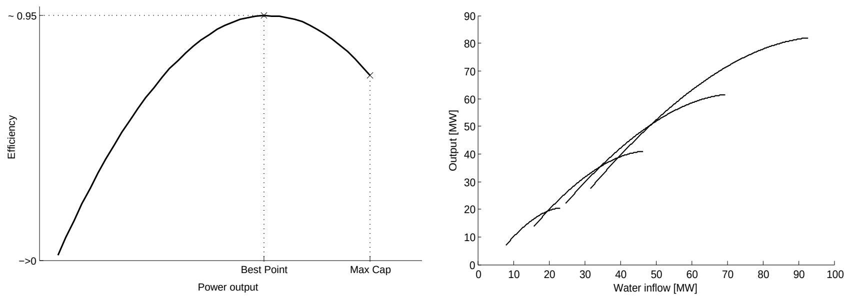
For many producers run-of-river power plants are a big contributor to total output. As the name implies, the producer has no storage capabilities for run-of-river water. The marginal opportunity cost of this water is therefore considered to be zero. Yet hydro power producers must include bids from these power plants in their day-ahead bidding. The forecasted flow of water for tomorrow should then be bid in at a price around zero. The reason why the price should not necessarily be zero is mainly due to the feed-in fee, which represents a direct marginal cost. In this empirical study we have tried to avoid water systems containing run-of-river power plants, however there are instances where bids for this type of power occur. Nevertheless, they can be spotted and taken out of the equation when analyzing the bidding for reservoir power.

Concession power Hydropower producers in Norway are committed to delivering up to 15 % of the electricity production to the local municipality and the state government at an estimated cost price set by the government (NVE, 2001). This obligation is known as concession, or compulsory power. The arrangement ensures that the local society benefits from the economic surplus generated by the electricity production and trade. For 2011 the concession power price is set at 106.8 NOK/MWh (Regjeringen, 2010). For certain producers this compulsory power may be evident in the bidding through buying power at very low prices, simply for the possibility over covering the obligation through purchases at a favourable price level compared to producing it themselves.

2.2 Internal premises for bidding

The marginal water value The most important parameter when bidding hydropower is the value of an additional unit of water in a reservoir, the marginal water value, or simply the water value. This also translates into the opportunity cost of not being able to sell power from this water at a later stage. As a fully correct water value calculation is very complex, most producers use specialized software developed by external actors to perform the calculation. Some do their own simplified calculations in customized computer programs or as simple functions of the reservoir levels. We do not have exact water values for all three producers, nor will we create our own models for estimating them. However, through deduction it is possible to see the water values from the bids.

Efficiency curves The bidding problem also relates to the efficiency of the power plants and each separate turbine. The efficiency at which a turbine runs is given by its power output in MW divided by the inflow of water, also in MW: $\eta = w/q$. Modern turbines and generators are at their best point of production able to convert up to 95 % of the kinetic energy to electric power (Doorman, 2009). Both above and below best point the efficiency usually drops a few percentage points, illustrated with the concave curve in Figure 2.1a. Naturally, producers want to run their turbines at best point for as much of the time as possible to best utilize the water in their reservoirs. Though, as the spot price rises so should the producer's willingness to produce above best point production. On the other hand, due to high start-up costs or regulations on minimum flow, a producer might also end up producing below best point.



(a) Efficiency curve for a single turbine related to power output.

(b) Combined efficiency curves for one to four identical turbines.

Figure 2.1: Efficiency and output curves for one and several combined turbines. The single efficiency curve shows the actual efficiency $\eta = w/q$ relative to the power output while the multiple turbine curve shows output relative to the water inflow.

When bidding for power stations with multiple turbines, producers also have to consider the combined efficiencies of two or more turbines. Depending on the efficiency curves of the turbines, it might be better to run three turbines at a certain point of production than two, or vice versa. Figure 2.1b shows actual combined output of one to four identical turbines relative to water inflow. At a given flow of water, producers naturally want to get the highest possible power output in return. In this case we see a slight overlap when using one and two turbines, and an increasing overlap when including more turbines. Where the curves cross each other is where the producer wants to transition from using one more (or one less) turbine. We will use the term efficiency curve for both types of curves shown in Figure 2.1 as they both are able to display the efficiency of turbines.

Start-up costs If a producer starts a turbine from a stand-still, both direct and indirect costs will occur. The direct costs come in the form of spilled water and potential extra wages for employees, while the indirect costs are wear and tear on the equipment and a risk for a failure in the start-up procedure. Although very low compared to other power production technologies, start-up costs still have to be accounted for when running a hydro plant. In terms of bidding this is one of the reasons why block bids are sensible.

Remedy for bad bidding Large hydropower producers have many power stations, some with multiple turbines. After Nord Pool Spot clears the market, producers are given a total production commitment according to the spot price and their submitted bids. At this point they usually run some sort of deterministic optimization software, for instance SHOP developed by SINTEF (Belsnes et al., 2003). Based on expected future events, this helps them allocate the realized volume commitment across their turbines in a way that minimizes the total value of the water used to fulfill the assigned production volume (Fleten and Kristoffersen, 2007). Thus, the possibility of post commitment production allocation favours big producers with many turbines. In a sense, it eases off some of the pressure on bidding optimally as, if lucky, even bidding poorly might in the end result in a production scheme where every turbine runs at a highly efficient output level.

3 Bid analysis

In the following section we present the empirical analysis of the bids. First we give a summary of the bids and other information we have received in 3.1. In 3.2 we analyze the bids based on the underlying factors found in Section 3.2. Finally, in Section 3.3 we show the maximum potential increase in income for each producer, by always running maximum production capacity in the hours with the highest spot prices. Due to the complex nature of the bids and producers' bidding premises, drawing definite conclusions on the producers' bidding behavior can be difficult. Thus the analysis can be seen as more qualitative than quantitative. Wherever possible, we try to give our findings in the least ambiguous way, and use numbers to give precise answers.

3.1 Presentation of bid data

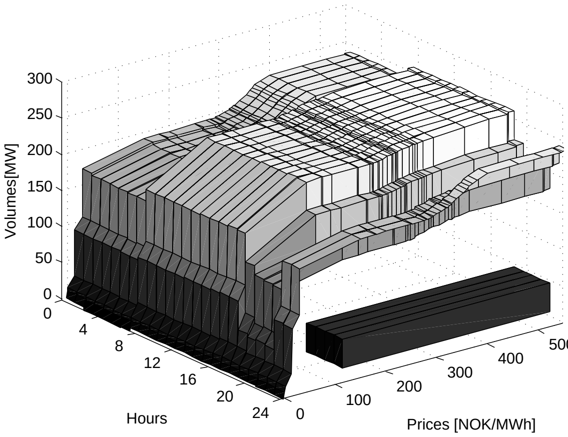
The bid data We have been so fortunate to be provided with extensive data sets from three medium to large Norwegian hydropower producers. We have received all variations of bids submitted by the producers to Nord Pool Spot for four two-week periods representing each season in 2011. Additionally, Producer A in particular has provided us with highly detailed data regarding all their cascades and power stations and thus enabled a more extensive analysis of their bidding. All producers have been very helpful answering our questions and clarifying all confusion to help overcome our own shortcomings in the subject. Table 3.1 gives a brief overview of the data we have received from the three producers.

Table 3.1: Overview of the data received from producers A, B and C. A cascade is two or more sequential reservoirs and power stations in the same hydrological system. H=regular hourly bids, B=block bids, L=linked blocks.

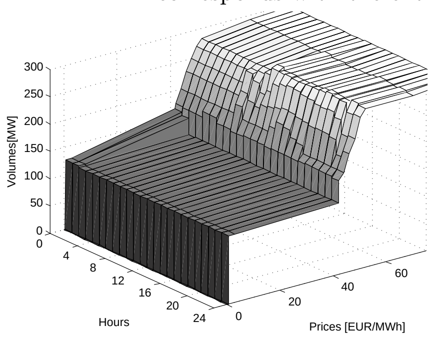
Producer	Scope	Indiv. power station bids	Bid types in use	Water values	Efficiency curves
A	All bids	Yes	H+B+L	Yes	Yes
B	Cascade	No	H+B	No	No
C	All bids	No	H+B	No	No

Anonymity Bid data is among the most sensitive information a hydropower producing company holds and must be treated as such. The bids and other information were submitted to us under confidentiality and with non-disclosure agreements. Thus we can not present data that might reveal the identity of a certain producer in the thesis.

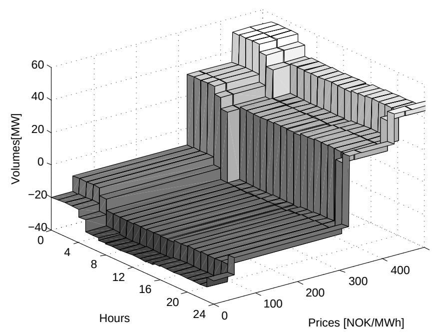
Plotting a large matrix The hourly bid is the most common type of bid used by Norwegian hydropower producers, and is submitted to NPS in matrices. The hourly bid matrix can be quite large, spanning 24 rows and up to 64 columns representing respectively the hours and the price points. Actual bids are presented in cut-outs of full matrices in Section 3.2 and as volume-price plots in Section 4.4 to illustrate and support the analyses. However to exemplify the dimensions, the matrices are plotted along three axes in Figure 3.1 below. We present three plots of these matrices, one for each company, and each matrix with different characteristics. For Producer A we also include a block bid.



(a) Bids a winter day for Producer A, including a block of 40 MW from 18:00 to 22:00 at price 150 NOK/MWh. The bid uses 37 price points in total. Most of the volume is bid at low price levels, after which the bid flattens out. We also see how the drop in hourly bid volume corresponds with the entrance of the block.



(b) Bids a weekend summer day for Producer B. The bids are very stable throughout the day, though affected by some noise. Volumes at the max. and min. prices are constant over the day.



(c) Both demand and supply bids a spring day for Producer C. At 320 NOK/MWh the bids shift from buying to selling. We also see distinct volume shifts at hour 6 and hour 21.

Figure 3.1: Three-dimensional plots of representative bids for Producer A, B and C, respectively. Volumes at z-axis, hours at y-axis, and respective currency prices at x-axis

3.2 Empirical evidence from bidding problem

We find that there are basically three decisions that a bidder faces in the day-ahead auction. These are deciding the volumes to bid, setting the exact price points you want to bid in, and lastly figuring out what type of bids to use. We analyze these bidding decisions based on the underlying factors given in the previous section, with each decision in a separate section. Naturally, all these decision are strongly interconnected. A producer would likely never set a price point without having an idea of which volume to connect it to. Even so, to obtain an analytical approach we see the three-way split as necessary and as the most sensible procedure.

3.2.1 Deciding the volumes

The production volumes found in a bid matrix or a block bid are naturally connected to the technical specifications of the turbines a producer controls. Depending on the price level and price expectations, a producer usually wants to run his turbines at the minimum level, at best point, at maximum capacity or somewhere in between the latter two. These levels of production are fixed and do not change unless the producer decides to physically alter the design of the power station. Submitting sensible bids thus consists mainly of setting a limited set of volumes at strategic price points. However, a number of other elements come into play when setting the volume points.

