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Optimization of offshore natural gas field development

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MASTERKONTRAKT
- uttak av masteroppgave

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Oppgavetekst/Problembeskrivelse The purpose of this thesis is to develop a solution approach for an offshore natural gas field development problem and to test the performance of this approach using real field data. The goal of the offshore natural gas field development problem is to maximize the total net present value for a natural gas field development over the field life time, typically several decades. Main contents: 1. Acquire real cost and reservoir data for testing. 2. Decide which problem formulation should used. 3. Implement the model in XpressMP 4. Investigate solution approach(es) for handling integer variables and multiple common constraints	
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4. Underskrift

Student: Jeg erklærer herved at jeg har satt meg inn i gjeldende bestemmelser for mastergradsstudiet og at jeg oppfyller kravene for adgang til å påbegynne oppgaven, herunder eventuelle praksiskrav.

Partene er gjort kjent med avtalens vilkår, samt kapitlene i studiehåndboken om generelle regler og aktuell studieplan for masterstudiet.

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Preface

This Master Thesis is written as the final work completing the study of Industrial Economics and Technology Management at the Norwegian University of Science and Technology (NTNU). It combines the fields of operations research and petroleum technology, allowing a large range of the knowledge acquired during the author's studies to be used.

I would like to thank supervisor Mikael Rönnqvist and co-supervisor Vidar Gunnerud for valuable input and advice during the development of the thesis work. Further, I would like to thank my friends on the study of Industrial Economics and Technology Management. Without you the world would be a much less entertaining place.

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Gaute Rannem Johansen

Abstract

In this thesis the target is to find the optimal development solution of an offshore natural gas field. Natural gas is increasing in importance as an energy source. Whilst most of the large oil fields have been developed, there are still several major natural gas deposits that may be developed. In addition, there are also smaller offshore natural gas fields that may be put into production. Finding an optimal development solution for these resources will increase the availability of natural gas.

The objective of the mathematical model presented in this thesis is to maximize the total net present value of an offshore natural gas field development. The model does this by finding the optimal combination of investment decisions of the necessary natural gas field infrastructure. Infrastructure included in the model includes the number of wells to be drilled, flowlines, production facilities, energy infrastructure on site and transport infrastructure to the customers. The model also decides whether gas sales agreements should be made with selected customers and the natural gas production in all time periods.

This offshore natural gas field development problem is formulated as a mixed integer linear programming problem. Piecewise linearization is used to increase the accuracy of the reservoir model and to find the energy demand for a given natural gas flow rate and pressure difference. The linearization makes the model easier to solve than if it was formulated as a non-linear program.

Branch and bound was chosen as the solution method for the implementation of the model. The model has been implemented in the Mosel programming language, using Xpress-MP as the solver.

Results from testing of the model on three different test cases indicate promising potential for the utilisation of the model. Optimal solutions were found in less than six minutes for all of the test cases, and close to optimal solutions were found quickly in the global search.

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Nomenclature

DCQ	-	Daily Contract Quantity
FLNG	-	Floating LNG
FPSO	-	Floating Production, Storage and Offloading
IP	-	Integer Programming
LNG	-	Liquefied Natural Gas
LP	-	Linear Programming
MEG	-	Monoethyleneglycol
MILP	-	Mixed Integer Linear Programming
MINLP	-	Mixed Integer Nonlinear Programming
MIP	-	Mixed Integer Programming
NCS	-	Norwegian Continental Shelf
NGL	-	Natural Gas Liquids
NPD	-	Norwegian Petroleum Directorate
NPV	-	Net Present Value
NLP	-	Non-Linear Programming
PI	-	Productivity Index
PIP	-	Pure Integer Programming
SOS1	-	Special Ordered Set of type 1
SOS2	-	Special Ordered Set of type 2
TLP	-	Tension Leg Platform
WI	-	Wobbe Index

1 Introduction

1.1 History of natural gas production

Mankind has known natural gas for thousands of years. The temple in Greece where the famous Oracle of Delphi lived was built where seepage of natural gas from underground created a burning flame. This occurred approximately 1000 B.C. [NaturalGas, 2011a]. Around 500 B.C the Chinese built pipelines out of bamboo shoots in order to utilise the natural gas. The natural gas was used to boil saltwater, making it drinkable.

In the late 18th and early 19th century gas manufactured from coal, so-called coal gas, was being used for street lighting in Britain. This is one of the first major utilisations of natural gas. In 1821 the first intentional natural gas well in the United States was dug in Fredonia, New York, when a man named William Hart dug a 27-foot well where natural gas was seeping to the surface to obtain a higher flow rate. Already in 1869 the first offshore drilling rig patent was given. Despite this, onshore fields would remain the main natural gas fields for the next century.

In this early period of petroleum production natural gas was not considered particularly useful. Although some pipelines were built, they were not very efficient. Due to this lack of transport alternatives, natural gas was usually vented to the atmosphere, burnt on site or left in the ground when found alone before World War II. In the 1920s an effort was put into building a pipeline infrastructure in the United States. After World War II advances in pipeline technology allowed an extensive pipeline network to be built in the United States. This opened new possibilities for utilisation of natural gas in the United States.

Figure 1.1 shows the total natural gas production in the world from 1970 to 2009, using the data from the BP Statistical Review of World Energy June 2010 [BP, 2010]. The natural gas production has been steadily increasing, until 2009. In 2009 the global natural gas production declined for the first time on record. This was largely due to declining consumption because of the financial crisis.

1.2 History of natural gas production in Norway

In 1959 the giant Groningen gas field was discovered in the Netherlands [Whaley, 2011]. This made oil companies interested in exploration activities in the North Sea. The Norwegian government were surprised when they were approached by

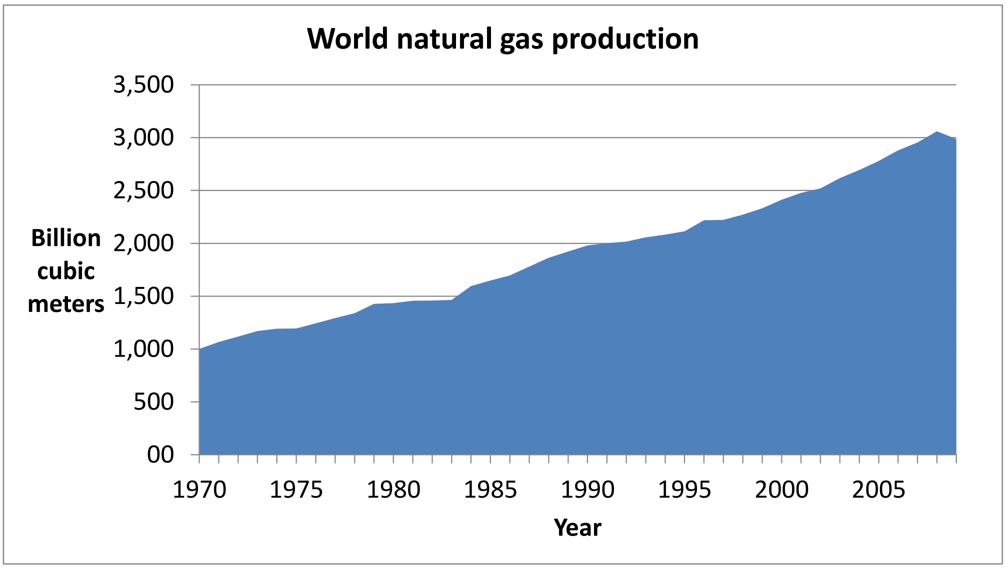

Figure 1.1: Historical world natural gas production [BP, 2010]

Phillips Petroleum Company, asking for the rights of exploration activities on the Norwegian Continental Shelf (NCS). In 1958 Norges geologiske undersøkelse had announced in a letter to the Ministry of Foreign Affairs that it was unlikely that oil could be found by the Norwegian coast [NRK, 2011].

Oil and gas exploration activities in Norway commenced in the 1960s. The first oil field was discovered in 1967, when the Balder field was found. However, this field was not considered profitable back then and it was not developed until 30 years later. In 1969 the first giant field, Ekofisk, was discovered by Phillips Petroleum Company. Petroleum production on the NCS started in 1971 from this field. More giant discoveries followed in the 1970s, such as Statfjord, Gullfaks, Oseberg and Troll. The first major natural gas find was the Frigg field, which was found in 1971 [Tønnesen, 2011]. It was the world's largest offshore gas field when it was discovered [Frigg, 2011].

As these fields were brought on stream throughout the 80s and 90s, Norway rapidly became a major oil and natural gas producer, as can be seen in Figure 1.2 which shows the historical petroleum production on the Norwegian Continental Shelf.

These fields found in the North Sea in the 70s have been the backbone of the Norwegian petroleum production. Exploration in the Norwegian Sea and Barents Sea followed in the 1980s, with several major discoveries. An extensive network of oil and gas pipelines have been built to exploit these resources, as well as the

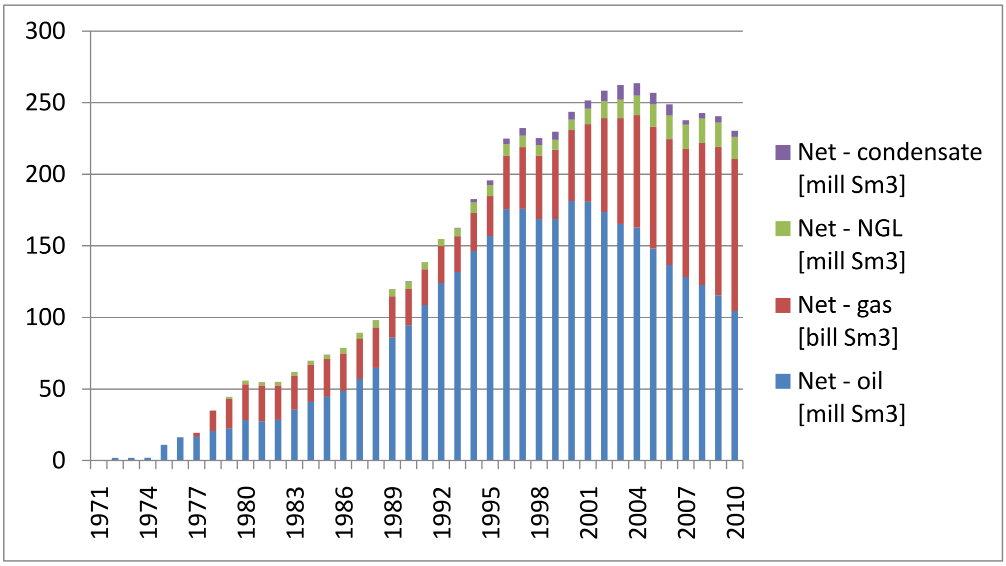

Figure 1.2: Oil and natural gas production in Norway [NPD, 2011]

northernmost liquefied natural gas (LNG) plant in the world at Melkøya outside Hammerfest.

Natural gas exports from Norway started in 1977 when the Norpipe pipeline from the Ekofisk field to Germany was opened. The second field that started exporting natural gas from Norway was the Frigg field. Production started from the British part of this field in 1977, and from the Norwegian part in 1978. The gas was transported by the Vesterled pipeline to St. Fergus in Scotland. Following the development of these two pipelines, the Norwegian natural gas production remained relatively stable at a production level of around 25 billion standard cubic meters per year until the mid 1990s. The historical Norwegian natural gas production is illustrated in Figure 1.3.

The giant Troll field was discovered in 1979 by Norske Shell. This field contains more than 40 per cent of the total gas reserves on the NCS [Statoil, 2011]. The development of this and other natural gas fields in the late 1990s led to a significant increase in the total natural gas production in Norway, as seen in Figure 1.3.

The natural gas production in Norway is mainly exported to the United Kingdom and continental Europe by pipelines. Receiving terminals have been built in the United Kingdom, France, Belgium and Germany. From these terminals the natural gas may end up in other countries after being transported through the existing distribution network. The distribution of where Norwegian gas exported by pipeline is sold is illustrated with Figure 1.4.

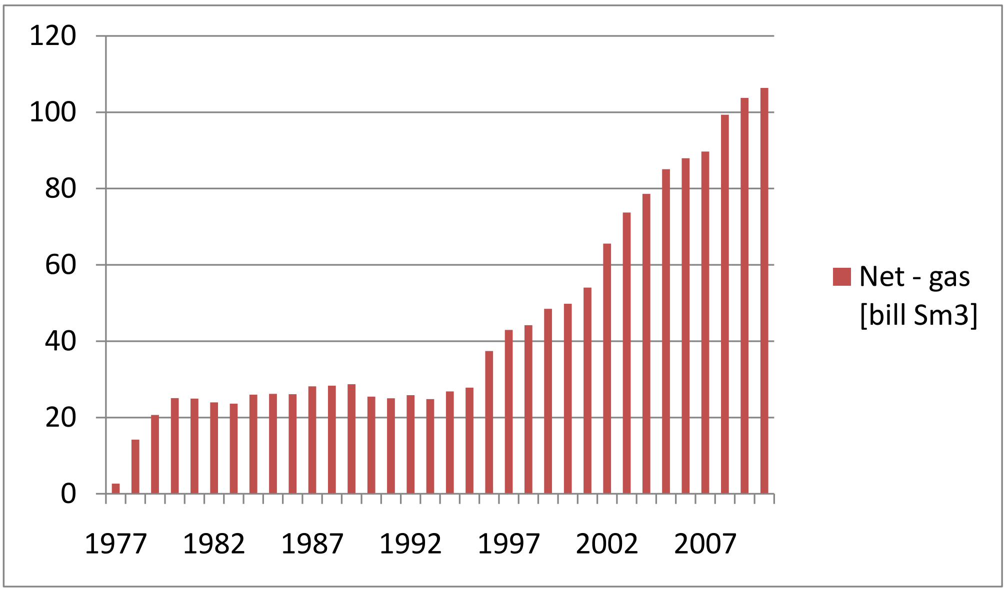

Figure 1.3: Natural gas production in Norway [NPD, 2011]

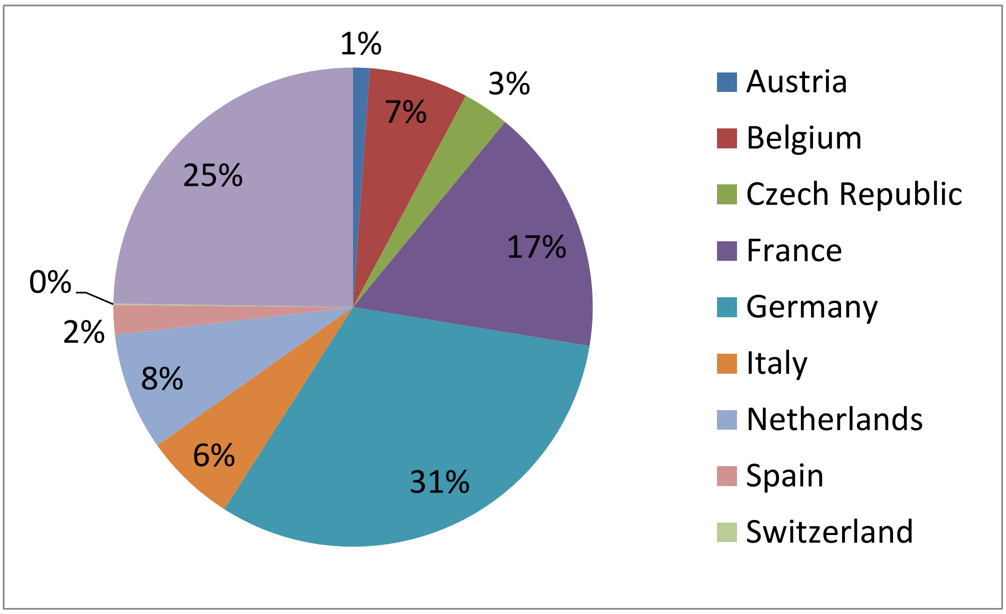

Figure 1.4: Recipients of Norwegian gas exports by pipeline [BP, 2010]

The construction of the LNG plant at Melkøya outside Hammerfest has allowed natural gas from the Snøhvit field to be produced. This natural gas is mainly exported to Spain and the United States, although other countries also buy some of this natural gas supply. Figure 1.5 illustrates where LNG exports from Norway are sold.

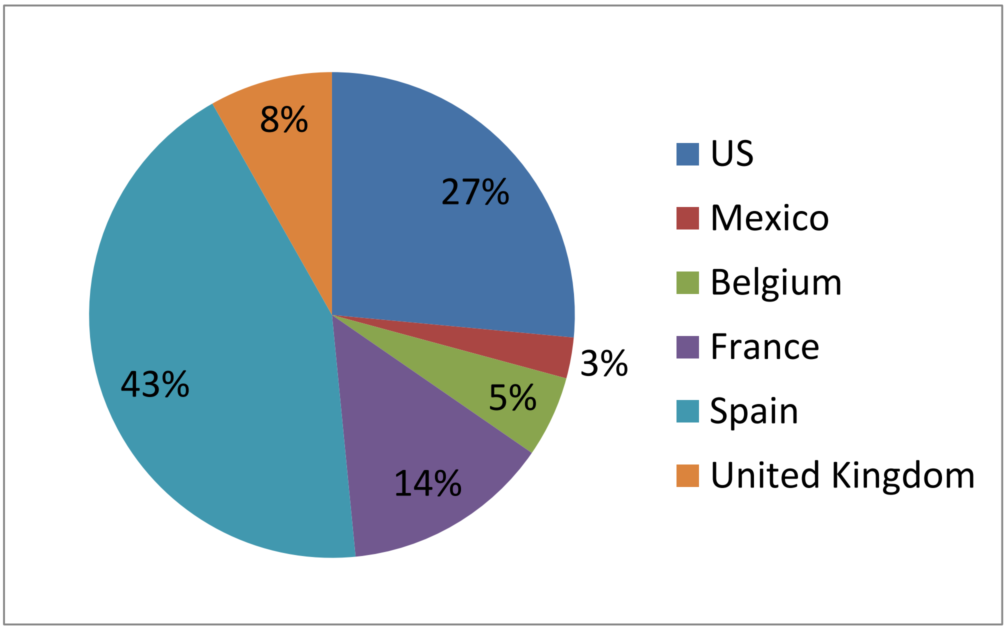

Figure 1.5: Recipients of Norwegian exports of LNG [BP, 2010]

In recent years no giant fields have been found, with Ormen Lange in 1997 being the last giant discovery. However, exploration activity is still high, with a record high number of 65 exploration wells being drilled in 2009. In 2009 oil and gas companies invested about 136 billion NOK in field developments, exploration, transport infrastructure and onshore process facilities. Today 22 per cent of Norway's gross domestical product comes from the oil and gas industry. Oil and natural gas is responsible for approximately half of the total value of exports from Norway. In 2008 Norway was the world's second largest exporter of natural gas and the sixth largest exporter of oil. In 2010 the natural gas production surpassed the oil production for the first time in Norway. It is expected that Norwegian oil production will continue to decline, but the total petroleum production will be relatively stable as natural gas production is still increasing. Concerns have however been raised about Norway's ability to sustain natural gas production after 2020 [Lindeberg, 2011]. More natural gas discoveries are needed to maintain a high level of natural gas exports to Europe.

Today many fields are developed as tie-backs to existing infrastructure. Some examples of this is the Hyme development that will connect to the Njord platform, and Morvin that is tied-back to Åsgard. This is in contrast to the large concrete platforms that were used to develop the old, giant fields such as Frigg and Troll. Cooperation between different fields is important to develop marginal resources. Utilising existing infrastructure also contributes to increased petroleum recovery from old fields, as the petroleum from new fields makes it profitable to continue production from the old fields. Finding an optimal development plan is increasingly important to make these marginal projects profitable.

1.3 Natural gas fundamentals

Natural gas is an odourless mixture of light hydrocarbons, water, nitrogen, hydrogen sulphide, carbon dioxide and other trace components. The main component in natural gas is methane (CH_4). Natural gas is the cleanest fossil fuel when comparing CO_2, NO_x, SO_2 and particulate emissions on a per energy unit basis [NaturalGas, 2011b]. The composition of natural gas varies from field to field, and can show large differences. Table 1.1 shows examples of the gas composition of natural gas from four different natural gas fields.

Table 1.1: Natural gas composition of some natural gas fields [Gudmundsson, 2010]

Molecule fraction	Troll, Norway	Sleipner, Norway	Draugen, Norway	Groningen, Netherlands
Methane	93.070	83.465	44.659	81.29
Ethane	3.720	8.653	13.64	2.87
Propane	0.582	3.004	22.825	0.38
Iso-Butane	0.346	0.250	4.875	0.15
N-Butane	0.083	0.327	9.466	0.04
C5+	0.203	0.105	3.078	0.06
Nitrogen	1.657	0.745	0.738	14.32
Carbon dioxide	0.319	3.429	0.720	0.89

The behaviour of natural gas as the pressure and temperature changes is often visualised with a so-called phase envelope. The phase envelope describes which phase a natural gas mixture is in at a given pressure and temperature. This concept will not be discussed further in this thesis, but two properties related to the phase envelope need to be mentioned.

These are the cricondenbar pressure and the cricondentherm temperature. The cricondenbar is the highest pressure on the phase envelope curve. Above this pressure the natural gas will be in a single dense phase at all temperatures. If the temperature is lowered the gas mixture will become more dense and liquid-like, but no phase transition will happen and no condensation of liquid will occur.

Similarly, the cricondentherm is the highest temperature on the phase envelope. As long as the temperature of the natural gas mixture is above the cricondentherm, it will never be in liquid phase.

These two properties are important related to natural gas specifications, which are described in Chapter 1.7.

1.4 Forming of natural gas

Natural gas is formed during a long process, lasting millions of years. Several conditions need to be met if natural gas is to be formed. Exploration for natural gas involves looking for three types of rock: source rock, reservoir rock and cap rock. These rocks are necessary for the formation of a natural gas reservoir. The following explanation of how natural gas is formed is mainly based on Morgan Downey's book on oil [Downey, 2009].

Source rock is rock laden with kerogen, which is a solid dark waxy rock. Kerogen is the portion of naturally occurring organic matter that is nonextractable using organic solvents [Schlumberger, 2011]. Kerogen typically constitutes of algae and plant material. It is mostly formed by sedimentation of marine material. An example of a source rock is the Kimmeridge Clay (Draupneskifer in Norwegian). Kimmeridge Clay is the main source rock of the oil deposits in the North Sea [SNL, 2011]. If this source rock is buried deep enough, the heavy hydrocarbon molecules in the kerogen will, over time, be cracked into lighter hydrocarbons. These lighter hydrocarbons are oil and natural gas. This process where crude oil and natural gas are formed from the source rock is called maturation.

The amount of heat that is necessary to form oil and natural gas from kerogen is reduced when the pressure increases. Both the temperature and pressure increases with depth underground. Thus, the deeper the kerogen is buried, the higher the temperature and pressure, and the lighter the oil that is formed will be. If the kerogen is buried even deeper, all the oil will be cracked into methane. The deeper level where all crude oil formed at shallower depths is cracked into methane is known as the gas window. How deep this level is varies around the world.

When the less dense crude oil and natural gas are formed, it will migrate upwards towards the surface. If no suitable reservoir rock and cap rock are in place, the oil and gas will migrate all the way to the surface. Here it will evaporate, be eaten by bacteria or oxidize. Most of the oil that has been formed throughout history has escaped in this manner.

