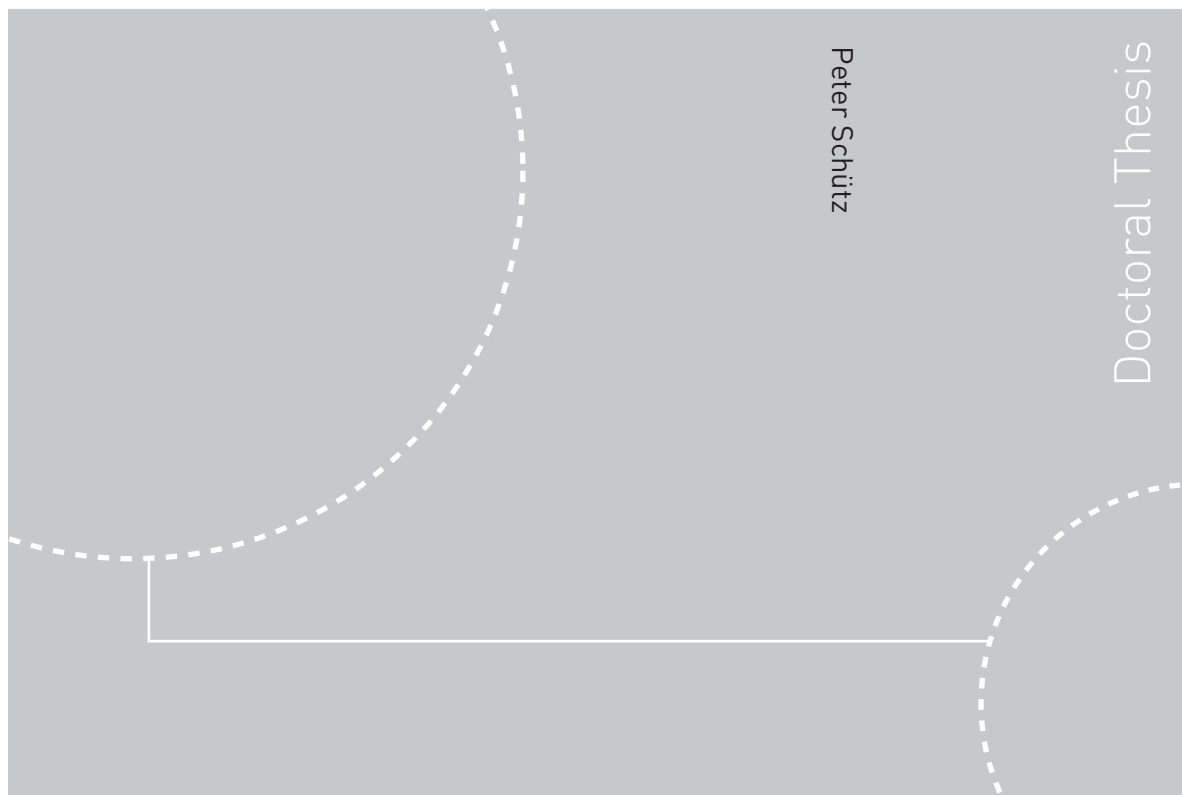


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Peter Schütz  
**Managing Uncertainty and Flexibility  
in Supply Chain Optimization**

Peter Schütz

# Managing Uncertainty and Flexibility in Supply Chain Optimization

Thesis for the degree of philosophiae doctor

Trondheim, June 2009

Norwegian University of  
Science and Technology  
Faculty of Social Sciences and Technology Management  
Department of Industrial Economics and Technology  
Management



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When I started working on this thesis quite some time ago, I did not know what to expect. Looking back, this period reminds me of a roller coaster ride: the past years had their ups and downs, but were exciting all the way to the finishing line. However, this thesis would have never been completed, had it not been for all the people who accompanied me these years.

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Peter Schütz

Trondheim, March 2009



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# Introduction

This thesis presents and discusses optimization models for various aspects of supply chain management. It focuses on the challenges imposed by uncertain information about the future and dynamics in the decision-making process. A dynamic decision-making process is characterized by a sequence of decisions, such that later decisions depend both on the previous ones and any additional information that becomes known in between making the two decisions (see e. g. Edwards 1962). In order to capture this dynamic structure and to model the uncertainty present in the problem, the models are formulated as two-stage stochastic programming problems (Kall & Wallace 1994, Birge & Loveaux 1997).

The research in this thesis has been financed by the Norwegian Research Council through the SMARTLOG-project. In SMARTLOG, NTNU, SINTEF, and Marintek worked together with partners from the Norwegian industry to enhance the competitiveness by creating knowledge and improving the understanding of how to design, develop, and control dynamic value chains.

The thesis consists of two parts. The first part is this introduction. Relevant aspects of dynamic decision-making are discussed in Section 1.1. A short overview over the field of quantitative supply chain management is given in Section 1.2. The existing literature on selected fields of supply chain optimization is presented in Section 1.3. I also indicate how this thesis contributes to the existing literature. A summary over the papers included in this thesis can be found in Section 1.4. Part 2 consists of the aforementioned four papers.

## 1.1 Dynamics of the Decision-Making Process

In a dynamic environment, decisions are made at different points in time. Later decisions are based on the consequences of decisions made earlier and relevant information that becomes known in between the two decisions. The information that becomes available can be the realization of some random variable, e. g. demand, according to a given probability distribution (Edwards 1962). Two-stage and particularly multi-stage stochastic programming (see e. g. Birge & Loveaux 1997) try to capture these dynamics by modeling the different points in time when decisions have to be taken and the information available at that time as stages. In a deterministic setting, this stage structure has no meaning as



no additional information becomes available. It only represents the sequence in which the decisions have to be made.

This thesis focuses on two questions that are a direct result of the sequential nature of the decision-making process. The first one is how decisions made at different points in time relate to each other. This also includes the economic consequences of the decisions to be made, e. g. should one forgo short-term profit opportunities in favour of some long-term profit goals. In Section 1.1, I give some motivation for the use of long-run and short-run cost functions to examine the relationship between strategic and operational decisions in a setting where uncertainty is revealed in between the decisions.

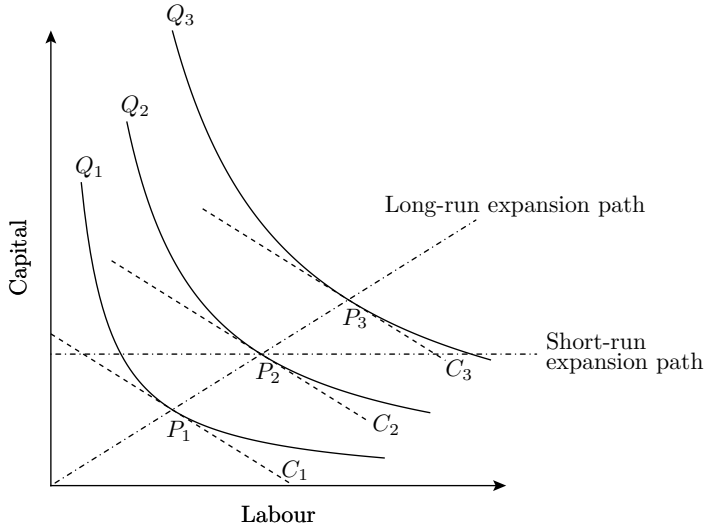
The second question is how to cope with the uncertainty in the decision-making process. It is a key characteristic of a dynamic decision-making process that new information, predictable or not, may become available in between making decisions (Edwards 1962). A short introduction on how to use flexibility to handle this uncertainty about the future inherent in the decision-making process is given in Section 1.1.

## **Combining Long-term And Short-term Decisions**

Ever since Anthony (1965) published his framework for managerial decisions, it has become common to consider decisions as part of a hierarchical system. Higher level decisions constrain lower level decisions, whose feedback in turn is used to evaluate the quality of the higher level decisions (Bitran & Tirupati 1993). A simple example of such a hierarchical system is the problem of investing in production equipment (e. g. a facility or number of machines) that has to be operated in order to satisfy future customer demand. Clearly, this problem consists of two decisions, made one after the other: the first one is how much capacity to invest in and the second one of how many units to produce to satisfy the demand. The horizon for the decisions and the data on which the two decisions are based on are different, though. While the investment decision has a planning horizon related to the lifetime of the equipment invested in, the production decision is related to observed demand and may actually change from day to day. In the following I show, how the long-run total cost function and the short-run total cost function depend on each other and how they can be combined to link long-term decisions to short-term decisions.

According to microeconomic theory, a firm will choose the combination of input factors that allows production of the desired output at minimum costs. The minimum cost combinations of input factors for given levels of production results in the long-run total cost curve (see e. g. Mathis & Koscianski (2002) or Perloff (2004)). To illustrate this, consider a product with two input factors, for example capital and labour. The three isoquants in Figure 1.1 are the technologically

efficient combinations of the two input factors to produce the quantities  $Q_1$ ,  $Q_2$ , and  $Q_3$  respectively. The economically efficient combinations of capital and labour to produce these quantities are given by the points  $P_1$ ,  $P_2$ , and  $P_3$ , where the isocost curves  $C_1$ ,  $C_2$ , and  $C_3$  are tangents to the corresponding isoquants (assuming linear costs for the components). These minimal cost combinations constitute the long-run expansion path. Thus, in order to produce  $Q_2$  in the long-run, one would choose the combination of capital and labour as given by  $P_2$ .



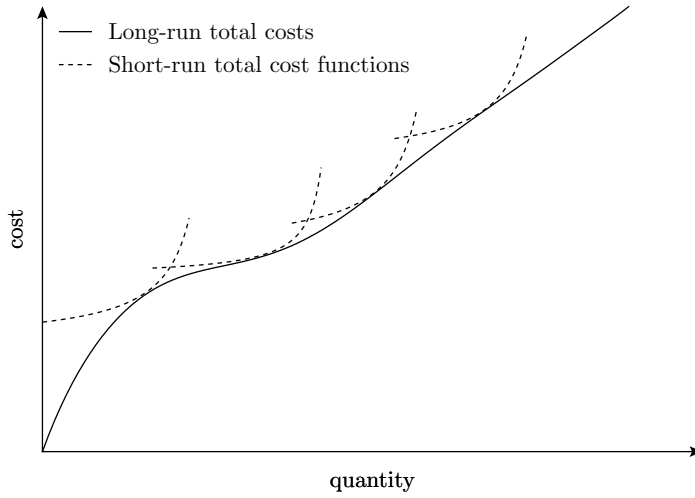
**Figure 1.1:** Long-run and short-run expansion paths

In the short-run, it is no longer possible to vary all input factors. Consider capital (e. g. the investment in a number of machines) as the fixed input in the example above. The decision-maker has implemented the combination of capital and labour as given by  $P_2$ . Demand however, is varying and the quantity produced deviates from  $Q_2$  in order to meet demand. The output can only be increased or decreased by adjusting the factor labour, creating the short-run expansion path in Figure 1.1. One can see from this figure that the costs for producing quantities  $Q_1$  and  $Q_3$  on the short-run expansion path are higher than the costs for the same quantities on the long-run expansion path.

Both the long-run expansion path and the short-run expansion path translate into total cost functions. The typical S-shape of long-run total cost curves results from a long-run marginal cost function that is decreasing and then increasing in the production interval. The resulting long-run total cost curve then exhibits economies of scale as the average cost function is decreasing. This type

of marginal cost function can for example be found in the meat producing industry (Kern 1994). In the natural monopoly case, the marginal cost approaches the average cost from below without crossing it. Usually, diseconomies of scale will eventually lead to a situation where average costs start rising.

For each investment in a set of machines, a short-run cost function is assigned, which is tangent to the long-run cost function at that capacity. The short-run total costs represent the costs of operating the machines, e. g. to satisfy uncertain demand. These cost functions are convex under the assumptions that the marginal returns of the variable input factors are diminishing. The relationship between long-run total costs and short-run costs is depicted in Figure 1.2.



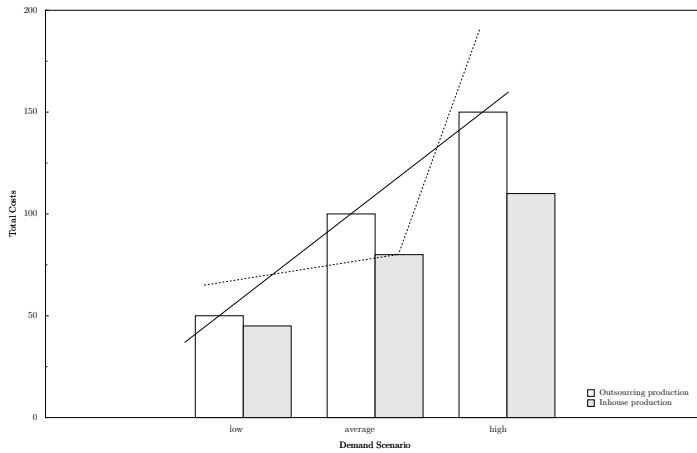
**Figure 1.2:** Long-run and short-run total facility cost function

In a stochastic setting, where investments in machines have to be made before demand is known, the connection between the long-run costs and short-run costs is as follows: The first-stage decision is to decide upon how many machines to invest in. This decision is based on the long-run total cost function and thereby implicitly decides the second-stage short-run cost function. Once the machines are installed, production is assigned in order to satisfy demand in the second-stage. The second-stage cost function is then the short-run total cost function, i. e. a deviation of the production level from the installed machine/labour-capacity is more costly than the long-run total costs. The short-run costs are not needed in the deterministic case: demand is known and there is no need to deviate from the long-run cost function.

## Using Flexibility To Handle Uncertainty

Uncertainty is widely recognized as one of the most important challenges in supply chain management. Supply chains need to react to changes in the supply of raw materials, demand for finished products, their prices, production costs and capacities. This uncertainty affects the planning and decision-making processes on all levels of the supply chain. Flexibility is frequently suggested as one way of coping with variations.

Upton (1994) defines flexibility as "the ability to change or react with little penalty in time, effort, cost or performance." To illustrate how flexibility can be used to handle uncertainty, consider the following simple example inspired by Wallace (1998). Assume that a company has to decide whether to produce a certain component itself or buy it from a supplier. The decision has to be made while the finished product is still developed, i. e. before demand for the component is known. The company believes that future demand for the finished product, and thus also for the component, is either low, average or high. Examining the total costs of producing the component inhouse for each of the possible demand scenarios and comparing it to the costs of outsourcing production reveals that it is always cheaper to produce inhouse than to outsource (see Figure 1.3).



**Figure 1.3:** Costs of inhouse production vs. outsourcing

Based on these numbers, it seems natural to decide for inhouse production. The remaining question is then how much capacity to invest in. Assume that the company installs capacity for the average demand scenario. As this decision

is made before true demand is known, they examine the expected costs of this solution. The dashed line in Figure 1.3 shows how the costs change if a demand scenario other than the average one is realized. They also analyze the expected costs of outsourcing production based on the solution for average demand (see the solid line in Figure 1.3 for the changes in outsourcing costs), and find that outsourcing, in expectation, is actually cheaper than inhouse production. The reason for this is that outsourcing production is the more flexible decision. Variations in demand can be handled at a lower cost than inhouse production.

The example above is very simple. Its main purpose is not to perfectly describe a real-world situation, but to illustrate how flexibility can be used to deal with uncertainty in the decision-making process. Here, deciding to outsource production, the company pays a premium for acquiring flexibility: the costs of outsourcing are higher than those of inhouse production for any given scenario. The company will however benefit from the decision in case demand deviates from the given demand scenario.

It is important observation that flexibility usually does not come for free, flexible decisions are more expensive than the inflexible ones. There are many different types of flexibility that can be used in a supply chain. Examples include volume flexibility, i. e. excess capacity, and the possibility to change delivery amounts and times, also known as delivery flexibility (Sabri & Beamon 2000). A common feature of all types of flexibility is however that it has to be designed into the supply chain and the necessary investments have to be justified by the potential benefits (Bertrand 2003).

In order to value flexibility correctly, it is important to explicitly consider the uncertainty present in the decision-making problem. In a deterministic setting, flexibility will not be needed and it has no value. Flexible decisions can have a large value in a stochastic setting due to the costs of correcting bad decisions once the uncertainty is revealed. Stochastic programming can explicitly value flexible decisions (see e. g. Christiansen & Wallace 1998, Fleten, Jørgensen & Wallace 1998), it models the sequential structure of a dynamic decision process and accounts for the fact that information is becoming available over time.

## **1.2 Quantitative Supply Chain Management**

The field of Supply Chain Management has attracted a lot of interest from the research community over the past 15-20 years, (see e.g. the reviews by Lambert & Cooper 2000, Ganeshan, Jack, Magazine & Stephens 1999 or Thomas & Griffin 1996). Early motivation for the concept of supply chain management however, can already be found in the work by Clark & Scarf (1960), who develop optimal policies for multi-echelon inventory systems. Another early example is the paper by Geoffrion & Graves (1974), that is among the first to use an optimization-based

approach for designing a multi-commodity distribution system. Since then, Operations Research has been applied to many areas of supply chain management. The handbooks by de Kok & Graves (2003) and Graves, Rinnooy Kan & Zipkin (1993) provide a good overview over possible applications.

The research in quantitative supply chain management can be distinguished in two approaches, based on who manages the supply chain: The first approach assumes a dominant member, managing the whole supply chain as a single entity, whereas the second approach focuses on cooperation and coordination of activities between agents, e. g. using contracts and/or incentive schemes (see the discussion in e. g. Ganeshan et al. 1999, Tomasgard & Høeg 2005). The research in the field of Operations Research usually follows the first approach, see e.g. the models for distribution and inventory planning (Simchi-Levi, Kaminsky & Simchi-Levi 2007), production planning and inventory management (Clark & Scarf 1960, Hax & Candea 1984) or also the more recent models on supply chain design (see e.g. Santos, Ahmed, Goetschalckx & Shapiro 2005). Ernst & Powell (1998) follow the second approach, discussing the use of incentives to coordinate the activities of a manufacturer and a retailer. The use of contracts and pricing scheme to coordinate a supply chain is described in Lariviere (1999), Tsay (1999) or Cachon & Lariviere (2005). See Cachon (2003) for a more detailed discussion on the use of contracts in supply chain coordination. Cachon & Netessine (2004) review game theory approaches to supply chain management.

This thesis takes the first approach, modeling the whole supply chain as controlled by a single, centralized decision-maker. Following the taxonomy by Ganeshan et al. (1999), the thesis can be classified as research using quantitative models to address supply chain management. The first three papers included here belong to the research on competitive strategy, dealing with decisions on the design of the supply chain and location of plants. The fourth paper is part of the research on operational efficiency, particularly the category on production, planning and scheduling.

## 1.3 Existing Literature and Research Contribution

In this section, I give a short introduction to some of the research literature relevant for this thesis and indicate how this thesis extends the existing literature. A more detailed overview over the specific research literature can be found in each of the individual papers. The papers will be discussed in the next section.

First, I will discuss the literature on facility location, before moving on to the literature on supply chain design. An overview over literature on operational supply chain planning completes this section.

## **Facility Location**

Facility location models address the problem of locating facilities in order to satisfy customer demand. The objective is usually to minimize the total costs, i. e. the sum of the costs of opening a facility plus the costs of satisfying customer demand from these facilities. These problems have been studied extensively since the 1950s. See Baumol & Wolfe (1958) or Cooper (1963) for some early examples on how to formulate and solve capacitated facility location problems. A good survey of research with focus on solution methods can be found in Labbé & Louveaux (1997). Snyder (2006) and Louveaux (1993) provide overviews over the literature on facility location under uncertainty.

In both the standard uncapacitated and capacitated facility location problem, the costs of opening a facility are just fixed setup costs (see e. g. Hax & Candea 1984). In combination with linear variable costs (i. e. constant marginal costs), this approach models economies of scale by simply distributing the fixed costs over a greater amount of units produced. In the real-world however, both facility costs and marginal costs often depend on the size of the facility (Haldi & Whitcomb 1967, Norman 1979, Kern 1994). Even diseconomies of scale, increasing average costs caused by increasing marginal costs, are common (Baumol, Panzar & Willig 1982, Mathis & Koscianski 2002).

Economies of scale and also diseconomies of scale have been represented using a staircase cost function in deterministic facility location problems (FLSC), see for example Holmberg (1984, 1994), Holmberg & Ling (1997) or Harkness & ReVelle (2002). The modular capacitated plant location model (MCPL) can be interpreted as a generalization of the FLSC, see e. g. Correia & Captivo (2003) or Correia & Captivo (2006). Both Holmberg & Ling (1997), Harkness & ReVelle (2002), Correia & Captivo (2003) and Correia & Captivo (2006) have used Lagrangean relaxation to solve the facility location problem. Other examples of deterministic models are Soland (1974), who develops an algorithm for a facility location problem with concave facility costs and concave transportation costs. Domschke & Voß (1990) consider a multi-product facility location model with concave production costs.

Even though a considerable number of researchers has studied facility location problems under uncertainty, few have examined the case of economies of scale. Balachandran & Jain (1976) explicitly address both uncertain demand and economies of scale. They present a formulation for a facility location model with a general, piecewise linear facility cost function and uncertain demand.

Papers 1 and 2 consider an application where the facility costs exhibit economies of scale. The cost curve has a typical S-shape, often found in long-run cost curves. Paper 1 considers the case of deterministic demand, whereas Paper 2 presents a model formulation for uncertainty in both demand and costs. We develop a solution method in each of the papers based on a piecewise linear approximation

of the facility cost function and Lagrangean relaxation. The solution methods are applied to a case from the Norwegian meat industry and we can show that they are capable of solving large-scale real-world applications.

The main contributions of paper 1 are the development of an efficient solution method for facility location problem with piecewise linear facility costs and its application to a case from the Norwegian meat industry. Paper 2 extends the setting of the first paper, including uncertainty in demand and costs. It also shows how long-run and short-run cost functions can be used in a two-stage stochastic programming model to link the first-stage and the second-stage decisions.

## Supply Chain Design

The field of supply chain design has received a lot of attention from the Operations Research community. Starting with the paper by Geoffrion & Graves (1974) on the design of a multicommodity distribution system, numerous optimization-based approaches for supply chain design have been proposed. The reviews by Melo, Nickel & Saldanha-da-Gama (2009), Vidal & Goetschalckx (1997) and Geoffrion & Powers (1995) provide a good overview over the different approaches.

Notable applications of supply chain design models include, amongst others, Arntzen, Brown, Harrison & Trafton (1995) who describe supply chain optimization for Digital Equipment Corporation. Camm, Chorman, Dill, Evans, Sweeney & Wegryn (1997) present the efforts to restructure the product-sourcing and distribution of Procter & Gamble's operations in North America. A model for redesigning the supply chain of Elkem's silicon division, at the time the world's largest producer of silicon metal and ferrosilicon, can be found in Ulstein, Christiansen, Grønhaug, Magnussen & Solomon (2006).

These applications follow the traditional approach of assuming that all input data, e. g. costs of raw materials and production or market prices for finished products, for the supply chain design problem is known with certainty. Given a planning horizon of several years, this assumption is not very likely to hold. An early example of research acknowledging this fact is the paper by Eppen, Martin & Schrage (1989). They develop a capacity planning model for General Motors using a scenario approach for uncertain demand. It has however not been before the end of the 1990s that researchers started to focus on supply chain design under uncertainty. MirHassani, Lucas, Mitra, Messina & Poojari (2000) discuss a supply chain design problem under uncertain demand. In order to solve the problem, they discuss the use of scenario analysis and Benders decomposition in a parallel implementation. A similar capacity planning problem is considered by Lucas, MirHassani, Mitra & Poojari (2001). They develop a solution method based on Lagrangean relaxation and scenario analysis. Alonso-Ayuso, Escudero, Garín, Ortuño & Pérez (2003) present a supply chain planning model with



binary first-stage decisions and continuous second-stage decisions. They solve the problem using an algorithmic approach based on Branch-and-Fix. Santoso et al. (2005) solve a supply chain design problem with uncertain demand using a solution method combining Sample Average Approximation (SAA) and Benders decomposition. They also present computational results from a real-world application.

Paper 3 presents an application from the Norwegian meat industry. We formulate and solve a large-scale supply chain design problem with uncertain demand. Main focus of the analysis is how the level of detail in modeling the operations of the supply chain affects the strategic design decisions. We explicitly distinguish between long-term uncertainty, e. g. uncertainty in the total demand level, and short-term uncertainty, e. g. weekly variations in demand. In order to properly represent short-term uncertainty, operational decisions have to be modeled in detail in the second-stage. The paper shows that detailed operational decisions in the second stage can affect strategic decisions, requiring more capacity than aggregate decisions in the second stage.

## **Operational Supply Chain Planning**

One of the first papers on supply chain planning is the one by Clark & Scarf (1960), developing optimal policies for multi-echelon inventory systems. The field of operational supply chain planning however, covers more than just inventory management. Production planning and scheduling, lotsizing, inventory management, and transportation are all important topics for the operational level of supply chain planning. The reviews by Drexel & Kimms (1997), Bhatnagar, Chandra & Goyal (1993) and Goyal & Gunasekaran (1990) on lot-sizing and scheduling, multi-plant coordination, and multi-stage production-inventory systems, respectively, provide an overview over relevant literature for operational planning.

The focus here is on production planning in a supply chain setting. A recent example for an application in this area is the paper by Denton, Forrest & Milne (2006), presenting an application of supply chain optimization at IBM's semiconductor business. They use a mixed-integer programming model for planning production, inventory, as well as internal and external shipments. Bredström, Lundgren, Rönnqvist, Carlsson & Mason (2004) develop a supply chain planning model for the pulp mill industry. The model includes all levels of the supply chain starting with the harvesting of trees, transportation of trees to the pulp mills, production scheduling at the mills, and distribution of pulp from the mills to the customers. These examples follow the traditional deterministic approach to operational planning. Uncertainty, even though an important factor in many supply chains, has not been considered. One of the first papers on operational

supply chain management under uncertainty is the one by Escudero, Galindo, García, Gómez & Sabau (1999). They develop a modeling framework for industrial production based on manufacturing, assembly, and distribution. Other examples of operational supply chain planning under uncertainty include Alonso-Ayuso, Escudero & Ortuño (2005) and Tomasgard & Høeg (2005). Alonso-Ayuso et al. (2005) formulate several models for production planning, machine selection and assignment, as well as scheduling and sequencing under uncertainty. Tomasgard & Høeg (2005) examine the supply chain in the Norwegian meat industry and develop a linear two-stage stochastic programming model for operational planning.

A topic closely related to the presence of uncertainty is flexibility. Deterministic models do not need flexible decisions and will never choose a flexible decision unless it comes for free. It is well understood that stochastic programming models are able to value, and hence favour, flexible decisions (see e.g. Christiansen & Wallace 1998, Fleten et al. 1998). There is however very little discussion about whether or not flexible operational decisions are needed if the supply chain itself is flexible and decisions can be revised. Paper 4 examines the need for operational decision flexibility, studying the results from both deterministic and stochastic planning models for different levels of flexibility in the supply chain. To my knowledge, it is the first paper that discusses the need and the value of decision flexibility for a production system with various degrees of flexibility already inherent in the supply chain.

## 1.4 Papers

The papers have been submitted to different scientific journals, having different layouts and requiring different referencing styles. For collecting the papers in this thesis, I standardized the layout and formatting of references. The content of the papers has not been modified.

### **Paper 1: Location of Slaughterhouses Under Economies of Scale**

We present a method for modeling economies of scale in uncapacitated facility location problems. The non-convex, non-concave objective function is approximated by a piecewise linear function. Using Lagrangean relaxation, the problem becomes separable in locations and we solve the resulting subproblems using a method originally developed for solving continuous knapsack problems. We apply the solution method to a case from the Norwegian meat industry and show that it is suitable for solving real-world problems.

The modeling of the problem as well as the development and implementation of the solution method was mainly done by the other three authors. My contribution was in making the code platform-independent, generating the case data, and performing the calculations. I also wrote some parts of the paper.

Co-authors: John van den Broek, Leen Stougie, and Asgeir Tomasgard.

The paper is published as Van den Broek, J., Schütz, P., Stougie, L. & Tomasgard, A. (2006), ‘Location of slaughterhouses under economies of scale’, *European Journal of Operational Research* **175**(2), 740–750.

## **Paper 2: Stochastic Facility Location with General Long-Run Costs and Convex Short-Run Costs**

This paper can be seen as a natural extension to Paper 1, introducing stochasticity in both costs and demand into the problem. The problem is formulated as a two-stage stochastic programming model. The first-stage and second-stage cost functions are derived from the microeconomic long-run and short-run cost functions respectively. We extend the solution method developed in the first paper for the case of uncertainty in both costs and demand. We allow for general first-stage cost functions, the second-stage costs however have to be convex. The solution method is successfully tested for real-world problem from the Norwegian meat industry.

I have formulated the model, implemented the solution method, and written large parts of the paper. In addition, I had an equal part in developing the solution method for the stochastic problem.

Co-authors: Leen Stougie and Asgeir Tomasgard.

The paper is published as Schütz, P., Stougie, L. & Tomasgard, A. (2008), ‘Stochastic facility location with general long-run costs and convex short-run costs’, *Computers & Operations Research* **35**(9), 2988–3000.