Other mouths to feed Bilateral agreements include the obligation to deliver concession power, as well as directly to industrial consumers. We can therefore find bids in the bid matrices where producers bid to purchase power at low price levels. Of course, if the price of electricity is negative, the producer would receive a direct profit from delivering on the bilateral agreements. Table 3.2 shows us how Producer C ensures fixed delivery on his agreements while bidding to stay at a constant, and most likely best point, production level. We deduce this by the following: The volumes at the far left are the volume sizes of the bilateral commitments. As they change from hour to hour, so do the volumes further right, at higher price points. If the purchase volumes increase, the bids to sell decrease, meaning that if the price is high, e.g. above 350 NOK/MWh, the producer wishes to produce and deliver the volume himself. We see that at a spot price of 50 NOK/MWh, Producer C finds it profitable to produce 9.6 MW of the power commitment and purchase the rest in the spot market. If the spot price reaches 321 NOK/MWh he wants to produce the entire commitment and sell power in the spot market as well. The total output level should remain constant at 9.6 MW at a price above 50 NOK/MWh and at 50.8 MW above 321 NOK/MWh. For example in hour 3, if the spot price were above 321 NOK/MWh, Producer C would produce 31.2 MW for the spot market and an additional 19.6 MW on bilateral agreements, totalling 50.8 MW. All in all, producers must adapt their volume points in the spot market to their obligations outside the spot market.

Table 3.2: Bids from Producer C to deliver on bilateral agreements seen by purchase bids at low price points. These agreements also affect the bidding at the other price points.

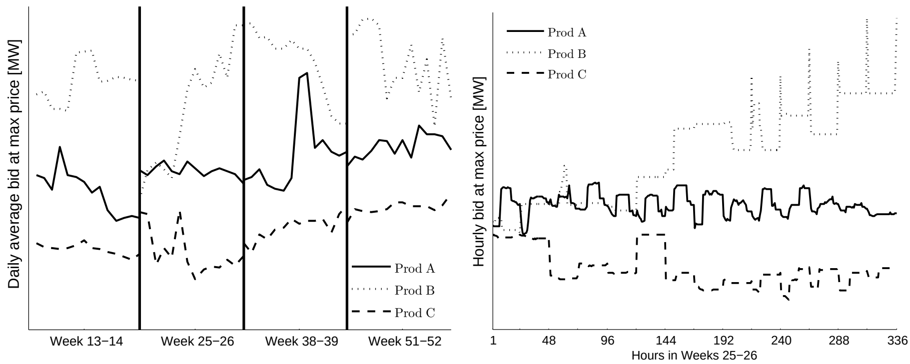
Hour	-2100	49	50	320	321	349
3	19.6	19.6	10.0	10.0	-31.2	-31.2
4	19.5	19.5	9.9	9.9	-31.3	-31.3
5	24.4	24.4	14.8	14.8	-26.4	-26.4
6	31.4	31.4	21.8	21.8	-19.4	-19.4

Manual refueling Purchase bids at negative or low prices can also represent bids to run pump stations. The pumps then transport water from a lower reservoir or a river to a higher reservoir connected to a power station below. For instance, a producer could have the lower reservoir at an altitude of 500 meters above sea level (masl), with the higher reservoir at 600 masl. Given a power station connected to the higher reservoir at 100 masl, the net gain of altitude would be 400 m. We remind the reader here that a common practise is to generate hourly bids for separate power stations before aggregating all bids to the final bid matrix submitted to NPS. Table 3.3 shows Producer A's bid matrix for an individual power station connected to such a system. The bids show a willingness to pump at prices under 29 NOK/MWh, and up to 180 NOK/MWh in some hours.

Table 3.3: Producer A's bid matrix for an individual power station to pump water into higher reservoir, seen by bidding to purchase at low prices.

Hour	-2100	29	180	181	500	511	21000
2	9	9	9	0	0	0	0
7	9	0	0	0	0	0	0
9	9	0	0	0	0	-2	-2
21	9	0	0	0	0	0	0

Variations in maximum production capacity It might seem natural for a producer to bid the combined technical capacity of all its turbines to Elspot at the highest price point of 2000 EUR/MWh. However, this is not exactly the case. We assume that the producers want to produce as much power as they can at the maximum price of 2000 EUR/MWh. The maximum production capacity then becomes the sum of the hourly bid at maximum price and the submitted block bids in an hour. This capacity can vary greatly, even in shorter periods of time such as a week, or even a day. Figure 3.2 shows variations in maximum capacities, with Figure 3.2a showing daily average maximum output over the weeks. Variations on hourly intervals for week 25-26 are shown in Figure 3.2b. There are a number of reasons for the observed variation. Producers commit production in other markets besides day-ahead spot as they see it best to reduce risk and maximize profits, selling for example bilaterally to power intensive industry, or setting aside capacity to the regulating capacity options market (RKOM) in hopes of high prices. Additionally, hydropower producers in Norway must deliver concession power at at a varying level. In certain periods a producer can experience reservoirs that are empty, or near-empty, to further disrupt the total output capacity. Maintenance work is another reason. Finally, the maximum output also depends on the head of water which varies with the reservoir level. All in all, these factors strongly affect producers' ability to deliver to Elspot. As we can see the deviations for all producers are quite high.



(a) Daily average max. capacity, all weeks (b) Hourly maximum capacity, week 25-26

Figure 3.2: Maximum production capacity to Elspot for producers A, B and C, showing high variations both within bi-weekly and daily time perspectives.

An indication of setting aside production capacity to RKOM is found when looking at Producer A's maximum capacity hourly bids for a winter week when the RKOM is at its most active. Over the week, Producer A's maximum hourly bid volumes were consistently higher outside the 05:00 to 23:00 time interval, the time when producers set aside capacity for RKOM. Table 3.4 gives Producer A's weekly average capacity in the time space within and outside of RKOM.

Table 3.4: Producer A having a higher maximum hourly bid capacity outside the RKOM time window, indicating that he sets aside capacity to produce for RKOM.

Time window	05:00 to 23:00	23:00 to 05:00
Weekly average capacity	273 MW	308 MW

Rounding off your power Producers have different practises when it comes to rounding off the volumes in their bids. Some producers choose to bid at 0.1 MW volume ticks, while other choose to round off their bids to half or whole MWs. When trying to bid optimally, it would seem best to submit as precise bids as possible to be able to fine-tune operations and thus squeeze out even more profits. Producer A bids at whole MWs except for some of its smaller power stations, while Producers B and C bid in tenths of MWs. Producer A explains the reasoning for submitting whole MW bids is that the precision in electricity production and grid delivery is rougher than to have measurability in

0.1 MWs. Thus it might be of little value to bid in volume ticks of 0.1 MW. This leads us to the question of how much can be gained by being able to produce and thus bid more precisely? For Producer B we did the opposite; how much would he lose, or gain, by rounding off his hourly bids? The table below shows the results of a simple analysis where we simply round off all the hourly bids submitted by Producer B for his water system for the 8 weeks in 2011. The rounding off affects the total yearly production, thus instead of looking at total income, we measure the success by looking at the average price received when rounding off or not. Not surprisingly, we see very little difference between two precision levels in bidding. The results show us a minuscule fall in the average price received of 0.002 NOK/MWh, or 0.005 %, bidding in whole MWs.

When rounding off, one must also consider the possible loss in efficiency. Assuming that a producer moves away from best point production when rounding off his bids to whole MWs, the expected difference between best point and the rounded value is 0.25 MWs, and is at most 0.5 MWs. Looking at Producer A's efficiency curves, we find that this usually represents a loss of efficiency of less than 0.05 %. Thus it constitutes a potential greater influence on the profitability than the average price variation, but still a perhaps negligable difference.

Table 3.5: Average price achieved for Producer B over 8 weeks when bidding at different volume precision levels, showing a slight loss of bidding at whole MWs.

Precision level	Whole MWs	Tenths of MWs
Income	16 976 948 NOK	16 975 690 NOK
Total output	416 924 MWh	416 874 MWh
Average price achieved	40.719 NOK/MWh	40.721 NOK/MWh

The rounding off comes more into play when producers bid for small power stations and round off. For Producer A, there are instances of rounding off and submitting bids in the size of less than 5 MWs. On the other hand, the less the size of the bid, the easier it becomes to smooth out the error in post commitment optimization and production reallocation, e.g. using SHOP.

3.2.2 Setting the right price points

Once a producer has established which volumes are sensible to bid, he must figure out the right prices to connect them to. The marginal water value mentioned in the introduction and Section 2 is a typical price point producers would want to bid their best point volume at. However that is necessarily not enough. The producers also have to account for the fact the Nord Pool Spot will interpolate their bids between consecutive price points as well as the fact that costs for feeding power onto the grid often are not included in the water values. Additionally, at sufficiently high price levels, producers will want to produce above best point, and thus have to consider the drop in efficiency relative to the increase in price points.

Avoiding and exploiting interpolation Most often, the spot price set by Nord Pool Spot will not equal any of the price points chosen by the producer. The exact hourly bid commitment will then be an interpolated value between volumes of the two neighboring price points. Interpolation between bid points can be unfavorable for the producers. Table 3.6 illustrates how Producer C bids in a certain manner to avoid volume interpolation. The price points in boldface show his marginal water values for four separate reservoirs when all marginal costs are accounted for. If the spot price exceeds these marginal water values Producer C will produce at exactly best point for the respective turbines, unless the spot price happens to land between the 1 NOK price gaps, thus interpolating the bids. This is accomplished by setting two price points at the smallest allowed interval apart combined with a sharp increase in volume. Notice also how the producer bids best point volume at the technical maximum price, implying that the turbine best point is calibrated to lie at maximum production.

Table 3.6: Producer C using neighboring price points to minimize the risk for interpolation of his hourly bids.

Hour	-2100	335	**336**	349	**350**	450	**451**	469	**470**	21000
10	27.9	27.9	27.9	27.9	-16.6	-16.6	-16.6	-16.6	-34.8	-34.8
11	28.0	28.0	28.0	28.0	-16.5	-16.5	-16.5	-16.5	-34.7	-34.7
12	27.7	27.7	27.7	27.7	-16.8	-16.8	-16.8	-16.8	-35.0	-35.0
13	27.8	27.8	27.8	27.8	-16.7	-16.7	-16.7	-16.7	-34.9	-34.9
14	27.8	27.8	27.8	27.8	-16.7	-16.7	-16.7	-16.7	-34.9	-34.9
15	27.4	27.4	27.4	27.4	-17.1	-17.1	-17.1	-17.1	-35.3	-35.3
16	26.8	26.8	26.8	26.8	-17.7	-17.7	-17.7	-17.7	-35.9	-35.9

Producers also purposely bid so as to control the interpolation between price points. Bids with noticeable gaps between price points and volume points can represent a linear approximation of the falling efficiency above best point. Table 3.7 below gives a real example of a bid matrix generated by Producer A for a single power station. The strategy in the bid matrix is to allow for interpolation whilst letting higher price levels weigh up for the loss of efficiency. Figure 3.3 further up illustrates this graphically. The efficiency η in Table 3.7 is relative to best point efficiency set at 1, and have been used to calculate the necessary price levels to compensate for respective efficiency loss of running above best point.

Table 3.7: Producer A setting strategic price points to allow and control interpolation, compensating for loss of efficiency above best point by using higher price points, single turbine.

