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The Real Options to Shutdown, Startup, and Abandon: Structural Estimation of Switching Costs

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Abstract

In this thesis we analyze the real options to shutdown, startup and abandon gas fired power plants. We assume that the plants' status for a given year is either operating, in standby or retired. Their status is further dependent on a stochastic two-factor model for the spark spread process. The analysis is made possible by data on operating status as reported annually to the US Energy Information Administration. We estimate the irreversible costs of switching by structural estimation of a real options model. The proposed model also indicates the spark spread triggers and option values for the switching decisions.

Sammendrag

I denne avhandlingen analyserer vi realopsjoner knyttet til å midlertidig stenge ned, starte opp og endelig legge ned gasskraftverk. Vi antar at statusen til et kraftverk i et gitt år er enten i drift, i standby eller nedlagt. Statusen er avhengig av en stokastisk to-faktor modell for kraftverkets spark spread. Analysen er muliggjort med data for kraftverkenes årlige status som hvert år rapporteres til US Energy Information Administration. Ved å benytte strukturell estimering av en realopsjonsmodell, estimerer vi de irreversible kostnadene som inntreffer ved en statusendring. Den foreslåtte modellen gir også teoretiske triggerverdier for når statusendringer bør inntreffe i henhold til realopsjonsteori, samt opsjonsverdiene som ligger i muligheten til å endre status.

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1 Introduction

In the presence of irreversibility and uncertainty, switching decisions - as typically made in natural resource industries - can be considered real options. Real options theory values the opportunity to invest in, shutdown, restart, or abandon production assets, as call and put options on the present value of the future cash flows of the assets. According to theory, the decision maker should delay incurring irreversible switching costs until a significant gap develops between the switching decision's expected payoffs and costs. Higher uncertainty induces an incentive to wait, and the gap between the expected payoffs and costs necessary to trigger the switching decision should widen. While the theory of real options is well established as a framework for prescribing optimal decision making, empirical verification of the theory remain scarce. This is mainly due to a lack of data.

Fleten, Haugom, and Ullrich (2012) examine empirically the real options to shutdown, startup, and abandon peak power plants. The exercise is made possible by a dataset with detailed information for 1,121 individual US power plants for the period from 2001 to 2009. Their work is close in spirit to that of Moel and Tufano (2002).

Our contribution is to provide a structural model for the data Fleten et al. (2012) are investigating and present estimates of the switching costs. The proposed model is built on the ideas from the *infinite resource case* in Brennan and Schwartz (1985). By means of structural estimation we estimate the costs of switching[1]. Moreover, we calculate the trigger levels at which switching decisions occur and the resulting option values.

The remainder of the thesis is structured in the following way. Section 2 gives a review of the literature. In section 3 an overview of the data is provided. Section 4 explains the methodology. The results are presented in section 5 and discussed in section 6. Section 7 concludes.

2 Literature review

The real options literature rationalizes how firms should time investments in the face of irreversible investment costs and uncertainty over the future rewards from the investments. Arrow (1968) introduces one of the first models for optimal capital policy concerning irreversible investments. Since then real options theory has primarily been applied to natural resource industries (see Brennan and Schwartz (1985)), in discussions of growth options as sources of firm value (compare Myers (1977), Kester (1984) and Pindyck (1988)), and as a technique to value projects (see McDonald and Siegel (1986) and Paddock, Siegel, and Smith (1988)). The considerable research within the field of real options has culminated in introductory textbooks, such as Dixit and Pindyck (1994) and Trigeorgis (1996). Despite the theoretical contributions to the literature and the vast number of applications that assume firms optimally make decisions in the presence of uncertainty, little empirical evidence has been provided on real options effects.

2.1 Empirical verification of real options theory

Empirical studies of real options explaining asset prices are found in Quigg (1993) and Davis (1996). These papers suggest that real options models are able to explain the ob-

[1]By switching costs, we refer to the restart, shutdown and abandonment cost. In addition, the maintenance costs of being in standby are included in this definition as they are a direct result of the decision to change status.

served market prices for land and minerals, respectively. Studies that empirically examine whether investments respond to changes in uncertainty typically focus on natural resource industries. Hurn and Wright (1994) and Dunne and Mu (2010) investigate the impact of price volatility on offshore oil field and refinery investments. Both papers use the historic realized variance of commodity prices as the uncertainty measure. In contrast, Kellogg (2010) uses implied volatility to measure expected price volatility. He finds that Texas oil companies reduce their drilling activity when volatility increases. The magnitude of this change is deemed to be consistent with real option theory. Kellogg (2010) further discusses why these real options effects are observed:

> *Why might these estimates of firms' responses to changes in expected volatility accord so well with theory? Given the small size of the majority of these firms, it seems unlikely that they are formally solving Bellman equations. However, they may have developed decision heuristics that roughly mimic an optimal decision-making process. Moreover, the firms have a strong financial incentive to get their decision-making at least approximately right*(Kellogg, 2010, p. 30).

Heggedal, Linnerud, and Fleten (2011) examine whether uncertainty with respect to the introduction of a market for renewable energy certificates affected the timing of investments in small hydro power plants in Norway. They conclude that investors holding a portfolio of licenses during the time period from 2001 to 2009 act in line with real options theory, and uncertain climate policy decisions delay their investment rate.

The predictions of the Brennan and Schwartz (1985) real options model are empirically evaluated in Moel and Tufano (2002). They examine the shutdown and startup decisions for 285 gold mines from 1988 to 1997. The empirical data is found to be well described by the real options model. In a similar way, Fleten et al. (2012) examine the real options to shutdown, startup, and abandon existing peak power plants, and find evidence of real options effects, particularly for shutdown and abandonment decisions. Their work differs from that of Moel and Tufano (2002) in three aspects. They examine peak power plants rather than gold mines, explicitly take into account the decision to abandon a plant, and include measures of regulatory uncertainty.

2.2 The real options to shutdown, startup and abandon

Brennan and Schwartz (1985) is the pioneering article on the joint decisions to invest and abandon. They establish a general model of the decision to shutdown, startup and abandon a mine producing a natural resource whose price are assumed to follow a geometric Brownian motion. Several authors have argued that mean-reverting price processes, instead of geometric Brownian motion based models, are more appropriate for commodities (see for example Cortazar and Schwartz (1994) and Smith and McCardle (1998)).

2.2.1 Spark spread models with short and long term uncertainty

Schwartz and Smith (2000) develop a two-factor model where the short term deviations are modeled with a mean-reverting process, and the equilibrium price evolves according to an arithmetic Brownian motion. Näsäkkälä and Fleten (2005) and Fleten and Näsäkkälä (2010) use this to model the spark spread, defined as the difference between the price of electricity and the cost of gas used for the generation of electricity, in order to value a gas fired power plant. By using the spark spread process they have a single reference price process for gas fired power plant investments. When electricity and gas processes are used separately there is no such reference price, compare Deng, Johnson, and Sogomonian (2001).

In the two-factor model for the spark spread, the short term deviations reflect non-persistent changes, e.g. changes in the demand resulting from variations in the weather. The equilibrium price reflects fundamental changes that are expected to persist, e.g. expectations regarding the discovery of natural gas (Näsäkkälä and Fleten, 2005). Other two-factor models with long and short term factors are found in Ross (1997) and Pilipović (1998). The spark spread is the sum of a short term deviation and an equilibrium price, and can be positive or negative. Näsäkkälä and Fleten (2005) thus express it as

$$S(t) = \chi(t) + \xi(t), \tag{1}$$

where $\chi(t)$ is the short term deviation from the equilibrium price $\xi(t)$.

The short term deviation reverts toward zero, following an Ornstein-Uhlenbeck process

$$d\chi(t) = -\kappa\chi(t)dt + \sigma_\chi dB_\chi(t), \tag{2}$$

The equilibrium price follows an arithmetic Brownian motion process

$$d\xi(t) = \mu_\xi dt + \sigma_\xi dB_\xi(t), \tag{3}$$

where κ, σ_χ, σ_ξ and μ_ξ are constants. The standard Brownian motions, $B_\chi(t)$ and $B_\xi(t)$, are correlated according to $\rho dt = dB_\chi(t)dB_\xi(t)$.

In Schwartz and Smith (2000) it is shown that the dynamics given in equations 1-3, imply that the spark spread is normally distributed. Hence, the expected value and variance are given by

$$E_t[S(t)] = e^{-\kappa(T-t)}\chi(t) + \xi(t) + \mu_\xi(T-t) \tag{4}$$

and

$$Var_t(S(t)) = \frac{\sigma_\chi^2}{2\kappa}(1 - e^{-2\kappa(T-t)}) + \sigma_\xi^2(T-t) + 2(1 - e^{-\kappa(T-t)})\frac{\rho\sigma_\xi\sigma_\chi}{\kappa} \tag{5}$$

2.2.2 The value of an operating plant

An increase in the variability of the spark spread increases the value of up and down ramping, making a peak power plant more valuable. On the other hand, uncertainty also delays investments, compare Dixit and Pindyck (1994). In their numerical example Näsäkkälä and Fleten (2005) illustrate the relative strengths of these opposite effects. The expression that is derived in Näsäkkälä and Fleten (2005) for the value of an ideal peak plant is given as

$$V\left(\chi(t),\xi(t)\right) = C\int_t^T e^{-r(s-t)}\left(\frac{\sqrt{Var_t(S(s))}}{\sqrt{2\pi}}e^{-\frac{(EC-E_t[S(s)])^2}{2Var_t(S(s))}}\right. \tag{6}$$
$$\left.+\left(E_t[S(s)]-EC\right)\Phi\left(\frac{E_t[S(s)]-EC}{\sqrt{Var_t(S(s))}}\right)\right)\mathrm{d}s-\frac{G}{r}\left(1-e^{-r(T-t)}\right)$$

where C is the capacity of the plant, EC is the emission cost, G is the fixed costs of running the plant, $(T-t)$ is the remaining lifetime of the plant, and Φ is the cumulative distribution function. The expected value and variance of the spark spread, $E_t[S(s)]$ and $Var_t(S(s))$, are given by equations 4 and 5, respectively.

Deng and Oren (2003) find that the operational characteristics affect the valuation of a power plant to different extents, depending on the operating efficiency of the power plant and the assumptions about the electricity and the generating fuel prices. The impacts of physical operating characteristics on the power plant valuation are generally found to be far more significant under mean-reversion models than they are under geometric Brownian motion price models (Deng and Oren, 2003).

2.2.3 Optimal switching of status

There is a large body of literature analyzing the problem of optimal switching, either from a theoretical perspective or aiming at specific applications. Optimal switching among a number of alternatives in response to changing economic conditions can be viewed as a set of linked or compound options. Each switch is an exercise of an option, and each switch yields an asset that combines a cash flow with the option of switching again. Consequently we have compound options that need to be priced simultaneously (Dixit and Pindyck, 1994).

One of the earliest works on optimal switching is by Mossin (1968) who develop a model where operating revenue follows a trendless random walk with upper and lower reflecting barriers. There is no possibility of abandonment in the model, but the optimal revenue levels at which it is optimal to shutdown and startup are calculated. The more general model of Brennan and Schwartz (1985) includes the possibility of shutting down as well as active operation and abandonment. According to Dixit and Pindyck (1994), the model of Brennan and Schwartz (1985) confuse the transition to the two states from an active state, by using the same lower threshold symbol for shutdown and abandonment. As maintenance costs are assumed to be zero in their numerical solution, abandonment will never be considered in the Brennan and Schwartz (1985) model. Hence, only switches between an operating and a suspended state are considered. A more coherent analysis is found in the subsequent work of Dixit (1989) where the entry and exit decisions are isolated from issues of lay-up or finite stocks.

Recent work on optimal switching includes that of Adkins and Paxson (2011). They study optimal replacement and abandonment decisions for real assets, when both revenues and costs are uncertain and deteriorate with age. A quasi-analytical model is derived that is thought sufficiently flexible to deal with other real options models involving two variables.