The reservoir rock needs to be in contact with the source rock and be porous enough to contain the natural gas. Contrary to common perception, oil and natural gas are not found in pool-like domes, but occupy the pores of sedimentary rocks. The two main types of reservoir rocks are sandstone and carbonates. Both of these are in place on the NCS. For example, Ekofisk is a carbonate reservoir, while Statfjord is a sandstone reservoir. If petroleum production shall be feasible, the reservoir rock also needs to be permeable enough to let the oil and gas pass through to a production well.

Finally, a cap rock is needed to keep the oil and gas in place. This cap rock may for example be shale, salt or anhydrite. The cap rock also needs to have a shape that can prevent the oil and gas from moving upwards. Some examples of this are anticline and fold traps, although other possibilities exist.

1.5 Petroleum exploration

Previously, the fundamentals about how natural gas is formed and the necessary conditions for a natural gas reservoir to exist was outlined. Petroleum exploration is the activity where one tries to identify and find out where these reservoirs are, and how much petroleum a reservoir contains. Several geophysical methods might be used to try to get know ledge of the conditions underground. These include gravity surveys, magnetic surveys, electro-magnetic surveys and seismic surveys, of which seismic imaging is the most well-known. By analyzing data from these surveys oil companies try to identify possible reservoirs. Confirming that a prospect from these analyses contains oil and/or natural requires a well to be drilled. If petroleum is found, additional wells might be drilled to get more information about the size of the reservoir. Exploration activities are expensive, in particular exploration drilling.

1.6 Natural gas classifications

Natural gas may occur as associated gas or non-associated gas. Associated gas is found together with oil, either as a gas cap or dissolved in the oil. Non-associated gas is natural gas with no underlying oil column. All the oil that has been formed has been cracked into gas for these kinds of reservoirs. This means that non-associated gas usually has a higher content of methane and lower fraction of heavier hydrocarbons than associated gas.

Natural gas can also be divided into sweet gas and sour gas. These terms are related to the content of hydrogen sulphide and carbon dioxide in the gas. Sour gas has a high content of at least one of these components. Typical limits are 2 volume percent for carbon dioxide and 1 volume percent for hydrogen sulphide for the gas to be deemed sour gas. Sour gas needs extra treatment to remove these components as they are highly corrosive.

The natural gas that has been found on the NCS is mostly sweet gas. Some sour gas fields exist on the NCS, an example is Sleipner from Table 1.1 and Snøhvit, which both have a high carbon dioxide content by Norwegian standards. Examples exist of high hydrogen sulphide or carbon dioxide content in natural gas. In Canada some wells produce natural gas with more than 30 percent H_2S content [Jahn et al., 2008], and the Natuna gas field in Indonesia contains as much as 70 percent CO_2 [Technology, 2011].

In addition to being divided into associated or non-associated gas and sweet or sour gas, natural gas is also categorised as being dry or wet gas. Dry gas is gas with a very high content of methane, which will never condense into liquid at normal ambient conditions. Sales gas is dry gas, as the possibility of liquid formation in the end-customers burners can be dangerous.

Wet gas, sometimes used interchangeably with rich gas, contains a larger amount of heavier hydrocarbons. These may condense during pipeline transport, processing or at the end user. Thus it needs further processing before being sold.

1.7 Natural gas specifications

Processing of natural gas is done to meet some specification, which may vary depending on in which part of the value chain the processing is done.

Transport specifications are necessary to ensure safe and reliable transport of natural gas from an offshore production facility to a processing plant for further processing. These are typically related to water content, outlet pressure at the platform and sour gas content.

The water content must be low enough to make sure that no liquid water will drop out during worst case conditions. This is at high pressure and low temperature, which can happen if the export pipeline is blown down. The water content specification is given as a water dew point in terms of pressure and temperature.

Necessary outlet pressure at the production facility is related to two factors. The outlet pressure must make sure that the natural gas is at the correct inlet pressure of the processing plant. Further, this must also make sure that the entire transport happens above the cricondenbar of the natural gas. This is assured by previous processing at the production facility. It is necessary to transport the gas above the cricondenbar pressure to avoid possible liquid hydrocarbon dropout in the pipeline

It is important to avoid liquid dropout in large parts of the value chain for several reasons. Hydrates may be formed, corrosion problems may occur and operational difficulties leading to safety risk may be experienced if liquids do form.

The sour gas transportation specification is mainly dependent on two factors. First of all it is important for corrosion control of the pipeline. Elevated levels of carbon dioxide or hydrogen sulphide may easily corrode the pipeline, especially if free water is present. Secondly, it is related to sales gas specifications. The amount of sour gas removal that is required due to this is also dependent on two external factors. An amount of sour natural gas that would otherwise need extensive sour gas removal may avoid this if mixing it with sweet natural gas from another field results in natural gas that still meets sales specifications. Some processing plants also contain a sour gas removal unit, lowering the need of sour gas removal offshore.

Sales specifications are important to ensure safe use of the natural gas for the end customer. They can be broadly divided into four areas.

The energy content of the natural gas is an important measure for billing purposes. The customer is interested in getting the correct energy amount, and the gas seller wants to get paid for supplying it. Gross calorific value is used as a measurement of the energy content in the natural gas.

Wobbe Index (WI) is used to measure the energy load of the natural gas. It is the main indicator of the interchange ability of different natural gas mixtures. Different countries often have different WI requirements.

Combustion characteristics describe the combustion kinetics during combustion of the natural gas. Two factors describe this. The soot index gives an indication of the risk of soot formation during combustion. The Incomplete combustion factor indicates the risk of carbon monoxide formation during the combustion process.

Finally there are specifications regarding impurities in the natural gas. These include for example mercury, water, nitrogen, hydrogen sulphide and carbon dioxide.

1.8 Natural gas field development

After an oil company has found a natural gas field the next task is to find the best way to develop that natural gas field. Many decisions have to be made, and one decision is likely to affect the outcome of other decisions. It is important to try to look upon the field development as a whole, rather than choosing the optimal solution for each problem separately. Otherwise it is likely that the total solution will be suboptimal. This in turn makes optimization and operations research an interesting tool to find a good solution for the whole field development problem. The following subsections will outline the different parts of a natural gas field development.

1.8.1 Wells

Wells need to be drilled if one wishes to produce the natural gas reserves in a given reservoir. Some production facilities have an integrated drilling facility to allow relatively easy drilling at all times. Other production platforms are dependent on moveable drilling rigs for the drilling of wells. The latter is often more expensive than the former. There are also different types of wells. The simplest type of well is the vertical well. These are drilled vertically down, and have a relatively short section where oil and gas is able to flow into the well. One way to reach more oil or gas is to drill horizontal wells. Technology advancements led to these becoming successful; as well trajectories could be accurately planned and followed. An even more advanced type of well is the multilateral well. This kind of well involves drilling several holes branching from a central borehole. These can be particularly attractive to drain remaining pockets of hydrocarbons in mature fields, and in subsea developments [Jahn et al., 2008].

Well costs can be a significant part of the investment costs of a natural gas field development. The day rate of a drilling rig may be around 2 million NOK/day, and even more for an advanced deep water drilling rig. Drilling a well may also take a long time, some exploration wells as long as three months [Salthe, 2011].

1.8.2 Subsea manifolds, templates and flowlines

Today, many fields on the NCS are developed as subsea field developments. Here, the equipment which is necessary if such a solution is chosen will be discussed. If the field is developed solely with wells drilled from a fixed platform, the wells can be connected to the production facility without the need for the following equipment.

The simplest form of subsea developments is with a single satellite well, connected to a production facility through a series of pipelines and umbilicals. This solution is typically chosen if the field to be developed is small and near a large field.

If the field is large enough to justify the drilling of several wells, it will soon become uneconomical to tie each well back to the production facility with separate flowlines. Subsea templates and manifolds are used in this case.

The subsea production template is generally recommended for use with six or more wells. It is commonly used when an operator has a firm idea of the number of wells that will be drilled in a certain location. All subsea facilities are contained within one protective structure [Jahn et al., 2008]. The subsea template is connected to the production facility with flowlines and umbilicals.

If several single wells or templates need to be connected to the production facility, it might be economical to use an underwater manifold system. Each well or template are connected to this manifold, which is then tied back to the production facility with a single set of pipelines and umbilicals. This saves expenses on flowlines and umbilicals that would otherwise be required. Large field developments may involve a large network of umbilicals and flowlines, in order to get the best solution.

In addition to flowlines bringing the natural gas towards the production facility, flowlines are also necessary to bring various chemicals that need to be injected into the natural gas flow. An example of this is the need to inject monoethyleneglycol (MEG) to avoid the formation of hydrates that would otherwise block the natural gas flow in the flowline. Other chemicals are also used, to avoid scaling and other issues.

1.8.3 Production facilities

Natural gas needs to undergo several treatments in order to meet a given transport or sales specification. At least parts of this happens at a production facility. The following description of different production facilities is mainly based on Jahn et al. [2008]s book on hydrocarbon exploration and production. Production facilities come in a wide range of variations. Depending on local conditions such as water depth, distance to shore and already existing nearby production infrastructure, several different types of platforms might be considered. Production platforms can be divided into two main types: fixed and floating platforms.

Fixed platforms can be used in relatively shallow water depths. The most common fixed platforms are steel jacket platforms and gravity-based platforms. Steel jacket platforms have been successfully used in both shallow and calm water, as well as in demanding North Sea conditions. Many fields on the NCS have platforms of this type, such as the Ekofisk and Valhall fields. Steel jacket platforms are used in water depths of up to 150 m [Jahn et al., 2008]. Figure 1.6 shows an example of a fixed steel jacket platform.

Figure 1.6: Huldra. A fixed platform. Photo: Kjetil Alsvik/Statoil

The concrete gravity-based platform of the Condeep type is possibly the best-known platform of the Norwegian public. Many of the largest fields on the NCS have been developed with one or more platforms of this type. Troll, Statfjord and Gullfaks are some of the fields that have at least one Condeep platform. Concrete platforms can be used in similar water depths as steel jacket platforms, and slightly deeper. The water depth around the Troll A platform, shown in Figure 1.7, is over 300 metres.

A field does not need to be developed with only one sort of platform. Indeed, both steel jacket and gravity-based have several times been used together to develop a field. For example, a gravity-based platform may hold heavy processing facilities whilst a steel jacket platform may be used as a wellhead platform.

As the water depth increases fixed platforms become infeasible. Floating platforms are able to operate in much greater water depths than fixed platforms. Many variations of the floating platforms exist, of which some will be described here.

Figure 1.7: Troll A platform. Photo: Øyvind Hagen/Statoil

Semi-submersible platforms have been used in water depths of over 1800 m. Pontoons at the base of the platform construction are filled with water, partially submerging the platform and thus giving it its name. The platform is moored to the bottom with anchor chains. Semi-submersible platforms have been widely used on the NCS. Snorre, Kristin and Gjøa are some of the fields that have been developed with semi-submersible platforms.

Floating storage, production and offloading vessels (FPSOs) are another possible development solution. They have been used in water depths of up to 2600 m. The biggest FPSO on the NCS to this date is expected to start producing in 2011, when the Skarv FPSO comes on stream. Skarv is a natural gas and oil field, with a majority of natural gas. Several other fields on the NCS have been developed with FPSOs, for example Norne and Åsgard.

SPAR platforms resemble large cylinders supporting a platform. These can be used in very deep water. They are tethered to the bottom with cables and lines. SPARs have been very popular in the Gulf of Mexico, but to the author's knowledge no SPAR platform is installed in Norway. The current plan for the development of the Luva natural gas field does however involve building the largest SPAR in the world.

Tension leg platforms (TLPs) fall somewhat in between fixed and floating platforms. They consist of a floating platform that is attached to the sea floor by long tension legs, allowing some movement from side to side, but keeping the platform relatively stable vertically. The Heidrun field is developed with a TLP.

Subsea production systems have been discussed earlier. Advances in multiphase technology have made it possible to develop a field with only subsea structures offshore in some cases. These are connected to an onshore processing plant by pipelines. All the processing happens at this plant. Ormen Lange is an impressive example of a subsea development. Located 120 km offshore at a water depth of 850 to 1100 m, the gas is processed at the Nyhamna plant, before being exported to the United Kingdom. The Snøhvit field is also a subsea development.

Finally, plans for floating LNG (FLNG) plants are currently being made by several oil companies. Recently, Shell announced that they have made the final investment decision on the Prelude FLNG project, making it the world's first FLNG facility [Shell, 2011]. FLNG plants resemble FPSOs, by having all gas processing and gas liquefaction at one vessel offshore. FLNG will make it possible to develop gas resources that were previously considered to be too far away from land for development.

1.8.4 Transport infrastructure

After some processing at the production facility the natural gas will be transported either to an onshore processing plant for further processing or directly to the customer. This depends on several factors such as gas composition and distance to the customer. There are two main ways of transporting the gas: through a gas pipeline or as LNG. The choice of how to transport the gas is mainly a matter of distance to the market, even if LNG offers some additional flexibility regarding where to transport the gas.

The transportation cost for LNG is characterised by that the transportation cost for short distances is very high compared to pipeline transport, but that the costs increase much slower than for pipeline transport as the transport distance increases. Over a certain distance LNG is the cheaper option.

During the past 40 years Norway has developed the world's largest system of underwater pipelines. This means that it will often be profitable to use this pipeline system, rather than building a new export pipeline to transport the natural gas to the market. The cost for transportation of natural gas is then reduced to an investment cost and operational cost for a new, smaller pipeline connecting to the existing pipeline and a tariff for using the existing pipeline.

If the natural gas field that is to be developed is large enough, or the capacity of the existing pipeline system is too small for a long enough time, it will anyway be necessary to build a new pipeline. Specialised pipeline laying vessels are used to do this.

1.8.5 Energy infrastructure

A production platform contains a lot of power consuming equipment. Pumps and compressors used for natural gas processing and other purposes can have very high power demand. This power needs to be acquired either by generating it on the platform or by importing it from another source.

Most natural gas field developments until now have generated the necessary power internally on the platform by the use of gas turbines. The necessary amount of fuel gas is taken from the produced natural gas after some processing, usually before the last step of compression for exports.

There are however cases where electricity is imported to the production facility. Several reasons might make this an attractive solution. In Norway there has been a political desire from the government that electricity should be supplied from the onshore grid, in an effort to reduce carbon dioxide emissions. It has often been required to investigate the possibility of importing electricity during the planning process of a field development.

Weight limitations on the production platform may also make import of electricity interesting. If there is a tax on carbon dioxide emissions, low electricity prices and high natural gas prices, importing electricity may become a cheaper solution than generating power by gas turbines. Subsea developments also need to import electricity, as one obviously cannot use gas turbines underwater.

Several field developments import electricity to cover some or all of the power demand. This includes amongst other the Troll and Gjøa natural gas field developments. The Goliat field in the Barents Sea will import about half of the power demand, and the Valhall field redevelopment includes electrification of the platforms.

Limited availability of power may restrict the field development. The maximum size of a gas turbine might for example be limited by weight or available space. It is also possible that nearby platforms or the onshore grid has a limited amount of available power.

1.8.6 Other infrastructure

There are many other decisions that have to be taken in addition to those regarding equipment that has been described previously. An important issue is how to avoid the formation of hydrates. It is possible to avoid this in several ways. Perhaps the most common hydrate prevention system in use on the NCS is the injection of MEG into the well stream. Other systems include methanol injection and electrical heating systems.

Usually some oil or condensate is produced together with the natural gas. These liquid products need to be stored, transported to the market and sold. The condensate may be transported with ships to the market, or by a dedicated condensate pipeline.

The processing system on the production facility contains several different pieces of equipment. Decisions need to be made for example regarding the use of turbines or Joule-Thomson valves to reduce pressure, what kind of separators that are to be used and how many compression steps that are needed.

1.9 Natural gas markets and field development

Natural gas field developments have usually required a long plateau production period compared to oil field developments. A gas field typically has a plateau production period of around 10 years, producing around two thirds of the reserves on plateau production. This can be compared to typical oil field plateau production period of 2-5 years. Three examples of natural gas production profiles are shown in Figure 1.8, 1.9 and 1.10. In these examples, Frigg has been closed down whilst Huldra and Troll are still producing.

Whereas a spot market has always existed for oil, gas sales traditionally require a contract to be agreed between the producer and a customer. This forms an important part of gas field development planning, since the price agreed between producer and customer will vary, and will depend on the quantity supplied, the plateau length and the flexibility of supply. Whereas oil price is approximately the same across the globe, gas prices can vary very significantly from region to region. To attain a good sales price for the gas the customer usually requires a reliable supply of gas at an agreed rate over many years.

Long term contracts are often linked to the oil price. An example of a long term contract is the Troll gas sales agreement, in which gas sales for a period of 30 years were agreed.

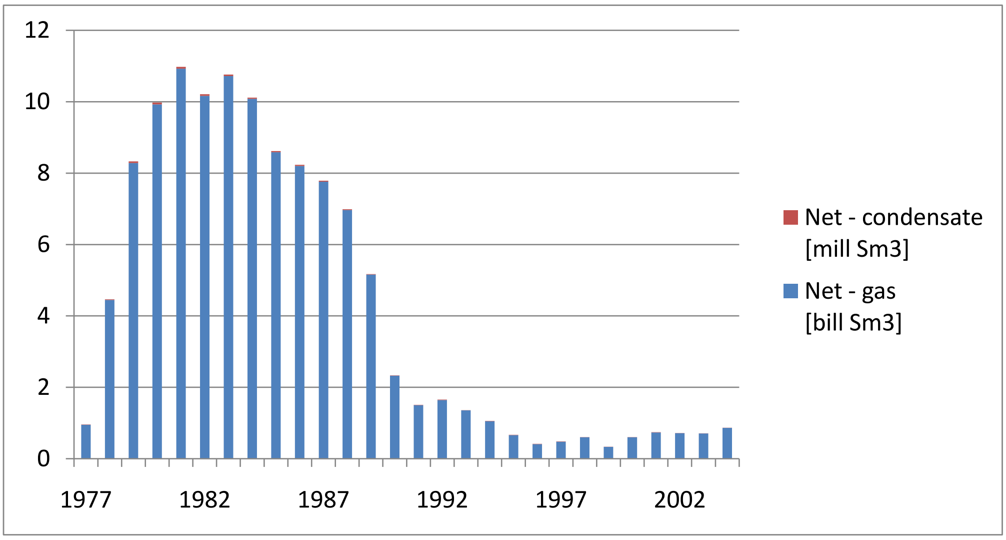

Figure 1.8: Production profile for the Frigg natural gas field [NPD, 2011]

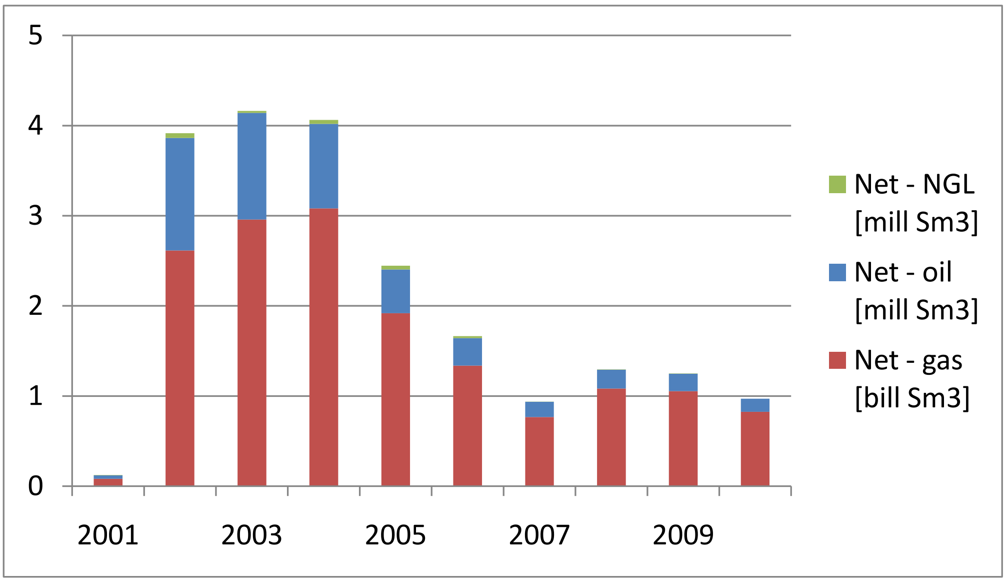

Figure 1.9: Production profile for the Huldra natural gas field [NPD, 2011]

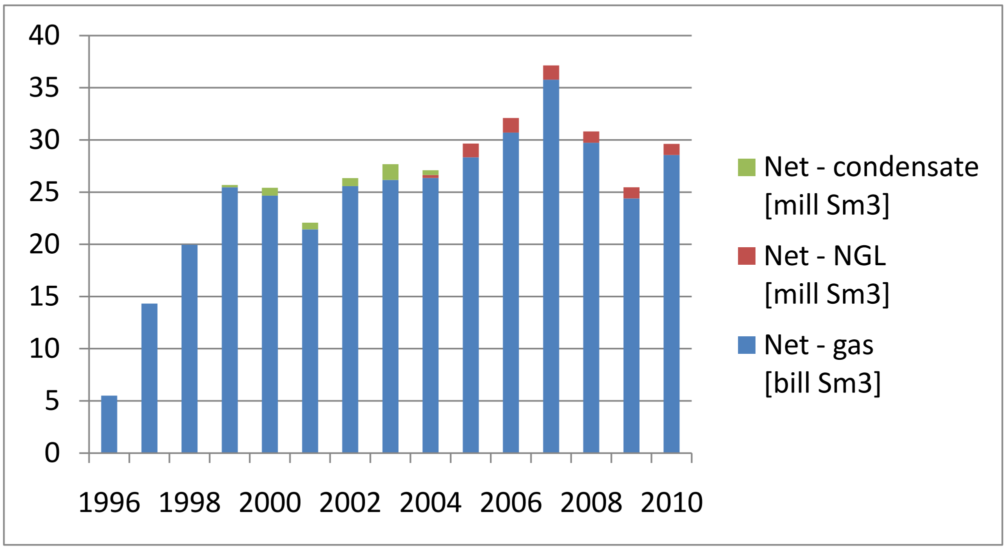

Figure 1.10: Natural gas, NGL and condensate production profile for the Troll field [NPD, 2011]

These contracts do not mean that production is completely stable over the whole contract period. Instead, a set of parameters are agreed upon such as:

Daily contract quantity (DCQ)	-	The daily production which will be supplied, usually averaged over a period such as a quarter year
Swing factor	-	The amount by which the supply must exceed the DCQ if the customer so requests
Take or pay agreement	-	If the buyer chooses not to accept a specified quantity, he will pay the supplier anyway
Penalty clause	-	The penalty which the supplier will pay if he fails to deliver the quantity specified within the DCQ and swing factor agreements

Although the natural gas spot market is increasing in importance in Europe, and more fields are developed without long term contracts, most of the gas on the NCS is still sold on long term agreements. Statoil sell almost 90 per cent of their natural gas on long term contracts linked to the oil price [Steensen, 2011]. The natural gas market in Europe has been increasingly deregulated, both at the distribution side in continental Europe and at the transport side.

1.10 Fluid mechanics

In order to determine the natural gas flow rates, one has to perform analyses with tools from the field of fluid mechanics. This section will present some of the most important relations in fluid mechanics.

The flow rate of any fluid is defined by equation (1.1):

$$\dot{Q} = \iint_A \vec{V} \cdot \vec{n} \mathrm{d}A \tag{1.1}$$

Where $\dot{Q}$ is the volumetric flow rate, $\vec{V}$ is the velocity, $\vec{n}$ is the normal vector of the surface, dA is the surface differential and A is the area over of which the flow rate is calculated.

