

Chapter 4

Accounting for Student Perspectives in Task Design

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4.1 Introduction

Mathematical tasks or sequences of tasks are, we may assume, designed to embody mathematical knowledge in ways that are accessible to students and to improve students' mathematical thinking. However, if we look beyond the intentions of those who design and select tasks and focus on the impact of students' perceptions of tasks on their mathematical learning, some important questions are raised. One of the aims of this chapter is to gain insights into students' perspectives about the meanings and purposes of mathematical tasks and to better understand how appropriate task design might help to minimize the gap between teacher intentions and student mathematical activity.

The title of the chapter is deliberately ambiguous; we attempt both to explore *accounts of* how students understand the meaning and purpose of the mathematical activity they undertake and to discuss how task design might *take account of* what we know about these perspectives. In Sect. 4.2 we explore research that indicates ways in which the perceptions of students may differ from the intentions of teachers and task designers and attempt to articulate more clearly the nature of those differences. Such research raises both theoretical and methodological challenges concerning how an observer can appreciate the student's point of view. In Sect. 4.3 we explore ways

in which task design that takes account of students' responses might reduce the discrepancies between the intentions of designers and/or teachers and students' perceptions of their activity and achievements. Finally, in Sect. 4.4 we raise some questions for further research.

4.2 Articulating the Gap Between Teachers' Intentions and Students' Perceptions and Responses

4.2.1 Students' Responses to Word Problems

One area in which research has focused on learners' perspectives is the common practice of setting mathematical tasks within "everyday" contexts. The use of contextualized word problems has a long history and indeed is so deeply embedded in school mathematics that such problems have become stereotypical of the experience of learning, and perhaps more significantly being assessed in, mathematics. Contexts are often used by teachers or designers in order to make learning easier by giving meaning to mathematical ideas and showing their usefulness. However, there is an ambiguity in the use of word problems, as they are also traditionally used in assessment, which suggests they are seen as more challenging than straightforward calculations; the lack of attention sometimes paid to the realism of the contexts chosen suggests that meaning and usefulness are lower priorities than the mathematical content (Ainley, 2012). In this section we try to understand the difficulties that arise when students are dealing with word problems; in Sect. 4.2.2 we return to the question of "meaning".

Research about learners' perceptions of the use of contexts in mathematical tasks has suggested that these can differ considerably from the intentions of those who designed them (Cooper & Dunne, 2000). Although designers may choose contexts to offer real-world models to think with or to illustrate the usefulness of mathematical concepts in real life, pedagogic practice may lead students to adopt "tricks" to bypass the contextual elements (e.g., Gerofsky, 1996; Verschaffel, Greer, & Torbeyns, 2006) or fail to appreciate the extent to which everyday knowledge is intended to be utilized in the mathematical task (Cooper & Dunne, 2000). Thus, despite the intentions of task designers, the use of contexts can serve to distract attention from the mathematics ideas. Tasks or task sequences which draw on real-world contexts, but which do not reflect the purposes for which mathematics is used in the real world, may be perceived by students as evidence of the gap between school mathematics and relevance to their everyday lives.

Although the peculiar conventions of word problems as a genre have been explored (Gerofsky, 1996) and the relative cognitive challenges posed by different styles and formats of problems extensively researched (e.g., Csikos, Sztitanyi, & Kelemen, 2012; Patkin & Gazit, 2011), the pedagogic value of such problems is

rarely questioned (Ainley, 2012). Much of the research in this field has demonstrated a considerable gap between the intentions of the teacher (or the task designer) and the activity of pupils. Many studies (e.g., Hershkovitz & Nesher, 1999; Verschaffel, 2002) have revealed that the strategies used by pupils (and sometimes encouraged by teachers) to answer word problems successfully involve ignoring the context in which the problem has been set. They involve identifying key features of the problem, particularly the numbers and words such as *altogether* which signal the operation, and moving as quickly as possible to a numerical calculation; the story context of the problem is seen as a distraction. Gerofsky (1996) paraphrases this approach as follows:

I am to ignore ... any story elements of this problem, use the math we have just learned to transform ... [it]... into correct arithmetic or algebraic form, solve the problem to find one correct answer.... (p. 39)

This problem-solving strategy, using a “translation” of key words into arithmetic operations (Hegarty, Mayer, & Monk, 1995) can be highly effective in terms of obtaining correct answers and achieving good test scores, even though it may bypass the intentions of the teacher by failing to give consideration to either the mathematical structure of the problem or the context.

It is unsurprising that pupils “perceive school word problems as artificial, routine-based tasks which are unrelated to the real world” (Verschaffel et al., 2006, p. 60), particularly given the lack of attention given to realistic content by some writers of word problems (Gerofsky, 1996). Pupils’ recognition of this lack of realism is demonstrated vividly in studies which have engaged pupils in creating their own problems, where the examples include inappropriate uses of decimals (for specifying numbers of sweets) or ownership of unrealistic numbers of items, such as household irons (Pimm, 1987).

Verschaffel (2002) argues that this tendency to disregard everyday knowledge arises from pupils’ experience of the culture and practice of mathematics classrooms. A more nuanced view is presented in a wide ranging study by Cooper and Dunne (2000) which provides evidence of a difficulty some children, and particularly those from lower socioeconomic groups, have in understanding the implicit rules of the mathematics classroom. They report observations of pupils who approached solving word problems by drawing on aspects of their everyday knowledge in ways which were not intended by the teacher or task designer. For example, when given a problem designed to be solved using simultaneous equations, set in the context of buying drinks and popcorn at the cinema, some pupils used the actual price that they have recently paid for a canned drink rather than using information given in the problem to work it out.

It appears that the use of problems which contextualize mathematics in “real-world” situations may serve to extend rather than to reduce the gap between teachers’ intentions and students’ responses. We now introduce two concepts which offer a model of the classroom context with the potential to shed light on the reasons for this gap.

4.2.2 *The Student's Situation*

In this section, we turn to the question of the “meaning” which can be constructed by the students in the situations that they encounter at school. We thus discuss the conditions in which it might be useful to use a material or “real” context. When the teacher is planning a task or managing this task during a lesson, he/she has some specific objectives; he/she knows more or less in advance what the goals are. In contrast, students try to adjust to what they perceive as the teacher’s objectives.

In order to better understand what the point of view of the students is, two concepts seem useful within the framework of the Theory of Didactical Situations in mathematics (Brousseau, 1997; Brousseau, Brousseau, & Warfield, 2014). *Didactic contract* points to the implicit interpretations that have been shared between students and teachers about a specific type of task or knowledge (see Chap. 2). *Milieu* refers to what the student is actually dealing with: concrete objects and elementary mathematical objects. The student’s situation, and hence their understanding of the task, is based on these two aspects (see Barquero & Bosch, 2015).

