This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TPWRD.2020.2965771, IEEE Transactions on Power Delivery

An Integrated Method for Generating VSCs' Periodical Steady-state Conditions and HSS-based Impedance Model

Chen Zhang, Marta Molinas, *Member, IEEE*, Sjur Føyen, Jon Are Suul, Takanori Isobe

***Abstract*—An integrated method for generating the system's periodical steady-state (PSS) conditions and the harmonic-state-space (HSS)-based impedance model is presented, referred to as the automatic-model-generation (AMG) method. This method is efficient for parametric impedance-based analysis (e.g. stability analysis) since it can precisely take the varying PSS into account and the algorithm of which can be readily implemented by the frequency-domain iteration. Application of this AMG method to the impedance acquisition and stability analysis of a single-phase grid-tied voltage-source-converter (VSC) along with experimental verifications is presented as an example. The presented results demonstrate how the AMG method can facilitate parametric stability assessments (e.g., under varying control parameters) in an efficient and accurate manner.**

***Index Terms*— converter, impedance, stability, periodical steady-state, Nyquist criterion**

I. Introduction

Voltage Source Converters (VSCs) are becoming ubiquitous in power systems for which the small-signal stability analysis is of significant importance. In this respect, the impedance-based method [1] is an effective tool for such analyses and is gaining popularity. However, impedance modeling for systems that cannot be readily represented by a time-invariant equivalent (e.g. the *dq*-model) is challenging, and this usually happens to systems with periodical steady-state (PSS) [2]. Since the PSS conditions are an essential part of the impedance modeling, they need to be calculated in accordance with operating conditions. This procedure is commonly fulfilled by time-domain simulations which is practical if the analysis is merely concerned with few specified conditions. As for parametric studies, impedance modeling using the simulated PSS conditions will become cumbersome and thus significantly compromises its efficiency in such analyses.

To address this issue, this letter presents an integrated approach for calculating the PSS conditions and the harmonic-state-space (HSS)-based impedance model, referred to as the *Automatic Model Generation* (AMG) method. Formulation and application of this method is presented in section II and III. Section IV draws the main conclusions.

II. Formulation of the AMG method

In principle, a nonlinear time-invariant system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}\left(\boldsymbol{x}, \boldsymbol{u}\right) \quad (1)$$

with constant input $\boldsymbol{u}$ can be solved iteratively for the steady-state operating point $\boldsymbol{x}_{\mathrm{ss}}$ by the Newton-Raphson method:

$$\begin{cases} \boldsymbol{f}\left(\boldsymbol{x}_{\mathrm{k}}, \boldsymbol{u}\right) + \dfrac{\partial \boldsymbol{f}\left(\boldsymbol{x}, \boldsymbol{u}\right)}{\partial \boldsymbol{x}}\Big|_{\boldsymbol{x}=\boldsymbol{x}_{\mathrm{k}}} \cdot \Delta \boldsymbol{x}_{\mathrm{k}} = 0 \\ \boldsymbol{x}_{\mathrm{k+1}} = \boldsymbol{x}_{\mathrm{k}} + \Delta \boldsymbol{x}_{\mathrm{k}} \end{cases} \quad (2)$$

According to (2), the solution is updated at each step using $\boldsymbol{x}_{\mathrm{k+1}} = \boldsymbol{x}_{\mathrm{k}} + \Delta \boldsymbol{x}_{\mathrm{k}}$, and this iterative process will be ended if the change of states is within a defined tolerance, i.e. $\boldsymbol{x}_{\mathrm{ss}} \approx \boldsymbol{x}_{\mathrm{k+1}}, \forall \left\| \Delta \boldsymbol{x}_{\mathrm{k}} \right\| < \sigma$.

For periodic input $\boldsymbol{u}$, since all the state-variables will be periodically time-varying at steady-state, i.e. PSS, the condition $\dot{\boldsymbol{x}}_{\mathrm{ss}} = 0$ for deriving the iterative solution (2) is no longer valid. Although equivalent time-invariant steady-state solutions are achievable for some specific systems by applying multiple Park transformations [3], the modeling process is complicated and is usually associated with model reductions.

