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A Strategic Tool for Competence Building within the Health Sector

With the Use of Nurse Rerostering

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Abstract

Despite high expenditures within the hospital sector, policy makers are facing a situation in which there is an emerging global nursing shortage along with misdistribution and poor utilization of nurses [1].Today hospitals are rigid regarding utilizing the competence across the wards, and the costs of absence are high because external nurses are hired in a one-to-one ration of absent nurses. Due to this one of the hospital management's most important assignment is to map their competence profiles and make a strategic plan of how to utilize the competence to the fullest, and how to further evolve the competence profile.

This thesis suggest a strategic approach to optimize the competence profile. The mathematical model presented for this competence building problem is a two stage stochastic model. The first stage is about building competence, by allowing an increase in competence by experience or bought competence and special expertise. The second stage is to evaluate the new competence profile by examining how the profile behaves in different absence scenarios. This evaluation were done by using a model of nurse rerostering with internall rotation and preference consideration.

Implementing the model in XpressMP and building a case based on a simplified version of a hospital gave room for computational studies. Both single test runs and extended analysis proved that by increasing the competence profile several rotations of nurses between the different wards were done. This amount of rotation further lead to lower cost of external hires and thus decreased the cost of absence. The hospital chose to invest in gained competence, instead of bought competence, which was a result of lower cost and that the nurses competence profile in this specific case allowed enough nurses to get competence by experience. Even though by more rotations the nurses gained mostly competence and special expertise within their belonging home wards. Together the main result of this thesis is the positive change in cost, by the increase of the nurses competence. Compared to the solution of working with the initial competence profile, the optimal profile found by the model proved to be more beneficial even though the building phase cause an extra yearly fixed cost by increased salary.

Preface

This study is conducted as part of MSc-studies at Norwegian University of Science and Technology, Faculty of Social Science and Technology Management, Department of Industrial Economics and Technology Management.

The study is a master thesis that builds on a previous study on "Nurse Rerostering - Optimization of nurse rerostering, by use of internal rotation and external substitutes"[2], conducted earlier in the MSc-studies. Some of the introductory chapters are congruent with the previously report, as this is considered new two the reader.

I would like to express thanks to my supervisors Henrik Andersson and Lars Magnus Hvattum, for valuable guidance and useful discussions during the study. I would also like to thank Sintef IKT, who gave me the possibility to work on this study and given me background information an support along the way. Especially thanks to Patrick Schittekat, Martin Stølevik and Tomas Nordlander.

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1 Introduction

Norway is a welfare state, and has put a lot of effort into building a solid and versatile health sector. Norway has the second highest health expenditure per capita, after the USA, according to OECD Health at a Glance 2009 [3]. Calculated from GNP (Gross National Product) and divided into categories, Norway stands out in two areas. It has the highest ratio of expenditures to long-term treatment and the lowest ratio of expenditures to medications.

Despite the high expenditure within the hospital sector, policy makers are facing a situation in which there is an emerging global nursing shortage along with misdistribution and poor utilization of nurses [1]. Major contributors to the nursing shortage include: the demographic change, the decline in enrollment at nursing schools, the changing work climate at hospitals and nursing homes, and the low status and salary associated with nursing. The current situation is characterized by an increasing demand and a decreasing supply [4, 5].

Scientists have been approaching this challenge of shortage for several years. Most of the focus have been on increase the efficiency of the nurse rostering assignment, both by time and recourses. This nurse rostering assignment is based on finding a feasible schedule, a work schedule for a given time horizon, that matches the nurses competences and their key position en each ward. The need to take into account individual preferences further increase the challenge of the process.

When focusing on nurse rostering the hospital works on an operational level, making the best of out of what they have on the ground level. Looking at the hospital from a different perspective might shed some light over other weaknesses and opportunities that can have a an impact on the utilization of resources.

> *"Strategic thinking focuses on finding and developing unique opportunities to create value by enabling a provocative and creative dialogue among people who can affect a company's direction."*[6]

The quotation above describes the basic idea behind strategic thinking. Putting the quote into a hospital context, the people who affect a company's direction are the nurses, the doctors, the patients and the management. How the parts interact has a high impact on how the hospital runs. If the employees are dissatisfied with the managements decisions, patients are complaining about the care taking, or the doctors are discontented about the nurses effort etc., the results will be a negative working environment and low work ethics. This will again lead to a disfavorable reputation and most importantly an *increase of employee absence*. Thus, as the quotation invite to, by enabling a provocative and creative dialogue between the management and the employees, the hospital can become more dynamic towards employees preferences, dissatisfaction and unforeseen situations.

Emergencies, sick-leave and other fluctuations in personnel requirements influence the ordinary schedule made by nurse rostering. As a result, it is necessary to make short-term adjustments to meet the demand. While the hospital management complain about expenses as a result to absence, the nurses are feeling the shortage in their own way. They get tired from working overtime as a result of high absent ratios, they complain about hospital directions, they long for more challenges and development within their work field and last but not least their lack of time to care for the patients. Thinking strategically, the two parties should join together and work for a creative outcome that benefits both parties.

This report take the nurse rostering assignment a step further, and suggest a solution to a consensus between the hospital management and the nurses in a strategical point of view. Today each department hires external substitutes, from a recruitment-agency, if there are nurses absent. This is very costly for the hospital and it might end up with nurses who are not familiar with the hospital and its routines, and thus might be less effective. A previous report by the author[2] (presented in further detail in the report) proves that by having the nurses rotate between the hospital wards, and not only work in the ward they are employed to, the daily cost of being flexible to fluctuation in absence can be reduced. By rotating the nurses, here referred to nurse rerostering, they expand their competence and are more able to step in for absent nurses. The report also proved that by having a higher competence level all over, the cost of absence could be reduced even more. The question then is: What is the optimal competence profile for the hospital?

To approach this problem, this report develops a strategic tool for competence building, by using the nurse rerostering assignment to evaluate the competence profiles. By investing in the nurses competence, and thus increasing the human capital, the hospital combines the assignment of listening to the nurses requests and handling the utilization of recourses as to reduce daily costs.

The main goal of this report is to formulate a mathematical model for competence building within a hospital, and thus find an optimal competence profile. A model like this becomes a two stage problem, where the first stage is to build a competence profile, and the second stage is to evaluate the given competence profile. A second goal is to make use of this model formulation to build a case study of how the competence building and costs are dependent on the uncertainty of future absence scenarios.

The report are divided into five parts. First there is a state of the art of previous work within the nurse rostering and nurse rerostering assignments. This is followed by a description of the hospital structure and out of that a problem definition. To understand the model of competence building a description of the rerostering model is necessary before the final mathematical formulation of the strategic competence building is done. To analyze the model a base case is presented, and the result of several computational studies of the case are executed and described. Finally a conclusion and some points on further work is presented.

2 Related Work

Despite the high expenditures within the hospital sector, policy makers are facing a situation in which there is an emerging global nursing shortage along with misdistribution and poor utilization of nurses. To approach this challenge, scientists have done research and studies on nurse rostering for several years. This chapter presents a state of the art of the development within nurse rostering.

2.1 Nurse Rostering

Every hospital needs to repeatedly produce shift schedules, called rosters, for its nursing staff. Rosters can be generated manually by the head nurse in each ward. However, scheduling nurses has always been difficult. One of the main reasons is that hospitals need to be staffed 24 hours a day over seven days a week, where the demand of nurses fluctuate over time. Usually, the head nurses spend a substantial amount of time developing rosters especially when there are many staff requests, and then even more time is consumed in handling ad hoc changes to current rosters. Because of tedious and time-consuming manual scheduling, and for various other reasons, the nurse rostering problem (NRP) or the nurse scheduling problem (NSP) has attracted much research attention [7].

Burke et al [7] presents a state of the art overview of nurse rostering up until 2004. In this article they presents different approaches to nurses. One important aspect they mention is that mathematical programming methods are appropriate for finding optimal solutions. However, their major limitation is that they are simply not appropriate (at least, on their own) for the enormous and complex search spaces that are represented by modern nurse rostering problems. Therefor, most researchers restrict the problem dimensions and consider a small set of constraints in their models, or they work with algorithms and heuristics. The article present and discuss the key approaches to the nurse rostering problem that have appeared in the scientific literature. The papers are grouped according to the type of method that is described. The five groups of approaches are (1) optimizing approaches - mathematical programming, (2) goal programming/multicriteria approaches, (3) artificial intelligence methods, (4) heuristics and (5) metaheuristic scheduling. More details of the different authors and their work can be studied in Burke et al [7].

Researchers has continued to produce articles about nurse rostering, and since the publication of Burkes [7] state of the art overview there has been over 40 publications. Some of these will be described in the next paragraphs.

2.2 Preference Scheduling

Preference scheduling is to schedule the roster after nurses requests, not only on the demands from the hospital directions or the nurses contract of employment. Some of these restrictions may be meeting each wards demand, restrictions prohibiting a nurse to work more than 4 night shifts in a row, more than 8 hours between shifts or for full time nurses to only work one weekend each third week. Some nurses might prefer only night shifts, or only day shifts, while other nurses prefer a combination or weekend-shifts. There are a lot of different preferences, and to handle them all is often an impossible task. As mentioned in the previous section it is also time-consuming and the preference scheduling may be more costly, though when taking all of these preferences into account the nurse rostering problem becomes more realistic.

Bard and Purnomo have done several studies on preference scheduling [8, 9, 10]. In [10] Bard and Purnomo present a column generation-based approach to solve the preference scheduling problem for nurses with downgrading. Here two approaches are investigated for substituting nurses with higher level skills and for those with lower level skills when there is sufficient idle time to do so. Idle time is usually due to scheduling constraints and contractual agreements that prevent a hospital from randomly assigning nurses to shift over the week. When the substitution is skill related, as here, it is often called downgrading. Since the assignment is a preference scheduling problem, the individual preferences are taken into account when constructing monthly rosters. Their conclusion is that the use of downgrading can lead to considerable reductions in the need for expensive outside nurses and much better schedules for the regular staff, as measured by preference satisfaction. The implications are higher morale and higher quality of service. Although the results show that both downgrading methods provide good solutions when two categories of nurses are scheduled at a time, it is pointed out that further investigation is needed to evaluate their relative effectiveness when three or more categories are involved.

Jaumard et al [11] describes one of the firsts exact method for preference scheduling, by use of a 0-1 column generation model with a resource constrained shortest path auxiliary problem for nurse scheduling. They consider a model consisting of a master problem which involves objectives and constraints, concerning the whole configuration of individual schedules and a sub problem whose formulation includes requirements specific to a single nurse. Each column in the constraint matrix of the master problem corresponds to a feasible schedule for a nurse. The sub problem is formulated, for a given nurse, as a resource constrained shortest path problem where the paths correspond to columns in the matrix formulation of the master problem. The assignment priorities are such that the regular workload of full-time and part-time permanent nurses is considered for assignment first, then overtime for part-time nurses, within the limit of a full-time schedule and finally floating nurses from the unit based availability list. Jaumar et al [11] argue that a sound advantage of this model with respect to a heuristic approach is its flexibility vis-a-vis changes in the scheduling environment.

2.3 Nurse Rerostering

Nurse rerostering is the operational side of nurse rostering, and is rescheduling based on a given roster. Nurse rerostering can be described as a short-term nurse scheduling in respond to daily fluctuations in supply and demand, in other words adjustments to the nurse roster on the basis of emergencies, sick-leave and other fluctuations in personnel requirements.

Warner [12] was one of the first to mention the need for rescheduling nurses to match short-term supply and demand. He defined the allocation decision that has to be made each shift to fine-tune the schedule, but did not develope a model or an algorithm. Siferd and Benton [13] however considered a day-to-day rescheduling based on information obtained from stochastic patient acuity models, where simulation was their primary tool.

Bard and Purnomo is, however, one of the first to present an article concerning only short-term nurse scheduling in response to daily fluctuations in supply and demand [9]. Prior to the start of each shift, the number of nurses who are scheduled to be on duty over the next 24 hours is compared with the number actually available. If shortage exist, decisions involving use of overtime, outside nurses and floaters has to be made to ensure that each unit has sufficient coverage. To address this problem Bard and Purnomo developed an integer programming model that takes the current set of rosters for regular and pool nurses and the expected demand for the upcoming time period as input, and the result is a revised schedule that makes the most efficient use of the available nurses. The model was tested by solving a range of problems for a 14 unit hospital, where all codes were written in C++ language and the IPs were solved with CPLEX 7.5 callable libraries. One of the weaknesses they found with this model was that it did not allow shifts to be split among units, say, in 4-hour blocks.

To remedy the weaknesses of their last model, Bard and Purnomo developed and introduce a new methodology for reactively scheduling nurses in light of shift-by-shift imbalances in supply and demand [8]. The problem associated with making the daily adjustments was formulated as an integer program and solved within a rolling horizon framework. The idea now was to consider 24 hours at a time, but to only implement the results for the first 8 hours. Initially attempts to solve 50-nurse problems with a commercial code proved to be unsuccessful and led to the development of a B&P algorithm. A set-covering-type integer program was used to find upper bounds and mixed-integer rounding cuts were used to tighten the relaxed feasible region. As part of the research, two heuristics were developed to find feasible solutions. The first was a tabu search and although extensive testing was done with various neighborhood definitions, list sizes, and diversification strategies, they were never able to achieve more than a 1 or 2% improvement. The second was a set-covering-type approach, which involved solving an integer program whose columns correspond to "good" schedules. This heuristic proved to be much more effective than tabu search and was incorporated in the B&P algorithm. Most problem instances with up to 200 nurses were solved within 10 minutes.

Lilleby [2] presents a nurse rerostering problem (NRSP) with internal rotation of nurses, which is basically the same as Bard and Purnomos use of the term floaters. The response to absence in Lilleby's NRSP model is based on rotating nurses between the different wards to cover some of the missing demand of competence. Today, absence is covered by hiring nurses one-to-one from recruitment agencies, but this is fairly expensive for the hospital. So by rotating the nurses, the gap between the actual present competence and the competence demand is decreased, and thus the competence is better utilized and the external hires needed is decreased. The model also considers nurse preferences, by making it favorable to rotate nurses which has a request of working at another ward.

The nurses competence is also one of the key aspects within the NRSP with internal rotation of the nurses. Seen from a different point of view, rotation of the nurses also leads to extra experience from other fields of expertise. When nurses increasing the knowledge of other expertise, the result is competence building. Competence building again leads to an increased human capital, which further makes it easier to rotate and adapt to absence in the next time period. Section 5 describes more in detail the aspect and results of Lillebys report, because of this thesis further use of the NRSP model.

3 Hospital Structure

This section will give an introduction to the authors understanding of a hospital structure. This is the basis of understanding the nurse rostering assignment.

