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► **To cite this version:**

Berie Kassa, Liping Ding. The use of variation theory in a problem-based task design study. Eleventh Congress of the European Society for Research in Mathematics Education, Utrecht University, Feb 2019, Utrecht, Netherlands. hal-02423413

HAL Id: hal-02423413

<https://hal.archives-ouvertes.fr/hal-02423413>

Submitted on 24 Dec 2019

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The use of variation theory in a problem-based task design study

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The aim of this pilot study is to test the usefulness of variation theory as a theoretical framework in guiding the researcher to design the main study of developing students' deep learning in mathematics. Both Marton's variation theory of learning and Gu's theory of teaching with variation were used in the design-based pilot study. Two teachers and 106 Grade four students from an elementary school in a large north-west city in Ethiopia participated in the pilot study. The pilot study data included transcripts of audio-recording of preparatory meetings (the first author with the two teachers) and students' interview, video-recording of lesson observation, and pre and post-tests. The pilot study topic was of learning concepts of area and perimeter of rectangles. We find that the theoretical framework of variation helped us to address the systematic use of patterns of variation and invariance in the problem-based task design and to analyze the three patterns of variation in the problem-based tasks implementation in the classroom learning.

Keywords: Variation theory, patterns of variation and invariance, problem-based task design.

Introduction

School mathematics curriculum has been significantly reformed from kindergarten to upper secondary school in Ethiopia. Consequently, school mathematics is now viewed from a dynamic and experimental perspective rather than only as a body of strict and logical knowledge already in existence (MOE, 2013). However, the national learning assessments have constantly shown that students have achieved poor results in all areas, and their achievement in mathematics is the least (MOE, 2013). For example, the fourth national learning assessment (FNLA) result has shown that the overall test performance of grade 8 students in 2013 was far below the national minimum achievement standard of 50%, and their achievement in mathematics is the lowest (the composite average became 25.65%), where 50% of the students scored at or below a score of 22.5% similar in both sampled grades 4 and 8 (MOE, 2013). According to the FNLA reports in 2013, the education of mathematics should receive special emphasis in all deliberations, for example, the use of teaching and learning resources, and professional development of teachers about innovative pedagogies.

The overall objective of this study is therefore to make a contribution to developing the quality of mathematics teaching and learning in Ethiopia. This pilot study is to test the usefulness of the variation theory as a theoretical framework in guiding the researcher to design and implement problem-based tasks to improve students learning in mathematics. In this paper, we mainly focus on the following two research questions: (1) *to what extent does the theoretical framework help to design the instruction of problem-based tasks?* (2) *to what extent does the theoretical framework help to analyze the implementation of such design in the classroom?*

Literature Review and Theoretical Framework

Marton's Variation Theory of Learning (VToL)

Ference Marton and his colleagues in Sweden and Hong Kong have developed Variation Theory of Learning (VToL) to describe how a learner might come to see, understand, or experience a given phenomenon in a certain way (Marton, 2015; Marton & Booth, 1997; Marton & Pang, 2006; Marton & Tsui, 2004). For instance, Marton and Booth (1997) emphasized that one cannot become aware of new aspects without becoming aware of differences. If certain aspects of a phenomenon are varied while its other aspects are kept constant, then those that vary will be noticed. Every concept, situation or phenomenon has particular or critical aspects, and it is important to identify the critical aspects and the focus of students' attention should always be directed to the critical aspects (Lo, 2012). In order for students to discern the critical aspects of a concept(s), they need to experience different forms of variation in which the critical aspects of the concept(s) will be varied and not varied. Moreover, VToL addresses that for learning to occur, some critical aspects of the object of learning must vary while other aspects remain constant. According to VToL, the specific knowledge/content or capability to be gained through learning is called the object of learning (Lo, 2012; Marton & Pang, 2006; Marton & Tsui, 2004). VToL takes the object of learning as the point of departure and claims that what is being learned is highly influenced by what actually comes to the forefront of students' attention. Kullberg, Runesson and Marton (2017, p. 561) explained that "when analyzing how the object of learning is handled during teaching, the 'intended', the 'enacted', and the 'lived' objects of learning are used to differentiate between the teacher's particular goal regarding what the students should learn, what is made possible to learn in the lesson, and what the learners actually learn". The theory further suggests that how students perceive a specific object of learning depends on what pattern of variation and invariance is provided by the teacher. Four patterns of variation and invariance are identified: contrast (i.e., recognizing differences between two values of an aspect), generalization (i.e., keeping the focused feature invariant and varying other out of focus aspects), separation (i.e., separating aspects with varying values from invariant aspects), and fusion (i.e., experiencing several critical aspects simultaneously) (Leung, 2012; Lo, 2012; Marton & Tsui, 2004). Marton (2015) further proposed a new pattern of variation in which only three of the four patterns, i.e., contrast, generalization, and fusion exist as the basic ones, which are observed by learners.