Hour	-2100	258	371	420	21000
1	0	-28	-31	-35	-36
2	0	-28	-31	-35	-36
.	.	.	.	.	.
24	0	-28	-31	-35	-36
Efficiencies relative to best point, η	-	1	0.98	0.97	0.967
Price pts to compensate for η $(258/\eta)$	-	258	263	266	267

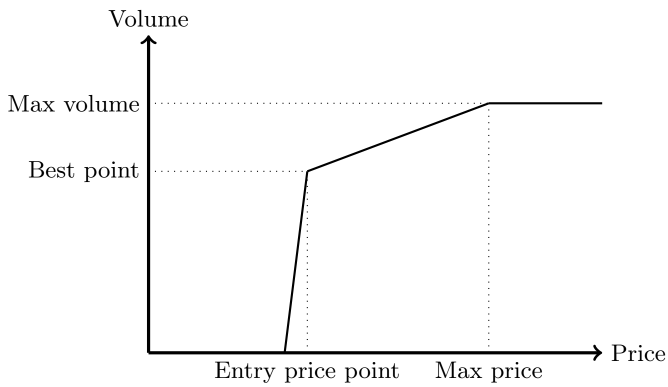


Figure 3.3: Graphical display of an hourly bid for a single turbine. The bid enters at the best point production level whereas the next price point hits at maximum production. The linearly increasing price between the two points should approximately weigh up for the loss of efficiency from best point to maximum production.

In Table 3.7 Producer A seems to want a disproportionally high premium to produce at maximum production. This sort of instance is found in most of the bid matrices for Producer A. The particular power station and reservoir in this example had a degree of filling over 90 %, along with the other reservoirs in the hydrological system. At this storage level, the turbines could run at full capacity for over 1700 hours without emptying the reservoir. Thus we find this bidding pattern even more peculiar. Even if the spot price should reach 10000 NOK/MWh the producer will be allocated closer to 35 than 36 MWhs, missing out on large profits however improbable. Producer A explains this by that they can have high price expectations to the more real-time Frequency controlled normal-run reserves market (FNR). Thus Producer A wants a large premium to dedicate all his production capacity to the spot market.

We find similar bidding patterns for Producer B, yet he charges a lesser premium to run at maximum capacity. For Producer C we find that he does not charge a premium over its best point production, simply because the maximum production level is at, or very close to, the best production for all its turbines (Table 3.6). This simplifies the bidding process and allows the producer to run at best point more frequently. Which output and inflow levels are best point is decided by the design and calibration of turbine and generator, but whether or not a design is optimal the producer in the day-ahead auction is a question we will not go into.

Table 3.8 shows us how many hours the realized volumes from the day-ahead were actually interpolated over the 8 weeks for producers A, B and C. For each of the 22 occurences of interpolation for Producer C, the spot price lies sandwiched in a 1 NOK gap between two neighboring price points, so the interpolation was clearly not intentional. Still, at 98 % of the time Producer C produces at or very near best point. Over a longer period of time undesirable interpolation is tough to avoid as long as the smallest price step allowed is 0.1 EUR/MWh, or 1 NOK/MWh. For producers A and B instances of interpolation are far more frequent, but determining the intentionality and general impact of this is more difficult than in the case of Producer C.

Table 3.8: Numbers and shares of interpolated and not interpolated bids realized over 8 weeks for Producer A, B and C.

Producer	A	B	C
# of interpolated hourly bids	777 58 %	917 68 %	22 2 %
# of not interpolated hourly bids	567 42 %	427 32 %	1322 98 %

Entry- and break-even price points The lowest price point at which the producer starts bidding to supply electricity is from here on referred to as the *entry price point*. Below this point the producer will be offering zero supply and perhaps submit demand bids to cover other commitments cheaply. The entry price point should consist of what the producer sees as his marginal cost of production, including the marginal water value, the feed-in fees, etc. Producers often let portions of the water in their reservoirs run through their power stations no matter what the price level. The producers set the marginal value of this water to zero. Thus if they can produce electricity from it at a price above their power stations' direct marginal cost, they will. If not, they simply let water run through while disconnecting the generators. The price point at which the producers thus bid for a break-even production is henceforth known as the *break-even price point*. This should also include all marginal costs, however when using this point the producer sees the marginal water value as zero. At any time a producer employs the break-even price point this should also be the entry price point, as a producer should never bid to produce below the break-even price point.

Dual water values The producers often bid at the break-even price point for water in flexible reservoirs. Our study gives no conclusive answers to why they do this, but in interviews they state it is most often due to the reservoir situation in combination with the weather forecast. Strangely, we find there is plenty of available reservoir storage capacity in some periods when the producer bids for power at the break-even price point. This implies that they see that a certain amount of water must under any circumstance be released from the reservoir. Yet, they value the remainder of the water in the reservoir at a higher price. Thus they can be seen as having two marginal water values, one at zero and one usually in the range of 100's of NOK/MWh. Table 3.9 shows an example of Producer A's bids for an individual power station where there seems to be two water values in play. As in Table 3.7, Producer A charges a high premium for higher volumes. This particular power station is situated at the bottom of a cascade, so there should be no incentive to release water simply to be able to produce further downstream.

Table 3.9: Producer A seemingly operating with two marginal water values for a single flexible reservoir, indicated from the use of the break-even price point as well as much higher price points. Producer A bids to let water through at the break-even price point of 30 NOK/MWh while bidding to produce more at much higher prices. The high premiums above 100 NOK/MWh are in line with Table 3.7.

Hour	-2100	29	30	159	160	230	231	400	21000
1	0	0	-74	-74	-110	-110	-150	-165	-170
·	0	0	-74	-74	-110	-110	-150	-165	-170
24	0	0	-74	-74	-110	-110	-150	-165	-170

Taking feed-in fees into account Some producers are more exposed to high feed-in fees and have to take the fee more into consideration when bidding than others. In certain areas during the winter season the variable part of the fee can be as high as 20 % of the system price. Others experience the variable part being close to 0 % year around. The variable part of the feed-in fee can thus comprise a significant part of the direct marginal cost of production, and have a great impact on the entry- and break-even price points. Other marginal costs comprising the entry price point, such as wear and tear, are more or less fixed. Hence, we should be able to observe the changing feed-in fee reflected in changing entry price points. However, even though the marginal loss rate used to calculate the feed-in fee is known, it is also dependent on the unknown system price the next day. Producers can therefore at best use their price forecasts for the next day to predict the variable feed-in fee. We use the same price forecast as Producer A had in hand when setting price points for the next day. The marginal loss rates are multiplied with the average price forecast, respectively between 06:00–22:00 and 22:00–06:00, to obtain forecasted variable feed-in fees for daytime and for the night. In this example, the change in the forecasted variable feed-in fee over a day is well reflected in Producer A's bidding for an individual power station, as seen in Table 3.10 below.

Table 3.10: Change in variable feed-in fee reflected well in Producer A's entry price points. The average forecasted change in the variable feed-in fee is very near the change in Producer A's use of entry price point.

	Day	Night	Difference
Approx. marginal loss rates	-5 %	20 %	25 %
Entry price points	280	215	**65.0**
Average variable feed-fee	55.5	-12.4	**67.9**

This example illustrates how a producer ideally should match the variable feed-in fee with the entry price point. Yet, this very often not the case for the producers in this study. The marginal loss rate can vary significantly from week to week and within any weekday, thus should we see a corresponding change in the entry price points the producers employ in their bid matrices. An example of where the change in the variable feed-in fee is not taken into account is presented below in Tables 3.11 and 3.12. In the example, Producer A bids to produce at 1 NOK/MWh, below the feed-in fee alone and thus below his break-even point. Neither does he not change the entry price point according to change in the marginal loss rate from hour 6 to 7. This will cause a direct loss to the producer if the spot price should land below 22.1 NOK/MWh. Most likely is the feed-in fee not taken well into account in the higher price points either, thus causing a loss taking the marginal water value into account. The bid can therefore be said to be irrational. To improve the bid, the producer can simply bid at lower price points at night and at higher price points during the day according to the changing feed-in fees.

Table 3.11: Average feed-in fees seen by Producer A in relation to Table 3.12.

	Day	Night
Approx. marginal loss rate	3 %	-3 %
Average variable feed-in fee	14.1	-12.9
Average total feed-in fee	22.1	-4.9

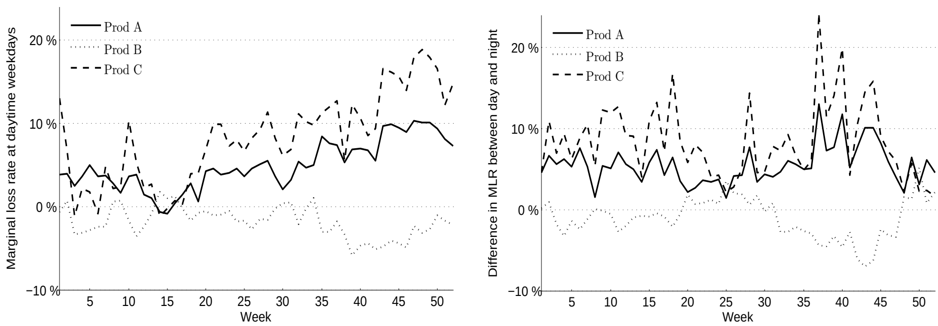
Table 3.12: Bid matrix showing irrational bidding as the entry price point (in boldface) is below the feed-in fee alone, shown in Table 3.11.

Hour	-2100	0	**1**	600	700	21000
1	0	0	-8	-8	-11	-11
·	0	0	-8	-8	-11	-11
24	0	0	-8	-8	-11	-11

We find that the producers utilize only a handful different break-even price points over the 8 weeks. To avoid confusion, we remind the reader here that when break-even price points are used, they will also act as entry price points. Producer B utilizes a single break-even price point of 0.6 EUR/MWh for every hour over the 8 weeks. Only for one day do we observe Producer C using a break-even price point outside the 44-50 NOK/MWh range. Producer A uses five different values, enabling him to better account for the variable feed-in fee. However, this indicates that all three producers employ estimates, or round-offs, on the feed-in fee. As the marginal loss rate changes within a weekday, so should the entry price point for the producer.

Figure 3.4a shows the marginal loss rates seen by three typical power stations, one for each producer, on weekdays in 2011. As we see, there is quite a difference between the three producers in the typical marginal loss rates they experience for their power stations. Producer C is very exposed, A is more moderately influenced, while B is very little affected by the marginal loss rates, and consequently the variable feed-in fee.

Figure 3.4b shows us the drop in the marginal loss rates from weekdays to weekend/nights for the same three power stations in 2011. Producer C often changes entry price point within the same day, and as we see from Figure 3.4b he absolutely needs to. Producer A is not quite as good as C in this aspect, only changing entry price points within a day when there is a very large difference in the marginal loss rate from day to night, usually at least 5 %. Producer B experiences quite stable marginal loss rates and understandibly does not see the same need to change his entry price points within a day. As we see from Figure 3.4b the difference between weekdays and weekend/nights is around zero year around.



(a) Marginal loss rates for the three producers on weekdays in 2011.

(b) Difference from weekdays to weekend and nights.

Figure 3.4: Weekday marginal loss rates for three power stations and the corresponding difference from weekdays to weekend and nights for 2011.