3.1 Introduction to Hospital Structure

The hospital consists of several wards. Each ward is either a general department, or it handles a special field, like orthopedy, heart, cancer etc. Both kinds have overnight stays for the patients. In most hospitals the nurses are employed at a specific ward, and this is where they principally do their work.

The 24-hour period in a day is divided into several shifts. Each ward has a set of shifts, and normally the shifts can vary at what time they start and end, and in duration. Some of the shifts usually overlap, because of the need for communication across the shifts. Typically the shifts are divided into three fixed time periods; *day shifts, evening shifts and night shifts.* The different shifts are covered by different nurses, and are summarized in a schedule, called a *roster*, which is made by the head nurse of the ward. Most hospitals have a time horizon of more than a month for their rosters, but this can vary from hospital to hospital. A lot of work is put into finding a feasible schedule that matches the nurses competences and their key position, and the need to take into account individual preferences further complicates the process.

Table 1 show an example of a roster for a specific ward. The letters indicates which shift the nurse has, day- (D), evening- (E) or nightshift (N).

3.1.1 Nurses Competence and Demand

The nurses have different competences. The expertise consists of two parts: (1) The first part of competence is a *hierarchic level* of competence for a specific ward. These levels describe experience, and an overview of what

Table 1: *Illustration of a roster at a ward*

WARD A							
Date	01.feb	02.feb	03.feb	04.feb	05.feb	06.feb	07.feb
Day	Mon	Tue	Wed	Thurs	Fri	Sat	Sun
Nurse 1	D	D	D			E	E
Nurse 2	D		D	D	D		
Nurse 3	E	E		E	E		
Nurse 4		E	E			D	D
...							
Nurse X	N	N	N			N	N

is expected to learn over time in the ward. The first time a nurse starts working at a ward, without earlier experience within the field, she or he (from now on: she) has a competence level of 0 at this ward. This is a general level, corresponding to the undergraduate program in nursing, that all of the nurses have at all wards. Later when she gains more experience at the ward she is employed, she goes up to level 1 and so on. The exception is the highest competence level, to gain this usually some sort of special education is needed. At the same time the same nurse can have the highest competence level in another ward, because the level is specified to each ward. How many levels the competence are split into depends on the hospital's directions. By moving nurses between the wards, it is possible for the nurses to get more experience and to be able to increase their competence in other areas.

(2) The second part of competence is *special expertise*, which consists of expertise in different equipment, extension of education, special experiences and so on. Some of the special expertise applies to all wards, but others may belong to a specific ward. Examples of this is how to handle oxygen supply and how to clean a gastronomical wound. Since there are so many different areas within the special expertise the total list becomes very long. It is specified for each nurse what kind of special expertise she has, and at which hospital or institution she has it from. Here it is no kind of grading, she either has the competence or not.

Table 2 illustrates an example of a nurse competence matrix, at a hospital that has several wards (A,B,...,G) with 3 levels of hierarchic competence (A1, A2, and A3, B1, B2...) and 50 special expertise (S1, S2,...S50). The second column indicates which ward the nurse is stationed in (A, B...). The competences each nurse holds are marked with an X, and the competences they wish to develop in is marked with a G, for goal. There are also need for an extra competence matrix from earlier work experiences at other hospitals or institutions.

Table 2: *Illustration of nurses and the competence matrix for input in scheduling assignment*

Nurse	**Ward**	**Competence (level 1-3)**							**Special expertise**				
		A0	A1	A2	B0	B1	B2	...	S1	S2	S3	...	S50
Nurse 1	A	X	G		X						X		
Nurse 2	A	X			X	X	G		X		G		
Nurse 3	B	X	X		X	G				G			G
....													
Nurse 300	G	X	X	X	X				X	X	X		

Nurse competence is a key aspect of nurse rostering. When making a nurse roster, the competence has to be considered. Each ward in a hospital needs different competences, and a variation of competence levels. When executing the nurse rostering assignment, there is a demand of nurses with different competence levels at each shift. For instance a ward needs some nurses who can handle coordination of the ward and difficult patients. Maybe the patients have a rare disease, or are in a critical state after an operation, or the patients needs a close attention of a doctor. These situations are in need of nurses with a high competence, who know how to react to changes and can give adequate answers to the patient. Other patients in the ward may only be in need of routine check-ups and a helping hand, which nurses with lower competence can handle. Handling different competences makes the nurse rostering even more complex and difficult.

The demand at each shift, ward and competence is expressed as a number of nurses. The peak of demand is usually around noon, after new patients have arrived. For each hierarchic level of competence, there is defined one or several nurses to cover the demand of different assignments at each time period. A nurse with a high competence level is able to work shifts that demand her own level, and all the levels below. Together, the nurses who work at the same ward and time period has to cover the set of special competences defined for this period. The demand for a special competence is mostly assignment related and not long lasting, so a nurse with both special expertise A and B can thus cover the need for both in the same shift.

Table 3 illustrated a demand matrix, for the same hospital example as in the competence matrix described above. This matrix indicates the demand for the specific competences at each shift, in each ward, for the upcoming planning horizon. The demand is given by number of nurses. Table 3 illustrates a demand matrix for the same hospital as in Table 2, with 3 competence levels (L1, L2 and L3), and 50 special expertises (S1, S2, ..., S50).

Table 3: *Illustration of the demand matrix for input in rostering assignment*

Skift	**Demand competence level**			**Demand: Special expertise**					
	L0	L1	L2	S1	S2	S3	...	...	S50
Ward A									
Shift 1	10	4	1			3	1	1	
Shift 2	4		1	1			1		1
...	...	...	...						
Ward B									
Shift 1	4	1	1			2		1	
Shift 2	2	0	1	1	1				1
...	...	...	...						

3.1.2 Absence

When some nurses are absent the workload becomes larger for the other nurses at the ward, but the most important aspect is that they might lack competence to carry out all their tasks throughout the shift. To manage this today, the hospital hires external substitutes through a recruitment agency. It is possible to hire nurses from any competence level, or with specific special competences, but the cost increases proportionally with the competence. Data from Akershus University Hospital (A-HUS) [14] says that generally there is a absence ratio of 8-14 %. To hire external substitutes to always cover the absence ratio becomes costly.

4 Problem Description

The main purpose of this thesis is to create a strategic tool that finds an optimal nurse competence profile for a hospital. Here the optimal competence profile is described as the most cost efficient combination of competence that makes the hospital flexible and less vulnerable to day-to-day situations like absence and nursing demand. Today hospital are rigid regarding utilizing the competence across the wards, and costs of absence are high because external nurses are hired in a one-to-one ratio with absent nurses. Thus one of the hospital management's most important tasks is to map their competence profiles and make a strategic plan of how to utilize the competence to the fullest, and how to further evolve the competence profile. To build a good competence profile the hospital has to invest in their human capital, however it is important to invest in the right way. Too much competence in one field of expertise will give a high long term cost, based on high salaries, while to low competence in a field of expertise will make the hospital less flexible and may increase variable costs of hiring external competence. The hospital also needs to be open for new ways of managing the staff. Rotation of the nurses between the wards, so that the nurses can experience and gain expertise in other fields, can in this situation be shown to increase the human capital as well as decrease the variable day-to-day cost.

This section presents how nurse rerostering can be utilized further, from functioning on an operational level to become part of the strategic tool to develope the competence profile of the hospital. In other words, a nurse rerostering problem (NRSP) with internal rotation can be utilized to evaluate different competence profiles, and thus be part of the strategic tool.

4.1 Short Term versus Long Term Scheduling

Nurse rostering has focused both on short term and long term scheduling [7]. A roster can be created by the month, by 3 months or more. The advantage of these rosters is the possibility to focus on a short term scheduling, and then create cyclical rosters by repetition.

Nurse rerostering has up until now only been handling short term planning. This is mainly because the focus is on short term adjustments to the roster, based on last minute fluctuations. Taking the NRSP model as a starting point, the nurse rerostering problem is executed for each shift or a day because of new information. However the model is made as to consider several time periods at the time. For instance, reported absence can be a three day sick-leave, which can be implemented in the model and adjustments can be made for a time period of the three shift instead of only the first shift. The disadvantage of scheduling for a wider time period is that the NRSP has to be executed before the next day anyway, because of new information, and thus the changes done the day before might not be optimal for todays knowledge of absence.

As the two paragraphs above describe, both nurse rostering problems are functioning on an operational level. Here, operational level implies that the nurse rostering has been executed by head nurses at the bottom of the managing hierarchical pyramid of the hospital and for each shift or a given time period. Developing the nurse rostering further, the focus can be drawn to a more strategical level - by being a part of the hospitals central managements tools of managing the staff.

4.2 Nurse Rostering on a Strategic Level

As described in section 1 strategic thinking is developing new and unique opportunities that can create value, by provocative co-operations [6]. By looking at nurse rostering from a strategic point of view, a new way of competence management can be created, and finding a right combination of competences can be of great value to the hospital.

4.2.1 Competences and Skills

Strategic thinking focus on how to map the strengths and weaknesses of a company, and the competence within a company is a large part of this [6]. Do the company have the right competence and human capital for the services they offer?

Within a hospital the employees competence is the largest resource the hospital has. This competence can both be the hospital's strengths and weaknesses, depending on the fit between the competence profile and the hospital services. Thus one of the hospital managements most important tasks is to map its competence profiles and make a strategic plan of how to utilize the competence to the fullest, and how to further evolve the competence. Evaluate their present competence profile and finding the optimal competence profile for the hospital services is though difficult and a time consuming tasks.

The long term results of increasing the nurses competence is however increased salary, which is not included in the NRSP. This long term cost is one of the reasons why it is important to balance the higher and lower competence levels. Having a high competence within all of the fields experiences will make the hospital extremely flexible to absence, but the advantage of being this flexible will not outweigh the high long term costs. However having a low competence profile, the hospital is very vulnerable for absence and will have a high variable cost from hiring external substitutes. This proves that keeping a balance between the two is important.

Evaluation of a competence profile

The competence profile evaluation is basically an analysis of how the competence profile will handle day-to-day situations. These day-to-day situations mainly imply different absence and demand scenarios. A proper evaluation will give an overview of in which fields of expertise the hospital is lacking competence, and in which fields it has enough of redundant competence.

Evaluation of a competence profile can be done manually by close co-operation and a dialogue between the management and the nurses. A co-operation like this can be productive by developing unique opportunities to create value, as the strategic thinking quotation [6] implies, but it can also create a unity between the different groups. A positive working environment is often equally important as a economic profit, because an employee who feels comfortable will perform better [15].

Even though a profile evaluation can be done manually, and create positive outcome, the evaluation will probably give a suboptimal result due to lack of resources to evaluate all the possible scenarios. Looking further into the field of operation research, an evaluation tool based on scenario analysis of a large number of scenarios can ease the management work. Looking closer at nurse rerostering, you can find previous work that can be utilized to evaluate different absence scenarios.

4.3 How to Build a Strategic Tool for Competence Building

A strategic tool based on operational research can be the solution to map and utilize the nurse competence profiles within a hospital, and further find the optimal competence profile. A transition from manually evaluation to automatically generated competence profiles will ease the work, and make more room for the management and nurses to focus on how to move from the present competence profile to an optimal competence profile. This will strengthen both the co-operation between the nurses and the management, as well as creating value by utilizing the competence resource to the fullest.

The process of building up a good competence profile for the hospital can be divided into two stages:

1. At the first stage a decision have to be made; the competence profile the hospital think is the most cost efficient. This decision has to be made without knowing how the future absence and demand situations will look.
2. The Second stage, the hospital experiences how the new competence profile acts to day-to-day situations. Is the competence as cost efficient as it thought?

How to choose the "best" competence profile is very difficult. In a given absence scenario, one specific competence profile is the optimal and in another absence scenario another competence profile is the optimal. However taking advantage of operational research and nurse rerostering, a given competence profile can be evaluated for several absence scenarios and an expected cost of absence can be calculated. This is what the NRSP model was created for. Creating different competence profiles for evaluation, the competence profiles can be compared to each other by summing up the cost of competence with investment and the expected evaluation cost of responding to different absence scenarios.

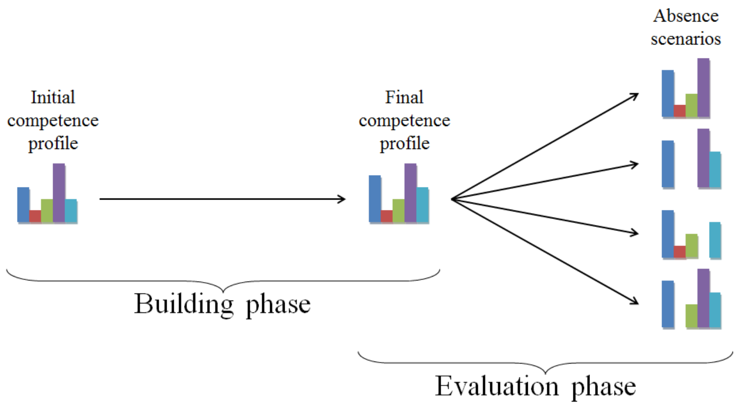

Figure 1: *Illustration of strategic tool of competence evaluation.*

The strategic tool can then be illustrated by Figure 1, with a decision done in stage one and the experience of the new competence done in stage two.

Between the present competence profile and the new, "optimal", competence profile the hospital have a building phase. This phase will be a time period set by the hospital, and the longer the time period the more can be done to the competence profile. Typical changes could be to increase the competence level by experience, push competence by paying for courses and by buying extra special expertise. After the building phase, the evaluation phase begins. This phase will do the evaluation described in 4.2.1, and find the value of the given competence profile.

5 Modeling Nurse Rerostering with Internal Rotation

In this section Lillebys [2] nurse rerostering problem (NRSP) with internally rotation and preference consideration, described in section 2.3, is further elaborated. The NRSP model is going to be a part of further development of a strategic tool of competence, and are therefor needed to understand the problem description properly.

5.1 Description of the NRSP

If nurses are absent in one ward, it is possible to lend a nurse from another ward which has better manpower. This will make the other ward more exposed for short-handedness, but the intention is to level out the lack of competence needed in each ward. There is also a possibility to rotate nurses, even if no one is absent. The purpose of this is to build individual competence, by letting the nurses get more experience in others areas and thus enhance the intellectual capital

The NRSP model has an optimizing approach, and is formulated as an integer problem (IP). The objective of the mathematical model is to find the minimal cost of rearranging the nurses within a set of time periods (here: shifts), together with hiring external substitutes from an recruitment agency where the demand of competence is not fulfilled. The focus is on cost, both direct and indirect cost. The direct cost depends on the externally hired nurses, where both hierarchic level of competence and special expertise matter on the level of cost. The indirect cost is an artificial cost for rotating the nurses within the hospital. This internal cost depends on the ward the nurse was originally signed to, which ward she end up working in, and her requests for competence building. If the nurse stays in the originally signed ward, the cost is 0. If she ends up at a different ward, and this ward is not on her requested list, the cost is positive (> 0). If she ends up at a different ward from the original, and this ward is what she requested, the cost is negative (< 0). By utilize a cost structure like this, the positive effect of meeting nurses requests are integrated in the model, and it reduces the chance of rotating nurses to other wards.

