



NTNU – Trondheim
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Production optimization in the salmon farming industry

Ordering smolt under uncertainty

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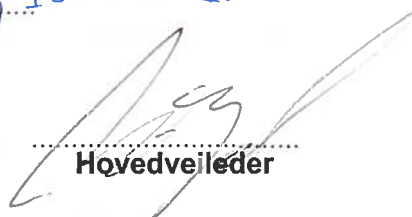
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Sammendrag

I løpet av det siste tiåret har lakseindustrien opplevd en kraftig vekst og bransjen har konsolidert. Dermed har kompleksiteten innenfor planlegging økt, og det har blitt behov for bedre planleggingsverktøy. I denne masteroppgaven har en stokastisk optimeringsmodell for bestilling og utsett av smolt, samt slakting av laks blitt utviklet basert på arbeidet til Hæreid (2011), Langan og Toftøy (2011) og Øveraas og Rynning-Tønnesen (2012). De viktigste forbedringene i denne oppgaven er en mer realistisk modellering av ferskvannsproduksjonen, en mer realistisk beslutningsprosess for bestilling og utsett av smolt og tidsavhengig aggregering av lokasjoner.

Bakgrunnen for denne oppgaven er det langsiktige planleggingsproblemet for smoltbestilling og utsett når flere produksjonsparametere er usikre. De viktigste kildene til usikkerhet i sjøvannsproduksjon er vekst, pris og dødlighet. Målet til modellen er å ta profitable beslutninger, men samtidig ta hensyn til usikkerheten. For å sikre løsbarehet med tilgjengelig hardware har modellen blitt forenklet. De viktigste forenklingene er at pris og fôrfaktor er implementert deterministisk. Modellen er implementert i tre versjoner; en deterministisk, en to-steps stokastisk og en tre-steps stokastisk utgave. Disse har blitt implementert for Marine Harvest Region Midt. Data har blitt innhentet fra tilgjengelig offentlig informasjon om Marine Harvest, og brukergrensesnittet er MS Excel.

Resultatet av å bruke stokastisk programmering er marginal sammenlignet med å bruke deterministisk programmering, men planene blir bedre med en stokastisk modell. Når en deterministisk modell brukes kan det føre til at store mengder smolt ødelegges. Dette skjer fordi modellen bestiller for mange smolt for scenariene der biomasseutviklingen blir større enn forventet. Den stokastiske modellen unngår destruksjon av smolt ettersom den tar hensyn til at alle scenariene kan inntreffe. Derfor ser stokastisk modellering ut til å være bedre egnet for langsiktig planlegging i lakseindustrien. I tillegg viser denne oppgaven at gjennomsnittlig slaktevekt bør justeres i korrelasjon med temperaturvariasjoner for å øke profitten. Ved å redusere den nedre grensen for slakting fra 5.5-6.5 kg til 4-6.5 kg, kan profitten øke med 450 millioner NOK over fem år. Modellen indikerer også at ferskvannsproduksjon ikke er like begrensende for saltvannsproduksjon som tidligere arbeider har hevdet.

Abstract

In the last decade the salmon farming industry has expanded rapidly and gone towards consolidation, thus the complexity of planning has increased. Therefore, the need for better planning tools has arisen. In this master thesis a stochastic optimization model for ordering and deployment of smolt and harvesting of salmon is developed based on the work of Hæreid (2011), Langan and Toftøy (2011) and Øveraas and Rynning-Tønnesen (2012). The most important improvements in this thesis are more realistic modelling of freshwater production, a more realistic decision process for ordering and deployment of smolt and time dependent aggregation of production sites.

The basis for this thesis is the long term tactical planning problem of making smolt delivery and deployment plans and harvest salmon, in an environment where several parameters regarding production are uncertain. The most important sources of uncertainty in seawater production are growth, price and mortality. The model aim is to make profitable decisions while consider these risks. All relevant constraints regarding production capacities and governmental regulations are considered, and all input data reflect a real salmon producer. In order to ensure solvability with the available hardware, some simplifications of the model have been made, the most important being deterministic price and feed conversion rate. The model is implemented in three versions; a deterministic, a two-stage and a three-stage stochastic model. These models have been implemented for Marine Harvest Region Mid. Data are collected from public available information about Marine Harvest, and the user interface is MS Excel.

The results improve marginally when stochastic programming is used, compared to deterministic. However, the plans made by the stochastic model are better. Using a deterministic model can result in a large amount of smolt being destroyed. This happens as the model will have ordered too many smolt for the scenarios where biomass development turns out to be higher than expected. The stochastic model avoids destruction of smolt, as it considers all possible scenarios. Stochastic programming therefore seems like a better tool for long term production planning in the salmon farming industry. Furthermore, this thesis shows that average harvest weight should be adjusted in correlation to seasonal temperature variations to increase profit. By lowering the lower bound for allowable harvest interval from 5.5-6.5 kg to 4.0-6.5 kg, the profit can increase by 450 million NOK in a five year period. Lastly, the model indicates that freshwater production is not as limiting for saltwater production as previous work suggests.

Preface

This master thesis is the final step of achieving a Master of Science at the Norwegian University of Science and Technology (NTNU). The degree specialization is Applied Economics and Optimization at the Department of Industrial Economics and Technology Management. This master thesis is written in cooperation with Marine Harvest ASA, SINTEF Technology and Society and SINTEF Fisheries and Aquaculture.

We would like to thank our supervisor, professor at NTNU and senior scientist at SINTEF Technology and Science, Asgeir Tomasgard for helpful contributions in our work. Also we would like to thank co-supervisor Peter Schütz at SINTEF Technology and Science for constructive discussions and feedback. In Marine Harvest we would first and foremost thank Eivind Osnes for taking a genuine interest in our work and for valuable information, and also his colleagues receive a thank for teaching us about the industry. Their cooperation has been a key part of gaining an in-dept insight to the industry, and receiving realistic input data for the model. We would also thank Jan Fredrik Helgaker for valuable feedback regarding the thesis. Our work has greatly benefited from all of the mentioned above.

Trondheim, June 2, 2012

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List of abbreviations

AR - Autoregressive

DET - Deterministic model

EDEV - Expected result of using the dynamic solution

EEV - Expected result of using the EV solution

ESV - Expected salmon value

EV - Expected value

EVPI - Expected value of perfect information

MAB - Maximum allowable biomass

MR - Møre and Romsdal

MS - Multi-stage stochastic model

RP - Recourse problem

T - Trøndelag

TS - Two-stage stochastic model

VSS - Value of stochastic solution

WS - Wait-and-see solution

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1 Introduction

Seafood was the 3rd largest export industry in Norway in 2009 after Oil and Gas (Norwegian Ministry of Fisheries and Coastal Affairs, 2010), and is an important contribution to the Norwegian Economy. Norway is by far the world's largest producer of farmed salmon with a production of 944 thousand tons in 2010, and which corresponds to 66% of the world production of farmed salmon in 2010 (Liu, Olaf Olaussen, and Skonhøft, 2010). In recent years the industry has matured and gone towards consolidation. Salmon farming is a biological industry, and salmon producers are experiencing large variations in biomass development and price, resulting in risk for the salmon producers. Production complexity and risk create a need for better operation systems in order to remain competitive and increase profit.

Salmon farming started in the early 1970s, and has attracted interest from researches since then. The main focus in research, in order to develop a sustainable industry, has been on the biological factors. As production has increased, research also started considering effectiveness of production. Several articles have discussed effectiveness in the industry, but few of them consider optimization of the production plan in a value chain perspective.

The first optimization models in aquaculture considered harvesting of salmon. Lillestøl (1986) investigated the problem of optimal timing of slaughtering in fish farming. A model based on price and feeding cost, aimed on profit maximization was developed. The model assumed that all fish were slaughtered at the same time, and the problem was solved for both one and several time periods. Bjørndal (1988) formulated an optimization model for harvesting of farmed fish, modelled in a microeconomic approach. The model analysed the optimal harvesting time for farmed fish due to growth and costs. As input parameters for modelling growth the model uses age, density and feed quantity. Costs considered were harvesting, feed and insurance. The model considers a one-time investment in fish. Arnason (1990) generalized the model developed by Bjørndal by proving interdependence between deciding optimal harvesting plan and feeding schedule. This relationship is important, as feeding is continuous throughout production, while output in terms of an optimal harvesting plan will come several time periods after feeding starts. Arnason stated that no general optimal feeding schedule exists, as it is strictly dependent on the growth function used in the model. Mistiaen, Strand, et al. (1998) further extended the microeconomic modelling of optimal feeding schedules and harvesting time, by making prices to be piecewise-continuous and weight-dependent. They stated that prices per kg

are not continuous, but rather constant for a given weight interval.

Forsberg (1999) developed a multi-period, deterministic, mixed-integer programming model for harvesting different fish cohorts. The aim of the model was to maximize overall profits considering growth and harvesting. Growth was based on feed type, fish size and water temperature, and the cohort was modelled with a size-range to represent the dispersal within the same fish generation. Harvesting was modelled with time windows, biomass, size-range of harvested fish, limitations for harvest operations and recovery time between harvest operations. The profit was modelled by sales revenue, set as an input parameter and costs regarding feed, transportation and slaughtering. The analysis showed that by sorting the salmon before slaughtering, the producer could increase overall profit by 10% compared to slaughtering the total cohort at the same time. This model has been frequently referred to in later literature, and is still among the most advanced optimization tools regarding harvesting. However, the model do not consider important production restrictions like MAB limitations and fallowing, as these regulations were introduced after the model was made.

Several studies compared optimization models and production at aquaculture farms. Pascoe, Wattage, and Naik (2002) concluded that there is a gap between aquaculture farmer practice and theory. They stated that optimization models are not sufficient in identifying the appropriate strategy given risk and uncertainty, so it is not possible to find out whether the farmers operate in optimal manners or not. As temperature and other important production variables are difficult to control within a realistic cost environment, modelling approaches dealing with these risks should be more appropriate tools. Forsberg and Guttormsen (2006) stated that fish farming traditionally have focused on either production planning or price forecasts, and conclude that the two topics must be considered together in order to make good harvesting decisions. Prices are closely related to optimal decisions, making it a great source of uncertainty in production planning models. Asche, Kumbhakar, and Tveterås (2007) tested whether cost or profit functions should be used in optimization of production technology, as cost functions are a lot more common than profit functions. For the salmon industry, the study proved that the cost function approach does not maximize profit. However, the study concluded that one cannot refute salmon farmers to be profit maximizers.

Fish Pool opened in 2006, and several studies have analysed risk related to salmon prices. Bergfjord (2009) concluded that salmon farmers consider future salmon prices as one of the most important sources of risk. Oglend

and Sikveland (2008) analysed the behaviour of salmon price volatility, and found that when prices are high, volatility increases and hence larger profits come with a trade-off of greater price risk.

In 2005 maximum allowable biomass (MAB) restrictions were introduced in Norway. Stikholmen (2010) analysed the efficiency in the aquaculture industry before and after the introduction of MAB-restrictions, but could not conclude in whether it has become more or less efficient. Langan and Toftøy (2011) developed an optimization tool where the scope was to make smolt ordering plans when maximizing MAB utilization of the saltwater facilities. The model considers uncertainty in growth and mortality of the salmon, and ensures that MAB limits set by the authorities are followed according to regulations. The model only considers the saltwater production, and the planning period of the model is five years. One of the findings of Langan and Toftøy was regarding the misconception of maximizing MAB utilization. While the industry benchmarks itself on how much biomass they can produce per MAB quota each year, they often speak of the minimizing deviation from the MAB restriction. Langan and Toftøy's work showed that minimizing deviation from the MAB is not the same as increasing the biomass output. Hæreid (2011) made a stochastic model with a one-year time horizon, where the objective was to decide which sales contracts the salmon production company should enter. Øveraas and Rynning-Tønnesen (2012) made an optimization model for tactical production planning in saltwater based on Langan and Toftøy (2011) and Hæreid (2011). The mathematical growth model developed by Hæreid (2011) was implemented for a five-year planning horizon, and documented to work satisfactory. Furthermore Øveraas and Rynning-Tønnesen (2012) improved the model by having feed costs dependent of growth in the objective function. The main conclusion from the work is that adjusting average harvested weight throughout the planning horizon is critical when optimizing salmon farming production, and profit can be increased by allowing slaughtering of salmon at lower weights. This holds independent of whether biomass or profit is to be maximized and whether uncertainty is taken into account or not.

In this work, a stochastic optimization model for long term tactical planning in saltwater production is developed. Focus is on the saltwater part of the value chain, from smolt ordering to harvesting of salmon. The main challenge in production planning is to make optimal smolt orders, smolt deployments and harvesting plans, in accordance with governmental regulations and production capacities. The model in this thesis considers uncertainty in growth, mortality and price which are the major uncertainties in saltwater

production.

This thesis combines the growth model of Hæreid (2011) with the production model of Langan and Toftøy (2011) and Øveraas and Rynning-Tønnesen (2012). In order to better reflect reality, the model in this thesis has been extended with the following improvements. The freshwater limitations that impose saltwater production have been modelled more realistically, and the decision process of the point in time where smolt is ordered, delivered and deployed is thoroughly modelled. The model is divided in a location specific part in the beginning of the planning horizon and an aggregated part in the later planning periods, to ensure a correct detail level for smolt deployment in the beginning of the planning horizon, and improve solution time. End of horizon conditions are improved to make the model take more realistic decisions in the last periods. Furthermore the model is implemented with more realistic input data for a salmon farmer, and a forecast method has been implemented to make temperature scenarios.

The first goal of this thesis is to describe the salmon production value chain and explore the sources of uncertainty in the production process. Second, a stochastic optimization model has been developed to deal with the uncertainty and the restrictions in the salmon farming industry. Finally the model was simplified and implemented as a deterministic, a two-stage and a three-stage stochastic model for Marine Harvest Region Mid to evaluate the value of stochastic programming. Marine Harvest Region Mid is the regional office of Marine Harvest that operates in Trøndelag and Møre and Romsdal in Norway.

This thesis is structured as follows. Chapter 2 gives an introduction to the salmon farming industry with relevant aspects of the production process and a discussion of uncertainty, while chapter 3 presents relevant stochastic programming theory. Chapter 4 gives an introduction to the stochastic optimization model developed in this thesis, while mathematical model is presented in detail in chapter 5. Chapter 6 presents the implementation of the model at Marine Harvest Region Mid. Chapter 7 covers the results of the implementation, and the conclusion will be presented in chapter 8. Conclusively chapter 9 contains a discussion of future extensions of the model.

2 Salmon farming industry

This chapter gives an introduction to the salmon production value chain in general. The main focus will be on the saltwater part of the production cycle. However, the freshwater production is also thoroughly presented to understand the limitations it imposes on saltwater production. The detail level will be high, so that this chapter can be used as a starting point for future studies on the salmon farming industry. Marine Harvest's production process will be presented, although this value chain will apply to most of the Norwegian salmon industry. Section 2.1 gives an historic overview, 2.2 gives a detailed description of the production cycle, 2.3 gives an introduction to relevant governmental regulations and in section 2.4 uncertainties in the saltwater part of salmon production are discussed.

2.1 Historic perspective

Norway has a long coastline, and through history Norwegians have made a living by harvesting fish. Traditionally salmon was harvested as a food source, but compared to other fish species like cod, salmon catch has been rather modest (Liu et al., 2010). In the late 1960s and early 1970s modern salmon farming started in Norway as a result of a decline in wild fishing (Hjelt, 2000). During the 1970s the industry experienced many breakthroughs in solving biological and technical bottlenecks such as smolt rearing and development of dry feed. But it was not before the 1980s that the real large-scale commercial production took off, as better fish health were obtained by healthier food and feed-technology in combination with lower labor costs. Hence production costs and prices were greatly lowered. Norway is by far the world's largest producer of farmed salmon, and total production volume was 944 000 tons in 2010, which corresponds to 66% of the world's total production (Liu et al., 2010). In Norway the industry has also experienced a development towards consolidation in recent years. In 1997 there were 70 companies producing salmon compared to only 25 in 2009, and this trend is expected to continue (Marine Harvest, 2010).

2.2 Production

The production of Atlantic Salmon can roughly be divided in two parts as in figure 2.1; freshwater and saltwater production. At the end of the saltwater production the salmon are transported by well boats to the slaughter house where the fish is harvested. Salmon is then sold in the market.

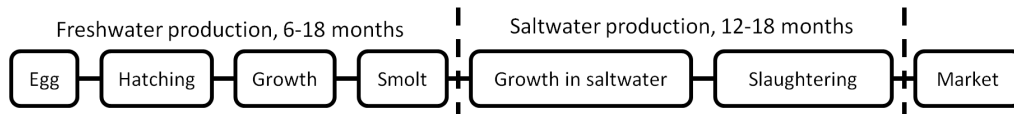


Figure 2.1: Value chain for salmon production (Marine Harvest, 2012).

2.2.1 Freshwater production

Production time in freshwater facilities is between 6-18 months, dependent on growth and which weight the fish is requested to have when deployed in the saltwater facilities. In the freshwater facilities the fish grows in large tubs onshore, and tank volume capacity is a limitation. When the fish have reached the right size, the smoltification process is started. Smolt is defined as the stage in the salmon life cycle when the fish is ready for the transition from freshwater to saltwater. If the fish does not get into saltwater within approximately two weeks, they will return to their freshwater state. Then the salmon will have to be resmoltificated, a situation which is undesirable as it increases mortality.

By controlling light and temperature conditions, initialization of the smoltification process can be manipulated. All freshwater facilities have light controlling equipment, but when it comes to controlling the temperature there are mainly two types of freshwater facilities; facilities with recycling of freshwater and facilities with through-flow of freshwater. Facilities with recycling can control the freshwater temperature, while the sites with through-flow are not controllable and depend on the natural freshwater temperature. This means that the freshwater facilities with recycling can deliver smolt at desired weights, within certain limits, throughout the year, whereas production at the locations without recycling is more unpredictable as the temperature here is uncertain. Smolt should not weigh less than 40 grams at delivery, because of mortality regarding transition to saltwater and mesh size in the marine farms, which cause a risk of escape (Marine Harvest, 2012). The upper limit on the weight of a smolt is set by law to 250 grams (Forskrift om drift av akvakulturanlegg, 2008).

In freshwater production the total loss is divided between uncontrolled and controlled losses, see below. The uncontrolled losses are caused by diseases, weak genes and more leading to mortality in the production, while the controlled losses are used as a buffer to compensate for fluctuations in the natural mortality. The buffer is a way to reduce risk for the freshwater producer. In years with a high natural mortality rate, the destruction rate of smolt is low

and vice versa. Hence the freshwater producer is a stable deliverer of smolt, and the deviation in number of smolt ordered and delivered is fairly small. However, there may be deviations in delivery time and size of the smolt.

Uncontrolled losses: Natural mortality in freshwater production; diseases, weak genes, smoltification and more	10-15%
Controlled losses: Destruction of fish used as a buffer for fluctuations in uncontrolled losses	10-15%
Total losses in freshwater	25-30%

In Marine Harvest Region Mid a rough smolt production plan for the next four to five years is available at all times. The plan specifies the total estimated number of smolt to be produced each year, and the freshwater facilities decide the number of eggs to produce based on the plan. In addition a more detailed smolt production plan is made for the next year. The detailed plan specifies the number of eggs that are to be set out within the next year, how many smolt that shall be produced at each weight interval and when these smolt are to be delivered in the next one to two years. When the smolt are ready to be delivered from the freshwater to the saltwater facilities, the saltwater production planners decide which smolt should be released at each saltwater location.

2.2.2 Saltwater production

Salmon are bred in a saltwater facility for 12-18 months, dependent on initial size, growth and harvest weight. Saltwater farms consist of several net cages where salmon are farmed. The number of fish per cage varies from 30 000 to 200 000, dependent on size and layout of the facility (Cermaq, 2012). Having several net cages enables the farmer to deploy smolt of different size and generation, while still maintaining traceability of the food production. It also gives flexibility concerning when the different deployments should occur and allows for sorting and controlled transfer of fish between cages.

To reduce the threat of diseases and lingering in the facilities, all locations must be fallowed for at least 2 months every second year. Fallowing means that the facility must be empty, so all fish must be harvested before fallowing and new smolt can first be released after the fallowing period. During this period all equipment must be cleaned and prepared for the next generation of smolt. Fallowing periods are either set in the spring or in the autumn. As a consequence, classification of facilities can be divided in four based on when the first batch of new smolt can be deployed; spring- even or odd years,

or autumn- even or odd years. In between fallowing periods smolt can be deployed at any time and any size, except in December, January and February due to generally cold water temperatures causing a high mortality rate (Marine Harvest, 2012).

Throughout the salmon life cycle, individual fish grows at different rates, and the school will therefore be ready for harvesting at different points in time. Hence there is a need for sorting the fish. When fish are sorted, the school is split between bigger and smaller fish. With the equipment available in 2012 perfect sorting of the fish is neither possible not economical favourable, but rough sorting methods are available. A salmon can be sorted several times, but the process stresses the fish which leads to higher mortality and deformities (Pickering, 1993). Frequent sorting is therefore undesirable.

In the end of the saltwater production process the fish is harvested. The fish is transported from the saltwater facility to the slaughter house in a well boat. A well boat is a special vessel for shipping living fish over large distances. It can also be used to transport young fish from the freshwater production site to the saltwater facility or transport salmon from one saltwater facility to another. Due to strict government regulations in avoiding contagion, the well boat can only transport salmons from one facility at a time. Additionally the well boat has to be cleaned thoroughly before it can pick up a ship load from a new facility (Marine Harvest, 2012). Marine Harvest Region Mid has a leasing contract for two well boats. The contract is based on a fixed yearly rental, and extra capacity can be rented in the market (Marine Harvest, 2012).

For Marine Harvest Region Mid the target weight for slaughtering is 6 kg, but fish is harvested between 2-8 kg. If the salmon is growing significantly over the target weight, it will be time consuming to slaughter it as the slaughter house machinery is designed for the weight interval of 3-6 kg. Furthermore there will be an increasing chance of the salmon becoming mature and able to reproduce, something that reduces the quality and price and also reduces the growth per kilogram feed. If the weight of salmon processed has a large variance, the slaughter house equipment has to be adjusted many times, which slows down the process. For salmon under 2 kg, the slaughter house will also be a bottleneck as they must be gutted by hand. Marine Harvest has one slaughter house in each of the four regions in Norway. In Region Mid the slaughter house at Ulvan can process approximately 70 000 salmons per day, and they aim on having an even production to ensure employment of the workforce. The slaughter house is also responsible of fulfilling contracts.

2.2.3 Production costs

Production costs per kilogram produced salmon are shown in table 1. The feeding cost is by far the largest, corresponding to 50% of the total production cost. The smolt production is about 10%, slaughtering 12% while other operation expenses amount to 14% and includes sorting and well boats. Costs related to mortality and disease will come in addition, but these are relatively unpredictable. Notice that the costs in table 1 are averages, and will vary with location, temperature, competence of the salmon farmer, disease outbreaks and other factors. Smolt production costs vary with the size of the smolt, and are given by equation 2.1 (Marine Harvest, 2012).

Estimated costs per kg produced fish		2008	2009	2010
Smolt cost per kg	NOK	2,09	1,94	2,28
Feeding cost per kg	NOK	9,76	9,85	10,64
Insurance cost per kg	NOK	0,15	0,14	0,15
Labour cost per kg	NOK	1,43	1,29	1,69
Historic depreciations per kg	NOK	1,06	0,99	1,18
Other operating costs per kg	NOK	2,88	2,90	3,20
Net financial costs per kg	NOK	0,93	0,38	0,28
Production cost per kg	NOK	18,31	17,48	19,42
Slaughtering costs (incl. transportation) per kg	NOK	2,33	2,35	2,83
Sum costs per kg	NOK	20,64	19,83	22,25

Table 1: Estimated costs per kg produced fish (Directorate of Fisheries, 2010b). The numbers for 2010 are incomplete.

$$\text{Smolt cost (NOK per smolt)} = 3.35 * \text{Weight (grams)} + 4.25 \quad (2.1)$$

2.3 Governmental regulations

In order to make salmon farming a sustainable industry the Norwegian Government has developed strict regulations. The purpose of the legislation is to ensure food safety, promote competition and profitability of the aquaculture industry within the limits of sustainable development, and to ensure health and welfare of the fish (Forskrift om drift av akvakulturanlegg, 2008). In Norway the legislation is executed by the Ministry of fisheries and coastal affairs, the Directorate of Fisheries, the Norwegian Food Safety Authority, the Norwegian Coastal Administration and local governments. This section will focus on the most relevant regulations regarding salmon farming in fresh-

and saltwater production facilities. The reader is referred to Forskrift om drift av akvakulturanlegg, 2008, for a complete list of regulations concerning operation of aquaculture facilities.

2.3.1 Regulations in the freshwater production

Freshwater facilities are regulated through licenses, where each freshwater facility is allocated a yearly amount of feed. The year goes from January to December. The feed limit can be transformed into a biomass limit by using a feed conversion rate. Therefore the feed regulation indirectly regulates the total yearly biomass produced in the freshwater facility. For the freshwater facilities with through-flow, availability of freshwater is another main limitation in smolt production. Local authorities control the use of freshwater; hence this limitation is location specific. For the freshwater facilities with recycling, availability of freshwater is not a limitation as the water consumption is significantly smaller.

2.3.2 Regulations in the saltwater production

Since 1973 biomass licensing has been practiced for saltwater facilities in Norway, and a company cannot operate in the industry without one or more licenses. Laksetildelingsforskriften, 2005, is the regulatory framework that is used today. The limit of biomass for one license is 780 tons, except for the counties Troms and Finnmark where the limit is 900 tons. A license is linked to a region, and the sum of the licenses within the region gives the region's maximum allowable biomass, MAB. Within a region, one license can be linked to a maximum of four sites, while two or more licenses can be combined and linked to a maximum of six sites. While the allocation of licenses seems like an intricate affair, in practice the salmon producers only need to make sure that the total biomass within a region does not exceed the regional MAB at any point in time. In some special cases, salmon producers are also allowed to move licenses between regions (Marine Harvest, 2012).

In addition to regional MAB, each location has a location specific MAB designated by the authorities. As the different sites can reach their locational MAB at different times during a two year period, the regional MAB can be better exploited throughout the year if the total locational MAB aggregated from every site within a region is larger than the regional MAB. Furthermore the density of salmon in a net pen cannot exceed 25 kg/m³. If either regional MAB, locational MAB or the density regulation is exceeded, the government can fine the producer and issue forced harvesting to reduce biomass.

The Regulation of Abatement of Sea Lice in Aquaculture Facilities determines when a saltwater facility needs to be emptied due to fallowing. This regulation defines lice zones as geographical areas where all locations have to be fallowed at the same time. Each lice zone has a specific date for fallowing, which reoccur each second year. Enforcing coordinated fallowing in large areas is one of the most effective ways of combating sea lice, as it removes potential host for the parasite (Forskrift om bekjempelse av lus i akvakulturanlegg, 2009).

Competition has led to regulations on market shares. One company can control maximum 25% of the available MAB licenses and total national biomass, but they have to seek permission from the Ministry of Fisheries and Coastal Affairs if they want to exceed 15%. In the spring of 2012 Marine Harvest ASA controls 25% of the licenses in Norway (Marine Harvest, 2012).

2.4 Uncertainties in saltwater production

The main uncertainties in saltwater production are fish growth, loss of fish and price. Temperature is the main uncertain parameter affecting growth, while losses are caused by diseases, mortality and escape. Furthermore prices fluctuate significantly in the short run due to uncertain demands and the salmon farmer's limited opportunity to respond to price changes.

2.4.1 Growth

Salmon growth depend on the fish wellbeing and is controlled by the following biological and physical parameters: water temperature, oxygen concentration, salinity, pH, ammonia and carbon dioxide content, fish density, lighting conditions, feed, disease and more.

Oxygen concentration, salinity, pH, ammonia and carbon dioxide content are connected to water quality. In the saltwater facilities the water quality is sustained by natural currents, and the locations are selected to ensure high water quality. Therefore the parameters regarding water quality are usually within acceptable values (Marine Harvest, 2012).

Fish density and lighting conditions are to a large degree controllable. Thorarensen and Farrell (2010) state that salmon density up to at least 80 kg/m^3 does not limit the growth or survival of Atlantic salmon, provided that water quality is maintained within acceptable limits. The density regulation of 25

kg/m³ means that fish density will never be a direct limitation for growth in Norwegian fish farms. The use of artificial lighting can be utilized to stabilize and increase growth by extending the photo period throughout the year.

The feed conversion rate, which is the ratio of fish food consumed to the weight gained, varies from location to location depending on temperature, genetics, diseases and more. It can nearly be as low as one for Atlantic salmon, meaning that for every kilogram of fish food consumed the salmon will grow one kilogram (Thorarensen and Farrell, 2010). By using video surveillance the fish farmer can feed the salmon until they stop eating to ensure maximum utilization of this potential.

However, water temperature is not controllable, as the saltwater facilities are open systems. Salmons are coldblooded and temperature is one of the governing factors for growth. The temperature is therefore the main uncertain parameter regarding growth. The specific growth rate for Atlantic salmon increases with increasing temperature, up to an optimum temperature for growth at about 14-16 degrees Celsius, beyond which it decreases (Thorarensen and Farrell, 2010). Optimal temperature for growth increases with fish size, whereas optimal temperature for feed conversion efficiency decreases with fish size (Handeland, Imsland, and Stefansson, 2008).

In the coastal water where saltwater facilities are located the temperature depends of oceanic and local conditions. Oceanic water follows a relatively fixed seasonal pattern due to high thermal capacity, which makes temperature change considerably slower than in air. In near shore water the local weather conditions become more dominant. This includes rain and freshwater runoff from land, air temperature, wind and clouds, which can quickly change the temperature in the uppermost layer of water where salmon are kept (Sætre, 2007). The uncertainty and difficulty related to long-term weather forecasting is one of the governing factors that makes the future growth of salmon uncertain.

The uncertainty in temperature can be observed by looking at historical data. Figure 2.2 shows the monthly mean temperature for the average of all the fish farms in Sør-Trøndelag in the period 1998-2006. The seasonal variations are clearly visible, with low temperatures in the winter months from January to April, and high temperatures during the summer from June to October. The variability in temperature for different years is greater in the summer than in the winter months.

