Random wave-driven drag forces on near-bed vegetation in shallow water based on deep water wind conditions

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Abstract

This article provides a simple analytical method giving estimates of random wave-driven drag forces on near-bed vegetation in shallow water from deep water wind conditions. Results are exemplified by using a Pierson-Moskowitz model wave spectrum for wind waves with the mean wind speed at the 10 m elevation above the sea surface as the parameter. The significant value of the drag force within a sea state of random waves is given, and an example typical for field conditions is presented. The method should serve as a useful tool for assessing random wave-induced drag force on vegetation in coastal zones and estuaries based on input from deep water wind conditions.

Keywords: Random waves, drag force, drag coefficient, shallow water, Pierson-Moskowitz spectrum, aquatic vegetation, coastal water, estuaries.

1. Introduction

Estuaries and coastal zones are generally characterized by shallow water depths where the flow is caused by surface waves and currents. The environments in estuaries and costal zones are vulnerable due to the combined action of waves and currents and the effect this has on changing the hydrodynamic and sediment transport processes, and consequently on coastal erosion. The role of seagrass for coastal protection has recently been addressed by Paul[1]; also identifying the existence of knowledge gaps regarding the support that seagrass can provide for sandy shorelines protection. Furthermore, Nowacki et al.[2] found that the wave-induced bottom shear stress is about 15% less in the presence of vegetation than the bottom shear stress due to higher waves that would occur without vegetation; clearly documenting the damping effect of vegetation on waves in an estuary.

A commonly used tool in coastal protection work is coastal flow circulation models including parameterizations of many flow mechanisms; e.g. the wave damping due vegetation is often represented in terms of a bulk drag coefficient formulation. However, a variety of drag coefficient formulae beneath wave conditions are available in the literature, and there is no consensus on how wave damping due to vegetation shall be taken into account; see Henry et al.[3] for a critical review of existing drag coefficient formulations for waves as well as a literature review up to that date. Recent works include those of Luhar and Nepf [4], Hendersen et al.[5], Nowacki et al.[2] and Paul[1]; also including literature reviews.

By following Mendez and Losada[6] the maximum horizontal drag force per unit volume acting on plants during a wave-cycle is

$$F_m = \frac{1}{2}\rho C_D b\, N U^2 \tag{1}$$

Here it is assumed that the drag force is the main fluid force component acting on plants, and is given in terms of a Morison-type equation when sway motion of the plants as well as vertical forces and vertical force component are neglected; U is the maximum horizontal velocity during the wave-cycle, ρ is the fluid density, b is the plant width corresponding to the plant area per unit height of each plant normal to U, N is the number of plants per unit area, and C_D is a bulk (depth-averaged) drag coefficient (see Fig. 1). It should be noted that Eq. (1) is based on neglecting the relative velocity between fluid and plant; but Eq. (1) is also used for flexible plants by adjusting C_D for rigid plants.

In this article the Sànches-Gonzàles et al.[7] C_D-formula is adopted, which as a compromise between simplicity and accuracy can be approximated by

$$C_D = cKC^d \, ; (c, d) = (15.6, -1) \tag{2}$$

The original coefficients were $(c, d) = (22.9, -1.09)$ and valid in the range $15 < KC < 425$, obtained as a best fit to flume test results for regular and irregular waves over artificial seagrass. Here $KC = UT / b$ is the Keulegan-Carpenter number, $T = 2\pi / \omega$ is the wave period, and ω is the cyclic wave frequency.

