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MASTERKONTRAKT
- uttak av masteroppgave

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Studieprogram	Hovedprofil
Industriell økonomi og teknologiledelse	**Anvendt økonomi og optimering**

3. Masteroppgave

Oppstartsdato	Innleveringsfrist
16. jan 2012	**11. jun 2012**

Oppgavens (foreløpige) tittel
Optimization of investment decisions and production planning in aging offshore petroleum fields

Oppgavetekst/Problembeskrivelse
The purpose of this work is to analyse and develop a solution approach for optimal long-term petroleum investments and production.

The problem is an investment decision where new production technology or other infrastructure reconfigurations are evaluated in order to maximize the net present value of a producing asset. The time horizon is typically the remaining period of the production license, or the estimated remaining lifetime of the field itself.

Main contents:
1. Describe the planning problem
2. Formulate one or more models for the problem
3. Implement one or more of the models using appropriate software
4. Compute computational studies on the model(s)
5. Discuss the applicability of the model(s) in practical decision making

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4. Underskrift

Student: Jeg erklærer herved at jeg har satt meg inn i gjeldende bestemmelser for mastergradsstudiet og at jeg oppfyller kravene for adgang til å påbegynne oppgaven, herunder eventuelle praksiskrav.

Partene er gjort kjent med avtalens vilkår, samt kapitlene i studiehåndboken om generelle regler og aktuell studieplan for masterstudiet.

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Sted og dato

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Originalen lagres i NTNUs elektroniske arkiv. Kopi av avtalen sendes til instituttet og studenten.

Preface

This thesis is written as a part of the Master Degree in Industrial Economics and Technology Management with a specialization in Managerial Economics and Operations Research at the Norwegian University of Science and Technology (NTNU) during the spring semester of 2012. The work has been strongly motivated by an internship at Petrobras' research center CENPES in Rio de Janeiro during the summer of 2011 and by meetings with petroleum engineers at the Petrobras University in January 2012. Some of the ideas presented in this thesis are based on the preliminary work done in TIØ 4500 during the fall 2011.

The paper is prepared in Microsoft Word 2010, while the numerical analysis and the generation of figures and graphs is performed in Microsoft Excel 2010. The optimization model is written in Mosel and run using Xpress-MP optimizer as commercial solver. Parameters for the optimization model are generated in Schlumberger's multiphase flow simulator PIPESIM.

I would like to thank my supervisor Associate Professor Henrik Andersson for constructive discussions and excellent guidance. I further want to thank co-supervisor PhD Vidar Gunnerud at the Department of Engineering Cybernetics for vital guidance and input to my work, and the IO Center at NTNU for giving me the chance to go to Brazil and work with Petrobras. I am particularly grateful for the time Alex Teixeira has devoted to me, giving supervision and guidance during my stay at CENPES.

Trondheim, June 11th 2012

Vegard Storvold

Picture on cover page is from FMC's catalogue "Oil and Water Subsea Separation: Petrobras Marlim", retrieved from www.fmctechnologies.com

Summary

This thesis suggests that an optimization model can be used as decision support when different subsea solutions are evaluated in order to increase hydrocarbon recovery and production lifetime of a field. The Marlim field in Campos Basin outside the coast of Brazil is used as a case, which is a field described as an aging field with increasing water production and declining profitability. The objective for the model is to maximize the net present value of a production asset by evaluating the installation of subsea boosters, subsea water separators and alternative routing solutions. The case consists of a well cluster of four wells that is routed through a subsea manifold connected to a FPSO through a riser. This case is similar to many of the production assets in the Marlim field, making the model applicable on other cases in the field as well.

To solve this problem, a mixed integer linear program is developed. The nonlinear functions are approximated with piecewise linearization by a SOS2 formulation, where the set of breakpoints are generated in a multiphase flow simulator. Binary variables with time period indexes are introduced to say what type of technology should be installed, and when it should be installed. The time horizon in the model is ten years with a yearly resolution.

The problem turns out to be quite challenging to solve if the detail level in the linear approximations and the time periods becomes too high. The problem is however solved nicely for a detail level found to be sufficient to provide useful results. The model suggest that there is a large potential for improving an asset's NPV by installing new subsea processing equipment for enhanced production. The improvement consists of increased production and better usage of today's infrastructure and topside installations, as well as increased system flexibility to mitigate uncertainty.

Sammendrag

Denne oppgaven viser at en optimeringsmodell kan brukes som beslutningsstøtte når ulike subseaslosninger vurderes for å øke utvinning og levetiden til et offshore petroleumsfelt. Marlim-feltet i Campos Basin utenfor kysten av Brasil brukes som eksempel. Dette er et aldrende felt med økende vannproduksjon og minkende lønnsomhet. Målet for modellen er å maksimere nåverdien av en produksjonsenhet ved å evaluere installasjon av undervannsboostere, vannseparatorer og ulike rutingløsninger. Caset består av fire brønner som rutes gjennom en subseamanifold koblet til en FPSO gjennom en riser. Dette caset er likt mange av de produksjonsenhetene vi finner i Marlim-feltet, noe som gjør modellen anvendbar for flere liknende case.

En lineær blandet heltallsmodell er formulert for å løse dette problemet. De ikke-lineære funksjonene er approksimert med stykkevis linearisering av type SOS2, hvor settene av brekkpunkter er generert ved hjelp av en flerfasestrømsimulator. Binære variabler med tidsindekser er innført for å si hva slags teknologi som skal installeres, og når det skal installeres. Tidshorisonten i modellen er ti år med en årlig oppløsning.

Problemet viser seg å være ganske utfordrende å løse dersom detaljnivået i de lineære approksimasjonene og tidsperiodiseringen blir for høyt. Problemet er imidlertid fint løsbart for et detaljnivå som er tilstrekkelig for å gi nyttige resultater. Modellen antyder at det er et stort potensial for forbedring av NPV ved å installere nytt subsea prosesseringsutstyr. Forbedringen består av økt produksjon og bedre benyttelse av dagens infrastruktur og plattforminstallasjoner, samt økt fleksibilitet i systemet for å håndtere usikkerhet.

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1 Introduction

Petroleum production has existed in Brazil for many years. The first explorations and field developments found place onshore, while offshore activities has become more important in the later years. The onshore petroleum production is expected to stay at today's production levels for some years, while an enormous growth is prospected in the offshore production. The expected growth is mainly driven by the discovery of so called pre-salt petroleum formations, found very deep into the ground of the Brazilian continental shelf. Today Brazil is the 11th largest oil producer in the world. Within 2020, it is expected to be one of the top five (The Economist, 2011).

From a Norwegian perspective the offshore activity in Brazil is of particular interest because of its similarities to Norway's petroleum industry. In addition, increasing future activity on the Brazilian shelf is expected to introduce new challenges and new areas for collaboration, and Norwegian companies are expected to play an important role in this venture. This thesis will address challenges connected to Petrobras' offshore petroleum production in the Marlim field in the Campos Basin, looking at the P-35 platform in particular. This field is considered as a brown field, meaning that the field has been operational for a considerable time, and has now started to reach the depletion phase. The main challenge for the production companies is to maintain production levels and profitability.

The major decisions for the exploration strategy are taken up-front, and are in some sense irreversible, especially when talking about offshore developments. Space and weight limitations on the platforms make new installations difficult. New technology has in the later years made petroleum engineers think differently about this; both reconfiguration of the existing subsea architecture and investments in new subsea equipment might start to be both operationally and economically realizable (Singh & Hannaford, 2007). Case studies show potential for applying subsea processing equipment to significantly improve the commercial figures of an aging production asset, as well as improving production performance. The technology of subsea systems does also provide the basis on which future small fields and deepwater fields can be economically developed. High lifting costs may be compensated by the introduction of subsea technology that improves the production performance of the production systems (Goodfellow Associates, 1990). Ribeiro et al. (1996) argue that the potential of subsea processing on deepwater field developments may be overlooked by engineers and managers. High investment costs and a high degree of uncertainty seem to prevent investments from being realized, especially if the net present value of the project is positive. Case studies show that up-front investments in infrastructure and boosting technology are minor compared to the benefits they could later bring.

The traditional approach to strategic planning in the oil and gas industry is to manually generate and evaluate different scenarios and rank them due to their performance (Erlingsen, Strat, Gunnerud, Nygreen, & Dìes, 2012). This may result in a complex and difficult searching process, and optimal solutions may be overlooked. Nystad (1988) argues that models and simulators used in such search processes are often far too detailed and computer time demanding in order to evaluate strategic and extensive economic problems. Such models may also direct too much attention to the mechanistic mathematics of the problem, making the model nonflexible to changes in the bigger strategic problem.

As a producing field is maturing, changes in reservoir characteristics and well performance may result in declining production and unfavorable changes in production flow composition. Investments in new subsea equipment can be done in order to improve the production and the net present value of the producing asset. This value is dependent on the investment scheme, the production plan and the total amount of recoverable hydrocarbons. Choosing the right type of equipment and the optimal combination of these is not trivial, as possible solutions reach a very high number. Solving this decision problem as an optimization problem will provide suggestions for solutions that could be further investigated, and in that sense make an important contribution to the decision making process.

The next chapter gives a more detailed presentation of a typical offshore petroleum production system and the most important technical aspects of it. The mathematical model is presented in Chapter 3 and 4, where Chapter 3 describes the problem and how it can be modeled mathematically while Chapter 4 presents how nonlinearities are reformulated in order to obtain a mixed integer linear formulation of the problem. It is also described how the multiphase flow simulator PIPESIM is used to generate the parameters needed for such a formulation. Chapter 5 presents the implementation and results of the model. Some analysis and discussion of the results and the properties of the model are presented afterwards. Chapter 6 gives the conclusions, while ideas for further work are presented in Chapter 7. A complete summary of the mathematical model is found in the Appendix section, together with the parameter values used in the model runs.

2 Background

This chapter gives a more comprehensive presentation of a typical offshore production asset, and provides the reader with a deeper insight and understanding of the production principles and its dynamics. The production system presented in this chapter is very close to a real and specific production asset operative in the Marlim field. It is however representative for the general principles and physics which almost all offshore production systems is based upon. The methodology and mathematical model presented later should therefore be applicable for similar production systems and petroleum fields.

2.1 Terms and Units

Figure 2.1 shows a typical well topology of one of the producing wells in the Marlim field. The piping from the bottom hole to the seabed is called *tubing*, while piping along the seabed are called *flowlines* and piping from the seabed up to the surface is called *risers*. When a well is drilled into a reservoir, the hydrocarbons start flowing through the well perforations and up to the surface if the *reservoir pressure* is higher than the *bottom hole pressure* experienced in the bottom of the well. The flowrate is determined by this pressure differential and a performance index unique to that particular well. The flow from several wells can be routed into a common subsea manifold. A *wellhead pressure* is defined at the outlet of the flowline before the flow enters the manifold, while a *manifold pressure* is defined inside the manifold. Further, a *topside pressure* is defined at the riser outlet. All pressures presented in this project are given in *bara*, which equals 100 kPa, and is the pressure relative to absolute vacuum.

The flow from a hydrocarbon reservoir consists typically of several types of natural gas, condensates, liquid hydrocarbons, water and other types of nonhydrocarbon components such as CO_2 and mercury. Specifications of the multiphase flow are usually simplified in practical modeling and described by only three phases, namely oil, gas and water. The following volume fractions are often used to describe this composition:

Gas and oil: GOR – the gas to oil ratio [$\frac{amount\ of\ gas\ (scf)}{amount\ of\ oil\ (stb)}$]

Water: Watercut – the volumetric fraction of the liquid that is water [$\frac{water\ (stb)}{total\ liquid\ (stb)}$]

A multiphase composition of oil, gas and water can then be completely described by the watercut and the GOR, illustrated in Figure 2.2. Here, *scf* stands for standard cubic feet, while *stb* stands for standard barrel. These parameters describe the component fractions at *standard conditions*, defined as 1 bara pressure at 15.5 C°. Production rates are also described at standard conditions, given in volume per day, i.e. *stb/d* or *scf/d*. It is common to use the notation *mmscf/d* for a million scf/d.

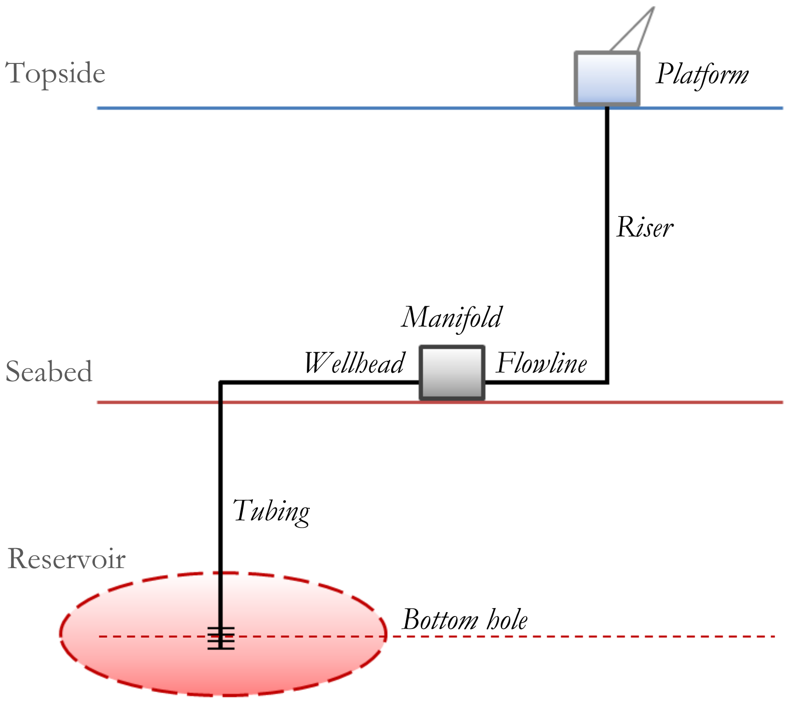

Figure 2.1: Well topology with key components

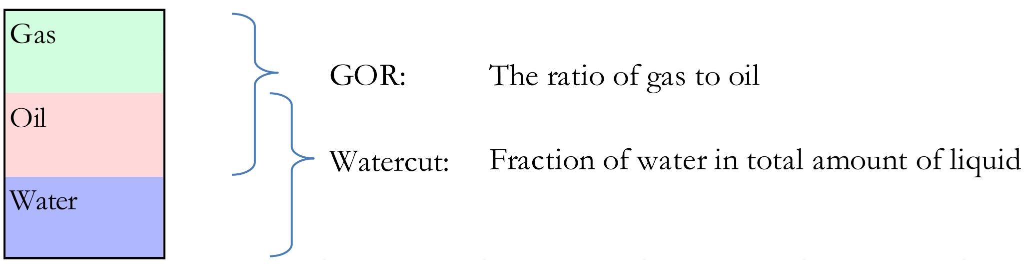

Figure 2.2: A multiphase composition described by watercut and GOR

Reservoir contents can be defined as *resources* or *reserves*, where resources describe the initial hydrocarbons in place while reserves are the amount of the resources that is economically extractable. The amount of reserves is dependent of a field's recovery factor, which typically is 60% of the total resources in place (Gudmundsson, 2011). This factor is strongly dependent on the properties of the reservoir, such as perforation characteristics and drive mechanisms.

2.2 Production Principles

Hydrocarbons flow from the reservoir and up to the topside as long as the pressure differential is large enough to drive the flow through the rock formations and up the production system. Challenges may occur when certain pressure and temperature conditions affect the multiphase flow properties in unfavorable ways, or when the reservoir enters more difficult production phases. The following sections present the main principles and concerns when dealing with flow assurance.

2.2.1 Production Flow

Pushing the flow through the bottom hole perforations requires a lot of work, and results in a large pressure drop between the reservoir and the bottom hole inside the tubing. Further pressure loss is experienced when the flow is elevated through the system as the hydrostatic pressure is declining. Pressure loss is also related to friction between the pipe wall and the production flow. Figure 2.3 shows how the pressure declines inside the tubing and flowline of one of the asset's wells from the reservoir to the inlet of the manifold. Notice how the pressure gradient is decreasing after the point where *artificial lift gas* is injected into the tubing. This will be further described in later sections.

The production engineers are able to control the day to day flow rates by using *chokes*. Chokes are usually installed at the manifold inlet and at the topside inlet and make it possible to increase either the wellhead pressure or the topside pressure in order to reduce the flow. Controlling the system pressures and the production is crucial, as sudden changes in pressure conditions can result in harmful blowouts or other unwanted well behavior. Coking of the flow is also necessary if some of the system capacities are constraining the production.

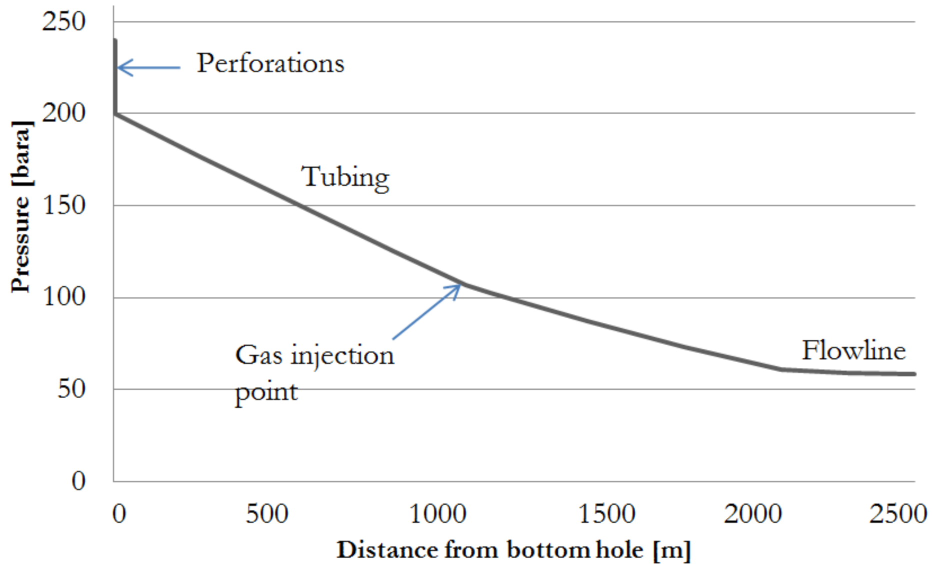

Figure 2.3: Pressure in the production line as a function of distance

Based on estimates of the pressure drop across the tubing and the flowline, and given the production characteristics of the formation and the production performance of the well, the curve of wellhead pressure p^{WH} against production rate for an individual well can be obtained. This curve is named the *wellhead pressure curve* from now on. The production flow is mixed with other production flows in the manifold and sent to the platform, or routed directly to the platform in its own riser. In both cases, based on estimates of the pressure drop across the flowline and the riser, and given the outlet pressure at the topside platform, the curve of the required inlet pressure at the manifold against production rate can be obtained. This curve is named the *back-pressure curve*.

Figure 2.4 presents the wellhead pressure curve and the back-pressure curve for one of the wells at the asset we are considering. The point where the two curves intersect is the maximum system production point. From this we can deduce that the wellhead pressure has to be equal or higher than the required manifold or riser inlet pressure. The shaded area illustrates the area of which the well *can* produce if the wellhead pressure is chocked down. If no choking is used, the well is producing approximately 7000 stb/d of liquid, with a wellhead pressure of 50 bara. Different wells enter the manifold with different wellhead pressures, but the manifold pressure can off course not be higher than the lowest inlet pressure. This means that the well with the lowest wellhead pressure curve dictates the other wells, and decides where the intersection point is going to be. This implicates that the other wells has to be choked down and production is potentially lost.

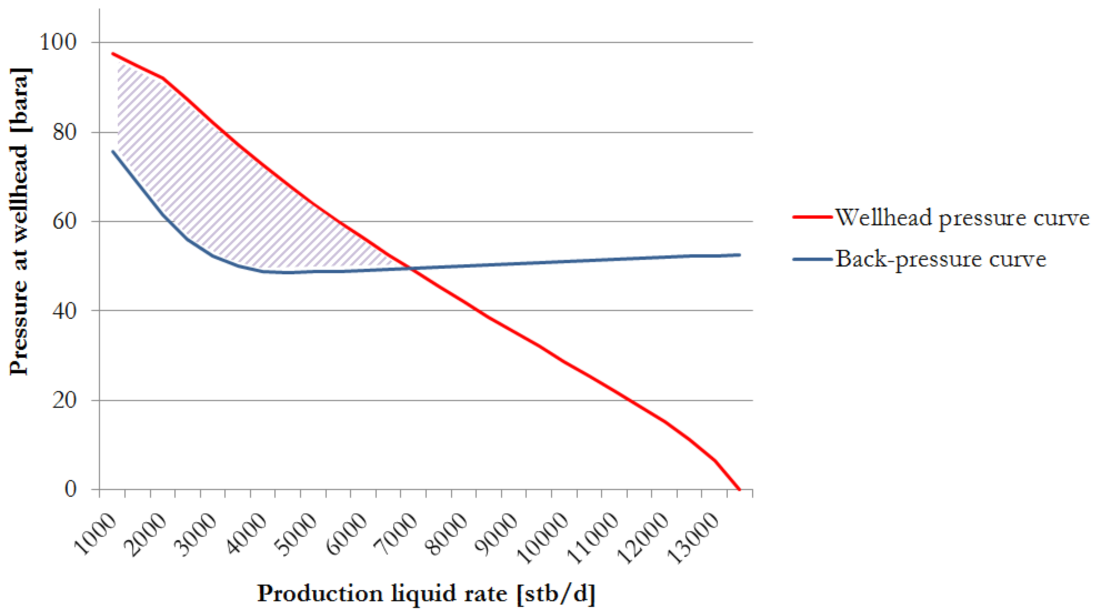

Figure 2.4: Production system analysis with wellhead pressure curve and back-pressure curve

2.2.2 Flow Assurance Challenges

Camargo (2004) suggests that the flow assurance problem in Brazilian deepwaters can be divided into two main challenges. The first is to maintain pressure differences in the system to assure production flow. Pumps and artificial gaslift are common technologies to enhance production flow, together with gas and water reinjection in the reservoir to maintain reservoir pressure. The second challenge is related to formation of solids, such as asphaltene and hydrates, due to pressure changes and pipeline cooling. This can potentially lead to plugging of the pipelines or harm the production equipment.

Regarding the first challenge, water is definitely a problem, as the water increases the weight of the multiphase liquid column in the tubing and the riser. We want to maintain pressure loss as low as possible to sustain high production rates from the fields over time, and increasing watercuts are making this difficult. In addition, the water handling capacities on the topside platforms are constraining the total liquid handling capacity as the water production increases. The presence of liquid water in the multiphase flow does also increase the chance for hydrate formations (Gudmundsson, 2011).

2.2.3 Different Production Phases

The lifetime of an oil producing field can be divided into distinct phases that have different characteristics and challenges related to the system capacities. Fjøsne (2002) defines the most relevant phases for an aging field:

The mature phase capacity gap

The production is limited by the topside capacity of handling associated well fluids, such as water and gas. Topside modifications are difficult due to space and weight limitations, and production has to be reduced. A solution is to improve the well fluid composition upstream of the topside production facilities. This will involve some sort of subsea separation and processing. If high watercut in the well flow is a problem, a subsea water knock-out and re-injection system should be considered. If high GOR is limiting the topside gas handling capacities, a solution could be to install a subsea system for gas separation.

