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A Mixed Complementarity Model of European Energy Markets

Using equilibrium modeling to analyze the optimal price and trade volumes of energy commodities in Europe

Lars Harald Gundersen
Carl Magnus Skutle

Industrial Economics and Technology Management
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Supervisor: Asgeir Tomasgard, IØT
Co-supervisor: Ruud Egging, IØT

Norwegian University of Science and Technology
Department of Industrial Economics and Technology Management

NTNU
Det skapende universitet

MASTERKONTRAKT

- uttak av masteroppgave

1. Studentens personalia

Etternavn, fornavn **Gundersen, Lars Harald**	Fødselsdato **27. des 1986**
E-post **gundersen.harald@gmail.com**	Telefon **97786008**

2. Studieopplysninger

Fakultet **Fakultet for Samfunnsvitenskap og teknologiledelse**	
Institutt **Institutt for industriell økonomi og teknologiledelse**	
Studieprogram **Industriell økonomi og teknologiledelse**	Hovedprofil **Anvendt økonomi og optimering**
E-post **gundersen.harald@gmail.com**	Telefon **97786008**

3. Masteroppgave

Oppstartsdato **17. jan 2011**	Innleveringsfrist **13. jun 2011**
Oppgavens (foreløpige) tittel **A Mixed Complementarity Model of European Energy Markets** **Using equilibrium modeling to analyze the optimal price and trade volumes of energy commodities in Europe**	
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Hovedveileder ved institutt **Professor Tomasgard Asgeir**	Biveileder(e) ved institutt **Ruud Egging**
Merknader **1 uke ekstra p.g.a påske.**	

4. Underskrift

Student: Jeg erklærer herved at jeg har satt meg inn i gjeldende bestemmelser for mastergradsstudiet og at jeg oppfyller kravene for adgang til å påbegynne oppgaven, herunder eventuelle praksiskrav.

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Sted og dato

Harald Paulsen

....

Student

....

Hovedveileder

Originalen oppbevares på fakultetet. Kopi av avtalen sendes til instituttet og studenten.

NTNU
Det skapende universitet

MASTERKONTRAKT
- uttak av masteroppgave

1. Studentens personalia

Etternavn, fornavn	Fødselsdato
Etternavn, fornavn **Skutle, Carl Magnus**	Fødselsdato **04. sep 1986**
E-post **carlmagn@gmail.com**	Telefon **90193765**

2. Studieopplysninger

Fakultet **Fakultet for Samfunnsvitenskap og teknologiledelse**	
Institutt **Institutt for industriell økonomi og teknologiledelse**	
Studieprogram **Industriell økonomi og teknologiledelse**	Hovedprofil **Investering, finans og økonomistyring**
E-post **carlmagn@gmail.com**	Telefon **90193765**

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14/01/11

Sted og dato

Student

Hovedveileder

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Preface

This paper represents the Master's Thesis in Managerial Economics and Operations Research at the Department of Industrial Economics and Technology Management (IØT), at the Norwegian University of Science and Technology (NTNU).

The past five months at NTNU have been an immensely rewarding and intellectual interesting period. We were both excited to be part of this endeavor, but when we started in January, the union of our knowledge on the subject was not that different from the empty set, $\emptyset$, at least that was the way we felt. Five months is a short time to learn such a comprehensive subject, get ideas, investigate those ideas and articulate the findings. We did it even so. The present text is the fruition of our work.

We want to thank our advisor Professor Asgeir Tomasgard for his support, directions and feedback throughout the project. A special thanks goes to Senior Research Fellow Rudolf G. Egging for his positive attitude and his valuable GAMS workshops. We would also like to thank Kjetil T. Midthun and Marte Fodstad for their helpful comments and guidance during the work. Additionally, we appreciate the sweeping introduction Gaute R. Johansen gave us in LaTeX.

Harald Gundersen and
Carl Magnus Skutle

Trondheim, 14th June 2011

Abstract

Energy markets are complex networks of producers, exporters, traders and consumers characterized by different market structures in each sector. The infrastructural network connecting the markets plays an important role in determining the volume of the trade flows and the location of the final consumption.

The market players' behavior in an energy market can be described using a game theoretic approach where each player's decision depends on the other market players decisions. Over the last decades these ideas have evolved and there have been produced some material where markets for a single commodity are modeled, using ideas from game theory to describe the players' influence on each other's decisions.

However, little work has been done analyzing multi-commodity markets with the same set of tools. Based on existing literature that is written on single-commodity modeling, we have applied equilibrium programming with complementarity structure to describe the markets for electricity and natural gas in Northern Europe through a strategic market model. The complexity level is potentially high, so we decided to limit ourselves to a deterministic and myopic model without investment possibilities.

The problem is formulated through a strategic MCP model where each market participant solves an optimization problem connected through the market clearing conditions. Besides showing that the model is an MCP we implemented the model in GAMS and solved it for the gas and electricity market in Northern Europe.

Our results indicates that a Cournot model gives an adequate description of the electricity and gas market in Northern Europe, and that considerable changes in production, consumption, traded volumes and prices in one market can lead to price, quantity and welfare effects in markets far away from the initial cause. We can also register close links between the markets for electricity and natural gas, suggesting that agents' behavior in one commodity market might affect the other commodity market and vice versa.

Contents

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Nomenclature

BP	-	Bilevel Programs
BC	-	Base Case
CP	-	Complementarity Programs
EP	-	Equilibrium Programming
EPEC	-	Equilibrium Programs with Equilibrium Constraints
FDVI	-	Finite Dimensional Variational Inequalities
FOC	-	First Order Conditions
GAMS	-	General Algebraic Modeling System
GNE	-	Generalized Nash Equilibrium
KKT	-	Karush-Kuhn-Tucker
LCP	-	Linear Complementarity Problems
MP	-	Market Power
MCP	-	Mixed Complementarity Problems
MPEC	-	Mathematical Programs with Equilibrium Constraints
M/S	-	Market Share
MSNE	-	Mixed Strategy Nash Equilibrium
NCP	-	Nonlinear Complementarity Problems
nTPA	-	Negotiated Third Party Access
OTC	-	Over-The-Counter
PC	-	Perfect Competition
PSNE	-	Pure Strategy Nash Equilibrium
QVI	-	Quasi Variational Inequalities
RC	-	Reference Case
rTPA	-	Regulated Third Party Access
SB	-	Single-Buyer
TOP	-	Take-Or-Pay
TPA	-	Third Party Access
VI	-	Variational Inequalities

Chapter 1

Introduction

This chapter presents a brief introduction to the many challenges of modeling energy markets. Section 1.1 provides the background and motivation for the thesis while Section 1.2 introduces some of the modeling frameworks being used in market modeling. Section 1.2 is meant as a starting point for further analysis and does not go into the details in each of the modeling approaches. An overview of the thesis and its structural outline is given in Section 1.3.

1.1 Background

Choices made by market participants in an energy market can be described using fundamental game theory. Using a game theoretic approach, this thesis seeks to identify presence of market power in the gas and electricity markets of Northern Europe and analyze how strategic behavior will influence the market prices and volumes.

The European markets for gas and electricity have undergone some major changes over the last decades. In the 1990s and early 2000 they both went through a liberalization process that broke the monopolistic structures that had characterized the supply side of the industries. Despite initiatives from supranational organizations, the development towards two fully liberalized markets has been restrained. Lack of sufficient interconnections and transfer capacities are pointed out as some of the issues that needs to be solved to achieve the goal of the European Union.

The reasons of modeling the markets for gas and electricity instead of other energy carriers like oil, coal, renewables or nuclear energy are many. First of all, both markets share a lot of important characteristics. They are both partly liberalized markets that allows for an oligopolistic representation. In addition, they both have a reliable supply and a well-developed network for regional and international distribution. There has also been made a lot of research in modeling electricity and gas markets separately through single-commodity models. However, little work has been done in integrating both markets into the same model. Hence, we want to analyze both the gas and electricity market within a single modeling framework.

1.2 Modeling Approaches

While energy markets are complex, energy models are simplified representations of energy production and consumption, regulations, and producer and consumer behavior. There are different ways of modeling energy markets; each with its advantages and disadvantages. You can look at several trading regimes, various pricing strategies, different set of market actors' behavior, multiple commodities, simple or complicated nodal structure and the list goes on. The more you include, the more complexity you add to your model. The following paragraphs summarize and concretize three applicable frameworks being used in modeling energy markets: *optimization*, *equilibrium modeling* and *network analysis*.

1.2.1 Optimization

The first optimization techniques go back to the 19th century and the mathematicians' search for the steepest descent of a function. In the middle of the 20th century efficient algorithms of solving optimization problems developed and optimization was found a valuable tool for industry processes. Today, optimization is applicable to a large set of problems and plays an important role of planning and forecasting in nearly all types of industries.

Despite the multiple range of use, the general structure is the same: you want to *maximize* or *minimize* an objective function (1.2.1) of its argument(s) under a set of constraints (1.2.2). The complexity of the function and the constraints can vary from few variables and a linear structure, to numerous variables and nonlinear problems.

In any case, the basic setup can be expressed in the following form:

$$\max_{x} \quad f(x) \tag{1.2.1}$$

$$s.t. \quad g_i(x) \leq 0, \qquad \forall i \in \mathcal{I} \tag{1.2.2}$$

where x denotes the decision variable, $f(x)$ the objective function and $g_i(x)$ the i'th constraint.

There exists a lot of market models using an optimization approach for expressing energy markets. However, traditional optimization models fails when *market power* is included in the model (Harker et al. [1993]).

1.2.2 Equilibrium Modeling

Duality theory from optimization models forms the basis of the complementarity concepts that are used in equilibrium modeling. Complementarity structures have the favorable properties of handling equilibria situations, both generally and in particular game settings. This characteristic is especially useful dealing with economic models and game theoretic problems where states of equilibria is likely to occur.

The method is simple enough; by solving each and every individual's optimization problem within the complementarity system simultaneously, the resulting solution will be the equilibrium solution of the market game. Hence, the equilibrium solution goes beyond the solution of the individual optimization problem of each player, by giving the simultaneous solution to all agents in the game. In many situations the individualistic interests of each player cause the equilibrium solution not to be pareto optimal[1]. A way of dealing with this is by introducing rewards and penalties that change the players' incentives.

[1] A famous example of the inefficiency of equilibrium solutions includes *the Prisoner's Dilemma*. Here, we have two criminals, Bonnie and Clyde, who have been caught by the police. The police has only enough information to charge them for a minor fraud, but suspect that they are responsible for a major fraud, which will give them a harder punishment. The criminals are being separated and each of them is presented with two choices: Either testify against your partner or to remain silent. If both testify, they will get five years in jail each. On the other hand, if both choose to remain silent, they get one year in jail each. The last scenario follows if Bonnie testifies against Clyde, while at the same time Clyde remains silent. That would lead to Bonnie being set free (zero years in jail) and Clyde would get ten years in jail, and vice versa. The equilibrium solution will in this particular case be that both players testify resulting in five years in jail each. However, both players would be better off if they both had remained silent, giving both players only one year in jail. Accordingly, the equilibrium solution is not the pareto optimum.

Midthun [2007b] describes the general form the complementarity format of finding a vector x, assuming $f(x) \geq 0$, satisfying the complementarity condition $f(x)^T x = 0$.

For each element i of the vector x, either x_i or $f_i(x)$ must equal zero:

$$0 \leq x_i \quad \perp \quad f_i(x) \geq 0, \qquad \forall i \in \mathcal{I} \tag{1.2.3}$$

Accordingly, the variables x and $f(x)$ are called *complementary.*

A general maximization problem becomes:

$$\max_x \quad f(x) \tag{1.2.4}$$

$$s.t. \quad g_i(x) \leq 0, \qquad (\lambda_i), \quad \forall i \in \mathcal{I} \tag{1.2.5}$$

$$h_j(x) = 0, \qquad (\mu_j), \quad \forall j \in \mathcal{J} \tag{1.2.6}$$

$$x \geq 0 \tag{1.2.7}$$

where i and j are indexing the inequalities and equalities respectively.

The corresponding Karush-Kuhn-Tucker conditions (KKT) to the maximization problem are necessary and sufficient for optimality given that $f(x)$ is concave and the feasible solution space is convex.

$$\nabla f(x) + \sum_i \lambda_i \nabla g_i(x)^T + \sum_j \mu_j \nabla h_j(x)^T = 0 \tag{1.2.8}$$

$$0 \geq g_i(x) \quad \perp \quad \lambda_i \geq 0, \qquad \forall i \in \mathcal{I} \tag{1.2.9}$$

$$h_j(x) = 0, \qquad \mu_j \text{ free}, \qquad \forall j \in \mathcal{J} \tag{1.2.10}$$

where (1.2.8) makes sure the solution is stationary while (1.2.9) and (1.2.10) guarantees complementarity and feasibility. Note that the dual variable λ_i of the inequality (1.2.5) has to be greater or equal to zero, while the dual μ_j of equality (1.2.6) can take any real number.

The structure of the objective function and the constraints defines the character of the equilibrium problem. Figure 1.2.1 lists the most important subgroups and their relations. A rigorous discussion of each of them is given in Section 5.6.

Imperfect games where one or more players exert market power, are often described by complementarity conditions and equilibrium states. In this way each players' decisions are taken into account, formally expressed through the KKT conditions.

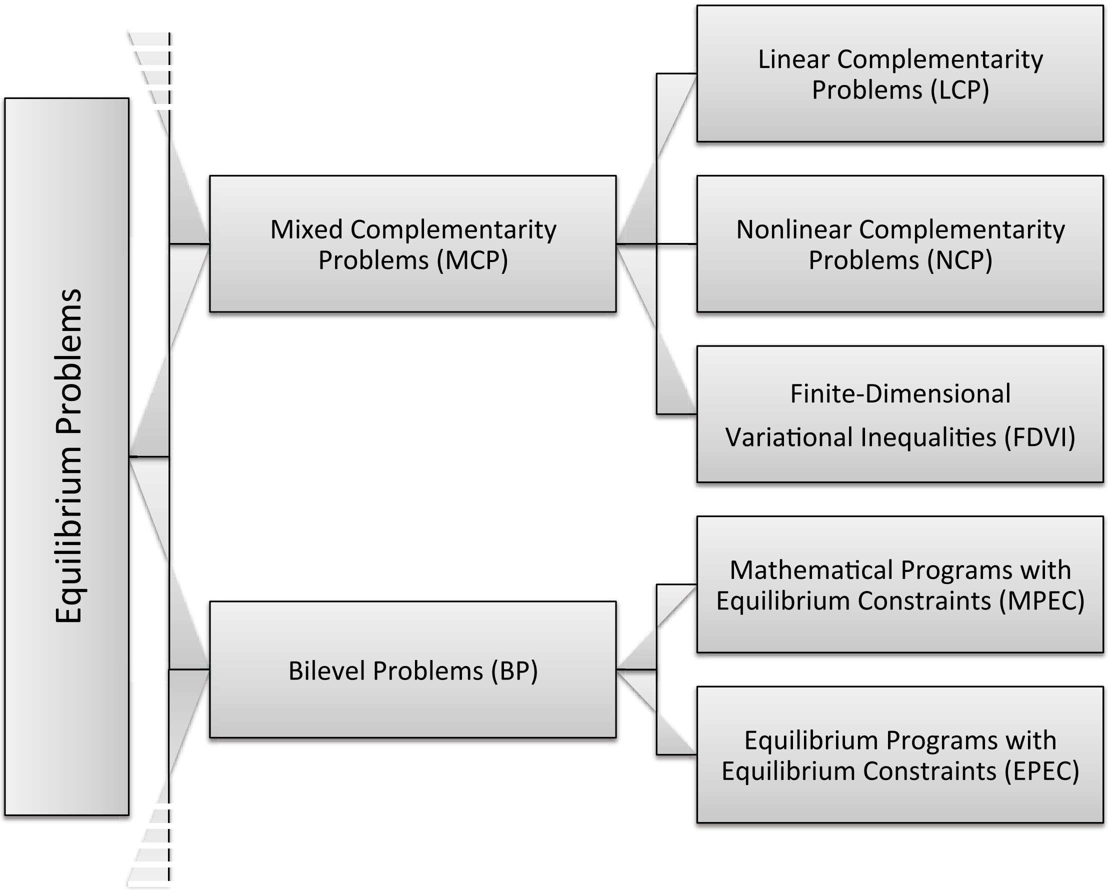

Figure 1.2.1: Important subgroups of equilibrium problems.

In general, solving the resulting system of equations and proving uniqueness and existence of the solution happens to be mathematically challenging. Because of these difficulties, many of the economical papers written on equilibrium modeling do not leave this subject a thought at all, and solely base the validity of their result on earlier papers written by mathematicians.

1.2.3 Network Analysis

A third commonly used approach of analyzing energy markets is by using graph theory fundamentals. By thinking of each interconnection between nodes as arcs you can create economical interpretations of network theorems and solve cost minimizing problems in an efficient way (Figure 1.2.2). Historically this method has been primarily applied in routing problems, but has also proved useful in modeling commodity markets.

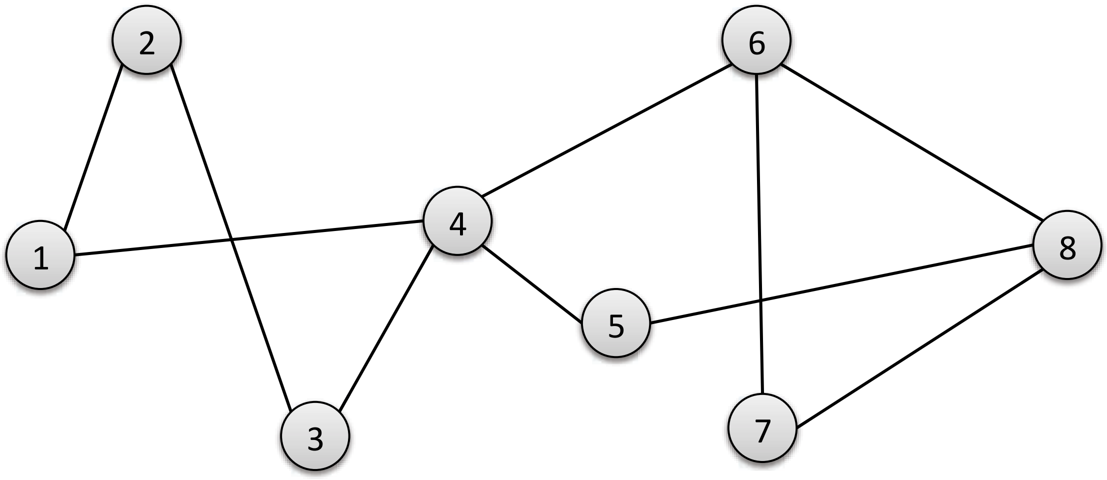

Figure 1.2.2: Non-directed graph with arcs and vertices.

1.3 Thesis Overview

This thesis introduces a two-commodity complementarity model analyzing the prices and volumes for natural gas and electricity in Northern Europe. Furthermore, we sought to investigate presence of market power, how the two commodities influence each other and how the market players act from a game theoretic perspective.

In order to understand the supply and demand side of the two markets, the value chains of natural gas and electricity were given a thorough study. This was crucial to get a realistic picture of the parameters that affects the commodity trade and an understanding of the interaction between the two value chains. We have also looked into the market situation for both commodities and identified different market structures that are present.

Moreover, a brief mapping of important game theoretic aspects has been carried out to understand the underlying concepts of strategic actions between market actors. Different model classes are listed with the accompanying properties, range of applications and solution approaches.

To get the fundamental modeling pieces in place we created a simplified model that describes many of the challenges of equilibrium programming. First of all, this basic model works as a starting point for the extended model, but it also give some interesting results concerning market power.

Furthermore, an MCP equilibrium model of the market for both natural gas and electricity were created based on the theoretical part of the thesis, the simplified model and existing research done in modeling each of the sectors. We found it necessary to make a large set of assumptions to get the complex system into a reasonable model, but we are still confident that the realism is safeguarded and the results are of importance.

The thesis opens with the analysis of the two value chains including a study of the respective market situations in Chapter 3 and Chapter 4. This is followed by some game theoretic interpretations in Chapter 5. Chapter 6 presents the simplified model followed by the MCP equilibrium model and the accompanying implementation in Chapter 7 and 8. Chapter 9 includes deliberations of the data sets, while the main results and the following discussion is written in Chapter 10 and Chapter 11 respectively. The conclusion is stated in Chapter 12 while appendices are given in Appendix A to C and represent the very end of the paper.

Chapter 2

Literature Review

The following chapter gives an overview of literature made on modeling energy markets with focus on game theoretic approaches in particular. Section 2.1 presents literature treating the mathematics behind equilibrium programming and theoretic fundamentals written on the subject. Applications of this theory on modeling commodity markets are described in Section 2.2, while Section 2.3 puts in concrete terms how this thesis extends the existing area of research.

2.1 Equilibrium Programming

Two of the most groundbreaking papers on equilibrium programming were those of Debreu [1952] and Arrow and Debreu [1954]. They proved existence of equilibria in competitive economies and presented one of the first theoretical papers describing equilibrium programming and its economic applications. Szidarovszky and Yakowitz [1977] and Novshek [1985] added some important contributions to the theory, modifying the conditions for existence and uniqueness of Cournot-Nash equilibria.

The complexity of the problem is often related to the problem class, which again is highly dependent on the structure of the problem. Harker [1991], Billups and Murty [2000], Dempe [2003], Midthun [2007a] and Cottle et al. [2009] give an overview over the problem classes within the complementarity format presenting characteristics and solution approaches for each of them. Harker et al. [1993] presents a nice summary over historical developments, relevant theory of equilibrium programming and its wide range of applications.

2.2 Modeling Commodity Markets

Despite the liberalization process in the European gas and electricity markets we can observe strategic behavior at different levels of the value chain. Berry et al. [1999] and Mathiesen et al. [1986] describe the market situations and explain why strategic behavior is likely to occur in gas and electricity markets, and why some players in the market game both can and will use market power.

The well-defined methods of deriving economical equilibria for monopolies and perfect competition situations fall short when it comes to cases with few firms operating in a single market. However, Murphy et al. [1982] demonstrated how oligopolistic equilibria can be determined through a mathematical approach and thereby describe oligopolistic market situations.

The oligopolistic theory is often represented by three market models: Cournot, Bertrand and Stackelberg. In the Cournot model, firms make simultaneous decisions about how much of a homogeneous good to produce. In the Bertrand model, firms make simultaneous decisions about what price to set for the homogeneous good. While in the Stackelberg model, one firm moves first, deciding how much to produce, and the other firm responds.

There have been written many papers on different non-cooperative games within various commodity markets. Hobbs [1986] compares the Bertrand and Cournot models and argue for their modeling relevance. The GASMOD model of Holz et al. [2008] uses the complementarity conditions to solve market equilibria. It states that Cournot competition on both the upstream and downstream market is the most realistic representation of today's European gas market, where suppliers generate a mark-up at the expense of the final customer. The World Gas Model of Egging et al. [2010] formulates the gas market as an equilibrium model allowing producers and traders to act non-cooperatively as of Cournot.

Jing-Yuan and Smeers [1999] demonstrates how mathematical programming approaches can be used to determine Cournot-Nash market equilibria in electricity markets. In the same way, Hobbs [2002] and Hobbs et al. [2002] argue for Cournot competition in electricity markets. Lise et al. [2006] develops a static computational game theoretic model where different market structures are compared, depending on the ability of firms to exercise market power.

Haftendorn and Holz [2008] analyzes the world market for steam coal through a complementarity model. Unlike the market for gas and electricity, it indicates that the Cournot model is not realistic. However, the reality implicates some form of market power in this market, possibly following a Bertrand model in a spatial setting.

Hobbs and Kelly [1992] analyzed the transmission pricing policies in the U.S. as a non-cooperative game using the two-stage Stackelberg model. In general, dynamic Stackelberg games are difficult to solve, and the modeling complexity is also higher than one-stage models.

2.3 Model Aspects

The numerical model described in this thesis differs from the majority of existing literature on the field. Instead of using a single-commodity approach we introduce a strategic multi-commodity model including both natural gas and electricity. To our knowledge, few or none strategic market models include this multi-commodity aspect.

Abrell and Weigt [2010] analyzes the interaction between the electricity and natural gas network applying a partial equilibrium approach resulting in an MCP model. This model could very well be solved as an optimization model, because of the absence of market power. As opposed to the MCP model of Abrell and Weigt [2010] we include the possibility of strategic behavior in the market game.

In several ways, the game theoretic model approach used in this thesis differs from traditional energy market modeling formulated as pure optimization problems. Firstly, the modeling process gets more complicated as some of the agents possess market power and will use this to gain profits. Secondly, the search for optimal solutions gets more difficult as we are seeking an equilibrium solution, a state where none of the agents have incentives of changing its strategy, rather than a global optimum. Thirdly, proving existence and uniqueness of solutions of complementarity problems can be mathematically challenging without making certain assumptions.

Chapter 3

Electricity

This chapter describes electricity, its characteristics, trade and market features. The value chain of electricity is investigated in Section 3.1. This is necessary to understand the dynamics of the electricity market and how a market participant in one of the stages will influence the other players in the value chain. Moreover, Section 3.2 surveys the current market and points out different market essentials present in Europe. Section 3.3 presents power exchanges, their function and presence of necessity in a liberalized market.

3.1 Electricity Value Chain

Figure 3.1.1: Electricity value chain.

Source: We live in a world where electricity is fully integrated, and people take easy power access for granted. However, electricity is not an energy resource in the same way as fuel, wood and water. You cannot drill for electricity in the Gulf of Mexico; neither can you dig for electricity in Russian mines, you cannot even send electricity through a windmill and expect it to rotate.

Electricity is based on a source of energy, including both non-renewable resources (coal, oil, gas, uranium) and renewable resources (wind, water, solar, geo). Fuels are usually transported through pipes, rails or by large vehicles to conversion facilities. Even so, for most of the renewables the power conversion takes place in generators at the exact location of the source.

Generation: At the conversion facility the source is transferred from its original form into electric power. Depending on the power source there are different methods in use to obtain high efficiency on the net power output. For fuels the most prevalent method is by using electromechanical generators driven by heat engines. The various generators deliver power into the same local grid where electricity from all producers merges.

Transmission: Transmission cover the transfer of electricity from power plants to smaller substations located near the end-user. These grids are usually transmitting high voltage currents to avoid energy losses in long-distance transfers.

Distribution: Distributors carry the electricity from the substations to the customers. These networks are usually carrying medium-voltage currents transforming them down to low-voltage current depending on the customer.

Retailing and Consuming: Electricity retailers charge the customers for the amount of electricity being used and therefore connect the customers to the rest of the value chain. However, if a consumer changes its retailer, this will only affect the price and terms of the deliveries - the physical power flow will remain the same.

3.2 Electricity Market

The electricity market is limited by the main feature of electricity, it can not be stored. Due to this property, it is necessary to transmit and retail the electricity immediately after production.