Demand at ward A and shift 1

Competence Level	Minimum demand	Aggregated minimum demand
1	10	15
2	4	5
3	1	1

Figure 2: *Example of aggregated demand of hierarchic competence levels. The demand of level 2 and 3 is aggregated to level 2, and the demand for level 1, 2 and 3 is aggregated to level 1. Data from Table 3*

The model operates with aggregated demand, which can be described with the following: The minimum demand of a hierarchic competence level is given by a number of nurses who has to hold the competence level in a specific shift, see Table 3. The roster given indicates which of the nurses who are working at the specific shift, but not which level they are assigned to. It is important to notice that nurses with a higher competence level also holds the lower levels, and can thus have the responsibility of a lower level. To include this in the model, there is a need for an aggregated demand. Figure 2 illustrates an example of aggregated demand, from the data in Table 3.

The nurses competence is also one of the key aspects within the NRSP with internal rotation of the nurses. Seen from a different point of view, rotation of the nurses also leads to extra experience from other fields of expertise. When nurses increasing the knowledge of other expertise, the result is competence building. Competence building again leads to an increased human capital, which further makes it easier to rotate and adapt to absence in the next time period.

The goal as a whole is to find a new match between nurses and wards, that levels out the lack of competence at a minimum cost while building up an intellectual capital after preferences from the nurses. The economical aspect is the strongest intensive to adjust todays arrangement, and will make the other aspects secondary goals. On the other hand is the secondary goals the nurses highest goal, and thus important to integrate in the assignment to get a practical and good solution.

The new roster will define which nurses who rotates, and to which ward. It will also define if any external substitutes are hired at any of the wards. External substitutes can be hired to any ward, and with any competence level and special expertise the hospital is lacking. This gives the wards an overview of the competence they hold, which in turn gives them an indication of what kind of instructions the substitutes need and tasks they can handle with guidance. After ended shift, the matrix with records of competence building is updated.

5.2 The Mathematical Model Formulation

Sets The sets describe classifications of nurses, wards and competence levels and the time periods. All the nurses working in the original roster, in time period t, are listed in set $\mathcal{N}_t$. When given the information of absent manpower, the wards are left with the present nurses in $\mathcal{N}_t^{\mathcal{P}}$. $\mathcal{L}$ is the set of hierarchic competence levels at the hospital, while $\mathcal{L}_{nw}$ is a set of the highest competence levels nurse n holds in ward w. All sets used in the model formulation is given below.

$\mathcal{T}$	set of all time periods/shift
$\mathcal{N}_t$	set of all nurses listed in the shift $t \in \mathcal{T}$
$\mathcal{N}_t^{\mathcal{P}}$	set of listed nurses who are present in shift $t \in \mathcal{T}$, $\mathcal{N}_t^{\mathcal{P}} \subseteq \mathcal{N}_t$
$\mathcal{W}$	set of wards
$\mathcal{L}$	set of competence levels
$\mathcal{L}_{nw}$	set of the highest competence levels for nurse n in ward w, $\mathcal{L}_{nw} \subseteq \mathcal{L}$
$\mathcal{E}$	set of special expertise
$\mathcal{E}_w$	set of special expertise that are demanded in ward w

Parameters The parameters used in this model are stated below. Notice that the parameter D_{twl}^{LA} is an aggregated minimum demand for the nurses at ward w and level l at shift t. C_{tnwl}^{I} presents an artificial cost for rotating nurses within the hospital, as described above. C^{EE}_{tws} is the extra cost of demanding the hired nurses to have special expertise w.

D^{LA}_{twl}	aggregated demand for nurses at ward w with level l at shift t
D^{E}_{tws}	demand for nurses with special expertise e in ward w at shift t
E^{COMP}_{ne}	$= 1$ if nurse n have the special expertise e, 0 otherwise
C^{I}_{tnw}	the artificial cost for having nurse n working at ward w at shift t
C^{E}_{twl}	cost for hiring a nurse with competence level l at ward w at shift t
C^{EE}_{tws}	additional cost for hiring a nurse with special expertise e at ward w at shift t

Variables The binary variable x_{tnwl} is used to indicate which ward and level the nurse works for in the final roster. This variable will also indirect imply if the nurse has been rotated or not, from her original ward. If the minimum demand cannot be fulfilled by rotating nurses internally, an external nurse has to be hired. The integer variable y_{twl} indicates how many, and at which ward and level, nurses is hired for the specific time period. z_{twe} is the number of nurses with special expertise e which is necessary to hire in externally to meet the minimum demand.

$$x_{tnwl} = \begin{cases} 1 & \text{if nurse } n \text{ ends up working in ward } w \text{ with level } l \text{ at shift } t \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} y_{twl} &= \text{number of nurses which are hired externally} \\ &\quad\ \text{to ward } w \text{ at shift } t \\ z_{twe} &= \text{number of nurses with competence level } l \text{ and special expertise } e, \\ &\quad\ \text{which are hired externally to ward } w \text{ at shift } t \end{aligned}$$

Complete model The objective together with the definitions stated above gives the following model.

$$min \; = \sum_{t\in\mathcal{T}}\sum_{w\in\mathcal{W}}\sum_{l\in\mathcal{L}} C^{E}_{twl} y_{twl} + \sum_{t\in\mathcal{T}}\sum_{w\in\mathcal{W}}\sum_{e\in\mathcal{E}} C^{E} E_{twe} z_{twe} + \sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}^{\mathcal{P}}_{t}}\sum_{w\in\mathcal{W}}\sum_{l\in\mathcal{L}} C^{I}_{tnwl} x_{tnwl} \quad (1)$$

$$\sum_{n\in\mathcal{N}^{\mathcal{P}}_{t}}\sum_{l'\in\mathcal{L}_{nw}:l'\geq l} x_{tnwl'} + \sum_{l'\in\mathcal{L}:l'\geq l} y_{twl'} \geq D^{LA}_{twl}, \qquad t\in\mathcal{T}, w\in\mathcal{W}, l\in\mathcal{L}, \quad (2)$$

$$\sum_{n\in\mathcal{N}^{\mathcal{P}}_{t}}\sum_{l\in\mathcal{L}:l\leq\mathcal{L}_{nw}} E^{COMP}_{ne} x_{tnwl'} + z_{twe} \geq D^{E}_{twe}, \qquad t\in\mathcal{T}, w\in\mathcal{W}, e\in\mathcal{E}_{w}, \quad (3)$$

$$\sum_{w\in\mathcal{W}}\sum_{l\in\mathcal{L}_{nw}} x_{tnwl} = 1, \qquad t\in\mathcal{T}, n\in\mathcal{N}^{\mathcal{P}}_{t}, \quad (4)$$

$$z_{twe} \leq \sum_{l\in\mathcal{L}} y_{twl}, \qquad t\in\mathcal{T}, w\in\mathcal{W}, s\in\mathcal{E}_{w}, \quad (5)$$

$$x_{tnwl} \in \{0,1\}, \qquad t\in\mathcal{T}, n\in\mathcal{N}^{\mathcal{P}}_{t}, w\in\mathcal{W}, l\in\mathcal{L}_{nw}, \quad (6)$$

$$y_{twl} \geq 0, \text{ integer}, \qquad t\in\mathcal{T}, w\in\mathcal{W}, l\in\mathcal{L}, \quad (7)$$

$$z_{twe} \geq 0, \text{ integer}, \qquad t\in\mathcal{T}, w\in\mathcal{W}, e\in\mathcal{E}_{w} \quad (8)$$

The objective function (1) consists of three terms. The first term captures the cost by hiring external manpower, the second term the extra cost of having the hired nurses holding special expertises, and the third term deals with the artificial cost of rotating the nurses within the hospital. There are two types of demand that has to be fulfilled. Constraints (2) ensure that the minimum demand is fulfilled in all the combinations of ward and the hierarchic competence levels, while constraints (3) ensure the minimum demand of special expertises in each ward. Constraints (4) make sure that each nurse only works in one ward at the time. To connect the two variables that undertake hiring of external manpower, constraints (5) ensure that only the actual nurses hired can have the special expertise they are in shorthand with. The hospital cannot hire in only special expertise, and thus the number of a specific special expertise has to be lower or equal to the number of hired nurses at a ward. One nurse can nevertheless hold several special expertises, as long as they are different from each other. Lastly, the constraints (6) - (8) define the variables respectively as binary, integer and non negative. The model is separable with regards to time period t. This indicates that each shift can can be solved on its own.

5.3 Results

Implementation of the mathematical model in Mosel Xpress, gave significant results. Two single example cases proved it beneficial to rotate the nurses internally before hiring external substitutes. The two cases had the same set of nurses absent, but the one had a higher aggregated competence profile. The cost result from the high competence profile were lower than the regular profile, which proved it possible to benefit from an investment in the nurses competence building. A cost analysis with variation in nurses absent ratio was also executed. With a solid data set with a thousand test for each percentage, analysis of the average real cost, for both the regular- and high competence profile were computed. The cost result from the high competence profile were lower than the regular profile, which proved it possible to benefit from an investment in the nurses competence building. The results are illustrated in Figure 3. The report concluded that, with a given regular competence profile, rerostering can reduce the costs of hiring external nurses with 53%, at an absence ratio of 10%. By increasing the competence level in several wards, and thus operating with a higher competence profile, the result became a cost reduction of 74% at the same absence ratio as with the regular profile. The results also supported the findings in the singe example cases, and proved it beneficial to have a high competence profile. The high competence profile gave the opportunity to rotate more nurses, and to thus hire external substitutes at lower competence levels. Compared with the cost of hiring external nurses in a scale of one to one, which illustrates todays procedure, it proved it cheaper to rotate and invest in the human capital. The question then is what is the *most* profitable competence profile?

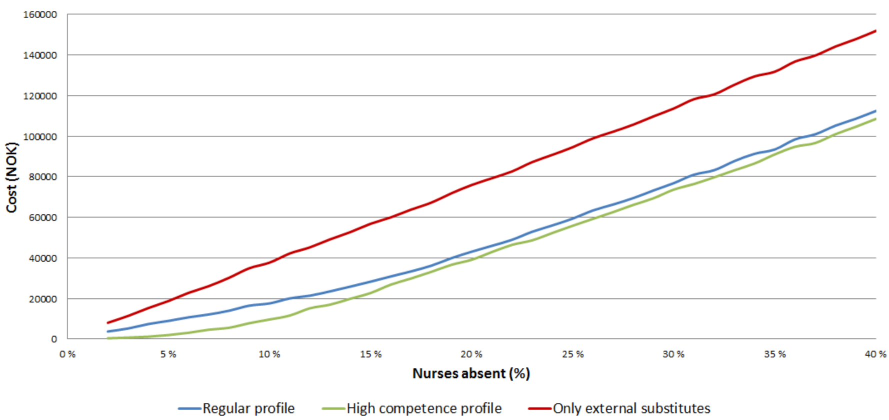

Figure 3: *Overview of the cost analysis of the regular profile, the high competence profile and only external substitutes. Absence ratio is in the interval 2% - 40%.*

6 Modeling Competence Building, with Nurse Rerostering

This section will first present stochastic programming and it's relevance to this problem. After that a presentation of the mathematical formulation of the strategic model for competence building within hospital management will be made. The model consists of two phases, as described in 4.3. The first phase is the building part, where the hospital can choose to invest in the nurses by increasing the competence and special expertise. The second stage is to evaluate the different building opportunities by computing the expected cost of several absent scenarios. In the mathematical model the building phase is described first and then it is merged with the evaluation phase, based on the NRSP model described in 5.2. The mathematical model is not a linear problem, as the variables in stage 2 are binary, so it becomes a mixed integer problem (MIP)

6.1 Stochastic Programming

Stochastic programming is a framework for modeling optimization problems that involve uncertainty. Whereas deterministic optimization problems are formulated with known parameters, real world problems almost invariably include some unknown parameters. Therefor when some of the variables in a linear program is better described as an uncertain, and thus random variable, as stochastic linear program may be a good fit for the model. Faced with random variables stochastic programming take advantage of the fact that probability distributions control the data are known or can be estimated. In many cases solution techniques for stochastic programs rely on statistical estimation and numerical approximation methods.

A *two-stage* linear programs is one of the widely applied and studied stochastically programming models. Recourse models, which is a more specific name for two-stage problems, result when some of the decisions must be fixed before information relevant to the uncertainties is available, while the rest of the decisions can be delayed until afterwards.In other words, the decision maker has to take some action in the first stage, after which a random event occurs affecting the outcome of the first-stage decision. A recourse decision can then be made in the second stage that compensates for any bad effects that might have been experienced as a result of the first-stage decision[16].

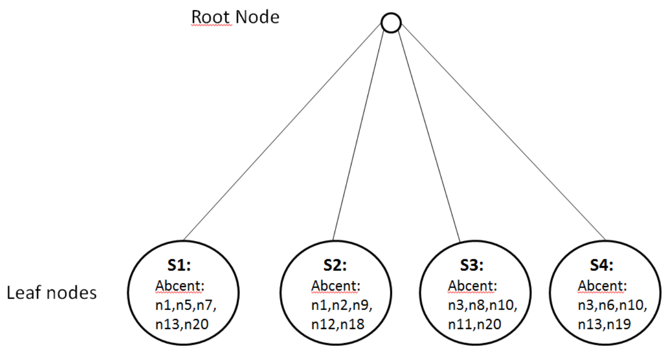

Figure 4: *Illustration of the strategic tools scenario tree. The leaf nodes are the scenarios with different nurses absent.*

Each recourse problem can be characterized by its "scenario tree" and its "scenario problems". A scenario are one specific, complete, realization of the stochastic elements that might appear during the course of the problem. The scenario tree are a structured distributional representation of the stochastic elements and the manner in which they may evolve over the period of time represented in the problem. A scenario problem is associated with a particular scenario and may be looked upon as a deterministic optimization problem [16].

Looking at the strategic tool of nurse competence, a two-stage program is the best method to use. The building phase becomes the first-stage decision in a recourse model - how to develop and invest in the initial competence profile. The evaluation becomes the second stage, where the uncertainty lies in not knowing how the upcoming day-to-day fluctuations in demand and absence is. These fluctuations can be described by different scenarios, like the scenarios described in the paragraph above. The belonging scenario tree is illustrated in Figure 4. The root node corresponds to the initial decision stage - how to build the new competence profile - where no specific information regarding the random variables has been obtained. The leaf nodes correspond to the final decisions required - the evaluation of the new profile - which are made after all available information has been obtained. Each scenario is an absence situation, with a given percentage of absence. Random nurses are picked out to be absent in each scenario, and the scenario problem is to cover the lack of competence and to meet the nurse demand in each specific scenario by rotating nurses and hire external nurses from recruitment agencies. The next section presents the stochastic two-stage program, and s will represent the scenarios with uncertain data associated with the decision problem. Decision variables, matrices, etc, are defined in the same manner as for traditional linear programming problems. However, elements that might vary with scenario will be identified by subscript s.