Utilization of price points Even though the water value in reality changes constantly, doing so would be unpractical and time consuming for producers. Though in theory, the more often a producer updates his water values, the greater is the possibility of mimicking the actual water values and thus bid more correctly and profitably. Producers operate with different practices for updating the value of the water in their reservoirs. On a minimum basis, the producers analyzed in this study update their water values each week. As the price points are very much influenced by the water value, producers should also update their price points when changing their water values. Producer B uses more unique price points per week and changes his price points more often, in some weeks from day to day, thus implying that he either explicitly or implicitly has changed his marginal water value. Producers A and C mainly stay with the same marginal water values for one week at a time, something which is again confirmed in the week-constant price points.

The utilization of price points is also connected to the variable feed-in fee. The producer most exposed to high and variable feed-in fees, Producer C, uses on average 50 % more price points during weekdays. The explanation is that the marginal loss rate stays constant over the weekend, while on weekdays it changes from day to night. Thus, Producer C has to take that change into account and use different price points accordingly. For Producers A and B we see no difference in the average number of price points from weekdays to weekend. However, in Figure 3.4b we see that Producer A is almost as exposed to large variations in the marginal loss rates from day to night. Similarly, Producer A should utilize a higher number of price points on weekdays then in weekends. Yet, he does not do so, even though he normally has a high number of spare price points to use.

We find a certain correlation between the number of price points used and the number of power stations they operate and bid for. On average over the 8 weeks, we find the number of price points used by the three producers to be close to 1 per power station. Naturally, the need for additional price points goes up with the number of power stations and reservoirs you control. Producers A, B and C all generate price points and bids for the individual power stations first, before aggregation towards the final hourly bid matrix. Producers with individual water values for each reservoir may want to use several unique price points for when bidding each power station. Thus if a producer control tens of power stations, the number of unique price points is quickly limited by the fact that NPS only allows 64 unique price points in the submitted bid matrix. Large producers therefore have to determine price points that can be used across several power stations, a possible challenge. Yet, even though the producers in the study are medium to large producers they rarely approach the limit in number of price points. We certainly find instances where the producer could add an additional price point to a bid matrix e.g. to avoid undesirable interpolation. See for example Table 3.7 where Producer A faces interpolation below best point for a spot price below 258 NOK/MWh.

Nonetheless, as NPS has set the maximum number of price points to 64, an interesting discussion, which we will not go into, is whether certain large producers could profit from submitting more than one matrix to allow for more unique price points per power station. The current fee to NPS is 15 000 EUR/year per bidding portfolio.

3.2.3 Choosing the bidtype

The choice between bidding hourly bids or block bids, with or without links, is perhaps the most complex decision the bidding responsibles have to deal with. And there is certainly no straight forward solution as to what gives the best outcomes. The reasoning behind allowing block bids is to give players with large start-up costs a predictable game plan for their production. An advantage mentioned by our set of partly block bidding producers is being able to set the exact best point production directly at the marginal water value. Simply put, an average spot price above the marginal cost gives the most efficient production, and a price below means no production. Such practise can be seen as rational and is a common way to submit block bids. For the hydro producers the start-up cost will not have that much of an effect, as it is a small cost relative to spot revenues. In reality the start-up cost will vary with the water value of spilled water, while the cost of dispatching an operator to the power station depends on the hour and type of day. However, all producers A, B, and C operate with fixed estimates on starting costs. They may vary slightly from station to station, yet average in the neighborhood of 400 EUR/start-up.

Furthermore, a common strategy is to combine hourly bids with block bids, submitting the block bid at best point and the marginal water value, while the hourly bids cover the production from best point to maximum production at higher price points. This is shown in an example below, taken from Producer A and shown in Table 3.13 and 3.14. As the hourly bids are priced above the block bid, they will most likely not be accepted should the block be rejected, thus avoiding an unfavorable production of only 2 MW.

Table 3.13: Producer A bidding a block to produce at best point, while using hourly bids for any production above (Table 3.14).

Start	Stop	Volume	Price	Block
00:00	07:00	-34	377	B0007

Table 3.14: Producer A submitting hourly bids for production above best point while using a block bid to produce at best point (Table 3.13).

Hour	-2100	370	412	450	500	21000
1	0	0	0	2	6	6
.	.	.	.	.	.	.
7	0	0	0	2	6	6

Producer A generally employs block bids when the price forecast for the next day is close to the marginal water value of a reservoir. Using only hourly bids set at the marginal water value, the producer must start or stop the turbines every time the spot price crosses the marginal water value. Thus the producer risks encountering several start-ups. By using block bids he can avoid this situation. Figure 3.5

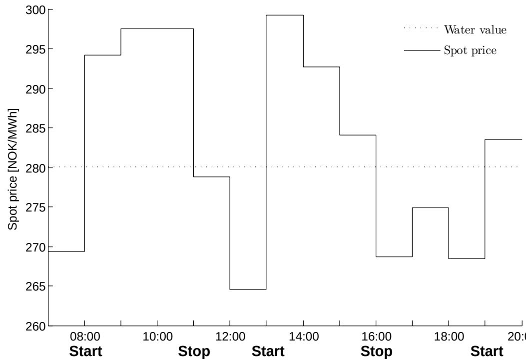


Figure 3.5: A fluctuating spot price crossing the marginal water value five times, causing several start-up and shut-down procedures if solely hourly bids are used.

shows a real example where Producer A submits a block bid for a power station connected to a reservoir with a marginal water value of 280 NOK/MWh for hours 8-20, as the price forecast for these hours averages at 287.43 NOK/MWh. In this particular case he avoids three start-ups and two shut-downs compared to if he had submitted solely hourly bids for the hours in question.

Nord Pool Spot gives participants the possibility of submitting up to 100 block bids per day, where the blocks can be any consecutive combination of minimum three hours. The latter implies there are $\sum_{n=1}^{22} n = 253$ possible combinations to choose from. However the producers rather use a few combinations of hours that give practical meaning with regards to peak and off-peak price hours, working shifts and feed-in costs, and stick to simple rules when making block bids. Blocks are usually submitted sequentially in time, with the overlap often taking place at the shift of the marginal loss rate at 06:00, or according to work shifts around 07:00–09:00. Table 3.15 shows a typical block bid submitted by Producer A, which directly relates to the marginal water value, best point production and feed-in fees for a certain power station.

Table 3.15: A typical set of block bids submitted by Producer A; successive in time, bidding at best point production while at marginal water value, the second block being linked to the first, and taking an increase in the marginal loss rate into account.

Start	Stop	Volume	Price	Marginal loss rate	Block	Linked to
00:00	07:00	-56	322	2 %	B0007	
07:00	24:00	-56	335	4 %	B0724	B0007

At 56 MW the turbine runs slightly above its best point, at 99.2 % of maximum efficiency. Producer A raises and lowers its bid price according to the variable feed-in fee, hence the variable price of the bids. If the variable feed-in fee is in the area of what the marginal water has taken into account, Producer A submits a block bid at the water value. If the variable feed-in fee is particurarly high or

low, Producer A changes the price point accordingly.

The second block bid, B0724, in the example in Table 3.15 is linked to the first block. Bidding this way gives the producer a higher expected commitment than if he were to block all 24 hours together, seeing that the first block still can be accepted alone. We find that only Producer A uses linked blocks among the three producers. Linked blocks can be useful for producers as they further extend the predictability block bids provide for producers. Linked blocks almost always lie connected to its mother block in time, either directly before or after. Most often it is also for the same power station, but not necessarily. The producer thus may want to deliver a constant output over time using two different power stations, and there might a waterway link between them.

In pure numbers, from a total of 459 blocks submitted by Producer A over the 8 weeks, 139 blocks, or 30 %, were accepted. 105 were linked to another block, but only 16 linked blocks were accepted. An additional 7 would have been accepted if not for their link. Furthermore, the 105 linked blocks had a total of 74 mother blocks, meaning each mother on average had 1.4 daughters. Table 3.16 summarizes the block bid data for Producer A.

Table 3.16: Block bid data over 8 weeks for Producer A, showing accepted and rejected blocks, number of linked bloks and accepted linked blocks and rejections due to their dependence on its mother block.

Total number of block bids	459
Accepted	30 %
Rejected	70 %
Total number of linked blocks	105
Number of mother blocks	74
Accepted linked blocks	16
Rejected due to link	7

In general, to bid in your entire capacity as a block bid might not turn out well. For a block bid that includes an hour with a price spike, the average spot price for the block hours may fall below the bid price. Thus the block bid is not accepted. However, by submitting the block bid, the producer has allocated capacity to produce if the block bid should be accepted, and will not have spare capacity enough to fully profit from the price spike through an hourly bid. An example of this regarding Producer B is shown below in Table 3.17 and Figure 3.6. Here the spot price averages to 33.54 EUR, thus declining the block bid at 34 EUR and inflicting Producer B a lost income opportunity of 1 446 EUR in hours 6 and 7. Assuming a start-up in hour 6 costing 400 EUR, the net foregone profit resulting from the price spike is 1 046 EUR. The lost profit is calculated as the spot price less the bid price multiplied with the block size. The bid price is thus assumed to be at the marginal water value including all marginal costs.

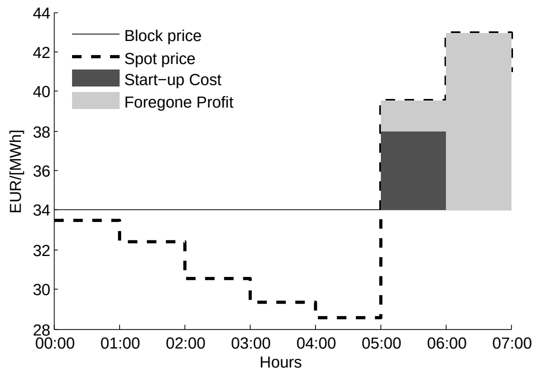


Figure 3.6: The allocated capacity to the rejected block bid causes a foregone profit of 1 046 EUR (Table 3.17), as Producer B fails to take full advantage of the price spike in hour 6 and 7 by using hourly bids.

Table 3.17: Producer B fails to capitalize fully on price spike (Figure 3.6) as production capacity has been allocated to the rejected block bid.

Block bid hours	1-7
Block volume	100 MW
Average spot price	33.82 EUR/MWh
Bid price	34 EUR/MWh
Foregone income hour 6,7	1446 EUR
Start-up cost hour 6	400 EUR
Foregone profit	1046 EUR

Share of bid types The application of bid types varies quite a lot between the three producers. Table 3.18 shows the volume share of the two main bidtypes submitted and realized over 8 weeks by the three producers. The hourly bids are the maximum hourly bid in each hour. The producers each submit a sizeable share of block bids, which drop quite a bit once the bids are actually realized in Elspot. Producer B submits 27 % of the total volume in block bids, but only 6 % are actually realized. Producer C is the most noteworthy in terms of the volume of block bids used, submitting 71 % of his capacity in the spot market in blocks. A total of 60 % was still realized and produced as block bids. Producer C gives no other reasons for submitting block bids than what we have already mentioned, mainly bidding blocks to avoid starts and stops. However, Producer C's largest power station comprises a sizeable portion of his total capacity and is almost exclusively bid in using blocks.

Table 3.18: Volume share of hourly bids and block bids submitted by and realized for Producers A, B and C over 8 weeks.