3.2.1 Market Principles

The physical features of electricity place limitations on the trading and make it necessary to induce some standards of trade. Today there are two main standards in use, the *Single-Buyer* (SB) and *Third Party Access* (TPA).

The SB model illustrated in Figure 3.2.1 is characterized by one, single buyer buying electricity at the lowest cost from producers and sells it to end-users. This model gives preferential treatment to the sole buyer and is therefore not often in use in Western Europe.

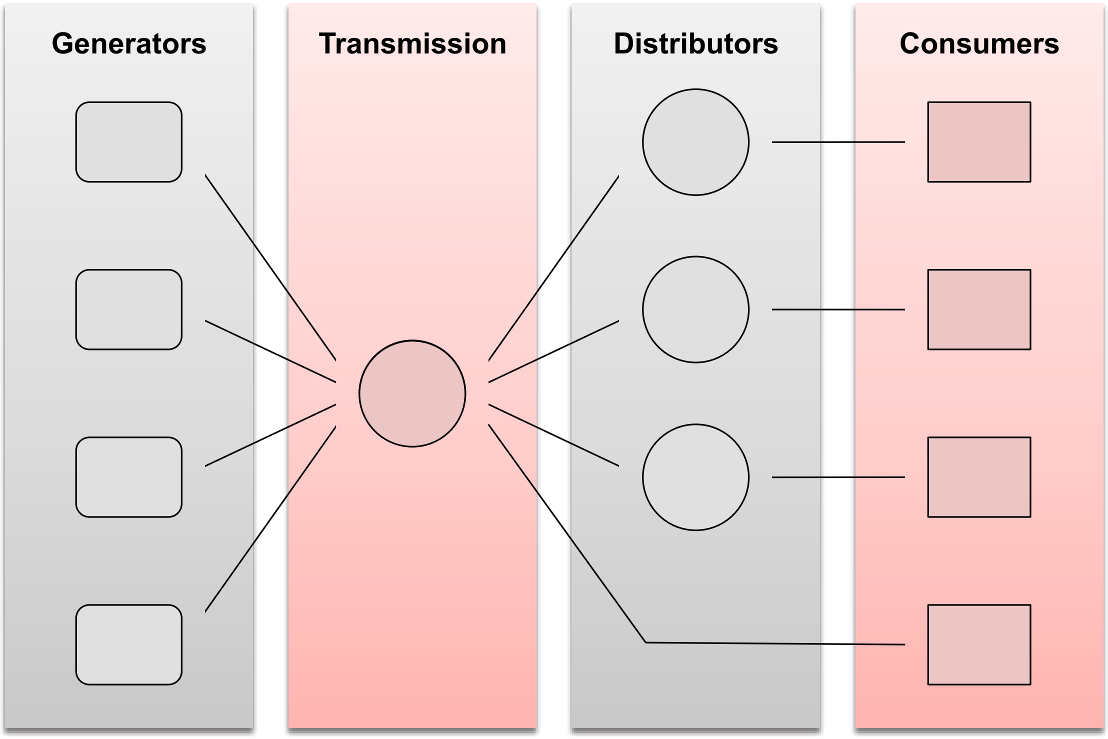

Figure 3.2.1: The single-buyer model for electricity trading.

A more common way of trade is through the TPA model that allows for third party access to the transmission grid (Figure 3.2.2). There are two ways of interpreting this practice. *Regulated Third Party Access* (rTPA) assumes that the grid access is based on regulated prices (assuming a regulatory authority), while *Negotiated Third Party Access* (nTPA) assumes negotiated prices.

3.2.2 From Closed to Liberalized Electricity Market

While trade with non-renewable resources has been executed for many years, electricity trading is a relatively new business. The fact that electricity is a non-storable energy carrier makes trade more challenging as demand and supply vary continuously over time. Today electricity is usually traded as a commodity on power exchanges, i.e. it is traded on the basis of price rather than quality.

Historically the power business has been dominated by vertically integrated companies controlling the following parts of the value chain; generation, transmission and distribution. Through large, centralized generating facilities, integrated with transmission and distribution systems, they acted as natural monopolists in the

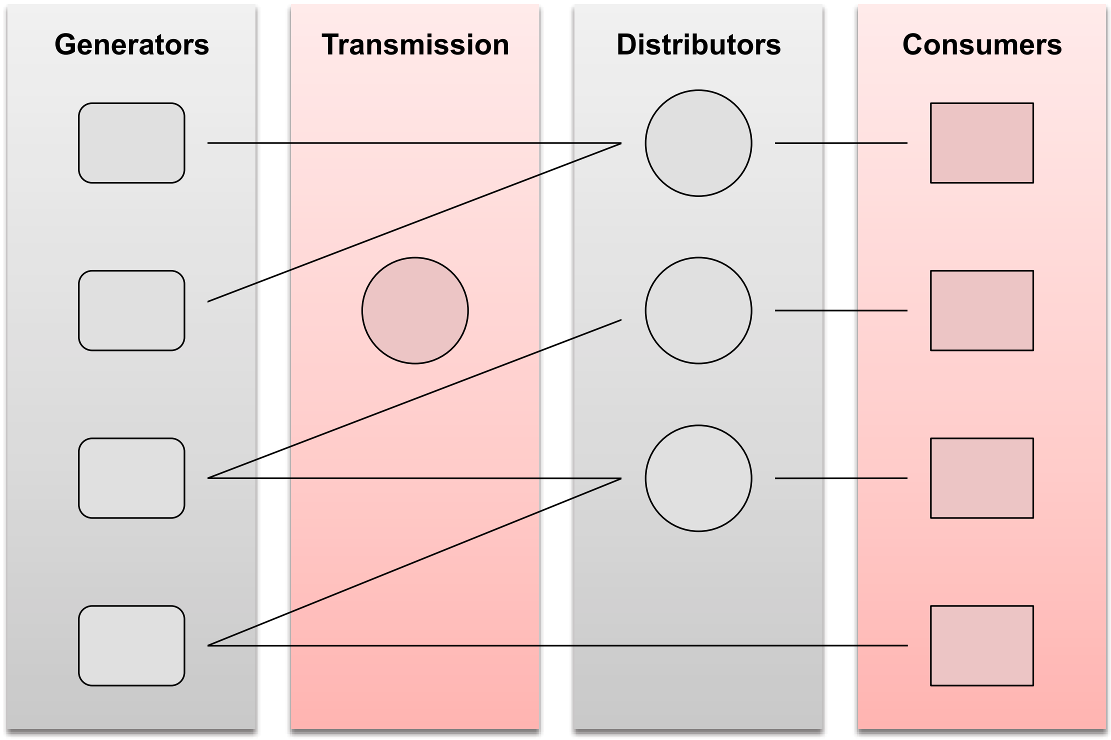

Figure 3.2.2: The TPA model for electricity trading.

market. This was based on the assumption that economies of scale would be the most efficient way of generating power. Simultaneously, governmental regulations controlled prices and operating procedures to protect customers from monopolistic abuses.

In the early 1980s several South American countries pioneered the electricity business by privatizing generation assets that government had failed to operate. This lead to a deregulation of the traditional governmental monopoly and opening up the supply of electricity to competition. At the same time, there were some major technical developments in the power industry. Cheap and highly efficient gas-fired power plants replaced many of the old coal-fired plants and new long-distance transmission technology gave customers more flexibility in choice of supplier. All this shifted the traditional system to a more competitive marketplace.

In 1990, Margaret Thatcher[1] adapted these concepts and slowly, but surely the whole Commonwealth experienced deregulated electricity markets. Even though the countries executed the concept differently, the basic ideas were the same: Sep-

[1] Margaret Hilda Thatcher was Prime Minister of the United Kingdom from 1979 to 1990 and Leader of the Conservatives from 1975 to 1990.

aration of the monopoly into a *wholesale electricity market* and a *retail electricity market*.

3.2.3 Wholesale Electricity Market

The wholesale market is based on trade between generators, grid owners and retailers, both for short-term purposes (spot prices) and for long-term deliveries (forward prices). Wholesale trading takes place both bilaterally, on the over-the-counter market (OTC), and through power exchanges. Over the last decade there have been some large end-users trying to reduce their overhead costs, buying directly (wholesale) from the generators. This will usually involve high investment costs and higher uncertainty, but for large-scale end-users it might turn out profitable.

3.2.4 Retail Electricity Market

After a trade in the wholesale market, the retailer re-prices the electricity and sells it to final consumers through the retail market. The customers can freely choose their supplier from competing retailers offering electricity to different prices and pricing regimes. Most common is the *fixed-price* regime where the supplier is constrained to a given price during the contract period, even though it might be high prices in the supply market. There are also contracts based on *variable pricing*, where the retailer can change the price after sending a warning to the customers. In addition, there exist some spot contracts that are connected to the wholesale spot prices.

3.3 Power Exchanges

As a response to the liberalization of the European electricity sector, numerous power exchanges has been put into operation (Table 3.3.1). The rationale behind them is to facilitate trading of short-term standardized products and promotion of market competition, information and liquidity. Theoretically power exchanges will also serve as a "neutral and easy access marketplace with negligible transaction costs, a neutral price reference, a safe counterpart, and a clearing and settlement provider" (Madlener and Kaufmann [2002]). In addition, the spot prices are important references for bilateral contracts for forward, future and options contracts.

Name	Country/Region
APX	Netherlands
Belpex	Belgium
Borzen	Slovenia
EEX	Germany
Elexon	Great Britain
EXAA	Austria
GME	Italy
HUPX	Hungary
Nord Pool Spot	Scandinavia
OMEL	Spain
OMIP	Portugal
OPCOM	Romania
OTE	Czech Republic
Powernext	France
SEMO	Ireland
TGE	Poland
UKPX / APX UK / UK IPE	United Kingdom

Table 3.3.1: European power exchanges.

3.3.1 Auctions

Most European power exchanges are organized by auctions[2]. There are various ways of promoting auctions (Table 3.3.2), but the basic idea is the same: the marketer receives bids from generators and demand bids from retailers or large end-users. The marketer then calculates the optimal strategy that minimizes cost but still meets the demand and the physical grid constraints.

In Europe, the most prevalent way of clearing contracts is through hourly double sided auctions where transactions are cleared at a given price at a fixed time. Both buyers and sellers submit their bids, which determine how much they are willing to sell and buy and to what prices. Depending on the power exchange there might be upper price limits and also mechanisms that limits price volatility and ensure continuity in price. In a sealed bid auction, your bid is only visible to the marketer and yourself. It is possible to submit the same bids over consecutive time periods, in so-called block bids.

[2] APX UK, UKPX, Borzen and EEX practice a *continuous* bidding system where all bidders have access to the order book.

Criteria	Type	Type
Number of bidding sides	One-sided	Two-sided
Objective function	Cost minimizing	Consumer payment minimizing
Pricing rule	Uniform pricing	Discriminatory pricing
Disclosure of bids	Open	Sealed

Table 3.3.2: Different types of auctions used in power trade (Madlener and Kaufmann [2002]).

Based on the bids from both producers and consumers, the marketer aggregates the bids to get a market demand and supply curve. If the initial auction price violates one or more of the conditions, one of the unfulfilled bids is removed, and the same calculation is repeated. This iteration will continue until there are only valid bids left in the auction.

Chapter 4

Natural Gas

The following chapter presents natural gas, its value chain and market characteristics. The value chain is analyzed in Section 4.1 while a historical overview of important developments in the European gas market is presented in Section 4.2. Section 4.3 describes the gas hubs and their role in a modern gas market.

4.1 Natural Gas Value Chain

Natural gas is considered one of the most important energy sources of tomorrow. With its abundant supply and environmental advantages natural gas is predicted to play an important role supplying Europe and the rest of the world with cleaner energy.

Natural gas is originally organic material transformed into gas through million years of high pressure and temperature. The gas can be found in oil fields as associated gas, in gas fields as non-associated gas and in coal beds as coal bed methane. Usually the *rich gas* contains components that are removed before the *dry gas* is available for end-users. The value chain of natural gas is represented in Figure 3.3.2.

Figure 4.1.1: Natural gas value chain.

Exploration and Production: The exploration part cover the process where natural gas is discovered and production facilities are planned. Seismological analysis radars the seabed and provide reason for field development. The production facilities can be put onshore, offshore and also on the seabed as subsea installations. Extracting the gas from the fields takes place on the production sights, where the gas enters from high-pressured reservoirs. There are primarily three different wells natural gas can be extracted from: crude oil-, gas- and condensate wells.

Processing: The rich gas enters the processing plants containing significant amounts of alkanes, CO_2, nitrogen, helium and hydrogen sulphides. In order to separate the methane (dry gas) from the other substances the rich gas is heated. This process removes contaminants and heavier hydrocarbons to meet the industry standards for transportation in high-pressure pipelines.

Transport and storage: Natural gas can be transported in high-pressure pipelines from the processing units to local distribution companies, or with LNG-carriers from liquefaction plants to import terminals. LNG[1] is usually preferred when large volumes are transported over long distances. The fact that the short-term demand for natural gas is subject to large fluctuations, there would be beneficial to store natural gas in low-demand periods and use the reserves in high-demand periods. There are several commercial storage methods in use today: injecting gas into abandoned oil and gas fields, liquefaction plants with storage facilities, and using the comprehensive pipeline network as storage.

Market: The gas enters import terminals from pipelines and the LNG-network. From here the gas is brought to the market and traded to the customers through local networks and priced according to different pricing regimes. A thorough review of the market for natural gas is presented in Section 4.2.

End-user: Most of the natural gas production in Europe is transformed to electricity in power plants. This fraction is expected to increase in the future, as the demand for green power gets even higher than today's level. However, natural gas is used for a variety of purposes in different market sectors. Power intensive industry use gas for heating, while other use it as a raw material in their industry processes. Commercial use of gas includes heating, lightning and cooling of public and private enterprises, like schools, hotels, restaurants and office buildings. The residential use of gas is mainly due to cooking, heating and cooling. With its low emissions and high efficiency there is also a potential for natural gas as fuel for vehicles in the transport sector.

[1]LNG (Liquefied Natural Gas) is natural gas in liquid phase at a temperature of -163°C. The volume is reduced 600 times, enabling efficient transport and storage.

4.2 Historical Perspectives of the European Gas Market

The first gas production in Europe started already early in the 19th century. This gas was exploited from coal and used as city lightning and later on in private households in the major cities. As a result, Northern Europe had a well-developed distribution network for coal gas in the beginning of the 20th century. In the 1930s Italy and France discovered their first natural gas reserves and some decades later, Netherlands, Germany and United Kingdom also started production of natural gas. However, the demand for natural gas did not accelerate until the 1960s, when the national demand exceeded the internal supply. This entailed the establishment of a comprehensive distribution network for natural gas, which in turn initiated an international natural gas market (Figure 4.2.1). Later on, Russia expanded their gas fields in Siberia and became an important gas distributor to the European gas market.

Figure 4.2.1: Development of the European gas network from 1970 (left) to 2000 (right) (Forum [2004]).

The first contracts were typically long-term contracts, to ensure payback on the capital-intensive pipeline investments. In that way, the risk linked to pipeline investments were minimal and made this type of contracts favorable. As the world faced high oil prices during the international oil crises in the 1980s, the incentives for natural gas exploration and distribution became even better. When the oil price again stabilized large volumes of natural gas were available and the gas price fell.

During the 1990s there was raised a growing concern regarding the competitiveness of European industry in a globalized market. As a response EU introduced Gas Directive I[2] (1998) and Gas Directive II[3] (2003). These directives were supposed to liberalize the market and create one single market for European gas.

4.3 The European Gas Hubs

Today's situation is a combination of monopolized markets and new competitive markets. At retail level, there are still countries where the consumers face price capping, restrictions and regulations from governmental institutions. However, at supply level, there remain no significant areas within the EU, where gas prices are subject to direct national intervention to cap prices (Melling [2010]). In order to achieve the vision of one single gas market, the local markets need to be integrated properly – creating interconnections between the markets and expanding storing facilities.

As mentioned, local markets organized as hubs characterize the European natural gas market. At these hubs, buyers and sellers meet and agree on price and volume on the contracts. The gas price can vary from hub to hub, and generally the pricing falls into three categories depending on the degree of regulation, the market liquidity and the competitiveness of the market:

1. Government-regulated prices (usually based on cost of service)
2. Price indexation to other fuels (usually oil-indexed pricing)
3. Spot market pricing in competitive gas markets

The increasing number of European hubs reflects the importance of natural gas. Currently, there are nine major hubs (Figure 4.3.1), with the National Balancing Point (NBP) in UK as the most mature and liquid and therefore closest related to the *ideal hub*. Through Interconnector and Balgzand Bacton Line, this hub was connected to the hubs in Zeebrugge (Belgium) and the Title Transfer Facility (Netherlands), two of the most important hubs on the continent. Other hubs are

[2]The first Gas Directive (98/30/EC) sought to create competitive markets by ensuring access to the network to third parties. This should be achieved by unbundling the transport and the marketing sections of the traditional, vertically integrated monopolists, monitored by independent regulatory authorities in each member state (Holz [2009]).

[3]The second Gas Directive, the so-called "Acceleration Directive" (2003/55/EC). It mandated regulatory third party access (TPA) for all existing infrastructures, including high-pressure transit pipelines for imports. The Acceleration Directive also commanded legal unbundling as the minimum level of unbundling and it reinforced the importance of the regulator (Holz [2009]).

emerging, but they are struggling by lack of supply liquidity and infrastructural obstacles as border crossing and capacity constraints.

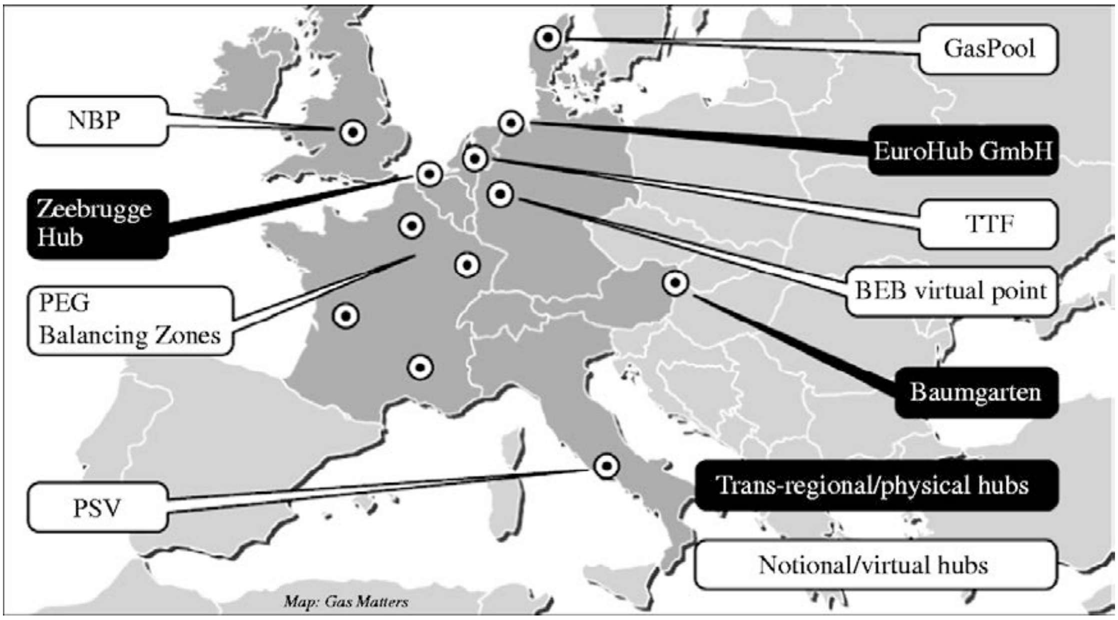

Figure 4.3.1: European gas hubs (Stern [2007]).

In the real world, ideal gas hubs will not occur. In theory, however, there exist ideal hubs with several important characteristics:

1. Easy access to natural gas and a large customer portfolio
2. Connections to other hubs through pipelines and/or LNG-routes
3. Tools to handle fluctuations in supply and demand
4. Low or no entry costs for new actors
5. Standardized contracts and financial intermediaries to avoid negotiating directly with producers and consumers
6. Offer financial instruments to minimize risk related to price fluctuations
7. Be liquid and create trust between the actors on both sides of the contract

According to Tomasgard et al. [2007] and Midthun [2007a] pricing according to oil indexation is the dominant method for long-term contracts[4] in continental Europe.

[4]Most of the gas produced in Europe has been committed to take-or-pay (TOP) contracts where the buyer agrees on taking a given volume of gas to import terminals for a fixed number of years.

Even though spot markets are developing in the Northwestern part of Europe, most of the continent is not yet ready for the development of traded markets (Melling [2010]).

Chapter 5

Game Theory

In this chapter, descriptions of relevant game theory and its wide range of appliances are provided. It presents theory needed to understand the complexity of equilibrium programming and serves as a fundament for discussions and decisions made in later chapters. Section 5.1 introduces the topic and presents the basic setup of a game. The equilibrium concept is explained in Section 5.2 focusing on the mathematical representation of an equilibrium. This is followed by the Cournot game in Section 5.3 while Section 5.4 extends the Cournot game through the Generalized Nash Equilibrium. Section 5.5 concerns the KKT conditions and their usability in market modeling whereas relevant model classes in Section 5.6 close the chapter.

5.1 Fundamental Concepts

Game theory has recently become a serious challenger to conventional economic theory. Although aspects of game theory have been examined for centuries, the first formal conception of the field were stated and structured in the famous work *Theory of Games and Economic Behavior* of John von Neumann and Oskar Morgenstern (Von Neumann and Morgenstern [1944]). Subsequent theorists such as John Nash and John Maynard Smith, has advanced the discipline and made game theory applicable to politics, inter-personal relationships, biology, philosophy, artificial intelligence and other fields. Game theory is especially useful describing situations and possible behavior where agents make strategic decisions to other agents' actions. An agent is faced with a set of moves that can be played and will form and play a strategy based on a best response to other agents strategies.

A game can be defined by a set of players $N = \{1, ..., n\}$. Each player $i \in \mathcal{I}$ has a number of actions, X_i, called player i's *pure strategies*. Naming the collection of all possible *pure strategies*:

$$X = X_1 \times X_2 \times ... \times X_n = \{(x_1, x_2, ..., x_n) | x_i \in X_i\}, \quad \forall i \in \mathcal{I} \tag{5.1.1}$$

This is simply the set of all possible strategy configurations in the game.

Moreover, each player's payoff function, denoted by $u_i(x)$ is defined. This payoff function takes into consideration not only player i's chosen strategy, but also the other actions made by the other players in the game.

Thus, the input in the payoff function takes a *pure strategy*:

$$x = x_1, x_2, ..., x_n \in X \tag{5.1.2}$$

and returns a number $u_i(x) \in \mathbb{R}$. This gives $u_i(x) : X \to \mathbb{R}$, where $u_i(x)$ is a function from the set of all pure strategies to the set of all real numbers, $\mathbb{R}$.

Another important aspect in game theory is the rationality assumptions, ensuring that each player would not intentionally make decisions that would leave them worse off:

1. Every player is rational in his or her choices (utility maximizing)
2. Every player knows that the other players are rational
3. Every player knows assumption 2
4. Every player knows assumption 3
5. ...

5.2 Concept of Equilibrium

Once a game is established there is a need for a method of predicting the actions of the different players. A way of assigning predicted strategies to games is named an *equilibrium concept*.

First, one might look for any *dominant strategies* among the players. A strategy x_i is dominant compared to other strategies, if it gives a higher payoff regardless of the actions the other players choose. We can express this formally: $x_i \in X_i$ is dominant if $u_i(x_i, x_{-i}) \geq u_i(x_i', x_{-i})$ for all $x_i' \in X_i$ and $x_{-i} \in X_{-i}$, where x_{-i}

denotes the set of all vectors of pure strategies with the i'th element removed (all players except player i). If $u_i(x_i, x_{-i}) > u_i(x'_i, x_{-i})$ there exists a *strictly dominating strategy.* If $u_i(x_i, x_{-i}) \geq u_i(x'_i, x_{-i})$ and for at least one $x_i \in X_i$ gives $u_i(x_i, x_{-i}) > u_i(x'_i, x_{-i})$, a *weakly dominating strategy* exists (Lamberson [2009]).

In other words, when a dominant strategy is discovered, this strategy will always be preferred regardless of the opponents actions – it is simply the best response in every situation.

In many games it will not necessarily exist dominant strategies, and therefore one cannot always use the above-mentioned technique. A weaker equilibrium concept is that of a *Pure Strategy Nash Equilibrium* (PSNE). A pure strategy x_i is a best response to $x_{-i} \in X_{-i}$ if $u_i(x_i, x_{-i}) \geq u_i(x'_i, x_{-i})$ for all x'_i. A profile of strategies $x \in X$ is a PSNE if x_i is a best response to x_{-i} for all i. Informally, given a strategy tuple $x \in X$ where none of the players has anything to gain in changing their strategy, there exists a PSNE.

PSNE is one type of Nash equilibriums; another one is *Mixed Strategy Nash Equilibrium* (MSNE). In mixed strategies you do not necessarily have to play one of the pure strategies with probability equal to 1. Instead you can play each of the pure strategies x_j with a probability $\mu_i(x_i)$ where $\sum_{j=1}^{n} \mu_j(x_j) = 1$. (5.2.1) states that the sum of player i's payoff from every strategy x multiplied by the probability of each player choosing x:

$$\sum_{x \in X} \Big(\prod_{j=1}^{n} \mu_j(x_j) \Big) u_i(x), \quad \forall i \in \mathcal{I} \tag{5.2.1}$$

An MSNE exists if each player is maximizing their expected payoff conditional on all the other players playing the mixed strategy specified by μ. Said in mathematical terms:

$$\sum_{x \in X} \Big(\prod_{j=1}^{n} \mu_j(x_j) \Big) u_i(x) \geq \sum_{x_{-i} \in X_{-i}} \Big(\prod_{j \neq 1} \mu_j(x_j) \Big) u_i(x'_i, x_{-i}), \quad \forall i \in \mathcal{I} \tag{5.2.2}$$

5.3 Cournot Competition

For now, only games with finite strategy spaces, but with the possibility of playing mixed strategies have been discussed. According to Cournot [1838] the Cournot model possesses a continuous strategy space. This model describes the competition between N firms in the same market and has the following features:

1. There is more than one firm and they all produce a homogenous product
2. Firms do not cooperate
3. Firms have market power, i.e. each firms output decision affects the good's price
4. The number of firms, N, is fixed
5. Firms compete in quantities and choose quantities simultaneously
6. The firms are economically rational, and seeks to maximize profit given their competitors decisions

Cournot solved this problem by assuming that each firm chooses its profit maximizing output by treating rival output as given and choosing the best response to the rival's actions.