The tail-end capacity gap

The production is declining, and processing and transport infrastructure are no longer fully utilized. Nearby satellite fields and marginal fields that were considered noneconomic to develop might become profitable by installing subsea processing technology at the remote field location enabling transport to the already existing production asset in order to utilize the excess capacity. The question is whether to build a new topside facility at the remote location, or to utilize excess capacity on nearby platforms. New technology can provide the system with the flexibility needed to fully exploit such tail-end production phases.

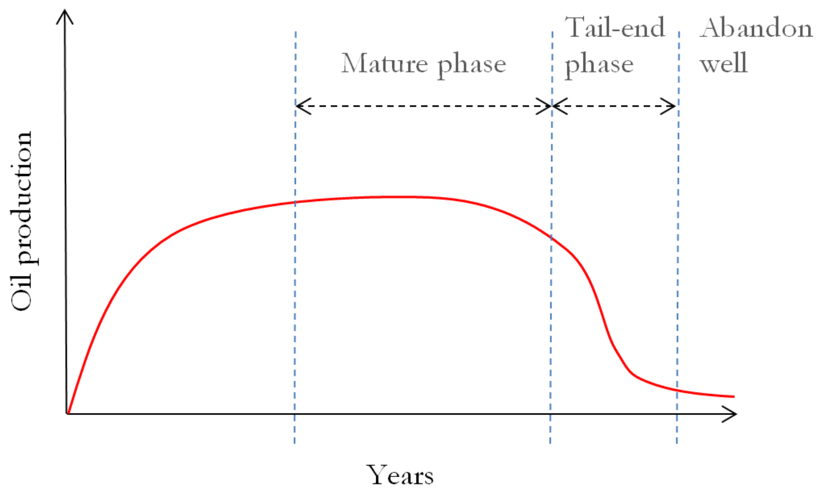

Figure 2.5: Illustration of an oil field's lifetime and production levels

2.3 The Marlim Field

The Marlim field was once considered the world's largest subsea development. According to Bampi & Costa (2010), the field has a history of more than 200 wells. Today, 129 wells are still active, together with 8 floating processing, storage and offloading units (FPSOs) owned and operated by Petrobras. The water depth at which the wells are operated ranges between 650 and 2600 meters. The field is producing both oil and gas, with water and sand as the most important nonhydrocarbon associated components. As Figure 2.6 shows, the Marlim field (MRL) is located in the eastern part of Campos Basin, about 110 km offshore the Rio de Janeiro state, and is a part of a larger complex of reservoirs, including the Marlim South (MLS) and the Marlim East (MLL) fields.

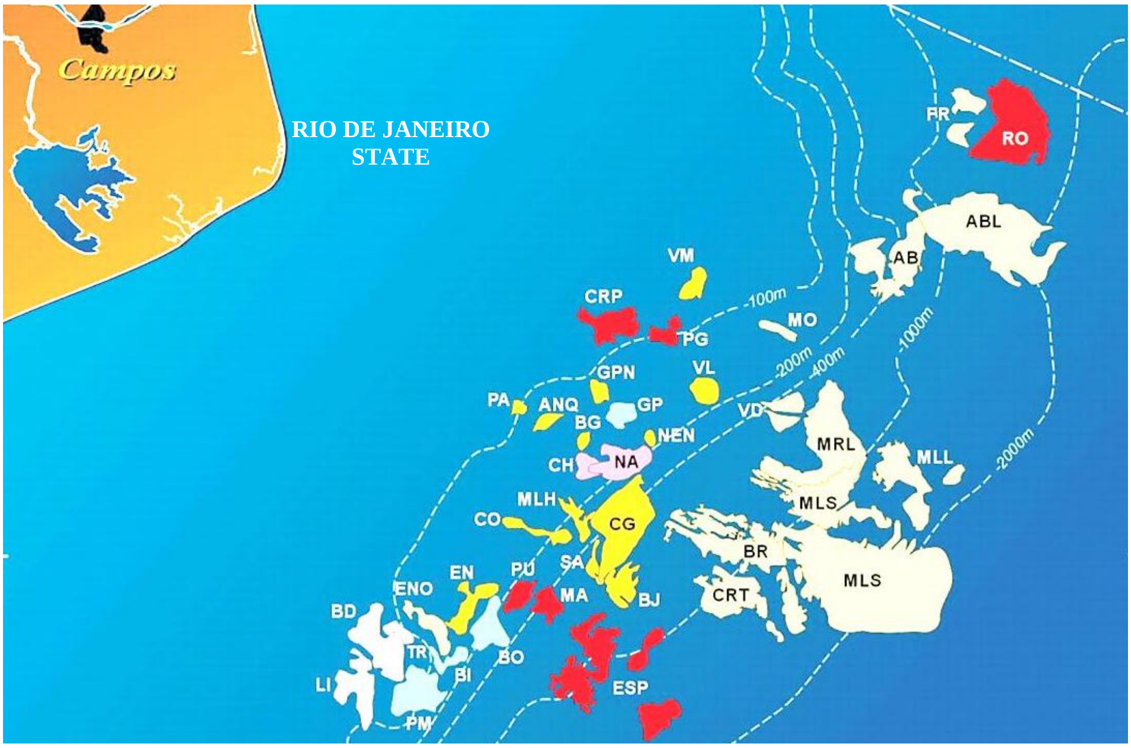

Figure 2.6: Layout of the Campos Basin and the Marlim field (GeoScienceWorld, 2009).

2.3.1 Properties and Characteristics

When the explorations started in 1985, the total amount of oil resources in the Marlim field was estimated to be 6 365 million stb. When the production started in 1991, a peak oil production of 430 000 stb/d and an economic lifetime of 20 years were projected. In 2002, it turned out that the peak oil production reached 650 000 stb/d (Bampi & Costa, 2010).

Improved seismic and new production technology has extended production prospects by many years. Today, projects in the Marlim field are expected to run until 2030 (Ribeiro, Steagall, Oliveira, Formiga, Kerber, & Jaeger, 2005).

Every reservoir is composed of a unique combination of geometric and geological rock properties, fluid characteristics and recovery mechanisms. It is normal to group reservoirs according to their primary recovery mechanism. Each drive mechanism has certain typical performance characteristics in terms of GOR, watercut, pressure decline rates and recovery factors. The Marlim field is primarily a *water driven* reservoir, where water moves into the pore spaces originally occupied by oil as the oil is withdrawn from the reservoir. The result is a very gradual reservoir pressure decline, around 0.07 bara per million barrels of oil, but an ever increasing watercut as more oil is withdrawn from the reservoir. The GOR is practically unchanged during the reservoir lifetime. As water provides a very efficient displacing mechanism, the reservoir recovery factor may become high, sometimes up to 75% (Ahmed, 2006).

There exist several types of driving mechanisms that provide the natural energy necessary for oil recovery. In addition to water drive, *gas drive* is a common mechanism in oil producing reservoirs. In this type of reservoir, the principal drive energy is a result of the liberation and the subsequent expansion of dissolved gas in the liquid phase. When the reservoir pressure falls below the bubble-point pressure, gas is liberated inside the rock formations, forcing the oil out of the pores. This results in a rapid and continuously pressure decline, together with increasing GOR. However, the water production stays at a low rate. Gas drive is regarded as a less efficient recovery mechanism than the water drive. The topside gas handling capacity is often the restricting factor for the production system in this case.

2.3.2 The Production Asset

The layout of the P-35 asset is presented in Figure 2.7, consisting of 2 subsea manifolds and 15 wells. 6 wells are so called satellite wells, which are producing directly to the platform through their own separate risers. The rest of the wells are linked to subsea manifolds that are routing the production flows through a shared riser. 7 wells are linked to manifold 1, and 2 wells are linked to manifold 2. The platform is a FPSO, with a topside manifold to route gas and liquid flows to one of the 3 separators A, B or Test.

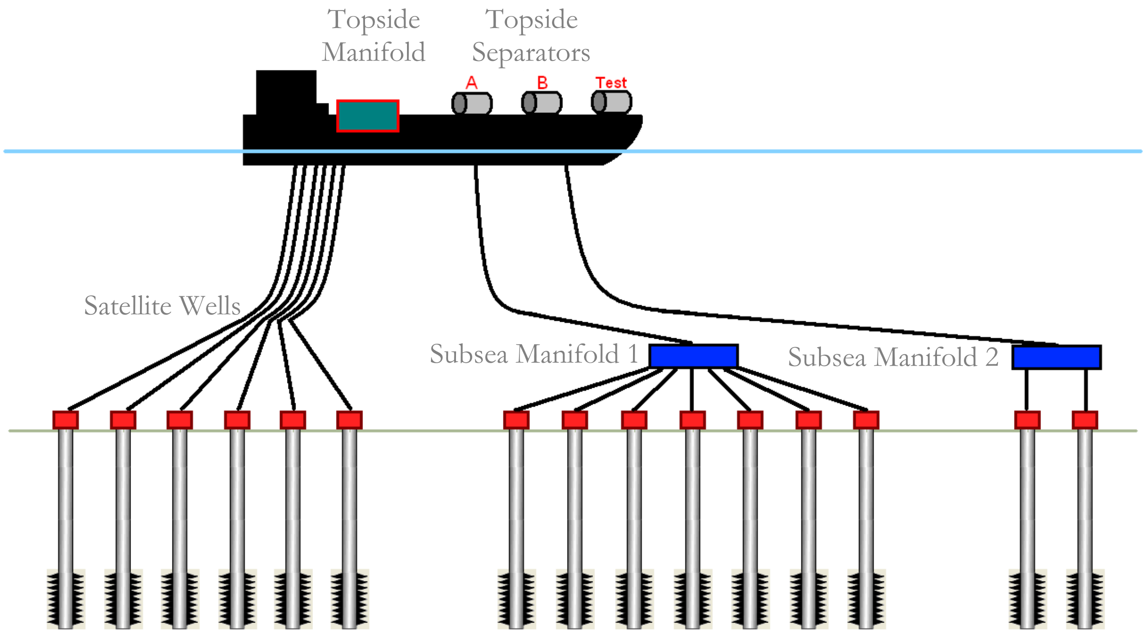

Figure 2.7: Conceptual layout of the P-35

As of today, the P-35 asset is not re-injecting any gas or water into the reservoir for reservoir pressure maintenance. This might become relevant in the coming years as a small decline in reservoir pressure might someday lead to a substantial drop in oil production. Combined with increasing water production and constrained topside liquid handling capacities, the P-35 together with many of the other Marlim assets are expected to face some tough challenges related to flow assurance in the coming years.

2.4 Introduction of new subsea equipment

Subsea equipment such as boosters and separators are believed to enhance and accelerate the production in both existing and not-yet developed fields. It is also possible that such equipment can increase the total amount of recoverable hydrocarbons and extend field lifetime. Another incentive for investing in such equipment is potentially fewer long-term deferred investments related to reconfiguration of infrastructure and system capacities (Elde, 2005). The planning horizon of such projects is typically the remaining lifetime of the field, and involves large investment costs and hopefully considerable new revenues over long time periods, making this a strategic planning issue (Ulstein, Nygreen, & Sagli, 2007).

Altered wellhead pressures or back-pressures can be accomplished by artificial gas lift, subsea boosting equipment, or even subsea water separators. The latter option also has the potential for water reinjection to gain reservoir pressure support.

2.4.1 Single Phase and Multiphase Boosters

Petrobras has large estimated potential reserves of heavy oil, and has launched several research programs to develop subsea boosting technology to make production from deep fields feasible. Examples of new booster developments are the Vertical Annular Separation and Pumping System (VASPS) and different applications of Electrical Submersible Pumps (ESP), designed to alter system pressures in a beneficial way (Peixoto et al., 2005).

Single phase boosting is considered as the traditional approach, and involves separating the gas from the liquid phase before the production flow is boosted, allowing the use of gas compressors and liquid pumps. Such phase specific boosters may handle higher flow rates and provide higher pressure differentials than multiphase boosters, and are usually more energy efficient (Ramberg, 2007). Figure 2.8 compares the single phase and multiphase performance for a typical subsea pump at a given speed. The location and steepness of the multiphase performance curve is highly dependent on the gas volume fraction in the liquid flow, while the single phase performance curve remains more or less the same as the gas and liquid are separated and boosted separately.

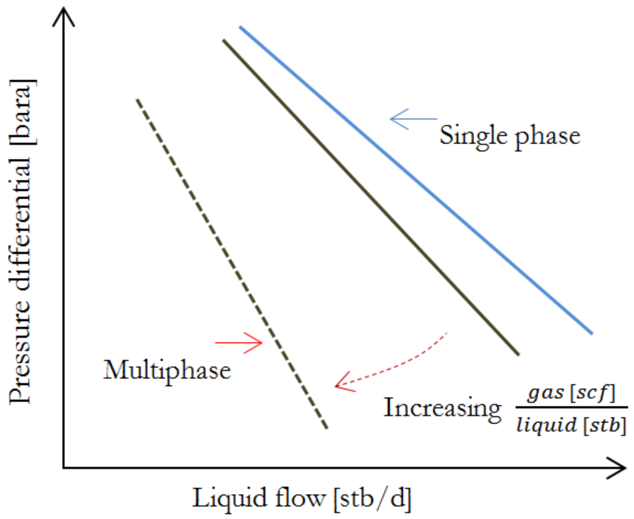

Figure 2.8: Single phase and multiphase performance for a subsea pump (Ramberg, 2007)

With the gas separated, different routing of the gas from the seabed (dotted lines in the figure) could be evaluated to obtain more efficient transport or better use of system capacities downstream, see figure 2.9.

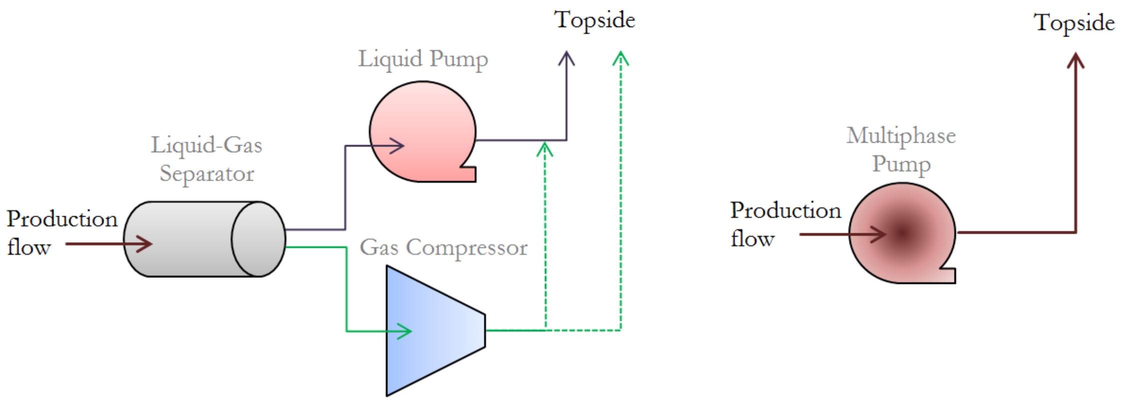

Figure 2.9: Traditional single phase approaches versus multiphase boosting

Multiphase boosting technology for the oil and gas industry has been in development since the early 1980s, and is now rapidly gaining acceptance as a tool to optimize multiphase production systems as the performance of such boosters are getting closer to single phase boosters. The majority of the installations have so far been done at onshore or offshore topsides, but subsea based installations have become more and more common in the later years (Rodrigues et al., 2005). Particularly for the development of satellite fields, multiphase boosting has been recognized as a promising technology: rather than separation, gas compression and liquid pumping, multiphase boosting enables the non-separated well stream to be boosted in a single machine. Besides the realized simplification of the production system, the potential cost reductions could make development of marginal fields economic.

2.4.2 Three Phase Separation

Within the next years, several Petrobras developments will be considering new technology to reduce lifting costs and to increase oil recovery. One particular interesting technology is the Oil Water Subsea Separation - SSAO in Portuguese terms. Among the eight platforms of the Marlim field two are natural candidates to host the production of the first SSAO for heavy oil in deepwaters, namely the P-35 and P-37 (Euphemio et al., 2007).

Subsea water separation equipment together with gas separation can perform an almost complete first stage processing of the production flow on the sea bed. The water is separated from the production flow and treated and reinjected into the producing reservoir to maintain reservoir pressure, or dumped into a deposit reservoir. Only re-injection into the producing reservoir will be considered as an option in this work. It is also an option to boost it and transport it elsewhere. In addition to the boosting effect from removing water from the production flow and thus reducing the density of the multiphase column in the riser, liquid handling capacity on the topside is freed. The gas and oil is separated in the process as well, and the application has then the same boosting and routing alternatives as in the case of the single phase boosting application, see figure 2.10 where the different options are illustrated with dotted lines.

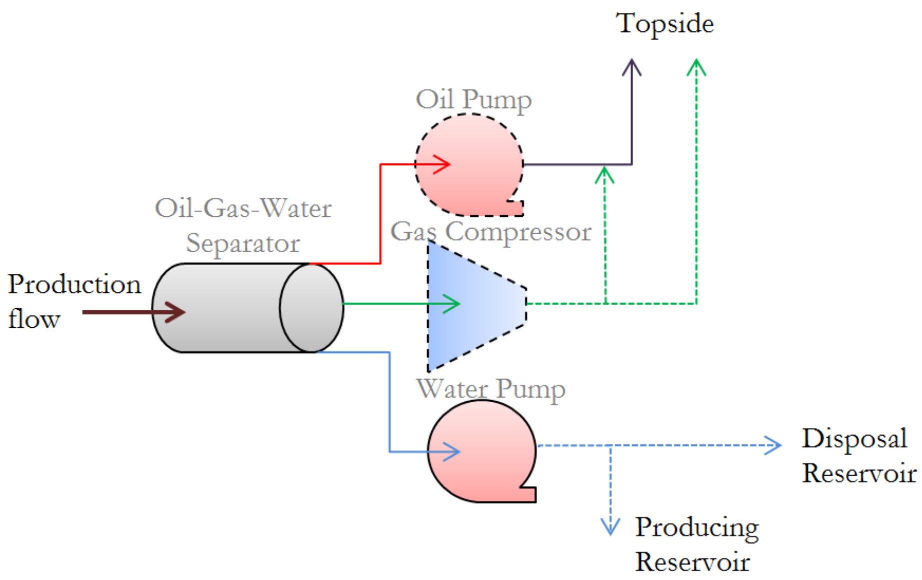

Figure 2.10: Three phase separation with boosting and routing options

2.4.3 Artificial Gas Lift

Gas lift is performed by compressing gas on the topside and reinjecting it into the tubing or the riser to reduce the density of the multiphase production flow, which in turn reduces the wellhead back-pressure and increases production. Gas lift is regarded as a relatively cheap and reliable lift technology, and is widely used in the Marlim field and similar fields around the world. The potential for increased production is somewhat limited, as the beneficial effects diminish rapidly after a certain level of gas injection. A conceptual layout of the process is shown in figure 2.11.

2.4.4 Investment Options

We have defined two boosting concepts, consisting of multiphase pumping or gas separation with gas compression and liquid pumping. Three phase separation is also an option, making it possible to treat the associated water on the sea floor. Artificial gas lift is already installed in our case, and is therefore not a part of our decision. For further reading on how gas lift can be optimized to maximize production in a steady state problem, the reader is referred to Dzubur and Langvik (2011). Rodrigues et al. (2005) conclude that artificial lift provided by subsea boosting equipment is potentially more effective than the equivalent gas lift application. The downside with the pump applications is that they are triggering high investment costs and more uncertainty associated with maintenance and equipment lifetime. Combinations of different equipment are of course possible, and could be installed simultaneously to reduce the operation costs related to logistics and system shut downs during the installation.

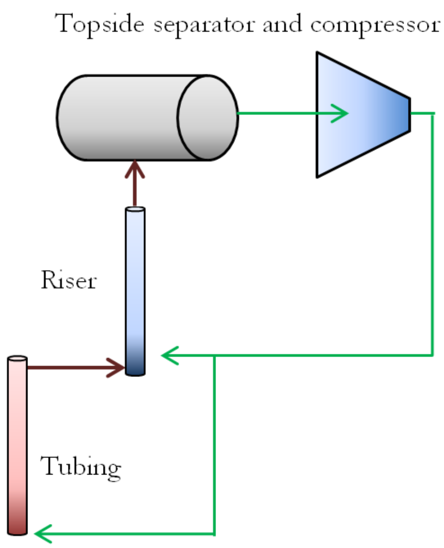

Figure 2.11: Conceptual layout of an artificial gas lift application

2.5 Applications of Optimization in the Petroleum Industry

We find applications of optimization and mathematical programing within a wide range of problems in the petroleum industry. Optimization methods can be applied to solve both extensive value chain problems over several time periods and small static problems that concern only limited parts of a production system. The problems can be solved by using a deterministic approach, or by introducing stochastic modeling to cope with uncertainty. This section presents some of the work done on different problems from the petroleum industry.

Nygreen & Haugen (2010) discuss various attempts on using optimization models for decision support in the Norwegian petroleum field and pipeline development from the last 30 years. The first applications were deterministic models, where discrete decisions were regarded as most interesting. An example of such modeling is found in Nygreen et al. (1998), where a mixed integer multiperiod model used for optimization in downstream investment planning is presented. It is a model that has been used by The Norwegian Petroleum Directorate in long-term field development since the early 1980s to find the most optimal timing of new field developments by maximizing the net present value of the field. Such deterministic models often assume fixed production for a single market. Later, stochastic models have been introduced to deal with uncertainty, especially related to liberalized markets and prices and demand. Tomasgard et al. (2007) describe a multiperiod optimization model for a Norwegian natural gas producer where transport of gas is optimized in order to maximize profit. The focus is on production and transportation of multiphase flows within the network, and splitting and blending of components to meet market demand. The problem is modeled as a mixed integer nonlinear problem, and then piecewise linearized by the use of Special Ordered Sets of type 2 (SOS2). The paper introduces the trading and risk management aspect of the planning problem and the use of stochastic programing to improve the quality of the model. Bittencourt (1997) uses the net present value to evaluate a long run reservoir development, and concludes that involving to many variables that affect the operational schedule may result in large and unsolvable problems. He suggests that heuristic search methods can be used in order to find nearly optimal solutions, or at least generate bounds for the decision.

Some work has been done to model upstream production systems as well, where the production performance of wells and flow to the topside platforms are described and modeled in more detail. Gunnerud (2011) presents a model where production from a reservoir is described by functions approximated by piecewise linearization in several dimensions. Several clusters of wells are evaluated, and the goal is to maximize oil production given the constraints of the system. Solving a detailed model of a rather complex production system is found to be challenging, and structure in the problem is exploited to decompose the problem into smaller sub-problems. The model focuses on solving short term operational problems in one time step, and assumes that reservoir conditions and well performances are constant. Erlingsen & Strat (2010) are introducing time steps and dynamics in the system conditions to evaluate long term investment decisions in a field with several reservoirs and production units. The reservoir dynamics are described by a simple deterministic tank model, where the production from each reservoir is determined by the amount of each phase that is present in the reservoir and the number of wells producing from it. This model is very general, and does not conserve the realism of how each single well is producing. It is however interesting to observe that downstream production aspects such as reservoir dynamics are evaluated together with upstream long term investment decisions as it was modeled by Nygreen et al. (1998). This report aims to develop a more realistic and detailed description of the upstream conditions, while finding the optimal timing of new investments by maximizing the net present value.

