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Managing End-user Flexibility in Electricity Markets

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Preface

This thesis has been prepared at the Department of Industrial Economics and Technology Management at the Norwegian University of Science and Technology (NTNU) for the defense of the Doktor Ingeniør degree. The work has been conducted during a four year period from January 2000 to February 2004. Stein W. Wallace has been the thesis advisor and Stein-Erik Fleten has been the co-advisor. The project has been founded by the Norwegian Electricity Industry Association (EBL) and the Research Council of Norway (NFR).

The main goal of this doctoral project has been to analyze how retailers and network operators would behave in a situation where the end-users have their electricity consumption metered by the hour. The focus has been on the Norwegian market, but some of the work may be useful for analysing similar situations in other countries as well. During the about four years that I have been working with this thesis, there have been many late nights at the office, many periods during which I have been frustrated because I felt I was getting nowhere and many hours of doubt whether it was a wise choise to commence doctoral studies. Fortunately, however, these doubts came after a couple of years, and then it was too late for second thoughts. Looking back today, I am really happy that I went through this. Not many people get the chance to spend four years working with only those problems that they find interesting. Also, now that my thesis is finished, and I see the concrete result of those four years of work, the negative things are almost forgotten and the doubts are gone.

I am very grateful for the excellent supervision from Stein W. Wallace. As it turned out, we have been based far away from each other most of the time, and I think this has been an extra challenge for both of us. I have learned that even modern solutions for communicating over distances can never fully compensate for the benefits of meeting face to face. Fortunately, however, I have had the chance to visit him at his workplace in Molde many times during the last two years. I have really apprechiated that Stein has given high priority to the supervision of my work on my every visit in Molde. Stein has always provided quick, thorough, and most insightful feedback on my work, as well as general guidance and good advice, whenever needed. He is a great supervisor and a very interesting discussion partner. I would also like to thank him for giving me the opportunity to take a doctoral degree on a very interesting problem.

I would also like to thank my co-supervisor Stein-Erik Fleten for valuable help and many interesting discussions. I have benefited greatly from Stein-Eriks knowledge on, and experience in working with, electricity related problems.

In 2001, I visited the Departmend of Engineering Science at the University of Auckland in New Zealand for almost a full year. During my stay, I had the pleasure of getting aquainted with Professor Andy Philpott, with whom I have been working closely both during my stay in Auckland and afterwards. As this was rather early in my doctoral studies, I had not done much research before I came to Auckland. Therefore, Andy's supervision has been very important for my work, which should be apparent from the fact that he has co-authored three of the papers that are included in this thesis. In addition, he has given valuable advice on Paper 1, which is singly authored. I am most grateful for his profound supervision, as well as his contribution to making the time in New Zealand enjoyable, both for my wife, Maria, and myself.

I would also like to thank the rest of the staff at the Department of Engineering Science for their hospitality. A special thanks to Mike O'Sullivan (Snr), who provided valuable inputs on thermodynamics for Paper 1.

I have been a very privileged doctoral student. Due to the generous funding from EBL and NFR, I have had the opportunity to attend several academic conferences around the world. Such conferences are very inspiring, because they provide an opportunity to meet colleauges from all over the world, and not least to see new places. I am most grateful for the funding, and I would hereby like to thank NFR and EBL for funding this project. A special thanks to those of EBL's member firms that have contributed the most. It is really inspiring to experience that the industry sees some value in funding research. Arne Utne has been my champion within EBL. He has always provided swift replies to any question that I may have had, and he has given me the chance to contribute to industry conferences that EBL has arranged. I am thankful for the work he has done administrating my project. I would also like to thank his predecessor, Knut Ola Aamodt.

During my time as a doctoral student, I have had the chance to work closely with representatives from the electricity industry. Getting some industry experience has helped me putting my work into context. It has been very inspiring. I would like to thank Glenn Grøtheim at Trondheim Energiverk's (TEV) trading division for providing me with an office space at TEV, to which I was welcome whenever I wanted to get some variation from my work at the university. Grøtheim has also functioned as the head of my project's steering committee, and I have really apprechiated his enthusiasm for my project. At TEV I was working closely together with Bjørn Hjulstad. I thank him for giving me interesting and challenging tasks. Also, thanks to the rest of the staff at TEV's trading division for their hospitality and for always being willing to answer my many questions.

I would like to thank Arnvid Sylte at TEV's network operator division for always being willing to answer any questions I might have had about network operation, and for providing feedback on my work. Three years ago I was working with energy economising for two weeks together with Jon Olav Hafsmo, also at TEV's network operator division. Thanks to him for making those two weeks interesting.

I was also working for Fjordkraft in Bergen for two weeks in the autumn of year 2000. I would like to thank Johannes Gjesdal and Arnstein Flaskerud for inviting me there, and for organizing my stay. While in Bergen, I got to know Petter Brunvold, with whom I have had many interesting discussions since then. I have very much apprechiated the insight about electricity markets in general, and electricity retail business in particular, that he has provided me with during these discussions.

Nils Jacob Berland possesses a lot of knowledge about hourly metering and two-way communication. I would like to thank him for many valuable and interesting discussions.

I would like to thank all my colleges at the department in Trondheim, and I would especially like to thank my good friend and office-mate Bård Karsten Reitan for many fruitful discussions. Also, thanks to our departmental secretaries, Guri Andresen and Jorid Øyen, for always being helpful and supportive.

Finally, I would like to thank my wife, Maria, for her support. In particular, her support during the laborious final weeks of writing has been very much apprechiated. I have had to leave Trondheim quite often to go to conferences and to Molde. Being away from my wife so much has been very difficult sometimes, and I am really thankful for her understanding. I am also grateful for her taking a year off to go with me to New Zealand.

Any errors in the thesis are, of course, entirely my responsibility.

Trondheim, February 13, 2004.

Erling Pettersen

Contents

Part I

Background

1 Introduction

Over the past couple of decades, electricity markets across the world have been deregulated. When the electricity systems were being developed, governments exercised extensive control over the industry to ensure reliable services. Today, most of the basic investments have been carried out, and more and more countries have chosen to let market mechanisms regulate the electricity systems. Different countries have, however, approached the deregulation in different ways. While production of electricity is subject to competition in all deregulated markets, there are differences as to how the wholesale markets and end-user markets are organized. An overview of energy optimization models dealing with uncertainty in both regulated and deregulated electricity markets is found in [5].

The primary focus of this thesis is to analyze how participants in the Norwegian electricity market would behave in a situation where the consumers are metered and charged by the hour. The thesis includes five papers, preceded by an introduction. The current part provides a background for the papers and puts them into a common framework. Another purpose of the background part is to point out the scientific contributions of the thesis, which, as we shall see, primarily lie in the application of known methods to new problems.

In Section 2 of the background part a description of the Norwegian electricity market is provided. The section starts with a very brief overview of when and why the market was opened up for competition. Next, we give a description of the wholesale market, with some emphasis on the regulating market (Section 2.1). The end-user market is the primary subject of this thesis, and an overview of this market is given in Section 2.2. Today Norwegian end-users are normally not metered and charged by the hour, as will be apparent from Section 2.2, but if they were, it is likely that this would induce more short-term flexibility in the end-user market. In Section 3 we give a brief overview of how hourly metering, coupled with some sort of two-way communication, may help achieve this. Also, we discuss some potential effects of such flexibility for consumers, retailers, network operators, environment and society (Section 3.2). Finally, in Section 4, we look at two alternative ways of making the end-users alter their load profiles, and

discuss pros and cons of the two solutions.
Each paper includes its own introduction, and for an overview of the contents of the papers, we refer to these.

2 The Norwegian electricity market

Norway was among the first countries to deregulate its electricity market. The deregulation was initiated with the passing of the Energy Act in 1990, and this act was effective as of January 1st 1991. The deregulation came about for two main reasons. Firstly, the legal framework was spread over a large number of laws, and a simplification of this legal framework was desired. Secondly, the previous regime had resulted in overinvestment, and the deregulation was intended to improve efficiency and profitability in the electricity sector. More on the deregulation of the Norwegian electricity market can be found in [4] and [15].
In the Norwegian market, production, transmission and retail of electricity have been split into three different business areas, and each category of firms has its specific role in the market. We refer to the introduction of Paper 2 in this thesis for a brief overview.

2.1 The wholesale market

The wholesale market is built up around Nord Pool, which is an electricity exchange that provides a spot market and a transparent exchange for electricity derivatives. Nord Pool was established in 1993 as a pure Norwegian market. Today, however, Nord Pool serves as a common electricity exchange for Denmark, Finland, Norway and Sweden, and these countries now form an integrated wholesale market where any producer in any country may deliver electricity to the entire region. The Nordic market has a large share of hydro plants[1], giving a mix of production technologies that provides great flexibility in terms of exchange.

[1]Norway has 99% hydro power, Sweden 48% and Finland 22%. Denmark has no hydro power.

The day-ahead market

All participants in the spot market at Nord Pool, that is all producers and retailers in the Nordic market, must submit their bids for sale and purchase of hourly contracts and block contracts covering all 24 hours of the next day. An hourly bid is a sequence of price/volume pairs for each specified hour while a block bid is an aggregated bid for several consecutive hours with a fixed price and volume. The bids must be submitted by noon on the day before delivery, and the hourly prices for the next day are published by Nord Pool at $3pm$.
After receiving the bids, Nord Pool will develop a supply curve and a demand curve for each hour by aggregating all bids. The point at which the supply curve and the demand curve intersect will determine what is called the system price, as depicted in Figure 1. The system price is the theoretical price obtained if there were no transmission bottlenecks within the region.

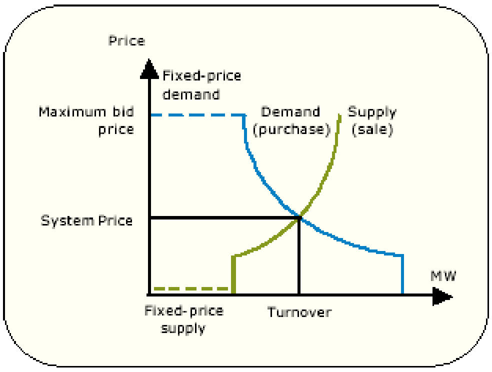

Figure 1: System price determination. Source: [3].

Sometimes the system price will lead to bottlenecks in the transmission grid. To handle such a situation the market has been divided into a pre-determined number of bidding areas. If there are problems with the transmission capacity between these areas, price mechanisms are used to relieve the bottlenecks. In areas where the demand cannot be met at the system price, the price will increase while the price will

decrease in areas with excess capacity. The spot price in each area is the system price adjusted for bottlenecks. Finland and Sweden each form single price areas. If transmission capacity constraints occure within the countries, the system operators solve the situation by using counter-trade. In Norway, line congestions are solved by dividing the country into pre-determined bidding areas, also called zonal pricing. In Denmark, there are two bidding areas, but they are not directly connected. Within each of those areas, the Danish system operator uses counter-trade to handle grid congestions. We refer to [2] for a discussion about different approaches to handling grid congestions.
The spot prices vary significantly between seasons, and as depicted in Figure 2, prices are typically higher in winter than in summer. Since electricity is the main energy source for space heating in Norway, the demand for electricty is much higher in winter than in summer. Also, fluctuations in inflow to the hydry reservoirs put the supply under pressure in the winter season. The increased demand coupled with the increased strain on supply make prices higher in winter than in summer. These effects were particularly extreme during the winter of 2002 and 2003, when low precipitation during the summer and the autumn preceeded a cold and early winter.

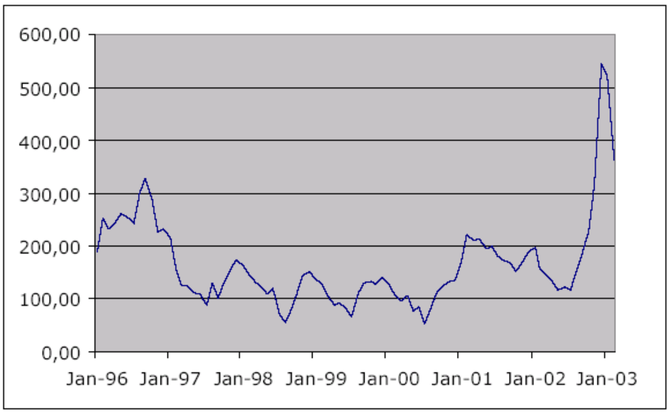

Figure 2: Average monthly spot prices in Oslo region in NOK/MWh. Source: [3]

The hourly spot prices may also vary significantly during one day. At night the demand for electricity is low, and a high share of the consumption may be generated by the cheapest generation sources. Normally, this means hydro power, but during periods of low reservoir levels, thermal plants may be the least expensive. At daytime, however, demand increases, and production from the more expensive plants is needed. The prices tend to peak at the start of the business hours at $8am$ and around $6pm$ when the residential end-users switch on their TV-sets and do the dishes after dinner. In Figure 3 four daily spot price patterns from the Oslo region are depicted. We observe that the intra-day prices may vary significantly some days, while the price patterns are flatter on other days. The intra-day variations tend to be more extreme in winter than in summer.

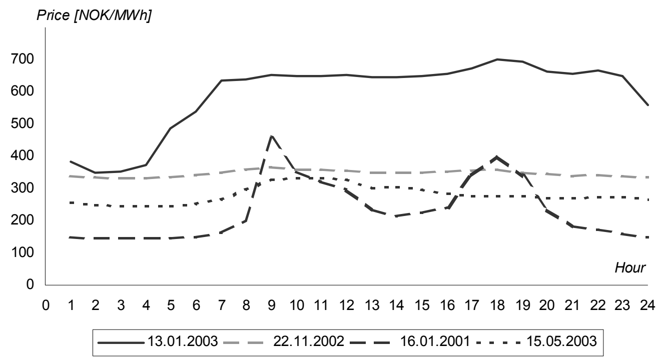

Figure 3: Four daily spot price patterns in Oslo region.

The regulating market

We have seen that Nord Pool is a day ahead market, and the spot prices are in reality forward prices that are calculated based on the expected consumption the next day. It is, however, not possible to know exactly the consumption one day ahead. Especially in Norway, where most buildings use electricity for space heating, the electricity consumption is highly dependent on weather, which is intrinsically random. Hence, the system operators need some tool to handle real-

time imbalances between supply and demand. The different countries have different systems for handling this, but here we will have a brief look at the regulating market, which is the system used in Norway. In [12] an econometric analysis is used to model the regulating market prices. This is one of the few published articles that analyze this market.
In the regulating market, those who are able to increase or hold back production if required, submit regulating bids before 7:30pm on the day before dispatch is to take place. Since the regulating market is supposed to treat real-time imbalances, the bidders are allowed to change their bids down to two hours before delivery. The majority of bidders in this market are producers[2]. About than 99% of Norwegian electricity production comes from hydro power, and hydro stations are able to alter their production level at very short notice. This is necessary, because participation in the regulating market requires the bidders to be able to increase or shut down production on fifteen minutes notice.
The producers submit bids for up-regulating power and for down-regulating power. As for the bids in the spot market, the regulating market bids are sequences of price/volume pairs. When submitting an up-regulation bid, the producer obliges himself to increase production by certain amounts at certain prices. Up-regulation power must be offered at a higher price than the spot price. Submitting down-regulation bids, means that the producer agrees to decrease production by certain amounts at certain prices. A down-regulation bid does, indeed, mean that a producer is willing to buy back some of the electricity he has sold into the spot market. Down-regulation power must be offered at a lower price than the spot price.
If the actual consumption of electricity in a certain hour within a spot price area[3] is higher than the consumption dispatched in the spot market, the market is regulated upwards. This is called up-regulation. In the opposite situation, when the actual consumption is lower than dispatched in the spot market, we get down-regulation. If the actual

[2] In addition, some large industrial enterprices submit regulating bids, if they are able to shut down production on short notice. By doing so, they make more power available in the market.

[3] The regulating price areas are the same as the spot price areas.

consumption is equal to, or very close to, the anticipated consumption we get no regulation.
It is important to notice that during hours of up-regulation the regulating price is always, and for all participants, higher than the spot price. If there is down-regulation, the regulating price is always, and for all participants, lower than the spot price and if there is no regulation the regulating price is always, and for all participants, equal to the spot price. In some countries, e.g. Sweden and Denmark, there is quoted one price for up-regulating power and one price for down-regulating power, but in Norway only one regulating price is quoted each hour in each of the price areas. The regulating price is published a couple of hours after dispatch, when the actual consumption is known.
The retailers are passive participants in the regulating market, because they are not able to make their consumers change their consumption on short notice[4]. If a retailer's customers during one hour consume less electricity than the amount ordered by the retailer in the spot market, then the retailer must sell the surplus into the regulating market, whether he likes it or not. On the other hand, if his customers use more than anticipated, he is forced to purchase the deficit in the regulating market.
As mentioned, it is the consumption observed within one entire area that determines whether the market is up-regulated or down-regulated. Hence, the retailers within one area will, in total, lose money if the market is regulated either way. This happens, because if the total consumption in the area is higher than the volume dispatched in the spot market, the regulating price is higher than the spot price, which means that the retailers must buy electricity at a price that is higher than they would if they had estimated consumption correctly. On the contrary, if the total consumption is lower than the spot market dispatch, the regulating price is lower than the spot price, meaning that the retailers must sell power into the market at a price below the spot price, (at which they bought the power). This also incurs a loss to the retailers.
It is important to understand that even though the market as a whole

[4]Experiments are currently being made where retailers submit bids to the regulating market. To do so, they must be able to cut the supply from some of their customers on short notice.

is, say, up-regulated, an individual retailer may still be down-regulated. This will happen if a retailer has overestimated the consumption of his consumers while the aggregated consumption estimate of all retailers in the area is underestimated (and vice versa for down-regulation). If this happens he must sell the excess consumption into the regulating market, and since the market is up-regulated, he receives a price that is higher than the spot price. This, in turn, means that the retailer makes a profit by selling at a higher price, some electricity that is bought at the spot price. Hence, an individual retailer may make a profit in this market, but on average retailers will suffer losses in the regulating market.

This discussion indicates that there may be opportunities for retailers to speculate in the regulating market by deliberately ordering more or less power in the spot market than they expect their customers to consume. If they believe that they are better than other retailers at guessing the direction of regulation, they could try to make extra profits by doing this.

In Paper 4 we study conditions under which it would actually be optimal for the retailer to speculate like this. The main scientific contribution of this paper is the modeling of demand-side behaviour in day-ahead electricity markets and the identification of conditions under which it is optimal for the retailer to speculate. On Nord Pool, the financial contracts are written on the spot price, and not on the regulating price. Thus, to maintain an efficient contract market, the prices and volumes dispatched in the spot market should as far as possible reflect the true conditions of the power system. Due to this, the Norwegian system operator, Statnett, prohibits demand-side speculation in the regulating market, and require that the retailers order their expected demand in the spot market. At the time of bidding, the actual consumption is however random, and it may be rather difficult for Statnett to assess whether or not a retailer has submitted an inaccurate bid on purpose. If consumers become more price-flexible in the short run, e.g. due to hourly metering getting more common (see Section 3), such a judgement may be even more difficult to make. The findings in Paper 4 suggest that the day-ahead structure of the wholesale market may be problematic.

Another scientific contribution of Paper 4 is a new application to the recently developed concept of a market distribution function (see [1]). While Paper 4 takes a rather theoretical approach to the problem of a retailer optimizing his bidding curve in an electricity market with a day-ahead structure, a more direct approach is taken in Paper 5. Today, as will be explained Section 3, demand is almost totally inflexible with respect to intra-day price variations. Therefore, the retailers normally do not bother to submit demand bids to Nord Pool that are price flexible. Instead, they simply bid a single volume that they agree to buy at any price (with volume on the horizontal axis, they bid a vertical curve). If the demand becomes price-flexible in the short run, this may change. In Paper 5 we develop a model that derives optimal offer stacks for a price-taking retailer whose customers' demand is price-flexible in the short run. The scientific contribution of this paper is the application of optimization methods on a new type of problem.

2.2 The end-user market

As part of the Energy Act of 1990, all end-users were allowed to have their electricity delivered by any retailer. During the first years after the deregulation, however, they were charged a fee for changing retailer, making it economically meaningful only for large consumers. Also, each retailer had to pay a fee to the local network operator in each area to which it delivered power, which made it less attractive for retailers to compete for customers away from their home market. The fees paid by the consumer and the retailer were to cover the administrative costs of the network owner, who had to deal with several retailers instead of just one like earlier. From 1997, however, new legislation made it possible for all end-users to change retailer at no cost. The administrative costs were from then on covered by the consumers' local network owners. This gave a slight increase in the network fees, but competition between the retailers was enhanced to the benefit of the end-users.

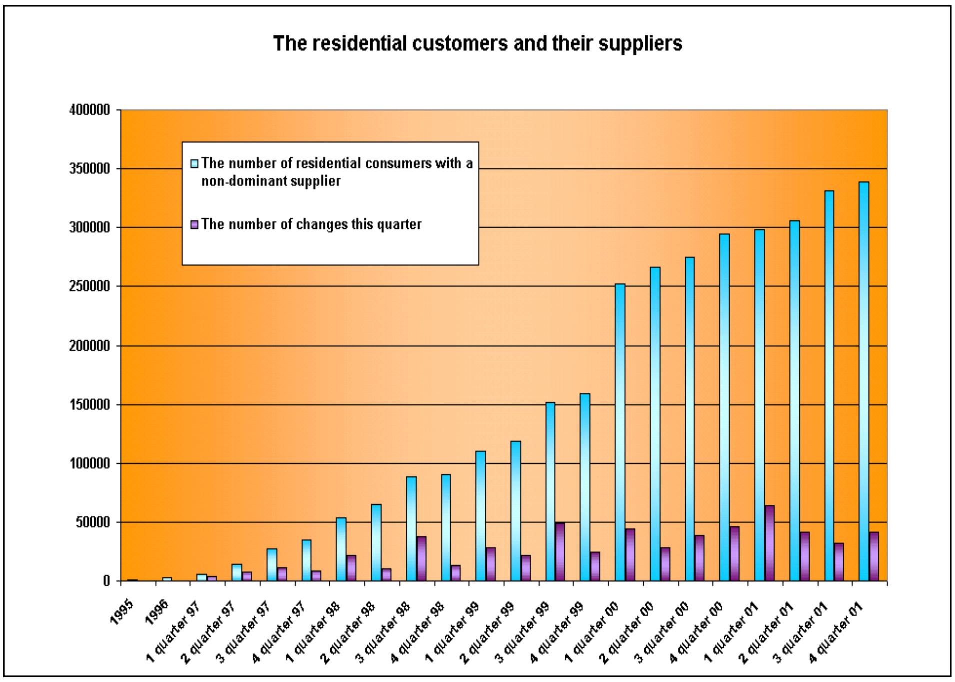

Figure 4: The residential customers and their suppliers. Source: [10].

From Figure 4, we clearly see how the liberalization of the end-user market has enhanced competition in the residential market, as it has become increasingly common for residential customers to change retailer. The light bars to the left show the number of customers buying power from a different retailer than the one dominating their area. The darker bars to the right show the number of residential customers that changed retailer during the quarter. (We have used the word retailer instead of supplier throughout the thesis). Before 1997, few residential customers were served by a different retailer than their local one. Since the legislation was passed in 1997, the number has increased every quarter, and the overall increase over the years is substantial. The increase in this number within each quarter is, however, smaller than the number of customers changing retailer within the quarter.

In Norway, there is more than two hundred retailers (according to [14]). Though not all of those retailers serve consumers nationwide, this has lead to the end-user market being characterized by fierce competition.

The competition is further enhanced by the invariable focus on electricity prices by the news media. As a result of this, the retailer's profit margins are generally low.

Metering and pricing of end-users

The end-users' consumption meters are normally read every two or three months to find the accumulated consumption over the period. The invoice for this consumption consists of two parts: The first is the transmission fee to the local network operator, and the second part of the invoice is the electricity payment, which is paid to the retailer according to whatever contract the customer has entered into with the retailer. For residential customers and small businesses, the transmission fee is normally charged in NOK/kWh, that is they are charged by energy consumption. Larger customers are sometimes charged both by peak load and energy consumption. The retailers always charge by energy consumption.
The end-users may choose from a handful of different contracts. The most risk averse consumers may prefer a fixed price agreement, under which the price is fixed for a long period of time, normally one, two or three years. This removes the risk of price movements for the consumer, but because risk is being priced in the market, the expected cost of such a contract is higher than for contracts with variable prices. An example of a variable price agreement is the so-called standard variable tariff, which is the most common agreement for residential end-users. Under such an agreement the retailer may change the price on two weeks notice. Hence, this contract provides some security towards short-term price movements for the end-user. However, this safety may be somewhat illusionary since two weeks is a relatively short notice. Consumers who are willing to be fully exposed to the wholesale market risk may choose a spot price contract, under which the end-user pays the average spot price over some period, typically one month, plus a mark-up. The size of the mark-up varies among the retailers. Since this contract gives the highest risk, it also gives the lowest expected costs. A special type of spot price contract is the price-ceiling contract, where the consumer pays the spot price plus a mark-up, but never more than a pre-determined price.

When consumers are metered every two or three month, the values obtained only show the total amount of energy consumed over the period, but they do not give any idea of the consumers' load profile within the period. If the end-user is on a spot price contract, or some other contract where the price varies within the three months[5], then the retailer would, however, like to know how much energy is consumed between price changes that have occurred during the period. This is estimated by utilizing the fact that each network node is equipped with meters capable of measuring the energy sold at that specific node, hour by hour. Then the assumption is made that all consumers connected to that node have the exact same load profiles. This means that if the average consumer in one area has a high consumption peak between 5*pm* and 6*pm*, it is assumed that all consumers in the area have a consumption peak in that hour. We say that the consumers are invoiced due to their adjusted load profile (ALP). ALP also determines the estimated profile on a weekly basis, monthly basis, or whatever time span that is relevant to the end-user's contract.
Usually hundreds of consumers are connected to the same node. This means that the profile of one single consumer has almost no impact on the total load profile at the node. Hence, with this way of metering consumption, the consumers have no incentives to shift load from peak to off-peak periods, even if they are on a pure spot price contract.

3 Short-term flexibility in the end-user market

Since the end-users are metered only a few times a year, and then charged based on their accumulated consumption during the period, the only way for end-users to save money, except from switching retailer, is by reducing their total electricity consumption. Hence, the incentive structure in the end-user market is directed only towards the energy consumption.
We have seen though, that the prices in the wholesale market vary

[5]About $70-75\%$ of Norwegian residential consumers have a variable price contract of some kind.

by the hour, and this suggests that capacity is a scarce resource that is being priced in the market. Because of the way consumption is metered today these short-term price signals in the wholesale market are not visible to the end users, and the end-users have no incentives to alter their load profiles according to the intra-day (or even intra-week or intra-month) price profiles. Rather, they actually do have such incentives. A somewhat bizarre effect of the ALP concept is that a well-informed end-user would understand that she should use more electricity while prices are high, and less while they are low, at least if she is connected to the same node as a high number of consumers. The end-user will, as mentioned, have almost no impact on the load profile at the node, so she will not lose anything by using a lot of electricity while it is cold, and the market places high value on energy, and rather save while prices are low. Her adjusted load profile will not become worse from this, but she may still benefit from higher comfort. In other words, short-term flexibility has no value for the end-users.

3.1 Hourly metering and two-way communication

To provide the end-users with proper incentives to alter their load profiles according to the price variations in the wholesale market, one would need to meter consumption differently. In many countries, metering devices that distinguish between peak and off-peak consumption are common. With such a system retailers and network operators may offer two-part tariffs and thereby, at least to some extent, move consumption away from time periods with potential capacity problems. In Paper 1, we refer to a range of studies on two-part tariffs, and they generally suggest that consumers are willing to alter their load profiles if incentivised to do so.

One drawback of the peak/off-peak metering devices is that they too are read only a few times a year. Hence, retail prices will normally not change from day to day, and the worst price peaks will still not be visible to the consumers. A more sofisticated solution than two-part tariffs would be to offer prices that vary according to the wholesale market variations. To make the price signals in the wholesale market visible to the end-users, and thereby provide them with incentives to react properly to these price signals, one would need to install metering devices that measure consumption with the same time resolution as the trading periods in the wholesale market. In the Nordic market this would require hourly metering.

Today, consumers with an annual consumption of more than $400,000kWh$ are required to be hourly metered[6], while smaller end-users may choose to be hourly metered. The cost of hourly metering equipment is (currently) too high in comparison to the potential economic savings, however, and therefore, rather few small end-users find it worthwhile. The share of consumers with hourly meters is likely to increase as the necessary equipment gets cheaper.

For hourly metering and pricing to have any purpose, the retailers and network owners need some medium to inform their customers about the hourly prices, plus some way of receiveing consumption data from them. This would require two-way communication between the utilities and the end-users. Often, when the term two-way communication is mentioned, people in the industry think of automated, perhaps Internet-based, solutions that enable them to submit prices to the consumers, and receive consumption data from the end-users in real time. Such a solution would clearly be the most efficient way of utilising the possibilities that may arise from hourly metering, because it would enable the utilities to change prices whenever the state of the power system suggest that it be necessary. The infrastructure for exchange of information between the utilities and end-users in real-time is currently not in place, however, and the cost of developing such a system solely to control load profiles would probably by far outweigh the gains. The utilities may want to wait for sufficient infrastructure to be built by other service providers, like broadband providers, or they may build the infrastructure themselves with a view to utilize it to provide a broader range of services to their customers.

Also, for this system to be efficient the end-users must be able to quickly respond to updated prices. Only large industrial enterprices are likely to find it cost efficient to employ personell to monitor price signals and take action according to them. For residential end-users, the savings would be too small to justify the cost of manually monitoring prices all along. Some years into the future, it may become

[6] The limit is about to be decreased to $100,000kWh$.

common for end-users to have fully automated solutions, so called smart-houses, installed in their homes to take care of this. Today, however, this is far from common, although it does exist. Currently smart-house solutions are far too expensive to justify the potential savings on the electricity bill, and they will perhaps always be. Smart-houses could, however, provide a range of services that make life easier for the owner, in addition to energy usage control, and these services may be the driving force of a possible evolvement of smarter homes. The saved energy costs may simply be a nice addition.

Although fully automated solutions for two-way communication together with smart houses would provide the most efficient infrastructure to induce short-term flexibility, we have argued that this is too expensive in the short run. There is, however, a range of less sophisticated possibilities that may give an adequate effect. The perhaps most rudimentary solution may be that the retailers and network owners inform about prices by mail, or through advertisements in newspapers, just like today, but with the important exception that they offer price profiles instead of flat prices. Since the daily price-patterns in the wholesale market are rather predictable, this would be better than nothing. Another possibility is to send the prices for the next day by SMS or e-mail every evening. Since the spot prices for the next day are known by then, this would enable retailers to take them into account in their price profiles.

When it comes to collecting consumption data, one may also think of several rather simple solutions. For instance, the consumer could once a year pick a chip which has stored last year's consumption data out of their metering device and mail it to the retailer, or plug it into their PC (if they have one), and e-mail the data. Since the consumers probably would like to be invoiced more than once a year, the data on this chip could form the basis for a yearly balance-settlement. Another possibility, which is being used in the city of Pasadena in the state of Texas in the US, is to now and then drive a car that is equipped with instruments to communicate with the metering devices through the relevant areas.

There is a range of possibilities for communication between the utilities and the end-users, and I have only mentioned those that occurred to my mind at the time of writing. More creative people may find out much more efficient ways of communicating that are cost-effective with today's technology. But inducing short-term flexibility does require hourly meters to be installed.

If the costs related to installing hourly meters are shared in a proper way between end-users, retailers and network owners, a full-scale installation of hourly meters may be cost-efficient even today. In my opinion this is perhaps being viewed as too expensive because the relevant economic agents believe that very sophisticated solutions for two-way communication and load control must be installed at the same time. That is wrong. Hourly meters could be installed now, and the means of communication may evolve over time.

Currently, one of the worlds largest energy utilities, Italian Enel SpA, is installing equipment for two-way communication and real-time consumption metering at all their twenty-seven million customers. The installation started in 2002 and is expected to be completed in 2005. The cost is estimated to about 85USD a customer, and they expect the investment to be paid back in four years. Enel SpA has not published too much information about the project, but a brief overview may be found in [11]. One of the subcontractors, Echelon Corporation, provides some information on their web pages (see [12]).

In Norway, a research project on hourly metering and two-way communication is currently being carried out by Sintef Energy Research. Two network operators, Skagerak Energinett and Buskerud Kraftnett, have established two-way communication and hourly metering devices at about ten thousand end-users, of which more than 75% are households. More information about this project may be found on Sintef Energy Research's web pages (see [13]).

Currently, Istad Nett AS, a network operator in Møre and Romsdal county, is offering hourly metering equipment, together with possibilities for remote reading of the meters, to their residential customers. They have set the price at 1500NOK each meter, which must be paid by the consumer. See [16] for more about this[7].

[7] The web page is only available in Norwegian.

3.2 Potential effects of increased end-user flexibility

In this section an overview of how increased end-user flexibility may affect market participants and the society.

Consumer

The most important change for the consumers is rather obvious: they get the chance to save money by altering their daily load profiles. The consumers' willingness and ability to utilize this would differ, though. At one extreme, consumers may show a great deal of concern about this, and go far in reorganizing their lives, and install advanced equipments for load control. They may save significant amounts. At the other extreme, some consumers may be completely unable to alter their load profiles, and some of those who are able to do so may not care. These consumers may have to pay more for electricity than they do today. The consumers would pay according to the burden they actually impose on the power system. This seems fair compared to todays situation, as the ALP concept actually benefits those with "bad" load profiles.

Another important change is that the increased potential for saving money also would expose the end-users towards more risk. At least those who choose pure hourly pricing agreements, would be fully exposed to the price-variations in the wholesale market. It is likely, however, that the retailers would design different contracts that reduce this risk for the end-users. For example, the end-user could order a pre-determined amount at a fixed price. If the actual consumption during an hour (or a block of hours) exceeds the ordered amount, the consumer would pay the hourly price (or an average price over a block of hours) for the excess consumption. Vice versa, if less than the ordered amount is being consumed, the consumer would be paid for this. This would provide the consumers with some security, while they are still exposed to the hourly prices.

There are several electricity consuming processes that are carried out in Norwegian homes that may be flexible. In Figure 5 we see that space heaters take the highest share of residential consumption. Since the houses are insulated, and heat is stored in walls and furniture, it is to some extent possible to heat rooms at low-price hours, and still have them pleasantly warm when needed. Also, reducing the inside temperature while at work will become more attractive than today, since prices are typically high during working hours. Heat will disappear from the house fast enough to make it unpleasant after a couple of hours. However, due to the high share of consumption used for space heating, there may be some money to save, even though space heating is not as flexible as for example water heating. Water heating is, as depicted in Figure 5, the second most energy consuming task in the homes. Water has a high heat capacity compared to the rooms, and the water boiler may be switched off for several hours before the water gets too cold. Water heating constitutes the highest savings potential for residential end-users.