The general structure is as follows: There are N firms, that individually and simultaneously choose their quantities, q_i. Each player i has an inverse demand function $p(Q)$ where $Q = \sum_{i=1}^{N} q_i$, and a cost function $c_i(q_i)$. As a result, the profit function for each player i becomes: $\Pi_i(q_i) = p(Q)q_i - c_i(q_i)$. Solving this game can be done by introducing *best response functions*, also called *reaction functions*. A best response is the strategy that produces the best outcome for a player, taking other players' strategies as given. The best response function for player i is found by maximizing the profit function with respect to his own strategy q_i. The intersection between the reaction functions constitutes the *Cournot-Nash Equilibrium*, which is simply the Nash Equilibrium when the strategies consist of quantities.

The *Cournot Game* can be illustrated with an example: Two firms, A and B, compete in the same market. q_A and q_B describe the quantities produced by each of the firms, while the inverse demand function is denoted by $p(Q) = a - Q$, where $Q = q_A + q_B$. Both firms have identical cost functions that follows: $c_i(q_i) = b \cdot q_i$. This results in the following profit functions for the two firms:

$$\Pi_A(q_A, q_B) = (a - q_A - q_B) \cdot q_A - b \cdot q_A \tag{5.3.1}$$

$$\Pi_B(q_A, q_B) = (a - q_A - q_B) \cdot q_B - b \cdot q_B \tag{5.3.2}$$

Differentiating each of the firms' profit functions with respect to their own produced quantity gives the *first-order conditions* (FOC):

$$\frac{\partial \Pi_A(q_A, q_B)}{\partial q_A} = a - 2q_A - q_B - b = 0 \implies q_A = \frac{a-b}{2} - \frac{q_B}{2} \tag{5.3.3}$$

$$\frac{\partial \Pi_B(q_A, q_B)}{\partial q_B} = a - 2q_B - q_A - b = 0 \implies q_B = \frac{a-b}{2} - \frac{q_A}{2} \tag{5.3.4}$$

Inserting (5.3.3) in (5.3.4) and solving for q_A and q_B gives:

$$q_A = q_B = \frac{a-b}{3} \quad \text{and} \quad p = \frac{a+2b}{3} \tag{5.3.5}$$

Accordingly the Cournot-Nash Equilibrium is $q^{eq} = (\frac{a-b}{3}, \frac{a-b}{3})$, that is, none of the players has anything to earn by deviating from their strategy.

Proving uniqueness of the solution requires a set of assumptions: The inverse demand function p_i and the cost functions c_i are twice differentiable: $\frac{\partial p_i}{\partial q_i}, -\frac{\partial c_i}{\partial q_i} < 0$ and $\frac{\partial^2 p_i}{\partial q_i^2}, -\frac{\partial^2 c_i}{\partial q_i^2} \leq 0$ in an interval $[0, \epsilon]$. Seeing that epsilon is given when $p(\epsilon) = 0$, and as it is unreasonable to have negative prices: $p(s) = 0$ for $s > \epsilon$. Szidarovszky and Yakowitz [1977] proved that, under these conditions, there exists exactly one equilibrium point.

5.4 Generalized Nash Equilibrium

In a standard form of a cooperative game it is usually assumed that the feasible set of the game is composed of the full Cartesian product of the individual strategy sets. In other words, it is assumed that the players can only affect the utilities of the other players, but not their feasible sets (Harker [1991] and Arrow and Debreu [1954]). However, in a *Generalized Nash Equilibrium* (GNE) each player's feasible set can depend on the rival players' strategies. In other words, they can share a common strategy set, $x \in \mathbb{R}^n$, which is usually assumed to be non-empty, closed and convex. A vector $x^* = (x_1^*, x_2^*, ..., x_n^*) \in X$ is called a GNE if:

$$u_i(x^*) \geq u_i(x_1^*, x_2^*, ..., x_n^*), \quad \forall x_i : (x_i, x_{-i}^*) \in X, \quad \forall i \in \mathcal{I} \tag{5.4.1}$$

Typical applications for GNE may be oligopoly models using joint resources and network problems with capacity constraints. According to Harker [1991] GNEs can be converted to quasi-variational inequalities (QVI). QVIs are difficult to solve, and no general uniqueness results exist for the QVI problem. It is possible to establish conditions that ensure uniqueness, but these conditions are often overly restrictive. Thus, uniqueness is a rarity for QVI problems.

5.5 KKT Conditions

In a general form, the nonlinear programming problem is to find a vector $x = (x_1, x_2, ..., x_n)$ so as to:

$$\max_{x} \quad f(x) \tag{5.5.1}$$

$$s.t. \quad g_i(x) \leq b_i, \quad \forall i \in \mathcal{I} \tag{5.5.2}$$

$$x \geq 0 \tag{5.5.3}$$

where $f(x)$ and the $g_i(x)$ are given functions of the n decision variables.

In nonlinear programming problems, where the objective function or the constraints are nonlinear, there are both *necessary* and *sufficient* conditions for optimality. Table 5.5.1 summarizes this for different problem structures.

Problem	Necessary conditions	Also sufficient if
One-variable constrained	$\frac{df}{dx} = 0$	$f(x)$ concave
Multivariable unconstrained	$\frac{\partial f(x_j)}{\partial x_j} = 0$	$f(x)$ concave
Constrained, non-negative	$\frac{\partial f(x_j)}{\partial x_j} = 0$	$f(x)$ concave
General constrained problem	KKT conditions	$f(x)$ and $g_i(x)$ concave

Table 5.5.1: Necessary and sufficient conditions for optimality for different problem structures.

For the general case, the KKT conditions are necessary for a solution to be optimal. Their basic result can be embodied in the following theorem stated in Hillier and Lieberman [2005]:

Theorem: Assume that $f(x), g_1(x), g_2(x), ..., g_m(x)$ are differentiable functions satisfying certain regularity conditions. Then $x^* = (x_1^*, x_2^*, ..., x_n^*)$ can be an optimal solution for the nonlinear programming problem only if there exist m numbers

$\lambda_1, \lambda_2, ..., \lambda_m$ such that all the following KKT conditions are satisfied:

$$\frac{df}{dx_j} - \sum_{i=1}^{m} \lambda_i \frac{\partial g_i}{\partial x_j} \leq 0, \quad \text{at } x = x^*, \qquad \forall j \in \mathcal{J} \tag{5.5.4}$$

$$x_j^* (\frac{df}{dx_j} - \sum_{i=1}^{m} \lambda_i \frac{\partial g_i}{\partial x_j}) = 0, \quad \text{at } x = x^*, \qquad \forall j \in \mathcal{J} \tag{5.5.5}$$

$$g_i(x^*) - b_i \leq 0, \qquad \forall i \in \mathcal{I} \tag{5.5.6}$$

$$\lambda_i [g_i(x^*) - b_i] = 0, \qquad \forall i \in \mathcal{I} \tag{5.5.7}$$

$$x_j^* \geq 0, \qquad \forall j \in \mathcal{J} \tag{5.5.8}$$

$$\lambda_i \geq 0, \qquad \forall i \in \mathcal{I} \tag{5.5.9}$$

Corollary: Assume that $f(x)$ is a concave function and that $g_1(x), g_2(x), ..., g_m(x)$ are convex functions (i.e. this problem is a convex programming problem) where all these functions satisfy the regularity conditions. Then $x^* = (x_1^*, x_2^*, ..., x_n^*)$ is an optimal solution if and only if all the conditions of the theorem are satisfied.

5.6 Model Classes

The structure of the objective function and the constraints define the character of a complementarity problem. The following section presents the most important groupings and their mathematical characteristics.

5.6.1 Variational Inequalities

The theory of *Variational Inequalities* (VI) is a powerful unifying methodology for the study of equilibrium problems. The theory provides tools for formulating a variety of equilibrium problems as VIs and provide qualitative analysis of the problems in terms of existence and uniqueness of solutions, stability and sensitivity analysis and efficient algorithms for computational purposes. Formally VIs can be formulated as:

Given a subset K of the Euclidean n-dimensional space $\mathbb{R}^n$ and a mapping $F : K \to \mathbb{R}^n$, the *Variational Inequality*, denoted *VI(K,F)* is to find a vector $x \in K$ such that (Facchinei and Pang [2003]):

$$(y - x)^T f(x) \geq 0, \quad \forall y \in K \tag{5.6.1}$$

The set of solutions to this problem is denoted *SOL(K,F)*. Nash Equilibriums with independent strategy sets can be viewed as VIs (Lions and Stampacchia [1967]).

5.6.2 Linear Complementarity Problems

If the constraints of the complementarity problem are exclusively linear and contains exogenous parameters, the problem can be regarded as an *Linear Complementarity Problem* (LCP). Mathematically the LCP consists of finding a vector $x \in \mathbb{R}^n$, such that (Cottle et al. [2009]):

$$x \geq 0 \tag{5.6.2}$$

$$q + Mx \geq 0 \tag{5.6.3}$$

$$x^T(q + Mx) = 0 \tag{5.6.4}$$

for a given vector $q \in \mathbb{R}^n$ and a matrix $M \in \mathbb{R}^{n \times n}$. The problem is simply denoted $LCP(q, M)$.

5.6.3 Nonlinear Complementarity Problems

As opposed to the linear requirements in the LCP formulation, *Nonlinear Complementarity Problem* (NCP) allows for nonlinearity in its set of constraints. Cottle et al. [2009] describes an NCP as finding a vector x such that:

$$\begin{aligned} x &\geq 0 \\ f(x) &\geq 0 \\ x^T f(x) &\geq 0 \end{aligned} \tag{5.6.5}$$

where $f(x)$ is a given mapping from $\mathbb{R}^n$ into itself.

5.6.4 Mixed Complementarity Problems

LCPs, NCPs and finite-dimensional VIs can be generalized to *Mixed Complementarity Problems* (MCP). An MCP consists of both complementarity conditions and equality/inequality constraints. In a multiplayer game, an MCP formulation has the unique properties of giving the solution to all players' optimization problems simultaneously. Formally the MCP can be written as:

Given the matrices $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{m\times m}$, $C \in \mathbb{R}^{n\times m}$, $D \in \mathbb{R}^{m\times n}$, $a \in \mathbb{R}^n$ and $b \in \mathbb{R}^m$. The MCP is to find vectors $u \in \mathbb{R}^n$ and $v \in \mathbb{R}^m$ such that (Cottle et al. [2009]):

$$a + Au + cv = 0 \tag{5.6.6}$$

$$b + Du + Bv \geq 0 \tag{5.6.7}$$

$$v \geq 0 \tag{5.6.8}$$

$$v^T(b + Du + Bv) = 0 \tag{5.6.9}$$

where (5.6.6) - (5.6.8) represent an LCP and (5.6.9) represents the complementarity condition.

According to Rutherford [2002] MCPs can incorporate mixtures of equations and inequalities and because of this express a variety of economic models for both markets and games.

An MCP can be formulated by deriving the KKT conditions of an optimization problem. In a multiplayer game, the aggregated KKT conditions will form an equilibrium problem that can be solved as an MCP.

5.6.5 Mathematical Programs with Equilibrium Constraints

Mathematical Programs with Equilibrium Constraints (MPEC) are present if the constraints itself is the result of an equilibrium problem and the objective function is a single players' optimization problem.

The two-stage Stackelberg game described in Von Stackelberg [1952] can be formulated as an MPEC where the top level is a dominant company and the bottom level is the rest of the market. The MPEC can be formulated as follows where y solves the MCP:

$$\begin{aligned} \max_{x} \quad & f(x, y) \\ s.t. \quad & (x, y) \in Z \\ & a \leq x \leq b \end{aligned} \tag{5.6.10}$$

MPECs are in general non-convex and non-differentiable problems. Thus, it is difficult and computationally challenging to find a global optimal point, as the FOCs are not sufficient for optimality (Midthun [2007b]).

5.6.6 Equilibrium Programs with Equilibrium Constraints

An *Equilibrium Program with Equilibrium Constraints* (EPEC) is a multi-leader follower game, where each leader is solving an MPEC. Thus, the objective function gives the solution to another equilibrium problem. The general EPEC can be formulated as:

$$
\begin{aligned}
\max_{z} \quad & f(z) \\
s.t. \quad \min(G(z), H(z) &= 0 \\
g(z) &\leq 0 \\
h(z) &= 0
\end{aligned}
\tag{5.6.11}
$$

An issue with EPECs is that they in general are non-convex problems, and existence of a solution is not necessarily guaranteed even under standard compactness assumptions. Although one should find a solution you cannot guarantee for uniqueness (Midthun [2007b]).

Chapter 6

Basic Production Model

This chapter presents a limited production model designed to illustrate market power and how it affects price, quantity and the social welfare in a market game. Section 6.1 opens with a discussion of assumptions and simplifications made in order to get an adequate description of the market situation. The model formulation, including notations, definitions and the sets of equations, is given in Section 6.2. Section 6.3 presents the results and some concluding remarks.

6.1 Assumptions and Simplifications

This model focuses on the fundamental pieces that is required to describe an energy market. The modeling work starts out at the very basic, with two producers meeting an external demand for natural gas. Additional complexity is added when nodes and competition among producers are assigned to the model. This single-commodity model will form the basis of a more comprehensive multi-commodity model described in Chapter 7 and contains important deliberations used in the subsequent chapters.

A production problem in its simplest form can be described by a single producer selling its products to a consumer. The producer seeks to maximize his profit of selling its products subtracted the cost involved in the production process. He faces a capacity constraint of production that limits his total production and a constraint forcing him to produce a positive amount of goods.

To make the model a bit more realistic, we add a nodal structure of two nodes in the network with a single producer located in each of the nodes (Figure 6.1.1). Each of the nodes contains a market for gas that is not interfering with each other. The producers can choose to sell the gas in either its own node or in the neighboring node. The two producers are the only market players present in this example. Depending on the market situation they can behave strategically against each other as of Cournot, i.e. produce the optimal amount of gas given the production of the other producer in the network.

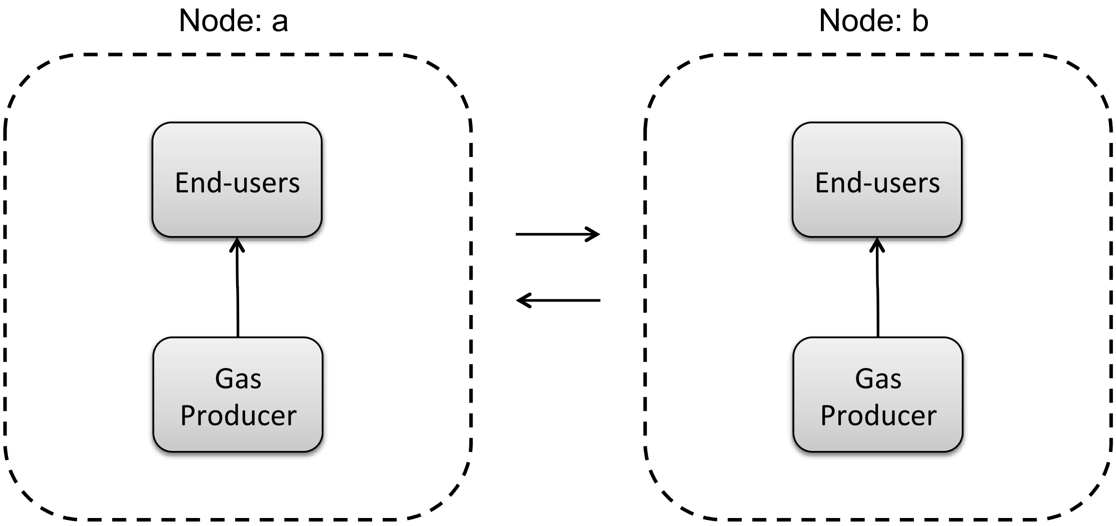

Figure 6.1.1: Nodal structure of the basic production model.

We also assume that the two markets for natural gas are characterized by their own inverse linear demand curve (Appendix A.1.1), where the total demand for natural gas in the node is aggregated into one single demand curve. Hence, these inverse demand curves represents the total demand of natural gas in the respective node.

This structure is of course a large simplification of the real world situation, but will serve as a good starting point for understanding the concept of strategic behavior.

6.2 Formulation of the Basic Model

This model intends to describe the fundamental mechanisms of equilibrium programming and how a small production example can be analyzed through a game theoretic framework.

6.2.1 Declarations

The following notations are valid for the basic model:

Indices

n	Nodes in the network. $(a, b) \in \mathcal{N}$
i	Producers in the network. $(g_1, g_2) \in \mathcal{I}$

Sets

$\mathcal{N}$	The set of all nodes in the network
$\mathcal{I}$	The set of all producers in the network

Data

$PRODCAP_i$	Capacity limit of production of producer i
INT_n	Intersection point of the inverse demand curve in node n
SLP_n	Slope of the inverse demand curve in node n
A_i	Linear cost factor in the cost function of producer i
B_i	Quadratic cost factor in the cost function of producer i
MP_{in}	Market power parameter of producer i in node n

Variables

q_{in}	Sold quantity of producer i to node n
λ_i	Lagrange multiplier of production constraint in producer i's problem
p_n	Price of natural gas in node n

Functions

Π_i	Profit function of producer i
C_i	Cost function of producer i

6.2.2 Production Cost, Revenues and Profit

The production involves different types of cost depending on each production process. For modeling purposes it is beneficial to avoid unnecessary complexity by combining variables. (6.2.1) aggregates all costs related to the production process into a single cost function that depends on the total production by each producer only. This will simplify the modeling and ease the search for relevant data.

$$C_a + C_b + \cdots + C_s \approx C = C_i\Big(\sum_n q_{in}\Big) \tag{6.2.1}$$

In order to guarantee optimality we need to assume that the aggregated production cost function denoted $C_i(q_{in})$ has either linear or quadratic properties. This will ensure convexity of the problem and the KKT conditions will provide optimal solutions (Section 5.5).

Thus, we assume the cost functions to have quadratic shape following (6.2.2):

$$C_i(q_{in}) = A_i \cdot \sum_n q_{in} + B_i \cdot \Big(\sum_n q_{in}\Big)^2 \tag{6.2.2}$$

The two producers' profits come from sales to the internal and external market subtracted the cost from production. This function will form the objective function in which both producers seeks to maximize. Due to the nature of both the demand and cost function, the objective function will have a quadratic shape.

$$\Pi_i = \sum_n p_n \cdot q_{in} - C_i\Big(\sum_n q_{in}\Big) \tag{6.2.3}$$

6.2.3 Optimization Program

The gas producer i maximizes profit in accordance with the rationality assumptions listed in Section 5.3. He gains profit from sales to customers at a price p_n subtracted the cost related to the production process. The production of gas is constrained by the production limit $PRODCAP_i$.

Based on the assumptions in Section 6.1 each producer i is given the following optimization program:

$$\max_{q_{in}} \quad \Pi_i = \sum_n p_n \cdot q_{in} - C_i\Big(\sum_n q_{in}\Big), \qquad \forall i \qquad (6.2.4)$$

$$s.t. \quad \sum_n q_{in} \leq PRODCAP_i, \qquad (\lambda_i), \qquad \forall i \qquad (6.2.5)$$

$$q_{in} \geq 0, \qquad \forall i, n \qquad (6.2.6)$$

where (6.2.4) represents the objective function of producer i, (6.2.5) the capacity constraint of production and (6.2.6) the fact all production has to be non-negative.

We convert the problem into a minimization problem to solve it as a complementarity model. By inverting the signs of the objective function and restructuring the constraints, the related minimization problem becomes:

$$\min_{q_{in}} \quad -\Pi_i = -\sum_n p_n \cdot q_{in} + C_i\Big(\sum_n q_{in}\Big), \qquad \forall i \qquad (6.2.7)$$

$$s.t. \quad PRODCAP_i - \sum_n q_{in} \geq 0, \qquad (\lambda_i), \qquad \forall i \qquad (6.2.8)$$

$$q_{in} \geq 0, \qquad \forall i, n \qquad (6.2.9)$$

Deriving the first-order conditions (FOCs) give the Karush-Kuhn-Tucker conditions of this minimization problem:

$$0 \leq -p_n - MP_{in} \cdot SLP_n \cdot q_{in} + \frac{\partial C_i\Big(\sum_n q_{in}\Big)}{\partial q_{in}} + \lambda_i \quad \perp \quad q_{in} \geq 0, \quad \forall i, n \quad (6.2.10)$$

$$0 \leq PRODCAP_i - \sum_n q_{in} \qquad \perp \quad \lambda_i \geq 0, \qquad \forall i \quad (6.2.11)$$

The KKT conditions will provide optimal solutions to the problem as long as the problem structure is strictly convex. The fact that we have convex cost functions, the negative of the profit functions are strictly concave and we are dealing with convex sets of constraints we will end up with an optimal solution to the problem. An essential feature of expressing strategic behavior, is the market power parameter MP_{in}. This parameter will be explained and given a thorough study in Chapter 7.

The market clearing condition for the producer i is formed by the inverse demand function of market n:

$$p_n(q_n) = INT_n + \sum_i SLP_n \cdot q_{in}, \quad (p_n \text{ free}), \qquad \forall n \qquad (6.2.12)$$

where p_n represents the dual variable of the market clearing condition in (6.2.12). As p_n is dual to an equality it is not restricted to positive. Consequently, sufficient input data is necessary to assure positive prices.

6.3 Results

We want to analyze the impact of market power on a small gas market and contrast the results to relevant theory. The model is run for several market scenarios including monopoly, perfect competition and a Cournot oligopoly. The following section highlights the results and points out implications used in Section 7. The input data are not reflecting real values, but are considered sufficient for the purpose this model serves.

Table 6.3.1 and Table 6.3.2 present the producers' and consumers' numbers for the chosen market structures. We know from theory in Appendix A.2 that relative to a perfectly competitive firm, a monopolist will restrict output in order to increase price. It can be shown that given the same demand and cost structures, the Cournot duopoly will choose a price/output combination that is between the monopoly and the perfectly competitive firm. Hence, the Cournot duopoly will, without collusion, restrict output to a level that elevates prices above the competitive amount, and thus allow the firms to earn above perfectly competitive (but not monopoly) profits. Table 6.3.1 and Table 6.3.2 confirm just that, showing that the volumes and prices of the Cournot duopoly are between the two market extremes.

	PC			Cournot			Monopoly		
Producer	Prod	M/S	PS	Prod	M/S	PS	Prod	M/S	PS
Producer 1	33	50%	0	25	50%	150	32	100%	534
Producer 2	33	50%	0	25	50%	150	-	-	-
Total	66	100%	0	50	100%	300	32	100%	534

Table 6.3.1: Production (GW), market share (%) and producer surplus (€) for different market structures.

Furthermore, we can verify the relationship between the producer and consumer surplus in the different scenarios. According to Appendix A.2 the monopolist will set marginal revenue equal to marginal cost and therefore induce a deadweight loss that extracts some of the consumer surplus. Perfect competition represents the opposite case where the producer sets price equal to marginal cost and maximizes consumer surplus. Both scenarios are verified by the results displayed in Figure 6.3.1.

	PC			Cournot			Monopoly		
Market	Cons	Price	CS	Cons	Price	CS	Cons	Price	CS
Market 1	33	17	534	25	25	300	16	34	133
Market 2	33	17	534	25	25	300	16	34	133
Total	66	-	1,068	50	-	600	32	-	266

Table 6.3.2: Consumption (GW), price (€/GJ) and consumer surplus (€) for different market structures.

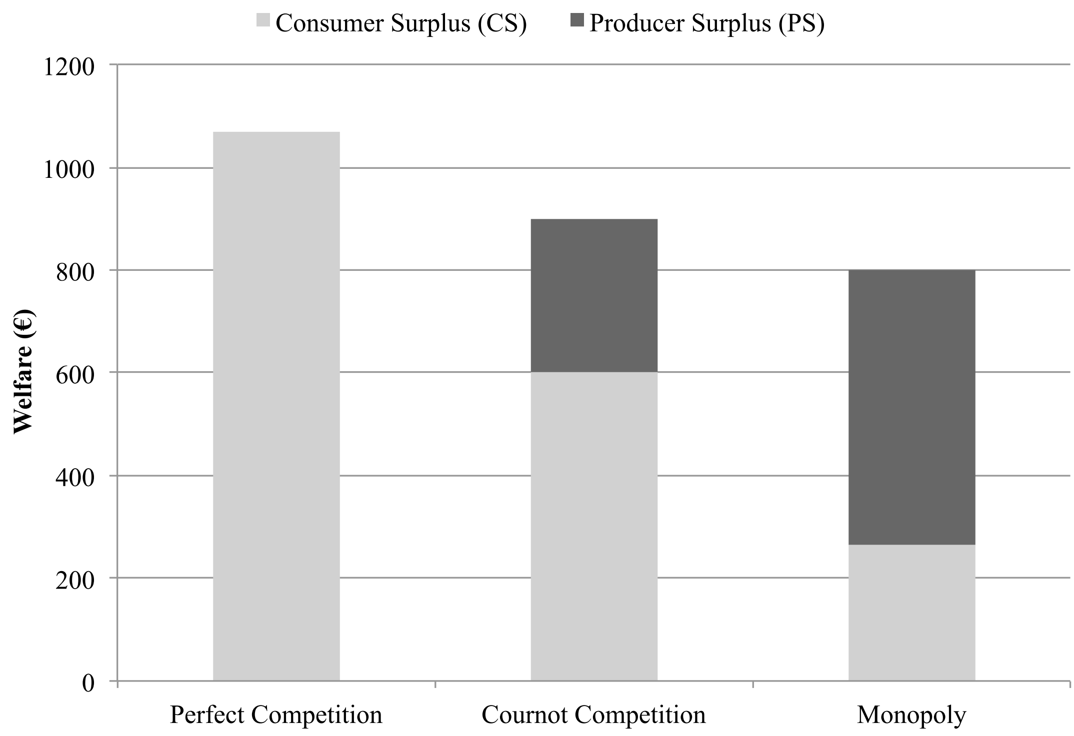

Figure 6.3.1: Welfare results (€) for each market structure in the basic production model.

Chapter 7

Strategic MCP Model of Gas and Electricity Markets

This chapter gives a complete strategic MCP formulation of a simplified gas and electricity market in Northern Europe. Similar to the previous chapter, Section 7.1 lists the assumptions and simplifications required to describe the market through a proper model. Section 7.2 presents the nodal network, definitions and declarations. Each players' optimization problem is described in Section 7.3.

7.1 Assumptions

Energy markets are difficult to model, considering the number of actors in the market and the complexity of their interaction with each other. When one player's optimization problem depends not only on his own decision, but also on other players' decisions a strategic problem is present. Both electricity and natural gas are network industries, meaning that they are highly dependent on the infrastructure and how efficiently it is used. This model aims at a comprehensive and accurate representation of the transmission network to capture these properties. Based on the current situation in Northern Europe, we assume two liberalized, but oligopolistic markets allowing for market power.

The model extends the simplified model stated in Chapter 6. It is based on myopic and deterministic assumptions, meaning that time dependency and stochasticity are not implemented, and future events are omitted. Hence, each player optimizes his action with respect to today's situation only and uncertainties are ignored in the decision-making. These assumptions are obviously not reflecting reality and how market actors actually make their decisions, but we consider it necessary to make these assumptions in order to simplify the modeling process sufficiently.