Producer	A	B	C
Hourly bids submitted	76 %	70 %	29 %
Block bids submitted	24 %	30 %	71 %
Hourly bids realized	87 %	94 %	40 %
Block bids realized	13 %	6 %	60 %

3.3 As good as it gets

To find the potential increase in income for the producers, we perform a simple analysis of a case where the producers are able to hit every price peak within each two-week period. In other words, we simulate having perfect price information for the entire period, and producing at maximum bidded capacity in the hours with the highest spot prices. Given the limited technical and hydrological data on two of the producers some simplifications were necessary. We assume the producers' capacity to deliver to the spot market is given by the sum of its hourly and block bids submitted for each hour. We aggregate the total MWhs of flexible electricity produced over each two week period, meaning we have subtracted bid volumes we consider less than fully flexible, i.e. bids at break-even price points. Thus the remaining MWhs should be 100 % flexible. These MWhs are then reallocated to the highest priced hours until the total amount is distributed, where we assume we achieve the same average efficiency of the turbines. This is of course a somewhat unfair analysis as it disregards both the start-up costs, the reservoir levels and the potentially lower efficiency achieved from always running at maximum capacity. Yet it does paint a picture, and to a certain degree, the analysis can give an indication of the producers' bidding performance. This is displayed in Table 3.19 below.

Table 3.19: Potential increase in actual realized income in each two-week period given complete knowledge of future price levels.

	Week 13-14	Week 25-26	Week 38-39	Week 51-52
Producer A	5.2 %	5.8 %	9.4 %	5.8 %
Producer B	1.2 %	7.3 %	11.1 %	3.2 %
Producer C	0.4 %	6.2 %	-	3.7 %
Average	5.4 %			

We see that the producers perform quite well, competing against *perfect* price information. The highest potential increase in income over the six periods is 11.1 %. On average over the 8 weeks, the potential increase for all producers was 5.4 % Over the 8 weeks, Producer A displayed a more stable performance than B and C. For Producer C in week 38-39 there was very low net trade due to low price levels, making this analysis less interesting. We observe that all producers on average performed best in week 13-14. Producers A and B had their weakest performance in week 38-39, and all had their second poorest results in week 25-26.. Clearly, there is some correlation between producers' performance. It appears to be easier to bid closer to optimality in the spring and winter weeks, than in the summer and autumn weeks. In Table 3.20 we have calculated the standard deviation of the area price relative to the average price in the respective periods. The price deviations indicate that the higher the volatility in prices, the more difficult it is to bid optimally and take advantage of the high prices in a period.

Table 3.20: Standard deviation of area price relative to average price in period, showing a certain correlation to the producers potential to increase income shown in Table 3.19.

	Week 13-14	Week 25-26	Week 38-39	Week 51-52
Producer A	8 %	23 %	41 %	28 %
Producer B	7 %	22 %	39 %	15 %
Producer C	6 %	23 %	41 %	28 %

As a final analysis in this subsection, we look at the producers' flexibility to submit large volume bids when desirable. Table 3.21 shows the number of hours the producers ran at the maximum bid capacity in each two-week period in the aforementioned potential income test. Assuming the producers want to produce as much as they can at the maximum price point, the maximum bid capacity is then the producers' hourly bid at maximum price added all block bids submitted in each hour. The number of hours run gives us an indication of the maximum production capacity relative to the size of the reservoirs and amount of inflow. A relatively large production capacity gives the producer more flexibility to bid large volumes at high price points. Thus gaining advantage of price peaks is possible, in preference to being forced to bid in a large volumes relative to power station capacity at low prices, simply to avoid flooding. On the other hand it also represents a lot of invested capital and higher maintenance costs. For a relatively smaller production capacity it implies the opposite; less tied up capital, but also less flexibility to exploit high price levels when bidding. From Table 3.21 we see how Producer A can submit large volume bids at high price point and can thus exploit high price levels when they occur. Producer C, however, is less flexible and is dependent on producing closer to maximum capacity more often. Producer B lies somewhere in middle between the other two. However, what are the optimal sizes of the turbines is a question we will not try to answer.

Table 3.21: Hours of running at maximum capacity over two weeks when all production is allocated to these hours, given as a percentage of all 336 hours in each two-week period. This gives an indication of the producers' flexibility for submitting large volume bids at high price levels.

	Week 13-14	Week 25-26	Week 38-39	Week 51-52
Producer A	29 %	54 %	27 %	38 %
Producer B	47 %	43 %	23 %	61 %
Producer C	82 %	79 %	-	65 %

4 Model for optimal bidding of hydropower

This section presents a model to optimize day-ahead bidding given price scenarios for a given period forth in time as well as reservoir levels at the period's start and end. The mathematical formulation of the model, with parameters, variables, constraints and objectives is given in the first Section 4.2. The results from running and empirical comparisons with the received bids will be presented in Section 4.4. First we give some background information and a few assumptions taken in the model.

4.1 Model background and assumptions

Stochasticity At the time of bidding, the prices for tomorrow as well as all future spot prices are unknown to the hydro producer. The same goes for the precipitation to come. There is, however, a correlation between tomorrow's prices and the prices to come, as well as between future prices and future rainfall. This model is formulated and implemented as a two-stage stochastic mixed-integer linear program. As opposed to a deterministic model where you model as if you know the actual outcomes of future events, a stochastic model explicitly accounts for the uncertainty through the use of scenarios with corresponding probabilities. We will refer to Shapiro and Philpott (2007) for a more detailed introduction to stochastic programming than what follows. A first stage decision is taken before you know the outcome of the stochastic parameters. Thus to assure that the first-stage decision does not anticipate the future, the parameters have to be independent of scenario output. The constraint that ensures this is called the non-anticipativity constraint. The variables that are decided after realization of stochastic outcomes are scenario dependent and known as recourse variables. Figure 4.1 illustrates one set of first stage variables, the scenario dependent realizations of stochastic parameters, as well as one set of the second stage recourse variables for this particular model.

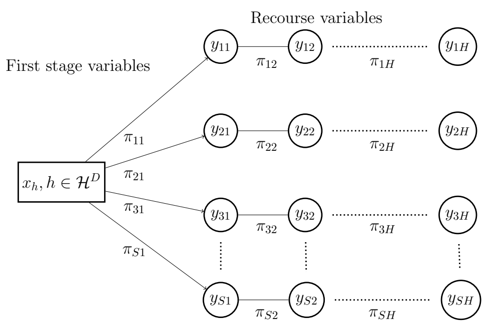


Figure 4.1: Scenario fan demonstrating how second stage recourse variable y_s is dependent on scenario s and first stage variable x. x is the bid volume, π_{sh} is the spot price for scenario s and hour h, while y_s is the volume commitment for scenario s. See Section 4.2 for details around the formulation and relation between the variables.

As the model is meant to support empirical studies on no longer time spans than 14 days, as opposed to actual decision support with longer time horizons, the inflow to the reservoirs is modeled deterministically. The model aims to use as much water as the empirical cases, while generating the maximum amount of income. The formulation that follows will model the prices as stochastic scenario series, scenario fans like the one illustrated in Figure 4.1, as opposed to multistage scenario trees. This implies that once you know the realization of the first 12-36 hours, second stage prices become deterministic. The motivation for such a formulation is that it allows the model to use more computer power and running time on the day-ahead price scenarios, compared to a formulation with continued scenario branching in the second stage. It is the stochasticity of tomorrow's prices that are of highest importance for the bidding. For the n'th day to come, the bidding decision will be taken at day $n-1$, for which you know the prices and thus have a better idea of what tomorrow's, n's, price will be. Note that the model easily can be run deterministically, through inputting the deterministic price forecast as the only scenario with probability 1.

Bid specifics The model will solely support supply bids in the form of regular hourly bids or block bids. The omitting of linked block bids and hourly flexible bids, as well as all demand bids, is related to the nature of the received bids and to assure the model gives more interpretable results. All bids are given positive volume values, since demand bids are not possible and it's easier to relate to positive numbers.

Notational conventions To help the readability of the model, indices are always defined by small single Latin letters. Sets have capital letters in a calligraphic font, with potential controlling subscripted index and/or superscripted letters to point out that it is a subset of special characteristics. Parameters are defined similar to sets, except with a regular font. All decision variables are lower case single Latin letters, with lower case controlling subscripts. For reference the units used on parameters and decision variables in the implementation are included in the descriptions that follow.

4.2 Mathematical formulation

Sets and indices The model has a generic formulation with regards to reservoirs, power stations and turbines. The following sets and indices are used consequently in the modeling.

$\mathcal{S}$:	Set of scenarios, indexed by s, sized to S
$\mathcal{H}$:	Set of all hours the model spans over, indexed by h, sized to H
$\mathcal{I}$:	Set of bidpoints, indexed by i
$\mathcal{B}$:	Set of possible blocks, indexed by b
$\mathcal{R}$:	Set of reservoirs, indexed by r
$\mathcal{K}$:	Set of power stations, indexed by k
$\mathcal{T}$:	Set of turbines, indexed by t
$\mathcal{E}_t$:	Set of efficiency segments for turbine t, indexed by e

Subsets The model is split up into one day-ahead part that is directly related to the bidding and one part for the coming days, that will not need bidding. Thus the hour resolution is also split up into two subsets corresponding to the two model types. The other subsets relate to the topography and positioning of reservoirs and turbines.

$\mathcal{H}^D \subseteq \mathcal{H}$:	Set of hours for the day-ahead bidding
$\mathcal{H}^L \subseteq \mathcal{H}$:	Set of hours for long-term part
$\mathcal{K}_r \subseteq \mathcal{K}$:	Subset of power stations that tap water from reservoir r
$\mathcal{K}_r^+ \subseteq \mathcal{K}$:	Subset of power stations directly above reservoir r
$\mathcal{T}_k \subseteq \mathcal{T}$:	Subset of turbines in power station k

Parameters The constants and coefficients given as parameters in the model are stated below. Notice that the price points for hourly bids and block bids are the same. Also note that the capacity of a turbine is time dependent.