To overcome this issue, the frequency-domain iteration [4] is adopted for better accuracy and the process of which is explained as follows.

First, the time-domain iterative solution of (1) at step *k* is

$$-\dot{\boldsymbol{x}}_{\mathrm{k}} - \Delta \dot{\boldsymbol{x}}_{\mathrm{k}} + \boldsymbol{f}\left(\boldsymbol{x}_{\mathrm{k}}, \boldsymbol{u}\right) + \underbrace{\frac{\partial \boldsymbol{f}\left(\boldsymbol{x}, \boldsymbol{u}\right)}{\partial \boldsymbol{x}}\Big|_{\boldsymbol{x}=\boldsymbol{x}_{\mathrm{k}}}}_{\boldsymbol{A}_{\mathrm{k}}(t)} \cdot \Delta \boldsymbol{x}_{\mathrm{k}} = 0 \quad (3)$$

By applying the Fourier expansions and the principle of harmonic balance, its frequency-domain iterative model is

$$\begin{cases} -\tilde{\boldsymbol{\mathcal{N}}}_{\mathrm{blk}} \tilde{\boldsymbol{\mathcal{X}}}_{\mathrm{k}} + \tilde{\boldsymbol{\mathcal{F}}}_{\mathrm{k}}\left(\boldsymbol{x}_{\mathrm{k}}, \boldsymbol{u}\right) + \left(-\tilde{\boldsymbol{\mathcal{N}}}_{\mathrm{blk}} + \tilde{\boldsymbol{\mathcal{A}}}_{\mathrm{k}}\right) \Delta \tilde{\boldsymbol{\mathcal{X}}}_{\mathrm{k}} = \boldsymbol{0} \\ \tilde{\boldsymbol{\mathcal{X}}}_{\mathrm{k+1}} = \tilde{\boldsymbol{\mathcal{X}}}_{\mathrm{k}} + \Delta \tilde{\boldsymbol{\mathcal{X}}}_{\mathrm{k}} \end{cases} \quad (4)$$

where $\tilde{\boldsymbol{\mathcal{X}}}_{\mathrm{k}}$, $\tilde{\boldsymbol{\mathcal{F}}}_{\mathrm{k}}\left(\boldsymbol{x}_{\mathrm{k}}, \boldsymbol{u}_{\mathrm{k}}\right)$ are vectors of Fourier coefficients of $\boldsymbol{x}_{\mathrm{k}}$ and $\boldsymbol{f}\left(\boldsymbol{x}_{k}, \boldsymbol{u}\right)$. $\tilde{\boldsymbol{\mathcal{A}}}_{\mathrm{k}}$ is the Toeplitz matrix of $\boldsymbol{A}_{k}\left(t\right)$ and $\tilde{\boldsymbol{\mathcal{N}}}_{\mathrm{blk}} = diag\left(-N\boldsymbol{I}\mathrm{j}\omega_{1}, ..., \boldsymbol{0}, ..., N\boldsymbol{I}\mathrm{j}\omega_{1}\right)$ is a diagonal matrix.

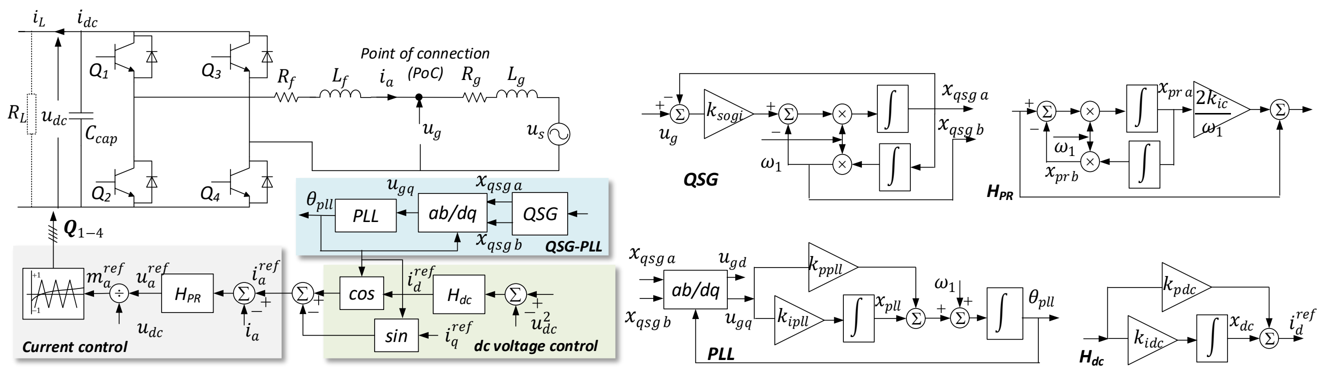
This work was supported by Norwegian University of Science and Technology (NTNU) under the grant NTNU Energy (81617922).