Gu's Teaching with Variation (TwV)

In parallel with Marton's theory of variation, a theory of mathematics teaching and learning, called Teaching with Variation (TwV) (*bianshi jiaoxue* in Chinese), has been developed by Gu Ling-yuan and his colleagues through a series of longitudinal mathematics teaching experiments in China (Gu, 1994). According to Gu's theory, which was highly inspired by the theory of cognitive constructivism, meaningful learning empowers learners to establish a significant and non-arbitrary connection between their new knowledge and their previous knowledge (Gu, Huang, & Marton, 2004). Classroom activities can be designed to help students to create this kind of connection by experiencing certain dimensions of variation. Gu's theory of TwV suggests that two types of variation are helpful for meaningful learning. One is called "conceptual variation", and the other is

“procedural variation” (Gu et al., 2004). Conceptual variation refers to understanding concepts from multiple perspectives. Procedural variation is progressively unfolding mathematics activities. That is, teaching process-oriented knowledge by enhancing the formation of concepts step-by-step, and experiencing problem-solving from simple problems to complicated problems (Gu et al., 2004).

Based on the above literature, we formed an initial analytical framework (see Table 1) to guide us in designing an instruction and implementing such design in the pilot study.

Theories	Theoretical terms	The use of the theoretical terms in both design and implementation of an instruction of problem-based tasks in the pilot study
Gu’s TwV (Gu et al., 2004)	<ul style="list-style-type: none"> • conceptual variation • procedural variation • non-arbitrary connection b/n new and prior knowledge • systematic and step-by step way of problem-solving 	1. Design of the instruction of the problem-based tasks: <ul style="list-style-type: none"> • Conceptual variation (concept and non-concept variation). • Procedural variation (varying methods, problem transformations, connection within and between the problems) (for details see finding one below).
Marton’s VToL (Marton, 2015; Marton & Tsui, 2004)	<ul style="list-style-type: none"> • object of learning (intended, enacted and lived object of learning) • critical aspects of the object of learning • variation and invariance • Three patterns of variation (contrast, generalization, and fusion) 	2. Implementation of such design <ul style="list-style-type: none"> • The analysis focused on how the designed tasks are planned (intended), taught (enacted), and learned(lived) (explained in the findings section). • Two critical aspects of an object of learning were designed (see Table 2). • Each critical aspect was brought into focal awareness through the systematic use of variance and invariance of values of each aspect, and all were experienced and discerned simultaneously (for details see finding two below).

Table 1: The analytical framework of Gu’s and Marton’s work of ‘variation’

Methodology

Research Design

This study was based on the principles of design research (Cobb, Jackson, & Dunlap, 2016). According to Cobb et al. (2016), design research is a research approach that has been increasingly appreciated in mathematics education research. It can help to construct the instructional ways of promoting learning of a particular concept, while regularly studying the development of that learning, considering all components of the instructional ways, including the context in which it is carried on. According to Cobb et al. (2016), some of the main characteristics of design research approach is its highly interventionist nature, iterative design and cyclic nature. Each cycle progresses in three phases: (i) preparation and design of teaching experiments; (ii) implementation of teaching experiments; and (iii) retrospective analysis that can lead to revisions and a new cycle (Cobb et al., 2016). In this pilot study, two teaching experiments, in two fourth grade classes of 106

students, were conducted in their regular classes, during a 45 minutes period for each class. The two classes were taught by the same teachers and the study design is illustrated in Figure 1.