The real gas law is especially important to describe the behaviour of natural gas when the pressure changes.

$$pv = zRT \tag{1.2}$$

Where p is the pressure, v is the specific volume of the gas, z is the compressibility factor, R is the universal gas constant and T is temperature. As the gas volume is dependent on temperature and pressure, the volume is often given at standard conditions. These standard conditions may actually vary, so it is essential to check what the standard conditions are for a given volume. An example of a standard condition can be 15 degrees C and 1 atm pressure. The relationship between volume at standard conditions and volume at some other condition is given as:

$$V_{s.c.} = V \left(\frac{p}{p_{s.c.}}\right) \left(\frac{T_{s.c.}}{T}\right) \left(\frac{1}{z}\right) \tag{1.3}$$

The gas flow equation for low pressure steady state flow can be shown to be:

$$q_{s.c.} = \frac{\pi k h T_{s.c}(p_R^2 - p_w^2)}{p_{s.c.} T \bar{\mu} \bar{z} [\ln \frac{r_e}{r_w} - \frac{1}{2}]} \tag{1.4}$$

Where:

$q_{s.c.}$	-	flowrate at standard conditions, m^3/s
k	-	permeability of the reservoir rock, m^2
h	-	height of producing formation, m
$T_{s.c.}$	-	temperature at standard conditions, K
p_R	-	reservoir pressure, Pa
p_w	-	well (flowing) pressure, Pa
$p_{s.c.}$	-	pressure at standard conditions, Pa
T	-	reservoir temperature, K
$\bar{\mu}$	-	fluid viscosity at average pressure, Pas
$\bar{z}$	-	fluid compressibility at average pressure, 1
r_e	-	distance to drainage boundary, m
r_w	-	wellbore radius, m

A productivity index, PI, can be defined by rephrasing this equation. As long as the viscosity and temperature remains constant, this productivity index will also remain constant.

$$PI = \frac{\pi k h T_{s.c}}{p_{s.c.} T \bar{\mu} \bar{z} [\ln \frac{r_e}{r_w} - \frac{1}{2}]} \tag{1.5}$$

By using this formulation, a simpler form of the gas flow equation can be given.

$$q_{s.c.} = PI(p_R^2 - p_w^2) \tag{1.6}$$

An even simpler gas flow equation is appropriate at high pressure conditions. At high pressure, the gas will behave in a similar way to liquids such as oil. The gas flow rate is then dependent on the pressure difference $p_R - p_w$, which is linear.

$$q_{s.c.} = PI(p_R - p_w) \tag{1.7}$$

Producing natural gas will lead to a pressure reduction in the reservoir. This will again limit the production potential of the reservoir. The pressure drop in the reservoir is linked to the production in an almost linear manner. The non-linearity occurs due to the z-factor of the gas changing because of the reduction in pressure. Equation (1.8) shows the relationship between the pressure p, compressibility z, initial gas resource G_i, initial pressure p_i, initial gas compressibility z_i and cumulative natural gas production, G.

$$\left(\frac{p}{z}\right) = \left(\frac{p_i}{z_i}\right)\left(1 - \frac{G}{G_i}\right) \tag{1.8}$$

This is often illustrated with a p/z-plot, which can have remarkably good fit to actual data as can be seen in Figure 1.11.

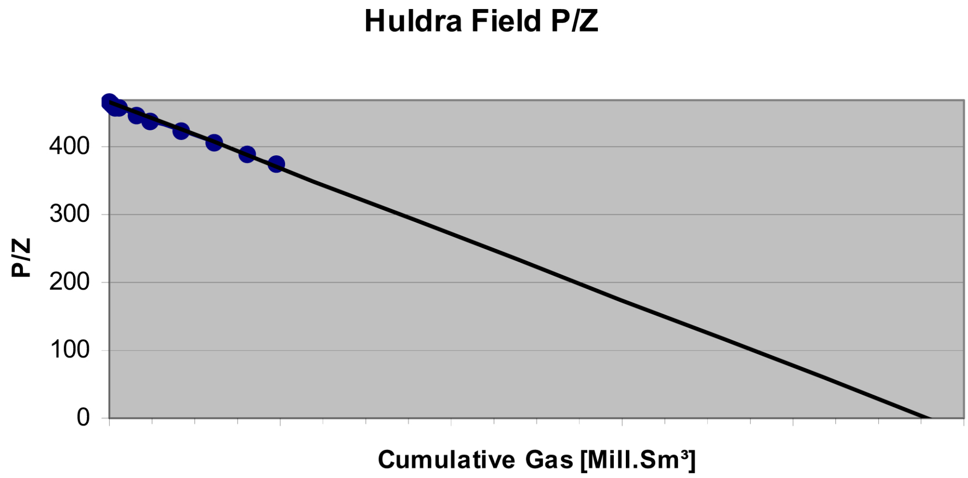

Figure 1.11: p/z-plot for the Huldra natural gas field [Saksvik, 2004]

1.10.1 Pressure drop in pipelines

Natural gas field developments usually contain several pipelines, in the form of flowlines and export pipelines. The pressure drop in these pipelines will affect the outcome of a project. Compressing natural gas is expensive energy-wise, so having low pressure loss in the export pipelines is important. Also, one wants to be able to keep the wellhead pressure as low as possible to achieve as high a production rate as possible. This means that the pressure loss in the flowlines should be low.

The total pressure loss in a pipeline is due to three effects: hydrostatic difference, acceleration and friction. As the hydrostatic pressure loss is given of external conditions and acceleration can be avoided in steady state conditions, the pressure loss because of friction is the most important of these three effects regarding optimization.

$$\Delta p = \Delta p_g + \Delta p_a + \Delta p_f \tag{1.9}$$
$$\Delta p_g = \rho g \sin \alpha \Delta L \tag{1.10}$$
$$\Delta p_a = \rho u \Delta u \tag{1.11}$$
$$\Delta p_f = f \frac{\Delta L}{d} \frac{1}{2} \rho u^2 \tag{1.12}$$

The equation for pressure drop due to friction, p_f is known as the Darcy-Weissbach equation.

This means that it will usually be favourable to have a large diameter pipeline. It is necessary to weigh this up against increased investment cost and, in the case of gas-condensate being transported, the liquid volume build-up in the pipeline.

Another way to reduce the pressure loss in the pipeline is by reducing the friction factor f. Assuming that the Reynolds number of the gas is higher than 10^5, this needs to be done by using a smoother pipe. This will be more expensive, but may be profitable due to the reduced pressure drop in the pipeline.

1.11 Previous work

Optimizing offshore oil and gas field developments have had the attention of oil companies for a long time. Different authors have approached the problem in various ways.

Frair and Devine [1975] formulate a deterministic MIP model where the number of platforms, platform capacity and location, assignment of wells to platforms, platform placing and well drilling schedule and production rate in each time period are determined. The number of platforms is fixed in the model, and then the model is solved for several numbers of platforms to determine the optimal number. Maximizing the after-tax profit is the objective in this model. In order to solve the model it is partitioned into one platform location subproblem and a well drilling, platform placement and reservoir production scheduling problem. These subproblems are then solved sequentially.

McFarland et al. [1984] formulate a deterministic non-linear programming model where the decision variables are the number of wells to be drilled, production rates, abandonment time and platform size. The reservoir is modelled as a zero-dimensional tank. A gas reservoir with water drive and a three phase oil reservoir are modelled. Reduced gradient methods are used to solve this non-linear model. The flow rates are modelled as being pressure dependent.

Haugland et al. [1988] present a MILP model that can propose platform capacity, where and when to drill wells and production from the wells in each time period. These decisions are taken simultaneously, as opposed to Frair and Devine's sequential approach. A two-dimensional reservoir model is used, and tax and royalty costs are ignored. The flow rates are modelled as being pressure dependent.

Iyer et al. [1998] present a MILP model where the decision variables are the choice of reservoirs to develop, selection from among candidate well sites, the well drilling and platform installation schedule, capacities of well and production platforms and production rates for each time period. The flow rates are modelled as being pressure dependent, and the non-linearity reservoir performance is handled by piecewise linear interpolation. Drilling rig availability constraints are also taken into account. The model is solved by a sequential decomposition approach, although the suggested algorithm does not guarantee optimality.

Jonsbråten [1998] develops a MIP model with the same decision variables as Haugland et al. However, unlike the previously mentioned articles, price uncertainty is taken into account, leading to a stochastic model. A two-dimensional reservoir model is used. A progressive hedging algorithm is used to find lower bounds for the problem, which converted to an implementable solution.

van den Heever and Grossmann [2000] present a multiperiod MINLP model. Decision variables include investment in production platforms, well platforms and wells, well drilling schedule and well production profiles in each time period. The non-linear reservoir performance, including reservoir pressures, gas to oil ratio and cumulative gas produced, is expressed directly as non-linear functions of the cumulative oil produced. Aggregation/disaggregation as well as dynamic programming is used in the solution algorithm.

van den Heever et al. [2000] develop a MINLP model. Their focus is on complex economic specifications such as taxes, tariffs and royalties. Although taking this into account leads to a better solution, the solution time increases by over an order of magnitude. This limits the size of the problems this model can handle.

van den Heever et al. [2001] continue to investigate MINLP models with complex economic specifications. A Lagrangean decomposition approach with subgradient updating of multipliers leads to more than an order of magnitude decrease in solution time. The results from this model are still significantly better than from models not taking the complex objectives into consideration.

Goel and Grossmann [2004] present a stochastic MILP model. The amount of petroleum reserves in the reservoir is considered to be uncertain. The stochastic solution approach leads to significant improvements in the NPV compared to the deterministic approach.

Goel et al. [2006] present a stochastic MILP model. Investment decisions include selecting which well platforms, production platforms and pipeline connections should be installed in each time period, and the capacities of the well platforms and production platforms. Production rates in every time period is also to be decided. Both the size and deliverability of the field are considered to be uncertain. The model is solved by a Lagrangean duality based branch and bound algorithm. The solutions are significantly better than those obtained in Goel and Grossmann [2004].

Barnes and Kokossis [2007] develop a deterministic MILP model that is quite different from those previously mentioned. Their objective is to minimize the CAPEX required to meet a target production profile. Earlier work has concentrated on maximizing profit, e.g NPV. The resulting model is decomposed into two parts: selecting an optimum drilling centre and optimizing the well selection. This reduces the complexity in solving the model.

Erlingsen and Strat [2010] present a deterministic MILP model for planning of petroleum production from several fields in an area. The objective is to maximize total NPV before tax. Decision variables include number of wells and manifolds, installations at the production site, choice of transport alternatives from the production field to the customer and how to provide energy to the site installations. Production rates and capacities are also determined, as well as possible cooperation between different fields. Methods used to solve the model include Branch and Bound, Dantzig-Wolfe decomposition and Branch and Price.

2 Theory

In this chapter the theoretical background for the mathematical model developed in this thesis will be presented. First, a short overview is given over linear programming (LP), integer programming (IP) and nonlinear programming (NLP). Then mixed integer nonlinear programming (MINLP) and mixed integer linear programming (MILP) are discussed. Finally, the branch and bound (B&B) algorithm is explained.

2.1 Mathematical programming models

2.1.1 Linear programming

Linear programming models are mathematical programming (MP) models where the objective function and constraints involve linear expressions. LP models are much easier to solve than NLP models. The reason for this is that the optimal solution of an LP model will always be at a vertex of the feasible region. If the form of the model is such that alternative solutions exist, an optimal solution will still be found at a vertex. The famous simplex algorithm only examines vertex solutions, rather than the generally infinite set of feasible solutions.

2.1.2 Integer programming

If one or more of the variables in the model needs to take an integer value, the resulting model is an integer programming model. If all the variables are integer variables, the resulting model is a Pure Integer Programming (PIP) model. If both integer and continuous variables are used, the model will be a Mixed Integer Programming (MIP) model. IP can be used to model a wide range of practical problems.

There are two main applications that involve the use of IP. The first application occurs in situations where it is only meaningful to make an integral amount of goods (cars, houses, ships etc.) or use an integral amount of resources(employees, machines etc.). IP can be used to model these decisions, although it can sometimes be good enough to use LP and round off the optimal solution values to the nearest integers. However, the main use of IP is to model decision variables which can only have the values 0 or 1. These variables model decisions where there is a clear yes or no answer. You either buy a car or you do not. This is a powerful way of modelling that is widely used.

IP models are in general much harder to solve than LP models. An IP model usually involves many times as much calculation as a similarly sized LP model due to the integer variables. Unlike even large LP models with thousands of variables and constraints which can almost certainly be solved within reasonable time limits, the same cannot be said for IP models. Several methods exist to solve IP models, of which cutting planes methods, enumerative methods, pseudo-boolean methods and Branch and Bound methods can be mentioned. The Branch and Bound method will be explained in Chapter 2.4.

2.1.3 Non-linear programming

The difference between linear programming and non-linear programming is quite simple. If the objective function and/or at least one constraint contain a non-linear expression, the resulting model will be a NLP model. These are, as IP models, much harder to solve than LP models. NLP models often arise in both economical and engineering situations. For example, there is often increasing or decreasing returns to scale for the profit or costs of a company.

Several factors give rise to the increased difficulty of solving an NLP model than an LP model. One of the problems is that the optimal solution is not necessarily found on a vertex of the feasible region, but may be found in the interior of the feasible region. When a solution has been found, it is also often difficult to determine whether it is a local or a global optimal solution.

2.1.4 Mixed integer non-linear programming

If a mathematical model contains both integer variables and non-linear constraints or objective function, the resulting model is a mixed integer non-linear programming (MINLP) model. Such models share the characteristic features from IP and NLP models. This means that MINLP models are in general very hard to solve, and quickly become intractable. To reduce the amount of computer time that is necessary to solve a model, or to be able to solve it at all, MINLP models are often converted into mixed integer linear programming (MILP) models. This is done by linearising the non-linear functions by the use of some conversion method.

2.1.5 Mixed integer linear programming

MIP models can be categorized as MILP or MINLP models. MILP models are those models which consist of linear expressions of integer and continuous variables in all the constraints and the objective function. These are harder to solve than pure LP problems, but usually easier to solve than MINLP problems.

Sufficiently small MILP problems can often be solved by a brute force approach, enumerating all possible solutions and choosing the best of these. As the problem size grows, this solution approach quickly becomes practically unsolvable, and special algorithms should be used to solve the problem.

2.2 Conversion of MINLP problems to MILP problems

As MILP problems are usually easier to solve than MINLP problems it is often appealing to use a MILP formulation of the problem. Care should be taken to ensure that this will result in a sufficiently good representation of the system that is being modelled. LP formulations are usually much easier to solve, but the linearity assumption must be warranted.

The easiest way of linearising a non-linear function is by fitting a linear function to the function values in the domain of the variable. A crude approach of doing this would be to linearise between the function values of the maximum and minimum values of the variable. A better linearisation can be found by linear regression of several function values. This will result in the single linear expression that diverts the least from the non-linear function. If the non-linear function doesnÕt divert too much from this linearisation, this might be a successful approach that still gives a good representation of the reality whilst reducing the complexity of the model.

If however the non-linear function is very non-linear for reasonable values of the variables, another approach must be used to ensure a sufficiently good representation of the reality. Piecewise linearisation of the non-linear function in question can often be a successful approach to achieve greater accuracy of the linearisation.

Several breakpoints are defined on the non-linear function curve. The non-linear function is linearised between these breakpoints. Function values between the breakpoint values are found by linear interpolation between the breakpoint values. The advantage with this approach is increased accuracy in the modelling, compared to using a single linear function that should give a reasonably good representation for all possible values. The calculation time will however increase compared to the single linear function approach. Both the accuracy and calculation time depends on the number of breakpoints. More breakpoints will increase accuracy, but lead to longer calculation time.

2.3 Special ordered sets

Special ordered sets is a concept that was introduced by Beale and Tomlin in 1979 [Beale and Tomlin, 1979]. A special ordered set of type 1 (SOS1) is a set of variables (continuous or integer) within which exactly one variable must be non-zero. A special ordered set of type 2 (SOS2) is a set of variables within which at most two can be non-zero. The two variables must be adjacent in the ordering given to the set [Williams, 2008].

Although it is possible to model the restrictions that a set of variables belongs to an SOS1 set or an SOS2 set using integer variables and constraints, it is however great computational advantage to be gained from treating these restrictions algorithmically [Williams, 2008]. The most common application of SOS1 sets is to modelling what would otherwise be 0-1 integer variables. The most common application of SOS1 is to modelling non-linear functions. A chain of linked SOS sets can be used to model non-separable functions without converting the model into one where the non-linearities are all functions of a single variable. Doing so may reduce the computational difficulty.

2.4 Branch and bound

Branch and bound search is a popular algorithm used to efficiently find optimal solutions for IP problems, including MILPs. The idea is to determine for large classes of solutions whether they are likely to contain optimal solutions or not, and doing so without explicit enumerations of all its members. Only the most promising classes have to be searched in detail. This section is largely based on Ronald L. Rardin's textbook on optimization [Rardin, 2000].

Branch and bound algorithms form classes of solutions and analyze whether these classes can contain optimal solutions by analyzing associated relaxations. More detailed enumeration ensues only if the relaxations fail to be definitive. Branch and bound searches through partial solutions, which are solutions that have some decision variables fixed, while other are free or undetermined.

A branch and bound search starts at an initial or root partial solution where all the variables are free. A partial solution is terminated in a branch and bound search if it either identifies a best completion or prove that none can produce an optimal solution in the overall model. If a partial solution cannot be terminated in a branch and bound search it is branched by creating two subsidiary solutions by fixing a previously free binary variable. One of the subsidiary partial solutions is the same as the current solution except that a binary variable is fixed to be 0, and the other partial solution fixes the same variable to be 1.

It is important to know how a search stops. A branch and bound search stops when every partial solution in the tree has been either branched or terminated. As long as some partials solutions remain, the branch and bound search must select an active partial solution to explore next. Several schemes to decide which partial solution to explore exist. The simplest scheme is called depth first. A depth first search selects at each iteration an active partial solution with the most components fixed. This means the "deepest" partial solution in the search tree, giving the name to this search scheme.

Branch and bound searches often try to take advantage of various relaxation methods. First the incumbent solution at any stage in a search is defined to be the best feasible solution known so far. The candidate problem to a partial solution is the restricted version of the model obtained when variables are fixed as in the partial solution. With these two properties defined, several results can be obtained.

First, it's known that the feasible completions of any partial solution are the feasible solutions to the corresponding candidate problem, and thus the objective value of the best feasible completion is the optimal objective value of the candidate problem. Also, if any relaxation of the candidate problem is infeasible, then the related partial solution can be terminated because it has no feasible completions. Then, if any relaxation of a candidate problem has optimal objective value no better than the current incumbent solution, the associated partial solution can be terminated because no feasible completion can improve on the incumbent solution. Thirdly, if an optimal solution to any constraint relaxation of a candidate problem is feasible in the full candidate, it is a best feasible completion of the associated partial solution. After checking whether a new incumbent has been discovered, the partial solution can be terminated.

The three properties described in the former paragraph provide powerful concepts that can be used to efficiently discard large amount of possible combinations of the variables at once. An example on this is the LP-based branch and bound search. This class of branch and bound search algorithms branch by fixing an integer restricted decision variable that had fractional value in the associated candidate problem relaxation. If several integer variables have fractional value the algorithm will usually branch by fixing the variable that's closest to an integer value.

In addition to the depth first scheme several other schemes to choose the next partial solution to explore. An example of this is the best first search, that at each iteration selects an active partial solution with the best parent bound. The parent node is the branched node. Parent bounds can also be used to terminate solutions. Whenever a new incumbent solution is discovered, any active partial solution with parent bound no better than the new incumbent solution value can immediately be terminated. Other schemes include the depth forward best back scheme, and the nearest child rule.

Parent bounds can also be used to estimate the error by accepting the incumbent solution. At any stage of a branch and bound search, the difference between the incumbent solution value and the best parent bound of any active partial solution shows the maximum error in accepting the incumbent as an approximate optimum. Thus you know how far from optimum you can maximum be by accepting the incumbent solution, and stopping the search there.

3 Mathematical model

In this chapter a new mathematical model for optimization of offshore natural gas field development will be presented. First the main assumptions and simplifications are given, before the model is presented stepwise through the natural gas value chain. The model follows a naming convention. Small letters are used for variables, and capital letters are used for constants. Indexes are small letters in subscript. Capital letters in subscript are part of variable or constant names.

3.1 Assumptions and simplifications

The model is developed for petroleum deposits that consist mainly of natural gas. Most natural gas fields will produce some liquids i.e. natural gas liquids (NGL), condensate or crude oil together with the natural gas. It is assumed that these liquids are produced as a constant fraction of the natural gas production. This is a simplification of the realities, as the liquids production will usually vary to some degree as the pressure in the reservoir declines. The need to develop a transport alternative and capacity to be able to sell these products is not explicitly formulated in the model. It can be taken into consideration to some degree by increasing the cost of the production facilities and transport alternatives for natural gas.

It is assumed that enough exploration activities have been done to reveal the main properties of the natural gas reservoir that is to be developed. Especially this includes the initial gas resource in place, pressure response to production and how productive the reservoir is probable to be.

The model is deterministic, not stochastic. This means that all values are assumed to be known with certainty. Indeed, this is a major assumption as most natural gas fields are affected to various degree by uncertainty. Most parts of the model will in reality have some degree of uncertainty related to them. The reservoir can be smaller or larger than what it is initially believed to be. The investment cost of the different parts of infrastructure is also assumed to be known. It is however not uncommon that field developments become more expensive than originally planned. An overview by the Norwegian Petroleum Directorate (NPD) in 2009 showed that field developments then were on average 13.8 per cent more expensive than when the plan for development was delivered. Furthermore, oil and gas prices that are used in the model are projected for the whole field life time which can be several decades. With oil prices rising above 140 dollar, falling below 40 dollar and rising towards 130 dollar in the course of three years, it is evident that projecting the prices 50 years ahead will be close to impossible.

It is assumed that there will always be additional demand for the natural gas in the market, and that the amount of natural gas supplied from the field will not have an influence on the market price. The model allows for a long term contract to be agreed with a customer, resulting in a higher price in return of meeting a given demand for several years.

The model that will be presented is a MILP model. It is assumed that costs that are given as constants are in fact constant, that linear relations used are in fact linear, and that linearisation of the non-linear functions gives a sufficiently good representation of the realities.

Furthermore, the model is developed with strategic decisions in mind, as opposed to the operational and tactical decisions. It is intended to be used in the early phase of offshore natural gas field developments to give an indication of what might be an optimal approach to the particular natural gas field in question. The user is assumed to be in position that has the power to make sure that the suggested decision from the model might be realised.

The time frame of the field development is divided into a given number of periods. All investment and production decisions are made at the start of a given period. This means that the reservoir pressure and flow rates are assumed to be constant within a time period. This is a simplification of the real conditions, as the reservoir pressure and thus the production potential of a natural gas reservoir will decline during the production in a time period. Thus the possible production in a time period will usually be slightly overestimated in the model. It will however be slightly counteracted by the fact that the reservoir pressure in following periods will be underestimated, as the additional production in previous periods will lead to additional decline in reservoir pressure.

The model consists of seven main parts, each taking into account a particular aspect of the natural gas value chain. These main parts are the reservoir model, wells, flowline infrastructure, production infrastructure, energy infrastructure, transport infrastructure and customers. The relationship between these can be illustrated with Figure 3.1.