One may think that all this is quite straightforward; the teacher sets the contract and the student’s milieu, and thus he/she makes his/her intention clear for the students. But it is not so simple! The didactic contract is mostly *implicit* and its features are never really fully decided by a teacher. Pupils become aware of some recurrent aspects relative to mathematical tasks, and these regular features evolve into some kind of implicit rules. The situation described by Cooper and Dunne in relation to contextualized problems might be seen as an example. The implicit contract adopted by the teacher or the task designer is that the real-world context provides a frame within which the mathematical problem is given meaning. It is not important that this context is genuinely realistic in its details (such as the actual prices of items) because it is the mathematics that is really important. Many pupils will derive from this the rule described by Gerofsky (1996) (see Sect. 4.2.1), and indeed this may be reinforced by aspects of pedagogy. However, others may not recognize the implicit didactic contract and focus attention on the context rather than the mathematics.

Often teachers are not aware of these “rules”, particularly when they are not mathematically correct. For instance, pupils may think that all equations always have a unique solution, because all the equations they have solved previously did have only one solution (e.g., $ax + b = 0$; $a \neq 0$). Thus, the didactic contract is at the core of implicit understandings between pupils and teachers and also plays a part in some gaps of understanding between teachers and pupils.

Other important differences in points of view may be generated by the *milieu* of the task itself. The teacher, when he/she plans a task, has her teaching goal in view, whereas the student has only the *milieu* and the contract in order to understand what the task really is about. One important point is that the *milieu* is never only material, even when pupils work with objects. For instance, in infant school a teacher may show a picture with three identical toy bears and ask: “how many bears are in the picture?” and a pupil may say “mine is different”. This seems not to be relevant for the task, because for the teacher, the bears are not interesting in themselves and only their quantity is interesting. But for this pupil, bears are the important things and the

quantity is not relevant since they are all the same! This kind of gap may originate from the fact that “how many” was only related to the contract “when the teacher says *how many* you have to count and say the last number”. *How many* does not trigger any feedback from the situation; only the teacher and some of the more advanced pupils know if the answer is correct or not. There is no reason inherent in the task for wanting to know how many bears there are. In contrast, if the question was “put in a basket the exact quantity of caps needed to give one cap to each bear”, counting the bears now has a purpose. The interest of the pupil may be on the bears, but even if she is distracted by the context, she might realize that in order to give the exact quantity of caps and make the distribution of the caps she has to do something, to develop a strategy. One of these strategies is to count the bears and count the caps until the last number pronounced is the same in both. In this case, the activity of counting is not only a response to the contract but prompted by the existence of feedback from the milieu. Thus, the design of the task itself creates the possibility for students and teachers to have the same interpretation of the task.

We can now turn back to the question of “meaning”, using the example of the bears. Often teachers use contexts which are emotionally important for young pupils because they want to trigger their interest, and sometimes this is wrongly understood as the way to give “meaning”. As we mentioned in Sect. 4.2.1, this may lead to serious misunderstanding. But material objects are necessary in order to build tasks which allow students to try their own procedures. Brousseau (1997) describes the *milieu* as the non-intentional part of the situation; objects have no intention, but they have properties which may give feedback if the situation is built to permit this feedback. That is how we analyze the difference between “how many bears?” and “put the exact quantity of caps in order to give one cap to each bear”. In the first situation, if the pupil does not know how to count or gives the wrong number, the teacher has to tell her that this is not the right number (or asks other pupils to do so). To give an explanation, the teacher has to count the bears, that is, to do the action in place of the child herself. In the second situation, if the pupil has put the wrong number of caps in her basket, she will realize that this is so when she is allowed to try the caps on. This feedback is given by the *milieu*, it cannot be confused with a moral judgment, as right or wrong are sometimes considered. The role of the teacher is thus completely different: he/she can interact with the student in order to prompt her to try another solution or to explain what she thinks might be the problem. The student can play the same “game” again and learn from this replay.

We now use the notions of contract and milieu to shed light on the development of our main questions.

4.2.3 When Student’s Milieu and Teacher’s Planned Milieu Are Not the Same: An Example

Different interpretations of the classroom situation by students have been discussed in Sect. 4.2.1 in the context of word problems, where a “real-world” interpretation of the problem by some students may lead to different views of what is required in

the same task. In this section, we develop an example of the same phenomenon of misunderstanding which does not originate from a word problem. This analysis (Comiti, Grenier, & Margolinas, 1995) has triggered a lot of French research about similar phenomena.

The question “is -1 the square of a number?” was part of a set of ten preliminary questions during the first lesson of a module on square roots for 15-year-old students. The (very experienced and competent) teacher expected no major difficulty for the students in answering this question.

The path of reasoning by the students, which was anticipated by the teacher, was:

- Try some possible candidates: $+1$, -1 calculate the square and obtain $+1$.
- Connect this fact to the known rule: minus by minus is plus.
- Answer: it is not possible, there is no number of which the square is -1 .

Thus, the teacher was quite surprised when Michael told her that he had a solution, which was “the negative square”. Because she didn’t understand fully what Michael had in mind, she asked him to go to the blackboard. Michael wrote thus:

$$-(1)^2 = -1$$

She was even more surprised when a lot of students declared that they agreed with Michael. In her expected view of the whole lesson, unexpected phenomena had occurred and the planned progression did not go smoothly.

Our analysis, which was based on the structure of the *milieu* (Brousseau, 1986, 1990; Margolinas & Steinbring, 1994), offers a way to understand Michael’s situation. Like every student, Michael was introduced to whole numbers in close relationship to their written form; “2” and “two” were in this sense *the same thing*, which means that “2” was not distinguished as a particular notation for the concept “two”. This is certainly normal for small integers, when you want children to fluently link * *, “two stars,” and 2. When decimal and rational numbers were introduced, Michael might have been told that 2, 2.0, and $4/2$ represent the same number, but he might have understood that 2 was the result (the “true” result) of 2.0 and $4/2$. What we want to stress is that Michael might not have had the occasion to *differentiate the number and its written signs*.

If we keep that in mind, we might understand Michael’s answer. We can infer his possible reasoning as follows:

- Write the possible well-formed expressions using the signs: “-”, “1”, “2”, and the brackets “()”; you can obtain $(-1)^2$, $-(1)^2$, and -1^2 (but not, e.g., $(-)^1^2$ which is not well formed)
- Calculate the result and see if some expressions are equal to -1 . You find two expressions: $-(1)^2$ and -1^2 which are the same if you consider that the brackets in the case of (1) are not useful
- Tell the teacher your answer

The objects of Michael's reasoning are not numbers but signs. The *milieu* Michael interacts with is totally different from the *milieu* which was anticipated by the teacher. Furthermore, this *milieu* is more familiar to other students who agree with him.

During the following part of the lesson, the students resisted the teacher's tentative attempts to reestablish the lesson on the basis she needed. For instance, one of the following questions was "is it possible for two different numbers to have the same square?"; 4 and -4 are proposed but some students considered these numbers as opposites but not different. If you refer to notation and not numbers, this is not so strange, for instance, "lion" and "Lion" are the same words, even if their writing is slightly different, which will be taken into account in certain circumstances (e.g., if the word "lion" is at the beginning of a sentence).

Furthermore, at the end of the lesson, the teacher asked the students to write the following sentences:

- a^2 is the square of a .
- a^2 is the square of $-a$.