## **Paper 3: Supply Chain Design under Uncertainty using Sample Average Approximation and Dual Decomposition**

This paper takes a supply chain perspective and studies how operating the supply chain affects the design decisions. We formulate the design problem as a two-stage stochastic programming problem with demand as the uncertain factor. To examine the effect of the operational decisions, we distinguish between long-term and short-term uncertainty. The results from using real-world data from the Norwegian meat industry show, that short-term variations in demand

should be considered when designing the supply chain. To solve the different problem instances, we develop a solution method that combines Sample Average Approximation with Dual Decomposition.

I have formulated the model, implemented the solution method, and written large parts of the paper. In addition, I had an equal part in developing the solution method.

Co-authors: Asgeir Tomasgard and Shabbir Ahmed.

The paper is accepted for publication as Schütz, P., Tomasgard A. & Ahmed, S. (2008), 'Supply chain design under uncertainty using sample average approximation and dual decomposition', *European Journal of Operational Research*, forthcoming.

#### **Paper 4: The Impact of Flexibility on Operational Supply Chain Planning**

We discuss the value of operational decision flexibility, storage flexibility, delivery flexibility, and volume flexibility in operational supply chain planning under uncertain demand. Comparing a stochastic approach to the expected value approach and planning using perfect information, we study the impact of the different flexibility types on the annual operational results in the value chain. The results of our analysis show that operational decision flexibility becomes more important as flexibility inherent in the supply chain is reduced. This paper provides useful insights in the relationship between uncertainty and the value of flexibility.

I formulated and implemented the model. I also wrote large parts of the paper.

Co-author: Asgeir Tomasgard.

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# Paper I

John van den Broek, Peter Schütz, Leen Stougie  
and Asgeir Tomasgard:

## **Location of Slaughterhouses Under Economies of Scale**

*Published in European Journal of Operational Research*



# Paper 1

## Location of Slaughterhouses Under Economies of Scale

### **Abstract:**

The facility location problem described in this paper comes from an industrial application in the slaughterhouse industry of Norway. Investigations show that the slaughterhouse industry experiences economies of scale in the production facilities. We examine a location-allocation problem focusing on the location of slaughterhouses, their size and the allocation of animals in the different farming districts to these slaughterhouses. The model is general and has applications within other industries that experience economies of scale.

We present an approach based on linearization of the facility costs and Lagrangean relaxation. We also develop a greedy heuristic to find upper bounds. We use the method to solve a problem instance for the Norwegian Meat Co-operative and compare our results to previous results achieved using standard branch-and-bound in commercial software.

**Keywords:** Location, Integer Programming, Non-linear Programming, Branch and Bound, Economies of scale

## 2.1 Introduction

In this paper we investigate a facility location problem with linear transportation costs and economies of scale in the operation of facilities. The problem arose at the Norwegian Meat Co-operative when they did a strategic restructuring of their business in year 2000. When this work started there were 25 cattle slaughterhouses in the company. Our task was to investigate the saving potential of reducing this number and investing in capacity in the remaining facilities in order to profit from economies of scale. As this was part of a long term strategic analysis, we were asked to consider all municipalities in Norway as possible locations of slaughterhouses without giving preference to existing slaughterhouses: the meat co-operative wished to know the saving potential between today's solution and the ideal solution if they were free to build up their structure from scratch. They were also interested in finding out if there are many alternative solutions with about the same cost as the optimal solution. Our results have been used in a strategic restructuring of the network of slaughterhouses for cattle in Norway.

The Norwegian Meat Co-operative is owned by a majority (37000) of the Norwegian farmers. The annual turnover is about 1200 million Euro. The market share for the company in Norway was in 2000 about 76% for slaughtering. Because the company is organized as a co-operative, it cannot refuse a request from one of its members for slaughtering animals. It is free to choose which slaughterhouse should serve the request. However, there exists an animal welfare restriction, forbidding animals to be on transport in a truck for more than 8 hours. The aim of the study was to suggest the optimal size and location of slaughterhouses and an allocation of animals to the slaughterhouses, given today's geographical distribution of the animal population.

The company faces a trade off between the number of slaughterhouses it owns and its transportation costs. The problem is an uncapacitated facility location problem. Fundamental parameters of such a problem are the costs of operating and owning the facilities and the unit transportation cost between customers and the facilities. The objective is to minimize total costs.

In the standard uncapacitated facility location problem (see e.g. Hax & Candea 1984), facility costs are just fixed set-up costs. In our case the facilities have cost functions with economies of scale: unit slaughtering costs are decreasing as the number of animals allocated to the slaughterhouse increases (see for example Mathis & Koscianski 2002, for a definition). The total cost curve of each of the slaughterhouses has a typical S-shape, often found in long run cost curves, see Figure 2.2: the function is concave in the first part, when marginal costs are decreasing, and convex towards the end, when marginal costs are increasing. At all points the marginal costs are lower than the average costs, leading to economies of scale. The transportation costs are best described by a cost function that is linear in the distance between the farmers and the slaughterhouses. As a result, the objective function is non-linear, non-convex and non-concave.

There is an extensive research literature on facility location problems. Such models have been used since the 50's and early 60's (see for example Baumol & Wolfe 1958, Cooper 1963), but recent advances in optimization technology and the integration into information systems with decent user interfaces have made their dissemination wider. In location theory it is useful to separate the literature into two classes: Firstly we have optimization problems where the purpose is to minimize the cost or maximize the profit of locating a set of facilities under constraints like capacities, number of facilities, distance to customers and so on. A good survey of research with focus on solution methods can be found in Labbé & Louveaux (1997). Secondly, we have models where location is modelled for competitive companies in the tradition of Hotelling. An overview of such models with different market assumptions and objectives can be found in Eiselt, Laporte & Thisse (1993) and Eiselt & Laporte (2000).

Our paper is within the first class. In structure the problem resembles the facility location problem with staircase costs (FLSC), see for example Holmberg (1984, 1985, 1994), Holmberg, Rönnqvist & Yuan (1999), or Harkness & ReVelle (2002), or the modular capacitated plant location problem (MCPL), see for example Correia & Captivo (2003) or Correia & Captivo (2004). Our problem diverges from these by having a continuous and differentiable objective function. However, the solution strategy that we employed to tackle the problem brings us within the framework of these papers, still exploiting the continuity, as we will explain later in this section.

The problem of the Norwegian Meat Co-operative was first investigated in Borgen, Schea, Rømo & Tomasgard (2000) in 2000. The problem instance has 435 possible locations and 435 demand points. In Borgen et al. (2000) it was formulated as a mixed integer program with a piecewise linear objective, and a standard branch-and-bound method was used in the solution procedure (using xpress-mp). It was not possible to solve the problem to optimality due to the size of the branching tree and weak lower bounds from the LP-relaxation. In fact, the best results in Borgen et al. (2000) were obtained when using the commercial software in combination with a simple heuristic which reduced the number of possible locations to 45. Even this reduced problem instance was only solved within 10% of optimality in 12 hours. Comparing this solution value with the best lower bound found on the original problem showed a gap of 27%. The purpose of the paper we present here is twofold. Firstly, we reduced the gap between the lower bound and the upper bound on the problem instance mentioned above and thereby find a good enough solution to the real life problem. Secondly, we find a more efficient solution method in terms of the time spent.

The poor performance of the approach in Borgen et al. (2000) is due to the piecewise linear approximation of the total cost curve in the facilities, which leads to a weak LP-relaxation, as the authors indeed mention. We therefore propose decomposing the problem using Lagrangean relaxation. The relaxation makes the problem separable in the facilities, and we use an efficient algorithm based on a solution method for continuous knapsack to solve the subproblems.

The technique of Lagrangean relaxation together with a heuristic to turn the Lagrangean relaxation solutions into feasible solutions for the original problem has been successfully applied already in the past on ordinary capacitated and uncapacitated facility location problems. We refer to the celebrated paper by Cornuejols, Fisher & Nemhauser (1977) and for other examples to Holmberg et al. (1999), Shetty (1990) or Nemhauser & Wolsey (1988). The piecewise linear relaxation brings the problem within the framework of staircase costs or modular costs mentioned above. The continuity of the objective function allows us to use another model for representing the choice of the capacities of the facilities than those used in Holmberg & Ling (1997) and in Correia & Captivo (2003).



In Holmberg & Ling (1997) Lagrangean relaxation on different constraints is employed, in fact leading to a rather weak relaxation as noticed also in Correia & Captivo (2003). Our Lagrangean relaxation resembles the one in Correia & Captivo (2003). For deriving a feasible solution from infeasible optimal solutions of the Lagrangean relaxations, we have another method than the one in the latter paper. Our method always produces a feasible solution, whereas the one in Correia & Captivo (2003) may fail to do so. Implementing this method (which we did in 2001, indeed independently from Correia & Captivo (2003)), led to satisfactory results for the Norwegian Meat Co-operative. We managed to reach a solution which is provably within 1% of optimal in 95 minutes of computing time on a PC.

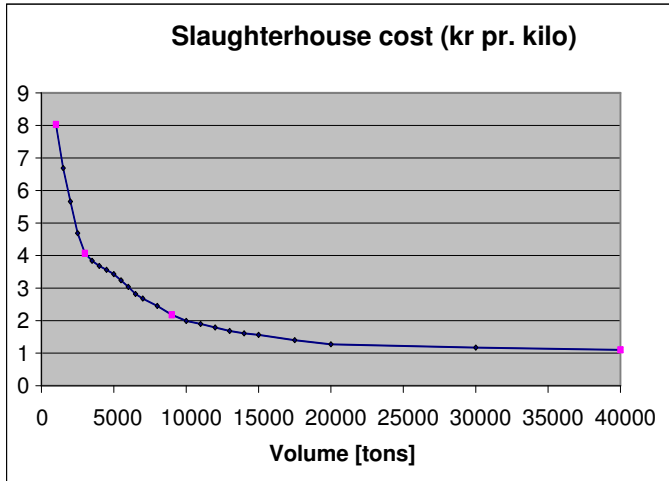
In Section 2.2 we give a detailed description of the problem we have solved for the Norwegian Meat Co-operative. We formulate the problem as a mixed integer program in Section 2.3. In Section 2.4 we present the Lagrangean relaxation, efficient solution of the subproblems, and a simple heuristic to generate feasible solutions from the infeasible solution of the Lagrangean relaxation. In Section 2.5 we show computational results. Apart from the results on the practical problem, we show additional computational tests on variations of the problem instance.

## **2.2 The Problem Data**

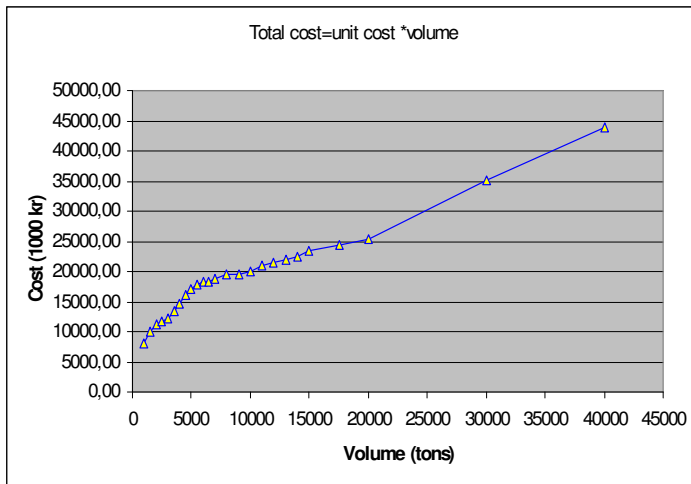
In this section we describe in detail the cost components of the problem and specify how we incorporate them in our model. We used unit cost data for slaughterhouses based on a German study (Kern 1994). The costs include fixed costs (capital cost, personal, insurance) and variable costs (energy, personal, water, cleaning, repairs, classification, material, waste management). The average cost function, cf. Figure 2.1, is close to convex and monotonically decreasing with volume, representing a situation with economies of scale. In Figure 2.2 the total cost curve of a facility is depicted. These functions are equal for all facilities. We approximate each of them by a piecewise linear function.

The transportation time is defined as the total duration from loading the first animal on the truck, until the last animal has left the truck at the slaughterhouse. It can be split in two parts:

- The collecting time: the time consumed while collecting the animals within the municipality, including driving time, stopping time and expected waiting time at the slaughterhouse before the truck is unloaded;
- The travelling time: the time of going from the municipality centre of the region where the animals are located to the municipality centre where the slaughterhouse is located.



**Figure 2.1:** Unit cost curve for slaughterhouses as a function of volume.



**Figure 2.2:** Total cost curve for slaughterhouses as a function of volume.

The collection time is approximated by the average time of filling up the car on a collection round-trip. As there was no indication of differences between the different regions we assumed equal velocity of the cars in all regions and no differences in collecting times or costs based on regions. The transportation operator is paid by the travelling distance, and has additional payment linear in the number of animals on the truck. Thus, the transportation costs are linear in the distance to the slaughterhouse and linear in the number of animals transported in the truck.

The choice between different car types is included in the model through preprocessing. We include a large car and a medium sized car in our analysis. It turns out that with our available cost data, large cars are always preferred to small cars, if feasible. The increased cost in driving from the slaughterhouse to the municipality and back is more than outweighed by the benefit of increasing the number of animals transported per trip. Still, the range of the large cars is limited. A larger car has to pick up more animals and thereby the collecting time increases. The 8 hour rule must be satisfied. This means that within some radius around a slaughterhouse the transportation costs are lower due to the ability to use larger trucks. Outside this range smaller cars must be used and the transportation costs increase slightly. We assume that cars pick up animals from one municipality only, and for each combination of slaughterhouse and demand municipality the best car size is found by preprocessing. This is reflected in the transportation cost and time matrices. In Norway there are 435 municipalities, hence the travel cost matrix and the travel time matrix will both have  $435^2$  elements defining the cost and the time needed to go between each pair of municipality centres.

## 2.3 The Mathematical Programming Model

In this section the problem is formulated as a mixed integer linear programming problem. The model resembles the model for the uncapacitated facility location problem (see e.g. Nemhauser & Wolsey 1988). The main difference is in the definition of the facility costs, as mentioned in the previous section. We approximate these by piecewise-linear non-convex, non-concave functions, modelled in a standard way by special ordered sets of type 2 (Williams 1999): using an ordered set of continuous variables, one for each breakpoint of the function, with value between 0 and 1. In a feasible solution at most two variables corresponding to neighbouring breakpoints have positive values adding up to 1. They specify a slaughterhouse volume as the convex combination of the two breakpoint volumes.

Let  $n$  be the number of points (municipalities) in the problem. We denote by  $K$  the number of breakpoints in the facility cost functions, by  $f_1, \dots, f_K$  the breakpoint volumes, and by  $p_1, \dots, p_K$  the unit slaughter costs at the breakpoints. The decision variables employed for representing the function in the special ordered

sets are  $y_{j1}, \dots, y_{jK}$  for a facility at location  $j$ ,  $j = 1, \dots, n$ . We define  $f_1 = 0$ . In this way  $y_{j1} = 1$  signifies that no facility is located at point  $j$ . To make sure that the fixed cost of installing a facility is represented,  $f_2$  is chosen small. Hence  $p_2$  becomes high as the fixed costs are shared only by these few units. Let  $y_j$ ,  $j = 1, \dots, n$ , be the  $K$ -dimensional vectors made up by all  $y_{jk}$ ,  $k = 1, \dots, K$ . We refer to Williams (1999) for a precise formulation of the constraints to represent a special ordered set of type 2. We represent the feasible set of values for  $y_j$  by  $Y_j$ ,  $j = 1, \dots, n$ . Let  $d_i$  denote the supply (number of animals to be slaughtered) in point  $i$ , and  $w$  the average weight of one unit of supply (one animal). By  $t_{ij}$  we denote the unit transportation cost from point  $i$  to point  $j$ . We define parameters  $a_{ij}$  as  $a_{ij} = 1$  if the transportation time from  $i$  to  $j$  is less than 8 hours (including the collecting time), and  $a_{ij} = 0$ , otherwise. We use the decision variables  $x_{ij}$  to denote the number of supply units transported from point  $i$  to point  $j$ . Although by its definition  $x_{ij}$  should be integer we allow it to take any non-negative real value, as we will discuss after giving the model. We call the following model MIP:

$$\min \quad \sum_{i=1}^n \sum_{j=1}^n t_{ij} x_{ij} + \sum_{j=1}^n \sum_{k=1}^K f_k p_k y_{jk}$$

$$\text{s.t.} \quad \sum_{j=1}^n x_{ij} = d_i, \quad i = 1, \dots, n, \quad (2.1)$$

$$\sum_{k=1}^K y_{jk} f_k = w \sum_{i=1}^n x_{ij}, \quad j = 1, \dots, n, \quad (2.2)$$

$$0 \leq x_{ij} \leq a_{ij} d_i, \quad i, j = 1, \dots, n, \quad (2.3)$$

$$y_j \in Y_j, \quad j = 1, \dots, n, \quad (2.4)$$

$$x_{ij} \in \mathbb{R}, \quad i, j = 1, \dots, n. \quad (2.5)$$

In the objective the term  $\sum_{k=1}^K f_k p_k y_{jk}$  is the slaughter costs at location  $j$ . The restrictions (1) enforce all supply from point  $i$  to be allocated to a facility. Restrictions (2) define the total volume allocated to the facilities. They also prohibit allocation to points without a facility. Restrictions (3) prohibit transportation on infeasible links. Restrictions (4) have been explained above. According to restrictions (5)  $x$  may take continuous values. This choice in the model is justified by the fact that, for any given  $y$ , the remaining problem is a bipartite transportation problem. Hence, if  $d_i$ ,  $i = 1, \dots, n$ , are integer and the breakpoints are integer multiples of  $w$ , then  $x$  will take integer values in any optimal solution.

There are no explicit binary variables in the model to indicate where facilities are to be located. This information is obtained from the values of the  $x$ -variables directly: no facility is located at point  $j$  if and only if  $x_{ij} = 0$  for all  $i$ . The information can also be derived from the values of the  $y$ -variables: no facility is located at point  $j$  if and only if  $y_{j1} = 1$ .

## 2.4 Lagrangean Relaxation

We define the Lagrangean relaxation by relaxing the supply constraints (1) in MIP. Introducing the vector  $\lambda = (\lambda_1, \dots, \lambda_n)$  of multipliers gives the Lagrangean subproblem:

$$\begin{aligned}
 LR(\lambda) = \min \quad & \sum_{i=1}^n \sum_{j=1}^n t_{ij} x_{ij} + \sum_{j=1}^n \sum_{k=1}^K f_k p_k y_{jk} + \sum_{i=1}^n \lambda_i (d_i - \sum_{j=1}^n x_{ij}) \\
 \text{s.t.} \quad & \sum_{k=1}^K y_{jk} f_k = w \sum_{i=1}^n x_{ij}, \quad j = 1, \dots, n, \\
 & 0 \leq x_{ij} \leq a_{ij} d_i, \quad i, j = 1, \dots, n, \\
 & y_j \in Y_j, \quad j = 1, \dots, n, \\
 & x_{ij} \in \mathbb{R}, \quad i, j = 1, \dots, n.
 \end{aligned}$$

We rewrite the objective function as

$$\min \sum_{j=1}^n \left( \sum_{i=1}^n t_{ij} x_{ij} + \sum_{k=1}^K f_k p_k y_{jk} - \sum_{i=1}^n \lambda_i x_{ij} \right) + \sum_{i=1}^n \lambda_i d_i.$$

Given  $\lambda$ , the last term is a constant, and therefore the objective is separable in  $j$ . We define  $LR(\lambda) = \sum_{j=1}^n g_j(\lambda) + \sum_{i=1}^n \lambda_i d_i$  with  $g_j(\lambda)$  the optimal value of the Lagrangean subproblem for location  $j$ :

$$\begin{aligned}
 g_j(\lambda) = \min \quad & \sum_{i=1}^n (t_{ij} - \lambda_i) x_{ij} + \sum_{k=1}^K f_k p_k y_{jk} \\
 \text{s.t.} \quad & \sum_{k=1}^K y_{jk} f_k = w \sum_{i=1}^n x_{ij}, \\
 & 0 \leq x_{ij} \leq a_{ij} d_i, \quad i = 1, \dots, n, \\
 & y_j \in Y_j, \\
 & x_{ij} \in \mathbb{R}, \quad i = 1, \dots, n.
 \end{aligned}$$

### Solving The Subproblem

We describe how the subproblem for each location  $j$  can be solved. The total slaughter cost is strictly monotonically increasing in the total weight. The unit increase in cost between the breakpoints  $k$  and  $k + 1$  is denoted by  $\alpha_k$ . We define the breakpoints of the cost function in terms of the number of animals as  $F_k = f_k/w$ ,  $k = 1, \dots, K$ , and we define  $F_{K+1} = \infty$ . These breakpoints may be fractional values, even if they are defined as a number of animals. The total slaughter costs in  $g_j(\lambda)$  at slaughterhouse  $j$  in these terms is then  $\sum_{k=1}^K w F_k p_k y_{jk}$ . The slope of the linear segment of the cost function between breakpoint  $k$  and

$k+1$  is denoted by  $\alpha_k$ : i.e.,  $\alpha_k = \frac{w(F_{k+1}p_{k+1}-F_kp_k)}{F_{k+1}-F_k}$ ,  $k = 1, \dots, K-1$ , and  $\alpha_K = wp_K$ .

Let us consider the subproblem for location  $j$ . Related to this location we define another  $K$  subproblems, one for each  $k = 1, \dots, K$ . For each  $i = 1, \dots, n$  let  $q_{ik} = t_{ij} - \lambda_i + w\alpha_k$  be the extra cost for bringing one more animal from location  $i$  to location  $j$ . For  $k = 1, \dots, K$  define

$$\begin{aligned} g_{jk}(\lambda) = \min \quad & \sum_{i=1}^n q_{ik}x_{ij} \\ \text{s.t.} \quad & F_k \leq \sum_{i=1}^n x_{ij} \leq F_{k+1}, \\ & 0 \leq x_{ij} \leq a_{ij}d_i, \quad i = 1, \dots, n, \\ & x_{ij} \in \mathbb{R}, \quad i = 1, \dots, n. \end{aligned}$$

Thus,  $g_j(\lambda) = \min_k g_{jk}(\lambda)$ ,  $j = 1, \dots, n$ . To find  $g_{jk}(\lambda)$  a method similar to the one for solving continuous knapsack problems is applied (Martello & Toth 1990). That method has to be adapted for the *lower* bounds on total capacity.

We order the points  $i$  in order of increasing  $q_{ik}$ :  $q_{1k} \leq \dots \leq q_{nk}$ . Start setting  $x_{ij}$ 's equal to their maximum value  $a_{ij}d_i$  in that order until for some order index  $(i_1)$  for the first time  $\sum_{l=1}^{i_1} a_{lj}d_l \geq F_k$ . If  $q_{(i_1)k} < 0$  set  $x_{(i_1)j} = a_{(i_1)j}d_{(i_1)}$ . Otherwise set  $x_{(i_1)j} = \lceil F_k \rceil - \sum_{l=1}^{i_1-1} a_{lj}d_l$ . In the latter case  $x_{lj} = 0$  for all  $l = i_1+1, \dots, n$  and the optimal solution is found. In the former case continue until for some index  $(i_2)j$  for the first time either  $q_{(i_2)k} \geq 0$ , in which case we set  $x_{(i_2)j} = 0$ , or  $\sum_{l=1}^{i_2} a_{lj}d_l > F_{k+1}$ , in which case we set  $x_{(i_2)j} = \lfloor F_{k+1} \rfloor - \sum_{l=1}^{i_2-1} a_{lj}d_l$ . In both cases set  $x_{lj} = 0$  for all  $l = i_2+1, \dots, n$ . Non-existence of an index  $(i_1)$  means that  $\sum_{i=1}^n a_{ij}d_i < F_k$ . In that case the problem is infeasible and we set  $g_{jk}(\lambda) = \infty$ .

## The Lagrangean Dual

The best lower bound on the optimal solution value of the original problem will be found by solving the Lagrangean dual problem (LD):

$$\max_{\lambda} LR(\lambda).$$

We do so using sub-gradient optimization (e.g. see Nemhauser & Wolsey 1988). The sub-gradient optimization routine that we have used is standard and often used for facility location problems. For example, it can be found in Holmberg et al. (1999).

The partial derivative of  $LR$  is given by

$$\delta_i = \frac{\partial LR(\lambda)}{\partial \lambda_i} = d_i - \sum_{j=1}^n x_{ij}(\lambda),$$

**Initialise:** Choose values for  $\epsilon_1 > 0$ ,  $\epsilon_2 > 0$ ,  $V$ ,  $V_1$  and  $\eta_0$ .  
 Set UB equal to the value of some approximate solution. Set  $LB \leftarrow -\infty$ .  
 Set  $v \leftarrow 1$ ,  $v_1 \leftarrow 1$ , choose starting point  $\lambda^{(1)}$ , and set  $\eta = \eta_0$ .  
**Iterate:** Until  $v = V$ ,

1. Determine  $LR(\lambda^{(v)})$ .  
 If  $LR(\lambda^{(v)}) > LB$ , set  $LB \leftarrow LR(\lambda^{(v)})$  and  $v_1 \leftarrow 0$ ;  
 Else, set  $v_1 \leftarrow v_1 + 1$ . If  $v_1 = V_1$  set  $\eta \leftarrow \frac{\eta}{2}$  and set  $v_1 \leftarrow 0$ ;
2. Derive a feasible solution  $x^G$  from  $x(\lambda^{(v)})$ , yielding value  $G^{(v)}$ . If  $G^{(v)} \leq UB$ , set  $UB \leftarrow G^{(v)}$ ,  $x^* \leftarrow x^G$  and set  $\eta \leftarrow \eta_0$ . If  $UB - LB < 1$ , stop: UB is the optimal solution value.
3. Calculate the gradient  $s^{(v)} = \nabla LR(\lambda^{(v)})$ , set step length  $t^{(v)} = \eta \frac{UB - LR(\lambda^{(v)})}{\|s^{(v)}\|^2}$ , and  $\lambda^{(v+1)} \leftarrow \lambda^{(v)} + t^{(v)} s^{(v)}$ .
4. If  $\|s^{(v)}\| \leq \epsilon_1$  or  $\|\lambda^{(v+1)} - \lambda^{(v)}\| \leq \epsilon_2$ , stop;  
 Else, set  $v \leftarrow v + 1$ .

**Output:** LB;UB; $x^*$ .

**Figure 2.3:** Subgradient algorithm.

with  $x_{ij}(\lambda)$  the optimal solution of the Lagrangean relaxation with multipliers  $\lambda$ . Hence, the gradient of  $LR$  is given by  $\nabla LR(\lambda) = (\delta_1, \dots, \delta_n)$ . For sake of completeness we present the subgradient algorithm in Figure 2.3 in pseudo-code. The algorithm involves determination of an upper bound on the optimal solution, the description of which is given below.

For given  $\lambda$ ,  $LR(\lambda)$  yields a lower bound on the optimal solution value, but in general the optimal solution of the Lagrangean relaxation is not a feasible solution to MIP. In Step 2 in the algorithm we use a heuristic to find an upper bound  $G^{(v)}$  by finding a feasible solution to MIP based on the solution of the Lagrangean relaxation in iteration  $v$ . In the next section we present this heuristic.

## An Approximate Solution Based On Lagrangean Relaxation

In general the optimal solution of the Lagrangean relaxation is not feasible to MIP. Here we show how to turn an optimal solution  $x_{ij}(\lambda)$  of the Lagrangean relaxation, given  $\lambda$ , into a feasible solution of MIP:

1. We start by setting  $x_{ij}^1 = d_i$  if  $x_{ij}(\lambda) > 0$ ,  $\forall i, j$ . We also introduce artificial variables  $z_j^1$ , which get value 1 if there exists an  $i$  such that  $x_{ij}^1 = d_i$ , and

- 0 otherwise. (These variables indicate if a facility is located at point  $j$  or not.)
2. If for  $i$  there are indices  $j$  such that  $x_{ij}^1 = d_i$ , then from among those  $j$  choose the one with minimum  $t_{ij}$ , index  $j^*$  say, and set  $x_{ij^*}^2 = d_i$  and  $x_{ij}^2 = 0, \forall j \neq j^*$ .
  3. If for  $i, x_{ij}^1 = 0 \forall j$  and there exists indices  $j$  for which  $z_j^1 = 1$  and  $a_{ij} = 1$ , then choose from the latter the one with minimum  $t_{ij}$ , index  $j^0$  say, and set  $x_{ij^0}^2 = d_i$  and  $x_{ij}^2 = 0, \forall j \neq j^0$ .
  4. If for  $i, x_{ij}^1 = 0, \forall j$  and no index  $j$  exists for which  $z_j^1 = 1$  and  $a_{ij} = 1$ , then set  $x_{ij}^2 = 0, \forall j$ .
  5. Set  $z_j^2 = 0$  if and only if  $x_{ij}^2 = 0, \forall i$ . Otherwise set  $z_j^2 = 1$ .
  6. We finish by greedily locating facilities for the set of  $I = \{i \mid x_{ij}^2 = 0, \forall j\}$ : For every  $j$  with  $z_j^2 = 0$  determine  $I_j = \{i \in I \mid a_{ij} = 1\}$ . Choose the set  $I_{j'}$  with highest cardinality (ties are broken arbitrarily), locate a facility in the corresponding point  $j'$ , i.e., change  $z_{j'}^2 = 0$  into  $z_{j'}^2 = 1$  and reset  $x_{ij'}^2 = a_{ij'} d_i, \forall i \in I_{j'}$ . Repeat this step as long as the set  $I$  is not empty.