2.3 Structural estimation of real options models

The main focus in the real options literature has been on normative models of strategic and operating flexibility in capital budgeting decisions under uncertainty (Gamba and Tesser, 2009). From a descriptive point of view, structural estimation could be applied to assess the empirical validity of a real options model, as is the case of financial derivatives models. Estimating structural models can however be computationally difficult. For example, dynamic discrete choice models are commonly estimated using the *nested fixed point algorithm* developed by Rust (1987). This requires solving a dynamic programming problem thousands of times during estimation and numerically maximizing a nonlinear likelihood function. Kellogg (2010) employs the nested fixed point routine as a means to empirically verify a real options model.

According to Gamba and Tesser (2009) the lack of descriptive contributions to the literature is generally due to two related challenges of real options models. Firstly, the underlying real assets are often not traded and their value is thus not observed. Secondly, some of the drivers may be unrelated to price but related to uncertainty in quantity. Gamba and Tesser (2009) propose an approach for structural estimation of real options models where the procedure allows for *unobserved heterogeneity* of firms. Unobserved heterogeneity implies that different firms, although in the same industry, may have different parameters for the same objective functions (Gamba and Tesser, 2009). As such, the approach is an extension of the nested fixed point algorithm.

One of the concerns with the nested fixed point algorithm is the excessive amount of computation demanded. Recent research proposes computationally simple estimators for structural models using a two-step approach that is computationally light and often requires minimal parametric assumptions. An example is *mathematical programming with equilibrium constraints* as described in Su and Judd (2011) and applied by Vitorino (2011). Since this approach is reliable and has speed advantages, structural models may become more accessible to a larger set of researchers (Su and Judd, 2011).

3 Data

Fleten et al. (2012) have collected detailed information on 1,121 individual US power plants for the period from 2001 to 2009, a total of 8,189 plant-year observations. This data is unique in its scope and level of detail, and is our main source of data. When appraising this data, we only consider the data for plants that are situated in the PJM interconnection and that run on natural gas. We select the 2004-2006 time frame due to availability of electricity futures data, compare the discussion in section 6.1.

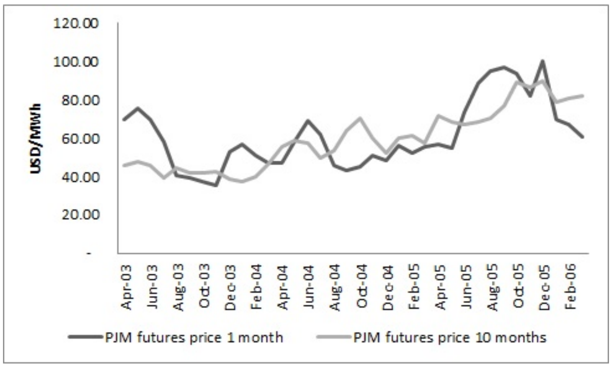
3.1 Futures prices of electriciy and natural gas

The spark spread parameters - κ, ρ, σ_χ, σ_ξ, μ_ξ, χ_0 and ξ_0 - should ideally be estimated on a continuous time basis as more and more price information will be available to the decision maker. Furthermore, since each generator has its own variable non-fuel cost and heat rate, the data series for forward prices of the spark spread should be calculated on an individual generator basis[2].

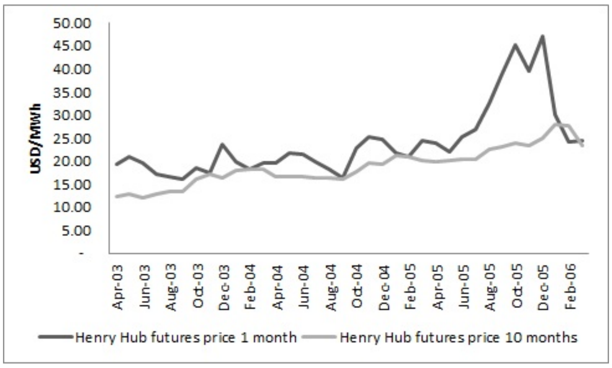
[2]The formulae for calculating the spark spread is S=S_e-$K_H S_g$-VOM where S_e and S_g are the prices of electricity and natural gas, respectively, and K_H is the heat rate of the generator given as the reciprocal of its efficiency. VOM denotes the non-fuel costs, i.e. the variable operation and maintenance costs.

However, given the resolution of data on the status changes in Fleten et al. (2012), it is sufficient to estimate the spark spread parameters at three instances in time, i.e. one for each of the reported time periods 2003-2004, 2004-2005, 2005-2006, compare tables 3 and 4. Since the Form 860[3] is collected by the Energy Information Administration in mid-February each year, see Fleten et al. (2012), we choose to estimate the spark spread parameters in April in the three time periods. We merge the generators into nine cost and efficiency specific groups, thereby limiting the number of times the spark spread parameters need to be estimated. There are 594 peak plants located in the PJM interconnection that run on natural gas, and these plants are distributed across the nine cost and efficiency specific groups.

Figures 1(a) and 1(b) show futures prices of electricity and natural gas[4], respectively, for the time period 2003-2006. These time series are used to calculate the time series for spark spread forward prices in figure 2(a) during the same time period. Figure 2(b) shows the spark spread forward curve as of April 2006.



(a) Historical futures prices of electricity from the PJM interconnection

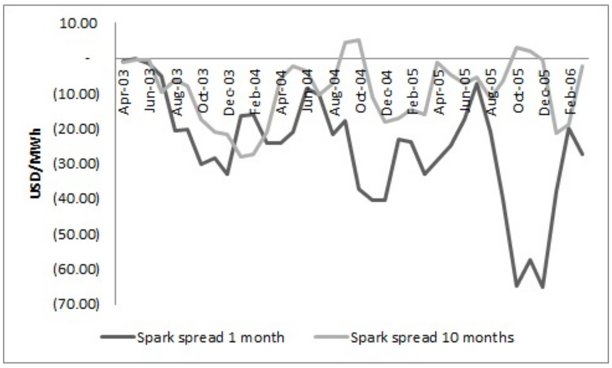


(b) Historical futures prices of natural gas from the Henry Hub

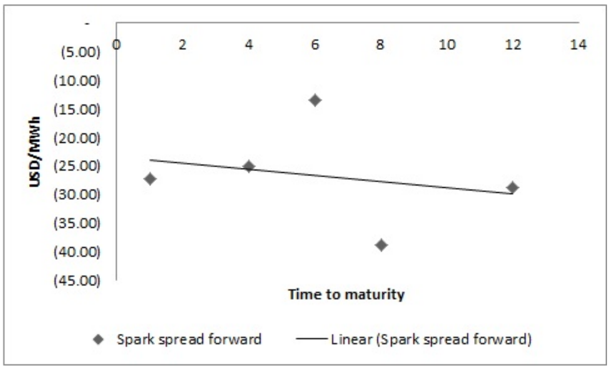
Figure 1: (a) shows the historical futures prices of electricity in the PJM interconnection from April 2003 to April 2006. (b) depicts the futures prices of natural gas from the Henry Hub from April 2003 to April 2006. Constracts maturing in 1 and 10 months are shown for both commodities.

[3]Form 860 contains detailed data for nearly every power plant in the United States, both existing and planned.

[4]Data from Ecowin Reuters and New York Mercantile Exchange.



(a) Historical forward prices of the spark spread



(b) Forward curve of the spark spread

Figure 2: (a) shows the historical forward prices of the spark spread for the PJM interconnection and Henry Hub natural gas from April 2003 to April 2006. (b) illustrates the linear trend in the forward curve of the spark spread as of April 2006. A plant efficiency of 29.3% and VOM of 4.6 \$/MWh are used to obtain the spark spread forwards.

We randomly select generator #6407 from the nine cost and efficiency specific groups. Generator #6407 belongs to a group with an efficiency of 29.3% and variable operation and maintenance costs VOM of 4.6 \$/MWh. The calculation of the spark spread forwards in figure 2 are based on the characteristics of this group. Even though we merge the generators into nine groups based on VOM and efficiency, the generators still have individual values for capacity C, fixed operation and maintenance costs G, and operating status (OP for operating, SB for standby, and RE for retired).

In the remainder of the thesis we report and discuss the results of generator #6407, thereby providing consistency and preserving space. Note that the results from the structural estimation come for all the plants in the dataset. The operating characteristics of generator #6407 are summarized in table 1.

Table 1: Operating characteristics of generator #6407

Parameter	Efficiency	VOM	G	C	Status
Unit	%	\$/MWh	m\$/yr	TWh/yr	2003-2004-2005-2006
	29.31	4.6	0.1638	0.3986	OP→ SB→ OP→ SB

Industry convention is to quote G in \$/kW-yr and C in MW, but in order to apply these parameters in equation 6 they are converted to m\$/yr and TWh/yr, respectively.

During the time period from 2001 to 2009, the interest rate on 10-year US Treasury bonds varies from 4% to 6% with a mean of 4.7%. Hence, we set the risk-free interest rate to 4.7%.

3.2 Status changes of US power plants

The plants in the dataset from Fleten et al. (2012) are collected from three US electricity hubs. Summary statistics are presented in table 2, while status changes for the plants are shown in tables 3 and 4.

Table 2: Plant Summary Statistics

	Age (yrs)	Size (MW)	Efficiency
NOBS	1,121	1,121	1,121
Mean	18.6	43.1	24.7%
Stdev	14.1	41.0	4.6%
Min	0	0.4	5.4%
Max	60	246.0	41.8%

Table 2 presents summary statistics for the age (to the nearest year), size (MW), and efficiency (%) of plants in the sample. The ages are calculated based upon the first year a plant appears in the sample.

Table 3: Shutdown: Transitions from OP to OP/SB by Year

from year	*to year*	OP	SB	Total
2001	2002	695	2	697
2002	2003	803	1	804
2003	2004	808	43	851
2004	2005	820	12	832
2005	2006	829	16	845
2006	2007	848	0	848
2007	2008	851	2	853
2008	2009	885	0	885
	Total	6,539	76	6,615

Table 3 shows the number of plants classified as operating (OP) in the *from year* and either operating (OP) or shutdown (SB) in the *to year*

Table 4: Startup and Retirement: Transitions from SB to OP/SB/RE by Year

from year	*to year*	OP	SB	RE	Total
2001	2002	60	221	1	282
2002	2003	47	198	1	246
2003	2004	9	143	49	201
2004	2005	22	153	13	188
2005	2006	1	158	6	165
2006	2007	6	173	0	179
2007	2008	32	139	2	173
2008	2009	7	127	6	140
	Total	184	1,312	78	1,574

The number of plants classified as shutdown (SB) in the *from year* and either operating (OP), shutdown (SB), or retired (RE) in the *to year* are accounted for in table 4.

4 Methodology

4.1 Estimating the spark spread parameters

Since neither the short term deviation $\chi(t)$ nor the equilibrium price $\xi(t)$ can be observed directly, estimates of the short and long term dynamics need to be inferred from data that are somehow dependent on the dynamics in question. Näsäkkälä and Fleten (2005) point out that the difference in electricity and gas futures prices is the risk-adjusted expected future spark spread value. Thus futures prices can be used to estimate the risk adjusted dynamics of short term deviations and equilibrium price. Since expected short term variations revert toward zero when the maturity increases, the long maturity futures will contain information about the equilibrium price. When the time to maturity is short, the short term variations may be nonzero. The difference of long and short maturity forwards therefore yields information about the short term dynamics (Näsäkkälä and Fleten, 2005).

The parameters for mean-reversion κ, correlation ρ, and volatility σ_χ and σ_ξ, are estimated from the forward price history with a Kalman filtering procedure[5]. The long-term drift μ_ξ is then estimated from long term forwards by means of a linear regression. Finally, the current short term deviation χ_0 and equilibrium price ξ_0 are chosen so that the expected value fits the forward curve. For a thorough review of the procedure, we refer to Schwartz and Smith (2000) and Näsäkkälä and Fleten (2005).

4.2 Valuation of an operating peak plant

For the purpose of parsimony and tractability in the analysis, the lifetime of the peak power plants are assumed to be infinite. Since power plants tend to be refurbished and upgraded, the engineered lifetime of approximately 25 years is often greatly extended in practise[6]. In principle, our analysis is equivalent to the *infinite resource case* of Brennan and Schwartz (1985). Infinite lifetime implies that the value of an operating plant is given by equation 6 when T approaches infinity. However, there is no analytical solution to this integral and in section 4.2.2 we thus resort to numerical methods. This is unfortunate as it reduces the speed of computation in the structural estimation, see section 4.4. In an attempt to bypass numerical solutions, we try to value an operating peak plant in accordance to the framework of McDonald and Siegel (1986).