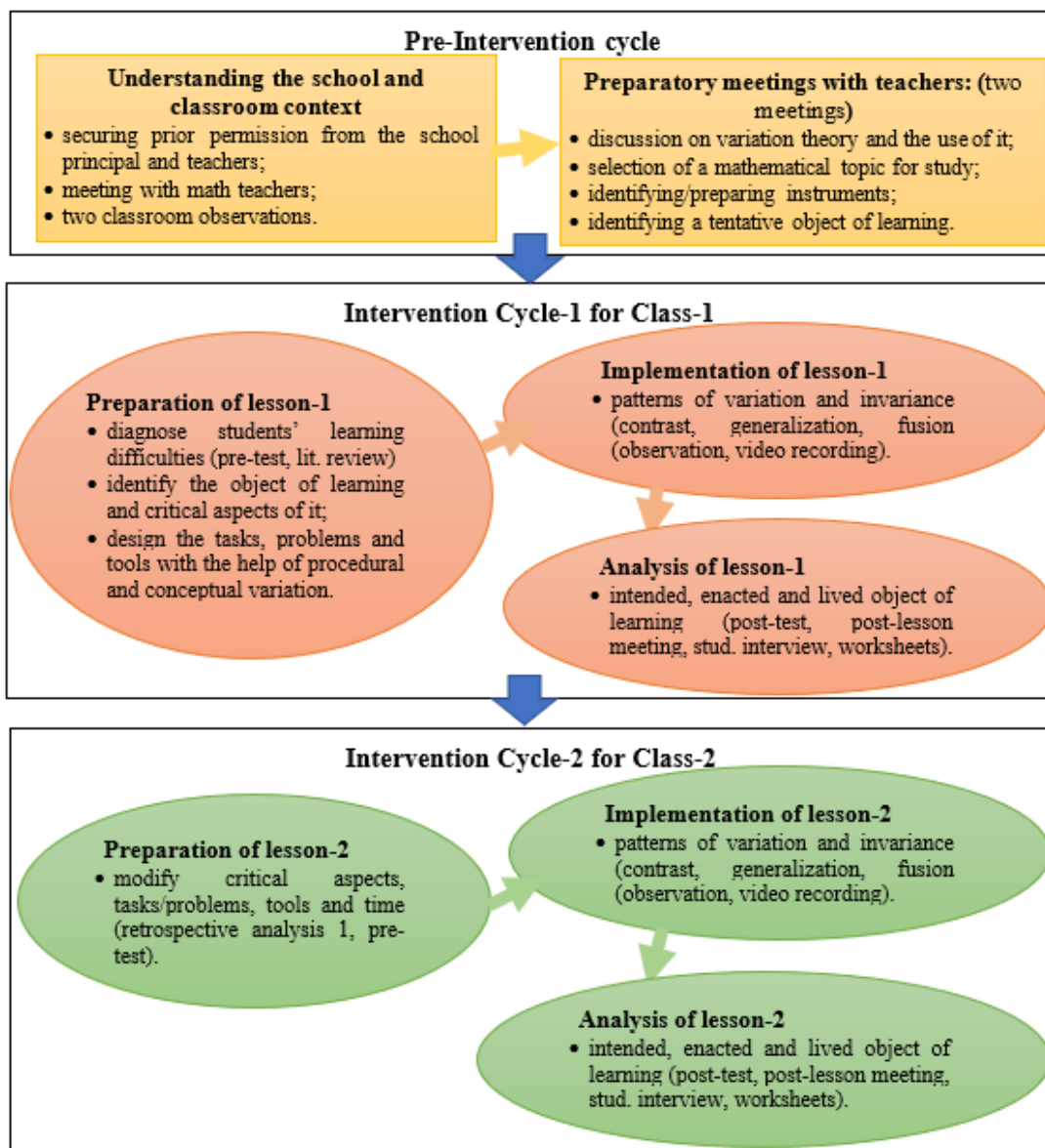


Figure 1: The research process in the pilot study

Data Collection

106 fourth-grade students from a governmental elementary school in a city in the north-west of Ethiopia participated in this pilot study. There are four mathematics teachers teaching in Grades 1-4 (1 male, 3 females) in the school. The researcher (first author), together with one experienced teacher and one beginner teacher, formed the pilot study group. In this study, the study group collected data through preparatory meetings, pre-tests and post-tests, lesson observations and video-recordings of lessons, students' worksheets and students' interview (See Figure 1).

Data Analysis Procedure

We followed the three phases of each cycle progresses (Cobb et al., 2016) in our data analysis. The data analysis mainly focused on how the object of learning was designed and implemented in terms of patterns of variation and invariance of critical aspects of the object of learning within and between units of teaching, such as the problem-based tasks (for details see the analytical framework in Table 1). The analyses were made by watching the video recorded lessons and meetings, repeatedly focusing on what was planned (the *intended*), what was possible for students to discern (the *enacted*) and what the students actually learned (the *lived* object of learning). In addition, pre and posttests, students' worksheets and students' interview were also analyzed to find out students' discernment of the object of learning. The analysis of the first class (see the intervention cycle 1 in Figure 1) is based on the data of the study group's initial lesson plan, namely the problem-based task design and implementation in the class. The analysis of the second class (see the intervention cycle 2 in Figure 1) concerns more on the usefulness of the redesign of tasks, the effective design and use of tools, and the impact on students learning. Given the space of this paper, we will chiefly focus on our research questions in the analysis of the first class.