2.4 Uncertainties in saltwater production

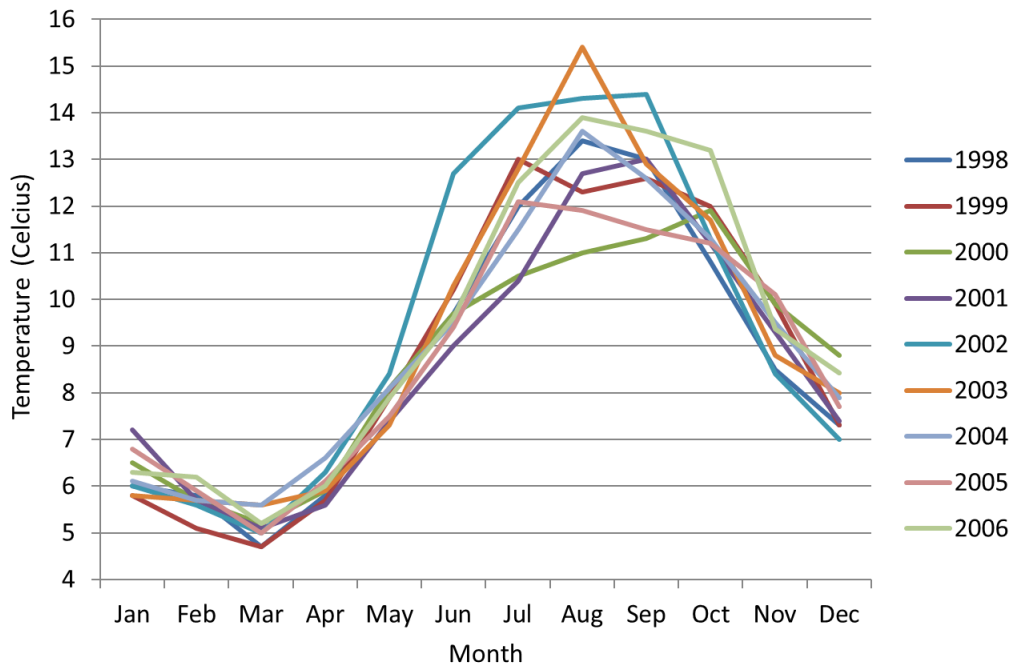


Figure 2.2: Monthly mean temperature for Marine Harvest's locations in Sør-Trøndelag, 1998-2006 (Marine Harvest, 2012).

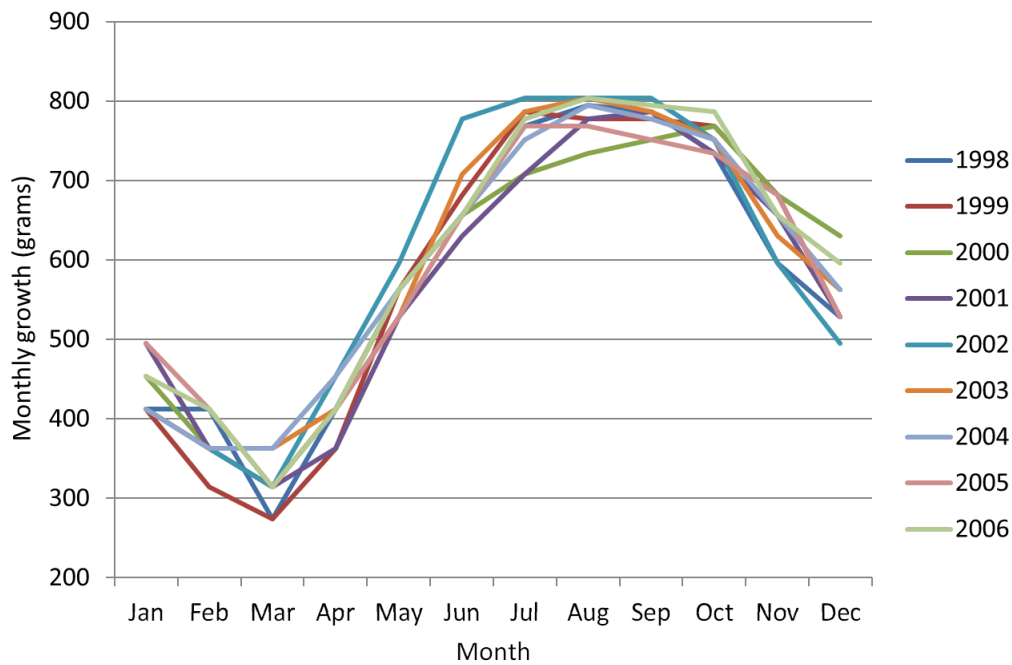


Figure 2.3: Monthly growth for a 5 kg salmon in a salmon farm in Sør-Trøndelag, 1998-2006 (Marine Harvest, 2012).

Figure 2.3 shows growth for Atlantic salmon calculated by the growth model used by Marine Harvest for the different temperature scenarios in figure 2.2 for the period 1998-2006. In comparison with the variance in temperature which is greater during the summer, the variance in growth from year to year is relatively stable. The great variances in temperature during the summer months do not give the same variance in growth, because water temperature and growth are not linearly dependent. Salmon has its optimal growth temperature at 14-16 degrees Celsius, and deviations in both directions from the optimal growth temperature leads to a lower growth rate. Therefore variations in temperature during summer are not as decisive for growth as in the winter. However, figure 2.3 still indicates that temperature is a critical input factor for growth, and fluctuations in growth due to variations in temperature from year to year is an uncertain factor for the salmon farmer.

2.4.2 Loss in production

The main causes of loss in saltwater production are mortality and escape. Diseases may either lead to a weak fish, that can still be produced at a lower quality, or it leads to mortality. In Trøndelag and Møre and Romsdal the average mortality rate during seawater production was 16.1% in 2009, which was below the Norwegian average of 22.3% in 2008 (Mattilsynet, 2011). Mortality amounts to 32,8 million fish in 2010 (Directorate of Fisheries, 2010a). On average about 80% of fish mortality occur before the salmon is 0.5 kg due to deformities, injuries from transportation and release, or fish not coping with transition to seawater (Marine Harvest, 2012). During seawater production a risk is that diseases might create mass death, and the diseases that cause the greatest risk of mass death are Infectious Pancreatic Necrosis (IPN), Infectious Salmon Anemia (ISA) and Gill Disease (GD). Further there are several diseases that reduce fish health, growth, quality and at worst the disease is fatal. In addition to those previously mentioned, the most important parasite and disease risks are Sea Lice, Pancreas Disease (PD), Heart and Skeletal Muscle Inflammation (HSMI) and Salmonid Rickettsial Septicaemia (SRS) (Marine Harvest, 2010).

Another risk of loss in salmon production is escape. In 2010 about 252 000 salmon escaped from Norwegian salmon farms (Rømmingskommissjonen for akvakultur, 2010). Fish escaping through holes in the net is approximately 65% of the reported cases of salmon escape (Hæreid, 2011). Of the reported cases in 2010, only 19% were classified as large escapes (>10 000 fish). However, large escapes contributed to 91% of all escaped fish. For a salmon producer escapes will in general be a less serious problem than mor-

tality. As escapes contribute to less than 1% of the total loss of production, mortality will be the governing uncertain factor for a large salmon producer like Marine Harvest. One of the most prominent problems with escapes are the fines and bad publicity that follows the incident.

2.4.3 Salmon price

In order to succeed in the salmon industry, understanding the dynamics of the market is an important factor. According to surveys done by Bergfjord (2009), Norwegian fish farming companies consider future salmon prices to be the most important source of risk.

Salmon is not a homogeneous product as the price varies for different types of salmon. Prices vary due to different quality levels, the size of the fish and whether the fish is fresh or frozen. In the short term salmon is sold either on contracts or in the spot market. Contracts can be negotiated for certain deliveries at a future point in time at a given price. The long-term contracts are negotiated for the next 1-2 years, while short-term contracts last for less than a year (Marine Harvest, 2012). Hence a production cycle for salmon is longer than the long-term contracts. In the short term the production depends critically on fixed factors such as available biomass, slaughter house capacity and more, thus there are limited opportunities to respond to price changes. A long production time makes the stock fixed in the short term. Strict government regulations regarding MAB and fixed production capacity gives the producer little or no production flexibility. Producers are therefore unable to adjust the production to the prices in the short run (Andersen, Roll, and Tveterås, 2008). In combination with a short time period from harvesting to consumption the short term price is both inelastic and uncertain.

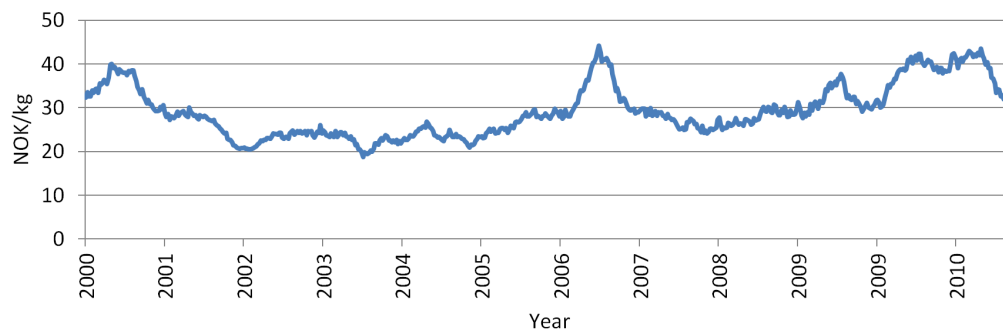


Figure 2.4: Average salmon price in NOK/kg, 2000-2010, (Statistisk sentralbyrå, 2011).

2 SALMON FARMING INDUSTRY

Over the previous years the prices of salmon have experienced large fluctuations, see figure 2.4. The average price of Norwegian whole salmon in the last decade has been 25 NOK/kg, with peaks at 19 NOK/kg and 44 NOK/kg. These huge fluctuations can mainly be explained by great variations in supply and demand. The total available stock of salmon in the market is to a large extent deciding the price. A reduction in stock will lead to an increase in the price, as the production cycle of salmon cannot immediately replace new fish. Contrary a high stock will lead to a reduction in the salmon price as there is not enough flexibility in the production system to avoid slaughtering of the salmon (Oglend and Sikveland, 2008).

It is difficult to estimate the future price of salmon since production time from spawn to slaughtering is about 2-3 years. In long term planning salmon farmers therefore give little attention to future salmon price when they are making decisions. Instead salmon production companies are aiming towards keeping the production as smooth as possible throughout the year to avoid large seasonal price fluctuations, and keeping the costs down Marine Harvest (2012).

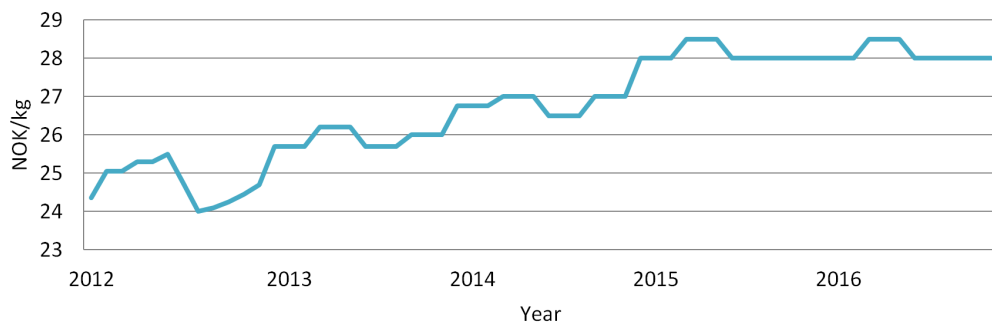


Figure 2.5: Price forecast from January 2012 to December 2016, closing date 10.01.2012 (Fish Pool, 2012).

After Fish Pool opened in 2006 the market has been more liquid. Fish pool gives out a monthly forward price for the next five years, figure 2.5. The forward price reflects the expectations of the Fish Pool's Members for this time period, and are assessed by contracts made as well as interests to buy or sell at Fish Pool (Fish Pool, 2012). The forward price does not differentiate between fish products and is therefore an average price for all fish sizes traded. Still a large share of the total volume of salmon are traded outside Fish Pool and the market still experiences a considerable growth, thus the market is not yet mature (Marine Harvest, 2012). However, Fish Pool is a reference

2.4 Uncertainties in saltwater production

market with the best available information regarding future contract trading, and their forward prices are used as a benchmark in the industry.

3 Stochastic programming

In the salmon farming industry the production planner makes decisions in a complex and uncertain environment. Uncertainty in growth, loss of production and price in combination with a long planning horizon greatly increases the risk for the salmon farmer. A stochastic programming model is a tool that can help the salmon farmer in making better decisions in this complex and uncertain environment. This chapter will introduce relevant theory on the subject of stochastic programming. Section 3.1 will introduce uncertainty and argue for why stochastic programming is preferred to deterministic in problems with uncertain parameters. Section 3.2 introduce recourse problems, section 3.3 explains the mathematical formulation and section 3.4 further describes evaluation of recourse models. After the modelling aspect of stochastic programming has been explained, section 3.5 gives an introduction to how uncertainty can be represented. It is assumed that the reader is familiar with basic principals in optimization and mathematical programming.

3.1 Modelling uncertainty

Randomness or uncertainty can be defined as lack of predictability of outcomes. Meaning, it is not known for certain what will happened in the future. Uncertainty introduces risk. In optimization research there are two dominating modelling approaches: deterministic and stochastic programming. Whereas deterministic models assume everything to be certain in the future, stochastic programming takes uncertainty into account. Stochastic models make more flexible solutions than deterministic as they include uncertainty in the model formulation by letting information become available at different points in time.

On the other hand stochastic models are more complex and more difficult to solve. Supporters of deterministic models claim that it is better to have a deterministic model with a solution than having a stochastic model without one, or that the solution time becomes too long. However, solving a problem, which is stochastic by nature with a deterministic approach, may lead to undesirable results. It is often impossible to include all wanted aspects of a comprehensive stochastic model while maintaining a short solution time and solvability. Understanding the uncertainties in the phenomenon is crucial when deciding which aspect to emphasize the most.

3.2 Recourse problems

When uncertainty is introduced to a model the point in time where decisions have to be made becomes very important. Does the choice have to be made today, or can it be postponed to a later point in time where more information is available? In deterministic models time steps such as hours, weeks, months or years are normally referred to as (time) periods. In stochastic models the term *stage* is introduced as well. A stage is a point in time where new and useful information is revealed (Kall and Wallace, 1997). The term "recourse" refers to the opportunity to adapt a solution to new information when it becomes available (Higle, 2005). It is therefore very important to allocate decisions to the correct time period, so that they can be made with the information that would be available at this point in time.

3.2.1 Scenario trees

To get a better understanding of how a recourse problem depicts uncertainty it is useful to look at a scenario tree, shown in figure 3.1. A scenario tree is a structured distributional representation of the stochastic elements and the manner in which they may evolve over the period of time represented in the problem (Higle, 2005). The circles in figure 3.1 represent nodes. There are one or more nodes in each time period, located vertically above time period t_1 to t_7 . In every node specific values for the *uncertain* or *stochastic parameters* are given. Therefore the number of nodes within the same time period depicts the number of possible futures that could transpire at this point in time.

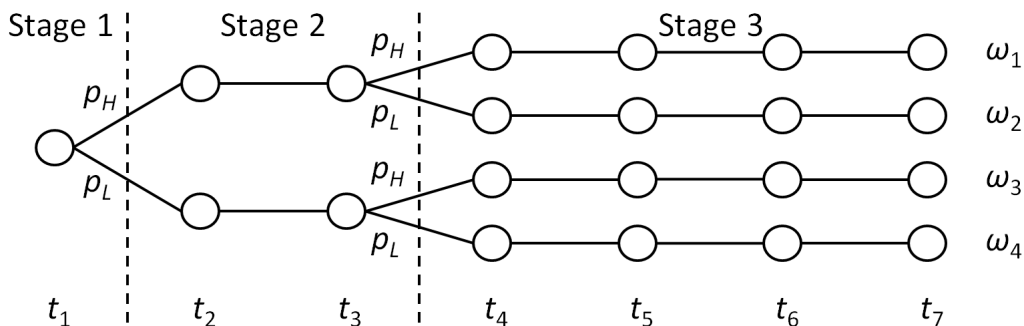


Figure 3.1: Three stage horizontal scenario tree.

The single node in the first time period is often referred to as the *root node*, while the ones in the last period are called *leaf nodes*. A scenario is defined by the path that can be drawn from the root node to a leaf node and represents one specific, complete course of events that can occur during the planning

horizon. The scenarios in figure 3.1 are denoted by ω_1 to ω_4 . The stages are separated by dashed lines, which represent revelation of new information. As the figure shows there is more than one scenario going through each node in all but the last stage, meaning that they are required to share common stochastic parameters and have equal choices made for all decisions in these periods. This is known as the non-anticipativity requirement.

When moving from one stage to the next there is a probability connected to each possible outcome, as for instance p_H and p_L in figure 3.1, which makes it possible to calculate the probability of occurrence for each scenario. As the paths splits, unique event transpires and the decisions made in a node is therefore directly dependent on all previous and possible succeeding nodes. However, the decisions in nodes within the same time period are indirectly dependent of each other, because they affect the optimal decisions that have to be made in the previous stages. In the time periods within a stage, perfect information is assumed, which can be used when making all the decisions that have to be made during that stage. All possible outcomes of the succeeding stage and their respective probabilities are known, but it is not known what scenario that will actually occur. This is the heart of recourse problems, making decisions today that will ensure the best expected results given all predicted futures and the ability to somehow adapt to each of these scenarios.

3.3 Mathematical formulation

The decisions that have to be made today, in the first stage, are often referred to as the "here and now" decisions, and are represented by the variable x . The decisions in the other stages are called recourse decisions, denoted by y_ω , because they can be decided knowing which scenario ω they are a response to. All possible scenarios ω are given by the set Ω , which contains every scenario in the scenario tree. Decision y_ω therefore adapts to the specific combination of x and ω . Since the recourse variables are scenario specific they can compensate for bad events, or exploit opportunities in the more optimistic scenarios. However, because all of the recourse variables are depended on mutual "here and now" decisions, the variable x must be chosen such that it allows flexibility to handle both worst and best case scenarios, while emphasizing all scenarios probability of occurring. This is the reason to why a stochastic model will be willing to pay for flexibility, while a deterministic model will not.

The structure of a recourse problem can have important implications for feasibility, possible solution methods and computational demand. Depending on

the problem's properties, it can be classified as either a general, simple, fixed or complete recourse problem. Only the general recourse will be presented here, and the reader is referred to Birge and Louveaux (1997) for details on the other subjects. A two-stage model will be used to introduce the implicit and explicit formulation, after which a multi-stage explicit formulation will be presented.

3.3.1 Implicit formulation

The *implicit formulation* of the stochastic model is also known as *compact form* or a *node formulation* (Birge and Louveaux, 1997). The reason for calling it implicit formulation is that the non-anticipativity requirement is ensured implicitly, by assigning the decision variables to the nodes of the scenario tree (Kall and Mayer, 2011). In the two-stage stochastic problem this means that the x only appears as one variable (independent of ω) in the implicit formulation, because the first stage decision is only represented by a single node. The number of second stage decision variables y_ω is however equivalent to the total number of scenarios because the second stage is the last one in the two-stage stochastic problem, as seen in figure 3.2.

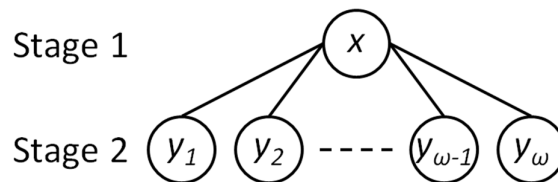


Figure 3.2: Two-stage vertical scenario tree with implicit formulation.

Higle (2005) presents a two-stage stochastic linear problem on a general form. This model has been modified by showing that y is dependent on ω . This is to emphasize that recourse decisions are scenario dependent and to ensure that the reader does not think there is any difference in how the implicit and explicit formulations handles recourse decisions. The implicit formulation is therefore stated as follows:

$$\begin{aligned} \min \quad & cx + E[h(x, \tilde{\omega})] & (3.1) \\ \text{s.t.} \quad & Ax \geq B \\ & x \geq 0 \end{aligned}$$

$$\begin{aligned} h(x, \tilde{\omega}) = \min \quad & g_\omega y_\omega & (3.2) \\ \text{s.t.} \quad & W_\omega y_\omega \geq r_\omega - T_\omega x \\ & y_\omega \geq 0 \end{aligned}$$

Problem 3.1 is the *first stage problem*, and problem 3.2 is known as the *second-stage problem*, *subproblem* or *recourse subproblem* (Higle, 2005). The term $E[h(x, \tilde{\omega})]$ in the first stage problem is referred to as the *value function* or *recourse function* (Birge and Louveaux, 1997). The uncertainty in the problem is governed by $\tilde{\omega}$, which is a discrete random variable with probability $p_\omega = \mathcal{P}\{\tilde{\omega} = \omega\}$ for each scenario $\omega \in \Omega$. An advantage of using the implicit formulation is that the information process in the problem is clearly visible, as the "here and now" decision (x) and occurring event (ω) are the premises when solving the recourse subproblem $h(x, \tilde{\omega})$ (Higle, 2005). However, the compact form has the disadvantage that in the case, when the underlying LP problem has some special structure (for instance, it is a transportation problem), this structure will be partially lost in the equivalent LP. The problem structure can be preserved by instead using the explicit formulation when making the LP equivalent (Kall and Mayer, 2011).

3.3.2 Explicit formulation

The explicit formulation is known by many different names; *extensive/full form* (Birge and Louveaux, 1997), *explicit/split-variable form* (Kall and Mayer, 2011) and *scenario formulation* (Higle, 2005). The LP problem presented in the previous section will look like the following with this formulation:

$$\begin{aligned} \min \quad & \sum_{\omega \in \Omega} (cx_\omega + g_\omega y_\omega) p_\omega & (3.3) \\ \text{s.t.} \quad & Ax \geq B \\ & T_\omega x + W_\omega y_\omega \geq r_\omega, \quad \omega \in \Omega \\ & x_\omega - x = 0, \quad \omega \in \Omega & (3.4) \\ & x_\omega, y_\omega \geq 0, \quad \omega \in \Omega \end{aligned}$$

The main change from the previous formulation is that the "here and now" decision has been split, making x_ω scenario specific. Figure 3.3 shows how this makes all the scenarios appear as parallel problems. If this was the only change made it would mean that each scenario could be solved separately, which implies that decision x_ω could be made with certain information about the occurrence of scenario ω . As there can only be made one "here and now" decision, still represented by variable x , the non-anticipativity requirement is explicitly taken into account by the constraint 3.4, which is represented with the first-stage nodes being connected in figure 3.3. The non-anticipativity constraint ensures that every x_ω variable must have the same value as x . Also, the object function 3.4 has been modified to weight each scenario with their respective probability of occurrence p_ω . These two changes ensure that

the optimal solution will be the same for the explicit and implicit formulations. There are more ways to represent the non-anticipativity constraint 3.4, and the specific choice of modelling is typically guided by the solution method to be used (Higle, 2005).

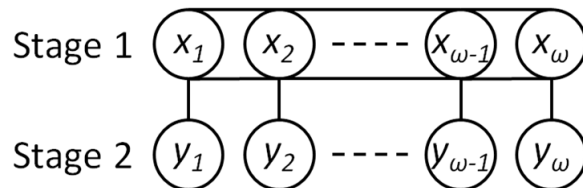


Figure 3.3: Two-stage vertical set of scenario problems linked with non-anticipativity constraint in first stage.

Even though the explicit formulation in figure 3.3 might look and behave like several different deterministic models linked together with the non-anticipativity constraint and a modified objective function, it is important to remember that it is usually not enough to add these features to a deterministic model to make it stochastic. For instance, in a deterministic model where demand is certain it would not be necessary to have separate variables for orders and sales, as one would never order something that would not be sold when demand is known. However, if the demand is uncertain and the actual demand is only revealed after orders have been placed, it is meaningful to treat ordering as a "here and now" decision and sales as recourse variables (Higle, 2005). Keeping this in mind when constructing a stochastic model, the explicit formulation allows the problem to be constructed for a single scenario, which can thereafter be expanded with the desired amount of scenarios with coherent stochastic parameters and probability.

The increased number of variables in the split-variable form makes it more computational demanding than the compact form. However, the explicit formulation has the advantage that the shape of the stochastic tree can be controlled solely by adjusting the non-anticipativity constraint and making sure the stochastic parameters are the same within the periods where scenarios share nodes in the scenario tree. The value of this controllability is not present in the formulations above, as there are only two time periods and two stages. However, in a problem with multiple time periods the explicit formulation makes it easy to change which time periods belong to which stage and adding new stages.

3.3.3 Multi-stage recourse modelling

The *multi-stage recourse problem* represents a planning situation where new information is revealed at several points in time during the planning horizon and decisions have to be made continuously based on the available information. This gives a "decide-observe-decide..." pattern which can be repeated numerous times (Higle, 2005). The multistage recourse problem can be represented in the explicit formulation as follows:

$$\begin{aligned}
 & \min \sum_{\omega \in \Omega} p_{\omega} \sum_{t \in \mathcal{T}} c x_{\omega}^t \\
 & \text{s.t.} \sum_{j=1}^t A_{\omega}^{tj} x_{\omega}^j \geq b_{\omega}^t, \quad t \in \mathcal{T}, \omega \in \Omega \\
 & \quad x_{\omega}^t - x_n^t = 0, \quad t \in \mathcal{T}(n), \omega \in \Omega(n), n \in \mathcal{N} \\
 & \quad x_{\omega}^t \geq 0, \quad t \in \mathcal{T}, \omega \in \Omega
 \end{aligned} \tag{3.5}$$

Here the first-stage and recourse variables are no longer separated using different variable names, instead it is the time period which decides which stage the variable belongs to through the non-anticipativity constraint 3.5. By introducing a set of envelopment n given by set \mathcal{N} , equation 3.5 ensures that each decision x_{ω}^t in time period $t \in \mathcal{T}(n)$ is equal in all scenarios given by $\omega \in \Omega(n)$. This is illustrated in figure 3.4, which is the explicit representation of figure 3.1, where the non-anticipativity constraints are represented by the connection of nodes within the same dashed envelopment and time period.

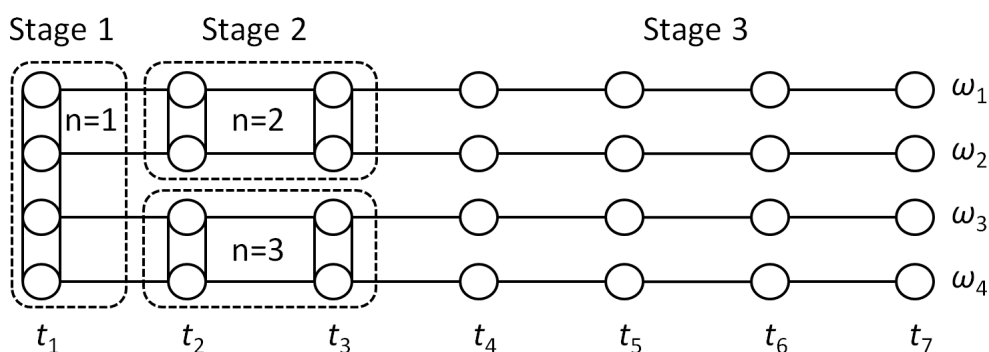


Figure 3.4: Figure 3.1 represented by a horizontal set of scenario problems with envelopments given by n .

3.4 Evaluation of recourse models

Most decision problems are certainly affected by randomness, but that is not the same as saying that the randomness should be introduced into the model (Kall and Wallace, 1997). Since stochastic models are much more computational demanding to solve compared to a deterministic version of the model, it is important to be able to evaluate the gain of including uncertainty in the formulation (Birge and Louveaux, 1997). Two methods for evaluation of recourse models are presented in this section; *the expected value of perfect information* and *the value of the stochastic solution*. The following theory will be presented in regards of a maximization problem.

3.4.1 Expected value of perfect information

The *expected value of perfect information* (EVPI) is defined as the difference in the expected objective value when making decisions with uncertainty compared to perfect information (Kall and Wallace, 1997). Another definition is that EVPI is a measure for the maximum amount a decision maker would be ready to pay in return for complete (and accurate) information about the future (Birge and Louveaux, 1997). The EVPI is calculated as the difference between the expected objective value of the wait-and-see solution and of the stochastic solution, shown in equation 3.6 with the notation used by Birge and Louveaux (1997).

$$EVPI = WS - RP \tag{3.6}$$

The *expected objective value of the stochastic solution* is defined as the solution to the recourse problem (RP), also known as the *here and now solution*. The *expected objective value of the wait-and-see solution* (WS) is the expected value of being able to solve every possible scenario with perfect information. Each scenario has to be solved separately with all information available in the first period and the objective value from each solution has to be weighted in accordance to their probability of occurrence. The probability has to be taken into account, because the decision maker will only know the actually occurring scenario after he has paid for the information, which means that the probability is still in effect before the transaction is completed. The WS model can be constructed by removing the non-anticipativity requirements from the RP model, which is easy to do if the explicit formulation is used. Removing the non-anticipativity constraints are then just as easy to do for a multistage as a two-stage recourse problem. When the WS model is solved it gives the scenario specific solution that would be best in hindsight of each scenarios occurrence. The WS solution is therefore a plan of action for each

scenario, and not a single implementable here-and-now decision as the RP solution is (Birge and Louveaux, 1997).

The EVPI can give an indication of the possible gain which can be achieved through the reduction of the uncertainty present in the problem. The calculation of the EVPI can therefore be useful when evaluating the potential value of new or improved forecast and decision support tools.

3.4.2 The value of the stochastic solution

Because stochastic models are more computational demanding than its deterministic counterparts, it is important to be able evaluate the improvements that comes at the expense of an increased solution time. The *value of the stochastic solution* (VSS) measures the expected objective value gain from using a stochastic model instead of a deterministic model run with mean values for the stochastic parameters (Birge and Louveaux, 1997). The deterministic *mean value or expected value solution* (EV) is the solution of a single scenario problem where all the stochastic parameters have been replaced with their expected values. The objective value of the EV can however not be directly compared with the RP solution, because the EV solution assumes that only the mean scenario can occur. Therefore, only the here-and-now decisions given by the EV solution would be meaningful in a setting where uncertainty transpires, because it is possible to change the recourse decisions once new information is available. Hence the EV solution might be infeasible in one or more scenarios. The *expected result of using the EV solution* (EEV) is defined as the expected objective value that would result from implementing the here-and-now decisions given by the EV. For a two-stage stochastic model the EEV model can be made from the WS model by fixating the here-and-now variables to the values given by the EV first-stage solution, whereas the second-stage variables can be adjusted for each scenario to achieve the best possible expected object value (Escudero, Garín, and Pérez, 2007). The VSS can thereby be calculated by equation 3.7:

$$VSS = RP - EEV \tag{3.7}$$

3.4.3 VSS in multistage models

Because the EEV is calculated using a modified WS model, complications arise when finding the VSS for a multistage model. In particular it is not clear which variables that should be fixed in the WS models (Escudero et al., 2007). Escudero et al. (2007) show that the two-stage calculation of the

EEV is trivial if it is used for a problem with more stages, and they proposed different approaches to calculate the EEV for multi-stage problems. If the first-stage decisions are fixated in the same way as for the two-stage calculation of EEV, it is possible that the EEV solution will outperform the RP solution. This is because there are no non-anticipativity constraints in the modified WS model. Therefore the VSS can become negative, which indicates that the two-stage method for calculation of the EEV is not suited for multistage stochastic problems. To evaluate VSS in multistage models Escudero et al. (2007) suggest two approaches to how the EEV can be calculated.