4.2.1 Quasi-analytical solution

McDonald and Siegel (1986) value a project as a simple contingent claim where the owner of the project has an infinite set of options on the underlying asset. Alternatively, the option values can be calculated using the standard Black-Scholes formulae, and by summing these values over time the project value can be obtained. In our case, the owner of an operating power plant has an infinite set of European call options on the spark spread. Instead of the contingent claims method, we use dynamic programming to solve for the value of the plant. Since the spark spread is modeled by a two-factor model, the value of

[5] A Kalman filter procedure recursively computes estimates for unobserved state variables based on observations that depend on these state variables. In Appendix A we have included the R code for the Kalman filter.

[6] For further justifications of the assumption about infinitely lived power plants, compare the discussion in Fleten and Näsäkkälä (2010). A convenient analytical alternative is to model the lifetime of the plant as random following a Poisson process, see e.g. Dixit and Pindyck (1994, chap. 6). In effect, this would raise the required return to equity.

the operating power plant is a function of short term deviation $\chi(t)$ and equilibrium price $\xi(t)$, i.e. $V(\chi(t), \xi(t))$ assuming the fixed costs of running the plant G can be neglected.

Multivariate Itô's Lemma[7] is used to expand $dV(\chi(t), \xi(t))$ and it gives the following relation

$$
\begin{aligned}
dV(\chi, \xi) = V_\chi(\chi, \xi)d\chi + V_\xi(\chi, \xi)d\xi + \frac{1}{2}V_{\chi\chi}(\chi, \xi)(d\chi)^2 \\
+ \frac{1}{2}V_{\xi\xi}(\chi, \xi)(d\xi)^2 + V_{\chi\xi}(\chi, \xi)d\chi d\xi
\end{aligned}
\tag{7}
$$

Inserting equations 2 and 3 into 7 gives

$$
\begin{aligned}
dV(\chi, \xi) = V_\chi(\chi, \xi)\left(-\kappa\chi dt + \sigma_\chi dB_\chi\right) + V_\xi(\chi, \xi)\left(\mu_\xi dt + \sigma_\xi dB_\xi\right) \\
+ \frac{1}{2}V_{\chi\chi}(\chi, \xi)\sigma_\chi^2 dt + \frac{1}{2}V_{\xi\xi}(\chi, \xi)\sigma_\xi^2 dt + V_{\chi\xi}(\chi, \xi)\rho\sigma_\chi\sigma_\xi dt
\end{aligned}
\tag{8}
$$

Taking expectations on both sides yields

$$
\begin{aligned}
E[dV(\chi, \xi)] = V_\chi(\chi, \xi)\left(-\kappa\chi dt\right) + V_\xi(\chi, \xi)\left(\mu_\xi dt\right) \\
+ \frac{1}{2}V_{\chi\chi}(\chi, \xi)\sigma_\chi^2 dt + \frac{1}{2}V_{\xi\xi}(\chi, \xi)\sigma_\xi^2 dt + V_{\chi\xi}(\chi, \xi)\rho\sigma_\chi\sigma_\xi dt
\end{aligned}
\tag{9}
$$

Bellman's principle of optimality states that over a time interval dt, the total expected return on the value of the project, $rV(\chi, \xi)dt$, is equal to the sum of its expected rate of capital appreciation and the dividends from holding the project, i.e.

$$
rV(\chi, \xi)dt = E[dV(\chi, \xi)] + \pi dt \tag{10}
$$

where we use the risk-free rate r as the discount rate (due to the estimation of risk-adjusted parameters in the spark spread dynamics) and denote the cash flow from an operating plant by π.

Equation 9 is inserted into equation 10, and we divide both sides by dt. Then

$$
\begin{aligned}
rV(\chi, \xi) = V_\chi(\chi, \xi)\left(-\kappa\chi\right) + V_\xi(\chi, \xi)\left(\mu_\xi\right) + \frac{1}{2}V_{\chi\chi}(\chi, \xi)\sigma_\chi^2 \\
+ \frac{1}{2}V_{\xi\xi}(\chi, \xi)\sigma_\xi^2 + V_{\chi\xi}(\chi, \xi)\rho\sigma_\chi\sigma_\xi + \pi
\end{aligned}
\tag{11}
$$

When rearranging the expression in equation 11 we have the following partial differential equation (PDE)

$$
\begin{aligned}
\frac{1}{2}\left(V_{\chi\chi}(\chi, \xi)\sigma_\chi^2 + V_{\xi\xi}(\chi, \xi)\sigma_\xi^2\right) + V_{\chi\xi}(\chi, \xi)\rho\sigma_\chi\sigma_\xi \\
+ V_\chi(\chi, \xi)(-\kappa\chi) + V_\xi(\chi, \xi)(\mu_\xi) - rV(\chi, \xi) + \pi = 0
\end{aligned}
\tag{12}
$$

[7] In mathematics, Itô's Lemma is used to find the differential of a function of a stochastic process. The lemma is widely employed in mathematical finance and its best known application is in the derivation of the Black−Scholes equation used to value options.

To the best of our knowledge, this PDE cannot be solved analytically. However, if we set the correlation ρ equal to zero[8], we can use the method of separation of variables to find the homogeneous solution of the form

$$V_{hom}(\chi, \xi) = J(\chi)K(\xi) \tag{13}$$

i.e. the homogeneous solution of the PDE is the product of a function of χ and a function of ξ.

After some differential calculus[9] equation 13 turns into

$$\begin{aligned} V_{hom}(\chi, \xi) = {} & C_1 \chi M\left(\frac{2\kappa + sepconst}{4\kappa}, \frac{3}{2}, \frac{\kappa\chi^2}{\sigma_\chi^2}\right) e^{\frac{\xi}{\sigma_\xi^2}\left(-\mu_\xi + \sqrt{\mu_\xi^2 - \sigma_\xi^2 sepconst + 2\sigma_\xi^2 r}\right)} \\ & + C_2 \chi M\left(\frac{2\kappa + sepconst}{4\kappa}, \frac{3}{2}, \frac{\kappa\chi^2}{\sigma_\chi^2}\right) e^{\frac{\xi}{\sigma_\xi^2}\left(-\mu_\xi - \sqrt{\mu_\xi^2 - \sigma_\xi^2 sepconst + 2\sigma_\xi^2 r}\right)} \\ & + C_3 \chi U\left(\frac{2\kappa + sepconst}{4\kappa}, \frac{3}{2}, \frac{\kappa\chi^2}{\sigma_\chi^2}\right) e^{\frac{\xi}{\sigma_\xi^2}\left(-\mu_\xi + \sqrt{\mu_\xi^2 - \sigma_\xi^2 sepconst + 2\sigma_\xi^2 r}\right)} \\ & + C_4 \chi U\left(\frac{2\kappa + sepconst}{4\kappa}, \frac{3}{2}, \frac{\kappa\chi^2}{\sigma_\chi^2}\right) e^{\frac{\xi}{\sigma_\xi^2}\left(-\mu_\xi - \sqrt{\mu_\xi^2 - \sigma_\xi^2 sepconst + 2\sigma_\xi^2 r}\right)} \end{aligned} \tag{14}$$

where C_1, C_2, C_3 and C_4 are arbitrary constants, and $0 \leq sepconst \leq 2r$ is the separation constant. $M(a, b, z)$ and $U(a, b, z)$ are the Kummer and Tricomi functions, respectively. These solve the Kummer equation given as

$$z\frac{d^2w}{dz^2} + (b - z)\frac{dw}{dz} - aw = 0 \tag{15}$$

with a regular singular point at 0 and an irregular singular point at ∞. It has two linearly independent solutions given by the Kummer $M(a, b, z)$ and Tricomi $U(a, b, z)$ functions. See Abramowitz and Stegun (1972, chap. 13) for definitions of the confluent hypergeometric functions.

It can be shown that the particular solution to equation 12 when $\rho = 0$ is

$$V_{part}(\chi, \xi) = C\left(\frac{\chi}{r + \kappa} + \frac{\xi}{r} + \frac{\mu_\xi}{r^2}\right) \tag{16}$$

where C is the capacity of the plant, χ is the short term deviation in the spark spread, κ is the rate of mean reversion, ξ is the equilibrium price, μ_ξ is the long term drift, and r is the risk-free interest rate.

In line with Dixit and Pindyck (1994, chap. 6) the value of the plant must be considered for two cases, depending on whether the spark spread $\chi+\xi$ exceeds emission costs E or not. When $\chi+\xi < E$ and either χ or ξ, or both, move towards -∞, the value of the option to resume operation tends towards zero. Conversely, when $\chi+\xi > E$ and either χ or ξ, or both, move towards ∞, the value of the option to suspend operation tends towards zero. See appendix B for a complete derivation of the quasi-analytical solution.

[8]The two-factor model of Pilipović (1998) also neglects correlation.

[9]Dockendorf and Paxson (2010) provide a discussion on how to solve for functions such as $J(\chi)$ in equation 13.

4.2.2 Numerical solution

An alternative to the quasi-analytical approach in section 4.2.1, is to obtain the value of an operating plant by solving the integral in equation 6 numerically. A step-by-step derivation of this expression is found in appendix C. In the real options model in section 4.3 we need to differentiate the expression in equation 6 with respect to ξ. After differentiating we have

$$\begin{aligned} V_{\xi(t)}\left(\chi(t),\xi(t)\right) &= \frac{dV\left(\chi(t),\xi(t)\right)}{d\xi(t)} \qquad (17)\\ &= C\int_t^T e^{-r(s-t)}\left(-\frac{(E_t[S(s)]-EC)}{\sqrt{2\pi}\sqrt{Var_t(S(s))}}e^{-\frac{(EC-E_t[S(s)])^2}{2Var_t(S(s))}} \right.\\ &+ \Phi\left(\frac{E_t[S(s)]-EC}{\sqrt{Var_t(S(s))}}\right) + \left.\frac{(E_t[S(s)]-EC)}{\sqrt{2\pi}\sqrt{Var_t(S(s))}}e^{-\frac{(EC-E_t[S(s)])^2}{2Var_t(S(s))}}\right)\mathrm{d}s\\ &= C\int_t^T e^{-r(s-t)}\left(\Phi\left(\frac{E_t[S(s)]-EC}{\sqrt{Var_t(S(s))}}\right)\right)\mathrm{d}s \end{aligned}$$

where C is the capacity of the plant, EC is the emission cost, $(T-t)$ is the remaining lifetime of the plant, and Φ is the cumulative distribution function. The expected value and variance of the spark spread, $E_t[S(s)]$ and $Var_t(S(s))$, are given by equations 4 and 5, respectively.

As is evident from equation 17 and the nature of the cumulative distribution function, an increase (decrease) in the equilibrium price ξ, will increase (decrease) the value of the operating plant $V(\chi(t),\xi(t))$. In other words, the higher the long term spark spread, the more valuable is the operating plant.

A numerical method suitable for solving the integrals in equation 6 and 17 is the Gauss Lobatto rules, see for example Weisstein (2012). In appendix D we have written the algorithm in C++[10].

4.3 Modeling the real options to shutdown, startup and abandon

When modeling the real options to shutdown, startup and abandon we assume that the switching decisions are made as a function of equilibrium price and occur instantly. In other words, the current short-term realization $\chi(t)$ is disregarded in the switching decisions. In theory, switching decisions under two-factor dynamics depend on both short and long term uncertainty. In practice however, short term effects diminish quickly and have insignificant bearing on the decision to switch operating status.

This assumption accords well with the logic applied for the investment decision in Näsäkkälä and Fleten (2005) and Fleten and Näsäkkälä (2010). The parameters governing

[10]This script can be called as a user-defined function when performing structural estimation in AMPL. Depending on the spark spread estimates, a smooth shape of the integrands in equations 6 and 17 is possible. Thus simpler numerical methods, such as composite Simpson's rule, can be applied without affecting the accuracy of the numerical solution. The advantage is that such simple algorithms can be implemented directly in AMPL, thereby avoiding the need for user-defined functions.

the short term variations, i.e. mean reversion κ and short term volatility σ_χ, still affect the value of the plant, compare section 4.2. Hence, the short term parameters have an indirect effect on the switching decisions.