3 Problem Description and Model

This chapter presents the production system and its properties in more detail, and suggests how this can be modeled mathematically in an appropriate way. The goal is to develop a model that is complex enough to conserve the realism of the system, but in the same time making it solvable by doing the right simplifications. The model is developed for solving a well cluster-case, where several wells are producing to a subsea manifold, and routed to the topside through a common riser. The model can then be used for evaluating well clusters similar to the ones at the P-35 asset. The number of wells that is connected to the manifold can easily be adjusted for different cases. If the number of wells is set to one, we are in fact evaluating a satellite well. A summary of the mathematical model is found in Appendix A.

3.1 Assumptions and Simplifications

The concept of methodology is to use a multiphase flow simulator to generate data that is used in an optimization model. To construct a production system in the multiphase flow simulator, several different assumptions and simplifications are done to make the simulator run properly. These simplifications are quite technical, and will not be discussed here. The main objective is to make a simulator able to generate production rates similar to the real asset. The absolute error is found to be 5%-10% compared to the real production rates at the P-35 when using today's production conditions as input in the simulator. The error is considered acceptable for further use in this project. A detailed description of the composition of hydrocarbons and associated components in the reservoir is used as parameters in the simulations, as the pipe flows are strongly affected by this composition. However, the production rates found by simulations are described only by three components; oil, gas and water. There are generally many factors that may affect the results in the simulation run, but most of them are threated in the simulator, and not in the optimization model.

Another challenge is to add the artificial gas lift to the volume of gas in the production lines. The injection gas can be considered as going in loops in the system; it is separated from the liquid and compressed at the platform before it is reinjected into the production lines and returned to the platform as lift gas. A limiting factor is the topside artificial gas compression capacity. The lift gas can therefore be regarded as a constant, given that the capacity constraint is active. The injection gas is therefore removed from the optimization model, but counted for in the simulator. Thus, gas production rates in the model are described *without* the constant amount of injection gas. Compressed gas can also be reinjected in the reservoir to maintain reservoir pressure, but as of today, no reinjection wells have been made.

The planning period is discretized into several time periods. This implies that no production flows or other conditions can be altered during a time period. Such a periodization of the reality is expected to give higher production rates than a continuous description, since unfavorable changes in the reservoir conditions are updated only once every period. This challenge can be handled by increasing the time resolution, while the obvious downside by doing so is that the problem is getting larger and harder to solve.

The main properties of both the boosters and the separators are chosen to be the efficiency and the production flow capacity. As there are numerous combinations of such properties, discrete sets of alternatives are defined for use in the model. The investment cost is defined by the equipment alternative, and covers the development-, purchasing- and maintenance cost. The cost associated with the installation operation itself depends on whether other equipment is installed during the same time period or not. The operation cost of the separator is linked to the amount of water treated, while for the boosters it is the amount of bara added to the discharged production flow.

3.2 Sets and Indices

The set of time steps is defined as T, where t is the corresponding index. We have earlier defined the multiphase flow from the reservoir as a composition of oil, gas and water. P is the set of these three phases, with p as the corresponding index. If the index p is set to o, g or w, it means that the phase oil, gas or water is treated in that particular formulation, respectively. The set of wells is defined as J, where j is the corresponding index. The set of three-phase separator alternatives is given as S, where s is the corresponding index, while F and f is the set and index for the different boosting alternatives. For readability, the notation with sets is left out in the summation symbols, and the sets are indexed with the corresponding small letter in the formulation.

3.3 Modeling the Production

System pressures, reservoir contents and flow rates are incorporated as variables in the optimization model, and will be updated in each time period to describe the dynamics in our problem. The system constraints are limiting the possible values of these variables. Other system properties are considered as constants or not included in the model at all, but handled by the flow simulator.

3.3.1 The Reservoir Model

As the fluids and gas are withdrawn from the reservoir, reservoir conditions may change. The pressure will often decrease, and the production flow will falter. In other cases, the influx of different phases such as water and gas or other components from surrounding formations will tend to maintain pressure. Reinjection of produced phases is also being used to maintain pressure. The reservoir is modeled as a simple tank, with the content of phase *p* at the beginning of time period *t*, q_{tp}^{RES}, and reservoir pressure at the beginning of time period *t*, p_t^{RES}, as the only properties. A typical inventory model is used to describe the state of the reservoir in time period $t+1$ as follows

$$q_{t+1,o}^{RES} = q_{to}^{RES} + (Q_o^{IFLUX} - \sum_j q_{tjo}^{W})D \qquad \forall t \in \{1 \dots |T|-1\} \tag{3.1}$$
$$q_{t+1,g}^{RES} = q_{tg}^{RES} + (Q_g^{IFLUX} - \sum_j q_{tjg}^{W})D \qquad \forall t \in \{1 \dots |T|-1\} \tag{3.2}$$
$$q_{t+1,w}^{RES} = q_{tw}^{RES} + \left(Q_w^{IFLUX} + q_{tw}^{I} - \sum_j q_{tjw}^{W}\right)D \qquad \forall t \in \{1 \dots |T|-1\} \tag{3.3}$$

where D is the number of days in a time period and Q_p^{IFLUX} is the daily natural influx of phase *p* into the reservoir. q_{tw}^{I} is the daily rate of re-injected water from the subsea separator in period *t*, while q_{tjp}^{W} is the daily production rate of phase *p* in well *j* in time period *t*. Reservoir content of phase *p* in $t = 1$ is given as the initial reservoir content Q_p^{INIT}.

The reservoir pressure in period *t* is modeled as a function of the contents in the reservoir in period *t*

$$p_t^{RES} = f\left(q_{to}^{RES}, q_{tg}^{RES}, q_{tw}^{RES}\right) \qquad \forall t \in T \tag{3.4}$$

The levels of different phases given in (3.1)-(3.3) have a linear relationship with the levels from the previous time step balanced with in- and outflows while the reservoir pressure given in (3.4) is expected to have nonlinear relations with the component levels in the reservoir.

As Section 2.1 showed, the composition of the reservoir contents q_{tp}^{RES} is usually described by watercut and GOR. Using the reservoir watercut and GOR as production data for all wells would be a convenient, but very inaccurate way to describe the production. Depending on well placement and other properties these values may vary between wells. It turns out that all wells are experiencing practically the same reservoir pressure. Since there is no obvious connection between the reservoir pressure and the watercut and GOR in a well, (3.4) does not need to depend on the well.

3.3.2 Production Flows

As we are considering a manifold constellation, the production flow has to be modeled for two different parts of the system. The first part is the flow from the reservoir to the manifold. The second is the flow from the manifold to the topside separator.

The topside pressure is in fact set by the operator, and regulated by chokes placed just at the end of each flow line that is entering the topside platform. In the same way, the wellhead pressure is "set" by the respective well and its attributes, and regulated down to subsea manifold pressure by chokes. In our formulation, the wellhead pressure refers to the outlet pressure of the flow line entering the manifold. To ensure production flow, the pressure in the flow line p_{tj}^{WH} must be greater than or equal to the pressure in the manifold p_t^{MAN}, and the following restrictions connecting well *j* and the manifold apply

$$p_t^{MAN} \leq p_{tj}^{WH} \qquad \forall t \in T, j \in J \tag{3.5}$$

This manifold pressure is dependent on the topside separator pressure P^{TOP}, which is assumed to be a constant, and the pressure loss Δp_t^R in the riser

$$p_t^{MAN} = P^{TOP} + \Delta p_t^R \qquad \forall t \in T \tag{3.6}$$

Elevation, pipe characteristics and other physical properties of the riser are given, so pressure loss in period *t* in the riser may be stated as a function of the multiphase flow streaming from the subsea manifold to the topside. This pressure loss is expected to be a nonlinear function of the daily production rates q_{tp}^R in the riser in period *t*

$$\Delta p_t^R = f(q_{to}^R, q_{tg}^R, q_{tw}^R) \quad \forall t \in T \tag{3.7}$$

The production flow in each well should be dependent on both reservoir pressure and wellhead pressure, as it is the difference in these two pressures that drives the production flow. The reservoir contents given in (3.1)-(3.3) define the fractions of the different phases in the flow, and should also be included in the function. The production flow of phase p from well j in period t can then be described by the following function

$$q_{tjp}^W = f_{jp}\left(p_{tj}^{WH}, p_t^{RES}, q_{to}^{RES}, q_{tg}^{RES}, q_{tw}^{RES}\right) \quad \forall t \in T, j \in J, p \in P \tag{3.8}$$

3.3.3 Flow Balances

We have now described the wells and the riser as separate systems. Flow balances, in addition to the pressure constraints in (3.5), are our way to connect these two systems

$$q_{tp}^R = \sum_j q_{tjp}^W \qquad \forall t \in T, p \in \mathrm{P} \tag{3.9}$$

Equation (3.9) simply states that the daily flowrate of phase p in the riser in time period t is equal to the sum of the daily flow rates from the wells. As new equipment can alter this equilibrium, new equations have to be introduced.

$$q_{to}^R = \sum_j q_{tjo}^W \qquad \forall t \in T \tag{3.10}$$
$$q_{tg}^R = \sum_j q_{tjg}^W - \sum_j q_{tj}^{W.E} - q_t^{M.E} \qquad \forall t \in T \tag{3.11}$$
$$q_{tw}^R = \sum_j q_{tjw}^W - q_t^I \quad \forall t \in T \tag{3.12}$$

where $q_{tj}^{W.E}$ and $q_t^{M.E}$ is the amount of gas from well j and the manifold that is exported directly from the seabed, respectively, while q_t^I is the amount of water treated and re-injected subsea. All of the oil entering the topside facility is assumed to be exported and sold. Gas entering the topside facility is either used for power generation, denoted q_t^{USE}, or exported and sold, given as q_t^{EXP}. The total amount of gas exported then becomes

$$q_t^{EXP} = \sum_j q_{tjg}^W - q_t^{USE} \qquad \forall t \in T \tag{3.13}$$

The water that is treated and purified at the topside facility is simply disposed into the sea.

3.3.4 System Capacity Constraints

We have already assumed that the platform gas compression capacity is an active constraint for the injection of gas lift. The topside gas handling capacity $Q^{G.TOP}$ is as far as we know not an active constraint today, but as it may become active in the future due to increased production or changes in the reservoir GOR the following constraints apply

$$q_{tg}^{R} \leq Q^{G.TOP} \qquad \forall t \in T \tag{3.14}$$

The topside is also constrained by a liquid handling capacity $Q^{L.TOP}$, giving the following constraints

$$q_{to}^{R} + q_{tw}^{R} \leq Q^{L.TOP} \qquad \forall t \in T \tag{3.15}$$

together with the topside water treatment capacity

$$q_{tw}^{R} \leq Q^{W.TOP} \qquad \forall t \in T \tag{3.16}$$

The oil production is constrained directly by (3.15), and indirectly by (3.16) as water is an associated product of the oil production. A dedicated topside oil treatment capacity is not believed to ever become a constraint, as this capacity should be highly prioritized when the platform is designed and build. The fact that the gas or water treatment capacity someday may constrain the oil production might not be considered in the early design phase.

3.4 Investment and Operating Decisions

Both single phase and multiphase boosters can be installed, each with different properties concerning boosting performance, investment cost and operation costs. We also have the option to install subsea water separators that perform a three phase separation and water re-injection, as well as alternative routing of gas. The investment decisions are modeled as binary variables, where a value equal to one means that the equipment is installed. Each binary variable has a time index, an equipment type index, and a well index if applicable. Installation of new equipment could be done on single wells or at the manifold. It is also possible to do combinations, e.g. install multiphase pumps at two out of four wells, in addition to a subsea water separator at the manifold. The different decisions with their binary variables are illustrated in Figure 3.1, together with the system pressures. We see that it is possible to install multiphase or a single phase boosters at each wells, as well as multiphase or single phase boosters or three phase separators at the manifold. The alternative subsea gas transport is illustrated with green arrows, while water re-injection is illustrated with a blue arrow.

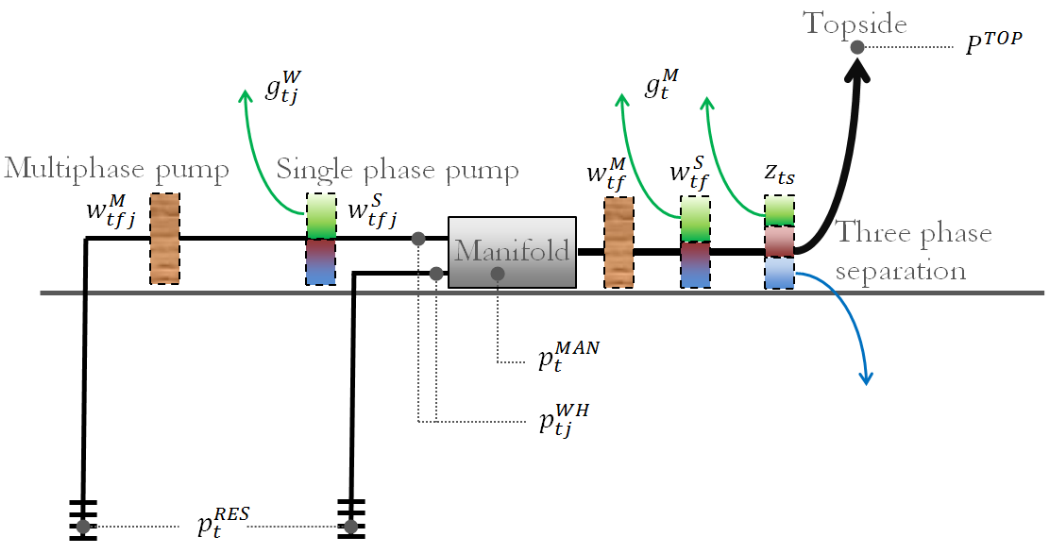

Figure 3.1: Overview of investment decisions and system pressures

If we consider the three phase separator, z_{ts} is a binary variable that equals 1 if a separator of type *s* is installed in time period *t*. We introduce another binary variable y_{ts} that equals 1 if a separator of type *s* is operating in time period *t*. The following constraint kicks the installation variable z_{ts} to 1 in the first time period where a separator of type *s* is operating

$$y_{ts} \leq \sum_{t_2 \in T | t_2 \leq t} z_{t_2 s} \qquad \forall t \in T, s \in S \tag{3.17}$$

Obviously, this formulation does not consider any installation lead time, and the equipment is therefore assumed to be operative in the same time period that the installation is done.

Similar formulations are done for the boosting equipment as well, where the binary variables w_{tjf}^{M} and w_{tjf}^{S} describe the installation of a multiphase and a single phase booster, respectively, of type *f* at well *j* in time period *t*. w_{tf}^{M} and w_{tf}^{S} describe the installation of a multiphase and a single phase booster, respectively, of type *f* at the manifold in time period *t*. $x_{tjf}^{M}, x_{tjf}^{S}, x_{tf}^{M}, x_{tf}^{S}$ are the corresponding operating variables.

Alternative subsea gas export from the well or the manifold is enabled in time period *t* when the binary variable g_{tj}^{W} or g_{t}^{M} is set to 1 and gas separation unit is installed, that is a single phase booster or a three phase separator, at well *j* or the manifold

$$g_{tj}^{W} \leq \sum_{f} x_{tfj}^{S} \qquad \forall t \in T, j \in J \tag{3.18}$$

$$g_{t}^{M} \leq \sum_{f} x_{tf}^{S} + \sum_{s} y_{ts} \qquad \forall t \in T \tag{3.19}$$

When g_{tj}^{W} or g_{t}^{M} are set to 1, a binary variable c_{t}^{ALT} is forced to 1, triggering a onetime investment cost C^{ALT} related to the upgrade of the subsea gas transport system

$$\sum_{t_2 \in T | t_2 \leq t} c_{t_2}^{ALT} \geq \frac{g_{t}^{M} + g_{tj}^{W}}{2} \qquad \forall t \in T, j \in J \tag{3.20}$$

As single phase boosting implies gas separation, this equipment is expected to cost more than the multiphase boosters. However, if a gas separation unit already is installed where we want to place another boosting unit, the investment cost should become the same for single phase and multiphase boosters. The extra cost of installing subsea gas separation $C^{G.S}$ is kicked in by a binary variable $c_{t}^{G.SM}$ that equals 1 if gas separation is installed in time period *t* at the manifold and similarly $c_{tj}^{G.SW}$ at well *j*, formulated as

$$c_{t}^{G.SM} \geq \left(w_{tf}^{S} - \sum_{t_2 \in T | t_2 < t} \sum_{f_2 \in F} w_{t_2 f_2}^{S} - \sum_{t_2 \in T | t_2 \leq t} \sum_{s_2 \in S} z_{t_2 s_2}\right)$$
$$\forall t \in T, s \in S, f \in F \tag{3.21}$$

$$c_{tj}^{G.SW} \geq \left(w_{tjf}^{S} - \sum_{t_2 \in T | t_2 < t} \sum_{f_2 \in F} w_{t_2 j f_2}^{S}\right) \qquad \forall t \in T, j \in J, f \in F \tag{3.22}$$

Another conditional cost is connected to the installation operation itself. If more equipment is installed in the same time period, costs associated with the installation operation such as transport and production shut-down is not expected to increase proportionally with the number of units being installed in that time period. The investment cost is therefore split in two; one associated with unit specific costs and one associated with the total installation operation. The unit specific cost is given as $C_{f}^{I.BST}$ for a boosting unit of type *f* and $C_{s}^{I.SUB}$ for type *s*. The common installation operation cost is given as C^{OP}, triggered by a binary variable c_{t}^{OP} that equals one if an installation operation is initiated in time period *t,* constrained by the following equation

$$c_{t}^{OP} \geq \frac{z_{ts} + w_{tf}^{M} + w_{tf}^{S} + w_{tjf}^{M} + w_{tjf}^{S}}{5} \qquad \forall t \in T, j \in J, s \in S, f \in F \tag{3.23}$$

3.5 Modeling the New Equipment

We have to model how the new equipment affects our system conditions. We will first consider the pressure conditions, then the mass balances. The formulations of the operational costs are also presented.

3.5.1 Single Phase and Multiphase Boosting

GOR Dependent Multiphase Boosting Effect

As described in Chapter 2.4.1, the multiphase pump effect in $[bara]$ is highly dependent on the volumetric gas fraction in the multiphase flow. This should be implemented in the model in some way. This is an important aspect when multiphase pumping is evaluated together with single phase boosting, where single phase boosting represents *ideal boosting* effect accomplished by separating the gas from the liquid phase before compression and boosting. The boosting effect for multiphase pumps is modeled linearly, see Figure 3.2, depending on the ideal boosting *efficiency* E_f^{BST} for type *f* and the amount of gas in the multiphase flow q_{tjg}^{W} at well *j* or q_{tg}^{R} at the manifold relative to the gas handling capacity $Q_f^{G.BST}$ for type *f*.

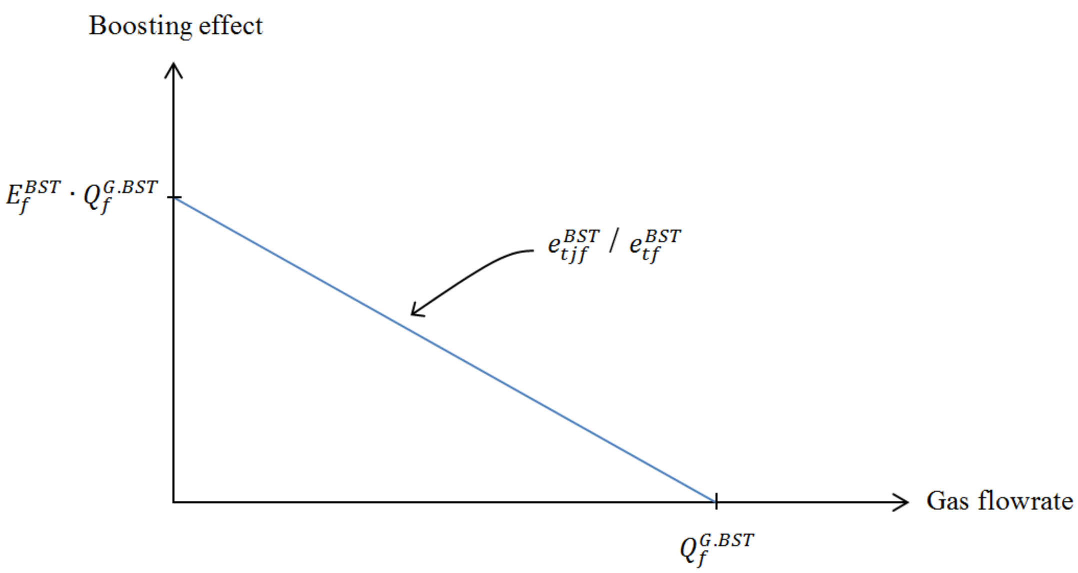

Figure 3.2: Relation between boosting effect and gas flowrate for a multiphase pump

This formulation makes the boosting effect dependent on the gas handling capacity and the gas flow through the pump, and not the relation between the gas and liquid in the multiphase flow, which is more realistic but more difficult to model linearly. This means that a pump can have the same efficiency for different GORs. However, as the pump got a liquid handling capacity $Q_f^{L.BST}$ as well, the liquid flow through the booster is constrained from becoming out of proportions compared with the gas flow. The booster gas and liquid constraints that applies are not stated here, but are found in the model summary in the appendix section.

The real boosting effect e_{tjf}^{BST} and e_{tf}^{BST} can be described as

$$e_{tjf}^{BST} = E_f^{BST} \cdot \left(Q_f^{G.BST} - q_{tjg}^{W}\right) \quad \forall t \in T, j \in J, f \in F \tag{3.24}$$

and

$$e_{tf}^{BST} = E_f^{BST} \cdot (Q_f^{G.BST} - q_{tg}^{R}) \quad \forall t \in T, f \in F \tag{3.25}$$

As the boosting effect e_{tjf}^{BST} and e_{tf}^{BST} is defined as non-negative variables, the gas flow q_{tjg}^{W} and q_{tg}^{R} is constrained by $Q_f^{G.BST}$ implicitly in (3.24) and (3.25) even if a booster is not installed. Equation (3.24) and (3.25) is therefore reformulated to deal with this inconvenient formulation

$$e_{tjf}^{BST} \leq E_f^{BST} \cdot \left(Q_f^{G.BST} - q_{tjg}^{W}\right) + E_f^{BST} \cdot Q_f^{G.BST} \cdot (1 - x_{tjf}^{M}) \cdot \mathrm{M} \quad \forall t \in T, j \in J, f \in F \tag{3.26}$$

$$e_{tjf}^{BST} \leq x_{tjf}^{M} \cdot \mathrm{M} \quad \forall t \in T, j \in J, f \in F \tag{3.27}$$

$$e_{tf}^{BST} = E_f^{BST} \cdot (Q_f^{G.BST} - q_{tg}^{R}) + E_f^{BST} \cdot Q_f^{G.BST} \cdot (1 - x_{tf}^{M}) \cdot \mathrm{M} \quad \forall t \in T, f \in F \tag{3.28}$$

$$e_{tf}^{BST} \leq x_{tf}^{M} \cdot \mathrm{M} \quad \forall t \in T, j \in J, f \in F \tag{3.29}$$

where M is large enough to avoid q_{tjg}^{W} and q_{tg}^{R} being constrained by the gas handling capacity of a booster that is not installed, for example the topside gas handling capacity.