For linear shallow water waves $U = \omega a / kh$, i.e. independent of the vertical coordinate z, where a is the linear wave amplitude, h is the water depth, and k is the wave number determined from the shallow water dispersion relationship as $k = \omega / \sqrt{gh}$, where g is the acceleration due to gravity. Thus, by combining this with Eqs. (1) and (2), the maximum wave-induced drag force per unit mass for shallow water regular waves becomes

$$f_m \equiv \frac{F_m}{\frac{1}{4\pi}\rho c b^2 N} = \sqrt{\frac{g}{h}}\,\omega a \tag{3}$$

Based on this result for regular waves in shallow water, the random wave-induced drag force estimation on near-bed vegetation in shallow water is organized as follows. Section 2 presents the drag force for random waves in shallow water. Section 3 gives example of results for a Pierson-Moskowitz model wave spectrum for deep water wind waves with the mean wind speed at the 10m elevation above the sea surface as the parameter, also providing an example representing realistic field conditions. Summary and conclusions are given in Section 4.

2. Random wave-induced drag force in shallow water

The wave-induced drag force on plants per unit mass for an individual random wave component with amplitude a_n and cyclic wave frequency ω_n at a shallow water depth *h* is given by that for regular waves in Eq. (3) as

$$f_n = \sqrt{\frac{g}{h}}\,\omega_n a_n \tag{4}$$

Now $a_n^2 = 2S(\omega_n, h)\Delta\omega$ where $S(\omega,h) = (h/2g)\omega^2 S(\omega)$ is the wave spectrum in shallow water (Massel[8], Section 7.3), $S(\omega)$ is the deep water wave spectrum, and $\Delta\omega$ is a constant separation between frequencies. Similarly, $f_n^2 = 2S_{ff}(\omega_n, h)\Delta\omega$ where $S_{ff}(\omega, h)$ is the spectrum associated with the wave-induced drag force on plants in shallow water. Thus, by substituting a_n and f_n in Eq. (4), it follows for an infinite number of frequency components that

$$\int_0^\infty S_{ff}(\omega,h)d\omega = \frac{1}{2}\int_0^\infty \omega^4 S(\omega)d\omega \tag{5}$$

Thus, Eq. (5) takes the form

$$m_{0f} = \frac{1}{2} m_4 \tag{6}$$

where m_{0f} is the zeroth spectral moment of $S_{ff}(\omega, h)$ and m_4 is the fourth spectral moment of $S(\omega)$. Here the spectral moments for deep water waves are defined as $m_n = \int_0^\infty \omega^n S(\omega) d\omega$; $n = 0,1,2,--$. A commonly used statistical quantity is e.g. the significant value of the wave-induced drag force defined as $H_{sf} = 4\sqrt{m_{0f}}$, which from Eq. (6) is obtained as

$$H_{sf} = 2\sqrt{2}\sqrt{m_4} \tag{7}$$

Consequently, the significant value of F_m in Eq. (3) is

$$H_{sF} = \frac{1}{4\pi} \rho c b^2 N H_{sf} \tag{8}$$

The most frequently used deep water model wave spectra, e.g. the Pierson-Moskowitz and JONSWAP spectra, behave as ω^{-5} for large ω, and consequently m_4 does not exist. However, for a narrow-band process Longuet-Higgins[9] showed that the relationship between the two spectral bandwidth parameters $\varepsilon^2 = 1 - m_2^2 / (m_0 m_4)$ and $\nu^2 = m_0 m_2 / m_1^2 - 1$ can be approximated by $\nu = \varepsilon / 2$, implying that m_4 can be expressed as

$$m_4 = \frac{m_2^2 / m_0}{5 - 4 m_0 m_2 / m_1^2} = \frac{\pi^4 H_s^2}{T_2^4 (1 - \nu^2)} \tag{9}$$

Moreover, for deep water waves the significant wave height H_s, the mean spectral wave period T_1 and the mean zero-crossing wave period T_2 are given as (Tucker and Pitt[10])

$$H_s = 4\sqrt{m_0} \tag{10}$$

$$T_1 = 2\pi \frac{m_0}{m_1} \tag{11}$$

$$T_2 = 2\pi \sqrt{\frac{m_0}{m_2}} \tag{12}$$

which is used giving the third term in Eq. (9). Thus, by combining Eqs. (7) and (9), Eq. (7) is rearranged to

$$H_{sf} = \frac{2\sqrt{2}\,\pi^2}{\sqrt{1-\nu^2}} \frac{H_s}{T_2^2} \tag{13}$$

According to the results in Appendix 1, Eq. (27) gives that Eq. (13) is valid for

$$h \geq (0.124\, H_s\, T_2^{-\frac{5}{2}})^{-\frac{4}{9}} \tag{14}$$

and $15 < KC_{sh} < 425$ (see Appendix 1).