Findings

Finding one and two below focus on reporting our retrospective analysis of the problem-based task design (namely the stage of the preparation and design of teaching experiments of the first lesson); and classroom implementation of such design in the first lesson.

Finding one: The usefulness of the variation theory in designing the problem-based tasks

In this pilot study, we focused on designing problem-based tasks for improving students' mathematical understanding of the concept of area and perimeter of rectangles. The problem-based tasks were designed by referring to both Gu's terms of conceptual and procedural variations and Marton's terms of variation/invariance (see Table 1). The sequence of tasks used in the classroom instruction were designed with reference to the two problems and critical aspects shown in Table 2.

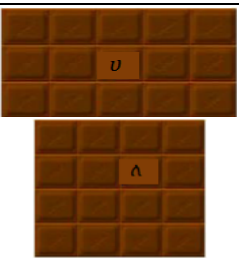
Object of learning	Critical aspects	Problems used to design sequences of tasks in the lesson
The relationship between area and perimeter of rectangles	(1) If the perimeter of rectangles is the same, then the area does not have to be the same.	(1) The figures below are two bars of biscuits. If the price of these biscuit bars is the same, which biscuit do you want to buy? Why do you choose that biscuit? What is your strategy for choosing the biscuit you want to buy? Explain your answer. Use the given grid paper to help you compare the shapes. <div style="text-align: right;">  </div>
	(2) A rectangle with dimensions closest together or the same (square) is the largest.	(2) A farmer has a 16 meters long fence and he plans to enclose a rectangular grazing field with it. Can you help the farmer to think of a way to enclose the largest grazing field? At what condition is the field smallest?

Table 2: The link between the problems and critical aspects of the object of learning

Conceptual variation was carried out to firstly emphasize certain concepts of rectangle such as its area and perimeter and hence the concepts could be more readily picked up for use in finding the relationship between area and perimeter of rectangles. In terms of procedural variation, the problems were transformed into simpler problems, so that students could solve them step by step in a progressive way, using different methods and that helped them to answer the original problem. In addition, the second more advanced problem was designed to scale up the methods/solution techniques used by students in the first problem (connecting new and prior knowledge). Students were also used their prior measurement techniques as a procedure to find area and perimeter of rectangles. Both procedural and conceptual variation was also carried out in connecting the abstract concept, for example, mathematical formula of area ($A = \text{length} \times \text{width}$), directly to its concrete correspondence (counting unit squares).

Marton's theoretical notion 'the object of learning' (Marton et al., 2004) was also used in the pilot study in that the study group aimed to develop students' learning of the relationship between the concepts of area and perimeter of rectangles. Moreover, the study group also applied the critical aspects of the object of learning to further understand students' learning (see Table 2). The two problems shown in Table 2 were designed to bring the two critical aspects into the focal awareness of learners so that they could discern them step by step. The two problems were also used to guide the design of the sequence of tasks in the classroom instruction.

Finding two: The usefulness of the theories of variation in analyzing the implementation of the designed tasks.

The implementation of the designed instruction was guided by both Marton's VToL and Gu's TwV. For example, the teacher used procedurally varied approaches in implementing each of the designed tasks, in a systematic and progressive way. At the beginning of each task, the teacher asked students to conjecture the answers to each problem individually and then to compare it with their colleague's work. The teacher then conducted a discussion with students on their answers without giving any hint on the correctness of students' suggestions and did not give them the correct answers. The teacher then asked students to start a group investigation of the posed problems in a small group. Students were also encouraged to use the different tailor-made resources made available by the teacher to discover patterns in their answering. Each task was finally concluded by a whole class discussion followed by summary given by the teacher.

Marton's (2015) term of 'contrast' (i.e., recognizing differences between two values of an aspect) was used to guide the task implementation. For example, students were asked to draw rectangles by fixing one dimension of the rectangle (say, length) and varying the other (width) and vice versa to find the area and perimeter of each using resources such as strings and unit square tiles. At this stage the concepts of perimeter and area of rectangles were discerned as area is the amount of surface of a region, and perimeter is the distance around the region. In addition, students were able to realize the change in area/perimeter due to the change in length/width of rectangles.