π_{sh} :	Area spot price for scenario s in hour h [EUR/MWh]
ρ_s :	Probability of scenario s
P_i :	Price at bidpoint i [EUR/MWh]
P_{bi} :	Price for block b at bidpoint i [EUR/MWh]
B_b^{start} :	The first hour of block b
B_b^{end} :	The last hour of block b
B_{sb}^{ave} :	Average spot price for block b in scenario s [EUR/MWh]
W_{ht}^{cap} :	Maximum output from turbine t in hour h [MW]
W_t^{min} :	Minimum output from turbine t [MW]
E_{te} :	Efficiency for turbine t and segment e [MW]
E_{te}^0 :	Efficiency constant for turbine t and segment e [Wh/m^3]
Q_{hk}^{min} :	Minimum flow for power station k in hour h [Mm^3/h]
C_{shk}^{feed} :	Feed-in fee for station k in scenario s and hour h [EUR/MWh]
C_t^{start} :	Start-up cost for turbine t [EUR]
R_r^0 :	Initial reservoir level of r [Mm^3]

R_r^{max} : Maximum reservoir level in r [Mm^3]
R_r^{min} : Minimum reservoir level in r [Mm^3]
R_r^{end} : Final end-of-period reservoir level in r [Mm^3]
F_{hr} : Inflow to reservoir r in hour h [Mm^3/h]

Decision variables The variables can be split up into two groups, the ones related to the bids and the ones related to the actual water flow. The x-variables are the sole variables not dependent on scenario, and thus the only first-stage variables. All other variables are second-stage recourse variables.

x_{hi} : Volume bid in hour h at bidpoint i [MW]
$\hat{x}_{bi}$: Volume bid for block b and bidpoint i [MW]
y_{sh} : Commitment for hourly bids in scenario s and hour h [MW]
$\hat{y}_{sb}$: Commitment from block bid b in scenario s [MW]
w_{sht} : Power output from turbine t in scenario s and hour h [MW]
q_{sht} : Flow through turbine t in scenario s and hour h [Mm^3/h]
$\hat{q}_{shk}$: Flow through power station k in scenario s and hour h [Mm^3/h]
l_{shr} : Reservoir level of r in scenario s and hour h [Mm^3]

$$\gamma_{sht} = \begin{cases} 1, \text{ if turbine } t \text{ is running in scenaro } s \text{ and hour } h \\ 0, \text{ otherwise} \end{cases}$$

$$\delta_{sht} = \begin{cases} 1, \text{ if turbine } t \text{ starts up in scenaro } s \text{ and hour } h \\ 0, \text{ otherwise} \end{cases}$$

Objective function The objective maximizes the total revenues from day-ahead and the period to come, less costs associated with start-ups and feed-in fees.

$$\mathcal{Z}^{dh} = \sum_{s \in \mathcal{S}} \rho_s \left(\sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}_k} \left(\sum_{h \in \mathcal{H}^D} w_{sht} \left(\pi_{sh} - C_{shk}^{feed} \right) - \delta_{sht} C_t^{start} \right) \right) \tag{4.1}$$

$$\mathcal{Z}^{long} = \sum_{s \in \mathcal{S}} \rho_s \left(\sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}_k} \sum_{h \in \mathcal{H}^L} w_{sht} \left(\pi_{sh} - C_{shk}^{feed} \right) \right) \tag{4.2}$$

$$\max \mathcal{Z} = \mathcal{Z}^{dh} + \beta \times \mathcal{Z}^{long} \tag{4.3}$$

Subject to:

$$x^h_{h(i-1)} \leq x^h_{hi}, s \in \mathcal{S}, h \in \mathcal{H}, i \in \mathcal{I}\backslash\{1\} \tag{4.4}$$

$$\begin{gathered} y_{sh} = x^h_{h(i-1)} + (\pi_{sh} - P_{i-1}) \times \frac{x^h_{ih} - x^h_{h(i-1)}}{P_i - P_{i-1}}, \\ s \in \mathcal{S}, h \in \mathcal{H}^D, i \in \mathcal{I}\backslash\{1\} | P_{(i-1)h} \leq \pi_{sh} \leq P_{ih} \end{gathered} \tag{4.5}$$

$$\hat{y}_{sb} = \sum_{i \in \mathcal{I} | P_{bi} \leq B^{ave}_{sb}} \hat{x}_{bi}, \qquad s \in \mathcal{S}, b \in \mathcal{B} \tag{4.6}$$

$$\sum_{b \in \mathcal{B} | B^{start}_b \leq h \leq B^{end}_b} \sum_{i \in \mathcal{I}} \hat{x}_{bi} + x_{hI} \leq \sum_{t \in \mathcal{T}} W^{cap}_{ht}, \qquad h \in \mathcal{H}^D \tag{4.7}$$

$$\sum_{t \in \mathcal{T}} w_{sht} = y_{sh} + \sum_{b \in \mathcal{B} | B^{start}_b \leq h \leq B^{end}_b} \hat{y}_{sb}, \qquad s \in S, h \in \mathcal{H}^D \tag{4.8}$$

$$w_{sht} \leq \gamma_{sht} W^{cap}_{ht}, \qquad s \in \mathcal{S}, h \in \mathcal{H}^d, t \in \mathcal{T} \tag{4.9}$$

$$w_{sht} \geq \gamma_{sht} W^{min}_{t}, \qquad s \in \mathcal{S}, h \in \mathcal{H}^d, t \in \mathcal{T} \tag{4.10}$$

$$\delta_{sht} \geq \gamma_{sht} - \gamma_{s(h-1)t}, \qquad s \in \mathcal{S}, h \in \mathcal{H}^d\backslash\{1\}, t \in \mathcal{T} \tag{4.11}$$

$$\delta_{s(1)t} \geq \gamma_{s(1)t} - \gamma_{s(24)t}, \qquad s \in \mathcal{S}, t \in \mathcal{T} \tag{4.12}$$

$$w_{sht} \leq W^{cap}_{ht}, \qquad s \in \mathcal{S}, h \in \mathcal{H}^l, t \in \mathcal{T} \tag{4.13}$$

$$w_{sht} \leq E^0_{te} + E_{te} q_{sht}, \qquad s \in \mathcal{S}, h \in \mathcal{H}, t \in \mathcal{T}, e \in \mathcal{E}_t \tag{4.14}$$

$$\hat{q}_{shk} = \sum_{t \in \mathcal{T}_k} q_{sht}, \qquad s \in \mathcal{S}, h \in \mathcal{H}, k \in \mathcal{K} \tag{4.15}$$

$$\hat{q}_{shk} \geq Q^{min}_{hk}, \qquad s \in \mathcal{S}, h \in \mathcal{H}, k \in \mathcal{K} \tag{4.16}$$

$$l_{s1r} = R^0_r - \sum_{k \in \mathcal{K}_r} \hat{q}_{s1k} + F_{1r} + \sum_{k \in \mathcal{K}^+_r} \hat{q}_{s1k}, s \in \mathcal{S}, r \in \mathcal{R} \tag{4.17}$$

$$\begin{aligned} l_{shr} = l_{s(h-1)r} - \sum_{k \in \mathcal{K}_r} \hat{q}_{shk} + F_{hr} \\ + \sum_{k \in \mathcal{K}^+_r} \hat{q}_{shk}, s \in \mathcal{S}, h \in \mathcal{H}\backslash\{1\}, r \in \mathcal{R} \end{aligned} \tag{4.18}$$

$$l_{shr} \leq R^{max}_r, \qquad s \in \mathcal{S}, h \in \mathcal{H}, r \in \mathcal{R} \tag{4.19}$$

$$l_{shr} \geq R^{min}_r, \qquad s \in \mathcal{S}, h \in \mathcal{H}, r \in \mathcal{R} \tag{4.20}$$

$$l_{s(H)r} = R^{end}_r, \qquad s \in \mathcal{S}, r \in \mathcal{R} \tag{4.21}$$

$$x_{hi}, \hat{x}_{bi}, y_{sh}, \hat{y}_{sb}, \bar{y}_{sh} \geq 0, \qquad s \in \mathcal{S}, h \in \mathcal{H}, i \in \mathcal{I}, b \in \mathcal{B} \qquad (4.22)$$

$$w_{sht}, q_{sht}, \hat{q}_{shk}, l_{shr} \geq 0, \qquad s \in \mathcal{S}, h \in \mathcal{H}, k \in \mathcal{K}, t \in \mathcal{T}, r \in \mathcal{R} \qquad (4.23)$$

$$\gamma_{sht}, \delta_{sht} \in \{0, 1\}, \qquad s \in \mathcal{S}, h \in \mathcal{H}, t \in \mathcal{T} \qquad (4.24)$$

The objective function, (4.1) to (4.3), sums up the income from all scenarios and multiplies it with the scenario probabilities, both for the day-ahead part and for the long term part. It also subtracts the costs associated with feed-in fees and start-up costs for the turbines. The latter would naturally push production away from day-ahead towards the rest the period, such as to avoid the costs of binary start-up variables. To make up for this shift we include a factor β, calibrated to compensate for this through assuring an equal output for all days, relative to the price forecast. The β adjustment may also justify for poorly balanced price scenarios, which can occur if the model is run for a small number of scenarios. An alternative formulation of the objective could take the total water value of reservoirs into account as well, similar to Fleten et al. (2011). However, we do not possess the representative end-term water values.

Constraint (4.4) is simply a rule given from Nord Pool that makes their problem easier to solve. It states that all bids have to be strictly non-decreasing, thus making sure a producer cannot bid totally stepwise constant bids, as discussed in Section 3.2.2. It also prohibits a decreasing hourly bid volume with rising prices, which otherwise might occur if block bids are bid in at a certain price or if participants are in possess of market power. Due to the previous constraint (4.4) the interpolation to the correct committed volumes is done as easily as in (4.5). Setting the commitment for each scenario based on scenario-independent bid variables also function as the non-anticipativity constraint of the stochastic model.

Equation (4.6) commits production from block bids if the price is below the average realized spot price. Constraint (4.7) makes sure the model never bid such that total volumes from hourly bids and block bids are greater than the combined turbine capacity in any hour. The sum of production in all turbines have to equal total commitment from hourly bids and block bids, expressed through (4.8).

Constraints (4.9) to (4.11) set the binary variables, while (4.12) says that hour 24 is related to hour 1. The latter states that if the turbine is not running in hour 24, then it needs to start to be able to run in hour 1. This is included to discourage the model from doing more start-ups in the earlier hours of the day than the later hours. And it is not so farfetched, seeing that if a turbine is running one night, it is not unlikely that it will run the next night as well.

A turbine cannot deliver more power than its capacity, (4.13). The conversion from output power to water flow through the turbine is simplified through the linearizations of efficiency in equation (4.14). Approximations e_t of the conversion rate from water flow to output power are given through the Y-axis intercept at E^0_{te} and a slope of E_{te}. See the next subsection 4.3 for an example of these linearizations. Constraints (4.15) and (4.16) sum the flow through all turbines in power station k and bounds it to be equal to or higher than an hourly dependent minimum flow.

(4.17) to (4.20) control the reservoir levels and (4.21) states how the reservoir levels in the final hour have to equal the input end-of-period reservoir levels. Notice how there is no explicit modeling of potential spill over reservoirs. As we will not analyze spill any further, it would only enter the model as an increase in the upwards unbounded q variable.

4.3 Input parameters

This subsection will elaborate on the generation of input parameters for the model runs that are included in the results.

Price scenarios We have received historical day-ahead price forecasts from SKM Market Predictor AS for all the days in question, denoted by $\pi_h, h \in \mathcal{H}^d$. Price scenarios have been constructed as a normal distribution based around these forecasts with a standard deviation σ_h equal to that of the area spot price for the respective hour 40 weekdays or 16 weekend days back in time. For the first day-ahead hour all scenarios will be normally distributed around the price forecast. Given that in a scenario s the price π_{sh} misses the forecast for hour h by

$$\Delta_{sh} = \pi_{sh} - \pi_h, \qquad s \in \mathcal{S}, h \in \mathcal{H}, \tag{4.25}$$

then the expectation for hour $h+1$ in same scenario s will be

$$\mathbb{E}[\pi_{s(h+1)}] = \pi_{h+1} + \frac{\Delta_{sh}}{\sigma_h}\sigma_{h+1}, \qquad s \in \mathcal{S}, h \in \mathcal{H} \backslash \{H\} \tag{4.26}$$

Thus we have

$$\pi_{s1} = \pi_h + \phi^{-1}(z_{s(1)}) \times \sigma_h, \qquad s \in \mathcal{S}, z_{s(1)} \in \mathcal{U}(0,1) \tag{4.27}$$

$$\pi_{sh} = \mathbb{E}[\pi_{sh}] + \phi^{-1}(z_{sh}) \times \sigma_h, \qquad s \in \mathcal{S}, h \in \mathcal{H} \backslash \{1\}, z_{sh} \in \mathcal{U}(0,1) \tag{4.28}$$

where the cumulative distribution function is

$$\phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp^{-x^2/2} \, dx \tag{4.29}$$

Since we only possess day-ahead forecasts, the forecast for days to come, $\pi_h, h \in \mathcal{H}^L$ have been assumed to equal the day-ahead forecast, $\pi_h, h \in \mathcal{H}^D$, for the remainder of the period, adjusting to weekends and weekdays according to average weekend versus weekday ratios for 2010. When going several days forward, we have also used a steadily increasing weight towards the forecasts to make sure the scenarios do not go way out of hand. An example of 500 generated price scenarios for day-ahead is displayed in Figure 4.2. Notice how one can easily recognize the actual price forecast, and the spread in scenarios being wider in the last than the first hour.