An important part of Paper 1 is the development of a model for a residential end-user minimizing the costs of space heating and water heating, when subject to hourly prices. By using some basic relations from thermodynamics we set up a linear program to minimize the costs. Though the model is quite simplistic, it does give a decent picture of the decision problem faced by the consumer. The utilization of simple optimization tools to model an hourly metered end-user's cost minimization is one of the scientific contributions in Paper 1. The model uses some of the same principles as that in [9], but our framework was published one year earlier (see [18]).

We have argued that water heating is more flexible than space heating. It is, however, possible to utilize the flexibility in water heating for space heating by using water borne heating. The consumers may install large water boilers, much larger than they have today, and have hot water sent through pipes to radiators that heat the rooms. This way, space heating would get the same flexibility as water heating, and the savings may be substantial. It is rather expensive, though, to completely change the heating system in a house. But new and renovated houses may choose such solutions. Hourly pricing, or at least some other kind of time-of-day tariff, is required to make such solutions profitable.

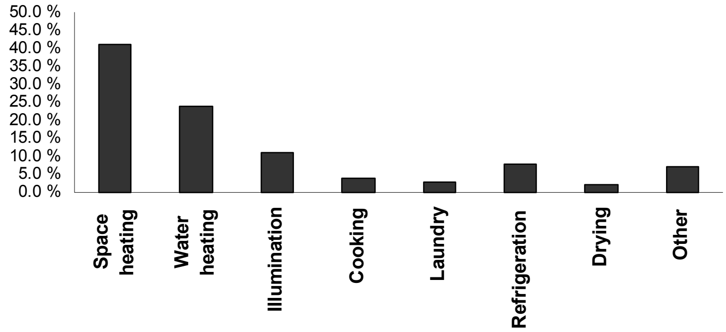

Figure 5: Distribution of residential electricity consumption. Source: [15].

Some of the other electricity consuming tasks mentioned in Figure 5 may also be flexible to some extent. It is possible to have dinner earlier, or to postpone it. The same goes for laundry and drying clothes. Refrigeration could be flexible to some extent if the fridge is "smart." The lights, however, must be switched on while people are in the room, so they carry no flexibility, at least if we look away from the possibility of leaving the house.

Some consumers may find it difficult to understand what is going on and why they are offered money - implicitly or directly - to alter their daily habits. Some insight is needed to really understand how the electricity market works, and many consumers, (even those who are rather well educated), do not have this insight. The news media focus rather invariably on electricity prices and the behaviour of the actors in the market, and even though the information is not always correct, the articles both reflect and form the general knowledge and opinion. If the focus from the news media is negatively minded towards the market participants, this may make consumers more sceptical to any action from the firms. Therefore, there is a potential risk that some consumers start wondering if they are being fooled in some way, even though they actually find themselves better off than they were before hourly metering was introduced. This suggests that retailers and network owners should be careful with respect to how the new possibilities are presented to their customers.

Retailer

Even though ALP may give an estimated consumption for each consumer during each hour, most small and medium-sized consumers do not have contracts where the prices vary from hour to hour. A consumer who is on a spot price contract, for example, is typically charged the monthly average of the spot prices quoted at Nord Pool. The retailer, however, must purchase the electricity from hour to hour. If a consumer's real load profile differs from the estimated one, the retailer may have to pay more than he should for the electricity delivered to this customer. This is a risk to the retailer, and this risk is being paid for, either by the retailer or by the consumer, so hourly metering and hourly pricing, by reducing this risk, increases overall welfare.
In addition to the advantage the retailer obtains from the reduced risk, hourly metering would also enable him to incentivise his customers to use power at cheaper hours. He could offer the customers some money for moving consumption from peak-price periods to off-peak periods. This way, the retailer will make a profit from sourcing cheap power, and the incentive payment means that he shares this profit with his customers.
In a perfect market, it would not be possible to make a profit by doing this, as all retailers would offer the same prices each hour, and this would be reflected in the spot prices. Hence, we assume in this thesis that we are not dealing with a perfectly competitive market. Such an assumption could be justified in at least two ways. Firstly, the prices do vary between the retailers today, indicating that the market is not perfectly competitive now. Coupled with the retailers' difficulties in making their business profitable, this suggests that competition is fierce, but not perfect. This imperfection is possibly due to the observation that many electricity consumers do not bother spending the time it takes to look around for the cheapest offer all the time. Even though the number of residential consumers not buying electricity from the dominant retailer in their area is increasing, the majority of Norway's households still deal with their local utility. We find no reason to believe that introducing hourly metering would change this.

Secondly, since hourly metering of small and medium sized consumers is rather uncommon today, some region would have to be the first to introduce this. The dominant retailer in that region could now utilize the fact that his customers' newly acquired short-term price flexibility would not be reflected in the wholesale market price (because the retailer will be a small participant in the wholesale market). Then, even if the end-user market were perfectly competitive before his local customers got hourly metered, he could now buy more electricity during cheap periods and less during expensive periods and share some of the profit with his consumers. This would mean that the retailer has a first mover advantage and utilizes a temporary market imperfection.

After the day-ahead spot market is settled, and the hours of delivery come closer, the retailer will have more updated information on weather conditions than he had when submitting his spot market bids. If the weather conditions seem likely to be different from what he believed back then, he will be likely to face a loss in the regulating market. If his consumers were hourly metered, and the appropriate means of communication between the retailer and his consumers were in place, he may try to reduce this loss by offering his consumers some money to alter consumption. This, together with the mentioned market imperfections, suggests that retailers could find it beneficial to control their customers' load profile.

We mentioned that Paper 1 includes a model of an hourly metered end-user's cost minimization. In this paper we also model the pricing decision of a retailer serving such a customer. The retailer decides a price profile to offer that maximizes his profit, while considering competition from other retailers and the consumer's cost minimization. This becomes a Stackelberg game where the retailer is the leader and the end-user the follower. It is implemented as a mathematical program with equilibrium constraints (MPEC), and an important contribution of this paper is the practical application of an MPEC. Also, the modeling of the pricing decision faced by a retailer serving an hourly metered and charged end-user is in itself a scientific contribution to the operations research disipline.

Network operator

As explained in Paper 2, the network owners' profit margins are regulated by the authorities. The framework is formulated by The Norwegian Water Resources and Energy Directorate (NVE). In the same paper we explain how the regulatory regime provides the network owners with incentives to save future investments.
The network operators are obliged by law to carry any realistic load needed to the consumers in their region. Therefore, high consumption peaks may require them to expand the grid just to carry the peaks, which is expensive. Having the consumers curtail their peak loads would therefore save costs for the network operators. If they, for example, need to connect a new housing estate to their network, lower consumption peaks may enable them to connect the new housing estate to an existing network node instead of building a new and expensive one. Therefore, the network operators may want to incentivise consumers to move consumption out of peak load periods.
In Paper 2 and Paper 3 we model the interaction between a network owner and a retailer who are both interested in having the same consumer shift load out of peak periods. Since both may benefit from this, they may both be willling to provide the consumer with incentives to do so. This leads to a game between the network operator and the retailer where both have to consider the other player's revenue function when deriving their own optimal behaviour. In Paper 2, we analyze this game in a situation with two load periods: a peak period and an off-peak period. In Paper 3 we extend this model to three load periods to analyze some issues for which two load periods are not enough to discuss. The contribution of those papers lie in the application of methods from game theory to a problem that to our knowledge has never been studied before.
One issue to discuss regarding the game model is how to measure if the consumer has actually shifted load. Let us assume that one day the consumer has switched off her electric water boiler in the peak period to heat the water in the off-peak period instead. If she, however, had planned for a party that day, she may have used the kitchen stove more than she normally does in the peak period. In this case she has actually shifted load by heating her water in the off-peak period, and

should therefore receive the incentive payment, but since she has used more electricity than she usually does for cooking in the peak period, her load shifting will not be visible on her consumption meter. There may be many ways of getting around this problem, and we will suggest one, without suggesting that this is the best one, but rather to show that it is possible.
Assume that the consumer signs a contract on the maximum load that is installed on her house. If she uses this amount of power all the time in one day she will pay a certain amount, P. Then if she one day uses less than the maximum load, she will receive a discount. This discount may be different in the peak period and in the off peak period (we now consider the two-period version), and then she will still have the incentive to shift load between those periods even though the load shifting, for whatever reason, may not show up on her consumption meter that day. To illustrate, let d_1 be the per unit discount offered in the off-peak period and d_2 the per unit discount offered in the peak period. If $d_1 > d_2$, the consumer will benefit by moving load from the peak period to the off-peak period. Furthermore let x_1 be her consumption in the off-peak period and x_2 the consumption in the peak period. Then her electricity bill for one day will equal $C - d_1 x_1 - d_2 x_2$. The incentive, d, "seen" by the consumer is $p = p_1 - p_2$.

Environment

There are some potential environmental benefits related to increased end-user flexibility. A reduction in peak load would mean that less power plants are needed to serve the peak hours. The peak load plants are often polluting thermal plants. The plants do not look very pretty either, and they absorb some space that could potentially be green areas. Hence, if increased end-user flexibility removes the need to build new plants, that would be good for the environment.
We have mentioned that increased end-user flexibility may remove some of the need for network operators to invest in more transmission capacity. Increased transmission capacity implies new and ugly lines, together with construction work out in the wilderness.
Though the most important environmental benefits arise from the reduced need for capacity expansion, energy consumption may be re-

duced as well. For example, hourly pricing would increase the amount of money saved by turning down the temperature during daytime. Also, we have mentioned smart houses; advanced technological solutions that automatically manage electricity consumption to minimize the electricity bill. A smart house (if programmed properly) would never forget to switch off lights in unoccupied rooms.

Society

In Figure 6 we have depicted how the enhanced short-term flexibility could even out the intra-day price fluctuations in the wholesale market. The solid-drawn black curves to the right, D_3 and D_4, in the figure show two possible demand curves during daytime, while the dashed black curves, D_1 and D_2, to the left show two possible demand curves during night-time. We have used iso-elastic demand curves. Assume that for some price P_0, the corresponding demand is D_0. Then, the demand at price P is derived by

$$D = D_0 \left(\frac{P}{P_0} \right)^{\eta}$$

where $\eta \leq 0$ is the elasticity of demand. By Figure 6 we see that D_1 and D_2 intersect at $(D_{0,night}, P_{0,night})$, where $D_{0,night}$ may be thought of as the night-time demand at the current weather conditions and some reasonable price $P_{0,night}$. With "reasonable price" we mean a price that is neither abnormally high nor abnormally low at the given weather conditions, season and time of day. Also $\eta_1 < \eta_2$, implying that D_1 is more elastic than D_2. In other words, D_1 and D_2 depict two situations where the physical conditions, that is weather and time of day, are equal, but D_1 shows a situation where the demand-side is more flexible, possibly because hourly metering has been introduced. During daytime the demand curves, D_3 and D_4 intersect at $(D_{0,day}, P_{0,day})$, where the $D_{0,day}$ may be thought of as the daytime demand at the current weather conditions and some reasonable price $P_{0,day}$. while $\eta_3 < \eta_4$, thus making D_3 more elastic than D_4. Hence, D_3 and D_4 also show two situations where the need for energy is equal, but D_3 shows a situation where the demand-side is more flexible. The fact that $D_{0,day} > D_{0,night}$ reflects that more energy is needed during daytime than at night.

The grey line, S_1, shows a possible supply curve. By studying the intersections between supply and demand, we observe that enhanced flexibility could increase prices at night, and decrease prices during daytime. Hence, the intra-day price fluctuations have been reduced in this case. For simplicity, we have chosen a linear supply curve, which is not very realistic, but in this case we may use it without loss of generality. The analysis may easily be generalized to a case with a strictly convex supply function.

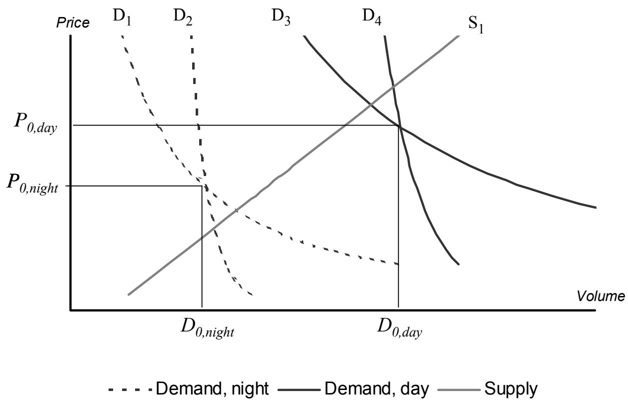

Figure 6: More flexibility on the demand-side could even out intra-day prices.

However, whether or not the intra-day prices even out, would depend on the shape of the supply curve. In Figure 7 the same demand curves are depicted, but the supply curves have changed. Both supply curves in Figure 7 are more elastic than that in Figure 6. The uppermost supply curve, S_2, in Figure 7 depicts a situation with shortage of supply, while the lower supply curve, S_3, depicts a situation where supply is under less pressure. For example, S_3 could reflect the beginning of a cold period, when the reservoir levels are still high. Then, S_2 could reflect the situation at the end of the same cold period: the demand is the same, but reservoir levels have run lower, increasing

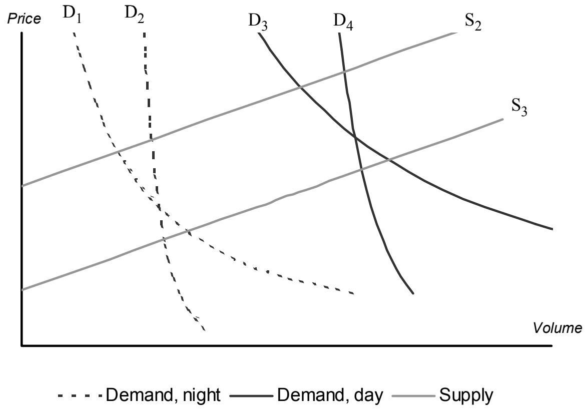

Figure 7: More flexibility on the demand-side may even out prices in general.

the value of water. We observe that at the end of the cold period, the prices have decreased both during day and night as demand has become more flexible. At the beginning of the cold period, though, the prices have increased. Hence, in this case, increased flexibility of demand has not evened out the intra-day prices, but the difference in prices have become lower between periods.

The situations analysed above indicate that enhanced end-user flexibility would even out the prices in the wholesale market. Today, when demand is rather inflexible, changes in supply and demand must be absorbed almost only by changes in price. With more flexible demand, as we have seen, changes in supply and demand may also be absorbed by changes in volume. Lower variability in the prices means less risk to whoever has to manage this risk, and reduced overall risk would increase social surplus (because risk has a price). Also, price fluctuations reflect that capacity is being priced by the market. Making the price signals visible to those who are able to act according to them, that is the end-users, would give more proper valuation of capacity. Today, flexibility clearly has some value to the generators, but it has (almost) no value in the end-user market.

In Figure 8 two demand curves, D_5 and D_6, that are of the same type as those in figures 6 and 7, are depicted together with a more realistic supply curve, S_4. This supply curve is strictly convex, with a vertical asymptote at the volume where all available generation plants produce at their capacity limits. In this special case we observe that D_6, the least flexible of the demand curves, does not intersect with the supply function. In this situation we would not get a market cross, and since the market price is determined by the intersection between supply and demand, this means that we would get no market price either. There is not enough generation capacity available to meet demand. If this happens, the authorities would have to ration power. Then, consumers with low priority, which means residential areas, will be disconnected to ensure that there is enough power for important institutions like industry, hospitals and schools. This has not happened for several decades, but during the winter of 2002/2003 some were actually worried that rationing could be needed. Fortunately, the authorities did not have to take such measures, but there is a risk that rationing may be needed if we get another long, cold and dry winter preceded by a summer with low precipitation. In Figure 8 we see that the more flexible demand-curve, D_5, does give a market cross. Hence, increased end-user flexibility would make rationing less likely.

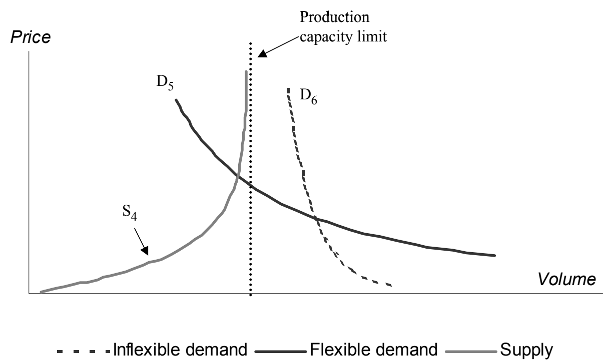

Figure 8: More flexible demand would make rationing less likely.

Increased end-user flexibility may be benefitial to society in even more ways: As explained in previous sections, hourly metering and pricing may save costs for both network operators, retailers and consumers. The saved costs would provide some social surplus. Also, high market prices reflect that the power system be under more pressure than while prices are low. This pressure would be relieved if the peak loads are reduced, which again reduces the chances of power outages. Outages are expensive to society.

In Norway, there is a net flow of inhabitants from rural areas to urban areas. Hence, the population in urban areas is increasing, which, ceteris paribus, increases the peak loads in the major cities. This is one of the main reasons why urban network operators find value in reducing peak load: a reduction in peak load may enable them to serve the increasing population without expanding the grid capacity.

The fact that the adjustment parameter for new investments is based inter alia on net immigration to the network area strengthens this effect. A network owner that serves a rural area will, however, not have this problem. If today's grid capacity is sufficient, the capacity will in general remain sufficient in the foreseeable future, and the network owner will not see any potential need for grid expansions. If he does not see any need for expansions, he will not see any value in peak load reductions. Hence, some rural network owners would not be willing to offer the consumers anything to make them curtail peak loads. If we consider the game described in papers 2 and 3 of this thesis, the retailer would in this situation have to "play" the game (which is no longer a game) alone.

Due to the concept of ALP, the consumers are today billed due to an overall average load profile. Therefore, an introduction of hourly pricing and metering will disbenefit those consumers who use more electricity than average in the peak periods and less than the average in the low load periods. Vice versa, consumers with the opposite profile would benefit from an introduction of hourly pricing. The losers will be those consumers who, for some reason, are at home during daytime, when the prices are high, while the winners will be those who work long days and therefore be able to turn down the room temperature during peak hours. Some of those who are at home on daytime, are people who, for whatever reason, do not work and therefore have rather low incomes. This is a potential problem for the politicians that argue in favour of hourly metering. Being held responsible by the news media for an old rentist couple having to freeze is not a boost for a political career. We have, however, argued that hourly metering and pricing may induce some social surplus. A way of dealing with this problem may therefore be to taxate some of this surplus and use the taxes to pay welfare energy payments to people who for whatever reason have to stay at home during peak price hours.

4 How to make the consumer curtail load

As we have seen, both retailers and network owners may find it valuable to gain some control over their consumers' daily load profiles. In this section we will discuss the pros and cons of two different ways of controlling load profiles; direct control by physically cutting off load and indirect control through price mechanisms.

4.1 Direct control by physically cutting off load

Retailers and network owners may achieve direct control of their customers' load profiles by installing devices that enable the utilities to cut off all, or parts of, their consumers' load from a remote central whenever needed. The utilities would then enter into an agreement that enable them to buy back power from the consumers under certain conditions. For example, they may pay the consumer an amount up front to make them willing to let the utilities cut load.

A major advantage of cutting off load from a central is that the utilities will always get the needed load reduction at the exact time when they need it. However, this alternative would take away the consumers' control over their own consumption, though it may be possible to develop systems to ensure that the consumers do not take any notice of this in a normal situation. For example, some customers would not care much if their space heaters and water heaters were switched off for a few hours while they are at work. On the other hand those at home during the day may find this unappealing. Perhaps they have visitors one day and therefore have taken a day off to treat them. Then, a water boiler that is switched off may cause some problems. Also, some customers may dislike that their consumption is controlled by others, perhaps especially by network operators, who by many may be looked upon as representing the authorities, and therefore regard this as an intervention into their private lives.

Also, building such a system could be rather expensive, as one would need to install some equipment both at the consumers and at the control centre. In addition comes the cost of administrating the system. Agreements with consumers to carry out load curtailment centrally will have to be rather rigid with respect to which appliances that may be cut off and when. A consumer who controls this on her own would be more flexible.

Another issue that makes this approach questionable is the fact that the retailer and the network owner may not always want to cut load at the same time. If, say, the network owner cuts off a substantial amount of supply, this could impose a huge regulating market loss on the retailer.

Another disadvantage of such a system is that there would be a possibility of cheating. Someone could, for instance, move a parallel line from the space heaters to the electricity network, and this would be close to impossible for the utilities to control. Engineers may find good ways of preventing such cheating, but dishonest customers will always find ways around this.

4.2 Indirect control through price mechanisms

An alternative way of making the consumers shift load, which I think is better, is to simply offer price profiles that make it beneficial for them to alter their daily habits. This way, the utilities may control load profiles indirectly through price mechanisms, but the actual load curtailment is carried out locally. This approach to inducing load shifting is what this thesis is about.

The main disadvantage of using price mechanisms is that the utilities would not know for sure how the consumers react to the offered prices, and therefore they cannot fully control their load profiles; the challenge is to estimate how they react. With this approach, the consumers would not lose control over their own consumption, and by using rather inexpensive timer devices they may control their own load profiles without much inconvenience. They may turn the space heaters on and off whenever they like - as long as they pay for it. This approach is far less expensive than central load curtailment, as only hourly metering equipment is required.

Today, it may seem obvious that load curtailment should be carried out centrally, because most consumers do not possess the necessary equipment to do it on their own without much hassle. Obviously, they have had no reason to invest in equipment enabling them to respond to short term price signals that are invisible to them.

If hourly metering is introduced, this is likely to change. Some will adapt to the new situation rather quickly because they find it interesting. Others will adapt more slowly, but in the end, people will adapt to this because they may save money by doing so. First off, consumers may acquire simple timer devices. A bit further ahead in time, more sophisticated solutions, like smart appliances or even fully integrated smart houses, may become more common. In Figure 3 a potential evolvement of end-user flexibility is scetched[8]. This may also be about introducing good habits at an early stage. If end-users get used to having a central body taking care of their load curtailment, they are less likely to actively take part themselves in the future.

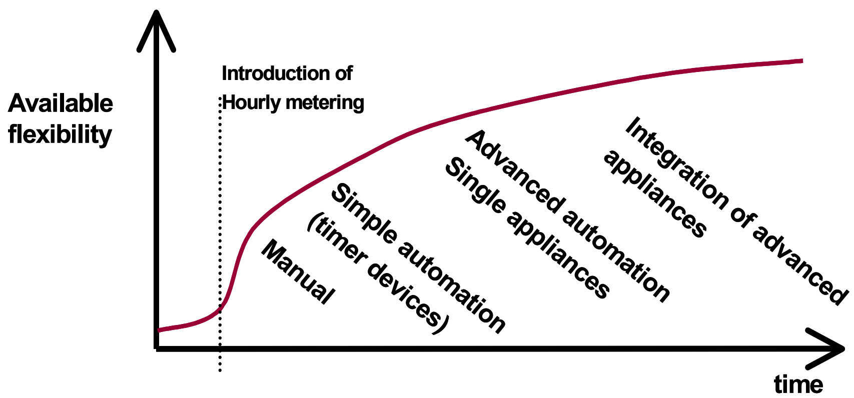

Figure 9: Possible evolvement of flexibility.

[8]The figure is taken from Bård Karsten Reitan's test lecture for the defense of the degree Doctor Ingeniør in March 2003.

4.3 Which solution is best?

For a network operator, central load curtailment is probably the best solution as they make most money by having as much energy as possible flow through the current grid, but preferably without network expansions. Being in control helps them achieve this. Also, after a while consumers may start thinking about using alternative energy sources for heating during peak hours, possibly making them less dependent on electricity. This is not in the interest of the network owners. In addition, both generators and retailers have no interest in a possible shift towards alternative energy sources. Retailers conduct marginal business and want as high volumes as possible. Fierce competition means that it is difficult to increase their margins, so they would dislike any shift towards different energy sources. Generators too, of course, would like to sell as much electricity as they can.

If hourly metering is introduced a market for flexibility enhancing equipment in the homes would emerge, together with a market for two-way communication services. We have mentioned that profit margins are rather low among retailers, and they may see this as a valuable new business opportunity. Also, network operators may want to add to their income from regulated services by selling such equipment.

By taking direct control over the consumers' load profiles from the beginning, the energy utilities may get into a position where they become the natural suppliers of such equipment. They may build up platforms, both technologically and market-wise, that deter entry of other suppliers of products and services related to end-user flexibility and two-way communication. This is okay, but currently the authorities are considering to enforce the installation of solutions that enable the power companies to cut load. Today, there is no market for such products and services, but such a market would emerge if hourly metering equipment is installed. By making laws that require central load curtailment solutions to be installed in all homes, the authorities could actually impose a monopoly situation in an area where market forces may provide cheaper and better solutions.

In my opinion, the authorities should enforce the installation of hourly metering equipment, and let the market develop products and services related to end-user flexibility.

As we have seen, the physical solution is probably the solution that would be preferred by the power companies. It may be beneficial to the industry, but not necessarily to society. I believe that even though the physical solution would trigger more load shifting in the short run, putting the trust in price mechanisms would be better in the long run.

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Part II

Articles

Article 1

A Stackelberg Game Model of Electricity Demand-Side Optimization

Erling Pettersen

Abstract

This paper presents a Stackelberg game model of strategic interaction between an electricity retailer and an end-user in an electricity pool market. The retailer offers a vector of 24 hourly prices to the end user who lives in a so-called smart house. The smart house minimises the electricity costs based on the offered prices taking into account the residents' habits. The consumption pattern chosen by the consumer must be purchased by the retailer in a risky wholesale market. We assume that the risk averse end-user is unwilling to take the risk in the wholesale market while the risk neutral retailer seeks to maximise expected profit. Thus, the price profile offered includes a risk premium. To model the house's cost minimization we have chosen to utilize an energy storage approach focusing on space heating and water heating, and these processes are carried out at the lowest possible cost. Our approach results in the house solving a linear program. This is convenient due to the highly non-linear and non-convex nature of the mathematical program with equilibrium constraints (MPEC), which constitutes the overall problem.

1 Introduction

Many countries have deregulated, or initiated the process of deregulating, the market for electricity. The first step often lies in the deregulation of the wholesale market and most OECD countries have carried out this process. The structure of the deregulated wholesale markets are, however, somewhat different from country to country.

After the wholesale markets have been deregulated, a possible next step is to deregulate the end-user markets. In Norway, for example, the end-user market was formally deregulated in 1991 and the individual consumers were allowed to change retailers. During the first years, however, the end-users were charged a fee for this, and changing retailer was economically meaningful only for large consumers. Some countries still have laws that allow only the largest consumers to change retailer. The Norwegian electricity market became fully liberalized in 1997 when all consumers were enabled to change retailer at no cost.

Today, the incentive structure in the liberalized Norwegian end-user market is directed only towards the consumer's total energy consumption. The vast majority of customers have their consumption metered a few times a year, and they are billed based on their accumulated consumption since the last metering. Hence, to seize customers from their competitors, the retailers have focused on offering as low energy prices as possible. One of the advantages of this system is that it makes it quite simple for the customers to compare the offers from different suppliers. This has caused fierce competition among the retailers, which in turn has resulted in decreasing energy prices for the consumers.

In the wholesale market, the prices vary from hour to hour. This suggests that power is a scarce resource that is being priced in the market. With today's system for metering and billing consumers, however, these prices are not made visible to the consumers[1]. The only way for a consumer to reduce her electricity bill, besides changing to a cheaper retailer, is to reduce her total consumption of energy. If the consumers had metering instruments installed that made them exposed to the short-term fluctuations in the wholesale electricity prices, they would also have incentives to alter their daily consumption habits in order to save costs. They could, for instance, do the laundry while prices are low, have electric water boilers switched off in peak hours and even pre-heat the living room while the prices are low[2]. This load shifting could very well make the total energy consumption increase but, unlike today, increased energy consumption could be to the benefit of the customer. It would also be beneficial to society as the prices are, at least in theory, computed to maximize social benefit. A potential drawback of this is that the prices could become less transparent to the consumers, which could make it more difficult for them to compare offers from different retailers, potentially leading to

[1] Only very large consumers have their consumption metered by the hour and therefore have incentives to adjust their consumption according to the short-term price fluctuations in the market.

[2] In Norway electricity is the main energy source for space heating.

market imperfections.
In some countries, the consumers are faced with two-part tariffs. That is, the day is divided into a peak and an off-peak period between which the prices are different. This partially makes the price fluctuation imposed by the scarcity of power visible to the consumers, and provides them with incentives to shift load. Some econometric studies have been carried out to investigate consumer responses to two-part tariffs. Asano et al [16] measure the effect of incentive payments on residential time-of-day (TOD) electricity demand in the Kyushu region in Southern Japan. Results based on an econometric model suggest that households tend to modestly shift their electricity consumption from peak to off-peak when offered incentive payments for load shifting. Filippini [8] considers the household on a micro level and expresses the household budget shares of peak and off-peak consumption as a function of electricity prices, real electricity expenditures and household characteristics. The results indicate that the demand for peak and off-peak electricity is elastic. There exist many more econometric analyses, which provide information on customer response to time-differentiated electricity tariffs, (see Aigner [1] and [2], Aigner and Ghali [3], Aigner and Hausman [4], Caves et al [5], Ham et al [10], Henley and Perison [11], Lawrence and Aigner [13], Mountain and Lawson [16], Tishler and Lipovetsky [20], Train and Mehrez[19]).
In order to make the daily price fluctuations completely visible to the consumers, the measuring instruments should be able to meter the consumption with the same time resolution as the trading periods in the wholesale market. In Norway, this means that consumption should be metered by the hour. Today, rather few households have installed instruments capable of metering the consumption in real time, but the number of real-time metered households is increasing and signals from network owners, who are the ones responsible for metering the consumers, indicate that in some years most households will have the necessary equipment installed. An interesting study on time-of-day (TOD) tariffs can be found in Hirschberg [12]. This paper presents an econometric model for estimating TOD substitution. The model employs the estimated second moment of demand to estimate a matrix of relative own- and cross-price elasticities, and as one of the examples a case of an individual household's electricity demand is presented. The

model is tested on a household from the control group in an experiment conducted by the Los Angeles Department of Water and Light to measure the impact of TOD prices on the demand for electricity. (See Manning et al [15] for a description of this experiment). The study finds that the matrix of relative elasticities appears to show bands of elasticities for sets of two-hour intervals and it is suggested that this regularity may be due to the use of a particular appliance.

Hirst [11] argues that customers that choose dynamic pricing would pay less for electricity over the long run. He also points out that if some customers chose dynamic rates, the loads would be reduced at times of high power prices, which would lower overall prices and thereby benefit all electricity consumers. Hirst [11] illustrates this by performing some simulations on data from the California Power Exchange. Customers that choose dynamic electricity rates will take more risk than do customers that pay traditional tariffs. The financial insurance aspects of electricity are new to most consumers. However, the consumers have plenty of experience with the concept of managing financial risk. Hirst [11] illustrates this by drawing analogies to other industries with comparable risk attributes, for example car insurance and financial markets.

An alternative to using an econometric approach to model the demand is to employ an engineering approach to the problem. In econometric approaches the emphasis is on using data to estimate parameters, while an engineering approach would attempt to investigate the physical properties of the process. In the case of load shifting, one would try to model how the consumer changes habits to accommodate the price fluctuations by figuring out what processes are flexible with regard to load shifting, and to what degree these processes may be moved over the day without making the value of lost comfort exceed the benefit from the reduced electricity costs. In this paper we have chosen an engineering approach to the problem.

The load shifting could be done in different ways. The simplest solution is for the consumer to turn the devices on and off manually. However, waking up at night to switch the water heater on would in most cases involve a comfort loss for the consumer that would not justify the financial savings. A simple, feasible, and much more convenient solution would be to connect time switches to electrical appliances. By adjusting the time switches to switch off some appliances in peak-hours, the load shifting could be accomplished without too much trouble for the consumer. To figure out when the appliances should be switched on and off does, however, require the consumer to know the price profile and, not least, to understand the physical properties of the appliances. For example, most consumers do not know for how long the water heater could be switched off without the water getting too cold. Many consumers would probably find it laborious having to look up the market prices each day, especially since the potential cost savings after all are quite small in the big picture.

At least in Scandinavia, it is becoming increasingly common for consumers to install advanced technological solutions capable of automatically carrying out the cost minimization automatically. Assuming that two-way communication between the retailer and the consumer is established, we may think of a system where the consumer receives a price vector from the retailer and, based on this vector and the consumer's consumption preferences, the cost of electricity consumption is minimized. These solutions are also designed to turn heating devices on and off, implying that the residents' inconvenience of regulating the load levels, and therefore an implicit transaction cost, is removed. Within this framework, the retailers will be able to provide consumers with proper price signals, to which the consumers will be able to react in an efficient manner. As it is believed that the market for such solutions, sometimes referred to as "smart houses," will increase over the next years, we choose this advanced approach to load shifting.

An example of a smart-house model of demand-side response is found in Boertjes et al [5]. They construct a working solution to automatic comfort management in a large-scale real-time pricing environment. Scale methods are used to formalize the concept of comfort and develop a procedure in order to convert this comfort scale into cardinal utility functions. Their simulation results demonstrate how computational agents, which are features of a smart house, deliver higher comfort at lower cost.

Hämäläinen and Mäntysaari [9] develop a model for a residential consumer who optimizes her use of electricity for space heating under TOD pricing. In their examples, they use a three-part tariff. They model the decision as a multicriteria problem where the consumer maximizes the living comfort while minimizing the heating costs under a given price profile. In addition to the economic cost criterion they include an environmental criterion by making the consumer minimize the total amount of energy consumed. They model the space heating by viewing the house as an energy storage. We also model the house as an energy storage, though in a slightly different way. Our method of modeling the space heating was introduced in Pettersen [18], which was a preliminary version of the current paper, in which we also model the heating of water. We have not included any environmental criterion, but focus solely on cost and comfort as we believe that rather few households would be willing to save energy if that makes them pay more. In our opinion, the most realistic way of making households save energy is by increasing energy prices.

While Boertjes et al [5] treat the electricity prices as fixed input and give a rather detailed description of the demand-side response, we choose a more rudimentary approach to the processes underlying the consumer's demand, but focus on the pricing decision facing a retailer serving a smart-house consumer.