The motivation for omitting the short term realization when switching decisions are made is due to the fact that status switches are long term decisions, and switching of operating status is never made due to non-persistent spikes in the price process[11].

An operator decides whether the peak power plant should be operating, in standby, or retired. When the plant is operating its strategic value is the sum of the value of the operating plant and the option to shutdown. A plant that is in standby has a strategic value that consists of two options - an option to startup and an option to abandon operation. There is also a continuous maintenance cost related to being in standby, which we denote by M. When a plant is retired there is no going back, and the operation is thus permanently shut down. In figure 3, a principle drawing is made on how the real options model may work over time.

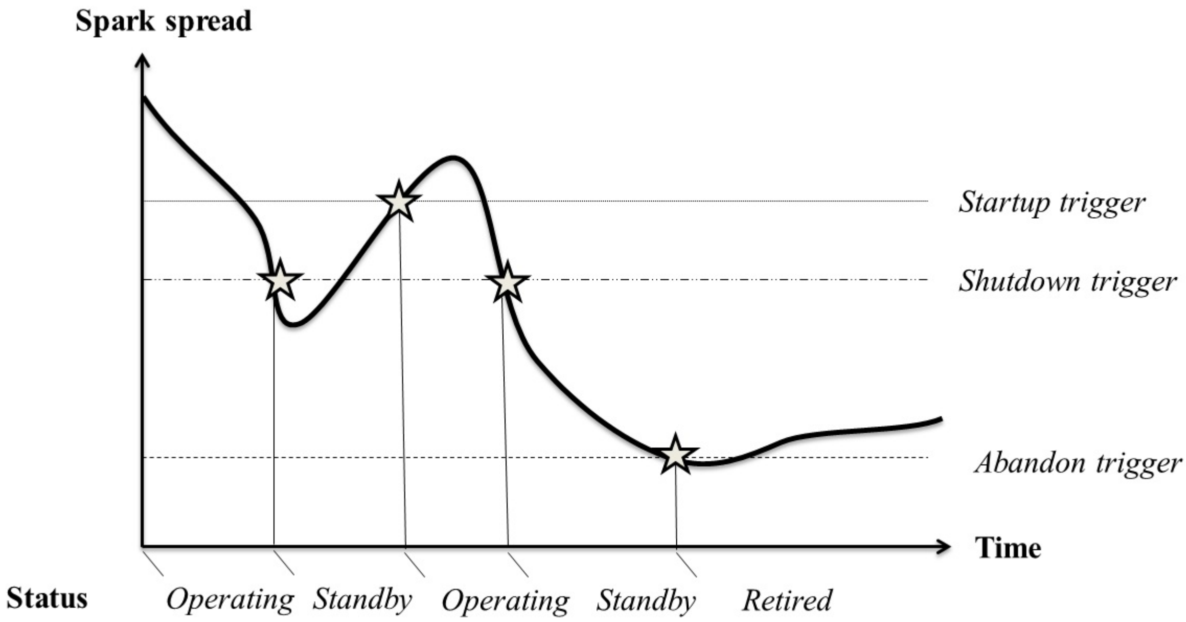


Figure 3: The real options to shutdown, startup and abandon - principle drawing

The switching costs are denoted by E, R, and S, representing the costs of shutting down, restarting and abandonment, respectively. While the interpretations of E and R are as positive costs, S can be considered a salvage value, allowing it to be both negative and positive. Figure 4 depicts the sequence of switching costs E, R, and S as well as the continuous maintenance costs M. Note that switching from operating to retired status will never occur as this is inherent in the dataset. We dropped the few occurrences of OP→RE as being driven by something other than spark spread economics.

[11]Consequently, the spark spread is only governed by the equilibrium price process given in equation 3. In discrete time this process is equivalent to $\Delta\xi = \mu_\xi \, \Delta t + \sigma_\xi \, \sqrt{\Delta t} \, \varepsilon$ where $\varepsilon \sim N(0, 1)$.

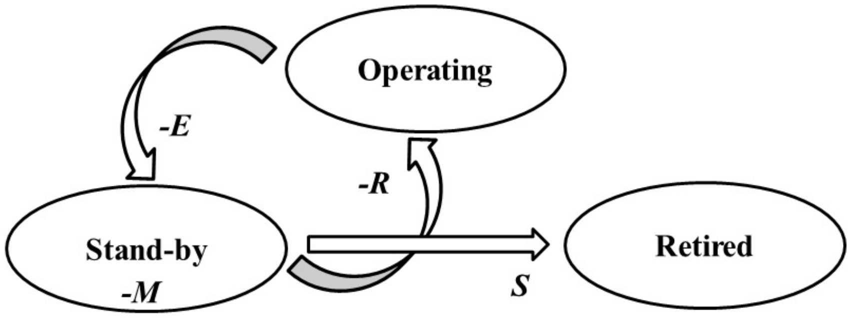


Figure 4: Sequence of switching costs

According to the framework of Dixit and Pindyck (1994, chap. 7) three value-matching and three smooth-pasting conditions can now be formulated. At the boundary, the value-matching conditions ensure that the values in two adjacent states are equal, and the smooth-pasting conditions are first order necessary conditions that avoid the optimal solution being at a kink. Assuming for now that we know the costs, M, E, R and S, we have a system of six equations with six unknowns. The six unknowns are the three trigger values for equilibrium price, $\xi^{shutdown}$, $\xi^{startup}$, and $\xi^{abandon}$, and the three arbitrary constants related to the three option values. The option values can be derived and analyzed using a contingent claims or a dynamic programming approach as is done in appendix E.

The switching decisions are made on the basis of equilibrium price. Hence, the shutdown decision is evaluated at the spark spread value $S(t) = \xi^{shutdown}$ in time t and bounded by

$$V(\chi,\xi) + B_2 e^{\beta_2 \xi} = D_1 e^{\beta_1 \xi} + D_2 e^{\beta_2 \xi} - \frac{M}{r} - E \tag{18}$$

$$\frac{d(V(\chi,\xi) + B_2 e^{\beta_2 \xi})}{d\xi} = \frac{d\left(D_1 e^{\beta_1 \xi} + D_2 e^{\beta_2 \xi} - \frac{M}{r} - E\right)}{d\xi} \tag{19}$$

where B_2, D_1 and D_2 are arbitrary constants that are to be determined. β_1 and β_2 are the positive and negative root, respectively, of the fundamental quadratic

$$\frac{1}{2}\sigma_\xi^2 \beta^2 + \mu_\xi \beta - r = 0 \tag{20}$$

The left-hand side of the value-matching condition, see equation 18, is the sum of the value of the operating plant and the option to shutdown. At $S(t) = \xi^{shutdown}$ this sum must equal what the operator receives in the standby state minus the switching cost, i.e. the sum of the option to restart and the option to abandon minus maintenance costs and the shutdown cost, which is the right-hand side of equation 18. The subsequent smooth-pasting condition ensures that the equality holds when differentiating with respect to equilibrium price.

The decision to startup is evaluated at $S(t) = \xi^{startup}$ in time t and constrained by

$$D_1 e^{\beta_1 \xi} + D_2 e^{\beta_2 \xi} - \frac{M}{r} = V(\chi, \xi) + B_2 e^{\beta_2 \xi} - R \tag{21}$$

$$\frac{d\left(D_1 e^{\beta_1 \xi} + D_2 e^{\beta_2 \xi} - \frac{M}{r}\right)}{d\xi} = \frac{d\left(V(\chi, \xi) + B_2 e^{\beta_2 \xi} - R\right)}{d\xi} \tag{22}$$

The left-hand side of equation 21, is the sum of the option to restart and the option to abandon minus maintenance costs. At $S(t) = \xi^{startup}$ this sum must equal what the operator receives in the operating state minus the switching cost, i.e. the sum of the value of the operating plant and the option to shutdown minus the restart cost, which is the right-hand side of equation 21. The smooth-pasting condition makes sure that the equality holds when differentiating with respect to equilibrium price.

The decision to abandon is evaluated at $S(t) = \xi^{abandon}$ in time t and bounded by

$$D_1 e^{\beta_1 \xi} + D_2 e^{\beta_2 \xi} - \frac{M}{r} = S \tag{23}$$

$$\frac{d\left(D_1 e^{\beta_1 \xi} + D_2 e^{\beta_2 \xi} - \frac{M}{r}\right)}{d\xi} = \frac{d\,(S)}{d\xi} \tag{24}$$

In equation 23 the left-hand side is the sum of the option to restart and the option to abandon minus maintenance costs. At $S(t) = \xi^{abandon}$ this sum must equal what the operator receives in the retired state, i.e. the salvage value, which is the right-hand side of equation 23. In the smooth-pasting condition the equality holds when differentiating with respect to equilibrium price.

Taking derivatives and inserting trigger values of equilibrium price, we have the following system of equations

$$V(\chi, \xi^{shutdown}) + B_2 e^{\beta_2 \xi^{shutdown}} = D_1 e^{\beta_1 \xi^{shutdown}} + D_2 e^{\beta_2 \xi^{shutdown}} - \frac{M}{r} - E \tag{25}$$

$$V_\xi(\chi, \xi^{shutdown}) + \beta_2 B_2 e^{\beta_2 \xi^{shutdown}} = \beta_1 D_1 e^{\beta_1 \xi^{shutdown}} + \beta_2 D_2 e^{\beta_2 \xi^{shutdown}} \tag{26}$$

$$D_1 e^{\beta_1 \xi^{startup}} + D_2 e^{\beta_2 \xi^{startup}} - \frac{M}{r} = V(\chi, \xi^{startup}) + B_2 e^{\beta_2 \xi^{startup}} - R \tag{27}$$

$$\beta_1 D_1 e^{\beta_1 \xi^{startup}} + \beta_2 D_2 e^{\beta_2 \xi^{startup}} = V_\xi(\chi, \xi^{startup}) + \beta_2 B_2 e^{\beta_2 \xi^{startup}} \tag{28}$$

$$D_1 e^{\beta_1 \xi^{abandon}} + D_2 e^{\beta_2 \xi^{abandon}} - \frac{M}{r} = S \tag{29}$$

$$\beta_1 D_1 e^{\beta_1 \xi^{abandon}} + \beta_2 D_2 e^{\beta_2 \xi^{abandon}} = 0 \tag{30}$$

The structural estimation is constrained by the above real options model.

4.4 Structural estimation

According to Rust (1987), the idea of structural estimation is to employ a micro-theoretic model to derive aggregate investment parameters from individual optimizing behavior. Rust (1987) further provides a strategy for maximum likelihood estimation of single-agent dynamic programming models, known as the nested fixed-point algorithm. Su and Judd (2011) argue that structural estimation of economic models is an important technique for analyzing economic data but that the technique - as described in Rust (1987) - is often viewed as computationally difficult. They claim that even though optimization is used heavily in econometrics, there is a large gulf between current practice in econometrics and the methods found in the mathematical programming literature. Thus Su and Judd (2011) propose a mathematical programming approach where structural parameters and endogenous economic variables are chosen so as to maximize the likelihood of the data. The optimization problem is subject to the constraints that the endogenous economic variables are consistent with an equilibrium for the structural parameters. When formulated in this way, it is evident that we have to define the likelihood (the objective function) and the equilibrium constraints, and submit this problem to one of the state-of-the-art optimization solvers[12](Su and Judd, 2011).

In sections 4.4.1 - 4.4.3 we perform the structural estimation by means of a mathematical programming method. Inspired by the real options model in Brennan and Schwartz (1985), Gamba and Tesser (2009) find the switching costs of the gold mines through structural estimation. As discussed in section 4.2, the Brennan and Schwartz (1985) model is in essence similar to our real options model. While Gamba and Tesser (2009) maximize the value of the mines by finding the implicit sequence of optimal decision making, we formulate the real options explicitly, compare the system of equations in section 4.3. Not only will this allow us to estimate the switching costs, but also the trigger levels and option values to shutdown, startup and abandon. To be able to compare the two approaches, we briefly formulate our model in the framework of Gamba and Tesser (2009) in appendix F.

