On Reliability Challenges of Repairable Systems Using Hierarchical Bayesian Inference and Maximum Likelihood Estimation

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Abstract

Failure modelling and reliability assessment of repairable systems has been receiving a great deal of attention due to its pivotal role in risk and safety management of process industries. Meanwhile, the level of uncertainty that comes with characterizing the parameters of reliability models require a sound parameter estimator tool. For the purpose of comparison and cross-verification, this paper aims at identifying the most efficient and minimal variance parameter estimator. Hierarchical Bayesian modelling (HBM) and Maximum Likelihood Estimation (MLE) approaches are applied to investigate the effect of utilizing observed data on inter-arrival failure time modelling. A case study of Natural Gas Regulating and Metering Stations in Italy has been considered to illustrate the application of proposed framework. The results highlight that relaxing the renewal process assumption and taking the time dependency of the observed data into account will result in more

precise failure models. The outcomes of this study can help asset managers to find the optimum approach to reliability assessment of repairable systems.

Keywords: Repairable system, Failure modelling, Time dependency, Hierarchical Bayesian Analysis, Maximum Likelihood Estimation

Nomenclature

Subscripts

α	shape parameter	T	successive failure times
β	scale parameter	n	total number of failures
$f(t)$	probability density function	$MTTF$	Mean Time To Failure
$F(t)$	cumulative distribution function	$E[N(t)]$	expected number of failures
t	time (sec)	$\lambda(t)$	rate of failure limit
MLE	Maximum Likelihood Estimation	HBM	Hierarchical Bayesian modelling
$\bar{T}$	mean of the inter-arrival times	P	pressure (kPa)
PR	Perfect Repair	MR	Minimal Repair

1. Introduction

1.1. Background and literature review

Failure time modelling of repairable components has attracted a great deal of attention owing to the high level of risk associated with the failure events occurring within process industries. Using statistical inference, different probability distributions are adopted to model the rate of occurrence of failures (ROCOF). These probability distributions are characterized by one or more parameters. The parameter estimation process may be implemented based on different assumptions regarding maintenance strategies including Perfect Repair (PR) or Minimal Repair (MR).

PR represents an ideal model in which the time between successive failures of a given system are independent and identically distributed (iid) random variables. Although PR is recognized as the most applied assumption by a number of researches (Nandi, Toliyat et al. (2005), Quy, Vrijling et al. (2006), Toroody, Abaiee et al. (2016a), Quy, Vrijling et al. (2008), Louit, Pascual et al. (2009) Toroody, Abaiee et al. (2016b) and Barabadi, Barabady et al. (2014)), neither of these have accounted for the system to be "as bad as old" after repair. In the present paper, it is illustrated that analyzing failure times given PR (also known as renewal process) often yields improper results and subjects to a significant level of uncertainty.

A comprehensive reliability analysis must include a time dependent study, if the system is degrading or improving. Therefore, an ongoing effort on reliability assessment based on MR are carried out (Majeske 2007, Slimacek and Lindqvist 2016, Antonov and Chepurko 2017, Peng, Shen et al. 2018, Sheu, Liu et al. 2018). Li et al. (2017) used two recurrent-event change-point models arisen from a non-homogeneous Poisson process (NHPP) to find the time of change in driving risk. In another recent study, Pesinis and Tee (2017) presented a model for reliability analysis of failure data based on NHPP incorporated with a robust structural reliability model. Furthermore, an extensive review of PR, MR and probabilistic knowledge elicitation made with a wide range of engineering applications is presented by Crow (1975), Asher and Feingold (1984) and most recently by Ross (2014) and Modarres, Kaminskiy et al. (2016).

In the present paper, two mathematically robust and efficient approaches are implemented to represent the state of system after repair. Maximum Likelihood Estimation (MLE) and Hierarchical Bayesian Modelling (HBM) are established based on actual data in order to predict the likelihood of studied failures, given PR and MR assumptions. The capability of HBM in modelling the variability of non-stationary data and the correlation between nonlinear data via open source Markov Chain Monte Carlo (MCMC) sampling software packages, i.e., OpenBUGS (Spiegelhalter et al. 2007), have resulted in its widespread use in engineering applications, e.g. probabilistic risk assessment and condition monitoring (Behmanesh, Moaveni et al. 2015, Chitsazan, Nadiri et al. 2015, Yu, Khan et al. 2017, Mishra, Martinsson et al. 2018). Recently, Abaei et al. (2018) developed an HBM for safety assessment of vessels crossing shallow-waters based on time-domain hydrodynamic simulations. There is also a great deal of methods developed based on MLE that show the applicability of this method in risk and reliability assessment of complex engineering systems, examples of which are structural degradation modeling, risk-based maintenance planning, geotechnical risk assessment, etc. (Straub 2009, Arzaghi et al. 2017, Abaei et al. 2018, Leoni et al. 2018, BahooToroody et al. 2019). Nielsen and Sørensen (2017) estimated the remaining useful lifetime (RUL) of a wind turbine to calibrate a Markovian deterioration model based on MLE approach.