In this paper, a plausible model of domestic demand-side response is developed. The bulk of domestic electricity consumption in Norway is used for water and space heating, and therefore we have focused on these two processes in our model. Using this smart-house model of demand-side response, we consider the contracting decision made by a retailer that sells electricity to such a consumer in a competitive market environment. The retailer must decide on a price vector to pass on to the consumer who, in her turn, based on the offered price vector, minimizes the daily cost of electricity consumption. This interaction is modelled and implemented as a Stackelberg game model.

In Section 2 our model of domestic demand-side response is built up. In Section 3 we present a game model of strategic interaction between a retailer and the consumer modelled in Section 2. In Section 4 we give a brief conclusion.

2 A model of domestic demand-side response

End-users utilize electricity for many different processes: refrigeration, space heating, water heating, cooking and laundry to mention a few. Some of these processes are more flexible than others when it comes to changing the time the processes are accomplished. For example, we may think of the laundry as a process that can be moved to a time of the day when the electricity prices are low while lights are on when the residents are in the room no matter how high the price is. Other important examples of flexible processes are space heating and water heating, and in this paper we focus on these two processes.

When the house is heated up, the space heaters may be switched off for some time before it again gets uncomfortably cold. This is so because the walls are insulated and heat will be stored in furniture, walls and ceilings. Therefore, the consumers may choose to preheat the house when the electricity is cheap to, as far as possible, avoid having the space heaters switched on in peak price hours. In most houses, though, the possibilities for preheating will be quite limited. If the temperature outside is, say, below the freezing point, it will not be practically possible to have the space heaters switched off for several hours. Anyway, there is some potential for utilizing the intra-day price fluctuations by preheating. Space heating constitutes a major share of the electricity consumption in Norwegian homes. Even if the potential for load shifting is not that great when it comes to space heating, controlling the temperature in the house will be an important feature of the smart house. By knowing the residents' habits, the smart house would ensure that the house is as warm as required when the residents are at home and awake, and let the house cool down a bit at night and while the residents are away from home. This way the smart house could provide the consumer with significant energy savings in addition to the savings stemming from the load shifting.

To model the flexibility of space heating we think of the house as an energy storage. When space heaters are switched on, energy flows into the system and, since it is colder outside than inside (by assumption), there will be some energy-flow out of the system. The flow out of the system is, however, delayed by the fact that the house is insulated and that heat will be stored in furniture, walls and ceilings. Mathematically the energy storage is described by the following ordinary differential equation:

$$\beta\frac{dE_a(t)}{dt} = x_a(t) - \alpha[E_a(t) - E_{out}(t)] \tag{1}$$

where
x_a is the load over the space heaters
$E_a(t)$ is the energy level at time t
$E_{out}(t)$ is the outside energy level (a proxy for the outside temperature). By assumption, $E_{out}(t) < E(t)$
α and β are positive parameters.
Discretising the differential equation (1) and rearranging it to get the variables on the left hand side gives

$$\beta E_a(t) + (\alpha\Delta t - \beta)E_a(t-1) - \Delta t x_a(t) = \alpha\Delta t E_{out}(t) \tag{2}$$

where Δt is the length of a time step, measured in hours. The equation (2) is a constraint in the consumer's cost minimization problem.
The outside energy level, E_{out}, varies over the day and has its peak at around 2pm and its minimum at around 5am. The temperature profile is shown in Figure 1.

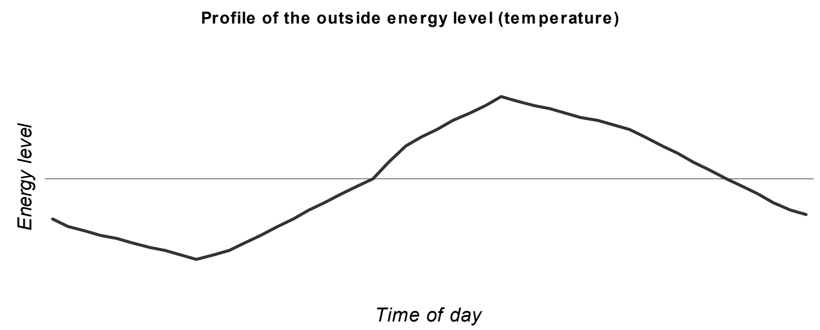

Figure 1: Outside temperature. Midnight is to the very left and to the very right in the figure. Noon is in the middle.

The residents have some comfort requirements that serve as constraints in our model. The comfort requirements reflect the fact that the consumers want the house to be comfortably warm when they are home and awake. These constraints are modelled by specifying required energy levels for each hour of the day, $E_{a,\min}$. The house must maintain the energy storage so that this requirement can be met at all times. The required energy levels in this model are depicted by the bars in Figure 2. The residents of the house would like the house to be warm when they get up in the morning, and therefore, the required energy level is high in one morning hour. When the residents are at school and/or at work, the house needs just enough energy to keep the potted plants alive and to ensure that family members could come home earlier than planned without freezing. In the evening, however, the house should be warm and pleasant. At night, when the residents are asleep, the house may be a bit cooler. There is also an upper limit to how warm it may get before it gets uncomfortable. This limit is depicted by the straight line in Figure 2.

We assume that the requirements are deterministic and do not change in the short run. In reality however, the comfort requirements would be, at least to some extent, stochastic. The residents may come home from work earlier or later than planned. They may get an opportunity to sleep late some mornings, or they could go to bed earlier than usual. This randomness is perhaps not that much of a problem for the residents or for the smart house. If the residents wanted to lie in one morning, one may think of several ways for a smart house to adjust to this without too much trouble for the residents. However, the retailer, which is supposed to offer a price vector to the consumer one day in advance, will get additional uncertainty to deal with in the pricing decision.

Also, the outside temperature is in reality stochastic. If it gets colder than expected, the consumers will use more electricity for space heating and the consumption pattern may also change. Again, this would not be that much of a challenge to the smart house, which presumably has temperature sensors outside, but the retailer would have to deal with the temperature uncertainty. On the other hand, the weather uncertainty will be reflected by the randomness in the wholesale prices. The idea of this paper is to build a plausible model of the demand-side response that can be used to look at the interaction between the retailer and the end user.

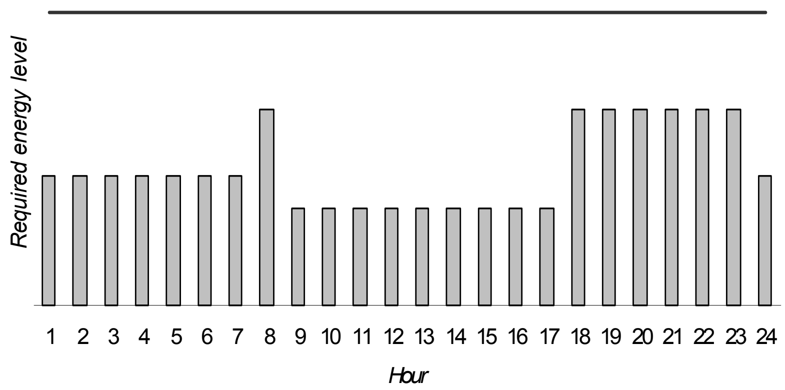

Figure 2: Comfort constraints. The minimum requirements are shown by the bars, while the maximum level is shown by the straight line above the bars.

In this setting, energy level serves as a proxy for temperature. In Equation (1) the energy level could be replaced with temperature and instead of specifying required energy levels, minimum temperature requirements could be used. Such an approach would indeed give a more accurate picture of the space heating process. Doing this would, however, require a much more detailed description of the house. We would need to know how big the house is, how it is furnished, how well insulated the walls are and so on. The goal here is however to build a model that gives a general picture of the process of space heating.
To model the flexibility in the heating of water we could have used a model similar to that used for space heating. Then, we would change the parameters in Equation 1 and add a term to describe the tapping of water from the storage. However, the physical properties of water and air are quite different, and water has a much higher ability to store heat. This means that water heating is more flexible than space heating when it comes to load shifting. Also, using a similar model

would require us to have the heat outflow from the water boiler depend on the room temperature. Then, the two models would be linked together, and this could make the model unnecessarily complicated without really giving much more useful insight. The heat outflow from the water boiler will probably not be significantly different at a room temperature of 15 °C than 21 °C, and besides, the water boilers are often placed in cool cellars with more or less constant temperatures. Thus, to model the water heating we have used an approach built on the same principles as for the space heating, but the model is a bit more simplistic.

Let $E_w(t)$ be the energy level in the water at the end of time period t. $E_w(t)$ will depend on the energy level at the end of period $t-1$, the load in period t, the amount of heat that is lost to the surroundings during period t and the amount of water that has been tapped from the boiler during period t. Then, the water heater is described by the following equation

$$E_w(t) - E_w(t-1) - \Delta t x_w(t) = -h_{outflow} - w(t) \qquad (3)$$

where

$x_w(t)$ is the load over the water heater in time period t

$h_{outflow}$ is the rate of heat outflow from the water heater, which is constant

$w(t)$ is the heat loss due to water being tapped from the water boiler in period t.

For health reasons, the water may not hold temperatures below a certain level for several hours, and at this temperature level, we let $E_w(t) = 0$. However, right after a lot of hot water has been tapped from the boiler, the water will often hold temperatures below this level. This is not hazardous for short time periods. Hence, we have the constraint $E_w(t) \geq E_{w,\min}(t)$, where $E_{w,\min}(t) = 0$ in most time periods, but for an hour or two after the boiler has been tapped we allow negative values of $E_w(t)$.

The consumer's objective is to minimize her cost of electricity consumption. We let $\mathbf{p}$ be the vector of prices offered by the retailer. p_i is an element of $\mathbf{p}$ expressing the electricity price in time period i. Here, $i = 1, 2, \ldots, 24$. Then, the optimization problem for the consumer becomes a linear program

$$\min \left\{ \Delta t \sum_{i} \sum_{t \in i} p_i \left(x_a\left(t\right) + x_w\left(t\right) \right) \right\} \tag{4}$$

subject to

$$
\begin{aligned}
\beta E_a(t) + (\alpha \Delta t - \beta) E_a\left(t-1\right) - \Delta t x_a\left(t\right) &= \alpha \Delta t E_{out}\left(t\right) && \text{(5a)}\\
E_a\left(t\right) &\geq E_{a,\min}\left(t\right) && \text{(5b)}\\
E_a\left(t\right) &\leq E_{a,\max} && \text{(5c)}\\
x_a\left(t\right) &\leq x_{a,\max} && \text{(5d)}\\
E_w\left(t\right) - E_w\left(t-1\right) - \Delta t x_w\left(t\right) &= -h_{outflow} - w\left(t\right) && \text{(5e)}\\
E_w\left(t\right) &\geq E_{w,\min}\left(t\right) && \text{(5f)}\\
E_w\left(t\right) &\leq E_{w,\max} && \text{(5g)}\\
x_w\left(t\right) &\leq x_{w,\max} && \text{(5h)}\\
E_a\left(t\right), x_a\left(t\right), x_w\left(t\right) &\geq 0 && \text{(5i)}
\end{aligned}
$$

where
Constraints (5a) are the heat storage equations for space heating.
Constraints (5b) are the comfort constraints.
Constraints (5c) prevent the rooms from getting to warm. $E_{a,\max}$ represents the maximum level of heat allowed.
Constraints (5d) ensure that the consumer does not plan to switch on more load for space heating than physically possible. $x_{a,\max}$ represents the total possible load over the space heaters.
Constraints (5e) are the heat storage equations for water heating.
Constraints (5f) state that the water should not be colder than the minimum allowed level.
Constraints (5g) state that the water may not get to warm. $E_{w,\max}$ represents the maximum level of heat allowed in the water at any time.
Constraints (5h) ensure that the consumer does not plan to switch on more load for water heating than physically possible. $x_{w,\max}$ represents the maximum load over the water boiler.
Constraints (5i) are non-negativity bounds. Note that $E_w\left(t\right)$ may be negative.

Note that for the first time period of the day, $E_a(t-1)$ and $E_w(t-1)$ will denote the energy levels in air and water, respectively, at the end of the previous day.

2.1 Example

Figure 3 shows the spot prices in the Nordic wholesale market on a winter day. To illustrate how the model performs, we first offered this price profile to the consumer. Next, we offered a flat rate of 23.07, which is the volume weighted average of the prices in Figure 3, depicted by the straight line in the figure. When offered a flat price, the cost minimization is equivalent to minimizing the total energy consumption, and this is what a smart house would do if it were not hourly metered.

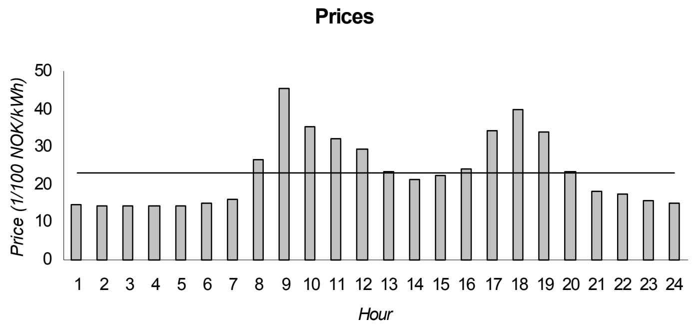

Figure 3: Spot prices.

Figure 4 shows the load profile for space heating under the two pricing schemes. The white bars show the space heating-load when the consumer is offered the variable price, while the grey bars show the load profile when the flat price is offered. In this example we have chosen $\Delta t = \frac{1}{2} hours$ in Equations (2) and (3). Hence, the consumer decides a load for space heating for each half hour throughout the day. In hour number 14, for example, the space heaters are switched on during the entire hour no matter which price profile that is offered. In hour number 8 the space heaters are switched on during the entire hour when

the flat rate is offered, but when the variable rate is offered, the space heaters are switched on only the last thirty minutes.
We see that when offered the variable price the consumer starts heating the house a little bit earlier in the morning compared with the case where the flat price is offered. The variable tariff makes the consumer use less energy in hour number eight and more in hour number seven, which makes sense since the price is higher in hour number eight. A similar effect can be observed in the afternoon when the variable price gives a small peak in hour number sixteen since the price is lower in that hour than in the next.

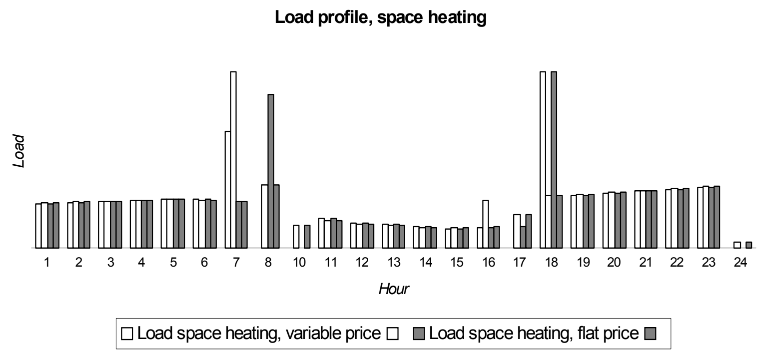

Figure 4: Load profile, space heating.

Figure 5 shows how the consumer manages her water boiler under the two pricing schemes. We observe a similar pattern as for space heating. When offered a flat rate, the water is heated just in time for it to be consumed without violating any constraints. When the consumer faces the variable tariff, the water is being heated at the lowest possible prices. In the morning, for instance, we observe how the consumer starts heating water for the morning bath earlier when offered the variable tariff than when the flat rate is offered. The pre-heating of water starts much earlier than the pre-heating of the air, because the water has a higher ability to store heat.

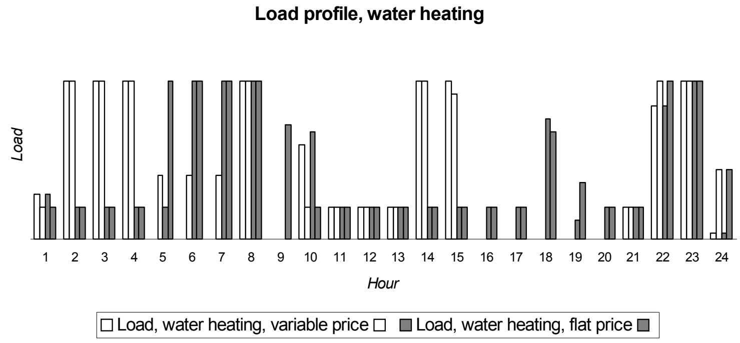

Figure 5: Load profile, water boiler.

Table 1 shows the consumer's costs and total energy consumption under different pricing schemes and circumstances. When consumers pay an average price (as they tend to do without real-time metering) they seek to minimize total daily consumption. This gives a consumption of $77.35kWh$. When charged at the flat price, this costs $NOK17.85$[3], but $NOK17.65$ when variable prices are charged. This cost can be reduced if the consumer is able to respond to the variable prices. Cost minimization at the variable price makes the household pay $NOK16.57$ for higher total energy consumption. Thus, the household will make some savings by utilizing the price profile and use energy when the price is low compared with if it just minimizes the overall consumption.

The integrated Nordic electricity system is a combined hydro-thermal power market. In normal circumstances hydropower is produced at a lower marginal cost than thermal power. Thus, if end users were able to react to price signals as efficiently as described, more hydropower and less thermal power would be consumed. This represents an improvement in overall welfare even though more energy is consumed, since the prices are (at least in theory) computed to maximize society's benefit.

To investigate the value related to electricity cost savings of installing smart house solutions, we have offered the variable price to a non-smart (or "dumb") house. The situation today is that consumers tend

[3] $1NOK \approx 0.14USD$. (NOK=Norwegian Kroner.)

to keep a steady temperature in the house throughout the day, even if they are asleep or not at home for several hours. Also, the residents have the water boiler running at a steady load all the time. This gives a consumption of $87.45kWh$, which costs $NOK20.08$.

	Cost(NOK)	**Consumed energy** (kWh)
Flat price, min. costs	17.85	77.35
Variable price, min. consumption	17.65	77.35
Variable price, min. costs	16.57	78.66
Variable price, "dumb" house	20.08	87.45

Table 1: Cost and energy consumption.

3 A model of strategic interaction between a retailer and an hourly metered consumer

The existence of real-time metered consumers with the ability to respond to price signals as described in the previous section introduces some new possibilities and challenges for the retailer. The possibilities may include a variety of new products, which could be designed to meet the desires and preferences of groups of customers or even individual customers. An important challenge would be for the retailer to anticipate the consumer's reaction to whatever product she is offered. The model presented in this section considers a retailer operating in a competitive environment and one end user purchasing electricity from the retailer. The purpose of the model is to study the strategic interaction between the retailer and the end user. First, the retailer decides a price profile to offer the end user and next the end user makes a consumption decision. Hence, the presented game is a Stackelberg-type game with the retailer as leader and the end user as follower.

3.1 Some assumptions

In this subsection we explain some important assumptions underlying the study. First we present our assumptions regarding the electricity prices. Next a few necessary assumptions are made for the players.

Electricity prices

We assume that the retailer operates in a deregulated electricity market. A common feature of deregulated electricity markets is that there will be a forward price and a final price. The forward price will reflect the expectation of the final price some time in advance while the final price reflects the marginal cost of delivering electricity to a certain area. In New Zealand, for example, a contract for differences would set the forward price. When the consumption at all nodes is known, the Independent System Operator publishes the final price. In the integrated Nordic market the system is a bit different. Here, the spot price, which is published by the electricity exchange Nord Pool one day in advance, could be thought of as a forward price. The regulating price, which is published some hours after the dispatch, could be thought of as the final price. Applying our model to the Nordic market would require some slight changes to the modelling, though, since the notions forward price and final price are not completely accurate for the spot price and regulating price, respectively.
In this study we have assumed that there are eight equally likely scenarios for the final price. The forward price is the expectation of these eight scenarios.

Assumptions on the players

We assume that the demand is derived from disutility minimizing behavior by the consumer. Let $x(t) = x_a(t) + x_w(t)$. The disutility from electricity expenses is expressed as

$$D(\boldsymbol{\pi}, \mathbf{x}) = \exp\left(\gamma \Delta t \sum_i \sum_{t \in i} \pi_i x(t)\right)$$

where
D is the value of the disutility

$\boldsymbol{\pi}$ is the price vector offered by the retailer
$\mathbf{x}$ is the consumption vector (the sum of consumption for water heating and space heating
γ is a positive parameter.
The disutility function is convex, implying that the consumer is risk averse. When the consumer is offered a deterministic price, minimizing disutility is equivalent to minimizing costs.
The retailer is assumed to be risk neutral and therefore its goal is to maximize expected profit. We also assume that the retailer has perfect knowledge of the parameters in the consumer's disutility minimization problem.

3.2 A Stackelberg-type game model

The retailer's task is to decide a price vector $\boldsymbol{\pi}$ to offer the consumer. Based on $\boldsymbol{\pi}$, the consumer decides a consumption profile $x(t, \boldsymbol{\pi})$. The retailer must purchase $x(t, \boldsymbol{\pi})$ in the wholesale market at a random price $\widetilde{p}\,(t)$. Since the retailer is risk neutral, his objective is to maximize expected profit and hence, the retailer's objective is to maximize

$$E\,(profit) = \Delta t \sum_i \sum_{t \in i} \pi_i x\,(t, \pi\,(\cdot)) - \Delta t \sum_i \sum_{t \in i} \overline{p}_i x\,(t, \pi\,(\cdot))$$

where
π_i is the price offered by the retailer in hour i
$\overline{p}_i = E\,[\widetilde{p}_i]$ is the forward price in hour i
To account for competition we assume that the consumer may choose to take the risk in the wholesale market instead of accepting the price profile offered by the retailer. Hence, to ensure that the consumer prefers the retailer's offer to the wholesale market, the retailer should offer a price profile, which satisfies the following constraint

$$D\,(\boldsymbol{\pi}, \mathbf{x}\,(\cdot, \boldsymbol{\pi})) \leq E\,(D\,(\mathbf{p}, \mathbf{x}\,(\cdot, \mathbf{p}))) = k \tag{6}$$

In Equation 6 the left hand side represents the consumer's disutility from accepting the price profile offered by the retailer. The right hand side is a constant expressing the expected disutility from purchasing power directly from the wholesale market. Hence, Equation 6 says

that the retailer must offer a bundle that makes the consumer at least as happy as if she took the risk in the wholesale market herself. It is perhaps not realistic to think of a small end user acting in the wholesale market on her own. However, the rationale behind this way of modelling competition is that if the retailer does not offer prices that satisfy the constraint 6, some other retailer will, and therefore the retailer must keep the prices sufficiently low to avoid losing the customer. Besides, we need a constraint to model competition, because otherwise the retailer would offer infinitely high prices since we have not included the possibilities for the consumer to change her comfort requirements or switch to different energy sources.
Furthermore the retailer knows that the consumer, whatever price profile she is offered, will choose a consumption pattern that minimizes her costs of electricity consumption. For the solution to be optimal for the retailer we need a set of constraints to ensure that the solution be optimal also for the consumer. As we have seen, for the consumer minimizing disutility is equivalent to minimizing costs. Therefore, the consumer solves a linear program with the prices offered by the retailer as input parameters. Hence, to ensure that the solution to the game model is optimal for the consumer, the primal and dual of the consumer's cost minimization problem is written down together with the complementary slackness conditions. The full model to be solved is then expressed by the objective function (7) and the constraints (8) on the next page.
The program (7) - (8) is a Mathematical Program with Equilibrium Constraints (MPEC). We refer to Luo et al [14] for a comprehensive treatment of MPEC models.
According to Tin-Loi and Que [19] there are three features of MPEC models that make them difficult to solve. Firstly, the equilibrium constraint is modelled by writing down the complementarity slackness conditions of the LP. However the complementarity constraints are disjunctive, making the feasible region a union of finitely many closed sets. Secondly, the feasible region of the MPEC may be non-convex even if all functions defining it are "nice". Thirdly, the feasible region may not be connected. These three difficulties are expected to cause problems in computing optimal solutions.

$$\max \left\{ \Delta t \sum_{i} \sum_{t \in i} (\pi_i - \overline{p}_i)\, x\,(t, \pi\,(\cdot)) \right\} \tag{7}$$

subject to

$$D\,(\boldsymbol{\pi}, \mathbf{x}\,(\cdot, \boldsymbol{\pi})) \leq E\,(D\,(\mathbf{p}, \mathbf{x}\,(\cdot, \mathbf{p}))) \tag{8a}$$
$$\beta E_a(t) + (\alpha \Delta t - \beta) E_a\,(t-1) - \Delta t x_a\,(t) = \alpha \Delta t E_{out}\,(t) \tag{8b}$$
$$E_a\,(t) \geq E_{a,\min}\,(t) \tag{8c}$$
$$E_a\,(t) \leq E_{a,\max} \tag{8d}$$
$$x_a\,(t) \leq x_{a,\max} \tag{8e}$$
$$E_w\,(t) - E_w\,(t-1) - \Delta t x_w\,(t) = -h_{outflow} - w\,(t) \tag{8f}$$
$$E_w\,(t) \geq E_{w,\min}\,(t) \tag{8g}$$
$$E_w\,(t) \leq E_{w,\max} \tag{8h}$$
$$x_w\,(t) \leq x_{w,\max} \tag{8i}$$
$$\beta y_a\,(t) + (\alpha \Delta t - \beta)\, y_a\,(t+1) + y_{a,\min}\,(t) + y_{a,\max}\,(t) = -\lambda_a\,(t) \tag{8j}$$
$$-\Delta t y_a\,(t) + y_{x_a,\max}\,(t) - \Delta t \pi_i = -\lambda_{x_a}\,(t)\,,\ t \in i \tag{8k}$$
$$y_w\,(t) - y_w\,(t+1) + y_{w,\min}\,(t) + y_{w,\max}\,(t) = 0 \tag{8l}$$
$$-\Delta t y_w\,(t) + y_{x_w,\max}\,(t) - \Delta t \pi_i = -\lambda_{x_w}\,(t)\,,\ t \in i \tag{8m}$$
$$\lambda_a\,(t)\, E_a\,(t) = 0 \tag{8n}$$
$$\lambda_{x_a}\,(t)\, x_a\,(t) = 0 \tag{8o}$$
$$\lambda_{x_w}\,(t)\, x_w\,(t) = 0 \tag{8p}$$
$$\pi_i \geq 0 \tag{8q}$$
$$E_a\,(t)\,, x_a\,(t)\,, x_w\,(t)\,, y_{a,\min}\,(t)\,, y_{w,\min}\,(t) \geq 0 \tag{8r}$$
$$\lambda_a\,(t)\,, \lambda_{x_a}\,(t)\,, \lambda_{x_w}\,(t) \geq 0 \tag{8s}$$
$$y_{a,\max}\,(t)\,, y_{x_a,\max}\,(t)\,, y_{w,\max}\,(t)\,, y_{x_w \max}\,(t) \leq 0 \tag{8t}$$

where
Constraint (8a) is the competition constraint.
Constraints (8b) - (8i) are the constraints of the consumer's cost minimization problem.
Constraints (8j) are the dual constraints related to $E_a\,(t)$.
Constraints (8k) are the dual constraints related to $x_a\,(t)$.
Constraints (8l) are the dual constraints related to $E_w\,(t)$.

Constraints (8m) are the dual constraints related to $x_w(t)$.
Constraints (8n) - (8p) are the complementarity slackness conditions.
Constraints (8q) - (8s) are non-negativity bounds.
Constraints (8t) are negativity bounds.
$y_a(t)$ are the shadow prices of the space heating equations.
$y_{a,\min}(t)$ are the shadow prices of the comfort constraints.
$y_{a,\max}(t)$ are the shadow prices of the upper bound on air energy level.
$y_{x_a,\max}(t)$ are the shadow prices of the upper bound on the load over the space heaters.
$y_w(t)$ are the shadow prices of the water heating equations.
$y_{w,\min}(t)$ are the shadow prices of the constraints stating that the water should not be colder than the minimum allowed level.
$y_{w,\max}(t)$ are the shadow prices of the bounds stating the maximum heat level of the water.
$y_{x_w,\max}(t)$ are the shadow prices of the upper bound on the load over the water boiler.
$\lambda_a(t)$, $\lambda_{x_a}(t)$ and $\lambda_{x_w}(t)$ are the slack variables of the dual constraints related to $E_a(t)$, $x_a(t)$ and $x_w(t)$, respectively.

MPEC models have received some attention in the mathematical programming literature. Tin-Loi and Que ([19]) compare some algorithms for solving such models and find that a smoothing method gives the best performance. The goal here however, has not been to find sophisticated methods finding globally optimal solutions to MPECs. The model presented here has been solved by writing down the program in GAMS. Then the model was solved hundreds of times from different starting locations using the sequential quadratic programming solver named snopt.

3.3 An example of a solution to the game model

In our search for an optimal solution we may use the fact that we already know one local optimum. Since the end user is risk averse, the retailer can make a positive expected profit by providing insurance to the end user. A trivial way of doing this would be to offer the expected wholesale price plus a risk premium in the form of a markup ε_1 on the final price. This policy is common in today's deregulated electricity markets. Since the retailer has complete information on how the end user will respond to any price profile offered, the retailer may maximize his expected profit by choosing the mark-up ε_1 that exactly makes the end user indifferent between the retailer's offer and the risky wholesale market. In this case the optimal mark-up is found to be $\varepsilon_1 = 35.7 øre/kWh$[4], giving an expected profit of $NOK28.033$. Hence, any solution from the game model must give an expected profit of at least this amount to be a candidate for a global optimum.

We have also solved the model offering the consumer a flat price; the volume weighted average price over the day plus a mark-up ε_2 in all hours. In this case the optimal mark-up is found to be $\varepsilon_2 = 33.1 øre/kWh$, giving an expected profit of $NOK25.579$.

We will now give an example of a solution to the model. In this solution the objective value, that is the retailer's profit, was $NOK28.604$. It has been stated that the model has been solved many times from different starting points, and $NOK28.604$ was the best objective value from all the solution attempts. This value was obtained several times, but each time the decision variables had different values. Hence, this amount could be the best obtainable result for the retailer, but there are several ways of obtaining it.

The profit derived from the game model indicates that it is possible for the retailer to do only slightly better than when offering the forward price plus a mark-up. Compared to the profit obtained from offering the flat price, however, the profits obtained from offering both the forward price plus ε_1 and the price profile from the game model are significantly higher. This indicates that having the opportunity to offer hourly differentiated rates, and thereby providing the consumer with incentives to shift load, makes the retailer better off.

The reason why we have chosen the solution presented here, is that it is "nice" in the sense that it is not very dissimilar to the forward price profile, and from a marketing point of view, such a profile could be easier to sell to the consumer. This is because the forward price profile (at least in the Nordic market) is quite similar from day to day. Hence, we find it likely that the consumer would feel more comfortable knowing that the price profile does not change significantly from one day to another. Since we have only focused on water heating and space

[4] $1 øre = 1/100 NOK$

heating, this argument does not make much sense in this model. This is because the smart house sees to that the space and water are heated at the lowest possible cost no matter how "wild" the price profile is. If all electricity-consuming processes were taken into account, however, there would be some things that must be done manually (cooking for example) and the consumer would perhaps like to carry out such processes due to daily routines and not due to price profiles that change every day. In other words, we find that this is a price profile that a retailer realistically could be considering.

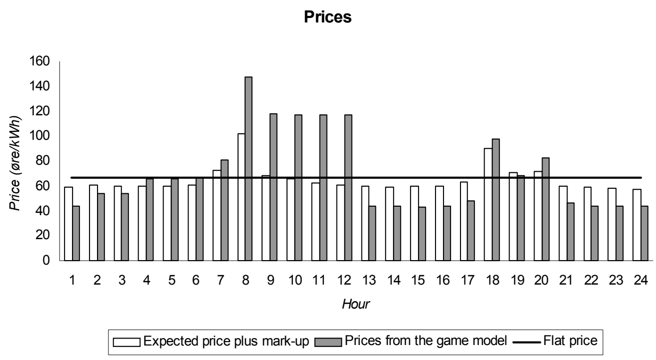

Figure 6: The columns show the prices from the game model (grey) compared to the expected price plus a mark-up ε_1. The line represents the volume weighted average price plus a mark-up ε_2. The latter is the price profile that is offered by the retailer if the consumer were not hourly metered and a flat price would have to be offered.

Figure 6 shows the prices from a solution to the game model together with the forward prices (plus the constant mark-up ε_1). We see that the prices from the game model in this solution have peaks in about the same periods as the forward price plus mark-up alternative. An interesting observation is that the game model gives substantially higher prices in hours $9 - 12$. During these hours, the consumers are away from home, and do not need to have much heat in the rooms. However, the consumer is forced to have the water boiler switched on during these hours to have the water re-heated after the morning bath, and

therefore, the retailer secures much of his income during these hours. Overall, however, the price profiles are not very different.
Figure 7 shows the load profile over the space heaters at the prices from Figure 6. The white columns represent the consumer's load profile at the forward price plus mark-up while the grey columns represent the profile at the prices from the game model. There are four columns for each hour, two for each price profile. This means that the consumer makes decisions for each half hour, and $\Delta t = \frac{1}{2} hours$. We see that in the game model the retailer has been able to move nearly all consumption for space heating out of hour 8 by offering a higher price in that hour. A similar effect is observed in hour 18. Apart from the mentioned differences early in the morning and late in the afternoon, the profiles are quite similar. This was expected due to the similarities of the price profiles.

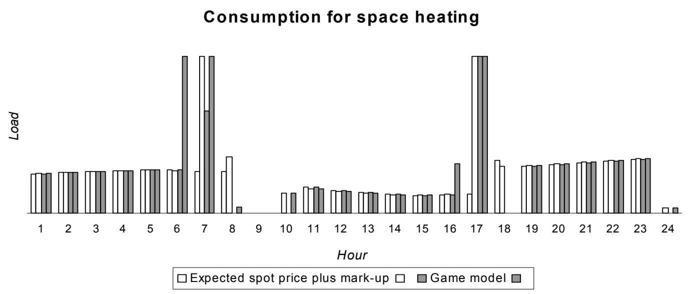

Figure 7: Load profile for space heating at the prices from the game model compared to the profile at the expected price plus a mark-up ε.

Figure 8 shows the load profile over the water heater. The profiles are almost identical, which is not surprising due to the similarity of the price profiles.

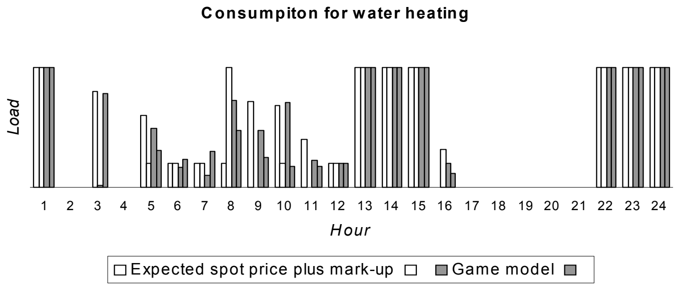

Figure 8: Load profile for water heating at the prices from the game model compared to the profile at the expected price plus a mark-up ε.