We have chosen to represent the gas and electricity market in Northern Europe by the nodes of Norway, Germany and Great Britain. Ideally, we would include all countries in the region, but considering the increased complexity of the modeling task and the purpose this model serves, we find it sufficient to limit ourselves to the three countries above. There are several reasons to choose Norway, Germany and Great Britain over the others countries of Northern Europe. Firstly, they are large contributors to the trade of both commodities in the region. Secondly, they are interconnected by a well-developed infrastructural network allowing for intensive trade between the countries. Thirdly, they are highly dependent of each others behavior, both on the supply and demand side of the market. Thus, we are confident that the reality and dynamics of the market situation will be preserved modeling those countries.

The relevant oligopolistic representation will differ from industry to industry but is likely to be based on the variable that cannot be altered in the short-run. From Appendix A.2 we know that Cournot competition is to be expected when firms make output decisions that are hard to change. These usually involve capacity, quality or location specific decisions where prices adjust to these decisions. Bertrand competition represent the opposite scenario, where firms commit to prices that are hard to change or when quantities can be adjusted quickly. The fact that capacity constraints limit price competition while excess capacity encourages price competition, indicates that a Cournot representation is preferable of expressing oligopolistic features in the following model.

As shown in Table 7.1.1, we assume that all of the players are profit maximizers and that we are allowing for strategic behavior (Cournot) between the producers in the network, satisfying the related features. As mentioned in Section 5.3, we distinguish between pure and mixed strategies in the decision making. Because of the numerical difficulties mixed strategies cause, we do not allow the players to operate with mixed strategies, but merely in pure strategies. We are also assuming that every player acts rationally and has complete information about the other players. This is an essential feature of multiplayer games and a necessary assumption to make in order to express strategic behavior mathematically (Section 5.1).

We do not allow for shared strategy sets where players compete on the same set of resources, e.g. a gas producer in Norway cannot produce from the same gas fields as a German gas producer. Instead we are assuming that each player operates from individual sets of resources. Accordingly we are not facing a Generalized Nash

Equilibrium making the problem less complicated to solve (Section 5.4). We are also assuming that there are no market leaders present in the game, implicating that the model is not an MPEC nor an EPEC.

Analyzing each and every consumer individually would be impractical to model. Instead we have chosen to aggregate the consumers into larger groups within each sector. This makes the model more comprehendible, without losing too much information as we would by aggregating all of the end-users into one large group. We have divided the consumers into two categories depending on their consumption pattern: *Household* and *Industry*. Each group in each node has their own demand function with respect to either *gas* or *electricity*.

We assume the inverse demand functions $p(q)$ and the cost functions $C(q)$ to be twice differentiable:

$$\frac{\partial p(q)}{\partial q}, -\frac{\partial C(q)}{\partial q} < 0 \quad \text{and} \quad \frac{\partial^2 p(q)}{\partial q^2}, -\frac{\partial^2 C(q)}{\partial q^2} \leq 0 \tag{7.1.1}$$

Hence, we can guarantee for uniqueness and existence of a solution (Section 5.3).

Similar to Egging et al. [2008], we implement the conjectural variation parameter $MP_{finj} \in [0, 1]$ to express market power in the model. A value of $MP_{finj} = 0$ characterizes perfectly competitive behavior by a player, which means that the player's action will not affect the market outcome. However, if $MP_{finj} = 1$ we will have a Cournot behavior where player i consider all the other players $-i$

Actor	Role	Comment
Natural Gas Producer	Produce gas and sell to end-users and electricity producers.	Can exert market power by behaving strategically against other gas producers.
Electricity Producer	Produce electricity and sell to end-users.	Can exert market power by behaving strategically against other electricity producers.
Network Operator	Assign transfer capacities to gas and electricity producers.	Perfectly competitive. Assigns capacities on a marginal willingness to pay basis.
End-user	Consume gas and electricity.	Divided into a gas and electricity market for both households and industry in each node.

Table 7.1.1: Market actors and their position in the market.

quantities as fixed. Values of the parameter in the interval: $0 < MP_{finj} < 1$ are not well explained in theory, but they can have relevance in order to decide on deviation from the market extremes. Details on different market structures and its characteristics is given in Appendix A.2.

7.2 Model Formulation

The model intends to analyze strategic choices made from different actors through a simplified value chain representation of the energy market in Northern Europe. Using fundamentals of equilibrium programming we can examine price and volumes effects of gas and electricity in the region.

7.2.1 Declarations

The following notations apply to the multi-commodity model presented in this chapter. Note that both markets, nodes and fuels are represented with multiple indices. Accordingly, a transfer from one node to another is indexed by (i, j). For the same reason we write (n, m) between two markets. The same notations applies for the fuels included in the model.

Indices

(n, m)	Markets in the network $(HH(NG), HH(EL), IN(NG), IN(EL)) \in \mathcal{N}$
(i, j, l)	Nodes in the network. $(NO, DE, GB) \in \mathcal{I}$
(f, g)	Fuels in the network. $(NG, EL) \in \mathcal{F}$

Sets

$\mathcal{N}$	The set of all markets in the network
$\mathcal{I}$	The set of all nodes in the network
$\mathcal{F}$	The set of all fuels in the network

Data

$ARCCAP_{fij}$	Capacity limit of commodity f on arc (i, j)
$PRODCAP_{fi}$	Capacity limit of energy production of producer (f, i)
$SALESCAP_{finj}$	Capacity limit of sales from producer (f, i) to market n in node j
$TRADECAP_{fij}$	Capacity limit of trade from producer (f, i) to node j
$CONVCAP_{fi}$	Converting capacity of of producer (f, i)
INT_{ni}	Intersection point of the inverse demand curve of market n in node i
SLP_{ni}	Slope of the inverse demand curve of market n in node i
A_{fi}	Linear cost factor in the cost function of producer (f, i)
$CONV$	Conversion factor in converting gas to electricity
$ARCCOST_{fij}$	Unit cost of transporting commodity f on arc (i, j)
MP_{finj}	Market power parameter of producer (f, i) in market n in node j

Variables

q_{fi}^{PROD}	Produced quantity of producer (f, i)
q_{finj}^{SALES}	Sold quantity of producer (f, i) to market n in node j
q_{fij}^{TRADE}	Traded quantity from producer (f, i) to node j
x_{fi}^{TRADE}	Producer (f, i)'s received trade
q_{fij}^{FLOW}	Physical flow of commodity f on arc (i, j)
λ_{finj}	Lagrange multiplier of sales constraint in producer (f, i)'s problem
ρ_{fi}	Lagrange multiplier of production constraint in producer (f, i)'s problem
κ_{fij}	Lagrange multiplier of trading constraint between producer (f, i) and node j
ϕ_{fi}	Lagrange multiplier of flow conservation constraint in producer (f, i)'s problem
ω_{fij}	Lagrange multiplier of the arc limit constraint of commodity f on arc (i, j)
τ_{fi}	Lagrange multiplier of the capacity converting constraint of producer (f, i)
bp_i	Border price of gas in node i
$tfee_{fij}$	Lagrange multiplier of the flow equation of f on arc (i, j)
p_{ni}	Price of energy of market n in node i

Functions

Π_{fi}^{PR}	Profit function of producer (f, i)
Π_{fij}^{NO}	Profit function of network operator (f, i, j)
C_{fi}^{PR}	Cost function of producer (f, i)
C_{fij}^{NO}	Cost function of network operator (f, i, j)

7.2.2 Network Description

Nodes and Markets

We have summarized the market situation and the interaction between the players in Figure 7.2.1. As the figure illustrates, the countries of Norway, Germany and Great Britain constitute the nodes in the network. This partitioning with respect to countries is used by Huppmann et al. [2009] and gives a detailed description of the energy trade. The model describes a snapshot of the market situation without time dependency. Hence, all trading is made on the spot market for both electricity and gas. The fact that most of the gas produced in Europe has been committed to take-or-pay (TOP) contracts would be impractical to implement as the time perspective is omitted.

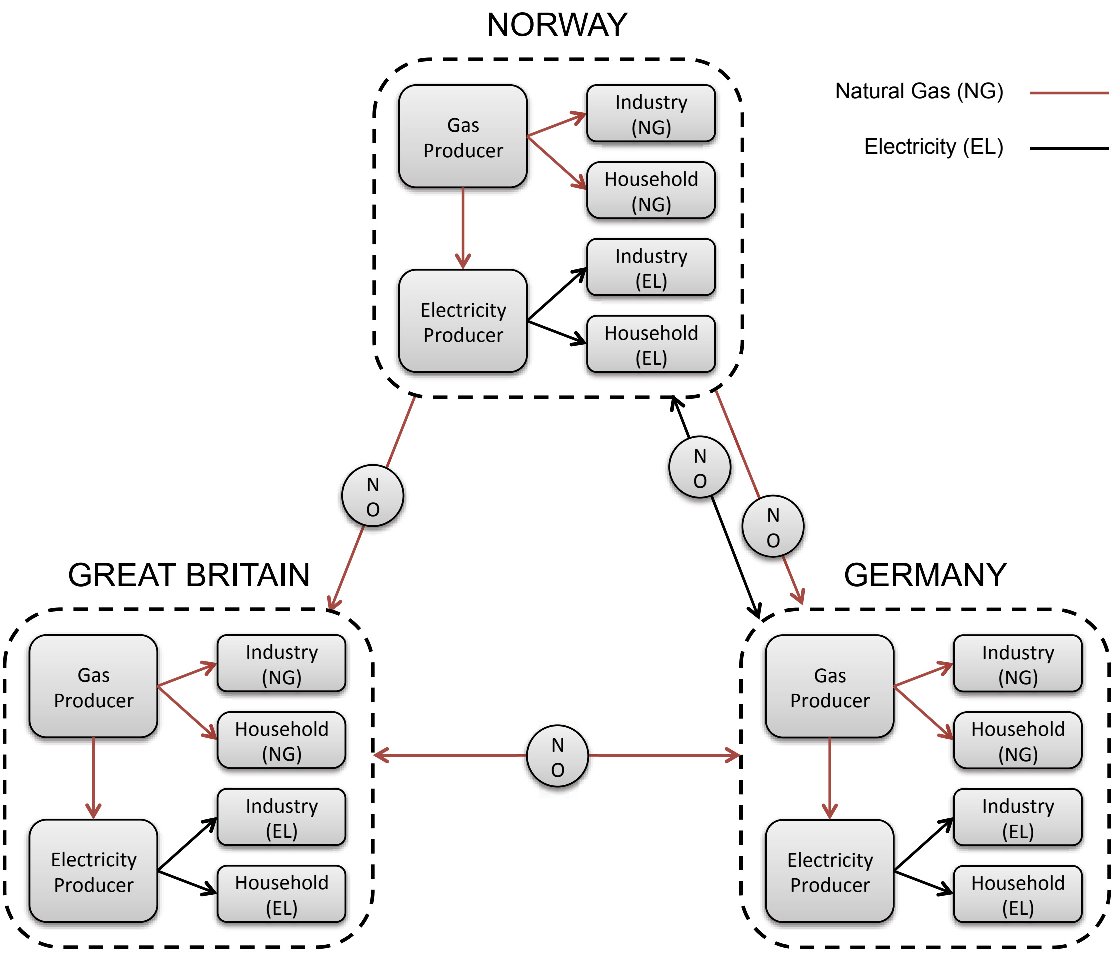

Figure 7.2.1: Markets, nodes and actors and the interaction between them.

According to the geographical location of the nodes and the present trade situation, each node can have the following activities: *production*, *consumption*, *import* and *export*. We allow for only one producer of of each commodity in each node. This is justified by observations in several countries of Europe, such as GdF in France and Gazprom in Russia that represents for the majority of the gas production in its country. We find somewhat the same pattern in the electricity market, where EdF, as an example, act as the dominant electricity producer in France.

Arcs

Each node in the network is connected to neighboring nodes through arcs. The existing gas and electricity networks in Northern Europe form the basis of this network and serve the flow between the nodes. The current LNG-flow between Norway, Germany and Great Britain is negligible compared to gas trade through pipelines[1]. As a result, we do not include the possibility of transporting LNG from export terminals to import terminals among countries in the network. This justifies the assumption of a myopic model as it would be meaningless to implement the storage features of LNG without including a time horizon.

In cases where the neighboring nodes are not directly connected to the other nodes through existing arcs, the model allows for transition through other nodes to facilitate trade. This allows for two-way gas transfer between Great Britain and Germany even though they are not directly connected by arcs. In this case, Netherlands and Belgium act as dummy nodes allowing for transfer between the countries, subjected to a transfer fee. The same applies for electricity transfer from Germany to Norway where Sweden and Denmark acts as transitioning dummy nodes. The arc capacity will be determined by the minimum capacity on the arc from the exporting node, via the dummy node(s) and into the importing node.

The model does not include local distribution to consumers within a node. This means that gas and electricity that is consumed in a node is not location specific. It is consumed *somewhere* within the node by one of the consuming sectors, but we do not know the exact location.

[1]BP [2009] indicates that Norway was the only country in the network that exported LNG. A transfer of 0.33 GJ of LNG from Norway to Great Britain is roughly one percent of the corresponding pipeline trade.

7.3 Optimization Problems

The following section addresses each players' optimization problem with belonging assumptions. In a multiplayer game like energy markets, we can aggregate the KKT conditions of each player and combine these with each of the players' market clearing conditions to form a set of equations. This gives us a market equilibrium model, which can be transformed into an MCP and solved (Section 5.6.4). In principle, a model like this could be solved by optimization but the presence of market power makes it necessary to transform it into an MCP model and solve it accordingly.

7.3.1 Producer

The production is done separately by each of the producers in the respective nodes, i.e. none of the production fields are operated by multiple producers. That being the case the producer (f, i) operates alone in its own node and assumed to be a profit-maximizing player with the possibility of exerting market power. As Cournot player he will behave strategically against the other producers in the network and decide on his decision variables based on the other producers' decisions.

His revenues comes from sales to end-users q_{finj}^{SALES} in market n at a price p_{nj} and sales to other producers q_{fij}^{TRADE} at a price bp_j. Ergo, electricity producers can buy a quantity of x_{fi}^{TRADE} gas at a price bp_j and convert it to electricity, penalized by the conversion factor $CONV$, resulting in a net amount of energy $x_{fi}^{TRADE} \cdot CONV$. Thus, electricity producers are assumed to produce from all available resources in his node except from natural gas, that he can buy from the gas producer.

The producer (f, i) will experience a cost related to the production process denoted by $C^{PR}(q_{fi}^{PROD})$. He will also face a cost of transferring q_{finj}^{SALES} to market n at transfer fee $tfee_{fij}$. The cost of converting gas to electricity is included in a lower efficiency factor $CONV$.

In order to guarantee optimality we need to assume that the production cost $C^{PR}(q_i^{prod})$ has either linear or quadratic properties. This will ensure convexity of the problem and the KKT conditions will provide optimal solutions (Section 5.5). Golombek et al. [1995] claims that this is a reasonable assumption to make in the gas and electricity industry. Quadratic cost data are hard to derive, thus we assume the cost function to have the following linear shape:

$$C^{PR}(q_{fi}^{PROD}) = A_{fi} \cdot q_{fi}^{PROD}, \quad \forall f, i \tag{7.3.1}$$

where A_{fi} is a constant in the cost function.

We also assume that producer and exporter are closely related and acting as one single actor (represented as the producer) in the network. The fact that most of the large-scale gas producers has their own export-unit within the firm, makes this is a fair assumption. Even though each unit serve different purposes, they are tied under the same umbrella. Hence, the producing unit and the exporting unit do not behave strategically against each other.

Based on the assumptions above, each producer (f, i) is given the following optimization program:

$$\begin{aligned}
\max_{\substack{q_{fi}^{PROD}\\ q_{fij}^{TRADE}\\ q_{finj}^{SALES}\\ x_{fi}^{TRADE}}} \quad \Pi_{fi}^{PR} = & \sum_{j\in\mathcal{I}} q_{fij}^{TRADE} \cdot bp_j - \sum_{j\in\mathcal{I}} q_{fij}^{TRADE} \cdot tfee_{fij} \\
& + \sum_{j\in\mathcal{I}} \sum_{n\in\mathcal{N}} q_{finj}^{SALES} \cdot p_{nj} - \sum_{j\in\mathcal{I}} \sum_{n\in\mathcal{N}} q_{finj}^{SALES} \cdot tfee_{fij} \qquad (7.3.2) \\
& - x_{fi}^{TRADE} \cdot bp_i - A_{fi} \cdot q_{fi}^{PROD} \qquad \forall f, i
\end{aligned}$$

$$\begin{aligned}
s.t. \quad q_{fi}^{PROD} &\leq PRODCAP_{fi}, && (\rho_{fi}), \quad \forall f, i && (7.3.3) \\
q_{finj}^{SALES} &\leq SALESCAP_{finj}, && (\lambda_{finj}), \quad \forall f, i, n, j && (7.3.4) \\
q_{fij}^{TRADE} &\leq TRADECAP_{fij}, && (\kappa_{fij}), \quad \forall f, i, j && (7.3.5) \\
x_{fi}^{TRADE} &\leq CONVCAP_{fi}, && (\tau_{fi}), \quad \forall f, i && (7.3.6) \\
q_{fi}^{PROD} &= \sum_{j\in\mathcal{I}} \sum_{n\in\mathcal{N}} q_{finj}^{SALES} + \sum_{j\in\mathcal{I}} q_{fij}^{TRADE} && && \\
&\quad - CONV \cdot x_{fi}^{TRADE}, && (\phi_{fi}), \quad \forall f, i && (7.3.7) \\
q_{fi}^{PROD}, q_{finj}^{SALES}, q_{fij}^{TRADE}, x_{fi}^{TRADE} &\geq 0, && \forall f, i, n, j && (7.3.8)
\end{aligned}$$

where (7.3.2) represents the profit function. (7.3.3) limits the total production. (7.3.4) ensures that sales to market n in node j do not exceed its capacity. (7.3.5) makes sure each gas producer does not exceed the capacity of selling to electricity producer in node j. The latter two restrictions are included to link the producers, markets and nodes rather than constraining the sales and trade. As a result, the upper limits of (7.3.4) and (7.3.5) take large values. (7.3.6) limits the converting capacity of an electricity producer. (7.3.7) ensures that the total production by each producer (f, i) and the received quantity from gas producer (g, j) to electricity producer (f, i) equals the total sales to all markets n i nodes j and the traded quantity of gas from producer (f, i) to electricity producer (g, j). (7.3.8) forces the production, sales and trade and received trade to be positive.

For the same reasons as of Section 6.2.3, we convert the problem into a minimization problem to solve it as a complementarity model:

$$\min_{\substack{q_{fi}^{PROD} \\ q_{fij}^{TRADE} \\ q_{finj}^{SALES} \\ x_{fi}^{TRADE}}} \quad -\Pi_{fi}^{PR} = -\sum_{j\in\mathcal{I}} q_{fij}^{TRADE}\cdot bp_j + \sum_{j\in\mathcal{I}} q_{fij}^{TRADE}\cdot tfee_{fij}$$

$$- \sum_{j\in\mathcal{I}}\sum_{n\in\mathcal{N}} q_{finj}^{SALES}\cdot p_{nj} + \sum_{j\in\mathcal{I}}\sum_{n\in\mathcal{N}} q_{finj}^{SALES}\cdot tfee_{fij} \qquad (7.3.9)$$

$$+ x_{fi}^{TRADE}\cdot bp_i + A_{fi}\cdot q_{fi}^{PROD} \qquad \forall f, i$$

$$s.t. \quad PRODCAP_{fi} - q_{fi}^{PROD} \geq 0, \qquad (\rho_{fi}), \quad \forall f, i \qquad (7.3.10)$$

$$SALESCAP_{finj} - q_{finj}^{SALES} \geq 0, \qquad (\lambda_{finj}), \quad \forall f, i, n, j \qquad (7.3.11)$$

$$TRADECAP_{fij} - q_{fij}^{TRADE} \geq 0, \qquad (\kappa_{fij}), \quad \forall f, i, j \qquad (7.3.12)$$

$$CONVCAP_{fi} - x_{fi}^{TRADE} \geq 0, \qquad (\tau_{fi}), \quad \forall f, i \qquad (7.3.13)$$

$$q_{fi}^{PROD} + CONV\cdot x_{fi}^{TRADE} - \sum_{j\in\mathcal{I}}\sum_{n\in\mathcal{N}} q_{finj}^{SALES} - \sum_{j\in\mathcal{I}} q_{fij}^{TRADE} = 0, \qquad (\phi_{fi}), \quad \forall f, i \qquad (7.3.14)$$

$$q_{fi}^{PROD}, q_{finj}^{SALES}, q_{fij}^{TRADE}, x_{fi}^{TRADE} \geq 0, \qquad \forall f, i, n, j \qquad (7.3.15)$$

Deriving the KKT conditions of the minimization problem is necessary for the MCP formulation. According to Section 5.5 the KKTs of this formulation are sufficient for optimality because of concave properties of the objective function and the convex set of constraints:

Given the market power assumption described in Section 7.1, we implement the factor $MP_{finj} = \frac{\partial p_n(q_{finj}^{SALES})}{\partial q_{finj}^{SALES}}$ representing the market power of producer (f, i) in market n of node j. If the producer operates as a Cournot player this factor will be multiplied with the slope of the demand curve of market n:

$$\frac{\partial p_n}{\partial q_n} = \frac{p^{REF}}{q^{REF}}\cdot\frac{1}{\varepsilon} = SLP_n\cdot MP_{finj} \qquad (7.3.16)$$

where (q_n^{REF}, p_n^{REF}) denotes the reference point for the demand in node n and ε the price elasticity of demand.

For a price-taker however, the market power factor will take the value zero, meaning that the contribution of producer (f, i) will not change the price.

As a result, the KKT conditions for the producer become:

$$
\begin{aligned}
&0 \leq A_{fi} + \rho_{fi} - \phi_{fi} && \perp \quad q_{fi}^{PROD} \geq 0, && \forall f, i && (7.3.17)\\
&0 \leq -p_{nj} - MP_{finj} \cdot q_{finj}^{SALES} \cdot SLP_{nj} && && &&\\
&\quad + tfee_{fij} + \lambda_{finj} + \phi_{fi} && \perp \quad q_{finj}^{SALES} \geq 0, && \forall f, i, n, j && (7.3.18)\\
&0 \leq -bp_j + tfee_{fij} + \kappa_{fij} + \phi_{fi} && \perp \quad q_{fij}^{TRADE} \geq 0, && \forall f, i, j && (7.3.19)\\
&0 \leq bp_i - CONV \cdot \phi_{fi} + \tau_{fi} && \perp \quad x_{fi}^{TRADE} \geq 0, && \forall f, i && (7.3.20)\\
&0 \leq PRODCAP_{fi} - q_{fi}^{PROD} && \perp \quad \rho_{fi} \geq 0, && \forall f, i && (7.3.21)\\
&0 \leq SALESCAP_{finj} - q_{finj}^{SALES} && \perp \quad \lambda_{finj} \geq 0, && \forall f, i, n, j && (7.3.22)\\
&0 \leq TRADECAP_{fij} - q_{fij}^{TRADE} && \perp \quad \kappa_{fij} \geq 0, && \forall f, i, j && (7.3.23)\\
&0 \leq CONVCAP_{fi} - x_{fi}^{TRADE} && \perp \quad \tau_{fi} \geq 0, && \forall f, i && (7.3.24)\\
&0 = q_{fi}^{PROD} + CONV \cdot x_{fi}^{TRADE} && && &&\\
&\quad - \sum_{j \in \mathcal{I}} \sum_{n \in \mathcal{N}} q_{finj}^{SALES} - \sum_{j \in \mathcal{I}} q_{fij}^{TRADE}, && \quad (\phi_{fi} \text{ free}), && \forall f, i && (7.3.25)
\end{aligned}
$$

where ρ_{fi}, λ_{finj}, κ_{fij} and τ_{fi} are shadow prices of the capacity constraints in (7.3.3), (7.3.4), (7.3.5) and (7.3.6) and represents the objective value of an extra unit of capacity.

The total trade balance between the producers has to be maintained at equilibrium. This forms the market clearing condition in (7.3.26):

$$
\sum_{f \in \mathcal{F}} \sum_{i \in \mathcal{I}} q_{fij}^{TRADE} - \sum_{f \in \mathcal{F}} x_{fj}^{TRADE} = 0, \qquad (bp_j \text{ free}), \quad \forall j \qquad (7.3.26)
$$

where the dual variable bp_j represents the border price of node j. This price is dual to an equality, hence it is not restricted to be positive - it can be negative, positive and even zero. In order to ensure positive prices the input data has to be of sufficient quality.

7.3.2 Network Operator

The network operator (f, i, j) is responsible for assigning pipeline and grid capacities to the exporting producers. Similar to Egging et al. [2010] we assume Third Party Access to the pipeline and grid network implying that the capacities are allocated on a willingness-to-pay basis for the producers. We simplify our model by ignoring friction and pressure drops in the pipeline network and effect losses in the transmission grid. A good interpretation of this can be found in Midthun [2007a].

The network operator experience a cost of transporting electricity and gas between the nodes and markets. The cost $C^{NO}_{fij}(q^{FLOW}_{fij})$ is assumed to be linear depending on the volumes of transfer.

$$C^{NO}_{fij}(q^{FLOW}_{fij}) = ARCCOST_{fij} \cdot q^{FLOW}_{fij}, \quad \forall f, i, j \tag{7.3.27}$$

where $ARCCOST_{fij}$ is the unit cost of transfer depending on the distance between two nodes and the way of transfer.

The network operator is maximizing his profit from assigning an amount q^{FLOW}_{fij} to a price $tfee_{fij}$ subtracted the cost related to operation $C^{NO}_{fij}(q^{FLOW}_{fij})$.

We limit the model to one-directional arcs , even though both pipelines and transmission grids theoretically could be bi-directional. In cases where the arcs are bi-directional we model it as two one-directional arcs with separate capacity limits.

The network operator's optimization problem becomes:

$$\max_{q^{FLOW}_{fij}} \quad \Pi^{NO}_{fij} = q^{FLOW}_{fij} \cdot tfee_{fij} - ARCCOST_{fij} \cdot q^{FLOW}_{fij}, \quad \forall f, i, j \tag{7.3.28}$$

$$s.t. \quad q^{FLOW}_{fij} \leq ARCCAP_{fij}, \quad (\omega_{fij}), \quad \forall f, i, j \tag{7.3.29}$$

$$q^{FLOW}_{fij} \geq 0, \quad \forall f, i, j \tag{7.3.30}$$

where (7.3.28) represents the profit function. (7.3.29) limits the flow of f on arc (i, j) and (7.3.30) ensures positive flows in the network.

