

**Simulating Fretting Fatigue using the Burridge-Knopoff-Pad model.**

Master Project

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### **Abstract**

In this project a new approach to fretting fatigue has been started based on the Burridge-Knopoff-Pad model implemented by H. Ferre and improved K. Standnes. The aim of this project is to expand the current model and to start experimenting with changing parameters related to the drivers of the system affects fatigue. The simulation is working well, although the material used is highly theoretical.

# 1 Introduction

## 1.1 Goal

This project is aimed at simulating fretting fatigue and understanding what parameters impact fatigue life, with a focus on the parameters of an external driver. Using an already established model created for studying theoretical seismicity by R. Burridge and L. Knopoff, incremented and implemented by H. Ferre and improved by K. Standnes.

## 1.2 Motivation and Background

Nearly all materials used today are prone to fatigue some more than others, this makes understanding fatigue important for our society, as long as humans have been using materials there should always have been fatigue related problems. However it's only lately that the structures and vehicles we've built have been fast enough to impose lethal danger if something were to fail. Aerospace is a prime example of why fatigue is important, as if something fails while you are in the air, it will likely end in catastrophe. What makes fatigue withing aerospace important is that efficiency, being "green" and sustainable is hard to combine when you don't want fatigue. Making parts bigger solely to avoid fatigue would jeopardise the efficiency, while making the parts light enough would make the risk of fatigue too great. There has to be a balance between the two, and understanding fatigue is of utmost importance to achieve this.

## 1.3 Approach

Fretting fatigue is a type of fatigue occurring when two contacting surfaces are moving relative to each other. The BK-model was published in 1967 and consisted of two surfaces making "contact" through a lot of blocks and springs. Herman Ferre used and altered this model to include a pad for simulating brake squeal, Karsten Standnes made a lot of UX improvements to this code as part of his project and master thesis and is where this thesis starts. The approach used in this thesis is to interpret and modify the existing code to support the features needed for simulating fatigue. This includes altering the movement of the driver from a linear ramp to a cyclical behaviour, having variable contact friction. And since we are introducing a cyclical load damping should be implemented to reduce the chance of having standing waves. When this has been achieved a simple parameter study of the driving forces will be done. Since the C++ program only saves the positions of the blocks a separate script has to be created for interpreting the data and showing it in a nice way, this was done in MatLab.

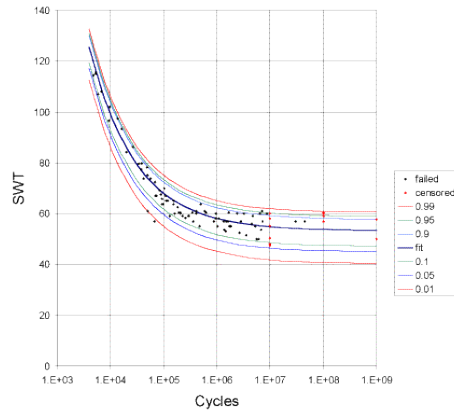


Figure 1: Example S-N diagram, this particular one is for Ti-6Al-4V.

## 2 Background and Literature

### 2.1 Fatigue

Fatigue is a failure mode that occurs due to cyclical or varying loads over long periods of time. These cyclical loads will slightly alter the microstructure with each cycle and over time cracks will form, which creates stress concentrations around the forming crack increasing the speed of the crack growth until failure occurs. The reason this failure mode is especially dangerous is because it can occur during normal operation. This is because of the loads being substantially lower than the strength of the material.

In fatigue analysis an ideal cyclic load can be described as a sinus function with a given stress amplitude  $\sigma_a$  and a mean stress  $\sigma_m$ . At this point a lot of testing is required as the model is phenomenological without a complete physical description. After testing a lot of samples for a metal or alloy with predetermined dimensions and surface roughness an S-N diagram is created, showing the correlation between  $\sigma_a$  and the number of cycles before failure  $N$ .

Since fatigue analysis is a completely empirical way of estimating fatigue life the only way to obtain these S-N diagrams is by testing multiple samples of a given material at set cyclical loads until failure, noting this down and applying statistics to determine the actual values. Usually there are enough samples done at each load that creating a "Bell Curve" can be done, resulting in a S-N diagram looking like the example shown in fig. 1. The values obtained this way is only valid for the material tested, and since some of these tests can run beyond  $10^6$  cycles, for example setting the frequency to 1 cycle/s this one test would run for 11 days nonstop, this makes fatigue testing expensive and doing estimates digitally can be of great value.

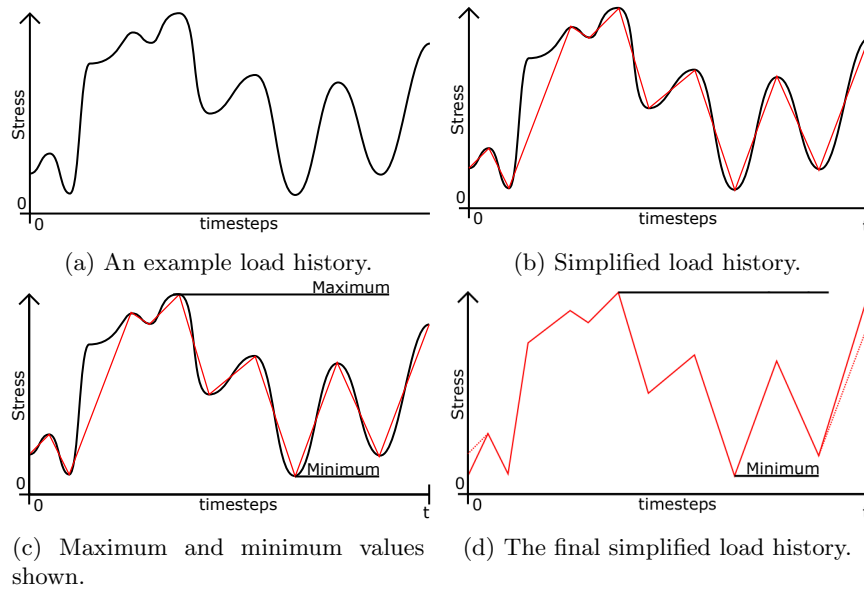


Figure 2: Figures showing how simplifying the load history is done.

## 2.2 Rainflow analysis

Since load histories in the real world rarely take the form the ideal cyclic load described over some sort of conversion is required. A common way of doing this is by performing a "Rainflow Analysis". This type of analysis simplifies a load history into "microcycles" or "subcycles" which are ideal cyclic loads and by summing the up these an estimate for fatigue life is obtained.

The Rainflow Analysis process starts by extracting "Peaks and Valleys" from the load history, the initial load history and the steps are shown in figs 2a through 2d. Once all the "Peaks and Valleys" have been found one can determine a minimum and maximum value for the load history, then the simplified versions first and last point is set to either of these, as can be depicted in fig. 2d.

Once the simplified load history have been obtained the next step is taking four consecutive points and determining if these points can represent a microcyclical load. This occurs if the load at point 2 and 3 are within the boundaries created by point 1 and 4. The next step is to determine the mean value and amplitude for this microcyclical load. The mean value is defined as the average stress of point 2 and 3 while the amplitude is the absolute value of the difference between point 2 and 3 divided by two. These values are now added to a list of microcycles and the points 2 and 3 are removed from the simplified load, simplifying it further. This process is repeated until there are two points left. When this iterative approach is completed and the list is done, a damage calculation can be done by applying a correction for the mean stress, f.ex. Hempel-Morrow correction as shown in eq. 1, where  $\sigma_{ar}$  represents the amplitude after correc-



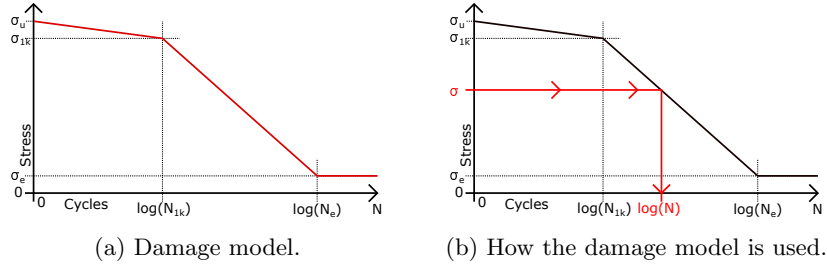


Figure 3: Figures showing the damage models implemented into the rainflow analysis.

Variable	Value
$\sigma_u$	0.0001
$\sigma_{1k}$	$0.9 * \sigma_u$
$\sigma_e$	$0.1 * \sigma_u$
$\log(N_{1k})$	3
$\log(N_e)$	6

Table 1: Table showing the values used in the damage calculation.

tion,  $\sigma_a$  is the current amplitude,  $\sigma_m$  is the mean stress and  $\sigma_u$  is the ultimate tensile stress of the material.

$$\sigma_{ar} = \frac{\sigma_a}{1 - \frac{|\sigma_m|}{\sigma_u}} \quad (1)$$

$$D = \frac{1}{N} \quad (2)$$

The last step of the algorithm for calculating fatigue is summing up the damage, this is done a simple theoretical damage shown in fig. 3a. The values used in the damage calculation are shown in table 1. These values are commonly used with the exception of  $\sigma_u$  which is a material parameter. During my project thesis a lot of time was spent on determining a value for  $\sigma_u$  and in the end a value of 0.0001 was chosen. The last step is adding up the damage of all the microcycles, eq. 2 shows the formula for the damage of one of the microcycles. Finally do a summation of all these partial damage values  $D$  and this is the fatigue of your stress history.

### 2.3 Burridge-Knopoff-Pad

The Burridge-Knopoff-Pad model is based on the simpler Burridge-Knopoff model created by R.Burridge and L.Knopoff which was created as a theoretical model of seismicity. The original BK model is one-dimensional consisting of a set number of blocks connected by elastic springs to each other, these blocks

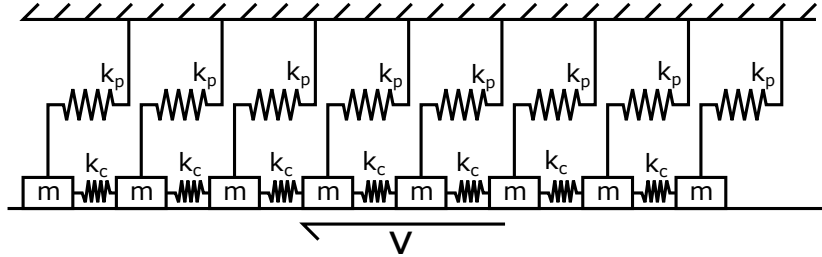


Figure 4: The Burridge-Knopoff model as was introduced originally.

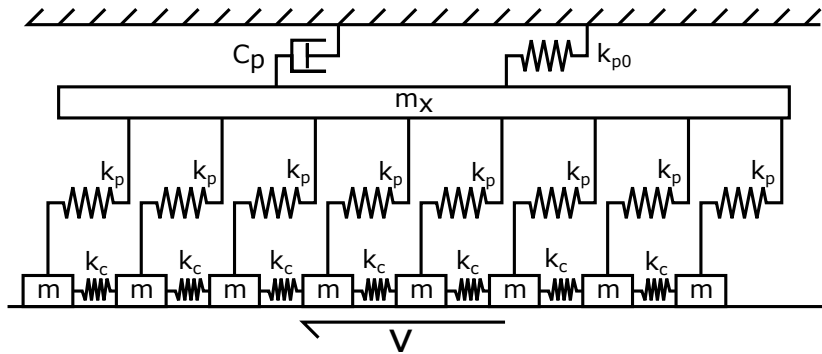


Figure 5: The Burridge-Knopoff Pad as modified by H. Ferre.

represent the surface interfacing with the moving surface. They are also connected to a fixed surface through another spring, as seen in fig. 4. The model then sets the lower surface to move parallel to the fixed surface at a given speed.