6.2 Mathematical Formulation

Sets The sets describe classifications of nurses, wards, competence levels and the scenarios. All the nurses working in the original roster, at the given shift, are listed in set $\mathcal{N}$. When given the information of absent manpower in each scenario s in set $\mathcal{S}$, the wards are left with the present nurses in $\mathcal{N}_s^{\mathcal{P}}$. $\mathcal{L}$ is the set of hierarchic competence levels at the hospital, while $\mathcal{E}$ is a set of the special expertises. All sets used in the model formulation is given below.

$\mathcal{N}$	set of all nurses listed in the given shift
$\mathcal{N}_s^{\mathcal{P}}$	set of listed nurses who are present in the evaluation shift in scenario s $\mathcal{N}_s^{\mathcal{P}} \subseteq \mathcal{N}$
$\mathcal{W}$	set of wards
$\mathcal{L}$	set of competence levels
$\mathcal{E}$	set of special expertise
$\mathcal{E}_w$	set of special expertise that are demanded in ward w
$\mathcal{S}$	set of scenarios in the evaluation phase

Parameters The parameters used in this model are stated below. There are 6 cost parameters, C_n^G, C_n^U, C_n^M, C_{nw}^{ADMIN}, C_{wl}^E, C_{we}^{EE}, used in the objective function. The three first cost parameters belong to stage 1, while three remaining belongs to stage 2. The next parameters are divided by the two stages as well. First the parameters concerning the initial competences, and maximum levels of competence builing for each nurse, and each ward. Second the parameters specific for each scenario are the absent nurses, the minimum demands of nurses and the probability for each scenario. Notice that the parameter D_{wl}^{LA} is an aggregated minimum demand for the nurses at ward w and level l at the given shift, and D_{we}^{E} is the demand of the special expertise, both in each scenario s.

C_n^G	the additional cost for increased competence level for nurse n, in terms of one time period
C_n^U	the additional cost for buying nurse n extra competence, in terms of one time period
C_n^M	the additional cost for buying nurse n special expertice, in terms of one time period
C_{nw}^{ADMIN}	administration cost of having nurse n rotated to ward w
C_{wl}^{E}	cost for hiring a nurse externally with competence level l at ward w
C_{we}^{EE}	additional cost for hiring a nurse externally with special expertise e at ward w
A_{ns}	$= 1$ if nurse n is absent in scenario s, 0 otherwise
Q_{nw}	the initial competence level that nurse n has in ward w
G_{nw}	maximum competence levels nurse n can gain with extra experience at ward w
G_n^{MAX}	the total maximum of gained competence levels for nurse n

U_{nw}	maximum competence levels the hospital can buy nurse n at ward w
U_n^{MAX}	the total maximum of bought competence levels for nurse n
O_n^{MAX}	the total maximum of bought special expertise for nurse n
D_{wl}^{LA}	aggregated demand for nurses at ward w with level l
D_{we}^{E}	demand for nurses with special expertise e in ward w
E_{ne}^{E}	$= 1$ if nurse n have the special expertise e, 0 otherwise
P_s	probability of scenario s

Variables The integer variables g_{nw} and u_{nw} is variables in the building phase and describe how many levels of competence nurse n gains or get bought in ward w, while the binary variable o_{nw} register if nurse n gains a special expertise or not. The binary variable x_{nwls} is used to indicate which ward and level the nurse works for in the final roster. This variable will also indirect imply if the nurse has been rotated or not for each scenario s, from her original ward. k_{nwes} is a variable created to linearize variable m_{ne} and x_{nwls} in the demand restriction, and register if nurse n is works in ward w and hold the special competence e in scenario s. If the minimum demand cannot be fulfilled by rotating nurses internally, an external nurse has to be hired. The integer variable y_{wls} indicates how many, and at which ward and level, nurses is hired for the specific time period. z_{wes} is the number of nurses with special expertise e which is necessary to hire in externally to meet the minimum demand.

g_{nw}	$=$	gained competence level(s) for nurse n in ward w
u_{nw}	$=$	bought competence level(s) for nurse n in ward w
q_{nw}	$=$	new competence level for nurse n in ward w
o_{ne}	$=$	1 if bought special expertise e to nurse n , 0 otherwise
m_{ne}	$=$	new value of special expertise: 1 if nurse n has the expertise e, 0 otherwise

k_{nwes}	=	1 if nurse n works at ward w in scenario s and has the special expertise e, 0 otherwise (linearization variable)
x_{nwls}	=	1 if nurse n ends up working in ward w with level l at the given time period and scenario s, 0 otherwise
y_{wls}	=	number of nurses which are hired externally to ward w at the given time period and scenario s
z_{wes}	=	number of nurses with competence level l and special expertise e,which are hired externally to ward w at the given time period and scenario s

Complete model The objective together with the definitions stated above gives the following model.

$$min \ = \sum_{n\in\mathcal{N}}\sum_{w\in\mathcal{W}} C_n^U g_{nw} + \sum_{n\in\mathcal{N}}\sum_{w\in\mathcal{W}} C_n^P u_{nw} + \sum_{n\in\mathcal{N}}\sum_{e\in\mathcal{E}} C_n^M o_{ne}$$
$$+\sum_{s\in\mathcal{S}}\left(P_s\left(\sum_{w\in\mathcal{W}}\sum_{l\in\mathcal{L}} C_{wl}^E y_{wls} + \sum_{w\in\mathcal{W}}\sum_{e\in\mathcal{E}} C_{we}^{EE} z_{wes} + \sum_{l\in\mathcal{L}} C_{nw}^{ADMIN} x_{nwls}\right)\right) \tag{9}$$

$$Q_{nw} + g_{nw} + u_{nw} = q_{nw}, \qquad n\in\mathcal{N}, w\in\mathcal{W} \tag{10}$$

$$E_{ne}^E + o_{ne} = m_{ne}, \qquad n\in\mathcal{N}, e\in\mathcal{E} \tag{11}$$

$$g_{nw} \le G_{nw}, \qquad n\in\mathcal{N}, w\in\mathcal{W} \tag{12}$$

$$u_{nw} \le U_{nw}, \qquad n\in\mathcal{N}, w\in\mathcal{W} \tag{13}$$

$$\sum_{w\in\mathcal{W}} g_{nw} \le G_n^{MAX}, \qquad n\in\mathcal{N} \tag{14}$$

$$\sum_{w\in\mathcal{W}} u_{nw} \le U_n^{MAX}, \qquad n\in\mathcal{N} \tag{15}$$

$$\sum_{e \in \mathcal{E}} o_{ne} \leq O_n^{MAX}, \qquad n \in \mathcal{N} \tag{16}$$

$$q_{nw} \geq 0, integer, \qquad n \in \mathcal{N}, w \in \mathcal{W} \tag{17}$$

$$g_{nw} \geq 0, integer, \qquad n \in \mathcal{N}, w \in \mathcal{W} \tag{18}$$

$$u_{nw} \geq 0, integer, \qquad n \in \mathcal{N}, w \in \mathcal{W} \tag{19}$$

$$m_{nw} \in \{0, 1\}, \qquad n \in \mathcal{N}, w \in \mathcal{W} \tag{20}$$

$$o_{nw} \in \{0, 1\}, \qquad n \in \mathcal{N}, w \in \mathcal{W} \tag{21}$$

$$\sum_{w \in \mathcal{W}} \sum_{l \in \mathcal{L}} x_{nwls} = 1 - A_{ns}, \qquad n \in \mathcal{N}_s^{\mathcal{P}}, s \in \mathcal{S} \tag{22}$$

$$\sum_{l \in \mathcal{L}} l x_{nwls} \leq q_{nw} \qquad n \in \mathcal{N}_s^{\mathcal{P}}, w \in \mathcal{W}, s \in \mathcal{S} \tag{23}$$

$$-\sum_{l \in \mathcal{L}} x_{nwls} + k_{nwles} \leq 0 \qquad n \in \mathcal{N}_s^{\mathcal{P}}, w \in \mathcal{W}, e \in \mathcal{E}, s \in \mathcal{S} \tag{24a}$$

$$-m_{ne} + k_{nwes} \leq 0 \qquad n \in \mathcal{N}_s^{\mathcal{P}}, w \in \mathcal{W}, e \in \mathcal{E}, s \in \mathcal{S} \tag{24b}$$

$$\sum_{n \in \mathcal{N}_s^{\mathcal{P}}} \sum_{l' \in \mathcal{L}: l' \geq l} x_{nwl's} + \sum_{l' \in \mathcal{L}: l' \geq l} y_{wl's} \geq D_{wl}^{LA}, \qquad w \in \mathcal{W}, l \in \mathcal{L}, s \in \mathcal{S} \tag{25}$$

$$\sum_{n \in \mathcal{N}_s^{\mathcal{P}}} k_{nwes} + z_{wes} \geq D_{we}^{E}, \qquad w \in \mathcal{W}, e \in \mathcal{E}_w, s \in \mathcal{S} \tag{26}$$

$$z_{wes} \leq \sum_{l \in \mathcal{L}} y_{wls}, \qquad w \in \mathcal{W}, e \in \mathcal{E}_w, s \in \mathcal{S} \tag{27}$$

$$x_{nwls} \in \{0,1\}, \qquad n \in \mathcal{N}^{\mathcal{P}}, w \in \mathcal{W}, l \in \mathcal{L}, s \in \mathcal{S} \tag{28}$$

$$y_{wls} \geq 0, \text{ integer}, \qquad w \in \mathcal{W}, l \in \mathcal{L}, s \in \mathcal{S} \tag{29}$$

$$z_{wes} \geq 0, \text{ integer}, \qquad w \in \mathcal{W}, e \in \mathcal{E}_w, s \in \mathcal{S} \tag{30}$$

$$k_{nwes} \in \{0,1\}, \qquad n \in \mathcal{N}_s^{\mathcal{P}}, w \in \mathcal{W}, e \in \mathcal{E}, s \in \mathcal{S} \tag{31}$$

The objective function (9) consists of three terms. The first term is the extra salary for the nurses who gained competence by experience. The second term is both the cost of bought competence and the extra salary for the nurses who was invested in by the hospital, while the third term is the cost of bought special expertise. All these three terms belongs to the models building phase. The forth term captures the cost by hiring external manpower, the fifth term the extra cost of having the hired nurses holding special expertises, and the sixth term deals with the administration costs by having nurses rotated to other wards. These three terms belongs to the evaluation phase, and they are all specific to each scenario. To find the expected cost of the all the scenarios, the sum of each scenario costs are multiplied with the probability of the scenario.

Building Constraints: Constraints (10) and (11) keeps track respectively of the building of competence and special expertise. The constraints (12) and (13) ensures that the gained, bought and lost competence levels do not exceed the maximum level for each ward and nurse, while constrains (14) and (16) ensures that each nurse do not exceed the number of possible gained and bought competence levels and bought special expertise. These constraints defines the time horizon of the building phase. The higher the maximum levels, the longer is the time horizon. Last in the building phase part is the constraints (17)- (21) defines the variables respectively as integer, binary and non negative.

Links and Evaluation Constraints: Each nurse can only work at one ward in the given shift, if she is present for the specific scenario, which is what constraint (22) ensures. To link the building phase of competence and the evaluation phase, constraint (23) restrict the nurse to work in a ward with a higher competence level than she holds in the new competence profile. Constraints (24) is a linearization of the two variables m_{ne} and x_{nwls}, and links the building phase of special expertise with the demand of special expertise in each ward. In the demand constraint of special expertise, the two variables had to be multiplied, and since this is a mixed integer linear program a multiplication of two variables is not allowed. However since the two variables is binary, a linearization is possible, and is done by creating a new variable k_{nwes} to be the multiplied result. Constraints (25) ensure that the minimum demand is fulfilled in all the combinations of ward, the hierarchic competence levels and scenario, while constraints (26) ensure the minimum demand of special expertises in each ward and scenario. To connect the two variables that undertake hiring of external manpower, constraints (27) ensure that only the actual nurses hired can have the special expertise they are in shorthand with. The hospital cannot hire in only special expertise, and thus the number of a specific special expertise has to be lower or equal to the number of hired nurses at each ward and scenario ward. One nurse can nevertheless hold several special expertises, as long as they are different from each other. Lastly, the constraints (28)-(31) define the variables respectively as binary, integer and non negative.

7 Base Case

For further evaluation of the problem, and to be able to analyze results, a case of a hospital is presented as a basis for computational studies. Two different cases were evaluated, both in which are simplified versions of a hypothetical complete hospital. One case is with a small number of nurses and the other with double the amount of nurses. Both numbers indicates the size of the wards, and thus the number of patients. The smaller case is from here on described as the *base case* and will be the main case of computations, while the other case will be described as the *medium case* and will only be used for some computational comparing. Both cases focuses on one shift, a general day-shift. For this shift a specific amount of nurses are assigned, which is defined by the roster. The hospital has requirements for a minimum demand of nurses at each ward, competence level and special expertise, and the roster is made at a former stage to fulfill both demand of minimum nurses and the competences needed at each shift.

7.1 General Assumptions

When creating the cases, there had to be made some assumptions. The main assumptions regard the upper limits of competence in the building phase, the scenario structure, the minimum demand, the competence- and special expertise-matrix and the cost structure.

As Table 4 shows, the base case consist of 4 wards, and has 26 employed nurses on the given shifts roster. Table 7 describes further information about the minimum demand of nurses in the four wards. Looking at the minimum demand for each ward, the wards seam to be the same size. Assuming a distribution of 2-3 patients for each nurse, though this depends of course on the state of the patients. The different wards has about 15 patients each.

The medium case consist of 4 wards and 56 employed nurses on the given shift, which indicate a number of patients close to 140. The general information is presented in Tables 4 and 7, together with the base case.

Table 4: *A general overview of the hospital base case*

	HOSPITAL CASES	
	BASE CASE	MEDIUM CASE
Wards:	4	4
Nurses employed:	26	51
Hierarchic competence levels in each ward:	4	4
Special expertises:	10	10

7.1.1 Upper Limits of Competence Building

As mentioned earlier the hospital can decide on how long the time period of building the new competence profile. There are 6 parameters that can indicate how long the time period is. The first one decides how many competence levels each nurse can gain by experience in each ward, G_{nw}. The second parameter decides how many competence levels that can be bought each nurse in each ward, U_{nw}. The next three sets the upper limit of how many competence levels each nurse can get in total, both for the gained and bought competence and as well the special experience, G_n^{MAX}, U_n^{MAX} and O_n^{MAX}. The longer the time horizon, the more competence can be invested in, and thus the upper limits will be higher.