The related minimization problem follows as:

$$\min_{q^{FLOW}_{fij}} \quad -\Pi^{NO}_{fij} = -q^{FLOW}_{fij} \cdot tfee_{fij} + ARCCOST_{fij} \cdot q^{FLOW}_{fij}, \quad \forall f, i, j \tag{7.3.31}$$

$$s.t. \quad ARCCAP_{fij} - q^{FLOW}_{fij} \geq 0, \quad (\omega_{fij}), \quad \forall f, i, j \tag{7.3.32}$$

$$q^{FLOW}_{fij} \geq 0, \quad \forall f, i, j \tag{7.3.33}$$

For the same reasons as of the producer, we derive the KKT conditions for the network operator's minimization problem:

$$0 \leq -tfee_{fij} + ARCCOST_{fij} + \omega_{fij} \quad \perp \quad q_{fij}^{FLOW} \geq 0, \qquad \forall f,i,j \qquad (7.3.34)$$

$$0 \leq ARCCAP_{fij} - q_{fij}^{FLOW} \quad \perp \quad \omega_{fij} \geq 0, \qquad \forall f,i,j \qquad (7.3.35)$$

The fact that the flow of f on arc (i,j) has to equal the trade and the sales by the producers, forms the market clearing conditions for the network operator:

$$q_{fij}^{FLOW} = q_{fij}^{TRADE} + \sum_{n \in \mathcal{N}} q_{finj}^{SALES}, \qquad (tfee_{fij} \text{ free}), \quad \forall f,i,j \qquad (7.3.36)$$

where the dual $tfee_{fij}$ of the market clearing can take both negative and positive values.

7.3.3 End-User

End-users are located in each node and divided into four categories of consumption: Household (NG), Household (EL), Industry (NG) and Industry (EL). Each of the categories forms a market n in its own node and is assumed to have a linear inverse demand functions of gas or electricity depending on the consuming sector.

The demand curves are aggregated such that each consumer group represents the total demand of its group within the node. The curves are assumed to be defined around a reference point of demand (q_n^{REF}, p_n^{REF}) based on empirical measures of price and consumption. As a result, the aggregated inverse demand curves of each sector will take the following form:

$$p_{ni} = INT_{ni} + SLP_{ni} \cdot Q_{ni}^{SALES}, \quad \forall n,i \qquad (7.3.37)$$

where Q_{ni}^{SALES} represent the total sales of all producers to market n in node i.

Chapter 8

Implementation

The following chapter presents important aspects of the implementation of the model. Section 8.1 describes relevant software that is capable of expressing problems of the complementarity format. The implementation is described in Section 8.2 while specifications of the hardware is given in Section 8.3.

8.1 Software

The model requires software and a solver capable of finding optimal solutions to MCP problems. The General Algebraic Modeling System (GAMS) modeling language is a high-level modeling system designed for mathematical optimization that allows for modeling in the complementarity format. The way GAMS is constructed, GAMS lets the users concentrate on the modeling instead of the solution methodology.

The GAMS modeling language combined with the PATH solver support the complementarity class of problems and can be used to find an optimal solution to the problem. According to Ferris and Munson [2000], the PATH solver is based upon a generalization of the classical Newton's method plus a linearization solved using a code related to Lemke's method. There exist several solvers capable of finding optimal solutions to MCP problems, but PATH has proven its considerable success on similar problems earlier and is therefore desirable. An introduction to the GAMS syntax is given in Rutherford [1995] and Rutherford [1999] while a detailed description of PATH and its range of application can be found in Dirkse and Ferris [1995].

8.2 Implementation and Solution Approach

By combining the KKTs and the market clearing conditions in each of the players' optimization problems we will end up with a market equilibrium model with an MCP structure (Section 5.6.4). The fact that all cost functions are assumed to be convex, the negative of the profit functions are strictly concave and we have convex sets of constraints gives us an optimal solution to the problem.

The implementation of the model was done through the graphical user interface GAMS IDE (Integrated Development Environment) that links the problem to the chosen solver. GAMS IDE allows for editing, debugging and running the model through the same graphical interface. By modeling in high-level languages such as GAMS, the similarities between the syntax and the mathematical structure of the problem are many, which eases the job of coding the model.

The model is purely deterministic, so it was sufficient to list the sets, parameters, variables and equations in GAMS. Furthermore, the equations are paired up with the corresponding dual variables from the mathematical description. GAMS do not directly solve the problem, but interfaces with the external solver PATH that provides the results. If the compilation is done successfully, GAMS IDE provides a solution summary indicating whether or not the solution is optimal. All of the equations and variables with corresponding values are listed in the solution summary. As the model is deterministic it is sufficient to run the model once for each set of input data. The input data varied for the different problem instances so several iterations of the model were run, with the results manually documented. The GAMS code of the model is included in Appendix B.2.

8.3 Hardware

All problem instances of the model are run on a Asus Eee PC with an Intel Atom CPU N270 1,60 GHz Processor and 0,99 GB RAM. This was considered sufficient, as the computational requirements by running the model were rather low.

Chapter 9

Data Sets

The following chapter contains relevant data sets applied for solving the model of Section 7. Section 9.1 provides data and reasoning for the Base Case based on the current situation in the gas and electricity markets of Northern Europe. Data for different problem instances are listed in Section 9.2.

9.1 Current Situation and Base Case

Most of the data used in this model is based on the situation in 2009. We do not consider seasonal variations over the year, thus annualized data sets are desired. This alternative will not comprise peaks in consumption, production and trade, but has its advantages of being representative for the whole year.

Production capacities for both natural gas and electricity are collected from the International Energy Agency (IEA) (IEA [2010]) and based on statistics of 2009. Converting capacities are assumed to be 5 percent more than current conversion data listed in IEA [2010]. Cost of production estimates are collected from Observatoire Méditerranéen de l'Energie (OME) (OME [2002]) and the IEA. In cases with lack of data we assume the production costs to be 25 percent of the average market price within the node. Both data sets are presented in Table 9.1.1.

Network capacities between the nodes are taken from the European Network of Transmission System Operators (ENTSO), ENTSO-G [2010] and ENTSO-E [2010]. Table 9.1.2 summarizes the capacity balance between the nodes for the different commodities.

Cross-border tariffs between each node in the electricity network are based on

Region	Natural Gas		Electricity		
	Production capacity	Production cost	Production capacity	Production cost	Convert. capacity
NO	134.95	2.28	17.10	8.25	0.09
DE	17.02	2.57	74.20	7.99	25.39
GB	94.35	2.05	45.34	7.88	44.84

Table 9.1.1: Production capacities (GW), converting capacities (GW) and unit production costs (€/GJ).

FINERGY [2003]. The electricity tariffs are depending on both the amount of energy and the number of border crosses between the exporter and the buyer. Transport fees for gas are based on average tariffs between countries in Europe and have the same structure as for electricity. Table 9.1.2 displays the transport costs of trading a commodity on a specific arc in the network.

From	To	Pipeline		Transmission grid	
		Transport capacity	Transport cost	Transport capacity	Transport cost
NO	DE	37.21	0.24	2.65**	1.25
	GB	81.14	0.24	-	-
DE	NO	-	-	2.10**	1.25
	GB	36.52*	0.71	-	-
GB	NO	-	-	-	-
	DE	13.39*	0.71	-	-

*Includes transition through Netherlands and Belgium

**Includes transition through Denmark and Sweden

Table 9.1.2: Network capacities (GW) and cross-border tariffs (€/GJ) between countries in the network. (-) represents connections without existing arcs.

Historically, the electricity production from natural gas has been done by so-called single-cycle gas turbines generally converting the heat energy from combustion into electricity at efficiencies of 35 to 40 percent. Today there are present high efficiency power plants using technologies such as Natural Gas Combined Cycle (NGCC) or Combined Cycle Gas Turbine (CCGT) that ensures efficiencies over 50 percent depending on the size and layout of the installation. According to Song [2010] it is fair to assume conversion efficiencies about 58 percent in Northern Europe. Due to lack of conversion cost data we assume the loss of energy of converting from gas to electricity to be a cost itself. Accordingly, the efficiency factor of 58 percent includes the cost of conversion.

The inverse demand curves of each market are derived using a reference point (q_n^{REF}, p_n^{REF}) for each of the consuming segments (the calculation is given in Appendix A.1.1). Looking at the reference consumption and reference price for 2009 (Eurostat [2010]) under the assumption of a fixed elasticity of demand ε, the demand curves are estimated. We assume the elasticity of demand to take the value $\varepsilon = -1$ for every natural gas market in each of the nodes. This value is rather low compared to the estimates Bernstein and Griffin [2005] made for the American gas market, but are justified by Holz [2009] in the natural gas markets of Europe. Empirical estimates by Halvorsen and Larsen [2001] concludes with a similar value of the price elasticity of demand in the residential market for electricity in Norway. Lack of sufficient data makes us use this value for the electricity markets for both industry and households in the other nodes. Table 9.1.3 presents the estimated demand curves of each market in the network.

Region	Natural Gas				Electricity			
	Industry		Household		Industry		Household	
	SLP	INT	SLP	INT	SLP	INT	SLP	INT
NO	−7.30	18.24	-	-	−2.82	49.62	−10.57	82.44
DE	−0.20	11.26	−0.26	21.46	−1.03	52.62	−2.72	73.38
GB	−0.20	17.96	−0.27	23.08	−0.75	51.16	−2.41	76.72

Table 9.1.3: Slope and intersection point (€/GJ) of the inverse demand curves in each market. (-) represents a nonexistent market.

In the BC we assume all producers to be profit maximizing Cournot players acting strategically against other producers in the network. This assumption is based on the fact that the European gas and electricity market are both partly liberalized with possible presence of market power. The network operators are assumed to be price takers assigning capacities on a willingness-to-pay basis of the strategic producers. Thus, the $6 \times 12-$ matrix in Table 9.1.4 describing market power by the producers in their respective markets, takes the following form:

The matrix is to be read as producer (f, i) (represented by the rows) has market power in market n in node j (represented by the columns). The elements of the matrix takes either the value 0 or 1 depending on the strategic influence of the player. For the BC all feasible elements are set equal to 1, describing a Cournot situation.

Producer	NO				DE				GB			
	HH		IN		HH		IN		HH		IN	
	NG	EL	NG	EL	NG	EL	NG	EL	NG	EL	NG	EL
NO (NG)	1	-	1	-	1	-	1	-	1	-	1	-
NO (EL)	-	1	-	1	-	1	-	1	-	-	-	-
DE (NG)	1	-	1	-	1	-	1	-	1	-	1	-
DE (EL)	-	1	-	1	-	1	-	1	-	-	-	-
GB (NG)	-	-	-	-	1	-	1	-	1	-	1	-
GB (EL)	-	-	-	-	-	-	-	-	-	1	-	1

Table 9.1.4: Representation of market power in the BC of the model. A value of 1 represents Cournot behavior while 0 represents a price taker. In situations where there are no arcs connecting a producer to a market or the producer of obvious reasons cannot sell in the particular market, the cells are marked by (-).

9.2 Problem Instances

By altering several parameters we can analyze interesting sensitivity effects and test the stability of our results. The analysis is divided into two main sections where Section 9.2.1 tests different combinations of market power structures to fit the current market situation in Northern Europe, while Section 9.2.2 gives a sensitivity analysis of some important input parameters.

9.2.1 Market Power

One of the main features of the model is the possibility of analyzing the presence of market power in the energy markets. By constructing different market scenarios and comparing it with 2009 values, referred to as the Reference Case (RC), the best fitted market model can be observed.

Using the strategic properties of the model, we can illustrate three main market structures being PC, Cournot oligopoly and a monopoly. However, there might be situations where only some of the players exert market power, while some of the smaller players acts as a competitive fringe, having no impact in the final price. In addition, we can construct scenarios where the market power parameter takes values different from 0 or 1, lying somewhere in between.

By altering the market power parameter (MP_{finj}) for each producer-market combination we can obtain endless market scenarios. Testing for all combinations would of course be impossible. Thus, it is reasonable to try out scenarios (Table 9.2.1) that seem plausible. In two of the scenarios we have chosen to set the market power parameter equal to 0.5. The reason for this is the possibility of exerting partial market power in a market, without acting as a true Cournot player, but neither as a price-taker. Table 9.1.4 highlights the altered parameters that are changed during runs to find the combination closest to today's situation.

Scenario	Description
Scenario 1	The German gas producer has no market power, neither in domestic nor foreign markets.
Scenario 2	All producers have market power domestically, but not in foreign markets.
Scenario 3	All producers have limited market power (MP = 0.5) both in domestic and foreign markets.
Scenario 4	All producers have limited market power (MP = 0.5) domestically, but act as price takers in foreign markets.

Table 9.2.1: Scenarios created by altering the market power parameter to fit the situation of 2009 (RC).

9.2.2 Sensitivity Analysis

A sensitivity analysis is important to test the robustness of the results and to examine how the model responds to a change in input parameters. The parameters are altered one by one to observe the changes in output for each change.

As earlier mentioned (Section 7.3.1), two of the restrictions in the model serve a different purpose than the rest, by linking players together rather than constraining their activities. The sales restrictions of (7.3.4) are included to make sure that the gas producers have access to the gas markets and electricity producers to the electricity markets. Furthermore, the trade restrictions of (7.3.5) are included in the model to ensure that only gas producers can sell to electricity producers and that electricity producers can convert the gas into electricity. Hence, small changes of $SALESCAP_{finj}$ and $TRADECAP_{fij}$ will not affect the objective function.

For that reason, the restrictions that are interesting to investigate are those of production capacity (7.3.3), conversion capacity (7.3.6) and arc capacity (7.3.29).

		Natural Gas		Electricity		
From	To	Transport capacity	Production capacity	Transport capacity	Production capacity	Convert. capacity
NO	DE	37.21	134.95	2.65**	17.10	0.09
	GB	81.14		-		
DE	NO	-	17.02	2.10**	74.20	25.39
	GB	36.52*		-		
GB	NO	-	94.35	-	45.34	44.84
	DE	13.39*		-		

*Includes transition through Netherlands and Belgium

**Includes transition through Denmark and Sweden

Table 9.2.2: Capacity limits (GW) that are altered to analyze sensitivity effects in prices and volumes. (-) represents connections without existing arcs.

By varying those constraints, we can examine both domestic and foreign effects in the output. Table 9.2.2 presents the parameters that are changed and examined.

Chapter 10

Results

This chapter presents results and considerations from the different problem instances presented in Section 9. Section 10.1 presents results from the Base Case, while results obtained from adjusting the strategic influence to fit the current market situation are presented in Section 10.1.1. Section 10.2 treats the sensitivity analysis of some of the most important input parameters.

10.1 Base Case

The following results are based on the data from the Base Case (BC) of Section 9.1. Recall that the BC is based under the assumption that all producers act as Cournot players and behaves strategically against other producers in an oligopolistic market. The BC is compared to a perfectly competitive (PC) scenario to analyze the different market characteristics and a Reference Case (RC) of 2009 (Eurostat [2010]) to see how it compares to actual data.

According to Figure 10.1.1 we can identify significantly lower prices in the PC scenario compared to BC and RC. As each producer in the PC case will set price equal to marginal cost it will generally face lower prices than a Cournot firm. The exception being the gas markets in Germany where the prices in PC are higher than in the RC. This can be explained due to the low capacity of gas production in Germany. As a result, Norway is exporting at full capacity to Germany in both PC and BC. If the model was extended with more exporting gas nodes, Germany would have increased import of natural gas and the PC prices would decrease. The fact that Germany imports substantial amounts of gas from both Russia and Netherlands supports this argument (BP [2009]).

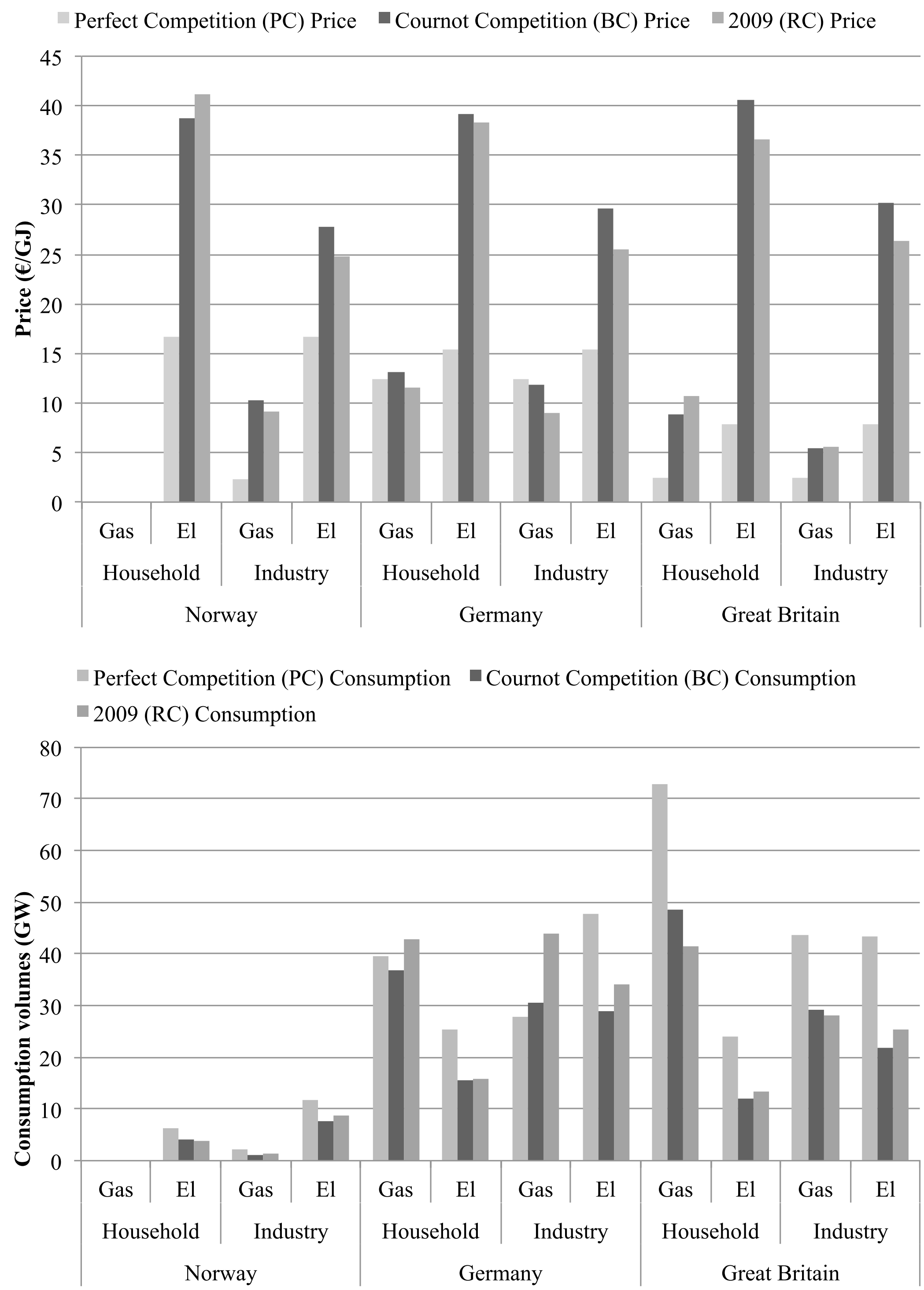

Figure 10.1.1: Prices (€/GJ) (top) and consumption volumes (GW) (bottom) for the Base Case (BC) in each of the markets compared to PC and 2009 values (RC).

BC prices in Germany are on average higher than values of RC. As reasoned above this is due to the fact that Germany in RC import large amounts of energy from nodes outside the network. These volumes are not compensated for in the BC and the results are increased prices.

Due to the inverse relationship between price and consumption, we can identify the reverse pattern for consumption as for the prices. Because of low prices in the PC case, the volumes of consumption are relatively high compared to RC. For the same reasons the BC will on average (not in the gas markets of Germany) have lower consumption because of higher prices, thus fitting the RC better than the PC scenario.

Looking at consumption and prices for the different markets, we see that BC matches the values from the RC fairly well. The fact that the prices and consumption of the BC fits the RC better than the PC scenario indicates that market power might be present as of 2009.

Figure 10.1.2 presents production, export and import for the three cases described above. Interestingly, the PC scenario describes the traded volumes better than the BC. This is due to the fact that the high consumption in PC forces the trade to be higher between the countries than it normally would be. Thus, the high volumes of trade will exceed the actual trade between the countries and level the numbers of RC that include trade with countries outside our predefined network. As a result, the PC scenario gives a unfounded good match with the export and import numbers of RC. Even though the BC deviates substantially on export and import numbers compared to the RC, the realism might be better preserved in the BC scenario even so.

Looking at the welfare effects of the different scenarios we can identify big variations in consumer surplus (CS) and producer surplus (PS) comparing PC and BC. Figure 10.1.3 illustrates this, showing that CS is larger in PC compared to BC, while the opposite is the case for PS. Nevertheless, the social surplus (SS) is larger in PC confirming that a perfectly competitive market produces the highest allocative efficiency. Appendix A.2 discusses this and provides supporting theory regarding welfare calculations. Another interesting observation is the welfare gains to expect by liberalizing the market for both natural gas and electricity. Because of the good match between BC and RC it is reasonable to assume similar results comparing social welfare across the scenarios.

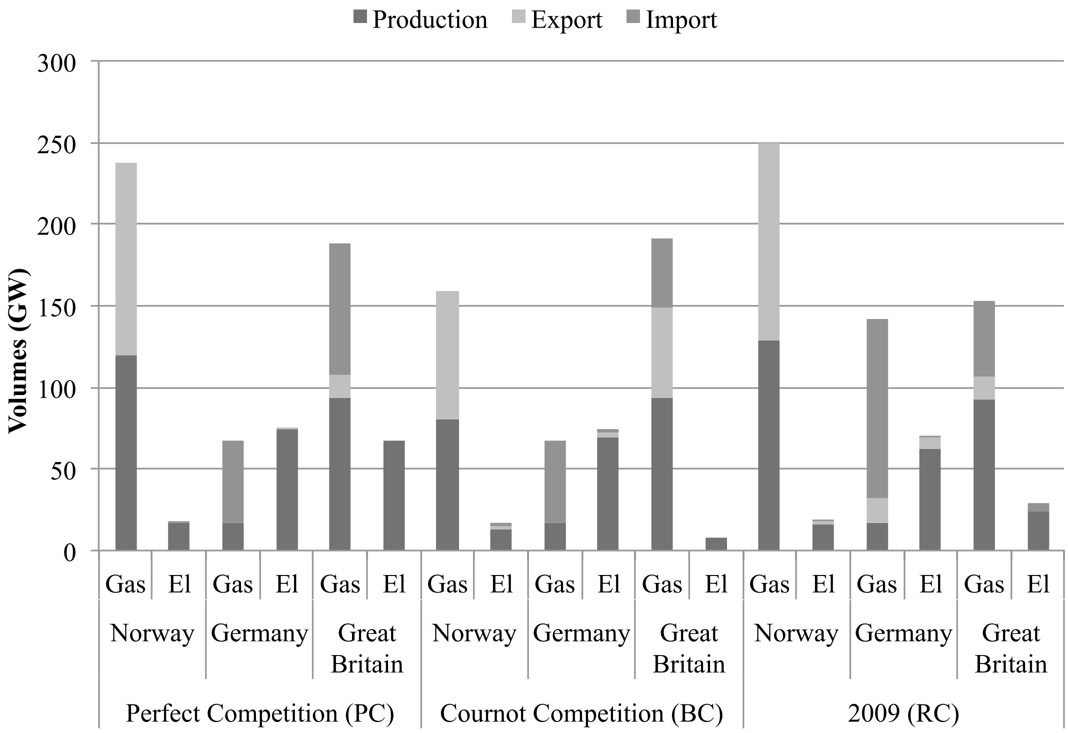

Figure 10.1.2: Production (GW), export (GW) and import (GW) of gas and electricity in each node.

10.1.1 Market Power

According to Section 9.2.1 we do not test for all combinations of market power but limit ourselves to plausible scenarios, listed in Table 9.2.1. A graphical presentation of the results for each scenario is given in Appendix C.

In Scenario 1 the German gas producer was assumed to be a price taker in all markets, both regional and foreign. Because of Germany's limited production of natural gas the impact of reducing his market power is minimal, resulting in an almost similar outcome as of the BC. Compared to the BC we get a slightly higher consumption in the German gas market for households and a similar reduction of consumption in the German gas market for industry.

The second scenario differs more from the RC than the BC. The Norwegian prices remain nearly the same, but in the German and British gas markets we can observe considerable changes. This could imply that only Norwegian producers possess market power in their own node, while this is not true for the other countries.

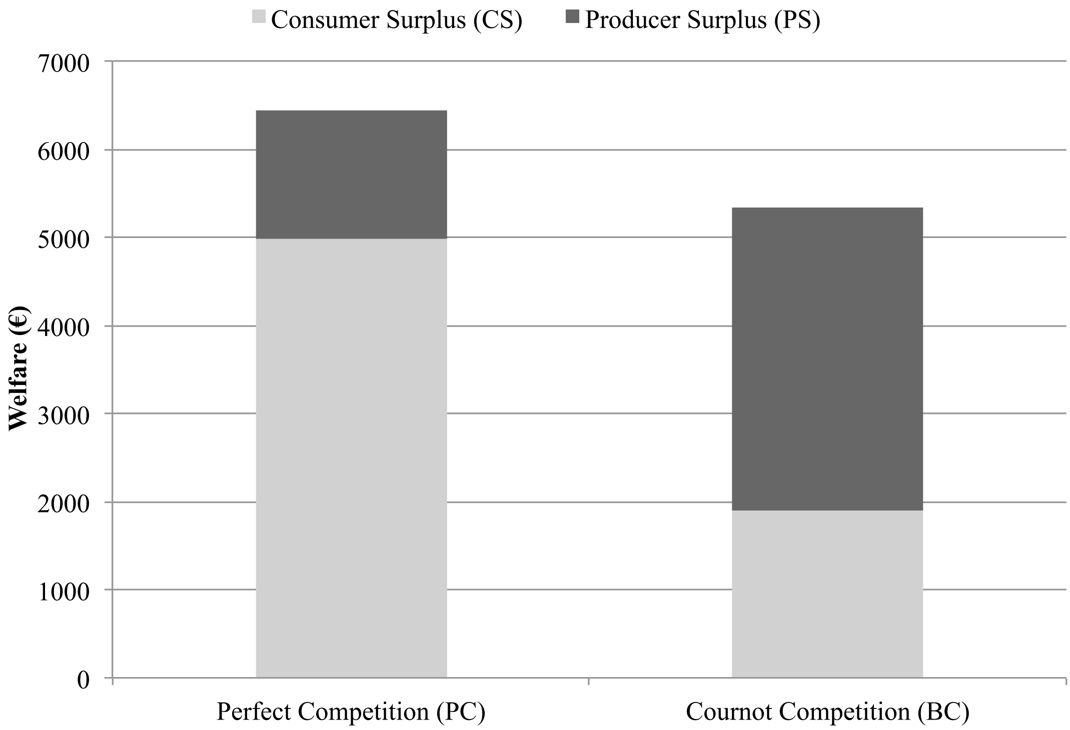

Figure 10.1.3: Welfare results (€) for BC compared to PC.