The Burridge-Knopoff-Pad was an expansion of the original model, now including a pad with a given mass between the blocks and fixed surface. This new model introduced by H. Ferre can be seen in fig. 5. These models work on a principle of "waves" of blocks slipping across the surface and then sticking to the moving surface again. Tension builds up over time as driver moves and at some point one of the blocks start to slip, starting a chain reaction moving through the entire system, until all blocks are at rest again and tension continues to build. It's an example of a simple yet complicated system, since understanding the governing principles can be considered easy. While actually solving the equations require some computational power as a lot of simple calculations have to be done.

The differences between the models are small as stated by H. Ferre, and they give the same results with a few anomalies where the BK-Pad model doesn't have a drop in friction.

## 2.4 The Preliminary Code

The code as given to me works by having an input file, the "parameters.yaml" file and an executable, the program itself. The input file contains all the important parameters and allows changing them without having to recompile the code, which saves a lot of time and making the code easier to use. Once the program starts it parses the input file and sets all the parameters, creating the necessary matrices and vectors needed to run the simulation. It then uses an iterative approach with timesteps using a 2nd order Runge-Kutta scheme. Lastly it saves the files as .csv files which are easily parseable by other programs. These timesteps are small increments in time specified in the input file, a good tip is to use the "save\_interval\_dt" function to save storage space.

## 2.5 Contact Friction

In this simulation it is assumed that Hertzian Contact theory applies and that this is the reason for fretting fatigue. Hertzian contact theory has some assumptions, most notably that the strains are small and within the elastic limit and the surfaces are continuous and non-conforming. This is fortunate as the same conditions foster fatigue. This type of friction contact can be modelled as a function of the radius  $r$  if the surfaces contacting are spherical. As the model used for simulating is one-dimensional the function  $f(r)$  describing the contact friction reduces down to the simpler function  $f(x)$ . The function  $f(x)$  is trying to mimic is shown in fig. 6. The current function governing friction can be seen in eq. 3. A simple yet effective way of changing the behaviour of the friction in the system is to create a separate function and multiply this with the already existing function for  $\phi$ ; eq. 3. This simple method works well as the friction is a function of the normal load on the contact point. and by multiplying the function of the normal load and friction; the Hertzian function. With  $\Phi$ , which is connected to shear forces. The result of this total friction should be able to model this.

$$\phi(y) = \begin{cases} F_0[-1, 1] & y = 0 \\ F_0 \frac{1-\sigma}{1+\frac{|y|}{1-\sigma}} & y \neq 0 \end{cases} \quad (3)$$

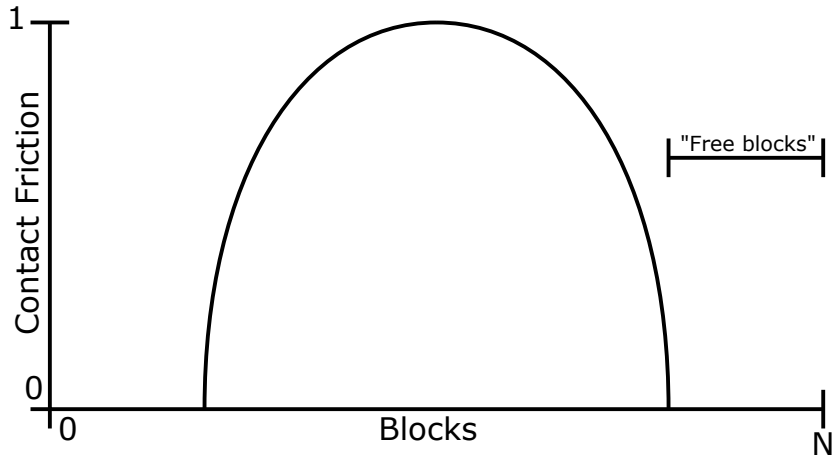


Figure 6: An example Hertzian contact friction.

### 3 Implementation into the existing code

#### 3.1 Varying Friction across the surface

To allow for Varying friction we need to be able to assign a scaling factor to each block individually, this scaling factor is then multiplied by the already existing function for friction "phi", so we have a superposition of the two functions. This is a simple way of changing the friction of individual blocks, without rewriting the entire existing code. If you set a blocks individual friction to 0 you have a free block, which means it will only be driven by the pad and adjacent blocks.

A new function has been created; "CalculateFrettingFrictionFactors". It returns a vector of values determined by the input parameter "friction\_profile" and "free\_blocks", as of now the profile have 2 options and can easily be expanded further if required. The two profiles are "circle" and "parabola". The parabola is the function  $-x^2 + 1 = 0$ , where x runs from  $-1$  to  $1$ ; circle has the function  $x^2 + y^2 = 1$  where x still takes values from  $-1$  to  $1$ . There is also a "DEBUG" option which will return a vector consisting only of ones, mathematically negating the newly implemented function. The free.blocks parameter is the total amount of blocks not in contact with the driver, this is a whole (and preferably even) number.

#### 3.2 Different Waveforms Representing movement

As the current driver behaviour is limited to a linear ramp up ending with a constant velocity the conditions for we lack the reciprocating motion that is so central in fatigue. To avoid changing the existing functioning code it was decided to implement different options for driving the simulation, this new driver would have a cyclical behaviour if needed take on different waveforms. Examples of

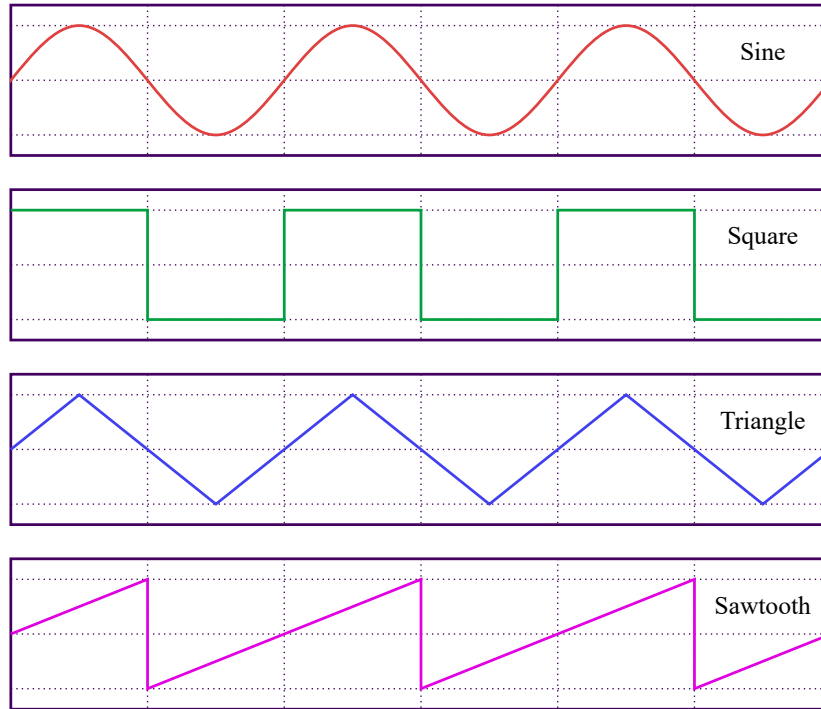


Figure 7: Examples of different Waveforms, Y-axis represents amplitude, or in this case driver speed, the X-axis are the timesteps.

such waveforms can be seen in fig. 7. These waveforms would represent the horizontal velocity of the driver, as this was the easiest conversion from the existing code. To do this there was created a function "sliderSpeedWaveform" that takes in the current timestep and returns the speed of the current driver speed. The remaining parameters such as what waveform, the amplitude, mean value and shift are all set in the parameters.yaml file. This allows for an easy way to change and interact with the simulation without having to change the actual code.

### 3.3 Damping

Damping to the system was introduced by modifying and adding extra elements into the already existing function "function\_u" where the forces on each block is calculated, what forces that were included in the original code and the new ones added for damping can be seen in list below. The added contributions are in bold. Worth noting is that all of these contribution can be toggled individually using the DEBUG parameters in the parameters.yaml file. There are also options for setting the values of the two different dampers in the same file.

- Contribution from neighbouring springs.
- Contribution from stationary springs.
- Contribution from friction.
- **Contribution from Block-Block dampers.**
- **Contribution from Block-Pad dampers.**

### 3.4 New Parameters

The new parameters are shown in table. 2.

Parameter name	Explanation
friction_profile	Can either be "circle" or "parabola".
free_blocks	Number of free blocks, distributed evenly.
damp_coeff_BB	Damping coefficient between neighbouring blocks.
damp_coeff_PB	Damping coefficient between block and pad.
use_waveform_movement	Boolean value.
wave_amplitude	Amplitude for the waveform movement.
wave_mean	Mean value for the waveform movement.
wave_period	Period for the waveform movement.
wave_shift	Time-shift for the waveform movement.
waveform	Used for choosing what waveform is wanted.
use_amplitude_as_max_val_for_start	Boolean used to limit the random starting positions of blocks.

Table 2: Table over the new parameters with a short explanation and what they represent.

### 3.5 Eigenvalues

It was discovered during simulations that different frequencies gave interesting results and knowing the eigenfrequency of the system might aid in explaining some of this behaviour. It was therefore decided to implement the functionality of finding the eigenvalues of the system using the inbuilt eigenvalue solver in Armadillo. To use this solver 2 matrices have to be created; one of these representing the masses of the system, and the other one representing the connecting springs, it is assumed that the dampers contributions to the overall stiffness in the system are small, and they are therefore neglected. The matrix representing the masses of the system can be seen in eq. 4, and since the mass of all the blocks currently are the same, they can be represented by a common value  $m$ , the pads mass is represented by  $M$ . The stiffness matrix have been constructed in a similar manner, here connecting springs are placed in row/column that corresponds to the springs, as denoted. Luckily all the springs between the blocks are identical so these can be assigned  $k$ , the springs connecting the pad and blocks are also identical, which means it can be represented by  $K$ , as seen in eq. 5.