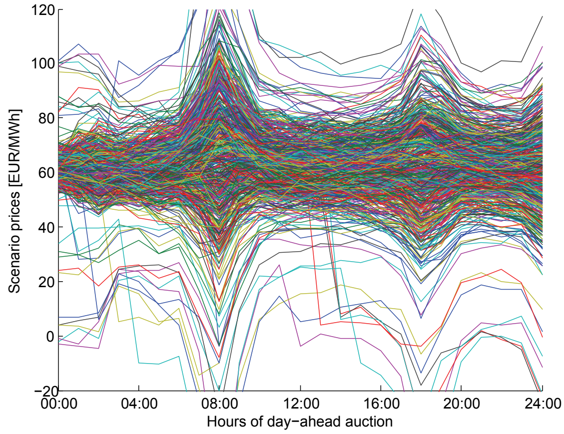


Figure 4.2: Plot of 500 generated price scenarios for day-ahead NO3 spot prices, April 5th 2011. Notice the easily recognizable price forecast, and the spread in scenarios being wider in the last than the first hour.

Efficiency curves The input to the efficiency curves, modeled through constraint (4.14), is based on the producers' records of measured water flow versus power output. The need for a linear formulation is taken care of by linearly approximating the efficiency curves for each turbine. The efficiency curve is usually concave within the turbine's operating range. Using several lines e consisting of a y-axis intercept E^0_{te} and a slope E_{te} to model each curve, good approximations of the efficiency curves are obtainable for each individual turbine t. The slope of the line stretching from origo represents the best point conversion rate. This is demonstrated above in Figure 4.3. Each turbine is also modeled with a minimum output W^{min}_t, which in the figure is where the thick unbroken curve begins.

Feed-in fees When running for Monday through Thursday the marginal loss rates, C^{mlr}_{hk}, in the model are assumed to be the same for the coming week as in the current week, while from Fridays the exact rates for coming week are used. The latter is consistent with actual information flow. The fixed part of the feed-in fee will always be set to 8 NOK/MWh, and can potentially be excluded from the following if only the variation is of interest.

$$C^{feed}_{shk} = \pi_{sh} \times C^{mlr}_{hk} + C^{fixed}, \qquad s \in \mathcal{S}, h \in \mathcal{H}, k \in \mathcal{K} \tag{4.30}$$

Price points The price points in the model are set somewhat pragmatically using efficiency curves and actual water values when such are available, while also making sure the density of price points around likely price outcomes is higher than other areas of the total price range. If the user needs to use certain extreme price points and still prefers converging and interpretable results, it is also necessary to make sure that all neighboring price points get a scenario price in between. Otherwise, the model will be indifferent as to whether bidding there or not. The probabilities for these specific scenarios can however be set to go towards zero.

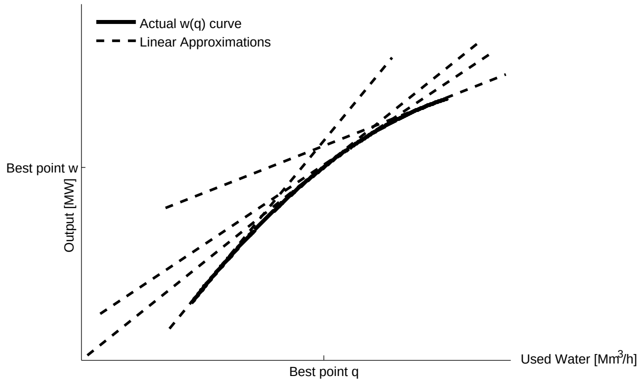


Figure 4.3: Linear approximations e_t of efficiency curve formulated as conversion from water flow to output power. Y-axis intercept at E^0_{te} and slope of E_{te} from equation (4.14).

4.4 Implementation and results

For testing the model it is implemented in the Xpress-Mosel modeling language with the Xpress-IVE environment and solved through the use of Xpress-MIP Optimizer 64-bit v3.2.3, all parts of the Xpress Optimization Suite version 7.2.1 from FICO. The computer that has been running the model has an Intel® Core™ i7-2600 (4 x 3.4GHz) Processor and 16 GB of RAM. This has been sufficient for the testing we performed, and in general there has been little focus on minimizing solution time for the model. Typical solution time lies around 1000 seconds for 300 scenarios and 336 hours, i.e. two weeks. The input with regards to bids, reservoir levels, inflow, and technical specifications are taken from Producer A and implemented in the model for testing. All print results, and most of the testing, have been done on a hydrological system with 4 reservoirs and 5 power stations. The goal for the testing is to quantify findings from Section 3.2 as well as to find whether there is need for an optimal bidding model to solve at all.

The Mosel implementation in its simplest form is digitally attached as the "BiddingRevealer.mos" and "BiddingRevealer.htm" for reference. However note that no input data are provided, due to the nondisclosure agreements made with the producers.

In 4.4.1 the model is run for two consecutive days, 25.09 and 26.09, binding the total water usage per power station to equal that of the actual bids. The bids generated are tested and compared towards the actual bids both on a set of price scenarios and on actual spot prices.

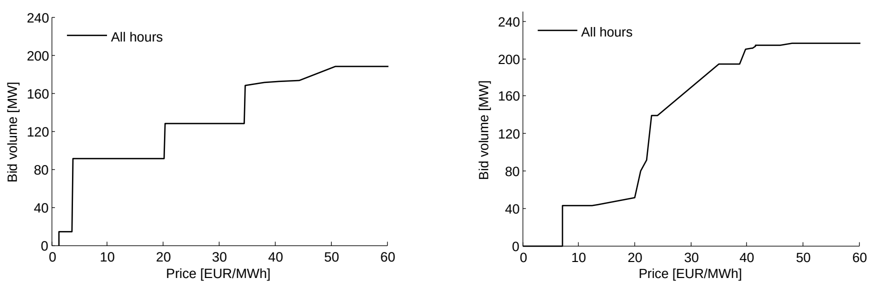
4.4.1 Comparison of generated bids to actual

To test the model against the actual bids we have limited the time span to two days. We have used the actual bids and efficency curves to calculate actual water usage for each of the five power stations. These usage values have been input as Q^{usage}_{kH} to the model, where constraint (4.21) that binds the end-of-period reservoir levels has been replaced by:

$$\sum_{h\in\mathcal{H}} q^{K}_{shk} = Q^{usage}_{kH}, \qquad s \in \mathcal{S}, k \in \mathcal{K} \tag{4.31}$$

The possible blocks in the model have been limited to the same hours as used by Producer A, and the natural inflow have been set to zero. The results from running the model for two days are first shown individually in the following.

25.09.2011 - Actual bids and generated bids from the model
The aggregated actual power station bids and the model-generated bids are displayed below in Figure 4.4, respectively. The bids generated came out a little noisy, but rounded they can be said to be equal for all hours. The price axes have been cut from 2000 EUR/MWh to make the figures readable. Worth noting first and foremost is the difference in entry price points. Whereas the actual bids place the first volume of 14MW already at 1.4 EUR, the model generates its first bid volume of 40MW at 7.20 EUR. The marginal loss rate for the likely power station with a capacity of 15 MW, is 9.4 % this weekend. This implies a total feed-in cost of $C^{feed} = \pi \times C^{mlr} + C^{fixed} = 1.4 \times 0.094 + 1.1 = 1.23$ EUR/MWh produced if the spot price turns out to be 1.4 EUR. Most likely this reservoir has a marginal water value above 0.17 EUR/MWh, and thus the bid is irrational. The generated bids are not that easily split up into power station bids, and thus the same analysis can not be done. However through searching the model for a scenario price neighboring 7.20 EUR, we see that the total feed-in costs are 67.04 EUR, implying an average average realized feed-in cost of 1.68 EUR/MWh. This implies a water value of 5.52 EUR/MWh. And with all reservoirs being far from full, and an average price forecast of 24.9 EUR, this too is irrational bidding. However, due to the binding water flow constraint (4.31), this sort of instance might occur.



(a) Actual hourly bids, 25.09.2011

(b) Generated hourly bids, 25.09.2011

Figure 4.4: Bids for Sunday 25.09.2011 from Producer A and the bidding model, respectively. The price axes have been cut from 2000 EUR/MWh to make the figures readable.

The figures also show that the model generates a higher hourly bid at maximum price. This comes from the fact that in addition to the hourly bids, the producer has also submitted block bids for this day, given in Table 4.1. The model has not used any block bids, so adding in block volumes, the two bids equal approximately at a maximum volume of 217 MW. However the actual bids only max to 217 MW for the first 9 hours of the day. We can not find any good reasons why the bid volumes should drop to 215 MW for the remainder of the day, and thus see it as irrational bidding behavior.

Table 4.1: Actual block bids of 25.09.2011 for the hydro power system in question. Notice how the three block bids do not equal in volume, even though the hourly bids are constant throughout the day.

Start	Stop	Volume	Price	Block
00:00	09:00	-23	33.5	B0009
09:00	20:00	-21	35.5	B0920
20:00	24:00	-21	35.5	B2024

Actual and generated bids input in a model run with stochastic prices
Through an average day the price will reach neither 1.4 nor 2000 EUR/MWh. Thus we run the actual bids through the same model input that generated the comparing bids, but with new price scenarios, to see how they perform under stochasticity. The results are displayed in 4.2. The actual bids end up with a little less output, but not enough to justify the loss in spot income. The average price achieved is 24.95 EUR/MWh, compared to the generated bids' 26.79 EUR/MWh. We also see that the start-up costs are greater for the actual bids than the generated ones, even though the former use block bids. This can interpreted to the fact that the model optimizes bidding for all five stations combined, whereas the actual bids have been constructed as bids for the individual stations. When the model then optimize, given the bids as input, nothing in the model specifies that block bid A belongs to station A etc., and thus the model might find other production allocations which yield other cost allocations.

Table 4.2: Running results from inputting bids as parameters to the model with 300 price scenarios. Note that the price scenarios are not the same that generated the bottom row bids. All numbers are averaged from the scenario realizations. Output is given in MW, and the other numbers in EUR.