Giving all producers an MP-parameter of 0.5, as done in Scenario 3, suggests that the producers are somewhere between price takers and price makers. The results from this scenario deviates substantially from the RC. Compared with the BC, this supports the hypothesis stating that producers in Northern Europe act as Cournot players.

Furthermore, the fourth scenario has large discrepancies compared to RC, even more than Scenario 3. Again, this gives reason to believe that the Northern European gas and electricity market is not fully liberalized.

By comparing the four scenarios with the BC and RC, we conclude that the Northern European energy market share a lot of characteristics with the Cournot oligopoly. There are also some interesting remarks following from the analysis. The results from Scenario 1 suggests that whether or not the German gas producer possesses market power, he will not influence the prices and volumes notably. Moreover, we observe that producers are price makers outside of their own nodes, not price takers as the results from Scenario 2 indicates. Scenario 3 and 4 further support the overall conclusion that the BC gives the best representation of how the market power is distributed within Northern Europe.

10.2 Sensitivity Analysis

To better understand the drivers of our results from the BC we carry out a sensitivity analysis of some important input parameters. By looking at the dual variables it is possible to identify which of the restrictions that are binding at the optimal solution of the BC and how a unit change of the capacities will affect the objective function. As of Section 9.2.2 we found that the restrictions of production capacity (7.3.3), conversion capacity (7.3.6) and arc capacity (7.3.29) are interesting to investigate. By varying those constraints, both domestic and foreign effects in the output can be examined. The following discussion is based on the results illustrated in Appendix C.

Production Capacity (ρ_{fi})

The German gas producer holds the highest value of the dual to the production constraint (ρ_{fi}). According to Table 9.2.2, Germany has a relatively small production capacity when it comes to gas, which is reflected by the value of the dual. If the production capacity of the German gas producer increases, he sells more to the German gas markets, thus lowering the prices and increasing consumer surplus in Germany. That being the case, he would get a higher market share in the German markets which again increases his own profit. This will reduce the profits of both the Norwegian and British gas producers, but also reduce the Network Operator's profit seeing a reduction in the marginal willingness-to-pay, ($tfee_{fij}$).

Conversion Capacity (τ_{fi})

In BC, both duals of the Norwegian and British electricity producers' constraints are strictly positive. By increasing the conversion capacity of the Norwegian electricity producer, he will buy and convert more gas from the Norwegian gas producer while simultaneously lowering his own production. Net production remains the same, but the costs of production is reduced, thus increasing his profit. The prices in the Norwegian market will remain the same.

A similar change of the British conversion capacity, results in a different reaction. The Norwegian gas producer will increase his production and export to the British electricity producer. He increases his sales to the domestic electricity market causing a growth in consumer surplus while at the same time earning a bigger profit.

Arc Capacity (ω_{fij})

An increase of the arc capacity between Great Britain and Germany will cause the particular gas flow to increase, thus making the gas market in Germany more competitive, again resulting in lower prices and increased consumer surplus in the

German market. Allowing the British gas producer to sell more into the German markets increases his profits, but at the same time reduces the market power and profits of the German gas producer. The Norwegian gas producer responds to this by increasing his sales to the British market, who in turn has become more attractive, now that the British gas producer is exporting more of his own production.

Concluding remarks

An increase in the production capacity can turn out to be favorable for both the consumers and the investing producer. The producer is able to sell and possibly earn more profits, but will also create lower prices, thus increasing consumer surplus. However, the investment costs by expanding production capacity may exceed increased future earnings on the investment. This is not confirmed by our results, but could be investigated by a farsighted model.

Moreover, increased conversion capacities favors first and foremost the electricity producers, causing lower production costs. Positive effects in consumer surplus can also be observed, as seen in Figure C.0.8 and C.0.10.

Higher arc capacities makes it easier to access markets in foreign nodes. This encourages greater competition, in turn making the market more liberalized. From Appendix A we know that this increases consumer surplus and decreases the total producer surplus, but the net effect of social surplus is positive. The results displayed in Figure C.0.12 verifies these claims.

Chapter 11

Discussion

This chapter provides an evaluation of the model stated in Chapter 7. A brief discussion of the weaknesses and limitations of the model is given in Section 11.1, followed by suggestions for further work and model improvements in Section 11.2.

11.1 Limitations

The model possess a lot of simplifications and assumptions made in order to reduce complexity and allowing us to find a unique solution.

Assuming a myopic view simplifies the decision-making, but excludes possibilities of expansion, investments and other decision strategies that may be more profitable for the players in the long-run. Choosing a deterministic model rather than a stochastic one eases the modeling process, but can cause the players to make decisions they otherwise would not make because of uncertainty of the outcome.

Furthermore, the players are not allowed to play mixed strategies. This could have added more realism to the model and would have been relatively easy to implement. The rationale for not including it in the model is because of the difficulties of solving it numerically.

Intentionally, we have chosen to ignore regional distribution within in each node. The domestic distribution network is unquestionably important and will influence the international flow of both gas and electricity. This shortcut can lead to some inaccuracies in the results, but we are confident that the overall conclusion will remain the same.

The classification of the players and how they interact is simplified compared to a real world situation. We have only included producers, responsible of producing gas and electricity and sell to end-users or other producers (EL), and network operators, responsible for transporting the commodities between the nodes. We believe this representation of the market participants do not lower the quality of the end results significantly, compared to a more detailed classification of the roles and their interaction. In addition, we have only allowed for one producer of each commodity in each node. This will not preserve regional accuracies, but the big picture would not be affected to a large extent.

Pricing in the end-user market by inverse demand functions is a fair assumption to make. Assuming that electricity producers buy gas at a price equal to a border price of the importing node is realistic in the sense that it reflects their marginal willingness-to-pay.

Moreover, we decided to only include Norway, Great Britain and Germany as nodes in the model. Naturally, including more countries would have created a better representation of the market interaction, but it could also have created additional complexity. Given the same model structure, including a fourth node, for instance, would force one of the nodes to act as a transit node as well as a production node, causing challenges in expressing the arc capacity restrictions. In that case, the links between two nodes are not exclusive between two countries. This might change the model structure ending up with a GNE instead of an MCP. GNE's are generally harder to solve and can cause trouble showing uniqueness of a solution.

The way the model is formulated, it does not capture the interaction between the different markets sufficiently. By taking use of cross-price elasticities in the pricing in each market this aspect could be dealt with. A consumer with the flexibility of choosing between both gas and electricity will of course compare the market prices for each commodity and select the one that offers him the greatest utility. The rationale for not including this in the model is due to lack of quality data concerning cross price elasticities. If such data were to appear, it would be relatively easy to implement this in the model, thus increasing the accuracy of the results.

11.2 Further Work

The proposed model of this thesis can be further developed to bring new aspects into the model.

A natural way of expanding the model would be to include more nodes to earn a more realistic representation of the European energy market. Considering the difficulties of adding a node without changing the overall class of the model, the representation of trade should be reevaluated.

Including traders, LNG-storage operators and exporters would give the model a more precise picture of the two markets (See Egging et al. [2010] for implementation). The same applies for allowing more than one player of each type in the three nodes, e.g. letting more than one electricity producer operate in Norway. Both assumptions would create a more realistic representation of the market interaction, but would also be more demanding to model.

Besides, including more commodities, e.g. coal or/and oil, could be a next step creating a broader perspective, capturing cross relations between the new markets. Input data with quality is a necessity if this were to be implemented.

Incorporation of time dependency would be a natural extension, causing the players to think farther ahead in contrast to the myopic view of this model. This would allow for investment decisions that are not necessarily profitable in the short-run, but can be profitable in a long-term perspective. Interesting implementations could be possible investments in pipeline capacity, increased production capacity or LNG-storing facilities. Thus, including a time perspective would give the model a planning and forecasting dimension useful for creating future predictions and estimates.

Given the deterministic assumption, the model produces the same results for every set of input data. In a real world situation a lot of uncertainties are present, bringing out a need for a stochastic model, with the ability of handling uncertainty. This would require more computational power, but would be a big step towards a more realistic representation of the energy market.

Chapter 12

Conclusion

This thesis presents a strategic complementarity model focusing on analyzing prices and volumes of the gas and electricity markets of Northern Europe. Using fundamentals of equilibrium programming, the model can examine strategic choices made by different actors through a simplified value chain representation of the two markets.

The game theoretic modeling approach allows for representation of market power and thereby create different scenarios based on the strategic influence of each market participant. The model is formulated as a Mixed Complementarity Problem where each player solves his optimization problem within the complementarity system, giving the simultaneous solution to all players in the game.

Despite the liberalization process the markets for gas and electricity have undergone, the results indicates that market power is present on the supply side of each market. Comparing different market structures, our model results suggests that a Cournot representation of the suppliers gives the best match of the current situation as of 2009. However, the welfare gains of further liberalization are large comparing the Cournot representation with a perfectly competitive scenario.

The results indicates that increased capacities of production can turn out favorable for both the consumers and the investing producer. The producer will increase his sales and possibly earn higher profits, but will also create lower prices, thus increasing the social welfare. However, the value of investments are not considered in this thesis due to the underlying myopic assumption.

Bottlenecks in the network, where both grid and pipelines are on capacity, can be identified. The sensitivity analysis intimates that an infrastructural expansion will benefit both consumers and producers by increased profits and lower prices, suggesting that network investments are desirable from a social perspective.

The results of the model reflects simplified representations of the actual market situation in Northern Europe. The model is based on a large set of assumptions made in order to clarify the modeling and guarantee for solution. Further research should move from a myopic analysis to a farsighted and time-dependent model allowing for investments and possible stochastic implementation. In addition, an extended nodal network would provide more realism into the model completing the actual trade balances between the countries. However, the modeling results prove significant potential of using the complementarity format to describe market players' behavior in a market setting.

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Appendices

Appendix A

Economic Theory

The next sections presents theoretic material needed to understand the economics of modeling energy markets. The theory is presented in short and is meant to be demonstrative, rather than supplementary and therefore not cover all the aspects of each subject. Section A.1 focuses on external demand, its characteristics and how demand can be represented mathematically. Section A.2 provides basic theory on various market structures and explains the underlying assumptions.

A.1 External Demand

There are many ways of interpreting external demand for products or services. There are fixed-price scenarios where the consumers' willingness to pay is the same regardless of the quantity. This representation is not reflecting Average Joes behavior, but can be descriptive for theoretical purposes. A more reputable way of expressing demand is by a downward sloping inverse demand curves. In this situation the willingness to pay decreases as the quantity increases. This property is referred to as the *law of demand*[1] and are valid for nearly all scenarios[2].

A.1.1 The Inverse Demand Curve

Inverse demand curves are characterized by price on the vertical axis and quantity on the horizontal axis. Depending on the complexity of the model the demand curve can have numerous mathematical properties. (Figure A.1.1).

The linear inverse demand curve is commonly expressed the following way:

$$P(Q) = a + b \cdot Q \tag{A.1.1}$$

where $P(Q)$ denotes the price of the good as a function of the quantity demanded Q. The constant a represents the intersection with the vertical axis, i.e. the price where the quantity demanded is zero and the slope b of the demand curve describes how a change in price will affect the quantity demanded.

[1]The law of demand states that the amount demanded of a commodity and its price are inversely related, given that the income of the consumer, prices of the related goods, and tastes and preferences of the consumer remain unchanged.

[2]Exceptions are Veblen and Giffen goods.

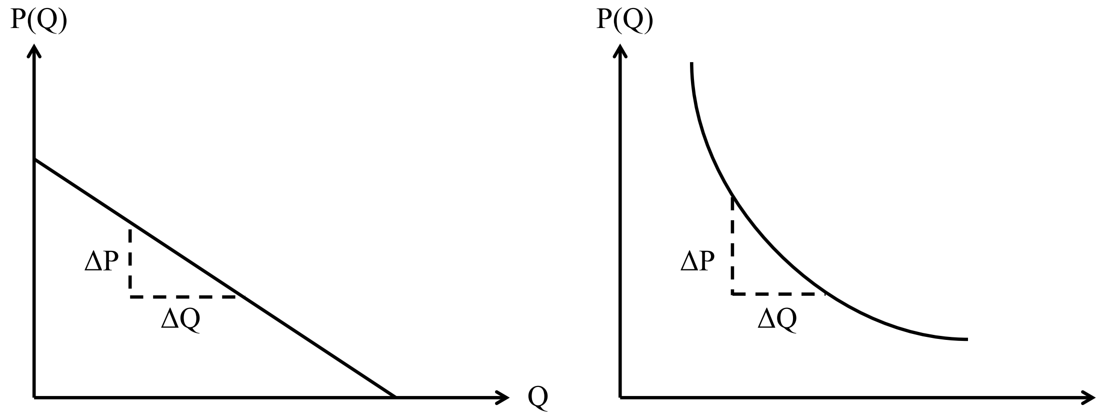


Figure A.1.1: Downward sloping linear inverse demand curve (left) and a quadratic inverse demand curve (right).

A.1.2 Elasticities

Important aspects of an economical analysis are considerations on how a change in one factor will affect another factor. Elasticities are measures of just that and plays an important role in analyzing and comparing different market situations.

Price Elasticity of Demand

The slope $b = \frac{\Delta Q}{\Delta P}$ multiplied with the base at a given quantity and price ($\frac{P}{Q}$) forms the *price elasticity of demand* denoted by ε:

$$\varepsilon = \left(\frac{\Delta Q}{\Delta P}\right) \cdot \left(\frac{P}{Q}\right) \tag{A.1.2}$$

This measure shows the *responsiveness* of the quantity demanded of a good or service to a change in price. As opposed to the slope, the price elasticity of demand (PED) is independent of the units used for P and Q, and therefore a more comparable measure. Figure A.1.2 shows different elasticity levels with the corresponding demand scenario. D_1 forms a *perfectly inelastic* demand curve, a situation where the customers simply has to buy the product regardless of its price (e.g. narcotics). Moreover, D_2 illustrates a *perfectly elastic* demand curve where the customers face a large number of perfect substitutes of the product (e.g. nails, spikes). While the two preceding examples shows extremities, D_3 represents a more realistic demand curve where PED increases when moving from left to right. In the upper left corner we have a *relatively elastic* PED meaning that a percentage change in quantity demanded is greater than that in price. At the middle point we have a demand that is *unit elastic* while the lower right region shows a *relatively inelastic* demand were a percentage change in quantity is less than that in price.

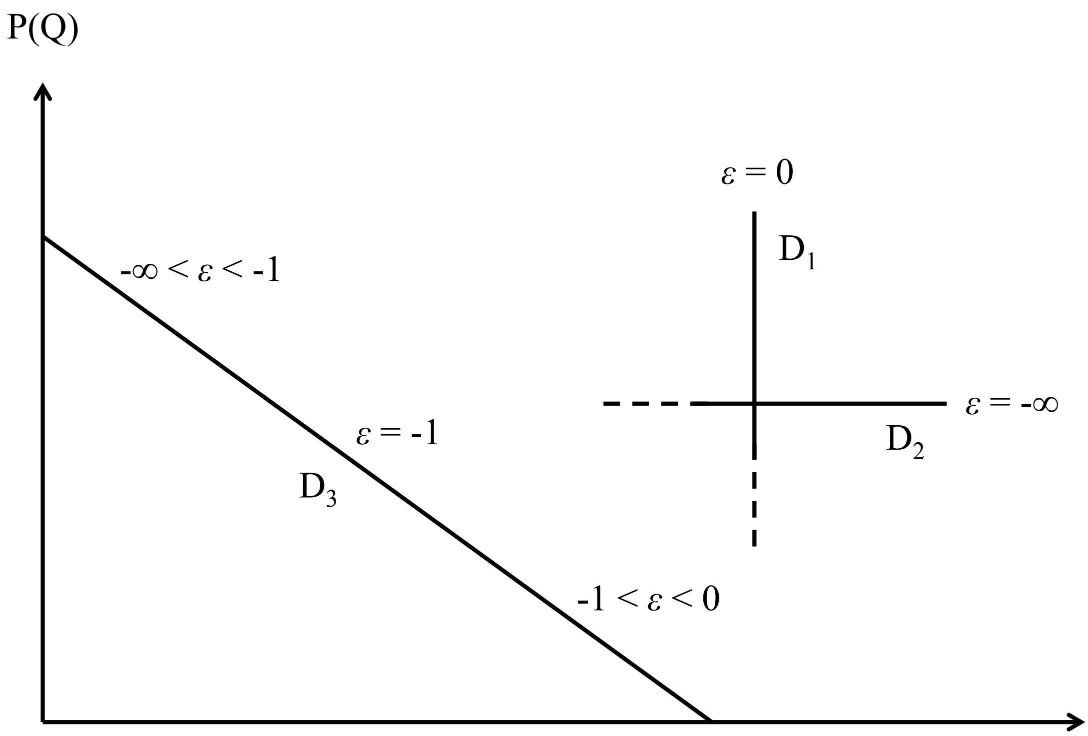


Figure A.1.2: Price Elasticity of Demand for different inverse demand curves.

Estimation of Inverse Demand Curves

Estimating inverse demand curves can be done by applying the relationship between the price elasticity of demand and the inverse demand curves. Assuming a reference point on the demand curve (Q^{REF}, p^{REF}) we have:

$$p^{REF} = a + b \cdot Q^{REF} \tag{A.1.3}$$

where p^{REF} and Q^{REF} are constants.

Recall in the general case, the demand is expressed through the inverse demand curve of (A.1.4).

$$p(Q) = a + b \cdot Q \tag{A.1.4}$$

Using the definition of the price elasticity of demand and the inverse demand curves, the following relationships can be expressed:

$$Q = -\frac{a}{b} + \frac{1}{b} \cdot p, \qquad \varepsilon = \frac{\partial Q}{\partial p} \cdot \frac{p}{Q} = \frac{1}{b} \cdot \frac{p}{Q} \tag{A.1.5}$$

$$b = \frac{p^{REF}}{Q^{REF}} \cdot \frac{1}{\varepsilon}, \qquad a = p^{REF} - b \cdot Q^{REF} \tag{A.1.6}$$

Inserting A.1.6 into A.1.4 gives the following estimation of the inverse demand curve:

$$p = p^{REF} - b \cdot Q^{REF} + \frac{p^{REF}}{Q^{REF}} \cdot \frac{1}{\varepsilon} \cdot Q \tag{A.1.7}$$

A.2 Market Structures

There are several ways individuals and companies can interact with each other, named market structures. Generally there are four different structures, each representing an abstract characterization of a type of real market. Whilst *Perfect Competition* and *Monopoly* form the extremes, *Oligopoly* and *Monopolistic Competition* are competitive scenarios that lie between somewhere in between.

A.2.1 Perfect Competition

In a market described by perfect competition there exist a large number of firms producing a homogenous product, where all market influents are perfectly informed and there are no barriers or entry fees for both seller and buyer. The firms demand curve is perfectly elastic ($\varepsilon = -\infty$) meaning that price is the same regardless of the quantity. Each firm will maximize its profit by setting the price equal to marginal cost. There are no long-term profits (or losses) to obtain, but in the short-run firms may earn profits (or losses). Hence the firms are price takers and do not excess market power.

Perfect competition has the desirable properties of forcing firms to sell at the lowest possible price while producing at the lowest possible costs. It can be argued that perfect competition is a desired social goal because it results in a social welfare maximum. However, many economists criticize the passive interpretation of the agents and point out different actions as advertising, innovation and design that characterize most industries. They substantiate the criticism with historical data showing that many firms earn above-average long-run-profits even in industries with many firms.

A.2.2 Monopolistic Competition

Monopolistic competition and perfect competition share many of the same characteristics; there exists many firms of equal size, entry and exit barriers are not present and the long-term profits are low. However, there are one important difference: the monopolistic competitor produces differentiated rather than homogenous products. The different products are substitutes but because of branding and advertising the monopolistic competitor set his product apart from the competition. Buyers are willing to pay more for a product or service because they believe it is much better than their other choices. A successfully producer will therefore experience a demand curve that becomes more vertical or inelastic.

Monopolistic competitive markets often experience price discrimination, a method to increase economic profits by selling the same good or service at a number of different prices. The greater the price discrimination, the greater the profits because buyers lose some of their consumer surplus. If price discrimination were carried to its logical conclusion, we would have perfect price discrimination and buyers would lose all of their consumer surplus.

The consumer surplus is the benefit to a consumer of buying a good at the equilibrium price. It measures the difference between a buyer's willingness to pay for a good and the price the buyer actually pays. Equivalently the producer's surplus is the benefit to a producer of selling a good at the equilibrium price. In mathematical terms the consumer and producer surplus takes the following form (under the assumption of linear demand curves):

$$CS = \int_{P_{eq}}^{P_{max}} Q(P)dP = \frac{1}{2}Q_{eq}(P_{max} - P_{eq}) \tag{A.2.1}$$

$$PS = \int_{P_{min}}^{P_{eq}} Q(P)dP = \frac{1}{2}Q_{eq}(P_{eq} - P_{min}) \tag{A.2.2}$$

$$SS = CS + PS = \frac{1}{2}Q_{eq}(P_{max} - P_{min}) \tag{A.2.3}$$

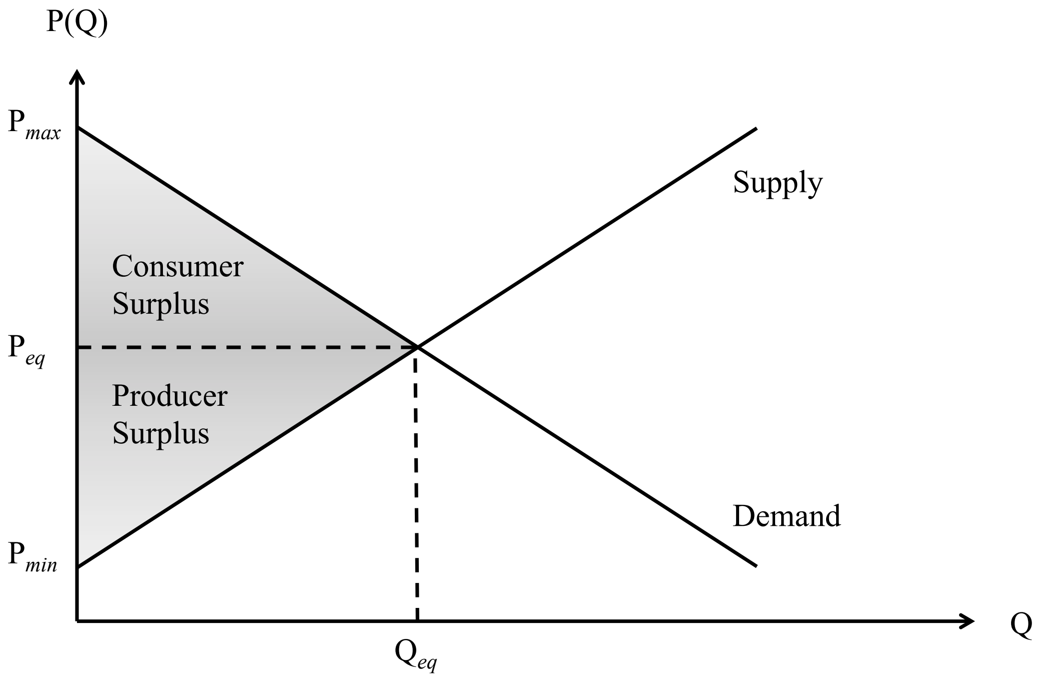


Figure A.2.1: Gains from trade represented by demand and supply curves.

A.2.3 Oligopoly

An oligopoly is a market structure in which there are a relatively small number of firms competing in a market and where some barriers to entry exist. The essential feature of competition in oligopoly markets is that firms recognize their mutual interdependence. As a consequence, firms know that their decisions will affect their rivals and vice versa. This is the essence of strategic thinking: competitors must determine how rivals will respond to their actions. Classical oligopoly theory was first addressed in the early 1800s and only recently, the analysis has been augmented by game theory.

The first models to explicitly recognize the interdependence of decision making in oligopoly situations were those of Cournot [1838], Bertrand [1883] and Von Stackelberg [1934]. In the Cournot model, two firms make simultaneous decisions about how much of a homogeneous good to produce. In the Bertrand model, two firms make simultaneous decisions about what price to set for the homogeneous good. While in the Stackelberg model, one firm moves first, deciding how much to produce, and the other firm responds. A supplementary study of the Cournot model and its characteristics is given in Section 5.3.

A.2.4 Monopoly

You can have total control of the market situation, acting as monopolist in your own market. By definition, a monopolist faces no rivals; hence a monopoly firm is the industry and the monopolist's demand curve is the industry demand curve. There are only one seller, exist no substitutes and even in the long-run the monopolist controls all access to the market. A successful monopoly would face a relatively inelastic demand curve. Hereby, a low coefficient of elasticity is indicative of effective barriers to entry.

From a static perspective, monopoly is socially undesirable because output is restricted. However, it is argued that monopolists are somewhat more inventive and innovative, so that from a dynamic perspective the negative effects of monopoly are reduced or even reversed. The Schumpeterian Hypothesis, first stated in Schumpeter [1992], suggests that firms with market power are more likely to engage in innovative activity or to invest in R&D.

Profit maximizing monopolists will produce output where marginal revenue (always less than price) equals marginal cost. This will typically result in less production and a higher price compared to perfect competition. Whether a monopolist earns profits depends on the position of its cost curves relative to demand.