Having created the matrices, the next step is using the armadillo function "arma::eig\_pair", which returns a complex vector with all the eigenvalues of the matrices given. Last step is to save it to an external file, this file has been given the name "eigval\_csv".

$$\begin{bmatrix} m_1 & 0 & 0 & \dots & 0 \\ 0 & m_2 & 0 & \dots & 0 \\ 0 & 0 & m_3 & \dots & 0 \\ \dots & \dots & \dots & m_n & 0 \\ 0 & 0 & 0 & 0 & M \end{bmatrix} \rightarrow \begin{bmatrix} m & 0 & 0 & \dots & 0 \\ 0 & m & 0 & \dots & 0 \\ 0 & 0 & m & \dots & 0 \\ \dots & \dots & \dots & m & 0 \\ 0 & 0 & 0 & 0 & M \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} 0 & k_{12} & 0 & 0 & \dots & k_{1P} \\ k_{21} & 0 & k_{23} & 0 & \dots & k_{2P} \\ 0 & k_{32} & 0 & k_{3n} & \dots & k_{3P} \\ 0 & 0 & k_{n3} & 0 & \dots & k_{4P} \\ \dots & \dots & \dots & \dots & 0 & k_{nP} \\ k_{P1} & k_{P2} & k_{P3} & k_{P4} & k_{Pn} & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & k & 0 & 0 & \dots & K \\ k & 0 & k & 0 & \dots & K \\ 0 & k & 0 & k & \dots & K \\ 0 & 0 & k & 0 & \dots & K \\ \dots & \dots & \dots & \dots & 0 & K \\ K & K & K & K & K & 0 \end{bmatrix} \quad (5)$$

## 4 Standard parameters

Parameter	Value	Parameter	Value
dt	Varying	friction_profile	"circle"
seed	Varying	free_blocks	40
num_events	1	damp_coeff_BB	Varying
N	100	damp_coeff_PB	Varying
max_time	Varying	use_waveform_movement	true
slider_speed	0.0	wave_amplitude	Varying
increment	Varying	wave_mean	0
interval	0	wave_period	Varying
file_name	test_solutions	wave_shift	0
progress_indicator	true	waveform	"sin"
m_F0	1	use_amplitude_as_max_val_for_start	true
m_alpha	0.5		
m_sigma	0.01		
m_mass_x	100		
m_scale_mass	1		
m_zeta	0.0833		
m_k_P0	100		
m_scale_P	1		
m_scale_C	0.01		
m_t	0.0		
m_v0	0.0001		
m_u_min	0		
block	[]		
start_speed_continuous	0.1		
end_speed_continuous	0.2		
save_interval_dt	Varying		
threshold_speed	0.1		

Table 3: Table showing values used for the simulations.



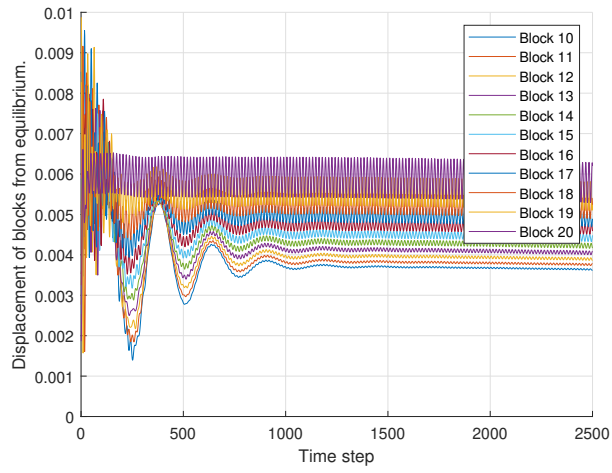
## 5 Initial Simulations and Results

### 5.1 Initial tests with cyclical movement and damping

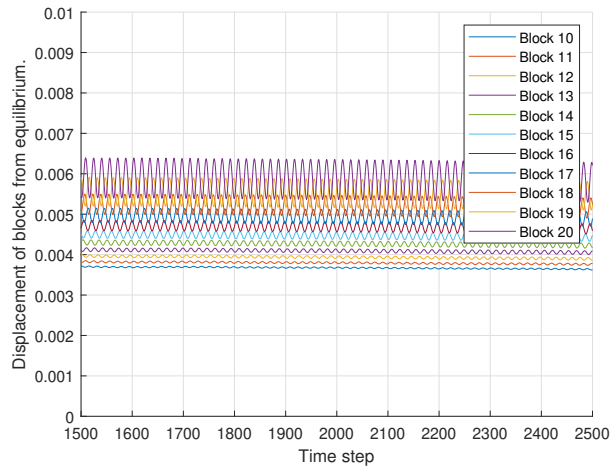
The initial test after implementation were done with the parameters found in tab. 4; case A. The values for damping and movement at this point were not that important so some values were arbitrarily chosen to see if the simulation behaved as expected. The results can be seen in fig. 8 and 9. From these results you can clearly see the chaos as the simulation starts, this is because of the randomised initial positions of the blocks, keep in mind that only blocks 10 to 20 are shown in these figures, the reasons being to keep bloat down and still attempting to capture the behaviour of the system. From this test blocks 20 to 80 are in contact with the driver and appears to be stuck for the majority of the simulation and they are therefore not interesting. Blocks 1 to 10 move in much the same way as 10 and are therefore not that interesting either. The transition area between the outer "still" blocks and the blocks moving in unison with the driver is interesting, as this area is physically deforming the material allowing for cracks to grow. Looking at fig. 9b we can see how the outer blocks are indirectly moved by the driver, and that the movement dissipates as a result of the damping, both behaviours wanted and expected. Another interesting point captured in this same figure is the delayed or lagging movement of the free blocks.

### 5.2 Changing Damping Coefficients

The next step was to figure out some sensible values for the damping coefficients, a lot of different simulations were done for the Block-Block coefficient. The values for the three runs used for deciding on a fitting value is shown in table 4, the corresponding simulations can also be seen in figs 10a and 10b. Looking at the response from the blocks and the movement of the free blocks it was decided that a damping coefficient of 1 would be too large, and 0.1 looked more promising as it clearly removed energy from the system, but without having a too big impact on the movement. Whilst deciding on the Block-block coefficient the Pad-Block was set to 1, so there was damping between the blocks and the pad. To determine a fitting value for this coefficient multiple runs were done, included in this report are 3 cases; D, E and F all with their respective parameters in table 4. If you look at the figures shown in fig. 11 the initial value of 1 seems like an alright value. Looking at fig. 11a where the coefficient was set to 0.1 the system tends to settle but the amount of residual noise from the initial positions is too large. If the value is set to 10 the movement of the free blocks are way to limited, as in the blocks aren't allowed to be driven by the inner most blocks, which means that the damping is too severe. This method of determining or doing qualified guesses is not great, but since we have a theoretical material with no known real world counterpart, it's sufficient enough to validate the simulations.



(a) Displacement shown for entire simulation.



(b) Displacement once steady state has been achieved.

Figure 8: Figures showing the displacement of blocks 10-20 for case A. Figure 8a shows the first simulation with damping and cyclic movement working.

Case	A	B	C	D	E	F
dt	0.001	0.001	0.001	0.001	0.001	0.001
seed	105	105	105	105	105	105
max_time	50	50	50	50	50	50
increment	0.000025	0.000025	0.000025	0.000025	0.000025	0.000025
save_interval_dt	20	20	20	20	20	20
damp_coeff_BB	-0.05	-0.1	-1	-0.1	-0.1	-0.1
damp_coeff_PB	1	1	1	0.1	1	10
wave_amplitude	0.01	0.01	0.01	0.01	0.01	0.01
wave_period	50	50	50	50	50	50

Table 4: Table showing the varying parameters from run to run.

Case	G	H	I	J	K
dt	0.001	0.001	0.001	0.001	0.001
seed	105	105	105	105	105
max_time	150	150	150	150	150
increment	0.000025	0.000025	0.000025	0.000025	0.000025
save_interval_dt	20	20	20	20	20
damp_coeff_BB	-0.1	-0.1	-0.1	-0.1	-0.1
damp_coeff_PB	1	1	1	1	1
wave_amplitude	0.01	0.01	0.01	0.01	0.01
wave_period	25	50	500	2500	5000

Table 5: Table showing the varying parameters from run to run.

### 5.3 Different Driving Frequencies

Since this system is driven by a periodic function, researching the effects of changing the frequency will help in understanding the system. The only changes in parameters from earlier simulations is that the "max\_time" has been increased to 150. As studying lower frequencies will require more time to show multiple periods of movement. A table of the parameters used for the cases where studying friction can be found in table 5. The resulting displacements can be seen in the figs 12 to 16. There were also done another set of runs at a higher amplitude to aid in recreating "stick slip" behaviour.

### 5.4 Eigenfrequencies

The result from the Eigenfrequencies is a list .csv file. Looking into this file we have 1 interesting value, which is assumed to be the lowest eigenfrequency of the system. This frequency is 151.518 and is assumed correlated to having the blocks move in unison and as extension of the pad, there are also a lot more values in the .csv file but all these values range from  $-0.1$  to  $0.1$  and are considered related to eigenfrequencies of the blocks.

Case	L	M	N	O	P
dt	0.001	0.001	0.001	0.001	0.001
seed	105	105	105	105	105
max_time	150	150	150	150	150
increment	0.000025	0.000025	0.000025	0.000025	0.000025
save_interval_dt	20	20	20	20	20
damp_coeff_BB	-0.1	-0.1	-0.1	-0.1	-0.1
damp_coeff_PB	1	1	1	1	1
wave_amplitude	0.1	0.1	0.1	0.1	0.1
wave_period	25	50	500	2500	5000

Table 6: Table showing the varying parameters from run to run.

Case	Q	R	S	T	U
dt	0.001	0.001	0.001	0.001	0.001
seed	105	105	105	105	105
max_time	150	150	150	150	150
increment	0.000025	0.000025	0.000025	0.000025	0.000025
save_interval_dt	20	20	20	20	20
damp_coeff_BB	-0.1	-0.1	-0.1	-0.1	-0.1
damp_coeff_PB	1	1	1	1	1
wave_amplitude	1	1	1	1	1
wave_period	25	50	500	2500	5000

Table 7: Table showing the varying parameters from run to run.

## 6 Discussion

### 6.1 Damping

Looking at figure 8a it is clear that the damping implemented is having an effect on the simulation and judging from the first simulations the system seems underdamped. You could argue that having the system critically damped would be the best as critical damping often is sought after, but at this early stage it is not known what represents a real material. Therefore it is left underdamped so that the simulation and connections between the blocks doesn't become too stiff, which in turn won't allow for individual movement of the blocks. If the blocks can't move individually, it would be pointless to run fatigue analysis as the all the blocks combined acted as another pad.

If you look at figures 11a, 11b and 11c you can clearly see how the Block-Pad damper is removing energy from the system. In the first figure not enough energy seems to be removed, this is with a coefficient of 0.1, too low of a value. Looking at the the third figure, fig. 11c, the system seems far too stiff as the free blocks aren't affected enough by the inner blocks. This would lead to a fatigue analysis with a really high "fatigue exposure" or "risk of fatigue" where the transition from free blocks to driven occurs as fatigue occurs where there are

cyclical loads/strains in the material. This means that a Pad-Block coefficient of 10 is too high. A coefficient of 1 was then tested, with promising results. The movement can be seen in 11b and it seems to avoid both pitfalls mentioned earlier.

As an important side note; the coefficient for the Pad-Block dampers should have a higher value than for the Block-Block damper, if not the blocks might start moving in unison. And since the Pad already has a damper connected to a fixed location if the damper between the Pad-Blocks were removed the blocks would vibrate for "eternity".

## 6.2 Eigenfrequencies

The result from doing the eigenfrequency analysis gave a value of 151.518 which coincidentally lies between 50 and 500, which are 2 of the frequencies chosen for the simulations. Meaning that for these simulations the probability of standing waves are the higher than for the others.

## 6.3 Driver parameters and system behaviour

Comparing how frequency impacts the simulation it is clear that the system responds quite differently when exposed to different frequencies. When the frequencies are high and amplitudes low as seen in fig. 12a and 12b The inner most blocks move in unison with the driver, while the free blocks have a hard time keeping up with the movement. This behaviour will create a very small area that is exposed to fatigue, and this is clearly seen in 27a. Two massive spikes where the blocks transition from being driven to free. Looking at the behaviour of Case H, where the frequency has been halved a lot more of the energy is transferred to the free blocks, spreading the fatigue over a greater number of blocks. This behaviour persists even if the amplitude is changed, this can be seen in figures 27b and 27c. If this is an unrealistically high frequency is hard to tell, but the system is clearly not "fast" enough to respond to these frequencies.