C. Zhang, M. Molinas, J. A. Suul are with the Department of Engineering Cybernetics, NTNU, 7034, Trondheim, Norway (email: chen.zhang@ntnu.no; marta.molinas@ntnu.no; jon.are.suul@ntnu.no).

Sjur Føyen is with the Department of Electric Power Engineering, NTNU, 7034, Trondheim, Norway, email: foyen.sjur@ntnu.no.

T. Isobe is with the Faculty of Pure and Applied Sciences, University of Tsukuba, Ibaraki 305-8573, Japan (email: isobe.takanori.gf@u.tsukuba.ac.jp).

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TPWRD.2020.2965771, IEEE Transactions on Power Delivery



(a) structure of overall system

(b) Control blocks

Fig. 1 Schematic of a single-phase grid-VSC system

Based on (4), the Fourier coefficients of steady-state $\boldsymbol{x}$ (i.e. $\tilde{\mathcal{X}}_{ss}$) is obtained if this condition $\tilde{\mathcal{X}}_{ss} \approx \tilde{\mathcal{X}}_{k+1}, \forall \left\|\Delta\tilde{\mathcal{X}}_{k}\right\| < \sigma$ is met. Moreover, the characteristic matrix of an HSS model (i.e. $\tilde{\mathcal{A}}_{k}$) can be obtained exactly from the same iteration.

This trait clearly indicates that the PSS conditions along with the HSS model can be derived in one unified algorithm, i.e. the frequency-domain iteration. Then, by simply inserting the Laplace variable s, the final HSS model with updated PSS conditions can be generally formulated as [2]:

$$\Delta\tilde{\mathcal{X}} = \left(s\mathcal{I} + \tilde{\mathcal{N}}_{blk} - \tilde{\mathcal{A}}_{k}\right)^{-1} \cdot \Delta\tilde{\mathcal{U}} \tag{5}$$

Next, it will demonstrate how this AMG method is applied for the impedance acquisition and parametric stability analysis of a single-phase grid-VSC system.

III. Application of the AMG Method to a Single-phase Grid-VSC System

A. Calculation of the PSS conditions

According to the configuration in Fig. 1, the state-space model of the evaluated single-phase VSC can be represented by

$$\dot{\boldsymbol{x}}_{c} = \boldsymbol{f}\left(x_{c}\right) + \boldsymbol{B}_{c} \cdot u_{a} \tag{6}$$

where $\boldsymbol{x}_{c} = [u_{dc}, i_{a}, x_{dc}, x_{pra}, x_{prb}, x_{qsga}, x_{qsgb}, x_{pll}, \delta_{pll}]^{T}$ is the state vector; $\boldsymbol{B}_{c}$ is a constant input matrix; the output current can be extracted from $\boldsymbol{x}_{c}$ by defining a constant matrix $\boldsymbol{C}_{i}$ = [0 1 $\boldsymbol{0}_{1x7}$].