Secondly, Marton's (2015) term of 'generalization' (i.e., keeping the focused *aspects* invariant and varying other out of focus aspects) was focused in the classroom. The analysis of students recording sheet showed that most students discovered the formula for finding the area and perimeter of

rectangles easily, and they were also able to understand how area formula is related to counting unit squares covering the rectangular region. Some of students' correct findings and generalizations presented in class and written in the record sheet were: the perimeter of a rectangle equals the sum of all of its sides and this can be calculated as: $\text{perimeter of a rectangle} = (2 \times \text{length} + 2 \times \text{width})$; the area of a rectangle is the total number of unit grid-squares inside the rectangles and this can be calculated as: $\text{area of a rectangle} = \text{length} \times \text{width}$. Students were also able to compare rectangular shapes with fixed perimeter. At this stage students discerned the first critical aspect, that is, if the perimeter of rectangles is the same, then the area does not have to be the same. Most students were able to change their conjecture for problem one that the biscuit with largest side (the first one) was chosen to buy. In the recording sheet, most students wrote a generalization that rectangles with the same perimeter can have different area and vice versa.

Finally, Marton's (2015) term of 'fusion' (i.e., experiencing several critical aspects simultaneously) was highlighted. For instance, the relationship between the two concepts was shown by varying certain aspects (length and width) while keeping an aspect fixed (perimeter of rectangles). At this stage students were able to identify the largest/smallest rectangle among all rectangles with the given fixed perimeter. Most students were also changed their conjecture on identifying the largest grazing field with the given fixed length fencing material in problem two. In the recording sheet, most students wrote a generalization that a rectangle with a fixed perimeter would have the largest area when its length and width are equal (a square).

Discussions and Conclusions

In this paper, we reported our retrospective analysis (Cobb et al., 2016) of using the theory of variation in both the designed instruction and the implementation of such design in the pilot study (see Table 1 & Figure 1). Our data analysis shows that the joint consideration of the two different but compatible frameworks of variation (see Table 1) helped the study group to focus on the systematic use of patterns of variation and invariance in the design and analysis of the implementation of the problem-based tasks in the classroom learning. In particular, it enables the researcher to analyze the implementation of such design to see how the students may experience and discern the critical aspects of the object of learning and to improve their learning in mathematics. The retrospective analysis of the pilot study would lead the researcher to revisions and new cycles for the main study. In particular, we aim to develop a more sufficient understanding of interrelationship of Marton's terms of 'the critical aspects of the object of learning', and the patterns of variation and invariance (Marton, 2015) and Gu's terms of conceptual and procedural variation (Gu et al., 2004) with the design-based approach (Cobb et al., 2016), to enable us to design and implement the problem-based tasks in the main study for engaging students in mathematical learning in a systematic process and at the same time to develop their own learning in mathematics.

The analysis of the pilot study also showed that the implementation of problem-based tasks with patterns of variation and invariance helped students to improve their understanding of the critical aspects of the object of learning (the relationships between area and perimeter of rectangles) significantly. After the implementation of the first lesson, as can be evident from the researcher's observation and video recording of lessons, most students were actively engaged in individual and

group discussions and some students were able to grasp the concept quickly and began to make interesting conjectures from experiencing variations in the tasks. For example, most students showed progress in answering each question of the post-test with correct explanations. It was found that there were percentage increases of students understanding in discerning all the critical aspects (49% and 26.4% of students provided correct explanations, 47.2% and 56.6% of students provided partially correct answers, in posttest questions related to the first and second critical aspects respectively), and paired-samples *t*-test result informed that it was statistically significant (with the sig. level, 0.01) when it is compared with their pre-test results. In addition to their significant progress in the post-test, during the post-lesson interview students said that they had achieved a better understanding of area and perimeter of rectangles by actively involving in the tasks. However, there are also few students who do not discern the critical aspects (3.8% and 17% of students were not able to answer the first and second questions of the posttest respectively). This is supported by the researcher's qualitative analysis result that the intended, enacted and lived object of learning sometimes did not coincide. According to Kullberg et al. (2017), well-designed tasks and examples, and patterns of variation (differences) and invariance(sameness) in teaching, however, are insufficient and does not guarantee learning, it can, at best, make it possible for learning to happen. From our analysis, the two theories can, however, offered tools that the study group can use to focus on the mathematical content taught, students' understanding of it and how to enable possibilities for learning.

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