In the base case these limits has been set to quite low values, that indicates a shorter time horizon (maybe 3-6 months). Each nurse is set to maximum gain and be bought *one* competence level in each ward, $G_{nw} = U_{nw} = 1$. The total maximum limit for each nurse in all the wards together is set to *one* gained competence by experience, and *two* bought competence levels, $G_n^{MAX} = 1$ and $U_n^{MAX} = 2$. The difference is because competence increased by experience takes more time than competence gained by for instance a course. The maximum amount of bought special expertise is set to *two*, $O_n^{MAX} = 2$.

Table 5: *Illustration of a competence matrix, where the segment is taken from the base case matrix. The x indicates competence the nurses hold.*

		Ward A				Ward B			
		level:				level:			
	Nurses	1	2	3	4	1	2	3	4
Ward A	1	x				x			
	2	x					x		
	3	x	x			x			
	4	x	x			x			
	5	x	x	x			x		
	6	x	x	x		x			
	7	x	x	x	x	x			
Ward B	8	x				x			
	9	x				x			
	10	x	x			x	x		
	11					x	x	x	
	12	x	x			x	x	x	
	13	x				x	x	x	x

7.1.2 Scenarios, Probability and Absence

The stochastic model includes scenarios in stage 2. The scenarios, as presented in section 6.1, describes different situations of absence at the given shift. The higher the number of scenarios the more accurate can the picture of a real world be described in the model, and thus the result of the competence building can be more robust. Generally in the computational studies of the cases 10 scenarios is used, but this vary between the different studies. An interval between 10 and 30 is used in the computational studies. The probability of each scenario is the same, and thus calculated by dividing 100% on the number of scenarios. The number of scenarios is reported for each study.

When the roster is set, the "expected" scenario is that all the given nurses are present, but this is often not the case. According to [14] the general absence ratio is between 8-14%, but since in these cases it is only operated with one absence ratio, this is set to 10%. There are one exception from this absence ration in the computational studies, and that is section 8.3.2.

Table 6: *Illustration of a special expertise matrix, where the segment is taken from the base case matrix. The x indicates the special experience the nurses hold.*

		Special Competence									
	Nurses	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
Ward A	1										
	2										
	3		x								
	4						x				
	5							x		x	x
	6		x		x						x
	7	x	x	x					x	x	
Ward B	8										
	9		x					x		x	
	10				x						x
	11		x	x					x		
	12	x		x	x		x			x	
	13	x	x		x						

When defining which of the nurses who is absent, the nurses are pixed randomly by each nurse having the 10% probability to be absent. When this is done for all of the scenarios given in the problem, it forms a matrix, which from now on will be called the absence-matrix.

7.1.3 Competence

The competence structure for both cases are the same. Both hospitals has 4 hierarchic competence levels, 0-3, where 0 is the level all nurses hold when graduating from the three-year undergraduate program in nursing. Level 1-3 is obtained as described in the chapter of the hospital structure (3). In the four wards combined there is a total of 10 kinds of special expertise. Some special expertise are specific to a ward, while others are expertise needed in all the wards. The competence matrix and the special expertise depends on the number of hierarchic levels and expertise, which is set for the specific case. The competence matrix, concerning both cases, was made with some assumptions regarding how many nurses was holding the different competence levels. As written in the section 3, all the nurses have the basic level 0 at all wards. It was assumed that upgrading from level 0-1 was the easiest step on the hierarchic levels, and thus only a few is still on level 0 in their principal department. From level 1 to level 2 a higher degree of competence building is needed, but when working regular hours at the same ward this competence is still not hard to accomplish. At the highest level of competence there are only one or two nurses needed. In the base case it was hard to follow these assumptions because of the small number of nurses, but the medium case were fairly adequate to these rules of thumbs. Some nurses were also given extra competence in different wards than their home ward, because of assumed earlier experience. Table 5 and Table 6 show respectively a segment from the base case competence matrix and matrix of special expertise. Similar tables are made for the medium case as well, though it is not presented here.

Table 7: *Overview of the demand for both the base case and the medium case wards*

Wards	Base Case		Medium Case	
	Scheduled Nurses	Minimum demand	Scheduled Nurses	Minimum demand
A	7	6	13	12
B	6	6	12	12
C	7	6	14	12
D	6	6	11	10

Table 8: *Minimum demand for each competence level at a general day-shift, for both the base case and the medium case*

	LEVEL							
	Base Case				**Medium Case**			
WARD	0	1	2	3	0	1	2	3
A	2	2	1	1	3	4	3	2
B	2	1	2	1	3	4	3	2
C	1	2	2	1	3	4	3	2
D	2	2	1	1	2	4	2	2

Table 9: *Minimum demand for each special expertise at a general day-shift, for the base case.*

	DEMAND OF SPECIAL EXPERTISE									
WARD	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
A	1	3	1	0	0	0	0	1	1	2
B	1	3	1	3	0	0	0	1	2	0
C	1	3	1	0	3	0	0	1	3	0
D	1	3	1	0	0	2	2	1	2	0

7.1.4 Demand

The distribution of the minimum demand of nurses between the different hierarchic competence levels is done by putting one nurse to be the head nurse which is responsible for coordinating and communication with the doctors.To have the head-nurse position, it is realistic to presume that the nurse has to hold a competence level of 3. The rest of the team consists of nurses with competence levels 0, 1 and 2. It is further assumed that level 0 and 1 does most of the work, and thus the largest fraction of the rest of the team consists of nurses with these levels. Which team that need the highest competence level all together depends again on the patients in the ward. When having a larger amount of nurses in a ward, a general way of dividing the nurses within the ward is to form two teams and handle each team as described above. Table 7 and 8 indicates the assumed minimum demand, for respectively the aggregated demand for each ward A-D and the detailed demand for each competence levels 0-3 in each ward.

Table 9 describes the demand of special experience for each ward. The numbers indicate that S1-S3, S8 and S9 are general experience since all the wards has a demand for these, while S4-S7 and S10 are ward specific to the ward with demand for these. Notice that ward D has both S6 and S7. A similar table is also given for the medium case, although this is not presented here.

7.1.5 Cost Structure

There are six different cost parameters, and they are divided into the two stages of the model.

Costs in the building phase
There are three types of cost in the building phase:
1) The first is the cost of increasing the nurses competence by letting the nurses experience the field of other wards, or by having enough extra experience to increase their level in the home ward. Gained experience and competence over a longer time period will lead to an increase in salary. It is assumed that a general yearly salary is close to 350.000 NOK, and for each competence gained an increase of 10.000 NOK per year. The focus is on the extra cost of gained competence, since this is the hospitals investment in this phase. The original salary can be seen as sunk cost, as they already are payed for. To be able to compare the cost with the evaluation phase, the cost in the model have to be scaled to the salary per shift. A simplified calculation of scaling down to this cost it is assumed that the shift cost is $38,50$ NOK. The calculation has been made like this: 10.000 NOK / 52 Weeks / 5 days per week.

2) The second cost is the cost of increase the nurses competence by buying extra experience, for instance by sending them to courses. Bought competence will result in the same increase in salary, but the difference from the cost of gained competence is that for courses fixed fees has to be considered, hence bought competence. The fixed cost is assumed to be 30.000 NOK, which together with the increase of salary of 10.000 NOK makes a shift cost of $153,90$ NOK.

3) The third cost is the cost of increasing the special expertise. It is assumed that the special expertise is not a competence that can be gained by experience, so the nurse has to take a course. The is assumed a cost of 10.000 NOK for each course, and an increase of salary of 5.000 NOK. This makes a shift cost of $57,70$ NOK. Table 10 gives an overview of the costs of the building phase.

Costs in the evaluation phase

Table 10: *Overview of the building costs. All the costs are in Norwegian kroner (NOK)*

BUILDING COST				
Type of cost	Yearly increase of salary	Cost of course	Sum	Cost per shift
Gained competence	10.000		10.000	38,50
Bought competence	10.000	30.000	40.000	153,90
Bought special expertise	5.000	10.000	15.000	57,70

There are three types of costs in the evaluation phase as well.
1) The first cost is an administration cost of rotation nurses between wards. This administration cost can be seen as the real cost of rotation because of administration of salary and invoicing and communication between the wards, which will demand more from the head nurses. Although the cost can also be seen as an artificial cost used in the calculation to prevent symmetry in the results of the evaluation phase. It seems reasonable that the first alternative is the correct interpretation, and the report will thus include this administration cost in the result of the expected cost of scenarios. The cost should be a small number, because of small adjustments in the administration, and has thus been set to 100 NOK.

2) The second cost is the cost of hiring an external nurse, to fulfill the demand of nurses in each scenario. When hiring external nurses the hourly wage is substantially higher than the hourly wage of the employed nurses by the hospital. Generally the recruitment agency charge double or triple the wage at the hospital, to cover the administration costs and to gain profit. This case make following assumptions regarding the level of wages. The basic wage is set to 350 NOK/hour, at the lowest competence level. For the additional competence levels, they in addition charge 20 , 30 and 50 NOK/hour for respectively level 1, 2 and 3. This is regardless of the ward they are hired to. When working a general day-shift of 8 hours, the total

cost for each level 0-3 is thus 2800, 2960, 3040, 3200 NOK/nurse.

3) The third cost is the additional cost of having the externally hired nurses with a set of special expertise. In this case the extra hourly cost of one special expertise is set to 20 NOK/hour, which gives in total for an 8 hour shift 160 NOK per special expertise. This cost is simplified, and is thus regardless of the type of special expertise. As an example, a nurse is hired externally to ward A, with competence level 2 and with 4 special expertises. This will in total cost 3280 + 4 x 160 = 3920 NOK. Compared to a nurse employed at the hospital, with an hourly wage around 180 NOK/hour which gives a total cost of 1440 NOK/nurse , proves that this the cost of an externally hired nurse is almost triple. Table 11 gives an overview of the costs in the evaluation phase.

Table 11: *Overview of the evaluation costs. All the costs are in Norwegian kroner (NOK)*

EVALUATION COST	
Type of cost	Cost
Administration cost per rotation	100
Cost of hiring external nurse at the different levels	2800, 2960, 3040, 3200
Extra cost of each hired special expertise	160

8 Computational Studies

To analyze the base case, consisting of 4 ward and 26 nurses, the model was implemented in the optimize program Xpress MP. Studies were done on both single test runs and extended analyses to get a wide understanding of how the mathematical model proved to behave. The computations are divided into three parts. The first part is technical aspects of the model for both the base case and the medium case and the second part describes a wide analysis of the base case results of one test run. The third and last part is four extended analyses of the base case.

8.1 Technical Aspects of Optimization

Technical aspects of optimization is a description of the variables, restrictions and the computational time. These aspects are sensitive to parameters in the model, which will be presented in the next subsections.

8.1.1 Variables and Restrictions

Variables and restrictions establish the size of the problem. Since the problem is a two stage problem, the variables can also be divided into two; the number of variables in stage 1 and in stage 2. Table 12 describes the number of variables and restrictions of a test run of a 30 scenario base case.

Table 12: *Number of variables and restriction of a test run of a 30 scenario base case.*

A 30 Scenario Base Case	
Restrictions:	64524
Variables:	42880
Variables in stage 1:	29696
Variables in stage 2:	13184

The absence-matrix, described in section 7.1.2, is one aspect of the model that can have an effect on the number of variables and restrictions. Since the absent-matrix is made by each nurse having a probability to be absent, and thus is randomly chosen, the absence-matrix is different for each test run. This consequently alters the number of nurses absent in each scenario. A series of test runs with 10 scenarios were executed to see what kind of effect the absence-matrix had on the variables and restrictions. Figure 5 show the result, and proves that both variables and restrictions decrease with increasing number of total absent nurses. The reason for this decrease in variables and restrictions is that both the variables defining the working ward for each nurse (x_{nwls}), and the linearization variables linking the working ward with the nurses special expertise (k_{nwes}), has been declared dynamic in Xpress MP. This indicate that the variables will not be created unless the appointed nurse in the given scenario is present, and therefor the more nurses absent the less variables of x_{nwls} and k_{nwes} will be created.

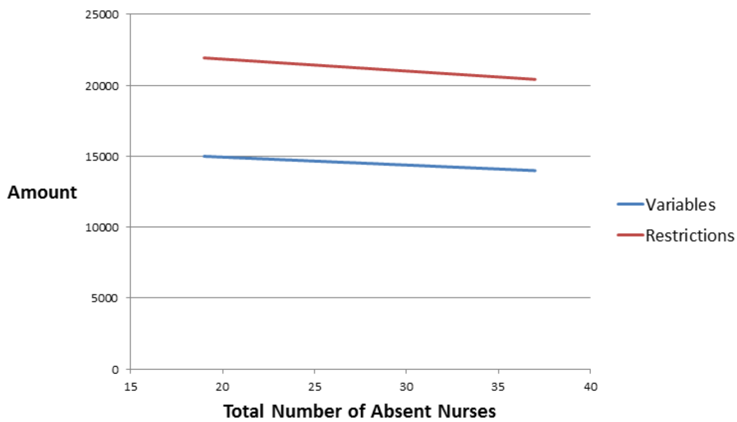

Figure 5: *Number of variables and restriction in relation to variation in total number of absent nurses*

Another parameter that decides how many variables and restrictions there are is how many scenarios is set for a test run. The more scenarios, the larger absence-matrix and the more the amount of created variables depending on scenarios will increase. This indicates that all of the variables in stage 2 will increase. Table 13 is based on the mean value of three different scenario possibilities (10, 20 and 30) run 20 times each, and proves that the variables and restriction does increases with the number of scenarios.

Table 13: *Variables and restriction in relations to variation in scenarios. The values are mean values of 20 test runs per scenario.*

Scenarios	Variables	Restrictions
10	14 580	21 315
20	28 299	42 272
30	41 880	63 024

8.1.2 Computational Time

Computational time is also one of the technical aspects of the computational studies. The expected parameters that can influence the computational time is the number of scenarios, the number of absent nurses and how close the computation gets to the optimal results. By making several computations with different scenarios, the different times could be presented statistically.

A 100 test runs were made of the base case with 10, 20 and 30 scenarios. Several computations were made to discover which parameters were influencing the computational time. The parameters that showed no trend on the time results were: number of absent nurses, the objective value and the amount of possible gained competence levels. The one parameter that gave results were the number of scenarios. Table 14 describe the mean value of time, for each of the scenarios with the 100 test runs. It also show the interval of time. Notice that the upper level of 20 and 30 scenarios is 36 000 seconds. The reason for this specific time is that 36 000 seconds were set as a time restriction on the computation. The computational time for 5 and 3 percent gap from the best bound of the objective value were also registered in the result. These values showed that for all the test runs which reached the time restriction, all of the runs with 20 scenarios came beneath a 3% gap, while the runs with 30 scenarios came between a 5 % and 3 % gap.