Based on the AMG method, the frequency-domain iterative model of (6) at the step k can be written

$$\begin{aligned}&\tilde{\mathcal{N}}_{c,blk}\tilde{\mathcal{X}}_{c,k} - \tilde{\mathcal{F}}_{c,k} = \\ &\left(-\tilde{\mathcal{N}}_{c,blk} + \tilde{\mathcal{A}}_{c,k}\right)\Delta\tilde{\mathcal{X}}_{c,k} + \tilde{\mathcal{B}}_{c,k}\left(\tilde{\mathcal{U}}_{a,k} + \Delta\tilde{\mathcal{U}}_{a,k}\right)\end{aligned} \tag{7}$$

and the frequency-domain iterative model for the Thevenin equivalent grid can be similarly derived as

$$\left(\tilde{\mathcal{N}}_{g,blk}L_{g} + R_{g}\mathcal{I}\right)\tilde{\mathcal{C}}_{i}\left(\tilde{\mathcal{X}}_{c,k} + \Delta\tilde{\mathcal{X}}_{c,k}\right) = \tilde{\mathcal{U}}_{g,k} + \Delta\tilde{\mathcal{U}}_{g,k} \tag{8}$$

where $\tilde{\mathcal{C}}_{i}$ is the Toeplitz matrix of $\boldsymbol{C}_{i}$.

Combining (7) and (8), a closed-loop frequency-domain iterative model can be formulated, based on which the Fourier coefficients $\tilde{\mathcal{X}}_{c,k}$ can be solved iteratively. For this process, corresponding time-domain waveforms are obtained by applying the inverse Fourier transform. For illustration, the steady-state dc voltage and ac current waveforms obtained from the AMG method are compared with simulated waveforms from the PSCAD/EMTDC in Fig. 2. As seen from the figure, the AMG results are well-matched with the simulations, proving that the frequency-domain iteration of the AMG is effective and accurate.

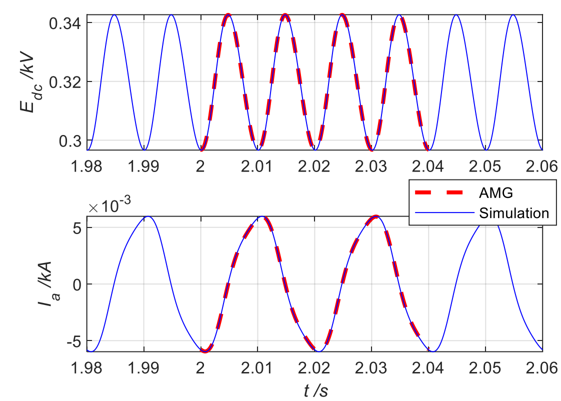


Fig. 2 Comparison of the calculated and simulated steady states of the dc voltage and ac current (I_{qref} = -6 A, R_{L} = 1e5 ohm)

B. Generation of the HSS-based impedance model and its SISO equivalent representation

1) Generation of the HSS-based impedance model

Since the state-matrix $\tilde{\mathcal{A}}_{c}$ is generated in the same iteration, then the HSS-based VSC admittance can be derived as

$$-\Delta\tilde{\boldsymbol{I}}_{a} = \underbrace{-\tilde{\mathcal{C}}_{i}\left(s\mathcal{I} + \tilde{\mathcal{N}}_{c,blk} - \tilde{\mathcal{A}}_{c}\right)^{-1}\tilde{\mathcal{B}}_{c}}_{\tilde{\mathcal{Y}}_{c}(s)} \cdot \Delta\tilde{\mathcal{U}}_{a} \tag{9}$$

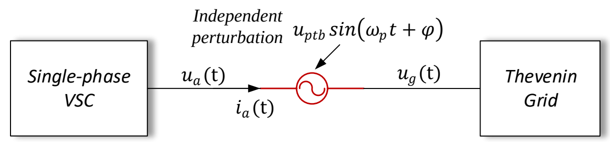
The HSS-based grid impedance is straightforwardly given as

$$\Delta\tilde{\mathcal{U}}_{g} = \underbrace{diag\left(Z_{-N}\left(s\right),...,Z_{0}\left(s\right),...,Z_{N}\left(s\right)\right)}_{\tilde{\mathcal{Z}}_{g}(s)} \cdot \Delta\tilde{\boldsymbol{I}}_{a} \tag{10}$$