The objective of the model is to find the maximum net present value (NPV) of an offshore natural gas field development project, pre-tax. Although taxes and royalty schemes will in reality affect the optimal solution, it is chosen to disregard this in the model. The reason for this is that the tax regime of a petroleum producing region will often change during a field development's life time. As recently as the spring of 2011, the oil tax rate was raised in the United Kingdom, apparently

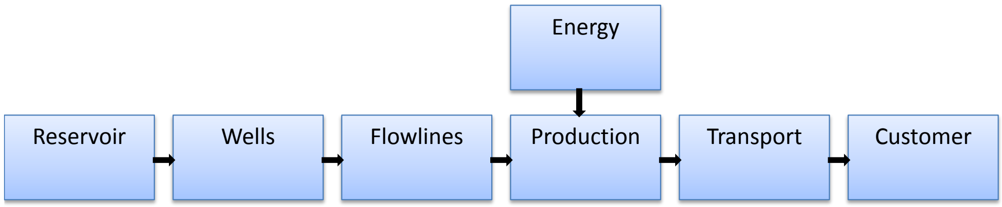

Figure 3.1: Mathematical model structure

surprising oil companies and putting field developments into jeopardy.

3.2 Reservoir model

The first part of the model is the reservoir model. This part of the model is very important, as the revenue of a field development project is dependent on how much natural gas that may be produced at a given time. The reservoir is modelled as a pressurised tank, with a set of characteristic properties.

G_0	-	initial gas resource in place
P_{R0}	-	initial reservoir pressure
P_{Z0}	-	initial p/z factor (pressure divided by compressibility)
P_{IND}	-	productivity index
P_{ZDROP}	-	reduction in p/z factor by producing one unit of gas
L_V	-	liquids produced with one unit of gas
P_{DMAX}	-	maximum pressure differential between reservoir and wellbore flowing pressure

The field development will happen over some time frame. Instead of having a continuous time variable, the time frame will be divided into several discrete time periods. This introduces the need to define a time index, as well as a set of time periods. The number of time periods will vary from one field development to another.

t	-	index for time period t
$\mathcal{T}$	-	set of all time periods

The gas flow from the reservoir in each time period is one of the main outputs from the model.

q_t	-	natural gas flow from the reservoir in time period t

Having defined these parameters, the reservoir model can be developed. First, an obvious constraint is that the total gas production from the natural gas field cannot exceed the initial amount of gas in place.

$$\sum_{t \in \mathcal{T}} q_t \leq G_0 \tag{3.1}$$

As explained earlier, producing natural gas from the reservoir will reduce the reservoir pressure and change the compressibility of the gas. The p/z factor decreases linearly with the production, so the pressure decline will normally be non-linear. This non-linearity will be linearized with the help of piecewise linearisation and SOS2 sets. The necessary sets, variables and constants to make such a formulation are presented below.

Sets and indices

k	-	index for breakpoint k related to p/z factor in reservoir
$\mathcal{K}$	-	set of all breakpoints related to p/z factor in reservoir

Data

P_{Pk}	-	value for pressure at breakpoint k
P_{Zk}	-	value for p/z factor at breakpoint k

Variables

p_{Rt}	-	reservoir pressure in time period t
p_{Zt}	-	p/z factor in time period t
δ_{kt}	-	weighting of breakpoint k in time period t

The linear relation between drop in the p/z factor and gas production can then be given. In the first time period the p/z factor and the pressure must be equal to the initial p/z factor and initial reservoir pressure.

$$p_{Z0} = P_{Z0} \tag{3.2}$$

$$p_{R1} = P_{R0} \tag{3.3}$$

$$p_{Zt} = p_{Z(t-1)} - P_{ZDROP} q_{t-1} \ \forall t \in \mathcal{T} \tag{3.4}$$

Next, this must be linked to a drop in the reservoir pressure. This is done by linking it to a piecewise linearisation of the non-linear relationship to the pressure. The p/z must be equal to a weighting of the p/z values in the breakpoints, similarly the pressure must be equal to the weighting of the pressure value of the same breakpoints.

$$p_{Zt} = \sum_{k \in \mathcal{K}} P_{Zk}\delta_{kt} \qquad \forall t \in \mathcal{T} \tag{3.5}$$

$$p_{Rt} = \sum_{k \in \mathcal{K}} P_{Pk}\delta_{kt} \qquad \forall t \in \mathcal{T} \tag{3.6}$$

In addition, one has to make sure that δ_{kt} forms a SOS2 set in all time periods. The sum of the weighting variables must also equal 1 in all time periods.

$$\sum_{k \in \mathcal{K}} \delta_{kt} = 1 \ \forall t \in \mathcal{T} \tag{3.7}$$

$$\delta_{kt} \text{ is SOS2 } \forall t \in \mathcal{T} \tag{3.8}$$

Referring to the chapter on fluid mechanics, the maximum natural gas flow from a reservoir could be expressed as a productivity index multiplied with a pressure difference between the reservoir pressure and the wellbore flowing pressure, p_{WFt}. The form of this relationship depended on the reservoir pressure. High pressure reservoirs can be modelled with a simple linear relation, whilst lower pressure reservoirs must be modelled with a second degree expression.

$$q_t \leq P_{IND}(p_{Rt} - p_{WFt}) \ \forall t \in \mathcal{T} \tag{3.9}$$

$$q_t \leq P_{IND}(p_{Rt}^2 - p_{WFt}^2) \ \forall t \in \mathcal{T} \tag{3.10}$$

Only one of equations (3.9) and (3.10) will be used in a given problem. If (3.9) is appropriate, then the formulation is fairly straightforward. In the low pressure case, (3.10) needs to be linearised, for example by the use of SOS2 sets and piecewise linearisation. In the rest of this thesis it is assumed that the reservoir pressure of the natural gas field that is to be developed is high enough to use equation (3.9).

Finally, P_{DMAX} is the maximum allowed pressure differential between the reservoir pressure and the flowing wellbore pressure. This will restrict the maximum possible gas flow rate. The effect of this is described in Chapter 3.4.

3.3 Wells

In order to produce the natural gas from the reservoir wells have to be drilled. It is assumed that all wells are equal, and that the wells have a fixed capacity that is constant throughout the field life time.The parameters and variables related to wells is presented below.

Data

W_{MAXt}	-	maximum amount of wells drilled in time period t
C_{APW}	-	gas production capacity of a well
C_{DECW}	-	decommissioning cost for a well
C_{INVWt}	-	investment cost for a well in time period t
$P_{DROPTUB}$	-	pressure drop in well tubing

Variables

n_{Wt}	-	number of new wells drilled in time period t
w_{Wt}	-	total number of wells available in time period t

One cannot drill a half well, although in reality it would be possible to drill a slightly smaller well that might be a bit cheaper. The model is simplified by demanding that only an integer amount of wells is drilled, and the aforementioned assumption of fixed well capacity. The investment cost of a well is high, but might vary with time. When the offshore drilling activity is high, the well cost will rise, whilst they might fall when the activity level is lowered. It is also possible to achieve lower rates if a long term contract with the drilling rig owner is agreed.

$$n_{Wt} \text{ is integer} \qquad \forall t \in \mathcal{T} \tag{3.11}$$

$$n_{Wt} = w_{Wt} - w_{W(t-1)} \qquad \forall t \in \mathcal{T}\backslash\{1\} \tag{3.12}$$

$$n_{W1} = w_{W1} \tag{3.13}$$

The number of new wells in a time period cannot exceed the maximum number of new wells for that time period.

$$n_{Wt} \leq W_{MAXt} \; \forall t \in \mathcal{T} \tag{3.14}$$

The effect on the objective function will be the sum of investment costs, and decommissioning costs. Decommissioning costs arise at the end of the field's life time, when the wells that have been drilled need to be plugged and abandoned. This must be done with drilling rigs, so it can be quite expensive. The model is simplified by assuming that the decommissioning of wells will happen in the last time period, rather than when the rest of the field is abandoned. As the objective of the model is to find the NPV of the project, the costs are multiplied with the discounting factor for the given time period, D_{Ft}.

$$\sum_{t \in \mathcal{T}} D_{Ft} C_{INVWt} n_{Wt} \tag{3.15}$$

$$D_{F(max(\mathcal{T}))} C_{DECW} w_{W(max(\mathcal{T}))} \tag{3.16}$$

Here, the notation $max(\mathcal{T})$ means that the values of D_{Ft} and w_{Wt} in the last time period t are used. The model does not try to maximize the set $\mathcal{T}$.

The gas flow from the reservoir is restricted by the amount of wells that have been drilled. This can be expressed with the following equation.

$$q_t \leq C_{APW} w_{Wt} \ \forall t \in \mathcal{T} \tag{3.17}$$

The parameter that has not been described yet, $P_{DROPTUB}$, is the pressure drop in the well from bottomhole to the wellhead. This is modelled in a simplified way, by assuming that it will be constant. In reality, it will depend on the flowrate and wellhead pressure. The effect of this pressure drop will be described in Chapter 3.4 .

This concludes the well part of the total model. The drilling of wells constitutes a significant part of the total investment cost of a project, so choosing the optimal number of wells is important.

3.4 Flowlines

From the wells the natural gas flows through pipelines to a production facility. These pipelines are known as flowlines. This subsea system will also typically include one or more manifolds, and umbilicals to supply electrical and hydraulic power to control the wells, templates and other equipment. This is not explicitly modelled, but can be considered by increasing the flowline costs to include this aspect. New sets, indices, data and variables are defined to model the flowlines.

Sets and indices

f	-	index for flowline alternative f
$\mathcal{F}$	-	set of all flowline alternatives

Data

C_{INVFft}	-	investment cost for flowline alternative f in time period t
C_{OPFft}	-	operational cost for flowline alternative f in time period t
P_{DROPFf}	-	pressure drop in flowline alternative f
P_{CHOKE0}	-	initial pressure drop over choke valve
C_{APFf}	-	capacity of flowline alternative f

Variables

i_{IFft}	-	investment variable for flowline alternative f in time period t
i_{AFft}	-	availability variable for flowline alternative f in time period t
p_{WHt}	-	wellhead pressure in time period t
p_{WFt}	-	wellbore flowing pressure in time period t
p_{CHKt}	-	pressure drop over choke valve in time period t

The investment variables, i_{IFft}, and availability variables, i_{AFft}, are binary variables that are introduced to keep track of when an investment happens and when a flowline is available. This is done to make sure that the correct investment cost is used, and that operational costs are paid in periods where a flowline alternative is available and active. The investment cost is related to procurement and offshore installation of the flowline, whilst the operational cost includes for example pipeline maintenance, pigging of the pipeline and scale removal to ensure good operational performance of the flowline. The investment and availability variables are defined below.

$$i_{IFft} = \begin{cases} 1 & \text{if flowline alternative } f \text{ is installed in period } t \\ 0 & \text{else} \end{cases} \tag{3.18}$$

$$i_{AFft} = \begin{cases} 1 & \text{if flowline alternative } f \text{ is active in period } t \\ 0 & \text{else} \end{cases} \tag{3.19}$$

It is assumed that the investment cost is taken when the flowline alternative is installed. Or stated in another way: the flowline is installed immediately after the investment has been done. Thus i_{IFft} can be thought of both as an investment and an installation variable for the flowline alternative f. The same thing applies for i_{AFft}. i_{AFft} can be considered both to indicate that a flowline alternative is available or that it is active and being used.

The investment and availability variables are linked with the following set of equations.

$$i_{IFft} \geq i_{AFft} - i_{AFf(t-1)} \qquad \forall f \in \mathcal{F},\ t \in \mathcal{T} \backslash \{1\} \tag{3.20}$$

$$i_{IFf1} = i_{AFf1} \qquad \forall f \in \mathcal{F} \tag{3.21}$$

Only one flowline alternative can be installed at a given natural gas field. This is enforced with the following constraint.

$$\sum_{f \in \mathcal{F}} \sum_{t \in \mathcal{T}} i_{IFft} \leq 1 \ \forall f \in \mathcal{F},\ t \in \mathcal{T} \tag{3.22}$$

The natural gas flow from the reservoir is constrained by the capacity of the flowline alternative that is available.

$$q_t \leq \sum_{f \in \mathcal{F}} C_{APFf} i_{AFft} \ \forall t \in \mathcal{T} \tag{3.23}$$

Pressure drop in the flowlines, P_{DROPFf}, will also have an effect on the maximum natural gas flow rate. This will be part of a larger equation that will be presented in the next section. For now, only the way of calculating which pressure drop that is effective is presented with the following equation.

$$\sum_{f \in \mathcal{F}} P_{DROPFf} i_{AFft} \ \forall t \in \mathcal{T} \tag{3.24}$$

Between the wellhead and the inlet separator a choke valve will be installed. The main motivation behind including this in the mathematical model is to be able to increase the accuracy of the values of the pressure variables in the model. It may also reduce the feasible region, lowering the solution time.

Two constraints are defined for the pressure drop over the choke valve here. The first requires that the pressure drop over the choke valve is equal to an initial pressure drop. Also, to avoid the simple solution of setting this pressure drop equal to zero in all time periods except the first one, it is assumed that the pressure drop over the choke valve cannot decrease faster than the reservoir pressure. In addition, the pressure drop over the choke will have an influence on the wellhead pressure. This relation will be presented in Chapter 3.5.

$$p_{CHK1} = P_{CHOKE0} \tag{3.25}$$

$$p_{CHK(t-1)} - p_{CHKt} \leq p_{R(t-1)} - p_{Rt} \;\forall t \in \mathcal{T}\backslash\{1\} \tag{3.26}$$

The flowing wellbore pressure, p_{WFt}, is equal to the wellhead pressure p_{WHt} plus the pressure drop in the tubing.

$$p_{WFt} = p_{WHt} + P_{DROPTUB} \;\forall t \in \mathcal{T} \tag{3.27}$$

As mentioned in Chapter 3.2, there is a maximum allowed pressure difference between the flowing wellbore pressure p_{WFt}, and the reservoir pressure p_{Rt}. This maximum pressure difference is P_{DMAX}. This needs the following constraint to be defined.

$$p_{Rt} - p_{WFt} \leq P_{DMAX} \;\forall t \in \mathcal{T} \tag{3.28}$$

The reasoning behind Equation (A.25) is that producing the reservoir at a too high flow rate may for example damage the reservoir, reducing the productivity and the amount of natural gas that may be produced.

The total discounted cost related to flowlines can now be defined.

$$\sum_{f \in \mathcal{F}} \sum_{t \in \mathcal{T}} D_{Ft} C_{INVFft} i_{IFft} + \sum_{f \in \mathcal{F}} \sum_{t \in \mathcal{T}} D_{Ft} C_{OPFft} i_{AFft} \tag{3.29}$$

Any decommissioning cost for flowlines is neglected. This is because pipelines and umbilicals can in general be left on the seabed according to the NPD.

3.5 Production infrastructure

In this section, the necessary sets, indices, data and variables to be able to model the production infrastructure are presented. Several different types of production facilities may feasible for a given natural gas field development. Some of the main properties of a production facility are used in the model, as modelling all the parts of a production facility would result in a prohibitively large problem.

Sets and indices

m	-	index for production infrastructure alternative m
$\mathcal{M}$	-	set of all production infrastructure alternatives

Data

C_{INVPmt}	-	investment cost for infrastructure alternative m in time period t
C_{OPPmt}	-	operational cost for infrastructure alternative m in time period t
C_{DECPmt}	-	decommissioning cost for infrastructure alternative m in time period t
P_{INm}	-	inlet pressure for infrastructure alternative m
C_{APPm}	-	capacity of infrastructure alternative m
P_{INREDm}	-	possible reduction in inlet pressure for infrastructure alternative m
$C_{INVINREDmt}$	-	investment cost for reduction of inlet pressure for infrastructure alternative m in time period t

Variables

j_{IPmt}	-	investment variable for production infrastructure alternative m in time period t
j_{APmt}	-	availability variable for production infrastructure alternative m in time period t
j_{DPmt}	-	decommisioning variable for production infrastructure alternative m in time period t
j_{IRmt}	-	investment variable for inlet pressure reduction for production infrastructure alternative m in time period t
j_{ARmt}	-	availability variable for inlet pressure reduction for production infrastructure alternative m in time period t

The investment variables, j_{IPmt}, and availability variables, j_{APmt}, can be thought of in the same way as the variables for the flowlines. They are defined in a similar way below. In addition, decommissioning variables j_{DPmt} are used to indicate whether a production infrastructure alternative is decommissioned in a given time period.

$$j_{IPmt} = \begin{cases} 1 & \text{if production infrastructure alternative } m \text{ is installed in period } t \\ 0 & \text{else} \end{cases} \tag{3.30}$$

$$j_{APmt} = \begin{cases} 1 & \text{if production infrastructure alternative } m \text{ is active in period } t \\ 0 & \text{else} \end{cases} \tag{3.31}$$

$$j_{DPmt} = \begin{cases} 1 & \text{if production infrastructure alternative } m \text{ is decommissioned} \\ & \text{in period } t \\ 0 & \text{else} \end{cases} \tag{3.32}$$

The relationship between these binary variables is also similar to those for the flowline variables. Two sets of constraints couple these variables.

$$j_{IPmt} \geq j_{APmt} - j_{APm(t-1)} \ \forall m \in \mathcal{M}, \ t \in \mathcal{T} \backslash \{1\} \tag{3.33}$$

$$j_{IPm1} = j_{APm1} \ \forall m \in \mathcal{M} \tag{3.34}$$

This relationship forces an investment to be done if one wishes to open a production infrastructure alternative. A resembling relationship is defined between the decommissioning variables and the availability variables.

$$j_{DPmt} \geq j_{APm(t-1)} - j_{APm(t)} \qquad \forall m \in \mathcal{M}, \ t \in \mathcal{T} \backslash \{1\} \tag{3.35}$$

$$j_{DPm1} = 0 \qquad \forall m \in \mathcal{M} \tag{3.36}$$

Equation (3.36) is defined a such way, as it makes no sense to decommission a field in the first time period.

Next, it is assumed that it is only interesting to invest at most in one production infrastructure alternative throughout the planning period.This will be true in most cases, although some fields are redeveloped with new platforms to increase the field life time after a considerable amount of time. The following constraint makes sure that at most one production infrastructure is invested in.

$$\sum_{min\mathcal{M}} \sum_{t\in\mathcal{T}} j_{IPmt} \leq 1 \tag{3.37}$$

The natural gas flow from the reservoir cannot be higher than the processing capacity on the production facility that is active and available.

$$q_t \leq \sum_{m\in\mathcal{M}} C_{APPm} j_{APmt} \; \forall t \in \mathcal{T} \tag{3.38}$$

Another important characteristic is the inlet pressure on the production facility, P_{INm}. A low inlet pressure will result in a higher possible natural gas flow from the reservoir than a high inlet pressure. It will however result in a higher energy consumption for the processing of the natural gas. Natural gas fields are often developed in such a way that the inlet pressure will be quite high initially, and then be lowered when the reservoir pressure has fallen after some time of production. This comes at a cost, as the processing facilities will usually have to be modified to some extent. It will however lead to higher production rates, and higher recovery rate. The model allows for one such inlet pressure reduction to happen.

The variables related to inlet pressure reduction, j_{IRmt} and j_{ARmt}, are defined in the following way.

$$j_{IRmt} = \begin{cases} 1 & \text{if inlet pressure reduction for production infrastructure} \\ & \text{alternative } m \text{ is installed in time period } t \\ 0 & \text{else} \end{cases} \tag{3.39}$$

$$j_{ARmt} = \begin{cases} 1 & \text{if inlet pressure reduction for production infrastructure} \\ & \text{alternative } m \text{ is active in time period } t \\ 0 & \text{else} \end{cases} \tag{3.40}$$

They are coupled with the following set of constraints.

$$j_{IRmt} \geq j_{ARmt} - j_{ARm(t-1)} \; \forall m \in \mathcal{M}, \; t \in \mathcal{T} \backslash \{1\} \tag{3.41}$$

$$j_{IRm1} = j_{ARm1} \; \forall m \in \mathcal{M} \tag{3.42}$$

It is only interesting to reduce the inlet pressure of the active production infrastructure alternative. A constraint is needed to ensure that the model does not reduce the inlet pressure at production alternatives that are inactive, which would lead to an incorrect solution. This is done by demanding that the inlet pressure reduction variable, j_{ARmt}, cannot be higher than the activity variable j_{APmt}.

$$j_{ARmt} \leq j_{APmt} \ \forall m \in \mathcal{M}, \ t \in \mathcal{T} \tag{3.43}$$

Together with the binarity requirement of the investment variable and equation (A.37), this also means that only one reduction of the inlet pressure can happen.

Now, all the necessary variables relating to the pressure in the reservoir, inlet pressure at the production facility and pressure drop in the flowlines have been defined. This allows for another constraint relating to maximum natural gas flow from the reservoir to be defined. The inlet pressure at the production facility can be derived to be the following.

$$\sum_{m \in M} (P_{INm} j_{APmt} - P_{INREDm} j_{ARmt}) \ \forall t \in \mathcal{T} \tag{3.44}$$

Another relation involving the wellhead pressure is defined.

$$p_{WHt} \geq \sum_{m \in M} (P_{INm} j_{APmt} - P_{INREDm} j_{ARmt}) + \sum_{f \in \mathcal{F}} P_{DROPFf} i_{AFft} + p_{CHKt} \ \forall t \in \mathcal{T} \tag{3.45}$$

The reason for this being a greater than or equal to relation is to avoid infeasible solutions in the early time periods. Both the wellhead pressure and the initial pressure drop over the choke valve are fixed in the first time period, not necessarily the same.

Assuming that the natural gas reservoir is a high pressure reservoir, equation (3.9) can be used. Equation (3.24) gives the pressure drop in the flowlines. Combined with the reservoir pressure, p_{Rt}, productivity index P_{IND} and Equation (3.44) and (A.24), the maximum natural gas flow rate can be derived.

$$q_t \leq P_{IND}(p_{Rt} - p_{WFt}) \ \forall t \in \mathcal{T} \tag{3.46}$$

The total contribution to the objective function from the investments and operation of the production infrastructure can then be stated. All the costs are multiplied with the discounting factor D_{Ft} for the corresponding time period.

$$\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} D_{Ft}(C_{INVPmt}j_{IPmt} + C_{OPPmt}j_{APmt} + C_{DECPmt}j_{DPmt})$$
$$+\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} D_{Ft}C_{INVINREDmt}j_{IRmt} \tag{3.47}$$

3.6 Transport infrastructure

After some processing on a production facility, the natural gas must be transported to the market or customer by a pipeline or by LNG ships if liquefied into LNG. Sets, indices, data and variables are defined to describe the transport infrastructure part of the model.