Given this approximate solution, a set of facilities is opened (those with  $z_j^2 = 1$ ), and supply is allocated as suggested by  $x_{ij}^2, \forall i, j$ . At iteration  $v$  in the algorithm from Figure 2.3 we denote this solution value by  $G^{(v)}$  and define  $G = \min_v G^{(v)}$ .

This solution can be improved by fixing the locations of the facilities installed according to  $z^2$  and using a general purpose branch-and-bound code to get the best possible allocation given these locations, yielding a solution value denoted by  $G_S$ .

## 2.5 The Computational Results

We present here the most important results from solving the problem instance with 435 location points and 435 demand points. In addition to the original problem instance, we ran the algorithm with several related cases. A case is described by a combination of scenarios for demand ( $D$ ) and transportation costs ( $C$ ). The dataset  $(D, C) = (0, 0)$  represents the original problem instance. The demand scenarios  $D = 1 \dots 10$  are generated randomly from a multivariate normal distribution with expectation equal to the original animal population and without correlation between the different regions. The cost scenarios consider an increase in transportation costs of 20% and 40% as well as a decrease of 20% and



40% with respect to the original cost data. The results from the calculations are shown in Table 2.1 below. The costs are given in NOK 1000. The gap is defined as  $\frac{UB(G_S) - LB}{UB(G_S)}$ .

In the computational experiments we use 6 breakpoints to approximate the facility cost curve. The points given in (*tons/year*, *kr/kilo*) are (rounded from the nearest multiple of  $w$ , the average weight of an animal): (0, 0), (1, 8000), (1000, 8.03), (3000, 4.07), (9000, 2.18), (40000, 1.10). The number and the values of the breakpoints have been chosen in direct consultation with the Norwegian Meat Co-operative, already by Borgen et al. (2000). Choosing the same ones also facilitates comparison to the first attempt to tackle the slaughterhouse problem in Borgen et al. (2000).

To set parameters in the algorithm, we performed some first test runs, which indicated that the best choice for  $\eta_0$  was around 2, and for both  $\epsilon_1$  and  $\epsilon_2$  it was  $1.0 \cdot 10^{-20}$ . We set the maximum number of iterations to  $V = 25000$ . Test runs showed that the initial values for the dual multipliers did not have much influence on the performance. We used  $V_1 = 225$  to limit the number of iterations without improvement, and report the best results in Table 2.1.

**Table 2.1:** Computational results

Dataset		$LB$	$UB(G)$	$UB(G_S)$	Runtime	Gap (%)
$D$	$C$					
0	0	241467	244162	243787	1:33:55	1.0%
0	+20%	259672	262013	261918	1:32:00	0.9%
0	+40%	277727	280067	279975	1:31:21	0.8%
0	-20%	222826	228464	226375	1:34:32	1.6%
0	-40%	204036	209641	207998	1:33:16	1.9%
1	0	238487	238595	238571	1:31:49	0.0%
2	0	234211	246894	242547	1:30:54	3.4%
3	0	244099	252300	250484	1:32:59	2.5%
4	0	241412	252578	248077	1:32:24	2.7%
5	0	243834	247560	247176	1:31:45	1.2%
6	0	230981	246829	243451	1:20:25	5.1%
7	0	248278	258753	256339	1:26:23	3.1%
8	0	244406	254396	252288	1:33:35	3.1%
9	0	243119	254074	250583	1:30:43	3.0%
10	0	245017	250707	249476	1:33:46	1.8%

The best results from the previous attempt to solve the original problem instance in Borgen et al. (2000) was a solution with 11 slaughterhouses, a solution

value of 258101 and an optimality gap of 27%. The best lower bound found for the original problem instance using our approach is 241467. The best feasible solution value( $G$ ) found by the 6-step method described in Section 2.4 is 244162. The best solution found after improvement as described at the end of Section 2.4 has value  $G_S = 243787$ . Hence, within running times of approximately 95 minutes the gap can be reduced to around 1%. Compared to previous results, our method shows substantial improvements both in running time, lower bound and upper bound.

Equally important is the effect on the solution itself. The method is able to reduce the optimal number of slaughterhouses from 11 to 9 for all problem instances based on the original animal population<sup>1</sup> ( $D = 0$ -instances). The best feasible solutions for demand scenarios  $D = 1, 5, 10$  have 9 slaughterhouses, whereas the method finds solutions with 10 slaughterhouses for the remaining datasets.

The transportation costs account for 37.8% of the total costs in the original problem instance. A raise in the costs of transportation of 40% results in a share of about 45% of the total costs. Decreasing the transportation costs by 40 % reduces the share to approximately 27% of the total costs. We also analyzed the impact of changing the transportation costs when it comes to the sizes and locations of slaughterhouses. This is shown in Table 2.2 where the best solutions are given in terms of the chosen locations and their size. Locations are numbered from 1 to 435 and the sizes of the slaughterhouses are given in tons. The level of the transportation costs has only a small influence on the geographical location of the slaughterhouses and the allocation of animals is also changing only slightly. Solving the transportation problem given the locations from the original problem instance, but with the alternative transportation costs, results in similar solutions in terms of costs. All our analyses show that there are many alternative solutions with almost the same cost level. What matters seems to be the number of slaughterhouses, which should be as low as possible.

## 2.6 Conclusions

We have shown how to model economies of scale in uncapacitated facility location problems. Our problem instance is motivated from an application where the purpose is to decide location and size of a set of slaughterhouses. Still the formu-

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<sup>1</sup>In fact, the best solution also has a tenth slaughterhouse, where only a single animal was allocated. This comes from the fact that the animal is located on a remote island. The facility cost of installing a single animal facility is about 2 million kroner in the model. Still this is lower than the alternative cost of shifting the whole slaughterhouse structure along the cost to bring this animal within the 8 hour limit for transportation to the closest facility. In practice, of course, this is solved by a local barbecue.

**Table 2.2:** Slaughterhouse location and size (in tons)

Case (0,0)		Case (0,+20%)		Case (0,+40%)		Case (0,-20%)		Case (0,-40%)	
Location	Size	Location	Size	Location	Size	Location	Size	Location	Size
79	17386	79	17382	79	17382	79	18577	79	18577
184	12930	184	12930	184	12930	184	13108	184	13108
238	6972	238	7031	238	7,031	225	7401	225	7401
291	16097	291	14563	291	14563	280	7272	280	7272
339	12741	323	12477	323	12477	330	17566	330	17647
363	1872	363	3615	363	3615	363	4074	363	3992
395	1385	395	2008	395	2008	395	1385	395	1385
415	1585	415	962	415	962	415	1585	415	1585
432	400	432	400	432	400	432	400	432	400

lation is general and can be viewed as an extension of the classical uncapacitated problem.

When the non-convex and non-concave objective function is approximated by a piecewise linear function using specially ordered sets, we seem to get a weak LP-formulation. By using Lagrangean relaxation we are able to improve the lower bound. Also by implementing a simple greedy heuristic we manage to find feasible solutions from the infeasible Lagrangean solutions. The use of Lagrangean relaxation reduces solution time for the problem and improves the quality of the solutions dramatically.

There is reason to believe that the approach shown in this paper will perform even better if the bounds are used in a branch-and-bound scheme. However, our main interest was to solve this real life problem close to optimality to make the company happy. This happened far before the gap was reduced to 1.0%, and results from this research have been important input in the restructuring process of the company.

The various runs also showed that a lot of different solutions exist to the problem with little difference in solution values, but with different locations in the solution.



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## Paper II

Peter Schütz, Leen Stougie and Asgeir Tomasgard:

# **Stochastic Facility Location with General Long-Run Costs and Convex Short-Run Costs**

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## Paper 2

# Stochastic Facility Location with General Long-Run Costs and Convex Short-Run Costs

### Abstract:

This paper addresses the problem of minimizing the expected cost of locating a number of single product facilities and allocating uncertain customer demand to these facilities. The total costs consist of two components: firstly linear transportation cost and secondly the costs of investing in a facility as well as maintaining and operating it. These facility costs are general and non-linear in shape and could express both changing economies of scale and diseconomies of scale. We formulate the problem as a two-stage stochastic programming model where both demand and short-run costs may be uncertain at the investment time. We use a solution method based on Lagrangean relaxation, and show computational results for a slaughterhouse location case from the Norwegian meat industry.

**Keywords:** Facility location, Stochastic programming, Lagrangean relaxation, Economies of scale

## 3.1 Introduction

Mathematical programming approaches to model and solve facility location models have been extensively studied since the 1950's (Baumol & Wolfe 1958, Cooper 1963). In this paper we deal with two issues that have been analyzed separately to some degree, but rarely in combination: non-linear facility costs and stochasticity in costs and demand. From a model perspective our work is a generalization of an early paper by Balachandran & Jain (1976) addressing both these issues. Our work may also be seen as a natural extension of Van den Broek, Schütz, Stougie & Tomasgard (2006) where we look at the deterministic variant of the model.

Before we move on to a description of the structure of the paper, we will give a short overview of relevant literature. Traditionally, the facility costs are treated as fixed set-up costs and linear variable costs (see e. g. Kuehn & Hamburger 1963, Hax & Candea 1984, Louveaux 1993). This is a situation where marginal costs are constant and the economies of scale come from sharing the fixed cost on more

units. In real-world applications however, both the fixed part of facility costs and the marginal costs often depend on the size of the facility (Haldi & Whitcomb 1967, Norman 1979, Kern 1994). Usually the degree to which economies of scale are experienced changes with volume and even diseconomies of scale are common (see e. g. Baumol, Panzar & Willig 1982, Mathis & Koscianski 2002).

Changing economies of scale and diseconomies of scale have been represented by means of a deterministic facility location problem with staircase costs (FLSC), see for example Holmberg (1984, 1994),

Holmberg & Ling (1997) or Harkness & ReVelle (2002). The modular capacitated plant location model (MCPL) can be interpreted as a generalization of the FLSC, see e. g. Correia & Captivo (2003) or Correia & Captivo (2006). Other deterministic approaches are Soland (1974) which develops an algorithm for a facility location problem with facility costs that are concave in the amount produced and transportation costs that are concave in the amount shipped and Domschke & Voß (1990) with a multi-product facility location model with concave production costs. Van den Broek et al. (2006) present an application from the Norwegian meat industry.

Good overviews over the literature on facility location under uncertainty can be found in the reviews by Louveaux (1993) and Snyder (2006). Some examples are listed here: Louveaux & Peeters (1992) present a two-stage stochastic programming problem with uncertainty in demand, selling prices, as well as in production and transportation costs, while Laporte, Louveaux & van Hamme (1994) include also establishment of transportation channels between a facility and customers. In Eppen, Martin & Schrage (1989) a two-stage formulation for capacity expansion is presented, and a multi-stage capacity expansion problem under uncertain demand is presented by Ahmed, King & Parija (2003).

In our two-stage approach we model the first stage decision as a design capacity interval rather than a fixed point. Inside this interval the variable cost in the second stage is linear and equal to the long-run cost. Volumes outside this interval may be produced subject to a piecewise linear short-term cost function in the second stage. Hence our approach can be seen as a hybrid between the capacitated and uncapacitated problem.

We decompose the problem using Lagrangean relaxation (Geoffrion 1974, Shapiro 1979). Relaxing the demand constraints makes the problem separable in the facilities and we apply an efficient algorithm based on a solution method for the continuous knapsack problem to solve the subproblems. Some earlier examples where Lagrangean relaxation has been used in combination with a heuristic to solve deterministic facility location problems are Cornuejols, Fisher & Nemhauser (1977), Shetty (1990), Beasley (1993), or Holmberg, Rönnqvist & Yuan (1999). Lagrangean relaxation has also been used for solving the deterministic FLSP and the MCPL, see e. g. Holmberg & Ling (1997), Harkness & ReVelle (2002),

Correia & Captivo (2003) and Correia & Captivo (2006). Our algorithm for the stochastic model is a modification of the one used by Van den Broek et al. (2006) for the deterministic version of our problem formulation. The solution method we choose allows for solving problems of sizes met in real life cases.

In Section 3.2 we provide the stochastic programming formulation for a facility location problem with a non-linear, non-convex, non-concave objective function, uncertain short-run costs and uncertain demand. Our solution method is presented in Section 3.3. A full-size real life case from the Norwegian meat industry is described in Section 3.4. Computational results for this problem are shown in Section 3.5. We present conclusions in Section 3.6.

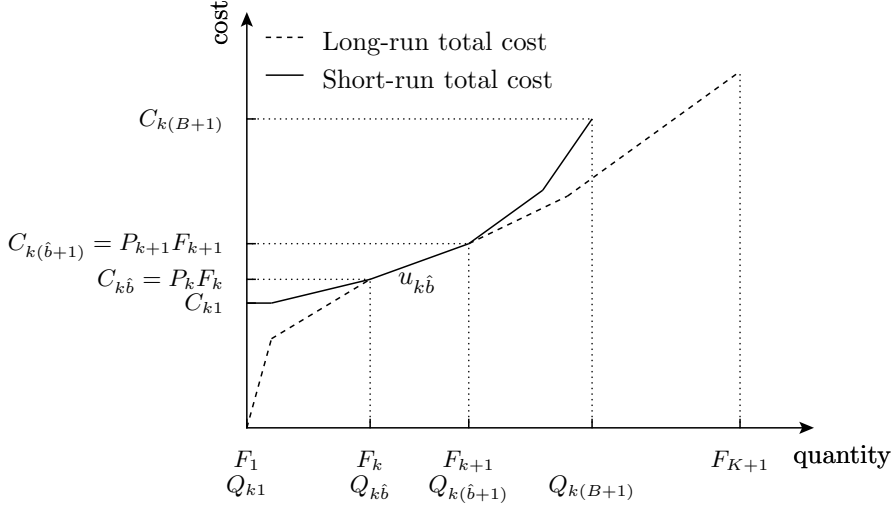
## 3.2 The Mathematical Programming Model

We provide a two-stage stochastic programming formulation for a facility location problem with non-convex non-concave facility costs, linear transportation costs and uncertain demand. We approximate both the first-stage facility cost function and the second-stage facility cost function by piecewise linear functions. The cost functions that are underlying the first-stage and second-stage decisions are the long-run and the short-run cost function, respectively (see e. g. Mathis & Kosciński (2002) or Perloff (2004)). The typical S-shape of long-run total cost curves results from a long-run marginal cost function that is decreasing and then increasing in the production interval. The resulting long-run total cost curve then exhibits economies of scale as long as the average cost function is decreasing.

In the short-run, costs are higher than the costs for the same quantities on the long-run function. To each installed capacity, a short-run cost function is assigned which is tangent to the long-run cost function at that capacity. The short-run total costs represent the costs of operating a facility given the installed capacity. These short run cost functions are convex under the assumption that the marginal returns of the variable input factors are diminishing. The relationship between long-run total costs and short-run costs is depicted in Figure 3.1.

We approximate the first-stage facility cost function by  $K$  linepieces,  $k = 1, \dots, K$ . The breakpoint volumes are given as  $F_1, \dots, F_{K+1}$ , with  $P_1, \dots, P_{K+1}$  being the corresponding total per unit costs of production.  $F_1$  and  $P_1$  are both 0. The first-stage decision is to determine the designed capacity interval for the facilities. The designed capacity is described by the lower and upper capacity limit of the chosen linepiece  $k$  on the first-stage facility cost function. After the facilities are opened, production is assigned to the open facilities in order to satisfy demand in the second-stage.

There is one second-stage cost function for each linepiece  $k$  in the first stage. Each of these short-run cost functions consists of  $B$  line-segments, thus having  $B + 1$  breakpoints. We denote the breakpoint volumes of this function by  $Q_{kb}$ .



**Figure 3.1:** Approximated first-stage and second-stage facility cost function

The total costs at each breakpoint are given by  $C_{kb}$ . The slope of a linepiece of the second stage cost function is given as  $u_{kb}$ . We define  $Q_{k\hat{b}} = F_k$  and  $Q_{k(\hat{b}+1)} = F_{k+1}$ . Note that  $C_{k\hat{b}} = P_k F_k$ , and  $C_{k(\hat{b}+1)} = P_{k+1} F_{k+1}$ , because at the designed facility capacity interval decided by the first-stage investment decision, the short-run cost curve is tangent to the long-run curve.

In the following we assume that all facilities have the same cost function. This is not a necessary assumption for the modelling and decomposition approach we have chosen, but we have not tested the quality of the heuristic used to find upper bounds in cases where they are different.

## A Two-stage Recourse Formulation

We introduce the following notation for the two-stage stochastic programming problem:

- Sets

$\mathcal{I}$	Set of customer locations.
$\mathcal{J}$	Set of possible facility locations.
$\mathcal{S}$	Set of scenarios.
$\mathcal{K}$	Set of linepieces of the first-stage cost function.

$\mathcal{B}$  Set of breakpoints of the second-stage facility cost function.

- Indices and superscripts

$i$  Customer location.  
 $j$  Possible facility location.  
 $s$  Scenario.  
 $k$  Linepiece of the first-stage cost function.  
 $b$  Breakpoint of the second-stage facility cost function.

- Parameters, constants, and coefficients

$T_{ij}$  Cost of serving one unit of demand at customer location  $i$  from facility location  $j$ .  
 $L_{ij}$  1 if demand at customer location  $i$  can be served from facility location  $j$ , 0 otherwise.  
 $Q_{kb}$  Production volume at breakpoint  $b$  of the second-stage facility cost function given linepiece  $k$  of the first-stage cost function.  
 $C_{kb}^s$  Costs at breakpoint  $b$  in scenario  $s$  of the second-stage facility cost function given linepiece  $k$  of the first-stage cost function.  
 $D_i^s$  Demand at customer location  $i$  in scenario  $s$ .  
 $p^s$  Probability of scenario  $s$ .

- Decision variables

$y_{jk}$  1 if linepiece  $k$  is chosen as a first-stage capacity decision at facility location  $j$ , 0 otherwise.  
 $x_{ij}^s$  Amount of customer demand at location  $i$  satisfied from facility location  $j$  in scenario  $s$ .  
 $\mu_{jkb}^s$  Weight of breakpoint  $b$  for linepiece  $k$  at location  $j$  in scenario  $s$ .

We also define an artificial breakpoint of the facility cost function  $F_0 = 0$  and  $P_0 = 0$ , such that choosing line piece  $k = 0$  means that no facility is opened (We already defined that  $F_1 = 0$  and  $P_1 = 0$ ).

With this notation we get the following formulation for our two-stage stochastic facility location problem:

$$\min \sum_{s \in \mathcal{S}} p^s \mathcal{Q}^s(y) \quad (3.1)$$

subject to

$$\sum_{k \in \mathcal{K}} y_{jk} = 1, \quad \forall j \in \mathcal{J}, \quad (3.2)$$

$$y_{jk} \in \{0, 1\}, \quad \forall j \in \mathcal{J}, k \in \mathcal{K}. \quad (3.3)$$

The second-stage problem is given as

$$\mathcal{Q}^s(y) = \min \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} T_{ij} x_{ij}^s + \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \sum_{b \in \mathcal{B}} C_{kb}^s \mu_{jkb}^s \quad (3.4)$$

subject to

$$\sum_{j \in \mathcal{J}} x_{ij}^s = D_i^s, \quad \forall i \in \mathcal{I}, s \in \mathcal{S}, \quad (3.5)$$

$$x_{ij}^s \leq L_{ij} D_i^s, \quad \forall i \in \mathcal{I}, j \in \mathcal{J}, s \in \mathcal{S}, \quad (3.6)$$

$$\sum_{i \in \mathcal{I}} x_{ij}^s = \sum_{k \in \mathcal{K}} \sum_{b \in \mathcal{B}} Q_{kb} \mu_{jkb}^s, \quad \forall j \in \mathcal{J}, s \in \mathcal{S}, \quad (3.7)$$

$$\sum_{b \in \mathcal{B}} \mu_{jkb}^s = y_{jk}, \quad \forall j \in \mathcal{J}, k \in \mathcal{K}, s \in \mathcal{S}, \quad (3.8)$$

$$x_{ij}^s \geq 0, \quad \forall i \in \mathcal{I}, j \in \mathcal{J}, s \in \mathcal{S}, \quad (3.9)$$

$$\mu_{jkb}^s \geq 0, \quad \forall j \in \mathcal{J}, k \in \mathcal{K}, b \in \mathcal{B}, s \in \mathcal{S}. \quad (3.10)$$

Restrictions (3.2) ensure that only one capacity interval is chosen for each location in the first stage. The objective function of the second-stage problem (3.4) is given as the sum of transportation and production costs. Constraints (3.5) force all demand at location  $i$  to be assigned. Constraints (3.6) only allow assignment of demand to locations where the demand can be satisfied. Constraints (3.7) and (3.8) ensure that demand is allocated to open facilities only. Restrictions (3.8) also link the correct second-stage cost function to the first-stage decision. (3.9)-(3.10) are the non-negativity constraints.

In this formulation we allow for uncertainty both in the right hand side in the demand and in the objective in the short-run facility costs. The decomposition approach we choose and the following sections also support both these uncertainties. The heuristic used to find upper bounds in Section 3.3 is also general in this respect, though better suited for the situation where only demand is uncertain. The presented case in Section 3.4 has only demand uncertainty.

### 3.3 Lagrangean Relaxation

We define  $\lambda$  as the vector of Lagrangean multipliers associated with demand constraints (3.5), relax these constraints and get the following Lagrangean sub-

problem:

$$LR(\lambda) = \min \sum_{s \in \mathcal{S}} p^s \left[ \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} (T_{ij} - \lambda_i^s) x_{ij}^s + \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \sum_{b \in \mathcal{B}} C_{kb}^s \mu_{jkb}^s + \sum_{i \in \mathcal{I}} \lambda_i^s D_i^s \right]$$

subject to (3.2)-(3.3) and (3.6)-(3.10).

For a given  $\lambda$ ,  $\sum_{s \in \mathcal{S}} \sum_{i \in \mathcal{I}} p^s \lambda_i^s D_i^s$  is constant. The problem is therefore separable in  $j$ . We write  $LR(\lambda) = \sum_{j \in \mathcal{J}} g_j(\lambda) + \sum_{s \in \mathcal{S}} \sum_{i \in \mathcal{I}} p^s \lambda_i^s D_i^s$  with  $g_j(\lambda)$  being the optimal value of the Lagrangean subproblem for each location  $j$ :

$$g_j(\lambda) = \min \sum_{s \in \mathcal{S}} p^s \left[ \sum_{i \in \mathcal{I}} (T_{ij} - \lambda_i^s) x_{ij}^s + \sum_{k \in \mathcal{K}} \sum_{b \in \mathcal{B}} C_{kb}^s \mu_{jkb}^s \right] \quad (3.11)$$

subject to

$$\sum_{k \in \mathcal{K}} y_{jk} = 1, \quad (3.12)$$

$$x_{ij}^s \leq L_{ij} D_i^s, \quad \forall i \in \mathcal{I}, s \in \mathcal{S}, \quad (3.13)$$

$$\sum_{i \in \mathcal{I}} x_{ij}^s = \sum_{k \in \mathcal{K}} \sum_{b \in \mathcal{B}} Q_{kb} \mu_{jkb}^s, \quad \forall s \in \mathcal{S}, \quad (3.14)$$

$$\sum_{b \in \mathcal{B}} \mu_{jkb}^s = y_{jk}, \quad \forall k \in \mathcal{K}, s \in \mathcal{S}, \quad (3.15)$$

$$y_{jk} \in \{0, 1\}, \quad \forall k \in \mathcal{K}, \quad (3.16)$$

$$x_{ij}^s \geq 0, \quad \forall i \in \mathcal{I}, s \in \mathcal{S}, \quad (3.17)$$

$$\mu_{jkb}^s \geq 0, \quad \forall k \in \mathcal{K}, b \in \mathcal{B}, s \in \mathcal{S}. \quad (3.18)$$

### Solving the Subproblem

The first-stage decision is to choose the designed capacity of the facility to open at location  $j$ , which corresponds to choosing a linepiece  $k$  of the piecewise linear long-run facility cost function. Once the linepiece  $k$  is chosen, the second-stage facility cost function is convex piecewise linear with  $B$  linepieces, having strictly increasing slopes  $u_{kb}$ . The slope is given as  $u_{kb} = \frac{C_{k(b+1)} - C_{kb}}{Q_{k(b+1)} - Q_{kb}}$ .

When considering the problem (3.11)-(3.18) for each linepiece  $k \in \mathcal{K}$  separately,  $g_j(\lambda)$  becomes separable in scenarios.



The subproblem  $g_{jk}^s(\lambda)$  for a given location  $j$ , linepiece  $k$  and scenario  $s$  is:

$$g_{jk}^s(\lambda) = \min \sum_{i \in \mathcal{I}} (T_{ij} - \lambda_i^s) x_{ij}^s + \sum_{b \in \mathcal{B}} C_{kb}^s \mu_{jkb}^s \quad (3.19)$$

subject to

$$x_{ij}^s \leq L_{ij} D_i^s, \quad \forall i \in \mathcal{I}, \quad (3.20)$$

$$\sum_{i \in \mathcal{I}} x_{ij}^s = \sum_{b \in \mathcal{B}} Q_{kb} \mu_{jkb}^s, \quad (3.21)$$

$$\sum_{b \in \mathcal{B}} \mu_{jkb}^s = 1, \quad (3.22)$$

$$x_{ij}^s \geq 0, \quad \forall i \in \mathcal{I}, \quad (3.23)$$

$$\mu_{jkb}^s \geq 0, \quad \forall b \in \mathcal{B}. \quad (3.24)$$

Problem (3.19)-(3.24) is of the same type as the Lagrangean subproblem for a given facility solved in Van den Broek et al. (2006) for deterministic facility location problems with a general objective. Their subproblem is a continuous knapsack problem with lower and upper capacity bounds and a linear objective, while we have a continuous piecewise linear objective with  $B$  segments in (3.19). Still we can adapt the method described by Martello & Toth (1990) for solving continuous knapsack problems with linear objective function to the algorithm to find  $g_{jk}^s(\lambda)$  as described in Figure 3.2.

In the initialization of the algorithm, the customer locations  $i$  are first sorted according to increasing marginal costs of serving one additional unit of demand. When we start out, we are at the first linepiece of the short run cost function ( $b = 1$ ). The marginal cost of serving another unit of demand in the facility is equal to the slope of the linepiece,  $u_{k1}$ . Then the marginal cost for the first units allocated to facility  $j$  from region  $i$  may be expressed as  $T_{ij} - \lambda_i^s + u_{k1}^s$ .

At Step 1 the customers are allocated to the facility at location  $j$  in this order. This continues until either all customers are allocated (a), or the marginal cost of all remaining potential customer regions  $i$ ,  $T_{ij} - \lambda_i^s + u_{kb}^s$  become positive, i.e. the objective function can no longer be improved by serving customers using facility  $j$  (b). In case the upper breakpoint of the linepiece is reached (c), only part of the demand for this unit will be met and the rest will met using the next linepiece of the second-stage cost function.

In Step 2 we calculate the objective function of the knapsack problem. The first element of the sum is the value of the customers served within the capacity of the previous linepiece, the next element is the value of all customers locations we decided to serve on the current linepiece. The third element corrects the objective function value in case the demand of the last customer on the previous

**Initialize:** Set  $g_{jk}^{s0} := 0$ ,  $b := 1$ , and  $i_0 := 1$ .

Define  $q_i^s := T_{ij} - \lambda_i^s + u_{k1}^s$ ,  $\forall i$ .

Sort the locations  $i$  in order of increasing  $q_i^s$ :  $q_1^s \leq \dots \leq q_n^s$ .

**Repeat:** Until  $b > B$ ,

1. Set  $x_{ij}^s := L_{ij} D_i^s$ ,  $i = 1, \dots, n$ , until either

- (a)  $x_{ij}^s = L_{ij} D_i^s$ ,  $\forall i$ ,

or for the first time for some index  $(i_b)$ ,

- (b)  $q_{i_b} > 0$ , or

- (c)  $\sum_{m=1}^{i_b} x_{mj}^s > Q_{k(b+1)}$ .

If (a): Set  $b := B$  and  $i_b := n$ . The solution is optimal.

If (b): Set  $x_{mj}^s := 0$ ,  $m = i_b, \dots, n$  and  $b = B$ . The solution is optimal.

If (c): Set  $x_{i_b j}^s := Q_{k(b+1)} - \sum_{m=1}^{i_b-1} x_{mj}^s$ .

2. Calculate  $g_{jk}^{sb} = g_{jk}^{s(b-1)} + \sum_{m=i_{b-1}}^{i_b} q_m^s x_{mj}^s - q_{i_{b-1}}^s \left( Q_{kb} - \sum_{m=1}^{i_{b-1}-1} x_{mj}^s \right)$

3. If  $b < B$ : update  $q_m^s := T_{mj} - \lambda_m^s + u_{k(b+1)}^s$ ,  $m = i_b, \dots, n$ . The sequence of locations  $i$  is not changed.

4. Set  $b := b + 1$ .

**Calculate:**  $g_{jk}^s(\lambda) = g_{jk}^{sB} + C_{k1}^s$

**Output:**  $g_{jk}^s(\lambda)$  is the solution to (3.19)-(3.24).

**Figure 3.2:** Solution algorithm for knapsack problem

linepiece spans across the two linepieces. In Step 3, we update the costs of serving additional customers, as we are moving to a new linepiece with slope  $u_{b+1}$  of the second-stage facility cost function, Step 4. This is the main modification of the original algorithm in Martello & Toth (1990). It comes from the fact that in our case the capacity limit is not strict, rather the unit cost changes whenever we move from one of the linepieces of the second-stage cost function to another (see Figure 3.1). This update works because the cost update never changes the ordering of customer regions  $i$ , just the cost level. The continuous knapsack is still filled up using the units with lower unit cost first and we serve only the customers with negative unit cost.