When the period is set to 50 the system is able to respond in a different way, where the free blocks move a lot more. Which spreads the fatigue loads over a greater area, this can be seen in figs 28b and 28c. In these two runs the amplitude is 0.1 and 1 respectively and the result is as expected. Two areas around the transition where the risk of fatigue is high. However when the amplitude is 0.01 a third spike appears closer to the middle. Looking closer at the behaviour this appears to be related to some form of standing wave within the material, which means more damping in this case might have been necessary. A simulation with a Pad-Block coefficient of 10 was run to look closer at this anomaly and the results stayed the same. The reason for this behaviour is therefore assumed related to eigenvalues and with a low amplitude the outer block create some form of standing wave that meets in the middle causing the spike.

Setting the period to 500 really shows how the free blocks "lag" behind, this couldn't be clearer looking at fig. 14b. Even though the frequency has been significantly reduced, the 3-spike behaviour persists. The reasons are fairly

similar to those above, and since we've passed the eigenfrequency of the system it makes sense that we have a duplicate as we are roughly three times slower than the eigenfrequency, while before we were three times faster. When the amplitude is increased this simulation behaves differently as the frequency is still assumed too high to have the blocks stick to the driver. This becomes even clearer looking at fig. 29c. Here there isn't really any major areas subjected to fatigue as the entire system still appears to be moving in unison.

Taking the frequency down even further the system starts being able to respond to the movement of the driver, as seen in figs. 15a, 20a and 25a. Looking at the block movement and having an idea about how the fatigue will look is significantly more difficult as the behaviour is more complex. Therefore working backwards from the fatigue figures and trying to figure out how the system responds is easier than the other way around. Considering fig 30a the behaviour looks a lot like the same for the earlier cases with the same amplitude, the outlier being fig. 27a. The only reason for this is that with these low amplitudes less "slipping" of the driven blocks occur and the free blocks influence the simulation by creating ripples. When you look at fig. 25b you can clearly see some inconsistencies in the movement and the only reason for this is assumed to be the blocks Sticking and Slipping to the driver. Looking at the resulting fatigue calculations, fig. 30c, for this case reveals a noisy yet defined wide peak.

For the lowest frequency in this thesis; cases K, P and U, the same wide peak appears with 2 different amplitudes, they are also much more defined than in the Case T. Although there is a much more defined peak, there also appears to be a "shear" or jump. The reason for this shear is not known, but looking at the movement of the outer blocks in fig. 21b, there is definitively stick-slip events. In case U the material behaves in an entirely new way, as can be seen in 26a, the reason for this behaviour is unknown but should definitively be looked further into.

Comparing this to a real world scenario, it makes sense that if the frequencies are high enough the outer free blocks start to oscillate with the driver, creating fatigue zones in the transitional area. At these higher frequencies the system responds in as you can expect and understand, but as the amplitude increases and the frequency decreases, the system starts moving in more complex ways, which are harder to relate to a real world scenario.

## **7 Conclusion**

As a conclusion to this thesis it is definitely possible to simulate fretting fatigue with the BK-Pad model, and by changing both the frequency of the driver and it's amplitude the resulting fatigue exposure can vary drastically. Most noteworthy is that there appears to be a threshold of sorts for the amplitude. If the amplitude is below this value the system will never experience any form of stick-slip behaviour. However since we have no real world counterpart as of now, these theoretical results would need to be confirmed by real world tests to validate the behaviour of the simulation.

## **8 Summary and recommendations for further work**

### **8.1 Summary**

In this thesis a further expansion of the BK-Pad model has been utilized to simulate a system where fretting fatigue should exist. It was then run a short parameter study of the driver how it affected the fatigue of the system.

### **8.2 Performing a larger Parameter analysis**

Since a lot of focus was on implementing and expanding the current model, there were a lot of parameters that weren't explored during this thesis. Doing a wider parameter analysis would vastly improve the knowledge around the model and might also uncover pitfalls or flaws, that haven't been discovered in this thesis.

### **8.3 Multi-spring Fatigue Analysis**

The post-processed fatigue analysis used in this thesis was performed only using the springs between the blocks, doing a more involved fatigue analysis including the spring connected to the pad would be interesting to do as well.

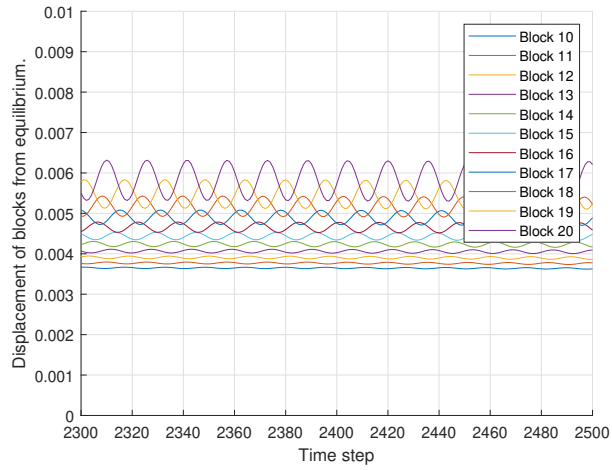
### **8.4 Comparing to a real world test**

As this thesis is purely theoretical trying to create a real world test that could validate the results should be an end goal.

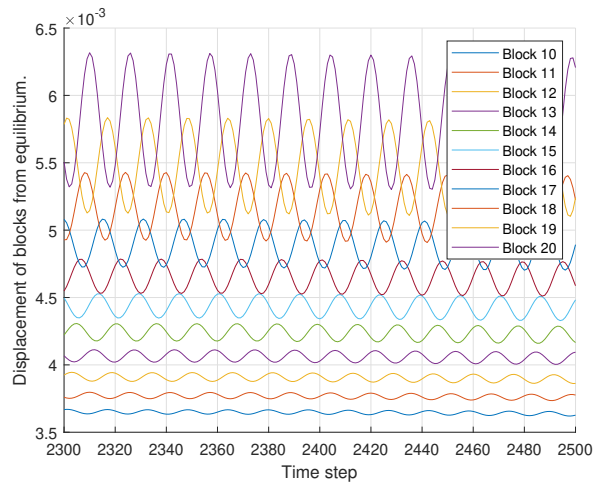
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<https://statistical-engineering.com/random-fatigue-limit/>
- [2] Omegatron *Sine, Square, Triangle and Sawtooth Waveforms*. Wikipedia, Feb 2020.  
<https://en.wikipedia.org/wiki/Waveform#/media/File:Waveforms.svg>
- [3] R. Burridge, L. Knopoff. *Model and theoretical seismicity*. Bulletin of the Seismological Society of America, June 01, 1967  
<https://pubs.geoscienceworld.org/ssa/bssa/article-abstract/57/3/341/116471>
- [4] Ferre, Herman Nylund *The Burridge-Knopoff-Pad model: A New Model for Studying Noise Generation & Brake Dynamics* Master thesis, Department of Mechanical and Industrial Engineering, Norwegian University of Science and Technology, February 2018
- [5] Standnes, Karsten *Simulating Noise Generation by Friction* Specialization Project, Department of Mechanical and Industrial Engineering, Norwegian University of Science and Technology, 2018



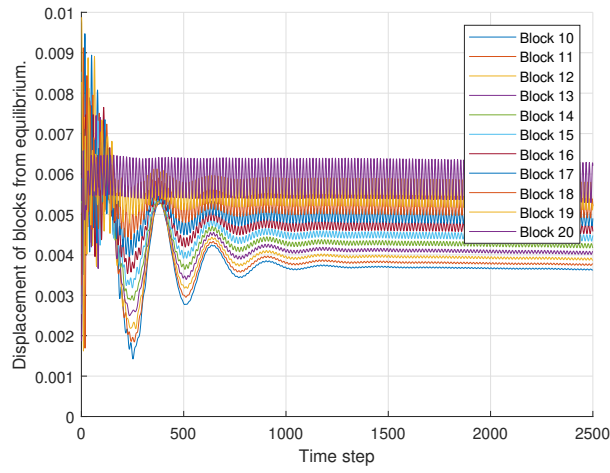


(a) Further zoomed in.

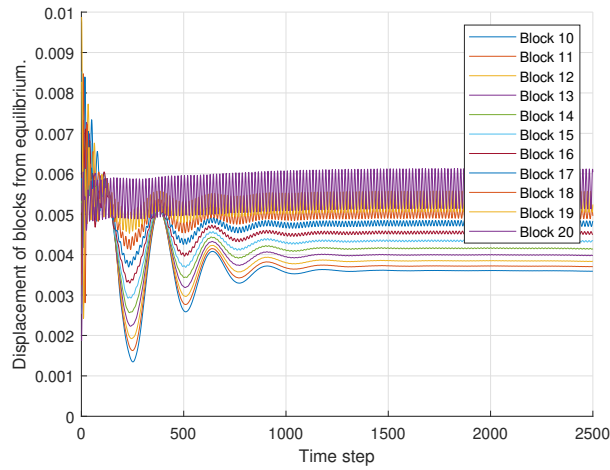


(b) Same as fig. 9a but with a scaled y-axis.

Figure 9: Figures showing the displacement of blocks 10-20 for case A.

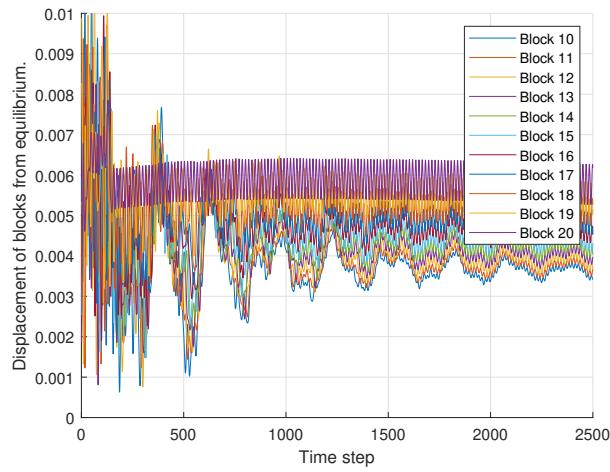


(a) Displacements of blocks for Case B.

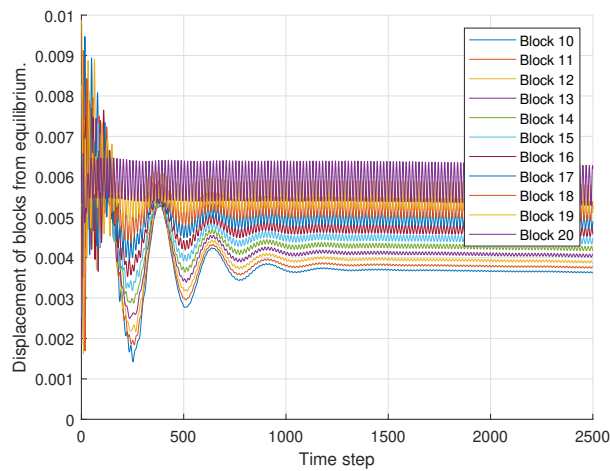


(b) Displacements of blocks for Case C.