Approach A: The value of the stochastic solution in s

The greatest difficulty with the modified WS method, used for the computation of the VSS for a two-stage stochastic problem, is the lack of non-anticipativity constraints. Therefore it is natural to try and calculate the VSS based on some sort of extended RP model. However, as there are multiple stages, the point in time where the decision maker switches from using a deterministic to a stochastic model affects the VSS. In Escudero's paper t is referred to as both time period and stage. This is because all examples and scenario trees in the paper have an equal number of time periods and stages, meaning that stage t and time period t refers to the same node. The notation and equations have been modified to fit a multistage RP problem with more time periods than stages, as RP model 3.5 and figure 3.4 represents. The *expected result in s of using the expected value solution* is denoted by EEV_s for stage $s = 2, \dots, S$. It is defined as the optimal objective value of the RP model, where the decision variables until stage $s - 1$ are fixed at the optimal solution given by the EV model (Escudero et al., 2007). EEV_s is defined as switching from a deterministic to a stochastic model in stage s . It would only be meaningful to switch right before new information is revealed, when moving from one stage to the next, since all information is certain within a stage. The modified EEV_s model looks as follows:

$$EEV_s = \begin{cases} RP \text{ model} \\ s.t. x_\omega^t = \bar{x}^t, t \in T(s-1), \omega \in \Omega \end{cases} \quad (3.8)$$

Here, \bar{x}^t are the optimal values obtained by solving the expected value problem. The set of scenarios, $\omega \in \Omega$, are defined as before. The equation is only defined for $t \in T(s-1)$, as the decision variables are only locked for all time periods in all stages prior to stage s . If the definition of EEV_s is extended to $s = 1$, then EEV_1 would equal the RP model and solution. As EEV_s is dependent of stage s , the measure VSS adopts a subscript s . VSS_s , the

value of the stochastic solution in s for multistage models, is then defined as:

$$VSS_s = RP - EEV_s, \quad s = 2, \dots, S \quad (3.9)$$

VSS_s is then the cost of ignoring uncertainty until stage s . It would equal making all decisions with a deterministic model using the average values for the stochastic parameters from the first stage up until stage s . VSS_S , where S is the last stage, would be comparable to the VSS for a two-stage problem as no uncertainty is taken into account during the planning horizon.

The greatest problem when calculating the EEV_s is the locking of decisions to a deterministic solution when the stochastic parameters vary in the RP model. It will often lead to infeasible solutions for EEV_s . All decisions from stage 1 to $s - 1$ are locked to values given by the EV solution, which only gives a single set of decision for an average scenario for the entire planning horizon. Thus the probability of infeasibility increases when s is increased, as more decisions are locked. To handle this problem Escudero et al. (2007) defines the *feasible expected value in s of using the solution of the average scenario solution*, denoted by $E\hat{E}V_s$. $E\hat{E}V_s$ is the optimal value of the RP model, where the decision variables until stage $s - 1$ are fixed to zero if they are fixed to zero in the EV solution. That is,

$$E\hat{E}V_s = \begin{cases} RP \text{ model} \\ s.t. x_\omega^t \leq \bar{x}^t M_t, t \in T(s - 1), \omega \in \Omega \end{cases} \quad (3.10)$$

where M_t is a sufficiently large constants so that none of the added constraints are restrictive unless \bar{x}^t is zero. The $E\hat{E}V_s$ can be thought of as the solution one would get if an EV model was used to decide which options should be left open and which to close, and then use the available information to decide how much of each open option that should be utilized. This method does not guarantee feasibility, but introduces more flexibility regarding the variation in stochastic parameters. Also, neither the EEV_s nor the $E\hat{E}V_s$ gives a realistic representation of how a decision maker would act when utilizing a deterministic model with a rolling horizon, in a setting where new information is revealed throughout the planning period. These two methods locks up unnecessary many decisions based on information in the first stage.

Approach B: The dynamic value of the stochastic solution

If a decision maker uses a deterministic planning tool in an uncertain environment, it would not be meaningful to make more decisions than the ones absolutely needed before new information is revealed, even though the EV

model gives a solution for the whole planning horizon. Once new information is revealed, the EV model can be run with updated information. Choices already made, information about transpired events and a new expected scenario given by the average stochastic parameter will then be updated in the remaining stages. As the estimates for the stochastic parameters are updated before the deterministic model is resolved, the process becomes dynamic. This adds more precise information, as the average values are only calculated from the subset of scenarios which could occur in the following stages, and also ensures that the solution is non-anticipative (Escudero et al., 2007).

To be able to calculate the expected value of using a deterministic model in this way, Escudero et al. (2007) simplify the scenario tree by representing all time periods within a stage by a single node. All decisions within a stage can be made as soon as uncertainty is revealed in the first period of a stage. These nodes are referred to as *scenario groups*, to emphasize that they represent scenarios grouped together by non-anticipativity constraints. The set G contains all scenario groups g . All scenario groups within the stage s is given by G_s . Scenarios belonging to the same scenario group g share equal stochastic parameters and decisions up to that stage, and these scenarios are given by the set Ω_g . Lastly, $\pi(g)$ points to the immediate ancestor scenario group of node g . All these notations are exemplified in figure 3.5.

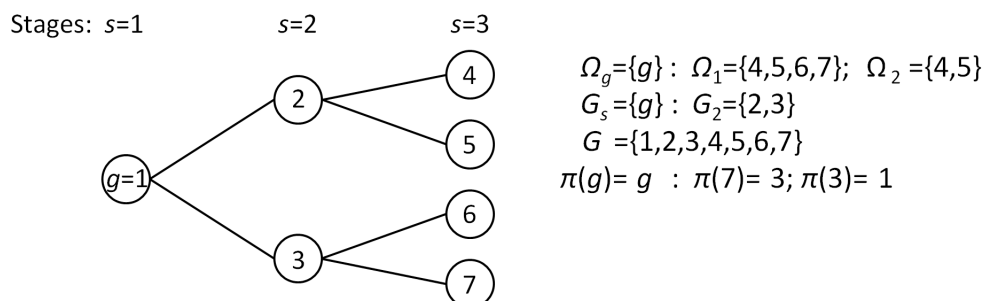


Figure 3.5: Simplified scenario tree with examples of the different notations used by Escudero et al. (2007).

With these notations it is possible to divide the scenario tree into sub-trees for each scenario group g , which gives a set of expected value sub-problems (EV_g) for each sub-tree. The optimal object value of EV_g is denoted by Z_{EV}^g . By solving the root node problem, EV_1 , and fixating the optimal first-stage decision (\bar{x}^1) for all succeeding sub-trees, the EV_g 's in stage two (given by $g \in G_2$) can successively be calculated. Continuing to solve the EV_g 's subsequently for each stage, while locking in the decisions from the ancestor scenario groups, makes it possible to calculate the expected result in s of

using the dynamic solution of the average scenario, denoted by $EDEV_s$:

$$EDEV_s = \sum_{g \in G_s} p_g Z_{EV}^g, \quad s = 1, \dots, S \quad (3.11)$$

where p_g represents the likelihood of the scenario group g , calculated as $p_g = \sum_{\omega \in G_s} p_\omega$. Finally, the dynamic value of the stochastic solution, VSS^D , is defined as the value of ignoring uncertainty throughout the planning period:

$$VSS^D = RP - EDEV_S \quad (3.12)$$

3.5 Representing uncertainty

Theory about stochastic programming has been presented, and an important part of making a stochastic model is to find values for the stochastic parameters that represent the uncertainty of the problem. Scenario generation methods are used to make scenario trees, and forecasting methods describe how information about a distribution is used to estimate future values. The theory presented here is used to generate temperature scenarios, the most governing factor for growth.

3.5.1 Scenario generation

As explained in section 3.2.1 a scenario tree is a discrete description of the possible future realizations of the parameters in a stochastic problem, and the probability of this outcome to occur in the future. In most cases the scenario tree approximates a continuous distribution, with complex interactions, that evolve over time in a complex way, and the randomness is partly external and partly internal (Wallace, 2002). A scenario tree needs to be in a reasonable size while still representing the future in a satisfactory way, and therefore a good scenario tree can be difficult to make.

Kaut and Wallace (2007) divide the most important pure scenario-generation methods in five groups; conditional sampling, sampling from specified marginals and correlations, moment matching, path-based methods and optimal discretization. The major difference between the groups is the available information of the distribution that the scenario tree should be made from. When the distribution functions of the marginals are not known, moment matching is used. In this method the marginals are described by their moments; mean, variance, skewness, kurtosis etc. For a detailed description of all the methods the reader is referred to Kaut and Wallace (2007).

3.5.2 Forecasting methods

While the scenario generation methods describe the discretization of possible outcomes, they do not consider how to get the information. The methods simply assume that some information is known today, and the future relies on this information. Nevertheless, what information is available and how this information should be emphasized, are preliminaries in the scenario tree generation process. Accordingly a method that analyses how known information today can be used to predict possible future values is needed. Such methods are called forecasting methods.

Forecasting is a wide field, and there have been developed numerous methods. Here we will present a time-series method; the autoregressive (AR) process. The AR method is chosen because it is simple. Also it takes into account the high thermal capacity of water, which means that the temperature in the next period is affected by the current temperature. In addition, a way to handle seasonality and divergent variance in a time-series is introduced. For a more comprehensive introduction to forecasting methods the reader is referred to Hiller and Lieberman (2001).

An autoregressive model

Time-series methods use data from the past to predict the future. An autoregressive model is a forecasting method where the current value of the variable that is to be predicted, y , only depends on the previous values it has taken plus an error term (Brooks, 2008). Equation 3.13 shows an autoregressive model of order p , AR(p), written in compact form.

$$y_t = \mu + \sum_{i=1}^p \varphi_i y_{t-i} + u_t \quad t \in \mathcal{T} \quad (3.13)$$

Here y_t is the value that is to be forecasted by the model and y_{t-i} is the value of y in previous periods. p is the number of previous time periods that will affect the value of y and u_t is the white noise disturbance term. φ_i is a measure of how strong the correlation between y_t and y_{t-i} is. μ is a constant in the forecast.

Seasonality

Many datasets seem to undergo episodes in which the behaviour of the series changes compared to that exhibited previously. Seasonality are periodic changes in a dataset due to the weather, campaigns, timing of activities and

more, and are so well documented that their existence cannot be doubted (Brooks, 2008). One simple way for coping with this effect, and to examine to which extent seasonality is present, is to introduce dummy variables in the regression equations (Alexander, 2008a). The number of dummy variables should be constructed to model the seasonality; twelve for monthly data and so on. Equation 3.14 illustrates a times series with dummy variables capturing monthly seasonality.

$$y_t = \gamma_1 D1_t + \gamma_2 D2_t + \dots + \gamma_{12} D12_t + u_t \quad t \in \mathcal{T} \quad (3.14)$$

In equation 3.14 y_t is the temperature in month t . $D1_t$ is the dummy variable for January, taking the value of 1 for all Januarys and 0 otherwise, $D2_t$ is the dummy variable for February, taking the value of 1 for all Februarys and 0 otherwise, and so on. u_t is the white noise error in the estimate. $\gamma_t, t=1\dots 12$ can be interpreted as the average sample seawater temperature in month t .

Divergent variances in a dataset

Many forecasting methods, including the AR-model, assume that the volatility does not change over time. This is not always the case, as the temperature data in figure 2.2 in section 2.4.1 has a higher variance during summer. Figure 3.6 illustrates a time series with a periodic shift in the volatility. A method to include a volatility shift in an AR-model is to use a normal mixture model to characterize the variance of the dataset (Alexander, 2008b).

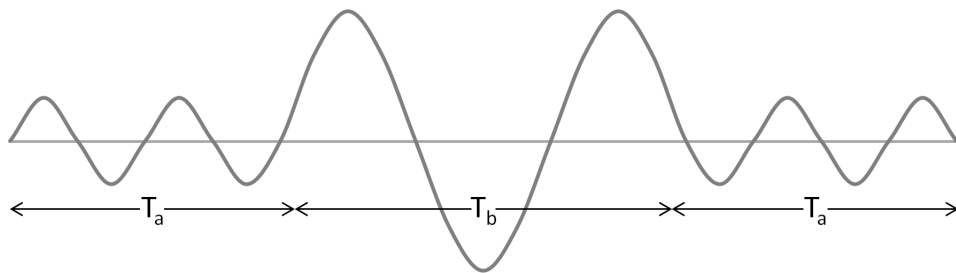


Figure 3.6: A dataset with a periodic shift in variance.

A normal mixture AR-model is based on a periodic shift where the volatility in that period can be expressed by one of several states. For a simple two-state normal mixture AR-model, the volatility is either σ_{1t}^2 or σ_{2t}^2 . The two variance components can be expressed by equation 3.15 and 3.16, and equation 3.17 gives the relationship between the two.

$$\sigma_{1t}^2 = a, \quad t \in T_a \quad (3.15)$$

$$\sigma_{2t}^2 = b, \quad t \in T_b \quad (3.16)$$

$$\sigma^2 = \pi\sigma_{1t}^2 + (1 - \pi)\sigma_{2t}^2 = \begin{cases} a, & t \in T_a \\ b, & t \in T_b \end{cases} \quad (3.17)$$

Here σ_{1t}^2 is the variance in the dataset for all time periods $t \in T_a$, while σ_{2t}^2 is the variance for $t \in T_b$. π is the periodic shifting variable that controls which state the model is operating in, and it is 1 in the first state and 0 in the second. σ^2 is the complete definition of the variance in all time periods t .

4 Model introduction

Uncertainties in the salmon farming industry have been presented in chapter 2, and in chapter 3 theory regarding stochastic programming have been described as a tool to deal with uncertainty. This chapter introduces the stochastic optimization model in this thesis. The aim of the model and simplifications regarded planning of salmon production in the real world is presented without any use of mathematical notation. Firstly the objective of the model is described in section 4.1. Section 4.2 introduces the model scope and how the model is limited from the real world problem, while section 4.3 presents the planning horizon. Finally section 4.4 gives a summary of the input data of the model.

4.1 The model objective

The main uncertain factors in salmon farming are growth, losses in production and price. Stochastic optimization models are tools to deal with uncertainty in production planning. Therefore a stochastic optimization model will be made to help the production planner. The aim of the model is to support the planner with decisions regarding optimal smolt orders, smolt deployments and harvesting. At the same time the model considers the various constraints that exist in the salmon farming industry. Because of the long production time of smolt, the planner must commit to orders that might be delivered up to two years later. The production planner also needs to decide the allocation of the pre-ordered smolt that will be delivered during the next year.

Traditionally biomass maximization has been the main objective by most salmon farmers, and today maximizing biomass output is still among the most common objective in long term production planning. However, salmon producers are profit maximizers. The opening of Fish Pool in 2006 gave producers increased access to market data, making it easier to make price forecasts and thereby increasing the use of profit maximization. The model maximizes profit corresponding to maximizing revenue minus costs. Fixed costs are not modelled, as they do not influence optimal decisions.

4.2 The model scope

The scope of the model focus on the saltwater production, thus slaughtering, sales, smolt orders, smolt deployment, growth and loss of production will be modelled. In the model mortality is the only loss of production parameter,

as escapes are assumed to be a part of mortality. Production is assumed to be location specific and net pens will not be modelled. Therefore the density regulation of net pens is also disregarded, and meeting this limitation is considered to be an operational task.

In the industry fish are classified based on quality, gene type, vaccines, feed type, weight and more. Quality of salmon meat is generally high and gene type, vaccines and feed type are controllable. However, weight is governed by uncontrollable factors affecting growth, like temperature, disease and other mentioned in section 2.4.1. Therefore weight is the most important salmon characteristic in growth modelling (Marine Harvest, 2012). Thus only the weight is modelled, and growth rate of the salmon is a result of fish weight and external factors.

As weight is the only salmon characteristic in the model, salmon price vary with weight only. This is a simplification as prices in reality are dependent of several parameters as described in section 2.4.3. However, this simplification is reasonable as weight is the most important parameter for price, given that quality is maintained. At fish Pool the salmon price is given for different weight intervals. Furthermore, salmon are assumed to be sold in the spot market; hence the model will not have the possibility of selling on contracts.

Necessary parts of freshwater production will be included in the model so that smolt order limitations are correctly handled. All freshwater facilities are assumed to have recycling of water, so availability of freshwater will not be a limitation. Also it is assumed that the freshwater producer has perfect control of the smoltification process, and can deliver smolt at any time. Furthermore it is assumed that the freshwater facilities operate as one aggregated unit, as the model only considers overall freshwater production. Lastly freshwater mortality is not included, as this is factored in by the smolt producers when they make plans in order to meet the demand.

In saltwater production, a rough sorting of fish is possible. Therefore fish can only be harvested at a wide weight interval. In the model perfect sorting is assumed, and fish can be harvested at precise weights. Hence uncertainty in fish weight when harvesting is omitted in the model.

4.3 Planning horizon

In this model the planning horizon is five years. A production cycle for one salmon is about 2-3 years from placing a smolt order to slaughtering, and

fallowing is done at each location during either spring or autumn every second year. With a five year production cycle, the model will therefore be able to have at least one full production cycle at all locations. The advantage of having a planning horizon over five years is that the future consequences of smolt ordered and deployed today will be modelled.

The uncertainty introduced with a long planning horizon set limits to how far into the future it is reasonable to plan harvesting and sales. On a five years perspective demand and prices for salmon, as well as mortality and growth, are impossible to accurately forecast. This introduces uncertainty into the planning tool, which implies that a specific growth, mortality and price scenario will have to occur for a slaughter plan to be valid. Nevertheless, tactical planning in this industry is done on a five years perspective. Scenario specific long term slaughtering plans given by the model will therefore have limited value beyond ensuring credibility for the smolt order and deployment plan.

The model is implemented with a rolling horizon, which means that the model can be run with real time information at any point in time. With a rolling horizon the model can be updated when the planner gets new information. The model output is a plan for smolt ordering and smolt deploying on a 1-3 years perspective. It will be suitable to run the model yearly, as smolt are ordered on a yearly basis and the freshwater facilities traditionally do not change their smolt production after it has started. Nevertheless a rolling horizon can be useful if extreme scenarios occur, as the planner can run the model and make the best of the situation.

4.4 Input data

Based on the introduction above, a brief presentation of the input parameters will be given.

Regulatory

Allocation of MAB licenses to sites is disregarded, as it is considered to be an operational task. Therefore, MAB licenses are only handled as regional MAB. Furthermore the model considers locational MAB limitations. For the freshwater facility the yearly feed license is implemented as a yearly maximum biomass, with a feed conversion rate changing feed to biomass.

Capacities

Minimum and maximum slaughtering capacities are given for the slaugh-

ter house. In addition a capacity factor for slaughtering fish at different weights is introduced, as small and large fish need extra slaughter capacity. A weight reduction factor is implemented due to gutting, so sales weight reflects salmon meat properly. The well boat is implemented with a maximum capacity. Also a maximum biomass is given for the freshwater facility so production volume capacity is not exceeded. This MAB limit is self-imposed by the smolt producer, and is therefore not part of the regulatory constraints.

Stochastic data

The biomass development of the salmon is based on input data for growth rate and fish mortality. Growth is modelled by a growth rate, while mortality is modelled by two survival rates; at release and throughout production. The growth rate and the survival rates are stochastic in the model, making the biomass development for future periods uncertain. Prices for fish and the feed conversion rate will also be uncertain. Lastly the probability of each scenario is stochastic.

Costs

Costs are deterministic input data, given as smolt cost, feeding cost and caring cost. Caring costs are related to ensuring the fish's wellbeing. In addition there are penalty costs for emergency harvest, exceeding the maximum well boat capacity and exceeding the minimum slaughtering capacity. Penalty costs reflect the cost of renting additional capacity.

Initial biomass

Input data for the initial biomass of fish in the saltwater facilities in the beginning of the planning period is given. Also, pre-ordered smolt is included, as they represent the initial biomass in the freshwater facility.

5 Model formulation

A linear multistage stochastic optimization model will now be presented. Firstly, section 5.1 explains decision structure and how growth is pre-processed and modelled. This section also explains how the problem is aggregated to improve solution time and how the end of horizon problem is handled. In section 5.2 all sets, parameters and decision variables will be defined. Then section 5.3 will give a detailed mathematical formulation of the problem with objective function and constraints.

5.1 Important model features

Accurately modelling the situation faced by a salmon producer who seeks to optimize profits requires an overwhelming degree of detail. Therefore, assumptions and simplifications are essential in making the task feasible and developing a computationally soluble model. Relevant notations are introduced during the discussions in this section, as it strengthens the connection to and understanding of the mathematical model in section 5.3.

Decision structure

Long term production plans are normally made once a year in the salmon farming industry (Marine Harvest, 2012). In January each year the planner makes decisions regarding future planning, which equals the first period in the model. Only the decisions that need to be implemented during the following year have to be made, as all other decisions can be postponed. Because of the long production time of smolt, the smolt that are going to be delivered during the upcoming year are already in production in the freshwater facilities. Delivery date, amount and size of the smolt for the next year can therefore not be changed. The only decisions that can and have to be made regarding these pre-order smolt, are how they are going to be distributed amongst the available saltwater facilities, referred to as either the deployment or release decision. This one year long deployment plan is sent to and implemented in the saltwater facilities. As each facility knows the weight, date and amount of smolt that will be released during the upcoming year, necessary preparations can be made to enable the implementation of the plan.

The freshwater facilities on the other hand need to know what type of smolt they should start to produce during the upcoming year. The smolt delivery plan specifies the delivery date, weight and amount of smolt that are to be delivered to the saltwater production. However, the production planner cannot order smolt that would require start of production in the freshwater

facilities prior to the current date. Also, it is not necessary to commit to an order that requires production start after January next year, as a more informed decision can be made in next year's smolt delivery plan. This is illustrated in figure 5.1. Figure 5.1 shows lead time in smolt production for different smolt delivery weights. As all the four smolt types in the figure must start production in 2012, ordering will have to be done in January 2012. The figure specifies the order point t' , point for deciding release \tilde{t} , and delivery and release point t for a smolt with delivery weight 250 g. Even though all the smolt in the figure have to be ordered in the same period, the point where releases have to be decided are not the same. The release decision is taken in January of the year of delivery and release.

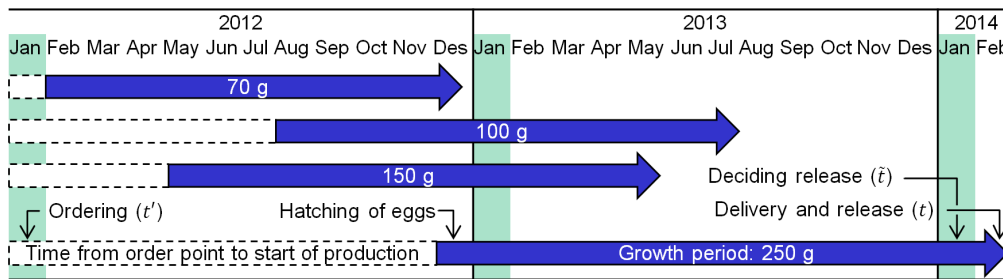


Figure 5.1: Smolt production lead time

The freshwater facilities have high controllability of the number of smolt produced due to buffers and controlled destruction of smolt, as explained in section 2.2.1. As the difference is normally smaller than the error of counting the fish, it is assumed that the number of smolt ordered is the same as the number of smolt delivered (Marine Harvest, 2012). The model is further simplified by assuming that smolt is delivered at the correct weight and date, according to the smolt delivery plan. These simplifications make the smolt order and delivery equal. The smolt delivery variable is defined as smolt delivered in time period t , ordered in time period t' , and is also referred to as the smolt order variable. This variable only exists for the order and delivery time periods that are interconnected through the lead time of smolt production. Next, the smolt deployment variable is defined as smolt deployed in period t , where the decision regarding deployment is made in period \tilde{t} . Non-anticipativity regarding smolt delivery and deployment is then handled for the point in time when the decisions are made. This leaves harvesting and sales as the only decisions made on a month to month basis. All actions regarding ordering, delivering, releasing, harvesting and sales are assumed done in the beginning of each month.

Classification of fish

All salmon are defined as part of a fish class f in the set of fish classes \mathcal{F} . The set of fish classes represents a discretization of fish weight, which in reality is continuous. In the beginning of each period, every fish in fish class f has a weight given by V_f . This weight would in reality represent a mean weight for a given interval between an upper and lower boundary. By assuming that all fish in fish class f weights exactly V_f , it is easier to model weight and growth. Lastly, by assuming that all fish in fish class f grow at the same rate; it is possible to model growth in a soluble way.

As producers and the market classify fish somewhat differently, salmon need to be distinguished for two purposes; as part of a fish class f and as part as part of a sales class p in the set of sales classes \mathcal{P} . Sales classes are weight intervals defined by the market. All fish classes f that are contained within a sales class p , are given by the subset \mathcal{F}_p . Therefore, all fish are defined by a specific fish class f , while at the same time being part of the weight interval given by sales class p . For example, for a fish in fish class f weighing $V_f = 4.25$ kilograms, f will be an element in the set \mathcal{F}_p which contains all fish classes with a V_f between 4.00 and 5.00 kilograms.

Modelling growth in saltwater facilities

As all salmon have been defined by a discrete weight V_f and fish class f , it is possible to make a weight dependent growth model. In a period, the fish grows a given amount of kilograms and is moved into the fish classes with the appropriate V_f . In each period, all fish belong to a fish class f . The number of individual salmon in fish class f after harvesting and release of smolt in period t , location i , region r and scenario s is given by variable n_{firs}^t . Number of salmon n_{firs}^t is given at the start of the period, as it will change during the period due to mortality and growth. Mortality is handled through the stochastic survival rate ε_{firs}^t , while harvesting and release is modelled by decision variables w_{firs}^t and y_{firs}^t respectively, all indexed the same way as n_{firs}^t . The model will be able to harvest fish directly from fish class f , as perfect sorting is assumed. Although the number of fish is an integer number, the magnitude of the number of fish and uncertainty in counting methods for fish both in freshwater and saltwater facilities make integer constraints less important. Therefore, all variables are allowed to take on real values, thereby avoiding the complexity related to integer programming.

The following growth model was developed by Hæreid (2011). The growth in kilograms in period t for fish in fish class f in location i , region r , scenario s is given by the stochastic parameter σ_{firs}^t . Parameter σ_{firs}^t is stochastic

due to the uncertainty related to salmon growth. The weight at the end of a period t for fish in fish class f in location i , region r , scenario s is given by adding the growth during t , σ_{firs}^t , to the weight at the beginning of the period, V_f . Because of the discretization of weigh, the value $(V_f + \sigma_{firs}^t)$ will usually fall between two weights given by the set of fish classes.

$$V_{\underline{f}} \leq (V_f + \sigma_{firs}^t) \leq V_{\bar{f}} \quad (5.1)$$

Here, $V_{\underline{f}}$ is the weight of a fish in fish class \underline{f} , the fish class with defined weight closest to $(V_f + \sigma_{firs}^t)$ from below. $V_{\bar{f}}$ is the weight of a fish in fish class \bar{f} , the fish class with defined weight closest to $(V_f + \sigma_{firs}^t)$ from above.

As the model keeps track of biomass using discrete weights connected to fish classes f , all fish must be distributed into new fish classes in a fashion that properly represents the total biomass development during the period. This is ensured by distributing the fish between the two classes \underline{f} and \bar{f} , given by equation 5.1. The distribution is done based on how $(V_f + \sigma_{firs}^t)$ compares to $V_{\bar{f}}$ and $V_{\underline{f}}$, as shown in equation 5.2. A linearized split, where $\delta_{f\bar{f}rs}^t$ is the share of fish class f that is distributed to fish class \bar{f} , and $\delta_{f\underline{f}rs}^t$ is the share of fish class f that is distributed to fish class \underline{f} is formed.

$$\delta_{f\bar{f}rs}^t = \frac{V_{\bar{f}} - (V_f + \sigma_{firs}^t)}{V_{\bar{f}} - V_{\underline{f}}} \quad (5.2)$$

$$\delta_{f\underline{f}rs}^t = \frac{(V_f + \sigma_{firs}^t) - V_{\underline{f}}}{V_{\bar{f}} - V_{\underline{f}}}$$

Equation 5.1 and 5.2 are part of the data preprocessing, and not the model itself. They are included to ease the understanding of how $\delta_{f\bar{f}rs}^t$ is calculated. A visual representation of the growth model is given in figure 5.2.



Figure 5.2: Growth model for saltwater facilities (Langan and Toftøy, 2011).

Being able to model biomass development accurately requires a combination of sufficiently large time resolution and a detailed enough partitioning of fish weight. The reason is that if the growth of fish in fish class f during period t in location i , region r , scenario s , σ_{firs}^t , is such that $\underline{f} = f$ for δ_{firs}^t , a share of the fish will remain in f . If this continues throughout the planning period, some of the fish will get stuck in fish class f . This would never occur in reality and is therefore unacceptable. Increasing the length of periods or increasing the number of fish classes within \mathcal{F} would make it less likely that fish stop growing in the model. Øveraas and Rynning-Tønnesen (2012) showed that this is not a problem when 82 fish classes and a 1 month resolution are used.

Modelling growth in freshwater facilities

The scope of the model focus on saltwater production, but the smolt deliveries cannot be unlimited as the freshwater facilities must be able to produce them. The main limitations in freshwater production are the available volume for keeping salmon at all times and the yearly amount of feed to be used, giving a yearly biomass limitation. If most of the smolt are to be delivered in consecutive time periods, the strain on the volume capacity would be much higher than if the delivery dates were evenly spread throughout the year. The complexity of this relation is most correctly handled with a simple freshwater growth model. The assumption that smolt ordered equals smolt delivered makes the growth of smolt independent of scenarios.

The number of smolt ordered in period t' to be delivered in time period t is given by the decision variable $o_{firs}^{t't}$, which is specified for smolt class f in the set of smolt classes \mathcal{C} , region r and scenario s . The set of smolt classes \mathcal{C} is a proper subset of fish classes \mathcal{F} , and the smolt class and fish class share the same index f to emphasize this. Therefore, smolt class f and fish class f have the same weight V_f , and smolt class f is only defined for the possible delivery weights of smolt. Releasing smolt into the saltwater facilities can therefore be seen as adding fish in the smallest fish classes, as the lowest fish class weights are the delivery weights of smolt.

The growth model used for saltwater facilities has the advantage that it accounts for all fish in every fish class in every period. This allows for easy modelling of harvesting, as fish may be removed at any weight at any time. It is however computational demanding, due to the large amount of variables needed to account for the fish. In the freshwater facilities, the number of possible "harvest" weights are limited by the set of smolt classes \mathcal{C} , as it defines

the possible delivery weights of smolt. Therefore growth in freshwater can be modelled using an array $V_{ft}^{\hat{t}}$, representing the weight in kilograms of how much a smolt in smolt class f contributes to the freshwaters facility's maximum biomass capacity in time period \hat{t} , when the smolt is to be delivered in time period t . This is illustrated in figure 5.3, where period length is one month. Here $V_{70g,11}^{\hat{t}}$ is the weight contribution of a 70 grams smolt to be delivered in period 11, while $V_{100g,12}^{\hat{t}}$ is the weight contribution of a 100 grams smolt to be delivered in period 12. For both $V_{70g,11}^{\hat{t}}$ and $V_{100g,12}^{\hat{t}}$ the weight contribution in the freshwater facilities from the delivery period 11 and 12 is zero. This is because delivery is done in the beginning of the period, while the limitation of total biomass in the freshwater facilities is checked at the end of the period after growth has taken place. Up until period 3 the weight is also zero, as the spawn does not start feeding before period 3 to achieve its target weight and delivery date. Lastly, in period 10 the weight differs, due to the feeding schedule being adjusted such that $V_{70g,11}^{\hat{t}}$ is exactly 70 grams the month before delivery.