Sets and indices

r	-	index for transport infrastructure alternative r
$\mathcal{R}$	-	set of all transport alternatives

Data

C_{INVTrt}	-	investment cost for transport alternative r in time period t
C_{OPTrt}	-	operational cost for transport alternative r in time period t
C_{TARrt}	-	transportation tariff for transport alternative r in time period t
P_{OUTr}	-	necessary outlet pressure for transport alternative r
C_{APTrt}	-	capacity of transport alternative r in time period t

Variables

l_{ITrt}	-	investment variable for transport alternative r in time period t
l_{ATrt}	-	availability variable for transport alternative r in time period t
q_{Trt}	-	natural gas transported in transport alternative r in time period t

The investment and availability variables for transport alternatives are defined in the same way as for flowlines and production infrastructures.

$$l_{ITrt} = \begin{cases} 1 & \text{if transport infrastructure alternative } r \text{ is installed in period } t \\ 0 & \text{else} \end{cases} \tag{3.48}$$

$$l_{ATrt} = \begin{cases} 1 & \text{if production infrastructure alternative } r \text{ is active in period } t \\ 0 & \text{else} \end{cases} \tag{3.49}$$

The coupling constraints between the investment and availability variables are also on the same form as those for flowlines and production infrastructures.

$$l_{ITrt} \geq l_{ATrt} - l_{ATr(t-1)} \ \forall r \in \mathcal{R}, \ t \in \mathcal{T} \backslash \{1\} \tag{3.50}$$

$$l_{ITr1} = l_{ATr1} \ \forall r \in \mathcal{R} \tag{3.51}$$

It is assumed that it is only interesting to invest in one transport alternative throughout the natural gas field's life time.

$$\sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} l_{ITrt} \leq 1 \tag{3.52}$$

As for the flowlines, the maximum natural gas flow rate is constrained by the maximum capacity of the transport alternative,C_{APTr}. The maximum gas flow rate will be the sum of the available alternatives.

$$q_t \leq \sum_{r \in \mathcal{R}} C_{APTrt} l_{ATrt} \ \forall t \in \mathcal{T} \tag{3.53}$$

In addition, the flowrate of gas in a given transport alternative cannot exceed that alternative's capacity.

$$q_{Trt} \leq C_{APTrt} l_{ATrt} \ \forall r \in \mathcal{R}, \ t \in \mathcal{T} \tag{3.54}$$

The total contribution to the objective function from the infrastructure part of the model consists of three parts. There will be an investment cost for building the transport alternative, and an operational cost for operation and maintenance of the transport alternative. If a transport alternative involves connecting to an existing pipeline, a transport tariff will have to be paid for the use of this pipeline. The transport tariff is paid per unit of natural gas transported. All the costs have to be multiplied with the discounting factor D_{Ft}.

$$\sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} D_{Ft}(C_{INVTrt} l_{ITrt} + C_{OPTrt} l_{ATrt} + C_{TARrt} q_{Trt}) \tag{3.55}$$

The outlet pressure of a given transport alternative r, P_{OUTr}, will have an effect on the energy consumption on the production facility. This effect is presented in the next section.

3.7 Energy infrastructure

Energy needs to be supplied to the production facility to power pumps, compressors and other equipment. This energy can be acquired by generating power by the use of gas turbines on the production facility, importing it from nearby platforms with excess capacity or by importing it from an onshore electricity grid. The sets, indices, data and variables that are necesseray to model the energy infrastructure are presented below.

Sets and indices

e	-	index for energy alternative e
a	-	index for breakpoint a related to natural gas flow rate
b	-	index for breakpoint b related to pressure
$\mathcal{E}$	-	set of all energy alternatives
$\mathcal{A}$	-	set of all breakpoints related to natural gas flow rate
$\mathcal{B}$	-	set of all breakpoints related to pressure

Data

C_{INVEet}	-	investment cost for energy alternative e in time period t
C_{OPEet}	-	operational cost for energy alternative e in time period t
C_{ELet}	-	electricity price for energy alternative e in time period t
G_{USAGEe}	-	gas usage for generating power for energy alternative e
S_{ENet}	-	available energy supply from energy alternative e in time period t
E_{Nab}	-	energy required with breakpoint a and b
Q_{ab}	-	value for flow rate in breakpoint a, b
P_{DSab}	-	value for pressure difference in breakpoint a, b

Variables

p_{DIFFt}	-	pressure differential between inlet and outlet of the production infrastructure in time period t
u_{IEet}	-	investment variable for energy alternative e in time period t
u_{AEet}	-	availability variable for energy alternative e in time period t
v_{RENt}	-	energy requirement in time period t
g_{ENet}	-	energy generated in energy alternative e in time period t
λ_{abt}	-	weighting of breakpoint a, b in time period t
μ_{at}	-	sum of weighting variables b in breakpoint a in time period t
η_{bt}	-	sum of weighting variables a in breakpoint b in time period t

The investment and availability variables are defined in the same way as for flowlines, production infrastructure and transport infrastructure.

$$u_{IEet} = \begin{cases} 1 & \text{if energy infrastructure alternative } e \text{ is installed in period } t \\ 0 & \text{else} \end{cases} \tag{3.56}$$

$$u_{AEet} = \begin{cases} 1 & \text{if energy infrastructure alternative } e \text{ is active in period } t \\ 0 & \text{else} \end{cases} \tag{3.57}$$

Energy investment and availability variables are linked in a similar way to the other investment and availability variables.

$$u_{IEet} \geq u_{AEet} - u_{AEe(t-1)} \ \forall e \in \mathcal{E}, \ t \in \mathcal{T} \backslash \{1\} \tag{3.58}$$

$$u_{IEe1} = u_{AEe1} \ \forall e \in \mathcal{E} \tag{3.59}$$

However, unlike the flowlines, production infrastructure and transport infrastructure, it is assumed that it might be interesting to invest in two different energy infrastructures. It may for example be possible to use gas turbines in the plateau period, and then import electricity after the natural gas production and energy requirement has declined for some time. This is expressed with the following constraint.

$$\sum_{e \in \mathcal{E}} \sum_{t \in \mathcal{T}} u_{IEet} \leq 2 \tag{3.60}$$

The energy requirement is modelled in a simplified way. It is assumed to be a function of the difference between inlet and outlet pressure of the production facility and the natural gas flow rate. This function is assumed to be equal for all production facilities. Equation (3.44) gives the inlet pressure that is to be used in the calculation. In addition the correct outlet pressure from the production facility, P_{OUTr}, must be used. Thus, the transport infrastructure has an influence on the energy requirement, as for example a long distance pipeline will usually demand a higher outlet pressure, resulting in a higher energy consumption at the production facility. Now, the pressure difference can be calculated.

$$p_{DIFFt} = \sum_{r \in \mathcal{R}} P_{OUTr} l_{ATrt} - \sum_{m \in \mathcal{M}} (P_{INm} j_{APmt} - P_{INREDm} j_{ARmt}) \ \forall t \in \mathcal{T} \quad (3.61)$$

As seen in the set of equations above, the pressure difference will be higher if the inlet pressure is reduced.

The energy requirement will be calculated with a piecewise linearization of a function of both the pressure difference and the natural gas flow rate. The sum of the weighting variables, λ_{abt}, need to be equal to one in each time period.

$$\sum_{a \in \mathcal{A}} \sum_{b \in \mathcal{B}} \lambda_{abt} = 1 \ \forall t \in \mathcal{T} \quad (3.62)$$

The piecewise linearisation is done with a chain of linked SOS2 sets, as described by Williams.

$$\mu_{at} = \sum_{b \in \mathcal{B}} \lambda_{abt} \qquad \forall t \in \mathcal{T} \quad (3.63)$$

$$\eta_{bt} = \sum_{a \in \mathcal{A}} \lambda_{abt} \qquad \forall t \in \mathcal{T} \quad (3.64)$$

These sets of variables will be subject to SOS2 constraints.

$$\mu_{at} \text{ and } \eta_{bt} \text{ is SOS2 } \forall t \in T \quad (3.65)$$

The value of the pressure differential and the natural gas flow rate must be equal to the breakpoint values multiplied with the value of the respective weighting variables.

$$q_t = \sum_{a \in \mathcal{A}} \sum_{b \in B} Q_{ab} \lambda_{abt} \qquad \forall t \in \mathcal{T} \tag{3.66}$$

$$p_{DIFFt} = \sum_{a \in \mathcal{A}} \sum_{b \in B} P_{DSab} \lambda_{abt} \qquad \forall t \in \mathcal{T} \tag{3.67}$$

With the value of the weighting variables decided by the pressure differential and natural gas flow rate, the energy requirement can be found by multiplying the same values of the weighting variables with values for the energy requirement in the breakpoints.

$$v_{RENt} = \sum_{a \in \mathcal{A}} \sum_{b \in B} E_{Nab} \lambda_{abt} \ \forall t \in \mathcal{T} \tag{3.68}$$

This energy requirement must be supplied from the energy alternatives.

$$v_{RENt} = \sum_{e \in \mathcal{E}} g_{ENet} \ \forall t \in \mathcal{T} \tag{3.69}$$

The energy supplied from an energy alternative cannot exceed the available supply from that energy alternative. The energy supply might be constrained of several reasons, for example the capacity of a gas turbine, amount of excess generation capacity at a nearby platform or available electricity from an onshore grid. A constraint is necessary to take this into account.

$$g_{ENet} \leq S_{ENet} u_{AEet} \ \forall e \in \mathcal{E}, \ t \in \mathcal{T} \tag{3.70}$$

Depending on which energy infrastructure alternative that is chosen, some of the natural gas production might be used to generate this power. The amount of gas that is used for power generation depends on the efficiency of the gas turbines, and whether the energy is imported or not. If the electricity is imported, the gas usage for power generation will obviously be zero. The amount of natural gas used for power generation needs to be subtracted from the total natural gas production to find the amount of gas that is available for exports.

$$q_t = \sum_{r \in \mathcal{R}} q_{Trt} + \sum_{e \in \mathcal{E}} G_{USAGEe} g_{ENet} \ \forall t \in \mathcal{T} \tag{3.71}$$

The contribution to the objective function from the energy infrastructure part of the model consists of the investment cost of the chosen energy alternative, operational costs for the same energy alternative, and possibly a cost for electricity if the energy is imported. As usual, the costs are multiplied with a discounting factor to get the NPV.

$$\sum_{e \in \mathcal{E}} \sum_{t \in \mathcal{T}} D_{Ft}(C_{INVEet} u_{IEet} + C_{OPEet} u_{AEet} + C_{ELet} g_{ENet}) \tag{3.72}$$

3.8 Customers

The last part of the model takes into account the customers that the natural gas will be sold to. As mentioned in the Theory chapter, natural gas has to a large degree been sold on long term contracts. This is an aspect that the model takes into account, although natural gas in Europe is increasingly sold on the spot market. The sets, indices, data and variables for this part of the model are defined below.

Sets and indices

c	-	index for customer c
$\mathcal{C}$	-	set of all customers

Data

R_{Gct}	-	gas price from customer c in time period t
D_{Gct}	-	demand from customer c in time period t
R_{GSt}	-	spot market gas price in time period t
R_{Ot}	-	oil price in time period t

Variables

q_{Cct}	-	gas sold to customer c in time period t
q_{CSt}	-	gas sold to the spot market in time period t
z_{Cc}	-	activity variable for customer c

If a gas supply agreement is signed with a customer, it will be necessary to meet the customer's demand for an agreed amount of years. An indicator variable, z_{Cc}, is used to show whether an agreement has been made with a customer c.

$$z_{Cc} = \begin{cases} 1 & \text{if a sales agreement has been agreed with customer } c \\ 0 & \text{else} \end{cases} \tag{3.73}$$

If a gas supply agreement is signed, the demand has to be met in all years for that agreement.

$$q_{Cct} = D_{Gct} z_{Cc} \; \forall c \in \mathcal{C}, \; t \in \mathcal{T} \tag{3.74}$$

Now, it is assumed in the model that there is a spot market that has a demand for any additional gas that is supplied to the market. Thus, there must be a balance between the natural gas transported in the transport alternatives, which must be equal to gas sold to customers on long term agreements plus gas sold on the spot market.

$$\sum_{r \in \mathcal{R}} q_{Trt} = \sum_{c \in \mathcal{C}} q_{Cct} + q_{CSt} \; \forall t \in \mathcal{T} \tag{3.75}$$

The customer part of the model contains the revenue generating part of the objective function. The revenue will come from natural gas sales sold to specific customers, natural gas sold on the spot market and sale of liquid products produced with the natural gas. The revenue from sales of liquids is calculated as the oil price in a given time period t multiplied with the amount of liquids produced with one unit of natural gas, L_V, times the natural gas production q_t. These terms are discounted with the discounting factor D_{Ft}.

$$\sum_{c \in \mathcal{C}} \sum_{t \in \mathcal{T}} D_{Ft} R_{Gct} q_{Cct} + \sum_{t \in \mathcal{T}} D_{Ft} R_{GSt} q_{CSt} + \sum_{t \in \mathcal{T}} D_{Ft} R_{Ot} L_V q_t \tag{3.76}$$

3.9 Objective function

The objective function is the last part of the model. It consists of the revenue from natural gas and liquids sales, investment, operational and decommissioning costs for production infrastructure, investment and decommissioning costs for wells, investment and operational costs for flowlines, investment, operational and tariff costs for transport infrastructure and investment, operational and electricity costs for energy. The objective function is formed by combining equations (3.15), (3.16), (3.29), (3.47), (3.55), (3.72) and (3.76).

$$
\begin{aligned}
\max\ z = & \sum_{c \in \mathcal{C}} \sum_{t \in \mathcal{T}} D_{Ft} R_{Gct} q_{Cct} + \sum_{t \in \mathcal{T}} D_{Ft} R_{GSt} q_{CSt} + \sum_{t \in \mathcal{T}} D_{Ft} R_{Ot} L_V q_t \\
& - \sum_{e \in \mathcal{E}} \sum_{t \in \mathcal{T}} D_{Ft} (C_{INVEet} u_{IEet} + C_{OPEet} u_{AEet} + C_{ELet} g_{ENet}) \\
& - \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}} D_{Ft} (C_{INVTrt} l_{ITrt} + C_{OPTrt} l_{ATrt} + C_{TARrt} q_{Trt}) \\
& - \sum_{m \in \mathcal{M}} \sum_{t \in \mathcal{T}} D_{Ft} (C_{INVPmt} j_{IPmt} + C_{OPPmt} j_{APmt} + C_{DECPmt} j_{DPmt}) \\
& - \sum_{m \in \mathcal{M}} \sum_{t \in \mathcal{T}} D_{Ft} C_{INVINREDmt} j_{IRmt}) \qquad (3.77) \\
& - \sum_{f \in \mathcal{F}} \sum_{t \in \mathcal{T}} D_{Ft} C_{INVFft} i_{IFft} \\
& - \sum_{f \in \mathcal{F}} \sum_{t \in \mathcal{T}} D_{Ft} C_{OPFft} i_{AFft} \\
& - \sum_{t \in \mathcal{T}} D_{Ft} C_{INVWt} n_{Wt} \\
& - D_{F(max(\mathcal{T}))} C_{DECW} w_{W(max(\mathcal{T}))}
\end{aligned}
$$

4 Implementation

In this section the implementation of the mathematical model presented in chapter 3 will be described. This includes the choice of software, solution method and structure of the model implementation in the software. The complete code for the software implementation can be found in Appendix B.

4.1 Software

The mixed integer linear progreamming model presented in Chapter 3 is implemented using the FICO Xpress Optimization Suite software. It is written in the Mosel programming language. Xpress was chosen to be used mainly because of the author being familiar with the Mosel programming language from courses at NTNU, as well as the fact that Xpress has been successfully used on similar MILP models before.

4.2 Solution method

The MILP model is solved with the built-in MIP optimizer in Xpress. The MIP optimizer uses a Branch and Bound algorithm to solve MIP problems. Several methods are used in the optimizer to make the solution process more efficient. A presolve algorithm is used to reduce the problem size and solve time. Cuttting plane strategies are used to improve the bounds and to reduce the size of the global search. Multiple LP algorithms are available for the initial LP relaxation and node solution. Heuristics are also included.

4.3 Model structure

The model structure of the implementation follows the structure of the mathematical model. Input data is structured into eight input data files. These are included into the main model file, where the objective function and constraints are declared. The model then optimizes the objective function, which is to find the maximum NPV of the offshore natural gas field development. After finding the optimal solution, some of the most important decision variables are printed to the screen.

5 Test cases

In this section the main properties of the three test cases the model has been used on are presented. The test cases are based on publicly available data on two offshore natural gas fields on the NCS, recently announced rates of drilling rigs on the NCS and public data from the NPD. The reservoir and cost data are mainly based on data from the Kristin and Troll natural gas fields. These are considered sufficient to get a general impression of the usefulness of the model in different situations. All the test cases are considered to be high pressure fields, allowing the linear formulation for fluid flow to be used. In all the test cases the investment costs are 10 per cent higher in time period 1 and 2, and 5 per cent higher in time period 3. This imitates that there is a lack of construction capacity in the market at the moment, and that it might be possible to achieve lower prices by waiting some years.

5.1 Test case 1

Test case 1 (TC1) is a small natural gas field. It is not large enough that a separate development is feasible. Nearby production, transport and energy infrastructure exists, so the utilisation of these facilities will be essential for this field development. The time frame of this field development will be around 30 years. A time resolution of 1 year per time period is used for this field. The main properties of this test case are given in Table 5.1.

Table 5.1: Main properties of Test Case 1

Property	Symbol	Value	Unit
Initial gas resource	G_0	8	GSm^3
Initial reservoir pressure	p_{R0}	900	bar
Number of time periods	$\mathcal{T}$	30	-
Flowline alternatives	$\mathcal{F}$	4	-
Production infrastructure alternatives	$\mathcal{M}$	5	-
Transport infrastructure alternatives	$\mathcal{R}$	3	-
Energy infrastructure alternatives	$\mathcal{E}$	2	-
Customer alternatives	$\mathcal{C}$	1	-
Number of variables	-	2911	-
Number of constraints	-	2044	-

Table 5.2 shows the different flowline, production, transport and energy infrastruc-

ture alternatives that are considered in the test case.

5.2 Test case 2

Test case 1 (TC2) is a medium size natural gas field. It is large enough to justify a stand alone development, although nearby production and transport infrastructure exists that may also be used. The total gas resource is large enough to suggest a 20-30 year life time of the field development. Weighing the uncertainty of the costs and natural gas prices far into the future against the need of being able to take decisions regularly, a time resolution of 1 year per period is chosen. The main properties of this field are given in Table 5.3.

Table 5.4 and Table 5.5 shows the different flowline, production, transport and energy infrastructure alternatives that are included in the model for TC2.

5.3 Test case 3

Test case 3 (TC3) is a giant natural gas field. This natural gas field is so large that many development alternatives exist and should be considered in the model. Some infrastructure exists nearby, which can be used to some degree. As the reserves of this natural gas field are so large, the planning period can easily exceed 50 years. A 1 year per time period time resolution is chosen for this field as well. The main properties of TC3 are given in Table 5.6.

Table 5.7 and Table 5.8 shows the different flowline, production, transport and energy infrastructure alternatives that are included in the model for TC3.

Table 5.2: Infrastructure alternatives in Test Case 1

Flowline infrastructure	
Alternative	Description
1	High capacity. High pressure drop
2	High capacity. Low pressure drop
3	Low capacity. High pressure drop
4	Low capacity. Low pressure drop
Production infrastructure	
Alternative	Description
1	Tie-back to production facility A. Low capacity. Slow ramp up in capacity
2	Tie-back to production facility A. Medium capacity. Medium ramp up in capacity
3	Tie-back to production facility B. High capacity. Medium ramp up in capacity
4	Tie-back to production facility B. High capacity Quick ramp up in capacity
5	Tie-back to production facility B. High capacity.. Quick ramp up in capacity. Capacity available 1 year earlier than alternative 4.
Energy infrastructure	
Alternative	Description
1	Low investment cost. High electricity price.
2	High investment cost. Low electricity price
Transport infrastructure	
Alternative	Description
1	Low investment cost. High operational cost. High transport tariff. Slow ramp up in capacity
2	Medium investment cost. Medium operational cost. Medium transport tariff. Medium ramp up in capacity.
3	High investment cost. Low operational cost. Low transport tariff. Quick ramp up in capacity.
Customer	
Alternative	Description
1	Natural gas demand 0.3 GSm^3 in time period 3 - 12. 10% higher gas price.

Table 5.3: Main properties of Test Case 2

Property	Symbol	Value	Unit
Initial gas resource	G_0	60	GSm^3
Initial reservoir pressure	p_{R0}	900	bar
Number of time periods	$\mathcal{T}$	50	-
Flowline alternatives	$\mathcal{F}$	4	-
Production infrastructure alternatives	$\mathcal{M}$	6	-
Transport infrastructure alternatives	$\mathcal{R}$	5	-
Energy infrastructure alternatives	$\mathcal{E}$	4	-
Customer alternatives	$\mathcal{C}$	4	-
Number of variables	-	5854	-
Number of constraints	-	4154	-

6 Results

In this section the results from optimisation of the offshore natural gas field developments of Test Case 1, Test Case 2 and Test Case 3 are presented.

6.1 Hardware and software specifications

All the test cases were tested using the same hardware and software specifications. These are summarized in Table 6.1.

6.2 Test Case 1

Test Case 1 is the smallest test case in terms of the number of time periods, variables and constraints. The main results from optimization of TC1 are given in Table 6.2.

As seen in Table 6.2, an optimal solution was found in reasonable time, with the optimality gap requirement being a gap less than 0,1 %. The Xpress presolve algorithm was able to reduce the problem size to 1742 constraints and 2602 variables. A total of 8 possible integer solutions were found. The first integer solution was found after 4,4 seconds, with an optimality gap of 281,624 %.

The natural gas production profile of the optimal solution of Test Case 1 is illustrated in Figure 6.1. Time periods without natural gas production in the end of the planning period are omitted from the illustration.

Table 5.4: Infrastructure alternatives in Test Case 2

Flowline infrastructure	
Alternative	Description
1	Low capacity. Low investment cost. Low operational cost. High pressure drop
2	Medium-low capacity. Medium-low investment cost. Medium-low operational cost. Medium-high pressure drop.
3	Medium-high capacity. Medium-high investment cost. Medium-high operational cost. Medium-low pressure drop
4	High capacity. High investment cost. High operational cost. Low pressure drop

Production infrastructure	
Alternative	Description
1	Semisubmersible platform. Medium capacity. Medium investment cost. Medium operational cost
2	Semisubmersible platform. Medium capacity. High investment cost. Low operational cost
3	Semisubmersible platform. High capacity. Medium investment cost. Medium operational cost
4	Semisubmersible platform. High capacity. High investment cost. Low operational cost
5	Subsea tie-back to nearby infrastructure. Variable, low capacity. Low investment cost. Low operational cost
6	Subsea tie-back to nearby infrastructure. Higher capacity than alternative 5. Higher investment and operational cost than alternative 5

Table 5.5: Infrastructure alternatives in Test Case 2

Energy infrastructure	
Alternative	Description
1	Gas turbine. Low capacity. Low investment cost.
2	Gas turbine. Medium capacity. Medium investment cost.
3	Gas turbine. High capacity. High investment cost.
4	Importing electricity. Medium capacity. Medium investment cost.
Transport infrastructure	
Alternative	Description
1	Tie-in to pipeline A. Low capacity.
2	Tie-in to pipeline A. High capacity.
3	Tie-in to pipeline B. Low capacity.
4	Tie-in to pipeline B. Medium capacity.
5	Tie-in to pipeline B. High capacity.
Customers	
Alternative	Description
1	Gas demand 1.5 GSm^3 in time period 2-12. 15% premium on gas price.
2	Gas demand 2 GSm^3 in time period 3-7. 10% premium on gas price.
3	Gas demand 1.3 GSm^3 in time period 11-20. 20% premium on gas price.
4	Gas demand 1.2 GSm^3 in time period 7-20. 17.5% premium on gas price.

Table 5.6: Main properties of Test Case 3

Property	Symbol	Value	Unit
Initial gas resource	G_0	1500	GSm^3
Initial reservoir pressure	p_{R0}	900	bar
Number of time periods	$\mathcal{T}$	70	-
Flowline alternatives	$\mathcal{F}$	4	-
Production infrastructure alternatives	$\mathcal{M}$	8	-
Transport infrastructure alternatives	$\mathcal{R}$	6	-
Energy infrastructure alternatives	$\mathcal{E}$	4	-
Customer alternatives	$\mathcal{C}$	7	-
Number of variables	-	9317	-
Number of constraints	-	6724	-

Table 5.7: Infrastructure alternatives in Test Case 3

Flowline infrastructure	
Alternative	Description
1	Low capacity. Low investment cost. Low pressure drop.
2	Medium-low capacity. Medium-low investment cost. Medium-low pressure drop.
3	Medium-high capacity. Medium-high investment cost. Medium-high pressure drop.
4	High capacity. High investment cost. High pressure drop.