If all of the gas is separated from the multiphase flow before it is boosted, the boosting effect becomes $E_f^{BST} \cdot Q_f^{G.BST}$, which is the boosting effect of a single phase booster of type f. This implicates that E_f^{BST} and $Q_f^{G.BST}$ is closely linked, and that these values have to be chosen carefully when we are defining the sets of pump alternatives.

Boosting Equipment's Effect on the Production System

If a booster is installed at the well equation (3.5) becomes

$$p_t^{MAN} \leq p_{tj}^{WH} + \sum_f (e_{tfj}^{BST} \cdot x_{tjf}^{M}) + \sum_f (E_f^{BST} \cdot Q_f^{G.BST} \cdot x_{tjf}^{S}) \qquad \forall t \in T, j \in J \quad (3.30)$$

This alteration will allow the wellhead pressure curve to move upwards, see Figure 3.3. If a booster is installed at the manifold equation (3.6) becomes

$$p_t^{MAN} = P^{TOP} + \Delta p_t^{R} - \sum_f (e_{tf}^{BST} \cdot x_{tf}^{M}) - \sum_f (E_f^{BST} \cdot Q_f^{G.BST} \cdot x_{tf}^{S}) \qquad \forall t \in T \quad (3.31)$$

This alteration will allow the back-pressure curve to move downwards.

Both type of installations are expected to result in higher production rates. Figure 3.3 illustrates that 20 bara boosting at the wellhead or at the manifold would increase the daily liquid production from about 7000 stb/d to 10000 stb/d, while boosting at both the wellhead and manifold would increase the production to almost 13000 stb/d.

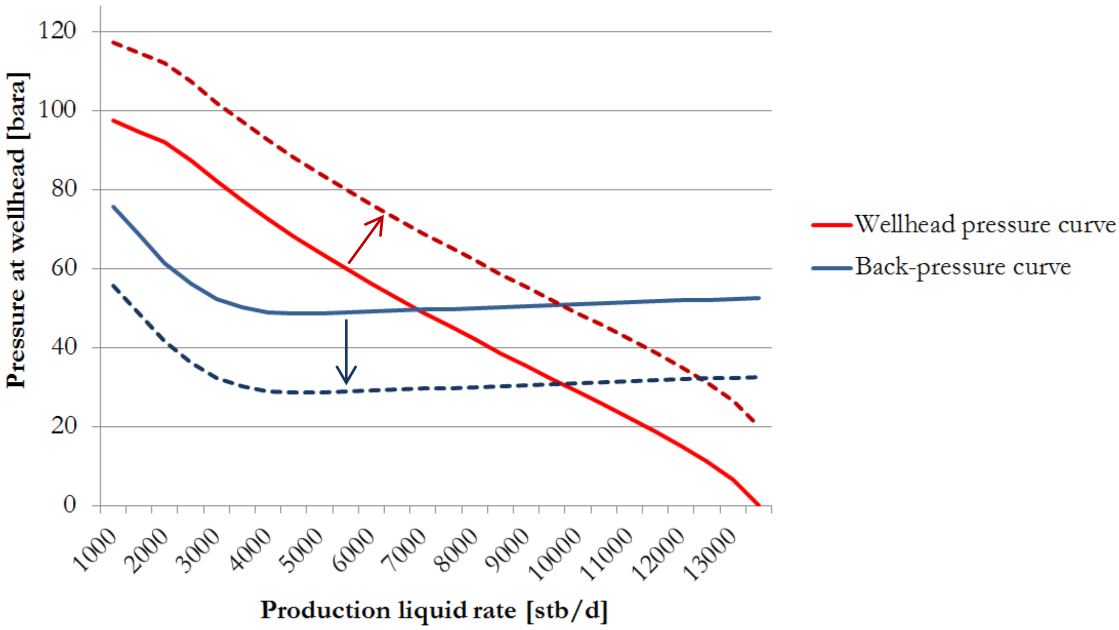

Figure 3.3: Booster effect on the wellhead pressure curve and back-pressure curve

3.5.2 Three Phase Separation and Water Re-injection

The re-injected water q_t^I presented in (3.12) is dependent on both the separation efficiency E_s^{SEP} and the water treatment- and re-injection capacity Q_s^{SEP} connected to separator type *s* in the following way

$$q_t^I = \sum_j q_{tjw}^W \cdot \sum_s (E_s^{SEP} \cdot y_{ts}) \qquad \forall t \in T \tag{3.32}$$

$$q_t^I \leq \sum_s Q_s^{SEP} \cdot y_{ts} \qquad \forall t \in T \tag{3.33}$$

This means that the flow of water *through* the separator and to the topside is not constrained by the installation of a separator,

3.5.3 Alternative Gas Export

When alternative gas export infrastructure is installed, the gas flow from the wells can be split and routed differently. The flow rate of gas $q_{tj}^{W.E}$ exported directly from well *j* in time period *t* is given as

$$q_{tj}^{W.E} \leq q_{tjg}^W \cdot g_{tj}^W \qquad \forall t \in T, j \in J \tag{3.34}$$

while the flow rate of gas $q_t^{M.E}$ exported directly from the manifold in time period *t* is given as

$$q_t^{M.E} \leq \sum_j q_{tj}^{W.M} \cdot g_t^M \qquad \forall t \in T \tag{3.35}$$

In addition to equation (3.11), the material balance is kept with $q_{tj}^{W.M}$, which is the amount of gas routed from well *j* to the manifold in time period *t*

$$q_{tj}^{W.M} = q_{tjg}^W - q_{tj}^{W.E} \qquad \forall t \in T, j \in J \tag{3.36}$$

3.5.3 Operational costs

Additional power is needed if we decide to install and run the new subsea equipment. The production asset is assumed to provide itself with power from gas driven power generators. Increased operational costs will therefore be accounted for through higher gas consumption, leading to reduced gas export and lower revenues. The operational costs, q_t^{USE}, are expressed as

$$q_t^{USE} = C_w^{TOP} \cdot q_{tw}^{R} + C_w^{SEP} \cdot q_t^{I}$$
$$+C^{BST} \sum_f (E_f^{BST} \cdot Q_f^{G.BST} \cdot \left(\sum_j \left(x_{tjf}^{M} + x_{tjf}^{S}\right) + x_{tf}^{M} + x_{tf}^{S}\right)) \quad \forall t \in T \quad (3.37)$$

where C_w^{TOP} is the topside water treatment cost in gas use per barrel of water, while C_w^{SEP} is the subsea water treatment and re-injection cost per barrel. C^{BST} is the gas use per ideal boosting effect $E_f^{BST} \cdot Q_f^{G.BST}$.

3.6 The Objective Function

A common economic decision tool is to investigate how the investment scheme affects the net present value of a project. In such a decision regime, any offshore project for improved oil recovery should deliver a positive contribution to the net present value if it is going to be realized. It should be fair to maximize the NPV of cash flows generated by sales of oil and gas, where costs affected by the new investments are the only costs. This simplifies the cost scheme and the objective function becomes

$$\max NPV = \sum_t \frac{SALES \cdot D - CAPEX}{(1+R)^t} \quad (3.38)$$

where

$$SALES = q_{to}^{L} \cdot S_o^{SALE} + q_t^{EXP} \cdot S_g^{SALE}$$

$$CAPEX = \sum_s (C_s^{I.SEP} \cdot z_{ts}) + C^{OP} \cdot c_t^{OP} + C^{G.S} \cdot (c_t^{G.SM} + \sum_j c_{tj}^{G.SW})$$
$$+ C^{ALT} \cdot c_t^{ALT} + \sum_f (C_f^{I.BST} \cdot (w_{tf}^{M} + w_{tf}^{S}) + \sum_j (C_f^{I.BST} \cdot (w_{tfj}^{M} + w_{tfj}^{S})))$$

S_p^{SALE} is the sales price for phase *p*, while R is the discount rate set by the project executive to reflect the required rate of return of the investment during one time period and *D* is the number of days in one time period. The operational costs are the amount of gas needed to generate enough power to run the equipment, both topside and subsea. Maintenance and other operational costs are assumed to be accounted for in the investment cost.

4 Solution Approach and Simulation

In the model in Chapter 3, there are several nonlinearities related to the model of the production asset. The production rates and the pressure properties are presented as functions of several variables. The water separation operation presented in (3.34) together with other formulations are also identified as nonlinear, as we are multiplying two or more variables in order to describe the process. We have several integer and binary variables as well, making this problem a mixed integer nonlinear problem, a MINLP. To make the problem easier to solve, linearization of the nonlinear functions is done in order to transform the problem into a MILP. When the problem is formulated as a MILP it can be solved with commercial solvers such as Xpress.

This section presents how the nonlinearities are handled, and introduce new formulations to the nonlinear functions that should be implemented in the model. It is also described how PIPESIM was used in order to generate the parameters needed for such a formulation.

4.1 Linearization of Functions

We identify four functions that are expected to make the optimization problem challenging to solve because of their nonlinear characteristics:

1.) The reservoir pressure as a function of reservoir contents in (3.4)

$$p_t^{RES} = f\left(q_{to}^{RES}, q_{tg}^{RES}, q_{tw}^{RES}\right) \quad \forall t \in T$$

2.) The pressure loss in the riser as a function of phase flow, given in (3.7)

$$\Delta p_t^R = f(q_{to}^R, q_{tg}^R, q_{tw}^R) \quad \forall t \in T$$

3.) Well production rates as a function of well head pressure, reservoir pressure and reservoir contents given in (3.8)

$$q_{tjp}^W = f_{jp}\left(p_{tj}^{WH}, p_t^{RES}, q_{to}^{RES}, q_{tg}^{RES}, q_{tw}^{RES}\right) \quad \forall t \in T, j \in J, p \in P$$

4.) We have made several nonlinear formulations by multiplying variables with each other. One example is the subsea water separation and re-injection operation.

4.1.1 Reservoir Pressure

The development in reservoir pressure over time is highly unpredictable and therefore challenging to model in an adequate way. This project will only implement a simple linear dependency between the reservoir contents and the reservoir pressure. Important for this model is that injection of water gives a positive contribution to the maintenance of reservoir pressure. Higher reservoir pressure gives higher production rates, and makes injection of water potentially profitable. The Marlim field, among most producing fields, is expected to experience a decline in reservoir pressure during the production lifetime, and a function supporting this should also be included in the reservoir pressure model. The function is stated as

$$p_t^{RES} = P^{BAS} + \sum_p Y_p \cdot q_{tp}^{RES} \qquad \forall t \in T \tag{4.1}$$

where Y_p is a factor multiplied with the contents of phase p in the reservoir in period t. The contribution to reservoir pressure is summed for all p, and added to a basal reservoir pressure P^{BAS}.

4.1.2 Pressure Loss

The function for pressure loss in the riser line is approximated with a piecewise linear function in three dimensions, similar to the approximation done by Gunnerud (2011). Breakpoints for the flow rate of each component in the riser are given in the sets *L*, *M* and *N*: for oil flow, gas flow and water flow respectively. *l*, *m* and *n* are the associated indices for the breakpoints. Q_{ol}^R is the flow of oil for breakpoint *l*, Q_{gm}^R is the flow of gas for breakpoint *m*, and Q_{wn}^R is the flow of water for breakpoint *n*. The flowrate in the riser can be expressed as a linear combination of these breakpoint values. The pressure loss Δp_t^R can now be expressed as a linear combination of ΔP_{lmn}^R, which is the pressure loss in the riser line for breakpoints *l*, *m* and *n*.

$$\Delta p_t^R = \sum_l \sum_m \sum_n \Delta P_{lmn}^R \cdot \gamma_{tlmn} \qquad \forall t \in T \tag{4.2}$$

$$q_{to}^R = \sum_l \sum_m \sum_n Q_{ol}^R \cdot \gamma_{tlmn} \qquad \forall t \in T \tag{4.3}$$

$$q_{tg}^R = \sum_l \sum_m \sum_n Q_{gm}^R \cdot \gamma_{tlmn} \qquad \forall t \in T \tag{4.4}$$

$$q_{tw}^R = \sum_l \sum_m \sum_n Q_{wn}^R \cdot \gamma_{tlmn} \qquad \forall t \in T \tag{4.5}$$

γ_{tlmn} is introduced as a weighting variable with the following restrictions

$$\sum_l \sum_m \sum_n \gamma_{tlmn} = 1 \qquad \forall t \in T \tag{4.6}$$
$$\gamma_{tlmn} \geq 0 \qquad \forall t \in T, l \in L, m \in M, n \in N \tag{4.7}$$

To obtain a correct description of the function, SOS2 restrictions are needed in every dimension. Three auxiliary weighing variables are introduced in order to define the SOS2 sets, restricted by the following equations

$$\rho_{tl} = \sum_m \sum_n \gamma_{tlmn} \qquad \forall t \in T, l \in L \tag{4.8}$$
$$\sigma_{tm} = \sum_l \sum_n \gamma_{tlmn} \qquad \forall t \in T, m \in M \tag{4.9}$$
$$\tau_{tn} = \sum_l \sum_m \gamma_{tlmn} \qquad \forall t \in T, n \in N \tag{4.10}$$
$$\rho_{tl}, \sigma_{tm}, \tau_{tn} \geq 0 \qquad \forall t \in T, l \in L, m \in M, n \in N \tag{4.11}$$

$$\rho_{tl}\ is\ SOS2\ for\ l \qquad \forall t \in T \tag{4.12}$$
$$\sigma_{tm}\ is\ SOS2\ for\ m \qquad \forall t \in T \tag{4.13}$$
$$\tau_{tn}\ is\ SOS2\ for\ n \qquad \forall t \in T \tag{4.14}$$

4.1.3 Well Production Rates

The function for production flow is approximated as a piecewise linear function in two dimensions combined with a linear term. To avoid a SOS2 formulation in five dimensions, which potentially can lead to very large problems, K_{jp}^{O}, K_{jp}^{W} and K_{jp}^{G} are used as slopes in a linear description of the reservoir content's effect on the production rates. For example, K_{jp}^{O} is interpreted as the change in flow rate of phase *p* from well *j* if the oil content in the reservoir changes one unit. This means that changes in the reservoir contents are assumed to have a linear relationship with the composition of the production flow that is withdrawn from the reservoir. *A* and *B* describes the sets of breakpoints in the SOS2 formulation for wellhead pressure and reservoir pressure, respectively, and *a* and *b* are the associated indices to the breakpoints.

$$q_{tjp}^{W} = \sum_a \sum_b Q_{jpab}^{W} \cdot \lambda_{tjab} + K_{jp}^{O}(Q_o^{INIT} - q_{to}^{RES}) +$$
$$K_{jp}^{W}(Q_g^{INIT} - q_{tg}^{RES}) + K_{jp}^{G}(Q_w^{INIT} - q_{tw}^{RES}) \qquad t \in T, j \in J, p \in P \tag{4.15}$$
$$p_{tj}^{WH} = \sum_a \sum_b P_a^{WH} \cdot \lambda_{tjab} \qquad \forall t \in T, j \in J \tag{4.16}$$
$$p_t^{RES} = \sum_a \sum_b P_b^{RES} \cdot \lambda_{tjab} \qquad \forall t \in T, j \in J \tag{4.17}$$

where Q^{W}_{jpab} is the production rates in well *j* of phase *p* for breakpoints *a* and *b*, P^{WH}_{a} is the wellhead pressure in well *j* for breakpoint *a*, and P^{RES}_{b} is the reservoir pressure for breakpoint *b*. The wellhead pressure and the reservoir pressure can be expressed as a linear combination of these breakpoint values. The well production rate q^{W}_{tjp} can now be expressed as a linear combination of Q^{W}_{jpab}, which is the well production rate for the breakpoints *a* and *b*.

λ_{tjab} is introduced as a weighting variable with the following restrictions

$$\sum_a \sum_b \lambda_{tjab} = 1 \qquad \forall t \in T, j \in J \tag{4.18}$$

$$\lambda_{tjab} \geq 0 \qquad \forall t \in T, j \in J, a \in A, b \in B \tag{4.19}$$

Two auxiliary weighing variables are introduced in order to define SOS2 sets

$$\alpha_{tja} = \sum_b \lambda_{tjab} \qquad \forall t \in T, j \in J, a \in A \tag{4.20}$$

$$\beta_{tjb} = \sum_a \lambda_{tjab} \qquad \forall t \in T, j \in J, b \in B \tag{4.21}$$

$$\alpha_{tja}, \beta_{tjb} \geq 0 \qquad \forall t \in T, j \in J, a \in A, b \in B \tag{4.22}$$

$$\alpha_{tja}\ is\ SOS2\ for\ a \qquad \forall t \in T, j \in J \tag{4.23}$$

$$\beta_{tjb}\ is\ SOS2\ for\ b \qquad \forall t \in T, j \in J \tag{4.24}$$

The production from each well can now be reduced by increasing the wellhead pressure. Sometimes we even want to close the production entirely. Now that the K-terms are introduced, choking the well would not be sufficient to achieve zero production of every phase from the well. We introduce an artificial flow, q^{ART}_{tjp}, together with a binary variable w^{RUN}_{tj} that is equal to 1 if well *j* is operating in time period *t*. The artificial flow is added on the left side of equation (4.15), giving

$$q^{W}_{tjp} + q^{ART}_{tjp} = \sum_a \sum_b Q^{W}_{jpab} \cdot \lambda_{tjab} + K^{O}_{jp}(Q^{INIT}_{o} - q^{RES}_{to}) + K^{W}_{jp}(Q^{INIT}_{g} - q^{RES}_{tg}) + K^{G}_{jp}(Q^{INIT}_{w} - q^{RES}_{tw}) \qquad t \in T, j \in J, p \in P \tag{4.25}$$

The following restrictions apply in order to be able to stop the well flow entirely

$$-M_p\left(1 - w^{RUN}_{tj}\right) \leq q^{ART}_{tjp} \leq M_p\left(1 - w^{RUN}_{tj}\right) \qquad \forall t \in T, j \in J, p \in P \tag{4.26}$$

$$q^{W}_{tjp} \leq M_p \cdot w^{RUN}_{tj} \qquad \forall t \in T, j \in J, p \in P \tag{4.27}$$

where M_p is set to be the topside capacity for the corresponding phase. The artificial flow will only appear when w_{tj}^{RUN} is set to zero, and has no effect on the material balance in the reservoir and in the production system.

4.1.4 Other Nonlinear Formulations

All of the nonlinear formulations can be made linear by taking advantage of some known system capacities, e.g. the topside gas handling capacity or the maximum boosting effect for a given multiphase pump. These capacities can be used to make big M formulations as described in Williams (1999). An example is given here for the subsea water separation and re-injection operation from equation (3.32). To see the rest of the reformulations, the reader is referred to the model summary in the appendix section.

We can take advantage of the known water handling capacity Q_s^{SEP}, and make a big M formulation that confine q_t^I in a linear manner where $M = Q_s^{SEP}$

$$q_t^I \leq \sum_s Q_s^{SEP} \cdot y_{ts} \qquad \forall t \in T \tag{4.28}$$
$$q_t^I \leq \sum_j q_{tjw}^W \cdot E_s^{SEP} + Q_s^{SEP} \cdot (1 - y_{ts}) \qquad \forall t \in T, s \in S \tag{4.29}$$

Constraint (4.28) is the same as the one presented in (3.32), and forces q_t^I to zero if no separator is installed. Constraint (4.29) is inactive for any case where y_{ts} is zero. If a separator is installed, (4.28) becomes active if $\sum_j q_{tjw}^W \cdot E_s^{SEP}$ is greater than Q_s^{SEP}. The material balance from (3.12) is also required to be sure that the big M formulation works.

4.2 Simulations

The PIPESIM software allows us to run different types of simulation operations on both single well systems and complete production networks. The manifold cluster considered in this project consists of 7 wells connected to one manifold and riser unit. Only 4 of the wells were producing at the time when production data was gathered, and the simulations and computations will therefore only consider a system where there are 4 producing wells. The simulation procedures and the optimization model can without extensive modifications also be applied for a case where all 7 wells are producing. The following sections describe how the well models are generated, and how simulations are carried out to generate parameters for the formulations presented in Section 4.1.

4.2.1 Generation of the Well Models

Each of the wells is modeled in PIPESIM as a vertical single branch well, and then imported in a master network model of the complete production system. The wells are modeled from the well perforations to the subsea manifold, while the flow line and riser from the subsea manifold to the FPSO are modeled in another file, as shown in Figure 4.1. All the required design data are typed into an Excel sheet. In order to generate the single branch files we use a communication tool called Open Link. By importing Open Link libraries into the Visual Basic editor in Excel, one can write a VBA code that generates the well models based on the data given in a spread sheet. This can be done for all the wells simultaneously. The Excel sheet provides a good overview of all the properties of the production asset, and alteration of the single branch files can easily be done by changing one or more values in the table, generating new files that replace the old ones.

4.2.2 Finding Data Sets for the SOS2-formulations

The breakpoint values used to express the pressure loss and the production rates in equations (4.2)-(4.5) and (4.15)-(4.17) are found by performing sensitivity analysis on the well models in PIPESIM. The K-values used in the production rate formulations are a linear simplification of how the production rates are changing when watercut and GOR is changed in the simulation runs for each well, where two points are enough to determine a gradient described by these K-values.

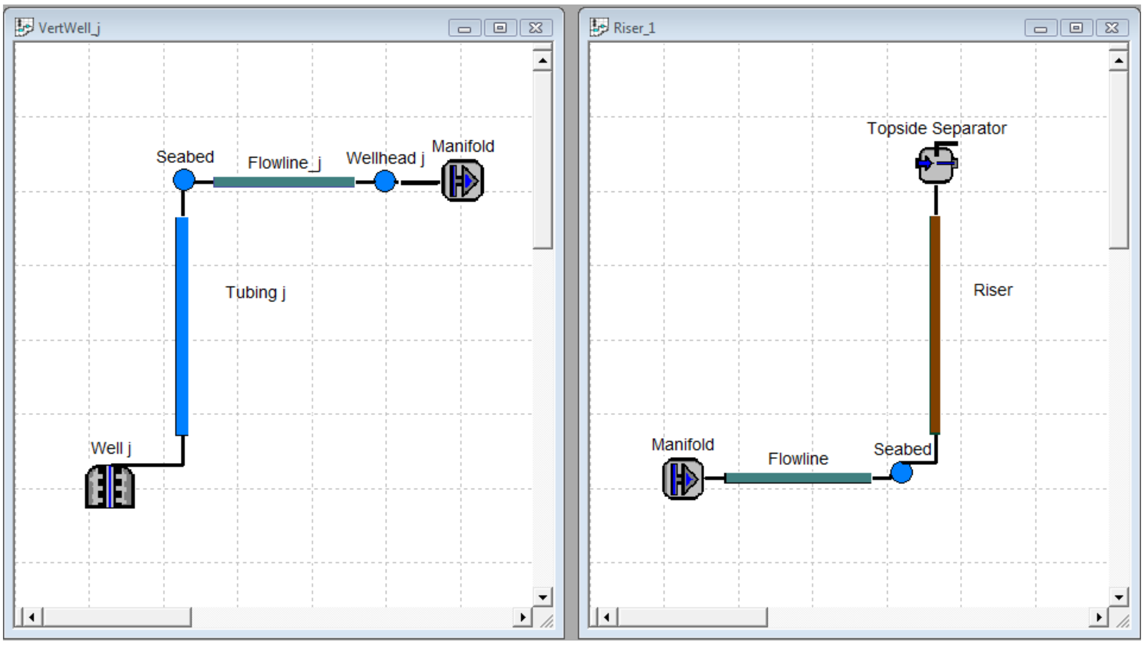

Figure 4.1: Templates for the well model and the riser model generated in PIPESIM

5 Results and Computational Studies

The model presented in Chapter 3 is a mathematical formulation of a simple system structure with complex properties associated with the physics and the dynamics of the different variables. We have used both continuous and binary variables in our description, and identified some nonlinear properties. Chapter 4 presented a solution strategy to make the problem solvable in the mixed integer linear solver Xpress. This chapter describes how the model is implemented and presents the solutions found for the planning problem. Some analysis and discussion of the results and the model properties are presented afterwards.