3. Example of results for a Pierson-Moskowitz spectrum

The Pierson-Moskowitz (PM) spectrum is chosen as the deep water wave spectrum to exemplify the results with the mean wind speed at the 10 m elevation above the sea surface as the parameter. The form of the PM spectrum is (Tucker and Pitt[10])

$$S(\omega) = \frac{A}{\omega^5} \exp(-\frac{B}{\omega^4}) \tag{15}$$

which for $n < 4$ has the spectral moments

$$m_n = \frac{1}{4} A B^{\frac{n}{4}-1} \Gamma(1-\frac{n}{4}) \tag{16}$$

where Γ is the gamma function. The original form of the PM-spectrum was given with $A = \alpha g^2$, $\alpha = 0.0081$, $B = 1.25\omega_p^4$, $\omega_p = 2\pi / T_p$, where ω_p and T_p are the spectral peak frequency and period, respectively, and with the mean wind speed at the 19.5 m elevation above the sea surface as the parameter. The formulation with the mean wind speed at the 10 m elevation with $U_{10} = 0.93\, U_{19.5}$ gives (Tucker and Pitt[10])

$$T_p = 0.785\, U_{10} \tag{17}$$

$$H_s = 0.0246\, U_{10}^2 \tag{18}$$

$$T_1 = 0.606\ U_{10} \tag{19}$$

$$T_2 = 0.557\ U_{10} \tag{20}$$

$$\nu = 0.425 \tag{21}$$

By substituting Eqs. (18), (20) and (21) in Eq. (13), Eq. (13) gives

$$H_{sf} = 2.45 \text{m/s}^2 \tag{22}$$

which according to Eq. (14) combined with Eqs. (18) and (20) is valid for

$$h \geq 0.040 U_{10}^2 \tag{23}$$

Moreover, Eq. (13) is also valid for $15 < KC_{sh} < 425$, where by using Eqs. (29) (in Appendix 1), (18) and (20):

$$KC_{sh} = 0.0080 \frac{U_{10}^{3.5}}{b\,h^{0.75}} \tag{24}$$

Similarly, it follows from Eq. (28) (in Appendix 1) that

$$H_{sh} = 0.0092 \frac{U_{10}^{2.5}}{h^{0.25}} \tag{25}$$

Further, consider $U_{10} = 15\,\text{m/s}$ as an example. Then it follows that:

- $h \geq 9.0\,\text{m}$ from Eq. (23)
- $H_s = 5.5\,\text{m}$ from Eq. (18)
- $T_2 = 8.4\,\text{s}$ from Eq. (20)
- $H_{sh} = 4.6\,\text{m}$ from Eq. (25) with $h = 9.0\,\text{m}$
- $KC_{sh} = 201$ from Eq. (24) with $h = 9.0\,\text{m}$ and exemplified with $b = 0.1\,\text{m}$; i.e. in the range of validity of KC_{sh}.

Further, from Eqs. (8) and (22) by taking (see Fig. 1) $\Delta h = 1\,\text{m}$, $\rho = 1027\,\text{kg/m}^3$, $N = 1$ (i.e. one plant)

$$H_{sF} = 31.2 N (= \text{kgm/s}^2)$$

It should be noted that the wave steepness in Eq. (30) (in Appendix 1) is $s_s = 0.050$, i.e. the criterion is strictly not fulfilled. However, the result should still serve as a first estimate for assessment of the random wave-induced drag on near-bed vegetation in shallow water.