1.2. Objective and organization

Different assumptions (e.g. MR, PR) and tools (e.g. MLE, HBM) lead to distinct results which are discussed here for the purpose of comparison and cross-verification. Accordingly, this paper aimed at presenting a comparison model for enabling industry on indicating the possible differentiation in failure assessment of random process under the assumption constraint. Consequently, the magnitude of the deviation value in different failure modelling approaches is highlighted. This would lead to identify the most efficient and minimal variance parameter estimator of failure modelling process. The developed framework in this study opens the door for the use of engineering researchers in risk analysis and reduction plan throughout the industries.

The remainder of present paper is structured as follows: the procedure of models specification and overview is presented in Section 2. Section 3 provides the application of the developed methodology in a Natural Gas Regulating and Metering Stations (NGRMS) in Florence, Italy. In Section 4, the results and discussions including a comparison of the investigated methods are presented, while Section 5 provides the concluding remarks of this research.

1.3. Assumptions

The outcomes of a maintenance plan, which is the condition of repaired systems, can be modelled stochastically throughout the operation. The division of repair categories is made based on a number of factors including whether the failure interarrival times are dependent over the asset operational time or not. The differentiation formed based on such factor is explained through the following assumptions:

1.3.1. Perfect Repair

The renewal process belongs to the class of stochastic point processes where inter-arrival times are assumed to be *iid* random variables. In this category, any repair originated by a failure in the system is assumed to be perfect and subsequently the system is said to be "as good as new". The expected number of failures, $E[N(t)]$, in time, t, is defined as renewal function given by Equation 1:

$$E[N(t)] = m(t), \qquad m(t) = F(t) + \int F(t-T)dm(T) \tag{1}$$

where T is successive failure times, $N(t)$ is the number of failure and $F(t)$ is the cumulative distribution function (CDF) of T. Measuring the changes of variables in both sides of Equation 1 with respect to the change of time results in Equation 2:

$$m'(t) = \lambda(t), \qquad \lambda(t) = f(t) + \int_0^t f(t-T)\lambda(T)dT \tag{2}$$

where $f(t)$ is the corresponding probability density function (PDF) of successive inter-arrival times, T. One of the most celebrated renewal processes including *iid* assumption is the Homogeneous Poisson Process (HPP) (Louit, Pascual et al. 2009, Barabadi, Barabady et al. 2014, Hajati, Langenbruch et al. 2015) which is recognized by the method presented in this paper.

1.3.2. Minimal Repair

Based on the HPP assumption, the failure rate will be independent of time. However, in reality the system condition in i^{th} time-step is dependent on its condition in time-step t_{i-1}. Relaxing the *iid* assumption leads to the Nonhomogeneous Poisson Process (NHPP) in which the system retains the "as bad as old" condition following a relatively instant repair action. Implementing MR, the observation process can be carried out either failure-truncated or stopped in a fix time. The calculation methods are similar in both mentioned approaches and here the failure-truncated case will be adopted as recommended by Kelly and Smith (2009). The expected number of failures through the specific time interval, $[t_n, t_{n+1}]$, $E[N(t)]$, is given using Equation 3.

$$E[N(t)] = \int_{t_n}^{t_{n+1}} \lambda(t)dt \tag{3}$$

where an appropriate functional form for ROCOF, $\lambda(t)$, must be determined to represent the expected number of failures, accordingly. For this purpose, power-law, log-linear and linear models are suggested in the literature (Kelly and Smith 2009, El-Gheriani, Khan et al. 2017).