Bids	Output	Spot income	Start-up costs	Feed-in costs	Profit
Actual	3 803	94 882	1 036	3 768	90 078
Generated	3 832	102 654	706	3 974	97 973

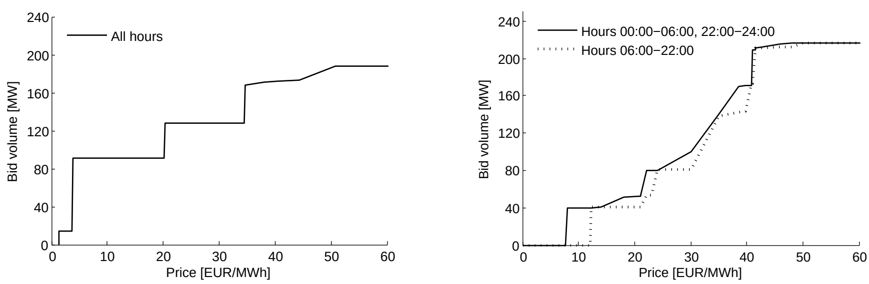
...and with actual spot prices
For reference we also include a similar table with a one-scenario run-through of actual spot prices, Table 4.3. We see that the generated bids still outperform the actual bids. The realized spot prices for 25.09 turned out to be quite a lot higher

than the forecast and thus the resulting numbers go up. The start-up costs are assumed to be equal 400 EUR/start-up for all turbines, thus the number of start-ups are easily recognized. Still, even though the empirical block bid is accepted, the generated bids achieve less start-ups through using hourly bids only.

Table 4.3: Running results from inputting bids as parameters to the model with actual spot prices for 25.09.2011. Output is given in MW, and the other numbers in EUR.

Bids	Output	Spot income	Start-up costs	Feed-in costs	Profit
Actual	3 895	118 775	1 200	4 567	113 008
Generated	3 944	120 557	800	4 637	115 120

26.09.2011 - Actual bids and generated bids from the model The resulting reservoir levels from 25.09.2011 was input as starting reservoirs for 26.09.2011. The aggregated actual power station bids as well as the bids generated by the model, are displayed below in Figure 4.5. The actual hourly bid from Producer A is identical to the bid from 25.09.2011. The bids generated on the other hand are split in two periods, displayed as the stipled line plot for 06:00–22:00 and the unbroken line for the other 8 hours. Now notice how the generated bids clearly differ in entry price points for the two plots. This perfectly reflects the fact that 26.09.2011 was a Monday and the marginal loss rates now vary intraday. The unbroken line consequently enters at an earlier price than the stipled line which implies a lower feed-in cost. This makes perfect sense, seeing that all the power stations in question had a lower marginal loss rate for the night hours than the day hours.



(a) Actual hourly bids, 26.09.2011 (b) Generated hourly bids, 26.09.2011

Figure 4.5: Bids for 26.09.2011 from from Producer A and the bidding model, respectively. The price axes have been cut from 2000 EUR/MWh to make the figures readable.

The figures show that the model still generates a higher hourly bid at maximum price. The difference in volumes are made up for through the first of the block bids given in Table 4.4. However, the irrationality continues also here as the volumes for the other blocks do not equal the first. The change from 09:00 to 07:00 in the bidded blocks reflect the change from weekend to weekday, which again can relate both to peak hours for prices and the actual work shifts of the power stations.

Table 4.4: Block bids at 26.09.2011 for the hydro power system in question. The volumes blocked are not equal for all hours.

Start	Stop	Volume	Price	Block
00:00	07:00	-23	33.5	B0009
07:00	20:00	-22	35.5	B0920
20:00	24:00	-21	35.5	B0920

Actual and generated bids input in a model run with stochastic prices
Once more, we run the actual bids through the same model input that generated the comparing bids, but with new price scenarios. The results on how the bids deal with new stochasticity are displayed in Table 4.5. Again the generated bids do better than the actual ones. It should be this way though, seeing that the optimal bids were generated using the same model and a price scenario set with the same properties. The average price achieved is 27.55 EUR/MWh, compared to the generated bids' 28.09 EUR/MWh. The start-up costs however, are in fact higher for the received bids than for the generated ones. The only conlusion we draw from this is that the model does not weigh start-up costs very heavily, neither should it, seeing that the estimated and used cost per start-up of 400 EUR is less than 0.4 % of the spot revenues.

Table 4.5: Running results from inputting bids as parameters to the model with 300 price scenarios. Note that the price scenarios are not the same that generated the bottom row bids. All numbers are averaged from the scenario realizations. Output is given in MW, and the other numbers in EUR

Bids	Output	Spot income	Start-up costs	Feed-in costs	Profit
Actual	3 762	103 633	1 744	4 116	97 773
Generated	3 799	106 731	2 001	4 239	100 491

...and with actual spot prices
A similar table with a one-scenario run-through of actual spot prices is shown in Table 4.6. Now the actual bids gives way more output than the generated ones. What happens is that the actual bids naturally hits the exact water usage values per station as they were set based on the bids and efficency curves. The generated bids on the other hand now used a too high price forecast in generating new price forecasts, so that when the spot realizes way below forecast the commitments become way too low. The model hits the water usage per station, as it must, but does it through spilling whatever water it cannot produce due to constraint (4.8). The average realized spot prices for 26.09 are 36.7 EUR/MWh and 37.0 EUR/MWh, respectively for actual and generated bids. The generated bids now do one more start-up, and we conclude that comparing actual bids to the model bids and getting unambiguous results is easier said than done.

Table 4.6: Running results from inputting bids as parameters to the model with actual spot prices for 26.09.2011. Output is given in MW, and the other numbers in EUR

Bids	Output	Spot income	Start-up costs	Feed-in costs	Profit
Actual	4 113	146 738	1 200	5 827	139 711
Generated	3 270	121 021	1 600	4 657	116 367

4.4.2 Stochastic versus deterministic runthrough

In Section 3.3 we showed that Producer A could have increased their week 13–14 period income by 5.2 % given complete knowledge of future price levels. Running a similar test for the single five-station cascade shows a 1.6 % potential in increased income. In the following, we test to see how much profit the model can realize through the same period, testing both with a 300-scenario model and a 1-scenario deterministic model with price forecasts. The model is run interatively for each day, first with price forecasts and free bid variables, then with actual prices and fixed bid parameters. The first day of running actual reservoir levels are used as initial levels, while constraint (4.21) binds the end-of period reservoir levels in hour $H = 336$. This initial run will generate bids that are put back into the model again, but now with actual prices for the first day. Now, the reservoir levels at $h = 24$ from this fixed-bid real-prices run are given as input to the next day's model run, where all other parameters are updated as the time span H is reduced by 24 hours. This way we run through the model a total of 14 times, every other time being with fixed bids and real prices. 64 price points are used and 40 different blocks are possible to bid at every day, and every stochastic run uses 300 price scenarios based around the day-ahead forecast.

Table 4.7: Results from running an iterative 14-day comparison between the stochastic and deterministic model.

	Stochastic			Deterministic		
	Average	**Total**	**St.dev**	**Average**	**Total**	**St.dev**
Price forecast	60.55	-	6.3 %	60.55	-	6.3 %
Actual spot price	60.52	-	6.2 %	60.52	-	6.2 %
Spot income	234.2	3 280	11 %	230.2	3 222	24 %
Feed-in cost	1.6	22.8	34 %	2.0	28.6	127 %
Start-up cost	1.3	18.1	8.3 %	1.0	14.0	60 %
Profit	231.3	3 239	13 %	227.1	3 180	22 %
# unique volumes	97	-	35 %	15	-	22 %
# unique price	49	-	14 %	11	-	33 %
# block bids	24	-	36 %	0	-	-
# blocks used	15	-	28 %	0	-	-
Avg. bids per block	2	-	17 %	0	-	-
Avg. vol. per block	18	-	66 %	0	-	-
Tot. vol. bid as block	1 832	25 648	31 %	0	-	-
Block commitment	1 399	19 592	38 %	0	-	-

The results in Table 4.7 show that total profit over 14 days is 1.9 % greater for the stochasticity-generated bids. Such a small number and with only one run-through is not enough to draw any conclusions. Yet we would like to point out our suspicion that the bidding design is such that we see the need to model short term price uncertainty to that great an extent. The stochastic model ends up realizing higher total start-up costs than the deterministic model. A possible explanation for this is again the fact that the stochastic model fixates a lot of total bid volumes to block bids, which may not be accepted in the deterministic real price run. In fact 23.6 % of the block volumes are rejected. Thus the model may need to shut down turbines as their volume was set aside for block production. The huge variance in daily feed-in costs can be explained a lot by the two weekends present. Excluding the weekends gives standard deviations of 15 % and 13 % for the stochastic and deterministic model, respectively.

Also worth noticing from Table 4.7 is the relatively higher standard deviation in the deterministic model run. The actual income increase compared to reality was minuscule 0.4 % and -0.2 %, for the stochastic and deterministic run-throughs respectively. We believe the improvement would have been higher if the prices and price forecasts had not been so stable. Another interesting result is that doing the same run for 7 days without allowing block bids gave just about the same profit, +0.4 %. This increase can be explained by the same logic used as in Section 4.4.1 with regards to locking away your bid capacity for what turns out to be rejected blocks in the test on real, not forecasted, prices.

As of the 1.6 % potential improvement shown in Section 3.3, a deterministic run of the model with actual spot prices gives an increase in income of 0.9 %, which confirms that the value of perfect information cannot be very high for this particular case. We have not run a tests through the entire 14 days with actual bids to find a profit for comparison, however it is likely that this potential increase would be higher.

5 Conclusion

When bidding in the spot market, we find that producers have to make three separate, yet strongly interconnected decisions. These are deciding the volumes, the prices and the bidtypes. Producers face a more complex reality than what is often assumed in academic models for bidding, and we find a number of factors that the producers have to take into consideration. The marginal water values and the turbines' efficiency curves can to some extent be controlled by the producers themselves. Other factors, for example regulations on waterways and spot market rules and regulations are exogenous factors that needs continuous adapting.

The most decisive factors when bidding are the marginal water value, feed-in fees, technical and hydrological characteristics, and bilateral agreements outside the spot market. We find that the producers take some of these factors well into account, and others not always so well. Among the things they consider well are the efficiency curves of the turbines and that they must choose to strategically interpolate or avoid it, as to hit suitable points of the efficiency curve. The feed-in fees are not taken as well into account. Producers' entry price points do not always adapt to changes in the variable feed-in fee over the course of a weekday, and some bids are submitted at price points below the feed-in fee alone. We also find that the producers do not fully utilize the range of price points allowed.

The producers perform quite well in their bidding, as indicated by the analysis of the maximum potential increase in spot revenues. On average over the 8 weeks, the potential increase for all producers was 5.4 %, competing against perfect price information. This comes both from the fact that price forecasts are generally good, and that the system design with several price-volume bids and uniform spot prices is well-functioning. We still find that their performance correlates with the standard deviation of the price levels, meaning it is more difficult to bid optimally when price variations are high.

The actual bids over two days perform quite good compared to what are supposed to be optimal bids from the model. However, a model will always be limited by the quality of input, and constructed price scenarios are not the same as perfect price information. Developing and testing our two-stage stochastic mixed-integer linear program has made us wonder how important great stochasticity in a model really is. When testing the stochastic model versus a deterministic variant, and iteratively backtesting on actual spot prices, the value of a stochastic solution does not seem to be especially high.

Further studies When generating bids for Elspot, Norwegian producers also consider the selling options in regulating and real time balancing markets. A study of how producers relate their bidding for Elspot to the weekly, intraday, and real-time markets would certainly be interesting. Due to the sensitivity of the bidding data, we were unsuccessful in obtaining bids from more than three hydro producers. A potentially larger empirical study can go deeper into how these producers bid in relation to each other, for example within the same price area, and according to price forecasts.

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