4 Conclusion

In this paper we developed a plausible model of domestic demand-side response to prices that vary over the day. The model shows how consumers could save money by utilizing the price profile. Real-time metering also gives the retailer the opportunity to influence the end users' load profile through the pricing of power. To study the interaction between a retailer and an end user we developed a Stackelberg game model. The model is a mathematical program with equilibrium constraints (MPEC) and such models are often difficult to solve. In this case, however, we have been able to obtain seemingly good solutions by using commercially available software. A trivial pricing policy for the retailer would be to offer the consumer the forward price plus a constant mark-up. The solutions indicate, however, that it is possible for a retailer to earn slightly more by using the opportunity to actively control the consumer's load profile.

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Article 2

An Electricity Market Game between Consumers, Retailers and Network Operators

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Abstract

We consider a simple game-theoretical model in which an electricity retailer and a network owner offer incentives to consumers to shift load from a peak period to an off-peak period. Using a simple example we compare the market outcomes from collusion with those from the equilibrium of a non-cooperative game, and examine the behaviour in this game when it is repeated in a situation in which agents have imperfect information.

Keywords: electricity distribution, markets, Nash equilibrium.

1 Introduction

Over the past couple of decades, most OECD countries have deregulated, or have started the process of deregulating, their electricity markets. Different countries have approached this deregulation in different ways (see [6]). Production of electricity is subject to competition in all deregulated markets, but there are differences as to how the wholesale markets and the end-user markets are organized.

The deregulation process in Norway started in 1990 and was initiated due to a desire to improve the efficiency and profitability of the electricity sector (see [4] and [15]). The formal legislation was effective as of January 1st, 1991, and from this date on, the wholesale market was fully liberalized. Also, end users were allowed to have their electricity delivered from any retailer, but the first years they were charged a fee for this, making it economically meaningful only to large end users. The end-user market was not fully liberalized until 1997, when all consumers were allowed to change retailer at no cost. We refer to [12] for a discussion of the development and effects of the rules and regulations that liberalized the Norwegian end-user market.

During the second half of the 1990's the markets in Sweden, Finland and Denmark were also deregulated and an integrated Nordic market emerged. These four countries now have a common wholesale market for electricity where any producer in any of the countries may deliver

electricity to the entire region. The market is built up around the electricity exchange Nord Pool, which provides a common spot market and a transparent exchange place for electricity derivatives.
In the Nordic market, production, transmission and retail of electricity have been split into three independent business areas. Briefly explained, the roles of these in the market are as follows:

- The *producers* produce electricity to be sold into the wholesale market.
- The *retailers* purchase electricity in the wholesale market, either through bilateral agreements with the producers or through Nord Pool. The purchased electricity is then sold to the end users. Hence, the retailers function as intermediaries between the wholesale market and the end users. Because of the large number of retailers, and partly also because of the invariable focus on the electricity prices from authorities and the news media, the competition among the retailers is fierce and most retailers experience very small profit margins.
- The *network operators* are responsible for transmitting the electricity from the production plants to the end users. The system operators are responsible for maintaining the main national grid that transmits electricity between regions. The lines that transport the electricity to the end users are the responsibility of local network operators. The local network operators have a monopoly on the transmission of electricity in their designated area and they are obliged to maintain a network that is capable of carrying the power needed at any time to all customers in their area at the same per kWh price for all customers. (In fact the network operators are allowed to charge different prices to different customer segments such as households, vacation homes, small businesses and large businesses, but the segments are defined by the authorities.) The network operators are financed through transmission fees paid by the end users, and to prevent them from enjoying monopoly profits, their profit margins are regulated by the Norwegian Water Resources and Energy Administration (NVE).

In the wholesale market, the prices of electricity vary from hour to hour. Usually the variations are more extreme in winter than in summer because in winter the water level in the hydro reservoirs may be low, while the cold weather puts transmission and production capacity under pressure in the peak periods. A high hourly price gives a signal that production capacity and/or transmission capacity are scarce resources at that time.

Some large consumers have their consumption metered by the hour and therefore have incentives to adjust their consumption to react to short-term price fluctuations in the market. The vast majority of end-users, however, do not have their consumption metered by the hour. Instead, they have their consumption metered four times a year, and they are billed based on their accumulated consumption over the last three months. This means that the incentive structure in the end-user market is directed only towards the consumer's total energy consumption. Hence, to attract customers from their competitors, retailers have focused on offering low energy prices. Since short term price fluctuations are not observed by a consumer (at least with current metering methods), the only way for her to reduce her electricity bill, besides changing to a cheaper retailer, is to reduce her total consumption of energy.

Since the retailers must purchase electricity at hourly varying prices in the wholesale market, and sell it to the consumers at flat prices, the current system of metering and billing introduces considerable risk to the retailers. This is because the consumers tend to use more electricity while the wholesale prices are high, but do not pay the market price for it. Customers with these consumption profiles would be expensive for a retailer. However, if the retailer were able to meter his customers' load by the hour, then he could provide incentives for them to shift load from peak to off-peak periods, and thereby make a profit from the ability to source cheaper power for his customers.

A similar opportunity exists for the network owner, even though his profit margins are regulated by the authorities. The regulatory regime was revised in 2001, and from 2002 the revenue cap RC for each local network operator is guided by the following formula:

$$RC = (OM + D + NL + RIC)\,(1 - EI) + NI. \tag{1}$$

Here
OM is operating and maintenance costs. The calculation of this parameter is based on the utilities' financial statements for the years 1996 through 1999.
D is depreciation.
NL denotes the network losses. The value of this parameter depends on the market price of power at the time the losses occur.
RIC is the expected return on invested capital as measured at the end of 1999.
EI is a rate denoting the efficiency improvement requirement. EI is set on an individual basis for each network operator and depends on how the network operator's efficiency increases compared to other network operators. This term provides an element of competition among the network operators.
NI is an adjustment parameter for new investments added to cover the need for new investments in the network. The calculation of this parameter is based on the nationwide increase in energy consumption and the number of new customers in the network operator's region.

The formula (1) is actually a simplification of the complete calculation of the revenue cap, which must satisfy other regulatory conditions (see [17], [18]). For example, the revenue cap is further constrained by a regulation stating that the arithmetic average of the return over a five year period must be between 2% and 20%.
For a network operator, a new investment will allow extra revenue from the terms RIC and D contributing to a relaxation of future revenue caps. Since this extra revenue is expected to accrue too late to fully compensate for the capital cost of the investment, an adjustment parameter NI is included in (1). The parameter NI is to provide the network operators with funds to undertake necessary grid expansions. A profound analysis of the adjustment parameter for new investments is given in [7]. (We also refer the reader to [3], which investigates possible peculiarities in the network operators' investment behaviour under the regulatory regime that applied from 1997 to 2001.)
For our purposes, it is important to note that network companies will benefit from the adjustment NI, regardless of whether investments are undertaken or not. The parameter NI is determined based on

nationwide and local increases in energy consumption. However the network operators are only obliged by law to carry any realistic load to the consumers in their region, where the needed network capacity required is determined by the maximum instantaneous load carried through the lines and transformers of the network. So consumers who can curtail their peak loads, or shift their loads from peak to off-peak periods, are desirable customers for network operators, who can benefit from NI, while avoiding or deferring the cost of investing in additional network capacity.

From this discussion we see that both the retailers and the network operators could benefit by encouraging consumers to shift load from peak to off-peak periods. One way of doing this may be to install equipment that allows retailers and/or network operators to physically cut load when there is danger of an extreme peak. The consumers should then get some sort of compensation for allowing this to happen. Another approach is to provide the consumers with economic incentives to cut load in peak periods. For example, this could be done by offering the customers hourly rates instead of flat rates, which would require that hourly metering instruments be installed. Customers would then react to the hourly differentiated rates by shifting load from peak to off-peak periods without the electricity utilities needing to physically cut off the supply.

The cost of hourly metering equipment is (currently) high in comparison to the potential economic savings. A cheaper approach is to install instruments that distinguish peak and off-peak consumption. With these, consumers can be offered two-part tariffs, normally a higher price in the peak period than in the off-peak period. See e.g. [1], [2], [5], [8], [10], [11], [13], [14], [16] and [19] for studies on consumer responsiveness to two-part tariffs.

We have seen that both retailers and network owners have incentives to offer consumers some sort of payment to make them shift load. The consumers, however, are only concerned about the total incentive payment that they receive, and they would not care about who provides the incentive. Therefore, if the retailer offers an incentive to a consumer, then there is an opportunity for the network operator to free ride on this. This will affect the incentive to be offered by the network operator. The question is then how large an incentive each of them

should offer to maximize their own profit.
In this paper we analyze a model in which a network owner and a retailer together offer a consumer an incentive to shift load out of a peak period and into an off-peak period. For simplicity, we analyze a simple case in which the day is divided into two periods. In Section 2 we give a formal description of the market participants by presenting our assumptions on the players' profit functions and the consumer's cost function. In Section 3 we compare the solutions to be found by the network owner and the retailer when offering independently with the solutions they obtain by colluding. In Section 4 we describe a Nash equilibrium for a single-period game based on the assumptions in Section 2, and we discuss the dynamic behaviour of this game if it is played repeatedly with incomplete information. In Section 5 we discuss strategic behaviour from the consumer's viewpoint.

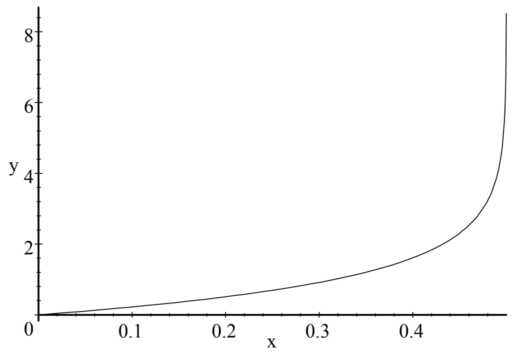
2 Description of the market participants

2.1 Consumer

The first type of participant in our model is the consumer, who is paid to shift load out of the peak period into the off-peak period. Suppose the consumer is offered an amount px to shift x units of demand out of the peak period. In the simplest model, the consumer has a cost function $f(x)$, for shifting load. We assume that f is a twice differentiable strictly convex function with $f(0) = 0$.
Example 1:
As an example, consider the following cost function

$$f(x) = \begin{cases} -\beta \log \frac{\alpha - x}{\alpha} & , \quad x \geq 0 \\ 0 & , \quad \text{otherwise} \end{cases} . \tag{2}$$

plotted below for $\alpha = 0.5$ and $\beta = 1$.



This cost function has some nice properties. First, $f'(x) > 0$ for positive x which makes sense since the consumer is likely to be more unhappy the more load that is shifted. Second, $f''(x) > 0$, indicating an increasing marginal cost. Third, the function has a vertical asymptote at $x = \alpha$. This asymptote makes sense because there will always be a limit to how much load that it is physically possible to shift.
Consider the problem faced by the consumer. To minimize the cost of shifting load, the consumer seeks to

$$\max_x \{px - f(x)\}.$$

Let the optimal solution to this problem be denoted $\bar{x}(p)$. Since $f''(x) > 0$, $\bar{x}(p)$ is the unique solution to

$$f'(x) = p. \tag{3}$$

This shows that the incentive $\bar{p}(x)$ required to induce an optimal shift in load of x is $f'(x)$. Since $\bar{p}'(x) = f''(x) > 0$, $\bar{p}(x)$ is strictly increasing in x.
At this point we shall make an additional assumption on the behaviour of the consumer. Although the price $\bar{p}(x)$ is increasing in x, there is a limit to how much load the consumer can shift, and it is clear that she will demand increasing incentives at the margin as this limit is approached. We assume therefore that $\bar{p}(x)$ is strictly convex. (When the third derivative exists, this amounts to the assumption that $f'''(x) > 0$.)

Differentiating (3) at the optimal solution gives

$$\bar{x}'(p) f''(\bar{x}(p)) = 1, \tag{4}$$

which shows that $\bar{x}(p)$ (the inverse of $p(x)$) is also a strictly increasing function of p.

Example 1 (continued)

For the cost function

$$f(x) = -\beta \log \frac{\alpha - x}{\alpha}$$

we obtain

$$\begin{aligned} f'(x) &= \frac{\beta}{\alpha - x}, \\ f''(x) &= \frac{\beta}{(\alpha - x)^2}, \\ f'''(x) &= \frac{2\beta}{(\alpha - x)^3}, \end{aligned}$$

and

$$\bar{x}(p) = \frac{-\beta + p\alpha}{p}.$$

Throughout this paper (apart from Section 5) we assume that the consumer reacts to the incentive offered by the other parties. In this sense she is a follower in a Stackelberg-type game.

2.2 Retailer

The second type of participant in the market is the retailer. He offers an incentive to the consumer to shift load. Shifts in load produce benefits for the retailer that can be modelled as a function $A(x)$ of the amount of load shifted. In our model we shall assume that A is (or may be approximated by) a concave increasing function with $A(0) = 0$. Suppose the incentive payment per unit of shifted load is s. Then the retailer seeks to

$$\max_s \{A(\bar{x}(s)) - s\bar{x}(s)\}.$$

This gives a first-order optimality condition of

$$\bar{x}'(s)A'(\bar{x}(s)) - s\bar{x}'(s) - \bar{x}(s) = 0.$$

Substituting for $\bar{x}'(s)$ using (4) we obtain

$$f''(\bar{x}(s))\bar{x}(s) = A'(\bar{x}(s)) - s. \tag{5}$$

As s varies the optimal response $\bar{x}(s)$ varies. Since $\bar{x}(s)$ is strictly increasing in s we may consider the optimal choice of s satisfying (5) as an equivalent choice of x satisfying

$$f''(x)x = A'(x) - s(x). \tag{6}$$

Since $s(x)$ is increasing and $A(x)$ is concave, the right-hand side of this equation is a decreasing function. Since the derivative of the left-hand side is

$$(f''(x)x)' = xf'''(x) + f''(x) > 0,$$

it follows that $f''(x)x$ is strictly increasing from 0 and so any solution to (6) will give a unique s, and therefore a unique $\bar{x}(s)$.
For simplicity we shall assume from now on that $A(x) = Ax$, yielding

$$f''(\bar{x}(s))\bar{x}(s) = A - s. \tag{7}$$

Observe that since $(A - s)\bar{x}(s) = 0$ at $s = 0$ and $(A - s)\bar{x}(s) \leq 0$ for $s \geq A$, we need only consider $s \in [0, A]$ in seeking an optimal value of s.

2.3 Network owner

Now consider a network owner offering an incentive to the consumer to shift load. Shifts in load produce benefits for the network owner that are assumed to be a concave function $B(x)$ of the amount of load shifted. (Observe that in reality B is likely to be a discontinuous function with jumps at points at which the network capacity needs to be expanded to meet load, but we choose to approximate this by a smooth function.) Suppose the incentive payment per unit of shifted load is t. Then the supplier seeks to

$$\max_t \{B(\bar{x}(t)) - t\bar{x}(t)\}.$$

The network owner has the same equations as the supplier, but the incentive offered by the network owner is t. This gives a first-order optimality condition of

$$\bar{x}'(t)B'(\bar{x}(t)) - t\bar{x}'(t) - \bar{x}(t) = 0.$$

Substituting for $\bar{x}'(t)$ we obtain

$$f''(\bar{x}(t))\bar{x}(t) = B'(\bar{x}(t)) - t.$$

For simplicity we shall assume from now on that $B(x) = Bx$, yielding

$$f''(\bar{x}(t))\bar{x}(t) = B - t. \tag{8}$$

Using the same argument of the previous section we may show that (8) has a unique solution t (with unique $\bar{x}(t)$).

3 Independence and Collusion

In this section we introduce the situation in which the retailer and network owner operate in isolation and compare this with the benefits to be obtained by colluding. We assume throughout that given the incentives offered by either the retailer or network owner, the consumer acts so as to minimize cost.

The retailer seeks to induce the consumer to shift load from the peak period. The optimal amount to offer is s satisfying

$$f''(\bar{x}(s))\bar{x}(s) = A - s.$$

Example 1: (continued)

As before let the cost function of the consumer be

$$f(x) = -\beta \log \frac{\alpha - x}{\alpha}.$$

Then at the optimal solution $\bar{x}(s)$, the marginal cost of the consumer equals the payment so

$$f'(x) = -\frac{\beta}{-\alpha + x} = s$$

giving
$$x = \frac{-\beta + s\alpha}{s}.$$
However if β is large and α and s are small then x will become negative. The optimal choice is
$$\bar{x}(s) = \max\{\frac{-\beta + s\alpha}{s}, 0\}. \tag{9}$$
Now we solve
$$f''(\bar{x}(s))\bar{x}(s) = A - s,$$
using
$$f''(x) = \frac{\beta}{(-\alpha + x)^2}.$$
Therefore
$$\frac{\beta}{(-\alpha + \bar{x}(s))^2}\bar{x}(s) = A - s.$$
Substituting (9) gives (assuming $s \geq \frac{\beta}{\alpha}$)
$$\frac{\beta}{\left(-\alpha + (\frac{-\beta+s\alpha}{s})\right)^2}\frac{-\beta + s\alpha}{s} = -s + \frac{1}{\beta}s^2\alpha,$$
so
$$\frac{1}{\beta}s^2\alpha = A,$$
giving
$$\bar{s} = \sqrt{\frac{A\beta}{\alpha}}.$$
Observe that $A \geq \frac{\beta}{\alpha}$ if and only if $\bar{s} \geq \frac{\beta}{\alpha}$. If $A \leq \frac{\beta}{\alpha}$then any incentive $s \in [0, A]$ will satisfy $s \leq \frac{\beta}{\alpha}$, and so $\bar{x}(s) = 0$. Therefore we can assume without loss of generality that $A \geq \frac{\beta}{\alpha}$. Under this assumption $\bar{s} \in [0, A]$ and
$$\bar{x}(s) = \frac{-\beta + \bar{s}\alpha}{\bar{s}}. \tag{10}$$
A similar analysis can be applied to the network owner. This yields
$$\bar{t} = \sqrt{\frac{B\beta}{\alpha}},$$

and assuming that $B \geq \frac{\beta}{\alpha}$,

$$\bar{x}(t) = \frac{-\beta + \bar{t}\alpha}{\bar{t}}.$$

To illustrate these formulae, consider an example where $A = 6$, $B = 7$, $\alpha = 0.5$ and $\beta = 1$. Acting independently, the retailer would offer $s = 3.464$, and the network owner would offer $t = 3.742$. This gives respectively

$$\bar{x}(s) = 0.2113$$

$$\bar{x}(t) = 0.2327$$

and respective profits for the retailer and network owner of 0.5359 and 0.7583.

Consider now the situation where the supplier and the network owner collude. They seek a joint incentive payment p that solves

$$\max_{p}\{A\bar{x}(p) + B\bar{x}(p) - p\bar{x}(p)\}.$$

Thus

$$p = \sqrt{\frac{(A+B)\beta}{\alpha}} = 5.0990,$$

using the choice $A = 6$, $B = 7$, $\alpha = 0.5$ and $\beta = 1$. This gives

$$\bar{x}(p) = \frac{-\beta + p\alpha}{p} = 0.3039,$$

and total profit equal to

$$A\bar{x}(p) + B\bar{x}(p) - p\bar{x}(p) = 2.401.$$

The joint incentive payment p can be divided by negotiation between the retailer and the network operator to give higher profits for each than those in the independent case. For example

$$s = 2.0495, \qquad t = 3.0495,$$

can be shown to give equal profits of 1.200 for the retailer and the network owner. The customer incurs a cost of

$$-\beta \log \frac{\alpha - \bar{x}(p)}{\alpha} = 0.936.$$

These values of s and t are both lower than what the retailer and network owner would offer in isolation, and yield higher profits.
Observe, however, that this situation does not represent a Nash equilibrium in the non-cooperative game played by the network operator and the retailer, because the network operator could reduce his incentive payment to $t = 2.2048$ and make more than he is currently making. Assuming that s remains at 2.0495, we get

$$\bar{x}(s+t) = \frac{-\beta + (s+t)\alpha}{(s+t)} = 0.2649,$$

so the network operator now makes a profit of $1.2704 > 1.200$.

4 Nash equilibrium

The analysis of the previous section leads us to a Nash equilibrium in the one-shot game in which the retailer and the network operator offer independently to induce a response from the consumer. The retailer offers the consumer an amount sx to shift x units of demand out of this period, and the network offers the consumer an amount tx to shift x units of demand out of this period. (The consumer will make $(s+t)x$ from this transaction.) The supplier then seeks to

$$\max_s \{A(\bar{x}(s+t)) - s\bar{x}(s+t)\},$$

and the network seeks to

$$\max_t \{B(\bar{x}(s+t)) - t\bar{x}(s+t)\}.$$

The first-order optimality conditions for the supplier (given a fixed offer of t from the network) are:

$$A\bar{x}'(s+t) - s\bar{x}'(s+t) - \bar{x}(s+t) = 0,$$

yielding

$$f''(\bar{x}(s+t))\bar{x}(s+t) = A - s.$$

Similarly, the first-order optimality conditions for the network (given a fixed offer of s from the supplier) are:

$$B\bar{x}'(s+t) - t\bar{x}'(s+t) - \bar{x}(s+t) = 0,$$

yielding

$$f''(\bar{x}(s+t))\bar{x}(s+t) = B - t.$$

Example 1 (continued):
We continue to use the consumer's cost function (2). This gives

$$-(s+t) + \frac{1}{\beta}(s+t)^2\alpha = A - s,$$

$$\frac{1}{\beta}(s+t)^2\alpha = A + t,$$

$$s = \sqrt{\frac{(A+t)\beta}{\alpha}} - t,$$

Similarly

$$t = \sqrt{\frac{(B+s)\beta}{\alpha}} - s.$$

Observe that since $s \in [0, A]$ and $t \in [0, B]$, the optimal solutions will in fact solve

$$\begin{aligned} s &= u(t) \\ t &= v(s) \end{aligned}$$

where

$$u(t) = \begin{cases} 0 & , \quad \sqrt{\frac{(A+t)\beta}{\alpha}} < t \\ \sqrt{\frac{(A+t)\beta}{\alpha}} - t & , \quad t \leq \sqrt{\frac{(A+t)\beta}{\alpha}} \leq A + t \\ A & , \quad \sqrt{\frac{(A+t)\beta}{\alpha}} > A + t \end{cases} , \tag{11}$$

$$v(s) = \begin{cases} 0 & , \quad \sqrt{\frac{(B+s)\beta}{\alpha}} < s \\ \sqrt{\frac{(B+s)\beta}{\alpha}} - s & , \quad s \leq \sqrt{\frac{(B+s)\beta}{\alpha}} \leq B + s \\ B & , \quad \sqrt{\frac{(B+s)\beta}{\alpha}} > B + s \end{cases} . \tag{12}$$

Let $X = [0, A] \times [0, B]$. If we consider the mapping $F : X \to X$ defined by

$$F((s,t)) = (u(t), v(s))$$

then it is clear that F is continuous and X is convex and compact. Therefore there always exists an equilibrium by Brouwer's fixed point theorem.
Suppose $A = 6$, $B = 7$, $\alpha = 0.5$ and $\beta = 1$. The players' response functions are shown in Figure 1. An equilibrium is given by $s_{eq} = 1.57, \quad t_{eq} = 2.57$.

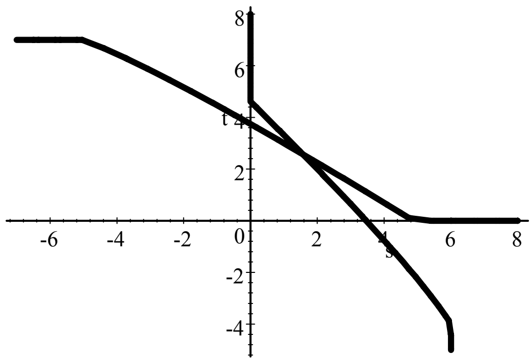


Figure 1: The players' response functions for $A = 6$, $B = 7$

Now

$$\bar{x}(s_{eq} + t_{eq}) = \frac{-\beta + (s_{eq} + t_{eq})\alpha}{(s_{eq} + t_{eq})} = 0.2585.$$

The supplier makes a profit of

$$A\bar{x}(s_{eq} + t_{eq}) - s_{eq}\bar{x}(s_{eq} + t_{eq}t) = 1.1449.$$

The network makes a profit of

$$B\bar{x}(s_{eq} + t_{eq}) - t_{eq}\bar{x}(s_{eq} + t_{eq}) = 1.1449.$$

The customer incurs a cost of

$$-\beta \log \frac{\alpha - \bar{x}(s_{eq} + t_{eq})}{\alpha} = 0.7277,$$

which is lower than the cost that they incur under collusion. The profits for the retailer and network owner are less than the profits (i.e.

1.2) to be made in collusion. A colluding retailer and network owner therefore find themselves in a Prisoner's Dilemma situation, in which it is profitable to renege from the collusion as long as the other player does not. The equilibrium strategy (when both players renege) has a poorer payout for both players.
As a second example of an equilibrium for the model we discuss, suppose $A = 30$, $B = 6$, $\alpha = 8$ and $\beta = 1$. The players' response functions are shown in Figure 2, where the equilibrium is $s_{eq} = 2, \quad t_{eq} = 0$.

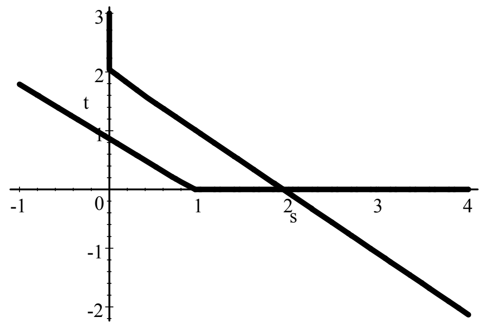


Figure 2: The players' response functions for $A = 6$, $B = 30$, $\alpha = 8$ and $\beta = 1$.

Figure 1 and Figure 2 illustrate some appealing features of our model that are true for all choices of parameters. First observe that $u(t)$ and $v(s)$ defined by (11) and (12) are both nonincreasing functions. This makes sense, as the optimal incentive to offer should not increase as the other agent offers more incentive. To show this, first observe that $u(t)$ is continuous, and is not constant only where $t \leq \sqrt{\frac{(A+t)\beta}{\alpha}} \leq A+t$. In this range

$$\sqrt{\frac{(A+t)\beta}{\alpha}} \leq A+t \Rightarrow \sqrt{\frac{\beta}{\alpha(A+t)}} \leq 1,$$

so

$$u'(t) = \sqrt{\frac{\beta}{4\alpha\,(A+t)}} - 1 < 0. \tag{13}$$

A similar argument shows that

$$v'(s) = \sqrt{\frac{\beta}{4\alpha\,(B+s)}} - 1 < 0, \tag{14}$$

if $s \leq \sqrt{\frac{(B+s)\beta}{\alpha}} \leq B+s$.

A second observation is that the players' response functions can intersect in at most one point. To see this consider the two curves plotted in each figure. The inverse of $u(t)$ (as plotted as a function of s) has two vertical sections and a downward sloping section. The slopes of these are all strictly more negative than the slope of $v(s)$ at the same s. This is because (11) and (12) imply

$$-1 < u'(t) < 0, \quad \text{and} \quad -1 < v'(s) < 0,$$

so

$$\left[\frac{du^{-1}}{ds}\right]_{s=u(t)} = \frac{1}{u'(t)} < v'(s).$$

It follows that the curves may intersect at most once.

The arguments above have shown the following proposition. For all strictly positive choices of A, B, α, and β there exists a unique Nash equilibrium in the noncooperative game played between the retailer and the network owner.

4.1 Repeated game

In this section we examine the case where the retailer and the network operator do not know anything about each other, but have perfect information about their own profit functions. The players alternate in offering incentives to the consumer and, at each offer, the offering agent chooses the incentive that maximizes his own profit given the incentive currently being offered by his opponent. Both players will continue offering as long as it is possible to make a decision that increases profit.

We assume that, at any stage of the game, neither player knows anything about the opponent's response function, apart from the opponent's most recent offer. This means that players do not use previous

plays of the game to infer their opponent's response function. They simply consider the total incentive currently offered to the consumer, and compute how they can change their contribution to this in order to maximize profit. They do not really understand that the opponent will respond by changing his incentive again.
In the Table 1 below we have presented some results from this process. We have assumed that the network owner offers first and that he also gains a slightly higher turnover per unit of load shifted. In this example $A = 6$, $B = 7$, $\alpha = 0.5$ and $\beta = 1$.

Period	s	t	*Retailer's profit*	*Network's profit*
1	0.000	3.742	1.396	0.758
2	0.672	3.742	1.457	0.891
3	0.672	3.245	1.304	0.919
4	1.055	3.245	1.322	1.004
5	1.055	2.959	1.240	1.014
6	1.274	2.959	1.246	1.066
⋮	⋮	⋮	⋮	⋮
29	1.569	2.571	1.145	1.145
30	1.570	2.571	1.145	1.145

Table 1: Results from alternating incentive offers

In period 1 the network owner makes his incentive decision, $t_1 = 3.742$, based on $s_1 = 0$. This gives the network owner a profit of 0.758, but observe that the retailer earns much more. This is so because the retailer gets the advantage of the load shifting, but does not pay anything for it. Now the game has started and, in period 2 the retailer realizes that he can make an even better profit by making a contribution to the incentive payment to the consumer. Therefore the retailer makes an incentive decision based on $t_2 = t_1 = 3.742$, giving $s_2 = 0.672$. Now, both players are actually better off than in period 1, but the network owner realizes that, given s_2, he would be even better off by offering $t_3 = 3.245$. This, however, makes the retailer even worse off than he has been in any earlier stage of the game. Therefore he offers

$s_4 = 1.055$ to catch up some of the lost profit. The game goes on and on like this (theoretically forever) until neither of the players can make a profit by changing his incentive offer.

We can make some interesting observations from the table. The first is that the results are affected by who offers first. In this example it is the network owner. If the retailer were to offer first then roles played by network and retailer in the following observations will be interchanged.

The second observation to make is that in this example the retailer's profit is decreasing towards equilibrium. His profit is, however, not monotonic decreasing. Each time the retailer plays, he will earn more in the following period, but when the network owner replies, the retailer will be worse off than he ever was. We see that the retailer, if he observed recent plays, would not continue play. Under these circumstances one might expect the network owner to realize that he would be better off if both players offered an incentive, and to progressively reduce his incentive down to a level (such as his equilibrium offer) that encourages the right response from the retailer.

Finally suppose we were to relax the assumption that neither player knows anything about the opponent's optimal response function, and provide the network operator with this knowledge. Therefore the network owner would prefer to offer $t_1^* = t_{eq} = 2.57$ in the first period. Under the assumptions of the game, the optimal response for the retailer will be $s^* = s_{eq} = 1.57$. Suppose now, however, that the retailer does not know (or is constrained from offering) his own optimal response. This would mean that the network owner might be offering t_1^* alone for some period of time, until the retailer responds. If there is no response, the network owner will prefer to offer t_1. His optimal offering strategy over time will depend on his model of the (possibly suboptimal) responses of the retailer.

5 Strategic consumer behaviour

In the discussion above we have assumed that the consumer's cost function $f(x)$ is known to the retailer and the network owner. Here we consider the situation in which this function is deduced by the retailer and network owner from observing behaviour of the consumer to the incentives offered by each. This raises the possibility of the consumer misrepresenting her true costs to these parties. For simplicity, we assume in this section that the retailer is the only agent offering incentives.

To model this suppose the consumer is represented by a single agent whose true cost function is

$$f(x) = -\beta \log \frac{\alpha - x}{\alpha},$$

where $\alpha = \beta = 0.5$. Suppose given an incentive s from the retailer, she were to shift load according to

$$g(x) = -\beta \log \frac{\alpha - x}{\alpha}.$$

where $\alpha = 0.5$ and $\beta = 1$. In other words she behaves as if her costs were twice as large.

Now we suppose that the retailer has experimented with different incentives to the extent that he has an accurate estimate of g, which he intends to use to compute an optimal incentive. As shown above, the optimal incentive given a response of g is

$$s = \sqrt{\frac{A\beta}{\alpha}} = 3.464,$$

assuming $A = 6$. The response of the consumer (still deceiving the retailer) is to shift a load of

$$x = \frac{-\beta + s\alpha}{s} = 0.2113.$$

Her true profit from this is

$$sx - f(x) = 0.457.$$

We compare this with the case where the consumer behaves according to her true cost function $f(x)$. Then $\alpha = \beta = 0.5$ gives

$$s = \sqrt{\frac{A\beta}{\alpha}} = 2.449$$

$$x = \frac{-\beta + s\alpha}{s} = 0.2959.$$

Her profit from this policy is

$$sx - f(x) = 0.277,$$

which is less than the profit she would make by misrepresenting her costs.

The conclusion here is that it is more profitable for the consumer to behave as though her costs were twice as high as they really were, since this induces the retailer to offer more inducement for her to shift load. So even if she does not optimize her profit (to maintain the deception) she is better off than if she were to optimize and reveal (over time) her true costs to the retailer.

As observed above, the retailer under this model will not offer the consumer any more than $s = A$, since this is the benefit he gets from a unit of load shifted. Thus there are bounds on how much profit the consumer can expect to extract from the retailer by inflating her true costs. For example suppose she were to choose to misrepresent her costs as

$$h(x) = -\beta \log \frac{\alpha - x}{\alpha}.$$

where $\alpha = 0.5$ and $\beta = 3$. Then the optimal offer from the retailer is

$$s = 6,$$

and the response of the consumer to this offer (while pretending to incur h) is to shift load of $x = 0$. The profit from this policy is clearly 0.

It is clear from the above discussion that for this model there is an optimal choice of β and α that will yield the best outcome for the consumer if they pretend to have cost function using that α and β. A more difficult question, that we leave unresolved, is the determination of an inflated cost function that produces the best response from the retailer.

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Article 3

An Electricity Market Game between Retailers and Network Operators - Multiple Load Periods

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Abstract

To enhance short-term flexibility in the demand side of the Nordic electricity market, network owners and retailers may provide consumers with incentives to curtail peak loads. The incentive given by the retailer will affect the optimal incentive for the network owner, and vice versa, since the consumers' reaction depends on the sum of the incentives. This strategic interaction between the retailer and the network owner is analyzed using a game-theoretical framework. The market outcomes from collusion are compared with those from the equilibrium of a non-cooperative game, and we examine the agents' behaviour when the game is repeated in a situation in which they have imperfect information. The game model is solved for three load periods, and the results are compared with those obtained for two load periods in an earlier paper.