When the solution is optimal or the overall capacity limit of the facility has been reached, the fixed costs are added to the objective function.

Now  $g_{jk}(\lambda) = \sum_{s \in \mathcal{S}} p^s g_{jk}^s(\lambda)$ , and subproblem (3.11)-(3.18) is solved by  $g_j(\lambda) = \min_k g_{jk}(\lambda)$ . The computational complexity of this procedure is  $O(|\mathcal{I}| \cdot |K| \cdot |\mathcal{S}|)$ .

## Calculating an Upper Bound

In order to find the best lower bound on the optimal solution value of the original problem, one has to solve the Lagrangean dual problem ( $LD$ ):

$$LD = \max_{\lambda} LR(\lambda).$$

We solve  $LD$  by a sub-gradient optimization method, which is commonly used for discrete problems. An early reference is Held, Wolfe & Crowder (1974). The procedure we use is a straightforward implementation identical to the one found in Holmberg et al. (1999).

Given  $\lambda$ ,  $LR(\lambda)$  yields a lower bound on the optimal solution value, but the optimal solution of the Lagrangean relaxation is in general not a feasible solution to the original problem. We use a heuristic to find an upper bound  $UB^{(v)}$  in each dual iteration  $v$  by constructing a feasible solution for the original problem. The heuristic for finding a feasible solution is presented in Figure 3.3. It starts from the optimal solution of the Lagrangean relaxation, by installation of facilities at locations  $j$ , which have  $y_{jk}(\lambda) = 1$  for  $k \geq 1$ , and allocation of demand for each scenario, given by  $x_{ij}^s(\lambda)$ . If not all demand is satisfied and there is no more capacity available subject to the short-run capacity limit  $Q_{k(B+1)}$  (see Figure 3.1), the heuristic first tries to expand capacity of open facilities. If this does not create enough total capacity then eventually the heuristic resorts to opening new facilities. The rules used for expanding and opening facilities are described in detail below. We introduce additional notation:  $MC_j$  for the maximum capacity available at location  $j$  and  $UC_j^s$  for the capacity used at location  $j$  in scenario  $s$  in the solution under construction. In the following we use  $y, MC, x^s$  and  $UC^s$

**Initialize:**

Set  $UB^{(v)} := 0$ ,  $x_{ij}^s := 0$ ,  $y_{jk} := y_{jk}(\lambda)$ ,  $MC_j := \sum_k y_{jk} Q_{k(B+1)}$ , and  $UC_j^s := 0$ ,  $\forall i \in \mathcal{I}, j \in \mathcal{J}, k \in K, s \in \mathcal{S}$ .

1. Define  $\mathcal{I}_1^s := \{i \mid x_{ij}^s(\lambda) > 0\}$ ,  $s \in \mathcal{S}$ .  
 For each scenario  $s$ : if  $\mathcal{I}_1^s \neq \emptyset$ , do  
**AssignToExisting**( $s, \mathcal{I}_1^s, x^s, MC, UC^s$ ).
2. Define  $\mathcal{I}_2^s := \{i \mid \sum_j x_{ij}^s < D_i^s\}$ ,  $\forall s$  and  $\mathcal{I}_2 = \bigcup_s \mathcal{I}_2^s$ .
3. While  $\mathcal{I}_2 \neq \emptyset$  do:
  - a) For each scenario  $s$ , if  $\mathcal{I}_2^s \neq \emptyset$ ,  
 do **AssignToExisting**( $s, \mathcal{I}_2^s, x^s, MC, UC^s$ ).
  - b) Update  $\mathcal{I}_2^s$  and  $\mathcal{I}_2$ .  
 If  $\mathcal{I}_2 \neq \emptyset$ :
    - i. Define  $\mathcal{J}_1 := \{j \mid \sum_{k>0} y_{jk} = 1, UC_j^s = MC_j\}$ .
    - ii. Do **ExpandExisting**( $\mathcal{I}_2, \mathcal{J}_1, x, MC, UC$ ).
 end If.
  - c) Update  $\mathcal{I}_2^s$  and  $\mathcal{I}_2$ .
  - d) If  $\mathcal{I}_2 \neq \emptyset$ : set  $\mathcal{J}_2 := \{j \mid y_{j0} = 1\}$ 
    - i. Do **OpenNew**( $\mathcal{I}_2, \mathcal{J}_2, y, x, MC, UC$ ).
    - ii. Update  $\mathcal{I}_2^s$  and  $\mathcal{I}_2$ .
 end If.
 end While.
4. Calculate

$$UB^{(v)} = \sum_{s \in \mathcal{S}} p^s \left( \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} T_{ij} x_{ij}^s + \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \sum_{b=1}^B y_{jk} \gamma_{kb}^s \left( C_{kb}^s + u_{kb} \left( \sum_{i \in \mathcal{I}} x_{ij}^s - Q_{kb} \right) \right) \right)$$

with  $\gamma_{kb}^s = 1$ , if  $Q_{kb} < \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} x_{ij}^s \leq Q_{k(b+1)}$ , 0 otherwise.

**Output:**  $UB^{(v)}$  is the cost of a feasible solution to problem (3.1)-(3.10).

**Figure 3.3:** Heuristic for upper bound

to denote the vectors of all  $y_j$ ,  $MC_j$ ,  $x_{ij}^s$  and  $UC_j^s$ , respectively, and  $x$  and  $UC$  to denote the vectors of all  $x^s$  and  $UC^s$ .

The subroutine **AssignToExisting** takes a set of customer locations  $\mathcal{I}$  and tries to assign the demand of these customers to existing facilities. For each customer  $i \in \mathcal{I}$ , the subroutine first determines the facility  $j$  that can satisfy demand at location  $i$  at lowest cost  $T_{ij}$ . It then assigns as much customer demand as possible to facility  $j$ . A detailed description of **AssignToExisting** is given in Figure 3.4.

```

AssignToExisting(Input:  $s, \mathcal{I}^s, x^s, MC, UC^s$ ; Output:  $x^s, UC^s$ )
While  $\mathcal{I}^s \neq \emptyset$ :
    1. Choose  $i \in \mathcal{I}^s$ .
    2. Define  $\mathcal{J} := \{j \mid \sum_{k>0} y_{jk} = 1, UC_j^s < MC_j, L_{ij} = 1\}$ .
    3. While  $\mathcal{J} \neq \emptyset$  do:
        a) Choose the location  $j^*$  with lowest transportation cost  $T_{ij}$ .
        b) Set  $x_{ij^*}^s := x_{ij^*}^s + \min \{MC_{j^*} - UC_{j^*}^s; D_i^s - \sum_j x_{ij}^s\}$ .
        c) Update  $UC_{j^*}^s := UC_{j^*}^s + \min \{MC_{j^*} - UC_{j^*}^s; D_i^s - \sum_j x_{ij}^s\}$ .
        d) If  $\sum_j x_{ij}^s = D_i^s$ : do  $\mathcal{I}^s := \mathcal{I}^s \setminus \{i\}$  else
             $\mathcal{J} := \mathcal{J} \setminus \{j^*\}$ .
    end While.
    4.  $\mathcal{I}^s := \mathcal{I}^s \setminus \{i\}$ .
end While.
    
```

**Figure 3.4:** Subroutine **AssignToExisting**

The subroutine **ExpandExisting** takes as input parameters the set  $\mathcal{I}$  of customers with unsatisfied demand and the set  $\mathcal{J}$  of facilities that have no more capacity available. It then determines the facility  $j \in \mathcal{J}$  that can serve most of the customers in  $\mathcal{I}$ , expands this facility, and assigns as much customer demand as possible to it. This subroutine is shown in Figure 3.5.

The subroutine **OpenNew**, described in detail in Figure 3.6, has as input the set  $\mathcal{I}$  of customer locations with unsatisfied demand and the set  $\mathcal{J}$  of locations without facility. It determines the location  $j' \in \mathcal{J}$  that can satisfy most customer demand and assigns this demand to location  $j'$ .

The subroutine then installs a maximum capacity at this facility such that the expected capacity usage is smaller than the installed capacity. If the used

---

```

ExpandExisting(Input:  $\mathcal{I}_2, \mathcal{J}_1, x, MC, UC$ ; Output:  $x, UC$ )
While  $\mathcal{J}_1 \neq \emptyset$  do
    1. For each  $j \in \mathcal{J}_1$  define  $\mathcal{I}_{2j} := \{i \in \mathcal{I} \mid L_{ij} = 1\}$ .
    2. Choose the set  $\mathcal{I}_{2j'}$  with highest cardinality.
    3. If  $\mathcal{I}_{2j'} \neq \emptyset$  do:
        a) Expand the facility at location  $j'$ , i. e. set  $y_{j'k} := 0$  and  $y_{j'(k+1)} := 1$ .
        b) Update  $MC_{j'} := \sum_k Q_{k(B+1)} y_{j'k}$ .
        c) For each  $i \in \mathcal{I}_{2j'}$  and for each scenario  $s$  do:
            i. Set  $x_{ij'}^s := x_{ij'}^s + \min \{MC_{j'} - UC_{j'}^s; D_i^s - \sum_j x_{ij}^s\}$ .
            ii. Update  $UC_{j'}^s := UC_{j'}^s + \min \{MC_{j'} - UC_{j'}^s; D_i^s - \sum_j x_{ij}^s\}$ .
            iii. If  $\sum_{j'} x_{ij'}^s = D_i^s, \forall s$  do:  $\mathcal{I}_2 := \mathcal{I}_2 \setminus \{i\}$  else  $\mathcal{J}_1 := \mathcal{J}_1 \setminus \{j'\}$ .
        end For.
    Else set  $\mathcal{J}_1 := \emptyset$ .
    end If.
end While.
    
```

**Figure 3.5:** Subroutine **ExpandExisting**

capacity exceeds the maximum capacity in one of the scenarios, we reset the allocation of demand and exit the subroutine. In case the newly opened facility is not enough to provide a feasible solution, step 3 of the heuristic is repeated with the new facility included. The heuristic returns a feasible solution,  $y_{jk}$  and  $x_{jk}^s$ , and a solution value  $UB^{(v)}$ .

**OpenNew**(Input:  $\mathcal{I}_2, \mathcal{J}_2, y, x, MC, UC$ ; Output:  $y, x, UC$ )  
While  $\mathcal{J}_2 \neq \emptyset$  do

1. For each  $j \in \mathcal{J}_2$  determine  $\mathcal{I}_{2j} := \{i \in \mathcal{I}_2 \mid L_{ij} = 1\}$ .
2. Choose the set  $\mathcal{I}_{2j'}$  with highest cardinality.
3. If  $\mathcal{I}_{2j'} \neq \emptyset$  do:  
For each  $i \in \mathcal{I}_{2j'}$  and for each scenario  $s$ :
  - a) Set  $x_{ij'}^s := D_i^s - \sum_j x_{ij}^s$ .
  - b) Update  $UC_{j'}^s := UC_{j'}^s + x_{ij'}^s, \forall s$ .
end For.
4. Choose  $k$  such that  $k$  is the smallest number for which  
 $E(UC_{j'}) < MC_{j'} = Q_{k(B+1)}$ . Do  $y_{j'k} := 1$  and  $\mathcal{J}_2 := \mathcal{J}_2 \setminus \{j'\}$ .
5. If  $\exists s' \in \mathcal{S}$  with  $UC_{j'}^{s'} > MC_{j'}$ : set  $x_{ij'}^{s'} := 0, UC_{j'}^{s'} := 0, \forall i, j, \mathcal{J}_2 := \emptyset$ .

end While.

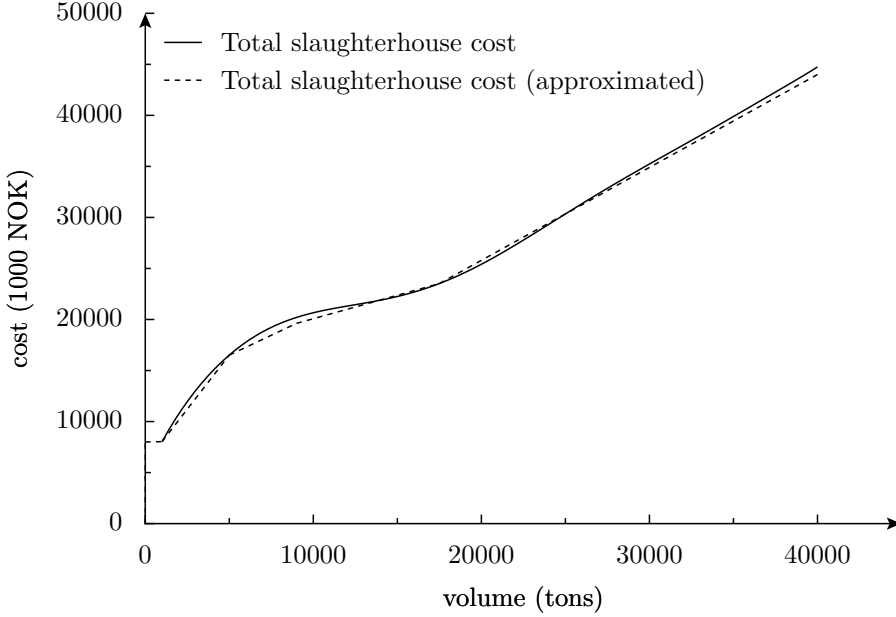
**Figure 3.6:** Subroutine OpenNew

When the locations are fixed, the resulting problem is a linear stochastic transportation problem. To improve the solution, every 100 iterations we use XpressMP to solve the stochastic transportation problem resulting from the open facilities in that iteration's heuristic solution. We also do this whenever a new best solution is found by the heuristic. The version of the algorithm where we do not try to improve the heuristic solution using XpressMP, only gives marginally worse results in empirical computational studies.

### 3.4 Case Description

In this section we present a case from the Norwegian meat industry regarding the location of slaughterhouses for cattle. The data used are the same as for the deterministic model in Van den Broek et al. (2006) and a more detailed

description of the case may be found there. We will here give a brief description of the case for completeness and also describe how we generate and test scenarios. Computational results for this stochastic model are presented in the next section. There are 435 possible locations for facilities, corresponding to municipalities in Norway. The facility cost function is equal for all facilities and represent the first-stage costs. It is depicted as the solid line in Figure 3.7 and is based based on a German study (Kern 1994).



**Figure 3.7:** Total facility costs for slaughterhouses as function of the volume.

We approximate the first-stage facility cost function by 6 linepieces. The breakpoints are given in  $(tons/year, NOK/kilo)$  and chosen as:  $(0, 0)$ ,  $(1.3, 6153.85)$ ,  $(1000, 8.03)$ ,  $(5000, 3.43)$ ,  $(9000, 2.18)$ ,  $(17500, 1.34)$ , and  $(40000, 1.1)$ . The piecewise linear function used to approximate the total cost function is the dashed line in Figure 3.7. In this case the short-run cost functions are assumed to be deterministic. The second-stage facility cost function is represented by a convex piecewise linear function with 3 linepieces. The second linepiece corresponds to the linepiece chosen in the first stage ( $y_{jk} = 1$ ) and has a per unit cost of  $u_{k2} = \frac{P_{k+1}F_{k+1} - P_k F_k}{F_{k+1} - F_k}$ , see Figure 3.1. The per unit cost of the first linepiece is given as  $u_{k1} = 0.75 \cdot \min \{u_{k1}, u_{(k-1)1}\}$  and on the last linepiece it is defined as



$u_{k3} = u_{k2} + u_{(k+1)2}$  for  $k < 6$  and  $u_{k3} = 5 \cdot u_{k2}$  for  $k = 6$ . In addition the upper limit on the capacity usage is set to  $Q_{k3} = 1.2 \cdot F_{k+1}$ .

Two important elements are transportation time and transportation costs. Transportation costs consist of two components. Firstly, there is the driving cost from the slaughterhouse to the region (municipalities) where animals are to be picked up and back to the slaughterhouse (travelling cost). Secondly, there is the cost of collecting animals in the region (collecting cost). For approximating these costs, Borgen, Schea, Rømo & Tomasgard (2000) estimated, based on empirical data from the Norwegian Meat Cooperative, the average distance driven, the average number of stops at farms, and the average time per stop. Based on this the transportation costs are linear in the distance to the slaughterhouse and linear in the number of animals transported in the truck.

Due to legal restrictions no more than 8 hours may pass since the first animal is loaded onto the truck, until the last animal has left the truck at the slaughterhouse.

This time is approximated in our model by the time to drive from the collection region to the slaughterhouse (travel time) plus the average time of filling up the truck on a collection round-trip (collecting time). For a large car the collection round-trip takes approx. 3.75 hours, limiting the maximum travel time to 4.25 hours. This is an approximation as the range of the car is based on the assumption that trucks will be filled up. The approximation may be improved by including smaller car types as well, allowing them to drive a longer distance (as they will on average pick up fewer animals). In the case presented here we also use a second car type with smaller capacity, higher costs and wider range. The collection time for the small car is approx. 3 hours, allowing a maximum travel time of 5 hours. Allocation of animal municipalities to slaughterhouse municipalities which do not satisfy the 8 hour rule is eliminated using the binary parameter  $L_{ij}$  which is set to 0 for infeasible combinations (in our data this is about 82% of the combinations).

We aggregate demand per year in the same 435 municipalities that are candidates for locations. Demand is here described as a farmer's demand to deliver animals to a slaughterhouse. We generate 3 groups of demand data sets drawn from a multivariate normal distribution with expectation equal to the original animal population of year 1999. The first group of data sets assumes that demand is varying on a national level, i. e. the demand in all municipalities is perfectly correlated. The second group considers regional demand variations. The municipalities are grouped into 4 regions (Northern Norway, Mid-Norway, Western Norway, and Southern Norway). Demand is perfectly correlated within a region, but uncorrelated between the different regions. The last data sets assume no correlation in demand between the different municipalities. For each of the three groups we consider 2 test problems, the first with a standard deviation equal to 50% of the expected demand, whereas the other has a standard deviation equal

to 20% of the expected value. We then generate two problem instances for each testproblem, one with 100 scenarios and one with 10 scenarios. To avoid unrealistic scenarios, any demand outcome in a municipality is restricted to be at most 2 times the expected demand in that municipality (higher outcomes are set to this limit). Also, negative demand outcomes are set to 0.

## 3.5 Computational Results

All calculations were carried out on a PC running a Linux kernel 2.6.11 with a 3GHz Intel Xeon processor and 6GB RAM. XpressMP 2004D was used as commercial solver whenever stochastic LP's were solved. Test runs indicated that the initial value for the Lagrangean multipliers has almost no influence on the results. The maximum number of iterations is set to  $V = 3000$ . Problem instances are described as a combination of the correlation level ((N)ational, (R)egional, or (U)ncorrelated), standard deviation  $\sigma$  of the demand dataset (50% or 20% of the expected demand), and the number of scenarios  $S$ . The initial step size parameter,  $\eta_0$ , and the number of iterations without improvement before reducing the step size parameter  $\eta$ ,  $V_1$ , were adjusted for each problem instance in order to produce reasonable results. The total costs are given in NOK 1000. Results are given in Table 3.1. The gap between lower bound given by the approximation of the Lagrangean dual and the upper bound given by the best solution found is defined as  $\frac{UB-LB}{LB}$ . We also show the expected value of the deterministic solution (EVDS) for each problem instance. This is the expected value of the second-stage solution when using the first-stage solution from the deterministic (year 1999) instance as input. A value of  $+\infty$  here means that the second stage was infeasible for the deterministic first-stage solution.

For comparison, when we solve the deterministic problem using the algorithm described in section 3.3, this results in a lower bound of NOK 233.2 million and a best feasible solution with cost NOK 241.8 million after 3000 iterations with a computation time of 38 minutes and 5 seconds. The optimality gap is 3.56%. Our method solves the stochastic problem instances within 10% of optimality in 3000 iterations for all but two instances and often the gap is around 7%. This is acceptable for practical purposes when solving large real life problem instances which was the target of our investigation. If better accuracy is required our approach may be integrated in a branch-and-bound scheme. If a time speed up is needed the algorithm is suitable for parallelization. The time used to find the solution in the current implementation increases linearly in the number of scenarios and in the number of iterations.

In the deterministic model, the facility costs account for approximately two thirds of the expected total costs. This ratio between facility costs and transportation costs appears to be the same for the stochastic problem instances.

Table 3.1: Computational results

Problem instance		$\sigma$	$S$	$\eta_0$	$V_1$	$LB$	$UB$	CPU-time	gap	$EVDS$
Corr.										
N	50%	10	1.75	350	222204	238722	3:31:41	7.43%	+∞	
N	50%	100	1.5	350	232780	249322	35:43:01	7.11%	+∞	
N	20%	10	2	125	232688	244620	3:39:37	4.88%	+∞	
N	20%	100	2	50	234583	248850	35:56:12	5.73%	+∞	
R	50%	10	0.9	20	226451	250460	3:29:30	10.60%	+∞	
R	50%	100	0.75	15	224403	253789	31:10:12	13.09%	+∞	
R	20%	10	1.75	100	228726	244972	3:28:44	6.63%	+∞	
R	20%	100	1.75	125	224558	242057	35:19:33	7.23%	+∞	
U	50%	10	1.75	125	226668	241970	3:33:16	6.75%	243377	
U	50%	100	1.75	150	227473	243880	35:36:58	7.21%	243203	
U	20%	10	2	125	231505	243063	3:33:16	4.76%	242224	
U	20%	100	1.75	125	229789	242543	34:47:57	5.26%	242166	

We would like to test the quality of the solutions obtained for the different problem instances and their robustness regarding the scenario representation. Therefore for each test problem we generated 10 new datasets for demand with 1000 scenarios each. We then tested how well the first-stage decisions of the deterministic problem, the 10-scenario instance and the 100-scenario instance performed for each of these 1000-scenario instances by solving the resulting stochastic transportation problems. The comparison can be found in Table 3.2 where we present the average objective function values for each of these first stage decisions over the new 1000 scenario sets. A value of  $+\infty$  means that the solution is infeasible for at least one of the 10 instances with 1000 scenarios.

Firstly, in Table 3.2 we see that the solutions obtained from the original problem instances with 100 scenarios are feasible for all new datasets. The solution from the deterministic problem is feasible for only two test problems. This is not surprising as we got the same result in Table 3.1 when we tried it as a solution for the 10 and 100 scenario instances. It is interesting to note that also the 10-scenario first-stage solutions become infeasible when we increase the size of the scenario trees, except for two of the test problems.

This leads us into a discussion of how well the stochastic models represent the real life decision problem. Clearly the number of scenarios used to represent uncertainty is important here. The cases with completely correlated and partly correlated demand have 1 and 4 stochastic variables respectively. Then the first-stage solutions based on 100 scenarios seems sound. The scenarios they are based on give a good enough description of the underlying uncertainty, as the first stage solution is also feasible and good for all the 1000-scenario instances. The 10-scenario solutions do not achieve this same robustness regarding changes in the scenario representation of the test problems.

If we look at the results for the uncorrelated cases in Table 3.2, we see that the deterministic solution is better than all the other solutions except for one. The reason is probably that when demand is uncorrelated, high demand in one municipality is more likely to be canceled out by low demand in another municipality within the same slaughterhouse operating region. The demand variance in a slaughterhouse region will in general be smaller than in the correlated cases. In our particular case the situation with uncorrelated demand therefore at first seems to be easier to solve as it can be argued that the deterministic solution is a good heuristic choice for the first stage solution. This is not true. In fact if one increases the number of scenarios used to represent uncertainty, also in the uncorrelated case we will see combinations where demand in different regions does not cancel out. In general the number of scenarios required to get a good representation of the uncertainty increases both with higher standard deviations for variables and when the variables are uncorrelated. The uncorrelated problem instances have 100 scenarios to represent 435 stochastic variables which is too

little to give a good enough description of the underlying uncertainty. We will in the uncorrelated case in general need more scenarios to capture situations with demand peaks in several municipalities within a slaughterhouse region. Results from the correlated cases suggest that the expected value solution will often lead to infeasibility in the second stage in such scenarios. One could argue though, that if one accepts a moderate probability of infeasibility, the solution from the deterministic problem may be a good heuristic for the uncorrelated cases.

### **3.6 Conclusions**

We have shown how to model and solve a facility location problem with a general piecewise linear objective for situations with changing economies of scale or diseconomies of scale, uncertain costs and uncertain demand. By means of a greedy heuristic, we generate feasible solutions from the solution of the Lagrangean subproblem. Based on sub-gradient optimization we solve the Lagrangean dual and achieve acceptable optimality gaps for real-life problems. For many practical situations, like the case we investigated for the Norwegian Meat Cooperative, the suggested Lagrangean relaxation and greedy approach presented here provide good enough solutions to be valuable as decision support in strategic processes.

These models have been used by the Norwegian Meat Cooperative in cooperation with the authors since 2000 in their strategic restructuring. The purpose of the work has been to reduce the number of slaughterhouses for cattle from an original number of 25 to utilize economies of scale. In the stochastic solutions presented here the typical number of slaughterhouses included is around 11. At the time of writing in 2006, the Norwegian Meat Cooperative operates 16 slaughterhouses for cattle. The analysis and the models have given the company an indication for the saving potential and also the number of slaughterhouses needed to satisfy the 8 hour rule. The models have provided information that many alternative solutions exist with approximately the same objective value, indicating that the number of slaughterhouses is maybe more important than the exact location. The stochastic model and results presented here has provided the insight that not all of these solutions are equally robust when it comes to demand variations and that solutions that are good on expectation, are not necessarily good or even feasible in individual scenarios. It shows the importance of solving the stochastic models and including enough scenarios, recognizing the variability in demand and correlations. The solutions provided are better than the ones we get from the deterministic problem both in terms of expected costs and robustness against shortfall situations when demand is stochastic.

**Table 3.2:** Average objective function values for the instances with 1000 scenarios when using the deterministic, 10-scenario and 100 scenario first-stage solutions.

Test problem		Deterministic		10 Scenarios		100 Scenarios	
		average	std. dev.	average	std. dev.	average	std. dev.
Corr.	N 50%	$+\infty$		$+\infty$		250054	0.83%
	N 20%	$+\infty$		243482	0.44%	245384	0.44%
R	50%	$+\infty$		$+\infty$		248152	0.87%
	20%	$+\infty^a$		$+\infty$		240890	0.30%
U	50%	242245	0.07%	241486	0.06%	242996	0.06%
	20%	241965	0.03%	$+\infty$		242331	0.03%

<sup>a</sup>Solution is feasible for one of the 10 instances



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## Paper III

Peter Schütz, Asgeir Tomasgard and Shabbir Ahmed:

# **Supply Chain Design under Uncertainty using Sample Average Approximation and Dual Decomposition**

*Accepted for publication in European Journal of Operational  
Research*



## Paper 3

# Supply Chain Design under Uncertainty using Sample Average Approximation and Dual Decomposition

### Abstract:

We present a supply chain design problem modeled as a sequence of splitting and combining processes. We formulate the problem as a two-stage stochastic program. The first-stage decisions are strategic location decisions, whereas the second-stage consists of operational decisions. The objective is to minimize the sum of investment costs and expected costs of operating the supply chain. In particular the model emphasizes the importance of operational flexibility when making strategic decisions. For that reason short-term uncertainty is considered as well as long-term uncertainty. The real-world case used to illustrate the model is from the Norwegian meat industry. We solve the problem by Sample Average Approximation in combination with Dual Decomposition. Computational results are presented for different sample sizes and different levels of data aggregation in the second stage.

**Keywords:** Supply chain design, Stochastic programming, Sample average approximation, Dual decomposition

## 4.1 Introduction

Supply Chain Management includes design of, planning for and operation of a network of suppliers, production facilities, warehouses, and distribution centers in order to satisfy customer demand. Strategic decisions regarding the design of supply chains affect the ability to efficiently serve customer demand. The design decisions should therefore not be taken without considering the effect on the operational decisions (Lee & Billington 1992). In this paper we examine the impact different modeling choices of the supply chain operations have on the strategic decisions regarding its design and structure.

The importance of supply chain design was recognized already in the early 1970's (see e.g. Geoffrion & Graves 1974). The early models however assume the parameters that influence the design decisions to be deterministic. For long planning horizons this assumption is unlikely to hold. Demands, prices for raw

materials, components and finished products, locations of markets, etc. are usually highly uncertain over the lifetime of the supply chain and thus require a supply chain that is robust and flexible enough to cope with the challenges of a changing environment. These challenges lead to an increased interest in stochastic programming models over the past 10-15 years.

Traditionally, the research literature focused on the facility location component of supply chain management. Schütz, Stougie & Tomasgard (2008) is a recent example of the facility location approach with uncertain demand, non-convex, non-concave first-stage costs and convex second-stage costs. For a broader overview over stochastic facility location, we refer to the reviews by Louveaux (1993) and Snyder (2006).