Table 14: *Computional time in relation to variation in scenarios. All of the time values are in seconds.*

Scenarios	Mean value Time of Time	Interval
10	685,50	12 - 8331
20	11412,70	228 - 36000
30	29883,40	1924 - 36000

The medium case were also run to check the computational time. Out of 10 test runs, only 4 of them reached a 5% gap to the best bound within the time restriction, and out of these only one reached a 3% gap. This indicates that running with the medium case would take to much time to be able to analyze several perspectives of the model. The following computations will thus be of the base case.

8.2 Analyzing the Base Case

By running the base case one time, a large amount of information could be drawn out. The single test run is done with 30 scenarios, so that the results will be more robust than by using only 10 scenarios. The different aspects of this section is detailed results for both the first and second stage of the model, describing connections between the stages, looking at the relaxation solution of stage two, study the 20 best solutions of this single run, computation of the relaxed version of the problem and calculate some relevant principles of stochastic programming.

8.2.1 General Information

Table 15 describes the general information of the single base case, where the objective value is in the top row. The number of variables and restrictions are the same as in Table 12, as it is the same case, with the same absence-matrix. The table also describes the time to attain a 5% gap between the objective value and the best bound and also the time to the optimal solution.

Table 15: *General information of the single run solution of the base case with 30 scenarios*

General information of solution	
Total Cost:	2141,33 NOK
Cost of gained competence	312 NOK
Cost of bought competence	0 NOK
Cost of bought special competence	138 NOK
Restrictions:	64524
Variables:	42880
Variables in stage 1:	29696
Variables in stage 2:	13184
Time to optimal solution:	36808,7 sec
Time to 5% gap of best bound:	9632,13 sec
Total number of absent nurses:	61
Number of gained competence:	8
Number of bought competence gained:	0
Nurses gained competence:	5, 10, 12, 15, 16, 19, 23 and 25
Number of competence bought:	0
Number of bought special experience:	6

8.2.2 Stage 1 - Hierarchical Competence

As can be seen from Table 15 there has been invested in 8 hierarchical competence levels gained by experience, while the number of bought competence levels are zero. The reason for this is that gained competence by experience is cheaper than bought competence levels, and the initial competence profile gives room for enough nurses to use experience as the method of competence building.

The nurses who increased their competence levels are divided between the different wards, which is illustrated in Table 16. Ward A, B, C and D has respectively 0, 2, 4, and 2 nurses employed at the ward gained competence levels. This indicates that level C is the ward with the most poorly initial competence profile, and thus in greatest need of more competence. The competence increase can also be described from the perspective of which hierarchical levels that has been invested in. The major competence increases is between the hierarchical levels 1-2, then 2-3 and only one has been from 0-1. More in detail for ward C with the most gained competence, the hierarchical levels in need are spread between the three possible leaps in levels. In ward B and D, there are one increase for both level 1 - 2 and 2 - 3. This result show that in this base case it is need for several nurses with competence 2 and some of competence 3, to be more flexible to daily fluctuations in absence.

Table 16: *Illustration of where the nurses gained competence from a perspective of which their home ward is.*

Ward who nurses gained competence	Home Ward: A	B	C	D	Sum
A	-	-	-	-	0
B	-	2	-	-	2
C	1	-	3	-	1
D	-	-	-	2	2

Looking further into which nurses who has gained competence, the results show that each nurse who has gained competence gains in one ward. Most of these nurses gain competence in their home wards, which is a natural place to increase their competence since this is where they work the most. Although one nurse is an exception to this result. Nurse 5 stand out by having gained competence in ward C, while her home ward is ward A. This would be reasonable if she has been rotated a lot to ward C and had use for a better competence in this ward. Although in total she has been rotated seven times, four out of these are to ward B, two to C and once to D. Thus the presumable reason of most rotation is not adequate since she is mostly rotated to ward B. Table 17 illustrates nurse five's results.

Although the eight nurses with increased competence who has been invested in, it is not given that all of these are present in all the scenarios. The mean value of absent nurses all over the 30 scenarios are 3 times. In

Table 17: *Illustration of nurse number five's gained competence level. X indicates in which ward she got the competence, and where she has been rotated to.*

Nurse: 5 Home Ward: A

Competence gained from level 1-2

Gained competence Ward

Ward mostly rotated to	**A**	B	**C**	D
A				
B			**X**	
C				
D				

* average rotated to B = 15 %

* average rotated to C = 7 %

this base case the results actually show that five of the eight nurses are only absent zero or once, which is a very good starting point for investing in these. The three nurses left are absent three to four times for all the 30 scenarios, which is close to the mean value, and therefore it is no reason as an argument for a poor investment.

8.2.3 Stage 1 - Special Experience

As Table 15 show, there has been invested in 6 special expertises. The table also show that only 5 nurses has got special expertise, which indicate that one of the nurses has acquired two different expertises. This nurse is number 11, who has got both special expertise S4 and S9. The different nurses, which by number also are presented in Table 18, have their home ward in three different wards. The distribution over the 4 wards is A(S10), B(S2,S4 and S9), C(S2 and S5) and D(none). The distribution show that nurses in Ward B are invested in the most, which is congruent with nurse 11 who has ward B as has her home ward.

Another aspect of describing the special expertise is to look at what kind of special expertise that is invested in. Is it in the expertise which are specific to the nurses home ward, or is it in the general expertises that works across the wards. Table 9 presented in section 7.1.3, indicate which special expertise are ward-specific or general. Which nurses who got the special expertise, and the overview of the type of special expertise are illustrated in Table 18. The results prove that there are a 50/50 partition between the general and ward specific special expertise. It also show that for each of the ward specific special expertise invested in, they all are in the same home ward as the nurse who got them belongs to. This indicates, as with the hierarchical competence, that the nurses are improving expertise in their home wards.

Table 18: *Overview of the special expertise the single base case invests in.*

Nurse number	Home ward	Special expertise	General or ward-specific expertise
3	A	S10	Ward A
11	B	S4, S9	Ward B and General
12	B	S2	General
15	C	S5	Ward C
16	C	S2	General

8.2.4 Stage 2 - Absence, Hiring and Rotation

Looking further into the solutions in stage 2, the relevant variables are the rotation of nurses and the hiring of external nurses. A description of the absence matrix is also of some interest.

The sum of nurses absent, counting all of the 30 scenarios, are 61. There are two perspectives that can describe the absent nurses. The first is: How often is each nurse absent? The average solution to this is that each nurse is absent in 2,4 times of the scenarios, and the interval of each nurse absent is [0-5]. There are 3 nurses who is never absent, thus 23 nurses are absent in at least one scenario, and the majority is that each nurse is absent 3 times. The second is: How many nurses are absent in each scenario? The average of absence in each scenario is 2 nurses. The interval of absent nurses for each of the 30 scenarios is between [0-6]. Only one scenario has 6 absent nurses, while the majority is 2 and 3 nurses absent per scenario. Comparing these results with the average value of nurses absent, most of them are lower.

Table 19: *Illustration of where the nurses are rotated to, from a perspective of which their home ward is.*

Ward rotated to \ Home Ward	A	B	C	D	Sum
A	-	1	4	2	7
B	12	-	9	0	21
C	2	2	-	0	4
D	2	4	7	-	13
Sum	16	7	20	2	45

There are different number of rotation in each of the scenario, and the interval is between 0 and 7. There are only one scenario that rotates 7 times, while the majority is 1-2 rotations per scenario. The only times there are no rotation is either when no nurses are absent in a scenario, or when there are one absent nurse. In the case of one absent nurse the extra manpower and competence at the given ward outweigh the absence so that no rotation or hire is needed. Table 19 illustrates how many nurses who has rotated from their home ward, to another ward. The numbers are aggregated for all the scenarios, and show that the ward most rotated to is ward B. The ward that ables the most nurses to rotate to other ward is ward C. This results indicate that ward B has the highest amount of absent nurses, while ward C has the least amount of absent nurses, which is the reality of the absence-matrix.

The external hiring of nurses from recruitment agencies are still in use, but for a small amount of scenarios. A total of 21 out of 30 scenarios do not hire external manpower at all, while the remaining 9 scenarios has 1-2 external hires. Comparing the number of external hires in these 9 cases, the number is lower than the amount of nurses absent for all of the cases, which is a positive result. The difference of hired nurses between the different wards are low, but the ward with the most hires is ward A, while the lowest hire is in ward C. This proves that they hire more external nurses in the ward with no competence building while they hire less in the wards invested into, which is a reasonable result. The number of hired nurses with extra special expertise is also low. The amount of hires of special expertise is actually only in 6 out of the 30 scenarios.

8.2.5 Stochastic Principles

When using stochastic programming to handle uncertainty, there are some principles that describe how the solution is valued and how it is compared to other methods of programming.

The Stochastic Solution (SS)
The stochastic solution is the solution produced by the mathematical competence model, since the model is built with stochastic programming.

The Wait and See solution (WSS)
If the decision maker of the problem waits until the uncertain variables becomes clear before he evaluate the problem, he works with perfect information. Thus with perfect information the problem becomes a deterministic model, which can be solved with a regular mixed integer program (MIP). The WSS is the expected value of all the scenario solutions as perfect information.

Expectation of the Expected Value (EEV)
In the EEV solution, the decision is based on the expected value of the uncertain variables. As an example, the decision maker is faced with two different outcomes 10 or -5. The expected value of these values is 2,5, and thus the stochastic model model sets the uncertain variable to 2,5 before executing the test runs for each scenario. In this model, the expected value is a more complicated decision. Finding it by the same method would be to find the the expected value of each nurse to be absent, but nurses can not be absent in for instance 0,5 times. They are either absent or not. Thus in this case the definition of the expected scenario is that all of the nurses

are present, which is what the schedule of nurses are based on. With no absent nurses the solution of the variables in stage 1 turns to zero, which means the same as no competence building. With this solution of stage 1, the solution of stage 2 is found for each of the scenarios. EEV is then the expected value of all the stage 2 solutions.

Expected Value of Perfect Information (EVPI)
EVPI is the difference in objective value between the solution based on perfect information and the stochastic solution. EVPI describes how much the decision maker is willing to pay for perfect information.

$$EVPI = SS - WSS \tag{32}$$

Value of Stochastic Solution (VSS)
VSS is the difference between the objective value of the stochastic solution and the expectation of the expected value, and thus describe how much the decision maker benefits from taking uncertainty into account by using stochastic programming, compared to handling the problem based on only expected value.

$$VSS = EEV - SS \tag{33}$$

The solutions to all the different principles computed by the model are presented in Table 20, while the principles calculated by the different solutions are presented here in the text.

Table 20: *Overview of solutions of stochastic principles.*

SOLUTIONS OF STOCHASTIC PRINCIPLES	
Type of Solution	Objective Value
SS	2141,33 NOK
WSS	1688,33 NOK
EEV	4409,33 NOK

EVPI is the difference between the SS and the WSS solutions, and the result became 453. This proves that the hospital are willing to pay 453 NOK to know the perfect information of absence for each shift. This is a logical result, compared to a value of zero or negative, because with those kind of values the perfect information should have bee known and thus a deterministic model would have been the best way of dealing with this problem.

VSS which is the value of the stochastic solution is as described above, the difference between the EEV and the SS solutions, and the result became 2268 NOK. This proves that by handling this problem with a stochastic model, compared to only the expectation of the expected value, the hospital decreases it's cost by half of the value. This is a high value, and thus indicate that the stochastic method is a good fit for this specific problem. The high value can though also be a result of the chosen definition of EEV, which was a different kind of interpretation of the original definition.

8.2.6 The 20 Best Solutions

When XpressMP execute the optimization, it works its way through several solutions until the objective value can not get any better. This indicates that it starts with a poor solution and results in the optimal solution. To comprehend how the development throughout the computation are, it is interesting to look into the optimizers 20 best solutions.

Extracting the 20 best solutions from the result of the base case, there were several aspects that were interesting to look into. How are the other solutions different from the optimal solution, is the objective value the same, is the amount of competence building the same and which wards and nurses increase their competence? Though when analyzing the results all of the above questions had the same answers: The numbers in all the 20 solutions were the same as the optimal solution, which means that all of the solutions from stage 1 were identical. The conclusion from this is that the difference lies in stage 2, and that all of the solutions of stage 2 are symmetric. Different nurses are rotated, and the external hires are in other wards and levels, but the numbers of rotated nurses and hired nurses are the same as in the optimal solutions. The underlying reason for this might come from the simple cost structure chosen for this base case. As the costs in stage 1 are now, they do not depend on which level of competence the nurses gains or for which kind of special expertise they get. An optimal cost structure may have a more detailed structure.

8.2.7 LP Relaxation of Stage 2

By relaxing variables that are set as integer or binary, an upper bound of the MIP can be found. Table 21 gives an overview of the results from relaxing the base cases variables in stage 2. These variables are: the variables indicating where the nurses present work in each scenarios (x_{nwls}), the variables indicating how many external nurses get hired in each scenarios (y_{wls}), the variables indicating how many special expertises the externally hired nurses hold (z_{wes}), and at last the linearization variable indicating if the nurse working at a specific ward holds a specific special expertise (k_{nwes}).

Table 21: *General information of the relaxed solution of the base case with 30 scenarios*

General information of solution	
Total Cost:	1 953,81 NOK
Cost of gained competence	117 NOK
Cost of bought special competence	161 NOK
Number of competence gained:	3
Nurses gained competence:	14,15,16
Number of competence bought:	0
Number of bought special experience:	7

As the table show, the cost are lower and can thus be seen as the best possible result of this base case, though this result is unattainable with the integer requirement in the regular problem. In addition to the difference in cost, the building phase show a change from the regular results. First of all the total amount of competence gained has decreased, while the amount of bought special expertise has increased.

Comparing which nurses who are invested in, two of the nurses are the same as in the regular result. The wards of gained competence for these nurses are all in ward C, which is their home ward. Compared to the regular results, it is the same ward that is invested in the most. The difference is that two, out of the three, competence levels gained are from 0-1. This show that the relaxed problem invest in the lower competence levels, while the regular problem proved to invest in the higher competence levels.

The variables with the most different solution is which nurses who has got special expertise. There are only three out of the six nurses who are the same, in both results. The specific expertise invested in are thus mostly the same, except from one extra (S1) in the relaxed solution. As well as the gained competence, the ward specific expertise are given to nurses with their belonging home-wards.

Looking at stage two the relaxation results has less rotation of the nurses. This is because of the relaxed situation and the possibility to be real numbers instead of integer. Comparing with the regular result, the difference lies in that the numbers are rounded up. The numbers of hired external nurses are thus the same.