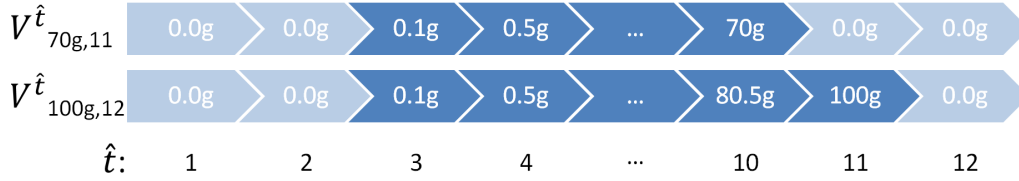


Figure 5.3: Growth model for freshwater facilities

Using the growth model presented in figure 5.3 does not increase the number of variables in the complete model, assuming there is only one freshwater facility, which excludes the need for a freshwater facility index on the smolt delivery variable. To ensure that the maximum biomass of the freshwater facility is not exceeded, a constraint checking the sum of all smolt delivery variables $o_{frs}^{t't}$ multiplied by the corresponding $V_{ft}^{\hat{t}}$ needs to be added for every time period \hat{t} and scenario s .

In addition to the biomass capacity, the freshwater facilities' total production within a year is limited by the total amount of feed that the government allows them to utilize within a year. The feed conversion rate specifies the relationship between feed and smolt weight. By assuming that the feed conversion rate in freshwater is one, the feed constraint is modelled by summing the number of smolt to be delivered within a year multiplied with its designated delivery weight, making sure it never exceed the yearly feed limit.

Reduction of problem size

As discussed in chapter 3, solving a stochastic model is time consuming. If production planners in the salmon farming industry are to use optimization software based on the model made in this thesis, the problem must be solvable within an acceptable timeframe. Avoiding the use of integer variables greatly simplifies the problem and reduces the solution time. However, the large number of variables needed to correctly model salmon farming makes the LP-problem computationally demanding. The model needs to have variables for every time period t , fish class f , region r and scenario s . All the cited variables, excluding the smolt delivery variables, are also given for every saltwater facility, location i . Reducing the sets of these indexes would reduce the problem size.

Time period t is given in the set of time periods \mathcal{T} . Time period t cannot represent a time period longer than a month, as the smolt delivery plans should be made with the possibility to deliver smolt at least once a month (Marine Harvest, 2012). Therefore, the set of time periods \mathcal{T} needs to contain every month in the planning period. Fish class f is given for set of fish classes \mathcal{F} . As discussed when the growth model in saltwater were presented, \mathcal{F} needs to be large enough to ensure a correct representation of growth. \mathcal{F} may only be reduced if time period t stretches over a longer period in time, but this not an option due to the delivery plan resolution. Region r is given by the set of regions \mathcal{R} . If \mathcal{R} is reduced, it would mean that the regional MAB licenses are used between regions. This is actually legal in some cases, but is viewed as an operational action that should not be taken into account in long term tactical planning. Scenario s is given by the set of scenarios \mathcal{S} . In a stochastic model the number of scenarios greatly influence the solution time. Therefore, the number of scenarios should be chosen as a trade-off between solution time and quality of stochastic solution in mind.

The number of locations within a region, given by set \mathcal{I}_r , can be reduced through aggregation, due to the fact that many saltwater facilities share similar properties. Within a lice zone, locations need to be fallowed at the same time. Due to geographical closeness, it is reasonable to assume that these locations have similar growth and rates of mortality; as long as no significant data indicates differently. The model could then be solved for all aggregated locations \mathcal{I}_r^A , where aggregated location \hat{i} has a locational MAB equal to all the locations within the lice zone it represents. This would give the same solution as for a non-aggregated model, although it would be less detailed. Aggregation is not a problem in the later part of the planning horizon, as the calculations made in these time periods are mainly done to evaluate the

feasibility of the first stage decisions. However, the model needs to tell the production planner exactly where to deploy the pre-ordered smolt, which will arrive during the first year of the planning period. Consequently, the model cannot be aggregated the first year. Therefore, aggregation of location must be time dependent. Time period T^A is defined as the time period from which locations are aggregated.

Aggregation of saltwater locations

Figure 5.4 illustrates how the time dependent aggregation is implemented. Since saltwater locations are aggregated from time period T^A , the problem needs two data sets for location MAB; MAB_i and MAB_i^A for non-aggregated and aggregated locations respectively. Also, biomass needs to be moved correctly from the non-aggregated locations to the aggregated locations in time period T^A . Hence, the linking set \mathcal{I}_{ir}^L is introduced. All non-aggregated locations i within an aggregated location \hat{i} in region r is given by the set \mathcal{I}_{ir}^L .

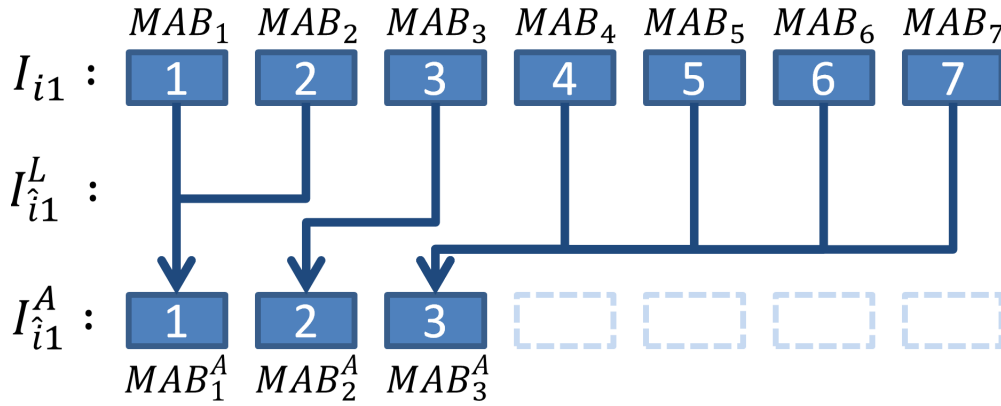


Figure 5.4: Illustration of aggregation in region 1 in time period T^A

The objective function and most of the constraints are only affected by the number of variables dependent on location being reduced. To ease the reading of this chapter, it is undesirable that all these equation are repeated twice; the only difference being whether they are defined for set \mathcal{I}_r or \mathcal{I}_r^A . By defining \mathcal{I}_r^A as a proper subset of \mathcal{I}_r , all location specific variables not included in \mathcal{I}_r^A are undefined from time period T^A . Therefore, when summing over the set \mathcal{I}_r , the elements of set \mathcal{I}_r^A are also included, meaning that only one version of the equation is necessary in most cases. Aggregation of locations is then the same as reducing the number of locations, where the changing properties are handled by the location MAB constraint and where correct biomass movement is performed using the biomass development in

saltwater constraints.

End of horizon problem

The end of horizon problem has to be handled in the model so that the model will not empty all the sites at the end of the planning horizon. Øveraas and Rynning-Tønnesen (2012) verified that the end of horizon constraint does not influence the first three years of planning, but the two last years will be greatly affected. There are several alternatives to deal with the problem, and the three following methods have been simultaneously implemented in the model.

Midthun (2007) modelled storing of gas in the pipeline system in the North Sea. He introduced a value function to give the gas a value at the end of the planning horizon, and thus give the model an incentive to store gas in the last periods. This problem is similar to what the salmon farmers face, as salmon must either be harvested and sold or kept in stock. An advantage of this approach is that the end of horizon value can vary with season, weight of salmon and the stock level of biomass in the last period. Nevertheless the method allows for limited flexibility and neglects the true value of storing (Midthun, 2007). In the model, values for biomass at the end of the planning horizon have been added.

Enforcing a minimum biomass in the last period will ensure that the model does not empty all locations. However, continued production is not only dependent of keeping salmon in the last period, but the composition of fish in the different weight classes. Therefore constraints for minimum regional biomass and minimum biomass in each sales class are introduced. Then the model is free to choose whether it will fulfil these using many smaller fish or fewer larger fish within each interval.

While the end of horizon constraints presented over ensure a minimum biomass in the last period, they do not guarantee sensible smolt release in the last years. The last year smolt delivery will be very different from previous years both in delivery data, weight and amount (Øveraas and Rynning-Tønnesen, 2012). Limiting the maximum biomass of smolt which can be released in each month in the last year, the smolt plan in these periods is controlled without completely predetermining it. Then months that would normally not be used for delivery are not available, and the total biomass delivered in each month would not be unreasonably high.

5.2 Notation

The following section presents all sets, indexes, constants, stochastic parameters and decision variables that are included in the model. Sets are denoted by capital, calligraphic letters with corresponding indexes in small, italic letters. Deterministic data, constants, are given by capital letters, while stochastic data are denoted by small, Greek letters. Decision variables in the model are small letters. Quantities refer to number of fish, while amounts describe number of kilograms. Prices are given in NOK per kg.

Sets

\mathcal{F}	Set of all fish classes
\mathcal{C}	Set of all smolt classes, $\mathcal{C} \subset \mathcal{F}$
\mathcal{F}_p	Set of fish classes f that is part of sales class p
\mathcal{P}	Set of all sales classes
\mathcal{I}_r	Set of non-aggregated locations i in region r
\mathcal{I}_r^A	Set of aggregated locations \hat{i} in region r after aggregation of locations, $\mathcal{I}_r^A \subset \mathcal{I}_r$
$\mathcal{I}_{\hat{i}r}^L$	Set linking non-aggregated location i to aggregated location \hat{i} in region r
\mathcal{R}	Set of all regions
\mathcal{S}	Set of all scenarios
\mathcal{H}	Set of all harvestable fish classes
\mathcal{Y}	Set of all years
\mathcal{T}	Set of periods
\mathcal{T}_y	Set of periods t in year y
\mathcal{T}^N	Set of periods t with no deployment
\mathcal{N}	Set of enveloped scenarios used in non-anticipativity constraints

Indexes

f, \hat{f}	Index for fish class f
p	Index for sales class p
i, \hat{i}	Index for location i
$t, t', \tilde{t}, \hat{t}$	Index for time period t
r	Index for region r
s, s'	Index for scenario s
y	Index for year y
n	Index for envelopment n

Deterministic Data

K_f	Caring cost per fish in fish class f
G_f	Purchasing cost per smolt in smolt class $f \in \mathcal{C}$
B	Feeding cost per kg feed
N_f^t	Number of pre-ordered smolt of fish class f to be delivered in period t
I_{Bfir}	Initial number of fish in fish class f in location i in region r
V_f	Weight in kilograms of a fish in fish class f
$V_{ft}^{\hat{t}}$	Weight contribution in kilograms of smolt in time period \hat{t} to the freshwaters facilities maximum biomass capacity, when the smolt is to be delivered in time period t with weight equal to fish class f
$L^{\hat{t}}$	Maximum allowable biomass in the freshwater facilities in time period \hat{t}
U_y	Total biomass of smolt available for delivery in year y
MAB_i^t	MAB in kilograms for non-aggregated location i in period t
$MAB_{\hat{i}}^{At}$	MAB in kilograms for aggregated location \hat{i} in period t
MAB_r	MAB in kilograms for region r
T^A	Time period from which locations are aggregated
\bar{S}_r	Maximum slaughtering quantity per period in region r
\underline{S}_r	Minimum slaughtering quantity per period in region r
Q_f	Weight reduction factor due to gutting for fish class f
C_f	Capacity used at the slaughter house for fish class f
W^t	Capacity of the well boat in period t
M^V_f	The value of keeping a kilogram of fish class f in the last period $ \mathcal{T} $
N_r	Minimum end of horizon share of the MAB limitation in region r
EOH_p	Number of fish in sales class p at the end of horizon
U^t	Upper limit for total biomass of delivering smolt in every period t in the last year
M^E	Penalty cost for performing emergency harvest
M^W	Penalty cost for exceeding the maximum well boat capacity
$M_{\underline{S}_r}$	Penalty cost for falling below the minimum slaughtering quantity in region r
M^S	Penalty cost for falling below the end of horizon condition regarding minimum weight in sales class p

Stochastic Data

ε^t_{firs}	The survival rate for a fish in fish class f at location i in region r in scenario s
φ^t_{firs}	The survival rate for a smolt in smolt class f released in location i in region r in scenario s .
$\delta^t_{\hat{f}firs}$	The share of fish class \hat{f} that has grown to become part of fish class f due to the growth in period t in location i in region r in scenario s
σ^t_{firs}	The growth in kilograms for fish class f in location i in region r in scenario s
ρ_s	The probability of scenario s
α^t_{ps}	Price per kilogram of fish in sales class p in time period t in scenario s
γ^t_{firs}	Feed conversion rate for fish class f at location i in region r in scenario s

Decision Variables

x^t_{ps}	Number of kilograms of fish in sales class p sold at price α^t_{ps} in scenario s
$y^{\tilde{t}t}_{firs}$	Number of smolt of fish class f released at location i in region r in scenario s in period t . Decided in time period \tilde{t} . Only defined for $f \in \mathcal{C}$ and where \tilde{t} and t are connected by the lead time of smolt deployment
w^t_{firs}	Number of fish of harvestable fish class f harvested at location i in region r in scenario s in period t . Only defined for $f \in \mathcal{H}$
n^t_{firs}	Number of fish of fish class f at location i in region r in scenario s at the beginning of period t
$o^{t't}_{frs}$	Number of smolt of fish class f at in region r in scenario s to be delivered in period t . Ordered in time period t' . Only defined for $f \in \mathcal{C}$ and where t' and t are connected by the lead time of smolt production
e^t_{firs}	Number of fish in fish class f emergency harvested from location i in region r in period t in scenario s
$m^t_{s \underline{s}_r}$	Deviation from the minimum slaughtering quantity in region r in period t in scenario s
$m^t_{s w}$	Deviation from the well boat capacity in period t in scenario s
$m_p s$	Deviation from the end of horizon condition regarding minimum weight in sales class p

5.3 Model formulation

The model will now be presented in detail. First the objective function will be described, and then constraints will be presented. The model is a stochastic multi-stage model, written using the explicit formulation. The number of stages and shape of the scenario tree can then easily be changed using the non-anticipativity constraints.

As \mathcal{I}^A_r has been defined as a proper subset of \mathcal{I}_r , summing over or defining an equation for \mathcal{I}_r includes all aggregated locations in \mathcal{I}^A_r . Therefore, only the locational MAB restriction and biomass development constraints need to be formulated to handle the aggregation from time period T^A , which eases the reading of this section.

Lastly, some variables are undefined for parts of sets used in some of the constraints. Consequently, the variables affected will be set to zero for these parts of sets after the constraint is presented. In the implementation, however, equations setting variables to zero are not included. The affected variables are instead undefined, to reduce the number of variables.

5.3.1 Objective function

The objective function maximizes profit from sales minus costs from production. Penalty costs for performing emergency harvesting, falling below the minimum capacity restrictions of the slaughter house, exceeding the capacity restrictions of the well boat and breaking the end of horizon constraint regarding minimum biomass are also included in the objective function. Finally a term for valuing salmon in the last period of the planning horizon is added.

$$\begin{aligned} \max z = & \sum_{s \in \mathcal{S}} \rho_s \left(\sum_{t \in \mathcal{T}} \left(\sum_{p \in \mathcal{P}} \alpha_{ps}^t x_{ps}^t - \sum_{r \in \mathcal{R}} \sum_{f \in \mathcal{F}} \sum_{t' < t} G_f O_{frs}^{t't} \right. \right. \\ & \left. \left. - \sum_{r \in \mathcal{R}} \sum_{i \in \mathcal{I}_r} \sum_{f \in \mathcal{F}} (n_{firs}^t (B \gamma_{firs}^t \sigma_{firs}^t + K_f) + M^E e_{firs}^t) \right) \right. \\ & \left. - \sum_{r \in \mathcal{R}} M_{\bar{s}_r} m_s^t \bar{s}_r - M^W m_s^t w) - \sum_{p \in \mathcal{P}} M^S m_p s + \sum_{r \in \mathcal{R}} \sum_{i \in \mathcal{I}_r} \sum_{f \in \mathcal{F}} M_f^V V_f n_{firs}^{|\mathcal{T}|} \right) \quad (5.3) \end{aligned}$$

In the objective function ρ_s is the probability for scenario s . The first seven terms are given for each scenario s and time period t . The first term represents income, and here x_{ps}^t represent the number of kilograms of salmon

in sales class p sold at sales price α_{ps}^t . In the second term G_f represent the purchasing cost of one smolt of smolt class f while $o_{frs}^{t't}$ is the number of smolt ordered in period t' to be delivered in time period t to region r . By summing over all orders made prior to t that are going to be delivered in period t , this term calculates the cost at delivery. The third term represents the cost of producing the salmon. Variable n_{firs}^t is the number of fish at location i in region r at the beginning of period t , B is the feeding cost per kg feed, γ_{firs}^t is the feed conversion rate for fish class f , location i in region r and σ_{firs}^t is the growth in kg for fish class f at location i in region r . K_f is the caring cost of keeping a fish in fish class f in a saltwater production site. The deviation variable e_{firs}^t represents the number of fish emergency harvested at penalty cost M^E . In the sixth term deviation variable $m_{s\bar{S}_r}^t$ allows the model to harvest less than the lower limit for slaughter house capacity at penalty cost $M_{\bar{S}_r}$ in region r . Deviation variable $m_{s_w}^t$ allows the model to exceed the well boat restriction at penalty cost M^W . The last two terms in the objective function also apply for all scenarios s . M^S is the penalty cost of breaking the end of horizon constraint concerning keeping a minimum weight in sales p in the last period, by utilizing the deviation variable m_{p_s} . Lastly M_f^V is the value of having a kilogram of fish class f in the last period $|\mathcal{T}|$, given by the product of fish class f 's weight V_f , and the number of fish in the last period $n_{firs}^{|\mathcal{T}|}$ in fish class f , location i and region r .

5.3.2 Constraints

In the model the following constraints are included.

Smolt delivery restrictions

The following restrictions set an upper limit to how much smolt that can be delivered.

$$\sum_{f \in \mathcal{C}} \sum_{r \in \mathcal{R}} \sum_{t > \hat{t}} V_{ft}^{\hat{t}} \left(\sum_{t' < t} o_{frs}^{t't} + N_{fr}^t \right) \leq L^{\hat{t}}, \quad \hat{t} \in \mathcal{T}, s \in \mathcal{S} \quad (5.4)$$

Equation 5.4 sets an upper limit for volume capacity at the freshwater facility, in each period \hat{t} and scenario s , indirectly limiting the smolt delivery in time period t . Here, $V_{ft}^{\hat{t}}$ is the weight contribution of smolt in kilograms to the freshwater facilities' maximum holding capacity at the end of time period \hat{t} , after growth in time period \hat{t} is accounted for. Weight contribution $V_{ft}^{\hat{t}}$ is defined for delivery time t and the delivery weight linked to smolt class f . $V_{ft}^{\hat{t}}$ is zero for every \hat{t} equal or larger than t , because the smolt is moved from freshwater to saltwater in the beginning of time period t . It is also zero

for all time periods before the fish starts to feed. Therefore, when summing over all delivery time periods t larger than \hat{t} , only the deliveries that would contribute to the biomass in time period \hat{t} are included. The smolt delivery variable $o_{frs}^{t't}$ is the number of smolt in smolt class \mathcal{C} ordered in period t' to be delivered in period t , region r , scenario s . By summing over all orders made prior to t , all delivered in period t are included. N_{fr}^t is the number of pre-ordered smolt currently in production in the freshwater facilities with delivery date t to region r . The combined maximum allowable biomass in the freshwater facilities, $L^{\hat{t}}$, is the product of volume capacity and maximum allowable density of fish.

$$\sum_{f \in \mathcal{C}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}_y} V_f \left(\sum_{t' < t} o_{frs}^{t't} + N_{fr}^t \right) \leq U_y, \quad y \in \mathcal{Y}, s \in \mathcal{S} \quad (5.5)$$

Equation 5.5 sets an upper limit for the total weight of all smolt delivered within each year y as a result of government regulations of feed licenses. It must hold for every scenario s . Set \mathcal{T}_y gives the time periods in year y . The delivery weight of smolt class f is given by V_f . Still, $o_{frs}^{t't}$ and N_{fr}^t is the number of smolt in smolt class \mathcal{C} in region r , in scenario s to be delivered in period t . By summing over all orders made prior to t , all delivered in period t are included. U_y is the maximum total weight of smolt that the freshwater facilities have licenses to produce within a year.

$$o_{frs}^{t't} = 0, \quad t' \in \mathcal{T}, t \in \mathcal{T}^N, f \in \mathcal{C}, r \in \mathcal{R}, s \in \mathcal{S} \quad (5.6)$$

Lastly, in equation 5.6 all smolt delivery variables are set to zero in the delivery time periods defined by \mathcal{T}^N , which represents the months where smolt should not be deployed in seawater.

Smolt release

The following constraint ensures the connection between delivered and released smolt, so that the number of smolt released in all locations is not greater than smolt delivered by the model plus initial smolt orders. Equation 5.7 is a less or equal constraint, because the salmon producer may choose to destroy delivered smolt instead of deploying it. The model might choose to utilize this option depending on whether the scenario specific growth and mortality are high or low, as smolt orders need to be placed over a year in advance of deployment.

$$\sum_{i \in \mathcal{I}_r} \sum_{\hat{t} \leq t} y_{firs}^{\hat{t}t} \leq \sum_{t' < t} o_{frs}^{t't} + N_{fr}^t, \quad f \in \mathcal{C}, r \in \mathcal{R}, t \in \mathcal{T}, s \in \mathcal{S} \quad (5.7)$$

Release variable $y_{firs}^{\tilde{t}}$ represents the number of smolt in smolt class \mathcal{C} released at location i , in region r , at time t in scenario s , decided in time period \tilde{t} . By summing over all deployment decisions made in and prior to t , all deployed in period t are included. Time period t is included in the sum, as decisions regarding smolt deployment can be taken in the same period as smolt is released; if smolt can be released in January. Delivery variable $o_{firs}^{t't}$ is the number of smolt in smolt class \mathcal{C} in region r , in scenario s ordered by the model in period t' to be delivered in period t . By summing over all orders made prior to t , all delivered in period t are included. N_{fr}^t is the number of pre-ordered smolt in smolt class \mathcal{C} in region r to be delivered in period t . N_{fr}^t will normally only have values in the two first years in the planning period, because of the lead time of smolt production, and will be zero for the rest of the planning horizon after the initial ordered smolts are released.

Initial biomass

Equation 5.8 governs the biomass in the first period. Initial biomass I_{Bfir} is the number of fish in fish class f which are in location i , in region r at the beginning of the planning period. At the beginning of period 1 the number of fish in location i , in region r in scenario s , n_{firs}^1 , equals the initial biomass after emergency harvesting. Emergency harvest variable e_{firs}^1 is included to ensure solvability. It is assumed that all release and harvesting in the first period have been done. Therefore, the model cannot release or harvest in the first period, which is assured by equations 5.13 and 5.14.

$$n_{firs}^1 = I_{Bfir} - e_{firs}^1, \quad f \in \mathcal{F}, i \in \mathcal{I}_r, r \in \mathcal{R}, s \in \mathcal{S} \quad (5.8)$$

$$y_{firs}^{11} = 0, \quad f \in \mathcal{F}, i \in \mathcal{I}_r, r \in \mathcal{R}, s \in \mathcal{S} \quad (5.9)$$

$$w_{firs}^1 = 0, \quad f \in \mathcal{F}, i \in \mathcal{I}_r, r \in \mathcal{R}, s \in \mathcal{S} \quad (5.10)$$

Biomass development in saltwater

The following constraints keep track of the development in biomass from one period to the next. This is done by keeping track of which fish class f each fish belongs to, and how growth affects the advancement of fish from one fish class f to another. In the biomass development constraint, n_{firs}^t is the number of fish in fish class in location i , region r at the beginning of time period t , scenario s , and it is determined by the following four elements:

1. The number of fish that are in fish class f after the biomass development due to growth and survival rate during period $t-1$.

2. The number of fish released in fish class f in period t , deduced after deployment survival rate.
3. The number of fish harvested in fish class f in period t .
4. The number of fish emergency harvested in fish class f in period t .

$$n_{firs}^t = \sum_{\hat{f} \leq f} \left(\delta_{\hat{f}firs}^{t-1} n_{\hat{f}irs}^{t-1} \varepsilon_{\hat{f}irs}^{t-1} \right) + \sum_{\tilde{t} \leq t} y_{firs}^{\tilde{t}t} \varphi_{firs}^t - w_{firs}^t - e_{firs}^t, \\ f \in \mathcal{F}, i \in \mathcal{I}_r, r \in \mathcal{R}, s \in \mathcal{S}, t \in \mathcal{T} \mid 1 < t < T^A \quad (5.11a)$$

$$n_{f\hat{r}s}^t = \sum_{\hat{f} \leq f} \sum_{i \in \mathcal{I}_{\hat{r}}^t} \left(\delta_{\hat{f}firs}^{t-1} n_{\hat{f}irs}^{t-1} \varepsilon_{\hat{f}irs}^{t-1} \right) + \sum_{\tilde{t} \leq t} y_{f\hat{r}s}^{\tilde{t}t} \varphi_{f\hat{r}s}^t - w_{f\hat{r}s}^t - e_{f\hat{r}s}^t, \\ f \in \mathcal{F}, \hat{i} \in \mathcal{I}_r^A, r \in \mathcal{R}, s \in \mathcal{S}, t = T^A \quad (5.11b)$$

$$n_{f\hat{r}s}^t = \sum_{\hat{f} \leq f} \left(\delta_{\hat{f}firs}^{t-1} n_{\hat{f}irs}^{t-1} \varepsilon_{\hat{f}irs}^{t-1} \right) + \sum_{\tilde{t} \leq t} y_{f\hat{r}s}^{\tilde{t}t} \varphi_{f\hat{r}s}^t - w_{f\hat{r}s}^t - e_{f\hat{r}s}^t, \\ f \in \mathcal{F}, \hat{i} \in \mathcal{I}_r^A, r \in \mathcal{R}, s \in \mathcal{S}, t \in \mathcal{T} \mid t > T^A \quad (5.11c)$$

Due to the aggregation of saltwater facilities in time period T^A three equations are needed to handle biomass development correctly. Equation 5.11a handles the biomass from time period 2 until time period $T^A - 1$, where locations are given by the non-aggregated set $i \in \mathcal{I}_r$. All parameters and variables are given for location i , region r and scenario s . In the first term $\delta_{\hat{f}firs}^{t-1}$ is the share of fish class \hat{f} , that during time period $t-1$ has grown to be part of fish class f . This share is multiplied by $n_{\hat{f}irs}^{t-1}$ which is the number of fish in fish class \hat{f} at the beginning of time period $t-1$ and $\varepsilon_{\hat{f}irs}^{t-1}$, the survival rate for a fish in fish class \hat{f} in time period $t-1$. The first term is the sum over all fish classes \hat{f} smaller than f , and so all the fish that grow to become part of f during $t-1$ are included. There is also a possibility that some fish do not grow out of \hat{f} , so the sum also have to include f itself.

Release variable $y_{firs}^{\tilde{t}t}$ is the number of smolt released and φ_{firs}^t is the survival rate of these smolt. Harvest variable w_{firs}^t represents the number of fish harvested, while e_{firs}^t is the number of fish emergency harvested. All these variables are specified for fish class f , location i , region r , scenario s and time period t . By summing over all deployment decisions made in and prior to t ,

all deployed in period t are included. The emergency harvest variable ensures that n_{firs}^t never exceeds any of the constraints related to the maximum value of n_{firs}^t .

Equation 5.11b takes care of the transition from non-aggregation to aggregation of location in time period $t = T^A$. This constraint exists for all aggregated location $\hat{i} \in \mathcal{I}_r^A$, and moves all fish from the non-aggregated location i into aggregated location \hat{i} by summing over the linking set $\mathcal{I}_{\hat{i}r}^L$. After time period T^A equation 5.11c handles biomass development in the same way as equation 5.11a, but only for aggregated location $\hat{i} \in \mathcal{I}_r^A$.

To ensure that the same aggregation applies to the objective function and all other constraints, all location specific variables not defined by \mathcal{I}_r^A are set to zero from time period T^A . This is done using equation 5.12.

$$n_{firs}^t, y_{firs}^{\tilde{t}t}, w_{firs}^t, e_{firs}^t = 0, \\ f \in \mathcal{F}, i \in \mathcal{I}_r \setminus \mathcal{I}_r^A, r \in \mathcal{R}, s \in \mathcal{S}, \tilde{t} \in \mathcal{T}, t \in \mathcal{T} \mid t \geq T^A \quad (5.12)$$

Lastly, since the biomass development restrictions are defined for all fish in fish class \mathcal{F} , both the release variable $y_{firs}^{\tilde{t}t}$ and the harvest variable w_{firs}^t must be set to zero for every fish class not included in smolt class \mathcal{C} and harvest class \mathcal{H} respectively, given in equation 5.13 and 5.14.

$$y_{firs}^{\tilde{t}t} = 0, \quad f \in \mathcal{F} \setminus \mathcal{C}, i \in \mathcal{I}_r, r \in \mathcal{R}, s \in \mathcal{S}, \tilde{t} \in \mathcal{T}, t \in \mathcal{T} \quad (5.13)$$

$$w_{firs}^t = 0, \quad f \in \mathcal{F} \setminus \mathcal{H}, i \in \mathcal{I}_r, r \in \mathcal{R}, s \in \mathcal{S}, t \in \mathcal{T} \quad (5.14)$$

Slaughtering capacity

The slaughtering capacity restrictions ensure that the number of fish harvested always is in the interval between the upper and lower capacity specifications. Equation 5.15 gives an upper limit to the slaughter house, reflecting installed capacity, while equation 5.16 provides a lower limit for slaughtering given by the aim of having a smooth harvesting profile. Both equations are given for each region r , scenario s and time period t .

$$\sum_{i \in \mathcal{I}_r} \sum_{f \in \mathcal{H}} C_f w_{firs}^t \leq \bar{S}_r, \quad r \in \mathcal{R}, s \in \mathcal{S}, t \in \mathcal{T} \quad (5.15)$$

$$\sum_{i \in \mathcal{I}_r} \sum_{f \in \mathcal{H}} C_f w_{firs}^t + m_s^t \underline{S}_r \geq \underline{S}_r, \quad r \in \mathcal{R}, s \in \mathcal{S}, t \in \mathcal{T} \quad (5.16)$$

In the first term C_f is the capacity factor for slaughtering fish class f at the slaughter house. For very small and very large fish $C_f > 1$, due to longer process times. Harvest variable w_{firs}^t is the number of fish harvested in fish class f at location i , region r , scenario s at time period t . \overline{S}_r and \underline{S}_r are the maximum and minimum capacities for the slaughter house in region r . In equation 5.16 the second term $m_s^t \underline{S}_r$ is the deviation variable for breaking the minimum slaughterhouse capacity, and it has penalty cost $M_{\overline{S}_r}$ in the objective function. The penalty cost for breaking the lower slaughtering level includes labour and extra costs due to uneven production. There is no need for a deviation variable in the maximum capacity restriction, as the emergency harvest variable in the biomass development constraint ensures solubility if the maximum harvesting capacity is binding.