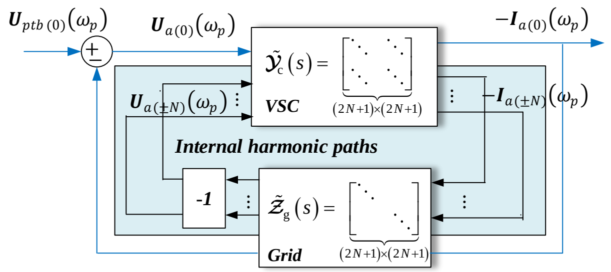
where $\Delta\tilde{\boldsymbol{I}}_{a} = [\Delta I_{a(-N)}(s),...,\Delta I_{a(0)}(s),...,\Delta I_{a(N)}(s)]^{T}$, similarly for $\Delta\tilde{\mathcal{U}}_{g}, \Delta\tilde{\mathcal{U}}_{a}$. If there are no independent voltage perturbations between the grid and VSC, then $\Delta\tilde{\mathcal{U}}_{g} = \Delta\tilde{\mathcal{U}}_{a}$ holds true.

The above HSS-based impedance must be truncated to be numerically tractable. In this study, $N = \pm 3$ is selected, however, the resulting HSS-based impedances are still of large dimensions which are 7x7 matrices. To make the

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TPWRD.2020.2965771, IEEE Transactions on Power Delivery



(a) Circuit representation of the grid-VSC system



(b) Harmonic domain block diagram of the perturbed grid-VSC system

Fig. 3 Illustration of the harmonic interactions of the single-phase grid-VSC system from a closed-loop perspective

implementation and analysis easier, a single-input and single-output (SISO) equivalent impedance will be further developed from the HSS-based impedance.

2) Derivation of the SISO equivalent model

The approach employed for developing the SISO equivalent model is based on [5]. Since the implementation is more complicated for a single-phase VSC which inherently exhibits more frequency couplings than the three-phase VSC discussed in [5], the procedure is briefly outlined in the following.

As shown in Fig. 3 (a), if an independent voltage perturbation is applied to the VSC, it will not only present the frequency response to the perturbation frequency but also give rise to many other frequencies. These frequencies will again be coupled to the VSC's inputs through the grid interaction, as indicated by the internal harmonic paths in Fig. 3(b). Since the above small-signal representation of the grid-connected VSC forms a closed-loop system and it is composed of 2(2N+1)+1 number of linear equations and 2(2N+1)+2 number of unknowns, the relationship between any two of the unknowns can be found by solving those linear equations. In this analysis, the following relationships are found

$$\begin{cases} -\Delta \boldsymbol{I}_{\mathrm{a}(0)}(s)=Y_{\mathrm{c}}^{\mathrm{SISO}}(s)\cdot\Delta \boldsymbol{U}_{\mathrm{a}(0)}(s) \\ \Delta \boldsymbol{U}_{\mathrm{g}(0)}(s)=Z_{\mathrm{g}}^{\mathrm{SISO}}(s)\cdot\Delta \boldsymbol{I}_{\mathrm{a}(0)}(s) \end{cases} \tag{11}$$

which are the VSC-SISO admittance and grid-SISO impedance. Notably, since all the frequency couplings are inherently considered in the calculation, the resulting SISO impedances are essentially low-dimensional equivalents to the HSS-based impedances, i.e. no loss of model accuracy.

3) VSC-SISO admittance validation

Next, the VSC-SISO admittance obtained from the AMG method will be compared with simulated and experimental frequency scanning. For this purpose, the single-tone injection-based method is adopted, i.e. only one frequency component is injected for each time interval. Then, this procedure is repeated to achieve the frequency scanning. Discrete Fourier analysis is conducted in MATLAB (with 50 kHz sampling rate and 5s data length) for impedance extraction. The main circuit and control parameters are listed in Table I.

TABLE I MAIN CIRCUIT AND CONTROL PARAMETERS

Circuit parameters		Control parameters
U_N = 200 V(rms)	L_g = 6.6 mH	H_{PLL}: k_{ppll} = 0.1, k_{ipll} = 100
I_N = 7 A (rms)	R_g = 0.258 ohm	H_{PR}: k_{pc} = 20, k_{ic} = 628
L_f = 3.3 mH	C_{cap} = 200 uF	H_{dc}: k_{pdc} = 5*10^{-5}, k_{idc} = 2.5*10^{-4}
R_f = 0.129 ohm	V_{dcN} = 320 V	k_{QSG} = 5.0
f_{sw} = 10 kHz	f_{sample} = 50 kHz	T_d = 50 us