Power-law model is one of the most common forms of ROCOF in reliability assessment (Abaei, Arzaghi et al. 2018) as it can predict the nonlinearity of the stochastic process with reasonable precision. The relationship for power law is given by Eq. (4).

$$\lambda(t) = \frac{\alpha}{\beta}\left(\frac{T}{\beta}\right)^{\alpha-1} \tag{4}$$

According to Arzaghi et al. (2018), the inter-arrival of times between successive failures, T, in the power-law process follows a Weibull distribution, $f(t,\beta,\alpha)$, with shape parameter, α, and scale parameter, β, given by:

$$f(t,\beta,\alpha) = \frac{\alpha}{\beta}\left(\frac{T}{\beta}\right)^{\alpha-1} \exp[-(T/\beta)^{\alpha}] \tag{5}$$

1.4. Parameter estimation methods

Observed data, manipulated information, and gathered knowledge are three consecutive steps of making inference. The effectiveness of a specific model must be examined, that is, how well the model fits the collected data. This question is answered through the process of parameter estimation. In case of reliability analysis, not only the assumptions but also the methods of parameter estimations are of high importance affecting the accuracy level of final results. Once the assumption is specified, and the data is observed, the mathematical method for estimating the parameter of interest should be established. In the present study, MLE and HBM as the most popular choices of model fitting in reliability assessment are utilized, as recommended by previous researchers (Mahadevan and Rebba 2005, Neil, Tailor et al. 2008, Rebba and Mahadevan 2008, Peng, Huang et al. 2013). A brief introduction to these methods can be found in the following sections.

1.4.1. Maximum Likelihood Estimation (MLE)

Assuming that vector $g = (g_1, \dots, g_n)$ is a random sample of an available data source, MLE is performed to predict the most likely data source that would yield the random sample, g. For this purpose, it is necessary to identify both the appropriate distribution of data source and its corresponding parameters. Consider that $x = (x_1, \dots, x_n)$ is a vector specified within the parameter space, therefore the PDF of the data vector, g, would be achieved by Equation 6, given by:

$$f(x_1, x_2, \dots, x_n|g) = f_1(x_1|g)f_2(x_2|g) \dots f_n(x_n|g) \tag{6}$$

1.4.2. Hierarchical Bayesian Model (HBM)

A summary of the process for performing inference using data and a probabilistic model is presented in Figure 1. As shown in this figure, the raw data are the collected values from a process. Evaluation of the data results in information and knowledge is obtained by gathering information. The process of making conclusion based on what once knows is referred to as inference. There is a need for models for obtaining information based on raw data. The models available for this purpose can be categorized into deterministic or probabilistic approaches (Kelly and Smith 2009). In this regard, probabilistic models are able to represent the uncertainty associated with available data where a HBM approach will assist in achieving the posterior distribution of the parameters of interest. HBM is carried out based on the Baye's theorem, given by Equation 7 (El-Gheriani, Khan et al. 2017):

$$\pi_1(\theta|x) = \frac{f(x|\theta)\pi_0(\theta)}{\int_\theta f(x|\theta)\pi_0(\theta)d\theta} \tag{7}$$

where θ denotes the unknown parameter of interest, $\pi_1(\theta|x)$ is the posterior distribution, and $f(x|\theta)$ is the likelihood function. HBM utilizes multistage prior distributions for the parameter of interest indicated by $\pi_0(\theta)$ (Abaei, Arzaghi et al. 2018) as follow:

$$\pi_0(\theta) = \int_{\emptyset} \pi_1(\theta|\varphi)\, \pi_2(\varphi) d\varphi \tag{8}$$

where, $\pi_1(\theta|\varphi)$ is the first-stage prior as the population variability in θ; φ denotes a vector of hyper-parameters, (e.g. $\varphi = (\alpha, \beta)$) , while α and β are the shape and scale parameters of a Weibull distribution, respectively. The uncertainty in φ is represented by $\pi_2(\varphi)$ as the hyper-prior distribution. The prior distribution, $\pi_0(\theta)$ is specified using generic data collected from different sources (numerical simulations, experiments or collected from industrial operations) to estimate the posterior distribution (Abaei, Arzaghi et al. 2018).

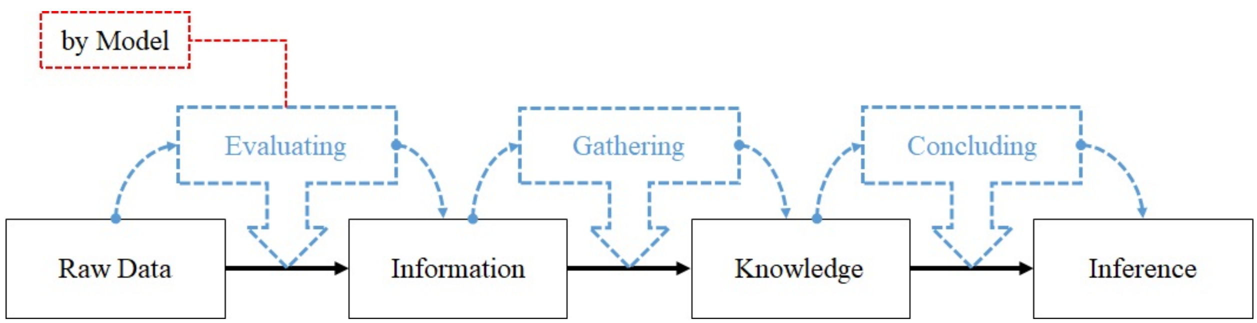


Figure 1. An overview of inference process and its key elements.