MAB

The maximum allowable biomass (MAB) constraints control that the biomass in the production sites never exceed the maximum level given by the authorities, neither on locational level nor on regional level. All MAB restrictions are checked at the end of each period, after harvesting, release, growth and mortality have been accounted for.

$$\begin{aligned} \sum_{f \in \mathcal{F}} (V_f + \sigma_{firs}^t) n_{firs}^t \varepsilon_{firs}^t &\leq MAB_i^t, \\ i \in \mathcal{I}_r, r \in \mathcal{R}, s \in \mathcal{S}, t \in \mathcal{T} \mid t < T^A \end{aligned} \quad (5.17a)$$

$$\begin{aligned} \sum_{f \in \mathcal{F}} (V_f + \sigma_{f\hat{r}s}^t) n_{f\hat{r}s}^t \varepsilon_{f\hat{r}s}^t &\leq MAB_i^{At}, \\ \hat{i} \in \mathcal{I}_r^A, r \in \mathcal{R}, s \in \mathcal{S}, t \in \mathcal{T} \mid t \geq T^A \end{aligned} \quad (5.17b)$$

Equation 5.17a and 5.17b restrict the MAB on locational level, in the non-aggregated and aggregated time periods respectively. The first term represent the biomass of a fish class f in the beginning of period t . The second term gives the growth of the fish within period t . V_f is the weight in kg for a fish in fish class f . The following parameters and variable are given for location i , region r and scenario s . Parameter σ_{firs}^t gives the growth in kilograms for fish class f in period t . Variable n_{firs}^t is the number of fish within the fish class f at the beginning of the time period t . Parameter ε_{firs}^t is the survival rate of for fish class f . MAB_i^t is the maximum allowable biomass at location

i , in period t , and it is zero in the time periods where the location needs to be followed. After aggregation time period T^A , when biomass is moved into the aggregated locations $\hat{i} \in \mathcal{I}_r^A$, the aggregated location MAB is given by summing up the non-aggregated location MAB of the linked facilities; $MAB_{\hat{i}}^{At} = \sum_{i \in \mathcal{I}_r^L} MAB_i^t$. The locations contained within an aggregated location need to have following in the same periods, to ensure that all location are empty when legislation demands it.

$$\sum_{i \in \mathcal{I}_r} \sum_{f \in \mathcal{F}} (V_f + \sigma_{firs}^t) n_{firs}^t \varepsilon_{firs}^t \leq MAB_r, \quad r \in \mathcal{R}, s \in \mathcal{S}, t \in \mathcal{T} \quad (5.18)$$

Equation 5.18 controls the MAB level on a regional level, and the variable, parameters and constant on the left hand side are the same as in 5.17a and 5.17b. However, the biomass is also summed up over all locations within a region. MAB_r is the maximum allowable biomass in region r . The regional MAB is not affected by the aggregation in time period T^A , as aggregation is only done within a region. The emergency harvest variable in the biomass development constraint ensures solubility for all the MAB restrictions, as it can lower n_{firs}^t in the biomass development constraint by paying penalty cost M^E .

Sales

The sales constraint makes a connection between fish classes and sales classes. It converts a fish from a fish class f into the right sales class p .

$$x_{ps}^t = \sum_{r \in \mathcal{R}} \sum_{i \in \mathcal{I}_r} \sum_{f \in \mathcal{F}_p \cap \mathcal{H}} Q_f V_f w_{firs}^t, \quad p \in \mathcal{P}, s \in \mathcal{S}, t \in \mathcal{T} \quad (5.19)$$

Here x_{ps}^t is the amount of kilograms of fish in sales class p in time period t in scenario s . Q_f is the weight reduction factor for a fish in fish class f , which reduce weight due to gutting, and V_f is the weight of a fish in fish class f . Variable w_{firs}^t is the number of harvested fish in fish class f in period t at location i , region r , scenario s . All fish classes f are summed over \mathcal{F}_p in intersect with \mathcal{H} , which is the set of fish classes f that belongs to sales class p and harvest set \mathcal{H} .

Well boat

The well boat restriction makes sure that the number of fish harvested and released in a period t does not exceed the well boat capacity.

$$\sum_{r \in \mathcal{R}} \sum_{i \in \mathcal{I}_r} V_f \left(\sum_{f \in \mathcal{H}} w_{firs}^t + \sum_{f \in \mathcal{C}} \sum_{\bar{i} \leq t} y_{firs}^{\bar{t}t} \right) - m_s^t w \leq W^t, \quad s \in \mathcal{S}, t \in \mathcal{T} \quad (5.20)$$

V_f is the weight of a fish in fish class f in the beginning of period t , w_{firs}^t is the number of fish harvested and y_{firs}^t is the number of smolt released in period t , location i , region r , scenario s , with f either given by harvest set \mathcal{H} or smolt set \mathcal{C} . Variable $m_{s,w}^t$ is the deviation variable for exceeding the well boat capacity in period t , scenario s , and is given a penalty cost in the objective function representing the cost of renting extra capacity. W^t is the maximum capacity of the well boats in period t .

End of horizon

In order to solve the end of horizon problem discussed in section 4.3 three constraints will be used. In addition, the value of keeping fish in the last period, M^V_f , has been added in the objective function.

$$\sum_{\hat{i} \in \mathcal{I}_r^A} \sum_{f \in \mathcal{F}} (V_f + \sigma_{f\hat{i}rs}^t) n_{f\hat{i}rs}^t \varepsilon_{f\hat{i}rs}^t \geq N_r MAB_r, \quad r \in \mathcal{R}, s \in \mathcal{S}, t = |\mathcal{T}| \quad (5.21)$$

The first end of horizon restriction assures that the amount of biomass in the system must equal or exceed a given share of the MAB limit on regional level in the end of the last period. In equation 5.21, the first term represent the biomass of a fish class f in the beginning of the last period $|\mathcal{T}|$, while the second term gives the growth of the fish within period $|\mathcal{T}|$. V_f is the weight in kg for a fish in fish class f in the beginning of period $|\mathcal{T}|$, and $\sigma_{f\hat{i}rs}^t$ gives the growth in kilograms for fish class f in location \hat{i} , region r , scenario s . Variable $n_{f\hat{i}rs}^t$ is the number of fish at location \hat{i} , region r , scenario s within the fish class f at the beginning of the time period $|\mathcal{T}|$. Parameter $\varepsilon_{f\hat{i}rs}^t$ is the survival rate of for fish class f at location \hat{i} , in region r in scenario s . MAB_r is the maximum allowable biomass in region r , and N_r is the share of MAB_r that must be in the system at the end of period $|\mathcal{T}|$.

$$\sum_{r \in \mathcal{R}} \sum_{\hat{i} \in \mathcal{I}_r^A} \sum_{f \in \mathcal{F}_p} V_f n_{f\hat{i}rs}^t + m_{p,s} \geq EOH_p, \quad p \in \mathcal{P}, s \in \mathcal{S}, t = |\mathcal{T}| \quad (5.22)$$

The second constraint specifies the minimum level of biomass for each sales class in the last period of the planning horizon. In equation 5.22 V_f is the weight in kg for a fish in fish class f in the beginning of period $|\mathcal{T}|$, and $n_{f\hat{i}rs}^t$ is the number of fish at location \hat{i} within the fish class f at the beginning of the time period $|\mathcal{T}|$. Deviation variable $m_{p,s}$ for sales class p has penalty cost M^S in the objective function. EOH_p is the minimum total weight of fish that must be in sales class p in period $|\mathcal{T}|$.

$$\sum_{f \in \mathcal{C}} \sum_{r \in \mathcal{R}} \sum_{t' < t} V_f o_{frs}^{t't} \leq U^t, \quad t \in \mathcal{T}^{|\mathcal{D}|}, s \in \mathcal{S} \quad (5.23)$$

Equation 5.23 constrains the total amount of biomass of smolt which can be delivered in each period t in the last year, given by $\mathcal{T}^{|\mathcal{Y}|}$. V_f is the weight of a fish in fish class f in the beginning of period t , and $o_{f r s}^{t' t}$ is the number of smolt ordered in period t' in fish class f , region r , scenario s to be delivered in period t . The upper limit U^t is the largest quantum the model should deliver. This equation can therefore make periods usually not used for delivery unavailable, while making sure biomass delivery is not unusually large in the other periods.

Non-anticipativity

The structure of the scenario tree and the relationship between stages, periods and scenarios is enforced by non-anticipativity constraints. The non-anticipativity constraints force variables in different scenarios s to be equal in a manner that is consistent with the information available in each period t . As figure 5.5 illustrates, the scenarios enveloped by n are given by $\mathcal{S}(n)$, while $\mathcal{T}(n)$ is the time periods of envelopment $n \in \mathcal{N}$. Set \mathcal{N} therefore controls the number of stages in the model, which can range from a one stage deterministic model to the upper limit of stages, where the number of time periods and stages are equal.

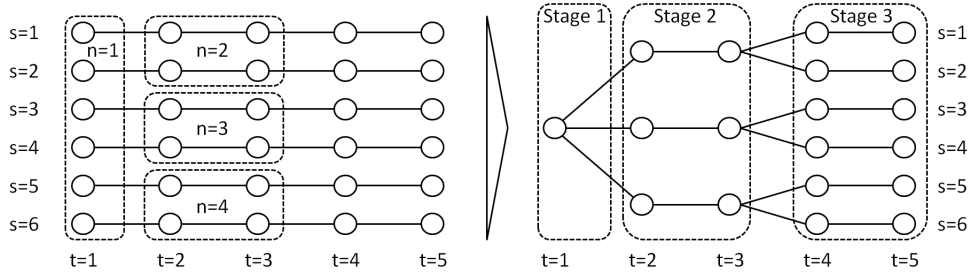


Figure 5.5: Non-anticipativity constraints in scenario formulation.

$$\begin{aligned}
 & \frac{1}{|\mathcal{S}(n)|} \sum_{s' \in \mathcal{S}(n)} (x_{ps}^t, w_{firs'}^t, n_{firs'}^t, e_{firs}^t, m_{s' \underline{s}_r}^t, m_{s' w}^t) \\
 & = (x_{ps}^t, w_{firs}^t, n_{firs}^t, e_{firs}^t, m_s^t \underline{s}_r, m_s^t w), \\
 & p \in \mathcal{P}, s \in \mathcal{S}(n), t \in \mathcal{T}(n), f \in \mathcal{F}, i \in \mathcal{I}_r, r \in \mathcal{R}, n \in \mathcal{N} \quad (5.24)
 \end{aligned}$$

Equation 5.48 ensures the non-anticipativity constraint for all variables except delivery and deployment. Here, variables with all indexes equal, except scenario s , within envelopment n are summed over scenario s' given by $\mathcal{S}(n)$ and divided by the number of scenarios within the envelopment, $|\mathcal{S}(n)|$. By

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setting this average value equal to each and every one of the variables that makes up the sum, non-anticipativity is enforced.

The non-anticipativity constraint for the smolt delivery variable is handled differently than the other variables, because ordering has to be made prior to the delivery of the smolt. This is handled by equation 5.25. Decisions regarding the smolt delivery variables need to be done based on the available information corresponding to the lead time before delivery, and therefore order time t' is given by $\mathcal{T}(n)$. As the lead time is dependent on the size of the smolt, variable $o_{f r s}^{t' t}$ only exists for the order time t' , delivery time t and smolt class f that are possible to achieve with the production time of smolt. Figure 5.6 shows an example of how this works. A large smolt delivery in time period $t=4$ must be ordered in stage 1, indicated in orange in the figure, while a small smolt delivery in $t=5$ can be ordered based on the available information in stage 2, indicated in blue. Therefore the blue smolt delivery will have to be the same in $s=1$ and $s=2$, while the orange smolt delivery has to be the same for every scenario. If the blue smolt delivery were to be delivered in $t=3$ instead of $t=5$, the order would have made in $t=1$. Then only the information in stage 1 would be available, thereby making it the same for every scenario in $t=3$.

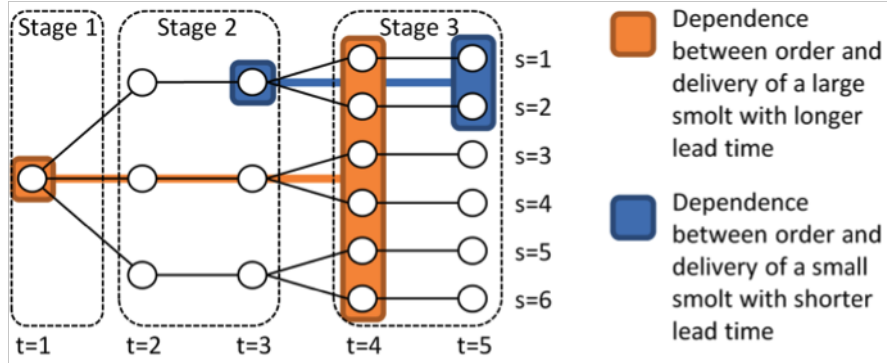


Figure 5.6: Illustration of special smolt delivery non-anticipativity constraints.

$$\frac{1}{|S(n)|} \sum_{s' \in S(n)} o_{f r s'}^{t' t} = o_{f r s}^{t' t},$$

$$r \in \mathcal{R}, f \in \mathcal{C}, t \in \mathcal{T}, t' \in \mathcal{T}(n), s \in \mathcal{S}(n), n \in \mathcal{N} \quad (5.25)$$

Lastly, the deployment of smolt are decided one year at the time. The non-

anticipativity constraint for smolt deployment, equation 5.26, is similar to the non-anticipativity constraint for smolt delivery. However, the smolt size does not influence when the decisions regarding deployment has to be made, and deployment decisions are made in time period \tilde{t} . The decision is the allocation of smolt delivered during the next year, which is not affected by the production time of smolt.

$$\frac{1}{|S(n)|} \sum_{s' \in S(n)} y_{firs'}^{\tilde{t}t} = y_{firs}^{\tilde{t}t},$$

$$i \in \mathcal{I}_r, r \in \mathcal{R}, f \in \mathcal{C}, t \in \mathcal{T}, \tilde{t} \in \mathcal{T}(n), s \in \mathcal{S}(n), n \in \mathcal{N} \quad (5.26)$$

5.3.3 The complete model

Finally, the complete model is presented along with non-negativity constraints.

$$\begin{aligned} \max z = & \sum_{s \in \mathcal{S}} \rho_s \left(\sum_{t \in \mathcal{T}} \left(\sum_{p \in \mathcal{P}} \alpha_{ps}^t x_{ps}^t - \sum_{r \in \mathcal{R}} \sum_{f \in \mathcal{F}} \sum_{t' < t} G_f o_{firs}^{t't} \right. \right. \\ & \left. \left. - \sum_{r \in \mathcal{R}} \sum_{i \in \mathcal{I}_r} \sum_{f \in \mathcal{F}} (n_{firs}^t (B \gamma_{firs}^t \sigma_{firs}^t + K_f) + M^E e_{firs}^t) \right) \right. \\ & \left. - \sum_{r \in \mathcal{R}} M_{\bar{s}_r} m_s^t \bar{s}_r - M^W m_s^t w - \sum_{p \in \mathcal{P}} M^S m_p s + \sum_{r \in \mathcal{R}} \sum_{i \in \mathcal{I}_r} \sum_{f \in \mathcal{F}} M_f^V V_f n_{firs}^{|\mathcal{T}|} \right) \end{aligned} \quad (5.27)$$

$$\sum_{f \in \mathcal{C}} \sum_{r \in \mathcal{R}} \sum_{t > \hat{t}} V_{ft}^{\hat{t}} \left(\sum_{t' < t} o_{firs}^{t't} + N_{fr}^t \right) \leq L^{\hat{t}}, \quad \hat{t} \in \mathcal{T}, s \in \mathcal{S} \quad (5.28)$$

$$\sum_{f \in \mathcal{C}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}_y} V_f \left(\sum_{t' < t} o_{firs}^{t't} + N_{fr}^t \right) \leq U_y, \quad y \in \mathcal{Y}, s \in \mathcal{S} \quad (5.29)$$

$$o_{firs}^{t't} = 0, \quad t' \in \mathcal{T}, t \in \mathcal{T}^N, f \in \mathcal{C}, r \in \mathcal{R}, s \in \mathcal{S} \quad (5.30)$$

$$\sum_{i \in \mathcal{I}_r} \sum_{\tilde{t} \leq t} y_{firs}^{\tilde{t}t} \leq \sum_{t' < t} o_{firs}^{t't} + N_{fr}^t, \quad f \in \mathcal{C}, r \in \mathcal{R}, t \in \mathcal{T}, s \in \mathcal{S} \quad (5.31)$$

$$n_{firs}^1 = I_{Bfir} - e_{firs}^1, \quad f \in \mathcal{F}, i \in \mathcal{I}_r, r \in \mathcal{R}, s \in \mathcal{S} \quad (5.32)$$

$$y_{firs}^{11} = 0, \quad f \in \mathcal{F}, i \in \mathcal{I}_r, r \in \mathcal{R}, s \in \mathcal{S} \quad (5.33)$$

$$w_{firs}^1 = 0, \quad f \in \mathcal{F}, i \in \mathcal{I}_r, r \in \mathcal{R}, s \in \mathcal{S} \quad (5.34)$$

$$n_{firs}^t = \sum_{\hat{f} \leq f} \left(\delta_{\hat{f}firs}^{t-1} n_{\hat{f}irs}^{t-1} \varepsilon_{\hat{f}irs}^{t-1} \right) + \sum_{\tilde{t} \leq t} y_{firs}^{\tilde{t}t} \varphi_{firs}^t - w_{firs}^t - e_{firs}^t, \\ f \in \mathcal{F}, i \in \mathcal{I}_r, r \in \mathcal{R}, s \in \mathcal{S}, t \in \mathcal{T} \mid 1 < t < T^A \quad (5.35a)$$

$$n_{firs}^t = \sum_{\hat{f} \leq f} \sum_{i \in \mathcal{I}_r^L} \left(\delta_{\hat{f}firs}^{t-1} n_{\hat{f}irs}^{t-1} \varepsilon_{\hat{f}irs}^{t-1} \right) + \sum_{\tilde{t} \leq t} y_{firs}^{\tilde{t}t} \varphi_{firs}^t - w_{firs}^t - e_{firs}^t, \\ f \in \mathcal{F}, \hat{i} \in \mathcal{I}_r^A, r \in \mathcal{R}, s \in \mathcal{S}, t = T^A \quad (5.35b)$$

$$n_{firs}^t = \sum_{\hat{f} \leq f} \left(\delta_{\hat{f}firs}^{t-1} n_{\hat{f}irs}^{t-1} \varepsilon_{\hat{f}irs}^{t-1} \right) + \sum_{\tilde{t} \leq t} y_{firs}^{\tilde{t}t} \varphi_{firs}^t - w_{firs}^t - e_{firs}^t, \\ f \in \mathcal{F}, \hat{i} \in \mathcal{I}_r^A, r \in \mathcal{R}, s \in \mathcal{S}, t \in \mathcal{T} \mid t > T^A \quad (5.35c)$$

$$n_{firs}^t, y_{firs}^{\tilde{t}t}, w_{firs}^t, e_{firs}^t = 0, \\ f \in \mathcal{F}, i \in \mathcal{I}_r \setminus \mathcal{I}_r^A, r \in \mathcal{R}, s \in \mathcal{S}, \tilde{t} \in \mathcal{T}, t \in \mathcal{T} \mid t \geq T^A \quad (5.36)$$

$$y_{firs}^{\tilde{t}t} = 0, \quad f \in \mathcal{F} \setminus \mathcal{C}, i \in \mathcal{I}_r, r \in \mathcal{R}, s \in \mathcal{S}, \tilde{t} \in \mathcal{T}, t \in \mathcal{T} \quad (5.37)$$

$$w_{firs}^t = 0, \quad f \in \mathcal{F} \setminus \mathcal{H}, i \in \mathcal{I}_r, r \in \mathcal{R}, s \in \mathcal{S}, t \in \mathcal{T} \quad (5.38)$$

$$\sum_{i \in \mathcal{I}_r} \sum_{f \in \mathcal{H}} C_f w_{firs}^t \leq \bar{S}_r, \quad r \in \mathcal{R}, s \in \mathcal{S}, t \in \mathcal{T} \quad (5.39)$$

$$\sum_{i \in \mathcal{I}_r} \sum_{f \in \mathcal{H}} C_f w_{firs}^t + m_s^t \underline{s}_r \geq \underline{S}_r, \quad r \in \mathcal{R}, s \in \mathcal{S}, t \in \mathcal{T} \quad (5.40)$$

$$\begin{aligned} \sum_{f \in \mathcal{F}} (V_f + \sigma_{firs}^t) n_{firs}^t \varepsilon_{firs}^t &\leq MAB_i^t, \\ i \in \mathcal{I}_r, r \in \mathcal{R}, s \in \mathcal{S}, t \in \mathcal{T} \mid t < T^A & \end{aligned} \quad (5.41a)$$

$$\begin{aligned} \sum_{f \in \mathcal{F}} (V_f + \sigma_{firs}^t) n_{firs}^t \varepsilon_{firs}^t &\leq MAB_i^{At}, \\ \hat{i} \in \mathcal{I}_r^A, r \in \mathcal{R}, s \in \mathcal{S}, t \in \mathcal{T} \mid t \geq T^A & \end{aligned} \quad (5.41b)$$

$$\begin{aligned} \sum_{i \in \mathcal{I}_r} \sum_{f \in \mathcal{F}} (V_f + \sigma_{firs}^t) n_{firs}^t \varepsilon_{firs}^t &\leq MAB_r, \\ r \in \mathcal{R}, s \in \mathcal{S}, t \in \mathcal{T} & \end{aligned} \quad (5.42)$$

$$x_{ps}^t = \sum_{r \in \mathcal{R}} \sum_{i \in \mathcal{I}_r} \sum_{f \in \mathcal{F}_p \cap \mathcal{H}} Q_f V_f w_{firs}^t, \quad p \in \mathcal{P}, s \in \mathcal{S}, t \in \mathcal{T} \quad (5.43)$$

$$\sum_{r \in \mathcal{R}} \sum_{i \in \mathcal{I}_r} V_f \left(\sum_{f \in \mathcal{H}} w_{firs}^t + \sum_{f \in \mathcal{C}} \sum_{\bar{t} \leq t} y_{firs}^{\bar{t}} \right) - m_s^t w \leq W^t, \quad s \in \mathcal{S}, t \in \mathcal{T} \quad (5.44)$$

$$\begin{aligned} \sum_{\hat{i} \in \mathcal{I}_r^A} \sum_{f \in \mathcal{F}} (V_f + \sigma_{firs}^t) n_{firs}^t \varepsilon_{firs}^t &\geq N_r MAB_r, \\ r \in \mathcal{R}, s \in \mathcal{S}, t = |\mathcal{T}| & \end{aligned} \quad (5.45)$$

$$\sum_{r \in \mathcal{R}} \sum_{\hat{i} \in \mathcal{I}_r^A} \sum_{f \in \mathcal{F}_p} V_f n_{firs}^t + m_p s \geq EOH_p, \quad p \in \mathcal{P}, s \in \mathcal{S}, t = |\mathcal{T}| \quad (5.46)$$

$$\sum_{f \in \mathcal{C}} \sum_{r \in \mathcal{R}} \sum_{t' < t} V_f o_{frs}^{t't} \leq U^t, \quad t \in \mathcal{T}^{|\mathcal{V}|}, s \in \mathcal{S} \quad (5.47)$$

$$\begin{aligned}
 & \frac{1}{|S(n)|} \sum_{s' \in S(n)} (x_{ps'}^t, w_{firs'}^t, n_{firs'}^t, e_{firs}^t, m_{s' \underline{s}_r}^t, m_{s' w}^t) \\
 & = (x_{ps}^t, w_{firs}^t, n_{firs}^t, e_{firs}^t, m_s^t \underline{s}_r, m_s^t w), \\
 & p \in \mathcal{P}, s \in \mathcal{S}(n), t \in \mathcal{T}(n), f \in \mathcal{F}, i \in \mathcal{I}_r, r \in \mathcal{R}, n \in \mathcal{N}
 \end{aligned} \tag{5.48}$$

$$\begin{aligned}
 & \frac{1}{|S(n)|} \sum_{s' \in S(n)} o_{frs'}^{t't} = o_{frs}^{t't}, \\
 & r \in \mathcal{R}, f \in \mathcal{C}, t \in \mathcal{T}, t' \in \mathcal{T}(n), s \in \mathcal{S}(n), n \in \mathcal{N}
 \end{aligned} \tag{5.49}$$

$$\begin{aligned}
 & \frac{1}{|S(n)|} \sum_{s' \in S(n)} y_{firs'}^{\tilde{t}t} = y_{firs}^{\tilde{t}t}, \\
 & i \in \mathcal{I}_r, r \in \mathcal{R}, f \in \mathcal{C}, t \in \mathcal{T}, \tilde{t} \in \mathcal{T}(n), s \in \mathcal{S}(n), n \in \mathcal{N}
 \end{aligned} \tag{5.50}$$

$$\begin{aligned}
 & x_{ps}^t, y_{firs}^{\tilde{t}t}, w_{firs}^t, n_{firs}^t, o_{frs}^{t't}, e_{firs}^t, m_s^t \underline{s}_r, m_s^t w, m_p s \geq 0, \\
 & p \in \mathcal{P}, s \in \mathcal{S}, t \in \mathcal{T}, t' \in \mathcal{T}, \tilde{t} \in \mathcal{T}, f \in \mathcal{F}, i \in \mathcal{I}^{\mathcal{R}}, r \in \mathcal{R}
 \end{aligned} \tag{5.51}$$

6 Model initialization

This chapter will look at the initialization of the mathematical model presented, where the model will be implemented for Marine Harvest Region Mid. Section 6.1 will introduce the model implementation, while section 6.2 will present the model simplifications. Section 6.3 and 6.4 present the structure of the two-stage and multistage stochastic models respectively. The stochastic mortality is explained in section 6.5. Section 6.6 shows how temperature forecasts have been made, while section 6.7 introduces the data set used. Finally section 6.8 gives a short presentation of the many utilizations of the model in planning and analysis for Marine Harvest. The purpose of the implementation is not to provide Marine Harvest with smolt delivery and deployment plans. Rather, this chapter and the next intend to exhibit how the model can be used to solve a realistic salmon farming planning problem, and the results can be used as a starting point for discussing existing practice.

6.1 Model implementation

Three versions of the model in the previous chapter are now presented; a deterministic, a two-stage stochastic and a three-stage stochastic. By comparing the results of using these three models to solve a realistic long term salmon planning problem, the model in the previous chapter can be assessed, and the effects of the uncertainty present in salmon farming further understood. Due to the size of the problem, it is natural to start with a deterministic model, adding stochastic properties and additional stages afterwards. In the deterministic (DET) model uncertainty regarding price, salmon growth and mortality are disregarded. Whereas in the two-stage stochastic (TS) and multistage stochastic (MS) model decisions regarding smolt deliveries and deployments have to be made before information about the occurring scenario is revealed. The DET model is needed to find the value of stochastic solution (VSS) for the TS and MS model by optimizing for expected values.

All the models are written in Mosel, implemented in Xpress-IVE and solved by Xpress Optimizer. All input and output data are handled by Microsoft Excel using the MMODBC module. Excel is used by production planners today, and using a familiar interface will ease the approval of the optimization tool. Planning can be done without knowledge about optimization or Xpress-IVE. The model is built to ensure that all parameters the operator needs to control can be changed using Excel. The computer used is specified below.

Operating system	Microsoft Windows 7 Enterprise 2009
Processor	Intel® Core™ i7-2600 CPU 3.40 GHz
Memory (RAM)	16 GB

6.2 Simplifications

It is assumed that growth is only dependent on temperature. This is done as the growth table made by Skretting, which is used in this thesis, is only dependent on fish weight and seawater temperature. If a more advanced growth table or function were available, other factors for growth could have been taken into account when generating the stochastic growth.

In addition, the stochastic parameters for price and feed conversion rate are made deterministic to reduce the problem size and making the problem solvable. However, temperature and mortality are stochastic parameters, but due to limited hardware capacity temperature and survival rate will not be independent. This means that all temperature forecast will be randomly assigned to each mortality scenario. Due to lack of data, temperature and mortality are regarded as being the same for every location in each individual scenario.

The total number of scenarios is nine for both the TS and the MS mode, and is chosen to ensure an acceptable solution time for repeated runs of the model and to make sure the problem does not become unsolvable due to computer memory shortage. All scenarios are equally likely, with the same probability of occurrence. It is possible to increase the number of scenarios to either include more stochastic parameters or to increase the number of scenarios, if more computational power is available.

To have a detailed enough representation of the biomass development, smolt delivery and release plans, the planning period resolution is set to one month.

6.3 Two-stage stochastic model

The TS model has two stages, where uncertainty occurs after decisions regarding smolt deliveries and deployments are made in the first period. The scenario tree for the two-stage problem is shown in figure 6.1. In the first node the information available is common for all of the scenarios. The second node represents the first period in the second stage, where all information regarding the temperature and mortality in the remaining periods is available.

Therefore the non-anticipativity constraints only apply to decisions taken in the first period. The smolt delivery and release plans that have to be made in the first period are equal in all scenarios. This applies to smolt deliveries that would require acquiring of eggs within the first year and smolt release in the first year. Remaining decisions regarding smolt delivery and release are not affected by the non-anticipativity constraints. Harvest plans may be adapted to maximize the object function in each scenario from the second period and throughout the rest of the planning period. Saltwater facilities are aggregated in period $T^A = 13$, January the second year, after the first year release plan has placed the pre-ordered smolt in non-aggregated facilities.

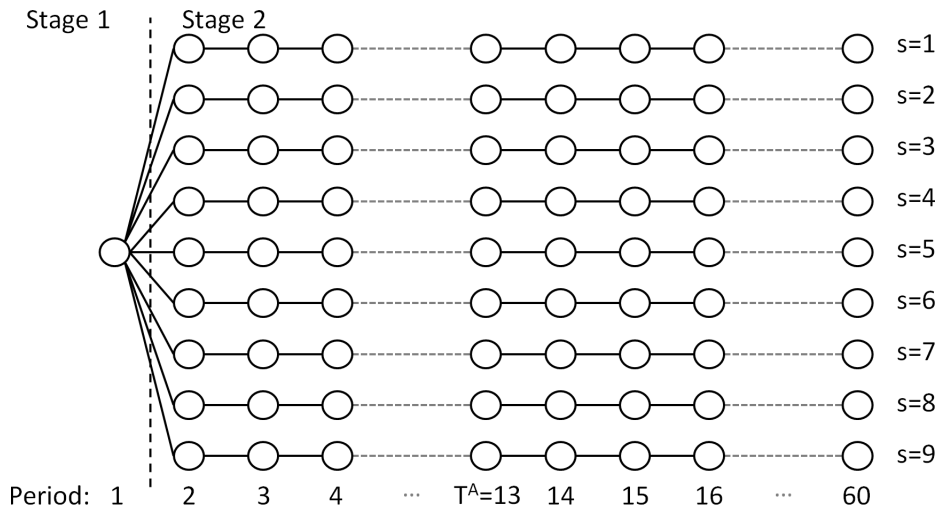


Figure 6.1: Illustration of the two-stage scenario tree.

6.4 Multistage stochastic model

The multistage stochastic model has three stages, which means that information is revealed two times during the planning horizon. First uncertainty is revealed in the second period, after decisions regarding smolt delivery and release have been made in the first period. The smolt delivery and release plans that needs to be made in the first period have the same timespan as in the TS model, but in the MS model there are only three possible outcomes that can occur in the second stage. The second stage lasts for 12 months, before all the remaining information is made available in period 14. Due to the three possible outcomes in period 14, the number of scenarios in total is 9. Period 14 is chosen as the start of the third stage, as it is February of the second year; making the decision process equal to the first year. Thereby,

the multistage stochastic model represents the characteristic of long term production planning. As the production planner only needs to finalize smolt release plans for the first year, saltwater facilities are aggregated from period 13. The multistage scenario tree is shown in figure 6.2.