4.4.1 Objective function

The objective function may involve a maximum likelihood estimator as in equation F.13 in appendix F. In order to find such an estimator, we need to assume a distribution for the error terms, and we also need to figure out where to put the error terms. A possibility is to include normally distributed error terms in conjunction with the switching costs, i.e. the structural parameters. However, since there are few time observations per plant, we formulate an alternative objective function where we minimize penalties. See appendix G for the AMPL code.

A penalty δ is introduced whenever the data deviates from the real options model in section 4.3. We minimize these penalties $\delta_{t,a,i} \geq 0,\ i = 1, \ldots, 6$ across time t and plants a. As such, the objective function is related to the method of least absolute deviation. The logic is explained in the pseudo code below.

[12]In this thesis we use AMPL, a comprehensive and powerful algebraic modeling language for nonlinear optimization problems, and we let AMPL communicate with the nonlinear solver KNITRO, see for instance Fourer, Gay, and Kernighan (1993) and Byrd, Nozedal, and Waltz (2006).

$$
\begin{aligned}
\underset{\delta_{t,a,i}}{\text{Minimize}} \quad & \sum_{t=2}^{T_a}\sum_{a=1}^{P}\sum_{i=1}^{6}\delta_{t,a,i} \\
\text{subject to} \quad & \text{if } OP_{t-1} \rightarrow OP_t \\
& \qquad \text{then } \delta_{t,a,1} = max(\xi_{t,a}^{shutdown} - \xi_{t,a}, 0) \\
& \text{else if } OP_{t-1} \rightarrow SB_t \\
& \qquad \text{then } \delta_{t,a,2} = max(\xi_{t,a} - \xi_{t,a}^{shutdown}, 0) \\
& \text{else if } SB_{t-1} \rightarrow OP_t \\
& \qquad \text{then } \delta_{t,a,3} = max(\xi_{t,a}^{startup} - \xi_{t,a}, 0) \\
& \text{else if } SB_{t-1} \rightarrow SB_t \\
& \qquad \text{then } \delta_{t,a,4} = max(\xi_{t,a} - \xi_{t,a}^{startup}, 0) \text{ and } \delta_{t,a,5} = max(\xi_{t,a}^{abandon} - \xi_{t,a}, 0) \\
& \text{else} \\
& \qquad \text{then } \delta_{t,a,6} = max(\xi_{t,a} - \xi_{t,a}^{abandon}, 0)
\end{aligned}
$$

As an example, consider what happens to plant a that is observed to be in standby (SB) in $(t-1)$ and operating (OP) in t. If the equilibrium price $\xi_{t,a}$ of plant a is above plant a's trigger level for startup $\xi_{t,a}^{startup}$ at time t, nothing happens and the penalty $\delta_{t,a,3}$ is zero by default. In other words, the plant operator decides to restart the plant and behaves in accordance with real option theory. However, if the equilibrium price $\xi_{t,a}$ of plant a is below plant a's trigger level for startup $\xi_{t,a}^{startup}$ at time t, a penalty $\delta_{t,a,3}$ is introduced. The size of the penalty is the deviation of $\xi_{t,a}$ from $\xi_{t,a}^{startup}$, i.e. the plant operator decides not to restart the plant and acts contrary to real options theory.

4.4.2 Additional constraints

In addition to the system of equations in section 4.3 there are additional constraints that need to be defined.

Since option values never can be negative, the following constraints are introduced for the option coefficients

$$B_2 \geq 0 \tag{31}$$
$$D_1 \geq 0 \tag{32}$$
$$D_2 \geq 0 \tag{33}$$

For the trigger values, the logical relationship below must hold

$$\xi^{startup} \geq \xi^{shutdown} \geq \xi^{abandon} \tag{34}$$

Uncertainty induces an incentive to wait. Thus there exist relations between the static NPV trigger values and the dynamic trigger values that emanate from uncertainty in the future cash flows. See for instance Dixit (1992) for perspectives on these hysteresis effects. Due to the value of waiting when there is uncertainty, the following relations are in force

$$\xi^{shutdown} \leq \xi^{NPV,stop} \tag{35}$$
$$\xi^{startup} \geq \xi^{NPV,start} \tag{36}$$

The expressions for $\xi^{NPV,stop}$ and $\xi^{NPV,start}$ can be found by setting uncertainty to zero, i.e. $Var_t(S(t)) = 0$, for the value of an operating plant, see equation 6. Remember that no uncertainty means that the option values will be zero, compare appendix E. Using this, the two NPV trigger values can be derived, and their value is determined by the following expressions

$$C\left(\frac{\chi_t}{r+\kappa} + \frac{\xi^{NPV,stop}}{r} + \frac{\mu_\xi}{r^2} - \frac{EC}{r}\right) - \frac{G}{r} = -\frac{M}{r} - E \tag{37}$$

and

$$-\frac{M}{r} = C\left(\frac{\chi_t}{r+\kappa} + \frac{\xi^{NPV,start}}{r} + \frac{\mu_\xi}{r^2} - \frac{EC}{r}\right) - \frac{G}{r} - R \tag{38}$$

The switching costs are constrained by

$$M \geq 0 \tag{39}$$
$$R \geq 0 \tag{40}$$
$$S \in \mathbb{R} \tag{41}$$

Since there is no NPV trigger value for the abandonment decision, we constrain the salvage value S from above. As explained in figure 4, our real options model do not consider changes from an operating to a retired status. An economic constraint that underpins the absence of this status change, is the upper boundary on the salvage value. Given that the firm is operating, the value of shutting down must always be greater or equal to the salvage value. If not, it would never be optimal to switch to standby. Hence, we have the following boundary on the salvage value when the plant is operating

$$S \leq D_1 e^{\beta_1 \xi} + D_2 e^{\beta_2 \xi} - \frac{M}{r} - E \tag{42}$$

4.4.3 Assumptions and approximations

Consistent with industry standards, we assume that both the switching cost of shutting down E and the emission costs EC are zero. Furthermore, we assume that the restart cost R, maintenance costs M and salvage value S depend on the linear relations below

$$R = r_1 + r_2 C + r_3 SB_{time} \tag{43}$$
$$M = m_1 + m_2 C \tag{44}$$
$$S = s_1 + s_2 C + s_3 Eff \tag{45}$$

where C is the capacity of the plant, SB_{time} is the number of time periods the plant has been in standby, and Eff is the efficiency of the plant. r_1, r_2, r_3, m_1, m_2, s_1, s_2 and s_3 are the structural parameters to be estimated.

The parameters affecting restart cost are size and the number of time periods the plant has been in standby. So the larger the size and the longer the time in standby, the higher is the cost of restarting the plant. This intuition relies on positive values of r_2 and r_3. We include the constant r_1 to account for the potential effect from other explanatory parameters.

When the plant is in standby, the maintenance costs are dependent on the size of the plant and a constant term. For positive values of m_2, larger plant size gives higher maintenance costs.

The salvage value depends on the size and efficiency of the plant. Larger plant size and higher efficiency result in higher salvage value as long as s_2 and s_3 are positive. The constant term s_1 is included to account for the costs of disabling the generator. The age of the plant is not included as an explanatory parameter as we believe the efficiency parameter accounts for the same effects, i.e. new generators generally have higher efficiency than old ones.

Since the normal cumulative distribution function is not predefined in AMPL, we use the following approximation from Johnsen, Kotz, and Balakrishnan (1994)

$$\Phi\left(\frac{E_t[S(s)]-EC}{\sqrt{Var_t(S(s))}}\right) \approx \frac{e^{2\times 0.7988\left(\frac{E_t[S(s)]-EC}{\sqrt{Var_t(S(s))}}\right)\left(1+0.04417\left(\frac{E_t[S(s)]-EC}{\sqrt{Var_t(S(s))}}\right)^2\right)}}{1+e^{2\times 0.7988\left(\frac{E_t[S(s)]-EC}{\sqrt{Var_t(S(s))}}\right)\left(1+0.04417\left(\frac{E_t[S(s)]-EC}{\sqrt{Var_t(S(s))}}\right)^2\right)}} \tag{46}$$

The appropriate numerical method for solving the integrals in equations 6 and 17 is the Gauss Lobatto rules. However, it is more convenient to solve these integrals by means of the composite Simpson's rule directly in the optimization problem. The difference in the numerical values obtained by these two approaches, is neglectable.

5 Results

5.1 Spark spread parameters

We estimate the spark spread parameters for nine groups with varying VOM and efficiency. Generator #6407 belongs to one of these, and the results as of April 2004, April 2005 and April 2006 are summarized in table 5 for this group. The estimated parameters for the remaining groups are found in appendix H.

Table 5: Spark spread parameters for generator #6407

Parameter Unit	r	κ	ρ	σ_χ \$/MWh	σ_ξ \$/MWh	μ_ξ \$/MWh	χ_0 \$/MWh	ξ_0 \$/MWh
2004	0.047	4.406	-0.650	30.027	11.129	0.473	-9.231	-16.360
2005	0.047	2.008	-0.820	33.068	18.688	0.190	-5.613	-20.920
2006	0.047	1.859	-0.867	40.006	22.287	-0.532	6.345	-29.600

We expect to see higher uncertainty in the short term than in the long term spark spread values. Generally, we observe that the volatility in short term deviation σ_χ is approximately twice as high as the volatility in equilibrium price σ_ξ. From 2004 to 2006 we see a significant increase in both short and long term uncertainty. This time period is characterized by high levels and high volatility in the natural gas prices. The high prices of natural gas explain the relatively low estimated levels of the spark spread $(\chi_0 + \xi_0)$ during the three years.

A mean reversion parameter κ of 1.86 imply that the short term variations are expected to halve in about 4 months[13]. Higher mean reversion rate means shorter half-time. From 2004 to 2006 we observe a decrease in the rate of mean-reversion, i.e. an extended impact from short term deviation χ_0 on the expected spark spread value, see equation 4.

The correlation ρ between the mean-reverting and arithmetic Brownian motion process is always negative and decreases during the time period from 2004 to 2006. Negative correlation means that the short term deviation tends to move in opposite direction of the equilibrium price. The long term drift μ_ξ decreases during the same time period, and in 2006 it turns negative. At this location and point in time, the market was pessimistic about the future development of equilibrium price.

5.2 Switching costs from structural estimation

The main objective with the structural estimation is to estimate the restart cost, maintenance costs and salvage value for each plant in the dataset. Estimates of the structural parameters are given in table 6 and for generator #6407 the resulting switching costs - R, M and S - are given in table 7. The relationships between the structural parameters and the switching costs are defined in section 4.4.3.

Table 6: Estimated structural parameters

r_1	r_2	r_3	m_1	m_2	s_1	s_2	s_3
0.000	0.000	0.000	0.110	10.337	-4.175	492.724	12.872

As we can see from table 7, the restart cost R hits the lower boundary of zero. This estimate implies that the generator does not experience any cost when going from the standby to the operating state. Based on industry experience, we expect the restart cost to vary up to an upper boundary of \$1.00m depending on the generator's size and the number of years since it switched into the standby state[14].

Table 7: Estimated switching costs R (m\$), M (m\$/yr) and S (m\$) for generator #6407

R	M	S
0.000	4.230	195.989

For generator #6407 the estimated maintenance costs are \$4.23m/year. The maintenance costs are the yearly expenses of keeping a generator in the standby state. The price

[13] The maturity in which short term deviations are expected to halve is given by $T_{0.5} = -ln(0.5)/\kappa$.

[14] Thanks to assistent professor Carl J. Ullrich at Virginia Tech for providing insights into the industry. Dr. Ullrich worked for seven years in the US electric power industry. His primary duties included capital budgeting analyses for power plants and economic analyses of various operational and maintenance strategies.

of a new generator is between \$150/kW and \$450/kW[15], so an upper boundary for a new generator with the characteristics of #6407 is then \$20.45m. Put in this perspective, the estimated maintenance costs are clearly too high. By looking at the two structural parameters, m_1 and m_2, we observe that the former is in line with our expectations whereas the latter is too high. The interpretation is that every plant incur a fixed cost of being in standby m_1 of \$0.11m plus a coefficient m_2 related to the size of the plant.