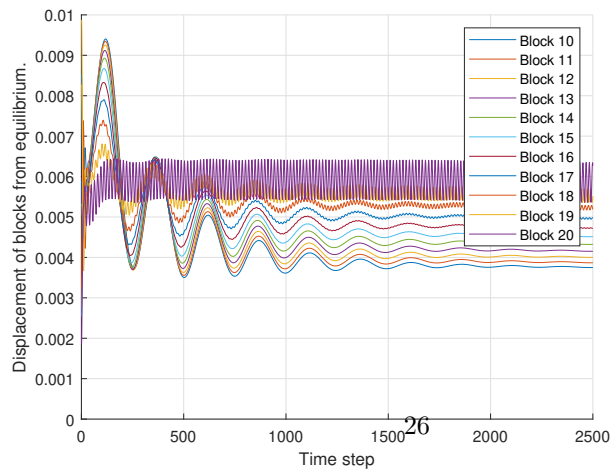
Figure 10: Some of the graphs used for determining the damping coefficient between blocks.



(a) Displacements of blocks for Case D.

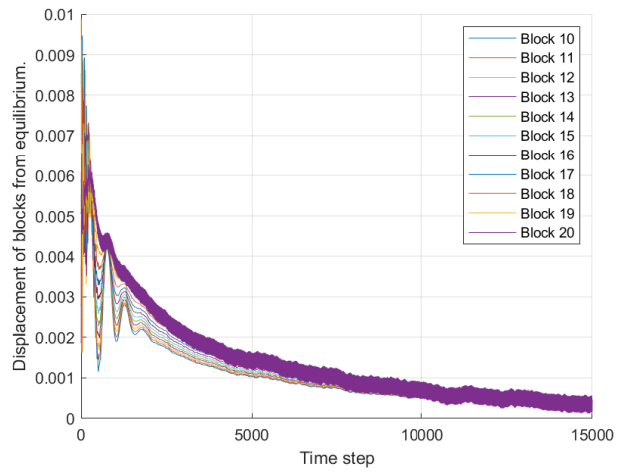


(b) Displacements of blocks for Case E.

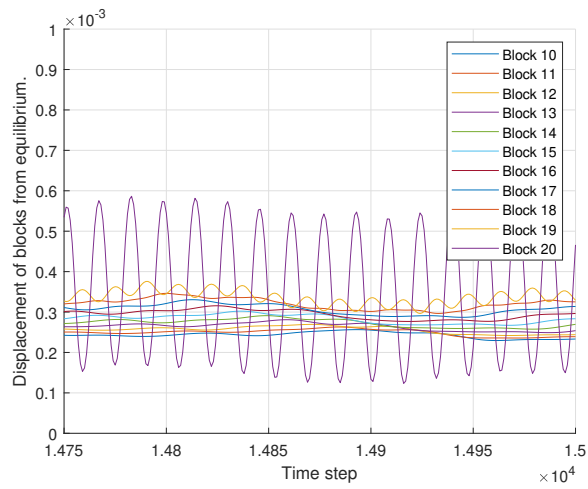


(c) Displacements of blocks for Case F.

Figure 11: Some of the graphs used for determining the damping coefficient between pad and blocks.

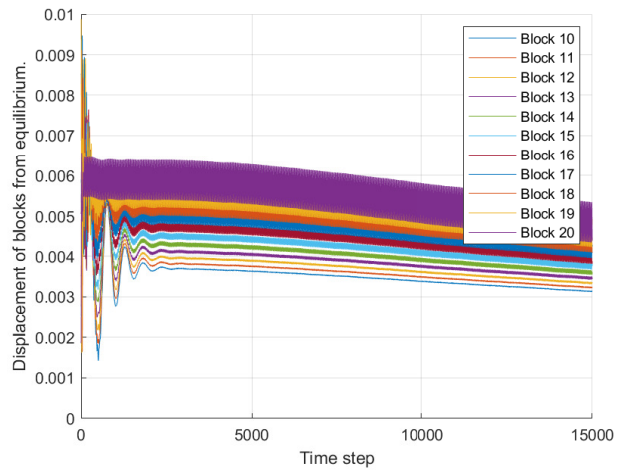


(a) Case G, full run.

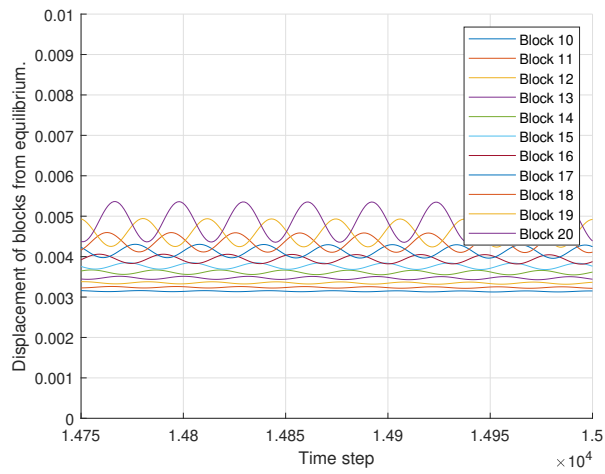


(b) Case G, Zoomed in.

Figure 12: Figures showing the simulation results of case G with an amplitude of 0.01

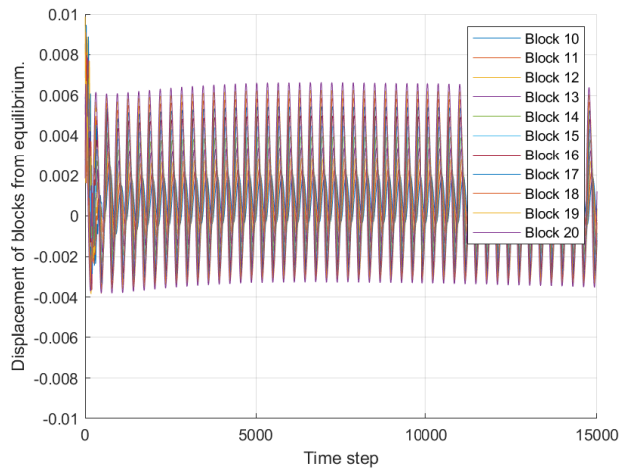


(a) Case H, full run.

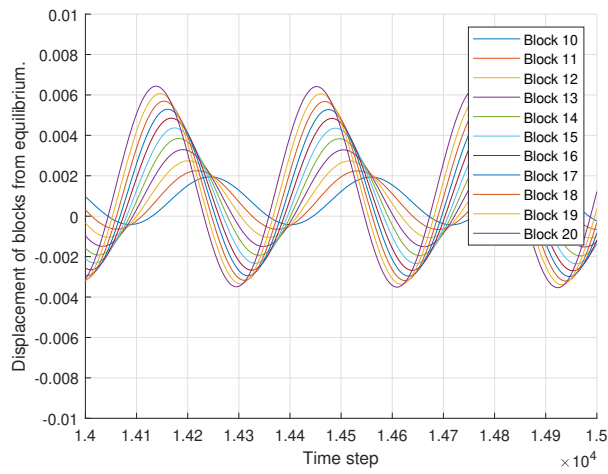


(b) Case H, Zoomed in.

Figure 13: Figures showing the simulation results of case H with an amplitude of 0.01

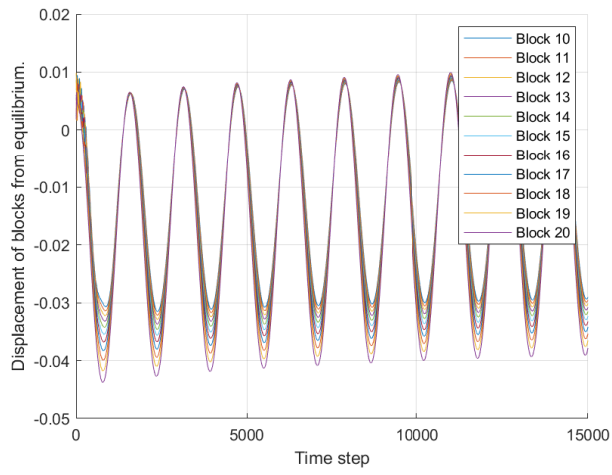


(a) Case I, full run.

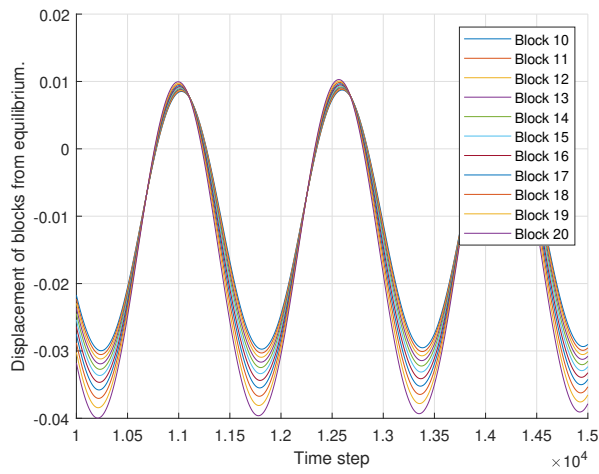


(b) Case I, Zoomed in.

Figure 14: Figures showing the simulation results of case I with an amplitude of 0.01

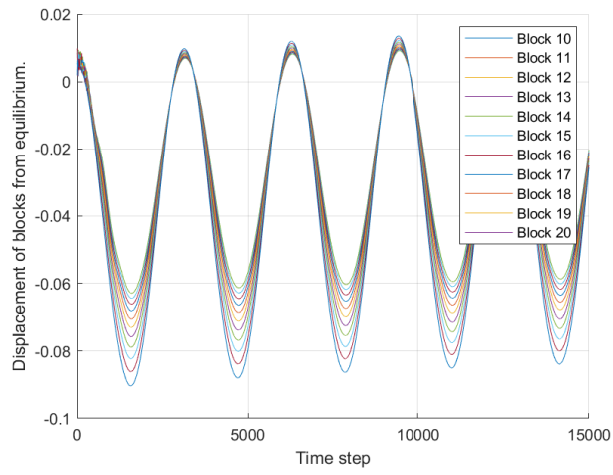


(a) Case J, full run.

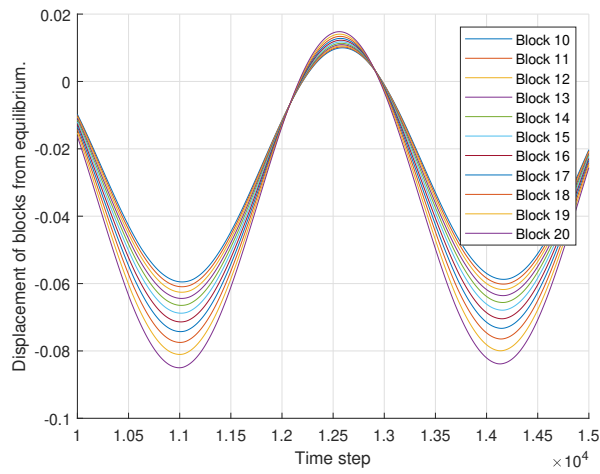


(b) Case J, Zoomed in.

Figure 15: Figures showing the simulation results of case J with an amplitude of 0.01



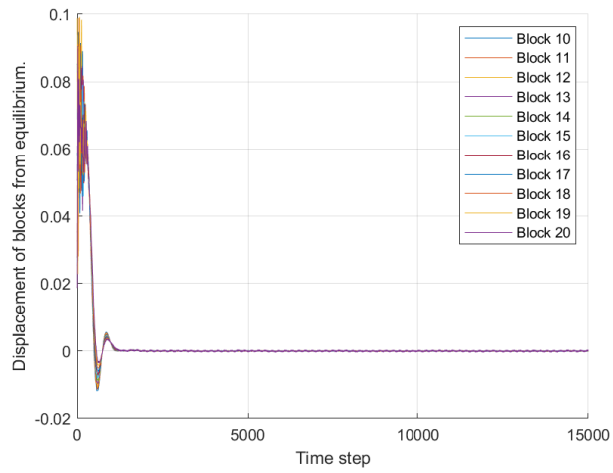
(a) Case K, full run.