We consider a model which covers several levels of the supply chain in contrast to the classical one level facility location models or location-allocation models. In this class of models, there is less literature on stochastic models. MirHassani, Lucas, Mitra, Messina & Poojari (2000) present a solution method for a supply chain design problem under uncertain demand that is based on scenario analysis. They also discuss the use of Benders decomposition in a parallel implementation. Lucas, MirHassani, Mitra & Poojari (2001) consider a similar capacity planning problem, but develop a solution method based on Lagrangean relaxation and scenario analysis. In Alonso-Ayuso, Escudero, Garín, Ortuño & Pérez (2003), a supply chain planning model is presented with binary first-stage decisions and continuous second-stage decisions. They use an algorithmic approach based on Branch-and-Fix to solve their problem. Alonso-Ayuso, Escudero & Ortuño (2005) present several models for supply chain design and production planning and scheduling. Different formulations for the problems are discussed. They also provide computational experience for the supply chain design problem. Santoso, Ahmed, Goetschalckx & Shapiro (2005) consider a supply chain design problem and a solution method based on Sample Average Approximation (SAA) and Benders decomposition. Computational results from a real-world case are also presented. Our paper extends this literature both in terms of solution method and model scope, as explained below.

Our case is from the Norwegian meat industry, based on cooperation with Nortura ([www.nortura.no](http://www.nortura.no)) and studies the effect on the strategic decisions of including the operation of the supply chain under uncertain and highly variable demand. The case is based on work carried out in 2005 and 2006 in restructuring their production network. In Tomasgard & Høeg (2005), the operational aspects of the same supply chain are modeled, but no computational results are presented. That paper does not look into strategic decisions regarding locations and capacities; the supply chain structure is fixed. Schütz et al. (2008) present a stochastic one level facility location model for the same supply chain. There, the

operational part is aggregated into a single time period and the model handles only a single commodity and only the first part of the supply chain.

The main contribution of this paper is to model and solve a stochastic multi-commodity supply chain design problem with a detailed description of operational consequences from the strategic decisions. It has the same level of detail on the operational side as Tomasgard & Høeg (2005) and handles in addition strategic decisions regarding location and capacities at every level in the supply chain. Because demand in the meat industry can exhibit huge variations over short periods, the supply chain is not only subject to long-term demand uncertainty, but also considerable short-term demand uncertainty. The long-term uncertainty covers trends and the development of national demand for meat products over a longer time horizon, whereas the short-term uncertainty deals with weekly demand variations. We study in this paper alternative models where the level of detail used to describe the operational decisions varies and discuss the impact on the strategic decisions. To our knowledge, our paper is the first one to examine whether or not the level of aggregation in the second stage affects the first-stage solution in supply chain design models. Our analysis also examines how the use of a stochastic model influences the decisions as compared to a deterministic model.

Modeling operational decisions in the second stage increases the model size considerably. For our model we can only solve problem instances with three or four scenarios when using a single processor computer with 6 GB of memory. To solve the model we combine Sample Average Approximation (SAA) (Kleywegt, Shapiro & de Mello 2001) with dual decomposition (Carøe & Schultz 1999). A second contribution of the paper is therefore to investigate how the approach of combining SAA with dual decomposition scales when distributed processing on a higher number of processors is used to increase the number of scenarios in the formulation. We also examine if increasing the number of scenarios have effect on the quality of the first-stage solution.

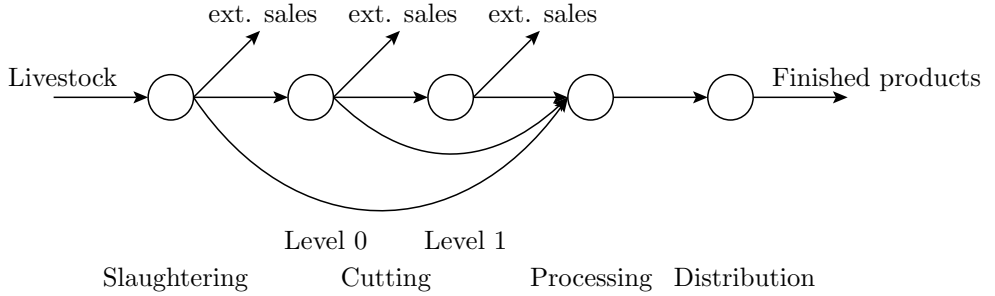
In Section 4.2, we present a supply chain model for the Norwegian meat industry. The scenario generation procedure and the different cases for uncertain demand are discussed in Section 4.3. In Section 4.4, we then give a generalized two-stage stochastic programming formulation for a supply chain design problem where the supply chain is modeled as a sequence of splitting and combining processes. The solution scheme is presented in Section 4.5. Computational results and an analysis of these results follow in Section 4.6. We conclude in Section 4.7.

## 4.2 The Supply Chain

The levels and the material flows of the supply chain used in our case are depicted in Figure 4.1. Slaughtering takes livestock as input and produces carcasses and intestines. It is followed by a two-step cutting process where the carcasses are split



into smaller parts. The fourth level is processing where the intermediate products from the previous levels are blended into finished products like sausages, ground meat, etc. The finished products are then shipped to the distribution centers where customer demand is satisfied. Intermediate products are sold to external customers or shipped to all downstream facilities that process or sell the product. Finished products are only sent to distribution centers.



**Figure 4.1:** Material flow in the supply chain of the Norwegian Meat Cooperative

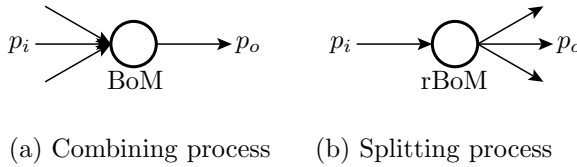
For both the cutting and the processing levels several recipes may be chosen in order to process or produce a given product. Which recipes to use at the processing level depends on the availability of the different intermediate products which in turn depends on the recipes chosen at the cutting level and eventually on the supplied livestock. A more detailed description of the supply chain and the different processes can be found in Tomasgard & Høeg (2002, 2005).

The part of Nortura's supply chain that we consider consists of 17 slaughterhouses, 18 plants at the cutting level, 9 processing facilities, and 9 distribution centers. The task is to redesign the processing level of the supply chain. Internal studies in Nortura revealed overcapacities in processing that have to be reduced. Thus, the first-stage decisions are to determine which of the existing 9 facilities should keep its processing unit in order to satisfy uncertain future demand. We look at 13 possibilities for locating different production equipment at the processing facilities. In the second-stage, we operate the production network in order to satisfy customer demand for 103 finished products. As input we use seven different animal types. For each of the animal types we have one recipe for the production of intestines at the slaughtering level. The carcasses are split into intermediate products at the first cutting level. We can choose among 35 recipes. At the second cutting level, we have a set of 87 recipes to choose from. With these recipes, both from the slaughtering and the cutting level, we produce 72 different intermediate products. All of the intermediate products can be used for

further processing or can be sold to external customers. At the processing level, we can choose from 169 recipes to produce 103 finished products.

The objective is to minimize the sum of annualized fixed facility costs and the expected annual operating costs of the supply chain. The first-stage costs are the fixed facility costs. These costs include capital cost, personnel and insurance, amongst others. We use annualized fixed costs in order to be able to compare them with the operating costs in the second stage. For our problem instances, the second-stage costs cover the variable costs of one year operating the production network. The operating costs include production, transportation, and shortfall costs. Production costs cover both direct and indirect costs like ingredients, packaging, personnel, and energy. The transportation costs are given by contracts between Nortura and the different transportation companies. They depend both on origin and destination as well as the pallet type used for transporting the products.

Our supply chain has as a set of facilities or plants which are possible locations for different production processes. The production processes can be different production lines, technologies or production phases. A production process is either a splitting processes or a combining process. Combining processes are common in manufacturing and assembling and often described by means of a Bill of Material (BoM). The BoM lists which and how many input products  $p_i$  are needed to produce one unit of output product  $p_o$  (see Figure 4.2a). The splitting processes are described in a similar manner. The process industry (e.g. the chemical and petroleum industry) uses a so-called reversed Bill of Material (rBoM) that defines in which output products  $p_o$  a unit of input product  $p_i$  can be split (see Figure 4.2b). The meat processing industry can be seen as an example of a supply chain consisting of subsequent splitting and combining processes (see e.g. Tomasgard & Høeg 2002, 2005). Our formulation allows for choosing from different BoMs for the same process. In our model, each BoM is assigned to the production of one specific output product in one specific process. Correspondingly each rBoM is assigned to the processing of one specific input product. For each location in our supply chain model we will have a set of possible production processes and for each process, a set of possible products.



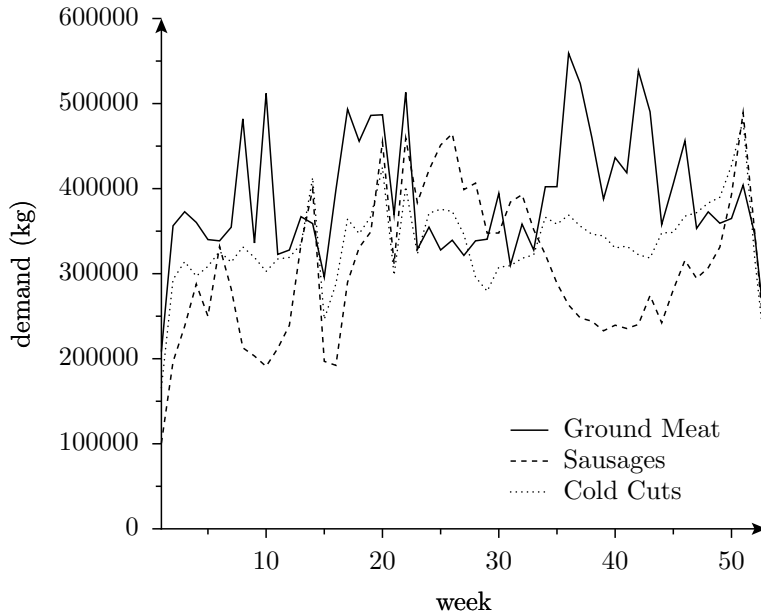
**Figure 4.2:** Combining and splitting processes

## 4.3 Modeling Uncertainty

Here we describe the modeling of short-term and long-term uncertainty, the temporal structure of the second-stage and the different demand cases.

### Short-term vs. Long-term Uncertainty

One of the main purposes of our analysis is to examine the importance of modeling the short-term operations and corresponding demand uncertainty when making strategic decisions. There can be high variations in weekly demand for meat products over the year. The products we consider in our problem instances belong to three different product groups. Weekly demand per product group is shown in Figure 4.3. We see that demand per product group changes by as much as 50% within 1-2 weeks, demand for a single product as used in our problem may exhibit even greater variations.



**Figure 4.3:** Weekly demand for three different product groups in 2004

We refer to weekly changes in demand as short-term variations. Some of the variations are due to seasonal changes in the demand pattern, but also daily variations, for example due to weather, can have a significant impact on demand.

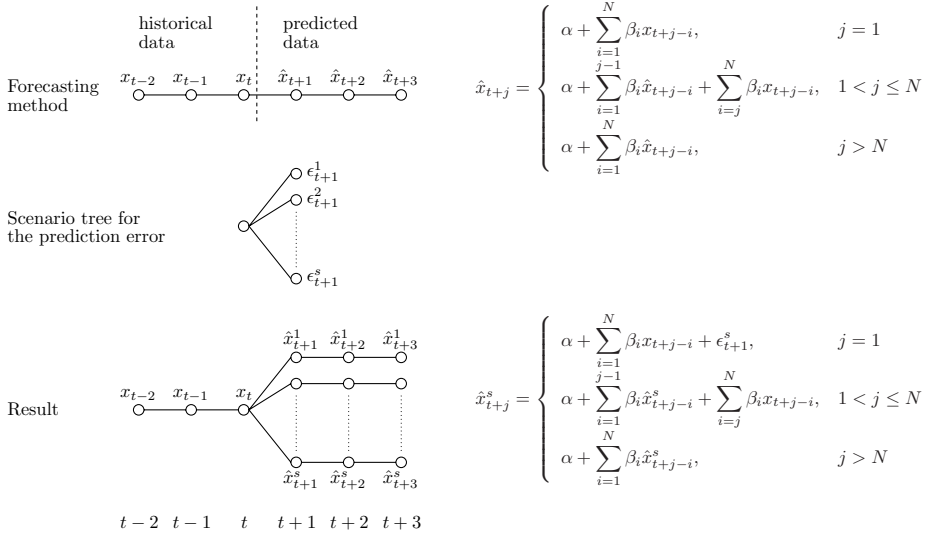
These short-term variations are neglected when demand is aggregated in longer time periods (or even a single period) in the second stage. The term long-term uncertainty is used to describe changes in the total level of demand over a longer time horizon like 5-10 years. This uncertainty is for example the market share of the company, the total market size or trends in demand for meat products.

In order to study the effect of short-term uncertainty and operational decisions on the strategic decisions, we have to generate scenarios for the short-term demand. We then combine these scenarios with scenarios for changes in the total demand level, i.e. the long-term uncertainty. The methods used to generate the demand scenarios are described below.

Short-term supply chain planning is based on demand forecasts. To capture the short-term uncertainty we use a methodology that combines forecasting and scenario-generation (Nowak & Tomaszgarczak 2007). We will briefly describe the steps for the short-term scenario generation here:

1. For each product, we parametrize an autoregressive process of  $N$ th order, AR( $N$ )-process, on historical data (see e.g. Hamilton 1994). For our problem instances, we use an AR(1)-process.
2. We then find the distribution of the historical prediction error for each product. We assume that the error term for a given product is i.i.d. between different time periods. The historically observed prediction errors then give us a multivariate discrete distribution for all the products' error terms.
3. On this empirical distribution, we perform a principal component analysis. For the  $P$  products we get  $P$  principal components and sort them by the share of variance they explain for the error distribution.
4. We calculate the first 4 moments of the empirical distribution of these principal components and choose the  $K$  principal components explaining the highest part of variance as stochastic in the further scenario generation process. For the  $P - K$  remaining components we use their expected value.
5. We then generate  $S$  scenarios for the  $K$  stochastic principal components using a moment matching procedure ensuring that the first 4 moments in the scenarios are the same as in the historical distribution. The scenarios are equally likely. The method used here is a modified version of the method developed by Høyland, Kaut & Wallace (2003).
6. The scenarios for the principal components are then transformed back into scenarios for the prediction error.
7. Finally, we combine the forecasting method with the scenario tree for the prediction error to get demand scenarios rather than pure prediction error

scenarios. This procedure is illustrated in detail in Figure 4.4: Using a deterministic AR(N)-process as forecasting method, predicted demand in period  $t + 1$  is given by the formula  $\hat{x}_{t+1} = \alpha + \sum_{i=1}^N \beta_i x_{t+1-i}$ . In our case, we add a realization of the error term  $\epsilon_{t+1}^s$  as represented in the scenario tree to the first prediction. Thus, the first prediction  $\hat{x}_{t+1}^s = \alpha + \sum_{i=1}^N \beta_i x_{t+1-i} + \epsilon_{t+1}^s$  is based on historical data and the error scenario  $\epsilon_{t+1}^s$ . When demand for several periods is predicted, the prediction may be based on both historical demand, predicted demand, and the error scenarios as shown in Figure 4.4.



**Figure 4.4:** Combining forecasting and scenario generation

The advantage of this method is that correlations between stochastic variables are taken care of through principal component analysis. Also, the principal component analysis makes it possible to reduce the number of stochastic variables used. In our case by using 150 principal components for 821 stochastic variables we still explain 93% of the variance in the multivariate error distribution.

For the long-term uncertainty we choose a different approach. For reasons of simplification, we model a possible change in long-term demand level without differentiating between the different demand regions. We also assume that an increase in total demand and a decrease in total demand are equally likely. We use a uniform distribution to describe the demand level, with today's demand as the

expectation. The scenarios for long-term uncertainty are generated by sampling a factor from a uniform distribution on  $[0.5;1.5]$ . By multiplying this factor with the demand in any demand scenario we model a change in the long-term demand level. Alternatively, a long-term trend, for example a general increase or decrease in demand level, would be easy to model by changing the interval.

## Demand Uncertainty

To properly catch all weekly demand variations, one would have to model the second-stage with 52 time periods. However, due to restrictions in both solution time and model size, this is not practical. We have to reduce the number of time periods in the second-stage by aggregating them while ensuring that we capture the effect of short-term demand variations.

Demand in the Norwegian meat market follows typical seasonal patterns: during the summer months, demand for barbecue products increases and both Easter and Christmas have distinct demand patterns. We therefore split the year in four seasons, each three months long. Usually, there is no correlation in demand between different seasons. We aggregate the first two months of each season into one period and model the last month of each season with a weekly time resolution (four weeks). This way, we capture the weekly demand variations around Easter, early and late summer, and Christmas in detail.

We describe here the problem instances based on three criteria: long-term uncertainty, short-term uncertainty, and aggregation.

**STU** The first problem instance we use is only taking short-term uncertainty into consideration. We choose to generate 200 scenarios for each of the 4-week periods using the scenario generation method described above. As we assume no temporal correlation between the different quarters, we can use any combination of these scenarios, resulting in a total of  $200^4$  possible demand scenarios. Demand in the cumulated two-month periods is represented by its expected value. No long-term uncertainty is modeled.

**LTU** The second case only takes long-term uncertainty into account. We use expected weekly demand to represent the demand variations over the year. This demand data is then multiplied with a random factor drawn from a uniform distribution on  $[0.5;1.5]$  to model the long-term changes in demand level. The scenarios may be viewed as possible demand realization at a point in the future where all scenarios are equally likely. Alternatively the scenarios may be viewed as realizations of yearly demand over a set of years. No short-term uncertainty is modeled.

**LSTU** The third case is a combination of long-term and short-term uncertainty. We use the scenario tree from STU and multiply each scenario with a

random factor that is uniformly distributed on  $[0.5;1.5]$ . This way, we capture both effects.

For each of these non-aggregated cases, we define a corresponding aggregated problem instances. In these aggregated problem instances (ASTU, ALTU, and ALSTU), we replace the 20-period second stage problem by a single period problem. All demand, supply, and capacities are cumulated.

## 4.4 Optimization Model

Let us introduce the following notation for our two-stage stochastic programming formulation:

- Sets

$\mathcal{F}_c$	Set of possible facility locations for combining processes.
$\mathcal{F}_s$	Set of possible facility locations for splitting processes.
$\mathcal{W}$	Set of possible warehouse locations.
$\mathcal{L}$	Set of all possible locations, $\mathcal{L} = \mathcal{F}_c \cup \mathcal{F}_s \cup \mathcal{W}$ .
$\mathcal{C}$	Set of customer locations.
$\mathcal{U}(j)$	Set of upstream locations able to send products to location $j$ , $j \in \mathcal{L} \cup \mathcal{C}$ .
$\mathcal{D}(j)$	Set of downstream locations able to receive products from location $j$ , $j \in \mathcal{L}$ .
$\mathcal{O}_c(j)$	Set of combining processes that can be performed at location $j$ , $j \in \mathcal{F}_c$ .
$\mathcal{O}_s(j)$	Set of splitting processes that can be performed at location $j$ , $j \in \mathcal{F}_s$ .
$\mathcal{O}(j)$	Set of all processes that can be performed at location $j$ , $j \in \mathcal{L}$ , $\mathcal{O}(j) = \mathcal{O}_c(j) \cup \mathcal{O}_s(j)$ .
$\mathcal{O}$	Set of all processes, $\mathcal{O} = \bigcup_{j \in \mathcal{L}} \mathcal{O}(j)$ .
$\mathcal{P}$	Set of products.
$\mathcal{P}_i(o)$	Set of input products for process $o$ , $o \in \mathcal{O}$ .
$\mathcal{P}_o(o)$	Set of output products of process $o$ , $o \in \mathcal{O}$ .
$\mathcal{B}(o, p)$	Set of (reversed) bills of materials that can be used for processing product $p$ in process $o$ , $o \in \mathcal{O}, p \in \mathcal{P}$ .
$\mathcal{S}$	Set of scenarios.
$\mathcal{T}$	Set of time periods.

- Indices and superscripts

$o$	Process index, $j \in \mathcal{L}, o \in \mathcal{O}(j)$ .
-----	--

$b$	Bill of materials index, $o \in \mathcal{O}, p \in \mathcal{P}, b \in \mathcal{B}(o, p)$ .
$j, k$	Location indices, $j, k \in \mathcal{L} \cup \mathcal{C}$ .
$p, q$	Product indices, $p, q \in \mathcal{P}$ .
$s$	Scenario index, $s \in \mathcal{S}$ .
$t$	Time period index, $t \in \mathcal{T}$ .

• Parameters, constants, and coefficients

$A_{p_o, p_i}^b$	Yield of product $p_o$ in case one unit $p_i$ is processed with reversed bill of materials $b$ , $o \in \mathcal{O}_s, p_i \in \mathcal{P}_i(o), p_o \in \mathcal{P}_o(o), b \in \mathcal{B}(o, p_i)$ .
$B_{p_o, p_i}^b$	Amount of product $p_i$ needed to produce one unit $p_o$ using bill of materials $b$ , $o \in \mathcal{O}_c, p_i \in \mathcal{P}_i(o), p_o \in \mathcal{P}_o(o), b \in \mathcal{B}(o, p_i)$ .
$C_{jt}^{os}$	Capacity of process $o$ at location $j$ in scenario $s$ at time $t$ , $j \in \mathcal{F}_c \cup \mathcal{F}_s, o \in \mathcal{O}, s \in \mathcal{S}, t \in \mathcal{T}$ .
$D_{jt}^{ps}$	Demand for product $p$ at customer location $j$ in scenario $s$ at time $t$ , $j \in \mathcal{C}, p \in \mathcal{P}, s \in \mathcal{S}, t \in \mathcal{T}$ .
$F_j^o$	Fixed cost of locating process $o$ at location $j$ , $j \in \mathcal{F}_c \cup \mathcal{F}_s, o \in \mathcal{O}$ .
$H_{jt}^{ps}$	Penalty for not satisfying one unit of demand of product $p$ at customer location $j$ in scenario $s$ at time $t$ , $j \in \mathcal{C}, p \in \mathcal{P}, s \in \mathcal{S}, t \in \mathcal{T}$ .
$P_{jbt}^{ps}$	Cost of processing one unit of product $p$ at location $j$ using (reversed) bill of material $b$ in scenario $s$ time $t$ , $j \in \mathcal{F}_c \cup \mathcal{F}_s, o \in \mathcal{O}(j), p \in \mathcal{P}, b \in \mathcal{B}(o, p), s \in \mathcal{S}, t \in \mathcal{T}$ .
$S_{jt}^{ps}$	Supply of product $p$ at location $j$ in scenario $s$ at time $t$ , $j \in \mathcal{F}_c \cup \mathcal{F}_s, p \in \mathcal{P}, s \in \mathcal{S}, t \in \mathcal{T}$ .
$T_{jkt}^{ps}$	Cost of transporting one unit of product $p$ from location $j$ to location $k$ in scenario $s$ at time $t$ , $j \in \mathcal{L}, k \in \mathcal{D}(j), p \in \mathcal{P}, s \in \mathcal{S}, t \in \mathcal{T}$ .
$i_{j0}^{ps}$	Initial inventory of product $p$ in scenario $s$ at location $j$ , $j \in \mathcal{W}, p \in \mathcal{P}, s \in \mathcal{S}$ .
$p^s$	Probability of scenario $s$ , $s \in \mathcal{S}$ .

• Decision variables

$v_{jt}^{ps}$	Inventory of product $p$ at location $j$ in scenario $s$ at period $t$ , $j \in \mathcal{W}, p \in \mathcal{P}, s \in \mathcal{S}, t \in \mathcal{T}$ .
$w_{jkt}^{ps}$	Amount of product $p$ transported from location $j$ to location $k$ in scenario $s$ at time $t$ , $j \in \mathcal{L}, k \in \mathcal{D}(j), p \in \mathcal{P}, s \in \mathcal{S}, t \in \mathcal{T}$ .
$x_{jbt}^{ps}$	Amount of product $p$ processed/produced at location $j$ using bill of material $b$ in scenario $s$ at time $t$ , $j \in \mathcal{F}_c \cup \mathcal{F}_s, o \in \mathcal{O}(j), p \in \mathcal{P}, b \in \mathcal{B}(o, p), s \in \mathcal{S}, t \in \mathcal{T}$ .



$y_j^o$	1 if process $o$ is located at facility location $j$ , 0 otherwise, $j \in \mathcal{F}_c \cup \mathcal{F}_e, o \in \mathcal{O}(j)$ .
$z_{jt}^{ps}$	Unsatisfied demand for product $p$ at customer location $j$ in scenario $s \in \mathcal{S}$ in period $t$ , $j \in \mathcal{C}, p \in \mathcal{P}, s \in \mathcal{S}, t \in \mathcal{T}$ .

We now model our supply chain as two-stage stochastic program with recourse. For reasons of simplicity, we denote the set of feasible combinations of first-stage decisions  $y_j^o$  by  $Y$ . The uncertain parameters in this formulation are the costs, supply, capacity, and demand. By  $\xi$ , we denote the vector of these parameters. For our problem we have  $\xi = (P, T, H, S, C, D)$ , where  $\xi^s$  is a given realization of the uncertain parameters.

$$\min \sum_{j \in \mathcal{L}} \sum_{o \in \mathcal{O}(j)} F_j^o y_j^o + \sum_{s \in \mathcal{S}} p^s \mathcal{Q}(y, \xi^s) \quad (4.1)$$

subject to

$$y \in Y \subseteq \{0, 1\}^{|\mathcal{L}| \cdot |\mathcal{O}|} \quad (4.2)$$

with  $\mathcal{Q}(y, \xi^s)$  being the solution of the following second-stage problem:

$$\begin{aligned} \mathcal{Q}(y, \xi^s) = \min & \sum_{j \in \mathcal{L}} \sum_{p \in \mathcal{P}} \sum_{o \in \mathcal{O}(j)} \sum_{b \in \mathcal{B}(o, p)} \sum_{t \in \mathcal{T}} P_{jbt}^{ps} x_{jbt}^{ps} + \\ & \sum_{p \in \mathcal{P}} \sum_{j \in \mathcal{L}} \sum_{k \in \mathcal{D}(j)} \sum_{t \in \mathcal{T}} T_{jkt}^{ps} w_{jkt}^{ps} + \sum_{p \in \mathcal{P}} \sum_{j \in \mathcal{C}} \sum_{t \in \mathcal{T}} H_{jt}^{ps} z_{jt}^{ps} \end{aligned} \quad (4.3)$$

subject to

$$S_{jt}^{ps} + \sum_{k \in \mathcal{U}(j)} w_{kjt}^{ps} = \sum_{o \in \mathcal{O}_s(j)} \sum_{b \in \mathcal{B}(o, p)} x_{jbt}^{ps} \quad j \in \mathcal{F}_s, p \in \bigcup_{o \in \mathcal{O}_s(j)} \mathcal{P}_i(o), t \in \mathcal{T}, \quad (4.4)$$

$$\begin{aligned} \sum_{o \in \mathcal{O}_s(j)} \sum_{q \in \mathcal{P}_i(o)} \sum_{b \in \mathcal{B}(o, q)} A_{p,q}^b \cdot x_{jbt}^{qs} &= \sum_{k \in \mathcal{D}(j)} w_{jkt}^{ps} \\ j \in \mathcal{F}_s, p \in \bigcup_{o \in \mathcal{O}_s(j)} \mathcal{P}_o(o), t \in \mathcal{T}, \end{aligned} \quad (4.5)$$

$$\begin{aligned} \sum_{p \in \mathcal{P}_i(o)} \sum_{b \in \mathcal{B}(o, p)} x_{jbt}^{ps} &\leq C_{jt}^{os} \cdot y_j^o \\ j \in \mathcal{F}_s, o \in \mathcal{O}_s(j), t \in \mathcal{T}, \end{aligned} \quad (4.6)$$

$$\begin{aligned} S_{jt}^{ps} + \sum_{k \in \mathcal{U}(j)} w_{kjt}^{ps} &= \sum_{o \in \mathcal{O}_c(j)} \sum_{q \in \mathcal{P}_o(o)} \sum_{b \in \mathcal{B}(o, q)} B_{q,p}^b \cdot x_{jbt}^{qs} \\ j \in \mathcal{F}_c, p \in \bigcup_{o \in \mathcal{O}_c(j)} \mathcal{P}_i(o), t \in \mathcal{T}, \end{aligned} \quad (4.7)$$

$$\sum_{o \in \mathcal{O}_c(j)} \sum_{b \in \mathcal{B}(o,p)} x_{jbt}^{ps} = \sum_{k \in \mathcal{D}(j)} w_{jkt}^{ps} \quad j \in \mathcal{F}_c, p \in \bigcup_{o \in \mathcal{O}_c(j)} \mathcal{P}_o(o), t \in \mathcal{T}, \quad (4.8)$$

$$\sum_{p \in \mathcal{P}_o(o)} \sum_{b \in \mathcal{B}(o,p)} x_{jbt}^{ps} \leq C_{jt}^{os} \cdot y_j^o \quad j \in \mathcal{F}_c, o \in \mathcal{O}_c(j), t \in \mathcal{T}, \quad (4.9)$$

$$v_{jt-1}^{ps} + \sum_{k \in \mathcal{U}(j)} w_{kjt}^{ps} = v_{jt}^{ps} + \sum_{k \in \mathcal{D}(j)} w_{jkt}^{ps} \quad j \in \mathcal{W}, p \in \mathcal{P}, t \in \mathcal{T}, \quad (4.10)$$

$$\sum_{k \in \mathcal{U}(j)} w_{kjt}^{ps} + z_{jt}^{ps} = D_{jt}^{ps} \quad j \in \mathcal{C}, p \in \mathcal{P}, t \in \mathcal{T}, \quad (4.11)$$

$$v, w, x, z \geq 0 \quad (4.12)$$

The objective function (4.1) is the sum of the first-stage costs and the expected second-stage costs. The first-stage costs represent the costs of installing a given process at location  $j$ . The objective function of the second stage (4.3) consists of three parts: firstly, the production costs, secondly, the transportation costs, and thirdly, the shortfall penalty for unsatisfied demand. Restriction (4.2) defines the feasible set for the binary first-stage variables. Constraints (4.4)-(4.6) describe the splitting processes. Constraints (4.4) ensure that the external supply and all products transported into the splitting node are processed. Restrictions (4.5) force all produced products to be transported to a downstream node. Restrictions (4.6) limit production to the available capacity in the splitting node. Constraints (4.7)-(4.9) describe combining processes in a similar way. Constraints (4.7) ensure that all products needed in the combining process are supplied at the combining node. Constraints (4.8) make sure all produced products are transported to downstream nodes and restrictions (4.9) take care of the capacity restrictions in the combining nodes. Restrictions (4.10) are the mass balance constraints for the inventory. With equations (4.11), we make sure that the sum of all products transported into a demand node and shortfall is equal to customer demand. Constraints (4.12) are the non-negativity constraints, the indices are omitted.