5.1 Implementation

The mathematical problem described in Chapter 3 and 4 is formulated as a MILP, and solved with Xpress-MP using the Mosel modeling language. Xpress-MP is widely used in optimization courses at the IØT department, and this is the main reason for its use in this work. In addition, the Xpress-IVE interface is good for understanding and learning with its graphical presentation of the optimization process.

The implementation consists of the parameters, variables and constraints presented in Chapter 3 and 4 for a case with 4 wells and 10 time periods. All datasets are written in *data_water driven.txt*, and loaded into the solver and solved directly in *code.mos*. The program is run on a HP dl165 G64 computer with two AMD Opteron 2.4 GHz processors and 24GB RAM, with a maximum program runtime set to 3600 seconds. The complete input dataset can be found in Appendix B: Data Tables, and electronically together with the program code in Appendix C: Electronic Attachments.

5.1.1 Choosing the Sets of Breakpoints in the Production Rates Formulation

The piecewise linearization describes the production rates in each well for every combination of reservoir pressure and wellhead pressure, together with the pressure drop for every combination of flow rates of oil, gas and water that flows through the riser. The algorithm interpolates linearly between the neighboring breakpoints if a pressure drop or a production rate lies between any of the breakpoints.

Figure 5.1 presents oil production as a function of wellhead pressure for each well given a reservoir pressure of 260 bara and the initial reservoir contents. We see that some wells are acting in a rather linear way, while others are more difficult to linearize. The wells are however expected to be producing in the more linear parts between 20 and 80 bara wellhead pressure during the decision period.

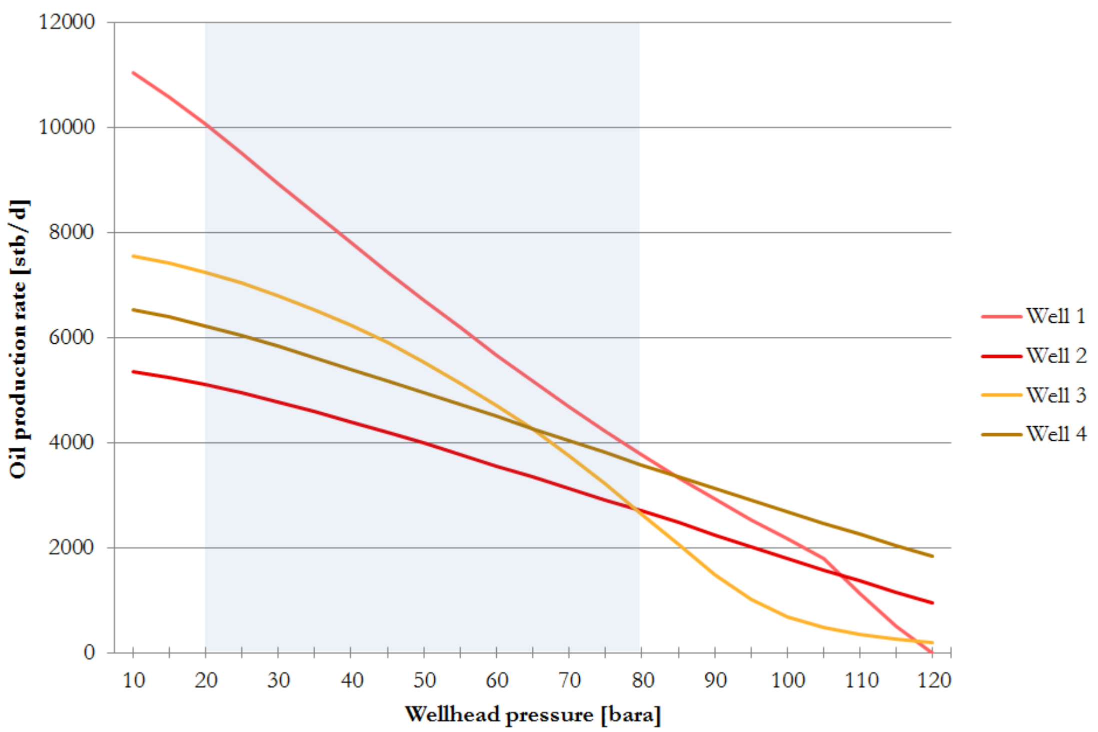

Figure 5.1: Oil production for each well at p_t^{RES} = 260 bara with initial reservoir contents

A breakpoint for zero production seems not to be necessary, since the formulation allows us to turn off a well by the use of w_{tj}^{RUN}. A maximum wellhead pressure at 80 bara is chosen as this is assumed to provide enough choking to sufficiently reduce the production from any of the four wells during the decision period. A minimum wellhead pressure is chosen to be 20 bara, as lower pressures in the subsea pipes might lead to dangerous conditions due to the high hydrostatic pressure on the outside. The range of wellhead pressures is evenly discretized into four breakpoints, and set A is defined as

$$A \in \{20,40,60,80\} \quad [bara] \tag{5.1}$$

We also have to choose breakpoints for the reservoir pressure. The relation between oil production rates and reservoir pressure is found to be rather linear within the range of wellhead pressures defined by A. The basal reservoir pressure P^{BAS} is set to 200 bara, and defines the minimum reservoir pressure possible in our formulation, while 260 bara is just above the reservoir pressure of today, which is assumed to be the maximum reservoir pressure that will be experienced. This range is evenly discretized into three breakpoints. Figure 5.2 shows how oil production from well 1 is affected by changes in the reservoir pressure, and how breakpoints are placed when B is defined as

$$B \in \{200,230,260\} \quad [bara] \tag{5.2}$$

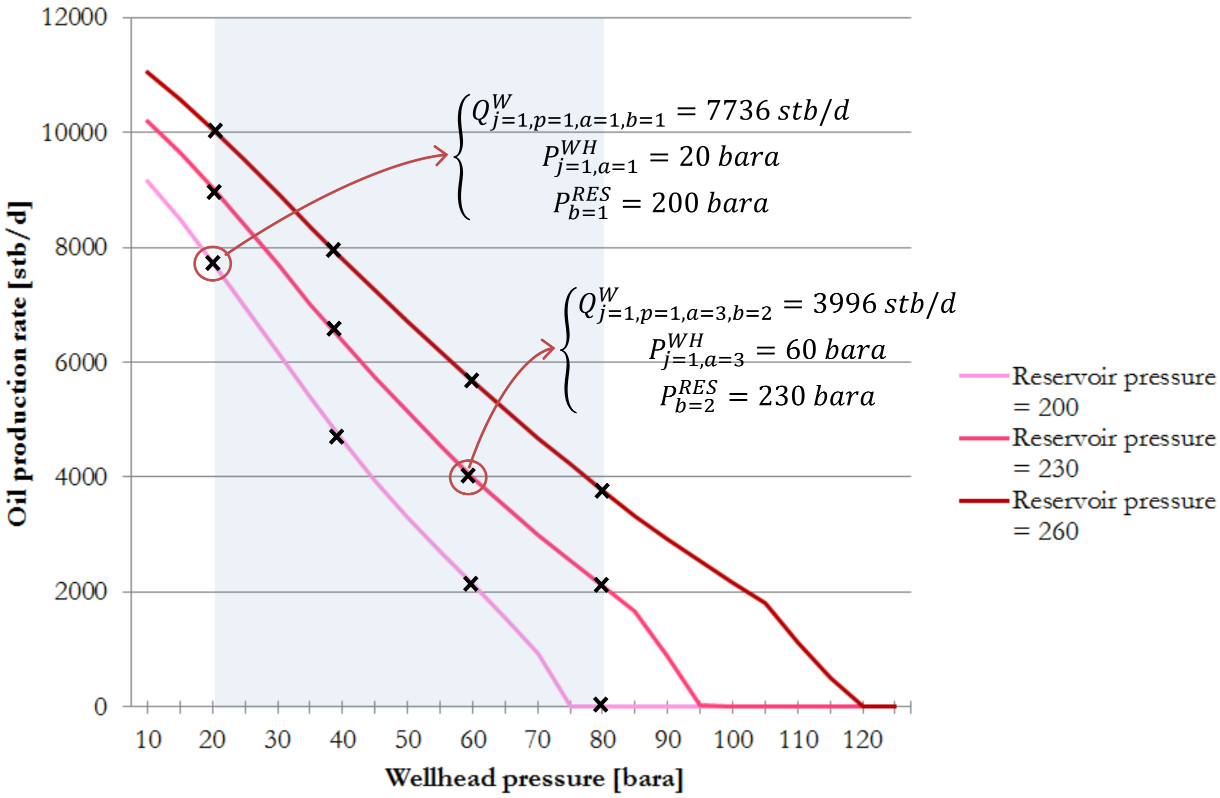

Figure 5.2: Oil production rates for well 1 at different reservoir- and wellhead pressures

5.1.2 Choosing the Sets of Breakpoints in the Pressure Loss Formulation

Choosing breakpoints in the pressure loss formulation is somewhat easier since we only have one riser. But again, one more dimension in the SOS2 formulation means that the problem size is more sensitive to the number of breakpoints for the flow rates of oil, gas and water in L, M and N. Figure 5.3 illustrate one of many function plots, where the pressure loss for different flow rates of oil and water for a fixed gas flow of 12 mmscf/d is presented.

The pressure loss function appears to be more difficult to linearize with few breakpoints, as the function plots have shown signs of nonlinear behavior such as nonsymmetrical properties and sudden drops in the function surfaces. The topside capacities for liquid, water and gas helps us limit the range of the flowrates in the riser, while initial model runs show approximately the range in which our production system is producing. The flow of oil is expected to lie somewhere between 25000 and 10000 stb/d, while the topside water treatment capacity $Q^{W.TOP}$ is constraining the maximum flow of water, which is assumed to lie between 15000 and 20000 stb/d. Several different function plots have been made and investigated, and the area defined by this information shows to be quite easy to linearize for gas flow rates of 12, 6 and 0 mmscf/d.

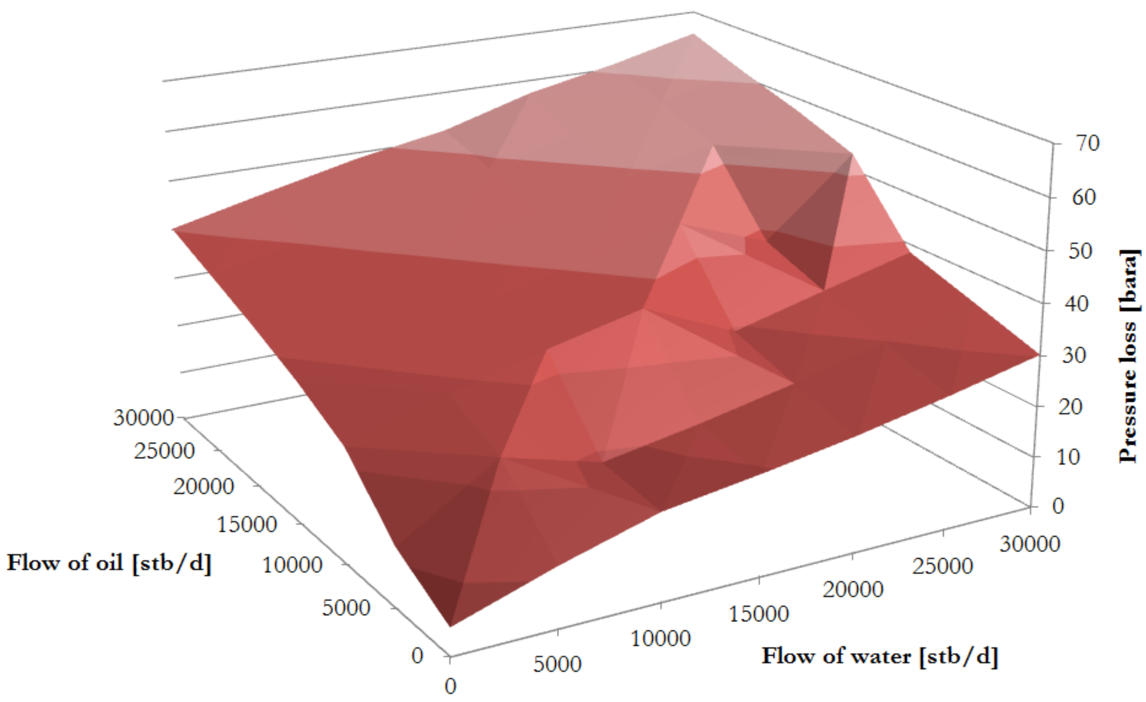

Figure 5.3: Pressure loss for different flowrates of oil and water with q_g^L = 12 mmscf/d

Few breakpoints is expected to be a key factor in order to keep the solution time down, and a rather rude approximation with three breakpoints in each dimension is chosen. Figure 5.4 presents the simplified function surface for a gas flow of 12 mmscf/d, where the following sets of breakpoints are chosen

$$L \in \{30000,10000,0\} \quad [\frac{stb}{d}] \tag{5.3}$$

$$M \in \{12,6,0\} \quad [\frac{mmscf}{d}] \tag{5.4}$$

$$N \in \{20000,10000,0\} \quad [\frac{stb}{d}] \tag{5.5}$$

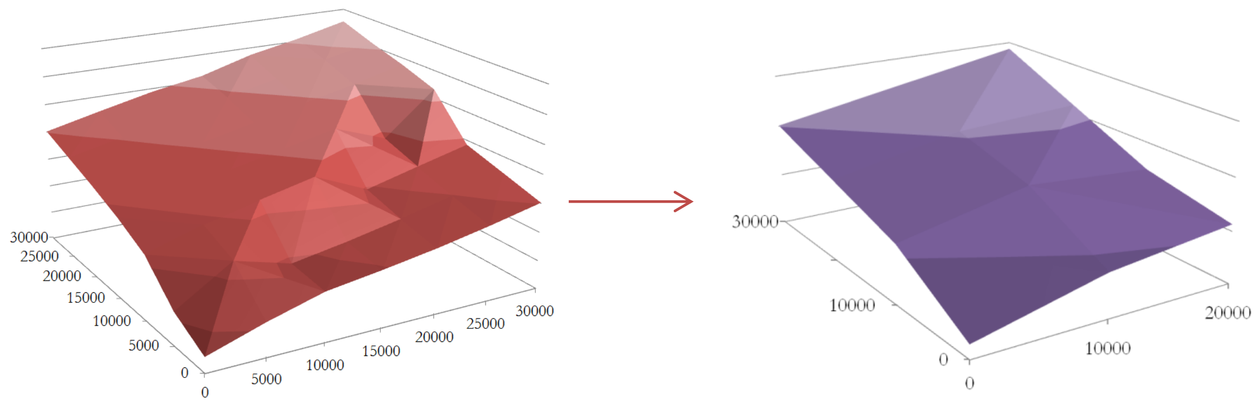

Figure 5.4: The simplified description of pressure loss used in the model runs

5.1.2 Determination of Other Parameters

In addition to the data tables generated in PIPESIM, the case also needs to be defined by values for investment and operating costs, sales prices, discount rates and other parameters. Some values are possible to find based on simple research and estimates, while others may be more challenging to determine. The petroleum sales price is set to today's market prices for all time steps in the decision period.

In the study of Singh & Hannaford (2007) subsea separation and gas compression are investigated to reduce the back-pressure experienced at the wells. Subsea separation of gas was estimated to cost approximately \$1M 100, while subsea compression would cost additionally \$1M 100. The capacity of this equipment is expected to be way too large for our case, so we design a portfolio that better suits our expected needs with lower capacities and lower prices. The bigger the portfolio, the better the suggestions for optimal investment scheme will become, but the size of the portfolio is also limited by problem solvability.

Table 5.1 presents the portfolio that was chosen to be available in the model runs. We remember that C denotes the investment costs, Q the flow capacities and E the efficiencies of the equipment. The subsea three phase separator is mainly defined by the water treatment capacity and its separation efficiency, while the boosters are defined by both liquid and gas capacity as well as boosting efficiency. The effect in bara is set to $Q_f^{G.BST} \cdot E_f^{BST}$ = 7.5 bara for all booster types in the portfolio. Ramberg (2007) suggests that there is a tradeoff between liquid handling capacity and the boosting effect. The portfolio presented in this paper is therefore chosen to describe one type of booster where the flow capacities increase with the cost while the boosting effect remains the same.

Subsea three phase separator and injector

s	1	2	3	4	5	6	7	8	9
$C_s^{I.SUB}$ [$1M]	50	60	70	55	65	75	60	70	80
$Q_s^{W.SUB}$ $[\frac{stb}{d}]$	14	14	14	16	16	16	18	18	18
$E_s^{W.SUB}$ [-]	0.7	0.8	0.9	0.7	0.8	0.9	0.7	0.8	0.9

Subsea boosters

f	1	2	3	4	5	6	7	8
$C_f^{I.BST}$ [$1M]	20	25	30	35	70	80	90	100
$Q_f^{L.BST}$ $[\frac{stb}{d}]$	13	15	18	20	36	42	48	54
$Q_f^{G.BST}$ $[\frac{mmscf}{d}]$	3	4	5	6	10	11	12	13
E_f^{BST} $[\frac{bara}{mmscf}]$	2.50	1.88	1.50	1.25	0.75	0.68	0.63	0.58

Table 5.1: Available equipment portfolio

5.2 Results

Several combinations of subsea processing systems are possible, and the final selection will depend on the multiphase flow composition, pressure conditions and production system topography. The number of possible solutions is potentially very high, and the searching process for good solutions is far from intuitive. The model is run for two different reservoirs: a water driven and a gas driven. It is also run with different time horizons and different variants of equipment portfolio, as well as different sets of breakpoints in the SOS2-formulations. All of the cases are based on the same production system, which is assumed to produce from an isolated reservoir without any influence from other assets. The Xpress solver is able to run the model close to optimality for most of the problems within one hour, and the following sections will present some of the results from these runs.

5.2.1 Solution for a Water Driven Reservoir

This case consists of 4 wells producing from a water driven reservoir. The reservoir conditions are changing with time, and are dependent on the production from each well, natural influx and the amount of water re-injected into the reservoir. Results from the program run are presented in Figure 5.5 and 5.6 showing the production plan for each well and the flow of components to the topside compared to the solution for a case where no new investments are done.

The solution suggests installing one single phase booster of type 7 and one of type 8 at the manifold in year one, together with a three phase separator of type 8. Well 1 should get a single phase booster of type 2 and 3 installed in year one, together with a single phase booster of type 4 in year two. A single phase booster of type 1 and 2 should be installed at well 4 in year one, while well 3 should get a single phase booster of type 2 in year one and one of type 3 in year two. No alternative gas transport is suggested.

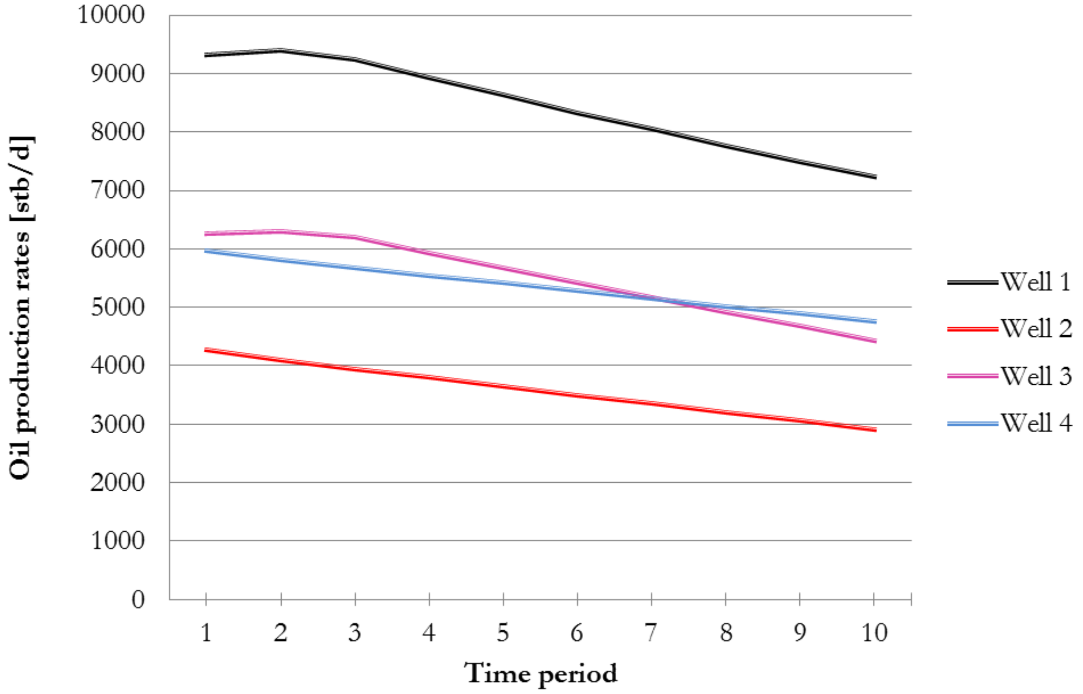

Figure 5.5: Production plan for a water driven reservoir

Compared to a production system without investment options, the total oil production is increased by 66% during the decision period by enabling installation of new equipment. The NPV is increased from $1M 3376 to $1M 4809 resulting in a $1M 1433 surplus, an increase of 42%.