Appendix B

GAMS Coding

B.1 GAMS Code of Basic Production Model

```
Sets

n            Nodes in the network
             /a, b/

i            Producers in the network
             /g1, g2/

Alias(i,j)
Alias(n,m)
;

Parameters

PRODCAP(i)   Capacity limit of production of producer i
             /p1 50,p2 50/

INT(n)       Interception point of the inverse demand curve in node n
             /a 50, b 50/

SLP(n)       Slope of the inverse demand curve in node n
             /a -1,b -1/

A(i)         Linear cost factor in the cost function of producer i
             /p1 1, p2 1/

B(i)         Quadratic cost factor in the cost function of producer i
             /p1 0.5, p2 0.5/

MP(i,n)      Market power param. of producer i in node n
             /p1.a  1,p1.b 1,p2.a  1,p2.b  1/
;

Positive variables

q(i,n)       Sold quantity of producer i to node n

lambda(i)    Lagrange multiplier of prodconstraint of producer i

p(n)         Price of natural gas in node n

z            Profit function of producer i
;
```

```
Equations

profit              Defines objective function

cap_constr(i)       Capacity limit of production of producer i

price(n)            Price in node n

stat_q(i,n)         First order condition (FOC) of producer i
;

profit..            z
                    =e=
                    sum(i,sum(n,p(n)*q(i,n))
                    -(A(i)*sum(m,q(i,m))
                    - B(i)*sum(m,q(i,m))*sum(m,q(i,m))))
;

cap_constr(i)..     CAP(i)
                    =g=
                    sum(n,q(i,n))
;

stat_q(i,n)..       -p(n)
                    - MP(i,n)*SLP(n)*q(i,n)
                    + A(i)
                    + 2*B(i)*q(i,n)
                    + lambda(i) =g= 0
;

price(n)..          p(n)
                    =e=
                    INT(n)
                    + SLP(n)*sum(j,q(j,n))
;

Model production
                     /stat_q.q, cap_constr.l,price.p/
;

solve production using mcp
;

display q.l, q.m,p.l
;
```

B.2 GAMS Code of MCP Model of the European Energy Markets

```
Sets

n               Markets in the network
                /IG, HG, IE, HE/

i               Nodes in the network
                /NOR, GB, DE/

f               Fuels in the network
                /gas, el/

$ There are four markets in each of the three nodes of NOR, DE and GB
$ IG is Gas market for industry
$ HG is Gas market for households
$ IE is Electricity market for industry
$ HE is Electricity market for households

Alias(n,m)
Alias(i,j,l)
Alias(f,g)
;

$ The following units are in GJ, GW and Euros
$ The values are changed during runs

Parameters
INT(m,i)            Intersection point of IDC in market m in node i
                    /IG.NOR 18.24, HG.NOR 0, IE.NOR 49.62, HE.NOR 82.44,
                     IG.GB 11.26, HG.GB 21.46, IE.GB 52.62, HE.GB 73.38,
                     IG.DE 17.96, HG.DE 23.08, IE.DE 51.16, HE.DE 76.72/

SLP(m,i)            Slope of IDC of market m in node i
                  /IG.NOR -7.3, HG.NOR 0, IE.NOR -2.819, HE.NOR -10.569,
                   IG.GB -0.2, HG.GB -0.259, IE.GB -1.032, HE.GB -2.718,
                   IG.DE -0.205, HG.DE -0.27, IE.DE -0.752, HE.DE -2.413
                      /

A(f,i)              Linear cost factor in the of producer (f,i)
                    /gas.NOR 2.28, gas.GB 2.05, gas.DE 2.57,
                      el.NOR  8.25, el.GB  7.88, el.DE  7.99/

CONV                Conversion factor in converting gas to electricity
                    /0.58/
```

```
CONVCAP(f,i)       Converting capacity of of producer (f,i)
                   /el.NOR  0.09
                    el.GB   44.8
                    el.DE   25.4/


PRODCAP(f,i)       Capacity limit of production of producer (f,i)
                   /gas.NOR 135, gas.GB 94, gas.DE 17,
                    el.NOR 17, el.GB 45, el.DE 74/

TRADECAP(f,i,j)    Capacity limit of trade from producer (f,i)
                   to node j
                   /gas.NOR.NOR      inf
                    gas.NOR.GB       inf
                    gas.NOR.DE       inf
                    gas.GB.NOR       inf
                    gas.GB.GB        inf
                    gas.GB.DE        inf
                    gas.DE.NOR       inf
                    gas.DE.GB        inf
                    gas.DE.DE        inf/

Table

SALESCAP(f,i,n,j) Capacity limit of sales from producer (f,i)
                  to market n in node j

                        IG.NOR  HG.NOR  IE.NOR  HE.NOR  IG.GB   HG.GB
                              IE.GB   HE.GB   IG.DE   HG.DE   IE.DE
                             HE.DE
                  gas.NOR inf     inf     0       0       inf     inf
                         0       0       inf     inf     0       0
                  el.NOR      0       0       inf     inf     0       0
                            inf     inf     0       0       inf     inf
                  gas.GB  inf     inf     0       0       inf     inf
                         0       0       inf     inf     0       0
                  el.GB       0       0       inf     inf     0       0
                            inf     inf     0       0       inf     inf
                  gas.DE  inf     inf     0       0       inf     inf
                         0       0       inf     inf     0       0
                  el.DE       0       0       inf     inf     0       0
                            inf     inf     0       0       inf     inf
;
```

```
Table

ARCCAP(f,i,j)     Capacity limit of commodity f on arc (i,j)
                            NOR    GB       DE
                   gas.NOR  inf    81.5     37.2
                   gas.GB   0      inf      13.4
                   gas.DE   0      36.5     inf
                   el.NOR   inf    0        2.65
                   el.GB    0      inf      0
                   el.DE    2.1    0        inf
;

Table

ARCCOST(f,i,j)    Unit cost of transporting commodity f on arc (i,j)
                         NOR     GB       DE
                   gas.NOR 0       0.24     0.24
                   gas.GB  0       0        0.71
                   gas.DE  0       0.71     0
                   el.NOR  0       0        1.25
                   el.GB   0       0        0
                   el.DE   1.25    0        0
;

Table

MP(f,i,n,j)       Market power param. of producer (f,i)
                  in market n in node j
                         IG.NOR  HG.NOR  IE.NOR  HE.nor  IG.GB   HG.GB
                               IE.GB   HE.GB   IG.DE   HG.DE   IE.DE
                             HE.DE
                   gas.NOR    1    1       1       1       1       1
                            1       1       1       1       1       1
                   el.NOR     1    1       1       1       1       1
                            1       1       1       1       1       1
                   gas.GB     1    1       1       1       1       1
                            1       1       1       1       1       1
                   el.GB      1    1       1       1       1       1
                            1       1       1       1       1       1
                   gas.DE     1    1       1       1       1       1
                            1       1       1       1       1       1
                   el.DE      1    1       1       1       1       1
                            1       1       1       1       1       1
;


Variables
```

```
bp(i)                       Border price of gas in node i
phi(f,i)                    Dual of of flow conservation constraint of
                            producer (f, i)
tfee(f,i,j)                 Dual of the floweq. of f on arc (i,j)
;

Positive variables

q_sales(f,i,n,j)            Sold quantity of producer (f, i)
                            to market n in node j
q_prod(f,i)                 Produced quantity of producer (f, i)
q_trade(f,i,j)              Traded quantity from producer (f, i)
                            to node j
x_trade(f,i)                Received trade of producer (f, i)
q_flow(f,i,j)               Physical flow of commodity f on arc (i,j)
rho(f,i)                    Dual of production constraint
                            of producer (f, i)
kappa(f,i,j)                Dual of trading constraint between
                            producer (f, i) and node j
omega(f,i,j)                Dual of arclimit constraint of
                            commodity f on arc (i, j)
lambda(f,i,n,j)             Dual of sales constraint
                            of producer (f, i)
tau(f,i)                    Dual of capacity converting constraint
                            of producer (f, i)
zPR(f,i)                    Profit function of producer (f, i)
zNO(f,i,j)                  Profit of the Network Operator fij
p(m,j)                      Price of energy of market n in node i
;

Equations

profit(f,i)                 Defines objective function
                            of the producer (f, i)
prod_const(f,i)             Production capacity constraint
price(n,i)                  Price in market n in node j
sellimit(f,i,n,j)           Sales constraint between producer (f, i)
                            and market (n, j)
arclimit(f,i,j)             Arc constraint of fuel f on arc (i,j)
tradelimit(f,i,j)           Tradelimit between (f, i) and j
stat_qtrade(f,i,j)          First order condition (FOC) of q_trade
stat_xtrade(f,i)            First order condition (FOC) of x_trade
stat_sales(f,i,n,j)         First order condition (FOC) of q_sales
stat_prod(f,i)              First order condition (FOC) of q_prod
mb(f,i)                     flow out = flow in - mass balance
mc(i)                       Market clearing condition
FOC_NO(f,i,j)               First order condition (FOC) of Network Op
floweq(f,i,j)               Floweq of f on arc (i, j)
conv_constr(f,i)            Converting capacity constraint
```

```
NOProfit(f,i,j)           Profit of the network operator
;

profit(f,i)..             zPR(f,i) =e=
                            sum(j,q_trade(f,i,j)*bp(j))
                          + sum(n,sum(j,q_sales(f,i,n,j)*(p(n,j)
                          - tfee(f,i,j))))
                          - x_trade(f,i)*bp(i)
                          - sum(j,q_trade(f,i,j)*tfee(f,i,j))
                          - A(f,i)*q_prod(f,i)
;

prod_const(f,i)..         PRODCAP(f,i) - q_prod(f,i) =g= 0
;

sellimit(f,i,n,j)..       SALESCAP(f,i,n,j) - q_sales(f,i,n,j) =g= 0
;

tradelimit(f,i,j)..       TRADECAP(f,i,j) - q_trade(f,i,j) =g= 0
;

conv_constr(f,i)..        CONVCAP(f,i) - x_trade(f,i) =g= 0
;

mb(f,i)..                 sum(n,sum(j,q_sales(f,i,n,j)))
                          + sum(j,q_trade(f,i,j))
                          - q_prod(f,i)
                          - CONV*x_trade(f,i)
                          =e=  0
;

stat_prod(f,i)..          A(f,i)
                          + rho(f,i)
                          - phi(f,i)
                          =g= 0
;

stat_sales(f,i,n,j)..     - p(n,j)
                          - MP(f,i,n,j)*SLP(n,j)*q_sales(f,i,n,j)
                          + tfee(f,i,j)
                          + phi(f,i)
                          + lambda(f,i,n,j)
                          =g= 0
;
```

```
stat_qtrade(f,i,j)..        - bp(j)
                            + tfee(f,i,j)
                            + phi(f,i)
                            + kappa(f,i,j)
                            =g= 0
;

stat_xtrade(f,i)..          bp(i)
                            - CONV*phi(f,i)
                            + tau(f,i)
                            =g= 0
;


price(m,i)..                p(m,i)=e=
                            INT(m,i)
                            + SLP(m,i)*sum(f,sum(j,q_sales(f,j,m,i)))
;

mc(j)..                     sum(f,sum(i,q_trade(f,i,j)))
                            - sum(g,x_trade(g,j))
                            =e= 0
;

NOProfit(f,i,j)..           zNO(f,i,j)=e=
                            (tfee(f,i,j)
                            - ARCCOST(f,i,j))*(q_trade(f,i,j)
                            + sum(n,q_sales(f,i,n,j)))
;

arclimit(f,i,j)..           ARCCAP(f,i,j)
                            - q_flow(f,i,j)
                            =g= 0
;

FOC_NO(f,i,j)..             -tfee(f,i,j)
                            + ARCCOST(f,i,j)
                            + omega(f,i,j)
                            =g= 0
;

floweq(f,i,j)..             q_flow(f,i,j)
                            - q_trade(f,i,j)
                            - sum(n,q_sales(f,i,n,j))
                            =e= 0
;
```

```
model stratmcp
                  /stat_qtrade.q_trade, stat_sales.q_sales,
                   stat_xtrade.x_trade, stat_prod.q_prod,
                   mc.bp, mb.phi, prod_const.rho,
                   tradelimit.kappa, sellimit.lambda,
                   arclimit.omega, price.p,
                   floweq.tfee,FOC_NO.q_flow,profit.zPR,
                   conv_constr.tau,NOprofit.zNO/;

solve stratmcp using mcp
;

display
q_trade.l, q_trade.m, x_trade.l, x_trade.m,
q_sales.l,q_sales.m,q_prod.l,q_prod.m,
bp.l, mc.l, p.l,mb.l,tfee.l,zPR.l,lambda.l,phi.l,zno.l;
```

Appendix C

Results of Sensitivity Analysis

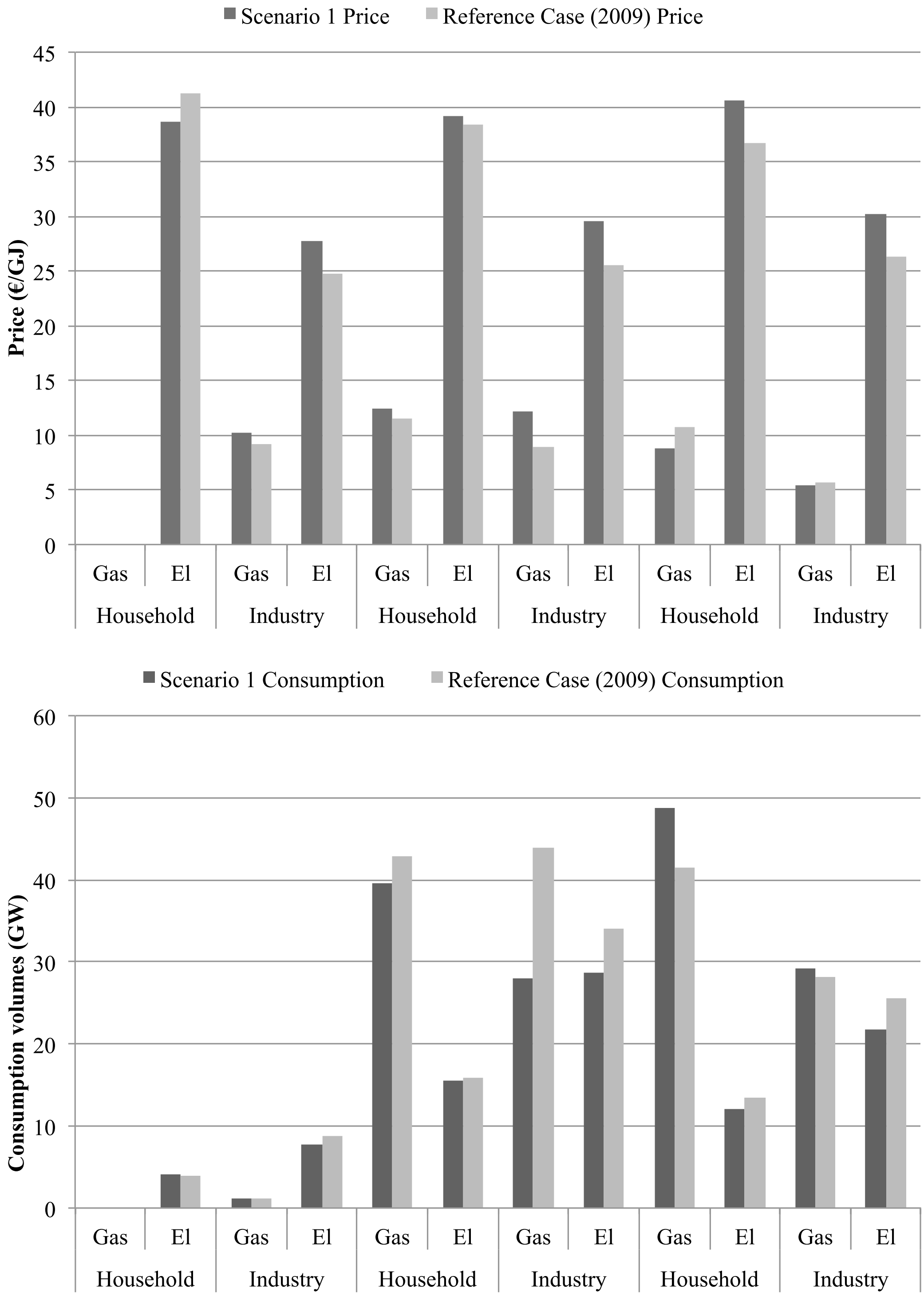


Figure C.0.1: Prices (€/GJ) (top) and consumption volumes (GW) (bottom) for Scenario 1 in each of the markets compared to 2009 values (RC).

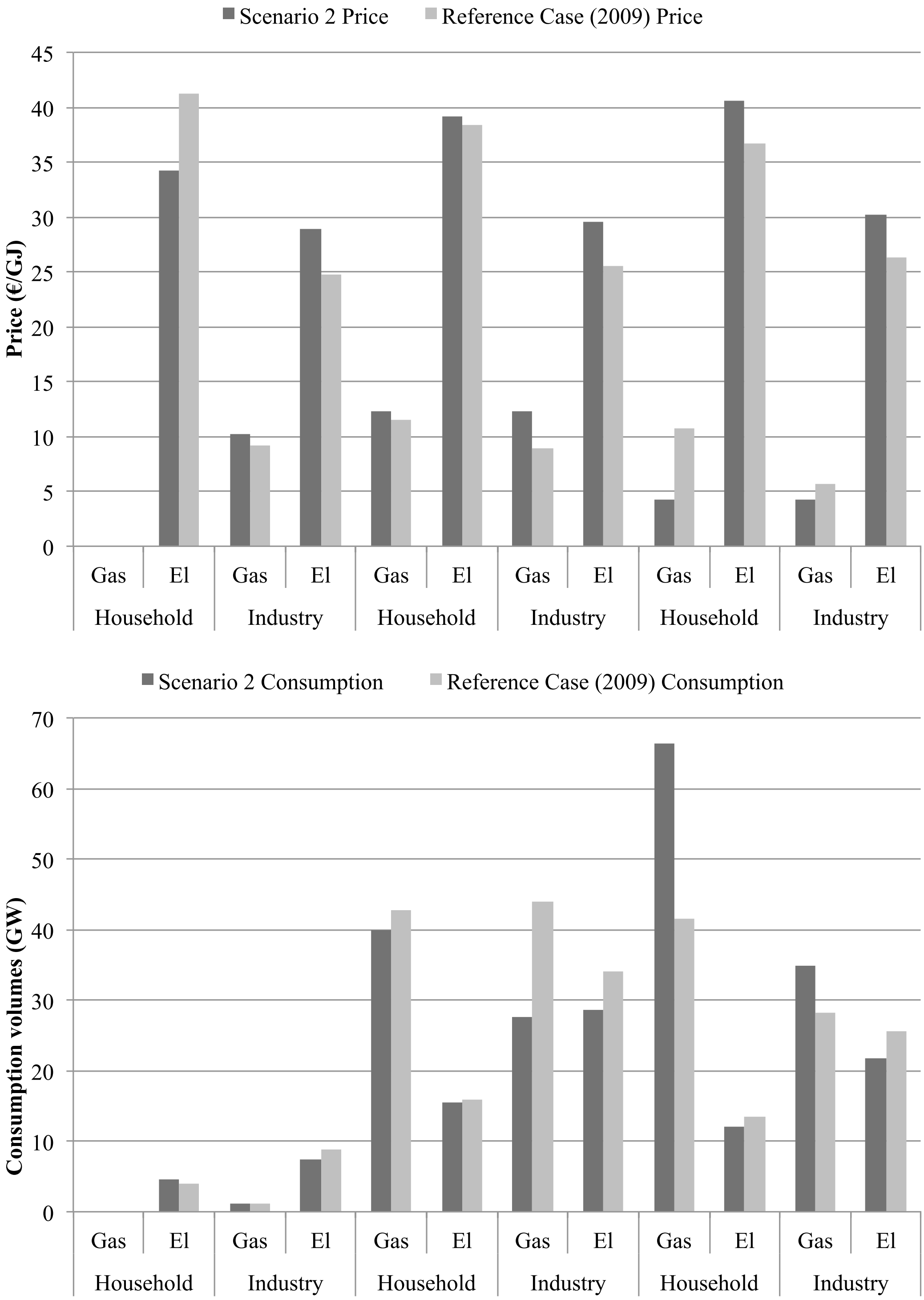


Figure C.0.2: Prices (€/GJ) (top) and consumption volumes (GW) (bottom) for Scenario 2 in each of the markets compared to 2009 values (RC).

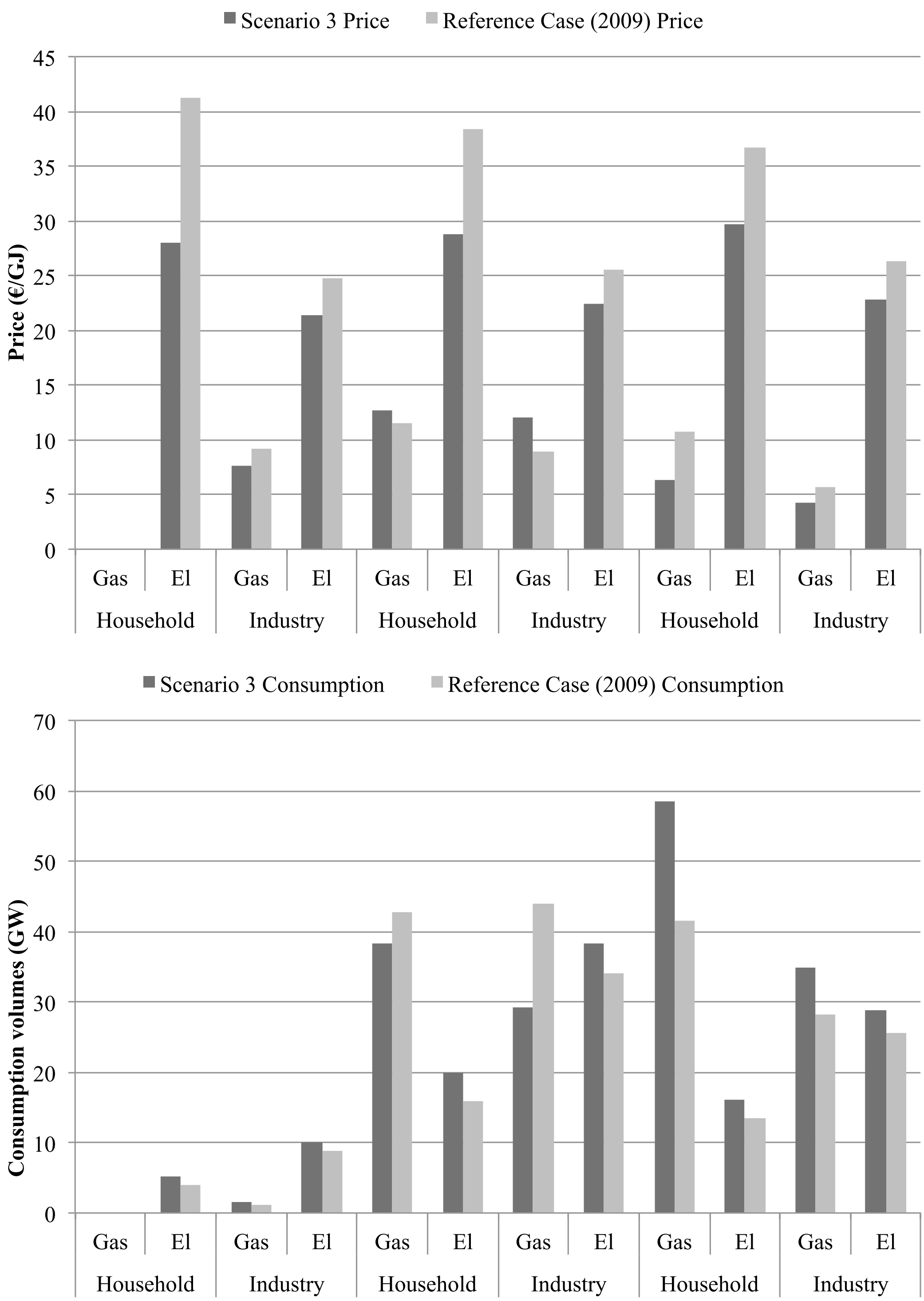
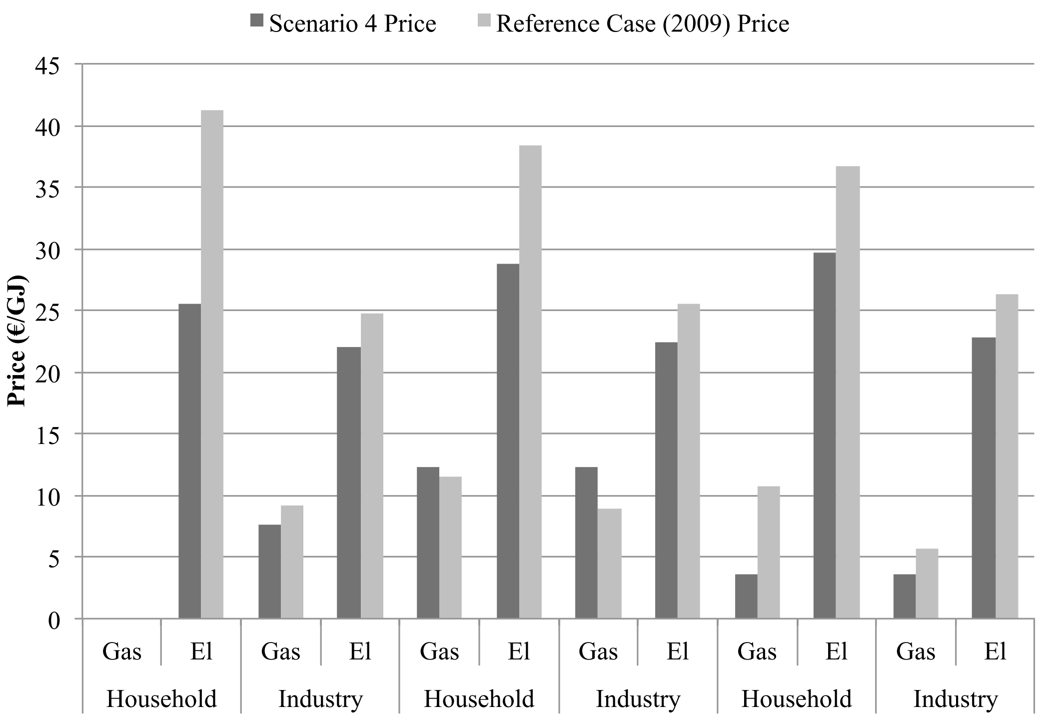


Figure C.0.3: Prices (€/GJ) (top) and consumption volumes (GW) (bottom) for Scenario 3 in each of the markets compared to 2009 values (RC).



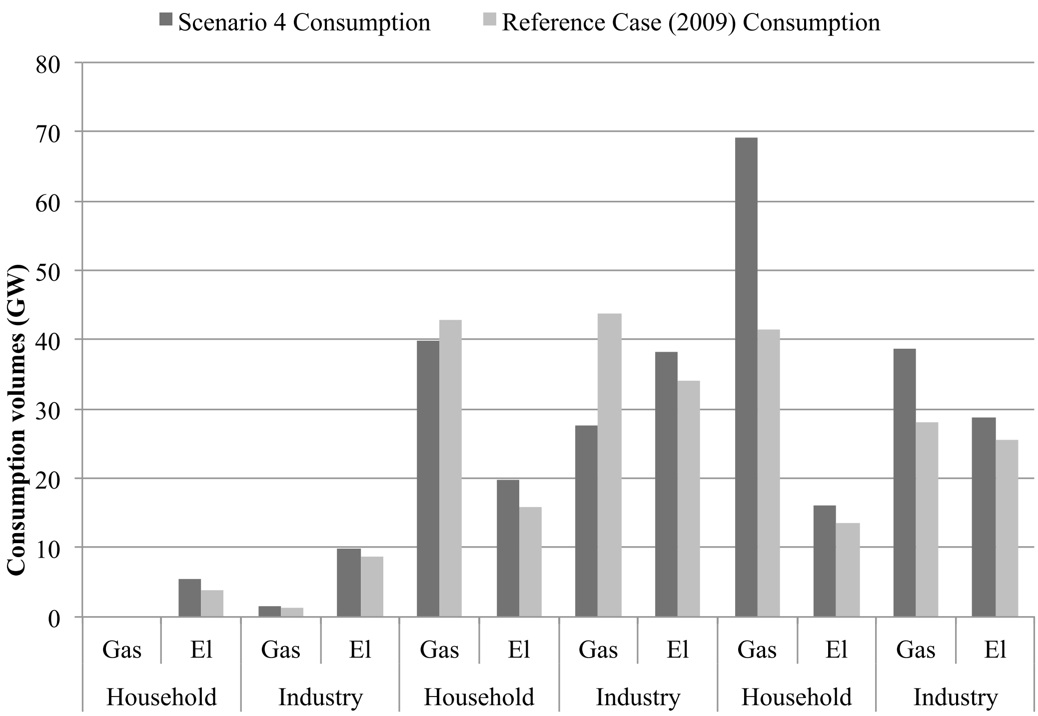


Figure C.0.4: Prices (€/GJ) (top) and consumption volumes (GW) (bottom) for Scenario 4 in each of the markets compared to 2009 values (RC).