## 4.5 Solution Scheme

For the model (4.1)-(4.12) off-the-shelf solvers can typically solve instances with 3-4 scenarios (the amount of memory is in our experience the limit). A typical problem instance in a practical case would have thousands of scenarios. We

handle this using Sample Average Approximation (Kleywegt et al. 2001) and dual decomposition (Carøe & Schultz 1999). These procedures are described in the following subsections.

### Sample Average Approximation

We use Sample Average Approximation (SAA) to reduce the size of problem (4.1)-(4.12) by repeatedly solving it with a smaller set of scenarios. We generate random samples with  $N < |\mathcal{S}|$  realizations of the uncertain parameters and approximate the expected recourse costs by the sample average function  $\frac{1}{N} \sum_{n=1}^N \mathcal{Q}(y, \xi^n)$ . The problem (4.1)-(4.12) is then approximated by the following SAA problem:

$$\min_{y \in Y} \left\{ \hat{g}(y) := \sum_{j \in \mathcal{L}} \sum_{o \in \mathcal{O}(j)} F_j^o y_j^o + \frac{1}{N} \sum_{n=1}^N \mathcal{Q}(y, \xi^n) \right\}. \quad (4.13)$$

The optimal solution of (4.13),  $\hat{y}_N$ , and the optimal value,  $v_N$ , converge with probability one to an optimal solution of the original problem (4.1)-(4.12) as the sample size increases (Kleywegt et al. 2001). Assuming that the SAA is solved to an optimality gap  $\delta \geq 0$ , we can estimate the sample size  $N$  needed to guarantee an  $\varepsilon$ -optimal solution to the true problem with a probability of at least  $1 - \alpha$  as

$$N \geq \frac{3\sigma_{\max}^2}{(\varepsilon - \delta)^2} (|\mathcal{L}||\mathcal{O}|(\log 2) - \log \alpha), \quad (4.14)$$

with  $\varepsilon \geq \delta$  and  $\alpha \in (0, 1)$ .

In (4.14),  $\sigma_{\max}^2$  is related to the variability of  $\mathcal{Q}(y^*, \xi)$  at the optimal solution  $y^*$  (see Kleywegt et al. 2001, for details). One would in practice choose  $N$  taking into account the trade-off between the quality of the solution obtained for the SAA problem and the computational effort needed to solve it. Solving the SAA problem (4.13) with independent samples repeatedly can be more efficient than increasing the sample size  $N$ . This procedure can be found in Santoso et al. (2005), but we include it here for the sake of completeness:

1. Generate  $M$  independent samples of size  $N$  and solve the corresponding SAA

$$\min_{y \in Y} \left\{ \hat{g}(y) := \sum_{j \in \mathcal{L}} \sum_{o \in \mathcal{O}(j)} F_j^o y_j^o + \frac{1}{N} \sum_n \mathcal{Q}(y, \xi^n) \right\}.$$

We denote the optimal objective function value by  $v_N^m$  and the optimal solution by  $\hat{y}_N^m$ ,  $m = 1 \dots M$ .

2. Compute the average of all optimal objective function values from the SAA problems,  $\bar{v}_{N,M}$  and its variance,  $\sigma_{\bar{v}_{N,M}}^2$ :

$$\bar{v}_{N,M} = \frac{1}{M} \sum_{m=1}^M v_N^m \quad \text{and} \quad \sigma_{\bar{v}_{N,M}}^2 = \frac{1}{(M-1)M} \sum_{m=1}^M (v_N^m - \bar{v}_{N,M})^2.$$

The average objective function value  $\bar{v}_{N,M}$  provides a statistical lower bound on the optimal objective function value for the original problem (4.1)-(4.12) (Norkin, Pflug & Ruszczyński 1998, Mak, Morton & Wood 1999).

3. Pick a feasible first-stage solution  $\bar{y} \in Y$  for problem (4.1)-(4.12), e.g. one of the solutions  $\hat{y}_N^m$ . With that solution, estimate the objective function value of the original problem using a reference sample  $N'$  as

$$\tilde{g}_{N'}(\bar{y}) := \sum_{j \in \mathcal{L}} \sum_{o \in \mathcal{O}(j)} F_j^o \bar{y}_j^o + \frac{1}{N'} \sum_{n=1}^{N'} \mathcal{Q}(y, \xi^n).$$

The estimator  $\tilde{g}_{N'}(\bar{y})$  serves as an upper bound on the optimal objective function value.

The reference sample  $N'$  is generated independently of the samples used in the SAA problems. Since the first-stage solution is fixed, one can choose a greater number of scenarios for  $N'$  than for  $N$  as this step only involves the solution of  $N'$  independent second-stage problems (4.3)-(4.12).

We can estimate the variance of  $\tilde{g}_{N'}(\bar{y})$  as follows:

$$\sigma_{N'}^2(\bar{y}) = \frac{1}{(N'-1)N'} \sum_{n=1}^{N'} \left( \sum_{j \in \mathcal{L}} \sum_{o \in \mathcal{O}(j)} F_j^o \bar{y}_j^o + \mathcal{Q}(y, \xi^n) - \tilde{g}_{N'}(\bar{y}) \right)^2.$$

4. Compute the estimators for the optimality gap and its variance. Using the estimators calculated in steps 2 and 3, we get

$$\text{gap}_{N,M,N'}(\bar{y}) = \tilde{g}_{N'}(\bar{y}) - \bar{v}_{N,M} \quad \text{and} \quad \sigma_{\text{gap}}^2 = \sigma_{N'}^2(\bar{y}) + \sigma_{\bar{v}_{N,M}}^2.$$

The confidence interval for the optimality gap is then calculated as

$$\tilde{g}_{N'}(\bar{y}) - \bar{v}_{N,M} + z_\alpha \left( \sigma_{N'}^2(\bar{y}) + \sigma_{\bar{v}_{N,M}}^2 \right)^{1/2}$$

with  $z_\alpha := \Phi^{-1}(1 - \alpha)$ , where  $\Phi(z)$  is the cumulative distribution function of the standard normal distribution.

## Dual Decomposition and Lagrangean Relaxation

Step (1) of the SAA algorithm outlined above involves solving a two-stage stochastic mixed-integer problem (4.13) with  $N$  scenarios. Even though the number of scenarios in this problem is considerably lower than in the original problem (4.1)-(4.12), it is still a large problem. To solve each of the SAA problems, we decompose the problem in scenarios (see e.g. Carøe & Schultz 1999). In order to do this, we introduce first-stage variables  $y^1, \dots, y^n$  for each scenario  $n = 1, \dots, N$  and add non-anticipativity constraints  $y^1 = \dots = y^n$  to the problem (Rockafellar & Wets 1991). For each  $j \in \mathcal{L}, o \in \mathcal{O}(j)$ , we implement the non-anticipativity constraints by the equation  $\sum_{n=1}^N K^n y_j^{on} = 0$  where  $K^1 = 1 - N$  and  $K^n = 1$  for  $n = 2, \dots, N$ .

We define  $\lambda$  as the vector of Lagrangean multipliers associated with the non-anticipativity constraints and relax these. The resulting Lagrangean relaxation is

$$LR(\lambda) = \min_{y \in Y} \left\{ \frac{1}{N} \sum_{n=1}^N \left( \sum_{j \in \mathcal{L}} \sum_{o \in \mathcal{O}(j)} (F_j^o y_j^{on} + \lambda_j^o K^n y_j^{on}) + \mathcal{Q}(y^n, \xi^n) \right) \right\}, \quad (4.15)$$

with  $\mathcal{Q}(y^n, \xi^n)$  being the solution to the second-stage problem (4.3)-(4.12) given realization  $n$  of the random parameters. Note that problem (4.15) is separable in scenarios.

We find the best lower bound for our problem by solving the Lagrangean dual

$$LD = \max_{\lambda} LR(\lambda).$$

To solve  $LD$ , we use cutting planes (Kelley 1961) in a bundle method with box constraints (see e.g. Lemaréchal 1986). Let  $k$  denote the superscript for the current iteration and let  $\nabla^k = \sum_{n=1}^N K^n y^{nk}$  be the subgradient of (4.15) with respect to  $\lambda^k$  in iteration  $k$ . Further, define  $L^k = LR(\lambda^k) - \lambda^k \cdot \nabla^k$  as the value of (4.15) without the Lagrangean penalty term. By  $\Delta$  we denote the allowed change in the Lagrangean multiplier. The Lagrangean multipliers are then updated solving the following linear problem:

$$\max_{\lambda^{k+1}, \phi} \phi \quad (4.16)$$

subject to

$$\forall i = 1, \dots, k : \quad \phi \leq L^i + \nabla^i \cdot \lambda^{k+1} \quad (4.17)$$

$$\lambda^{k+1} \leq \lambda^k + \Delta \quad (4.18)$$

$$\lambda^{k+1} \geq \lambda^k - \Delta \quad (4.19)$$

$$\phi \in \mathbb{R}, \lambda^{k+1} \in \mathbb{R}^{|\mathcal{L}| \cdot |\mathcal{O}|} \quad (4.20)$$

The Lagrangean multipliers are optimal once  $\phi$  is no longer changing. To speed up the process of finding the optimal Lagrangean multipliers, we solve the single-scenario subproblems only with an optimality gap  $\gamma \geq 0$  in the first iterations. We then reduce  $\gamma$  while we proceed with the iterative procedure and eventually solve all the subproblems to optimality. The idea behind this is that the solutions from the first iterations provide an initial search direction, whereas we need a better accuracy in the later iterations to determine the optimal value of the multipliers.

The solution  $y^k$  produced in iteration  $k$  is in general not feasible for the SAA problem (4.13) as the non-anticipativity constraints may be violated. Step (3) of the SAA procedure requires a feasible solution for the original problem (4.1)-(4.12), so we use a simple heuristic to turn the infeasible Lagrangean solution into a feasible, but possibly not optimal solution. We generate a feasible solution by fixing the binary first-stage variables at 1, if they are 1 in more than 50% of the optimal single-scenario solutions and 0 otherwise. This heuristic may produce feasible solutions far away from the optimal solution, thus increasing the necessary number of iterations. However, it finds good enough solutions for our problem instances. Once the first-stage variables are fixed, we solve the second-stage problem (4.3)-(4.12) for each scenario and get an upper bound on the optimal objective function value.

## Quality of the Solutions

We choose not to solve the Lagrangean subproblems to optimality during the first iterations. During these iterations, we may not get a lower bound on the optimal objective function value. In fact, the calculated lower bound may actually be above the optimal objective function value of the SAA problem. It may also be bigger than the upper bound from a feasible solution. However, we can still guarantee the quality of the feasible solution found.

If we solve all Lagrangean subproblems with an relative optimality gap  $\gamma \geq 0$  and stop the SAA procedure once the relative gap between upper and lower bound estimator is less or equal to  $\epsilon \geq 0$ , then the feasible solution providing the upper bound is a  $(\epsilon + \gamma + \epsilon\gamma)$ -optimal solution to the original SAA problem (4.13).

Let  $LR_\gamma$  be a the  $\gamma$ -optimal upper bound on the solution of the Lagrangean relaxation  $LR(\lambda)$  (4.15). With  $v_N$  being the optimal objective function value of (4.13) and  $\tilde{g}_{N'}(\bar{y})$  being an upper bound provided by the feasible solution  $\bar{y}$ , we get the following 3 cases:

1.  $LR_\gamma \leq v_N \leq \tilde{g}_{N'}(\bar{y})$ :  $LR_\gamma$  is a lower bound for  $v_N$ , i. e. the solution  $\bar{y}$  providing  $\tilde{g}_{N'}(\bar{y})$  is  $\epsilon$ -optimal for the SAA problem.

2.  $v_N \leq LR_\gamma \leq \tilde{g}_{N'}(\bar{y})$ :  $LR_\gamma$  overestimates  $v_N$ , but not by more than  $\gamma$ . The difference between  $LR_\gamma$  and  $\tilde{g}_{N'}(\bar{y})$  does not exceed  $\epsilon$ , thus the feasible solution  $\bar{y}$  is  $(\epsilon + \gamma + \epsilon\gamma)$ -optimal for the SAA problem.
3.  $v_N \leq \tilde{g}_{N'}(\bar{y}) \leq LR_\gamma$ : The difference between  $v_N$  and  $LR_\gamma$  is at most  $\gamma$ . This means, that  $\bar{y}$  is a  $\gamma$ -optimal solution for the SAA problem.

## 4.6 Computational Results

The calculations were carried out on a Linux cluster, running kernel 2.6.9 with each node consisting of two 1.6 GHz Dual-Core Intel Xeon 5110 processors and 8 GB RAM. The solution scheme is implemented in C++ using the library functions of the Message Passing Interface (MPI) for distributed processing and Xpress 2006 runtime libraries as solver for the SAA problems. Xpress 2006 is also used to solve the linear program updating the Lagrangean multipliers.

### Problem Instances

We choose to solve all cases using  $M = 20$  SAA problems. For the SAA problems, we use sample sizes of  $N = 20, 40$ , and 60 scenarios. Dual decomposition is combined with a simple heuristic as described in Section 4.5 for the non-aggregated cases. The heuristic is used every 5 iterations to generate a feasible solution to the SAA problem. We stop once the objective function value is within 5% of the lower bound estimator. The best feasible solution of each SAA is then stored as a candidate solution for valuation in the reference sample. For the aggregated cases, each SAA problem is solved to optimality, so we store the optimal solutions for evaluation in the reference sample. The size of the reference sample is set to  $N' = 1000$  scenarios.

Due to the size of the non-aggregated cases (STU, LTU, and LSTU), we use one processor core per single-scenario subproblem. The single-scenario subproblems have approx. 740000 variables and 165000 constraints. The aggregated problems (ASTU, ALTU, and ALSTU) are much smaller in size, so we can solve stochastic two-stage problems with 60 scenarios without having to decompose them. The aggregated problems with 60 scenarios have approx. 2825000 variables and 926000 constraints.

Solving the reference sample with a given first-stage solution provides a statistical upper bound on the optimal objective function value of the original problem. For the aggregated problem instances, the statistical lower bound is provided by the average of the  $M$  optimal objective function values of the SAA problems.

For the disaggregated problem instances, we normally get an optimality gap of less than 5% after 5-10 iterations when using a  $\gamma$ -optimal solution of  $LR(\lambda)$  to

calculate the estimator for the lower bound on the SAA problem. As  $\gamma > 0$  during these iterations, we cannot guarantee that this estimator is a true lower bound on the optimal objective function value of the SAA problem (see Subsections 4.5 and 4.5). To provide a valid lower bound on the SAA problem and a valid estimate for the optimality gap, we recalculate the lower bound estimator after the SAA procedure is completed using the lower bound on the objective function value of the Lagrangean relaxation. Test runs indicated that this procedure provides good enough solutions considerably faster than using the lower bound on  $LR(\lambda)$  directly or solving  $LR(\lambda)$  to optimality.

The statistical lower and upper bounds are shown in Table 4.1. We compare the results of the different problem instances with the solution to the expected value problem (EVP), i. e. the solution to the problem where the uncertain parameters are replaced by their expected value. We compute the upper bound provided by the EEV (see e.g. Birge & Loveaux 1997), the expected value of the EVP solution, by finding the expected value of implementing the EVP first-stage solution for the different cases. For each disaggregated case, we also give the upper bound when using the solution from the corresponding aggregated case (ASTU, ALTU, and ALSTU).

Firstly, we note that the feasible solutions from the SAA problems give an upper bound that is approx. 16% lower than the EEV. The value of the stochastic solution (VSS, see Birge & Loveaux 1997) is at least 180 mill NOK. Secondly, using the first-stage solution from the corresponding aggregated case as a solution for the disaggregated case works better than the solution from the EVP. However, the solutions for the aggregated case still give expected results that are approx. 50 mill. NOK worse than the best solutions from the disaggregated cases. Thirdly, the estimator for the lower bound is increasing in the number of scenarios while its variance is decreasing.

In Table 4.2, we present the estimator for the optimality gap as well as the upper and lower limit of the 90%-confidence interval for the best solution from solving the SAA problems with the different sample sizes. The estimator for the optimality gap is calculated by subtracting the lower bound (the SAA procedure) from the upper bound (the reference sample). The optimality gap for the EEV and the aggregated cases is calculated using the best lower bound from the SAA problems.

The confidence interval for the optimality gap is getting narrower as we increase the number of scenarios in the SAA problem. This is mainly due to a smaller variance of the lower bound. Thus, increasing the number of scenarios, we can give a better guarantee with respect to how close we are to the optimal solution. The results for the optimality gap indicate that the solutions produced by our scheme are good enough to be used in a practical application.



**Table 4.1:** Statistical lower and upper bounds of the SAA problems for  $M = 20$  and  $N' = 1000$ .

Case	$N$	Lower bound		Upper bound	
		average	$\sigma_{LB}$	average	$\sigma_{UB}$
STU	20	1182829	2784	1218230	1871
	40	1185931	2835	1217120	1947
	60	1186206	1920	1225730	1946
	EEV			1484790	1939
	ASTU			1269570	1950
LTU	20	1288111	37107	1384230	28545
	40	1331218	28018	1349840	28158
	60	1306378	25971	1381640	29665
	EEV			1630180	30180
	ALTU			1439330	30175
LSTU	20	1334359	44236	1327400	28671
	40	1310124	25211	1340130	28927
	60	1314995	22901	1382570	28540
	EEV			1569500	30660
	ALSTU			1383140	30595
ASTU	20	1118160	2396	1119390	1653
	40	1119700	2149	1116760	1657
	60	1119530	1496	1116250	1653
	EEV			1393740	1670
ALTU	20	1268770	47003	1298210	27811
	40	1244490	42382	1301490	28011
	60	1277340	24330	1305560	27699
	EEV			1513600	28456
ALSTU	20	1226480	31067	1235240	28100
	40	1249040	35042	1251810	27826
	60	1251200	27647	1246130	28574
	EEV			1443540	28875

**Table 4.2:** Estimated optimality gap and confidence interval with  $M = 20$  and  $N' = 1000$ .

Case	$N$	Estimated optimality gap			90% Confidence Interval			
		[1000 NOK]	%	$\sigma_{gap}$	min	%	max	%
STU	20	35401	2.99	3354	30573	2.58	40230	3.40
	40	31189	2.63	3439	26239	2.21	36139	3.05
	60	39524	3.33	2734	35589	3.00	43459	3.66
	EEV	298584	25.17	2728	294657	24.84	302512	25.50
	ASTU	93364	7.03	2737	79425	6.70	87304	7.36
LTU	20	96119	7.46	46817	28725	2.23	163513	12.69
	40	18622	1.40	39722	-38559	-2.90	75804	5.69
	60	75363	5.76	39428	18504	1.42	109162	8.36
	EEV	298962	22.46	41181	239682	18.00	358243	26.91
	ALTU	108112	8.12	41177	48837	3.67	167388	12.57
LSTU	20	-6959	-0.52	52715	-82843	-6.21	68926	5.17
	40	30006	2.29	38371	-25231	-1.93	85243	6.51
	60	67575	5.14	36592	14899	1.13	120251	9.14
	EEV	235141	17.62	53822	157662	11.82	312620	23.43
	ALSTU	48731	3.66	53785	-28645	-2.15	126207	9.46
ASTU	20	1230	0.11	2911	-2960	-0.26	5420	0.48
	40	-2940	-0.26	2714	-6847	-0.61	967	0.09
	60	-3280	-0.29	2229	-6489	-0.58	-71	-0.01
	EEV	274040	24.47	2721	270123	24.12	277957	24.82
ALTU	20	29440	2.32	54614	-49179	-3.88	108059	8.52
	40	57000	4.58	50802	-16131	-1.30	130131	10.46
	60	28220	2.21	36867	-24851	-1.95	81291	6.36
	EEV	236260	18.50	37439	182366	14.28	290154	22.72
ALSTU	20	8760	0.71	41890	-51543	-4.20	69063	5.63
	40	2770	0.22	44746	-61643	-4.94	67183	5.38
	60	-5070	-0.41	39635	-62125	-4.97	51985	4.15
	EEV	192340	15.37	39852	134972	10.79	249708	19.96

The average CPU-time per processor core for solving a single scenario in the SAA problem varies between 34 min (STU,  $N = 60$ ) and 62 min (LTU,  $N = 60$ ). For cases STU and LSTU the CPU-time is slightly decreasing in the number of scenarios, whereas it is slightly increasing for case LTU. The CPU-time for the aggregated cases is increasing in the number of scenarios per SAA problem. This is not surprising, as these problems are not decomposed. The CPU-time required for these problems increases for case ASTU from 46 min ( $N = 20$ ) to 206 min ( $N = 60$ ), for case ALTU from 32 min ( $N = 20$ ) to 148 min ( $N = 60$ ), and from 36 min ( $N = 20$ ) to 166 min ( $N = 60$ ) for case ALSTU. Evaluating a given candidate solution in the reference sample requires approximately 150 min for the disaggregated cases when using 20 processor cores. The candidate solutions of the aggregated cases need 10-15min of CPU-time.

## Solution Properties

When we compare the stochastic solutions to the expected value solution, we see that the solutions from the SAA problems open facilities with more capacity than the EVP solutions. The solutions of the disaggregated problem instances also install more capacity than the solutions of the aggregated cases. Case LTU is an exception, as the best solution from the SAA problems installs less capacity for ground meat than both the EVP solution and the ALTU solution. In Table 4.3, we show the amount of capacity installed per product group for the solution providing the lowest upper bound for each case.

**Table 4.3:** Installed capacities for the different product groups

Case	Installed Capacity (tons/year)		
	Ground Meat	Sausages	Cold Cuts
EVP	18216	14065	11381
STU	23100	23041	11381
LTU	17600	23041	11381
LSTU	18216	33645	11381
AEVP	18216	10604	7920
ASTU	18216	14168	7920
ALTU	18216	14168	7920
ALSTU	18216	14168	7920

Opening facilities with more capacity incurs higher fixed costs. The total costs of operating the value chain however are lower for the stochastic solutions that install more capacity. The additional capacity provides a flexibility in the second

stage that reduces the expected second-stage costs by more than the first-stage costs increase.

One of the purposes of solving several samples in the SAA approach is to find good candidates for first-stage solutions to be tested in the reference sample. The solution scheme produces 19 candidate solutions for case STU with  $N = 20$  and  $N = 40$ . When we solve this case with  $N = 60$ , we get 18 candidate solutions. For case LTU, the number of candidate solutions produced by our heuristic decreases from 20 for  $N = 20$  to 7 when  $N = 60$ . Case LSTU with combined long-term and short-term uncertainty is solved with 20 candidate solutions for  $N = 20$ , 15 candidate solutions for  $N = 40$ , and 20 candidate solutions for  $N = 60$ . The upper bounds from these candidate solutions do not vary a lot, indicating rather flat objective functions. The solution from the expected value problem however performs poorly in all cases. The aggregated cases are all solved to optimality and produce a single solution over all samples. The optimal solution of the aggregated cases is the same for all problem instances, independent of the number of scenarios or the type of uncertainty modeled.

## 4.7 Conclusions

In this paper, we have presented a supply chain design problem from the Norwegian meat industry. The mathematical formulation of the problem can be applied to any supply chain that consists of subsequent levels of splitting and combining processes. It is also possible to adapt the model for supply chains that consist only of splitting processes (e.g. the processing industry) or combining processes (e.g. manufacturing and assembly-based industries). We model both detailed operations of the supply chain and aggregated operations in the second-stage of the problem and examine the effect of this modeling choice on the first stage decisions. Due to the size of the non-aggregated problem instances, we use Sample Average Approximation in combination with dual decomposition to solve our problem. Comparing the results from both types of second-stage models to their corresponding expected value problem, we see that the first-stage decisions from the stochastic problems result in considerably lower costs than the solution of the EVP. We also note that the first-stage solutions from the aggregated problem instances have higher expected costs than those from the disaggregated problem instances in the disaggregated datasets, though not as high as the EVP solution.

The results also show that our solution scheme produces good first-stage solutions already for small sample sizes in the SAA problem. Increasing the sample size improves the lower bound on the optimal objective function value, thus improving the quality guarantee on the optimality gap. Due to the distributed processing of the solution scheme, we can increase the size of the SAA problems by using more processors. For our problem instances, we see that increasing the

number of scenarios in the SAA problems even reduces the total CPU-time, as the bigger problems produce fewer candidate solutions that have to be evaluated in the reference sample.

Even though we get good solutions for our problem instances, it might be worthwhile to investigate if a more sophisticated heuristic for finding feasible solutions produces even better results. We also need to reduce the runtime further in order to include more product families and add more facilities to the supply chain design decisions.

For the end-user in the meat industry, variations of this model have been used in the strategy process to examine structural decisions. A traditional supply chain model would not give the same insight as it lacks the operational detail. As the results show: high variations at the operational level will influence the strategic decisions. This cannot be captured properly in aggregated models. In addition, even when it comes to pure operational models, we do not know any alternative model that handles a combination of splitting processes and combining processes. This is essential in the meat industry, in order to capture the value creation of the cutting stage and the processing stage. Finally, one of the main advantages for Nortura in practical use has been the ability to examine a set of supply chain configurations that are almost equally good. This way, Nortura is more flexible with respect to the future design of the supply chain, knowing that the chosen design might not be optimal, but also that it will not be far away.

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## Paper IV

Peter Schütz and Asgeir Tomasgard:

# **The Impact of Flexibility on Operational Supply Chain Planning**

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## Paper 4

# The Impact of Flexibility on Operational Supply Chain Planning

### Abstract:

We study in this paper the effects of volume flexibility, delivery flexibility and operational decision flexibility in operational supply chain planning under uncertain demand. We use a rolling schedule to plan supply chain operations for a whole year. The planning horizon is 4 weeks with deterministic demand in the first week and predicted for the following 3 weeks. Using a case from the Norwegian meat industry, we compare the annual operating results of using a two-stage stochastic programming model to the deterministic expected value problem in order to discuss the impact of flexibility in the supply chain.

Supply chain planning, Flexibility, Stochastic programming, Demand forecasting

## 5.1 Introduction

Flexibility and robustness are terms often used in connection with companies that have to cope with uncertainty. Robust decisions are unaffected by uncertain events, whereas flexible decisions can be adapted to the new situation. Sabri & Beamon (2000) define two types of flexibility in supply chain management: the first one is volume flexibility or excess capacity. This flexibility allows for increasing or decreasing production according to realized demand. The second type is delivery flexibility, i. e. the ability to change both delivered amount and delivery date. Backlogging demand is a typical example exploiting this type of flexibility, postponing the delivery date. In comparison, volume flexibility is the supply chain's ability to handle different volumes (for example depending on strategic factors like installed capacities) and delivery flexibility is the ability to handle demand as a flexible instead of a fixed entity. These two types of flexibility are usually designed into the supply chain and are decided upon at a strategic level.

In addition, we discuss a third type of flexibility which we denote operational decision flexibility and a fourth type, storage flexibility. Operational decision flexibility is used to describe flexibility in the supply chain operations: e. g. the

assignment of jobs to machines is changed due to a breakdown, a different bill of material has to be used to produce the finished product, production volumes are assigned to another production facility to exploit economies of scale. These changes to the original production plan usually increase the production costs, but might be necessary to satisfy customer demand. Operational decision flexibility can also be used to exploit market opportunities, e. g. peaks in demand or high prices for finished products.

Storage flexibility is the ability to transfer raw material or finished products in time. Inventory held “just-in-case” to accommodate sudden peaks in demand or bottlenecks in production capacity is one way of using this flexibility (it might not be the most economic though). We can also utilize storage flexibility to balance seasonal variations in supply and demand. For example, we can increase the inventory of certain raw materials in times of cheap supply to reduce the procurement costs at periods with high prices for the raw materials. Storage flexibility is effectively limited by a product’s shelf life. We will discuss the relations between these types of flexibility and the effects of uncertainty.

It is well known that flexibility only has a value in the presence of uncertainty and that stochastic programming is able to explicitly value flexible decisions (see e. g. Christiansen & Wallace 1998, Fleten, Jørgensen & Wallace 1998). There is however little discussion about which types of flexibility have a value and under which circumstances. Schütz, Tomasgard & Ahmed (2008) for example, point out the value of volume flexibility in a supply chain design problem from the Norwegian meat industry. They compare the design decisions of the deterministic expected value problem to the two-stage stochastic programming problem. The results show that using volume flexibility reduces the costs of operating the supply chain by more than what is required to install the additional capacity. It is the purpose of this paper to create insights in the relationship between uncertainty and flexibility. We also discuss the conditions under which flexibility has a value.