The salvage value of generator #6407 is estimated to be \$195.99m, which is significantly higher than what we anticipate. If a decision is made to abandon the generator, there are costs associated with the disabling of the generator, and these expenses can be interpreted as s_1. The sign of s_1 is right, but the value of -\$4.18m is lower than expected. If a generator is abandoned, it is possible to sell the generator equipment and components in the second hand market. The maximum second hand value of a generator is approximately \$150/kW[16] resulting in an upper boundary of \$6.83m for generator #6407. This maximum value is significantly lower than the estimated second hand value of \$200.16m. Consequently, the estimated salvage values are not realistic.

For every year, more information will be available on electricity and gas futures prices. Thus, the spark spread parameters are estimated each year and we obtain yearly plant specific values for both the trigger values $\xi^{startup}$, $\xi^{shutdown}$ and $\xi^{abandon}$, and the option coefficients D_1, D_2 and B_2. Note that yearly changes in the trigger values are not a result of the spark spread process itself. These changes are rather a result of changes in the economic environment, such as interest rate and reserve margin changes, affecting the process' parameters. The real options trigger values for generator #6407 are given in table 8.

Table 8: Trigger values (\$/MWh) for generator #6407

		Trigger values		
		$\xi^{startup}$	$\xi^{shutdown}$	$\xi^{abandon}$
	2004	-20.165	-20.165	-20.447
Year	2005	-14.108	-14.108	-41.713
	2006	0.960	0.960	-42.790

We observe that the estimated shutdown and startup triggers are equal throughout the sample period. This makes sense when the optimal solution requires the restart cost to equal zero as the value-matching conditions at $\xi^{shutdown}$ and $\xi^{startup}$ are identical. In 2004 and 2005 we see that the trigger values for shutdown and startup are negative. At first glance, a startup trigger value that is negative may seem unlikely. However, operating peak plants have the flexibility to switch between being producing and idle from one day to another, and the operator takes day-to-day variations in the spark spread into account when deciding whether to produce or not. If the spark spread is negative, the production is typically suspended temporarily in order to avoid losses. The contribution of this flexibility is dealt with in the plant's value function through the variance of the spark spread in equation 6. Therefore, negative trigger values for the startup and shutdown

[15]Depending on size, manufacturer and technology, the price of a new generator varies from \$189.54/kW to \$407.62/kW based on 68 generators from General Electric, ABB, Alstom and Rolls Royce.

[16]A 171MW GE 7FA Gas Turbine Generator Set is for sale at The Utility Warehouse for \$25m, equaling \$146/kW. This is a modern generator, hence the upper price boundary of \$150/kW in the second hand market. The Utility Warehouse is a marketplace established in 1995 that connects over 300 buyers and sellers of generating equipment.

decisions are reasonable.

The abandonment trigger is negative throughout the sample period. A plant operating under a positive spark spread will provide a positive cash flow as long as the revenue exceeds the fixed operating costs. If we assume the fixed operating costs to be small, abandonment will occur only when the spark spread is negative. Hence, we never observe positive trigger values for the abandonment decision. The value of the abandonment trigger has to be lower than the shutdown trigger, which is evident from table 8. Generator #6407 experiences several status changes during the time period in the sample, including a change from standby to operating from 2004 to 2005. The economic reason for entering the operating state in 2005 is due to the relatively high value of the operating plant at that time, so that it is more valuable for the operator to be operating than staying in standby. It also becomes less likely to abandon the plant in 2005 because the operating plant is more valuable than the previous year. The abandonment triggers mirror this reasoning, as the trigger values decrease from -$20.45/MWh in 2004 to -$41.71/MWh in 2005.

Increasing volatility, see table 5, increases the value of the operating peak plant. At the same time, real options theory states that higher uncertainty gives a greater incentive to wait. During the time period 2004-2006 we thus expect to see increasing startup triggers, decreasing shutdown triggers, and decreasing abandonment triggers. For the estimated startup and abandonment triggers this is true. But as the shutdown triggers must equal the startup triggers when the restart cost is zero, the shutdown triggers increase in the event of higher uncertainty. A potential explanation is that the effect from increased plant value is relatively higher than the incentive to wait following an increase in the uncertainty.

The option values are given in table 10[17], and these are calculated from the estimated option coefficients in table 9. The startup option represents the value of the flexibility to restart operation from the standby state. When the plant is in standby and the spark spread is in the interval between the startup and the abandonment trigger, there is always a chance that the spark spread can improve and make operation profitable again. We observe that generator #6407 has an increase in the value of the startup option every year, from $161.68m in 2004 to $275.98m in 2006. During the same period there is an increase in the volatility of the spark spread. As volatility increases, the value of an option on the cash flows from the operating plant also increases. Thus there is consistency between our model and the results but the options are generally too valuable to be realistic. For comparison, consider the largest generator in our data sample. With the most advanced technology, the list price of this generator is $71m. To be realistic, the estimated option values cannot exceed the price of buying a new generator.

Table 9: Option coefficients (m$) for generator #6407

		Option coefficients		
		$D1$	$D2$	$B2$
	2004	281.742	61.077	39.746
Year	2005	311.975	62.556	22.662
	2006	271.604	82.316	20.770

[17]The startup, abandonment and shutdown option values are given by $D_1 e^{\beta_1 \xi^{startup}}$, $D_2 e^{\beta_2 \xi^{abandon}}$ and $B_2 e^{\beta_2 \xi^{shutdown}}$, respectively.

The shutdown option values the flexibility of going from operating to standby when the spark spread hits a level that results in negative cash flows from operating the plant. The shutdown option decreases for generator #6407, from $124.15m in 2004 to $81.18m in 2006. As the volatility increases, the value of operation increases, and thus the option value of giving up these cash flows decreases.

Table 10: Option values (m$) for generator #6407

		Option values		
		Startup	Abandon	Shutdown
	2004	161.676	124.149	80.792
Year	2005	242.003	81.888	29.665
	2006	275.981	81.177	20.482

The abandonment option values the flexibility to stop paying the continuous maintenance costs and receive a potential salvage value. We observe the same pattern for the abandonment option as for the shutdown option, i.e. it decreases from $80.79m in 2004 to $20.48m in 2006. The option to abandon the plant gets lower as the volatility increases and the value of the operating plant increases.

6 Discussion

6.1 Limitations in the data

The data on electricity futures prices are troublesome since trade volumes have been low and many of the contracts are discontinued. The US market for exchange traded electricity futures and options virtually collapsed in the wake of the Enron scandal. By February 2002, the New York Mercantile Exchange decided to delist all of its futures contracts due to lack of trading[18]. From April 2003 however, time series of peak futures contracts for the PJM interconnection are available, compare figure 1(a). As the spark spread model assumes complete markets, the illiquidity in the futures market will inevitably bias our model's results.

For practical reasons, we merge generators into cost and efficiency specific groups. The spark spread parameters are then estimated in April for each group and every year in the dataset. By selecting this particular month and dividing the generators into specific groups, we lose some information on the spark spread dynamics. In general, estimation of spark spread parameters on a yearly time basis may be problematic. Ideally, the spark spread parameters are estimated for every generator in continuous time. The data on operating status however, comes with yearly time resolution. We do not know at which time the switching decision actually takes place during a given year. In that respect, our choice of equilibrium price as profitability indicator is justified, as it is the long term trends in the spark spread that determine if a generator retires during the year or not. Whether the retirement occurs in April or October is in this case not of the essence.

[18]See NYMEX Notice number 02-57, Notice of Delisting of NYMEX Electricity Contracts (February 14, 2002)

6.2 Limitations in the model

The work of Fleten et al. (2012) suggest that the single most important profitability indicator for the plants is the reserve margin. Our model is market-oriented, implying that the only way these plants make money is through the markets for which we have price information. If there are no other markets or mechanisms by which peak plants can profit, then reserve margin serves as an indicator for future spark spread. An implicit assumption is thus that changes in futures prices impound projected reserve margin changes. Negative changes in reserve margin may emanate from retirement of power plants. We assume that such changes are reflected in higher futures prices for electricity and, perhaps less likely, in lower futures prices for natural gas. Thus, the values of the remaining generators increase.

However, if there are other markets for reserve capacity, the reserve margin not only signals future spark spread, but also other sources of revenue for a peaking plant. PJM has an Operating Reserve Mechanism to ensure adequate operating reserves and uses a Reliability Pricing Model to provide incentives for participants to build an efficient mix of generating resources. Consequently, the use of long term spark spread as profitability indicator may not be the only or most optimal choice.

In addition, there are alternatives to the two-factor model we apply for the spark spread process, see for instance Benth and Kettler (2011). We assume that the sum of the two factors in the model, short term deviation and equilibrium price, is normally distributed. Based on this assumption we derive the expression for the value of an operating plant. We pursue an analytical solution to this expression but there is no way of avoiding the use of numeric, compare the quasi-analytical solution in appendix B. Therefore, we need to employ numerical methods within the value-matching and smooth-pasting conditions, and this may cause technical issues to the optimization solver[19]. An analytical solution to the value of an operating plant is thus desirable, and requires the use of another spark spread process.

According to Deng and Oren (2003), the impacts of operating characteristics on the power plant valuation, compare equation 6, are far more significant under mean-reversion models than they are under geometric Brownian motion price models. This significance is also found to be increasing with decreasing plant efficiency. Since we consider plants that are less efficient than those evaluated in Näsäkkälä and Fleten (2005) and include a mean-reversion model for the spark spread, the values obtained for the operating plants may be too optimistic. In appendix C we derive the expression for the operating plant assuming that it is costless to temporarily suspend production, and that the generators optimally and immediately adapt to the observed spark spread. In reality, operators face operating challenges that potentially prevent them from making such ideal decisions. One way to adjust for these costs is to assign a higher value to the variable operation and maintenance costs VOM.

In section 4.4.3 we assume linear relations for the switching costs. There may be other functional forms for R, M and S that describe the cause and effect relationships in a more coherent manner. For example, the dependence of size in the expression for the restart cost is not necessarily linear. Instead, there can be a diminishing effect of size on the restart cost, e.g. through the expression $R = r_1 + r_2(1 - e^{-C}) + r_3 SB_{time}$. However, such changes to the model do not alter the fact that the estimated switching costs are unrealistic. Error terms can be added to equations for the switching costs in

[19] At some points the Hessian (matrix of second derivatives) will not be defined and we need to relax the accuracy of the numerical method to achieve stable results.

section 4.4.3 that allow us to formulate a maximum likelihood estimator. But the number of time observations in the dataset is too low for the method of maximum likelihood to be efficient.

6.3 Implications for further research

There are two implications from the results. The first implication concerns the use of structural estimation as a technique in econometric analysis. We have shown that it is possible to estimate the parameters of structural models by formulating an objective function that minimizes the difference between the observed data and the normative structural model. The optimization problem is constrained by the structural model and then submitted to a state-of-the-art solver. Indeed, this is in the spirit of Su and Judd (2011):

> *(...) the habit of restricting models to cases with closed-form solutions is unnecessary. There is no reason for economists to impose this burden on themselves instead of writing down the models they really want to analyze and solving them.* (Su and Judd, 2011, p. 17).

The real options model we find in section 4.3 consists of six equations with ten unknown parameters, i.e. the trigger values $\xi^{shutdown}$, $\xi^{startup}$ and $\xi^{abandon}$, the option coefficients B_2, D_1 and D_2, and the switching costs M, E, R, and S. There is no closed-form solution to this problem. But by performing structural estimation we are able to estimate the structural parameters for M, R, and S (assuming $E = 0$). Unlike Gamba and Tesser (2009), our real options model is formulated explicitly. Hence we can extract more information about the value of the real options and, perhaps more importantly, the timing of operation decisions.

The second implication from our work concerns the extent to which operators actually delay irreversible switching decisions following an increase in the uncertainty of their environment. According to real options theory, the shutdown trigger should decrease, the startup trigger should increase and the abandonment trigger should decrease due to an increase in the long term volatility. The results show that these effects are present for the startup and abandonment decision. Such sensitivity analysis may be a step towards empirical verification of real options theory.