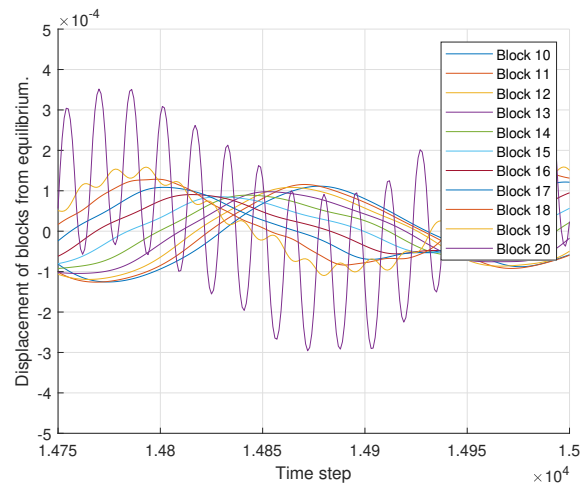
(b) Case K, Zoomed in.

Figure 16: Figures showing the simulation results of case K with an amplitude of 0.01



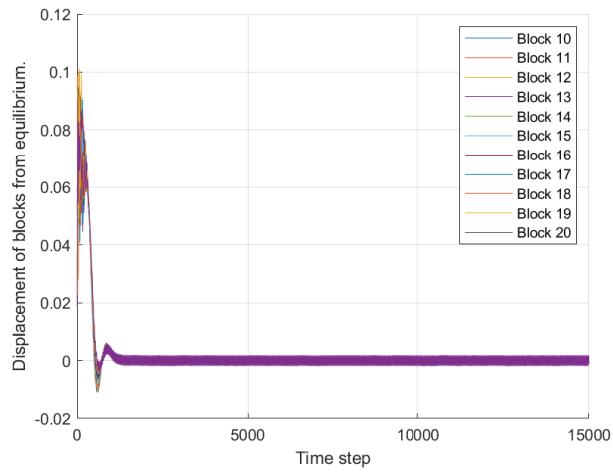


(a) Case L, full run.

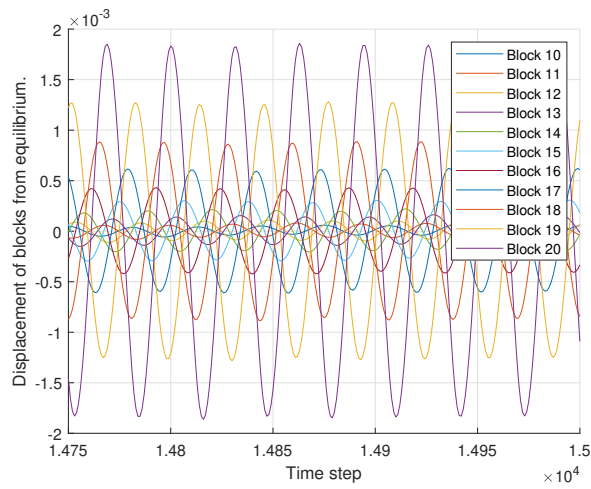


(b) Case L, Zoomed in.

Figure 17: Figures showing the simulation results of case L with an amplitude of 0.1

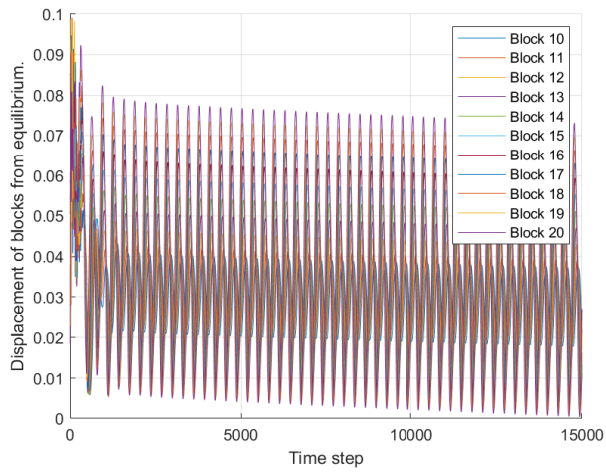


(a) Case M, full run.

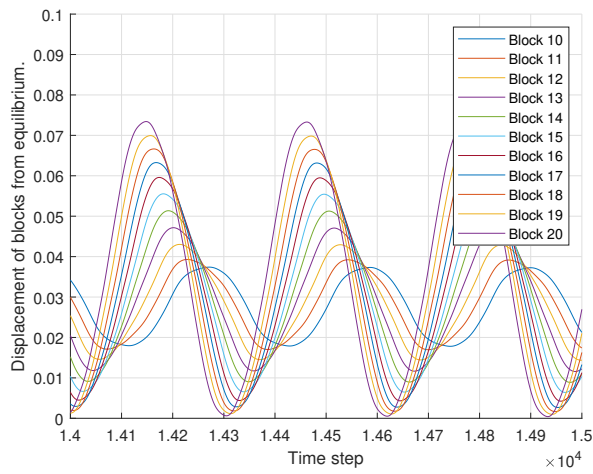


(b) Case M, Zoomed in.

Figure 18: Figures showing the simulation results of case M with an amplitude of 0.1

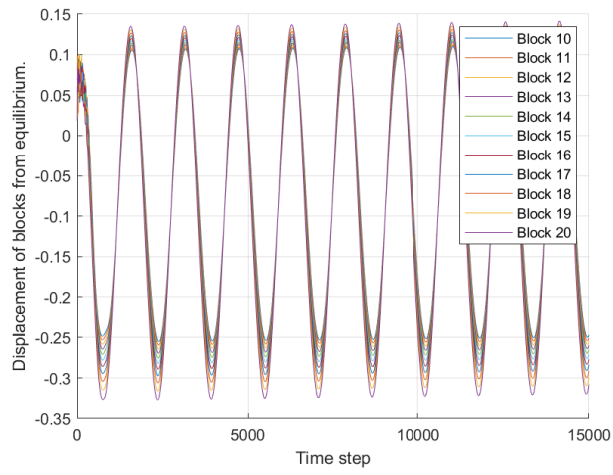


(a) Case N, full run.

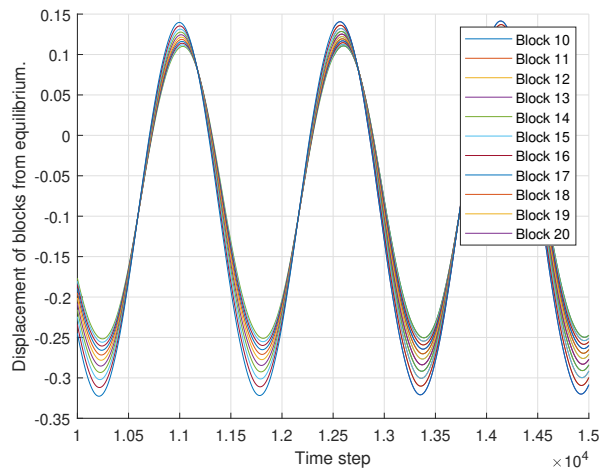


(b) Case N, Zoomed in.

Figure 19: Figures showing the simulation results of case N with an amplitude of 0.1

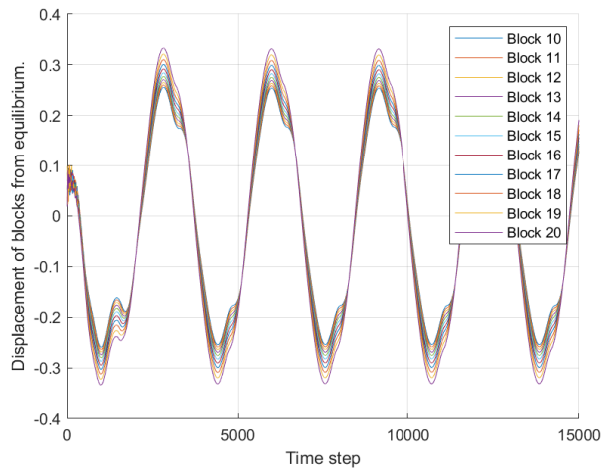


(a) Case O, full run.

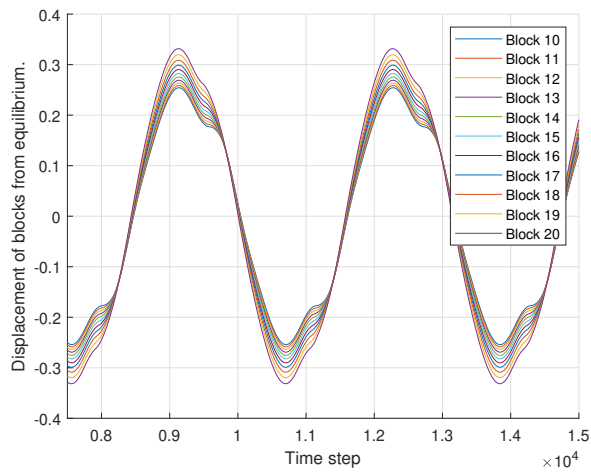


(b) Case O, Zoomed in.

Figure 20: Figures showing the simulation results of case O with an amplitude of 0.1

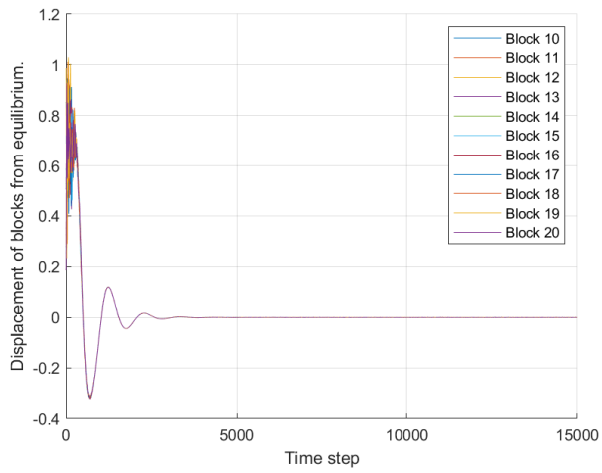


(a) Case P, full run.

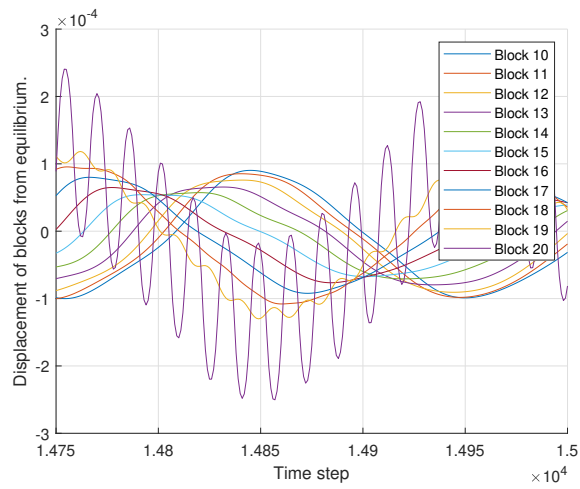


(b) Case P, Zoomed in.