The traditional approach to operational supply chain planning has been to use deterministic optimization (see e. g. Hax & Candea 1984, Bitran & Tirupati 1993). Escudero (1994) for example formulates and solves a deterministic linear model for production and inventory planning in a multi-commodity supply chain. More production-inventory models are discussed in the review by Goyal & Gunasekaran (1990). Deterministic optimization however cannot value flexibility correctly as the assumption of perfect foresight does not require flexible solutions.

The fact that deterministic models have problems capturing the real-world dynamics together with the availability of sophisticated solution algorithms and increased computational power, have lead to stochastic optimization becoming more popular. Escudero, Galindo, García, Gómez & Sabau (1999), Alonso-Ayuso, Escudero & Ortuño (2005), and Tomasgard & Høeg (2005) propose stochastic programming models for operational supply chain planning. Escudero

et al. (1999) propose a modeling framework for a supply chain with uncertainty in product demand, component supply cost, and delivery time. Alonso-Ayuso et al. (2005) formulate a multi-stage stochastic problem with significant delivery lead time. They consider product demand, production and supply costs, as well as available resource capacity as uncertain. The fraction of total unsatisfied demand that is lost is uncertain as well. Tomasgard & Høeg (2005) present a linear two-stage stochastic programming model for the supply chain found in the Norwegian meat industry.

Flexibility is often present in real-world supply chains, either as volume flexibility (excess capacity), delivery flexibility, or operational flexibility (i. e. the possibility of using different bills of materials in the production process). The papers mentioned above mainly provide models and solution methods, but do not explicitly discuss the issue of flexibility. To study the impact of flexibility inherent in the supply chain on the value of flexible operational decisions, we simulate the planning process for a whole year. We use a rolling schedule for weekly planning with a planning horizon of 4 weeks (see e. g. Baker & Peterson 1979). Based on real-world data from the Norwegian meat industry, we compare the annual operating results of four different planning models: a model with a deterministic demand forecast, one with a stochastic demand forecast, a model using perfect demand information with a rolling schedule and one with perfect demand information for the complete year in a single model. We use the results to examine the value of operational decision flexibility.

The traditional measures for comparing deterministic optimization models to stochastic ones are the expected value of perfect information (EVPI) and the value of the stochastic solution (VSS) (Birge & Loveaux 1997). As we use real demand to evaluate our decisions, instead of a set of demand scenarios, we cannot use these measures directly. The planning problem with perfect demand information for the complete year serves as our benchmark. Comparing the operational results from the other models to this one, we can actually calculate ex-post the value of perfect information. As we re-optimize the planning problem each week using a rolling schedule, we are only interested in the actual profits of implementing the first-stage decisions. In contrast to the VSS, we only consider the realized profits of both the stochastic planning model and the deterministic expected value problem. The value of the recourse function is not part our analysis. The difference in realized operational results is better described by the term "value of stochastic planning". Here, we refer to the difference in accumulated profits realized in the first week of each of the 52 planning problems.

We first describe the case from the Norwegian meat industry, i. e. the supply chain and the planning problem, in Section 5.2. The mathematical formulation for the two-stage stochastic programming problem is given in Appendix 5.A. The forecasting and scenario generation methods are presented in Section 5.3.

Computational results and the discussion of the impact of flexibility on these follow in Section 5.4. We conclude in Section 5.5.

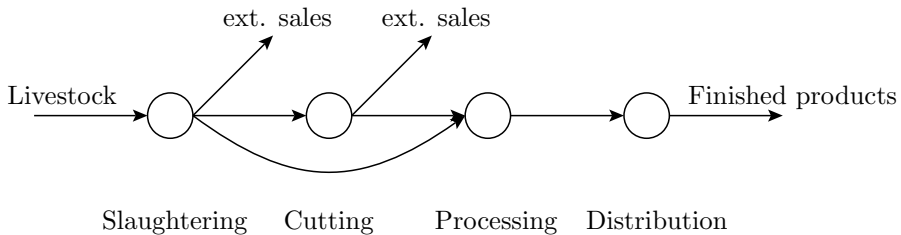
## 5.2 The Supply Chain Planning Problem

We consider a planning problem for a supply chain from the Norwegian meat industry. The task is to balance supply and demand on a weekly basis, ensuring that the right raw materials are available at the right production facilities in order to satisfy demand. The planning horizon is four weeks. Demand for the first week is known with certainty, whereas planning of production and material flow for the remaining three weeks is based on predicted demand. Decisions are implemented as rolling schedules (see e. g. Baker & Peterson 1979).

The two-stage stochastic programming formulation is based on the work by Schütz et al. (2008) and adapted to the operational planning problem. The complete formulation is given in Appendix 5.A.

### The Supply Chain

The supply chain we consider consists of 17 slaughterhouses, 16 cutting plants, 14 processing facilities, and 7 distribution centers. The material flow within the supply chain is depicted in Figure 5.1.



**Figure 5.1:** Material flow in the supply chain of the Norwegian Meat Cooperative

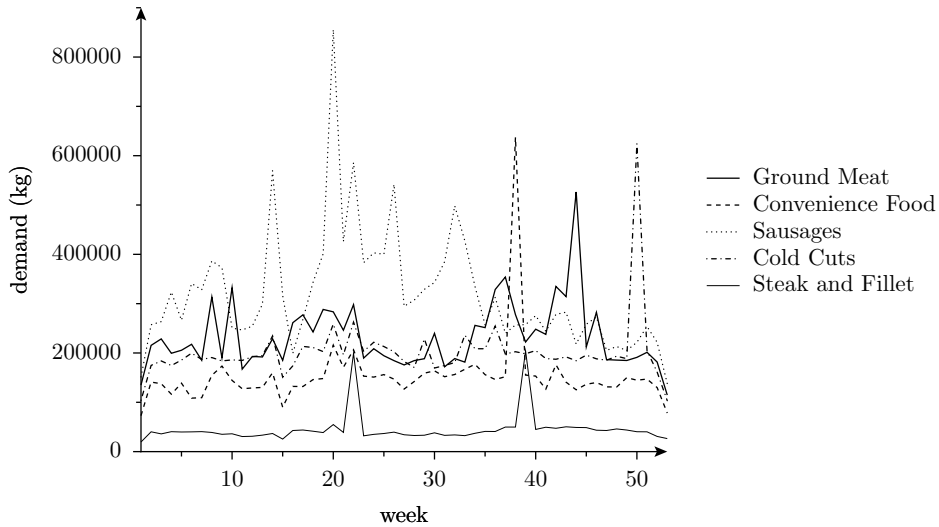
Livestock is delivered to the slaughterhouses which produce carcasses and intestines. These products are sent to downstream facilities: carcasses to the cutting plants and intestines to the processing facilities. The cutting plants produce intermediate products that are shipped to the processing plants. The processing plants transform the intermediate products into finished products, which are shipped to the distribution centers that satisfy customer demand. Intermediate products from the slaughterhouses and the cutting plants can be sold to exter-

nal customers as well. See Tomasgard & Høeg (2002, 2005) for a more detailed description of the supply chain.

## Product Data

We consider a set of 100 end-customer products, belonging to five different product groups (ground meat, convenience food, sausages, cold cuts, as well as steak and fillet). The products we have chosen represent approx. 50% of total sales. The supply of livestock is reduced to 50%, i. e. the amount of total sales represented by products in the dataset. We can choose among 125 recipes to produce these products. The recipes need 34 different intermediate products that are produced from six different types of livestock using one of 322 cutting patterns.

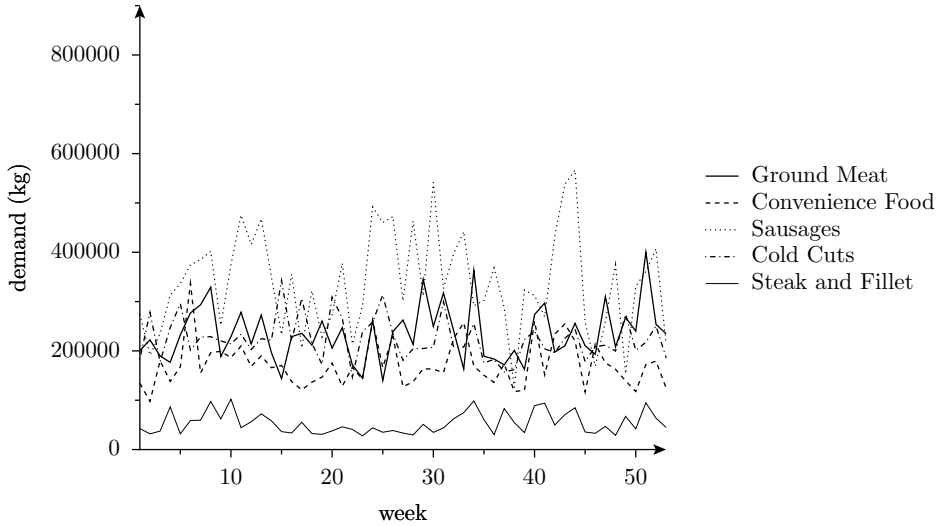
We test our model using two different time series for these products. The first time series is real demand data of 2004 (see Figure 5.2). Demand in this figure is aggregated into product groups, while the computations in this paper are based on single products. Still we see that demand is quite volatile for some groups. Demand for a single product may within 1-2 weeks change by far more than the 100% we observe in Figure 5.2. Other groups exhibit a relatively stable demand most of the year.



**Figure 5.2:** Real demand data for the different product groups in 2004

The second time series uses artificial demand. We apply the moment-matching method by Høyland, Kaut & Wallace (2003) to generate a discrete approximation

of the demand distribution for each product. The first two moments and the correlation correspond to real demand, but the third and the fourth moments are those of a normal distribution. We use 52 of the datapoints as time series for weekly demand of each product. The aggregated time series for the different product groups are shown in Figure 5.3. Most of the peaks of the real demand time series are removed, but also the periods with relatively stable demand disappear. Using the datasets with artificial data, we can study the model's behavior without the distinctive demand pattern from the meat industry.



**Figure 5.3:** Artificial demand data for the different product groups

The planning process is based on forecasting demand. We use an autoregressive process of 3rd order, AR(3)-process (see e. g. Hamilton 1994) as prediction model for both real and artificial demand. For the real demand time series, we perform an out-of-sample analysis as well as an in-sample analysis. For the out-of-sample analysis, we parameterize an AR(3)-process based on demand data from 2003. The in-sample analysis used 2004 demand data to estimate the parameters of the AR(3)-process. The prediction model of the in-sample analysis is overoptimistic with respect to the quality of the prediction, as we use data only available ex-post. We can however eliminate the risk of the AR(3)-process being wrong due to changes in the underlying stochastic process. For the artificial demand data, we perform only an out-of-sample analysis. The different combinations are summarized in Table 5.1.

**Table 5.1:** Summary of the different prediction models.

Dataset	AR(3)-parameter estimated for	Model tested against
03	2003 demand data	2004 demand data
04	2004 demand data	2004 demand data
AD	artificial data	artificial data (out-of-sample)

### 5.3 Demand Forecasts and Scenario Generation

Short-term supply chain planning is usually based on a combination of demand forecasts and already known demand. As historical sales data is easily available in most companies, time series analysis is commonly used to predict future demand. To model the short-term uncertainty, we use a methodology developed by Nowak & Tomasgard (2007) that combines forecasting and scenario generation. We present the main idea here.

Using an autoregressive process of  $N$ th order, AR( $N$ )-process, we can write demand in the next period as

$$x_{t+1} = \alpha + \sum_{i=1}^N \beta_i \cdot x_{t+1-i} + \epsilon_{t+1},$$

where  $x_t$  are the historical demand observations and  $\epsilon_{t+1}$  is the prediction error.

Assuming that all uncertainty of future demand is assigned to the prediction error  $\epsilon$ , we can split the forecast into a perfectly predictable part and the prediction error. Predicted demand for the next period can be rewritten as

$$\hat{x}_{t+1}(\omega) = \alpha + \sum_{i=1}^N \beta_i \cdot x_{t+1-i} + \epsilon_{t+1}(\omega),$$

with  $\omega$  being a realization of the uncertain events affecting future demand.

We then generate scenarios for the prediction error and create a scenario tree for future demand. This methodology is independent of the forecasting method and can therefore be applied to any method. A more detailed description of the scenario generation procedure can be found in Schütz et al. (2008). We include it here as well for the sake of completeness.

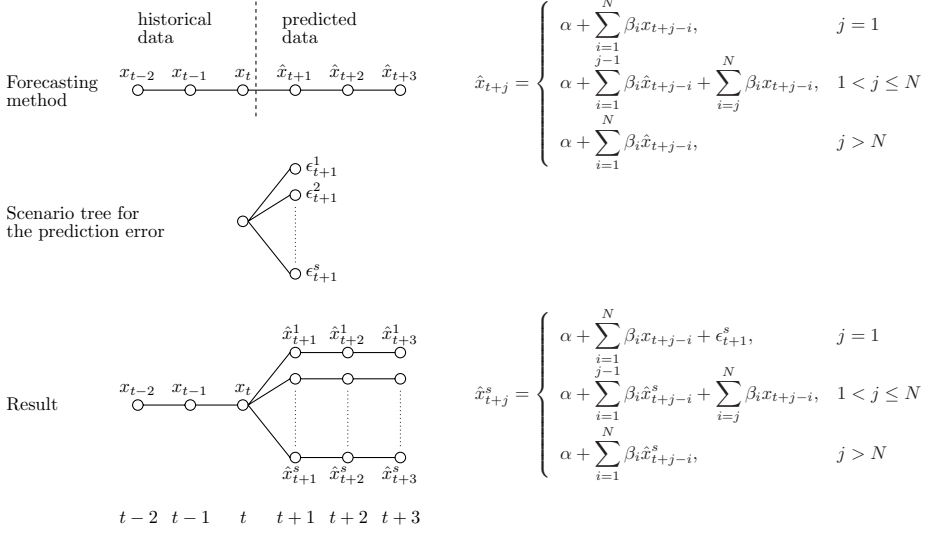
1. For each product, we parametrize an AR( $N$ )-process on historical data.
2. We determine the distribution of the historical prediction error for each product. We assume that the error term for a given product is i.i.d. between different time periods. Thus, we get a multivariate discrete distribution for all the products' error terms.



3. We then perform a principal component analysis on this empirical prediction error distribution. The  $P$  principal components for the  $P$  products are sorted by the share of variance they explain for the error distribution.
4. We calculate the first 4 moments of the empirical distribution of the  $K$  principal components explaining the highest part of variance. For the  $P - K$  remaining components we use their expected value.
5. We then generate  $S$  scenarios for the  $K$  stochastic principal components using a moment matching procedure ensuring that the first 4 moments in the scenarios are the same as in the historical distribution. The method used here is a modified version of the method developed by Høyland et al. (2003).
6. The scenarios for the principal components are then transformed back into scenarios for the prediction error.
7. Finally, we combine the forecasting method with the scenario tree for the prediction error to get demand scenarios rather than pure prediction error scenarios. This procedure is illustrated in detail in Figure 5.4: Using a deterministic AR(N)-process as forecasting method, predicted demand in period  $t + 1$  is given by the formula  $\hat{x}_{t+1} = \alpha + \sum_{i=1}^N \beta_i x_{t+1-i}$ . In our case, we add a realization of the error term  $\epsilon_{t+1}^s$  as represented in the scenario tree to the first prediction. Thus, the first prediction  $\hat{x}_{t+1}^s = \alpha + \sum_{i=1}^N \beta_i x_{t+1-i} + \epsilon_{t+1}^s$  is based on historical data and the error scenario  $\epsilon_{t+1}^s$ . When predicting demand for several periods, the prediction may be based on both historical demand, predicted demand, and the error scenarios as shown in Figure 5.4.

The correlations between the stochastic variables, i. e. the prediction errors, are taken care of through the principal component analysis. Using principal component analysis, we can also reduce the number of stochastic variables as only the most important principal components (those explaining most of the variance) will be considered in the moment-matching procedure.

For our problem instances, we use the 25 products with highest revenues as stochastic, i. e. we generate a scenario tree for the stochastic prediction error using the method described above. The other products are considered to be deterministic, i. e. their prediction error is 0. We generate 50 scenarios for 20 principal components. With these principal components, we explain 62% of the variance for the dataset based on real demand data and 72% of the variance for the dataset based on artificial demand data. The number of scenarios should be higher in order to give a better description of the demand uncertainty, but we



**Figure 5.4:** Combining forecasting and scenario generation

are limited by the amount of available memory of the computers used to solve the problem instances.

We generate one scenario tree for the prediction error and use the same scenarios for all runs of the planning model. These prediction error scenarios,  $\epsilon_{t+1}^s$ , are only added to the first demand forecast,  $\hat{x}_{t+1}^s$  (see Figure 5.4), in each optimization run. Predicted demand in the final scenario tree changes as the demand realizations used in the forecasting procedure are updated every time the planning horizon is rolled forward. It would have been possible to generate new scenarios for the prediction error before each new optimization run. In this case, the parameters of the AR(N)-process should be updated according to the new information as well.

## 5.4 Computational Results

The calculations were carried out on a computer with two 1.6 GHz Dual-Core Intel Xeon 5110 processors and 8 GB RAM, running Linux kernel 2.6.9. Xpress-MP 2007 is used to solve all planning problems.

## **Datasets and Problem Instances**

We tested our model with 3 different datasets. Two of these datasets are based on real demand data and tested against 2004 demand: the first set is using an AR(3)-process parameterized on demand data from 2003 and the second set where the prediction parameters are estimated based on demand data from 2004. The remaining set is based on artificial demand data. It would have been desirable to test the datasets against more real-world demand time series than just the one from 2004. This would have provided a broader basis for the discussion of the results presented in the next section. Unfortunately, we were not able to retrieve more demand data for the products in our datasets. An alternative would have been to generate synthetic time series and test the datasets against these. However, generating a synthetic time series reflecting the true behaviour of real-world demand is rather difficult. The results from the synthetic time series might not be comparable to real-world results.

We define 12 problem instances for each of the three parameter set/time series-combinations. These problem instances are characterized by the cutting capacity, the handling of unsatisfied demand, and the shortfall costs. We examine the effect of reducing the cutting capacity from 100% to 75% and eventually to 50%. We choose the cutting level of the supply chain to study this effect as it affects the ability to use and build inventory for certain raw materials. The handling of unsatisfied demand is modeled either as lost sales (LS) or as backlog (BL). Backlog introduces the possibility of moving delivery into periods with higher prices or a more suitable supply of raw materials. We penalize unsatisfied demand not at all or using shortfall costs equal to 25% of the market price.

We compare the results of 4 different problem types for each problem instance. The first problem type is the two-stage stochastic programming problem with 50 scenarios for the prediction error. The second problem type is the expected value problem (i. e. the prediction error is 0). The third and the fourth problem type use perfect demand information. Problem types 1-3 are implemented with a planning horizon of 4 weeks using rolling schedules to plan production for the whole year. The last problem type models the whole year in a single problem with a planning horizon of 56 weeks (the same period as for the problem types with rolling schedules). The sizes of the different problem types are summarized in Table 5.2.

## **Results**

We present the results of our calculations per problem instance. The problem instances are identified by the time series the forecasting method is parameterized on, the capacity available at the cutting level, the handling of unsatisfied demand, and the shortfall costs. Problem instance 03\_100\_LS\_0 for example, uses

**Table 5.2:** Model size of the different problem types.

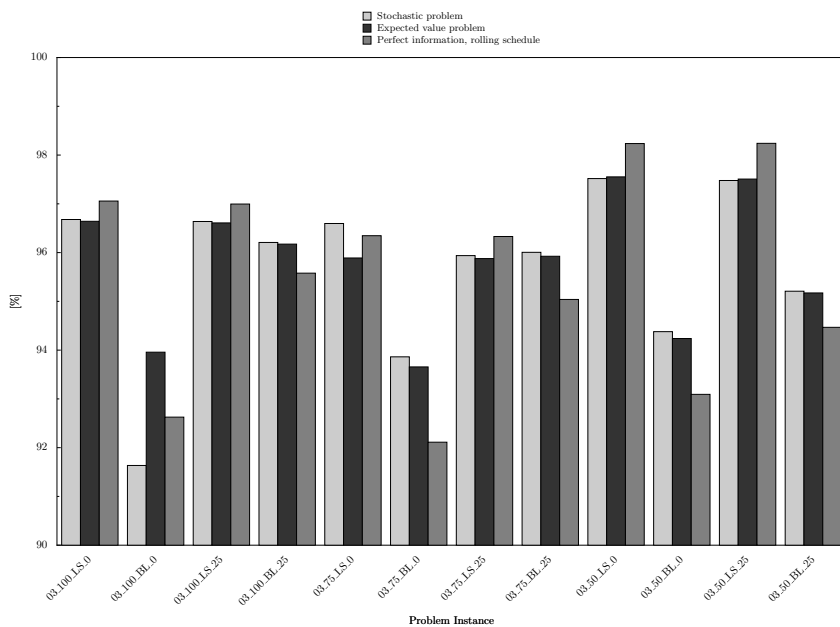
Problem type		Variables	Constraints
Stochastic problem, 50 scenarios	(stoch)	3.6 million	1.9 million
Expected value problem	(ev)	98000	50000
Perfect information, rolling schedule	(pir)	98000	50000
Perfect information, 1 year	(pi)	1.3 million	694000

a forecasting method parameterized on 2003 demand data, has 100% cutting capacity available, models unsatisfied demand as lost sales, and has no shortfall costs. Problem instance AD\_75\_BL\_25 denotes the instance with the forecasting method being parameterized on artificial data, 75% cutting capacity, backlog and a shortfall penalty equal to 25% of the market price.

For each problem instance, we compare the results of the four problem types described in the previous section. The results from the 1-year planning problem under perfect information are the benchmark for the other problem types, i. e. the stochastic problem, the expected value problem, and the planning problem under perfect information using rolling schedules. The results presented in this section are best suited to show the effects discussed below. We observe the same effects for all problem instances, although not all of them are as clear as the ones shown here. The complete results are given in Appendix 5.B.

Let us look at the operating results first, i. e. *revenues–production costs–transportation costs–inventory costs*. The shortfall penalty is not included here. Figure 5.5 shows the results for the different problem instances for the dataset based on parameterizing the forecasting method with demand data from 2003. We present the results of each problem instance relative to the corresponding benchmark. Therefore, we can only compare the results of the different problem types for a given problem instance. The bars cannot be used to compare the absolute operating profits of different problem instances.

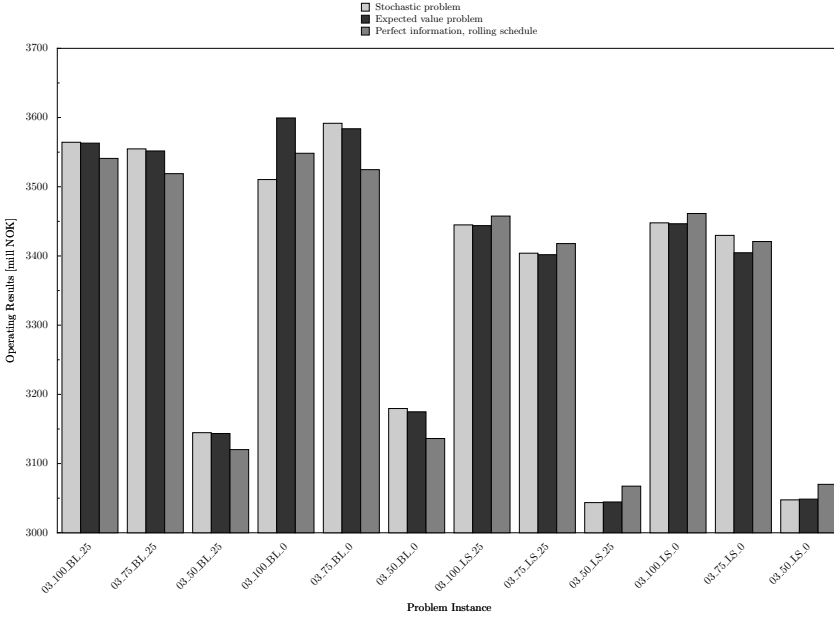
To examine whether a stochastic planning approach is better than a deterministic approach, we compare the accumulated operational results. At a first glance, we see that the differences between the stochastic problems and the expected value problems are small, often less than 0.5%. For most of the problem instances, the stochastic approach performs slightly better than the deterministic expected value approach. Looking at the results of the dataset based on artificial data, we observe basically the same results (see Appendix 5.B). This raises the question why the accumulated profits are that similar, despite the fact that demand is highly volatile. We will explore this issue in the following, in particular discussing the value of flexibility.



**Figure 5.5:** Operating results of different problem instances based on real demand data relative to their benchmark

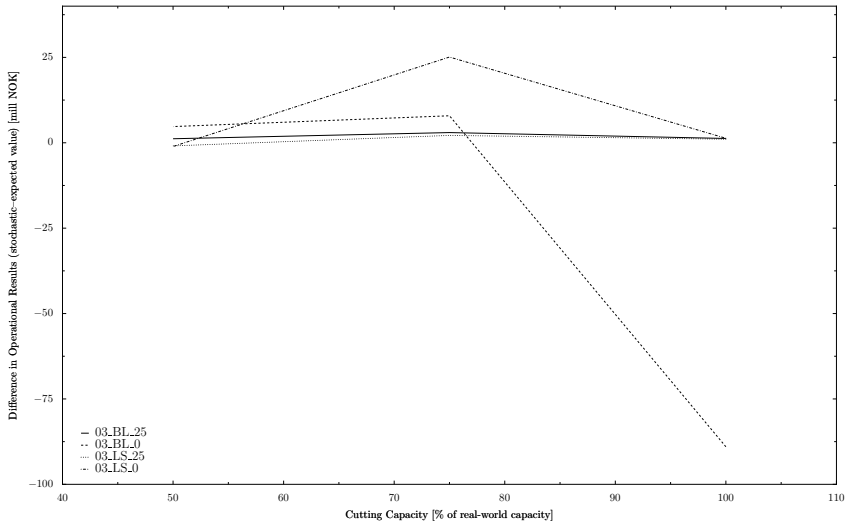
Firstly, comparing the absolute operating results for the dataset based on the parameterization on 2003 demand data (Figure 5.6), we see that the differences in absolute results when going from full cutting capacity to 75% cutting capacity are quite small. This behaviour is a sign of existing volume flexibility, i. e. excess capacity, in the system. This is again illustrated when comparing to the instances with 50% capacity where absolute income drops substantially as volume flexibility is gone.

Secondly, when comparing the relative values of Figure 5.5, there is a clear trend that the 50% capacity cases are closer to the benchmark. For all problem instances we observe that the inventory costs of the benchmark are 2-5 times higher than for the other problem types due to tactical inventory build-up. We also observe considerably higher sales revenues for the benchmark problems. From this we conclude that a planning horizon of 4 weeks is not long enough to plan tactical inventory build-up for future peaks in demand. Neither of the short term models with 4 week horizon are able to fully utilize storage flexibility. When volume flexibility is sufficiently low (like in the 50% cases), the value of the storage flexibility is reduced in the benchmark and the short-term models get closer in value.



**Figure 5.6:** Absolute operating results of different problem instances based on real demand data

Thirdly, we observe that the stochastic problem increases its advantage over the expected value problem when cutting capacity is reduced to 75% of the original capacity. The reduction in volume flexibility requires a higher degree of operational decision flexibility, thus favouring the stochastic solutions. In one case for example (03\_100\_BL.0 and 03\_75\_BL.0), the expected value problem outperforms the stochastic problem by more than 2% at full cutting capacity. With only 75% cutting capacity available, the stochastic problem achieves a higher operating profit than the expected value problem. When further reducing cutting capacity to 50% of the original capacity, we remove basically all volume flexibility, and as a consequence, there is hardly any difference in annual operating results of the stochastic problems and the expected value problem (see Figure 5.7). The reduced cutting capacity seriously limits the ability to exploit operational decision flexibility in order to satisfy customer demand and build tactical inventories. This is an interesting observation as it seems that even the deterministic model is able to take advantage of a situation with high volume flexibility, when the volume flexibility is decreased the stochastic model does better. It is worth noting that when the volume flexibility completely disappears, the operational decision flexibility has no value again and the deterministic model suffices.



**Figure 5.7:** The value of stochastic planning for different amounts of volume flexibility

Finally, delivery flexibility is defined as the ability to change delivery dates. This type of flexibility is modeled as backlog in our problem formulation. Pe-

nalizing unsatisfied demand with 25% of the market price removes much of this flexibility. Again we see a distinct advantage for the stochastic approach when moving from a situation with free, supply chain inherent delivery flexibility (e. g. no shortfall penalty, 03.100.BL.0) to a situation where the usage of delivery flexibility is costly (e. g. shortfall penalty equal to 25% of the market price, 03.100.BL.25). We observe that full delivery flexibility is needed for the stochastic model to utilize the operational decision flexibility when reducing volume flexibility (see Figure 5.7). In case both volume and delivery flexibility are limited, the stochastic model's ability to exploit operational flexibility is limited as well, and the value of a stochastic model decreases (relative to the deterministic). Interestingly, in the presence of delivery flexibility, both the stochastic and the expected value planning problems outperform deterministic planning based on perfect information for the next four weeks.