1 Introduction

As part of the Energy Act of 1990, the Norwegian end-user market for electricity was liberalized, and all end-users were allowed to have their electricity delivered by any retailer. During the first years after the deregulation, however, they were charged a fee for changing retailer, making it economically meaningful only for large consumers. Also, each retailer had to pay a fee to the local network operator in each area to which it delivered power, which made it less attractive for retailers to compete for customers away from their home market. The fees paid by the consumer and the retailer were to cover the administrative costs of the network owner, who had to deal with several retailers instead of just one like earlier. From 1997 new legislation made it possible for all end-users to change retailer at no cost. The administrative costs were from then on covered by the consumers' local network owners. This gave a slight increase in the network fees, but competition between the retailers was enhanced to the benefit of the end-users.

Distribution and retail of electricity have (by legislation) been split into two independent business areas. The end-users purchase energy from specialized retailers, while the distribution of electricity to the end-users is carried out by local network operators.

The retailers purchase electricity in the wholesale market and sell it to the end-users. Most retailers offer several different contracts to their customers, who choose the contract that best suits their risk preferences. Hence, the retailers function as intermediaries between the wholesale market and the consumers, helping end-users to handle the risk in the wholesale market, which they often do not have the competence or inclination to handle themselves.

While the retailers are subject to competition, the network operators have a monopoly on the distribution of electricity to their designated areas. The network owners' revenues stem from the transmission fees that are paid by the end-users, and regulations have been passed by the authorities to prevent the network owners from enjoing monopoly profits. An overview of these regulations is found in [2] and the references therein.

The end-users' consumption meters are normally read every two or three months to find the accumulated consumption over the period. The invoice for this consumption consists of two parts: The first is the transmission fee to the local network operator, and the second part of the invoice is the electricity payment, which is paid to the retailer according to whatever contract the customer has entered into with the retailer. The values obtained only show the total amount of energy consumed over the period, but they do not give any idea of the consumers' load profile within the period. This means that the incentive structure in the end-user market is directed only towards the consumers' total energy consumption.

Both the retailer and the network operator may see some interest in providing incentives directed towards the consumers' load profiles as well. For instance, the network owners may save future investments by having peak loads curtailed. Since the prices in the wholesale market vary over the day, the retailer may make extra profits by incentivising his customers to use more load while wholesale prices are low and less while they are high. More on this is found in [2]. In Norway, electricity is the main energy source for all energy consuming tasks carried out in the homes, including space heating, water heating and cooking. Hence, there should be a potential to obtain significant savings by

incentivising consumers to alter their load profiles.
Offering such incentives is not meaningful the way consumption is metered today, since they would have no way of knowing whether the consumers have actually reacted to the incentives. To make it worthwhile, the metering devices must be able to capture the changes in the end-users' load profiles over the day. In the Nordic market the wholesale prices vary by the hour, and hence, hourly meters would make these price signals visible to consumers. Today, consumers with an annual consumption of more than 400 000kWh are required to be hourly metered[1], while smaller end-users may choose to be hourly metered. The cost of hourly metering equipment is (currently) high in comparison to the potential economic savings, however, and therefore, rather few small end-users find it worthwhile. The share of consumers with hourly meters is likely to increase as the necessary equipment gets cheaper, which may make it increasingly interesting for retailers and network owners to incentivise the end users to alter their load profiles.
Let us consider two firms, a retailer and a network operator, offering the same consumer some price profile that varies over the day. The consumer makes her consumption decision based on total price, that is the sum of the prices offered by the retailer and the network owner in the different periods. If the retailer wants his customers to reduce their load by a certain amount during a time period, he would increase the price in that period. However, the network owner may also wish to increase the price in the same period, and if the retailer does not take this into account when setting his price, he will find the load in that period being reduced too much. On the other hand, if the network owner does not take the retailer's price increase into account, he will offer too high an incentive to achieve the result *he* wants.
This leads to a game between the network operator and the retailer where they must take the opponent's profit function into account when making the pricing decisions. It is worth noting that the players[2] do not have any interest in squeezing the opponent per se. The retailer

[1] The limit is about to be decreased to $100,000kWh$.

[2] Throughout this paper, the term "players" is taken to mean the retailer and the network owner. The consumer does not interact with anyone, and is regarded as a passive participant in this game.

does not care about the network owner's profit since they are not competitors within the same market. The retailer competes with other retailers, while the network owner is a regulated monopoly. A player will squeeze his opponent if he considers it profitable to himself (and he is allowed to by the autorities), but he will not do this because he has any incentives to hurt him.

In [2] a game of this type is considered. The day is divided into a peak period and an off-peak period, and the players decide on incentive payments to offer a consumer to have her shift load from the peak period to the off-peak period. The player standing to gain most from having load shifted will also offer the highest incentive.

If hourly metering is introduced, the day would be divided into 24 pricing periods. The network owner will primarily be interested in having load moved out of the peak load period. The retailer, however, would be interested in having load moved out of other periods as well. He may benefit from having load moved from any period to a period with a lower price, if such exists. This gives an asymmetry that is not captured in the two-period version, and one of the main purposes of this paper is to study this asymmetry.

We have divided the day into three periods, and this is enough to discuss the assymmetry in principle. The network owner offers an incentive to have the consumer shift load out of the peak period. The retailer offers two incentives: one to have load shifted out of the peak period and one to have consumption moved from the medium load period to the low load period.

In Section 2 we give a formal description of the market participants by presenting the players' profit functions and the consumer's cost function. In Section 3 we give a qualitative discussion of our findings, while the quantitative results are presented in Section 4. Section 5 provides a brief conclusion.

2 Model specification

Assume that we have three periods. Period 1 is the peak load period, Period 2 is the medium-load period and Period 3 is the low-load period. The network owner would like to have load shifted out of Period 1, but sees no reason to provide any incentive to make the consumer shift load out of Period 2. The retailer would like to have load shifted out of both Period 1 and Period 2.

2.1 Consumer

Before the game begins, the consumer has adjusted her consumption such that she may not shift any load from one period to another without losing some utility. Thus, her marginal utility of electricity consumption in each period are equal. This means that if the players want her to shift load, they will have to incentivise her to do so.
The consumer could shift load between any of the periods, but the players will only incentivise downwards shifts, that is, shifts from a period with high load to a period with lower load. Therefore, without loss of generality, we may assume that the consumer will have only three possibilities for shifting load: from Period 1 to Period 2, from Period 1 to Period 3 and from Period 2 to Period 3.
Shifting load involves a cost for the consumer. This cost comes from two sources. First, if load is shifted by preheating of rooms or water, more energy will be used than if this heating is carried out just in time. (As modelled in e.g. [3]). Then, the consumer must pay for the extra energy used. Alternatively, she could switch to a different energy source for heating. Secondly, shifting load by altering daily habits, such as doing the laundry or cooking meals at different hours than she normally does, is inconvenient for the consumer. This inconvenience has economic value to her. She experiences a disutility, and she must be paid to outweigh this disutility. In this model, we primarily focus on the load dimension, but the framework could be altered to take energy into account.
We represent the consumer's inconvenience by assigning a disutility function to each possible shift between two load periods. Let x_{ij}, $i = 1, 2$, $j = 2, 3$, $j > i$, be the amount of load shifted from period i to period j. We then denote the disutility functions $f_{ij}(x_{ij})$, $i = 1, 2$, $j = 2, 3$, $j > i$. The disutility functions are assumed to be separable, implying that the total daily disutility of load shifting is found by summing the disutilities from the three possible load shifts, $f_{12}(x_{12})$, $f_{13}(x_{13})$ and $f_{23}(x_{23})$. This means that we implicitly assume that each

load shift has a natural pair of periods. In Period 1, for example, the consumer will have identified one set of processes that may be moved to Period 2 and one (different) set that may be moved to Period 3. Within those sets she would have ordered the possible shifts by their cost, and when offered an incentive to move load out of Period 1, she will move the cheapest ones first.
Furthermore, she is offered an incentive q_i, $i = 1, 2$ from the players for moving load out of period i to a period with lower load. Let x_i be the total load shifted out of period i such that

$$\begin{aligned} x_1 &= x_{12} + x_{13} \\ x_2 &= x_{23} - x_{12}. \end{aligned}$$

Let Π_C denote the consumer's net profit from her actions. Then the consumer faces the following problem:

$$\max \Pi_C = \left\{ \sum_{i=1}^{2} \sum_{\substack{j=2 \\ j>i}}^{3} \left(q_i x_i - f_{ij} \left(x_{ij} \right) \right) \right\}.$$

Written out, this becomes

$$\begin{aligned} \max_{x_{12}, x_{13}, x_{23}} \Pi_C = \{ & q_1 \left(x_{12} + x_{13} \right) + q_2 \left(x_{23} - x_{12} \right) \\ & - f_{12} \left(x_{12} \right) - f_{13} \left(x_{13} \right) - f_{23} \left(x_{23} \right) \}. \end{aligned}$$

Observe that x_2 will be negative if $x_{12} > x_{23}$. This means that the consumer could actually be punished in Period 2 if she chooses to have a net load increase in that period. This happens because the load shifts are valued differently by the consumer and the players. The players observe the consumption of a homogeneous product (electricity) being moved between the periods. For them, it does not matter if the (possible) change in load in Period 2 stems from load being shifted from Period 1 to Period 2 or load being shifted from Period 2 to Period 3 or from both. They only observe that there has been a change. For the consumer, however, electricity is used for producing several heterogeneous services. Her disutility from load shifting does not only depend on the net change of load in a period, but also which processes are moved where.

By differentiating the consumer's profit function, we get the following first order conditions for the consumer

$$\frac{\partial \Pi_C}{\partial x_{12}} = q_1 - q_2 - \frac{df_{12}}{dx_{12}} = 0 \tag{1a}$$

$$\frac{\partial \Pi_c}{\partial x_{13}} = q_1 - \frac{df_{13}}{dx_{13}} = 0 \tag{1b}$$

$$\frac{\partial \Pi_C}{\partial x_{23}} = q_2 - \frac{df_{23}}{dx_{23}} = 0. \tag{1c}$$

Assume that the cost functions are on the following form:

$$f_{ij}(x_{ij}) = -\beta_{ij} \log \frac{\alpha_{ij} - x_{ij}}{\alpha_{ij}}$$

plotted in Figure 1 for $\alpha_{ij} = 10$ and $\beta_{ij} = 20$.

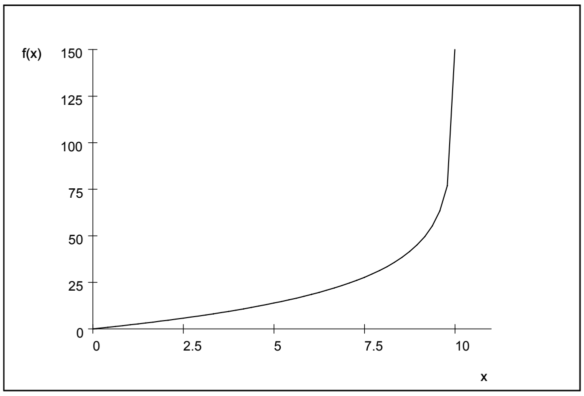


Figure 1: The consumer's disutility plotted for $\alpha_{ij} = 10$ and $\beta_{ij} = 20$.

This cost function has some nice properties. First, $f'(x) > 0$ for positive x, reflecting that the consumer be more unhappy the more load that is shifted. Second, $f''(x) > 0$, indicating an increasing marginal disutility. Third, the function has a vertical asymptote at $x = \alpha$. This asymptote makes sense because there will always be a limit to how much load that is physically possible to shift.

In addition to serving as a vertical asymptote, α_{ij} also influences the curvature of the disutility function. For convenience we now ignore the subscripts ij and let subscripts denote differentiators, i. e. $f_\alpha(x)$ denotes $f(x)$ differentiated wrt. α. Observe the following[3]:

$f_\alpha(x) = \frac{\beta x}{\alpha(x-\alpha)} < 0 \quad \forall \quad 0 < x < \alpha,$

implying that the higher the value of α, the lower the disutility of moving a given amount of load x.

$f_{x\alpha}(x) = \frac{-\beta}{(\alpha-x)^2} < 0 \quad \forall \quad 0 < x < \alpha,$

hence, the consumer's marginal disutility of moving consumption decreases with increasing values of α.

The parameter β_{ij} also affects the curvature of the consumer's disutility function. Again we ignore the subscripts ij and observe the following:

$f_\beta(x) = -\ln\frac{\alpha-x}{\alpha} > 0 \quad \forall \quad 0 < x < \alpha.$

We see that the disutility of moving a given amount of load x increases with increasing β.

$f_{x\beta}(x) = \frac{1}{\alpha-x} > 0 \quad \forall \quad 0 < x < \alpha,$

thus, the consumer's marginal disutility of moving consumption increases with increasing values of β.

As we will discuss later, we have not been able to find an analytical solution to the game model, and all the results presented in Section **??** have been obtained numerically. It could be that choosing a different disutility function would enable us to derive closed form solutions. However, an important goal of this paper is to evaluate the effects of the assymmetry of this three-period setup as compared to the symmetric two-period game in [2]. Therefore, we have chosen a disutility function similar to that in [2].

This gives us the following disutility functions for each possible load shift:

$$
\begin{aligned}
f_{12}(x_{12}) &= -\beta_{12}\log\frac{\alpha_{12}-x_{12}}{\alpha_{12}} \\
f_{13}(r_{13}) &= -\beta_{13}\log\frac{\alpha_{13}-x_{13}}{\alpha_{13}} \\
f_{23}(r_{23}) &= -\beta_{23}\log\frac{\alpha_{23}-x_{23}}{\alpha_{23}}
\end{aligned}
$$

[3] $x > 0$ by definition and there are no real solutions for $x > \alpha$.

Differentiating the disutility functions gives:

$$\frac{df_{ij}}{dx_{ij}} = \frac{\beta_{ij}}{\alpha_{ij} - x_{ij}}; i = 1,2; j = 2,3; j > i \tag{2}$$

Now, putting (2) into (1) and solving for x_{12}, x_{13} and x_{23} gives the following response functions for the consumer

$$x^*_{12}(q_1) = \frac{\alpha_{12}(q_1 - q_2) - \beta_{12}}{q_1 - q_2} \tag{3a}$$

$$x^*_{13}(q_2) = \frac{q_1\alpha_{13} - \beta_{13}}{q_1} \tag{3b}$$

$$x^*_{23}(q_3) = \frac{q_2\alpha_{23} - \beta_{23}}{q_2} \tag{3c}$$

The equations (3) represent the consumer's best response given incentive payments q_1 and q_2.

2.2 Retailer and network owner

First, we assume that the consumer's load shifting does not change the players' ranking of the periods with respect to which one is the peak period and which one is the low-peak period. This is an important assumption to justify that the network owner is providing incentives in just one period and not in all periods. If this was not the case, the network owner could be wary about the possibility that Period 2 (or even Period 3) could become the new peak period and therefore be willing to provide incentives in Period 2 as well. This assumption makes sense since the players' idea of peak period depends on the consumption patterns of all their customers, and not only on a single household. For the consumer, however, the ranking of the periods may be altered by the load shifting, but that does not matter.
The retailer offers incentives s_i, $i = 1,2$, to make the consumer shift load out of period i, while the network owner offers an incentive t to make the consumer shift load out of period 1. Hence, $q_1 = s_1 + t$ and $q_2 = s_2$.

Now, the equations (3) become

$$x_{12}^*(q_1) = x_{12}^*(s_1,t) = \frac{\alpha_{12}(s_1+t-s_2)-\beta_{12}}{(s_1+t-s_2)} \tag{4a}$$

$$x_{13}^*(q_2) = x_{13}^*(s_1,t) = \frac{\alpha_{13}(s_1+t)-\beta_{13}}{(s_1+t)} \tag{4b}$$

$$x_{23}^*(q_3) = x_{23}^*(s_2) = \frac{\alpha_{23}s_2-\beta_{23}}{s_2}. \tag{4c}$$

At a later stage, we will need the partial derivatives of the expressions above. They are

$$\frac{\partial x_{12}^*}{\partial s_1} = \frac{\partial x_{12}^*}{\partial t} = \frac{\beta_{12}}{(s_1-s_2+t)^2} \tag{5a}$$

$$\frac{\partial x_{12}^*}{\partial s_2} = \frac{-\beta_{12}}{(s_1-s_2+t)^2} \tag{5b}$$

$$\frac{\partial x_{13}^*}{\partial s_1} = \frac{\partial x_{13}^*}{\partial t} = \frac{\beta_{13}}{(s_1+t)^2} \tag{5c}$$

$$\frac{dx_{23}^*}{ds_2} = \frac{\beta_{23}}{s_2^2}. \tag{5d}$$

We are now ready to present the equations for the retailer and the network owner.

Retailer

Suppose that each unit of load shifted out of period i, $i=1,2$, saves A_i for the retailer. Then he seeks to

$$\max \Pi_s = \max_{s_i} \left\{ \sum_{i=1,2} \sum_{\substack{j>i \\ j\le 3}} x_{ij}^*(A_i - A_j) - x_{ij}^*(s_i - s_j) \right\}.$$

Written out, this becomes

$$\max_{s_1,s_2} \left\{ A_1(x_{12}^* + x_{13}^*) + A_2(x_{23}^* - x_{12}^*) - s_1(x_{12}^* + x_{13}^*) - s_2(x_{23}^* - x_{12}^*) \right\}.$$

This gives the following first order conditions for the retailer

$$\begin{aligned}\frac{\partial \Pi_s}{\partial s_1} &= (A_1 - A_2 - s_1 + s_2)\frac{\partial x_{12}^*}{\partial s_1} + (A_1 - s_1)\frac{\partial x_{13}^*}{\partial s_1} - x_{12}^* - x_{13}^* = 0 \\ \frac{\partial \Pi_s}{\partial s_2} &= (A_1 - A_2 - s_1 + s_2)\frac{\partial x_{12}^*}{\partial s_2} + (A_2 - s_2)\frac{dx_{23}^*}{ds_2} - x_{23}^* + x_{12}^* = 0.\end{aligned}$$

Network owner

Suppose now that each unit of load shifted out of period 1 saves B for the network owner. Then he seeks to

$$\max \Pi_t = \max_t \left\{ \sum_{j=2}^{3} Bx_{1j}^* - tx_{1j}^* \right\}.$$

Written out, this becomes

$$\max_t \{B(x_{12}^* + x_{13}^*) - t(x_{12}^* + x_{13}^*)\},$$

which gives the following first order condition for the network owner

$$\frac{\partial \Pi_t}{\partial t} = (B - t)\frac{\partial x_{12}^*}{\partial t} + (B - t)\frac{\partial x_{13}^*}{\partial t} - x_{12}^* - x_{13}^* = 0.$$

2.3 Finding a market equilibrium

The first order conditions for the retailer and the network owner form the following system of equations.

$$\begin{aligned}(A_1 - A_2 - s_1 + s_2)\frac{\partial x_{12}^*}{\partial s_1} + (A_1 - s_1)\frac{\partial x_{13}^*}{\partial s_1} - x_{12}^* - x_{13}^* &= 0 \quad (7a) \\ (A_1 - A_2 - s_1 + s_2)\frac{\partial x_{12}^*}{\partial s_2} + (A_2 - s_2)\frac{dx_{23}^*}{ds_2} - x_{23}^* + x_{12}^* &= 0 \quad (7b) \\ (B - t)\frac{\partial x_{12}^*}{\partial t} + (B - t)\frac{\partial x_{13}^*}{\partial t} - x_{12}^* - x_{13}^* &= 0 \quad (7c)\end{aligned}$$

Our task is to find a set of solutions, (s_1, s_2, t), that forms a Nash equilibrium. To find an analytical solution to this problem, one could replace the derivatives in the above system of equations with the equations (5a)-(5d) and solve the system with respect to s_1, s_2 and t. When

the equations are written out, however, we get highly nontrivial fourth order polynomials, and we have not been able to find an analytical solution to the problem with the software that we have available. Instead, we have analyzed the problem numerically.
The system of equations has been solved using AMPL [1], and the numerical solutions found indicate that there is only one non-zero solution to the model.

3 Discussion

In this section we will discuss some interesting qualitative aspects of the results we have obtained by solving the model. The quantitative results are presented in the next section.
In this asymmetric game the players' incentives are seemingly quite similar. Both players will benefit from load shifted out of the peak period, and the only difference between them is that the retailer may achieve some additional revenue from another load shift. Now, if the players see the same value in having load shifted out of the peak period, the intuitive result would be that they share the incentive payment equally in this period. The retailer's revenue from the load shift out of Period 2 would simply be a nice addition to him. This is, however, not the case. Even though the retailer sees no lower benefit than the network owner from load shifted out of Period 1, the retailer offers a significantly lower incentive in this period. The retailer knows that his opponent's only chance to produce a profit is to have load shifted out of the peak period. The retailer himself, however, is more flexible. Since full information is assumed, he knows what the network owner will do, and he knows that the network owner knows what he will do, and so on. He may use this knowledge to calculate his own optimal trade off between his two incentive payments. This is utilized by the retailer to squeeze the network owner, making the latter carry most of the burden in the peak period. If the network owner increases his incentive payment, the retailer reduces his payment with a somewhat smaller amount, keeps his incentive payment in Period 2 almost as it is, and still achieves an increased profit.
The retailer's ability to squeeze the network operator depends on the

difference in the wholesale price between the two highest peak periods. If the difference is small, the retailer's gain from having load shifted out of Period 2 is close to the gain from having load shifted out of Period 1. In such a situation, the retailer will concentrate his efforts on Period 2, forcing the network operator to increase his peak period incentive payment. When the spot prices in the two periods get very close, the retailer will even offer a negative incentive payment in the peak period. A negative incentive would imply that the consumer actually pays money to the retailer for shifting load. The consumer may consider this a bit weird. By doing some calculations, however, she will find that due to the incentive payment offered in the medium load period, she is still better off than she would have been if the retailer did not offer anything.

In this situation, the retailer's motivation for offering a negative incentive is not that he wants the consumer to stop shifting load out of Period 1. Consumption in Period 2 is still preferable to consumption in Period 1 for the retailer, and therefore the negative incentive payments is a result of the retailer being able to squeeze the network owner.

If the wholesale price in Period 2 decreases, however, the retailer will have to rely more and more on load shifted out of Period 1. The result is that the incentive payments in Period 1 even out. This effect is to be expected, since as the market price in Period 2 approaches zero, the game approaches the two period setting. In our main numerical example, discussed in the next section, the retailer sees a slightly higher gain from load reductions in the peak period than the network owner does. This means that as the game approaches the two period setting, the retailer will, at some point, start to offer a higher incentive payment than the network owner.

As the network owner's benefit from load shifts out of the peak period gets small compared to the retailer's benefit, the network owner may be the one to offer a negative incentive. Though this would be profit maximizing behaviour, it is likely that the autorities would regard this as an unacceptable exploitation of monopoly power. Negative incentive "payments" imply that the consumer pays the network operator to shift load, and in this situation, the consumer would clearly be better off if a zero incentive payment was offered. On the contrary, the authorities would probably not interfere if a retailer offers negative incentives. The retailers, being subject to competition, should be allowed to price the electricity the way they want, and lose customers that are unhappy with the offered price profile.

The retailer's ability to squeeze the network operator also depends on the consumer's willingness to shift load from the mid-peak period to the low peak period. As this willingness gets lower, the retailer must increase his incentive payment in Period 2 to try and keep up his advantage. The increased incentive payments combined with lower load shifts decreases the retailer's profit from this period. Also, the increased incentive payment in Period 2 makes load shifts from Period 1 to Period 2 less profitable for the consumer, and this amount will decline if the incentives in Period 1 are left unchanged. As the profit from the mid-low shift has declined, for the retailer, he will try to outweigh this by making more money from shifts out of the peak period. To achieve this, he increases the incentive payments in Period 1 in an attempt to keep up the load shifts from Period 1 to Period 2, and to increase the amount of load shifted from Period 1 to Period 3. Hence, the retailer's incentive payments in both periods increase substantially, while his profit decreases. The network owner, on his hand, only cares about the load shifted out of the peak period, and therefore he is not concerned with the customer being less willing to shift load out of the mid-peak. He dislikes, of course, that the retailer's increased incentive in Period 2 induces less load to be shifted out of Period 1, but he also sees that the retailer must increase his incentive in Period 1, which also benefits the network owner. The network owner's incentive payment increases only marginally, and his profit stays rather stable. This discussion shows that if the consumer is not very willing to shift load between the mid-price and low-price periods, much of the retailer's advantage, and therefore his ability to squeeze the opponent, disappears.

We have assumed that a single-period game of full information is being played. This is, however, a simplification. In the real world, the game would probably be dynamic, and the players would have to learn both their own, the consumer's and the opponent's profit function before the equilibrium is reached. The retailer would need to consider the behaviour of other retailers as well. It might be that they do not even start playing at the same time. One of the players may identify the opportunity to earn some extra profit by utilizing the consumer's flexibility and start offering incentives on his own. Then, after some time, the opponent will realize that he also benefits from this and will join the game. This could lead to a repeated game similar to the one described in Section 4.3.

4 Solutions and parametric analysis

Using the parameter set $A_1 = 22$, $A_2 = 10$, $B = 20$, $\alpha_{12} = 10$, $\alpha_{13} = 10$, $\alpha_{23} = 18$, $\beta_{12} = 20$, $\beta_{13} = 20$ and $\beta_{23} = 15$ we get the solution that will serve as the basis for our analysis. The values of the parameters have been chosen partly by trial and failure, testing different parameter sets to find one that gives sensible solutions, and partly by requiring that the parameter values themselves should be possible to relate, at least vaguely, to a real-world situation. As for the latter point, the interpretation of A_1, A_2 and B is quite straightforward. A_1 and A_2 represent the retailer's revenue from being able to source cheaper power to his consumers, which means that these parameters represent the difference in spot price between the periods. Here, the spot price in Period 1 is $0,22NOK/kWh$ higher than in Period 3, while the difference between Period 2 and Period 3 is $0,10NOK/kWh$. These values adequately reflect the intra-day spot price variations in the Nordic marked on a winter day. The value for B is a rough estimate of the network owner's long term marginal cost of grid expansions measured in NOK/kWh. Clearly, the network owner's long term marginal cost is load dependent, and not energy dependent. However, his motive is to decrease the overall peak load and not only the peak load at this single consumer. To achieve this, he should offer an incentive based on energy, not on load. If there are many customers, $1kWh$ moved by a single customer will on average give a peak load reduction of $1/T$ $[kW]$ in that period, if T is the number of hours within the period. Therefore, the parameter B is measured in NOK/kWh.
The values of α_{ij} and β_{ij} are parameters in the consumer's disutility function, and they are more difficult to interpret directly. The value

of α_{ij} gives the vertical asymptote of the utility function, which means that the consumer will never, no matter what incentive provided, move more than α_{ij} units of load from period i to period j. At some point, it will become practically impossible for the consumer to move more load between any two periods, and α_{ij} reflects this. This is not, however, the theoretical maximum amount of energy consumption moved between the periods. The theoretical maximum would be the total amount of energy used within period i. From Period 1, for example, $\alpha_{12} = 10$ is not the total energy consumption in Period 1 since the sum of consumption moved from Period 1 to Period 2 and Period 3, $x_{12} + x_{13}$, may very well be higher than α_{12}.
All this tells us that finding reasonable values of α and β is more or less a matter of trial and error. However, α_{ij} should have a reasonable size to make it possible for the consumer to move a significant amount of load between the periods. With the chosen parameter values, the maximum possible total amount of consumption moved out of Period 1 is $\alpha_{12} + \alpha_{13} = 20kWh$ per day and for Period 2 this amount is $\alpha_{23} = 18kWh$ per day.

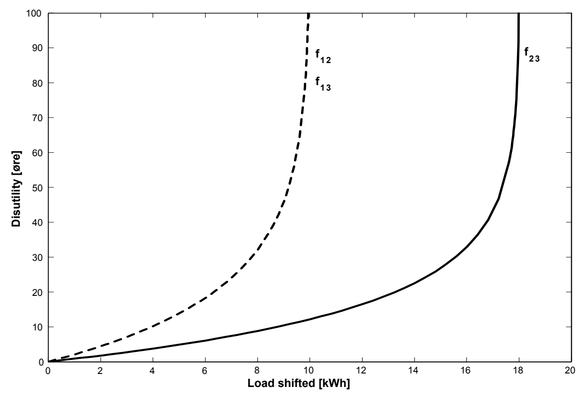


Figure 2: The consumer's disutility functions. The dashed line shows f_{12} and f_{13}, which are equal. The solid line shows f_{23}.

Figure 2 shows the consumer's disutility functions with the given parameters. The dashed line shows both f_{12} and f_{13}, that is, the disutilities of moving consumption out of Period 1. The solid line shows f_{23}, the consumers disutility from moving load from Period 2 to Period 3.

Retailer's incentive 1	s_1	$2.59øre/kWh$
Retailer's incentive 2	s_2	$2.75øre/kWh$
Network's incentive	t	$5.62øre/kWh$
Retailer's profit	Π_s	$3.15NOK$
Network's profit	Π_t	$2.00NOK$
Consumer's profit	Π_c	$0.65NOK$
Money paid to the consumer	$\Pi_c + \sum_{ij} f_{ij}$	$1.31NOK$
Consumption moved $1 \to 2$	x_{12}	$6.34kWh$
Consumption moved $1 \to 3$	x_{13}	$7.57kWh$
Consumption moved $2 \to 3$	x_{23}	$12.55kWh$
Disutility $1 \to 2$	$f_{12}(x_{12})$	$0.20NOK$
Disutility $1 \to 3$	$f_{13}(x_{13})$	$0.28NOK$
Disutility $2 \to 3$	$f_{23}(x_{23})$	$0.18NOK$

Table 1: The solutions obtained using the mentioned parameter set as input. $100øre = 1NOK$.

Table 1 shows the solution obtained by using the mentioned parameter set as input. We observe that even though $A_1 > B$, that is, the retailer is more interested than the network owner in having consumption moved out of Period 1, we get $s_1 < t$. In the two period case (see [2]), the retailer would choose a higher incentive than the network owner in this situation. In this asymmetrical three-period case, however, the opposite happens. The retailer has the chance to make a profit entirely on his own by incentivising the consumer to shift load out of Period 2, and is therefore in a position to squeeze the network owner to carry most of the burden in Period 1. In Section 4.1 we verify that this solution is indeed a Nash equilibrium.
Furthermore, we observe that the retailer makes more money than the network owner in this situation even though the retailer's total incentive payment is lower than that of his opponent. The retailer makes $3.15NOK$ a day, while the network owner makes $2.00NOK$ a day. If the game were played with the exactly same parameter values every day, this would add up to $1,150NOK$ a year and $730NOK$ a year, respectively, from this single consumer. If they offered incentives

to, say, 10,000 similar consumers, they would make $11,500,000 NOK$ a year and $7,300,000 NOK$ a year, respectively.
The consumer makes an economic profit of $\Pi_c = 0.65 NOK$ this particular day. This is the income from the incentive payments minus the disutility of moving consumption. The amount of money actually paid to the consumer is $1.31 NOK$, and this is the value of the observable effect on her electricity bill. She would receive $478 NOK$ a year. To receive this amount, the consumer each day moves $13.91 kWh$ out of Period 1, out of which $6.34 kWh$ is moved into Period 2 and $7.57 kWh$ is moved into Period 3. A consumption of $12.55 kWh$ is moved out of Period 2.

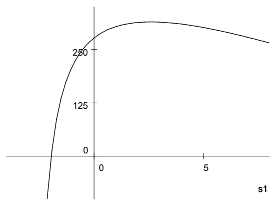
4.1 Verifying Nash equilibrium

We will now verify that the above solution is a Nash equilibrium, meaning that none of the players would be better off by choosing different incentive payments. In Figure 3.3(a) and Figure 3.3(b) the retailer's profit is plotted versus s_1 and s_2, respectively, with all other variables fixed to the values presented in Table 1. In Figure 3.3(c) the network owner's profit is plotted with respect to t with all other variables fixed. We observe that the retailer's profit peaks for $s_1 = 2.60$ and $s_2 = 2.75$, while the network owner's profit peaks at $t = 5.62$. This indicates that none of the players may do better by changing their incentive payment(s), suggesting that the presented solution indeed is a Nash equilibrium.

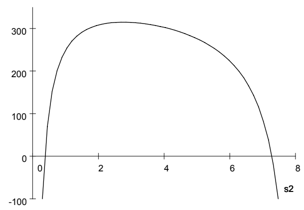
4.2 Collusion

If the players were allowed to collude, they would maximize the industry profit. Then, they would agree on a joint incentive payment p to be paid in Period 1. The retailer would still offer s_2 in addition to his share of p. The decision problem is then

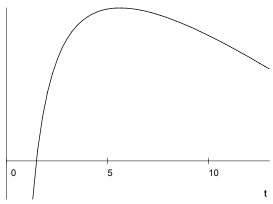
$$\max_{p,s_2}\{A_1\left(x^*_{12}+x^*_{13}\right)+A_2\left(x^*_{23}-x^*_{12}\right)$$
$$-p\left(x^*_{12}+x^*_{13}\right)-s_2\left(x^*_{23}-x^*_{12}\right)+B\left(x^*_{12}+x^*_{13}\right)\}.$$



(a) The retailer's profit plotted versus s_1.



(b) The retailer's profit plotted versus s_2.



(c) The network owner's profit plotted versus t.

Figure 3: Verifying Nash equilibrium.

Differentiating this with respect to the decision variables gives the following first order conditions

$$\frac{\partial \Pi_I}{\partial p} = (A_1 - A_2 + B - p + s_2)\,\frac{\partial x_{12}^*}{\partial p} + (A_1 + B - p)\,\frac{\partial x_{13}^*}{\partial p} - x_{12}^* - x_{13}^* = 0$$

$$\frac{\partial \Pi_I}{\partial s_2} = (A_1 - A_2 + B - p + s_2)\,\frac{\partial x_{12}^*}{\partial s_2} + (A_2 - s_2)\,\frac{dx_{23}^*}{ds_2} - x_{23}^* + x_{12}^* = 0.$$

Solving this gives

$$\begin{aligned} p &= 10.11862 \\ s_2 &= 2.7527 \end{aligned}$$

giving an industry profit of NOK 5.262. This is more than NOK 5.148 - the industry profit obtained when they operate in isolation. The consumer makes a profit of .930, and is also better off under collusion. Hence, in this case, allowing the players to collude would give an increase in overall welfare. The result for the consumer does, however, depend on the parameters in her disutility function. With much higher or much lower values of β_{ij}, for instance (with α_{ij} held constant), the consumer is worse off under collusion.
The joint incentive payment p can be divided by negotiation between the retailer and the network operator to give higher profits for each than those in the independent case. In this case, to make both players better off, we must have $3.1792 < s_1 < 3.9273$ (with a corresponding value of t between 6.93942 and 6.19132).
Observe, however, that this situation does not represent a Nash equilibrium in the noncooperative game played by the network operator and the retailer. Assume, for instance, that the players have negotiated $s_1 = 3.3411$ and $t = 6.7775$, giving profits of $\Pi_s = 3.238$ and $\Pi_t = 2.024$ for the retailer and the network owner, respectively. The retailer could break the agreement and offer $s_1 = 1.61037$ and $s_2 = 2.75105$, increasing his profit to $\Pi_s = 3.310$. The network owner would be left with a profit of $\Pi_t = 1.860$.
The network owner, on his hand, could also break the deal and offer $t = 4.98342$ increasing his profit to $\Pi_t = 2.104$ and thereby reducing the retailer's profit to $\Pi_s = 3.059$.
Hence, since the players could increase their profits by breaking the treaty, the collusive solution is not a Nash equilibrium.