Figure 21: Figures showing the simulation results of Case O with an amplitude of 0.1

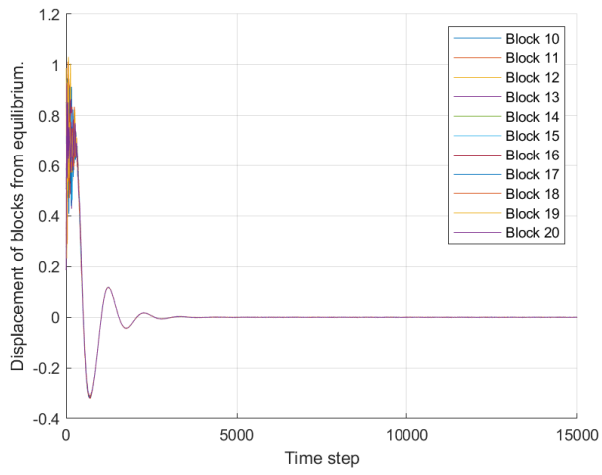


(a) Case Q, full run.

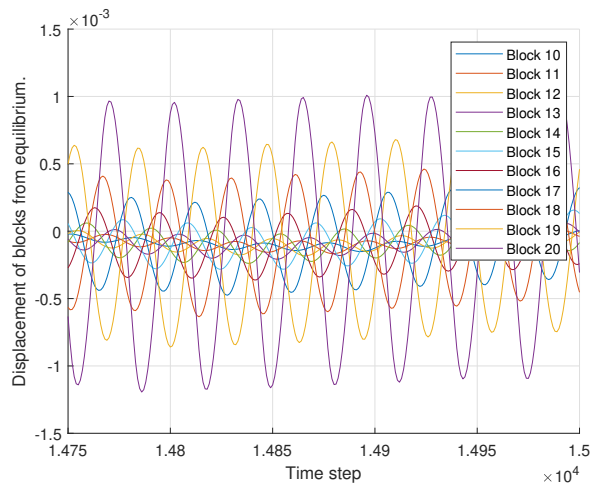


(b) Case Q, Zoomed in.

Figure 22: Figures showing the simulation results of case Q with an amplitude of 1

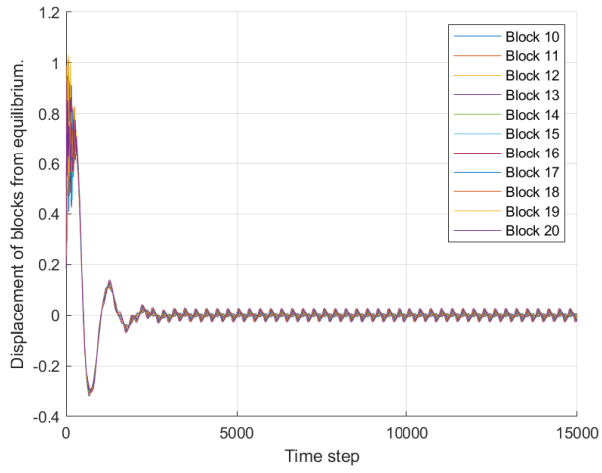


(a) Case R, full run.

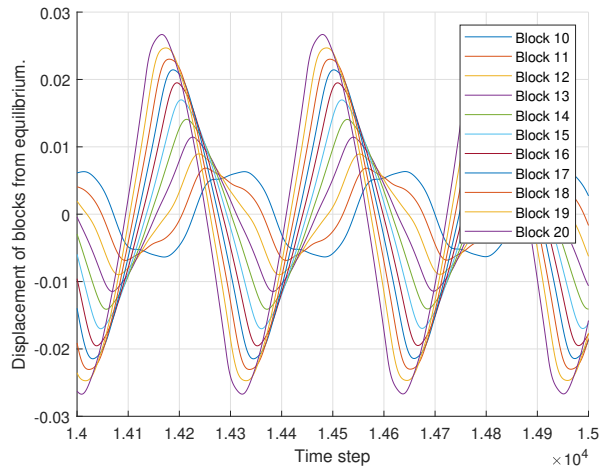


(b) Case R, Zoomed in.

Figure 23: Figures showing the simulation results of case R with an amplitude of 1



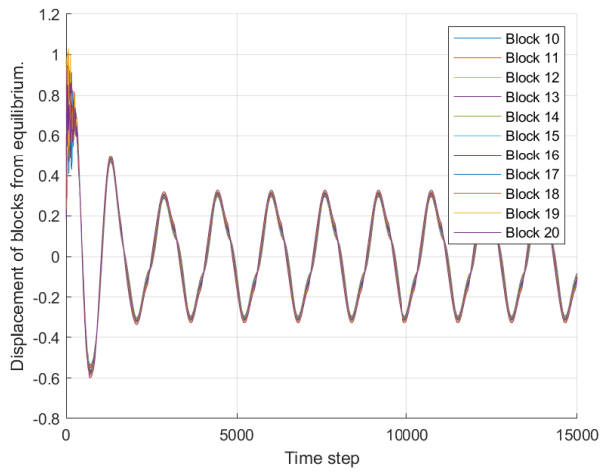
(a) Case S, full run.



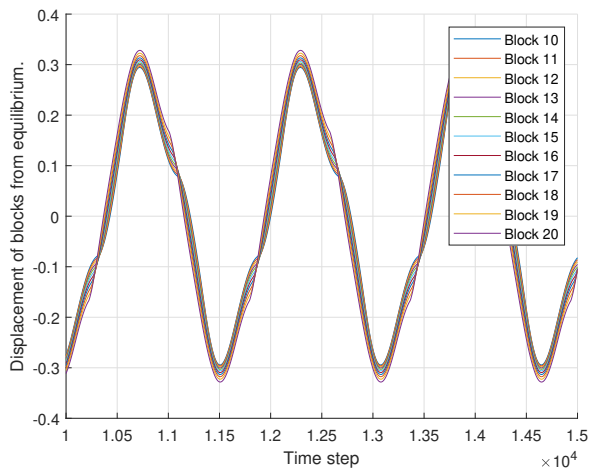
(b) Case S, Zoomed in.

Figure 24: Figures showing the simulation results of case S with an amplitude of 1



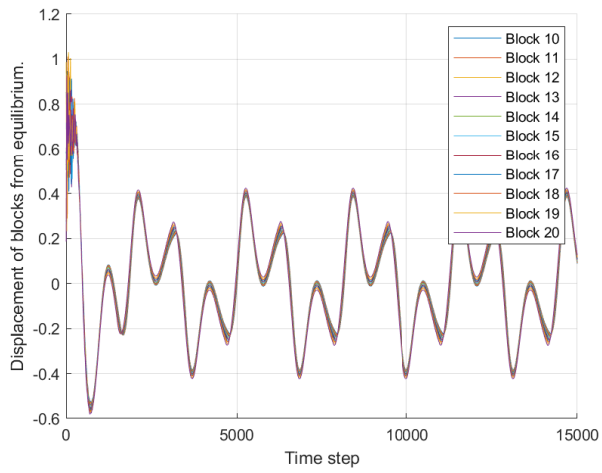


(a) Case T, full run.

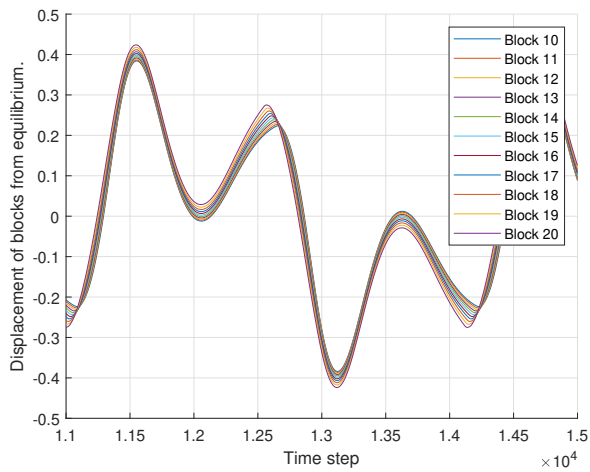


(b) Case T, Zoomed in.

Figure 25: Figures showing the simulation results of case T with an amplitude of 1

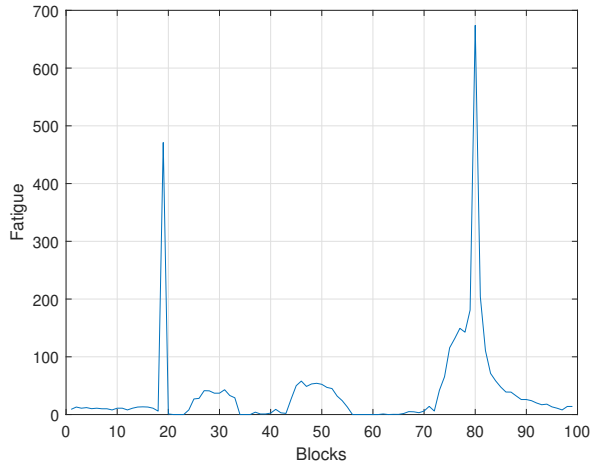


(a) Case U, full run.

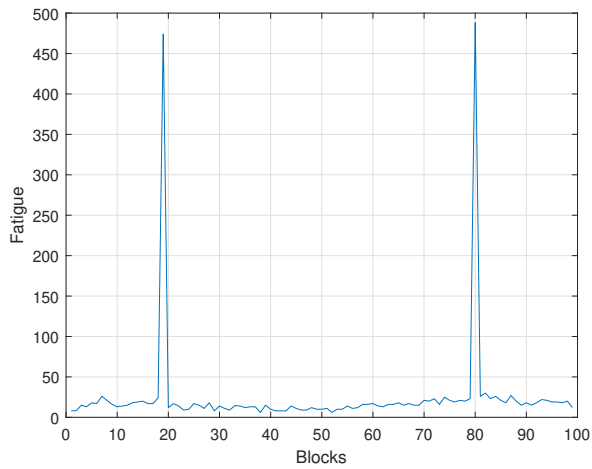


(b) Case U, Zoomed in.

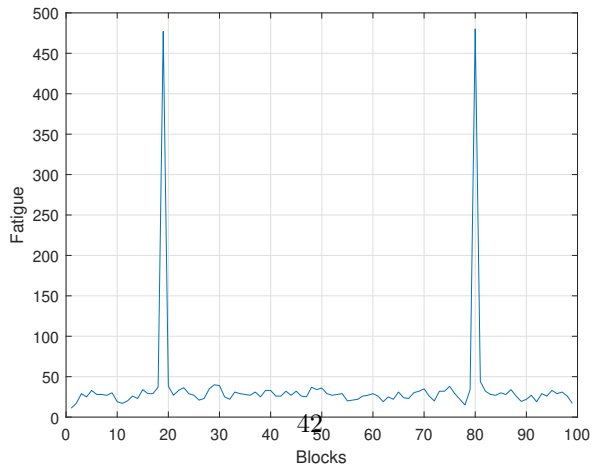
Figure 26: Figures showing the simulation results of case U with an amplitude of 1



(a) Fatigue for case G.

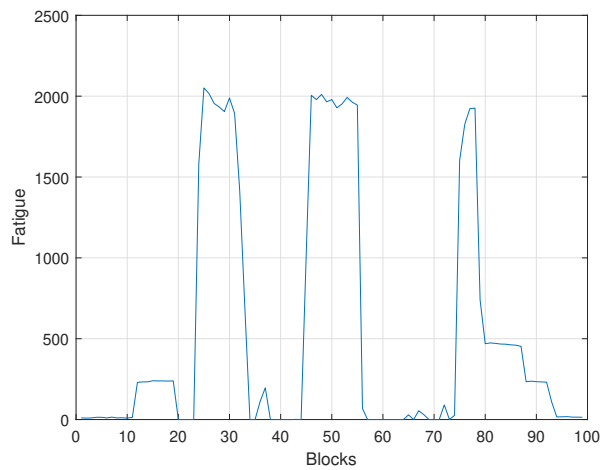


(b) Fatigue for case L.