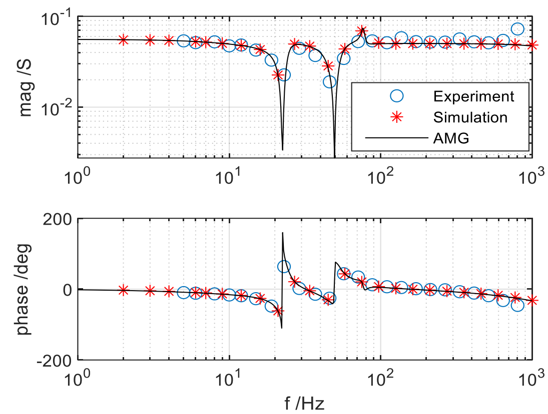


Fig. 4 VSC-SISO admittance validation (I_{qref} = -6 A, R_L = 1e5 ohm)

As shown in Fig. 4, the AMG-based results are basically consistent with the experimental results, although minor deviations are observed close to the highest frequencies. To alleviate the uncertainties in experiments, simulated results are also compared with the AMG-based results, it can be seen that a point-to-point match is achieved in the concerned frequency range, proving the validity of the AMG method.

C. AMG-based stability assessment

1) Example with parametric evaluations of stability margin

Based on the AMG method, a case study for evaluating the impacts of current and dc voltage controller parameters (specifically k_{pc} and k_{idc}, respectively) on the overall stability margin is presented. The stability margin is then defined as the *minimum distance* of the eigenlocus (i.e. $L_{\mathrm{gain}}(s)=Z_{\mathrm{g}}^{\mathrm{SISO}}\cdot Y_{\mathrm{c}}^{\mathrm{SISO}}(s)$) to the critical point (-1,0 j).

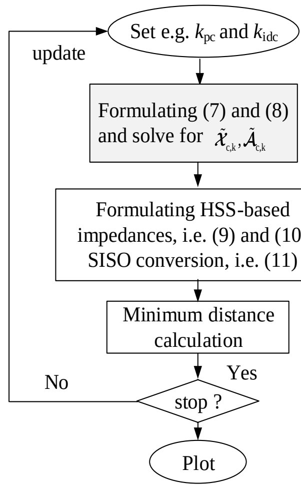
The procedure of applying the AMG approach for stability margin assessment is illustrated in Fig. 5 (a), and the corresponding results are plotted in Fig. 5 (b). As can be seen from the figure, the stability margin is decreased (i.e. smaller value of the minimum distance) when the proportional gain k_{pc} of the current controller is reduced. Under a specified current controller gain, the increase of the dc voltage controller integral gain k_{idc} will decrease the stability margin. And, this negative impact of k_{idc} is more evident for low values of k_{pc}.

This example clearly demonstrates the advantage of this AMG-based method in facilitating a systematic stability assessment, since the stability margin and the stability trends can be easily assessed by these curves.

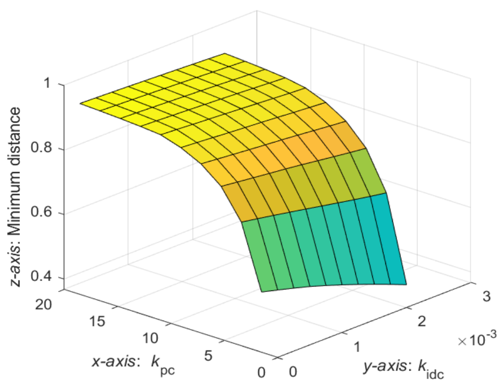
2) Experimental verification on the prediction of the marginally unstable condition using AMG

This section presents an experimental stability test focusing on the marginally unstable condition to consolidate the effectiveness of the AMG method on stability analysis. The

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TPWRD.2020.2965771, IEEE Transactions on Power Delivery