2. Model specification

In order to predict the condition of a system after it has undergone a repair, the precise and powerful mathematical approaches are established. A well-known parameter estimator (MLE) and the recent advances in Bayesian statistical methods (HBM) are accounted for revealing the gap between PR and MR. Based on the numbers of parameter required for characterization of distribution of failure time, Weibull and Exponential distributions are used to perform the analysis. Particularly, with a NHPP assumption, the time to failure cannot be characterized by an exponential distribution where $\alpha = 1$. So, a two-parameter Weibull distribution is required (Dar, Gowsami et al. 2015, Pesinis and Tee 2017). This is while for a HPP, Exponential distributions can be employed, as recommended by Hajati, Langenbruch et al. (2015) and Kumar and Chakraborti (2015).

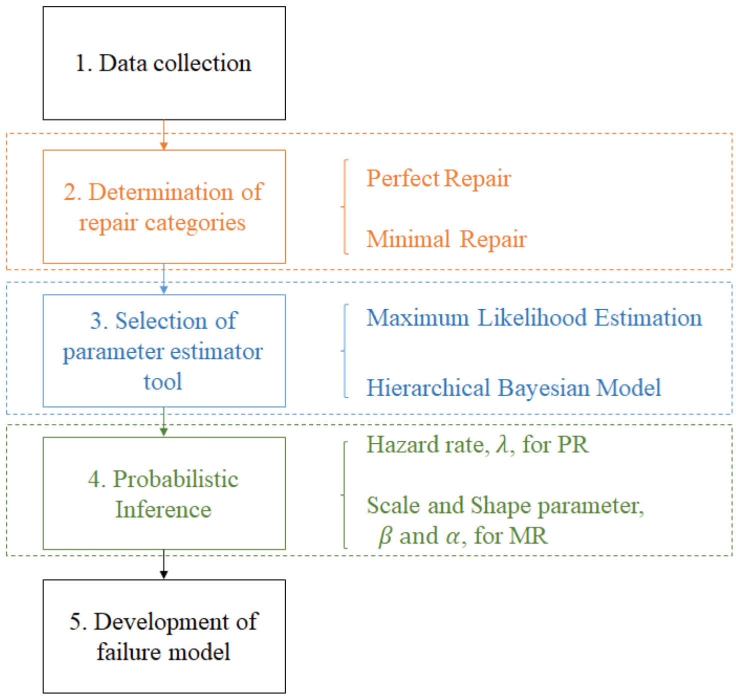


Figure 2. Developed framework for failure modelling based on different repair categories and parameter estimator tools.

2.1. Homogenous Poisson Process

2.1.1. Maximum Likelihood Estimation (MLE)

As discussed in Section 1.1.1, given a perfect repair condition, the probability distributions of failure inter-arrival times, denoted by T, are expressed by Eqs. (9) and (10):

$$F(t) = 1 - e^{-\lambda t} \tag{9}$$

$$f(t) = \lambda\, e^{-\lambda t} \tag{10}$$

where λ is defined as the rate parameter with a likelihood function expressed by Eq. (11):

$$L(\lambda) = \prod_{i=1}^{n} \lambda\, e^{-\lambda T_i} = \lambda^n e^{\left(-\lambda \sum_{i=1}^{n} T_i\right)} = \lambda^n\, e^{(-\lambda n \bar{T})} \tag{11}$$

where $\bar{T}$ is the mean of the inter-arrival times and *n* is the total number of failures observed. Thus, the maximum likelihood of rate parameter λ, is given by Eq. (12) (Ross 2014):

$$\hat{\lambda} = \frac{1}{\bar{\bar{T}}} = \frac{n}{\sum_i T_i} \tag{12}$$

It is anticipated that by using this approach to uncertainty modelling, the obtained exponential distribution would be a representative of the failure inter-arrival times during the future operations of studied system.