The students contradicted the teacher and asked for some brackets to be added in the last sentence:

- a^2 is the square of $(-a)$.

We interpret the students' response as follows: if you consider "the square" as "the sign ²" and apply this rule in writing, the teacher's second statement should be $-a^2$ is the square of $-a$, and since they know that a^2 is not equal to $-a^2$, the students want the teacher to add the brackets, giving $(-a)^2$ as the square, which they believe is equal to a^2 .

This case shows that it is difficult to define "a task". In fact, if the teacher had phrased her question differently, for instance, if the question had been "is it possible to multiply a number by itself and obtain the result -1 ?" the absence of any reference to the written signs for a square number might not have triggered the same misunderstanding. For the teacher, both questions are the same mathematical question because she considers numbers as theoretical objects, but for Michael and other students, these questions are not the same.

This case was the first description of a phenomenon which has been named as a *situation's bifurcation* (Margolinas, 2005) and investigated by different French and francophone researchers (e.g., Bloch, 1999; Clivaz, 2012). In this case study, we show that prior knowledge (and not only a lack of prior knowledge), when it is not what the teacher is expecting, can lead to a deep epistemological misunderstanding between student and teacher. But another phenomenon is interesting here, which is related to teacher's knowledge. In fact the task, which was considered very straightforward for the teacher, is ambiguous if you take into account students' knowledge about numbers. In this sense the task is ambiguous, but this could have been a wonderful occasion for the teacher to explain to the students what a square is (multiplication of a number by itself) and to open a mathematical discussion about writing and numbers. In a sense, this task might be considered as an example of a *missed*

learning opportunity (Bikner-Ahsbahs & Janßen, 2013, pp. 156–157) and a missed *emergent task*. The conditions described for a successful emergent task are that:

The teacher must:

- Have mathematical knowledge that extends the content of the lesson.
- Show interest in the students' learning processes.
- Be open for unusual ways on the part of the students. She must be willing to abstain from the planned course (Bikner-Ahsbahs & Janßen, 2013, p. 160).

In our observation of the “square of minus one” case, the teacher really showed interest in the students' learning processes and was open to deviate from the planned course (in fact the first part of the lesson took much longer than she had expected, and she accepted that), but she did not know that students of this level might confuse number and written signs. Thus, what she lacked was not exactly “mathematical knowledge” but “mathematical knowledge for teaching” (Ball, Hill, & Bass, 2005; Ball, Thames, & Phelps, 2008).

We link this with our previous discussions, because word problems might appear as a particular case of a more general phenomenon, which takes into account students' previous knowledge about the situation. When everyday knowledge is engaged, as it may be in the case of word problems, the risk of different interpretations of the problem is certainly higher, which might explain why this kind of phenomenon has been documented more frequently in “real-life”-based problems. Thus, the student's perspective changes the very definition of task.

4.2.4 *Influence of the Didactical Contract on the Definition of Task*

What we call a *task* has different possible definitions (see Chap. 2). Speaking solely about a *task* is a reduction of what the actual involvement of student and teacher implies, which is dealing with a situation which comprises a *milieu* and a *contract*. The situation which has been set up according to a particular design is constantly changing during class interaction: “the actions of teacher and student are mutually informing during the performance of a task” (Clarke & Mesiti, 2013, p. 173).

We have seen earlier that the way pupils interpret the *milieu* of the task can engage some pupils in a totally unexpected situation. If we now consider the *contract*, an exercise which seems to be a very straightforward task might instead develop into a completely open task depending on the *contract*. Clarke and Mesiti (*ibid.*, p. 178) offer an example from their observation in a Japanese school, in which “the seemingly simple pair of simultaneous equations $5x + 2y = 9$ and $5x + 3y = 1$ engaged the class for a 50-min lesson” (*ibid.*, p. 178). Discussing the lesson, the teacher emphasized prompting students' reflections on “what solving equations is all about” (*ibid.*, p. 178) and not only on the actual result. Clarke and Mesiti argue for a focus on “well-taught” mathematics rather than “good tasks”, taking into account the way students may involve themselves in mathematical tasks

and the way mathematical tasks are employed in order to maximize students' voice and agency (*ibid*, p. 181).

Moreover, tasks cannot be dissociated from the ways of working. As Coles and Brown (2013) claim: "In any task, as well as learning some mathematics, students are learning about *what learning mathematics is like* in this classroom; for us, the choice to use any task cannot be dissociated from a choice about *ways of working*" (p. 184).

The didactical contract being mostly implicit, it is not mainly shaped by what the teacher says explicitly but by what the teacher regularly encourages in the classroom. The implicit nature of the didactical contract implies that explanations from the teacher are not sufficient to generate a change of contract.

For instance, Johnson (2013) designed a sequence of four tasks for 7th grade pre-algebra students by adapting the well-known bottle problem developed by Swan (Swan, 1985). "Given the context of a bottle filling with liquid being dispensed into the bottle at a constant rate and a picture of a bottle, the bottle problem requires students to sketch a graph of the changing height of the liquid as a function of the changing volume" (Johnson, 2013, p. 212). To adapt this basic task, Johnson first reversed the activity by providing a graph and asking students to sketch the appropriate bottle and later by providing a computer environment which linked dynamic sketches of filling shapes to graphs as in Fig. 4.1.

The dynamic sketches were intended to foster students' consideration of relationships between covarying quantities. "The task sequence is designed to support students' progression in using nonnumerical quantitative reasoning to coordinate covarying quantities" (*ibid.*, p. 213).

However, the importance of calculation during a mathematics lesson, acquired during previous schooling, might have been too strong for some students:

The responses of two students, Navarro and Myra (who participated in different interview pairs), provide insight into the kind of difficulty students might have. When Navarro and Myra were presented with the filling triangle task, both of them attempted to determine

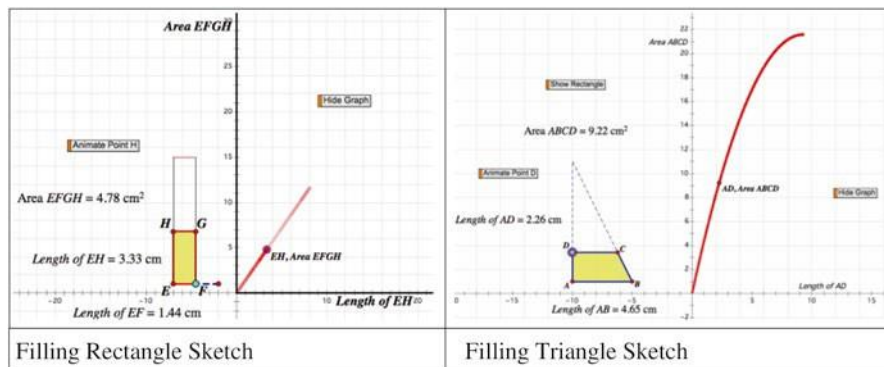


Fig. 4.1 Different sketches used by Johnson (Johnson, 2013, p. 213). (a) Filling *rectangle sketch*. (b) Filling *triangle sketch*

amounts of area. Even after prompting to not worry about making calculations, Navarro's persistence in trying to calculate amounts of area made it seem as if he depended on calculating amounts of areas to make such predictions. Unlike Navarro, after my prompt to not worry about how to calculate the area, Myra smiled and exclaimed "Oh, I get you now!" When I asked her to explain, she said "the area is getting bigger, but how much it increases is getting smaller." By no longer attempting to determine amounts of area, Myra was able to describe variation in how the area was increasing. (*ibid.*, pp. 217–218)

The prompting to not worry about calculation proved sufficient to trigger another kind of reasoning from Myra, but was not sufficient to trigger the same response from Navarro.