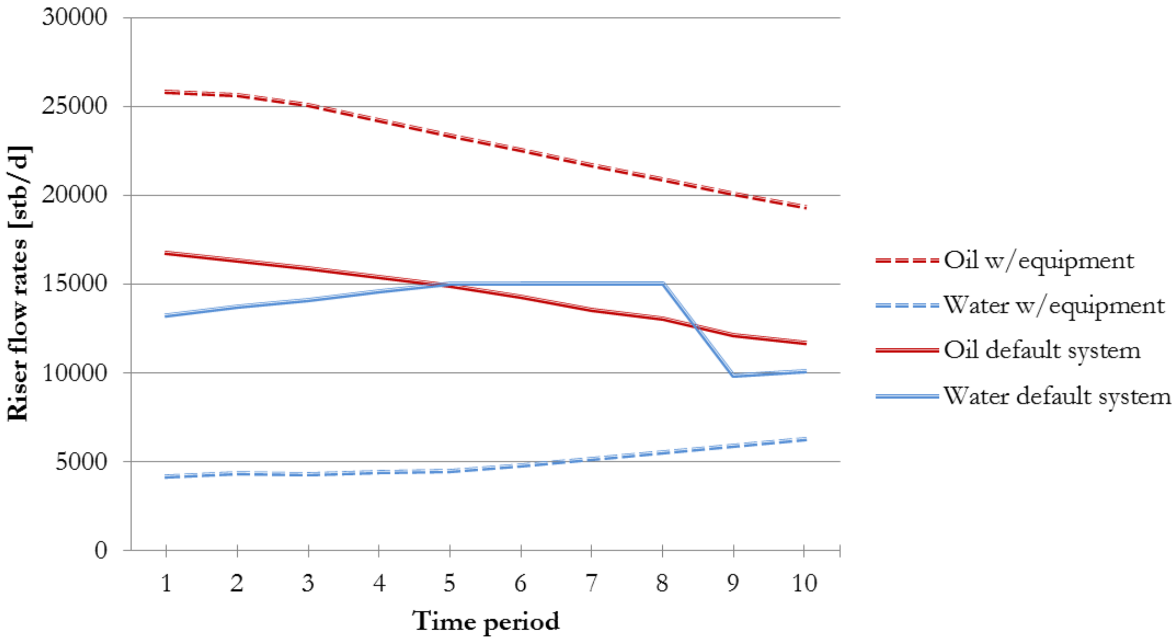

Figure 5.6: The flow of oil and water in the riser with and without new equipment

Another interesting result is how the wellhead pressures are regulated in the two different cases. From figure 5.6 we see that the topside water handling capacity, which is set to be 15000 stb/d in this case, is reached in year 5, making it necessary to choke the production down. We even decide to close well 2 entirely during time period 9 and 10 in order to free water handling capacity and to prioritize the better performing wells. Figure 5.7 illustrates how boosters and chokes are used to regulate the production in an optimal way for both cases. Boosting is here defined as a positive manipulation of the wellhead pressure, while choking is defined as a negative manipulation.

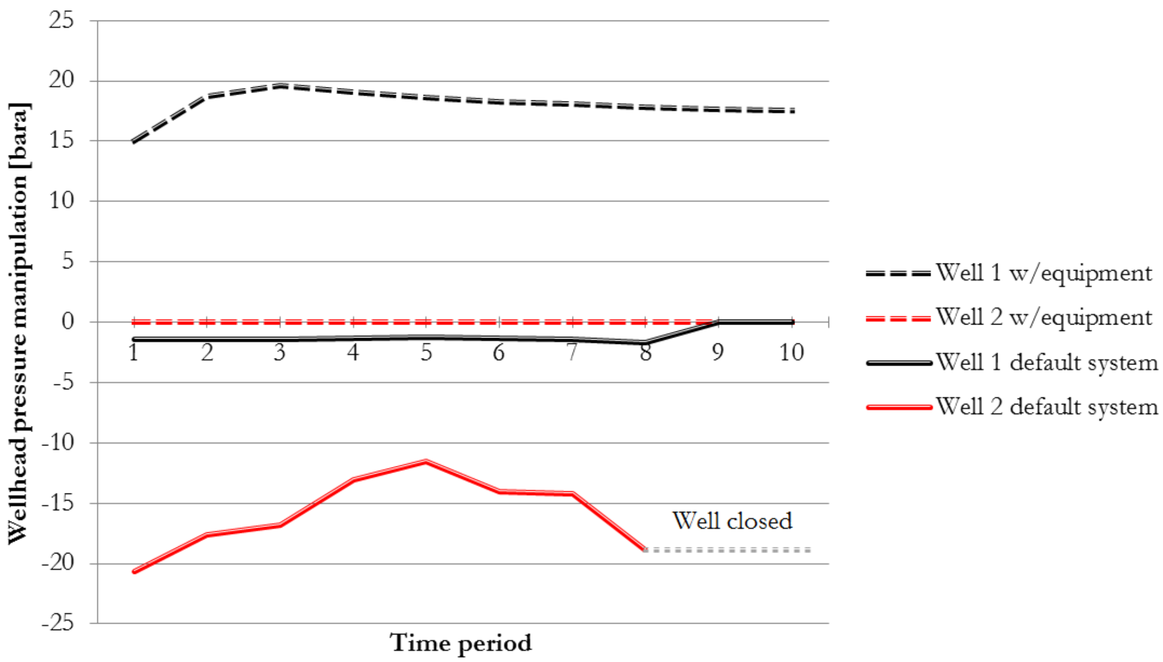

Figure 5.7: Use of boosting and choke at well 1 and 2 with and without new equipment

5.2.2 Solution for a Gas Driven Reservoir

As explained in Section 2.3.3, another common type of reservoir is the gas driven reservoir. Its characteristics are expected to introduce the production system to new challenges such as gas handling capacities and a more rapid decrease in reservoir pressure. By changing some of the parameters in the input data, the model can now be used to solve a different scenario. Results from the program run are presented in Figure 5.8, showing the production plan during the decision period.

The solution suggests installing one single phase booster of type 5, 6, 7 and 8 at the manifold in year one together with a three phase separator of type 9, while well 1 should get a single phase booster of type 3 installed in year one. Alternative gas routing is enabled from the manifold in year one. When new gas export pipes are installed and gas separation is done subsea, gas could be exported directly from the seabed. As we have chosen to install a single phase pump of type 5 at the manifold, the gas handling capacity will be reduced from the topside capacity of 12 mmscf/d down to the pump capacity of 10 mmscf/d. Excess gas is then exported directly from the sea bottom to avoid choking of the production due to limited gas capacities downstream the system. For the gas driven reservoir, the topside water capacity is no longer a constraint, and the watercut remains more or less the same during the production period, see Figure 5.9.

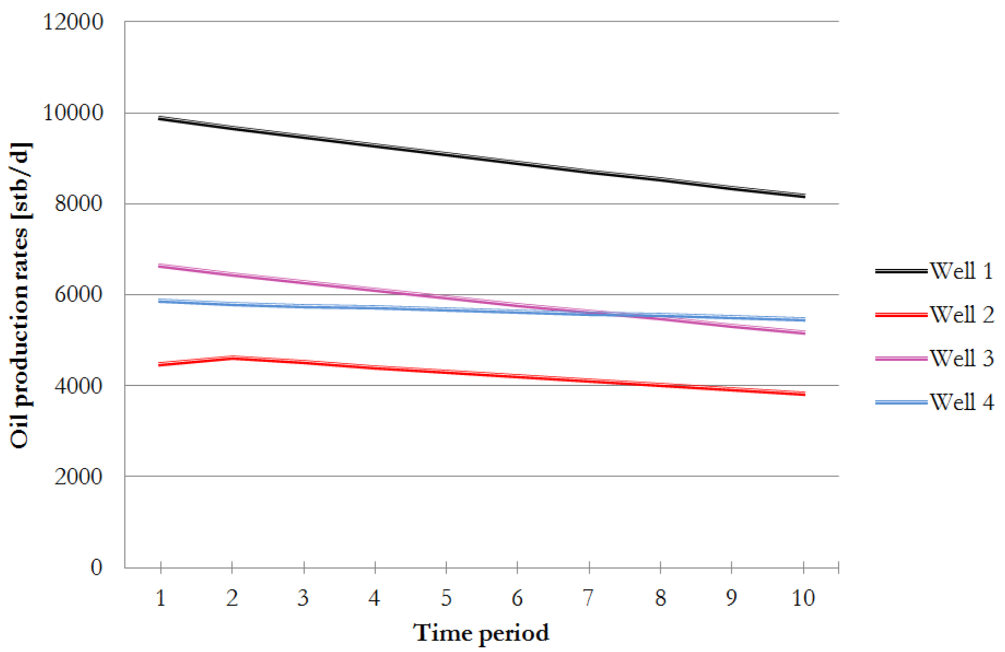

Figure 5.8: Production plan for a gas driven reservoir

Compared to a production system without investment options, the total oil production is increased by 52% during the decision period by enabling installation of new equipment. The NPV increased from $1M 3823 to $1M 5057 resulting in a $1M 1234 surplus, an increase of 32%.

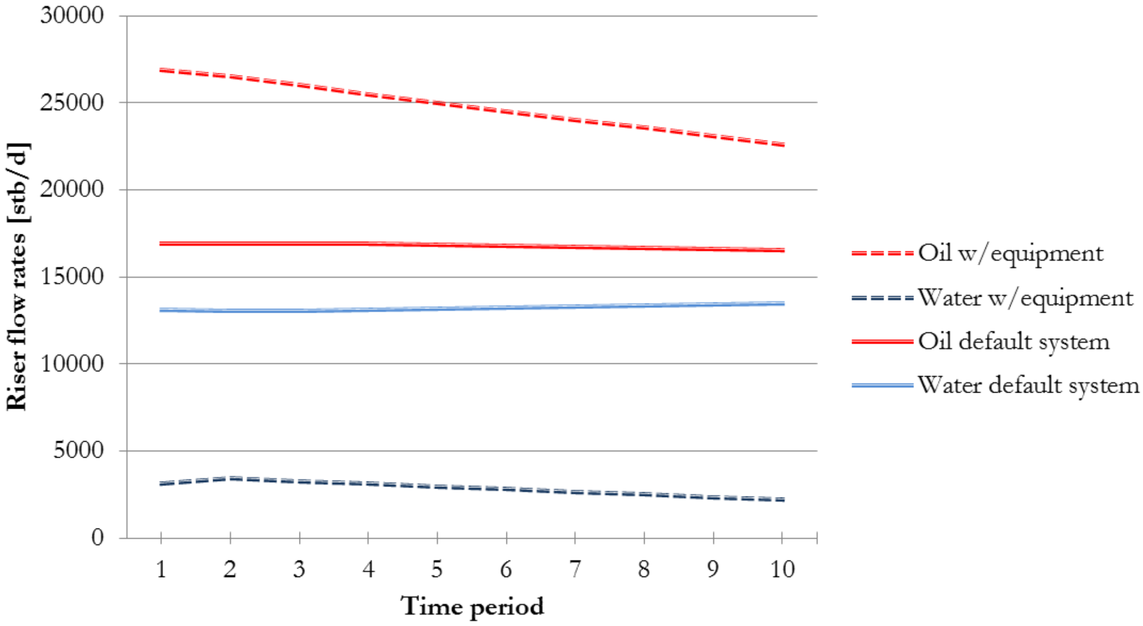

Figure 5.9: The flow of oil and water in the riser with and without new equipment

5.2.2 Other Results

The water driven case is solved again with equipment portfolios that are less efficient and more expensive in order to investigate the effects on the investment scheme and the resulting NPV. For the water separators investment costs were raised by 50% and the water treatment capacity reduced by 20%. The booster effect was reduced by 50%, from 7.5 bara to 3.75 bara, together with a 50% increase in the investment costs. The solution is found to be different than for the original case, where two water separators and two boosters smaller than in the original case are installed at the manifold in year one, while well 1 is prioritized and is the only well that gets boosters installed, this also in year one. Compared to a default production system, the total oil production is increased by 53%, while the NPV increased from \$1M 3376 to \$1M 4068 resulting in a \$1M 692 surplus, an increase of 20%.

Solutions are found for decision periods from 2 years up to 15 years as well. A program run with 15 time periods proves more difficult to solve and it is very difficult to make the optimality gap become lower than 13% even if we run the program for several hours. The model is also run with an increased number of breakpoints in the SOS2 formulations, making it possible to investigate the tradeoff between function approximation quality and problem solution time. Table 5.2 presents some results from these program runs

	NPV [$1M]	Run time [s]	Optimality gap
Original water driven	4809	3600	0.71 %
Gas driven	5057	1163	0.01 %
Poorer equipment	4068	3600	4.53 %
5 year decision period	2995	39	0.01 %
15 year decision period	5695	3600	15.88 %
2 × no. of breakpoints	4618	3600	16.69 %

Table 5.2: Results for different variants of the original water driven problem

5.3 Analysis and Discussion

The model provides us with detailed information about the system conditions during the decision period, as it is determining the values of all variables in the mathematical model of the system. These variables describe what decisions that should be made during the decision period. They also describe production conditions and the resulting production, making it possible to verify the solution and its realism. This could be done by comparing the results with similar studies, together with simulation runs in PIPESIM or other simulators. This section aims to analyze the results presented in the previous section, and discuss some of the properties of the model that is of importance for these results.

5.3.1 Solution Quality and Realism

The results suggest that new equipment should be installed primarily during the first time period. Choosing to make improving investments as fast as possible seems reasonable as long as the investment decision can be justified by surplus in the future cash flows. Maximizing NPV in offshore petroleum projects may have a tendency of resulting in decisions where as much as possible is produced as fast as possible, since the value of petroleum sold today is higher than petroleum sold ten years from now. The case presented in this project does not seem to be an exception. Low initial watercuts, together with rate dependent reservoir recovery are examples of technical aspects that are making this decision not so obvious. However, the new equipment may prove so beneficial that installing an optimal combination of them is never a question, as long as the technology exists and the price of it is somewhat reasonable. Changes in the price and cost regime or the technology itself might have an influence on the investment decision. A case where better and cheaper technology is introduced in later time periods might give the decision maker incentives to postpone the investment. Changes in sales prices is also expected to affect the decision, as decreased prices and decreased revenues may fail to defend the investment. Another obvious reason for postponing the investments is if the topside oil handling capacity is limited. In that case we have to wait until the production is low enough, i.e. we have enough slack in the oil handling capacity to handle the increased oil flow after an investment is done.

The model lacks the ability to reward the potential for improving the recovery factor. It says that new equipment has the ability to enhance the performance of the production system, which in turn results in higher production rates and potentially better conservation of reservoir pressure. Since the content of the reservoir is described as reserves, and not resources, the reservoir will run empty faster if the production rates increase. This will of course be a good solution if we are maximizing NPV, but not necessarily the best way to exploit the hydrocarbons present in the reservoir.

If we study the investments that are suggested for the original water driven case, we see that two booster units are installed at the manifold and at well 3 and 4, while three boosters are installed at well 1. This may indicate that the equipment portfolio is somewhat under dimensioned for our case when it comes to the pressure differential that the boosters provide, since the model finds it beneficial to install more than 7.5 bara pressure differential. The model allows only one installation of each equipment type, which explains why boosters of type 3 is installed simultaneously with boosters of type 2, when the best solution clearly is to install two boosters of type 2. As the smallest booster always will be the constraining unit for the flow, the larger boosters will only affect the total pressure differential. A formulation where only one booster of each type is allowed to be installed will therefore implicate that the first 7.5 bara installed will be cheaper than the next 7.5 bara if we exclude the one-time costs associated with extra gas separation. Table 5.3 explains this logic for well 1. The reason for waiting to install the booster of type 4 might be that the production is constrained during time period 1, and that the discounted investment cost in time period 2 proves easier to justify.

High GOR has obviously made multiphase boosters less attractive than single phase boosters, as the boosting effect is highly reduced for high flowrates of gas. Another reason might be that the extra cost of installing gas separation is low compared to the benefits of installing single phase boosters. These observations indicates that the performance of subsea multiphase boosters given in the data sets still are too low compared to the traditional single phase solution, and that further development of such boosters is required before it is suggested as a good investment option.

	Time period 1	Time period 2
Booster type installed	2 +3	4
Pressure differential	15 bara	22.5 bara
Investment costs	\$1M 25 + 30	\$1M 35
Gas separation costs	\$1M 20	-
Constraining booster unit	2	2

Table 5.3: Installation scheme for singe phase boosters at well 1

The objective function given in (3.38) is a simplified expression of the net present value of our production. Produced oil and gas is valued by a sales price, while water is priced with a topside treatment cost. Many of the other parameters used in the objective function is either challenging to determine a priori, or subject to a high degree of uncertainty. This is the reality for decision makers in the real world as well. The costs of innovative technology and new solutions may not be available, and the costs of temporary shut downs and installation operations are hard to determine in advance. The future income from petroleum sales is subject to a considerable level of uncertainty, depending on production plans, market prices and politics and other factors that are difficult to predict. The model presented in this paper is deterministic, and is not coping with this uncertainty. It will however be possible to investigate the solution's sensitivity to changes in the input data. Solutions can also indicate whether the equipment portfolios and their prices are realistic.

The quality of the piecewise linearization of the production rates can be found by comparing the SOS2-part of the formulation with the "real" production rates from PIPESIM. The PIPESIM rates are found for each time period by updating the reservoir pressure and wellhead pressure found in the model run for the water driven reservoir. Figure 5.10 shows the production rates without the linear K-formulation for well 3 compared with the rates from PIPESIM.

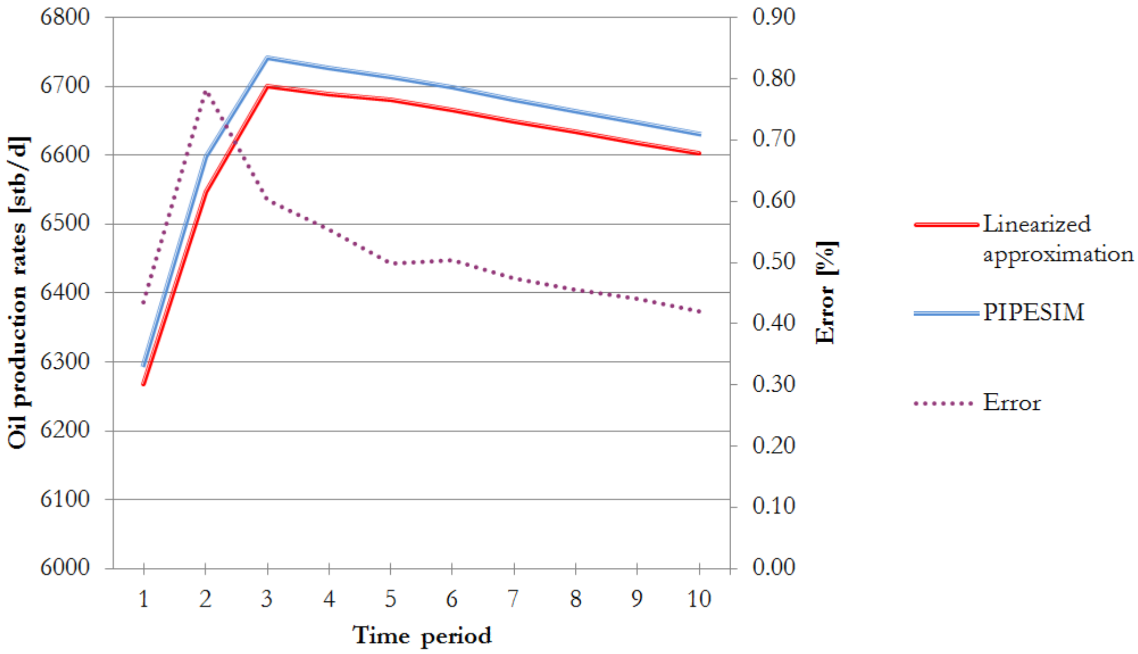

Figure 5.10: The SOS2-part of the production rates for well 3 compared with PIPESIM simulations

Remember from Figure 5.1 that well 3 was expected to be the most difficult function to piece-wise linearize. Despite this, the approximation is never more than 1% below the PIPESIM rates. It looks like the function we are linearizing has concave properties, as the approximated production rates always are lower than the original function. The use of K-terms will of course make the total formulation inaccurate, as they are used to manipulate the well production rates as the contents in the reservoir change. It is however important to notice how a rather rude linearization of the production rates can provide realistic results in a model where pressure conditions are the only variables.

A similar quality verification of the piecewise linearization is done for the pressure loss formulation. Figure 5.11 compares the pressure loss from the model run and the pressure loss from simulation runs done in PIPESIM, where production flows from the solution of the water driven reservoir is updated for every time period. Also here the difference is found to be rather small. It is hard to prove that the function for pressure loss has concave properties as it is described in four dimensions, but the error shown in Figure 5.11 is however suggesting that this is the case, as the linearized approximation always gives values lower than the PIPESIM simulations. Lower pressure loss results in higher production rates, which is a contrary effect compared to the production rates approximation presented in Figure 5.10. It is therefore hard to determine what the total effect of the two SOS2 formulations will be on the results, other than that they are rather small.

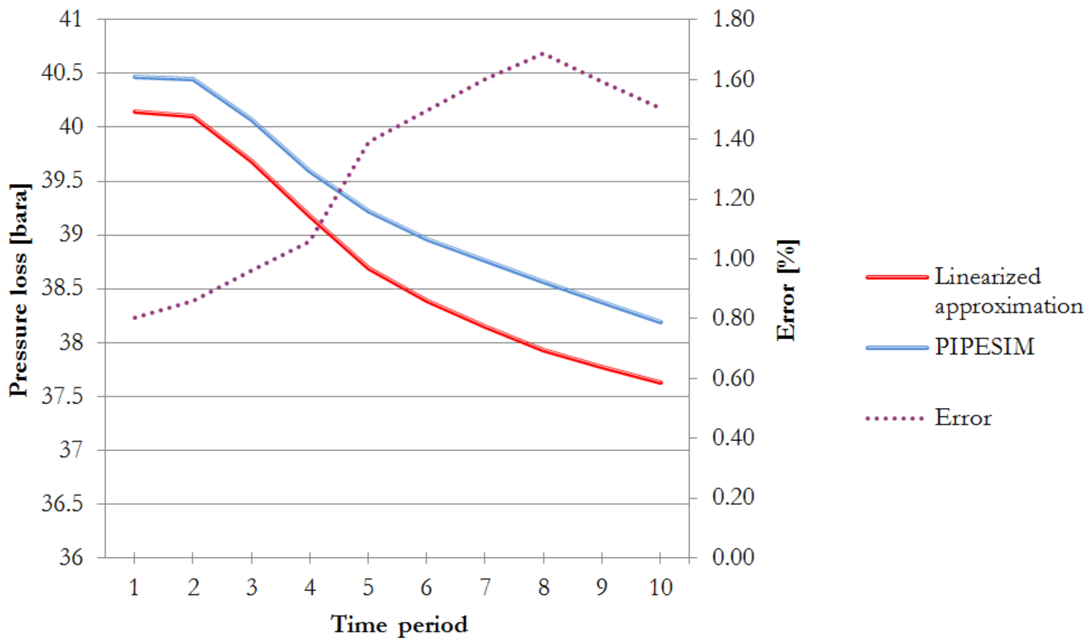

Figure 5.11: Comparison of the pressure loss found by the approximated function and PIPESIM simulations

From the observations done so far it seems like the model is able to describe the production system in an accurate way. The most important limitation is how the reservoir dynamics are described by a very simple tank model. We further assume that the content of the different phases are the only factors that affects the reservoir pressure and that production rates from the wells are a result of this reservoir pressure and the contents present in the reservoir tank. The reality is of course much more complicated. It is worth noticing that the description of the system downstream from the bottom hole to the topside is rather realistic, and that the model as a whole would improve considerably if the reservoir model is made more sophisticated.