2.1.2. *Hierarchical Bayesian Model (HBM)*

For a system with failure events that follow a Poisson distribution, the number of failures, x, can be modelled by Eq. (13):

$$f(x|\lambda) = \frac{(\lambda t)^x e^{-\lambda t}}{x!}\,, \quad x = 0,1,\ldots \tag{13}$$

where t is the exposure time and λ is the intensity of the Poisson distribution. For a HPP, a gamma distribution can be utilized to describe the variability of λ among the observed failure times. Therefore, given the hyper-parameters α and β, the first-stage prior distribution can be achieved by the Gamma distribution, as expressed by Eq. (14) (Siu and Kelly 1998):

$$\pi_1(\lambda|\alpha,\beta) = \frac{\beta^\alpha \lambda^{\alpha-1} e^{-\beta\lambda}}{\Gamma(\alpha)} \tag{14}$$

Diffusive Gamma distribution is applied independently to model the prior distribution of hyper-parameters, as suggested by El-Gheriani, Khan et al. (2017).

It should be noted that although the hyper-parameters are considered as independent random variables prior to any observations, they become dependent as soon as observations are introduced. According to Kelly and Smith (2009), this dependency is accounted for by the joint posterior distribution.

Once the model, including the prior distribution and likelihood functions, are developed, MCMC simulations are carried to predict the posterior distribution of the Gamma parameters, α , β. This results in the estimation of Exponential distribution with a rate parameter of λ, and its associated uncertainty.

2.2. Non-Homogenous Poisson Process

2.2.1. Maximum Likelihood Estimation (MLE)

The method of estimating the probability distribution of failure inter-arrival times, based on a MR assumption, is explained earlier in section 1.1.2. In order to obtain the ML of Weibull parameters, the recommended likelihood function is given by Eq. (15) (Asher and Feingold 1984):

$$L = \prod_{i=1}^{n} f(T_i) \tag{15}$$

where T_i is the time at which the i^{th} failure has occurred and n is the total number of failures. The ML of shape and scale parameters α, β are given by Eqs. (16) and (17) (Crow 1975):

$$\hat{\alpha} = \frac{T_n}{n^{1/\beta}} \tag{16}$$

$$\hat{\beta} = \frac{n-1}{\sum_{i=1}^{n-1} \ln\left(T_n/T_i\right)} \tag{17}$$

where T_n is the time at which last failure, n, has occurred.

2.2.2. Hierarchical Bayesian Model (HBM)

In order to reflect on the dependency of the inter-arrival times, T_i, a conditional probability must be established. This probability for the time interval $[t_{i-1}, t_i]$ can be expressed by Eq. 18 (El-Gheriani, Khan et al. 2017).

$$f(t_i|t_{i-1}) = f(t_i|T_i > t_{i-1}) = \frac{f(t_i)}{\Pr(T_i > t_{i-1})} \tag{18}$$

Consequently, the Weibull distribution and the corresponding likelihood function are given by Eqs. (19) and (20), respectively.

$$f(t_i|t_{i-1}) = \frac{\alpha}{\beta^{\alpha}}(t_i)^{\alpha-1} e^{\left[\left(\frac{t_{i-1}}{\beta}\right)^{\alpha} - \left(\frac{t_i}{\beta}\right)^{\alpha}\right]} \tag{19}$$

where $i = 2, \dots, n$.

$$f(T_1, T_2, \dots, T_n|\alpha, \beta) = f(T_1) \prod_{i-2}^{n} f(t_i|t_{i-1}) \tag{20}$$

where T_1 and T_n are the times of first and nth failure events. Furthermore, the uncertainty of parameters α and β are modelled by HBM representing the variability of failure inter-occurrence times. Similar to the HPP case, these parameters will become inter-dependent once observations are made. The likelihood function is not pre-programmed into MCMC sampling software packages, OpenBUGS. Based on the suggestions provided by Kelly and Smith (2009), the likelihood function, φ, which is a vector of n array can be assigned to the model. This function $\varphi = log(likelihood)$ as defined by Equation 21 adopts samples of α and β from the prior distribution in Equation 22 (Abaei, Arzaghi et al. 2018).

$$\varphi = \log(\alpha) - \alpha \times \log(\beta) + (\alpha - 1)\log(T_i) - (T_n/\beta)^{\alpha}/n \quad (21)$$

$$\begin{cases} \alpha \sim Gamma(0.0001, 0.0001) \\ \beta \sim Gamma(0.0001, 0.0001) \end{cases} \quad (22)$$

where T_n and T_i are the last and i^{th} observation of the failure times in the simulation, and n is the vector size. As similar to HPP, an independent diffuse is assumed for the prior distribution of hyper-parameters (El-Gheriani, Khan et al. 2017). The updated posterior distribution of the hyper-parameters obtained from the MCMC sampling using the observed data are inserted into the Weibull distribution, $f(t, \beta, \alpha)$, in order to estimate the PDF of failure under MR assumption.