More generally, given that the epistemological choices which have been made throughout the mathematics curriculum are generally built on deductive theory, it may be difficult, not only for students but also for teachers, to interpret a way of dealing with mathematics based on the modeling aspect of mathematics (Job & Schneider, 2013). This is particularly problematic when it is not possible for students to understand the deductive aspects of some mathematical knowledge at the beginning of their studies, which is the case in particular for calculus in high school.

These reflections lead to the consideration of extreme complexity in what a task is when we consider how the student might understand the task within a didactical contract. It also challenges researchers' analysis when their observations are too rapid; if they are not aware of the prevalent contract in the class, they might not understand fully the task-student-teacher interactions.

4.2.5 *Student's Perception of the Meaning and Purpose of the Task*

The way in which a student perceives the meaning and purpose of a task will have an impact on the aspects of the task he/she focuses on and the activity he/she undertakes in response to it. As has already been discussed in previous sections, the student's perception of the purpose of the task may be rather different from that of the teacher or the task designer. In the example in Sect. 4.2.2, although the teacher had designed a task about counting, the child saw the purpose of the task as talking about bears. We also see differences in the ways in which the meaning of a task may be interpreted by students and by teachers. Cooper and Dunne (2000) report students misinterpreting the role of everyday context within tasks, and thus failing to see the purpose as being about mathematical content. Further, given the same task, in the same classroom, within the same didactical contract, students may have different interpretations of the meaning of the task depending on their previous knowledge and experience, as illustrated in the case study in Sect. 4.2.3, and what they think is important for them when attending a mathematics course.

Accessing students' perceptions poses considerable methodological challenges. Gardner (2013, p. 194) uses a phenomenological approach to categorizing students'

Table 4.1 Outcome space for conceptions of statistics (Gardner, 2013, pp. 194–195)

<i>Conception 1: Statistics as facts or algorithms</i>	
Definition	Statistics is a class in which one states terms, evaluates expressions and formulas, solves equations, and makes and describes graphs
Approach	Write and study examples or facts the teacher presents, memorize formulas and procedures, manipulate a calculator, solve problems the way they are done in class
Capabilities	Do well on a statistics test, remember formulas and facts after a long period of time
<i>Conception 2: Concepts about and procedures for handling data</i>	
Definition	Statistics is the study of contextualized techniques for collecting, representing, and analyzing data
Approach	Write or state a contextual interpretation of graphs and numerical summaries, execute procedures with and without technology, relate personal experience and knowledge to statistical concepts, determine the appropriate statistical method for a given scenario
Capabilities	Explain or teach statistics to another person, read and understand statistics in media, use technology, know when it is appropriate to use a particular procedure or method
<i>Conception 3: Summarize, estimate, infer, and predict</i>	
Definition	Statistics is the study of processes used to estimate population attributes and to generalize or predict trends
Approach	Use multiple approaches; utilize technology to differentiate or discover trends; recognize when data need to be collected; explain assumptions, procedures, and results to others; assess the reliability of results; provide support for conclusions drawn or estimates made
Capabilities	Write or present a detailed analysis of an inference, estimate, or prediction that includes an assessment of assumptions, interpret statistical output from software, appreciate the practicality of statistics
<i>Conception 4: Adapting, restructuring, changing viewpoint</i>	
Definition	Statistics is a way to acquire knowledge about a population and illuminate trends to improve the quality of life, inform decisions, and change one's outlook of the world. It also comes with the responsibility to use and monitor ethical practices
Approach	Adapt to the variable nature of statistics, question the ethical treatment of subjects in studies involving humans or animals, employ the highest ethical standards and design principles, disseminate results of studies to illuminate attributes and inform decisions
Capabilities	Devise a plan of action to change policies or perceptions based on reliable study results, redefine one's understanding of statistics as new processes are learned, formulate theories, restructure one's view of the world

written responses, which she argues communicate their perceptions of the task's purpose. This approach is based on the determination of different possible conceptions about the chosen subject, in this case statistics (Table 4.1):

Data were collected from one section of a graduate course in data analysis and probability for preservice and inservice teachers. The task in Fig. 4.2 is an item from the course mid-semester examination. The item assessed the student's performance level on analyzing and reporting summarized data. (Gardner, 2013, p. 196)

27. Class data were collected on scores students made when playing the 1996 Bop It game. The statistical summaries are below. Write a short report summarizing the results.

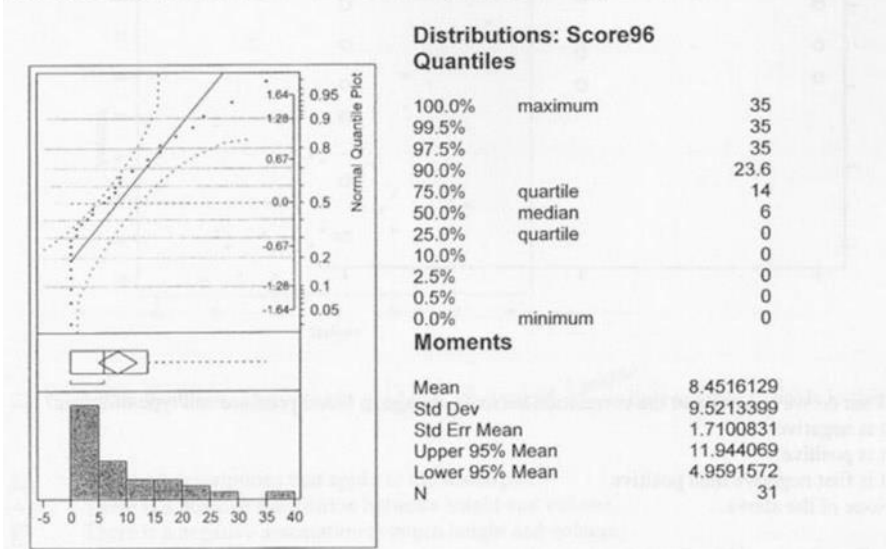


Fig. 4.2 Assessment task on descriptive statistics (Gardner, 2013, p. 196. Copyright Gardner 2007)

Gardner interprets the responses of three students as communicating their interpretation of the purpose of the task.