7 Conclusion

By applying structural estimation within the context of a real options model, we estimate the irreversible switching costs of status changes for gas fired power plants. The switching costs are dependent on the plant's capacity, time in standby and efficiency. The estimated restart cost is zero for all plants, implying that the plants do not incur any costs when going from standby to operating. The maintenance costs consist of a fixed cost plus a cost dependent on the plant's size. The estimated fixed cost is in line with our expectations with a value of \$0.11m/year. However, the cost dependent on the size of the plant is unrealistically high. There exists a second hand market for generator equipment, and we therefore assume the salvage value to be the sum of two parts. The first part is a cost related to the disabling of the generator. Secondly, there are potentially positive cash flows from sale of the equipment in the second hand market. The estimation shows that there is a cost related to the abandonment of a plant, although the absolute value of this estimate is too high to be realistic. The cash flows from a sale in the second hand market are positive but significantly higher than anticipated. In total, the estimated salvage value is not realistic.

Due to the estimated restart cost being zero, the trigger values for startup and shutdown are identical. Whereas the startup and abandonment triggers behave according to real options theory, the behavior of the shutdown trigger contradicts the theory when the uncertainty in the environment increases. This is due to the estimated restart cost of zero. The development of the estimated option values shows that real options effects are present. The scaling however, does not match our expectations based on industry experience.

The spark spread is modeled as the sum of two stochastic factors, where the first factor is the short term deviation following a mean-reverting process. The second factor, the equilibrium price, follows an arithmetic Brownian motion and is assumed to be the profitability indicator in our model. We estimate the spark spread parameters through a Kalman filter procedure and use linear regression to find the drift at given points in time. We find that it is not possible to obtain an analytical solution to our real options model with such a spark spread process. Even a quasi-analytical solution involves mathematics that requires the use of numeric to some extent. Numerical solutions of the real options model may result in technical problems to the structural estimation. Thus, a spark spread process that gives an analytical solution of the real options model is preferable in future analysis.

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A Kalman filter procedure (R code)

From Schwartz and Smith (2000) and Harvey (1990, chap. 3) we can obtain the transition equation for the state variables

$$x_i = c_i + G_i x_{i-1+\omega_i} \forall i = 1, ..., N \tag{A.1}$$

where

$$c_i = [0, \mu_\xi \Delta t_i] \tag{A.2}$$

$$G_i = \begin{bmatrix} e^{-\kappa \Delta t_i} & 0 \\ 0 & 1 \end{bmatrix} \tag{A.3}$$

The disturbance vectors ω_i, which are serially uncorrelated and normally distributed, have zero expected value and the following covariance matrix

$$Cov(\chi_{\Delta t_i}, \xi_{\Delta t_i}) = W = \begin{bmatrix} (1 - e^{-2\kappa \Delta t_i}) \frac{\sigma_\chi^2}{2\kappa} & (1 - e^{-\kappa \Delta t_i}) \frac{\rho \sigma_\xi \sigma_\chi}{2\kappa} \\ (1 - e^{-\kappa \Delta t_i}) \frac{\rho \sigma_\xi \sigma_\chi}{2\kappa} & \sigma_\xi^2 \Delta t_i \end{bmatrix} \tag{A.4}$$

The relationship between the state variables $x_i = [\chi_i, \xi_i]$ and price observations $y_i = [S_i^{T_1}, S_i^{T_2}]$ is given by the measurement equation

$$y_i = (d + F x_i) \tag{A.5}$$

where

$$d = (\mu_\xi T_1, \mu_\xi T_2) \, \forall i = 1, ..., N \tag{A.6}$$

$$F = \begin{bmatrix} e^{-\kappa T_1} & 1 \\ e^{-\kappa T_1} & 1 \end{bmatrix} \tag{A.7}$$

The likelihood function of a given parameter set - μ, κ, ρ, σ_χ, σ_ξ - with a prior mean and covariance matrix can by means of the transition equation A.1 and measurement equation A.5 be derived. See the attached CD for the complete R code of the Kalman filter procedure.

B Plant valuation using confluent hypergeometric functions

The boundary conditions imply that C_1, C_2, and C_4 must equal zero when $\chi+\xi < E$, and that C_1, C_2, and C_3 must equal zero when $\chi+\xi > E$. Whereas the limits for equilibrium price ξ are straight-forward to derive, the limits when short-term deviation χ approaches ∞ or -∞ deserve some explanation. First of all, by converting the Tricomi function into the parabolic cylinder function the limits can be treated in a more comprehendible manner. For the case of $\chi+\xi < E$, the value of the operating plant is given as

$$\begin{aligned} V_0(\chi,\xi) =& C_3\chi U\left(\frac{2\kappa + sepconst}{4\kappa}, \frac{3}{2}, \frac{\kappa\chi^2}{\sigma_\chi^2}\right) e^{\frac{\xi}{\sigma_\xi^2}\left(-\mu_\xi+\sqrt{\mu_\xi^2-\sigma_\xi^2 sepconst+2\sigma_\xi^2 r}\right)} \\ =& C_3\frac{\chi}{\sqrt{\frac{\kappa\chi^2}{\sigma_\chi^2}}} D\left(\frac{-sepconst}{2\kappa}, \sqrt{\frac{2\kappa\chi^2}{\sigma_\chi^2}}\right) \times \\ & e^{\frac{\kappa\chi^2}{2\sigma_\chi^2}} 2^{\frac{sepconst}{4\kappa}} e^{\frac{\xi}{\sigma_\xi^2}\left(-\mu_\xi+\sqrt{\mu_\xi^2-\sigma_\xi^2 sepconst+2\sigma_\xi^2 r}\right)} \end{aligned} \tag{B.1}$$

where $D(v;z)$ is the parabolic cylinder function. $D(v;z)$ tends towards ∞ for large negative values of z and towards zero for large positive z for all $v < 0$ (Dockendorf and Paxson, 2010). In figure 5 we plot the Tricomi function for relevant values of a and z and show that the same logic applies to this function.

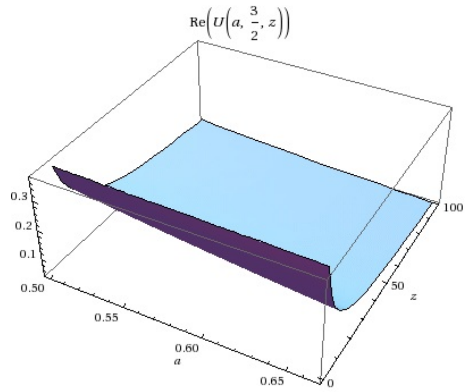

Figure 5: The Tricomi function for relevant values of a and z

Secondly, if equation B.1 is the solution for the case where $\chi+\xi < E$, it is implicitly assumed that $D\left(-sepconst/2\kappa, \sqrt{2\kappa\chi^2/\sigma_\chi^2}\right)$ converges to zero faster than $e^{\kappa\chi^2/2\sigma_\chi^2}$ approaches ∞, when χ goes to -∞. Numerical examples show that this indeed is the case.

By applying the same logic to $V_1(\chi,\xi)$, i.e. the value of the operating plant when $\chi+\xi > E$, the expression for the value of an operating plant is given by

$$V(\chi,\xi) = \begin{cases} V_0(\chi,\xi) = C_3\chi U\left(\frac{2\kappa+sepconst}{4\kappa}, \frac{3}{2}, \frac{\kappa\chi^2}{\sigma_\chi^2}\right) \times \\ e^{\frac{\xi}{\sigma_\xi^2}\left(-\mu_\xi+\sqrt{\mu_\xi^2-\sigma_\xi^2 sepconst+2\sigma_\xi^2 r}\right)} & \chi+\xi \le E \\ \\ V_1(\chi,\xi) = C_4\chi U\left(\frac{2\kappa+sepconst}{4\kappa}, \frac{3}{2}, \frac{\kappa\chi^2}{\sigma_\chi^2}\right) \times \\ e^{\frac{\xi}{\sigma_\xi^2}\left(-\mu_\xi-\sqrt{\mu_\xi^2-\sigma_\xi^2 sepconst+2\sigma_\xi^2 r}\right)} + C\left(\frac{\chi}{r+\kappa} + \frac{\xi}{r} + \frac{\mu_\xi}{r^2}\right) & \chi+\xi > E \end{cases} \tag{B.2}$$

In order to find the arbitrary constants C_3, C_4 and the separation constant *sepconst*, one value-matching and two smooth-pasting conditions are formulated and evaluated at $\chi+\xi=E$

$$V_0(\chi,\xi) = V_1(\chi,\xi) \tag{B.3}$$

$$\frac{dV_0(\chi,\xi)}{d\chi} = \frac{dV_1(\chi,\xi)}{d\chi} \tag{B.4}$$

$$\frac{dV_0(\chi,\xi)}{d\xi} = \frac{dV_1(\chi,\xi)}{d\xi} \tag{B.5}$$

After solving the three equations above, C_3, C_4 and *sepconst* can be inserted in equation B.2. We then have the expression for the value of an operating plant as a function of χ and ξ when ρ is zero.

C Derivation of plant value assuming normal distribution

A peak plant will only generate electricity when the spark spread exceeds emission costs. If the emissions costs are assumed to be zero, this means that the plant will be operating only when the spark spread is positive. The value of the plant at time t is the expected cash flows minus the fixed operational costs G

$$V\left(\chi(t),\xi(t),\kappa,\mu_\xi,\sigma_\chi,\sigma_\xi,\rho\right) = \int_t^T e^{-r(s-t)}\left(C \times c\left(\chi(s),\xi(s),\kappa,\mu_\xi,\sigma_\chi,\sigma_\xi,\rho\right) - G\right)\mathrm{d}s \tag{C.1}$$

where C is the capacity of the plant, $T-t$ is the remaining lifetime of the plant, and $c(s)$ is the expected value of spark spread exceeding emissions cost EC at time s

$$c\left(\chi(s),\xi(s),\kappa,\mu_\xi,\sigma_\chi,\sigma_\xi,\rho\right) = \mathbb{E}[max(S(s)-EC,0)|\mathbb{F}_s] = \int_{EC}^{\infty}(y-EC)h(y)\,\mathrm{d}y \tag{C.2}$$

In equation C.2 the term $h(y)$ is the density function of a normally distributed variable y, whose mean and variance are those of the spark spread at time s. See equations 4 and 5 for $E_t[S(s)]$ and $Var_t(S(s))$, respectively. Partial integration gives

$$
\begin{aligned}
\int_{EC}^{\infty}(y-EC)h(y)\,\mathrm{d}y &= \int_{EC}^{\infty}(y-EC)\frac{e^{-\frac{(y-E[S])^2}{2Var[S]}}}{\sqrt{2\pi Var[S]}}\,\mathrm{d}y \\
&= \left[y\frac{e^{-\frac{(y-E[S])^2}{2Var[S]}}}{\sqrt{2\pi Var[S]}}\left(\frac{-Var[S]}{(y-E[S])}\right)\right]_{EC}^{\infty} \\
&- \int_{EC}^{\infty}1\frac{e^{-\frac{(y-E[S])^2}{2Var[S]}}}{\sqrt{2\pi Var[S]}}\left(\frac{-Var[S]}{(y-E[S])}\right)\mathrm{d}y - \int_{EC}^{\infty}EC\frac{e^{-\frac{(y-E[S])^2}{2Var[S]}}}{\sqrt{2\pi Var[S]}}\,\mathrm{d}y \\
&= \frac{\sqrt{2Var[S]}e^{-\frac{(EC-E[S])^2}{2Var[S]}}}{2\sqrt{\pi}} + \frac{\sqrt{\pi}E[S]\times erf\left(\frac{\sqrt{2}(EC-E[S])}{\sqrt{Var[S]}}\right)}{2\sqrt{\pi}} + \frac{E[S]}{2} \\
&- \frac{\sqrt{\pi}EC\times erf\left(\frac{\sqrt{2}(EC-E[S])}{\sqrt{Var[S]}}\right)}{2\sqrt{\pi}} - \frac{EC}{2} \\
&= \frac{\sqrt{Var[S]}e^{-\frac{(EC-E[S])^2}{2Var[S]}}}{\sqrt{2\pi}} + (E[S]-EC)\left(\frac{1}{2} - \frac{erf\left(\frac{\sqrt{2}(EC-E[S])}{\sqrt{Var[S]}}\right)}{2}\right) \\
&= \frac{\sqrt{Var[S]}e^{-\frac{(EC-E[S])^2}{2Var[S]}}}{\sqrt{2\pi}} + (E[S]-EC)\left(1-\Phi\left(\frac{EC-E[S]}{\sqrt{Var[S]}}\right)\right) \\
&= \frac{\sqrt{Var[S]}e^{-\frac{(EC-E[S])^2}{2Var[S]}}}{\sqrt{2\pi}} + (E[S]-EC)\Phi\left(\frac{E[S]-EC}{\sqrt{Var[S]}}\right)
\end{aligned}
\tag{C.3}
$$

which is the last factor of the integrand in equation 6.