4. Summary

A simple analytical method which can be used to make preliminary assessment of random wave-induced drag forces on near-bed vegetation in shallow water based on deep water wave conditions is provided. The drag force formulation is based on using a slightly revised version of a bulk drag coefficient in terms of the Keulegan-Carpenter number valid for rigid plants, and it is used for random waves by transforming deep water waves to shallow water waves. The significant value of the drag force within a seastate expressed in terms of the sea state parameters significant wave height and mean zero-crossing wave period is given. Results are exemplified by applying a Pierson-Moskowitz model wave spectrum for wind waves with the mean wind speed at the 10 m elevation above the sea surface as the parameter. The validity of the results are given in terms of the Ursell number and the wave steepness within the sea state. An example is also provided typical for field conditions. The present method should be useful for estimating random wave-induced drag forces on near-bed vegetation in shallow water coastal zones and estuaries for well known wind conditions in deep water as input. Although simple, the strength of the proposed work is that it is a quick tool which can be applied e.g. in coastal protection work.

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Appendix 1. Wave parameters in shallow water

The Ursell number gives the ratio between nonlinear features of waves in terms of the wave steepness ka and the dispersive properties of the waves in terms of kh, defined as $U_R = ka/(kh)^3$ (Dean and Dalrymple[11]), where $U_R \leq 0.5$ for linear waves (Hedges[12]).

For linear harmonic waves approaching a straight coastline at normal incidence propagating over a gently sloping flat bottom the wave amplitude in shallow water is determined from that the energy flux is constant, i.e. $a = a_\infty / (2kh)^{1/2}$ (Dean and Dalrymple[11]), where deep water is used as a reference with $a_\infty = H_\infty / 2$ as the deep water wave amplitude, and H_∞ is the deep water wave height. Furthermore, by using the dispersion relationship in shallow water, $k = 2\pi / (T\sqrt{gh})$ where T is the wave period, the Ursell number in shallow water can be expressed as

$$U_{Rh} = 0.062 \frac{H_\infty T^{2.5}}{h^{2.25}} \tag{26}$$

By replacing H_∞ with H_s and T with T_2, the Ursell number for a sea state of random waves in shallow water is defined as

$$U_{Rhs} = 0.062 \frac{H_s T_2^{2.5}}{h^{2.25}} \tag{27}$$

and is taken to be valid for $U_{Rhs} \leq 0.5$.

Similarly, the significant wave height in shallow water is obtained as

$$H_{sh} = \frac{H_s}{2} \left(\frac{T_2}{\pi} \right)^{\frac{1}{2}} \left(\frac{g}{h} \right)^{\frac{1}{4}} \tag{28}$$

Further, for a sea state of random waves in shallow water, $KC = UT/b$ is rearranged to the Keulegan-Carpenter number

$$KC_{sh} = \frac{U_{sh} T_2}{b} \tag{29}$$

where $U_{sh} = (H_{sh}/2)\,(g/h)^{\frac{1}{2}}$ is the characteristic horizontal velocity for a sea state in shallow water. Here Eq. (29) is taken to be valid for $15 < KC_{sh} < 425$.

It should also be noted that according to Hedges[12] the upper limit of the wave steepness for linear waves in deep water is 0.04, i.e. $s = H_\infty / ((g/2\pi)T^2) < 0.04$. Thus, the wave steepness for a sea state in deep water is defined as

$$s_s = \frac{H_s}{\frac{g}{2\pi} T_2^2} \tag{30}$$

and should satisfy that $s_s < 0.04$.