In the first period, all decisions and stochastic parameters need to be the same in all of the scenarios. In the second stage however, three groups containing three scenarios each are connected through non-anticipativity constraints for the decision variables. This structure needs to be reflected in the stochastic parameters as well. As temperature and mortality are the only stochastic parameters after the simplifications, the structure of the TS and MS scenario trees must be taken into account when forecasting temperature and constructing mortality scenarios.

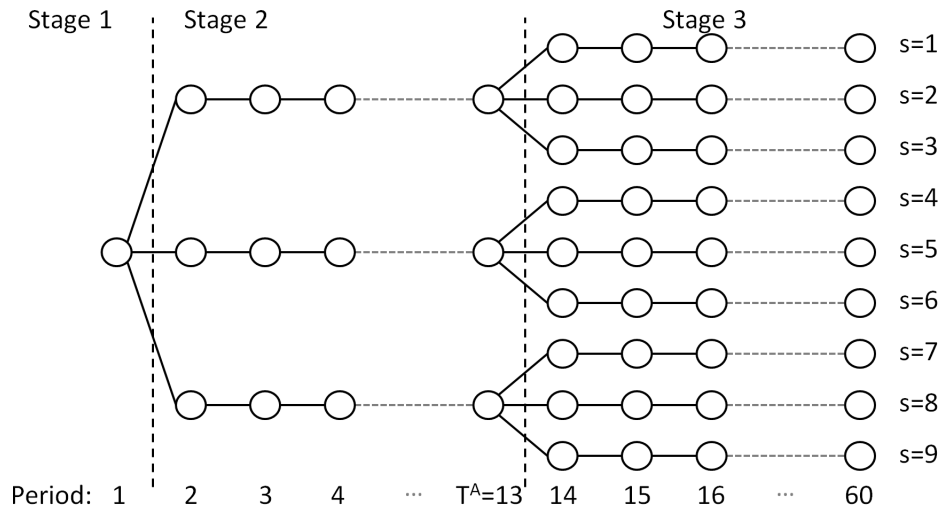


Figure 6.2: Illustration of the multistage scenario tree.

6.5 Stochastic mortality in saltwater

As mentioned in section 2.4.2, mortality is a great liability for salmon producers. 80% of mortality occurs before the fish reaches 0.5 kg which corresponds to the first 5 months after release. The average mortality rate in Trøndelag and Møre and Romsdal was 16.1% in 2009. These numbers form the basis to how mortality is implemented in the model. The saltwater growth model does not distinguish between fish of different releases once they have

been placed within a location. However, mortality could be dependent of fish size. Nevertheless, as no detailed data on how mortality is dependent of weight and temperature are available, mortality differs only based on whether it is related to release or growth. Therefore, survivability at release is given by φ^t_{firs} , while survivability each month is given by ε^t_{firs} . As the average production cycle in saltwater is 18 months, figure 6.3 shows the mortality distribution needed to achieve 80% mortality within the first 5 months.

Table 2 shows the implementation of mortality in the model, where the mortality at release and each month have been adjusted for the 80/20 split. Mortality is stochastic and can be 10%, 15% or 20%. This gives a low, normal and high mortality scenario, given in the table 2. In the two-stage stochastic model illustrated in figure 6.1 uncertainty is only resolved once. Here, mortality in the first period is normal, but in the second stage three scenarios have high mortality, three scenarios have normal mortality and three scenarios have low mortality. In the multistage model illustrated in figure 6.2 uncertainty is resolved twice. Here, the mortality in the first period is also normal, but when the scenarios split the mortality becomes high, normal or low. Therefore, the multistage model can have different mortalities in the second and third stage in the same scenario, while the mortality does not change throughout every scenario in the two-stage model after the second period.

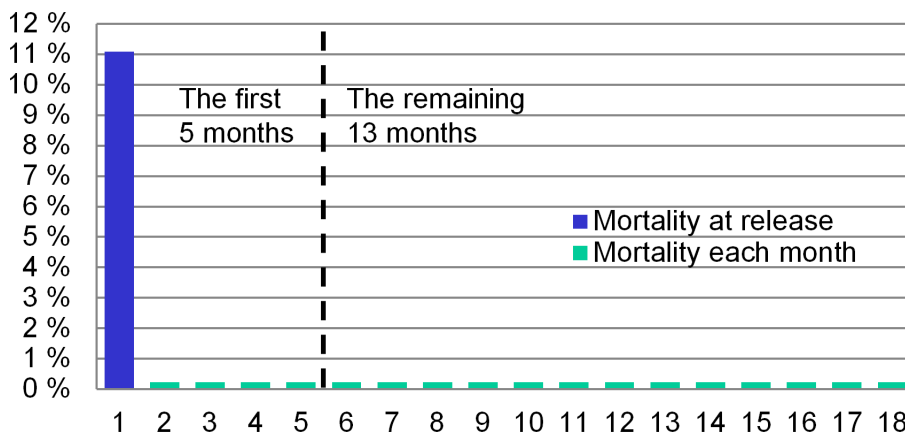


Figure 6.3: Modelled mortality, mortality at release and mortality each month.

Mortality scenario	Low	Normal	High
Mortality at release	7.38%	11.03%	14.77%
Mortality each month	0.15%	0.23%	0.31%
Aggregated mortality first 5 months, 80%	8.00%	12.00%	16.00%
Aggregated mortality last 13 months, 20%	2.00%	3.00%	4.00%
Total mortality during 18 months	10.00%	15.00%	20.00%

Table 2: Mortality scenarios based on 10%, 15% and 20% distributed between first 5 and last 13 months (80/20)

6.6 Forecasting seawater temperature

A forecasting method is used to generate temperature data for the stochastic model. As stated in section 3.5.2, forecasting is a diverse field and many methods have been developed. The main focus in this thesis is not to find the best suited forecast for future temperatures; rather it is to illustrate how forecasting and scenario generation can be combined. In the model the temperature has been predicted monthly for the next five years. Section 2.4.1 makes clear that there are seasonal trends in seawater temperature data, and historic data has proven to give a good indication of future realizations. Thus the temperature forecast will only consider historic measurements. An AR(1)-process will be used, as it performs better than an AR model of higher order on this specific data set.

6.6.1 Historic data vs a forecasting method

In the process of developing the forecasting method, one alternative was to use historic data directly, while the second option was to generate a forecast based on the historic data. The advantage of using the historic data as input directly is that it is easy, and does not require any pre-processing of the data. However, creating a forecast model has several benefits: It makes it possible to make as many scenarios as the model needs, and at the same time take all the measurements into consideration. The drawback is that it is time consuming to develop and implement the forecast. In the implementation a forecasting model for seawater temperature has been made, as forecasts are often used when planning.

6.6.2 Time series for temperature

Locational data measurement from Marine Harvest were not available, therefore the forecast have been made by using monthly average temperatures for

Marine Harvest Region Mid for the years 1998-2006, see figure 2.2. Ideally the dataset should be larger, but gaps in measurement periods and divergent measurement practices in the actual areas made it not worth considering using locational datasets.

6.6.3 Scenario-making procedure

To model the uncertainty in future temperature a methodology developed by Nowak and Tomasgard (2007) is used. The procedure of the method will here be described:

1. The temperature is forecasted with an autoregressive process of Nth order, AR(N)-process, that takes historical data as input. An AR(1)-model is used to predict future seawater temperatures. The AR(1)-model is parameterized so that the expected error is 0, hence it is unbiased.
2. The prediction error for the forecast in each period is calculated. The variance of the error is greater in August-October, and the error distribution is split in two distributions with ε_1 representing the error in November-July, and ε_2 representing the error in August-October.
3. For each error distribution the first four moments (expectation, variance, skewness, kurtosis) are estimated. Then S_i scenarios for each error ε_i is generated using a moment matching procedure assuring that the moments are the same as in the historical distribution. The scenarios are likely equal. The method used is developed by Høyland, Kaut, and Wallace (2003).
4. The forecasting method is combined with the scenario tree for the prediction error to get temperature scenarios. The procedure is illustrated in figure 6.4. First the AR(1)-method is used to predict future seawater temperatures. Then a scenario tree for the prediction error of the forecast is generated. Finally the forecast and the predicted error is combined to generate a scenario tree.

The advantage of using this method is that all the information from the temperature dataset will be analysed, and it is easy to generate the predefined number of scenarios. It should be emphasized that there are other ways to handle the split in the error distribution term. An alternative approach could be to try to remove the cause of the variance deviation instead of dealing with it subsequently. It could be done by choosing another forecast method, but the AR(1)-model performed adequately for the use in this thesis.

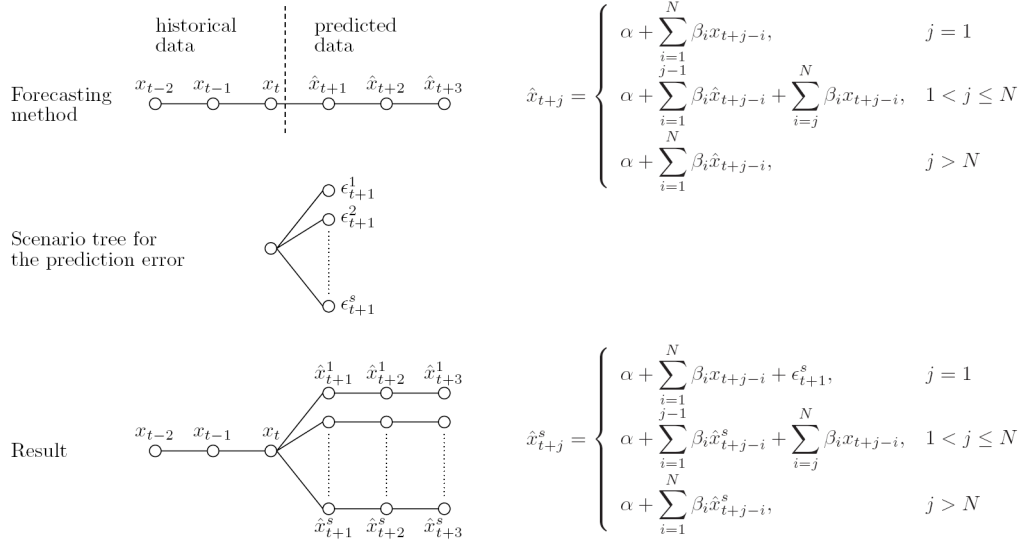


Figure 6.4: Combining forecasting and scenario generation (Schütz, 2009).

6.6.4 Temperature forecasts

The scenario-making procedure is used to make temperature forecasts for both the TS and MS model. Here the three-stage forecast will be presented, but the procedure for making the two-stage forecast is equal and the result can be found in the electronic documentation.

An important property of the scenario tree generated is that it should show the expected spread in temperature for the next five year period. This property is important as temperature is modelled as a stochastic parameter, and if the scenarios are very similar the model will act nearly as a deterministic one. Figure 6.5 shows the generation of a scenario tree where error terms are included in period 2 and 14 only. These two periods are the first once in the next stage of the three-stage model. In the first period, stage 1, the temperature is equal for all the scenarios, and in period 2-13, stage 2, the temperature in scenario 1-3 is equal and so on. This is due to the non-anticipativity constraints. From figure 6.5 it is clearly visible that the forecast goes back to a convergence value shortly after the error term is introduced. About 4 periods after the disturbance, the effect of it is unnoticeable. For the total of 60 periods of planning, this scenario tree will give very similar scenarios and the value of solving the model stochastically will be small. Therefore more disturbances were added.

Figure 6.6 shows a scenario tree where an error term is included in February

and August every year. The reason why August is chosen is that the error term is described by two distributions. Hence ε_1 will be added in February and ε_2 will be added in August. The difference in the nine scenarios in figure 6.6 is a lot greater than in figure 6.5 in most periods of the planning horizon. Furthermore the variance is greater in August than in February, which is a wanted property from the distributions. As the error is introduced regularly, they can add up over time making the error spread of the last year greater than the spread in the first. However, uncertainty increases the longer into the future a forecast tries to predict. This problem is not too prominent, as the values nearly go back to the convergence value after 4 months, and a new error is added every 6 months. The scenario tree of figure 6.6 is therefore used in the optimization model.

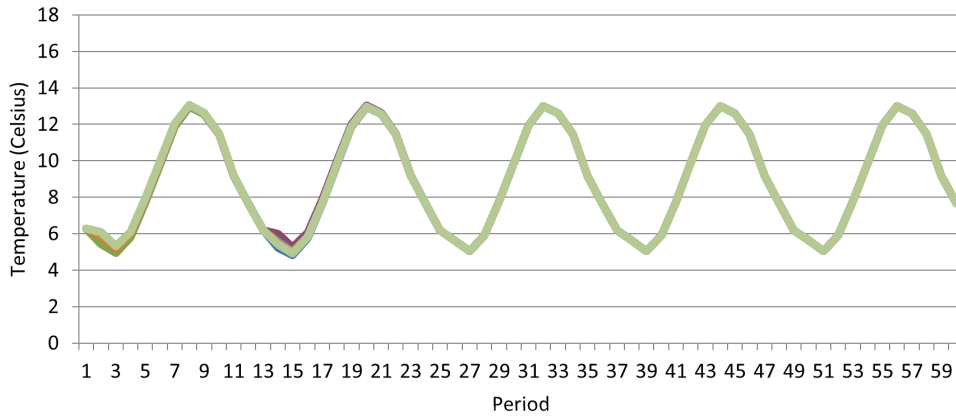


Figure 6.5: Generation of nine scenarios where an error terms are included in period 2 and period 14 only.

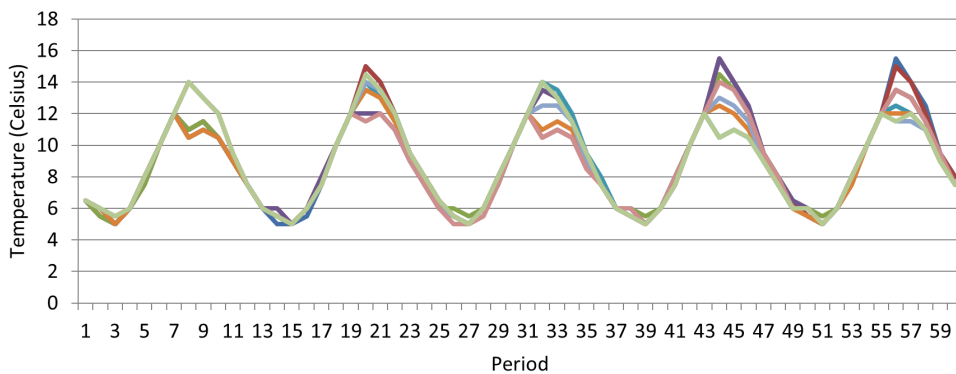


Figure 6.6: Generation of nine scenarios where an error term are included in period February and August every year.

6.7 Data sets

The data which have been used in the implementation of the model is based on public available information about Marine Harvest. Information about pre-ordered smolt, initial biomass and end of horizon criteria are generated by the model.

6.7.1 Resolution and planning horizon

The model has a planning horizon of five years. Resolution in the models is one month, with the first period being January in an even numbered year, for instance 2012. The resolution is chosen based on the level of detail wanted by Marine Harvest, and the length of the planning horizon is based on the discussion in section 4.3. Aggregation period $T^A = 13$, January in the second year, makes the deployment of pre-ordered smolt in the first year location specific.

6.7.2 Harvesting interval

Even though the target weight for harvesting is 6 kilograms in Region Mid, salmon is harvested between 2 and 8 kilograms dependent on the situation the planner faces. The average harvest weight in Region Mid is closer to 5 than 6 kilograms (Marine Harvest, 2012). When making input data regarding pre-ordered smolt and initial biomass, a slaughter interval of 5.5 to 6.5 kg is utilized. This is to represent that the previous decisions have been made to achieve the target weight of 6 kg. However, the actual harvest weight interval should be wider, as salmon are harvested at lower weights. Previous work has shown that lowering the allowable harvest weight from 5.5 to 4.0 kg yields an increase of 15% in biomass production throughout the planning period (Øveraas and Rynning-Tønnesen, 2012). Decreasing harvest weight further to 3.0 kg only gives an improvement of around 2 percentage points, while dramatically increasing problem size and solution time. Therefore, the 4.0-6.5 kg harvest weight is chosen in agreement with Marine Harvest. Reaching the target weight could be encouraged by lowering prices for the smaller sales classes, but this is not implemented because it would not reflect the actual salmon price.

6.7.3 Facilities and MAB restriction

The model has been implemented with the 42 locations Marine Harvest owns in region Mid, given by table 3. The table gives information about regional MAB, locational MAB and fallowing periods. Also, an aggregated location

Location	Aggreg.- number	Region	MAB _i (tons)	Following period Months	Year
Bremnessvaet	1	MR	5460	Jan, Feb	Even
Brettingen	1	MR	5460	Jan, Feb	Even
Leite	2	MR	3120	Jul, Aug	Even
Kornstad	3	MR	3120	Jul, Aug	Odd
Rokset	3	MR	3120	Jul, Aug	Odd
Storvikja	3	MR	3120	Not in use	
Nørholmen	1	MR	3120	Not in use	
Tennøya	4	T	3900	Jan, Feb	Odd
Mannbruholmen	4	T	7020	Jan, Feb	Odd
Grøttingsøy	4	T	5460	Jan, Feb	Odd
Slettholmene	4	T	3120	Jan, Feb	Odd
Langskjæra	4	T	3900	Jan, Feb	Odd
Ilsøya	5	T	3900	Jul, Aug	Odd
Gåsholmen	5	T	2340	Jul, Aug	Odd
Storbrannøya	5	T	1560	Not in use	
Lille Torsøy	5	T	5200	Jul, Aug	Odd
Lille Torsøy 2	5	T	3120	Jul, Aug	Odd
Korsholman	5	T	3120	Jul, Aug	Odd
Helsøya	5	T	3900	Jul, Aug	Odd
Osholman	6	T	3120	Jul, Aug	Even
Svellungen	6	T	3120	Jul, Aug	Even
Kåholmen	6	T	4680	Jul, Aug	Even
Heggvika	6	T	2340	Jul, Aug	Even
Grønnholmsundet	6	T	1820	Not in use	
Sengsholmen	6	T	1560	Not in use	
Veddersholmen	7	T	4680	Free	
Flatøya	7	T	2340	Free	
Tiltervågen	7	T	2340	Free	
Breidvika	8	T	5460	Jun, Jul	Odd
Indre Skjervøy	8	T	7020	Jun, Jul	Odd
Drogsholmen	8	T	2340	Jun, Jul	Odd
Svefjorden	8	T	2340	Jun, Jul	Odd
Almurden	8	T	3900	Jun, Jul	Odd
Estenvika	8	T	2340	Jun, Jul	Odd
Austvika	8	T	3120	Jun, Jul	Odd
Bjørgan	8	T	5460	Jun, Jul	Odd

Dalavika	6	T	1560	Jun, Jul	Even
Feøya	6	T	5460	Jun, Jul	Even
Bragstadsundet III	9	T	3900	Jan, Feb	Even
Kjelneset	9	T	4680	Jan, Feb	Even
Ølhammaren	9	T	2340	Jan, Feb	Even
Vedøya	9	T	3120	Jan, Feb	Even

Table 3: Marine Harvest Mid’s locations with given following periods in 2011.

Aggregation- number	Region	MAB^A_i (tons)	Following period Months	Year
1	MR	10920	Jan, Feb	Even
2	MR	3120	Jul, Aug	Even
3	MR	6240	Jul, Aug	Odd
4	T	23400	Jan, Feb	Odd
5	T	21580	Jul, Aug	Odd
6	T	20280	Jul, Aug	Even
7	T	9360	Free	
8	T	31980	Jun, Jul	Odd
9	T	14040	Jan, Feb	Even

Table 4: Aggregated locations, made from table 3.

number is given for each location, connecting them to the aggregated locations. The following periods are collected from the Regulation of Operations of Aquaculture Facilities (Forskrift om drift av akvakulturanlegg, 2008) and checked with Marine Harvest. In addition the regional MAB limits are 7800 tons for Møre and Romsdal (MR) and 32 760 tons for Trøndelag (T).

As table 3 shows, five of the locations are not in use today, but they have been implemented for easy inclusion in the model once they get approved and start operation. The MAB_i for facilities which are not in use are not added when calculating the MAB^A_i in table 4. In Bjugn the facilities are not affected by the Regulation of Abatement of Sea Lice in Aquaculture Facilities (Forskrift om bekjempelse av lus i akvakulturanlegg, 2009), and Marine Harvest is free to choose following period themselves, as long as it is reoccurring every second year. To try and balance out the number of locations with following in even and odd years, the following period for Bjugn is set to September and October in even numbered years. This seems like a reasonably good choice,

as it is in counter phase with the other following periods. Choosing following periods for the location in Bjugn could have been part of the optimization problem, but adding this to the model would greatly increase solution time and possibly make it insolvable.

6.7.4 Biomass development in saltwater

Biomass development is implemented the same way as done by Hæreid (2011). The growth model is based on the temperature and weight dependent growth table used by the food producer Skretting. Skretting's table gives the daily relative growth of Atlantic salmon, has 34 fish classes and integer temperatures ranging from 1 to 20 degrees Celsius. Hæreid expanded the original table to have 82 fish classes and 0.5 degrees increment by using interpolation. These 82 fish classes correspond to \mathcal{F} in the model. Hæreid also adapted the table to give the absolute growth in kilograms per month. An extract of the data is given in table 5. Note that the growth of the last fish class $f=82$ is zero. This is done to ensure that fish does not grow out of the discrete distribution. To avoid fish from being trapped in the final fish class, a costs connected to this fish class is implemented in section 6.7.7.

Fish f	Class V_f (kg)	Temperature (°C)								
		0,5	1,0	14,5	15,0	15,5	19,5	20,0		
1	0,03	0	0	0,04	0,04	0,04	0,03	0,03		
2	0,05	0	0	...	0,05	0,05	0,05	...	0,05	0,04
3	0,07	0	0	0,07	0,07	0,07	0,06	0,06		
...				...						
52	3,25	0,02	0,02	0,67	0,67	0,67	0,55	0,51		
53	3,38	0,02	0,02	0,68	0,69	0,68	0,55	0,52		
54	3,50	0,02	0,02	0,69	0,7	0,69	0,56	0,53		
55	3,63	0,03	0,03	0,7	0,71	0,7	0,56	0,53		
56	3,75	0,03	0,03	0,71	0,71	0,71	0,57	0,54		
...				...						
80	6,75	0,05	0,05	0,83	0,83	0,83	0,65	0,61		
81	6,88	0,05	0,05	...	0,83	0,83	0,83	...	0,65	0,61
82	8,00	0	0	0	0	0	0	0		

Table 5: Growth table showing growth of fish in saltwater in different fish classes f dependent on fish size and temperature, developed by Skretting and adjusted by Hæreid (2011).

6.7.5 Smolt limitations

The freshwater production faces many of the same types of restrictions as the saltwater production. Thus it is almost as complex to find the optimal smolt production as it is to find the optimal solution for the saltwater production. In contrast to the saltwater facilities, the freshwater production sites have a restriction regarding total biomass produced each year. Both the freshwater and saltwater production sites are limited by a maximum allowable biomass, due to volume capacity. The limits for yearly maximum biomass production for Marine Harvest Mid's freshwater facilities is given in table 6, showing that the total production of smolt cannot exceed 2650 smolt each year. The model is implemented with a maximum weight of smolts produced, 2650 tons per year.

Freshwater facility	Biomass production
Bessaker	1250 tons
Slørdal	300 tons
Terningen	600 tons
Nordheim	500 tons
Total	2650 tons

Table 6: Maximum allowable production of smolt each year for Marine Harvest Region Mid spring 2012.

The total biomass holding capacity for all freshwater facilities is 800 tons. In order to model the restrictions due to holding capacity in the freshwater facilities, the salmon growth in freshwater production has to be modelled. This can be done using Skretting's freshwater growth table, which is a table similar to table 5 with lower weights. Since the growth is dependent of the freshwater temperature, the growth pattern for a smolt would be dependent of both scenario and delivery date. By assuming that the freshwater facility utilizes water recycling, the temperature can be assumed to be constant at 14 degrees Celsius. This gives the growth pattern shown in table 7, which represents the weight in the end of each month. This growth pattern can then be used to create V_{ft}^t by having the growth pattern related to fish class f before delivery date t , while all other values are set to zero. The freshwater facility capacity limit is checked at the end of the period after growth has taken place, while delivery is done in the beginning of each month. Therefore, delivery takes place in the first month when the V_{ft}^t is zero. Lastly, the weights for the different smolt types have been adjusted such that it is

equal to the delivery weight in the end of the month before delivery.

Delivery weight: $V_f(\mathbf{g})$	70	100	150	250
Fish class (f)	3	4	5	7
Period (t)				
1	0.1	0.1	0.1	0.1
2	0.5	0.5	0.5	0.5
3	2.4	2.4	2.4	2.4
4	7.7	7.7	7.7	7.7
5	17.1	17.1	17.1	17.1
6	32.8	32.8	32.8	32.8
7	53.9	53.9	53.9	53.9
8	70.0	80.5	80.5	80.5
9	0.0	100.0	109.5	109.5
10	0.0	0.0	150.0	147.5
11	0.0	0.0	0.0	196.8
12	0.0	0.0	0.0	250.0
13	0.0	0.0	0.0	0.0

Table 7: Growth pattern with weights in grams of available smolt.

In the model, smolt have been implemented with four possible weights; 70 g, 100 g, 150 g or 250 g. These weights are chosen as they are spread out through the possible smolt weight interval of 40 g to 250 g, and they are used by Marine Harvest Region Mid. The production planner in Region Mid orders smolt for delivery to both Trøndelag and Møre and Romsdal, hence the smolt delivery variable is not dependent on region r in the implementation. Because smolt delivery and release are separated, the lead time in smolt production can be correctly implemented. By adding two months for hatching to the time it takes to reach the target weight and be delivered, the smolt order structure is given in table 8. Here, the first and last period for delivery of smolt ordered in the first and second stages are given. In the two-stage model, only the first stage smolt orders are bounded by the non-anticipativity constraint. In the three-stage model, the second stage smolt orders are placed in January of the second year, and are therefore bounded by their respective non-anticipativity constraints.

The ability to have pre-ordered smolt is implemented as input data, since new smolt deliveries only can be made as early the lead time allows. However, smolt delivery plans are highly sensitive information. If a salmon producer

Smolt size	First stage first delivery	First stage last delivery	Second stage first delivery	Second stage last delivery
70 g	Period 11	Period 22	Period 23	Period 34
100 g	Period 12	Period 23	Period 24	Period 35
150 g	Period 13	Period 24	Period 25	Period 36
250 g	Period 15	Period 26	Period 27	Period 38

Table 8: Possible delivery times for smolt growing in 14°C, when ordered in January.

gets hold of another producer’s smolt orders, it would give them the opportunity to adjust accordingly to increase profit on the others expense. No plans for smolt deliveries are therefore available. Instead, the pre-ordered delivery plan in table 9 was made by solving the MS model with a harvest interval of 5.5-6.5 kg, while allowing the model to deliver smolt prior to the first delivery period in table 8. Ordering smolt for delivery in the first year was then handled as first stage decisions. Lastly, release is prohibited in the months from December to February. These months are made unavailable on Marine Harvest’s request, as the seawater temperature is usually too low for deployment of smolt.

Period	Month	70g	100g	150g	250g
3	Mar	0	0	0	607
4	Apr	0	0	1 956	0
5	May	1 333	0	0	137
6	Jul	0	0	529	0
8	Aug	0	0	0	1 133
9	Sep	0	2 883	0	0
11	Nov	0	0	0	540

Table 9: Pre-ordered smolt deliveries [1000 smolt]

6.7.6 Implementation of deterministic salmon price

An important decision regarding input data is whether the future price of salmon should be implemented as deterministic or stochastic in the model. Earlier studies (Øveraas and Rynning-Tønnesen, 2012) proved that production was sensitive to price, and extreme values in price, both high and low, were governing for the production plans. Hence, a stochastic implementation

would make sure that an unlikely, extreme scenario would not be influencing the results more than necessary. On the other hand section 2.4.3 clarifies that making good future forecasts is difficult. In order to set the salmon price as a stochastic parameter, one should be able to make a good, stochastic representation of the salmon price for the next five years. As good stochastic price forecasts were not available for the next five years, and predicting future salmon prices are considered out of scope of this thesis, the future salmon price will here be implemented deterministic.

The Fish Pool forward price given in figure 2.5 represents the value of salmon sold on forward contracts. The forward price fits the model well, as prices are given monthly throughout the five year planning horizon. It is easily accessible, updated daily and based on information from the reference marked for future sales. Fish Pool’s forward price is also recommended by Marine Harvest as the price to be used in the model (Marine Harvest, 2012). Therefore, the data in figure 2.5 are chosen as price input data. The model has been implemented to handle different prices for different sales classes with 1 kg intervals. The sales classes are the same as the classification used by Fish Pool. All sales classes have been implemented with the same price forecast, as the forward price does not vary with weight. The price of fish below 3 kg is set to zero, as Marine Harvest Mid should not produce such small fish (Marine Harvest, 2012).

6.7.7 Costs

Smolt costs are given by equation 2.1, and the model has been implemented with the smolt costs given in table 10.

Smolt weight	Production cost per smolt (NOK/smolt)
70 grams	6.60 NOK
100 grams	7.60 NOK
150 grams	9.28 NOK
250 grams	12.63 NOK

Table 10: Prices of smolt at available delivery weights (Marine Harvest, 2012).

Feed costs are another cost parameter that is taken into account in the model. Prices of feed are normally negotiated on long term contracts, and are therefore sensitive information both for the feed producer and the salmon farmer.

Since data for feed cost were not available, it has been approximated using table 1, estimated by the Directorate for Fisheries. Here the feed cost per kg produced fish in 2010 was 10.64 NOK, and it has been approximated to 11 NOK in the model. However, as this feed cost is given per kg produced fish and not per kg feed, the feed conversion rate is included in the cost data. As discussed in section 2.4.1, the feed conversion rate will vary from location to location, with temperature, diseases and more. However, as no data for how the feed conversion rate varies with these factors are available, it is assumed to be constant. Therefore, both the feed cost and feed conversion rate are included in the constant feed cost of 11 NOK per kg growth.

The final cost modelled is the caring cost. It has been implemented to be able to give fish of every size a cost of ensuring the salmon's wellbeing, but has been set to zero due to lack of data. Instead, a penalty cost has been implemented for the last fish class. The reason is that fish in the final fish class do not grow in the model, and therefore do not have a feeding cost related to being in the production site. As it is undesirable to have fish that does not grow accumulating in the system, the penalty cost is set to $K_{82}=80$ NOK/fish in the model. This value is chosen because it is high enough to ensure that the model never uses fish class 82.