4.3 Repeated game

In this section we examine the case where the retailer and the network operator do not know anything about each other, but have perfect information about their own profit functions. The players alternate

in offering incentives to the consumer and, at each offer, the offering agent chooses the incentive(s) that maximize(s) his own profit given the incentive(s) currently being offered by his opponent. Both players will continue offering as long as it is possible to make a decision that increases profit.
We assume that, at any stage of the game, neither player knows anything about the opponent's response function, apart from the opponent's most recent offer. This means that players do not use previous plays of the game to infer their opponent's response function. They simply consider the total incentive currently offered to the consumer, and compute how they can change their contribution to this in order to maximize profit. They do not really understand that the opponent will respond by changing his incentive again.

Iteration	s_1	s_2	t	Retailer's profit	Network's profit
1	0.000	0.000	6.325	2.325	1.870
2	1.996	2.751	6.325	3.247	1.915
3	1.996	2.751	6.128	3.218	1.916
4	2.163	2.751	6.128	3.219	1.939
5	2.163	2.751	5.986	3.199	1.940
6	2.283	2.751	5.986	3.199	1.956
⋮	⋮	⋮	⋮	⋮	⋮
48	2.593	2.752	5.620	3.148	1.999
49	2.594	2.752	5.620	3.148	1.999

Table 2: Repeated game with the network owner making the first draw.

Table 2 shows how such a repeated game would evolve if the network owner begins. In Iteration 1 the retailer free rides on the network owner's incentive, but he realizes that he could make substantially more by offering incentives himself. Taking the network owner's incentive into account he figures out his own profit maximizing bundle of incentives. Now, in Iteration 3 the network owner reacts to the changed framework and changes his incentive again to make some more money. The game continues like this until equilibrium is reached after forty-nine iterations.

Observe that each change of incentive payment by the network owner reduces the retailer's profit. The retailer attempts to outweigh this by increasing his own contribution, but he never manages to fully compensate for the loss imposed on him by the opponent's action. The effect of this is that the retailer's profit decreases as the game evolves. Thus, if he knew how the game would progress, he should stop playing after Iteration 3. The network owner's profit, however, increases with each iteration, and he would be interested in pushing the game forward.

Table 3 shows how the game would progress if the retailer made the first move. Again, we observe that the player starting the game increases his profits as the game evolves, while the follower's profit decreases. In this case, the network owner would be better off not making any move at all if he knew what was going on, while in Table 2 the retailer would have done best by stopping after Iteration 3. The reason why the retailer should make a move even though he is the follower, is that unless he makes a move, he will not get any revenue for load shifts out of Period 2.

Iteration	s_1	s_2	t	Retailer's profit	Network's profit
1	7.286	2.759	0.000	2.394	2.567
2	7.286	2.759	1.596	2.553	2.665
3	5.969	2.756	1.596	2.602	2.429
4	5.969	2.756	2.731	2.727	2.476
5	5.025	2.754	2.731	2.753	2.318
6	5.025	2.754	3.543	2.849	2.341
⋮	⋮	⋮	⋮	⋮	⋮
62	2.594	2.752	5.619	3.148	1.999
63	2.594	2.752	5.620	3.148	1.999

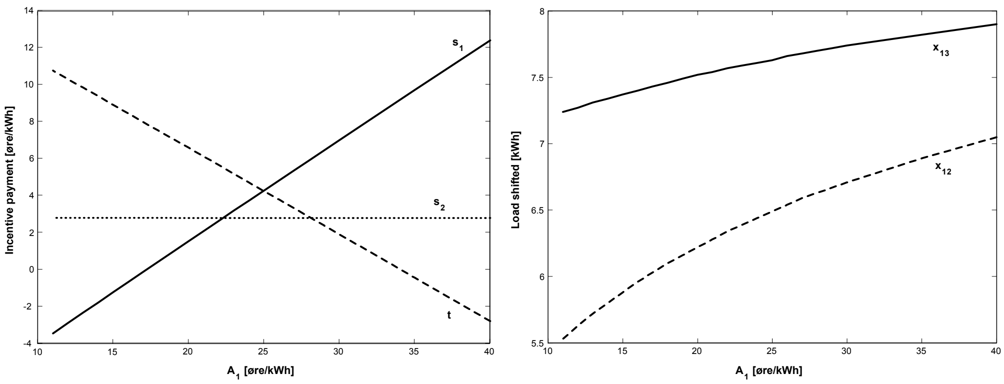
Table 3: Repeated game with the retailer making the first draw.

4.4 Parametric analysis

In this section we perform some parametric analysis to investigate how the results in the non-cooperative one-shot game change as some of the parameters are changed.

Varying A_1

Figure 4(a) shows how the players' incentive payments change with changes in A_1, the retailer's per unit benefit of load shifted out of Period 1. As expected, the more the retailer gains from load shifts out of Period 1, the more the retailer is willing to pay, while the network owner pays less. As A_1 decreases below about 17, s_1 becomes negative. This means that the retailer does not only free ride on the network owner in Period 1, but he also charges the consumer some of her profit.



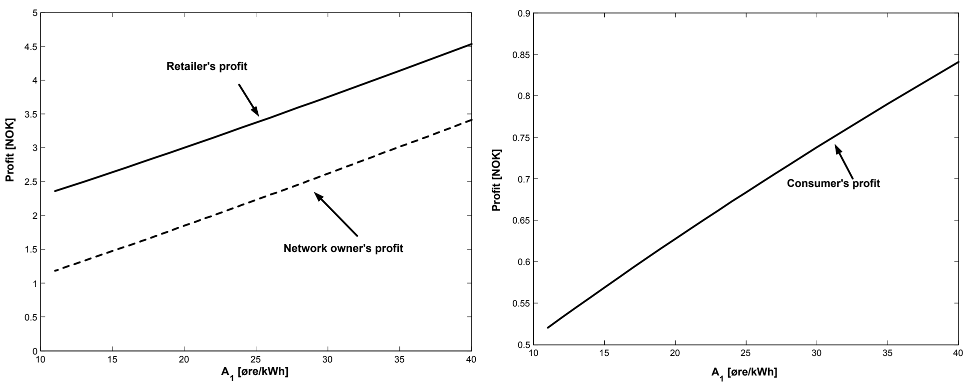
(a) Incentive payments wrt. A_1.

(b) Load shifts wrt. A_1.

Figure 4: Figure (a) shows how the players' incentive payments change with changes in A_1. The payments are not linear wrt. A_1. In the plot they look linear due to scale. Figure (b) shows how the load shifts out of Period 1 varies with changes in A_1.

As A_1 increases beyond about 34, t becomes negative. In the model this is okay, but we do believe that the authorities would put the foot down if the network owner used monopoly power this way. The retailer's incentive payment in Period 2 is almost not affected at all by variations in A_1. It increases very slowly as A_1 increases, but clearly, in this situation, the game is played in Period 1.

Figure 4(b) shows how the consumer's load shifts out of Period 1 are affected by changes in A_1. The load shift out of Period 2 is left out since it changes only marginally, which was expected due to the rather small changes in s_2. Since B is held constant, increasing A_1 means that the players' total gain from load shifts out of Period 1 increases. Therefore the sum $s_1 + t$ increases, which in turn, as expected, increases the amount of consumption moved out of this period. The higher the total incentive payment, the more she will gain by moving consumption. This effect is, however, counteracted by the increase in marginal disutility.



(a) The players' profits.

(b) The consumer's profit.

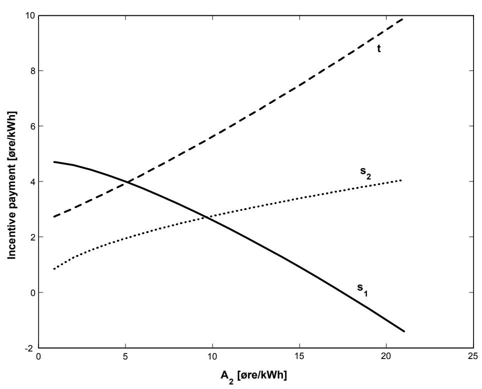
Figure 5: Profits with respect to A_1. Figure (a) shows the players' profits while Figure (b) shows the consumer's profit.

We have plotted the players' profits and the consumer's profit in Figure 5(a) and Figure 5(b), respectively. As was seen in Figure 4(a), the retailer pays more the higher A_1 gets, but the load shifts increase, making him better off with increasing A_1. The network owner pays a lower t, but more load is shifted out of the peak period, which obviously makes him earn more. The consumer is also better off as A_1 increases. She shifts more load, which increases her disutility, but this is outweighed by the increased total payment.
For changes in B, the network owner's per unit benefit of load shifted out of Period 1, t will increase and s_1 will decrease with higher B. The

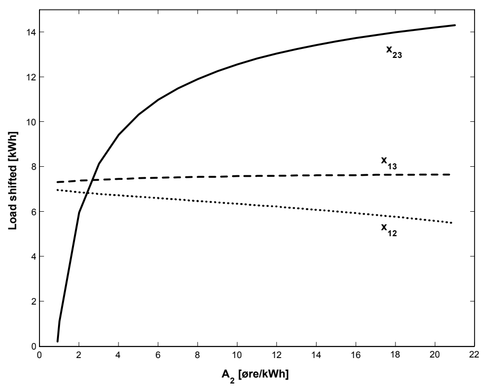
effects on load shifts and profits are more or less the same as those for variations in A_1.

Varying A_2

Figure 6(a) shows how the incentive payments change when A_2, the retailer's per unit benefit of load moved out of Period 2, changes. While s_2 was only very slightly affected by changes in A_1 and B, we see that changes in A_2 have significant impact on the incentive payments in Period 1. As A_2 increases, the retailer gives Period 2 higher priority, and therefore s_1 is reduced. The network owner is then forced to take a higher share of the burden in this period, so, t increases.



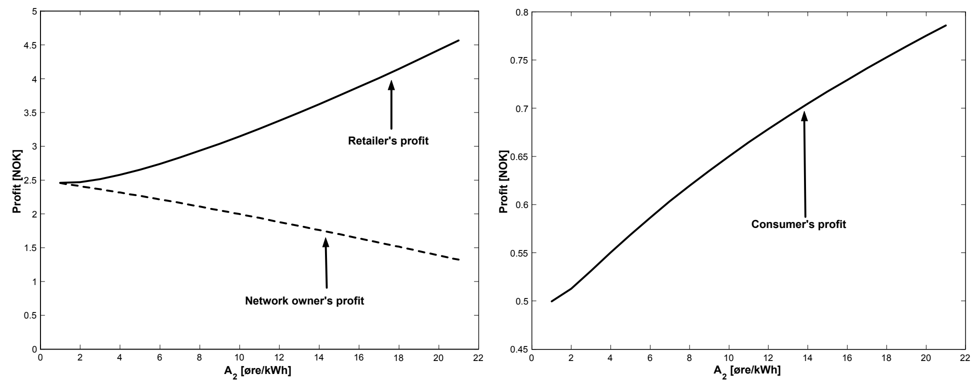
(a) Incentive payments wrt. A_2.



(b) Load shifts wrt. A_2.

Figure 6: Figure (a) shows how the players' incentive payments change with changes in A_2. The payments are not linear wrt. A_2. In the plot they look linear due to scale. Figure (b) shows how the load shifts out of Period 1 varies with changes in A_2.

Figure 6(b) shows how the consumer's load shifting changes with changes in A_2. As expected, higher A_2 gives higher x_{23} due to the increase in s_2. Also, the total amount of load shifted out of Period 1, $x_{12} + x_{13}$, decreases slightly. This is mainly due to a drop in x_{12}, which occures because s_2 increases, making load shifts into Period 2 more costly. x_{13} increases slightly because as s_2 increases, it becomes beneficial for the consumer to move a higher proportion of the load shifted out of Period 1 into Period 3 instead of Period 2.



(a) The players' profits. (b) The consumer's profit.

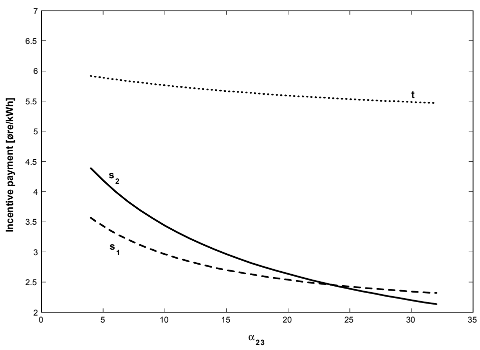
Figure 7: Profits with respect to A_2. Figure (a) shows the players' profits while Figure (b) shows the consumer's profit.

In Section 3 we discussed how the retailer's ability to squeeze the network owner increased as A_2 approaches A_1 and this point is made clear by Figure 7(a). The network owner must increase his incentive payment in an effort to keep the load shifts out of Period 1 stable as the retailer decreases s_1, and therefore, the network owner makes less money as A_2 increases. The retailer, on his hand, is better off, and so is the consumer.

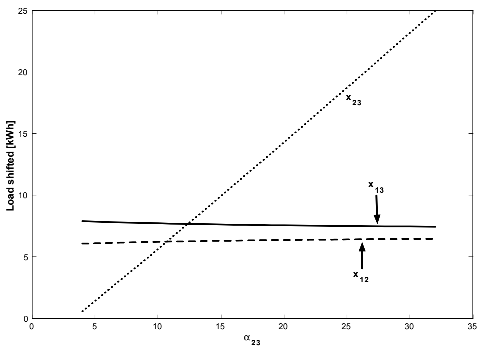
Varying α_{23}

We now let α_{23} vary to investigate how the players behave when the consumer's potential amount of load shifted from the medium load period to the low load period changes. In Figure 8 we have plotted how the incentives and the load shifts vary with α_{23}. As one would expect, the shift of load from Period 2 to Period 3 increases rapidly. In Figure 8(a) we see that s_2 decreases, making the shift x_{12} a bit more attractive compared to x_{13}, and therefore, slightly more of the load

shifted out of Period 1 goes to Period 2 and slightly less to Period 3 as α_{23} increases.
In Figure 9 the profits are plotted. Observe that while the network owner's profit remains rather stable, the retailer's profit increases significantly as the consumer gets more willing to shift load between the two periods in question. We also see how the consumer benefits from becoming more flexible.

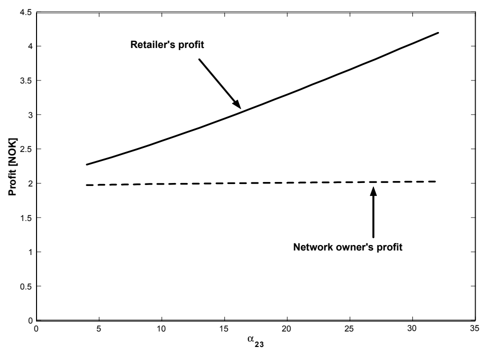


(a) Incentive payments wrt. α_{23}.

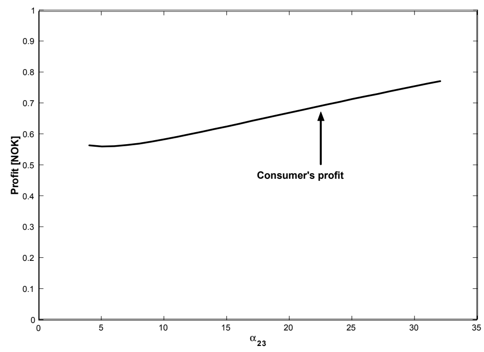


(b) Load shifts wrt. α_{23}.

Figure 8: Figure (a) shows how the incentive payments develop with respect to α_{23} while (b) shows how the different load change when α_{23} changes.



(a) The players' profits.

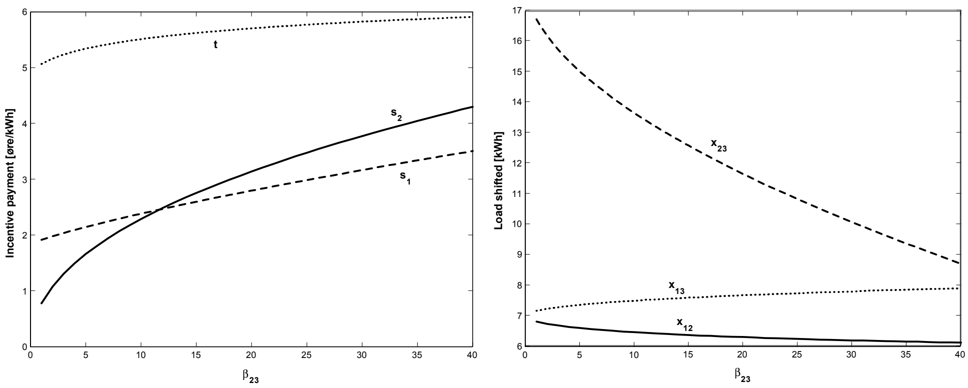


(b) The consumer's profit.

Figure 9: Profits with respect to α_{23}. The left figure shows the players' profits while the right figure shows the consumer's profit.

Varying β_{23}

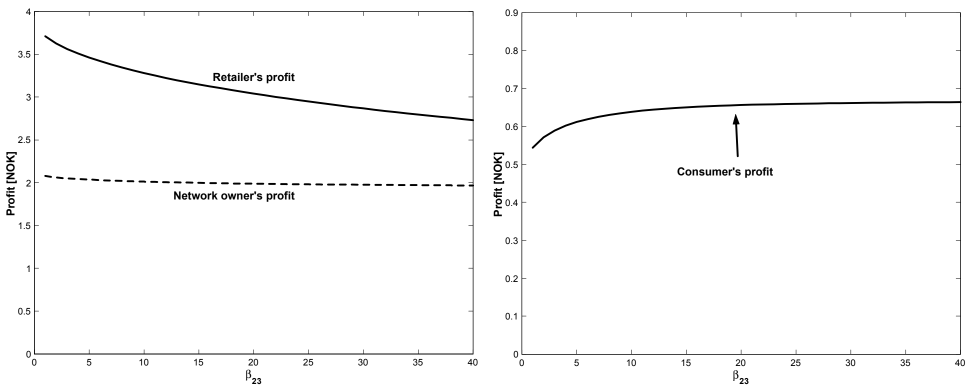
The parameter β_{23} affects the curvature of the consumer's disutility function $f_{23}(x_{23})$. The lower the value of this parameter, the higher the consumer's flexibility. This is illustrated in Figure 10(b) where we observe that the load shift x_{23} decreases rapidly with increasing values of β_{23}.
The retailer tries to outweigh the loss of flexibility by paying the consumer more for the load shift x_{23}, as seen in Figure 10(a). This, in turn, makes it less attractive to the consumer to shift load from Period 1 to Period 2, and therefore x_{12} decreases whith increasing β_{23}. The consumer compensates for this by increasing x_{13}, giving an almost negliable change in the sum $x_{12} + x_{13}$, which is the total shift out of Period 1.
As the consumer becomes less flexible, she is paid more, but shifts less load, as is apparent from Figure 10. Hence, as is shown in Figure 11(a), both the retailer and the network owner make lower profits with increasing β_{23}.



(a) Incentive payments wrt. β_{23}.

(b) Load shifts wrt. β_{23}.

Figure 10: Figure (a) shows how the incentive payments develop with respect to β_{23} while (b) shows how the different load shifts change when β_{23} changes.



(a) The players' profits.

(b) The consumer's profit.

Figure 11: Profits with respect to β_{23}. The left figure shows the players' profits while the right figure shows the consumer's profit.

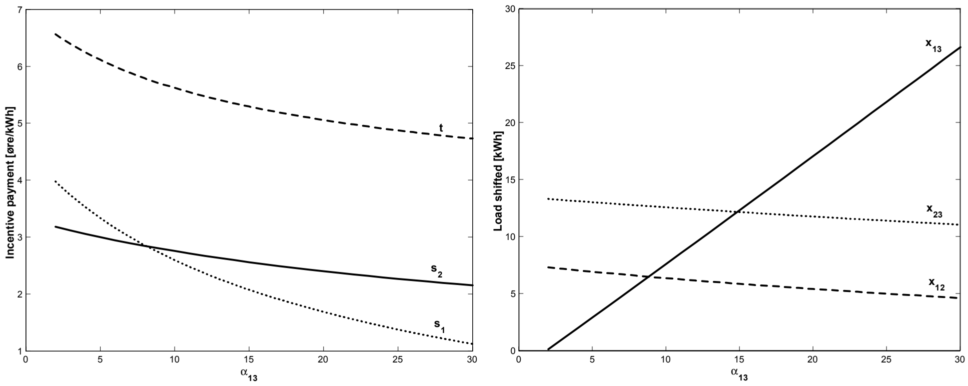
Interestingly, however, the consumer is better and better off the less flexible she becomes, as shown in Figure 11(b). So far, we have assumed that the consumer's cost function $f(x)$ is known to the retailer and the network owner. In the two load period version we analysed a situation in which this function was deduced by the retailer and network owner from observing behaviour of the consumer to the incentives offered by each. This raised the possibility of the consumer misrepresenting her true costs to these parties, and it was shown that, in that situation, the consumer would have incentives to pretend having a higher β than she really had. The result pictured in Figure 11(b) indicates that this might be case also for multiple load periods.

Varying α_{13}

Figure 12(a) shows how the incentives change with changes in α_{13}. It seems logical that s_1 and t decrease as α_{13} increases, since the players can achieve the same load reduction in Period 1 with lower incentive payments. Also, s_2 decreases with higher values of α_{13}. This happens because the consumer chooses to move a higher proportion of the load moved out of Period 1 into Period 3 instead of Period 2.

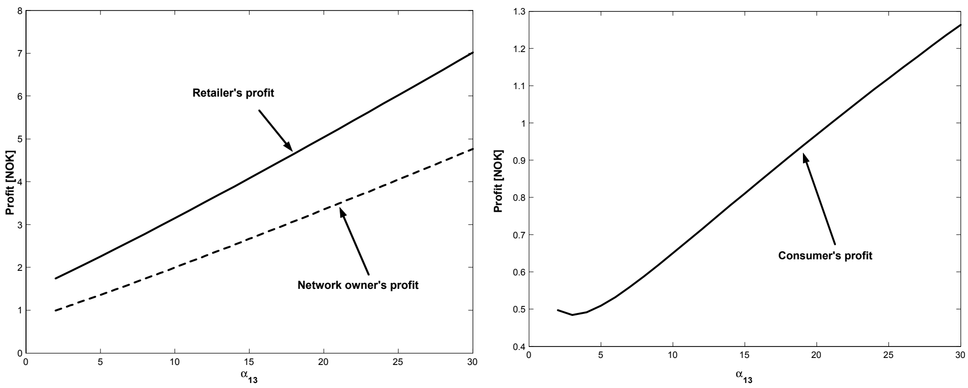
In Figure 12(b) we see that x_{13} increases rapidly as α_{13} increases, while x_{12} and x_{23} slowly decrease. The relationship between the increase in x_{13} and the decrease in x_{12} is rather obvious. The consumer chooses to move more of the load shifted out of Period 1 into Period 3 rather than Period 2 because moving into Period 3 becomes relatively more attractive. Also, the decrease in s_1 induces a lower x_{12}. The reduction in x_{23} is a result of the reduced s_2.

In Figure 13 we see that all market participants are better off with increasing α_{13}.



(a) Incentive payments wrt. α_{13}. (b) Load shifts wrt. α_{13}.

Figure 12: Figure (a) shows how the incentive payments develop with respect to α_{13} while (b) shows how the different load shifts change when α_{13} changes.

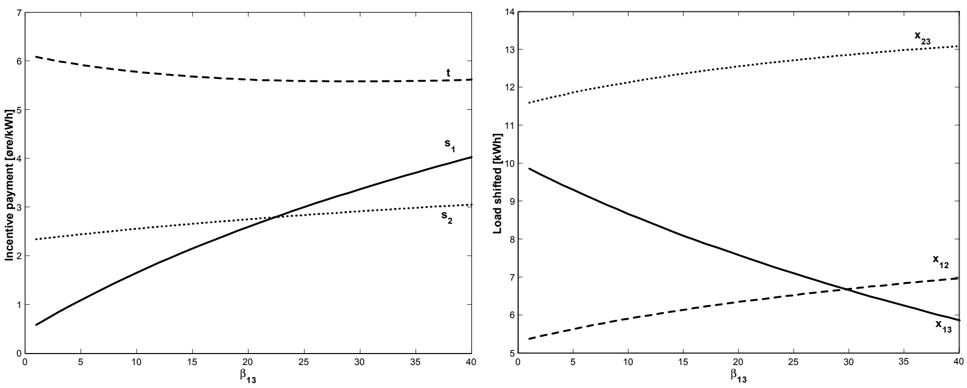


(a) The players' profits. (b) The consumer's profit.

Figure 13: Profits with respect to α_{13}. The figure to the left shows the players' profits while the figure to the right shows the consumer's profit.

Varying β_{13}

As β_{13} increases, the load shift x_{13} becomes less attractive, ceteris paribus, and, as visualized by Figure 14(b), x_{13} decreases rapidly for increasing β_{13}. The reduced x_{13} reduces the players' profits. The retailer tries to compensate for this by increasing s_2, as shown in Figure 14(a), to thereby get higher revenues from the increased amount of load shifted out of Period 2. This, however, makes the load shift x_{12} less attractive to the consumer, and to compensate for this, the retailer also increases s_1. The network owner takes some advantage of this by slightly reducing his payment, and as can be studied in Figure 15(a), he suffers a lower profit reduction than the retailer.

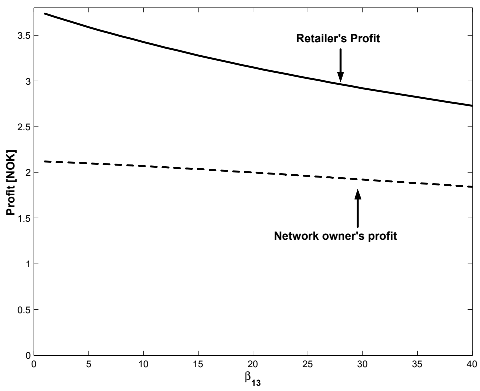


(a) Incentive payments wrt. β_{13}.

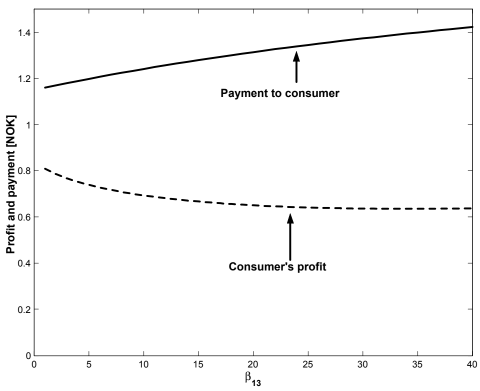
(b) Load shifts wrt. β_{13}.

Figure 14: Figure (a) shows how the incentive payments develop with respect to β_{13} while (b) shows how the different load shifts change when β_{13} changes.

Previously, we saw how the increase in β_{23} made the consumer better off. This is, however, not the case for increasing β_{13}. As we see in Figure 15(b), the consumer gets paid more, but this is outweighed by the increased disutility from shifting load from Period 1 to Period 3. As a result, the consumer gets worse off as β_{13} increases.



(a) The players' profits.



(b) The consumer's profit.

Figure 15: Profits with respect to β_{13}. The figure to the left shows the players' profits while the figure to the right shows the consumer's profit.

5 Conclusion

As an increasing number of households get their electricity consumption metered by the hour and get charged according to time-of-day tariffs, both the consumers and the utilities serving them will face new opportunities and challenges. The consumers may utilize the new tariffs to save money by altering their load profiles. Their challenge is then to find ways to do so that make the economic savings worth the potential loss of comfort.

The utilities would get the opportunity to make more money by designing price profiles that provide the consumers with incentives to change their daily habits with respect to electricity consumption. By doing so, the firms may gain some indirect control over their customers' consumption patterns. Their challenge is to design pricing schemes that add value to both the consumers and themselves.

One of the issues to be considered by the utilities, is that there are two firms who may be interested in controlling consumption patterns of end-users: the retailer and the network operator. In this paper we have developed a game model to investigate how they would interact in a situation where both firms are interested in offering incentives to make the consumer shift load. The model is based on [2], where a similar situation with two load periods is studied. In the current paper, we have increased the number of load periods to assess how the differences between the firms' cost structures may influence the results.

We find that the structure of the game does indeed change when there are three instead of two load periods. The retailer gets more flexible as he chooses a bundle of incentive payments, while the network owner chooses only one. The difference enables the retailer to squeeze the network owner, so that the latter covers a disproportionally high share of the costs in the peak period. The magnitude of this effect depends on the structure of the intra-day price movements in the wholesale market, and on the consumer's price elasticity of demand in the medium load period. The higher the difference between the wholesale price in the medium load period and in the low load period, the higher the retailer's advantage. Also, his advantage increases when the consumer's price elasticity in the medium load period increases.

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Article 4

Optimizing demand-side bids in day-ahead electricity markets

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Abstract

We consider a purchaser of electricity, bidding into a wholesale electricity pool market that operates a day ahead of dispatch. The purchaser must arrange purchase for an uncertain demand that occurs the following day. Deviations from the day-ahead purchase are bought in a secondary market. We study conditions under which the retailer should bid their expected demand, and derive conditions on the optimal demand curve that they should bid if the behaviour of the other participants is unknown, but can be modelled by a market distribution function.

1 Introduction

There has been much attention paid in recent years to optimizing the policies of generators who sell electricity in wholesale electricity pool markets. Much of this attention has focussed on equilibrium analyses that endeavour to quantify the extent of the market power generators might have (see e.g. [2], [3]). Although the effects of demand elasticity on the exercise of market power is well documented in Cournot models of electricity markets (see e.g. [2]), comparatively little attention has been paid to the effect of strategic demand bidding on market outcomes. An exception is the recent paper by Rassenti et al [6] who report on the results of demand behaviour in a set of market simulation experiments.

In this paper we use some simple optimization models to study the strategic behaviour of a large purchaser of electricity in a particular form of wholesale market, namely one in which the purchaser makes a day-ahead purchase bid, which is cleared against supply offers. The dispatch of power is then balanced in real time in a secondary regulating market on the day of dispatch.

An example of such a market is the Nord Pool in Scandinavia encompassing Norway, Denmark, Sweden and Finland. By noon each day, all generators and purchasers in the Nord Pool submit respective supply and demand curves giving production and purchase of electricity for the next day. Based on these bids, the spot prices for each hour of the

next day are derived. The regulating market is somewhat different for each of the countries in the region.
In this paper we focus on the particular form of the Norwegian regulating market, in which generators submit bids to increase or decrease the production level compared with their day-ahead market dispatch. If the market demand at the time of physical dispatch turns out to be higher than the quantity purchased in the day-ahead market, the market is said to be *up-regulated*. Thus, some generators must increase production so the demand can be met. In this case the market clearing model successively dispatches increases in generation from the offers with the lowest price. In the event of up-regulation, the regulating price is, by market design, always higher than the price in the day-ahead market.
On the other hand, if the market demand at the time of physical dispatch turns out to be lower than the quantity purchased in the day-ahead market, then the market is said to be *down-regulated*. In this case, some generators will have to reduce production, so effectively they buy back electricity to maintain the physical balance, and the market clearing model successively sells back generation to the generators offering the highest price. When the market is down regulated, the regulating price is always lower than the day-ahead price.
It is important to notice that even though the market as a whole might be down-regulated, an individual purchaser might be up-regulated. This situation, which we call *bidding against the market*, would occur if a purchaser bid for less power than he actually needed in the day-ahead market, while the market as a whole bought too much. Then, since the regulating price is lower than the day-ahead price, this purchaser would get to buy his excess demand at a lower price than if he had ordered the correct amount in the day-ahead market. Further details of the Norwegian regulating market can be found in Skytte [12].
A central theme of this paper is studying circumstances under which a purchaser ought to bid for his expected demand in the day-ahead market. We first examine a simple fixed-price model to see that bidding expected demand is not always optimal even in this case. In a more realistic non-cooperative equilibrium model we find in fact that purchasers should bid for less than their expected demand. This is because the regulating price is centred around the clearing price of

the day-ahead market, and by making this value small (by underbidding) the purchasers can effectively pay a smaller marginal price on each of two segments of their load. This underbidding can be seen in a more sophisticated model with uncertain demand, where an optimal demand curve to bid can be computed using market distribution functions.

The paper is laid out as follows. In the next section we look at a model for a single purchaser. We first prove a result that gives conditions under which the purchaser should bid their expected load when there is a fixed price in the day-ahead market. We then consider the case where the purchaser can influence the day-ahead price by bidding for supply from a known industry supply function. In this case it is optimal to buy less than the expected demand in the day-ahead market. In section 3 we extend the model of section 2 to n purchasers and construct a Nash equilibrium *a la* Cournot. In section 4, we present our most realistic model of participant behaviour. In this model the offers and bids of the other agents are assumed to be unknown, but can be represented by a market distribution function of a similar form to that introduced in [1]. This allows the optimality conditions for generators to be applied to a purchaser, to yield an optimal bid curve for the day-ahead market. We illustrate the procedure with a simple example.

2 A single purchaser model

We first consider the case where all generators offer at the same price $\overline{p}$ in the day-ahead market, and the market has a single purchaser who is to choose an amount x to order in the day-ahead market at this price. Following this, a random demand H is observed, and the purchaser must purchase the extra energy (or sell it back to the market) at the regulating price.

The regulating market price is determined by offers of generators into the regulating market. These take the form of non-decreasing supply functions passing through the point $(\overline{p}, x)$. The clearing price is determined by the inverse $\tau(\cdot)$ of the aggregate regulating supply function.