For one student, Anna, the purpose of the task might be described as “to determine whether she can recall facts about statistical summaries” (Gardner, 2013, p. 195). For another student, Byron, it might be described as “a means for him to communicate his understanding of concepts about data” (Gardner, 2013, p. 196). For a third one, Charles, the purpose was “a means for him to demonstrate his ability to summarize data and support the conclusion drawn” (Gardner, 2013, p. 197). Thus, in response to the same task, students not only answer differently but may be engaging in quite different kinds of activity.

4.3 Taking Account of Student Perspectives: How Task Design Might Reduce the Gap

In this section, we turn to the second interpretation of our chapter title and consider approaches to task design which might take account of the issues discussed in the previous section, concerning the gap which may arise between teachers’ (and task designers’) intentions and the responses of students. A theme which threads through this section is the challenge in moving beyond the idea that successful completion of a task is the end point and a valid proxy for mathematical learning. A number of the

studies we refer to draw on the work of Brousseau (1997) who described a pattern of interactions in which pupils draw on indications from the teacher's behavior, and other aspects of the situation, to find out what is required to complete a given task, and the teacher accepts this as evidence of learning. For example, Strømshag Måsøval (2013, 2015) describes a teacher posing easier and easier questions to students who were struggling to understand the requirements of a task, leading to completely different knowledge. This has been called the *Topaze Effect* by Brousseau (Brousseau, 1982) and *funneling* by Bauersfeld (1995) and reported widely (e.g., Mason, 2002). In the process of funneling questions toward simple answers, it is not always clear whether it is the teacher's desire for an answer or the students' lack of response that leads the process. Some authors (e.g., Wood, 1998) contrast this with *focusing* which is a deliberate interactive act by the teacher.

4.3.1 *Students' Expectations*

As we discussed in Sect. 4.2, a gap may occur between the intentions of the teacher and the perceptions of the students when the milieu can be interpreted in several ways and/or the didactic contract is disturbed, that is, when students have expectations which are not aligned to the teacher's expectations or intentions. In Sect. 4.2.3, we discussed an example in which students' previous learning led them to over-generalize in a way that the teacher had not anticipated. Deciding when it is, or is not, appropriate to generalize a mathematical idea can be challenging for students, and this may be a source of disconnection between teachers' and students' expectations.

Rote learning and algorithmic reasoning are very common at the core of the didactic contract, leading students to expect that this is the response that will be required. This poses a challenge for teachers who want to focus on developing creative reasoning.

Lithner et al. (2013) report a study concerned with introducing creative mathematical reasoning (see also Jonsson, Norqvist, Liljekvist, & Lithner, 2014). Despite carefully designed tasks, the students' expectation was that an algorithm would be provided. They also describe a typical response to the frustrations expressed by students in this situation in which "the teacher lets the teaching act collapse" (*ibid.*, p. 225), taking back the responsibility for the students' work by simplifying aspects of the task.

A similar phenomenon is reported by Calleja (2013), in an action research study which also aimed to move students toward more open problem solving. Calleja describes an interview with a student who frequently asked for help. The student commented that Calleja (as her teacher) responded in ways that made it easier for her to complete the tasks. Calleja reflects that he "evidently avoided the student's frustration and speeded up the task completion" (*ibid.*, p. 169), but at the expense of the deeper understanding, the tasks had been designed to support. Similar responses are reported in other studies which address student perspectives in relation to task design.

In selecting or designing tasks, it can be challenging for teachers to take account of students' expectations which differ from their own, and have the potential to present real obstacles to learning. Strømskag Måsøval (2013, p. 235) describes a rather confused conversation between a teacher and a group of students about a particular diagram in a numerical investigation designed by the teacher.

	(a) If this shape were part of a sequence of shapes, what would the next one look like?
	(b) What kinds of figurate numbers do you find in the bright and the dark areas and in the shape as a whole?
	(c) Express what the shape tells you about these numbers in terms of a mathematical statement.

Paul [is] insecure about what the teacher asks for; he wonders whether it is only the first element or it is the sequence of elements they are supposed to consider:

598. Paul: If we are supposed to see the connection, it is only this very shape we shall look at now? [Draws a curve with his pencil around the element given in the task.] It is not the next shapes we have made [points at the succeeding elements drawn in his notebook when he says "next"]?
599. Teacher: You may well look at it as it stands there [Pause 1–3 s] uh [Pause 1–3 s] [indecipherable]
600. Paul: Not further, ok.

The teacher's response in turn 599 I interpret as confirming that it is satisfactory that the students look at the element given in the task (a 5×5 square) as a basis for finding answers to Tasks 4b and 4c. It is plausible that the teacher takes this stance as a consequence of seeing the 5×5 square as a generic example. [...]

These general properties are however not addressed in the classroom situation. The teacher does not express to the students that he uses the 5×5 square in the sense of a generic example, nor does he use the term "generic". What I interpret as the teacher's implicit utilization of a generic example contributes to vagueness in the discourse: The stance taken by the teacher about the sufficiency of looking at one element of the shape pattern (genericity of the 5×5 square) is consistent with the formulation [of the following tasks], a correspondence which may be expected since the task is designed by the same teacher. Application of *singular* number in the noun "the shape" indicates that the shape presented in the task is seen as generic:

What kinds of figurate numbers do you find in the bright and the dark areas, and in *the shape* as a whole? [Task 4b, emphasis added] Express what *the shape* tells you about these numbers in terms of a mathematical statement. [Task 4c, emphasis added] (Strømskag Måsøval (2013, p. 235)

The teacher's intention, which was interpreted by Strømskag Måsøval as to offer a single diagram as a generic example, was not apparent to the students. This leads

to the students being unaware of the teacher's aim for the task. Mason and Pimm (1984) describe this process as an inherent implicit feature of a didactical situation; if the teacher offers an example, the learner is supposed to appreciate the general, and if the teacher offers a generality, the learner is supposed to be able to apply it to examples. Strømskag Måsøval exemplifies this phenomenon, suggesting that the teacher's focus on the mathematical content of the task, and his familiarity with the subject matter, led him to pay insufficient attention to the wording of the task, which also required students to "express what the shape tells you about these numbers in terms of a mathematical statement" (Strømskag Måsøval, 2013, p. 233), without any clear explanation about what "mathematical statement" may mean. Acknowledging the difficulties which may be presented when students' expectations lead them to look for algorithmic solutions rather than engaging in more open problem solving, a number of studies have developed task design approaches in deliberate contrast to traditional formats. Savard, Polotskaia, Frieman, and Gervais (2013) set out design principles for tasks which aim to promote holistic reasoning about mathematical structures. In particular, they state that "The task should not contain any explicit and immediate questions that could be answered by finding one particular number [...]. However the task should include an intriguing element" (*ibid.*, p. 272).

The example they present, designed for young children, shares many of the features of a traditional word problem, consisting of simple statements about the numbers of marbles three boys say that they own. However, the statements offered are incompatible, and the challenge of the task is to provide an argument for which statement is incorrect:

This is an example of a text proposed to students.

Peter, Gabriel and Daniel are playing marbles. Peter says, "I have 5 marbles." Gabriel says, "I have 8 marbles." Daniel says, "Peter has 4 marbles less than Gabriel."