Production infrastructure	
Alternative	Description
1	Semisubmersible platform. Low capacity. Low investment cost. Low operational cost. Low decommissioning cost.
2	Semisubmersible platform. Medium-low capacity. Medium-low investment cost. Medium-low operational cost. Low decommissioning cost.
3	Semisubmersible platform. Medium-high capacity. Medium-high investment cost. Medium-high operational cost. Low decommissioning cost.
4	Semisubmersible platform. High capacity. High investment cost. High operational cost. Low decommissioning cost.
5	Fixed platform. Low capacity. Low investment cost. Low operational cost. High decommissioning cost.
6	Fixed platform. Medium-low capacity. Medium-low investment cost. Medium-low operational cost. High decommissioning cost.
7	Fixed platform. Medium-high capacity. Medium-high investment cost. Medium-high operational cost. High decommissioning cost.
8	Fixed platform. High capacity. High investment cost. High operational cost. High decommissioning cost.

Energy infrastructure	
Alternative	Description
1	Gas turbine. Low capacity. Low investment cost.
2	Gas turbine. Medium capacity. Medium investment cost.
3	Gas turbine. High capacity. High investment cost.
4	Importing electricity. High capacity. Medium investment cost. Low operational cost.

Table 5.8: Infrastructure alternatives in Test Case 3

Transport infrastructure	
Alternative	Description
1	Low capacity. Low investment cost
2	Low plus capacity. Low plus investment cost.
3	Medium-low capacity. Medium-low investment cost.
4	Medium-high capacity. Medium-high investment cost.
5	High minus capacity. High minus investment cost.
6	High capacity. High investment cost.
Customers	
Alternative	Description
1	15 GSm^3 in time period 5-35. 15% premium on gas price
2	10 GSM^3 in time period 3-40. 10% premium on gas price
3	13 GSm^3 in time period 11-35. 20% premium on gas price
4	12 GSM^3 in time period 7-41. 17,5% premium on gas price
5	25 GSm^3 in time period 6-30. 27,5% premium on gas price
6	20 GSM^3 in time period 4-31. 20% premium on gas price
7	17 GSM^3 in time period 5-35. 25% premium on gas price

Table 6.1: Hardware and software specifications

Property	Specification
Operating system	Windows XP Professional SP3
Memory	4 GB RAM
CPU	Intel Core 2 Duo E6700 (2x2,6GHz)
Optimization software	Xpress Mosel Version 3.2.0
	Xpress Optimizer Version 21.01.00

Table 6.2: Results for Test Case 1

Property	Value	Unit
Objective function value	10594,1	million NOK
Total natural gas production	6,07	GSm^3
Number of wells drilled	3	-
Flowline infrastructure	2	-
Production infrastructure	4	-
Transport infrastructure	3	-
Energy infrastructure	1	-
Customers served	None	-
Inlet pressure reduction	In time period 6	-
Production start in time period	3	-
Solution time	84,8	seconds
Optimality gap	0,09	%

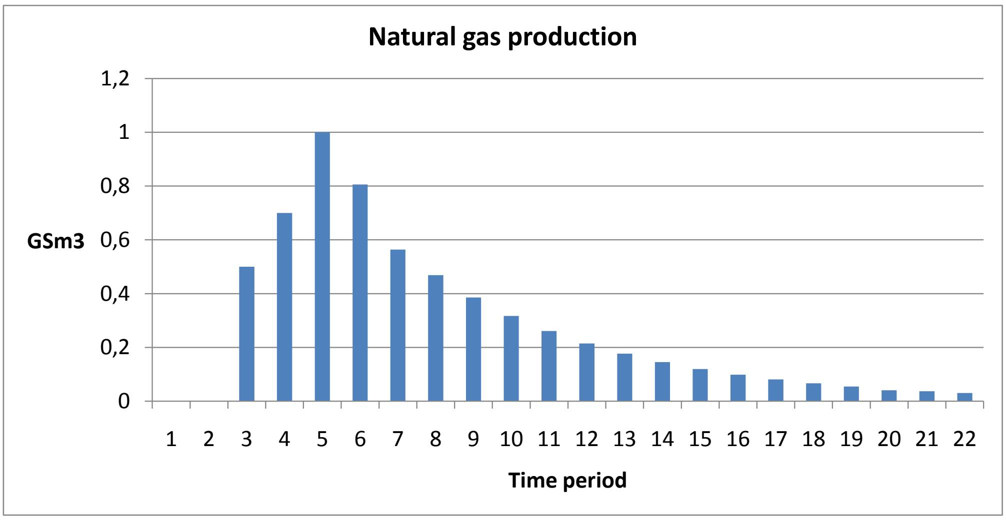

Figure 6.1: Natural gas production in Test Case 1

The natural gas production is ramping up in time periods 3 to 5, before declining until the end of production in time period 22. In time period 3 to 5 the natural gas production is constrained by the available capacity of the production facility. However, although additional capacity is available on the production facility from time period 6, the reservoir pressure has then declined so much that the production cannot be increased and will instead decline. Inlet pressure reduction in time period 6 allows for a higher production, but this is not sufficient to compensate for the decline in reservoir pressure.

6.3 Test Case 2

Test Case 2 is the middle case in terms of the amount of variables, constraints and time periods. The main results from optimization of Test Case 2 are given in Table 6.3.

Table 6.3: Results for Test Case 2

Property	Value	Unit
Objective function value	83353,8	million NOK
Total natural gas production	27,25	GSm^3
Number of wells drilled	9	-
Flowline infrastructure	3	-
Production infrastructure	1	-
Transport infrastructure	2	-
Energy infrastructure	4	-
Customers served	none	-
Inlet pressure reduction	from time period 6	-
Production start in time period	1	-
Solution time	320,8	seconds
Optimality gap	0,099	%

The presolve algorithm in Xpress was able to reduce the problem to one having 3553 constraints and 5298 variables. As for Test Case 1, the optimality gap requirement was less than 0,1%. The first integer solution was found after 30 seconds, with an optimality gap of 35,57%. In total 14 integer solutions were found.

The natural gas production profile of the optimal solution of Test Case 2 is illustrated in Figure 6.2. Time periods without natural gas production in the end of the field's life time are omitted from the illustration.

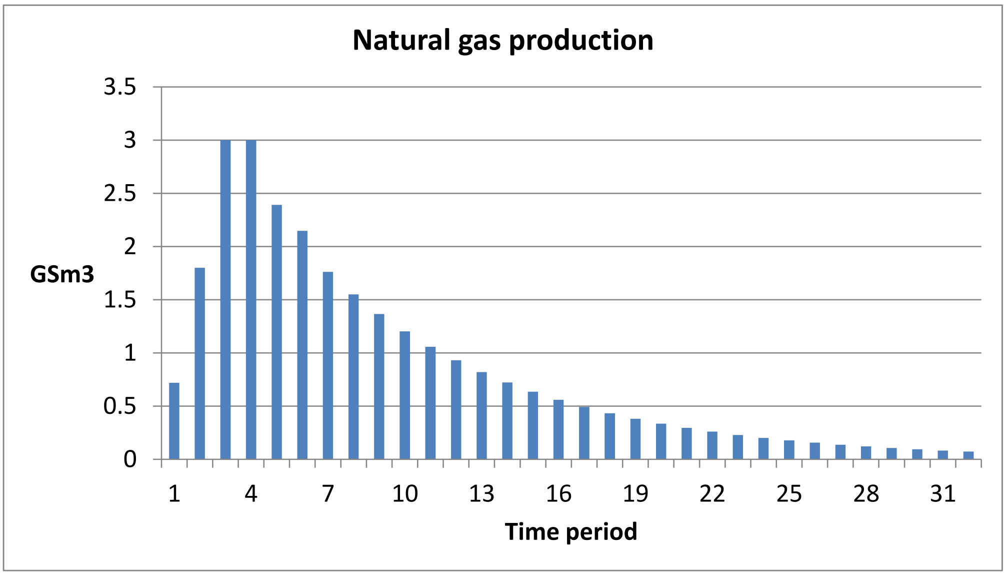

Figure 6.2: Natural gas production in Test Case 2

From Table 6.3 one can see that the medium size semisubmersible platform was chosen, consistent with this the flowline alternative with the same capacity as the semisubmersible was chosen together with the high capacity transport alternative.

In time period 1 and 2 the production is constrained by the amount of available wells. In these two time periods the production is ramped up towards the peak production rate that is achieved in time period 3 and 4. In these time periods the production is limited by the capacity of the production facility and flowlines. From time period 5 and onwards the natural gas production is in decline due to the reduction in reservoir pressure. The inlet pressure reduction in time period 6 offsets the decline to some degree, but is not sufficient to fully compensate for the decline in reservoir pressure. The last period with natural gas production is time period 32, and the field is decommissioned after that.

In Test Case 2 the chosen energy alternative is to import electricity, rather than generating it on the production facility, confirming the suitability of this alternative in some situations. No gas sales agreement is done with any of the customers in this case.

6.4 Test Case 3

Test Case 3 is the largest of the three test cases in this thesis. Test Case 3 involves both more variables, constraints and time periods than the two other test cases. The main results from optimisation of Test Case 3 are given in Table 6.4.

Table 6.4: Results for Test Case 3

Property	Value	Unit
Objective function value	$2,19 \cdot 10^6$	million NOK
Total natural gas production	976,65	GSm^3
Number of wells drilled	26	-
Flowline infrastructure	4	-
Production infrastructure	8	-
Transport infrastructure	6	-
Energy infrastructure	1, 4	-
Customers served	none	-
Inlet pressure reduction	in time period 18	-
Production start in time period	1	-
Solution time	136,5	seconds
Optimality gap	0,088	%

The presolve algorithm in Xpress was able to reduce the problem size down to 5723 constraints and 8363 variables. The optimality gap requirement was set to being less than 0,1%. The first integer solution was found after 25,9 seconds, with an optimality gap of 5,4%. 5 integer solutions were found in total.

The natural gas production profile of the optimal solution of Test Case 3 is illustrated in Figure 6.3. As for Test Case 1 and Test Case 2, time periods without natural gas production in the end of the field's life time are excluded from the illustration.

In this solution the production infrastructure alternative with the highest capacity is chosen. The highest capacity alternatives are also chosen for flowlines and transport infrastructure. From time period 1 to 5 the limiting factor is the amount of available wells. The natural gas production is then on a plateau from time period 6 to 20. The inlet pressure is reduced in time period 18, allowing the natural gas production rate to continue at plateau level from time period 18 to 20. In time period 21 the reservoir pressure has declined so much that the plateau cannot be continued, and production decline ensues until end of production in time period 70.

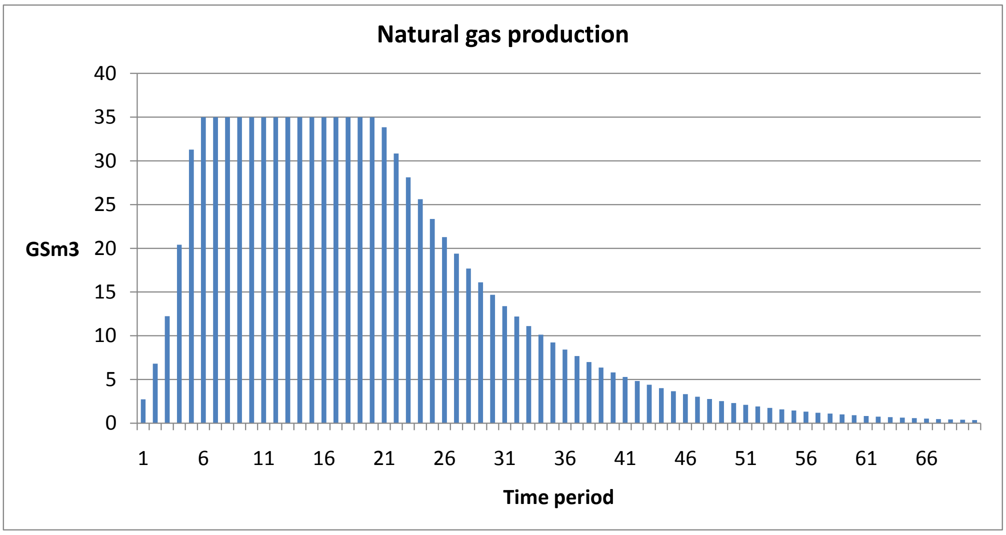

Figure 6.3: Natural gas production in Test Case 3

An interesting observation is that two energy alternatives are used in this solution for the development of Test Case 3. Electricity is imported in all time periods. This energy alternative has sufficient capacity to cover the energy requirement at peak production rate with the initial inlet pressure. However, as mentioned above, the inlet pressure is reduced in time period 18. The energy alternative that has been installed previously is then not able to cover the energy demand if the production rate is to be maintained. Thus, additional energy supply is needed and the low capacity gas turbine is installed in time period 18. The gas turbine is used until time period 22, when the production rate has declined slightly, enabling electricity imports to cover all energy demand yet again.

In this Test Case, seven customer alternatives are included in addition to the spot market. None of these alternatives are used in the optimal solution. An example of one of the energy alternatives is number 5, with a natural gas demand of 25 GSm^3 from time period 6 to 30. Although this alternative offers 27,5% higher natural gas price, it is still not optimal to accept this contract.

7 Discussion

In chapter 6 the results from optimization of the three test cases were presented. This chapter will discuss the applicability of the mathematical model presented in chapter 3 with the implementation of chapter 4. The main experiences from the outcome of optimization of Test Case 1, Test Case 2 and Test Case 3 will also be discussed. For easy comparison, some of the results from chapter 6 are summarised in Table 7.1.

Table 7.1: Comparison of test cases

	Test Case 1	Test Case 2	Test Case 3
Solution time	84,8s	320,8s	136,5s
Optimality gap	0,099%	0,098%	0,096%
Integer solutions found	8	14	5
First integer solution found after	4,4s	30s	25,9s
Optimality gap of first solution	281,62%	35,57%	5,40%

The implementation of the mathematical model in Xpress was able to find an optimal solution for all of the three test cases. As can be seen in Table 7.1, Test Case 1 has the shortest solution time and Test Case 2 the longest solution time. This is notable, as Test Case 3 is larger than Test Case 2 in terms of variables and constraints. This indicates that the form of the input data has a significant input on the solution time.

Even so, in terms of computation time, the model appears to do fairly well as the problem size increases. Test Case 3 has approximately four times as many variables and constraints as Test Case 1, and used approximately 61% more time to find the optimal solution.

The first integer solution was found fairly quick in all the test cases. This was apparently dependent on the amount of variables in the test cases, as it took almost seven times more time to find the first integer solution to Test Case 2 than to Test Case 1. Although Test Case 2 and 3 used more time to find the first integer solution, these first solutions had a lower initial optimality gap.The first integer solution to Test Case 3 had an initial optimality gap of only 5,4% compared to 281,962% for the first integer solution of Test Case 1. Figure 7.1, 7.2 and 7.3 shows the optimality gap of the integer solutions found to TC1, TC2 and TC3 at the time they were found.

Several integer solutions were found for all test cases. The objective value of the integer solutions to Test Case 1 are illustrated in Figure 7.4.

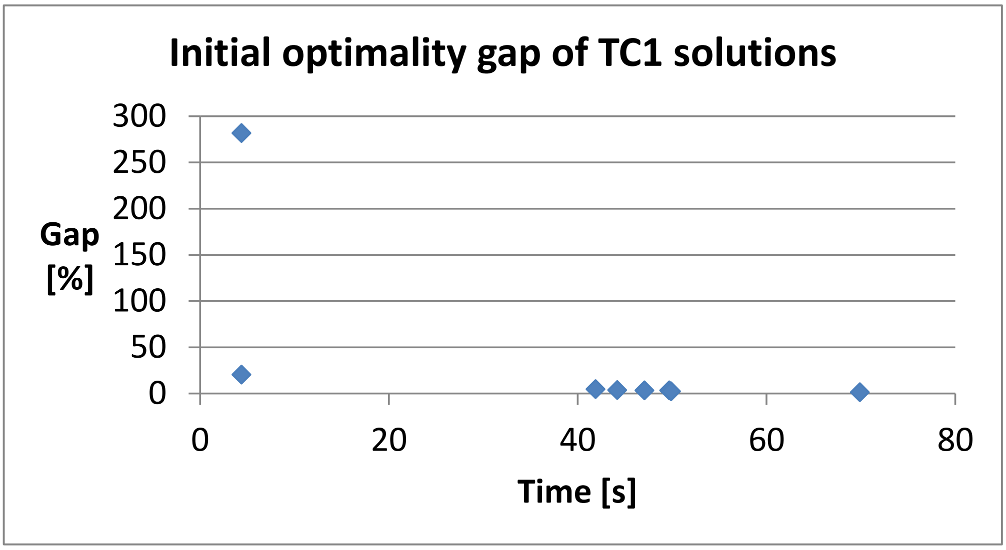

Figure 7.1: Initial optimality gap of integer solutions of Test Case 1

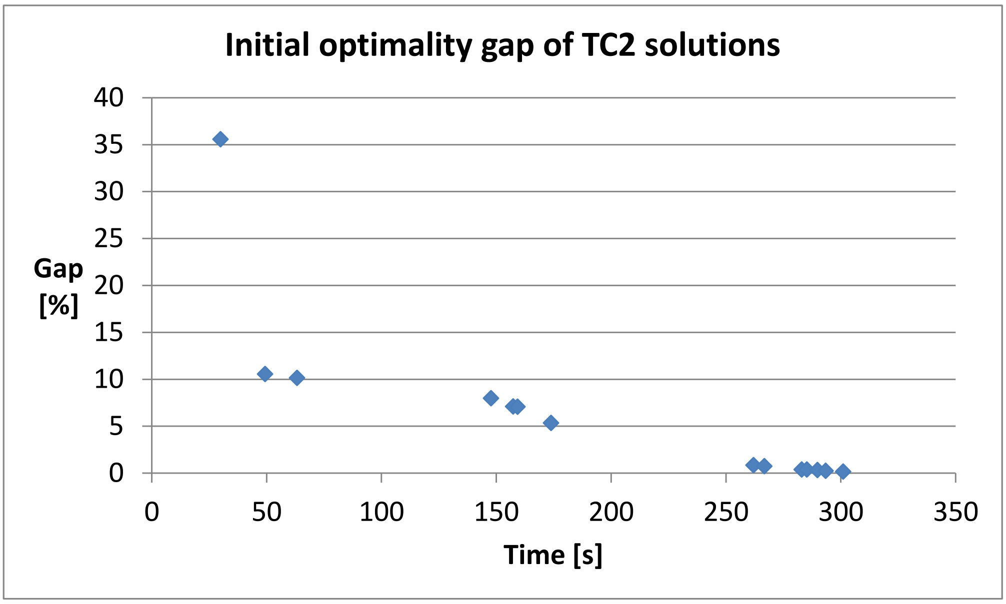

Figure 7.2: Initial optimality gap of integer solutions of Test Case 2

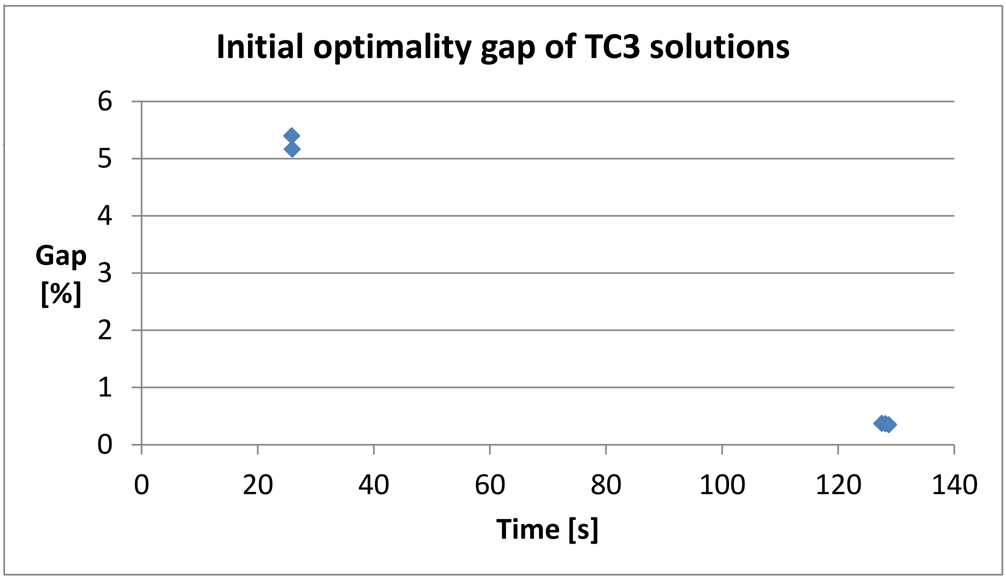

Figure 7.3: Initial optimality gap of integer solutions of Test Case 3

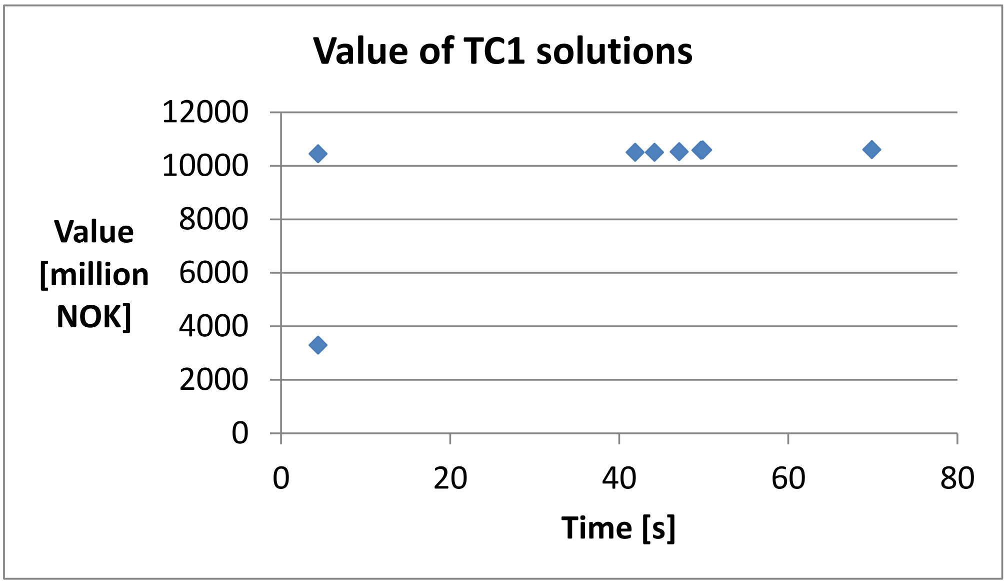

Figure 7.4: Objective function values of integer solutions of Test Case 1

The objective value of the first solution to Test Case 1 was far away from the objective value of the solution that was considered to be optimal, being 69% smaller. However, the second solution had an objective value that was only 1,4% smaller than the objective value of the optimal solution. Thus, a very good solution was found quickly for Test Case 1 after only 4,4 seconds. The rest of the computational time was used to find six slightly better solutions and generate bounds good enough to close the optimality gap.