The results from this study are comparable with similar studies done to uncover the boosting potential for offshore fields. Ribeiro, Camargo & Paulo (1996) performed a screening study for another Marlim field, where different types of subsea boosting technology where considered. Even though most of the technology was under development, the study showed a tendency of cost effectiveness. In the later years, several studies have shown that the boosting potential for offshore fields is substantial, and in many cases can result in increased production rates between 15% and 30% (Elde, 2005). Fjøsne (2002) describes a case where the production from a FPSO is approaching the mature phase. The topside liquid handling capacity is reached due to increased watecut, and topside modifications are difficult due to space and weight limitations. By installing a subsea three phase separator with water re-injection, the estimated increase in NPV is found to be about 20%. Similar potential for improvements in both production rates and economic results are found in our case, although they are a lot larger compared to the cases presented in this section. The realism of the exact figures is hard to determine, but the indications for improvements are nevertheless clear and should be considered as sensible information.

5.3.2 Problem Solvability

Even though most problem instances are solved to nearly optimality after one hour, a rather large amount of computational effort is required to find such solutions. The problem has proven harder to solve if the number of time periods is increased or the number of breakpoints in the SOS2 formulations is increased. A larger equipment portfolio is also expected to increase the solution time. It is hard to justify more breakpoints in the SOS2 formulations as this is not expected to improve the solution quality much, as we saw in Figure 5.10 and 5.11. This holds especially for the well production rates, as it is more relevant to alternate the reservoir formulation itself to provide a more accurate description, for example by the use of sophisticated reservoir simulators.

Judged by Figure 5.12, it seems like it is challenging to find good upper bounds, i.e. LP relaxations to the problem that confine the MILP solutions from above. Apparently, many good MILP solutions are found rather quickly by the solver, but it is difficult to say how good they are as the upper bound converges very slowly. For all we know, the optimal solution may be found after some minutes, while it takes hours before the upper bound can guarantee optimality.

The time periods' effect on the solution time is illustrated in Figure 5.13, where we see a rather exponential behavior in solution time as the time horizon with a yearly resolution is increased. The effect is expected to be similar if the time horizon is held constant, while the time resolution is increased. A problem with fifteen time periods has proven to be nearly impossible to solve to optimality, as the upper bound more or less stops to converge after a 3 hours run time.

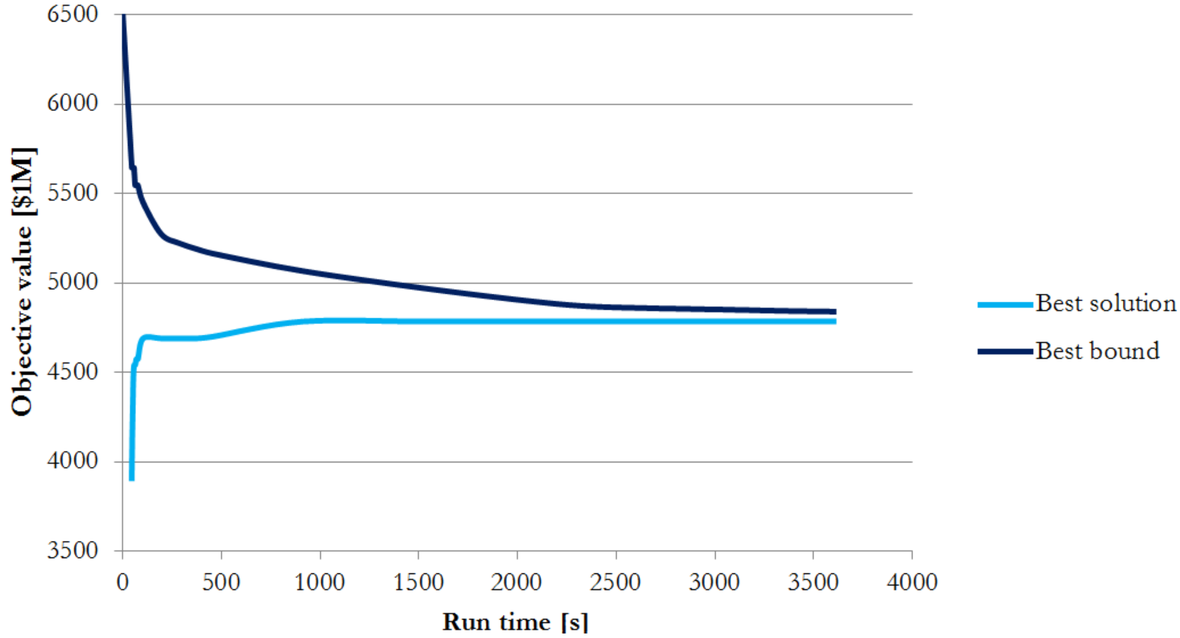

Figure 5.12: Graphical presentation of the solution search for the water driven case

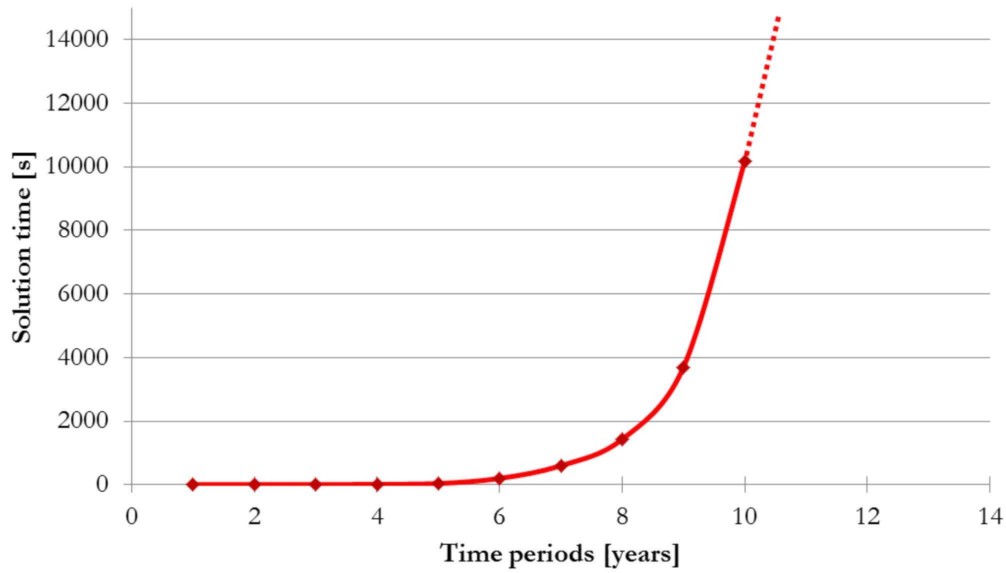

Figure 5.13: Solution time as a function of time periods with a yearly time discretization

The reservoir model is in any case a very rude model, and increased time horizon resolution is not believed to improve the realism of the solution, other than increasing the number of time periods and therefore the detail level of the production and investment scheme.

5.3.3 Applicability in Practical Decision Making

Newendorp (1975) argues that complex investment decisions in the petroleum industry should be analyzed using decision analysis techniques. He is also reminding us of a common misconception regarding the use of such techniques: decision analysis will not eliminate uncertainty or risk in decision making. For example, we can argue that a time horizon of ten years is a bit short for strategic decision making in offshore projects, and that a yearly resolution of the planning period might result in unrealistic investment schemes and production plans. Increasing the time horizon and the resolution will surely lead to a more complete and detailed solution. It is however very difficult to say anything about the *correctness* of the solution. The solution might even limit or mislead the decision makers more than it will guide them in the right direction. Another misconception is that sophisticated decision tools can replace professional judgment. Methodologies as the one presented in this paper are intended to supplement the decision making, and provide both engineers and economists with intelligible information about a rather complex and unintelligible decision problem.

The solution holds out expectations of substantial added value by doing different types of investments on the seabed. In the case studies of Elde (2005) results from investment scenarios similar to the ones presented in this paper are suggesting the same thing. It also argues that such investments potentially offer a reduction in risk as the new technology gives the operator a larger freedom to optimize the production. It makes the development strategy more flexible, as all long term developments are configurable by the installation of new equipment.

When different wells are connected to the same manifold, the production potential from some of the wells might be lost. This could be wells with lower reservoir pressure or higher watercuts than the other wells producing to the same manifold. Subsea processing equipment and alternative gas routing might then be used to alter the constraints in the system so that the production mix of the different wells is optimal. Subsea boosters can be used as “positive” chokes to balance the wellhead pressures, while subsea water separators free topside liquid handling capacities and reduce the density of the multiphase flow.

Besides verifying the cost effectiveness of the new equipment, the optimization model is determining the equipment characteristics, as well as estimating the number of equipment that would be necessary. Such information is expected to be very useful for potential manufacturers. The model is also verifying the impact of conceptual projects that could be used to exploit maturing and marginal fields in the best way possible, while determining the main technological constraints that has to be further developed in order to make such projects realizable.

The results from the model runs can be used to evaluate the potential value of a remaining production license period. Different license regimes exist in different countries, but they all introduce the operator to the question of whether to optimize production for a license period or the expected remaining lifetime of the reservoir. Different time horizons may result in different optimal solutions. We can see how the time horizon affects the solution in Figure 5.14. The production drop in year 11 is because of a temporary shutdown of well 2. This seems a bit odd, since the production continues as nothing has changed in year 12. This shutdown is likely to be a suboptimal solution, due to the optimality gap of 15.88% we have for the solution presented in Table 5.2. The total value of the investments done during the first five years is unlike for different time horizons in the planning problem. For the five-year horizon the total investments was found to be $1M 480, while it was $1M 615 for the ten-year horizon and $1M 745 for the fifteen-year horizon. This exemplifies the impact long term decision making might have on the solution compared to a shorter term. In practical decision making, planning horizons in offshore petroleum projects might never become as long as ten or fifteen years. This is mainly because of the high level of uncertainty, but also because of the rather small impact decisions done many years ahead has on the project today. Nevertheless, future income that is a direct result of decisions we make today, should be accounted for in our evaluations, even if the time horizon for our planning problem is just a few years. This will be further discussed in Chapter 7, Further Work.

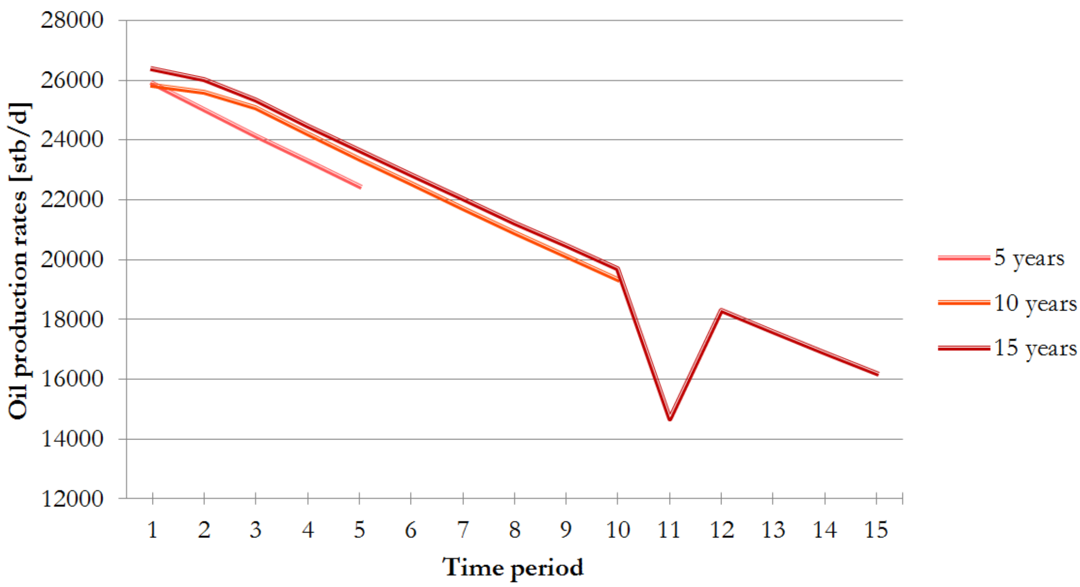

Figure 5.14: Optimal production for different time horizons in the planning problem

6 Conclusions

Several of the world's oil producing offshore fields are now entering advanced production phases with decreasing reservoir pressures and increasing water cuts, while improvements on the topside facility is difficult because of space and weight limitations. This project is looking at a production asset in the Marlim field in particular, and suggests that an optimization model can be used as decision support when different subsea solutions are evaluated in order to increase hydrocarbon recovery and production lifetime of a field.

The output from the model run presents a complete production scheme for every well in every time step. It also provides reservoir pressure and other production conditions as they develop over time. The strength of the model is that it uses real well models to generate parameters used in a quite simple description of a complicated reality. By doing this, some of the complexity in the production system is kept, and the results have proven to be quite realistic. The model suggests that there is a large potential for improving an asset's NPV by installing new subsea processing equipment for enhanced production. The improvement consists of increased production and better usage of today's infrastructure and topside installations, as well as increased system flexibility to mitigate uncertainty.

It is clear that the detail level in multi period models such as the one presented in this paper is greatly limited by solvability. It is therefore crucial to be able to prioritize which elements from the reality that should be covered and which elements that should be ignored. Knowledge about the production system and its properties is important in order to be able to do this, but knowing which results that are valuable for the decision maker is maybe even more important. Inaccuracies in the model's description of the real world is however not expected to be the main challenge in the decision making process because of the high level of uncertainty associated with the reservoir behavior and well performances, together with other factors such as market prices, legislation and politics. It is clear that results from such a model, no matter how detailed and sophisticated, have to be analyzed thoroughly and evaluated by expertise judgment.

7 Further Work

The model can quite easily be modified and used to evaluate different types of equipment. The challenge is to describe the processes linearly, and to connect the process to the system by introducing new variables and restrictions. The model can also be extended with several wells and well clusters. If the production system is extended with parts that is only mildly interconnected, or not connected at all, the idea of decomposition seems appealing.

If the model is made more detailed and complex in order to make it more realistic, it might get very hard to solve for many time periods. Different strategies can be applied to deal with this challenge:

Evaluate the forecasted value of future production

As we discussed in Section 5.3.3, a longer time horizon justifies more investments during the early years. If we modify the model to solve the planning problem with forecasted production rates after the first five time periods we will be able to solve the problem quite easy, se Figure 7.1. This will of course be under the assumption that all installations is done during the first five years of the planning period since production after year five is a forecast based on the first five.

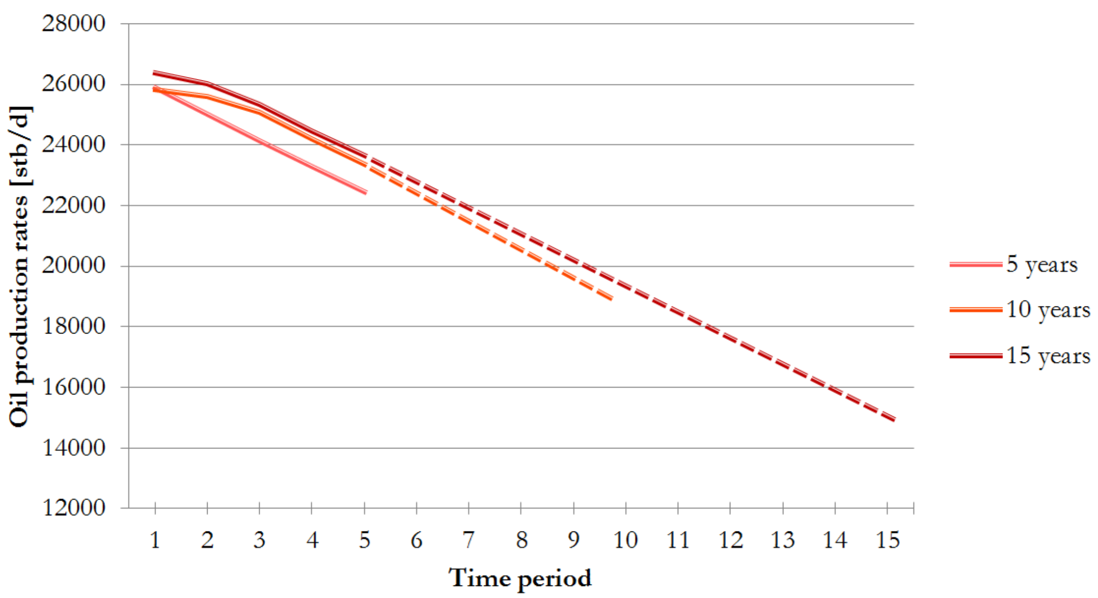

Figure 7.1: A possible solution where future production is forecasted after year five

Time discretization strategies

Since the uncertainty increases as time goes by, and the revenues are discounted for every year, the impact of decisions made for future time periods will decrease rapidly for every year we look further into the future. This fact justifies the use of increasing lengths of the time periods as time goes by, for example a yearly resolution the first four years, then every second year the next six years, and then every fifth year the next ten years. We are then able to run a problem with a twenty-year time horizon with only nine time steps, which could turn out to be quite solvable, even if the level of model complexity is increased.

Re-evaluation of the solution

As the planning period for offshore petroleum projects, for practical reasons mentioned in previous sections, might be no longer than five years, a re-evaluation of our decisions should be made when new information is available to secure optimal solutions for the next five-year period. Such updated evaluations can be done continuously, or after a certain period, such as a year. If we want to extend the planning horizon with ten, fifteen or even twenty years, the five-year problem could be run again, with the initial conditions set as they were in the end of year five in the first run. This approach will only evaluate the value of five years production in each run, unless evaluation of forecasted production after five years is implemented.

The complexity of predicting future hydrocarbon production profiles requires the use of reservoir simulators. Such a simulator should therefore be a part of the model when the objective function is evaluated. Furthermore, costs, sales prices and other parameters can be substituted in favor to more sophisticated models. Stochastic models can also be implemented, for example in the determination of market prices for petroleum products. Stochastic formulations are however expected to increase the problem size considerably, and might not be a good solution unless some other solution strategies are implemented to make the deterministic problem easier to solve.

To guarantee that the solution is feasible, the results from the optimization model can be tested in simulation software that is used in the operational decision making. Now that the optimal solution is found by the optimization model, it should be easy to verify this solution by more sophisticated simulators. The results from this test can be used to update the optimization parameters to generate a more accurate model. This especially holds for the reservoir model and the linear K-term in the production rate formulations.

Operations in PIPESIM and Xpress can be executed through commands in Excel. A simpler user interface where both input and output values is treated in the same Excel spread sheet can therefore be developed. This way, the optimization approach presented in this report is made more accessible for production engineers, economists and project executives during the decision process.

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Appendix A - Summary of the Mathematical Model

This section is a summary of the mathematical model presented in the previous chapters, and consists of a complete list of parameters, variables and constraints used to run the model. The values used as input data together with some of the output data are presented in Appendix B. The model is written in Mosel and implemented in Xpress. The Mosel code *code.mos* is found as an electronic attachment in Appendix C, together with the data tables in *data_water driven.txt.*

Sets and Indexes

P	- Set of components (*o* for oil, *g* for gas, *w* for water)
p	- Component index (oil, gas and water)
J	- Set of wells
j	- Well index
T	- Set of time periods
t	- Time speriod index
S	- Set of subsea separator types
s	- Subsea separator type index
F	- Set of booster types
f	- Booster type index

A	- Set of breakpoints *a* related to wellhead pressure
a	- Index for breakpoints related to wellhead pressure
B	- Set of breakpoints *b* related to reservoir pressure
b	- Index for breakpoints related to reservoir pressure

L	- Set of breakpoints *l* related to flow of oil in the riser
l	- Index for breakpoints related to flow of oil in the riser
M	- Set of breakpoints *m* related to flow of gas in the riser
m	- Index for breakpoints related to flow of gas in the riser
N	- Set of breakpoints *n* related to flow of water in the riser
n	- Index for breakpoints related to flow of water in the riser

Parameters

Capacities

$Q^{L.TOP}$ - Topside daily liquid handling capacity
$Q^{W.TOP}$ - Topside daily water handling capacity
$Q^{G.TOP}$ - Topside daily gas handling capacity
$Q_s^{W.SEP}$ - Subsea separator water handling capacity for separator type *s*
$Q_f^{L.BST}$ - Booster liquid handling capacity for type *f*
$Q_f^{G.BST}$ - Booster gas handling capacity for type *f*

Costs

C^{ALT} - Investment cost new subsea gas export lines
C^{OP} - Installation operation cost
$C^{W.TOP}$ - Gas used per stb of water treated topside
$C^{W.SEP}$ - Gas used per stb of water treated subsea and re-injected
C^{BST} - Gas used per boosted bara (ideal boosting)
$C_s^{I.SEP}$ - Investment cost of subsea separator alternative *s*
$C_f^{I.BST}$ - Investment cost of booster type *f*
$C^{G.S}$ - Extra investment cost for gas separation

SOS2 formulations

Q_{jpab}^{W} - Production rates in well *j* of phase *p* for breakpoints *a* and *b*
P_{ja}^{WH} - Breakpoint values for well head pressure in well *j* for breakpoint *a*
P_b^{RES} - Breakpoint values for reservoir pressure for breakpoint *b*
K_{jp}^{O} - Reservoir oil content factor for production rate of component *p* in well *j*
K_{jp}^{G} - Reservoir gas content factor for production rate of component *p* in well *j*
K_{jp}^{W} - Reservoir water content factor for production rate of component *p* in well *j*

ΔP_{lmn}^{R} - Pressure loss in the riser for breakpoints *l*, *m* and *n*
Q_{ol}^{R} - Breakpoint values for flow of oil in the riser for breakpoint *l*
Q_{gm}^{R} - Breakpoint values for flow of gas in the riser for breakpoint *m*
Q_{wn}^{R} - Breakpoint values for flow of water in the riser for breakpoint *n*

Others

D - Number of days in one time period
R - Internal rate of return
P^{TOP} - Topside separator pressure

S_p^{SALE} - Sales price of component *p*

$E_s^{W.SEP}$ - Subsea separator water separation efficiency for separator type *s*

E_f^{BST} - Ideal boosting factor in $\left[\frac{bara}{mmscf}\right]$ of type *f*

Q_p^{INIT} - Initial reservoir contents of component *p*

Q_p^{IFLUX} - Daily natural influx of component *p* into the reservoir in time period *t*

P^{BAS} - Basal reservoir pressure used in function for reservoir pressure

Y_p - Pressure coefficient for component *p* used in function for reservoir pressure

Variables

Binary variables

w_{tj}^{RUN} - Bin. Equals 1 if well *j* is running in time period *t*

y_{ts} - Bin. Equals 1 if a subsea separator of type *s* is operating in period *t*

z_{ts} - Bin. Equals 1 if a subsea separator of type *s* is installed in period *t*

x_{tf}^{M} - Bin. Equals 1 if a multiphase pump of type *f* is operating in period *t*

w_{tf}^{M} - Bin. Equals 1 if a multiphase pump of type *f* is installed in period *t*

x_{tjf}^{M} - Bin. Equals 1 if a multiphase pump of type *f* is operating at well *j* in period *t*

w_{tjf}^{M} - Bin. Equals 1 if a multiphase pump of type *f* is installed at well *j* in period *t*

x_{tf}^{S} - Bin. Equals 1 if a single phase pump of type *f* is operating in period *t*

w_{tf}^{S} - Bin. Equals 1 if a single phase pump of type *f* is installed in period *t*

x_{tjf}^{S} - Bin. Equals 1 if a single phase pump of type *f* is operating at well *j* in period *t*

w_{tjf}^{S} - Bin. Equals 1 if a single phase pump of type *f* is installed at well *j* in period *t*

g_{tj}^{W} - Bin. 1 if alternative routing of gas is enabled from well *j* in time period *t*

g_{t}^{M} - Bin. 1 if alternative routing of gas is enabled from manifold in time period *t*

c_{t}^{OP} - Common installation costs in time period *t*

$c_{t}^{G.SM}$ - Installation costs associated with gas separation at manifold in time period *t*