6.7.8 Slaughter house restrictions

Slaughtering is governed by the slaughter house capacity and minimum production due to contracts. The slaughter house at Ulvan receives salmon from both region Trøndelag and Møre and Romsdal, hence the slaughter house restrictions are shared by both regions in the implementation and are not dependent on r . Ulvan can slaughter about 70 000 fish per day, and operates 5 days a week so the total slaughter capacity is 1.4 million fish per month. This is therefore chosen as the upper limit for slaughtering. The lower limit for slaughter is set to zero. This allows the model to operate with more flexibility, but it is possible to give a wanted lower limit if the model does not produce enough salmon in certain periods. Weight reduction factor Q_f is set to 0.83, meaning that one gets 0.83 kilograms of fish meat per kilogram of harvested fish (Marine Harvest, 2012). As the harvesting interval does not include fish too small or too large to reduce slaughtering speed, C_f is set to one.

6.7.9 Well boat restrictions

For Marine Harvest Mid the well boats will never be a limiting factor for the model with the production capacity and authorities regulations in 2012 (Øveraas and Rynning-Tønnesen, 2012). Therefore, the well boat restriction is not included in the implementation for Marine Harvest Mid.

6.7.10 Initial biomass

As no data for initial biomass was available, the model created by Øveraas and Rynning-Tønnesen (2012) was used to generate a probable and reasonable data set. It was based on the strategy that Marine Harvest operates by; maximization of harvested biomass with a target weight between 5.5 and 6.5 kilograms. For a detailed explanation of how the initial biomass was created, the reader is referred to Øveraas and Rynning-Tønnesen (2012). The data set for initial biomass can be viewed in the electronic documentation.

6.7.11 End of horizon

The end of horizon constraints have been adjusted such that the decisions made by the model in the last year are similar to the ones in year three. This approach seems reasonable because the production cycle of salmon is two years and the results in year three are nearly unaffected by the end of horizon constraints. Therefore, the model does optimal decisions in year three. There are two specifications that should be met with the end of horizon constraints; firstly the biomass composition regarding weight in the last period of year five should be similar to the last period of year three and secondly the smolt delivery in each month of year five should not be very different from the smolt delivery in year three.

The end of horizon problem has been solved with three equations in the mathematical formulation; constraints for minimum biomass in each sales class and minimum utilization of MAB in the last planning period and constraints for maximum delivery of smolt in each month during the last year. In addition, the expected salmon value (ESV) was added in the objective function. These end of horizon conditions are tuned using the model.

The ESV estimate the value of one kilogram of salmon kept in seawater in the last period, based on expectations for future development of the salmon price after the planning horizon is over. In order to find an appropriate ESV the model has been run several times. In this thesis the ESV vary with sales class in order to give the model an incentive to deploy smolt in later periods

of the last year. The ESV for different sales classes is given in table 11.

The minimum biomass specification in each sales class in the last year is also given in table 11. Minimum MAB utilization after growth in the last period is set to 95%. These constraints, together with the ESV, will make sure that the model has the wanted biomass composition in year five. A problem with these three end of horizon specifications is that the model wants to order very large smolt batches to be delivered in certain months. In order to prevent this behaviour, the maximum biomass constraint for smolt delivery in each month of year five is used. The maximum limit is set to the maximum delivery of smolt for year 1 and 3 in month t , table 12.

Sales class	1	2	3	4	5	6	7
Weight (kg)	0-1	1-3	3-4	4-5	5-6	6-7	8
ESV (NOK)	26.7	19.7	19.7	21.7	13.7	13.7	
Minimum biomass (tons)	4134	12 053	14 525	0	0	0	0

Table 11: The expected salmon values and minimum biomass for each sales class in year five.

Month	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct
Max smolt delivery (tons)	272	291	217	434	48	685	0	0	470

Table 12: Maximum biomass for smolt to be delivered in each year in year five set as end of horizon criteria.

6.7.12 Deviation variables and penalty costs

Because the model is able to control everything regarding smolt ordering, deployment and salmon harvesting, the need for deviation variables is greatly reduced as the model will not deploy smolt that will cause infeasibility. The only restrictions that can cause an infeasible solution in this implementation are the initial biomass and the end of horizon conditions. An emergency harvest variable is created that allows additional slaughtering in every period with a large penalty cost. Also, the end of horizon constraint regarding minimum weight in the sales classes has its own deviation variables with a large penalty cost. This ensures that the solution will always be solvable.

6.8 Possible utilization of the model

There are many possible utilizations of the model. The main decisions that the model makes are smolt delivery plans, smolt deployment plans and harvesting plans that are scenario specific. The smolt delivery plans specify the number of smolt to order, the size of the smolt and the time of delivery. The smolt deployment plans are made for the first year of planning and describe when, where and how many smolt to release at the different locations. The smolt deployment plans are location specific for the first year, and for the remaining four years they apply to the aggregated facilities. The harvesting plans made by the model specify the number of harvested fish in each fish class, and are location specific for the first year and on an aggregated level for the last four years.

Furthermore the model describes the connection between the saltwater production and the rest of the value chain, and it can therefore be used to identify parts of the value chain with over- and under capacities. Input data regarding freshwater production and sales indicate how the saltwater production part is influenced by the rest of the value chain, and display what information that needs to be shared across the value chain. The model can also be used to model extreme events, for example losing all the fish in one location due to mortality.

Another possible utilization of the model is to compare existing operational practice with the models behavior. The objective value from the model will give the contribution margin for the total planning horizon, and for each month the cash flow is calculated. The harvesting interval can be adjusted, and the result from different harvest intervals can be compared. The production planner can also get the biomass development for all the sites and all the scenarios for the whole planning horizon.

The model can be adjusted by removing any restrictions and see how the production plans will alternate. The MAB, fallowing periods, freshwater facility, slaughter house and restricted slaughter months are examples of restrictions that can be detached. The model can also be run with other input data sets to see how sensitive the model is to different datasets. In particular price is an input data that can be interesting to change as it is uncertain.

7 Results

This chapter presents results of running the deterministic, two-stage and three-stage stochastic model for Marine Harvest Region Mid. Section 7.1 summarizes and analyses the results of running the models presented in chapter 6, and will be referred to as the main run. Section 7.2 will present results from different test cases with the three-stage model, where the model's behaviour given different parameter settings is tested.

As the model has been run several times to produce the data in this chapter, reduction of solution time was important when the model was implemented. The model developed in the project thesis of Øveraas and Rynning-Tønnesen (2012) had a solution time of one and a half hour. The model implemented in this master thesis has been expanded with freshwater growth, more detailed decision structure, stochastic mortality, three times as many scenarios and an additional stage. These additions have increased the complexity of the model. However, the solution time has been improved using the Xpress barrier function, which better utilizes the linear properties of the problem. The computational time of the three-stage stochastic model is 20 minutes. Implementing the time dependent aggregation further reduced it to 7 minutes. For comparison, the solution time of the DET model is 2 minutes. These solution times are acceptable, but the MS model is almost not solvable due to the memory capacity of the hardware.

7.1 Results from the main run

In the main run the DET, TS and MS models presented in chapter 6 have been implemented and solved. Stochastic models are computationally demanding, and it is therefore important to evaluate whether a stochastic model is necessary or if a deterministic is sufficient. Hence, the VSS and EVPI are calculated, and furthermore the effect of uncertainty in the model is discussed. Also results from the three-stage main run are presented; smolt delivery plans, smolt deployment plans, harvesting plans, biomass development and cash flow.

7.1.1 Valuation of the stochastic model

The value of stochastic solution (VSS) and the expected value of perfect information (EVPI) are as follows (Birge and Louveaux, 1997):

$$EVPI = WS - RP \tag{7.1}$$

$$VSS = RP - EEV \quad (7.2)$$

In order to calculate EVPI and VSS, the following values must be calculated. The solution found by the DET using average mortality and temperature gives the expected value (EV). The solution found by the TS and MS model are the here and now solutions defined as the recourse problem (RP). The wait and see solution (WS) is the solution to the TS and MS model if the non-anticipativity constraints are removed.

The expected value of using the EV solution when uncertainty is included is denoted EEV. However, calculating the EEV requires sequential runs, and the method for finding EEV for the two- and three stage models differ. For the two-stage model, the EEV is found by solving the TS model with the first stage decisions fixed to the values of the same decisions made by the EV model. To find EEV the non-anticipativity constraints are modified such that they are equal to the respective solution from the EV model.

The calculation of the EEV in the three-stage model is not as straight forward, and requires one of the two methods presented in section 3.4.3. Approach A, which is the easiest method to use, is an extension of the method used to find the EEV for a two-stage model. Here, some or all decisions made in the first and second stage are fixed to the EV solution values, while the ones not fixated are still governed by their non-anticipativity constraints. The problem with this method is that it does not take into account that new information becomes available in the second stage. If only the first stage decisions are locked, the EEV would be the result achieved by first utilizing a deterministic model and then switching to a stochastic model for the remaining stages. This would make the EEV better than it should be. On the other hand the EEV would be worse if also the second stage decisions are fixated to EV values, as these decisions could be re-optimized using the DET model once new information is available in the second stage. Therefore, this method is not ideal.

The second method for calculating the EEV for a multistage problem, approach B, resembles the process of utilizing a deterministic model with re-optimization when new information is available. However, the process is more cumbersome than approach A. First, the first stage decisions are found using the EV model. Afterwards a modified RP model is constructed, where the non-anticipativity constraints of the second stage are extended into the third stage. This equals a two-stage stochastic model with only three scenarios. In the time periods that originally belong to the third stage, the stochastic

parameters are equal to the average value of the scenarios in the third stage sub-trees. Solving this modified model with the first stage decisions fixed to the EV decisions, three sets of second stage decisions are obtained, as if they are solved separately with a deterministic model. Lastly, the MS model is run with first stage and second stage decisions fixed to the values of the corresponding scenario group decisions obtained in the previous iteration. A small error arises in approach B, as to the growth model only handles temperatures at a half degree accuracy. Thereby the average temperatures found in the three sub-trees are distorted. Nonetheless, approach B is chosen based on it being a better representation of the information and decision process.

The objective values for the five years planning horizon are presented in table 13. The highest profit is achieved by the EV solution, but the value cannot be directly compared to the others as it uses a single average scenario. The WS solution has the second highest profit, as all decisions are made with perfect information. However, when the smolt delivery plan made by the deterministic EV instance is used as input data in the EEV instance, the lowest profit is attained. The RP solution is higher than the EEV as the smolt delivery plan is made knowing that the nine different scenarios may occur, but it is lower than the WS because it may only make one smolt delivery and release plan for each scenario group.

Model instance	EV	EEV	WS	RP
Two-stage	3 758 700	3 718 820	3 746 470	3 730 250
Three-stage	3 758 700	3 686 410	3 742 530	3 720 940

Table 13: Five year objective values for the stochastic models [1000 NOK]

The calculated EVPI and VSS for the two-stage and three-stage stochastic models are given in table 14. The EPVI and VSS are not very large relative to the objective value, because all harvesting, delivery and deployment plans made from the first period in the last stage and onwards are done with perfect information regarding growth and mortality. However, a VSS of 34.5 million NOK indicates that the stochastic model makes better plans than the deterministic model. Increasing the number of stages would mean that harvesting and planning would have to be done without knowing which temperature and survivability scenario that would occur in the succeeding stages, but this would also require increasing the number of scenarios in the last period. Increasing the number of stages is likely to increase the VSS, as shown in table 14 where the VSS of the three-stage model is higher than that of the two-stage model.

Model instance	EVPI	EVPI (%)	VSS	VSS (%)
Two-stage	16 220	0.43%	11 430	0.31%
Three-stage	21 590	0.58%	34 530	0.93%

Table 14: Evaluation of the stochastic models [1000 NOK]

7.1.2 Effects of uncertainty

Table 15, 16 and 17 show the smolt delivery plans given by the EV, TS and MS respectively. They are given for the two first years only, as the delivery plans become scenario specific after this. The scenario specific smolt deliveries made by the WS instance may be found in the electronic documentation. TS and MS smolt deliveries have been made under uncertainty, while the EV and WS optimize the smolt deliveries to a specific temperature and mortality scenario. Light-gray numbers indicate that the smolt delivery is pre-ordered a year before the first period in the model due to smolt production lead time. The production planner would have to order the numbers colored black in stage one, while the dark-gray number represents a second stage decision that does not have to be made before January in the second year.

A significant difference between smolt deliveries made with and without uncertainty is found in the number of delivery dates utilized. In table 15, the DET model only orders smolt for delivery in June, September and November the second year. In addition, the DET model orders a large batch of 7.7 million smolt to be delivered in November year one; the earliest point of delivery of smolt ordered in the first stage. When the pre-ordered smolt were initialized with a harvest weight of 5.5-6.5 kg, the smolt order in November year one was only 2.0 million fish. Hence the large order in November in the DET model is made as the model is trying to change harvest strategy as quickly as possible. Moreover, the DET model utilizes the MAB and slaughter house restrictions better by increasing the number of fish in the system while harvesting at lower weights. Consequently, less smolt are ordered in the second year, as so many fish were released during the first year.

Both the TS and MS model, table 16 and 17, order less smolt during the five year planning horizon than the DET model. Both of these models face the same change in harvest strategy as the DET model. However, as they know mortality and temperature is going to vary, they do not order more smolt than they are able to keep within the possible forthcoming scenarios. When the EEV is calculated, the EV smolt orders are used in a stochastic

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setting. To be able to ensure feasibility with a larger number of smolt, the model utilizes the option to destroy smolt instead of deploying it. The expected average of destroyed smolt are 2 905 000 and 334 000 smolt in the EEV for the two-stage and three-stage setting respectively, most of which are destroyed in the scenarios with low mortality. When planning for uncertainty, the stochastic models order smolt for delivery in six rather than three months. By spreading the deliveries throughout the year, it is easier to adapt to changes in seawater temperature and mortality.

Period	Month	70g	100g	150g	250g	Yearly total
3	Mar	0	0	0	607	
4	Apr	0	0	1 956	0	
5	May	1 333	0	0	137	
6	Jun	0	0	529	0	
8	Aug	0	0	0	1 133	
9	Sep	0	2 883	0	0	
11	Nov	7 741	0	0	0	16 858
18	Jun	6 396	0	0	0	
21	Sep	0	1 368	1 409	0	
23	Nov	5 865	0	0	0	15 039

Table 15: EV and EEV smolt deliveries first two year [1000 smolt], pre-ordered, first stage and second stage decisions

When comparing the smolt delivery plans made by the TS and MS model in the first stage, these are quite similar when it comes to size of smolt and delivery time. In May, June, August and November, larger numbers of 70 gram smolt are ordered. In April and September smolt size is larger, while the total amount is smaller than in the other months. However, the number varies as the MS model is trying to adapt to the unresolved uncertainty in the third stage. As table 14 indicates, the VSS is tripled when the third stage is added. Therefore, the remaining analysis of effects of uncertainty will only look at the three-stage case.

Total MAB utilization obtained by planning with the deterministic model and the stochastic model is given in figure 7.1 and 7.2. In the MS result, the average MAB utilization is 37 305 tons, which is slightly higher than the EEV result of 37 164 tons. Another difference between the two figures is the variation in total MAB utilization between scenarios from period 26 to 33. In figure 7.2 the mortality and temperature difference between scenarios

Period	Month	70g	100g	150g	250g	Yearly total
3	Mar	0	0	0	607	
4	Apr	0	0	1 956	0	
5	May	1 333	0	0	137	
6	Jun	0	0	529	0	
8	Aug	0	0	0	1 133	
9	Sep	0	2 883	0	0	
11	Nov	4 400	0	0	0	13 517
16	Apr	0	0	1 889	24	
17	May	3 694	0	0	8	
18	Jun	2 795	0	0	0	
20	Aug	2 640	0	0	0	
21	Sep	0	1 310	0	708	
23	Nov	2 999	0	0	0	16 068

Table 16: TS smolt deliveries first two year [1000 smolt], pre-ordered, first stage and second stage decisions

Period	Month	70g	100g	150g	250g	Yearly total
3	Mar	0	0	0	607	
4	Apr	0	0	1 956	0	
5	May	1 333	0	0	137	
6	Jun	0	0	529	0	
8	Aug	0	0	0	1 133	
9	Sep	0	2 883	0	0	
11	Nov	5000	0	0	0	14 117
16	Apr	0	0	1 678	70	
17	May	3 452	0	0	0	
18	Jun	2 719	0	0	0	
20	Aug	2 648	0	0	0	
21	Sep	0	1 037	0	817	
23	Nov	4 070	0	0	0	16 491

Table 17: MS smolt deliveries first two year [1000 smolt], pre-ordered, first stage and second stage decisions

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have a much larger impact than in figure 7.1, where the MAB utilization in these periods is similar in each scenario. This is caused by the difference in number and delivery date of the first stage smolt orders, as most of the biomass in periods 26 to 33 are composed of fish from these releases. As the deterministic model orders large quantities of smolt, the MAB utilization will be higher in scenarios with lower survivability rates and temperatures. However, in scenarios with higher survivability and temperature the model will have to destroy smolt to ensure that the MAB restrictions are not broken in the earlier periods. Hence the MAB utilization is lower than it potentially could be from period 26 to 33 in the deterministic model. The fact that the deterministic model utilizes fewer delivery dates also increases the destruction of delivered smolt.

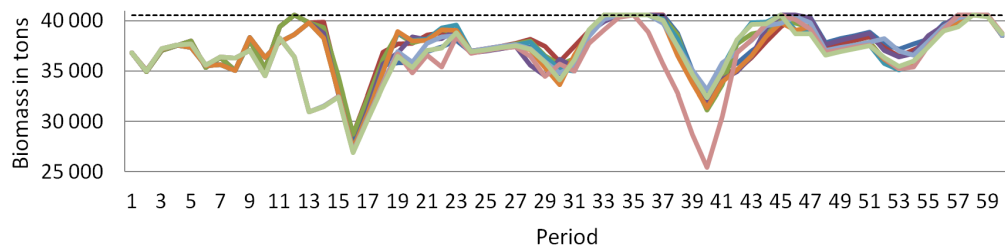


Figure 7.1: EEV results: Total MAB utilization in tons for both regions in all 9 scenarios.

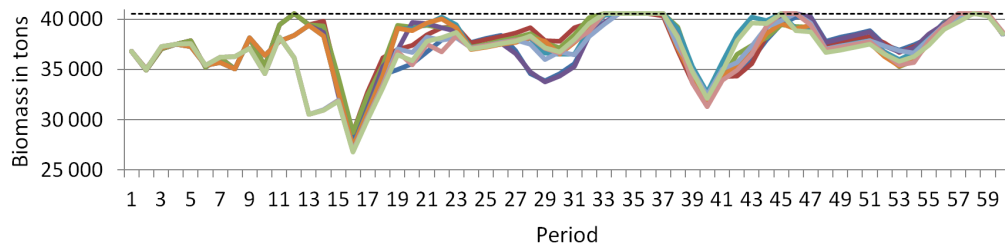


Figure 7.2: MS results: Total MAB utilization in tons for both regions in all 9 scenarios.

From period 36 to 42, the MAB utilization in scenario 8 drops significantly below the other scenarios in figure 7.1. This is because a significant amount of smolt is destroyed to ensure solvability in the scenario with high temperature and low mortality. The second stage smolt delivery plans that are made by the stochastic model avoid this dip in figure 7.2. From period 48 and onwards, which is the last year of the planning horizon, the difference between the EEV results and the MS results are much smaller. Here, decisions are

7.1 Results from the main run

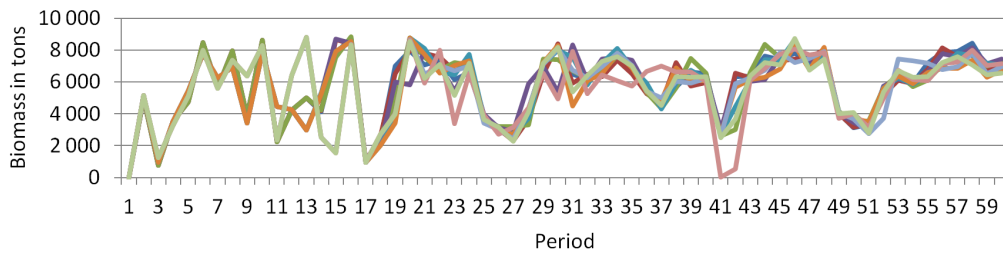


Figure 7.3: EEV results: Harvested biomass in tons for both regions in all 9 scenarios.

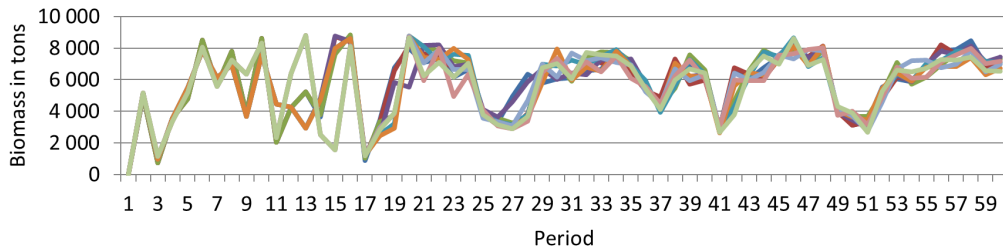


Figure 7.4: MS results: Harvested biomass in tons for both regions in all 9 scenarios.

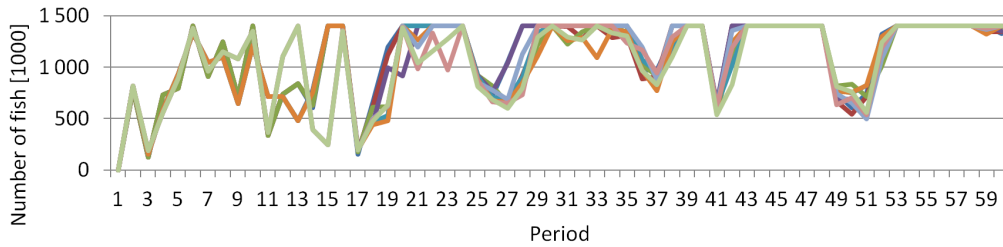


Figure 7.5: MS results: Harvested number of fish in thousands for both regions in all 9 scenarios.

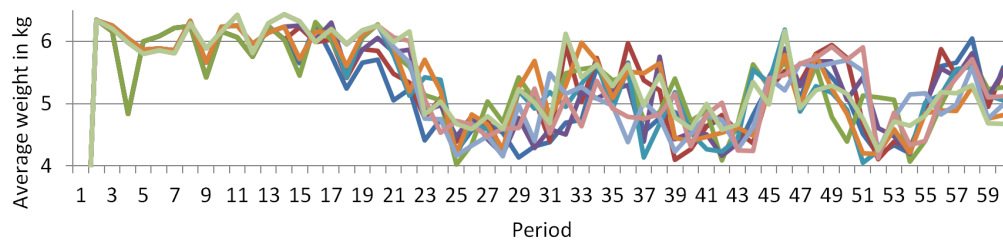


Figure 7.6: MS results: Average harvest weight for both regions in all 9 scenarios.

made with perfect information, which means that both the deterministic and stochastic model would make the same decisions. The small differences are caused by the different decisions being made earlier in the planning period.

Figure 7.4 and 7.3 show the harvested biomass in tons in each scenario when planning is done with and without uncertainty. Harvested biomass is the product of number of fish harvested and their average harvest weight, which is only shown here for the MS results in figure 7.5 and 7.6 respectively. The total harvested biomass in figure 7.3 and 7.4 follows a similar pattern, where the number of fish is affected by mortality, and average weight is affected by temperature. The seasonal variation in temperature causes the model to harvest significantly less during the months of January through March, as can be seen in periods 25 to 27, 37 to 40 and 49 to 51. In these months, the average harvest weights in figure 7.6 are at their lowest, while they increase in the warmer months when the slaughter house reaches maximum production, as seen in figure 7.5. This corresponds to previous work, which showed that average harvest weight should be adjusted in correlation to seasonal temperature variation to increase profit when MAB and slaughtering amount are limiting factors (Øveraas and Rynning-Tønnesen, 2012).

	EEV	MS	Difference
Total biomass harvested(tons)	3 051 879	3 078 866	0.88%
Total number of fish harvested(1000)	575 218	585 104	1.72%
Average harvest weight(kg)	5.31	5.26	- 0.82%

Table 18: Scenario average values regarding harvest for EEV and MS during the five year planning period.

Production follows a relatively similar pattern regardless of scenario. The main reason is that the salmon price has been modelled deterministically, and harvesting is planned with perfect information within each stage. However, planning for uncertainty regarding mortality and growth lead to a higher biomass output in the MS implementation, as table 18 illustrates.

7.1.3 Results from the three-stage main run

The biomass development for the aggregated facilities in period 13 to 60 for scenario 1 is shown in figure 7.7 for Trøndelag and figure 7.8 for Møre and Romsdal. For period 1 to 12 a similar pattern emerges, but these months are omitted because the 42 disaggregated facilities in year 1 make the graphs

unreadable. The aggregation of facilities is done based on the facilities' following periods. Each region meets its MAB limit several times during the planning horizon. The reason why the MAB utilization is better in Trøndelag than in Møre and Romsdal, is that there are more facilities with following periods at different times in Trøndelag, 6 compared to 3. Therefore the facilities can meet locational MAB limits at different times during the year. This indicates that it may be profitable to open facilities in Møre and Romsdal with following periods in anti-phase with the already existing locations. The biomass output and MAB utilization factor may then be increased for this region.

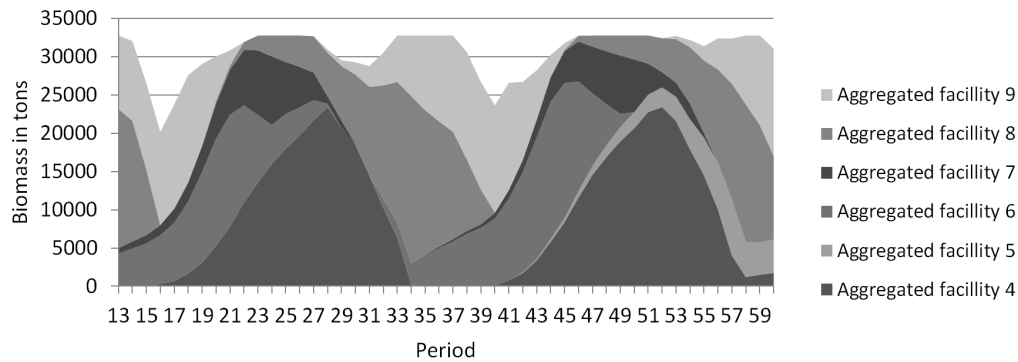


Figure 7.7: Biomass development for scenario 1 in Trøndelag for period 13 to 60.

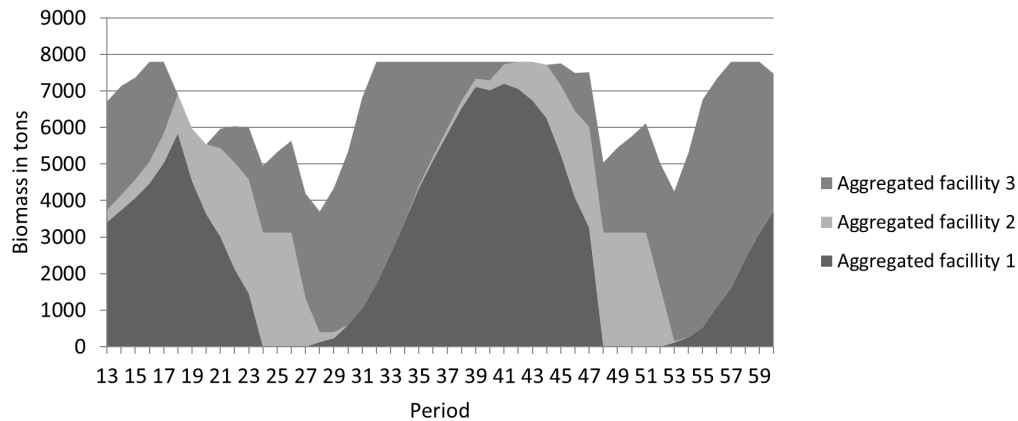


Figure 7.8: Biomass development for scenario 1 in Møre and Romsdal for period 13 to 60.

In the model, the most binding restrictions are the regional MAB constraint

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and the slaughter house capacity. Figure 7.5, 7.7 and 7.8 show that the maximum limit for these constraints are obtained in several periods. The freshwater facility on the other hand is only binding in some periods. Table 19 shows that the yearly capacity in the freshwater facility is much larger compared to the need of the value chain. In Marine Harvest Region Mid the yearly release of smolt in 2010 was 1 547 tons (Marine Harvest, 2012), while the model releases more smolt than 1 547 tons in year 1, 3 and 4.

Year	1	2	3	4	5
Maximum biomass production	2 650	2 650	2 650	2 650	2 650
Average biomass production	1 709	1 474	1 681	1 806	968

Table 19: Maximum production of smolt and average production of smolt in total each year in the freshwater facility [tons].

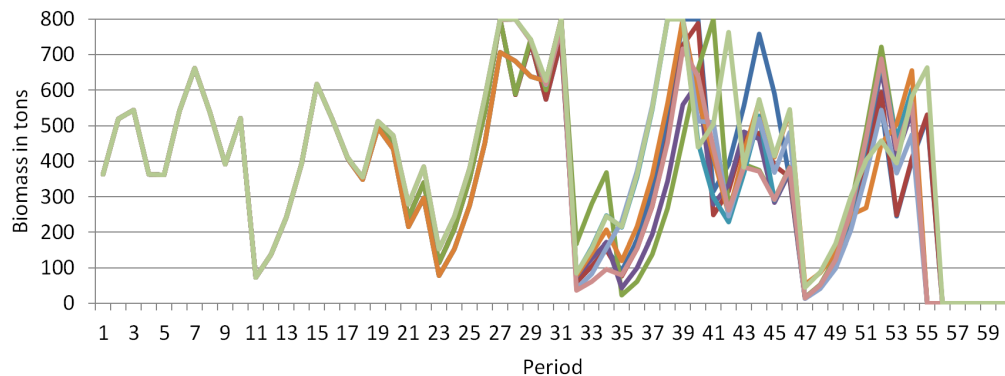


Figure 7.9: Biomass development in the freshwater facility during the planning horizon in all 9 scenarios [tons].

Figure 7.9 displays the monthly biomass development in the freshwater facility, and the monthly maximum holding capacity of 800 tons of biomass is only reached in certain periods. Furthermore it should be pointed out that because of the end of horizon problem the freshwater biomass is zero from period 56. The biomass development in the freshwater facility fluctuates more from month to month than the biomass development in saltwater. This is similar to how both biomass developments look like in reality, due to the freshwater production delivering in batches, while the saltwater production has a much more continuous harvesting.

Period	70g	100g	150g	250g
5	1184	0	0	137

Table 20: Smolt deployment plan in thousand smolt for location 39, year 1.

Table 20 gives the smolt deployment plan for location 39 in the first year of planning. The plan specifies the number of smolt, size and period of release. Similar plans are created for all of the 42 locations in Marine Harvest Region Mid. For location 39, smolt are released in one batch, and for most locations smolt are released in only one or two batches.

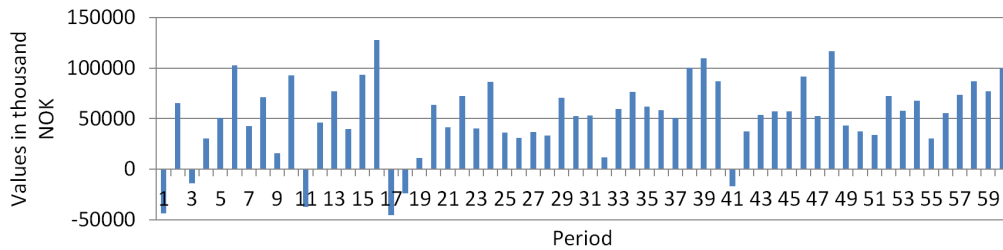


Figure 7.10: Average cash flow for all 9 scenarios.