3. Application of methodology

In order to demonstrate the application of the developed method and establish a comparison among the employed models, a practical example from the degradation process of Natural Gas Regulating and Metering Stations (NGRMS) operating in Italy is considered as the case study.

3.1. Scenario development

NGRMS is installed in a distribution network and supplied with natural gas flow through a (number of) transmission pipeline(s). Pressure reduction and gas flow measurements are the fundamental duties of these facilities that consist of five main sections including the inlet, filter, metering, regulator and outlet. In order to prevent any interruptions in the process caused by failures events, the redundant line is set up. A schematic of NGRMS is illustrated in Figure 3.

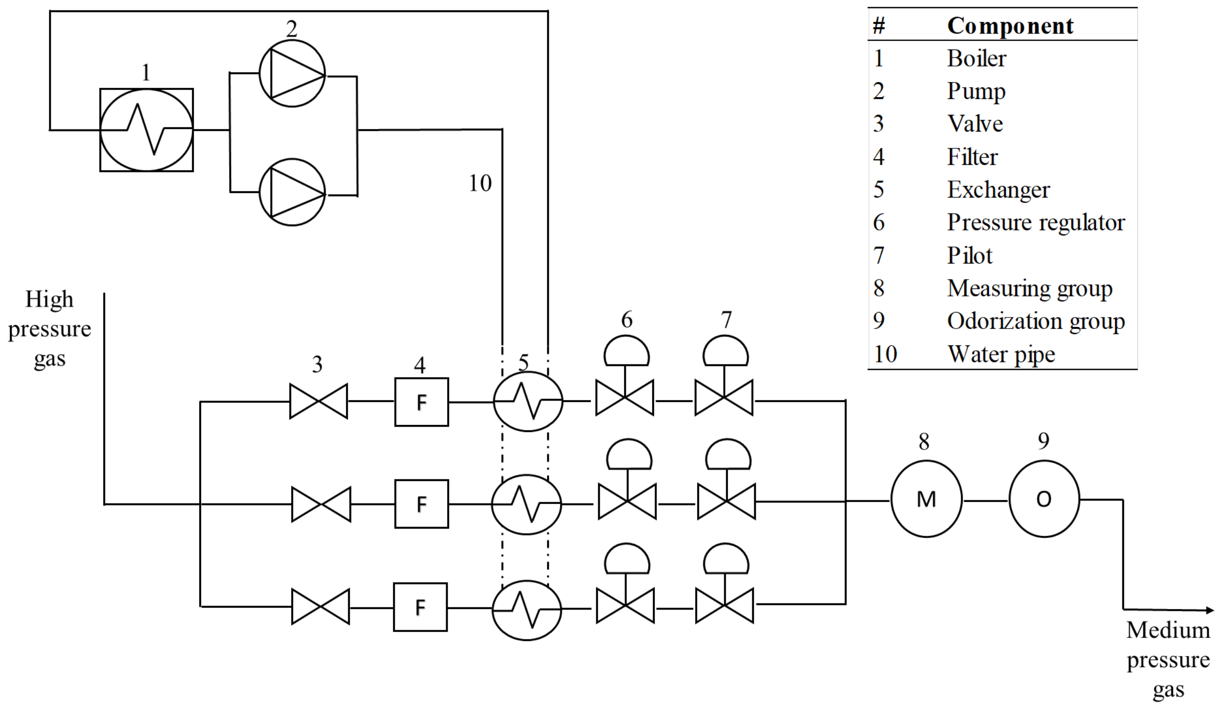
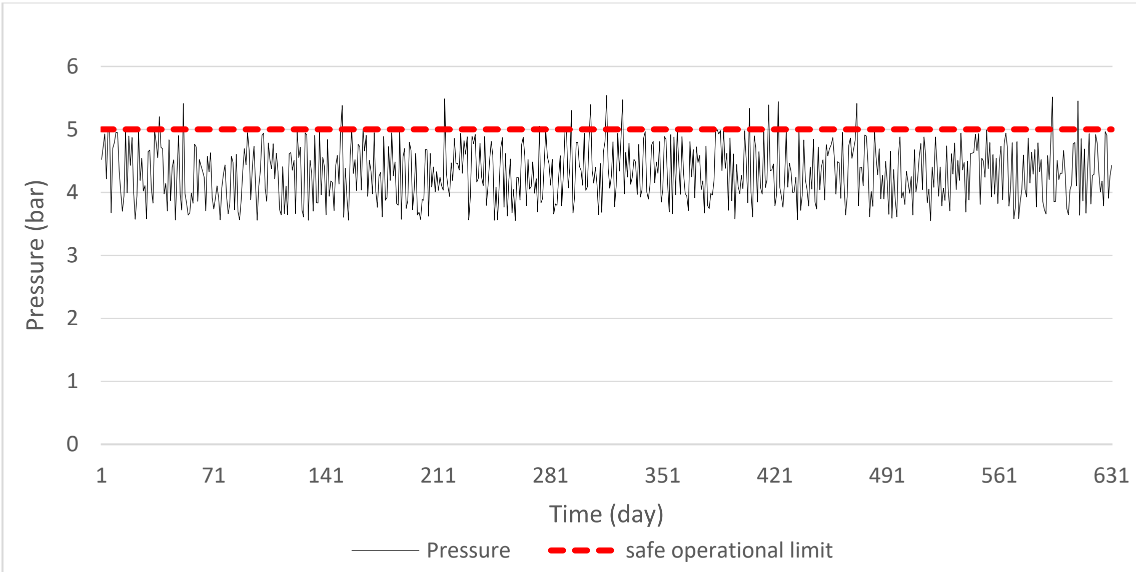
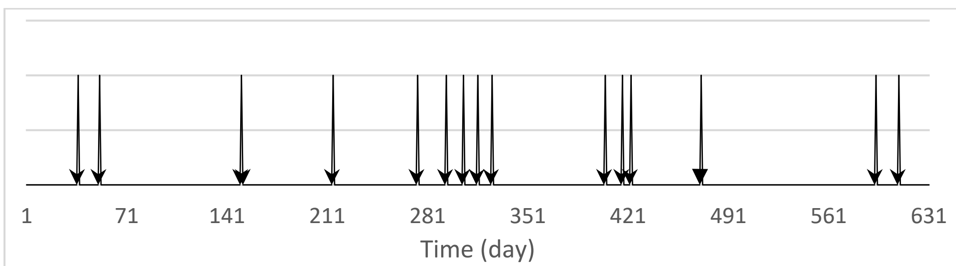