We introduce this text as a strange situation or as a situation where one of the persons made a mistake. Students are invited to explain why the text is unrealistic and how it can be corrected considering different quantities involved. (*ibid.*, p. 273)

This situation is thus not one in which the students have to act in order to get the right feedback from the milieu, but a validation situation (Brousseau, 1997; Brousseau et al., 2014) where the students exchange arguments which can be tested through the milieu (in this case through the manipulation of marbles). Reviewing the outcomes of the study, Savard et al. (2013) comment on the progress made by the children in solving problems which required holistic analysis and also on the challenges for teachers in resisting the pressure to revert to more traditional formats with a single "answer".

One response to the problem of mismatch between students' expectations and the teacher's intentions may be for the teacher to state more explicitly the kinds of behaviors that are expected in response to the task. But there is a tension here: if the teacher's aim is for students to develop creative, flexible, and independent mathematical thinking, specifying particular desirable behaviors may be counterproductive.

Coles and Brown (2013) capture this tension by saying that "the more the desired behaviors in students are specified, the less these behaviours are likely to emanate from the students' own awareness" (p. 184). Drawing on theoretical perspectives

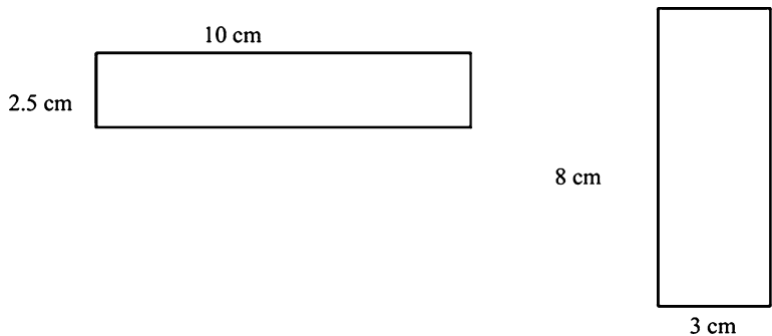


Fig. 4.3 Two contrasting examples (Coles & Brown, 2013, p. 187)

from enactivism and the development of mathematical thinking, Coles and Brown describe and exemplify design principles which underpin an approach to helping students develop patterns of thinking over an extended period:

These shapes [See Fig. 4.3] are ‘two contrasting examples’ [...]. With this image on the board, the teacher asks students, ‘what is the same and what is different’ [...]. (*ibid.*, p.187)

Coles and Brown’s description encompasses both the reflective task design, which teachers and task designers undertake in planning for teaching, and emergent design which takes place “in the moment” in the classroom, in response to the activity of students. Research addressing student perspectives in task design focuses on both of these forms of design, which we discuss in the next two sections.

4.3.2 Reflective Task Design

In this section, we discuss research studies focusing on ways to take account of student perspectives in task design which takes place in a *reflective* space away from the classroom (see also Chap. 9). In these studies, the design may be led by researchers, task designers, teacher educators, or teachers and often by a team combining individuals with different or dual roles (e.g., Coles & Brown, 2013; Lin & Tsai, 2013; Radonich & Yoon, 2013). Many of these studies draw explicitly or implicitly on the methodology of design-based research (Design-Based Research Collaborative, 2003), in which each iteration of task design is based on conjectures drawn from the analysis of student responses (Calleja, 2013; Lithner et al., 2013).

The development of hypothetical learning trajectories (Simon, 1995) or thought experiments in which the designer anticipates student learning is an important tool in such design (see Chap. 2 of this volume). Calleja (2013) and Palhares, Vieira, and Gimenez (2013) report on studies in which a priori analysis of the mathematical knowledge is involved, and students’ likely responses are used to develop sequences of tasks. Palhares et al., however, conclude that “[c]ognitive analysis seems not to

be enough to decide about ordering [of tasks]” (p. 247), in this case of tasks designed to promote algebraic thinking in young students. They found that students who started with a set of sequential tasks seemed to be more capable of establishing distant generalization than a group who started with structural tasks, and to retain their performance more. Therefore, they highlight the need for further cycles of design development in the light of the children’s responses in order to improve the design of task sequences rather than single tasks.

A common theme in studies about reflective task design is the use of aspects of design to direct students’ attention to certain mathematical features, particularly in situations in which previous research has indicated students’ tendencies to respond in other, less productive, ways. A simple change in a mathematical question can make a big difference in the cognitive demand for the students. For instance, Sullivan and Lilburn (2004) argue that you can transform a straightforward and closed question, like “ $731-256 = ?$ ”, into what they consider a *good question*: “Arrange the digits so that the difference is between 100 and 200” (*ibid.*, p. 4). Thus, the transformed task is now less about the procedure of subtraction and more about estimation. There are (at least) 6 possible answers without moving digits from one number to the other in the initial question. The revised question emphasizes the point that a small change in the question can prompt quite different thinking from the students.

Another example is given by Johnson (2013) who describes the principles she used in the design of a sequence of covariation tasks. In previous studies (Johnson, 2012a, 2012b), she had found that students tended to focus on change in variables as though they were independent, and so she included the use of an animation with prompt questions such as “what changes and what stays the same?” and the requirement for students to predict characteristics of graphs and other representations in order to focus their attention on relationships rather than the results of calculations (Johnson, 2013, p. 214). In Chap. 3 of this volume, further ways in which teachers can adapt cognitive demand are described, particularly focusing on moving learners toward appreciating generalities rather than following procedures (Knott, Olson, Adams, & Ely, 2013).

The use of a nonstandard version of a familiar task is also at the center of Lin and Tsai’s (2013) study which aimed to develop conjecturing by primary school students:

The task designed by the teacher was to ask students to make a conjecture and verify whether it is true. The statement is that “*In any two figures, if the area of one figure is bigger than the other, then the perimeter of the figure is greater than the other, too. Do you agree? Why? Show your work on the grid paper.*” The task for conjecturing is initiated from a false statement. (*ibid.*, p. 252)

Here, the use of a false conjecture about the relationship between the area and perimeter of a pair of figures was used to focus students’ attention on the ways in which conjectures can be explored and tested rather than on the calculation of specific values.

Coles and Brown (2013) highlight the importance of focusing students’ attention on making distinctions and include the use of two contrasting examples, with prompt

questions similar to those used by Johnson, and the introduction of appropriate mathematical language and notation to record the distinctions identified by students, within their list of task design principles. The starting point for one of the examples they offer, also concerning area and perimeter, is shown earlier in Fig. 4.3. Pupils are asked to consider what is the same and what is different about the rectangles. When attention is focused on the area of the rectangles, pupils notice that in one rectangle the values of the area and the perimeter are the same. This is designated as an *equable* shape, providing pupils with a new distinction with which to work in their exploration. The rationale for this approach is not based specifically on evidence of difficulties arising in the study of area and perimeter but rather on the *enactivist* approach which equates learning with the ability to act differently based on changing perceptions of distinctions in a particular sphere of action (Maturana & Varela, 1987). Coles and Brown offer the metaphor of a wine taster to illustrate this view. As the wine taster's palate develops to make finer distinctions, he/she is able to act differently on the basis of this perception.