The objective function values of the integer solutions to Test Case 2 are illustrated in Figure 7.5

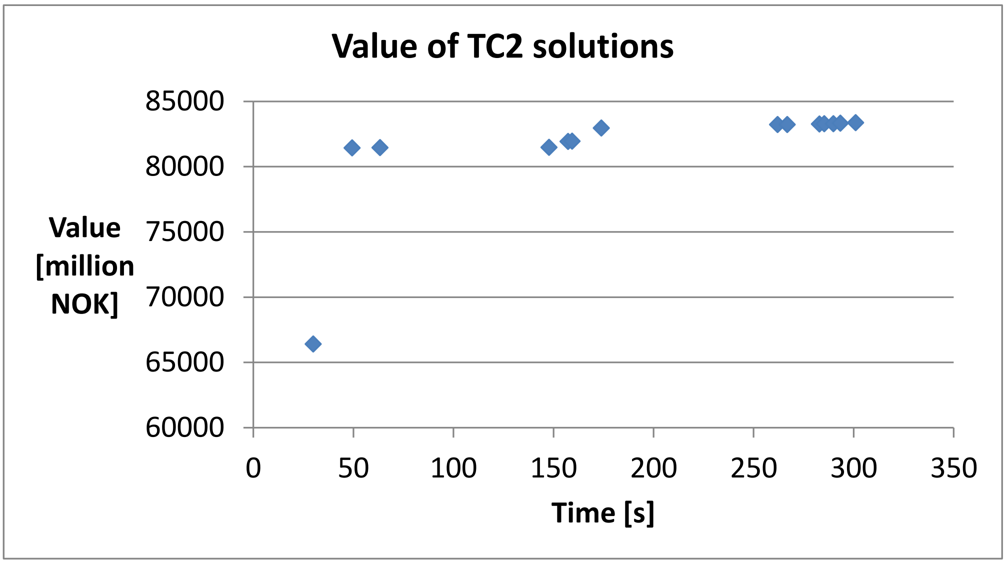

Figure 7.5: Objective function values of integer solutions of Test Case 2

Test Case 2 was the test case that had the longest solution time. It was also the test case that used most time to find the first integer solution. The objective function value of the first integer solution was 20,3% smaller than the optimal solution. The second integer solution was however only 2,3% smaller than the optimal solution. So even for this case, a quite good solution was found already after 49,4 seconds of solve time.

The objective function values of the integer solutions to Test Case 3 are illustrated in Figure 7.6

Test Case 3 is remarkable by having a very low initial optimality gap and high objective function value of the first solution. This solution had an objective function value that was only 0,3% lower than that of the optimal solution. The rest of the solution time was used to find four slightly better solutions and generate good

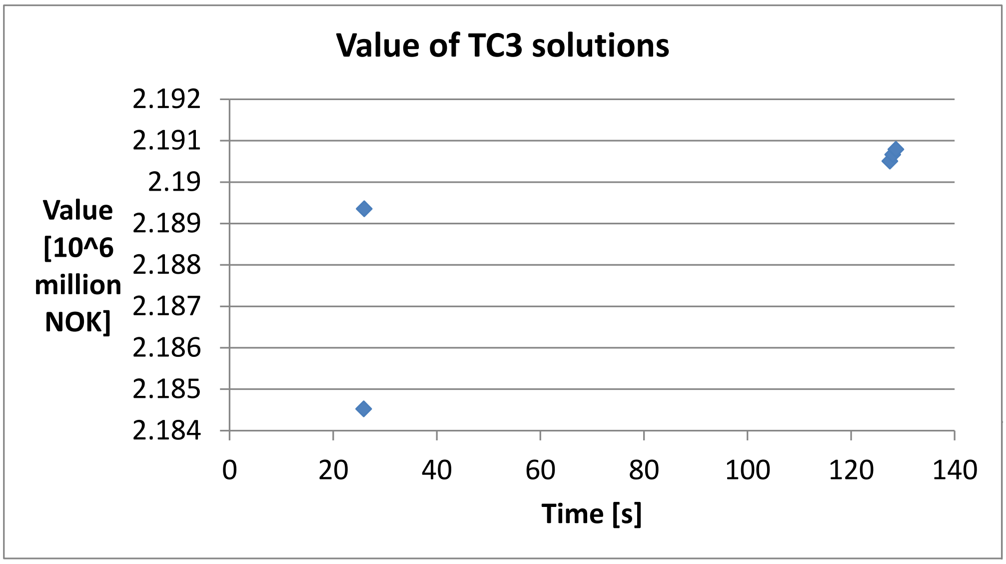

Figure 7.6: Objective function values of integer solutions of Test Case 3

enough bounds to close the optimality gap to below the optimality gap requirement.

The test cases performed consistently in terms of taking into account the various aspect included in the model. Both well availability, production, flowline and transport capacity and available energy supply may effect the production profile and investment decisions. The reservoir model, although limited, appears to give reasonable results.

It must however be stressed that the results here are for three sets of input data. If the structure of the input data changes, large variability in the solution time might be observed. One might either get a very short solution time if there is a development solution that is obviously better than all other solutions. On the other hand, at least the built-in MIP solver in Xpress may for example encounter difficulties in closing the optimality gap in some cases. During testing it has been observed that it has been hard to generate better bounds after a certain time for some input data. Fairly good solutions with single digit optimality gaps found it hard to generate good enough bounds to reach the optimality gap requirement.

8 Conclusion

With natural gas increasing in importance both in Norway and globally, it is essential to develop the available natural gas resources in an an optimal way. The mathematical model that has been developed may ease the process of finding the best way of developing a given natural gas field.

Although simplifications have to be made to create a usable model, the model as a whole is rather accurate. Infrastructure investment decisions have been modelled as choosing between predefined sets of alternatives, with a given cost and capacity structure. The reservoir model, being important for the revenue generation of a field development, has been modelled quite precisely with piecewise linearization. The performance of the reservoir model is satisfying.

Results from testing of the mathematical model by implementing it in Xpress indicate that the model may be used for both giant and marginal natural gas field developments. As the model size increased, the solution time increased, but not dramatically. The solution time is dependent on the input data. The medium size test case took more than twice as long time to solve to optimality compared to the large size test case. For the three test cases that were defined, optimal solutions within the optimality gap requirement were found within six minutes for all the test cases with the hardware specifications that were used. Good solutions were found early in the global search for all test cases.

The main usability of this mathematical model would be in the early phase of offshore natural gas field developments. The model may be used to quickly get an impression of what will be the optimal development solution for a given natural gas field. Running the model several times with slightly different values of the input parameters can be used to identify a few promising development candidates. More detailed engineering would then need to be done to ensure that the suggested solution would be a possible and optimal development plan.

9 Further work

Although the model presented in this thesis has shown promising performance in the test cases, the author is able to identify several areas in which improvements may be made with further work.

One of the drawbacks of this mathematical is that it is purely deterministic. In reality there will usually exist uncertainty regarding for example reservoir volume, gas price and productivity of the reservoir. A stochastic formulation of the problem could take one or more of these areas of uncertainty into consideration, allowing a more robust solution to be chosen.

The investment and operational costs for the different infrastructure alternatives are assumed to be constant, disregarding any dependence on the capacity or production rates. It would be possible to model these costs in a more accurate way.

The model assumes that the liquids production is a constant fraction of the natural gas production. The reservoir model could be enhanced by allowing this fraction to change, as will often be the case in reality. As the revenue from liquids sales may be significant, taking this into account may lead to increased quality of the suggested development solutions.

In its current implementation output of the model is reported in Xpress, and changing input data is done by editing the input files with Xpress or a suitable text editor. The user-friendliness of the model could be increased by handling this through Microsoft Excel or similar software. This could ease the process of visualising the data with graphs and changing input data.

So far, the model has only been implemented and tested using the built-in MIP solver in Xpress. If the problem size grows significantly, it might be useful to investigate other solution methods than this branch and bound based solver.

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A Mathematical model

A.1 Sets and indices

t	-	index for time period t
$\mathcal{T}$	-	set of all time periods
k	-	index for breakpoint k related to p/z factor in reservoir
$\mathcal{K}$	-	set of all breakpoints related to p/z factor in reservoir
f	-	index for flowline alternative f
$\mathcal{F}$	-	set of all flowline alternatives
m	-	index for production infrastructure alternative m
$\mathcal{M}$	-	set of all production infrastructure alternatives
r	-	index for transport infrastructure alternative r
$\mathcal{R}$	-	set of all transport alternatives
e	-	index for energy alternative e
a	-	index for breakpoint a related to natural gas flow rate
b	-	index for breakpoint b related to pressure
$\mathcal{E}$	-	set of all energy alternatives
$\mathcal{A}$	-	set of all breakpoints related to natural gas flow rate
$\mathcal{B}$	-	set of all breakpoints related to pressure
c	-	index for customer c
$\mathcal{C}$	-	set of all customers

A.2 Data

G_0	-	initial gas resource in place
P_{R0}	-	initial reservoir pressure
P_{Z0}	-	initial p/z factor (pressure divided by compressibility)
P_{IND}	-	productivity index
P_{ZDROP}	-	reduction in p/z factor by producing one unit of gas
L_V	-	liquids produced with one unit of gas
P_{DMAX}	-	maximum pressure differential between reservoir and wellbore flowing pressure
P_{Pk}	-	value for pressure at breakpoint k
P_{Zk}	-	value for p/z factor at breakpoint k
W_{MAXt}	-	maximum amount of wells drilled in time period t
C_{APW}	-	gas production capacity of a well
C_{DECW}	-	decommissioning cost for a well
C_{INVWt}	-	investment cost for a well in time period t
$P_{DROPTUB}$	-	pressure drop in well tubing
C_{INVFft}	-	investment cost for flowline alternative f in time period t
C_{OPFft}	-	operational cost for flowline alternative f in time period t
P_{DROPFf}	-	pressure drop in flowline alternative f
P_{CHOKE0}	-	initial pressure drop over choke valve
C_{APFf}	-	capacity of flowline alternative f

C_{INVPmt}	-	investment cost for infrastructure alternative m in time period t
C_{OPPmt}	-	operational cost for infrastructure alternative m in time period t
C_{DECPmt}	-	decommissioning cost for infrastructure alternative m in time period t
P_{INm}	-	inlet pressure for infrastructure alternative m
C_{APPm}	-	capacity of infrastructure alternative m
P_{INREDm}	-	possible reduction in inlet pressure for infrastructure alternative m
$C_{INVINREDmt}$	-	investment cost for reduction of inlet pressure for infrastructure alternative m in time period t
C_{INVTrt}	-	investment cost for transport alternative r in time period t
C_{OPTrt}	-	operational cost for transport alternative r in time period t
C_{TARrt}	-	transportation tariff for transport alternative r in time period t
P_{OUTr}	-	necessary outlet pressure for transport alternative r
C_{APTrt}	-	capacity of transport alternative r in time period t
C_{INVEet}	-	investment cost for energy alternative e in time period t
C_{OPEet}	-	operational cost for energy alternative e in time period t
C_{ELet}	-	electricity price for energy alternative e in time period t
G_{USAGEe}	-	gas usage for generating power for energy alternative e
S_{ENet}	-	available energy supply from energy alternative e in time period t
E_{Nab}	-	energy required with breakpoint a and b
Q_{ab}	-	value for flow rate in breakpoint a, b
P_{DSab}	-	value for pressure difference in breakpoint a, b
R_{Gct}	-	gas price from customer c in time period t
D_{Gct}	-	demand from customer c in time period t
R_{GSt}	-	spot market gas price in time period t
R_{Ot}	-	oil price in time period t

A.3 Variables

q_t	-	natural gas flow from the reservoir in time period t
p_{Rt}	-	reservoir pressure in time period t
p_{Zt}	-	p/z factor in time period t
δ_{kt}	-	weighting of breakpoint k in time period t
n_{Wt}	-	number of new wells drilled in time period t
w_{Wt}	-	total number of wells available in time period t
i_{IFft}	-	investment variable for flowline alternative f in time period t
i_{AFft}	-	availability variable for flowline alternative f in time period t
p_{WHt}	-	wellhead pressure in time period t
p_{WFt}	-	wellbore flowing pressure in time period t
p_{CHKt}	-	pressure drop over choke valve in time period t

j_{IPmt}	-	investment variable for production infrastructure alternative m in time period t
j_{APmt}	-	availability variable for production infrastructure alternative m in time period t
j_{DPmt}	-	decommisioning variable for production infrastructure alternative m in time period t
j_{IRmt}	-	investment variable for inlet pressure reduction for production infrastructure alternative m in time period t
j_{ARmt}	-	availability variable for inlet pressure reduction for production infrastructure alternative m in time period t
l_{ITrt}	-	investment variable for transport alternative r in time period t
l_{ATrt}	-	availability variable for transport alternative r in time period t
q_{Trt}	-	natural gas transported in transport alternative r in time period t
p_{DIFFt}	-	pressure differential between inlet and outlet of the production infrastructure in time period t
u_{IEet}	-	investment variable for energy alternative e in time period t
u_{AEet}	-	availability variable for energy alternative e in time period t
v_{RENt}	-	energy requirement in time period t
g_{ENet}	-	energy generated in energy alternative e in time period t
λ_{abt}	-	weighting of breakpoint a, b in time period t
μ_{at}	-	sum of weighting variables b in breakpoint a in time period t
η_{bt}	-	sum of weighting variables a in breakpoint b in time period t
q_{Cct}	-	gas sold to customer c in time period t
q_{CSt}	-	gas sold to the spot market in time period t
z_{Cc}	-	activity variable for customer c

A.4 Objective function

$$
\begin{aligned}
\max z = &\sum_{c\in\mathcal{C}}\sum_{t\in\mathcal{T}} D_{Ft}R_{Gct}q_{Cct} + \sum_{t\in\mathcal{T}} D_{Ft}R_{GSt}q_{CSt} + \sum_{t\in\mathcal{T}} D_{Ft}R_{Ot}L_{V}q_{t} \\
&-\sum_{e\in\mathcal{E}}\sum_{t\in\mathcal{T}} D_{Ft}(C_{INVEet}u_{IEet} + C_{OPEet}u_{AEet} + C_{ELet}g_{ENet}) \\
&-\sum_{r\in\mathcal{R}}\sum_{t\in\mathcal{T}} D_{Ft}(C_{INVTrt}l_{ITrt} + C_{OPTrt}l_{ATrt} + C_{TARrt}q_{Trt}) \\
&-\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} D_{Ft}(C_{INVPmt}j_{IPmt} + C_{OPPmt}j_{APmt} + C_{DECPmt}j_{DPmt}) \\
&-\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} D_{Ft}C_{INVINREDmt}j_{IRmt}) \qquad (A.1) \\
&-\sum_{f\in\mathcal{F}}\sum_{t\in\mathcal{T}} D_{Ft}C_{INVFft}i_{IFft} \\
&-\sum_{f\in\mathcal{F}}\sum_{t\in\mathcal{T}} D_{Ft}C_{OPFft}i_{AFft} \\
&-\sum_{t\in\mathcal{T}} D_{Ft}C_{INVWt}n_{Wt} \\
&-D_{F(max(\mathcal{T}))}C_{DECW}w_{W(max(\mathcal{T}))}
\end{aligned}
$$

A.5 Constraints

$$\sum_{t\in\mathcal{T}} q_t \leq G_0 \qquad (A.2)$$

$$p_{Z0} = P_{Z0} \qquad (A.3)$$

$$p_{R1} = P_{R0} \qquad (A.4)$$

$$p_{Zt} = p_{Z(t-1)} - P_{ZDROP}q_{t-1} \ \forall t \in \mathcal{T} \qquad (A.5)$$

$$p_{Zt} = \sum_{k\in\mathcal{K}} P_{Zk}\delta_{kt} \qquad \forall t \in \mathcal{T} \qquad (A.6)$$

$$p_{Rt} = \sum_{k\in\mathcal{K}} P_{Pk}\delta_{kt} \qquad \forall t \in \mathcal{T} \qquad (A.7)$$

$$\sum_{k\in\mathcal{K}} \delta_{kt} = 1 \ \forall t \in \mathcal{T} \tag{A.8}$$

$$\delta_{kt} \text{ is SOS2 } \forall t \in \mathcal{T} \tag{A.9}$$

$$q_t \leq P_{IND}(p_{Rt} - p_{WFt}) \ \forall t \in \mathcal{T} \tag{A.10}$$

$$n_{Wt} \text{ is integer} \qquad \forall t \in \mathcal{T} \tag{A.11}$$

$$n_{Wt} = w_{Wt} - w_{W(t-1)} \qquad \forall t \in \mathcal{T}\backslash\{1\} \tag{A.12}$$

$$n_{W1} = w_{W1} \tag{A.13}$$

$$n_{Wt} \leq W_{MAXt} \ \forall t \in \mathcal{T} \tag{A.14}$$

$$q_t \leq C_{APW} w_{Wt} \ \forall t \in \mathcal{T} \tag{A.15}$$

$$i_{IFft} = \begin{cases} 1 & \text{if flowline alternative } f \text{ is installed in period } t \\ 0 & \text{else} \end{cases} \tag{A.16}$$

$$i_{AFft} = \begin{cases} 1 & \text{if flowline alternative } f \text{ is active in period } t \\ 0 & \text{else} \end{cases} \tag{A.17}$$

$$i_{IFft} \geq i_{AFft} - i_{AFf(t-1)} \qquad \forall f \in \mathcal{F},\ t \in \mathcal{T}\backslash\{1\} \tag{A.18}$$

$$i_{IFf1} = i_{AFf1} \qquad \forall f \in \mathcal{F} \tag{A.19}$$

$$\sum_{f\in\mathcal{F}} \sum_{t\in\mathcal{T}} i_{IFft} \leq 1 \ \forall f \in \mathcal{F},\ t \in \mathcal{T} \tag{A.20}$$

$$q_t \leq \sum_{f\in\mathcal{F}} C_{APFf} i_{AFft} \ \forall t \in \mathcal{T} \tag{A.21}$$

$$p_{CHK1} = P_{CHOKE0} \tag{A.22}$$

$$p_{CHK(t-1)} - p_{CHKt} \leq p_{R(t-1)} - p_{Rt} \; \forall t \in \mathcal{T} \backslash \{1\} \tag{A.23}$$

$$p_{WFt} = p_{WHt} + P_{DROPTUB} \; \forall t \in \mathcal{T} \tag{A.24}$$

$$p_{Rt} - p_{WFt} \leq P_{DMAX} \; \forall t \in \mathcal{T} \tag{A.25}$$

$$j_{IPmt} = \begin{cases} 1 & \text{if production infrastructure alternative } m \text{ is installed in period } t \\ 0 & \text{else} \end{cases} \tag{A.26}$$

$$j_{APmt} = \begin{cases} 1 & \text{if production infrastructure alternative } m \text{ is active in period } t \\ 0 & \text{else} \end{cases} \tag{A.27}$$

$$j_{DPmt} = \begin{cases} 1 & \text{if production infrastructure alternative } m \text{ is decommissioned} \\ & \text{in period } t \\ 0 & \text{else} \end{cases} \tag{A.28}$$

$$j_{IPmt} \geq j_{APmt} - j_{APm(t-1)} \; \forall m \in \mathcal{M}, \; t \in \mathcal{T} \backslash \{1\} \tag{A.29}$$

$$j_{IPm1} = j_{APm1} \; \forall m \in \mathcal{M} \tag{A.30}$$

$$j_{DPmt} \geq j_{APm(t-1)} - j_{APm(t)} \qquad \forall m \in \mathcal{M}, \; t \in \mathcal{T} \backslash \{1\} \tag{A.31}$$

$$j_{DPm1} = 0 \qquad \forall m \in \mathcal{M} \tag{A.32}$$

$$\sum_{min\mathcal{M}} \sum_{t \in \mathcal{T}} j_{IPmt} \leq 1 \tag{A.33}$$

$$q_t \leq \sum_{m \in \mathcal{M}} C_{APPm} j_{APmt} \; \forall t \in \mathcal{T} \tag{A.34}$$

$$j_{IRmt} = \begin{cases} 1 & \text{if inlet pressure reduction for production infrastructure} \\ & \text{alternative } m \text{ is installed in time period } t \\ 0 & \text{else} \end{cases} \tag{A.35}$$

$$j_{ARmt} = \begin{cases} 1 & \text{if inlet pressure reduction for production infrastructure} \\ & \text{alternative } m \text{ is active in time period } t \\ 0 & \text{else} \end{cases} \tag{A.36}$$

$$j_{IRmt} \geq j_{ARmt} - j_{ARm(t-1)} \; \forall m \in \mathcal{M}, \; t \in \mathcal{T} \backslash \{1\} \tag{A.37}$$

$$j_{IRm1} = j_{ARm1} \; \forall m \in \mathcal{M} \tag{A.38}$$

$$j_{ARmt} \leq j_{APmt} \; \forall m \in \mathcal{M}, \; t \in \mathcal{T} \tag{A.39}$$

$$p_{WHt} \geq \sum_{m \in M} (P_{INm} j_{APmt} - P_{INREDm} j_{ARmt)} + \sum_{f \in \mathcal{F}} P_{DROPFf} i_{AFft} + p_{CHKt} \; \forall t \in \mathcal{T} \tag{A.40}$$

$$q_t \leq P_{IND}(p_{Rt} - p_{WFt}) \; \forall t \in \mathcal{T} \tag{A.41}$$

$$l_{ITrt} = \begin{cases} 1 & \text{if transport infrastructure alternative } r \text{ is installed in period } t \\ 0 & \text{else} \end{cases} \tag{A.42}$$

$$l_{ATrt} = \begin{cases} 1 & \text{if production infrastructure alternative } r \text{ is active in period } t \\ 0 & \text{else} \end{cases} \tag{A.43}$$