$c_{tj}^{G.SW}$ - Installation costs associated with gas separation at well *j* in time period *t*

Flow rates

q_{tjp}^{W} - Flow rate of component *p* in well *j* in time period *t*

$q_{tj}^{W.M}$ - Flow rate of gas routed into manifold from well *j* in time period *t*

$q_{tj}^{W.E}$ - Flow rate of gas routed as export gas from well *j* in time period *t*

$q_{t}^{M.E}$ - Flow rate of gas routed as export gas from manifold in time period *t*

q_{t}^{I} - Water injected in period *t*

q_{tp}^{R} - Flow rate of component *p* in the riser in period *t*

q_{t}^{EXP} - Total export of gas in period *t*

Pressures

p_{t}^{RES} - Reservoir pressure in period *t*

p_{tj}^{WH} - Well head pressure in well *j* in period *t*

p_{t}^{MAN} - Manifold pressure in period *t*

Δp_{t}^{R} - Pressure loss in the riser in period *t*

Others

e_{tjf}^{BST} - Boosting effect for multiphase pump alternative *f* at well *j* in time period *t*

e_{tf}^{BST} - Boosting effect for multiphase pump alternative *f* at manifold in time period *t*

q_{t}^{USE} - Use of gas to run subsea equipment and topside processing in period *t*

q_{tp}^{RES} - Amount of component *p* in the reservoir in period *t*

Objective Function

$$\max NPV = \sum_{t} \frac{SALES \cdot D - CAPEX}{(1+R)^{t}}$$

where

$$SALES = q_{to}^{R} \cdot S_{o}^{SALE} + q_{t}^{EXP} \cdot S_{g}^{SALE}$$

$$CAPEX = \sum_{s} (C_{s}^{I.SEP} \cdot z_{ts}) + C^{OP} \cdot c_{t}^{OP} + C^{G.S} \cdot (c_{t}^{G.SM} + \sum_{j} c_{tj}^{G.SW})$$
$$+ C^{ALT} \cdot c_{t}^{ALT} + \sum_{f} (C_{f}^{I.BST} \cdot (w_{tf}^{M} + w_{tf}^{S}) + \sum_{j} (C_{f}^{I.BST} \cdot (w_{tfj}^{M} + w_{tfj}^{S})))$$

Constraints

Installation and operating decisions

w_{tj}^{RUN} *is binary*

z_{ts} *and* y_{ts} *is binary*

$$y_{ts} \leq \sum_{\varepsilon \in T|\varepsilon \leq t} z_{\varepsilon s} \quad \forall t \in T, s \in S$$

w_{tf}^{M} *and* x_{tf}^{M} *is binary*

$$x_{tf}^{M} \leq \sum_{\varepsilon \in T|\varepsilon \leq t} w_{\varepsilon f}^{M} \quad \forall t \in T, f \in F$$

w_{tjf}^{M} *and* x_{tjf}^{M} *is binary*

$$x_{tfj}^{M} \leq \sum_{\varepsilon \in T|\varepsilon \leq t} w_{\varepsilon jf}^{M} \quad \forall t \in T, f \in F, j \in J$$

w_{tf}^{S} *and* x_{tf}^{S} *is binary*

$$x_{tf}^{S} \leq \sum_{\varepsilon \in T|\varepsilon \leq t} w_{\varepsilon f}^{S} \quad \forall t \in T, f \in F$$

w_{tjf}^{S} *and* x_{tjf}^{S} *is binary*

$$x_{tfj}^{S} \leq \sum_{\varepsilon \in T|\varepsilon \leq t} w_{\varepsilon jf}^{S} \quad \forall t \in T, f \in F, j \in J$$

g_{t}^{M} *is binary*

$$g_{t}^{M} \leq \sum_{f} x_{tf}^{S} + \sum_{s} y_{ts} \quad \forall t \in T$$

g_{tj}^{W} *is binary*

$$g_{tj}^{W} \leq \sum_{f} x_{tjf}^{S} \quad \forall t \in T, j \in J$$

c_{t}^{ALT} *is binary*

$$\sum_{t_2 \in T|t_2 \leq t} c_{t_2}^{ALT} \geq \frac{g_{t}^{M} + g_{tj}^{W}}{2} \quad \forall t \in T, j \in J$$

c_t^{OP} *is binary*

$$c_t^{OP} \geq \frac{z_{ts} + w_{tf}^{M} + w_{tf}^{S} + w_{tjf}^{M} + w_{tjf}^{S}}{5} \qquad \forall t \in T, j \in J, s \in S, f \in F$$

$c_t^{G.SM}$ *is binary*

$$c_t^{G.SM} \geq \left(w_{tf}^{S} - \sum_{t_2 \in T | t_2 < t} \sum_{f_2 \in F} w_{t_2 f_2}^{S} - \sum_{t_2 \in T | t_2 \leq t} \sum_{s_2 \in S} z_{t_2 s_2} \right) \quad \forall t \in T, s \in S, f \in F$$

$c_{tj}^{G.SW}$ *is binary*

$$c_{tj}^{G.SW} \geq \left(w_{tjf}^{S} - \sum_{t_2 \in T | t_2 < t} \sum_{f_2 \in F} w_{t_2 j f_2}^{S} \right) \quad \forall t \in T, j \in J, f \in F$$

Topside gas, liquid and water capacity constraints

$$q_{to}^{R} + q_{tw}^{R} \leq Q^{L.TOP} \qquad \forall t \in T$$

$$q_{tw}^{R} \leq Q^{W.TOP} \qquad \forall t \in T$$

$$q_{tg}^{R} \leq Q^{G.TOP} \qquad \forall t \in T$$

Booster Capacity Constraints

Liquid

$$q_{tjo}^{W} + q_{tjw}^{W} \leq Q_f^{L.BST} \cdot x_{tjf}^{M} + Q_f^{L.BST} \cdot (1 - x_{tjf}^{M}) \cdot M^{L} \qquad \forall t \in T, f \in F, j \in J$$

$$q_{tjo}^{W} + q_{tjw}^{W} \leq Q_f^{L.BST} \cdot x_{tjf}^{S} + Q_f^{L.BST} \cdot (1 - x_{tjf}^{S}) \cdot M^{L} \qquad \forall t \in T, f \in F, j \in J$$

$$q_{tjo}^{W} + q_{tjw}^{W} \leq Q_f^{L.BST} \cdot x_{tf}^{M} + Q_f^{L.BST} \cdot (1 - x_{tf}^{M}) \cdot M^{L} \qquad \forall t \in T, f \in F$$

$$q_{tjo}^{W} + q_{tjw}^{W} \leq Q_f^{L.BST} \cdot x_{tf}^{S} + Q_f^{L.BST} \cdot (1 - x_{tf}^{S}) \cdot M^{L} \qquad \forall t \in T, f \in F$$

Where M^L *is set to topside liquid handling capacity.*

Gas

$$q_{tjg}^{W} \leq Q_f^{G.BST} \cdot x_{tjf}^{M} + Q_f^{G.BST} \cdot (1 - x_{tjf}^{M}) \cdot M^{G} \qquad \forall t \in T, f \in F, j \in J$$

$$q_{tjg}^{W} \leq Q_f^{G.BST} \cdot x_{tjf}^{S} + Q_f^{G.BST} \cdot (1 - x_{tjf}^{S}) \cdot M^{G} \qquad \forall t \in T, f \in F, j \in J$$

$$q_{tjg}^{W} \leq Q_f^{G.BST} \cdot x_{tf}^{M} + Q_f^{G.BST} \cdot (1 - x_{tf}^{M}) \cdot M^{G} \qquad \forall t \in T, f \in F$$

$$q_{tjg}^{W} \leq Q_f^{G.BST} \cdot x_{tf}^{S} + Q_f^{G.BST} \cdot (1 - x_{tf}^{S}) \cdot M^{G} \qquad \forall t \in T, f \in F$$

Where M^G *is set to topside gas handling capacity.*

Mass balances

Reservoir

$$q_{tp}^{RES} = Q_p^{INIT} \qquad \forall t = 1, p \in P$$

$$q_{t+1,o}^{RES} = q_{to}^{RES} + \left(Q_o^{IFLUX} - \sum_j q_{tjo}^W \right) D \qquad \forall t \in \{1 \ldots |T| - 1\}$$

$$q_{t+1,g}^{RES} = q_{tg}^{RES} + \left(Q_g^{IFLUX} - \sum_j q_{tjg}^W \right) D \qquad \forall t \in \{1 \ldots |T| - 1\}$$

$$q_{t+1,w}^{RES} = q_{tw}^{RES} + \left(Q_w^{IFLUX} + q_{tw}^I - \sum_j q_{tjw}^W \right) D \qquad \forall t \in \{1 \ldots |T| - 1\}$$

Oil

$$q_{to}^R = \sum_j q_{tjo}^W \qquad \forall t \in T$$

Gas

$$q_{tj}^{W.M} = q_{tjg}^W - q_{tj}^{W.E} \qquad \forall t \in T, j \in J$$

$$q_{tj}^{W.E} \leq q_{tjg}^W \cdot g_{tj}^W \qquad \forall t \in T, j \in J$$

$$q_t^{M.E} \leq \sum_j q_{tj}^{W.M} \cdot g_t^M \qquad \forall t \in T$$

$$q_{tg}^R = \sum_j q_{tjg}^W - \sum_j q_{tj}^{W.E} - q_t^{M.E} \qquad \forall t \in T$$

$$q_t^{USE} \leq q_{tg}^R \qquad \forall t \in T$$

$$q_t^{USE} = C_w^{TOP} \cdot q_{tw}^R + C_w^{SEP} \cdot q_t^I$$

$$+C^{BST} \sum_f E_f^{BST} \cdot Q_f^{G.BST} \cdot \left(\sum_j \left(x_{tjf}^M + x_{tjf}^S \right) + x_{tf}^M + x_{tf}^S \right) \qquad \forall t \in T$$

$$q_t^{EXP} = \sum_j q_{tjg}^W - q_t^{USE} \qquad \forall t \in T$$

Water

$$q_t^I \leq \sum_s Q_s^{SEP} \cdot y_{ts} \qquad \forall t \in T$$

$$q_t^I \leq \sum_j q_{tjw}^W \cdot E_s^{SEP} + Q_s^{SEP} \cdot (1 - y_{ts}) \qquad \forall t \in T, s \in S$$

$$q_{tw}^R = \sum_j q_{tjw}^W - q_t^I \qquad \forall t \in T$$

Artificial flow

$$q_{tjp}^{ART} \ is \ free$$

$$q_{tjp}^W \leq M_p \cdot w_{tj}^{RUN} \qquad \forall t \in T, j \in J, p \in P$$

$$-M_p\left(1 - w_{tj}^{RUN}\right) \leq q_{tjp}^{ART} \leq M_p\left(1 - w_{tj}^{RUN}\right) \qquad \forall t \in T, j \in J, p \in P$$

Where M_p *is set to be the topside capacity for the corresponding phase* p.

Pressures

$$p_t^{MAN} \leq p_{tj}^{WH} + \sum_f (e_{tjf}^{BST} \cdot x_{tjf}^M) + \sum_f (E_f^{BST} \cdot Q_f^{G.BST} \cdot x_{tjf}^S) \qquad \forall t \in T, j \in J$$

$$p_t^{MAN} = P^{TOP} + \Delta p_t^R - \sum_f (e_{tf}^{BST} \cdot x_{tf}^M) - \sum_f (E_f^{BST} \cdot Q_f^{G.BST} \cdot x_{tf}^S) \qquad \forall t \in T$$

$$e_{tjf}^{BST} \leq E_f^{BST} \cdot \left(Q_f^{G.BST} - q_{tjg}^W\right) + E_f^{BST} \cdot Q_f^{G.BST} \cdot (1 - x_{tjf}^M) \cdot \mathrm{M} \qquad \forall t \in T, j \in J, f \in F$$

$$e_{tjf}^{BST} \leq x_{tjf}^M \cdot \mathrm{M} \qquad \forall t \in T, j \in J, f \in F$$

$$e_{tf}^{BST} = E_f^{BST} \cdot (Q_f^{G.BST} - q_{tg}^R) + E_f^{BST} \cdot Q_f^{G.BST} \cdot (1 - x_{tf}^M) \cdot \mathrm{M} \qquad \forall t \in T, f \in F$$

$$e_{tf}^{BST} \leq x_{tf}^M \cdot \mathrm{M} \qquad \forall t \in T, j \in J, f \in F$$

Where M is set to be the topside gas handling capacity.

Others

$$p_t^{RES} = P^{BAS} + \sum_p Y_p \cdot q_{tp}^{RES} \qquad \forall t \in T$$

SOS 2 formulations

Pressure loss

$$\Delta p_t^R = \sum_l \sum_m \sum_n \Delta P_{lmn}^R \cdot \gamma_{tlmn} \qquad \forall t \in T$$

$$q_{to}^R = \sum_l \sum_m \sum_n Q_{ol}^R \cdot \gamma_{tlmn} \qquad \forall t \in T$$

$$q_{tg}^R = \sum_l \sum_m \sum_n Q_{gm}^R \cdot \gamma_{tlmn} \qquad \forall t \in T$$

$$q_{tw}^R = \sum_l \sum_m \sum_n Q_{wn}^R \cdot \gamma_{tlmn} \qquad \forall t \in T$$

$$\sum_l \sum_m \sum_n \gamma_{tlmn} = 1 \qquad \forall t \in T$$

$$\gamma_{tlmn} \geq 0 \qquad \forall t \in T, l \in L, m \in M, n \in N$$

Three auxiliary weighing variables are introduced in order to define SOS2 sets:

$$\rho_{tl} = \sum_m \sum_n \gamma_{tlmn} \qquad \forall t \in T, l \in L$$

$$\sigma_{tm} = \sum_l \sum_n \gamma_{tlmn} \qquad \forall t \in T, m \in M$$

$$\tau_{tn} = \sum_l \sum_m \gamma_{tlmn} \qquad \forall t \in T, n \in N$$

$$\rho_{tl}, \sigma_{tm}, \tau_{tn} \geq 0 \qquad \forall t \in T, l \in L, m \in M, n \in N$$

$$\rho_{tl} \; is \; SOS2 \; for \; l \qquad \forall t \in T$$

$$\sigma_{tm} \; is \; SOS2 \; for \; m \qquad \forall t \in T$$

$$\tau_{tn} \; is \; SOS2 \; for \; n \qquad \forall t \in T$$

Well production rates

$$q_{tjp}^{W} + q_{tjp}^{ART} = \sum_{a}\sum_{b} Q_{jpab}^{W} \cdot \lambda_{tjab} + K_{jp}^{O}(Q_{o}^{INIT} - q_{to}^{RES})$$

$$+K_{jp}^{W}(Q_{g}^{INIT} - q_{tg}^{RES}) + K_{jp}^{G}(Q_{w}^{INIT} - q_{tw}^{RES}) \quad t \in T, j \in J, p \in P$$

$$p_{tj}^{WH} = \sum_{a}\sum_{b} P_{a}^{WH} \cdot \lambda_{tjab} \qquad \forall t \in T, j \in J$$

$$p_{t}^{RES} = \sum_{a}\sum_{b} P_{b}^{RES} \cdot \lambda_{tjab} \qquad \forall t \in T, j \in J$$

$$\sum_{a}\sum_{b} \lambda_{tjab} = 1 \qquad \forall t \in T, j \in J$$

$$\lambda_{tjab} \geq 0 \qquad \forall t \in T, j \in J, a \in A, b \in B$$

Two auxiliary weighing variables are introduced in order to define SOS2 sets:

$$\alpha_{tja} = \sum_{b} \lambda_{tjab} \qquad \forall t \in T, j \in J, a \in A$$

$$\beta_{tjb} = \sum_{a} \lambda_{tjab} \qquad \forall t \in T, j \in J, b \in B$$

$$\alpha_{tja}, \beta_{tjb} \geq 0 \qquad \forall t \in T, j \in J, a \in A, b \in B$$

$$\alpha_{tja} \; is \; SOS2 \; for \; a \qquad \forall t \in T, j \in J$$

$$\beta_{tjb} \; is \; SOS2 \; for \; b \qquad \forall t \in T, j \in J$$

Appendix B – Data Tables

This section presents the complete dataset used to run the original water driven reservoir case. Multi-dimensional parameters such as K_{jp}^{W} is presented in matrixes, read horizontally for the first index, in this case *p*, and then vertically for the next index, in this case *j*.

Sets

$|T| = 10$ $\quad |A| = 4$ $\quad |L| = 3$

$|J| = 4$ $\quad |B| = 3$ $\quad |M| = 3$

$|P| = 3$ $\quad |N| = 3$

$|S| = 9$

$|F| = 8$

Capacities

$Q^{L.TOP} = 30000\ \frac{stb}{d}$

$Q^{W.TOP} = 15000\ \frac{stb}{d}$

$Q^{G.TOP} = 12\ \frac{mmscf}{d}$

$Q_s^{W.SEP} = \frac{stb}{d}[14000\ \ 14000\ \ 14000\ \ 16000\ \ 16000\ \ 16000\ \ 18000\ \ 18000\ \ 18000]$

$Q_f^{L.BST} = \frac{stb}{d}[13200\ \ 15600\ \ 18000\ \ 20400\ \ 36000\ \ 42000\ \ 48000\ \ 54000]$

$Q_f^{G.BST} = \frac{scf}{d}[3\ \ 4\ \ 5\ \ 6\ \ 10\ \ 11\ \ 12\ \ 13]$

Costs

$C^{ALT} = \$1M\ 160$

$C^{OP} = \$1M\ 50$

$C^{W.TOP} = 0.00001\ \frac{mmscf}{stb}$

$C^{W.SEP} = 0.00002\ \frac{mmscf}{stb}$

$C^{BST} = 0.1\ \frac{mmscf}{bara}$

$C_s^{I.SEP} = \$1M\ [50\ \ 60\ \ 70\ \ 55\ \ 65\ \ 75\ \ 60\ \ 70\ \ 80]$

$C_f^{I.BST} = \$1M\ [20\ \ 25\ \ 30\ \ 35\ \ 70\ \ 80\ \ 90\ \ 100]$

$C^{G.S} = \$1M\ 20$

SOS2 formulations

$Q^{W}_{jpab} = [$

j=1:				*j=2:*		
7736.17	9024.08	10055.90		3807.33	4524.52	5099.41
4650.73	6376.16	7808.07	$\frac{stb}{d}$	2779.70	3688.35	4394.71
2132.73	3996.41	5670.86		1660.03	2704.00	3552.22
0.00	2116.49	3762.93		672.88	1705.59	2696.40
3.25	3.79	4.22		1.74	2.07	2.33
1.95	2.68	3.28	$\frac{mmscf}{d}$	1.27	1.69	2.01
0.90	1.68	2.38		0.76	1.24	1.62
0.00	0.89	1.58		0.31	0.78	1.23
1584.52	1848.30	2059.64		8046.12	9561.79	10776.72
952.56	1305.96	1599.24	$\frac{stb}{d}$	5874.40	7794.69	9287.46
436.82	818.54	1161.50		3508.17	5714.43	7507.00
0.00	433.50	770.72		1422.02	3604.47	5698.37

j=3:				*j=4:*		
4391.70	5840.66	7245.64		5535.12	5923.99	6219.12
2677.62	4551.87	6231.69	$\frac{stb}{d}$	4374.37	4954.74	5405.90
681.09	2539.71	4713.96		3145.32	3897.61	4499.26
182.51	623.69	2647.18		2004.28	2855.61	3578.44
2.35	3.12	3.88		2.58	2.77	2.90
1.43	2.44	3.33	$\frac{mmscf}{d}$	2.04	2.31	2.52
0.36	1.36	2.52		1.47	1.82	2.10
0.10	0.33	1.42		0.94	1.33	1.67
4005.44	5326.95	6608.36		3378.12	3615.45	3795.57
2442.11	4151.51	5683.59	$\frac{stb}{d}$	2669.71	3023.91	3299.25
621.19	2316.33	4299.35		1919.61	2378.74	2745.92
166.45	568.83	2414.35		1223.23	1742.80	2183.94]

$P^{WH}_{a} = [20\ 40\ 60\ 80]\ bara$

$P^{RES}_{b} = \left[200\ 230\ 260\right]\ bara$

$K_{jp}^{O} = [$

0	$\frac{stb}{d \cdot mmstb}$	0.000200	$\frac{stb}{d \cdot mmscf}$	-10.76	$\frac{stb}{d \cdot mmstb}$
0	sim.	0.000219	sim.	-10.57	sim.
0		0.000520		-10.31	
0		0.000456		-10.57	]

$K_{jp}^{G} = [$

0.0520	0.000014	0.0106	
0.0183	0.000019	0.0176	
0.0298	0.000021	0.0637	
0.0166	0.000012	0.0101	]

$K_{jp}^{W} = [$

-10.52	-0.0010	10.71	
-9.26	-0.0010	10.25	
-15.27	-0.0010	10.26	
-9.01	-0.0010	10.88	]

$$Q_{ol}^{R} = \frac{stb}{d}[30000 \quad 10000 \quad 0]$$

$$Q_{gm}^{R} = \frac{mmscf}{d}[12 \quad 6 \quad 0]$$

$$Q_{wn}^{R} = \frac{stb}{d}[20000 \quad 10000 \quad 0]$$

$\Delta P_{lmn}^{R} = [$

57.30	48.79	40.20	
68.51	59.69	49.88	
86.58	81.68	76.70	
29.42	32.45	22.76	
42.92	37.49	26.13	
79.27	78.25	72.80	
21.34	16.79	8.95	
29.73	17.57	12.77	
81.23	80.62	77.01	] $bara$

Others

$D = 365$

$R = 10\ \%$

$P^{TOP} = 15\ bara$

$S_p^{SALE} = [100 \frac{\$}{stb} \quad 3000 \frac{\$}{mmscf} \quad 0]$

$E_s^{W.SEP} = [0.7\ \ 0.8\ \ 0.9\ \ 0.7\ \ 0.8\ \ 0.9\ \ 0.7\ \ 0.8\ \ 0.9]$

$E_f^{BST} = \frac{bara}{mmscf}[2.50\ \ 1.88\ \ 1.50\ \ 1.25\ \ 0.75\ \ 0.68\ \ 0.63\ \ 0.58]$

$Q_p^{INIT} = [150\ mmstb \quad 80000\ mmscf \quad 105\ mmstb]$

$Q_p^{IFLUX} = [0\ \frac{stb}{d} \quad 2\ \frac{mmscf}{d} \quad 30000\ \frac{stb}{d}]$

$P^{BAS} = 200\ bara$

$Y_p = [0.150000\ \frac{bara}{mmstb} \quad 0.000215\ \frac{bara}{mmscf} \quad 0.145000\ \frac{bara}{mmstb}]$

Appendix C – Electronic attachments

The following files are attached to this report electronically:

1. PDF-version of the report
2. Mosel code of the mathematical model
3. Data files for water driven and gas driven reservoirs for program runs in Xpress
4. Read-me file describing how to implement the different cases in Xpress