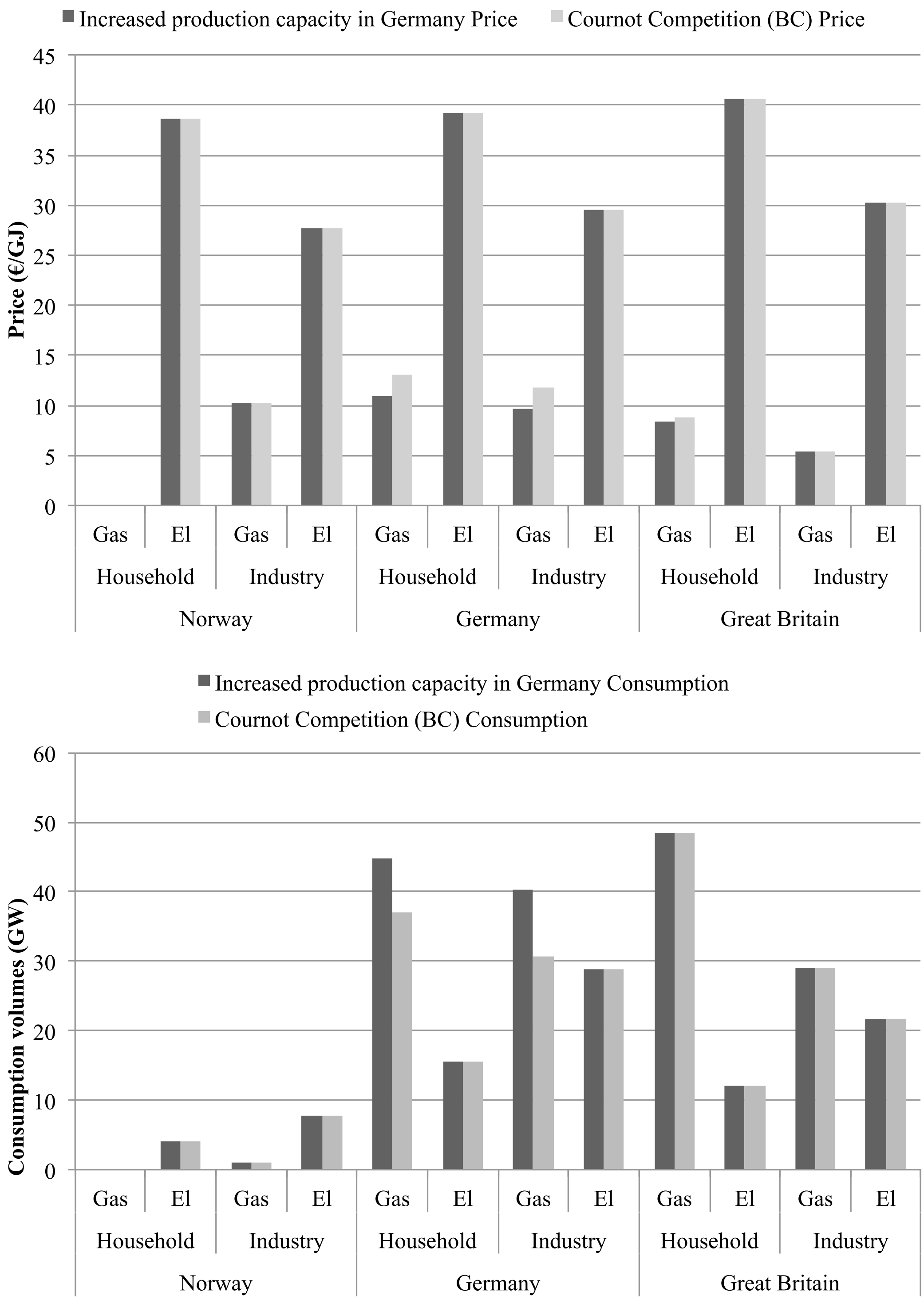


Figure C.0.5: Prices (€/GJ) (top) and consumption volumes (GW) (bottom) for increased production capacity in Germany.

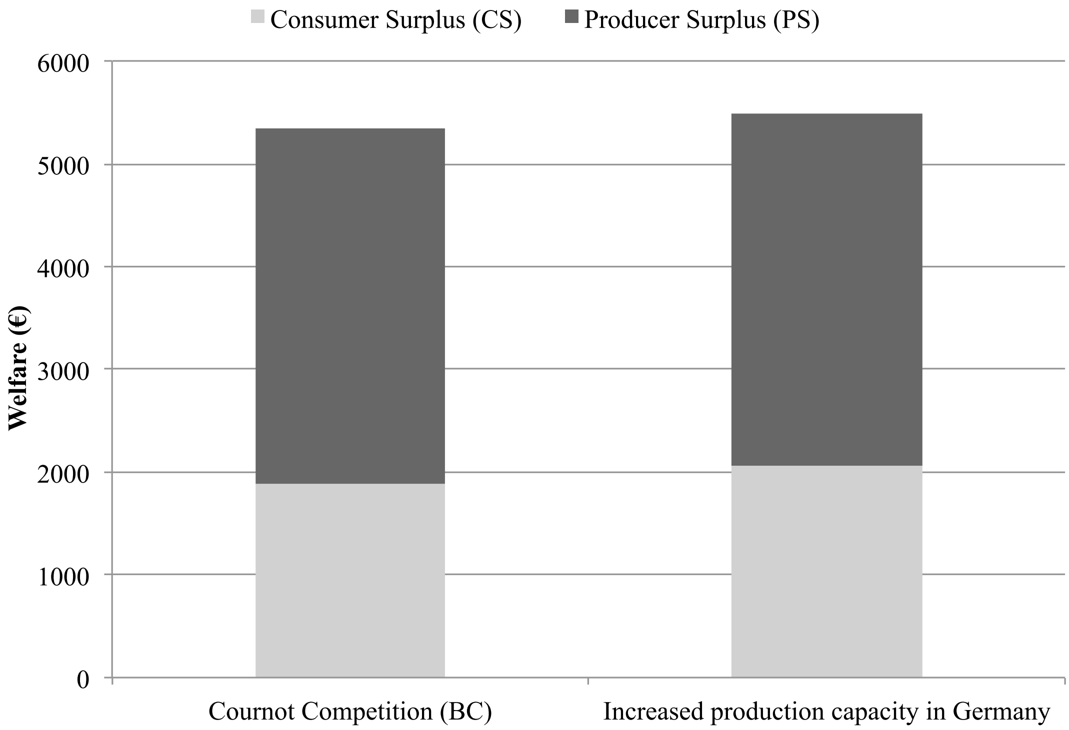


Figure C.0.6: Welfare results (€) for increased production capacity in Germany.

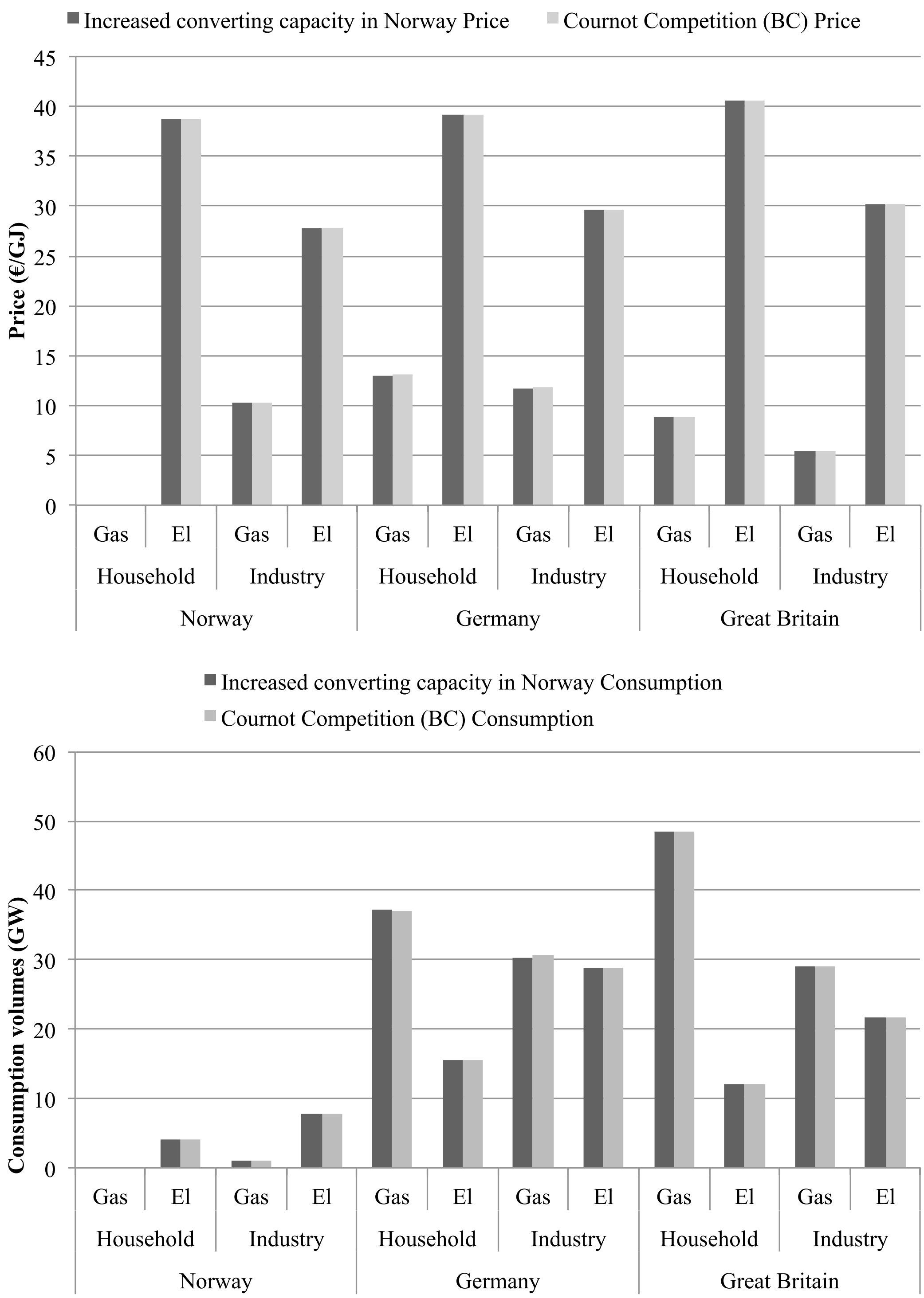


Figure C.0.7: Prices (€/GJ) (top) and consumption volumes (GW) (bottom) for increased conversion capacity in Norway.

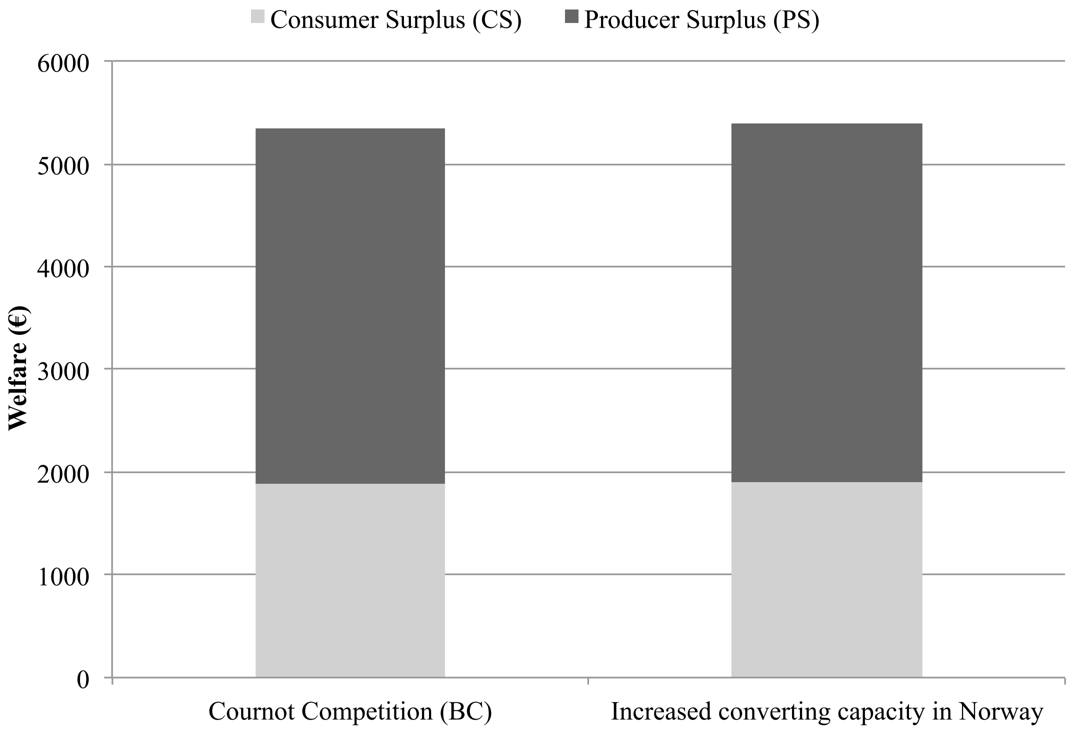


Figure C.0.8: Welfare results (€) for increased conversion capacity in Norway.

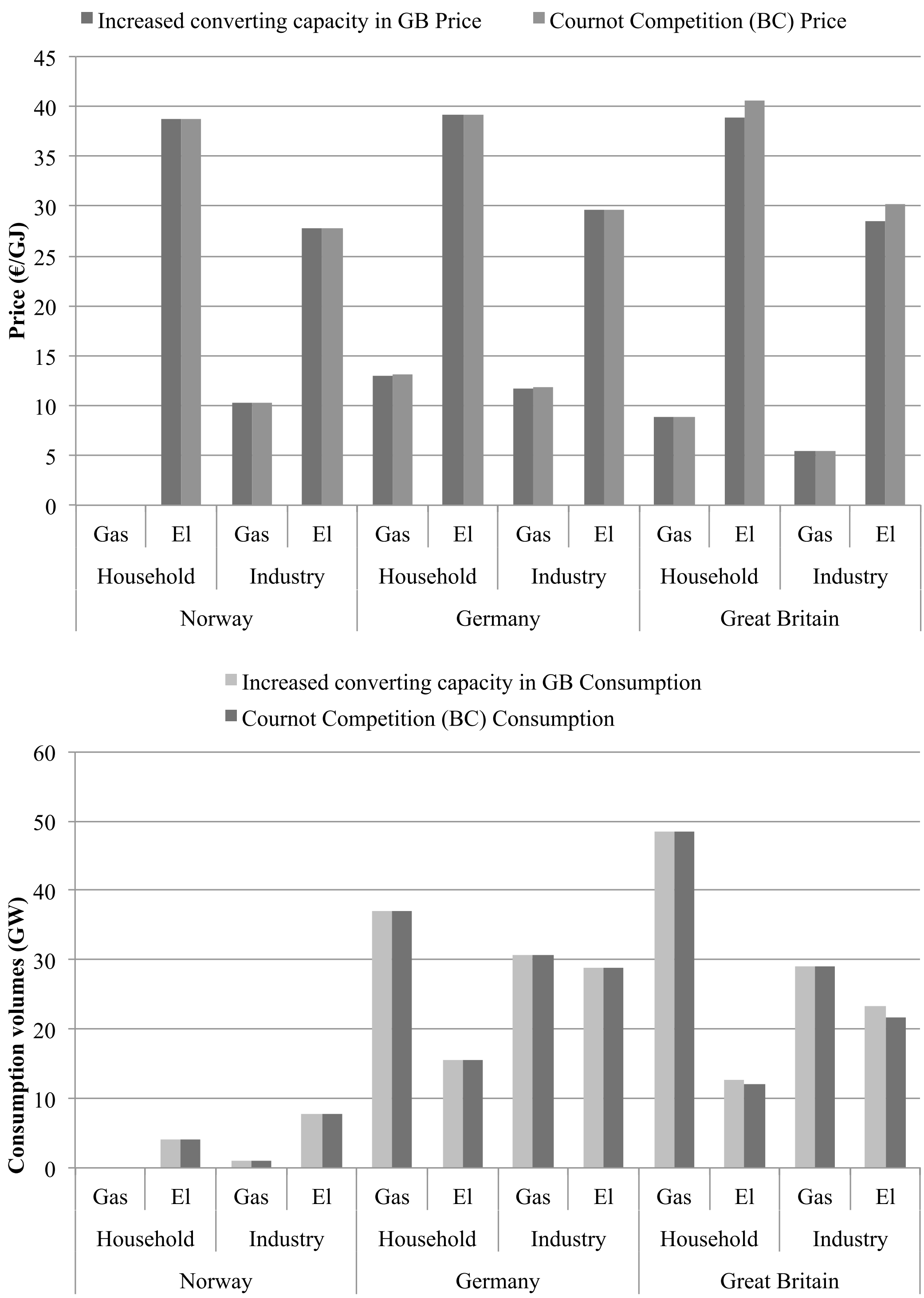


Figure C.0.9: Prices (€/GJ) (top) and consumption volumes (GW) (bottom) for increased conversion capacity in Great Britain.

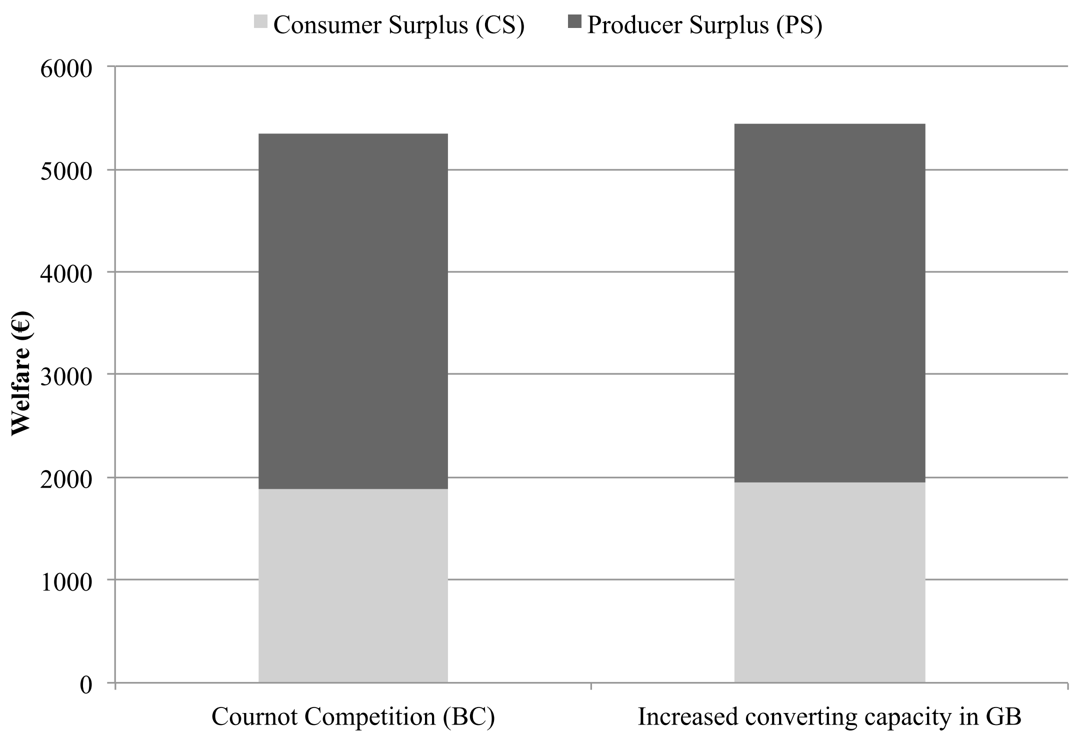


Figure C.0.10: Welfare results (€) for increased conversion capacity in Great Britain.

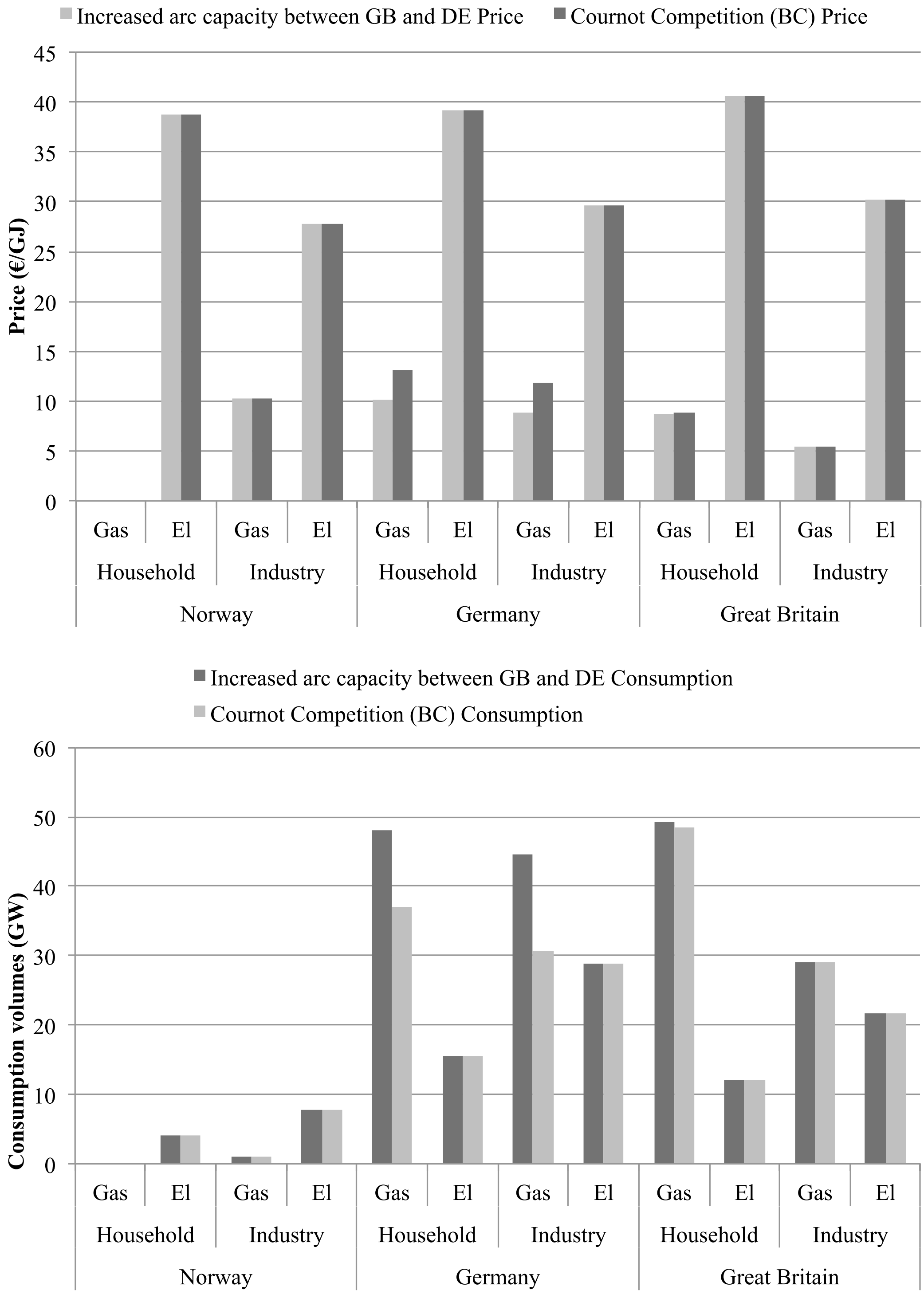


Figure C.0.11: Prices (€/GJ) (top) and consumption volumes (GW) (bottom) for increased arc capacity between Great Britain and Germany.

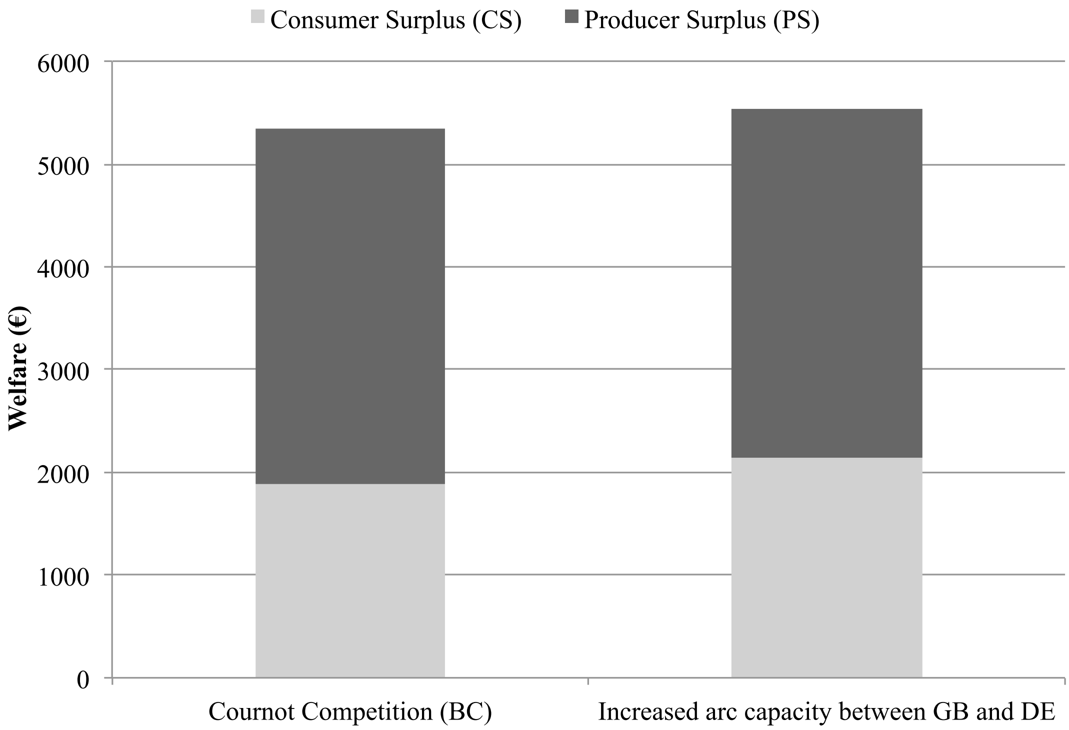


Figure C.0.12: Welfare results (€) for increased arc capacity between Great Britain and Germany.