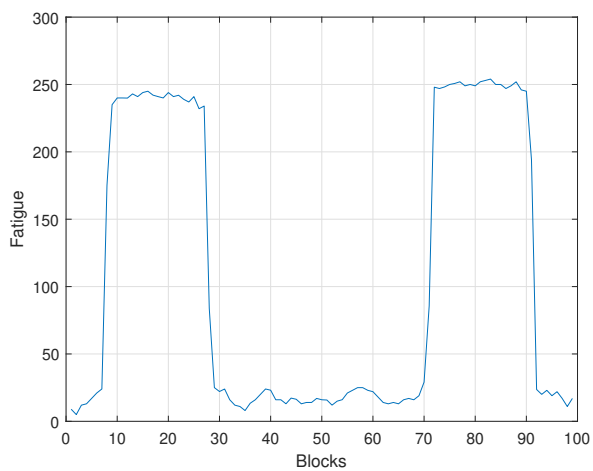


(c) Fatigue for case Q.

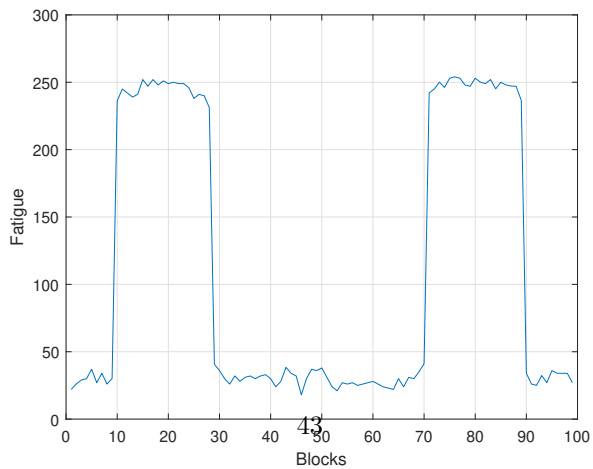
Figure 27: Figures showing the fatigue exposure of cases with a period of 25.



(a) Fatigue for case H.

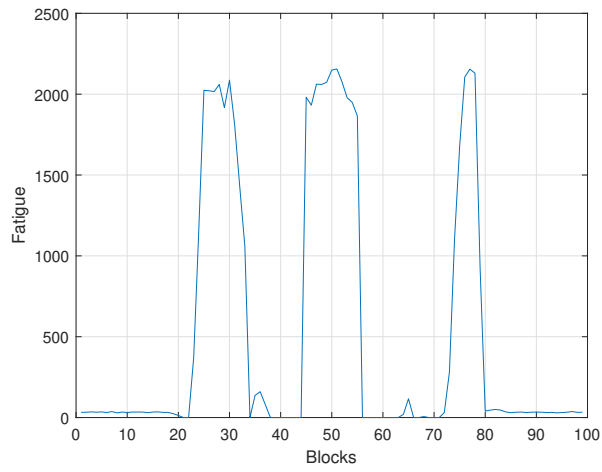


(b) Fatigue for case M.

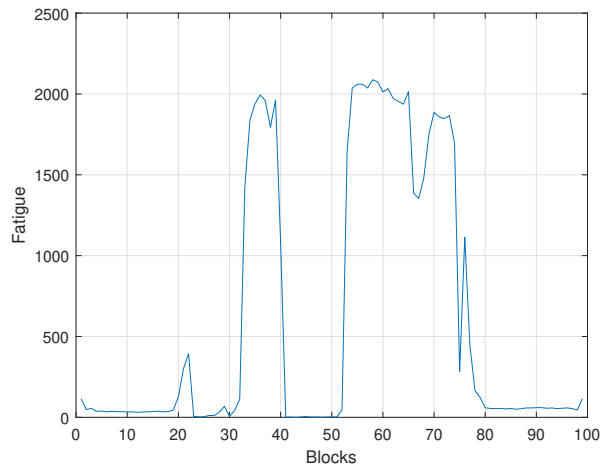


(c) Fatigue for case R.

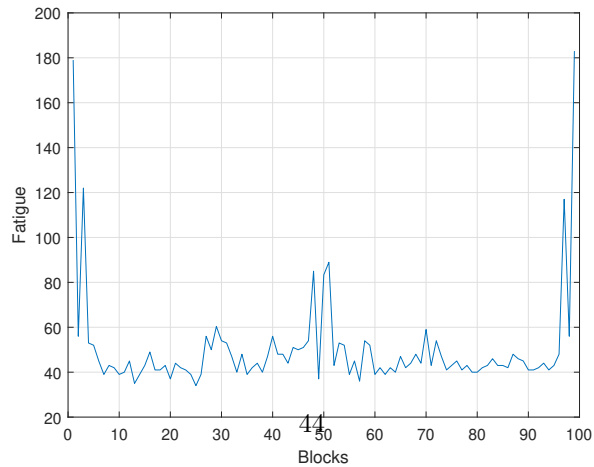
Figure 28: Figures showing the fatigue exposure of cases with a period of 50.



(a) Fatigue for case I.

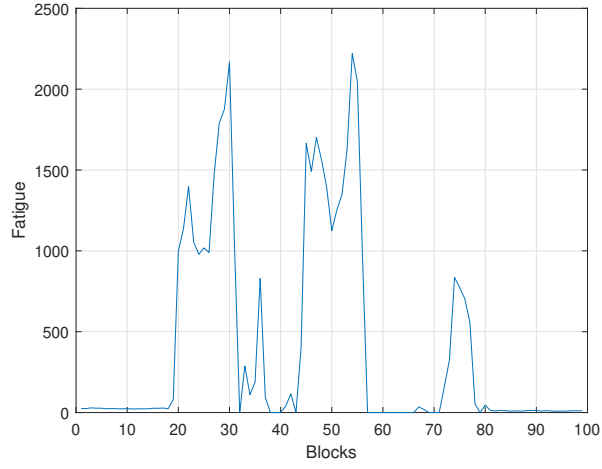


(b) Fatigue for case N.

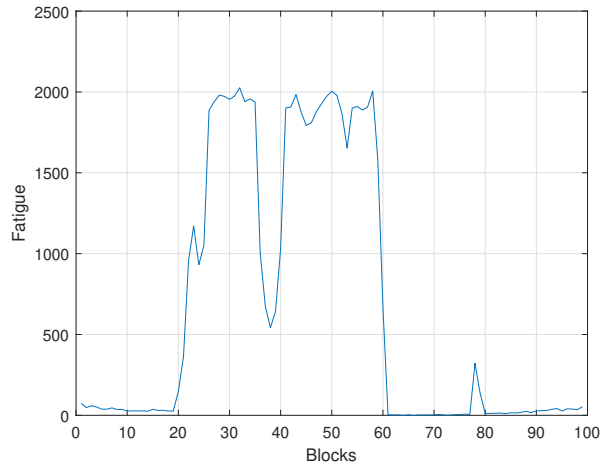


(c) Fatigue for case S.

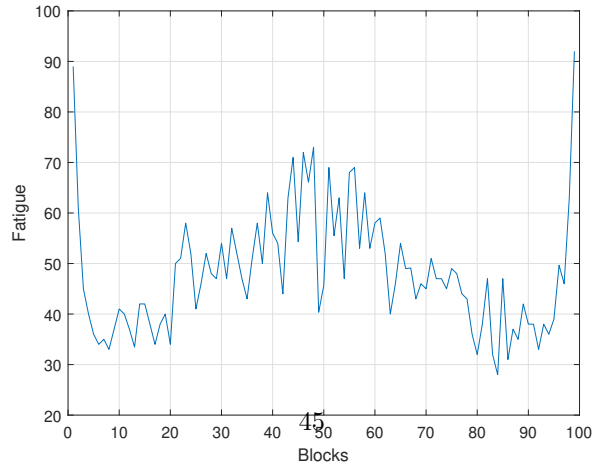
Figure 29: Figures showing the fatigue exposure of cases with a period of 500.



(a) Fatigue for case J.

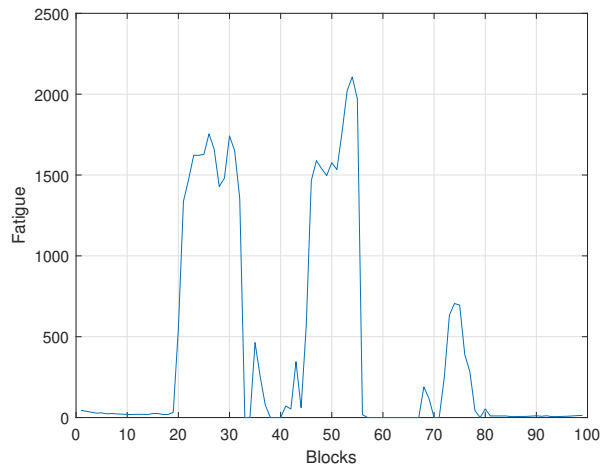


(b) Fatigue for case O.

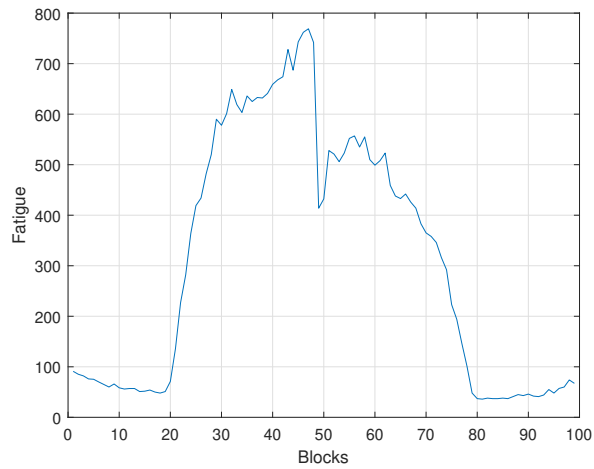


(c) Fatigue for case T.

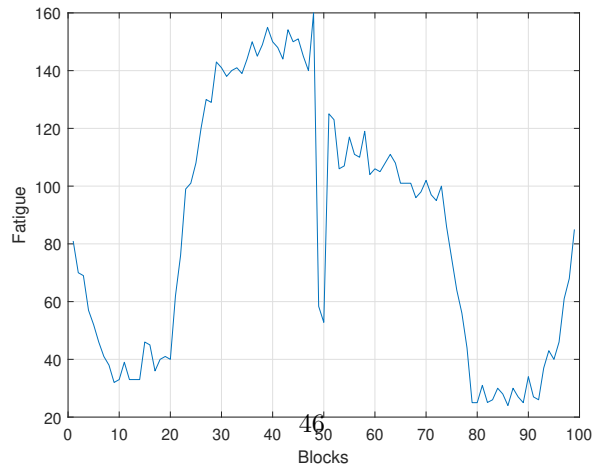
Figure 30: Figures showing the fatigue exposure of cases with a period of 2500.



(a) Fatigue for case K.



(b) Fatigue for case P.



(c) Fatigue for case U.

Figure 31: Figures showing the fatigue exposure of cases with a period of 5000.