D Numerical integration (C++ code)

The composite Simpson's rule is applied to find a numerical value of the operating plant. This method is adequate when the integrand is relatively smooth. This is the case for our value function, and by adjusting the number of integration points n, we are able to obtain an accurate value for the peak plant. In our estimations we use a time period from $t = 0$ to $t = 200$ with $n = 500$ integration points. As the Simpson's rule estimation requires a lot of calculation, there are other alternative methods available that can reduce the processing time of the algorithm. With our sample it is sufficient to apply the Simpson's method.

For a larger dataset however, it may be appropriate to use a more effective numerical approach for the plant value. One alternative is to use the Gauss Lobatto quadrature, which is similar to Gaussian quadrature except that it includes the endpoints of the integration interval. Furthermore, it is accurate for polynomials up to degree $2n$Ű3 where n is the number of integration points. See Abramowitz and Stegun (1972) or Weisstein (2012) for a detailed description. The C++ code for the Gauss Lobatto quadrature are found on the attached CD, and could be exported to AMPL if the processing time with the Simpson's rule takes too long.

E Valuation of switching options

When the power plant is *in standby*, compare figure 3, the operator has two options - either to restart or abandon operations. In order to value these options, denoted F_0, we make use of the dynamic programming approach, see Dixit and Pindyck (1994, chap. 4). Since we assume that the pricing model given by equations 1- 3 describes the spark spread dynamics adjusted for risk, the options must satisfy the following Bellman equation

$$rF_0(\xi)dt = E[dF_0(\xi)] - Mdt \tag{E.1}$$

Applying Itô's Lemma and taking the expectation result in the following differential equation for the option value

$$\frac{1}{2}\sigma_\xi^2 \frac{d^2F_0(\xi)}{d\xi^2} + \mu_\xi \frac{dF_0(\xi)}{d\xi} - rF_0 - M = 0 \tag{E.2}$$

The solution to the differential equation in E.2 is the linear combination of two independent solutions plus any particluar solution. That is

$$F_0(\xi) = D_1 e^{\beta_1 \xi} + D_2 e^{\beta_2 \xi} - \frac{M}{r} \tag{E.3}$$

where D_1 and D_2 are positive and unknown parameters to be determined. β_1 and β_2 are the positive and negative roots of the fundamental quadratic equation that is found by inserting $F_0(\xi) = De^{\beta\xi} - M/r$ into E.2, and hence given as

$$\beta = \frac{-\mu_\xi \pm \sqrt{\mu_\xi^2 + 2\sigma_\xi^2 r}}{\sigma_\xi^2} \tag{E.4}$$

When the plant is *operating* the same reasoning applies, except that B_1 must equal zero to satisfy its boundary condition, i.e. the option to shutdown move towards zero as the spark spread approaches ∞. In mathematical terms this option value is

$$F_1(\xi) = B_2 e^{\beta_2 \xi} \tag{E.5}$$

where β_2 is the negative root of the fundamental quadratic in equation E.4.

F Implicit real options approach

In the natural resource investment model of Brennan and Schwartz (1985), the mine produces a single commodity whose spot price follows a stationary stochastic process. The production entails a variable operation and maintenance cost, and the revenues are subject to taxation. While the mine is idle, a maintenance cost is paid. Operations can be suspended and restarted, and both switching decisions incur a fixed and non-recoverable cost. The decision to abandon the mine is irreversible. When the mine is operating, the natural resource of the mine is depleted, and a completely exhausted mine is thus abandoned.

In our real options model, see section 4.3, we neglect taxation and assume an infinite resource. With a few adjustments to the Brennan and Schwartz (1985) model, we can therefore employ the framework of Gamba and Tesser (2009). The formal valuation model and the econometric specification are given below.

The valuation model The overall state is defined by x=(ξ,m) where ξ is the equilibrium price and m is the operating mode, i.e. {*operating*, *standby*, *retired*}. The maintenance cost of being idle is given as M, and the fixed and variable operation and maintenance costs are denoted by G and VOM, respectively. The decision set is DS={*operating*, *standby*, *retired*}. The firm's decision, denoted d, is on the operating mode, $d \in DS(x)$.

The cash flow function is given as

$$CF(x) = \begin{cases} C\xi\Delta t - G\Delta t & \text{if } m = operating \\ -M\Delta t & \text{if } m = standby \\ 0 & \text{if } m = retired \end{cases} \tag{F.1}$$

Note that C is the capacity of the plant, and that equilibrium price ξ is estimated using VOM.

The payoff from decision d in state x=(ξ,m) is

$$g(x, d) = CF(\xi, d) - SC(\xi, d) \tag{F.2}$$

where $SC(\xi, d)$ denote the switching costs, i.e. $SC(\xi, operating)$=R is the startup cost, $SC(\xi, standby)$=E is the shutdown costs and $SC(\xi, retired)$=S is the abandonment cost.

Given a decision d at x, the state evolution function is

$$x' = f(x, d) = (\xi', m') \tag{F.3}$$

where m'=d and ξ' follows the exogenous dynamic given in equation 3.

The firm's problem is to find the schedule of contingent decisions for current and future dates that maximizes the expected discounted rewards. As before, the model has infinite horizon and a stationary discount factor $\beta \in$ (0,1) is given. The stationary transition probability is

$$p(dx'|x, d) = \begin{cases} p(d\xi'|\xi) & \text{if } x' = f(x, d) \\ 0 & \text{otherwise} \end{cases} \tag{F.4}$$

The value of the plant, Val, is the fixed point of the Bellman operator

$$Val(x) = \max_{d \in DS(x)} (g(x, d) + \beta \mathbb{E}_{x,d}[Val(f(x, d))]) \tag{F.5}$$

and the related optimal decision schedule is

$$arg \max_{d \in DS(x)} (g(x, d) + \beta \mathbb{E}_{x,d}[Val(f(x, d))]) \tag{F.6}$$

where $\mathbb{E}_{x,d}$ is the expectation with respect to $p(\cdot|x, d)$.

The econometric specification There is a sequence of observations (x_t^a,d_t^a) for t=1,...,T_a and $a \in A$, that is, for all time periods and plants. In discrete time the equilibrium price is

$$\xi' = \xi + \mu_\xi \Delta t + \sigma_\xi \sqrt{\Delta t} \eta \quad \text{(F.7)}$$

where $\eta \sim N(0,1)$. Gamba and Tesser (2009) assume that there is an unobservable stochastic process $\{\varepsilon_t\}$ so that through additive separability

$$g(x,d) + \varepsilon(d) \quad \text{(F.8)}$$

is the actual payoff at x from d. Assumptions concerning conditional independence are borrowed from Rust (1987) and will not be restated here. The Bellman operator is now

$$Val(x) = \max_{d \in DS(x)} (g(x,d) + \varepsilon(d) + \beta v(x,d)) \quad \text{(F.9)}$$

which has transition probability

$$Pr(dx', d'|x,d) = P(d'|x)p(dx'|x,d) \quad \text{(F.10)}$$

where $P(d'|x|)$ is the choice probability. Since the parameters of the exogenous stochastic process are known, the state transition probability $p(dx'|x,d)$ can be dropped from equation F.10. The error term ε is assumed to have a generalized extreme value distribution, with parameters (a,b), i.i.d. over all possible choices. Hence, the fixed point problem is

$$v(x,d) = \mathbb{E}_{x,d}\left[b \log \sum_{d' \in DS(x')} e^{\frac{1}{b}(g(x',d') + \beta v(x',d'))} \right] \quad \text{(F.11)}$$

and the choice probability of d conditional on x is

$$P(d|x) = \frac{e^{\frac{1}{b}(g(x,d) + \beta v(x,d))}}{\sum_{d' \in DS(x')} e^{\frac{1}{b}(g(x,d') + \beta v(x,d'))}} \quad \text{(F.12)}$$

where v is the unique fixed point of the Bellman operator.

The decision model is known up to the value of some structural parameters, θ. Given the sample of observations, the maximum likelihood estimator reduces to

$$\hat{\theta} = arg \max L(\theta) = arg \max \sum_{a \in A} \sum_{t=1}^{T_a} \log P(d_t^a | x_t^a, \theta) \quad \text{(F.13)}$$

Gamba and Tesser (2009) further incorporate unobserved heterogeneity into this framework. For numerical solutions and algorithms suitable for this purpose, we refer the interested reader to their work.

G Nonlinear programming (AMPL code)

Our structural model, compare section 4.4, is solved as a nonlinear optimization problem. The AMPL code can be found on the attached CD.

H Estimated spark spread parameters

Table 11: Spark spread parameters

Parameter Unit	r	κ	ρ	σ_χ \$/MWh	σ_ξ \$/MWh	μ_ξ \$/MWh	χ_0 \$/MWh	ξ_0 \$/MWh
Efficiency=16.0% and VOM=6.9 \$/MWh								
2004	0.047	11.423	-0.446	62.894	12.878	0.759	-14.057	-71.039
2005	0.047	6.467	-0.440	44.367	14.788	-0.010	-7.752	-90.891
2006	0.047	4.182	-0.595	50.226	17.864	-3.010	32.118	-125.782
Efficiency=18.0% and VOM=7.3 \$/MWh								
2004	0.047	10.110	-0.463	53.681	12.273	0.689	-12.877	-58.753
2005	0.047	5.165	-0.609	38.820	14.703	0.039	-7.229	-74.863
2006	0.047	3.753	-0.642	46.491	17.843	-2.404	25.812	-103.347
Efficiency=20.6% and VOM=8.2 \$/MWh								
2004	0.047	8.181	-0.502	43.782	11.739	0.619	-11.700	-46.770
2005	0.047	3.979	-0.597	34.826	14.976	0.088	-6.705	-59.130
2006	0.047	3.246	-0.700	43.050	18.032	-1.796	19.504	-81.200
Efficiency=21.3% and VOM=7.6 \$/MWh								
2004	0.047	7.633	-0.516	41.477	11.634	0.603	-11.420	-43.210
2005	0.047	3.743	-0.618	34.166	15.107	0.101	-6.585	-54.710
2006	0.047	3.106	-0.717	42.351	18.182	-1.657	18.052	-75.300
Efficiency=23.7% and VOM=7.5 \$/MWh								
2004	0.047	6.184	-0.563	35.850	11.417	0.552	-10.570	-33.870
2005	0.047	3.023	-0.693	32.713	15.834	0.134	-6.208	-42.670
2006	0.047	2.677	-0.769	40.501	18.881	-1.221	13.514	-58.650
Efficiency=26.0% and VOM=5.0 \$/MWh								
2004	0.047	5.292	-0.601	32.752	11.290	0.517	-9.973	-24.787
2005	0.047	2.549	-0.749	32.389	16.766	0.159	-5.941	-31.687
2006	0.047	2.330	-0.811	39.756	19.910	-0.912	10.303	-44.399
Efficiency=32.0% and VOM=4.9 \$/MWh								
2004	0.047	3.921	-0.682	28.722	11.363	0.444	-8.747	-11.390
2005	0.047	1.273	-0.913	39.286	25.563	0.210	-5.398	-14.400
2006	0.047	1.397	-0.918	43.482	26.955	-0.283	3.758	-20.460
Efficiency=34.9% and VOM=4.5 \$/MWh								
2004	0.047	3.551	-0.709	27.848	11.475	0.418	-8.314	-6.283
2005	0.047	1.203	-0.922	39.662	26.390	0.227	-19.770	-5.822
2006	0.047	1.247	-0.932	44.442	28.914	-0.060	1.444	-11.630