Figure 3. A schematic of Natural Gas Regulating and Metering Stations (Gonzalez-Bustamante, Sala et al. 2007)

A range of process variables characterize the health condition of the process in NGRMS, e.g. pressure, temperature and vibration. In this study pressure is considered as the variable of interest for the analysis of the degradation process. The failure of system is defined as an event where the value of pressure exceeds the desired safety limit of the operation. The recorded time series of operational, are illustrated in Figure 4. This figure also shows the observed failure times of system. It is worthwhile to mention that the random noise in operational pressure data is filtered from nonstationary and nonlinear raw data by adopting Empirical Mode Decomposition (EMD) method. The explanation of EMD is beyond the scope of this paper and readers are referred to the following researches for detailed discussions on this topic (Huang, Shen et al. 1998, Wu and Huang 2004, Wu, Huang et al. 2007, Li and Pandey 2017, BahooToroody, Abaei et al. 2019).



(a)



(b)

Figure 4. (a) Time series of pressure data collected from NGRMS (b) Time to failure for given pressure values.

As depicted in Figure 4, pressure values are recorded during a 631-days operation where 15 failure events have occurred. These data have been utilized in the analysis process.

3.2.Homogenous Poisson Process modelling

3.2.1. Maximum Likelihood Estimation (MLE)

The presented method in section 2.1.1 is applied on the pressure data based on an HPP assumption. The ML of rate parameter is estimated as $\lambda = 0.0246$ (per day) resulting in an exponential PDF of failure inter-arrival time illustrated in Figure 5. The specifications of this exponential distribution including its Mean Time To Failure (MTTF) are provided later in the summary of statistical analyses (see Table 2).