Although there are several differences in the solution, the major difference lies in the lower number of gained competence, which decrease the objective value with approximately 9%.

Taking the solution from stage 2 and implement it in the competence matrix, a second test run could be made. This test run were made with no competence building as to evaluate the relaxed solution without relaxed variables. The result became a higher objective value, at 2350 NOK (a 10% increase), which show that the relaxed solution is in fact a less beneficial solution. This solution indicate an upper value of the regular problem. The number of nurses rotated also show that there are less rotations compared with the optimal solution. This proves that by optimize the competence building the possibility to rotate increases and by that the cost of hire external nurses decreases. These results are illustrated in Table 22.

8.3 Extended Analyses

Extended analyses are computational studies based on several test runs of the model, to gain insight on results with statistical support. To limit the computational time, these studies are operating with 10 scenarios, unless another amount is specified.

Table 22: *Illustration of where the nurses are rotated to, from a perspective of which their home ward is. The solution is from the test run where the relaxed solution in stage 1, are fixed in the model.*

Ward rotated to \ Home Ward	A	B	C	D	Sum
A	-	1	0	0	1
B	3	-	12	0	15
C	1	1	-	1	3
D	4	1	7	-	12
Sum	8	3	19	1	31

8.3.1 Solutions with Variation in the Permission to Gain Competence

As the results of section 8.2 show that when allowing the nurses to gain competence there are no bought hierarchical competence. This is because the bought competence is more expensive. An interesting perspective is then how the solution reacts to variation in the possibility to gain competence (G_n^{MAX}). Six test runs with the same absence-matrix were executed with different G_n^{MAX}.

The results of these computational test runs showed that for the test run with G_n^{MAX}=0, the solution becomes to buy two competence levels. Instead of gaining it with experience. Compared with the solution in section 8.2 where they only invest in gained competence and where G_n^{MAX}=1. By investing in bought competence the objective value becomes higher than the other five solutions. Both of the bought competence levels were for nurses working in ward D, which also are in a different ward than the majority of nurses gained which competence in the solution in section 8.2. When the possibility to gain competence gets higher in test run [2-6], the objective value, the number of gained competence and the number of bought special expertise converge to the same solutions as in section 8.2. This proves that even by increasing the time horizon, as the value of G_n^{MAX} is an indicator for, the competence building do not change. The difference, from each solution are which nurses who actually gain their competence. Most of the nurses invested in are the same in each solution, both for the nurses who gained hierarchical competence and those who bought special expertise, but some of them change from test to test. This can indicate that there are some symmetry in the model, as also described in section 8.2.6.

After this computational study, it is interesting to look into how the problem reacts to variation in possible numbers of bought special experience (E_n^{MAX}). The same absence-matrix were used, and the parameter E_n^{MAX} changed one by one, from 0 - 7. This variation has some different results than the variation in G_n^{MAX}. First of all the objective function converge after two runs, instead of one. In this case the two first objective values are higher then the rest, which is a result of investing in more gained competence, to cover for the low possibility to buy special experience. Thus when increasing E_n^{MAX}, the number of nurses with gained competence decreases to a lower number while the number of bought special experience stays at the same amount. Comparing with the result of increased G_n^{MAX}, the result show that it is the same nurses who gets invested in. This goes for both nurses who gained hierarchical competence and who bought special expertise.

8.3.2 Cost of Absence in Relations to Absent Ratio

To compare the strategic competence building model with the project model described in section 5, a similar graph to Figure 3 was made with the stochastic model. The graph is a presentation of how the cost of absence, calculated by the project model, variate in relation to the absence ratio. A stochastic solution was found for the given percentage, and the result from stage 1 was further used as the new competence profile. Then the initial and optimal competence profile were run a thousand times for each increase of absent ratio. The mean value of the thousand test runs are presented in Figure 6. The values of the expected cost from the optimal competence profile also include the building expenses from the stochastic solution in all the percentage levels.

Figure 6: *Cost with variation in absence percentage. The cost of the optimal competence profile is included the building cost of the stochastic solution.*

The graph show that when having a higher and better competence profile, the cost of absence is lower, even though the building phase produce an extra yearly fixed cost by increased salary. The interval that seems to have the largest deviation between the two graphs are 7 - 13 %. This is positive result because this interval correlates closely with the real life absence ratio registered by A-HUS[14]. These findings support the conclusion in the project described in section 5, that by investing in the human capital, the hospital get more flexible to handle daily fluctuations in absence.

8.3.3 Stochastic Principles of Several Test Runs

To evaluate the results of the stochastic principles presented in 8.2.5, several test runs and calculations were executed for different absence-matrices. All of the test runs were done for 10 scenarios, and the results for the individual test runs are shown in Figures 7 and 8.

The first figure illustrate that all of the values are variable from each of the test runs, which indicates no coherence between the different solutions. This can be explained by the use of different and randomly picked scenarios.

Figure 8 illustrates the values of the EVPI and the VSS calculations, for the same test runs that are presented in Figure 7. The EVPI solutions show the most stable graph of both of the figures, and supports the solution of EVPI in section 8.2.5. VSS on the other hand is also very variable, though the values seems to fluctuate around 2300 NOK. All of the values show thus that the choice of using stochastic programming is a good fit for this specific problem.

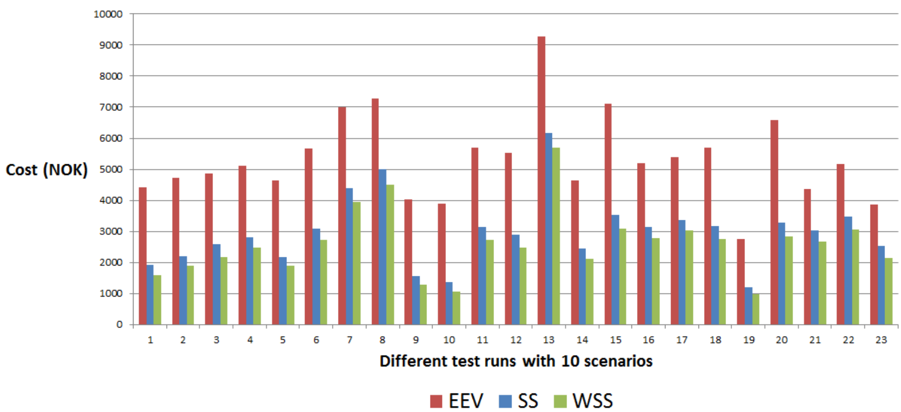

Figure 7: *SS, WSS and EEV for 23 individual test runs using 10 scenarios.*

As indicated above, the test runs show no coherence with each other. Presumably with a larger set of test runs the different solutions could show a more stable trend, though the independent solutions can also be the result of using 10 scenarios compared to 20 and 30 scenarios. Thus the result of using a larger set of scenarios would be an interesting aspect to look further into. This will be the task of the next section.

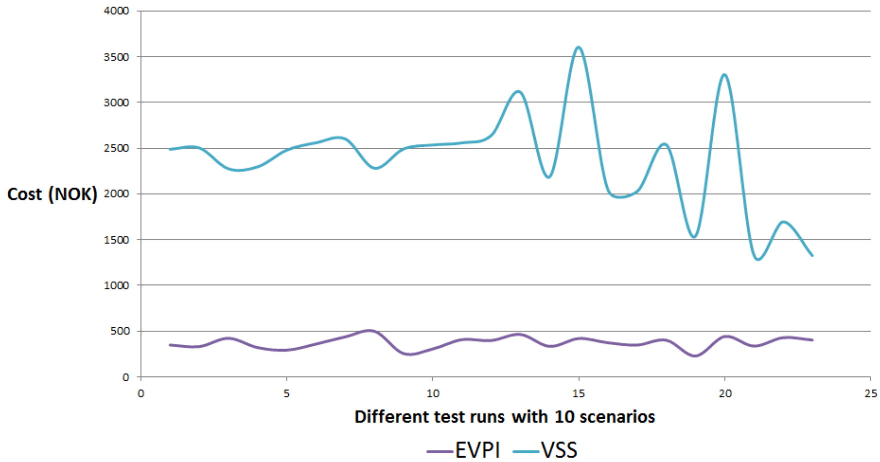

Figure 8: *Illustration of EVPI and VSS with different test runs using 10 scenarios.*

8.3.4 Out of Sample Solution

For the single base case the number of scenarios were set to 30, while most of the extended analysis were executed with 10 scenarios to reduce the computational time. As described in section 7.1.2, the more scenarios used the more accurate is the evaluation phase in this problem, because it increases the possibility of describing the actual absence scenario. Section 8.3.3 also question the accuracy of using only 10 scenarios. The question then is how many scenarios are enough? Is there a severe difference in the results by using 10, 20 or 30 scenarios?

To evaluate the accuracy of the different scenario options, an *out of sample test* was carried out. Ten solutions were found for each scenario option [10,20,30]. For each of these solutions 400 test runs were executed in the project model. The result became 12 000 solutions for each scenario option, which gave a reasonable basis for a statistical study. Table 23 show the mean cost of this post evaluation in the project model from section 5, and standard deviation for the same data set is described in column two. The results show that the post evaluation cost decreases with increasing scenarios, and the difference is larger between scenario 10 and 20 than for the interval 20 to 30, which implies a decreasing marginal cost. This show that by increasing the amount of scenarios the post evaluation becomes closer, and thus by having 20 or 30 scenarios it becomes more accurate then by the use of 10 scenarios. It is reasonable to presume that by increasing the scenarios further the post evaluation converge into the same values.

Table 23: *Out of Sample Solutions*

Scenarios	Post Evaluation cost	Standard deviation of the costs
10	3 476	3 811
20	3 326	3 824
30	3 304	3 773

Table 24: *Students t-test of the different data sets: The probability (P-value) of the two data sets indicates if the two data sets are from the same population.*

Data set	P-value
10/20	0,24 %
10/30	0,04 %
20/30	65,61 %

Since the gap between the values of 20 and 30 scenarios are small, compared to the gap between 10 and 20, it is reasonable to consider that maybe 20 scenarios are enough. To get a closer evaluation of this, a Students t-test of the three data sets were done. A Students t-test is a statistical method which examine if it is a significant difference between two data sets. In other words, the t-test show whether or not the two data sets come from the same population.

Statistical significance indicate the probability of the observed difference between data sets are due to chance. The P-value, as presented in Table 24, describes this probability for each pair of data sets. Before evaluating the values, the level of significance (α) has to be set as a fixed. This level has a purpose of making it possible to evaluate the results from the t-test. If the P-value is lower than α, the two data sets are significantly different. The level is set to the most common level, which is 5%. Table 24 show that for both the data pairs [10 / 20] and [10 / 30] the P-value is lower than the significance level. This indicate that the data set of 10 scenarios belong to a different population, and thus is significantly different from the two data sets of 20 and 30 scenarios. This can be matched with the indication made by the values in Table 23. The result also show that the data pairs [20 / 30] has a strong relation between each other. This close relation proves that 20 scenarios can give almost the same accuracy in the solution, as by using 30 scenarios, and may then be enough to describing the realistic absent scenario.

9 Conclusion

This report has described nurse competence building as a strategic tool to a better utilization of a hospitals resources, because one of the hospital managements most important assignment is to map its competence profiles and make a strategic plan of how to utilize the competence to the fullest. The mathematical model presented for this competence building problem is a two stage stochastic model. The first stage is about building competence, with an initial competence profile as a starting point. The choice of competence building in the model were between gained competence by experience, bought competence by courses and bought special expertises. The second stage is the evaluation of the building solution. This involve evaluation of different absence scenarios by the use of nurse rerostering.

Implementation of the mathematical model in Mosel Xpress, gave positive results. First a base case of a simplified hospital, with 4 ward and a total of 26 employed nurses, were made. Computational studies were done on both single test runs and extended analyses of the base case, to get a wide understanding of how the mathematical model proved to behave. A single test run of the case, with 30 absent scenarios, proved that the hospital benefited by investing in the nurse and giving them the opportunity to increase their competence. The result showed that 8 nurses gained competence by experience, while there were no bought competence. The reason for no bought competence in this specific case were the higher cost of of bought competence and that the competence structure of the nurses allowed enough competence gained by experience. The amount of bought special expertises were 6. Details of the competence building proved nurses mainly increased their competence in their home wards, and this included both gained competence and bought special expertise. Both single test runs and extended analysis proved that by increasing the competence profile more rotations of nurses between the different wards were done. This increased amount of rotation further led to lower cost of external hires and thus decreased the cost of absence.

To be able to compare the strategical tool with the rerostering model described in section 5, a cost analysis with variation in absence ratio were carried out. The results proved that by finding an optimal competence profile, the cost of responding to absence were reduced with comparison to the results of the initial competence profile. Even though the cost included the additional cost and investment cost of increasing the competence.

The four stochastic principles, WSS, EEV, EVPI and VSS were also computed for both the single test run of the base case and extended analysis of several individual test runs. The values of EVPI showed that the hospital were willing to pay approximately 450 NOK per shift to retrieve the perfect information of day to day absent scenarios, which proves that the model operates with uncertain variables. VSS were calculated to be 2268 NOK for the single base case, a value supported by the individual test runs. This proves that by handling this problem with a stochastic model, compared to only the expectation of the expected value, the hospital reduced the costs with 50%. This is a high value, and thus indicate that the stochastic method is a good fit for this specific problem.

To apprehend how the stochastic model responded to the amount of scenarios used, an out of sample test were carried out. The presumable result was that the accuracy of the solution improved by the number of scenarios, as the probability of describing the realistic absence increased. The question then was how many scenarios were enough. Post evaluation of different solutions from 10, 20 and 30 scenarios were analyzed, and the results showed that the values of 20 and 30 scenarios were more similar, compared to the 10 scenarios. A t-test confirmed that by using 10 scenarios, the solution accuracy were lower compared to the use of 20 and 30 scenarios. The t-test also showed that 20 scenarios gave almost the same accuracy as with 30 scenarios, which implies that in this specific problem 20 scenarios might have been good sufficient.

The final conclusion is that the hospital do benefit from optimizing the competence profile, by investing in the nurses and increasing the human capital. The cost of responding to fluctuations in absence decreases from the initial profile, even though an additional investing cost increased the fixed cost of the yearly salary.

Further work on this field is to do the computations with a higher amount of scenarios, to make the computations more accurate. It is also possible to include a nursing pool to lower the cost of external hires, by utilizing the already employed nurses even more. The "hiring" of nurses in the pool will probably be less expensive, even though the hospital would have to pay a fixed cost of having the nurses on call.