Potential negative consequences of the unintentional direction of students' attention are identified by Calleja (2013) in his discussion of one iteration of a task sequence, designed on the basis of hypothetical learning trajectories. He notes that the activity of some students appears to have been influenced by the title assigned to a task (*Investigate Pythagoras' Theorem*), which may have focused their attention on a limited range of mathematical content. This leads him to speculate that a more open title (*Investigate Right-Angled Triangles*) may encourage a more open approach, and, incidentally, avoid discouraging those who are as yet unfamiliar with Pythagoras.

A somewhat different perspective on the direction of students' attention through task design is at the heart of a design framework developed by Ainley and Pratt (Ainley, 2008; Ainley, Pratt, & Hansen, 2006) which also relies on detailed analysis of mathematical content to develop learning trajectories. This analysis focuses on the *utility* of mathematical ideas, that is, how and why the ideas are useful. They argue that understanding utility is a key component of mathematical thinking which is often overlooked. For example, teaching about measures of average may include both the procedures needed to calculate the mean and median and conceptual exploration of the nature of the measures, but fail to address how and why such measures can be used in solving problems. In order to create opportunities to experience the utility of mathematical ideas, and to focus students' attention on the power of using them, Ainley and Pratt design tasks which have a clear and immediate purpose for students within the context of the lesson. This might be designing a product, such as an efficient paper spinner or a computer-based model to generate data, or solving an engaging problem, in which the mathematical idea (i.e., the teacher's intended content for the task) is used in a meaningful way. This design framework offers a way in which the teacher's intentions (including a focus on the utility of the mathematical content) can be aligned with the students' activity, which is driven by the purposeful nature of the task, even though the two elements of purpose and utility remain quite distinct.

4.3.3 *Emergent Task Design*

An area of research which is complementary to the studies of reflective task design discussed in the previous section concerns the ways in which teachers develop tasks during the flow of classroom activity, in response to the actions of students. Bikner-Ahsbabs and Janßen (2013) use the term *emergent tasks* to refer specifically to situations in which “the teacher conceives the mathematical potential of a learning opportunity and translates it into a task” (p. 154) in such a way that students’ interest is maintained. To understand how to build on students’ questions in order to create emergent tasks in the moment is challenging for teachers, and in this section we discuss studies which both explore emergent task design and consider how teachers’ skills can be developed.

Bikner-Ahsbabs and Janßen (2013) build on previous research into the creation of interest-dense situations in mathematics lessons, that is, situations in which learners are deeply engaged with mathematical questions, developing successively deeper meanings and coming to see the importance of a mathematical object (Bikner-Ahsbabs, 2003). They explore how teachers exploit such situations by aligning emergent tasks to what they perceived as the students’ epistemic needs. The challenge for the teacher is, therefore, to understand a mathematical problem students have encountered within the interest-dense situation and to translate it into a task for the class:

Previous to the scene presented here the students had worked on the question how to divide a round licorice stick evenly among three persons. [...]

- 149. Rahel: yes Mister Kramer once more a stupid question, how does one GET the central point how did they GET that because that is so small.
- 150. S: that is just
- 151. T: that’s another problem right. that’s a practical problem (..) oh no, how does one even find the central point in such a small circle right’ (.) exactly. those are questions’
- 152. Anji: a very small compass right’
- 153. T: yes one can find out with the compass, only when one is just drawing the circle’ one has the central point. but when one has the circle already right’
- 154. Rahel: yes
- 155. L: that’s exactly what geometry works with.
- 156. Rahel: I know that
- 157. L: there are possibilities to find out a-n-d you can puzzle at home maybe someone finds a possibility’

[...] the teacher probably notices already that the missing central point poses an additional problem and wants to postpone it to an exercise. But the students insist on an immediate clarification by asking “but how” (can we exercise that). Rahel reacts by naming the difficulty in dividing the circle without knowing the central point (149). She grasps the epistemic gap and thus *sees* the mathematical *structure*. Now the teacher summarizes the two problems: How does one even find the central point in such a small circle? Commenting “those are questions” he documents wonder about the deep involvement of the students that he tries to take up. (*ibid.*, p.158)

From observations and analysis of occasions on which teachers are, or are not, able to identify suitable situations and develop emergent tasks, Bikner-Ahsbabs

and Janßen identify the three requirements for the teacher to meet this challenge successfully (previously noted in Sect. 4.2.3): sufficient mathematical knowledge to extend the content of the lesson, a genuine interest in students' learning, and a willingness to deviate from the planned lesson to follow unexpected directions in students' activity.

Emergent task design can build flexibly and effectively on student responses to support their engagement with mathematical thinking, but makes considerable demands on the teacher's ability to act "in the moment". It is, of course, not clearly separated from the more reflective design discussed in the previous section, but involves adaptation and adjustment of an initial task during the progress of a lesson. This performance of a task (Clarke & Mesiti, 2013) is shaped by interactions between the teacher and students but guided by the teacher's intentions. Presenting evidence from three lessons which form part of a wider international study, Clarke and Mesiti draw attention to the extended time and attention given to relatively straightforward mathematical tasks by teachers in Japan and China who construct their lessons around opportunities for students to discuss and report their reasoning. Clarke and Mesiti do not address the question of how teachers develop these skills, but this issue is made explicit by Coles and Brown (2013) in their discussion of the development of generic design principles within a school. These principles are used in reflective task design undertaken collaboratively among colleagues and, thus, come to inform the more spontaneous actions in the classroom: "Creating opportunities for students to make distinctions within mathematics can also become a habit for teachers and a normal way of both planning activity and informing decisions in the classroom" (Coles & Brown, 2013, pp. 191–192). Coles and Brown illustrate this through analysis of an example of emergent design in a lesson about area and perimeter (see Sect. 4.3.1).

In addition to the challenge for teachers, there is a potential threat to the coherence of the curriculum itself within emergent tasks. This difficulty might lead to very different didactical contracts in the class during the completion of these tasks. One contract includes covering curriculum; the knowledge to be learned is therefore determined in advance or at least has to be compatible with the curriculum and the coherent epistemological foundation of the knowledge. However, the mathematical content of emergent tasks, which follow the students' responses and offer an opportunity for students to engage in rich mathematical activity, may be less predictable.

4.3.4 *Open Tasks: Voice and Agency*

Underpinning much of the research we have discussed in this chapter is the recognition not just of the importance of accounting for student perspectives in task design, in order to reduce the gap between the intentions of teachers and the activity of students, but also of the significant role of student agency and voice in the development

of mathematical thinking. In this respect, our approach contrasts strongly with some other perspectives on learning in which the student is positioned as a rather passive recipient of information that is then processed into mathematical meaning. For example, cognitive load theory (e.g., Sweller, 1994) characterizes tasks according to the number of variables that learners have to manipulate in order to be successful and analyzes those that are essential for completion and those that are extraneous. The implication for design could be that only essential variables should be given, to simplify the load. This idea has to undergo considerable expansion to include the development of mathematical thinking, and students' capabilities in mathematizing situations, so that *load* has to include variables that, while not being essential in a mathematical sense, are germane to the situation.