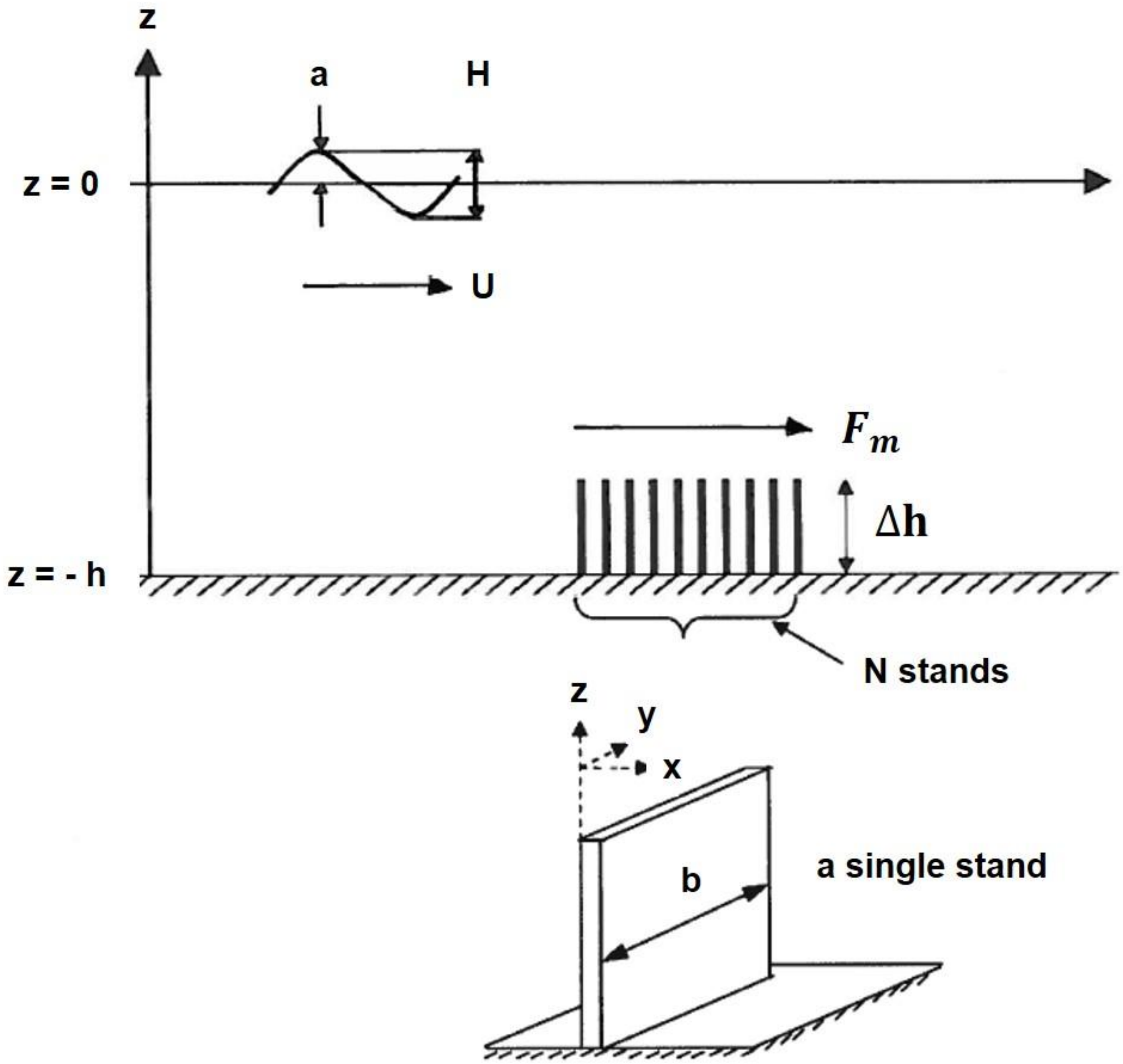

Fig. 1 Definition of a vegetation field where a is the wave amplitude, H is the wave height, U is the maximum horizontal velocity during the wave-cycle, F_m is the maximum horizontal drag force per unit volume acting on plants during a wave-cycle, Δh is the height of the stands (taken here as a unit height).