(A.44)

$$l_{ITrt} \geq l_{ATrt} - l_{ATr(t-1)} \; \forall r \in \mathcal{R}, \; t \in \mathcal{T} \backslash \{1\} \tag{A.45}$$

$$l_{ITr1} = l_{ATr1} \; \forall r \in \mathcal{R} \tag{A.46}$$

$$\sum_{r\in\mathcal{R}}\sum_{t\in\mathcal{T}} l_{ITrt} \leq 1 \tag{A.47}$$

$$q_t \leq \sum_{r\in\mathcal{R}} C_{APTrt} l_{ATrt} \ \forall t \in \mathcal{T} \tag{A.48}$$

$$q_{Trt} \leq C_{APTrt} l_{ATrt} \ \forall r \in \mathcal{R},\ t \in \mathcal{T} \tag{A.49}$$

$$u_{IEet} = \begin{cases} 1 & \text{if energy infrastructure alternative } e \text{ is installed in period } t \\ 0 & \text{else} \end{cases} \tag{A.50}$$

$$u_{AEet} = \begin{cases} 1 & \text{if energy infrastructure alternative } e \text{ is active in period } t \\ 0 & \text{else} \end{cases} \tag{A.51}$$

$$\tag{A.52}$$

$$u_{IEet} \geq u_{AEet} - u_{AEe(t-1)} \ \forall e \in \mathcal{E},\ t \in \mathcal{T} \backslash \{1\} \tag{A.53}$$

$$u_{IEe1} = u_{AEe1} \ \forall e \in \mathcal{E} \tag{A.54}$$

$$\sum_{e\in\mathcal{E}}\sum_{t\in\mathcal{T}} u_{IEet} \leq 2 \tag{A.55}$$

$$p_{DIFFt} = \sum_{r\in\mathcal{R}} P_{OUTr} l_{ATrt} - \sum_{m\in\mathcal{M}} (P_{INm} j_{APmt} - P_{INREDm} j_{ARmt}) \ \forall t \in \mathcal{T} \tag{A.56}$$

$$\sum_{a\in\mathcal{A}}\sum_{b\in\mathcal{B}} \lambda_{abt} = 1 \ \forall t \in \mathcal{T} \tag{A.57}$$

$$\mu_{at} = \sum_{b\in\mathcal{B}} \lambda_{abt} \qquad \forall t \in \mathcal{T} \tag{A.58}$$

$$\eta_{bt} = \sum_{a\in\mathcal{A}} \lambda_{abt} \qquad \forall t \in \mathcal{T} \tag{A.59}$$

$$\mu_{at} \text{ and } \eta_{bt} \text{ is SOS2 } \forall t \in T \tag{A.60}$$

$$q_t = \sum_{a\in\mathcal{A}} \sum_{b\in B} Q_{ab}\lambda_{abt} \qquad \forall t \in \mathcal{T} \tag{A.61}$$

$$p_{DIFFt} = \sum_{a\in\mathcal{A}} \sum_{b\in B} P_{DSab}\lambda_{abt} \qquad \forall t \in \mathcal{T} \tag{A.62}$$

$$v_{RENt} = \sum_{a\in\mathcal{A}} \sum_{b\in B} E_{Nab}\lambda_{abt} \ \forall t \in \mathcal{T} \tag{A.63}$$

$$v_{RENt} = \sum_{e\in\mathcal{E}} g_{ENet} \ \forall t \in \mathcal{T} \tag{A.64}$$

$$g_{ENet} \leq S_{ENet} u_{AEet} \ \forall e \in \mathcal{E},\ t \in \mathcal{T} \tag{A.65}$$

$$q_t = \sum_{r\in\mathcal{R}} q_{Trt} + \sum_{e\in\mathcal{E}} G_{USAGEe} g_{ENet} \ \forall t \in \mathcal{T} \tag{A.66}$$

$$z_{Cc} = \begin{cases} 1 & \text{if a sales agreement has been agreed with customer } c \\ 0 & \text{else} \end{cases} \tag{A.67}$$

$$q_{Cct} = D_{Gct} z_{Cc} \ \forall c \in \mathcal{C},\ t \in \mathcal{T} \tag{A.68}$$

$$\sum_{r\in\mathcal{R}} q_{Trt} = \sum_{c\in\mathcal{C}} q_{Cct} + q_{CSt} \ \forall t \in \mathcal{T} \tag{A.69}$$

B Mosel code

See the following pages for the Mosel code formulation of the mathematical model.

```
!*****************************************************************
!*  Model written by Gaute Rannem Johansen, NTNU, spring 2011    *
!*                                                               *
!*****************************************************************

model ModelName
uses "mmxprs"; !gain access to the Xpress-Optimizer solver

parameters
    data="inputhigh.dat"
end-parameters


declarations
        !Indexes and constants

        TIME: set of real
        PRODINF: set of real
        TRANSINF: set of real
        ENGINF: set of real
        CUSTOMER: set of real
        FLOWLINES: set of real

        PRODINV: array(PRODINF, TIME) of real
        PRODOP: array(PRODINF, TIME) of real
        DECOMPROD: array(PRODINF, TIME) of real
        TRANSINV: array(TRANSINF, TIME) of real
        TRANSOP: array(TRANSINF, TIME) of real
        TRANSTARIFF: array(TRANSINF, TIME) of real
        ENGINV: array(ENGINF, TIME) of real
        ENGOP: array(ENGINF, TIME) of real
        FLOWINV: array(FLOWLINES, TIME) of real
        FLOWOP: array(FLOWLINES, TIME) of real
        INLETREDINV: array(PRODINF, TIME) of real

        GASPRICE: array(CUSTOMER, TIME) of real
        ELPRICE: array(ENGINF, TIME) of real
        OILPRICE: array(TIME) of real
        WELLPRICE: array(TIME) of real
        DECOMWELL: real
        DEMANDGAS: array(CUSTOMER, TIME) of real
        GASUSAGE: array(ENGINF) of real
        ENSUPPLY: array(ENGINF, TIME) of real

        CAPWELL: real
        CAPPROD: array(PRODINF, TIME) of real
        CAPTRANS: array(TRANSINF, TIME) of real
        CAPFLOWL: array(FLOWLINES) of real

        MAXWELL: array(TIME) of real
        DISCOUNTF: array(TIME) of real
        GASAMOUNT: real

        DELTAP: real
        PNULL: real
        PINLET: array(PRODINF) of real
```

```
        PRODINDEX: real
        LIQVOLUME: real
        PDROPFLOWLINE: array(FLOWLINES) of real
        PINLETRED: array(PRODINF) of real
        POUTLET: array(TRANSINF) of real

        !SOS2-dimensions
        II: set of real
        JJ: set of real

        KK: set of real
        !SOS2-data
        EN: array(II, JJ) of real
        Q: array(II, JJ) of real
        PR: array(II, JJ) of real

        PZ: array(KK) of real
        PPZ: array(KK) of real
        PZNULL: real

        GASPRICESPOT: array(TIME) of real
        PDROPTUBING: real
        PDIFFMAX: real
        PCHOKEINI: real
end-declarations

declarations
        !Variables


        q: dynamic array(TIME) of mpvar
        j_AP: dynamic array(PRODINF, TIME) of mpvar
        j_IP: dynamic array(PRODINF, TIME) of mpvar
        j_DP: dynamic array(PRODINF, TIME) of mpvar
        l_AT: dynamic array(TRANSINF, TIME) of mpvar
        l_IT: dynamic array(TRANSINF, TIME) of mpvar
        u_AE: dynamic array(ENGINF, TIME) of mpvar
        u_IE: dynamic array(ENGINF, TIME) of mpvar
        n_W: dynamic array(TIME) of mpvar
        w_W: dynamic array(TIME) of mpvar
        q_T: dynamic array(TRANSINF, TIME) of mpvar
        p_R: dynamic array(TIME) of mpvar
        p_DIFF: dynamic array(TIME) of mpvar
        z_C: dynamic array(CUSTOMER) of mpvar
        q_C: dynamic array(CUSTOMER, TIME) of mpvar
        q_S: dynamic array(TIME) of mpvar
        v_REN: dynamic array(TIME) of mpvar
        g_EN: dynamic array(ENGINF, TIME) of mpvar
        i_AF: dynamic array(FLOWLINES, TIME) of mpvar
        i_IF: dynamic array(FLOWLINES, TIME) of mpvar
        j_IR: dynamic array(PRODINF, TIME) of mpvar
        j_AR: dynamic array(PRODINF, TIME) of mpvar

        mu: dynamic array(JJ, TIME) of mpvar
        lambda: dynamic array(II, JJ, TIME) of mpvar
        eta: dynamic array(II, TIME) of mpvar
```

```
        p_Z: dynamic array(TIME) of mpvar
        p_WH: dynamic array(TIME) of mpvar
        p_WF: dynamic array(TIME) of mpvar
        p_CHK: dynamic array(TIME) of mpvar

        delta: dynamic array(KK, TIME) of mpvar

end-declarations

declarations
        !Constraints
        invKon: dynamic array(PRODINF) of linctr
        betajAPKon: dynamic array(PRODINF, TIME) of linctr
        sigmajAPKon: dynamic array(PRODINF, TIME) of linctr
        omegaAlphaKon: dynamic array(TRANSINF, TIME) of linctr
        zetaKhiKon: dynamic array(ENGINF, TIME) of linctr
        iotaThetaKon: dynamic array(FLOWLINES, TIME) of linctr
        ksiOmikronKon: dynamic array(PRODINF, TIME) of linctr
        kapBoring: dynamic array(TIME) of linctr
        nyeBronner: dynamic array(TIME) of linctr
        kapBronnstrom: dynamic array(TIME) of linctr
        kapPlattform: dynamic array(TIME) of linctr
        kapReservoar: linctr
        kapTransport: dynamic array(TIME) of linctr
        transportSplit: dynamic array(TIME) of linctr
        transportOption: dynamic array(TRANSINF, TIME) of linctr
        pressureReservoir: dynamic array(TIME) of linctr
        kapPressure: dynamic array(TIME) of linctr
        demandCustomer: dynamic array(CUSTOMER, TIME) of linctr
        gasBalance: dynamic array(TIME) of linctr
        energyRequirement: dynamic array(TIME) of linctr
        energyGas: dynamic array(TIME) of linctr
        energyGeneration: dynamic array(ENGINF, TIME) of linctr
        transportLimit: linctr
        muLambda: dynamic array(JJ, TIME) of linctr
        etaLambda: dynamic array(II, TIME) of linctr
        kapFlowline: dynamic array(TIME) of linctr
        invFlowlKon: linctr
        invEnKon: linctr
        prodLimit: linctr
        pdiffKon: dynamic array(TIME) of linctr

end-declarations

!Initializes from input data
initializations from data
    TIME PRODINF TRANSINF ENGINF CUSTOMER PRODINV PRODOP DECOMPROD
    TRANSINV TRANSOP TRANSTARIFF ENGINV ENGOP GASPRICE OILPRICE ELPRICE
WELLPRICE DECOMWELL DEMANDGAS
    GASUSAGE ENSUPPLY CAPWELL CAPPROD CAPTRANS MAXWELL DISCOUNTF GASAMOUNT
DELTAP PNULL PINLET PRODINDEX
    LIQVOLUME II JJ EN Q PR FLOWLINES PDROPFLOWLINE FLOWINV FLOWOP CAPFLOWL
PINLETRED INLETREDINV POUTLET
    KK PZ PPZ  PZNULL GASPRICESPOT PDROPTUBING PDIFFMAX PCHOKEINI
end-initializations

!Creates variables
```

```
forall(t in TIME) create(q(t))
forall(t in TIME) create(n_W(t))
forall(t in TIME) n_W(t) is_integer
forall(t in TIME) create(w_W(t))
forall(p in PRODINF, t in TIME) create(j_AP(p, t))
forall(p in PRODINF, t in TIME) j_AP(p,t) is_binary
forall(p in PRODINF, t in TIME) create(j_IP(p, t))
forall(p in PRODINF, t in TIME) j_IP(p,t) is_binary
forall(p in PRODINF, t in TIME) create(j_DP(p, t))
forall(p in PRODINF, t in TIME) j_DP(p, t) is_binary
forall(r in TRANSINF, t in TIME) create(l_AT(r, t))
forall(r in TRANSINF, t in TIME) l_AT(r, t) is_binary
forall(r in TRANSINF, t in TIME) create(l_IT(r, t))
forall(r in TRANSINF, t in TIME) l_IT(r, t) is_binary
forall(e in ENGINF, t in TIME) create(u_AE(e, t))
forall(e in ENGINF, t in TIME) u_AE(e, t) is_binary
forall(e in ENGINF, t in TIME) create(u_IE(e, t))
forall(e in ENGINF, t in TIME) u_IE(e, t) is_binary
forall(ff in FLOWLINES, t in TIME) create(i_AF(ff, t))
forall(ff in FLOWLINES, t in TIME) i_AF(ff, t) is_binary
forall(ff in FLOWLINES, t in TIME) create(i_IF(ff, t))
forall(ff in FLOWLINES, t in TIME) i_IF(ff, t) is_binary
forall(p in PRODINF, t in TIME) create(j_AR(p, t))
forall(p in PRODINF, t in TIME) j_AR(p, t) is_binary
forall(p in PRODINF, t in TIME) create(j_IR(p, t))
forall(p in PRODINF, t in TIME) j_IR(p, t) is_binary
forall(r in TRANSINF, t in TIME) create(q_T(r, t))
forall(t in TIME) create(p_R(t))
forall(t in TIME) create(p_DIFF(t))
forall(c in CUSTOMER) create(z_C(c))
forall(c in CUSTOMER) z_C(c) is_binary
forall(c in CUSTOMER, t in TIME) create(q_C(c, t))
forall(t in TIME) create(v_REN(t))
forall(e in ENGINF, t in TIME) create(g_EN(e, t))
forall(ii in II, jj in JJ, t in TIME) create(lambda(ii, jj, t))
forall(jj in JJ, t in TIME) create(mu(jj, t))
forall(ii in II, t in TIME) create(eta(ii, t))

forall(t in TIME) create(p_Z(t))
forall(kk in KK, t in TIME) create(delta(kk, t))

forall(t in TIME) create(q_S(t))
forall(t in TIME) create(p_WH(t))
forall(t in TIME) create(p_WF(t))
forall(t in TIME) create(p_CHK(t))

!Defining constraints

!3.1
kapReservoar := SUM(t in TIME) q(t) <= GASAMOUNT
!3.2
p_Z(1) = PZNULL

!3.3
pressureReservoir(1) := p_R(1) = PNULL

!3.4
```

```
forall(t in 2..getsize(TIME)) p_Z(t) = p_Z(t-1) - DELTAP * q(t-1)

!3.5
forall(t in TIME) p_Z(t) = SUM(kk in KK) delta(kk, t) * PZ(kk)

!3.6
forall(t in TIME) p_R(t) = SUM(kk in KK) delta(kk, t) * PPZ(kk)

!3.7 & 3.8
forall(t in TIME) do
    convexto(t) := SUM(kk in KK) delta(kk, t) = 1
    makesos2(union(kk in KK) {delta(kk, t)}, SUM(kk in KK) PZ(kk) * delta(kk,
t))
end-do

!3.9, 3.46
forall(t in TIME) q(t) <= (p_R(t) - p_WF(t)) * PRODINDEX

!3.12
forall(t in 2..getsize(TIME)) nyeBronner(t) := n_W(t) = w_W(t) - w_W(t-1)

!3.13
nyeBronner(1) := n_W(1) = w_W(1)

!3.14
forall(t in TIME) kapBoring(t) := n_W(t) <= MAXWELL(t)

!3.17
forall(t in TIME) kapBronnstrom(t) := q(t) <= w_W(t) * CAPWELL

!3.20
forall(ff in FLOWLINES, t in 2..getsize(TIME)) iotaThetaKon(ff, t) :=
i_IF(ff, t) >= i_AF(ff, t) - i_AF(ff, t-1)

!3.21
forall(ff in FLOWLINES) iotaThetaKon(ff, 1) := i_IF(ff, 1) = i_AF(ff, 1)

!3.22
invFlowlKon := SUM(ff in FLOWLINES, t in TIME) i_IF(ff,t) <= 1

!3.23
forall(t in TIME) kapFlowline(t) := q(t) <= SUM(ff in FLOWLINES) (i_AF(ff,t)
* CAPFLOWL(ff))

!3.25
p_CHK(1) = PCHOKEINI

!3.26
forall(t in 2..getsize(TIME)) (p_CHK(t-1) - pCHK(t)) <= (p_R(t-1) - p_R(t))

!3.27
forall(t in TIME) p_WF(t) = p_WH(t) + PDROPTUBING

!3.28
forall(t in TIME) p_R(t) - p_WF(t) <= PDIFFMAX

!3.33
```

```
forall(p in PRODINF, t in 2..getsize(TIME)) betajAPKon(p, t) := j_IP(p, t) >=
j_AP(p, t) - j_AP(p, t-1)

!3.34
forall(p in PRODINF) betaGammaKon(p, 1) := j_IP(p, 1) = j_AP(p, 1)

!3.35
forall(p in PRODINF, t in 2..getsize(TIME)) sigmajAPKon(p, t) := j_DP(p, t)
>= j_AP(p, t-1) - j_AP(p, t)

!3.36
forall(p in PRODINF) sigmaGammaKon(p, 1) := j_DP(p, 1) = 0

!3.37
prodLimit := SUM(p in PRODINF, t in TIME) j_IP(p, t) <= 1

!3.38
forall(t in TIME) kapPlattform(t) := q(t) <= SUM(p in PRODINF) (j_AP(p, t) *
CAPPROD(p, t))

!3.41
forall(p in PRODINF, t in 2..getsize(TIME)) ksiOmikronKon(p, t) := j_IR(p, t)
>= j_AR(p, t) - j_AR(p, t-1)

!3.42
forall(p in PRODINF) ksiOmikronKon(p, 1) := j_IR(p, 1) = j_AR(p, 1)

!3.43
forall(p in PRODINF, t in TIME) j_AR(p, t) <= j_AP(p, t)

!3.45
forall(t in TIME) p_WH(t) >= SUM(p in PRODINF) (PINLET(p) * j_AP(p, t) -
PINLETRED(p) * j_AR(p, t)) +
                              SUM(ff in FLOWLINES) (PDROPFLOWLINE(ff) *
i_AF(ff, t)) + p_CHK(t)

!3.50
forall(r in TRANSINF, t in 2..getsize(TIME)) omegaAlphaKon(r, t) := l_IT(r,
t) >= l_AT(r, t) - l_AT(r, t-1)

!3.51
forall(r in TRANSINF) omegaAlphaKon(r, 1) := l_IT(r, 1) = l_AT(r, 1)

!3.52
transportLimit := SUM(r in TRANSINF, t in TIME) l_IT(r, t) <= 1

!3.53
forall(t in TIME) kapTransport(t) := q(t) <= SUM(r in TRANSINF) (l_AT(r, t) *
CAPTRANS(r, t))

!3.54
forall(r in TRANSINF, t in TIME) transportOption(r, t) := q_T(r,t) <= l_AT(r,
t) * CAPTRANS(r, t)

!3.58
forall(e in ENGINF, t in 2..getsize(TIME)) zetaKhiKon(e, t) := u_IE(e, t) >=
u_AE(e, t) - u_AE(e, t-1)
```

```
!3.59
forall(e in ENGINF) zetaKhiKon(e, 1) := u_IE(e, 1) = u_AE(e, 1)

!3.60
invEnKon := SUM(e in ENGINF, t in TIME) u_IE(e, t) <= 2

!3.61
forall(t in TIME) pdiffKon(t) := p_DIFF(t) = SUM(r in TRANSINF) (POUTLET(r) *
l_AT(r, t)) -
                                                   SUM(p in PRODINF)(PINLET(p) *
j_AP(p, t)  - PINLETRED(p) * j_AR(p, t))

!3.62, 3.63, 3.64 & 3.65
forall(t in TIME) do
    convex(t) := SUM(ii in II, jj in JJ) lambda(ii, jj, t) = 1
    forall(jj in JJ) muLambda(jj, t) := mu(jj, t) = SUM(ii in II) lambda(ii,
jj, t)
    makesos2(union(jj in JJ) {mu(jj, t)}, SUM(ii in II, jj in JJ) EN(ii, jj)
* mu(jj, t))
end-do
forall(t in TIME) do
    forall(ii in II) etaLambda(ii, t) := eta(ii, t) = SUM(jj in JJ)
lambda(ii, jj, t)
    makesos2(union(ii in II) {eta(ii, t)}, SUM(ii in II, jj in JJ) EN(ii, jj)
* eta(ii, t))
end-do

!3.66
forall(t in TIME) q(t) = SUM(ii in II, jj in JJ) lambda(ii, jj, t) * Q(ii,
jj)

!3.67
forall(t in TIME) p_DIFF(t) = SUM(ii in II, jj in JJ) lambda(ii, jj, t) *
PR(ii, jj)

!3.68
forall(t in TIME) energyRequirement(t) := v_REN(t) = SUM(ii in II, jj in JJ)
lambda(ii, jj, t) * EN(ii, jj)

!3.69
forall(t in TIME) energyGas(t) := v_REN(t) = SUM(e in ENGINF) g_EN(e, t)

!3.70
forall(e in ENGINF, t in TIME) energyGeneration(e, t) := g_EN(e, t) <=
u_AE(e, t) * ENSUPPLY(e, t)

!3.71
forall(t in TIME) transportSplit(t) := q(t) = SUM(r in TRANSINF) q_T(r, t) +
SUM(e in ENGINF) g_EN(e, t) * GASUSAGE(e)

!3.74
forall(c in CUSTOMER, t in TIME) demandCustomer(c, t) := q_C(c, t) = z_C(c) *
DEMANDGAS(c, t)

!3.75
```

```
forall(t in TIME) gasBalance(t) := SUM(r in TRANSINF) q_T(r, t) = SUM(c in
CUSTOMER) q_C(c, t) + q_S(t)

TotalProd := SUM(t in TIME) q(t)

!Objective function
TotalProfit :=  SUM(t in TIME) (q(t) * LIQVOLUME * OILPRICE(t) *
DISCOUNTF(t)) +
                SUM(c in CUSTOMER, t in TIME) q_C(c, t) * GASPRICE(c, t) *
DISCOUNTF(t)+
                SUM(t in TIME) q_S(t) * DISCOUNTF(t) * GASPRICESPOT(t) -
                SUM(t in TIME) n_W(t) * WELLPRICE(t) * DISCOUNTF(t) -
                SUM(p in PRODINF, t in TIME) (j_AP(p, t) * PRODOP(p, t) +
j_IP(p, t) * PRODINV(p, t) + j_DP(p, t) * DECOMPROD(p, t)) * DISCOUNTF(t) -
                SUM(r in TRANSINF, t in TIME) (l_AT(r, t) * TRANSOP(r, t) +
l_IT(r, t) * TRANSINV(r, t)) * DISCOUNTF(t) -
                SUM(r in TRANSINF, t in TIME) (q_T(r, t) * TRANSTARIFF(r, t)
* DISCOUNTF(t)) -
                SUM(e in ENGINF, t in TIME) (u_AE(e, t) * ENGOP(e, t) +
u_IE(e, t) * ENGINV(e, t) + g_EN(e, t) * ELPRICE(e, t)) * DISCOUNTF(t) -
                w_W(getsize(TIME)) * DECOMWELL * DISCOUNTF(getsize(TIME)) -
                SUM(ff in FLOWLINES, t in TIME) (i_AF(ff, t) * FLOWOP(ff, t)
+ i_IF(ff, t) * FLOWINV(ff, t)) * DISCOUNTF(t) -
                SUM(p in PRODINF, t in TIME) (j_IR(p, t) * INLETREDINV(p, t))
* DISCOUNTF(t)


    setparam("XPRS_miprelstop", 0.001)
maximize(TotalProfit)
writeln("Begin running model")
writeln("Objective value = ", getobjval)
forall(p in PRODINF, t in TIME | getsol(j_AP(p,t)) > 0) writeln("j_AP(", p,
", ", t, ") = ", getsol(j_AP(p,t)))
forall(p in PRODINF, t in TIME | getsol(j_IP(p,t)) > 0) writeln("j_IP(", p,
", ", t, ") = ", getsol(j_IP(p,t)))
forall(p in PRODINF, t in TIME | getsol(j_DP(p, t)) > 0) writeln("j_DP(", p,
", ", t, ") = ", getsol(j_DP(p, t)))
forall(r in TRANSINF, t in TIME | getsol(l_AT(r,t)) > 0) writeln("l_AT(", r,
", ", t, ") = ", getsol(l_AT(r,t)))
forall(r in TRANSINF, t in TIME | getsol(l_IT(r,t)) > 0) writeln("l_IT(", r,
", ", t, ") = ", getsol(l_IT(r,t)))
forall(e in ENGINF, t in TIME | getsol(u_AE(e,t)) > 0) writeln("u_AE(", e, ",
", t, ") = ", getsol(u_AE(e,t)))
forall(e in ENGINF, t in TIME | getsol(u_IE(e,t)) > 0) writeln("u_IE(", e, ",
", t, ") = ", getsol(u_IE(e,t)))
forall(r in TRANSINF, t in TIME | getsol(q_T(r, t)) > 0) writeln("q_T(", r,
", ", t, ") = ", getsol(q_T(r, t)))
forall(t in TIME | getsol(q(t)) > 0) writeln("Gas production in period ", t,
" = ", getsol(q(t)))
forall(t in TIME | getsol(n_W(t)) > 0) writeln(getsol(n_W(t)), " wells
drilled in period ", t)
writeln("Total gas production : ", getsol(TotalProd))

writeln("End running model")


end-model
```