Within the Theory of Didactical Situations, which positions the student as actively mathematizing, agency has been modeled through the concept of *adidactical* situations. This concept derives from the idea that in ordinary life, in *non-didactical* situations (e.g., trying to float in the water in a swimming pool), we acquire some implicit knowledge in the interaction with the milieu. This kind of knowledge is directly useful (you float or not) and meaningful in the situation. The idea of Brousseau was to study a sort of image of these non-didactical situations, which he named *adidactical*, designed to allow the acquisition of predetermined knowledge. Thanks to this design, the student is offered the possibility of trying her own procedure in order to succeed in dealing with the adidactical situation and to encounter the determined knowledge in a situation that becomes meaningful for the student. Therefore, Brousseau's idea is clearly inserted within a contract where the knowledge to be learned is entirely determined in advance and where the a priori analysis, which includes the careful study of the hypothetical learning trajectory, is the basis of the design. The epistemological aspects of this kind of design are crucial; the study of the knowledge and the situation in which this knowledge is useful is at the core of anticipating learning through adidactical situations.

Design approaches that aim to encourage student agency may vary considerably. Clarke and Mesiti (2013) describe lessons which start from relatively closed tasks but which are developed and extended around students' responses. The development of reasoning and argument may be stimulated by presenting students with situations which contain faulty mathematical statements or conjectures (Lin & Tsai, 2013, Savard et al., 2013). The design principles described by Coles and Brown (2013) start from offering a closed task, which is then developed through inviting students to focus on distinctions. Bikner-Ahsbabs and Janßen (2013) base their account of an example of an emergent task on the (apparently simple) initial task of dividing a strip of paper into three equal pieces—an open-ended task intended to initiate discussion about fractions. Other studies have focused on the use of more open, contextualized tasks in order to encourage and value students' independent creative activity.

In a design-based research study, Calleja (2013) developed a design framework which included structured, semi-structured, and unstructured (open) tasks in order to support students to move from the experience of traditional teaching to progressively

become familiar with “the social experiences of mathematical inquiry, discussion and communication” (p. 166). This framework acknowledges both the importance of student agency and the challenge that this may present to their expectations.

Radonich and Yoon (2013) utilize a design framework based on model-eliciting activities (Lesh, Hoover, & Kelly, 1993), in which real-world problems are presented to students as a stimulus for mathematical modeling. The work of Lesh and his colleagues has generated many examples of this kind of work, as has the Realistic Mathematics Education tradition (see de Lange, 2015). The example offered by Radonich and Yoon is particularly interesting:

The problem begins with a comic that tells how the Renaissance artist, Giotto, gained the pope’s attention by drawing a perfect freehand circle [...]. After reading the comic, students are asked to draw their own freehand circles and choose the best among them. Next, they watch a short YouTube video of a mathematics teacher who professes to be the world’s freehand circle drawing champion and appears to draw a perfect freehand circle.

[...] Students then meet the problem statement, which introduces them to a client, Bonnie, who is holding a circle drawing competition at the local Pancake House. The students are asked to work in teams of three to develop a method for ranking circle attempts from most circular to the least circular, which Bonnie can use to judge the circle drawing attempts on the night of the competition. Students are asked to test their method on some examples of circle attempts [...], but their method must also work for any circle attempt that could be drawn on the night of the competition. The student teams write their final method in the form of a letter to Bonnie. (*ibid.*, p. 261)

Although clearly located in the mathematical topic of circle properties, the openness of the task allows for students to develop and justify a wide range of models, which may draw on different theorems, challenging the expectation that there will be a single correct approach. The main focus of the project that Radonich and Yoon report is, however, not the initial performance of the task, but how the teacher might effectively build on the wide range of activity students have engaged in to ensure some curriculum coherence. Their approach involves presenting back to the class an example of the work from a group of students carefully chosen to present a good, but incomplete, model. The challenge for the class is then to test and improve this model, allowing the teacher to focus on reinforcing particular mathematical content. The use of students’ work in this context is a deliberate challenge to students’ expectations that teacher-presented ideas are better than their own, but Radonich and Yoon acknowledge the potential sensitivities in doing this, emphasizing that “effort will be required to create a culture where discussing student work is a natural and safe part of the teaching and learning process” (p. 266).

4.4 Conclusion: Some Topics for Future Research

In this chapter, we have attempted to both account for the gap which can open up between the intentions of teachers and task designers and the experiences of students tackling mathematical tasks and to reflect research which attempts to

minimize this gap through innovative task design. What is revealed is a complex picture, which we believe is relatively under-researched. Although some evidence is presented within this chapter, there is relatively little research which addresses directly the ways in which students perceive the meaning and purpose of tasks. Most studies that do address this issue do so by inferring students' perceptions from their actions. There is a need for further research which uses alternative methods to understand student perspectives more fully, particularly in the context of innovative task design. We suggest that the areas for further research in aspects of task design identified elsewhere in this volume might also be enhanced by a parallel study of student perspectives.

In our chapter, we have highlighted the importance of the context (in particular the *contract*) in the different impacts that similar tasks might have on students. Word problems have been thoroughly investigated because these kinds of tasks, which are often linked to "real-world" contexts, are more susceptible to alternative interpretations from the students. This has contributed to deep interrogations of the validity of such kinds of tasks. However, the intention of any tasks which include some "real" setting might be wrongly understood, and even tasks which are only mathematical tasks are not immune to misunderstanding. It seems thus very important *to collect more data and to develop analysis which is focused on the effect of the contract on the variability of student's understanding of the tasks* (see Sarrazy, 2002 for an example of such a study). This topic might be very interesting to develop in intercultural research, because the kinds of contracts that develop in different countries might be different and thus reveal the nature of the contract itself.

Research in mathematics education has often been developed without any special *interest in the differential effect of the tasks on students*: who is benefiting from a certain kind of task and who is not? Qualitative methodology has been reported in our chapter, which helps us to understand that students might interpret tasks very differently. However, it would be useful if this question were to be addressed in quantitative methodology-based research. The statistical analysis needed to compare the progress of different students has to be specific and some has already been developed in some publications (e.g., Sarrazy & Chopin, 2010).

More generally, particularly helpful might be research that addresses the question: *are some tasks more robust than others?* The *robustness* of tasks might be intended as *resistant to changes from the teacher* but also *understandable and useful for all the students*.

Of course, the student's perspective, which is the purpose of this chapter, cannot be detached from the teacher's ability to develop and implement "good tasks" or "good teaching". We suggest that *emergent tasks* (Bikner-Ahsbabs & Janßen, 2013) *might be developed into a general concept*. In fact, interest-dense tasks might emerge from a lot of tasks; it is not yet clear if this is mostly due to the intrinsic quality of the initial task or due to efficient mathematical knowledge for teaching or both. The ability of teachers to observe students' procedures and to develop the task accordingly might need special attention.

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