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Appendix: Mosel code

```
model nurse_strategic
options explterm
uses "mmxprs", "mmsystem"; !gain access to the
!Xpress-Optimizer solver

parameters
Datafile = 'mininursedata.dat';
Scenariofile = 'ScenarioFile.dat';
end-parameters

!initializations from 'specialexpertise.dat'
!end-initializations

!optional parameters section

declarations
! parameters from data
N: integer ;
PROB: real ;
SPROB: real ;
NURSE: set of integer ;
WARD: set of string  ;
LEVEL: set of integer ;
WLEVEL: set of string ;
SPEC: set of string ;
SCENARIO: set of integer ;
NUMBSCEN: integer ;
G: integer ;
U: integer ;
V: integer ;
GCOST: integer ;
MCOST: integer ;
UCOST: integer ;
GTOT: integer ;
```

```
UTOT: integer ;
VTOT: integer ;
ETOT: integer ;
T:     integer ;
end-declarations

initializations from Datafile
N ;
WARD ;
LEVEL ;
SPEC ;
WLEVEL ;
PROB ;
!HA !SPROB ;
!HA !SCENARIO ;
NUMBSCEN ;
end-initializations

!initializations from Scenariofile
!T ;
!NUMBSCEN ;
!end-initializations


SCENARIO := 1..NUMBSCEN;

SPROB := 1/getsize(SCENARIO);

NURSE:=1..N ;

declarations
! arrays from data
ROSTER: array(NURSE,WARD) of integer ;
COMP: array(NURSE,WLEVEL)  of integer ;
ECOMP: array(NURSE,SPEC) of integer ;
DEMAND: array(LEVEL,WARD) of integer ;
```

```
EDEMAND: array(SPEC,WARD) of integer ;
AGG: array(LEVEL,WARD) of integer ;
ACOST: array(NURSE,WARD) of integer ;
OCOST: array(LEVEL,WARD) of integer ;
ECOST: array(WARD,SPEC) of integer ;
SPRESENT: array(NURSE,SCENARIO) of integer ;
Q: array(NURSE,WARD) of integer ;
ABSENT: array(NURSE,SCENARIO) of integer ;


! constraints
BUILDCOMP: array(NURSE,WARD) of linctr ;
MAXBUILDG: array(NURSE) of linctr ;
MAXBUILDU: array(NURSE) of linctr ;
MAXBUILDV: array(NURSE) of linctr ;
MAXBUILDE: array(NURSE) of linctr ;
LEVELDEMAND:  array(WARD,LEVEL,SCENARIO)  of linctr ;
SPECIALDEMAND: array(WARD,SPEC,SCENARIO) of linctr ;
ONEWARD: array(NURSE,SCENARIO) of linctr ;
WORKLEVEL: array(SCENARIO,WARD,NURSE) of linctr ;
CONNECTYZ: array(WARD,SPEC,SCENARIO) of linctr ;
BUILDSPEC: array(NURSE,SPEC) of linctr ;
LINEARONE: array(NURSE,WARD,SPEC,SCENARIO) of linctr ;
LINEARTWO: array(NURSE,WARD,SPEC,SCENARIO) of linctr ;
end-declarations


!Who is absent in scenario s??
forall (n in NURSE, s in SCENARIO) do
rand:= random ;
if (rand<=PROB) then ABSENT(n,s):= 1 ;
else ABSENT(n,s):=0 ;
end-if
end-do

!initializations to 'Absent833.dat'
```

```
!ABSENT ;
!end-initializations


initializations from Datafile
ROSTER ;
COMP ;
ECOMP ;
DEMAND ;
EDEMAND ;
ACOST ;
OCOST ;
ECOST ;
GCOST ;
UCOST ;
MCOST ;
G ;
U ;
V ;
GTOT ;
UTOT ;
VTOT ;
ETOT ;
end-initializations


declarations
! decision variables
x: dynamic array(NURSE,WARD,LEVEL, SCENARIO) of mpvar ;
y: array(WARD,LEVEL, SCENARIO) of mpvar ;
z: array(WARD,SPEC, SCENARIO) of mpvar ;
g: array(NURSE,WARD) of mpvar ;
u: array(NURSE,WARD) of mpvar ;
v: array(NURSE,WARD) of mpvar ;
q: array(NURSE,WARD) of mpvar ;
o: array(NURSE,SPEC) of mpvar ;
```

```
m: array(NURSE,SPEC) of mpvar ;
k: array(NURSE,WARD,SPEC,SCENARIO) of mpvar ;
end-declarations

!Q indicates which is the INITIAL highest
!level nurse n has at ward w
!
forall (n in NURSE) do
forall (w in WARD) do
forall (l in LEVEL) do
txt:=w+l;
if COMP(n,txt)=1 then Q(n,w):=l ;
end-if
end-do
end-do
end-do

countstagetwo := 0 ;

forall ( w in WARD, l in LEVEL, s in SCENARIO,
n in NURSE | ABSENT(n,s)=0) do ! because it's dynamic
create(x(n,w,l,s));
x(n,w,l,s) is_binary;
countstagetwo:=countstagetwo + 1 ;
end-do

forall (w in WARD, l in LEVEL,s in SCENARIO) do
y(w,l,s) is_integer;
countstagetwo:=countstagetwo + 1 ;
end-do

forall (w in WARD, e in SPEC, s in SCENARIO) do
z(w,e,s) is_integer;
countstagetwo:=countstagetwo + 1 ;
end-do
```

```
forall (w in WARD, n in NURSE) do
g(n,w) is_integer;
end-do

forall (w in WARD, n in NURSE) do
u(n,w) is_integer;
end-do

forall (w in WARD, n in NURSE) do
v(n,w) is_integer;
end-do

forall (w in WARD, n in NURSE) do
q(n,w) is_integer;
end-do

forall (e in SPEC, n in NURSE | ECOMP(n,e)=0) do
create(o(n,e));
o(n,e) is_binary;
end-do

forall (e in SPEC, n in NURSE) do
create(m(n,e));
m(n,e) is_binary;
end-do

forall (w in WARD,e in SPEC, s in SCENARIO,
n in NURSE | ABSENT(n,s)=0) do
create(k(n,w,e,s));
k(n,w,e,s) is_binary;
end-do

!Aggregated demand
forall (l in LEVEL, w in WARD) do
AGG(l,w):=0;
forall (i in LEVEL | i>=l) do
```

```
AGG(l,w):=AGG(l,w)+DEMAND(i,w);
end-do
end-do

setparam('xprs_verbose',true);
setparam('xprs_miplog',-10);
setparam('xprs_miprelstop',0);

!objective function (9)
GainCompCost:= sum(w in WARD,n in NURSE)GCOST*g(n,w);
BoughtCompCost:= sum(w in WARD,n in NURSE)UCOST*u(n,w);
BoughtSpecCost:= sum(n in NURSE, e in SPEC)MCOST*o(n,e);
ExpScenarioCost:= SPROB*(sum(w in WARD,l in LEVEL, s in
SCENARIO)OCOST(l,w)*y(w,l,s) +
sum(w in WARD, e in SPEC, s in SCENARIO)
ECOST(w,e)*z(w,e,s)) ;
AdministrationCost:= SPROB*(sum(s in SCENARIO, n in NURSE |
ABSENT(n,s)=0, w in WARD, l in LEVEL)
ACOST(n,w)*x(n,w,l,s)) ;
Cost:= GainCompCost + BoughtCompCost + BoughtSpecCost +
ExpScenarioCost + AdministrationCost;


! constraints (10)
forall (n in NURSE, w in WARD) do
BUILDCOMP(n,w):=Q(n,w)+g(n,w)+u(n,w)-v(n,w)=q(n,w) ;
end-do
!
forall (n in NURSE, e in SPEC) do
BUILDSPEC(n,e):=ECOMP(n,e) + o(n,e)= m(n,e);
end-do
!+o(n,e)

!Constraints (11-13)
!non-negativity and restrictions building part:
```

```
forall (w in WARD, n in NURSE) g(n,w)<=G ;

forall (w in WARD, n in NURSE) u(n,w)<=U ;

forall (w in WARD, n in NURSE) v(n,w)<=V ;

!Constraints (12-14)
forall (n in NURSE) do
MAXBUILDG(n):= sum(w in WARD)g(n,w) <= GTOT ;
MAXBUILDU(n):= sum(w in WARD)u(n,w) <= UTOT ;
MAXBUILDV(n):= sum(w in WARD)v(n,w) <= VTOT ;
MAXBUILDE(n):= sum(e in SPEC)o(n,e) <= ETOT ;
end-do


forall (w in WARD, n in NURSE) g(n,w)>=0 ;

forall (w in WARD, n in NURSE) u(n,w)>=0 ;

forall (w in WARD, n in NURSE) v(n,w)>=0 ;

!Constraints (9) ensure that the minimum demand is
!fulfilled in all the combinations of ward and
!the hierarchic competence levels,
forall (w in WARD, l in LEVEL, s in SCENARIO) do
LEVELDEMAND(w,l,s):= sum(n in NURSE, i in LEVEL |
ABSENT(n,s)=0 and l <= i)x(n,w,i,s) +
sum(i in LEVEL | i>= l)y(w,i,s) >=  AGG(l,w) ;
end-do

forall ( s in SCENARIO, n in NURSE | ABSENT(n,s)=0 ,
 w in WARD, e in SPEC) do
LINEARONE(n,w,e,s):= k(n,w,e,s) <= sum(l in LEVEL
)x(n,w,l,s) ;
end-do
```

```
forall ( s in SCENARIO, n in NURSE | ABSENT(n,s)=0 ,
w in WARD, e in SPEC) do
LINEARTWO(n,w,e,s):= k(n,w,e,s) <= m(n,e) ;
end-do

!constraints (10) ensures the minimum demand of special
!expertises in each ward.
forall (w in WARD, e in SPEC, s in SCENARIO) do
SPECIALDEMAND(w,e,s):= sum(n in NURSE | ABSENT(n,s)=0)
k(n,w,e,s) + z(w,e,s) >= EDEMAND(e,w) ;
end-do

!Constraints (11) make sure that each nurse only
!works in one ward at the time.
forall (n in NURSE, s in SCENARIO) do
ONEWARD(n,s) := sum(w in WARD, l in LEVEL) x(n,w,l,s) =
1 - ABSENT(n,s); ! HA
end-do

!constraint (12) makes sure that the workinglevel of
!nurse n is equal or less than the competence level n
!has after the building phase
forall(s in SCENARIO, w in WARD, n in NURSE | ABSENT(n,s)=0) do
WORKLEVEL(s,w,n):= sum(l in LEVEL) l * x(n,w,l,s) <= q(n,w) ;
end-do

!constraints (13)
!To connect the two variables that undertake hiring
!of external manpower, constraints (5)
!ensure that only the actual nurses hired can have the
!special expertises they
!are in shorthand with.
forall (w in WARD, e in SPEC, s in SCENARIO) do
CONNECTYZ(w,e,s):= sum(l in LEVEL)y(w,l,s) >=z(w,e,s) ;
end-do
```

```
!constraints(14-15)
!nonnegativity
forall (w in WARD, l in LEVEL, s in SCENARIO) y(w,l,s)>=0 ;

forall (w in WARD, e in SPEC, s in SCENARIO) z(w,e,s)>=0  ;

! HA
declarations
start: real;
Time3: real;
Time5: real;
PreRows: integer;
PreCols: integer;
end-declarations
start := gettime;

public function TimeCheck: boolean
setparam("XPRS_SOLUTIONFILE",0);
declarations
gap: real;
end-declarations

if (Time3 * Time5 = 0) then
gap := (getparam('xprs_mipobjval') -
getparam('xprs_bestbound')) / getparam('xprs_mipobjval');
if (gap < 0.05 and Time5 = 0) then
Time5 := gettime - start;
end-if
if (gap < 0.03 and Time3 = 0) then
Time3 := gettime - start;
end-if
end-if
setparam("XPRS_SOLUTIONFILE",1);
end-function

setcallback(XPRS_CB_OPTNODE, 'TimeCheck');
```

```
minimize(Cost);

writeln(strfmt("TotalCost",9),strfmt("GainedCost",11),
strfmt("BoughtCost",11),
strfmt("BoughtSpecialCost",18),strfmt
("ExpScenarioCost",17));
writeln(strfmt(getobjval,8),strfmt(getsol(GainCompCost),9),
strfmt(getsol(BoughtCompCost),10),
  strfmt(getsol(BoughtSpecCost),14),strfmt(getsol(
  ExpScenarioCost),18)) ;


writeln(strfmt("NumbGained",10),strfmt("NumbBought",11),
strfmt("NumbLost",10),strfmt("NumbSpecialExp",16)) ;
writeln(strfmt(sum(n in NURSE,w in WARD)getsol(g(n,w)),8),
strfmt(sum(n in NURSE,w in WARD)
getsol(u(n,w)),8), strfmt(sum(n in NURSE,w in WARD)
getsol(v(n,w)),8), strfmt(sum(n in NURSE,e in SPEC)
getsol(o(n,e)),12)) ;

writeln(strfmt('Ward',4),strfmt('Gained',7),strfmt('Bought',8)
,strfmt('Lost',6)) ;
forall(w in WARD) do
writeln(strfmt(w,3),strfmt(sum(n in NURSE)getsol(g(n,w)),6),
strfmt(sum(n in NURSE)getsol(u(n,w)),6),strfmt(sum(n in NURSE)
getsol(v(n,w)),5));
end-do

writeln(strfmt("nurse ",6),strfmt("numbspecial ",7));
forall(n in NURSE) do
special:= sum(e in SPEC)getsol(o(n,e)) ;
if special>0 then
writeln(strfmt(n,4),strfmt(special,7)) ;
end-if
end-do
```

```
writeln(strfmt("nurse ",6),strfmt("numbgained ",7));
forall(n in NURSE) do
gain:= sum(w in WARD)getsol(g(n,w)) ;
if gain>0 then
writeln(strfmt(n,4),strfmt(gain,7)) ;
end-if
end-do

writeln(strfmt("nurse ",6),strfmt("numbbought ",7));
forall(n in NURSE) do
bought:= sum(w in WARD)getsol(u(n,w)) ;
if bought>0 then
writeln(strfmt(n,4),strfmt(bought,7)) ;
end-if
end-do

writeln(strfmt("Absent nurse for all scenarios ",3));
write(strfmt("nurse ",6));
forall(s in SCENARIO)write(strfmt(s,3));
writeln;

forall(n in NURSE |sum(s in SCENARIO)ABSENT(n,s)>0) do
write(strfmt(n,5));
forall(s in SCENARIO)do
if (ABSENT(n,s)=1) then write(strfmt("1",3));
else write(strfmt("-",3));
end-if
end-do
writeln;
end-do
write(strfmt("sum ",5));
forall(s in SCENARIO)write(strfmt(sum(n in NURSE)
ABSENT(n,s),3));
writeln;
```

```
writeln("Total number of absent nurses : ",
sum(n in NURSE, s in SCENARIO)ABSENT(n,s)) ;

end-model
```