Since $\tau(x) = \overline{p}$, we may represent τ by

$$\tau(h) = \overline{p} + \delta\,(h - x)$$

where H is the (random) quantity dispatched (i.e. a demand realisation), and $\delta\,(\cdot)$ denotes the difference between the regulating price and the day-ahead price. Since H is a random variable, the regulating price will also be a random variable.
The purchaser now faces the problem of minimizing the cost of meeting this random demand. His optimization problem is then

$$\text{P:} \quad \min_x \left\{\overline{p}x + E\left[\left(\overline{p} + \delta\,(H - x)\right)(H - x)\right]\right\}.$$

Observe that this is equivalent to

$$\text{P:} \quad \min_x \left\{\overline{p}E[H] + E\left[\delta\,(H - x)\,(H - x)\right]\right\},$$

so the purchaser should seek x^* to solve

$$\bar{\text{P}}\text{:} \quad \min_x \left\{E\left[(H - x)\,\delta\,(H - x)\right]\right\}.$$

It is easy to see that when $\delta\,(y) = ay$ for some $a \geq 0$ we have

$$E\left[(H - x)\,\delta\,(H - x)\right] = aE\left[(H - x)^2\right]$$

and so the optimal choice of x is $E[H]$. To obtain this optimal policy for more general forms of the regulating market, we must place some conditions on the probability distribution of demand as shown by the following proposition.

Proposition 1 *Suppose $\delta(y)$ is an odd increasing function with $y\delta\,(y)$ convex, and H has a symmetric probability distribution around $E[H]$. Then the optimal solution to P is $x^* = E[H]$.*

Proof. Define $\bar{h} = E[H]$, $g(y) = y\delta\,(y)$, and suppose the probability distribution of H is defined by measure μ. Then the objective function of $\bar{\text{P}}$ becomes

$$\begin{aligned} \phi(x) &= E\left[(H - x)\,\delta\,(H - x)\right] \\ &= \int_{-\infty}^{+\infty} g\,(h - x)\,d\mu(h). \end{aligned}$$

Since $g(y)$ is convex (with left and right derivatives denoted by $g'_{-}(y)$ and $g'_{+}(y)$ respectively) by virtue of Proposition 4 in [5] we may compute the left and right derivatives of ϕ,

$$\begin{aligned}\phi'_{+}(x) &= \int_{-\infty}^{+\infty} -g'_{-}(h-x)\,d\mu(h)\\ \phi'_{-}(x) &= \int_{-\infty}^{+\infty} -g'_{+}(h-x)\,d\mu(h).\end{aligned}$$

A change of variable yields

$$\begin{aligned}\phi'_{+}(x) = \int_{-\infty}^{0} -g'_{-}\left(u+\bar{h}-x\right) d\mu(u+\bar{h})\\ + \int_{0}^{+\infty} -g'_{-}\left(u+\bar{h}-x\right) d\mu(u+\bar{h})\\ = \int_{-\infty}^{+\infty} -g'_{-}\left(u+\bar{h}-x\right) d\mu(u+\bar{h}),\end{aligned}$$

whence

$$\phi'_{+}(\bar{h}) = \int_{-\infty}^{0} -g'_{-}(u)\,d\mu(u+\bar{h}) + \int_{0}^{+\infty} -g'_{-}(u)\,d\mu(u+\bar{h}).$$

Now since $\delta(\cdot)$ is an odd function, g is even, and since $\mu(u+\bar{h})$ is symmetric about $u=0$,

$$\int_{-\infty}^{0} -g'_{-}(u)\,d\mu(u+\bar{h}) = \int_{0}^{+\infty} g'_{+}(u)\,d\mu(u+\bar{h})$$

giving

$$\begin{aligned}\phi'_{+}(\bar{h}) &= \int_{0}^{+\infty} g'_{+}(u)\,d\mu(u+\bar{h}) + \int_{0}^{+\infty} -g'_{-}(u)\,d\mu(u+\bar{h})\\ &= \int_{0}^{+\infty} \left(g'_{+}(u) - g'_{-}(u)\right) d\mu(u+\bar{h})\\ &\geq 0\end{aligned}$$

by the convexity of g. Similarly

$$\begin{aligned}\phi'_-(\bar{h}) &= \int_{-\infty}^{0} -g'_+(u)\, d\mu(u+\bar{h}) + \int_{0}^{+\infty} -g'_+(u)\, d\mu(u+\bar{h}) \\ &= \int_{-\infty}^{0} -g'_+(u)\, d\mu(u+\bar{h}) - \int_{-\infty}^{0} -g'_-(u)\, d\mu(u+\bar{h}) \\ &\leq 0,\end{aligned}$$

demonstrating that $\bar{h}$ is a local minimizer of ϕ.
Furthermore, if $x > \bar{h}$, then by convexity $g'_-\left(u+\bar{h}-x\right) \leq g'_-(u)$, so

$$\begin{aligned}\phi'_+(x) &= -\int_{-\infty}^{+\infty} g'_-\left(u+\bar{h}-x\right) d\mu(u+\bar{h}) \\ &\geq -\int_{-\infty}^{+\infty} g'_-(u)\, d\mu(u+\bar{h}) \\ &= \phi'_+(\bar{h}),\end{aligned}$$

and if $x < \bar{h}$, then $g'_+\left(u+\bar{h}-x\right) \geq g'_+(u)$, so

$$\begin{aligned}\phi'_-(x) &= -\int_{-\infty}^{+\infty} g'_+\left(u+\bar{h}-x\right) d\mu(u+\bar{h}) \\ &\leq -\int_{-\infty}^{+\infty} g'_+(u)\, d\mu(u+\bar{h}) \\ &= \phi'_-(\bar{h}),\end{aligned}$$

which shows that $\bar{h}$ gives a global minimum. ■

The convexity of $y\delta(y)$ is needed in this result as shown by the following example.

Example 1
Suppose the price in the regulating market is determined by

$$\delta(y) = \begin{cases} -1+e^{3y} & y \leq 0 \\ 1-e^{-3y} & y > 0 \end{cases}$$

so

$$g(y) = \begin{cases} -y+ye^{3y} & y \leq 0 \\ y-ye^{-3y} & y > 0 \end{cases}.$$

It is easy to verify that $g(y)$ is not convex. Let μ be a discrete measure with weight of $\frac{1}{2}$ located at $h = \frac{1}{2}$ and $h = \frac{7}{2}$. Then

$$E\left[(H-x)\,\delta\,(H-x)\right] = \frac{1}{2}g(\frac{1}{2}-x) + \frac{1}{2}g(\frac{7}{2}-x)$$

for which $x = E[H] = 2$ is a local maximum (not a minimum) as shown in Figure 1.

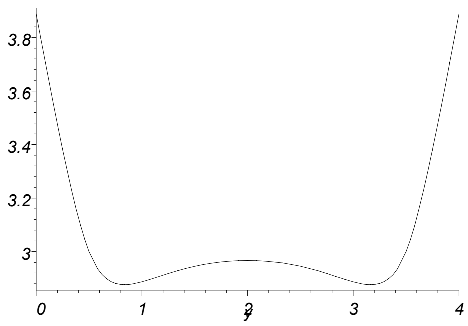

Figure 1: Plot of $E[(H-x)\delta(H-x)]$ for different bids x.

The symmetry of μ is also needed as shown by the following example.

Example 2
Suppose μ is a discrete measure with $\mu(0) = 0.1$ and $\mu(1) = 0.9$. Now let the price in the regulating market be determined by

$$\delta\,(y) = y^3.$$

We thus seek to minimize

$$\int_{-\infty}^{\infty} (h-x)^4\, d\mu(h) = 0.1x^4 + 0.9(1-x)^4$$

which has a minimum at $x = 0.67533$ rather than $x = E[H] = 0.9$. We conclude this section by considering the case where the demand side of the market consists of a single purchaser who is bidding into the day-ahead market. This market is supplied by n generators who offer supply functions, where we denote by $S_i(p)$ the quantity offered at price p in the day-ahead market by generator i. The purchaser will face a random market demand H the following day. If the amount bought on the day-ahead market is different from H, the difference must be offset on the regulating market. We assume in this section that the aggregate supply function (although perhaps not each S_i) is known to the purchaser, and is strictly increasing.

We observe that there is no advantage in the purchaser offering a demand curve. With perfect knowledge of $S(\cdot) = \sum_i S_i(\cdot)$ he can decide on a price p and choose any decreasing curve that passes through $(p, S(p))$. Equivalently, he can determine an optimum quantity x to buy, for which he will pay $p = S^{-1}(x)$. (In this model we assume that the generators do not game their offers in response to the purchaser's bids.)

For convenience let $T(\cdot) = S^{-1}(\cdot)$. If the purchaser selects x to buy in the day-ahead market then he will pay $C_s = xT(x)$. In addition he will face a cost in the regulating market, $C_r = (H - x)\tau(H - x)$, where $\tau(\cdot)$ is the regulating market price at demand H and day-ahead dispatch volume x.

The purchaser should choose x to solve

$$\text{P:} \quad \min_x \{xT(x) + E[(H - x)\tau(H - x)]\},$$

so setting

$$\tau(H - x) = T(x) + \delta(H - x)$$

gives

$$\text{P:} \quad \min_x \left\{\overline{h}T(x) + E[(H - x)\delta(H - x)]\right\}$$

where $\overline{h} = E[H]$.

It is interesting to observe that in contrast to the situation in which the day-ahead price is a known constant, the optimal offer is no longer $\overline{h}$ even if $\tau(\cdot)$ is linear. To see this suppose $\delta(y) = ay$, for $a > 0$. Then

$$\begin{aligned} E[(H - x)\delta(H - x)] &= aE\left[(H - x)^2\right] \\ &= aE\left[H^2\right] - 2a\overline{h}x + ax^2 \end{aligned}$$

Now, the purchaser solves

$$\min_{x} \left\{ \overline{h} T(x) - 2a\overline{h}x + ax^2 \right\}$$

Differentiating gives $\overline{h}T'(x) - 2a\overline{h} + 2ax = 0$, whereby

$$x^* + \frac{\overline{h}}{2a} T'(x^*) = \overline{h} \tag{1}$$

The purchaser therefore should purchase less than $\overline{h}$ in the day-ahead market. (The solution $\overline{h}$ for a constant price is recovered by setting $T'(x^*) = 0$.)

3 Many purchasers

In the previous section we looked at the behaviour of a single purchaser. In this section we use a similar framework to calculate the market outcomes in a market with n purchasers having random demands H_i, $i = 1, \ldots, n$. We assume Cournot conjectural variations, namely that each purchaser i bids x_i in the day-ahead market assuming that x_j, $j \neq i$ is fixed. We will assume that $T(x) = bx$ and the price in the regulating market to be

$$bx + a(H - x),$$

where $x = \sum_{i=1}^{n} x_i$ and $H = \sum_{i=1}^{n} H_i$.
In a Nash equilibrium each purchaser i chooses a quantity x_i to solve

$$\mathrm{P(n)}: \quad \min_{x_i} x_i bx + E[(H_i - x_i)(bx + a(H - x))]. \tag{2}$$

The objective function of P(n) can be written

$$\begin{aligned} f_i(x) &= x_i bx + E[H_i T(x) + H_i aH - H_i ax - x_i bx - x_i aH + x_i ax] \\ &= h_i bx + aE[H_i H] - h_i ax - x_i ah + x_i ax, \end{aligned}$$

where we let $E[H_i] = h_i$, $E[H] = h$. The optimal bid for each purchaser will satisfy

$$\frac{\partial f_i}{\partial x_i} = h_i b - ah_i - ah + ax_i + ax = 0,$$

which is guaranteed to be a minimum because

$$\frac{\partial^2 f_i}{\partial x_i^2} = 2a > 0.$$

Now $\sum_{i=1}^{n} h_i = h$, so summing over i gives

$$hb - ah - anh + ax + nax = 0.$$

Thus

$$x = h(1 - \frac{b}{a\,(1+n)})$$

and so the optimal bid for purchaser i is

$$\begin{aligned} x_i &= h_i + \frac{b}{(n+1)\,a}\,(h - (n+1)h_i) \\ &= (1 - \frac{b}{a})h_i + \frac{b}{a}\frac{h}{(n+1)}. \end{aligned}$$

Recall that for a single purchaser that the optimal bid x^* in the day-ahead market satisfies

$$x^* = \overline{h} - \frac{\overline{h}}{2a}T'\,(x^*) = h\left(1 - \frac{b}{2a}\right).$$

For n purchasers, the total amount bid in equilibrium is

$$\begin{aligned} \sum x_i &= h + \frac{b}{(n+1)\,a}\,(nh - (n+1)h) \\ &= h(1 - \frac{b}{(n+1)\,a}). \end{aligned}$$

So in aggregate, more demand will be bid for in the day-ahead market as the number of purchasers increases, and as $n \to \infty$, the limiting optimal bid for each purchaser is to bid their expected demand.

Observe that in this model the amount bid into the day-ahead market by each player depends only on the expected demand of each player. There is no dependence on the correlation between H_i and H. As a consequence of the linearity of the regulating market, this correlation appears only in the constant term $aE[H_iH]$ in the objective

function. A negative correlation means that purchaser i is likely to be down-regulated when the market as a whole is up-regulated, and up-regulated when the market is down-regulated. However, although he makes a windfall profit from such a market, his bidding strategy in the day-ahead market is independent of this correlation.
The model in this section also allows us to investigate a strategy of individual purchasers bidding against the market. Suppose we fix x_j, $j \neq i$, and assume for simplicity that $b = 0$. Then the optimal choice of x_i is given by

$$2x_i = (h - \sum_{j \neq i} x_j) + h_i(1 - \frac{b}{a})$$

so

$$x_i = h_i + \frac{1}{2}(\sum_{j \neq i} h_j - \sum_{j \neq i} x_j).$$

Thus if the purchasers apart from i bid less than their expected demand, then purchaser i should bid against the market (by bidding more than their expected demand). Of course in equilibrium, it is easy to see for this example that each purchaser will bid their expected demand, so bidding against the market is not an equilibrium strategy.
Finally, the degree to which purchasers reduce their bids depends upon the relative magnitudes of a and b. If $b > a$ then the variation in price with quantity is less dramatic in the regulating market, and so purchasers would want to purchase more at the margin in this market. On the other hand if $a > b$, then purchasers will tend to bid for relatively more in the day-ahead market.

4 Optimal bidding with a market distribution

In the model of the previous section, we assumed that the purchasers have perfect knowledge of the generator's supply functions in the day-ahead market, and constructed a Nash equilibrium in the single-shot game played against other purchasers. In practice the supply function offers of the generators will be chosen at the same time as the

purchaser's bids, and so one might seek a supply-function equilibrium for their offers. The game being played by generators is actually more complicated than this as generators choose both a day-ahead offer and an offer to the regulating market.
A less complicated analysis seeks an optimal bid curves from a single purchasers under the assumption that the generators' supply curves and the other purchasers' bid curves are not chosen strategically but are random. A single purchaser (say purchaser 1) then might seek a bid curve to offer that will yield a (random) day-ahead purchase outcome that in expectation minimizes his costs of purchasing to meet his next day's demand.
To model this situation we follow the approach of [1] and define a *market distribution* function ϕ, where $\phi(r, p)$ denotes the probability of the demand of purchaser 1 being fully met if he requests a quantity r at price p from the day-ahead pool. The probability $\phi(\cdot)$ is decreasing in r and increasing in p. Suppose now that the purchaser is to submit some demand curve $\mathbf{s} = \{(r(t), p(t)), 0 \leq t \leq T\}$ to the day-ahead market. The component $r(t)$, which is monotonic decreasing, traces the quantity component of the demand curve, while $p(t)$, which is monotonic increasing, traces the price component, as shown by the solid curve in Figure 2. The dashed line to the right shows all points for which $\phi(r, p) = \phi_0 = 0$. The dashed line to the left shows all points for which $\phi(r, p) = \phi_1 = 1$. From this, we observe that between ϕ_1 and ϕ_0 there is a probability measure that lies on $\mathbf{s}$ that defines the probability of the purchaser being sold quantity r at price p.

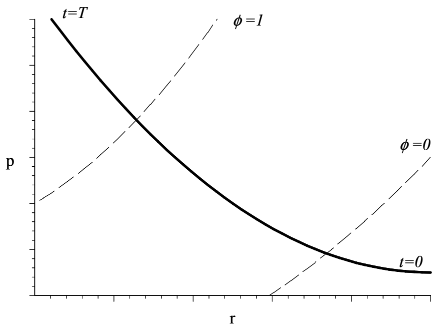

Figure 2: The solid line is the demand curve. The dotted line to the right corresponds to $\phi(\cdot) = 0$, while the dotted line to the left corresponds to $\phi(\cdot) = 1$.

Now, purchaser 1 seeks to minimize his total cost of purchasing electricity. The total cost is the sum of the cost in the day-ahead market and the cost in the regulating market. The optimal bidding curve will be the solution to

$$\min_{s} \int_{s} [rp + C(r,p)]\, d\phi(r,p) \tag{3}$$

subject to

$$\begin{aligned} s &= \{(r(t), p(t)), 0 \leq t \leq T\} \\ \frac{dr}{dt} &\leq 0 \ (r(\cdot) \text{ non-increasing}) \\ \frac{dp}{dt} &\geq 0 \ (p(\cdot) \text{ non-decreasing}) \\ 0 &\leq r(t) \leq q_M \end{aligned}$$

where $C(r,p)$ is the regulating cost and q_M is an upper bound on r. The market distribution function ϕ is different from the standard market distribution function introduced in Anderson and Philpott [1], but we can make use of their framework. To do so, we may relate ϕ to a standard market distribution function by a simple transformation. To do this we let $q = q_M - r$. The parameter q_M is an upper bound on both r and q. Also, we define $\psi(q,p) = \phi(q_M - q, p)$. This corresponds to reversing the graphs in Figure 2 giving the graph in Figure 3.

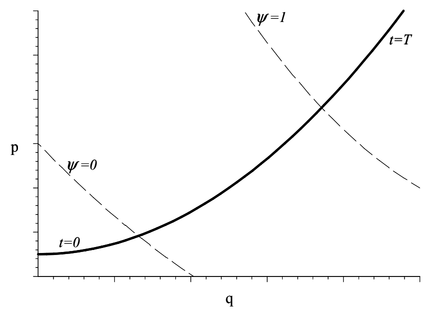

Figure 3: The plots in Figure 2 reversed. The dashed line to the right corresponds to $\psi = 1$, and the dashed line to the left corresponds to $\psi = 0$.

Observe that on the curve s in Figure 2 the measure $d\phi(r,p)$ is the same as $d\psi(q_M - r, p)$ on the curve s' depicted by the solid line in Figure 3. The objective function (3) is then equivalent to

$$\min_{s'} \int_{s'} \left[(q_M - q)\, p + C\,(q_M - q, p) \right] d\psi(q,p)\,. \tag{4}$$

Now we rewrite the expression (4) and get the following problem

$$\max_{s'} \int_{s'} \{-(q_M - q)\, p - C\,(q_M - q, p)\} d\psi(q,p) \tag{5}$$

subject to

$$\begin{aligned} s' &= \{q(t), p(t), \quad 0 \leq t \leq T\} \\ \frac{dq}{dt}, \frac{dp}{dt} &> 0 \text{ (q and p non-decreasing)} \\ 0 &\leq q(t) \leq q_M. \end{aligned}$$

We observe that this is equivalent to the problem of a generator maximizing the profit from an offer stack, where q is the quantity offered into the pool by the generator at price p as described in [1]. The market distribution function $\psi(q,p)$ denotes the probability of a generator not being fully dispatched at the price-quantity pair (p,q). We denote by $B(q,p)$ the integrand in the objective function (5). It is shown in [1] that the optimal solution to this problem must satisfy the first order condition

$$Z(q,p) = \frac{\partial B}{\partial q}\frac{\partial \psi}{\partial p} - \frac{\partial B}{\partial p}\frac{\partial \psi}{\partial q} = 0. \tag{6}$$

Computing the regulating cost in (3) is not entirely straightforward as the regulating price depends on the clearing price p in the day-ahead market and the amount of demand cleared in the day-ahead market. We shall denote by $C(r,p)$ the expected regulating cost for purchaser 1 conditional on its being dispatched r at clearing price p in the day-ahead market. To evaluate $C(r,p)$ we take expectations with respect to the conditional probability distribution of dispatch of the other purchasers.
To do this let $\delta(\cdot)$ be the difference between the regulating market price and the day-ahead price as a function of regulating market dispatch. Then the regulating price is $p + \delta(H - U(p) - r)$, where H is the total (random) demand of all purchasers, $U(p)$ is the total (random) day-ahead demand dispatched at price p to the other purchasers, and r is the amount of day-ahead demand that purchaser 1 is cleared at price p. The amount that purchaser 1 buys in the regulating market is $H_1 - r$, where H_1 is the (random) demand of purchaser 1.
Using this notation we obtain

$$C(r,p) = E_{H,H_1,U}[(p + \delta(H - U(p) - r))(H_1 - r) \mid (r,p)]$$

We then compute an offer curve s' that solves

$$\max_{s'} \int_{s'} \{-(q_M - q)\, p - C(q_M - q, p)\} d\psi(q,p).$$

The optimal bid curve will then be defined by $(r,p) = (q_M - q, p)$.
As an illustration of how one might compute an optimal demand curve

to offer in the day-ahead market, suppose $\delta(y) = y$. Then

$$\begin{aligned} C(r,p) &= E_{H,H_1,U}[(p + (H - U(p) - r))(H_1 - r) \mid (r,p)] \\ &= pE[H_1] - pr + E[HH_1|(r,p)] - E[H]r \\ &\quad - E[U(p)H_1|(r,p)] + E[U(p)|(r,p)]r - rE[H_1] + r^2. \end{aligned}$$

To simplify this expression we now assume that both H and H_1 are statistically independent of p, r, and $U(p)$, but H and H_1 may be correlated. This independence is quite a restrictive assumption. Although the assumption allows the total amount $U(p)$ bid by other purchasers in the day-ahead market to depend on $E[H]$, a single other purchaser might well base their contribution to $U(p)$ on the current day's observed demand level which will typically be correlated with H.

Under the independence assumption the function $C(r,p)$ simplifies to

$$\begin{aligned} C(r,p) &= r^2 + (-p - h + E[U(p)|(r,p)] - h_1)r \\ &\qquad + ph_1 + E[HH_1] - h_1E[U(p)|(r,p)] \\ &= r^2 + (-p - h + u(p) - h_1)r + ph_1 + E[HH_1] - h_1u(p), \end{aligned}$$

where we denote $E[U(p)|(r,p)]$ by $u(p)$, $E[H]$ by h, and $E[H_1]$ by h_1.

Example 3 (Single purchaser)

To illustrate how to derive an optimal bid in a particular case, we assume a single purchaser in a day-ahead market with $\psi(q,p) = \frac{qp}{4}$. For one purchaser, we have $H_1 = H$, and $u(p) = 0$. Now the integrand $-(q_M - q)p - C(q_M - q, p)$ becomes

$$\begin{aligned} -(q_M - q)p - \{&(q_M - q)^2 + (-p - h + u(p) - h_1)(q_M - q) \\ &+ ph_1 + E[HH_1] - h_1u(p)\} \\ &\quad = -(q_M - q)^2 + 2h(q_M - q) - ph - E\left[H^2\right] \end{aligned}$$

so we seek

$$\max_{s'} \int_{s'} \left(-(q_M - q)^2 + 2h(q_M - q) - ph - E\left[H^2\right]\right) d\left(\frac{qp}{4}\right)$$

As above

$$\begin{aligned} Z(q,p) &= B_q\psi_p - B_p\psi_q \\ &= (-2h + 2q_M - 2q)\,\frac{1}{4}q + \frac{1}{4}hp. \end{aligned}$$

Now suppose $E[H] = 1$, $E[H^2] = 2$, and $q_M = 2$. Then

$$Z(q,p) = \frac{1}{4}(2-2q)\,q + \frac{1}{4}p.$$

The curve $Z(q,p) = 0$ is plotted below in Figure 4, where $Z > 0$ above the curve. This curve does not completely specify the optimal solution to the problem with objective (5) as the optimal curve must be non-decreasing.

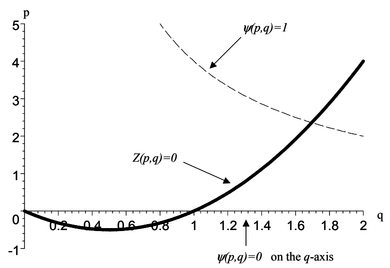

Figure 4: The curve $Z(q,p) = 0$.

However since $\psi(p,q) = 0$ for $p \leq 0$, the differential $d\psi = 0$ in this area, so the purchaser may choose any non-decreasing curve for $p \leq 0$, which corresponds to $q \leq 1$. The same argument goes for the area where $\psi(p,q) = 1$, which corresponds to $q \geq 1.6956$ and $p \geq 2.359$. Then, an optimal bidding curve would be

$$p(q) = \begin{cases} 0, & q \leq 1, \\ 2q^2 - 2q, & 1 < q \leq 1.6956, \\ 2.359, & q > 1.6956. \end{cases}$$

The optimal objective value of $p(q)$ is

$$\begin{aligned}
&\int_{s'} \left(-\left(q_M - q\right)^2 + 2h\left(q_M - q\right) - ph - E\left[H^2\right]\right) d\psi\left(q, p\right) \\
=\ & \int_{q=1}^{q=1.6956} \{-p(q) - 2q - (2-q)^2 + 2\} d\left(\frac{qp(q)}{4}\right) \\
=\ & \frac{1}{4}\int_{1}^{1.6956} (-(2q^2 - 2q) - 2q - (2-q)^2 + 2)(6q^2 - 4q) dq \\
=\ & -2.5323.
\end{aligned}$$

This gives the following optimal bid curve for the purchaser:

$$b(r) = \begin{cases} 2.359, & r < 0.3043, \\ 4 - 6r + 2r^2, & 0.3043 \leq r < 1, \\ 0, & r \geq 1, \end{cases}$$

which is plotted in Figure 5. The cost of this policy is 2.5323 (minus the objective function value that we maximized).

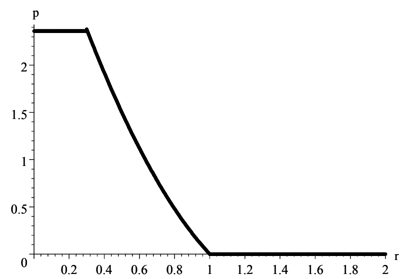

Figure 5: Optimal demand bid curve.

We can compare this policy with several other candidates. A potential strategy for the purchaser may be to bid the expected demand, $r = \overline{h} = 1$, in the day-ahead market. This is defined by the offer curve $q = q_M - r = 1$. The objective value of this curve is

$$\begin{aligned}
& \int_{s'} \{-p\overline{h} - 2q\overline{h} - (q_M - q)^2 + K\} d\psi(q,p) \\
= & \int_{p=0}^{p=4} \{-p - 2 - (2-1)^2 + 2\} d\left(\frac{p}{4}\right) \\
= & \frac{1}{4}\int_0^4 (-p-1)\, dp \\
= & -3.0,
\end{aligned}$$

giving an expected cost of 3.0 for the purchaser. Similarly if the purchaser chooses to buy the entire demand in the regulating market, then he would order $r = 0$ (or $q = 2$) in the day-ahead market. The objective value of this curve is also -3.0 giving a cost of 3.0. Bidding $r = 0.5$, in the day-ahead market, for example, gives an expected cost of 2.58.

5 Conclusions

In this paper we have seen that in most realistic circumstances a purchaser for electricity ought to bid for less than their expected demand in the day-ahead market. The central reason underlying this is that even with perfect knowledge of their demand, purchasers are likely to be better off buying their electricity in two marginally-priced tranches, rather than making a single bid. This behaviour is confirmed in both the equilibrium model and the model with uncertainty modelled using a market distribution function.

Note that we have ignored the risk attitude of purchasers in our models. Because the regulating market is typically more volatile than the day-ahead market (being susceptible to constraints and outages) purchasers might be unwilling to rely too heavily on sourcing their electricity from this market, and so in practice one might observe purchasers bids to be closer to their expected demand than we predict.

It is interesting to speculate on whether purchasers bidding only a proportion of expected demand in a day-ahead market poses problems for market designers. In addition to the market for physical delivery, Nord Pool serves as a transparent exchange place for electricity derivatives.

The derivatives are written on the day-ahead price, and not on the regulating price. Thus, to maintain an efficient market for electricity derivatives, the prices and volumes dispatched in the day-ahead market should as far as possible reflect the true conditions of the power system. Due to this, the Norwegian system operator, Statnett, prohibits demand-side speculation in the regulating market, and requires that the purchasers bid for their expected demand in the day-ahead market. At the time of bidding, the actual consumption is however random, and it may be rather difficult for Statnett to assess whether or not a retailer has submitted an inaccurate bid on purpose. If consumers become more price-flexible in the short run, e.g. due to hourly metering getting more common (see e.g. [4]), such a judgement may be even more difficult to make.

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Article 5

Constructing bidding curves for a price-taking retailer in the Norwegian electricity market

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Abstract

We propose a stochastic linear programming model for constructing piecewise linear bidding curves to be submitted to Nord Pool, the Nordic power exchange. We consider the case of a price-taking power marketer who supplies electricity to price-sensitive end users. The objective is to minimize the expected cost of purchasing power from the day-ahead energy market and the short term balancing market. The model is illustrated using a case study with data from Norway.

1 Introduction

The Nordic electricity market, comprising Denmark, Finland, Norway and Sweden has several hundred retailers, competing to serve a population of about 24 million. This amounts to 385 TWh of electricity per year[1] at an average price of 163 NOK/MWh[2]. The Scandinavian market for retail electricity is very competitive. Retailers get their margins from buying wholesale and selling to end-users. Retail electricity is a higly standardized and homogenous good, and both retail and wholesale prices are transparent, with different websites keeping track of around 100 retail companies' prices. The difference between the cheapest and most expensive is low, considering the uncertainty in input and competitors' prices, and menu costs associated with changing prices. Keeping the costs low is essential for long-term survival as a retailer, and our model should be able to contribute to that goal, by aiming at the lowest level of balancing costs over time.

Although some consolidation is likely to occur, the industry is expected to stay reasonably competitive for many years. Another expected development in this industry is the degree to which two-way communication technology will be adopted. This involves hourly metering of each subscriber and some form of letting the end user know

[1] Figures from 2000. Source: [14].

[2] Average wholesale spot price in Trondheim region, 10 March 1997 - 16 December 2003.

the current price of power. Over time, given large price differences over the course of a day, and occurrences of price spikes, such a development will induce households to become more price responsive than today. In anticipation of this, we propose an optimization model for a price-taking retailer with end users who will reduce load if prices are high, and increase load if prices are low.

Bidding in electricity markets has been studied by [1], [7] and others, for players with market power. Bidding for price-taking producers has been studied by, e.g. [5] and [3]. We study bidding for a retailer, and in contrast to previous studies, we use piecewise linear bids. An more general assessment of how a retailer with market power should behave in an electricity pool market with a day-ahead structure is found in [11].

Today, retailers usually submit price insensitive bids to the spot market at Nord Pool. This is due to the fact that consumers are not exposed to short term price fluctuations, with monthly (or less often) metering being the norm. The bids are more or less close to expected demand, which is estimated by statistical techniques. Demand estimation takes into account factors such as past demand and forecasted temperature, which is important since more than 50% of electricity use in the Norwegian households is for heating. See [2] for a load forecasting model.

Some managers are speculating in the balancing price becoming either higher or lower than the spot price. If a manager is able to guess correctly on e.g. the balancing price becoming being larger than the spot price, he will benefit from submitting a curve that asks for a quantity larger than expected load. However, it is the policy of the system operator, Statnett, to require all participants to submit bids in accordance with the load each participant expects. If e.g. a retailer is discovered to deviate from this rule, he will receive a formal request to change bidding practices. This has happened in the past. Exactly how and when the retailer continuing to break Statnett's policy will be penalized, is not known.

Although this research is motivated by the needs of a Norwegian electricity retailer, we believe that the bidding model also can be applied by power marketers elsewhere, given a pool accepting piecewise linear bids, and where there is a short term balancing market whose prices

affect the retailer costs whenever he experiences an imbalance between the load planned day-ahead and the realized load.
In Section 2 we expain the market rules relevant for demand side bidding in the Norwegian market. In Section 3 we specify a stochastic linear program to be solved by the retailer to obtain a bidding curve that gives an optimal combination of expected cost and risk, according to the retailer's preferences. In Section 4 we describe our data set and explain how we generated scenarios, while we present and discuss some solutions to our model in Section 5. In Section 6 a brief conclusion is given.

2 The bidding process

For a Norwegian retailer, there are two important markets for physical exchange of electricity: the day-ahead market at Nord Pool, "elspot", and the regulating market organized by Statnett, the independent system operator. In the day-ahead market, producers and retailers submit price-quantity bids for buying and selling electricity every day before noon. The bids are specified for each of the next 12–36 hours, and Nord Pool subsequently collects all bids and calculates market clearing prices for each hour and each price area. Each country in the region constitutes a price area, with Norway divided further into two or three areas, and Denmark divided into two areas. A retailer only submits bids for areas where he has customers.
The regulating market is used by the system operator to ensure real-time balance between supply and demand. Only producers with an ability to ramp up or down significantly on a 15 minute notice are allowed to participate[3]. Whenever there is a load greater than was committed in the day-ahead market, there is a need for up-regulation, and vice versa. Statnett has collected bids for such up- and down-regulations for each participant and chooses to use the cheapest feasible source for such ramping.
When submitting an up-regulation bid, the producer obliges himself

[3]Retailers that are able to cut of some of their customers on a 15 minutes notice, may also participate in the regulating market. Demand-side bidding in the regulating market is still rather immature, however, so we leave that out for now.

to increase production by certain amounts at certain prices on a 15 minute notice. Up-regulating power must be offered at a higher price than the spot price. This implies that in an hour of up-regulation, the price in the regulating market will be higher than the spot price. Submitting down-regulation bids, means that the producer agrees to decrease production by certain amounts at certain prices on a 15 minute notice. Down-regulating power must be offered at a lower price than the spot price, which implies that in an hour of down-regulation, the price in the regulating market will be lower than the spot price.

If the actual consumption within a price area is equal to, or very close to the volume dispatched in the spot market for that region, there is no need for regulation. The price in the regulating market then equals the spot price.

Now, assume that the market is being up-regulated. We have explained that during an hour of up-regulation the regulating market price is higher than the spot price. Since there are many retailers operating within a price area, however, there is a chance that not all retailers are regulated in the same direction.

A retailer whose customers have used less electricity than the volume that particular retailer was dispatched at in the spot market, will be down-regulated. This retailer will have to sell some power back to the regulating market at the regulating price, which is higher than the spot price. In other words, that retailer has in this case actually made a profit by ordering more electricity in the spot market than his customers actually needed, and then selling it back to the market at a higher price.

Other retailers in the same area may be up-regulated, however, if they have ordered less electricity in the spot market than their customers actually consumes. They will have to purchase the excess demand in the regulating market at a price that is higher than the spot price. These retailers would have been better off if they had been dispatched at a higher volume in the spot market.