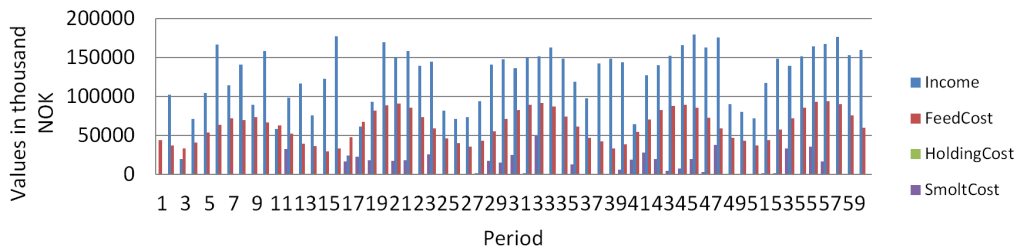


Figure 7.11: Average income and costs in 1000 NOK for all 9 scenarios.

Figure 7.10 shows the average cash flow for the whole planning horizon, whereas the income and costs from production are specified in figure 7.11. The smolt costs are calculated in the month where the smolt are delivered to the saltwater production, whereas the income comes in the month when the fish are harvested. Feeding costs fluctuate in accordance with seawater temperature, as salmon eat more when the temperature rises. Holding costs and feeding costs are continuous costs that occur in each month of production. Throughout the planning horizon the holding cost is zero as it is only given for fish at 8 kg, and slaughtering is prohibited for all fish over 6.5 kg. In some periods the cash flow is negative, figure 7.10. The cash flow in period

1 is negative due to no harvesting in this month. In the other periods with negative cash flow the harvesting amount is relatively small causing a low income. Figure 7.10 illustrate that optimal overall profit comes with negative cash flow in some months, and that lowering harvesting weight from period 20 gives a more even cash flow.

7.2 Results when changing parameter settings

The model has been run for different cases where the input data sets or the restrictions have been adjusted. For each case the adjustments from the three-stage main run will be described, and the results will be presented. Four test cases have been chosen, as they are interesting for Marine Harvest and increase the understanding of the model. In the first case the harvest interval is narrower, to compare Marine Harvest Region Mid's target weight with the main run interval of 4.0-6.5 kg. The second case will find out how the model acts if only the regulatory constraints are binding, and the third look into how sensitive the model is regarding price. The last case will find the potential value of salmon at different locations. All cases are run with the three-stage model.

7.2.1 Test case 1 - 5.5-6.5 kg harvest interval

In test case one, the harvest interval has been narrowed in from 4-6.5 kg to 5.5-6.5 kg. This case is introduced to compare Marine Harvest Region Mid's harvest strategy in 2012 of a target weight of 6 kg to a strategy where the harvest interval is widened.

	MS main run	Test case 1
Objective function	3.72 billion NOK	3.27 billion NOK
Average slaughter weight	5.26 kg	6.04 kg
Average MAB utilization	92.0%	85.2%

Table 21: The objective function, average slaughter weight and average MAB utilization factor for the MS main run where the harvest interval is 4-6.5 kg and test case 1 where the harvest interval is 5.5-6.5 kg.

When the harvest interval is extended from 5.5-6.5 kg to 4-6.5 kg, the objective function increase with 0.45 billion NOK, table 21. The increase in profit is a result of a 16% increase in biomass produced in the saltwater facilities during the planning horizon, which leads to an increase in the average MAB utilization factor from 85.2% to 92.0%. This is possible as the average slaughter weight is reduced from 6.04 kg to 5.26 kg. Table 22 shows that the

7.2 Results when changing parameter settings

Sales class	MS main run	Test case 1
4-5 kg	110 970	0
5-6 kg	119 186	93 973
6-7 kg	112 093	201 435

Table 22: Total biomass in each sales class in tons for the MS main run where the harvest interval is 4-6.5 kg and test case 1 where the harvest interval is 5.5-6.5 kg.

biomass production in the interval 6-7 kg is nearly halved in the MS main run compared to test case 1, in the 5-6 kg sales class production is slightly increased and for 4-5 kg production is significantly increased. In the MS main run the biomass harvested in each sales class is quite similar, as larger fish are harvested during summer and smaller during winter. The salmon farmer should not necessarily change harvest strategy to slaughter at lower weights, but the result can be used as a starting point of discussion.

7.2.2 Test case 2 - Only regulatory constraints

In test case two, all the capacity restrictions except the regulatory have been removed to see how the model would behave if capacities were not a limiting factor. The constraints that have been removed are the volume capacity in the freshwater facility restricting smolt deliveries, equation 5.4, release is allowed also in December, January and February, equation 5.6 and the maximum slaughtering capacity is removed, equation 5.15. Furthermore the slaughtering interval has been expanded to allow slaughtering from 3-8 kg, equation 5.14. Lastly, the end of horizon constraint for smolt, equation 5.23, is removed.

	MS main run	Test case 2
Objective value	3.72 billion NOK	4.17 billion NOK
Average slaughter weight	5.26 kg	4.05 kg
Average MAB utilization	92.0%	96.0%

Table 23: The objective value, average slaughter weight and average MAB utilization factor for the MS model and the MS model with only regulatory constraints.

Table 23 sums up the main results from test case 2. The MAB utilization and the objective function increase compared to the main run. The objective function increases with 0.45 billion NOK. Therefore, the increase in profit obtained in test case 1 by lowering the lower bound of the harvest

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weight interval from 5.5 kg to 4.0 kg, is the same as the increase obtained as the producer further lowers the allowable harvest weight to 3.0 kg while investing in unlimited production capacity. The main reasons are that the average slaughter weight goes down from 5.26 kg in the main run to 4.05 kg in test case 2, and that there are no limitations to the number of fish that can be harvested each month. Table 24 shows that there is a significant increase in the biomass slaughtered in the 3-4 kg sales class in test case 2 compared to the main run, whereas there is a decrease in all other sales classes. To increase the number of fish in the system, more smolt are released, given in table 25. The low increase in smolt delivery in year 5 in table 25 is a consequence of the end of horizon problem.

Sales class	MS main run	Test case 2
3-4 kg	0	221 329
4-5 kg	110 970	62 140
5-6 kg	119 186	39 033
6-7 kg	112 093	78 549

Table 24: Total biomass in each sales class in tons for the MS main run where the harvest interval is 4-6.5 kg and test case 2 where the harvest interval is 3-6.5 kg.

Year	MS main run	Test case 2	Difference[%]
1	14 117	15 803	11.9%
2	16 421	27 622	68.2%
3	15 514	28 098	81.1%
4	19 619	32 928	67.8%
5	13 320	13 845	3.9%

Table 25: The average total number of smolt delivered per year for the MS model and the MS model with flat price [1000 smolt].

Furthermore the increase in smolt delivery in test case 2 leads to a larger biomass production in the freshwater facility. Table 26 shows that the yearly maximum biomass production restriction in the freshwater facility is never binding. However, in year 4 the biomass production almost reaches the limit of 2650 tons. The production of smolt in year 4 is 1.68 times higher than in the main run, so there is a considerable over-capacity in yearly freshwater production.

7.2 Results when changing parameter settings

Year	1	2	3	4	5
Maximum biomass production	2650	2650	2650	2650	2650
Average biomass production	1827	2114	2615	2637	1161

Table 26: Maximum production of smolt and average production of smolt in total each year in the freshwater facility [tons].

Monthly biomass development in the freshwater facility is shown in figure 7.12. The maximum limit of 800 tons is exceeded several times in the planning period, particularly in year 3 and 4. The monthly biomass capacity restriction is therefore tighter than the yearly freshwater capacity restriction, but in general the over-capacity in freshwater production is large. As the monthly biomass capacity is a self-imposed limitation set by Marine Harvest, the total biomass could actually be exceeded in shorter periods (Marine Harvest, 2012).

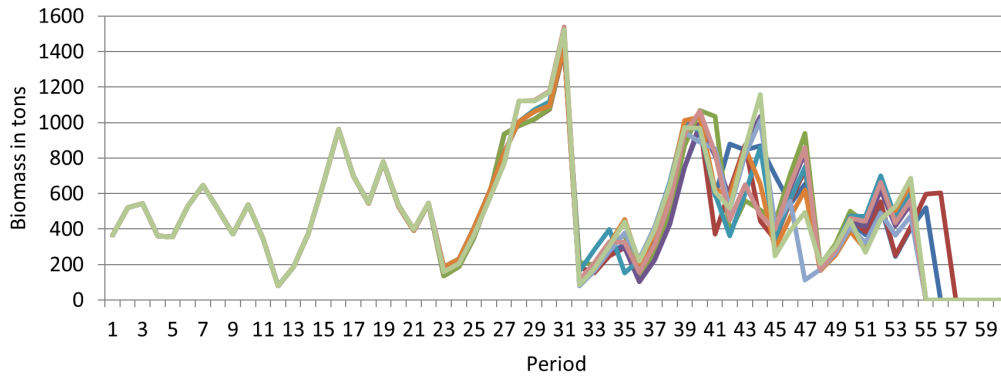


Figure 7.12: Biomass development in the freshwater facility during the planning horizon in all 9 scenarios [tons].

Lastly figures 7.13 and 7.14 give the number of harvested fish in thousands and average harvest weights. As the initial biomass and pre-ordered deliveries are made for target weight strategy of 6 kg, it is most profitable for the model to continue with this strategy in the beginning of the planning horizon. Even though the model can start ordering smolt for delivery from period 11 and onwards, it will not have built up enough numbers of fish in the system to change harvesting strategy before period 22. After period 24, the model stabilizes at an average harvest weight between 3 and 4 kg. The increased growth in the warmer months is utilized by increasing the number of harvested fish in these periods, as shown in figure 7.13. This indicates that it is most profitable to operate with a low average harvest weight and adjusting the number of fish harvested in correlation to seasonal temperature

variation, when only regulatory constraints are binding.

The monthly maximum slaughter house capacity implemented is 1 400 thousand fish, and for year 3 to 5 the number of harvested fish is close to this limit only in the winter months, whereas this limit is broken for the remaining periods as shown in table 7.13. The slaughter house restriction therefore seems to be the strictest of the non-regulatory constraints in the model.

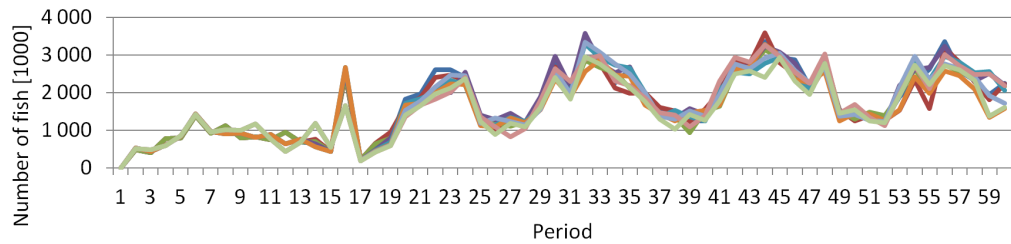


Figure 7.13: Monthly number of slaughtered fish in tons for all 9 scenarios.

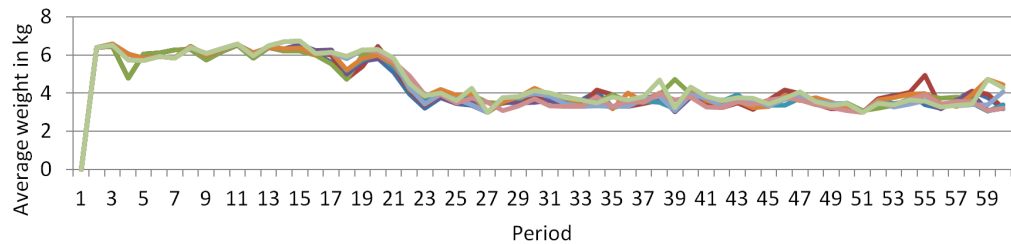


Figure 7.14: Average harvest weight in kilograms of slaughtered fish in all 9 scenarios.

7.2.3 Test case 3 - Adjusted input price

In test case three, the model has been run with different input prices to find out how sensitive it is to price. Input data from the main run have been adjusted to either have a flat price or an historic price. The model has therefore been run two separate times to get values for each price adjustment. From 2000-2010 the average price of salmon in Norway was 25 NOK/kg for all fish in the harvesting interval 4-6.5 kg (Statistisk sentralbyrå, 2011), and the flat price has been set to 25 NOK/kg. For the historic price run the average price for all fish in the harvesting interval 4-6.5 kg from 2002-2006 has been used, see figure 2.4. Still the model has full information about future price development, and the price is still implemented deterministic. Case three is relevant as price is an uncertain parameter that has been implemented

deterministically in the model, and testing the stochastic model with other price data will indicate how sensitive the model is to changes in price.

Table 27 gives the difference in smolt deliveries between the main run (MS model) and test case 3 with flat price. The largest deviation in yearly smolt delivery is -1.1%, which is less than the miscount of smolt at release, thus yearly deliveries are considered being the same. Also the harvest plan for the flat price run, figure 7.15, is very similar to the one from the MS model run, figure 7.4. One can therefore conclude that whether the model is run with the main run price forecast or with a flat price will have minor impact on the number of smolt delivered. The main reason for the small difference is that the forward price has small fluctuations as it lies between 24 NOK and 28.5 NOK for the whole planning horizon whereas the flat price is 25 NOK.

Year	MS main run	Test case 3 with flat price	Difference[%]
1	14 117	14 151	0.2%
2	16 421	16 290	-0.8%
3	15 514	15 502	-0.1%
4	19 619	19 397	-1.1%
5	13 320	13 327	0.1%

Table 27: The total number of smolt delivered per year for the MS main run and test case 3 with flat price [1000 smolt].

Figure 7.15 shows the harvested biomass in the planning horizon when the model is run with a flat price, while the harvested biomass for the run with historic prices is illustrated in figure 7.16. In both plans, the seasonality in the harvest plan is clearly visible. However, the harvest plans for the historic price, figure 7.16, become more volatile as they are influenced by price fluctuations. For instance, from period 29 to 33 the difference in harvest plans is clearly visible. In the flat price run the harvest pattern in this period is relatively even, figure 7.15, whereas there is a dip in the harvest pattern for the historic price run, 7.16. The dip in harvesting in the historic price run is caused by a price dip in the same period. Hence the conclusion is that larger fluctuations in price will influence the harvesting plans, which in turn influence the smolt delivery decisions. It is therefore likely that small adjustments in the deterministic input data for price will have a minor impact on the smolt delivery plan, while large deviations will have a great impact.

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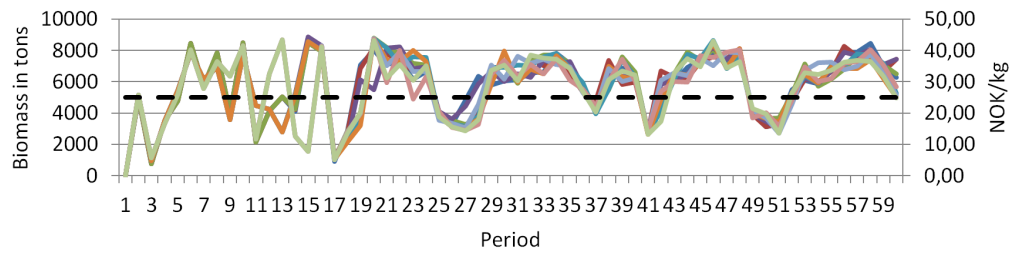


Figure 7.15: Harvested biomass in tons for all 9 scenarios with flat price. Biomass is given by the solid lines, price by the dashed line.

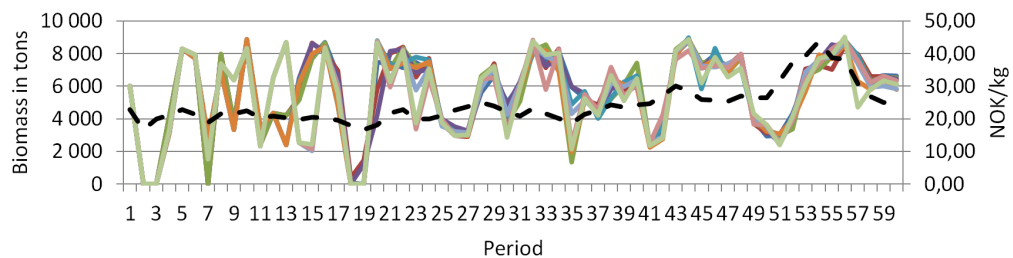


Figure 7.16: Harvested biomass in tons for all 9 scenarios with the historic price from 2002-2006. Biomass is given by the solid lines, price by the dashed line.

7.2.4 Test case 4 - Potential value of salmon

In test case four, the model has been tested to find the potential value of salmon. This value can for instance be used when making decisions regarding treatment of disease. The potential value of salmon in a specific location has been calculated by removing all the biomass in that location from the initial biomass. Then the model can adjust the smolt delivery plan, smolt deployment plan and harvest plans to get the most profit out of the remaining biomass in all other locations.

The input data from the main run have been adjusted so that the initial biomass in either location 9, 36, 3 or 4 in period 1 is zero. The model has therefore been run four separate times to get values for each location. The four locations have been chosen because 9 and 36 are in Trøndelag, while 3 and 4 are in Møre and Romsdal, and they have either a large or small average weight of fish. Potential value of salmon at a specific location is given by equation 7.3. Value of further growth is defined as the difference in potential value and sales value, given by equation 7.4.

7.2 Results when changing parameter settings

$$\begin{aligned} \text{Value}_{\text{potential}} &= \text{Objective function}_{\text{not removing fish}} \\ &\quad - \text{Objective function}_{\text{removing fish}} \end{aligned} \quad (7.3)$$

$$\text{Value}_{\text{further growth}} = \text{Value}_{\text{potential}} - \text{Income}_{\text{selling fish today}} \quad (7.4)$$

Location number		9	36	3	4
Number of fish	[million]	1.03	1.07	0.68	0.61
Average weight	[kg]	3.16	0.60	4.21	0.32
Value _{potential}	[NOK/fish]	79.58	56.39	102.58	54.93
Income _{Selling fish today}	[NOK/fish]	76.99	0	102.62	0
Value _{further growth}	[NOK/fish]	2.59	56.39	- 0.05	54.93
Value _{potential}	[NOK/kg]	25.17	93.35	24.34	171.11
Income _{Selling fish today}	[NOK/kg]	24.35	0	24.35	0
Value _{further growth}	[NOK/kg]	0.82	93.35	- 0.01	171.11

Table 28: Potential value of salmon

Table 28 gives the potential value, income from sale and value of further growth in NOK/fish and NOK/kg. Locations 9 and 3 have higher potential value given in NOK/fish than location 36 and 4, as average weight is larger here. However, for locations 9 and 3 the potential value is almost the same as the income value for selling the fish today. The model is able to adjust plans such that selling the fish today has a marginal effect on overall profit. For the fish at location 3, the value of further growth is negative. This means that it is better to remove the fish from the initial biomass and get paid accordingly, than to let it grow and selling it at a larger weight. This is due to the fact that a lot of the fish in location 3 weigh less than 4 kilograms. As shown in the main run, it is profitable to slaughter fish at lower weights in the colder months of January through March. Also, test case 2 showed that it is profitable to slaughter fish between 3 and 4 kilograms. The negative value of further growth is caused by the fact that the 4.0-6.5 kg harvest interval is broken when the fish is removed from the location. As delivery is not regionally dependent, more smolt can be released in region Møre and Romsdal when location 3 is emptied, increasing the overall profit.

When the values in table 28 are given in NOK/kg, the difference in potential value of biomass for small and large fish become comparable. In addition,

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the average weight in locations 36 and 4 are so low that the fish cannot be sold in the market. Therefore, the income values are zero for these locations. This causes the value of further growth to be significantly higher than for location 9 and 3. As the first possible delivery date of new ordered smolt is 11 month after the first period, the model is less able to adapt to the removal of smaller fish.

Test case 4 indicates that potential value of salmon is dependent on location, region, average fish weight, time period, fallowing and biomass in the other locations. The variation in value of further growth is considerable.

8 Conclusion

The aim of this thesis was to make a tactical planning model that makes smolt delivery and deployment plans and harvest salmon, while not breaking governmental regulations and production capacities, in an environment where several parameters are uncertain. The most important sources of uncertainty in seawater production are growth, price and mortality. A multistage stochastic model for production planning in seawater has been developed, and then implemented for Marine Harvest Region Mid. In the model growth and mortality have been implemented as stochastic parameters. Price has been implemented deterministically with the forward price from Fish Pool. An AR(1)-method has been used to forecast future temperatures, and all input data reflect a real salmon producer.

The value of using stochastic programming in production planning has been calculated. The objective value obtained from the tree-stage implementation is 0.93% better than using the deterministic implementation in an uncertain environment. Although the VSS is not very large, the plans made by the stochastic model are better than the ones made with a deterministic model. Using a deterministic model can result in a large amount of smolt being destroyed. This happens as the model will have ordered too many smolt for the scenarios where biomass development turns out to be higher than expected. The stochastic model avoids destruction of smolt, as it considers all possible scenarios. Stochastic programming therefore seems like a better tool for long term production planning in the salmon farming industry.

Furthermore this thesis has shown that average harvest weight should be adjusted in correlation to seasonal temperature variations to increase profit. Also, the freshwater facilities are a less limiting factor than previous work indicates. Additional test cases have been solved. The model shows that the objective function can increase with 450 million NOK when the allowable harvest weight interval is adjusted from 5.5-6.5 kg to 4-6.5 kg. If the model is solved with unlimited production capacity and a slaughter weight interval of 3.0-6.5 kg, the profit increases with an additional 450 million NOK. Regulatory MAB limits and slaughter house capacity seem to be the tightest restrictions in saltwater production. The model has also shown that the optimal harvesting plan becomes more volatile as prices fluctuation increase. Furthermore the model has been used to calculate the potential value of salmon and the value of further growth. The value of further growth is in the interval -0.05 to 56.39 NOK/fish for the cases in this thesis.

9 Future work

The growth model in the optimization model is based on a growth table from Skretting, which is only dependent on temperature and fish weight. Salmon producers continuously work on improving their growth models, because of the importance of accurately modeling growth, but they do not make them publicly available. Therefore, the optimization model should be implemented with the updated growth model that Marine Harvest uses today. Also a better forecast method could be implemented for temperature, and Marine Harvest should get location specific temperature data. The price forecast should also be updated if Marine Harvest has access to a better price forecast. Furthermore the freshwater facility could be implemented in more detail, and an independent optimization model for production in freshwater could be developed. The optimization model for freshwater could be implemented with an interface to the optimization model for saltwater production developed in this thesis. Also this optimization model for saltwater production in this thesis could be extended for all regions in Marine Harvest Norway.

In order to make the optimization model useful for Marine Harvest, a better user interface should be developed. A user interface for both input and output data is needed. This interface could be implemented with an interface to existing software or a new one could be built, so that the input data is automatically updated on the state of all locations. Better computer capacity should be used to decrease solution time and to make extensions of the model possible.

References

- C. Alexander. *Market risk analysis I: Quantitative methods in finance*. John Wiley & Sons Ltd, 2008a.
- C. Alexander. *Market risk analysis II: Practical financial econometrics*. John Wiley & Sons Ltd, 2008b.
- T. Andersen, K. Roll, and S. Tveterås. The price responsiveness of salmon supply in the short and long run. *Marine Resource Economics*, 23(4): 425–37, 2008.
- R. Arnason. *Optimal feeding schedules and harvesting time in aquaculture*. University of Iceland, Faculty of Economics and Business Administration, 1990.
- F. Asche, S. Kumbhakar, and R. Tveterås. Testing cost vs. profit function. *Applied Economics Letters*, 14(10):715–718, 2007.
- O. Bergfjord. Risk perception and risk management in norwegian aquaculture. *Journal of Risk Research*, 12(1):91–104, 2009.
- J. Birge and F. Louveaux. *Introduction to stochastic programming*. Springer Verlag, 1997.
- T. Bjørndal. Optimal harvesting of farmed fish. Center for applied research, Norwegian school of economics and business administration, 1988.
- C. Brooks. *Introductory Econometrics for Finance*. Cambridge University Press, 2nd edition, 2008.
- Cermaq. Sea water production - the salmon's life cycle – sea water. URL: <http://www.cermaq.com/portal/wps/wcm/connect/cermaqen/home-/fishfarming/Sea+water+production>, 2012.
- Directorate of Fisheries. *Forskrift om bekjempelse av lus i akvakulturanlegg*. Department of aquaculture, seafood og market, 2009.
- Directorate of Fisheries. Statistikk akvakultur. URL: <http://www.fiskeridir.no/statistikk/akvakultur/statistikk-for-akvakultur/laks-regnbueoerret-og-oerret>, 2010a.
- Directorate of Fisheries. Lønnsomhetsundersøkelse for matfiskproduksjon laks og regnbueørret. URL:

REFERENCES

- <http://www.fiskeridir.no/statistikk/akvakultur/statistiske-publikasjoner/loennsomhetsundersokelser-laks-og-regnbueoerret/loennsomhet-matfisk-laks-og-regnbueoerret>, 2010b.
- L. Escudero, A. Garín, and G. Pérez. The value of the stochastic solution in multistage problems. *Top*, 15(1):48–64, 2007.
- Fish Pool. Forward price from fish pool. URL: <http://fishpool.eu/iframe.aspx?iframe=forwardlist.asp&pageId=49>, 2012.
- O. Forsberg. Optimal harvesting of farmed atlantic salmon at two cohort management strategies and different harvest operation restrictions. *Aquaculture Economics & Management*, 3(2):143–158, 1999.
- O. Forsberg and A. Guttormsen. The value of information in salmon farming. harvesting the right fish at the right time. *Aquaculture Economics & Management*, 10(3):183–200, 2006.
- M. Hæreid. Allocating sales in the farming of atlantic salmon. Master’s thesis, Norwegian University of Science and Technology, 2011.
- S. Handeland, A. Inslund, and S. Stefansson. The effect of temperature and fish size on growth, feed intake, food conversion efficiency and stomach evacuation rate of atlantic salmon post-smolts. *Aquaculture*, 283(1-4):36–42, 2008.
- J. Higle. Stochastic programming: optimization when uncertainty matters. *Tutorials in Operations Research*, pages 1–24, 2005.
- F. S. Hiller and G. J. Lieberman. *Introduction to operations research*, pages 1009–1038. McGraw-Hill, 2001.
- K. Hjelt. Norwegian regulation system and the history of the norwegian salmon farming industry. In *Cage Aquaculture in Asia: Proceedings of the First International Symposium on Cage Aquaculture in Asia*, pages 1–17. Asian Fisheries Society and World Aquaculture Society - Southeast Asian Chapter, Bangkok, 2000.
- K. Høyland, M. Kaut, and S. W. Wallace. A heuristic for moment-matching scenario generation. *Computational Optimization and Applications*, 24: 169–185, 2003.
- P. Kall and J. Mayer. *Stochastic Linear Programming: Models, Theory and Computation*, volume 156. Springer, 2nd edition, 2011.

-
- P. Kall and S. W. Wallace. *Stochastic Programming*. John Wiley & Sons, 2nd edition, 1997.
- M. Kaut and S. Wallace. Evaluation of scenario-generation methods for stochastic programming. *Pacific Journal of Optimization*, 3(2):257–271, 2007.
- T. Langan and T. Toftøy. Produksjonsoptimering innenfor lakseoppdretten - planlegging under usikkerhet. Master’s thesis, Norwegian University of Science and Technology, 2011.
- J. Lillestøl. On the problem of optimal timing of slaughtering in fish farming. *Modeling, Identification and Control*, 7:199–207, 1986.
- Y. Liu, J. Olaf Olaussen, and A. Skonhoft. Wild and farmed salmon in norway-a review. *Marine Policy*, 2010.
- Marine Harvest. Salmon farming industry handbook, 2010.
- Marine Harvest. Data received from marine harvest. Contact: Eivind Osnes, Anders Jon Fjellheim, Eirik Talhaug, 2012.
- Mattilsynet. Regionalt tilsynsprosjekt, prosjekt overlevelse fisk. URL: http://www.mattilsynet.no/mattilsynet/multimedia/archive/00074/-Sluttrapport_Projek_74570a.pdf, 2011.
- K. T. Midthun. *Optimization models liberalized natural gas markets*. PhD thesis, Norwegian University of Science and Technology, 2007.
- Ministry of fisheries and coastal affairs. *Laksetildelingsforskriften*. Department of aquaculture, seafood og market, 2005.
- Ministry of fisheries and coastal affairs. *Forskrift om drift av akvakulturanlegg*. Department of aquaculture, seafood og market, 2008.
- J. Mistiaen, I. Strand, et al. Optimal feeding and harvest time for fish with weight-dependent prices. *Marine resource economics*, 13:231–246, 1998.
- Norwegian Ministry of Fisheries and Coastal Affairs. Facts about fisheries and aquaculture. URL: [http://www.regjeringen.no/upload/FKD/Brosjyrerdf,Grubbegata 1, P.O Box 8118 Dep., NO-0032 Oslo](http://www.regjeringen.no/upload/FKD/Brosjyrerdf,Grubbegata%201,%20P.O%20Box%208118%20Dep.,%20NO-0032%20Oslo), 2010.
- M. Nowak and A. Tomasgard. Scenario generation and forecasting. Working paper, Department of Industrial Economics and Technology Management, Norwegian University of Science and Technology, Trondheim, Norway, 2007.

REFERENCES

- A. Oglend and M. Sikveland. The behaviour of salmon price volatility. *Marine Resource Economics*, 23(4):487–506, 2008.
- S. Øveraas and C. Rynning-Tønnesen. Production optimization in the salmon farming industry - smolt ordering under uncertainty. Project thesis, Norwegian University of Science and Technology, 2012.
- S. Pascoe, P. Wattage, and D. Naik. Optimal harvesting strategies: practice versus theory. *Aquaculture Economics & Management*, 6(5-6):295–308, 2002.
- A. Pickering. Growth and stress in fish production. *Aquaculture*, 111(1-4): 51–63, 1993.
- Rømmingskommisjonen for akvakultur. Årsmelding 2010. URL: <http://www.rommingskommisjonen.no/uploads/aarsmelding2010.pdf>, 2010.
- R. Sætre. *The Norwegian coastal current - oceanography and climate*. Tapir Academic press, 2007.
- P. Schütz. *Managing Uncertainty and Flexibility in Supply Chain Optimization*. PhD thesis, Norwegian University of Science and Technology, 2009.
- Statistisk sentralbyrå. Average salmon price, 2000-2010. URL: http://www.ssb.no/vis/magasinet/norge_verden/art-2004-10-14-01.html, 2011.
- N. Stikholmen. Produktivitetsutvikling over tid i oppdrett av laks: en studie av perioden 2001-2008 med bruk av dea og malmquistindeks. 2010.
- H. Thorarensen and A. Farrell. The biological requirements for post-smolt atlantic salmon in closed-containment systems. *Aquaculture*, 2010.
- S. W. Wallace. Decision making under uncertainty: The art of modelling. Molde University College, 2002.