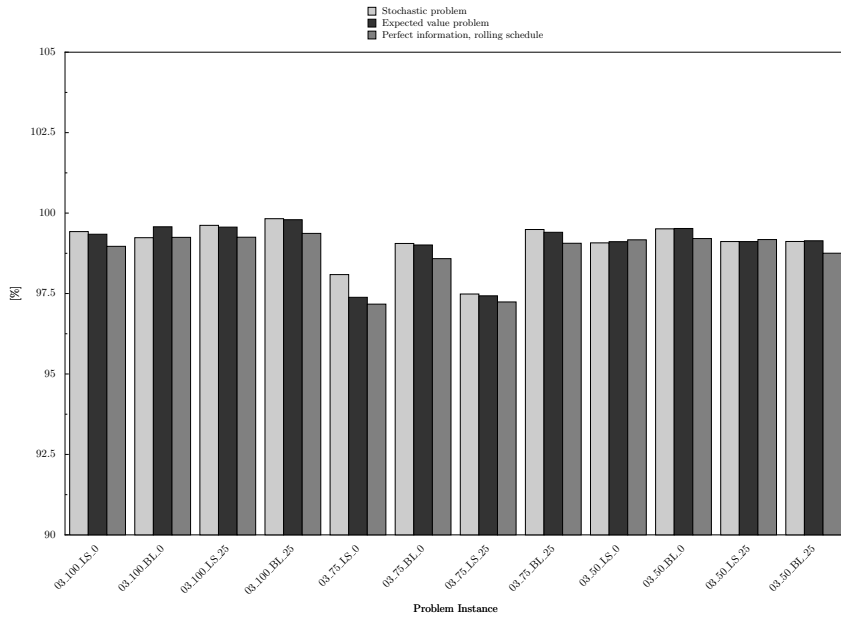
The costs are dominated by the production costs, accounting for approximately 94% of the total costs. Transportation costs stand for 5% of the total costs, with the remaining 1% being the inventory costs. When comparing the total costs of the different problem instances (see Figure 5.8 for problem instances based on real demand), we see that most of the real world instances vary slightly around 99% of the corresponding benchmark costs. It is also worth noting that some of the stochastic problems achieve higher operating results than the corresponding expected value problem, even though they incur higher costs. The problem instances based on artificial data exhibit a greater variability in total costs, but we observe similar results (see Appendix 5.B).

## 5.5 Conclusions

We have discussed the value of different types of flexibility in operational supply chain planning subject to uncertain demand. Even though it is well understood that flexibility only has a value in an uncertain environment, there has been little discussion about the value of different types of flexibility. We distinguish between flexibility already inherent in the supply chain (for example volume flexibility based on excess capacity) and flexibility from operational decisions (like choosing a bill of material). Based on real-world data from the Norwegian meat industry, we study the impact of volume flexibility, delivery flexibility, storage flexibility and operational decision flexibility on the operational profits of a supply chain. Our findings should be applicable to other industries as well.

The results of our analysis show that – given sufficient flexibility in the supply chain – a deterministic approach to supply chain planning may result in equally good (or better) results as a stochastic planning model. The main reason behind this is that flexibility already present in the production system can be used by both the deterministic and the stochastic models. Take for example volume





**Figure 5.8:** Costs of the different problem instances based on the 2003 parameterization relative to their benchmark

flexibility: given large excess capacity (and the absence of production lead time), volume flexibility can be used for make-to-order production. In that case, there is no need for flexible operational decisions and as such the stochastic model cannot provide additional value. Reducing volume flexibility, make-to-order is no longer possible and operational decision flexibility becomes more important. Flexible decisions now have a value as production plans may have to be changed in order to be able to satisfy demand. Once volume flexibility is removed, operational decision flexibility has little value as one has to focus on satisfying the most profitable orders first.

We observe a similar effect for delivery flexibility as well: if a lot of delivery flexibility is present in the supply chain, we can easily move demand to another time period. Thus we are able to manage bottlenecks in production and supply of raw materials by smoothing demand. Reducing delivery flexibility, we need flexible decisions to cope with peaks in demand. Once we remove delivery flexibility, we are no longer able to exploit operational decision flexibility.

The different types of flexibility are connected to each other. Their value depends on the presence of other types of flexibility and also on the planning horizon: when the planning horizon is long enough to value the build-up of tactical inventories (as in the benchmark problems), storage flexibility gets important as well, but the value is reduced if volume flexibility is too low to utilize it.

As a general conclusion, one should study the flexibility already inherent in the supply chain. If a lot of volume and delivery flexibility is present in the supply chain, operational decision flexibility usually has less value. Redesigning the supply chain, reducing both volume and delivery flexibility (which are expensive), can actually reduce its total costs. This implies using operational decision flexibility and storage flexibility instead. In this case, a stochastic model might be required for operational planning as it will provide flexible decisions. Hence, this can be viewed as transferring flexibility from the strategic level (capacities and delivery agreements) to the operational level (operations and storage).

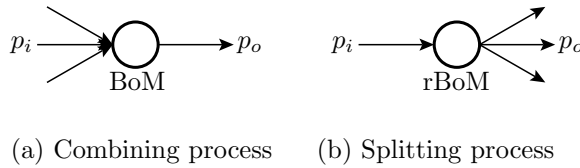
This paper provides some initial insight in the relationship between uncertainty and the need for flexibility. More research is required to further increase the understanding of the value of different types of flexibility and the conditions under which this flexibility is valuable.

## **5.A Appendix: The Model Formulation**

### **Modeling the Supply Chain**

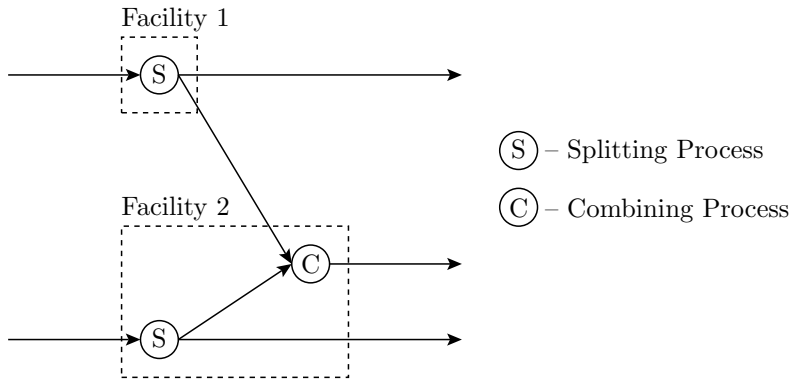
We model the supply chain as a sequence of production processes rather than a network of production facilities. The production process can be different production lines, technologies, or production phases. We distinguish between combining

processes and splitting processes. Combining processes are common in manufacturing and assembling and often described using a Bill of Material (BoM). The BoM lists the type and number of input products  $p_i$  required to produce one output product  $p_o$  (see Figure 5.9a). Splitting processes are described similar: The process industry uses a so-called reversed Bill of Material (rBoM) that defines in which output products  $p_o$  one unit of input product  $p_i$  can be split (see Figure 5.9b). The representation of the supply chain and the mathematical formulation are based on Schütz et al. (2008). The main differences in the model formulation lie in the objective function and the non-anticipativity constraints due to the focus on operational decisions.



**Figure 5.9:** Splitting and combining processes

If a facility houses several production processes, we include multiple nodes, one for each process, for this facility in our supply chain network. This increases the size of the network, but enables us to properly model the material flow in the network. An example of a supply chain consisting of three nodes with two production facilities, two splitting processes, and one combining process is shown in Figure 5.10.



**Figure 5.10:** Example for a simple supply chain with 2 facilities and 3 processes

Our formulation allows us to choose from different BoMs for a given production process. Each BoM is assigned to the production of a given product using a specified production process. Correspondingly, each rBoM is assigned to processing one specific input product.

## Mathematical Formulation

The operational supply chain planning problem is formulated as a two-stage stochastic programming problem (see e. g. Kall & Wallace 1994, Birge & Loveaux 1997). The objective is to maximize the expected profits over the planning horizon. The main focus in terms of decisions is on production levels, inventory build-up, and material flow between the facilities at the different levels of the supply chain.

Let us introduce the following notation for our problem formulation:

- Sets

$\mathcal{F}_c$	Set of possible facility locations for combining processes.
$\mathcal{F}_s$	Set of possible facility locations for splitting processes.
$\mathcal{W}$	Set of possible warehouse locations.
$\mathcal{L}$	Set of all possible locations, $\mathcal{L} = \mathcal{F}_c \cup \mathcal{F}_s \cup \mathcal{W}$ .
$\mathcal{C}$	Set of customer locations.
$\mathcal{U}(j)$	Set of upstream locations able to send products to location $j$ , $j \in \mathcal{L} \cup \mathcal{C}$ .
$\mathcal{D}(j)$	Set of downstream locations able to receive products from location $j$ , $j \in \mathcal{L}$ .
$\mathcal{O}_c(j)$	Set of combining processes that can be performed at location $j$ , $j \in \mathcal{F}_c$ .
$\mathcal{O}_s(j)$	Set of splitting processes that can be performed at location $j$ , $j \in \mathcal{F}_s$ .
$\mathcal{O}(j)$	Set of all processes that can be performed at location $j$ , $\mathcal{O}(j) = \mathcal{O}_c(j) \cup \mathcal{O}_s(j)$ .
$\mathcal{O}$	Set of all processes, $\mathcal{O} = \bigcup_{j \in \mathcal{L}} \mathcal{O}(j)$ .
$\mathcal{B}(o)$	Set of (reversed) bills of materials that can be used for process $o$ , $o \in \mathcal{O}$ .
$\mathcal{P}$	Set of products.
$\mathcal{P}_i(o)$	Set of input products for process $o$ , $o \in \mathcal{O}$ .
$\mathcal{P}_o(o)$	Set of output products of process $o$ , $o \in \mathcal{O}$ .
$\mathcal{N}$	Set of event nodes in the scenario tree.
$\mathcal{S}$	Set of scenarios.
$\mathcal{T}$	Set of time periods.

- Indices and superscripts

$b$	Bill of materials index, $b \in \mathcal{B}(o), o \in \mathcal{O}$ .
$j, k$	Location indices, $j, k \in \mathcal{L} \cup \mathcal{C}$ .
$o$	Process index, $o \in \mathcal{O}(j), j \in \mathcal{L}$ .
$p, q$	Product indices, $p \in \mathcal{P}$ .
$n$	Event node index, $n \in \mathcal{N}$ .
$s$	Scenario superscript, $s \in \mathcal{S}$ .
$t$	Time period index, $t \in \mathcal{T}$ .

- Parameters, constants, and coefficients

$A_{p_o, p_i}^b$	Yield of product $p_o \in \mathcal{P}_o(o)$ in case one unit $p_i \in \mathcal{P}_i(o)$ is processed with reversed bill of materials $b \in \mathcal{B}(o), o \in \mathcal{O}_s$ .
$B_{p_o, p_i}^b$	Amount of product $p_i \in \mathcal{P}_i(o)$ needed to produce one unit $p_o \in \mathcal{P}_o(o)$ using bill of materials $b \in \mathcal{B}(o), o \in \mathcal{O}_c$ .
$C_{jt}^o$	Capacity of process $o$ at location $j$ at time $t$ .
$D_{jt}^{ps}$	Demand for product $p$ at customer location $j$ in scenario $s$ at time $t$ .
$H_{jt}^p$	Shortfall penalty for one unit of demand of product $p$ at customer location $j$ at time $t$ .
$I_{jt}^p$	Cost of holding one unit inventory of product $p$ at facility $j$ in period $t$ .
$P_{jbt}^p$	Cost of processing one unit of product $p$ at location $j$ using (reversed) bill of material $b$ at time $t$ .
$S_{jt}^p$	Supply of product $p$ at location $j$ at time $t$ .
$R_{jt}^p$	Revenue of selling one unit of product $p$ at location $j$ at time $t$ .
$T_{jkt}^p$	Cost of transporting one unit of product $p$ from location $j$ to location $k$ at time $t$ .
$i_{j0}^{ps}$	Initial inventory of product $p$ at location $j$ in scenario $s$ .
$z_{j0}^{ps}$	Initial backlog of product $p$ at location $j$ in scenario $s$ .
$\pi^s$	Probability of scenario $s$ .

- Decision variables

$i_{jt}^{ps}$	Inventory of product $p$ at location $j$ in scenario $s$ at period $t$ .
$w_{jkt}^{ps}$	Amount of product $p$ transported from location $j$ to location $k$ in scenario $s$ at time $t$ .
$x_{jbt}^{ps}$	Amount of product $p$ processed/produced at location $j$ using bill of material $b$ in scenario $s$ at time $t$ .
$z_{jt}^{ps}$	Shortfall for product $p$ at customer location $j$ in scenario $s$ in period $t$ .

We now give a mathematical formulation of our supply chain design problem under uncertainty. Unsatisfied demand is modeled as lost sales:

$$\max \sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}} \pi^s \left( \sum_{p \in \mathcal{P}} \sum_{j \in \mathcal{C}} \sum_{k \in \mathcal{U}(j)} R_{jt}^p w_{kjt}^{ps} - \sum_{j \in \mathcal{L}} \sum_{p \in \mathcal{P}} \sum_{o \in \mathcal{O}(j)} \sum_{b \in \mathcal{B}(o)} P_{jbt}^p x_{jbt}^{ps} - \sum_{p \in \mathcal{P}} \sum_{j \in \mathcal{L}} \sum_{k \in \mathcal{D}(j)} T_{jkt}^p w_{jkt}^{ps} - \sum_{p \in \mathcal{P}} \sum_{j \in \mathcal{L}} I_{jt}^p i_{jt}^{ps} - \sum_{p \in \mathcal{P}} \sum_{j \in \mathcal{C}} H_{jt}^p z_{jt}^{ps} \right) \quad (5.1)$$

subject to

$$S_{jt}^p + \sum_{k \in \mathcal{U}(j)} w_{kjt}^{ps} = \sum_{o \in \mathcal{O}_s(j)} \sum_{b \in \mathcal{B}(o)} x_{jbt}^{ps} \quad j \in \mathcal{F}_s, p \in \bigcup_{o \in \mathcal{O}_s(j)} \mathcal{P}_i(o), s \in \mathcal{S}, t \in \mathcal{T}, \quad (5.2)$$

$$\sum_{o \in \mathcal{O}_s(j)} \sum_{q \in \mathcal{P}_i(o)} \sum_{b \in \mathcal{B}(o)} A_{p,q}^b \cdot x_{jbt}^{qs} = \sum_{k \in \mathcal{D}(j)} w_{jkt}^{ps} \quad j \in \mathcal{F}_s, p \in \bigcup_{o \in \mathcal{O}_s(j)} \mathcal{P}_o(o), s \in \mathcal{S}, t \in \mathcal{T}, \quad (5.3)$$

$$\sum_{p \in \mathcal{P}_i(o)} \sum_{b \in \mathcal{B}(o)} x_{jbt}^{ps} \leq C_{jt}^o \quad j \in \mathcal{F}_s, o \in \mathcal{O}_s(j), s \in \mathcal{S}, t \in \mathcal{T}, \quad (5.4)$$

$$S_{jt}^p + \sum_{k \in \mathcal{U}(j)} w_{kjt}^{ps} = \sum_{o \in \mathcal{O}_c(j)} \sum_{q \in \mathcal{P}_o(o)} \sum_{b \in \mathcal{B}(o)} B_{q,p}^b \cdot x_{jbt}^{qs} \quad j \in \mathcal{F}_c, p \in \bigcup_{o \in \mathcal{O}_c(j)} \mathcal{P}_i(o), s \in \mathcal{S}, t \in \mathcal{T}, \quad (5.5)$$

$$\sum_{o \in \mathcal{O}_c(j)} \sum_{b \in \mathcal{B}(o)} x_{jbt}^{ps} = \sum_{k \in \mathcal{D}(j)} w_{jkt}^{ps} \quad j \in \mathcal{F}_c, p \in \bigcup_{o \in \mathcal{O}_c(j)} \mathcal{P}_o(o), s \in \mathcal{S}, t \in \mathcal{T}, \quad (5.6)$$

$$\sum_{p \in \mathcal{P}_o(o)} \sum_{b \in \mathcal{B}(o)} x_{jbt}^{ps} \leq C_{jt}^o \quad j \in \mathcal{F}_c, o \in \mathcal{O}_c(j), s \in \mathcal{S}, t \in \mathcal{T}, \quad (5.7)$$

$$i_{jt-1}^{ps} + \sum_{k \in \mathcal{U}(j)} w_{kjt}^{ps} = i_{jt}^{ps} + \sum_{k \in \mathcal{D}(j)} w_{jkt}^{ps} \quad j \in \mathcal{W}, p \in \mathcal{P}, s \in \mathcal{S}, t \in \mathcal{T}, \quad (5.8)$$

$$\sum_{k \in \mathcal{U}(j)} w_{kjt}^{ps} + z_{jt}^{ps} = D_{jt}^{ps} \quad j \in \mathcal{C}, p \in \mathcal{P}, s \in \mathcal{S}, t \in \mathcal{T}, \quad (5.9)$$

$$\frac{1}{|\mathcal{S}(n)|} \sum_{s' \in \mathcal{S}(n)} \left( i_{jt}^{ps'}, w_{jkt}^{ps'}, x_{jbt}^{ps'}, z_{jt}^{ps'} \right) = \left( i_{jt}^{ps}, w_{jkt}^{ps}, x_{jbt}^{ps}, z_{jt}^{ps} \right) \quad n \in \mathcal{N}, s \in \mathcal{S}(n), t \in \mathcal{T}(n), \quad (5.10)$$

$$i, w, x, z \geq 0, \quad (5.11)$$

with  $\mathcal{S}(n)$  being the scenarios passing through event node  $n$  and  $\mathcal{T}(n)$  being the time period of node  $n$ .

The objective function (5.1) consists of five parts: first the revenues from satisfying demand at the customer nodes, second the production costs, third the transportation costs, fourth the inventory costs, and fifth is the shortfall penalty for not satisfying customer demand. Restrictions (5.2)-(5.4) describe the splitting process, and constraints (5.5)-(5.7) describe the processing stage of the supply chain. The first restriction ensures that all required input is either transported into the node or externally supplied. The second restriction forces all produced products to be transported to downstream nodes. The last constraint in each group takes care of the capacity restrictions. The mass balance for the inventory is given by (5.8). Equations (5.9) makes sure that the sum of products transported into a customer node and shortfall equals demand. We introduce non-anticipativity constraints (5.10) (see Rockafellar & Wets 1991) for each event node in the scenario tree where uncertainty is resolved. The final constraints (5.11) are the non-negativity constraints, the indices are omitted.

To backlog demand, we modify constraint (5.9) by adding unsatisfied demand of the previous period to the demand on the right hand side:

$$\sum_{k \in \mathcal{U}(j)} w_{kjt}^{ps} + z_{jt}^{ps} = D_{jt}^{ps} + z_{jt-1}^{ps} \quad j \in \mathcal{C}, p \in \mathcal{P}, s \in \mathcal{S}, t \in \mathcal{T}. \quad (5.12)$$

## 5.B Appendix: Results for all Problem Instances

Tables (5.5)-(5.4) contain the results for all problem instances and problem types. The instance-ID indicates first the dataset used to parameterize the forecasting method, then comes the percentage of the cutting capacity used, followed by the handling of unsatisfied demand and the shortfall penalty. The problem type denotes the expected value problem (ev), the two-stage stochastic programming problem (stoch), the perfect information using rolling schedules (pir), and the benchmark problem using perfect information for the whole planning period (pi). Results are given in mill. NOK.

**Table 5.3:** Computational results for the dataset based on parameterizing the forecasting method on 2003 demand data

Problem instance	Problem type	Revenues	Prod. costs	Transp. costs	Inv. costs	Shortfall penalty	Profit
03_100.BL_0	ev	4511	857	48	5	0	3599
03_100.BL_0	stoch	4418	856	46	5	0	3510
03_100.BL_0	pir	4457	857	49	2	0	3548
03_100.BL_0	pi	4746	856	48	10	0	3830
03_100.BL_25	ev	4479	861	51	2	2249	1313
03_100.BL_25	stoch	4480	862	51	2	2233	1330
03_100.BL_25	pir	4453	859	50	2	2452	1088
03_100.BL_25	pi	4622	861	51	5	2127	1577
03_100.LS_0	ev	4347	849	47	3	0	3446
03_100.LS_0	stoch	4349	850	47	3	0	3447
03_100.LS_0	pir	4358	847	47	2	0	3461
03_100.LS_0	pi	4472	853	47	6	0	3566
03_100.LS_25	ev	4348	852	48	3	111	3331
03_100.LS_25	stoch	4350	852	48	3	111	3333
03_100.LS_25	pir	4359	850	48	2	106	3350
03_100.LS_25	pi	4473	854	48	6	83	3481
03_75.BL_0	ev	4492	851	51	5	0	3583
03_75.BL_0	stoch	4500	852	51	5	0	3591
03_75.BL_0	pir	4429	850	52	2	0	3524
03_75.BL_0	pi	4744	854	51	11	0	3826
03_75.BL_25	ev	4462	853	54	3	2378	1172
03_75.BL_25	stoch	4466	854	54	3	2352	1202
03_75.BL_25	pir	4426	852	53	2	2610	908
03_75.BL_25	pi	4619	856	53	5	2165	1537
03_75.LS_0	ev	4288	832	48	3	0	3404
03_75.LS_0	stoch	4320	837	49	3	0	3429
03_75.LS_0	pir	4303	831	48	2	0	3420
03_75.LS_0	pi	4458	851	49	6	0	3550
03_75.LS_25	ev	4289	834	49	3	126	3275
03_75.LS_25	stoch	4292	834	49	3	126	3277
03_75.LS_25	pir	4304	834	50	2	121	3296
03_75.LS_25	pi	4459	853	51	7	84	3463
03_50.BL_0	ev	3977	753	45	2	0	3174
03_50.BL_0	stoch	3981	754	45	2	0	3179
03_50.BL_0	pir	3935	752	45	1	0	3136
03_50.BL_0	pi	4175	752	45	7	0	3368
03_50.BL_25	ev	3949	756	48	2	5532	-2389
03_50.BL_25	stoch	3950	756	48	2	5527	-2383
03_50.BL_25	pir	3923	754	47	1	5692	-2572
03_50.BL_25	pi	4116	761	47	3	5433	-2130
03_50.LS_0	ev	3837	743	42	2	0	3048
03_50.LS_0	stoch	3835	742	42	2	0	3047
03_50.LS_0	pir	3859	744	43	1	0	3070
03_50.LS_0	pi	3920	747	44	3	0	3125
03_50.LS_25	ev	3836	744	44	2	235	2809
03_50.LS_25	stoch	3835	744	44	2	235	2807
03_50.LS_25	pir	3860	746	45	1	229	2838
03_50.LS_25	pi	3921	748	46	4	210	2911



**Table 5.4:** Computational results for the dataset based on parameterizing the forecasting method on 2004 demand data

Problem instance	Problem type	Revenues	Prod. costs	Transp. costs	Inv. costs	Shortfall penalty	Profit
04_100.BL_0	ev	4513	858	48	5	0	3601
04_100.BL_0	stoch	4520	857	48	5	0	3608
04_100.BL_0	pir	4457	857	49	2	0	3548
04_100.BL_0	pi	4746	856	48	10	0	3830
04_100.BL_25	ev	4481	862	51	3	2226	1338
04_100.BL_25	stoch	4483	862	51	3	2212	1354
04_100.BL_25	pir	4453	859	50	2	2452	1088
04_100.BL_25	pi	4622	861	51	5	2127	1577
04_100.LS_0	ev	4337	851	47	4	0	3433
04_100.LS_0	stoch	4337	852	47	4	0	3433
04_100.LS_0	pir	4358	847	47	2	0	3461
04_100.LS_0	pi	4472	853	47	6	0	3566
04_100.LS_25	ev	4336	854	48	4	116	3312
04_100.LS_25	stoch	4338	855	48	4	115	3313
04_100.LS_25	pir	4359	850	48	2	106	3350
04_100.LS_25	pi	4473	854	48	6	83	3481
04_75.BL_0	ev	4497	852	51	5	0	3588
04_75.BL_0	stoch	4506	852	51	5	0	3596
04_75.BL_0	pir	4429	850	52	2	0	3524
04_75.BL_0	pi	4744	854	51	11	0	3826
04_75.BL_25	ev	4462	854	54	3	2366	1184
04_75.BL_25	stoch	4465	854	54	3	2341	1211
04_75.BL_25	pir	4426	852	53	2	2610	908
04_75.BL_25	pi	4619	856	53	5	2165	1537
04_75.LS_0	ev	4278	833	48	4	0	3391
04_75.LS_0	stoch	4280	836	49	4	0	3390
04_75.LS_0	pir	4303	831	48	2	0	3420
04_75.LS_0	pi	4458	851	49	6	0	3550
04_75.LS_25	ev	4279	836	49	4	130	3258
04_75.LS_25	stoch	4281	838	50	5	129	3257
04_75.LS_25	pir	4304	834	50	2	121	3296
04_75.LS_25	pi	4459	853	51	7	84	3463
04_50.BL_0	ev	3979	753	45	2	0	3177
04_50.BL_0	stoch	3983	753	45	2	0	3181
04_50.BL_0	pir	3935	752	45	1	0	3136
04_50.BL_0	pi	4175	752	45	7	0	3368
04_50.BL_25	ev	3950	756	48	2	5520	-2376
04_50.BL_25	stoch	3950	756	48	2	5513	-2369
04_50.BL_25	pir	3923	754	47	1	5692	-2572
04_50.BL_25	pi	4116	761	47	3	5433	-2130
04_50.LS_0	ev	3785	738	42	3	0	3001
04_50.LS_0	stoch	3787	738	42	3	0	3003
04_50.LS_0	pir	3859	744	43	1	0	3070
04_50.LS_0	pi	3920	747	44	3	0	3125
04_50.LS_25	ev	3785	739	44	3	248	2749
04_50.LS_25	stoch	3788	740	44	3	247	2753
04_50.LS_25	pir	3860	746	45	1	229	2838
04_50.LS_25	pi	3921	748	46	4	210	2911

**Table 5.5:** Computational results for the dataset based on artificial data

Problem instance	Problem type	Revenues	Prod. costs	Transp. costs	Inv. costs	Shortfall penalty	Profit
AD_100.BL_0	ev	4805	892	55	6	0	3851
AD_100.BL_0	stoch	4807	892	54	6	0	3854
AD_100.BL_0	pir	4744	891	55	2	0	3794
AD_100.BL_0	pi	5042	891	55	11	0	4083
AD_100.BL_25	ev	4771	896	57	3	2359	1453
AD_100.BL_25	stoch	4772	896	57	3	2354	1460
AD_100.BL_25	pir	4740	894	57	2	2584	1203
AD_100.BL_25	pi	4912	896	57	5	2285	1668
AD_100.LS_0	ev	4619	894	54	5	0	3664
AD_100.LS_0	stoch	4620	895	54	5	0	3664
AD_100.LS_0	pir	4653	886	54	2	0	3710
AD_100.LS_0	pi	4780	890	54	6	0	3828
AD_100.LS_25	ev	4619	897	55	5	126	3534
AD_100.LS_25	stoch	4634	904	55	6	125	3542
AD_100.LS_25	pir	4655	890	55	2	113	3593
AD_100.LS_25	pi	4782	891	54	7	88	3739
AD_75.BL_0	ev	4788	886	58	6	0	3837
AD_75.BL_0	stoch	4793	886	58	6	0	3841
AD_75.BL_0	pir	4710	882	58	1	0	3767
AD_75.BL_0	pi	5035	888	58	12	0	4075
AD_75.BL_25	ev	4748	884	60	3	2595	1204
AD_75.BL_25	stoch	4754	885	60	3	2551	1253
AD_75.BL_25	pir	4707	883	60	2	2835	925
AD_75.BL_25	pi	4912	886	60	6	2370	1587
AD_75.LS_0	ev	4544	872	56	5	0	3610
AD_75.LS_0	stoch	4545	875	56	6	0	3606
AD_75.LS_0	pir	4590	865	55	1	0	3666
AD_75.LS_0	pi	4772	886	58	8	0	3819
AD_75.LS_25	ev	4542	874	57	6	146	3457
AD_75.LS_25	stoch	4542	877	57	6	146	3454
AD_75.LS_25	pir	4591	868	57	1	130	3534
AD_75.LS_25	pi	4773	887	58	8	89	3728
AD_50.BL_0	ev	4139	768	49	2	0	3318
AD_50.BL_0	stoch	4141	769	49	2	0	3320
AD_50.BL_0	pir	4095	768	49	1	0	3276
AD_50.BL_0	pi	4335	768	49	6	0	3510
AD_50.BL_25	ev	4109	770	51	2	6746	-3461
AD_50.BL_25	stoch	4110	770	51	2	6744	-3458
AD_50.BL_25	pir	4083	769	51	1	6854	-3593
AD_50.BL_25	pi	4272	777	51	3	6677	-3237
AD_50.LS_0	ev	3964	756	47	3	0	3156
AD_50.LS_0	stoch	3965	757	46	3	0	3157
AD_50.LS_0	pir	4037	761	48	1	0	3227
AD_50.LS_0	pi	4094	765	48	3	0	3275
AD_50.LS_25	ev	3963	758	48	3	283	2868
AD_50.LS_25	stoch	3963	758	48	3	283	2868
AD_50.LS_25	pir	4038	762	49	1	264	2959
AD_50.LS_25	pi	4094	766	50	4	247	3025



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