A similar argument may be used to explain how a retailer that is up-regulated individually would find it benefitial if the market is down-regulated. Also, a down-regulated market would disbenefit retailers that are down-regulated individually. In other words, it is advantageous for a retailer to be regulated in the opposite direction of the

market.
The Norwegian approach to handling imbalances between the spot market volumes and the actual consumption is different from the approach used in Denmark, Finland and Sweden.

3 Model specification

We define β and π to be the regulating market price and the day-ahead price, respectively. Based on the brief explanation of the regulating market design given in the introduction, we see that during up-regulation $\beta - \pi > 0$, during down regulation $\beta - \pi < 0$ and when there is no regulation, $\beta - \pi = 0$.
The retailer's cost of purchasing electricity may be written as

$$C = C_d + C_r \tag{1}$$

where C_d is the amount paid in the day-ahead market and C_r is the amount paid in the regulating market. They can be expressed as

$$\begin{aligned} C_d &= \pi y && \text{(2a)} \\ C_r &= (\lambda - y)\,\beta && \text{(2b)} \end{aligned}$$

where λ is the demand and y is the volume knocked down in the day-ahead market. Observe that the regulating cost C_r is negative if $(\lambda - y) < 0$. One may consider including benefit from selling electricity to end users here. However, the benefit is the result of two exogenous variables: end user price and load. Bidding does not influence the benefit of selling electricity, therefore this benefit is not included.
Now, we combine the equations (2) and get the following cost

$$C = \pi y + (\lambda - y)\,\beta = \lambda\beta - y\delta$$

where $\delta = \beta - \pi$ is the difference between the regulating price and the spot price.
The first term, $\lambda\beta$ does not include any decision variables. Hence, the retailer should seek to

$$\min\{-y\delta\}$$

which is equivalent to minimizing the regulating market loss, $\gamma = (\lambda - y)\,\delta$. This relation denotes the money lost by purchasing the excess demand in the regulating market instead of in the spot market. Note that γ may well be negative, implying that the retailer may make extra profits in the regulating market.
Since the expected cost turns out to be negative in the case study, we have chosen to turn this around, so that the retailer's objective is to maximize the regulating market profit, or

$$\max y\delta. \tag{3}$$

The expression (3) is the main part of the objective function in our optimization model.

3.1 Bidpoints

To obtain a piecewise linear strictly decreasing curve for the day-ahead auction, which is consistent with the bidding rules on the Nord Pool day-ahead market, the retailer submits n price-volume pairs $(P_0, x_0), (P_1, x_1), \ldots, (P_n, x_n)$, where $P_0 \leq P_1 \leq \cdots \leq P_n$ and $x_0 \geq x_1 \geq \cdots \geq x_n$ to the pool. A linear interpolation between the pairs (P_i, x_i) and (P_{i+1}, x_{i+1}), $i = 1, \ldots, n-1$ gives the resulting $n-1$ line segments that decide at what price π and volume y the retailer is dispatched. Figure 1 shows an example of such a bidding curve with three line segments. If the day-ahead price is π^*, the retailer is dispatched at the volume y^*.

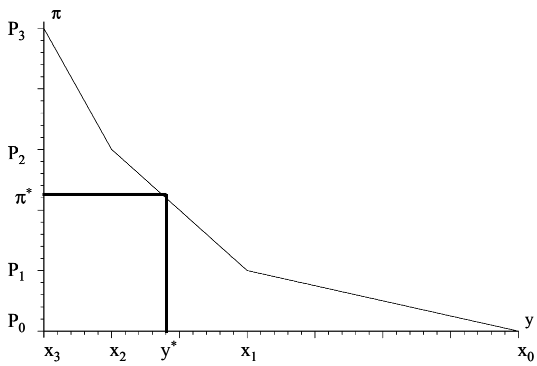

Figure 1: A possible bidding curve with three line segments. The retailer submits the points (x_i, P_i), $i = 0, \ldots, 3$ to the pool and the bidding curve emerges from a linear interpolation between those points.

The bidding curve is described by the following relation

$$\pi = \begin{cases} P_1 + \frac{P_0 - P_1}{x_0 - x_1}(y - x_1) & \text{if } x_1 < y \le x_0 \\ P_2 + \frac{P_1 - P_2}{x_1 - x_2}(y - x_2) & \text{if } x_2 < y \le x_1 \\ \vdots & \vdots \\ P_{n-1} + \frac{P_{n-2} - P_{n-1}}{x_{n-2} - P_{n-1}}(y - x_{n-1}) & \text{if } x_{n-1} < y \le x_{n-2} \\ P_n + \frac{P_{n-1} - P_n}{x_{n-1} - x_n}(y - x_n) & \text{if } x_n \le y \le x_{n-1} \end{cases} \tag{4}$$

Assuming that the retailer is a price taker, the relevant decision problem for him is to select volumes at which to be dispatched at at the different prices. As is apparent from the relation (4), the problem of selecting values of both P_i and x_i, $i = 1, \ldots, n$ is nonlinear.
To ease the solving of the problem, we make the model linear by fixing the price points $P_0, \ldots, P_n$ in advance. One plausible way of deciding pre-determined values of the price points is to require that

$$\Pr\{P_i \le \pi^* \le P_{i+1} \cup x_{i+1} \le y^* \le x_i\} = \frac{1}{n-1} \; \forall \, i \in \mathcal{I}. \tag{5}$$

Hence, the probability of being dispatched on any of the $n-1$ line segments is the same. This may be done by sorting the scenarios

generated by price, π_s, $s \in \mathcal{S}$, where $\mathcal{S}$ is the set of scenarios, and fix the values of P_i such that the number of scenarios between any price-volume pair is the same.
Since our problem is to select the volumes x_i corresponding to P_i, we rewrite the relation (4) to make it linear in x:

$$y_s = \begin{cases} \left(1 - \frac{\pi_s}{P_1 - P_0} + \frac{P_0}{P_1 - P_0}\right) x_0 + \left(\frac{\pi_s}{P_1 - P_0} - \frac{P_0}{P_1 - P_0}\right) x_1 \\ \hfill \text{if } P_0 \le \pi_s < P_1 \\ \left(1 - \frac{\pi_s}{P_2 - P_1} + \frac{P_1}{P_2 - P_1}\right) x_1 + \left(\frac{\pi_s}{P_2 - P_1} - \frac{P_1}{P_2 - P_1}\right) x_2 \\ \hfill \text{if } P_1 \le \pi_s < P_2 \\ \vdots \\ \left(1 - \frac{\pi_s}{P_n - P_{n-1}} + \frac{P_{n-1}}{P_n - P_{n-1}}\right) x_{n-1} + \left(\frac{\pi_s}{P_n - P_{n-1}} - \frac{P_{n-1}}{P_n - P_{n-1}}\right) x_n \\ \hfill \text{if } P_{n-1} \le \pi_s \le P_n \end{cases} \tag{6}$$

In the relation (6) we have included the subscripts s to point out that y and π are scenario dependent. As indicated, we will generate scenarios for the spot price π.
For each scenario the day-ahead price, π_s, will lie between two certain price-points P_i and P_{i+1}. All the other $n-2$ price points are irrelevant for computing the knocked down volume y_s for that scenario. Let $i(s)$ denote the largest i for which $P_i < \pi_s : i(s) = \sup_{i \in \mathcal{I}} \{i | P_i < \pi_s\}$. Hence, the point of dispatch, (y_s, π_s), will lie on a line segment on the bidding curve that is succinctly described by the linear interpolation between the price-volume pairs $\left(P_{i(s)}, x_{i(s)}\right)$ and $\left(P_{i(s)+1}, x_{i(s)+1}\right)$. The relation (6) can now be written as

$$y_s = \left(1 - \frac{\pi_s}{P_{i(s)+1} - P_{i(s)}} + \frac{P_{i(s)}}{P_{i(s)+1} - P_{i(s)}}\right) x_{i(s)} + \left(\frac{\pi_s}{P_{i(s)+1} - P_{i(s)}} - \frac{P_{i(s)}}{P_{i(s)+1} - P_{i(s)}}\right) x_{i(s)+1}$$

The corresponding volume y_s dispatched in each scenario $s \in \mathcal{S}$ is then derived by relation (6).

3.2 Consumption

As explained in the introduction, the goal of this paper is to present a model for a retailer whose customers' demand, λ, is price-flexible. A commonly used demand curve in the economics literature is a constant elasticity curve. Assume that for some price π_0, the customers have an expected demand $E[\lambda] = \lambda_0$, given the expected weather conditions. Then, the expected demand at price π is derived by

$$E[\lambda] = \lambda_0 \left(\frac{\pi}{\pi_0} \right)^{\eta}$$

where η is the price-elasticity.
Since the bidding curve for a certain hour is submitted to the pool between twelve to thirty-six hours ahead of the physical dispatch, the actual consumption is random. For example, the weather conditions may change, and even if they do not, the customers' behaviour will be stochastic[4]. We derive the consumption in each scenario by adding a prediction error term, ε_s, for which we generate scenarios. The consumption, λ_s, is then derived by the relation

$$\lambda_s = \lambda_0 \left(\frac{\pi_s}{\pi_0} \right)^{\eta} + \varepsilon_s. \tag{7}$$

The volume deviation in scenario s is then given by $\lambda_s - y_s$.

3.3 Risk

The retailer will be subject to two different risk factors which may influence his behaviour in the day-ahead market.
The first risk factor is the profit risk, which is simply the risk of making less money than desired on the end user sales. If both the volume deviation and δ are large and have the same sign, the retailer may lose a considerable amount of money during one single hour, potentially causing liquidity problems. However, the balance settlements

[4]The firm which supplies the most commonly used model for predicting consumption in Norway, asserts that the model has an average error of 2% when the weather conditions are known.

are made a few weeks after the physical dispatch, and then the regulating costs are charged for one full week. Hence, the actual invoice (or credit memo) from the system operator will include the regulating costs for 168 hours. If we assume that our model gives results that are optimal in the long run, we find it fair to assume that this risk is rather irrelevant. In the model presented here, we have assumed the retailer to be risk neutral. Later in this section, however, we suggest how to include this risk if so desired.

The second risk factor is the volume deviation risk. Since all derivatives are quoted with respect to the day-ahead price, the system operator wants the day-ahead market to reflect the physical conditions. In short; they do not like demand-side speculation in the regulating market. Therefore, they are inclined to take measures towards retailers that are suspected of bidding too high or too low volumes in the day-ahead market on purpose.

According to our data, $E[\delta] < 0$, which implies that the market is down-regulated on expectation. If it were not for the volume deviation risk, a risk neutral retailer would therefore order nothing in the day-ahead market and purchase the entire demand in the regulating market at a price that is likely to be lower than the day-ahead price. Hence, without possible volume deviation punishment, the day-ahead market would be rendered obsolete if the retailers are risk neutral.

An alternative way of modeling risk is by using shortfall costs, as e.g. in [6]. This may be done by defining variables, $w_{1s}^+, w_{2s}^+, \dots, w_{ms}^+$ and $w_{1s}^-, w_{2s}^-, \dots, w_{ms}^-$. When the retailer is up-regulated, $\sum_{m\in\mathcal{M}} w_{ms}^+ = \lambda_s - y_s$, and when he is down-regulated, $\sum_{m\in\mathcal{M}} w_{ms}^- = y_s - \lambda_s$. Now, we let $T_m^+ > 0$ and $T_m^- > 0$ denote the marginal cost of piece m on the volume deviation risk function for positive and negative deviations, respectively. Then, we may penalize volume deviations in the objective function by adding the term

$$-V \sum_{m\in\mathcal{M}} \left(T_m^+ w_{ms}^+ + T_m^- w_{ms}^-\right)$$

where V quantifies the retailer's aversion to the volume deviation risk. This term will penalize volume deviations more the higher they get, and if we define $T_1^\pm < T_2^\pm < \cdots < T_m^\pm$, the marginal penalty will increase with increasing deviations. Also, this would require the following constraints

$$\sum_{m \in \mathcal{M}} w_{ms}^{+} + y_s \geq \lambda_s \;\; \forall \;\; s \tag{8a}$$

$$\sum_{m \in \mathcal{M}} w_{ms}^{-} - y_s \geq -\lambda_s \;\; \forall \;\; s. \tag{8b}$$

$$0 \leq w_{ms}^{\pm} \leq W_m \; \forall \; m, s \tag{8c}$$

By using shortfall costs, the profit risk could also be plausibly modeled. We define the variables $u_{1s}, u_{2s}, \ldots, u_{ks}$, with the constraint

$$\sum_{k \in \mathcal{K}} u_{ks} \geq B - (y_s - \lambda_s)\, \delta_s$$
$$0 \leq u_{ks} \leq U_k \; \forall \; m, s$$

where B defines a target for the regulating market profit in each hour. If the regulating profit exceeds the budget, the constraint is not active. Next, we let C_k denote the marginal cost of piece k on the profit risk function. Then the term

$$-R \sum_{k \in \mathcal{K}} C_k u_{ks},$$

where R denotes the retailer's aversion to profit risk, is added to the objective function.
As mentioned, losses incurred during one hour is unlikely to be a big issue to the retailer. A loss in one hour may be levelled by a gain the next hour, and therefore, it may seem a bit irrelevant to impose an hour by hour budget. Nevertheless, big losses may be unpleasant even in just a single hour. Also, if the company has suffered heavy regulating market losses recently, it may want to take back lost ground. The retailer would perhaps want to change B on a regular basis, e.g. each day, based on previous performance.

3.4 Full model

Now, using the standard deviation approach to risk, the full model looks as follows.

$$\max \sum_{s \in \mathcal{S}} \rho_s \left\{ y_s \delta_s - V \sum_{m \in \mathcal{M}} \left(T_m^{+} w_{ms}^{+} + T_m^{-} w_{ms}^{-} \right) \right\} \tag{9}$$

subject to

$$y_s = \left(1 - \frac{\pi_s}{P_{i(s)+1} - P_{i(s)}} + \frac{P_{i(s)}}{P_{i(s)+1} - P_{i(s)}}\right) x_{i(s)}$$
$$+ \left(\frac{\pi_s}{P_{i(s)+1} - P_{i(s)}} - \frac{P_{i(s)}}{P_{i(s)+1} - P_{i(s)}}\right) x_{i(s)+1} \ \forall \ s \tag{10a}$$
$$\sum_{m \in M} w^+_{ms} + y_s \geq \lambda_s \quad \forall \quad s \tag{10b}$$
$$\sum_{m \in M} w^-_{ms} - y_s \geq -\lambda_s \quad \forall \quad s. \tag{10c}$$
$$0 \leq w^\pm_{ms} \leq W_m \ \forall \ m, s \tag{10d}$$
$$x_i \geq x_{i+1} \ \forall \ i \in \mathcal{I} \tag{10e}$$
$$x_i \geq 0 \ \forall \ i \in \mathcal{I} \tag{10f}$$

where we have the following indices and sets
s, $\mathcal{S}$ count the scenarios
i, $\mathcal{I}$ count the price-volume pairs,
the following stochastic parameters
π is the spot price
λ is the load over the retailer's customers
δ is the difference between the regulating market price and the spot price
ρ is the probability of scenario s,
the following deterministic parameters
P_i are the price points on the price grid, $P_i < P_{i+1}$,
V is the weight on volume deviation risk in the objective function
W_m is the size of segment m on the piecewise linear volume deviation cost function the following variables
x_i is the volume corresponding to P_i, $x_i \geq x_{i+1}$
y is the volume dispatched in the day-ahead market.

4 Scenario generation

We have generated scenarios for spot price, π, volume deviation, ε, and for the difference between the regulating market price and the spot price, δ.

For π and δ, we have used hourly spot prices and regulating prices for Trondheim region in the period 10 March 1997 - 16 December 2003. From these data we have removed the prices in weekends and holidays, in addition to the prices between 24:00 hrs and 07:00 hrs. We removed the mentioned data, because we would like to consider bidding for a normal day-time hour. We were then left with 25 500 entries for π and δ.
For volume deviation, ε, we have used measured and estimated consumption for the concession area of Trondheim Energiverk Nett AS[5] (TEV) for every hour during the period 1 October 2002 - 31 March 2003. We subtracted estimated consumption from measured consumption to obtain ε, so that $\varepsilon > 0$ means that the retailer has been up-regulated, while the retailer has been down-regulated if $\varepsilon < 0$. As this is a rather short time series compared to those for π and δ, we have chosen not to remove any data. This leaves us with 4 367 entries for ε. If we assume that our model is for a retailer serving the entire TEV consession area, this retailer was regulated in the opposite direction of the market in 1 518 of the hours, while he was regulated in the same direction in 1 983 of the hours. In 866 there was no regulation ($\delta = 0$). Some descriptive statistics of the data set are shown in Table 1. Prices are listed in NOK/MWh, while volume deviations are listed in MWh.

	π	δ	ε
Number of entries	25500	25500	4376
Highest entry	3840.6	1193.9	58.52
Lowest entry	17.5	−3740.6	−70.86
Mean	178.5	−4.13	−1.34
Std. dev.	106.15	57.48	16.33

Table 1: Some descriptive statistics of the data set.

According to [12] the regulating price strongly depends on the level of the spot price. Also, with such a huge data set, we find it natural to divide the data into subsets so that we may get a more robust

[5]Trondheim Energiverk Nett AS is the network operator serving Trondheim and Klæbu municipals, an area with 157 887 inhabitants as per 1 January 2003. Source: [13].

treatment of data that are in the outer edges of our sample. Therefore, we have divided the price data, π and δ, into three sets with respect to π. The first set is made up by the 15% lowest values of π together with their corresponding values of δ. The second set is made up by the middle 70% values of π together with the corresponding values of δ, while the remaining highest 15% of the values of π with their corresponding values of δ form the third set. We denote these sets $\mathcal{T}_1$, $\mathcal{T}_2$ and $\mathcal{T}_3$, respectively.
To generate scenarios, we need to estimate the first four moments of the variables' distributions. However, in about 21% of the 25 500 hours for which we have price data, there was no regulation, implying $\delta = 0$. This would significantly bias the estimates of the moments for δ, and therefore, we need to treat hours with no regulation separately. When there is regulation, that is $\delta \neq 0$, it is the absolute value of δ that counts towards the retailer's regulating cost. Hence, estimating moments for all $\delta \neq 0$ together, would give a mean which is too close to zero. One way of dealing with this could be to estimate moments based on $|\delta|$. However, both [12] and our own analysis indicate that that the characteristics of $|\delta|$ is influenced by the direction of regulation.
Due to the above mentioned reasons we divided each of the sets $\mathcal{T}_1$, $\mathcal{T}_2$ and $\mathcal{T}_3$ into three subsets according to the direction of the regulating price. That is, we divided each set into one subset for $\delta < 0$ (down-regulation), one subset for $\delta > 0$ (up-regulation) and one set for $\delta = 0$ (no regulation). We denote these subsets $\mathcal{T}_{l,h}$, for $h = a, b, c$, respectively,with $l = 1, 2, 3$. Hence, the subset $\mathcal{T}_{1,a}$ includes the price pairs for down-regulation among the 15% lowest spot prices.
As mentioned, our time series for ε is not large enough to get one ε for each pair of π and δ. Therefore, we have estimated only one set of moments for ε, which has been added to the nine series with pairs of π and δ. Since we are dealing with a price-taking retailer, we may assume that the volume deviation of this particular retailer has no influence on the prices. Hence, the fact that our volume data are a bit more sparesome than the price data should not disturb our analysis significantly, even though it would have been preferable to have equal time series of data for all three variables.
In Table 2 we have listed the number of entries in each subset. The table also shows some probabilities. The column $\Pr\{h|\mathcal{T}_l\}$ lists the

probability of the market being regulated in direction h, $h = a, b, c$ for set $\mathcal{T}_l$, $l = 1, 2, 3$. We see that the probability of down-regulation is significantly higher for the lowest 15% of spot prices, that is the set $\mathcal{T}_1$. The probability of up-regulation is significantly higher for the highest spot prices than for the lowest spot prices. The probability of no regulation seems quite stable.

The column $\Pr\{\mathcal{T}_{l,h}\}$ lists the probability of a randomly selected pair of price data, π and δ, belonging to the subset $\mathcal{T}_{l,h}$, $l = 1, 2, 3$ and $h = a, b, c$. This statistic is calculated as $\Pr\{\mathcal{T}_{l,h}\} = \Pr\{h|\mathcal{T}_l\} \Pr\{\mathcal{T}_l\}$, $l = 1, 2, 3$ and $h = a, b, c$, where $\Pr\{\mathcal{T}_1\} = \Pr\{\mathcal{T}_3\} = .15$ and $\Pr\{\mathcal{T}_2\} = .7$.

Subset	*Number of entries*	$\Pr\{h\|\mathcal{T}_l\}$	$\Pr\{\mathcal{T}_{l,h}\}$
$\mathcal{T}_{1,a}$	1729	0.452	0.0678
$\mathcal{T}_{1,b}$	1185	0.310	0.0464
$\mathcal{T}_{1,c}$	911	0.238	0.0357
$\mathcal{T}_{2,a}$	7234	0.405	0.284
$\mathcal{T}_{2,b}$	6885	0.388	0.270
$\mathcal{T}_{2,c}$	3730	0.209	0.146
$\mathcal{T}_{3,a}$	1347	0.352	0.0528
$\mathcal{T}_{3,b}$	1666	0.435	0.0653
$\mathcal{T}_{3,c}$	813	0.212	0.0319

Table 2: The number of entries in each subset, and probabilities.

Table 3 shows descriptive statistics for the variable δ. We observe that the mean δ at down-regulation is much lower when the spot price is high ($\mathcal{T}_{3,a}$) than otherwise. Also, the absolute value of the mean of δ is a bit higher for down-regulation than for up-regulation in all sets. The means are listed in NOK/MWh. Furthermore, we observe very high kurtosis for $\mathcal{T}_{2,b}$ and $\mathcal{T}_{3,a}$, which is due to some extreme events in these subsets. For the lowest spot prices, δ seems to be quite stable with all four moments relatively low. Obviously, all moments equal zero when there is no regulation.

Table 4 shows descriptive statistics for π. We observe that the variability of π seems to show of some more variation in for the set of high spot prices, $\mathcal{T}_3$. The high positive skewness and the high kurtosis

Subset	*Mean*	*Std. dev.*	*Skewness*	*Kurtosis*
$\mathcal{T}_{1,a}$	−28.8	13.0	−0.400	2.99
$\mathcal{T}_{1,b}$	23.2	18.1	1.77	7.51
$\mathcal{T}_{1,c}$	0	0	0	0
$\mathcal{T}_{2,a}$	−31.0	16.9	−2.05	12.9
$\mathcal{T}_{2,b}$	25.8	41.2	17.3	429
$\mathcal{T}_{2,c}$	0	0	0	0
$\mathcal{T}_{3,a}$	−76.5	187	−13.3	216
$\mathcal{T}_{3,b}$	38.2	68.0	6.50	56.1
$\mathcal{T}_{3,c}$	0	0	0	0

Table 3: Descriptive statistics for δ.

in this set indicates that some of the spot prices in this set are significantly higher than the mean. The mean spot prices are listed in NOK/MWh.

Subset	*Mean*	*St. dev.*	*Skewness*	*Kurtosis*
$\mathcal{T}_{1,a}$	83.8	21.1	−0.700	2.49
$\mathcal{T}_{1,b}$	83.1	22.0	−0.634	2.51
$\mathcal{T}_{1,c}$	82.6	23.3	−0.770	2.68
$\mathcal{T}_{2,a}$	158	35.7	0.955	3.24
$\mathcal{T}_{2,b}$	161	37.6	0.940	3.12
$\mathcal{T}_{2,c}$	161	37.6	0.798	2.89
$\mathcal{T}_{3,a}$	390	200	8.22	111
$\mathcal{T}_{3,b}$	363	129	3.22	18.8
$\mathcal{T}_{3,c}$	376	155	3.38	17.8

Table 4: Descriptive statistics for π.

Table 5 shows descriptive statistics for ε. The mean prediction error is measured in MWh.

To generate scenarios, we also need the correlations between the variables. In subset $\mathcal{T}_{2,a}$, the correlation between π and δ was estimated to $-.435$. Apart from that, the correlations were all rather low. We have chosen not to include the correlation matrices in the text.

Based on the descriptive statistics presented in tables 3, 4 and 5 we

	Mean	*St. dev.*	*Skewness*	*Kurtosis*
All subsets	-1.34	16.3	$1.87 * 10^{-4}$	$4.12 * 10^{-3}$

Table 5: Descriptive statistics for ϵ.

generated scenarios for each subset by using the method described in [8]. The advantage of this method is that it is quite simple since one only needs the first four moments of each variable, plus the correlations. This requires, however, that the distributions and the relationships between them may be succinctly described by those statistics. For most sets we generated 100 scenarios. For the sets $\mathcal{T}_{2,b}$ and $\mathcal{T}_{3,a}$, however, the algorithm used did not converge for 100 scenarios, possibly due to their relatively extreme skewnesses and kurtosis. For those, we generated 600 and 300 scenarios, respectively.

5 Solution

The specification of the volume deviation penalty function is necessarily ad hoc. We chose to work with the piecewise linear penalty function as shown in Figure 2.

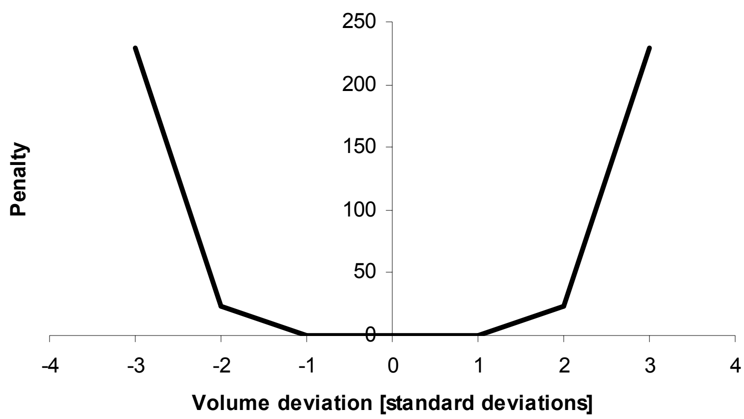

Figure 2: Volume deviation penalty function.

If the realized load is larger than one standard deviation from its expected value, one starts penalizing using a marginal penalty of 22.9, equal to the expected spot price. This holds for deviations up to two standard deviations. From two standard deviations and up, the marginal penalty is increased by a factor of ten.
We modelled and solved the linear program using XPRESS [4]. A typical problem was compiled and solved in about 16 seconds on a 1 GHz Pentium PC with 524 MB RAM. No special effort was made to make the code efficient with respect to solution time, as only a fraction of the time mentioned (ca. 0.5 s) is used in the dual simplex algorithm. This problem is a simple recourse problem for which there exists special purpose algorithms, see e.g. [9]. Such algorithms may become useful if the model is expanded.
An example of an optimal bidding curve is shown in Figure 3. This curve has 7 line-pieces. Also shown is the expected load curve and the load-price scenarios.

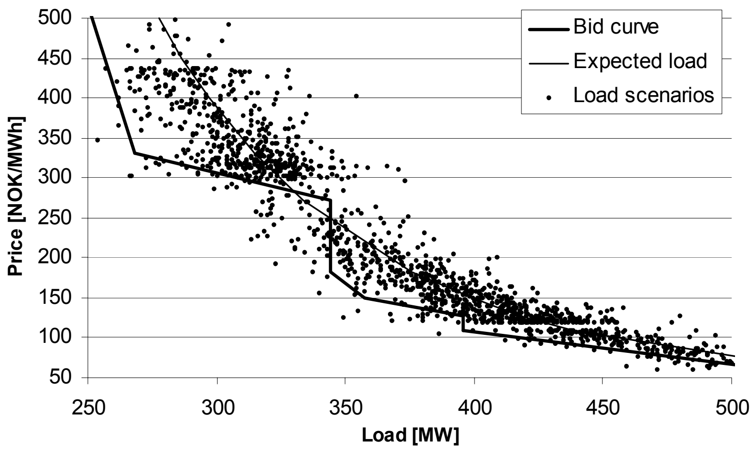

Figure 3: Example of optimal bid curve, expected load curve and load scenarios.

As mentioned, since $E[\delta] < 0$ in our data, the optimal curve asks for less power than the end users are expected to consume.
Using the data described in the previous section we selected $n = 64$

price points (63 line pieces) according to equation (bottom page 3). We do not know what the price elasticity of demand will be in a future with two-way communication. Thus we have constructed bidding curves with a range of elasticities from $\eta = -0.6$ to $\eta = 0$. See Figure 4:

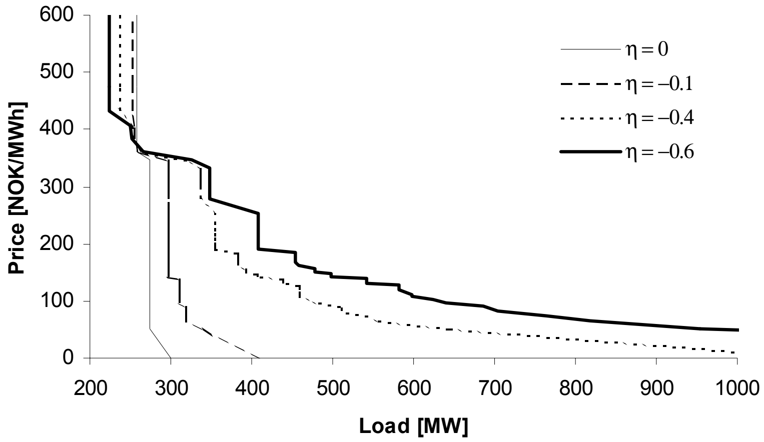

Figure 4: Bidding curves with varying price elasticity of demand.

Using a price elasticity of demand of $\eta = -0.3$, we show the effect of varying the number of fixed price points. See Figure 5:
If the number of price points is decreased, the result is a more crude bidding curve. Notice also that the bidding curves are cruder at prices that are less likely, e.g. above 500 NOK/MWh. This is reasonable and is due to equation (5).
Figure 6 shows the cumulative profit distribution, and illustrates the risk associated with bidding in this particular case. The expected profit is 212 NOK, a small amount. The risk is also relatively small, with a one-day 95% VaR of -1313 NOK.

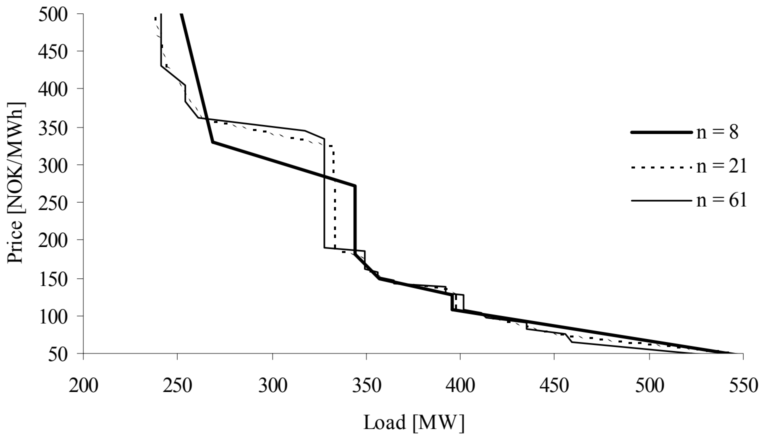

Figure 5: Bidding curves with a varying number of fixed price points.

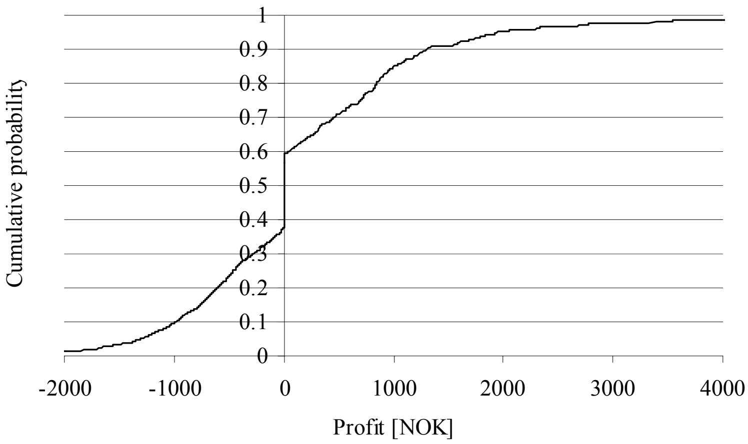

Figure 6: Cumuluative distribution function for profit.

Next we show the effect of varying the weight parameter V in the objective function, and also varying the number of fixed price points. See Figure 7:

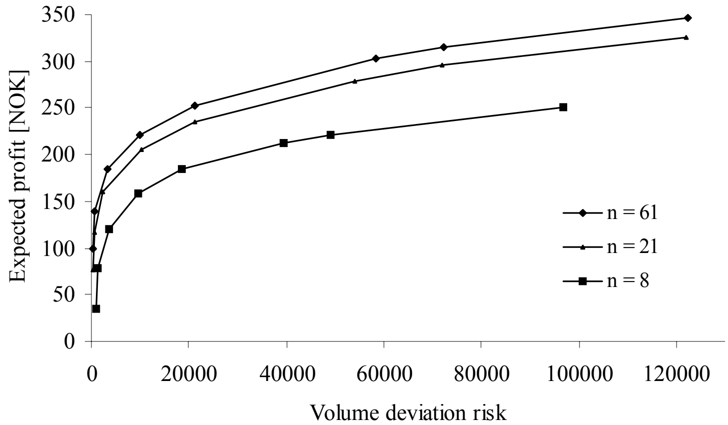

Figure 7: Tradeoff between expected profit and volume deviation risk.

Volume deviation risk is measured as the expected volume deviation penalty,
$\sum_{s\in\mathcal{S}} \rho_s \sum_{m\in\mathcal{M}} \left(T_m^+ w_{ms}^+ + T_m^- w_{ms}^-\right)$. We see that there is a tradeoff between expected profit and the expected penalty of the difference between volume bid and realized load. If the decision maker wants high expected profit, he has to submit a bid that deviates from the expected load. The larger the volume deviation, the higher the expected profit. From the figure we can also see that the retailer is better off by using many price points, and that the marginal benefit of increasing the number of price points is a decreasing function.

6 Conclusions and future research

In sum, we have suggested a model supporting the construction of a bidding curve that fits the rules of the Nord Pool day ahead market. The intended user is a retailer having end-users with price-sensitive demand. The case study indicates that the model is able to construct plausible bidding curves, and that there is a tradeoff between expected cost of purchasing electricity and risk of getting penalized from the system operator because bids deviate from expected load.

Model validation is left for future work. There is a growing number of tests one can make in order to learn about the quality of the model, e.g. regarding the generation of scenarios as explained by Kaut & Wallace (2003).

The model should be extended to support bidding for 24 hours simultaneously. A number of different issues will then emerge, for example how to account for possible dependency between stochastic variables across different hours.

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