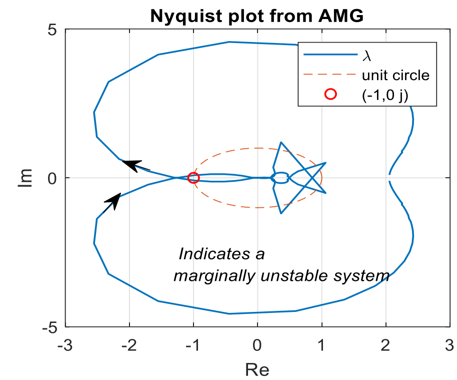


(a) Flow chart of the calculation process

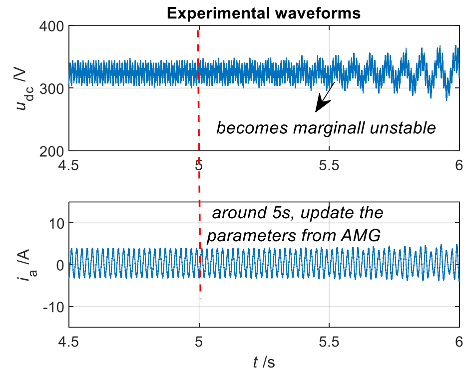


(b) Result of parametric stability margin analysis

Fig. 5 AMG-based parametric evaluation of stability margin



(a) Nyquist plot



(b) Experimental waveforms

Fig. 6 AMG-based stability analysis and experimental verification

applied procedure can be explained as follows: i) AMG-based stability analysis is performed to generate the Nyquist plot; ii) The VSC control parameters for a marginally unstable condition are identified (in this study, $k_{pc} = 1$ is found and other parameters in Table I remains unchanged); iii) This identified k_{pc} is applied to the experimental system to check if the predicted unstable phenomenon will occur or not.

Based on the above procedure, the Nyquist plot from the AMG is first given in Fig. 6 (a), it can be seen that the grid-VSC system will be marginally unstable under $k_{pc} = 1$. This identified current controller gain is applied to the experimental system at t = 5 s, and the resulting dc voltage and ac current waveforms are shown in Fig. 6 (b). Indeed, the results show how the VSC system is becoming marginally unstable as predicted when imposing the identified value of the control parameter. This stability test again justifies the effectiveness and accuracy of the AMG method.

IV. Conclusions

This letter proposes an integrated method for generating the VSCs' PSS conditions and the HSS-based impedance model and is referred to as the AMG method. The main features and novelties of this method are:

1) The PSS conditions along with the HSS-based impedance can be automatically and simultaneously generated by one unified algorithm.

2) Based on the trait of 1), this method can enable a systematic and accurate impedance-based stability assessment, avoiding the need of utilizing lengthy time-domain simulations to obtain the PSS conditions.

3) This AMG method is easily programmable and implementable in software, e.g. MATLAB.

4) Although this letter only presents the application of this method on the impedance acquisition and stability assessment of a single-phase grid-VSC system, it is directly applicable to other systems exhibiting PSS, aiming for an accurate and efficient impedance-based analysis.

References

[1] Sun. J. "Impedance-Based Stability Criterion for Grid-Connected Inverters," in IEEE Transactions on Power Electronics, vol. 26, no. 11, pp. 3075-3078, Nov. 2011

[2] Wereley, N. M. Analysis and control of linear periodically time varying systems, Diss. Massachusetts Institute of Technology, 1990.

[3] G. Bergna-Diaz, J. Freytes, X. Guillaud, S. D'Arco and J. A. Suul, "Generalized Voltage-Based State-Space Modeling of Modular Multilevel Converters With Constant Equilibrium in Steady State," in IEEE Journal of Emerging and Selected Topics in Power Electronics, vol. 6, no. 2, pp. 707-725, June 2018.

[4] M. Nakhla and J. Vlach, "A piecewise harmonic balance technique for determination of periodic response of nonlinear systems," in IEEE Transactions on Circuits and Systems, vol. 23, no. 2, pp. 85-91, February 1976.

[5] C. Zhang, X. Cai, A. Rygg and M. Molinas, "Sequence Domain SISO Equivalent Models of a Grid-Tied Voltage Source Converter System for Small-Signal Stability Analysis," in IEEE Transactions on Energy Conversion, vol. 33, no. 2, pp. 741-749, June 2018.