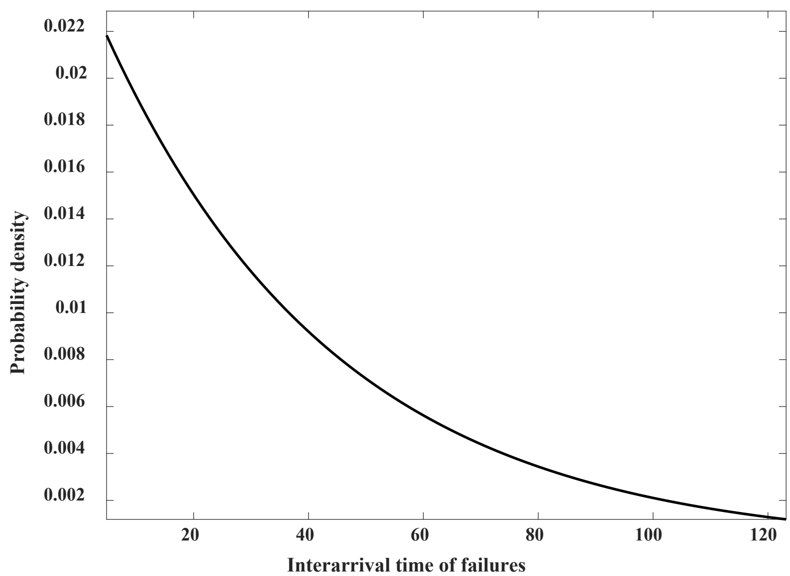


Figure 5. assigned Exponential probability distributions fitted on operational data given a perfect repair assumption.

3.2.2. Hierarchical Bayesian Model (HBM)

The failure rates of considered operation in Table 1, have been extracted from the pressure time-series. Three chains with over-dispersed initial values of α and β were used to ensure the convergence of simulations. In order to calculate the parameters of interest, the HBM was performed in OpenBUGS with 1000 burn-in iterations, followed by 300,000 iterations through each chain. The caterpillar plot of credible intervals of failure rate, λ, for all 12 interval is illustrated in Figure 6. The mean value of posterior predictive distribution of λ is 0.0243 per day (see Table 2). This average value of failure rate also incorporates an estimate of the interval-to-interval variability.

Table 1. Failure (pressure exceedance) rate data during NGRMS operation

Region	No. of Failures	Exposure time (day)
1	2	52
2	0	57
3	1	50
4	0	53
5	1	56
6	5	55
7	0	54
8	3	45
9	1	50
10	0	54
11	0	53
12	2	49

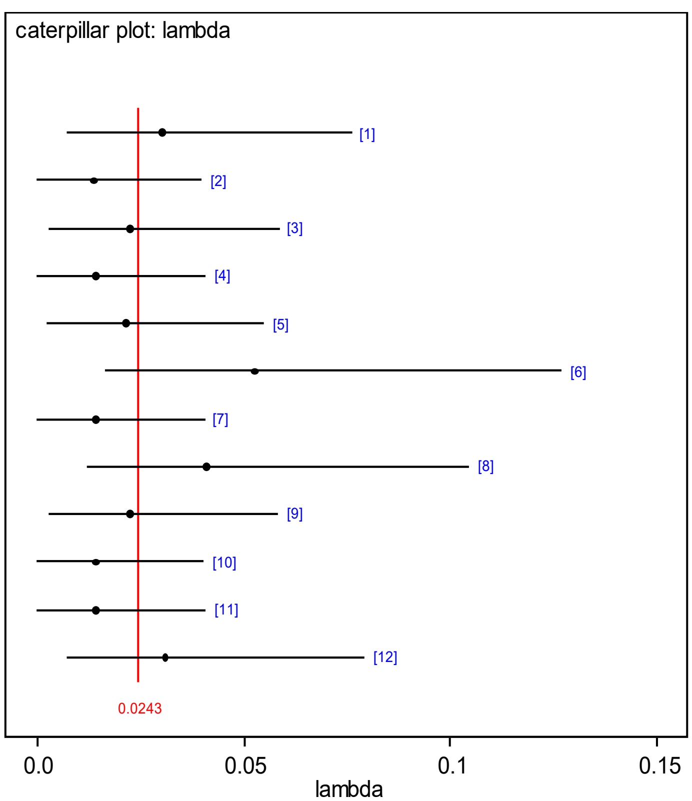


Figure 6. The posterior mean and 95% credible interval for pressure exceedance from safety limits over time given a PR assumption. Note: black dots are posterior means for each interval, the red line is average of posterior means.

3.3. Non-Homogenous Poisson Process modelling

3.3.1. Maximum Likelihood Estimation (MLE)

Given a NHPP, the failure inter-occurrence times, T, in the power-law process generate a Weibull distribution, $f(t, \beta, \alpha)$, with shape parameter, α, and scale parameter, β, which can be estimated using Equations (15-17) through the application of MLE. The maximum likely α and β are computed as 52.83 and 1.1069, respectively (for more details see Table 2). Figure 7 shows the resultant Weibull distributions of MLE on the available data.

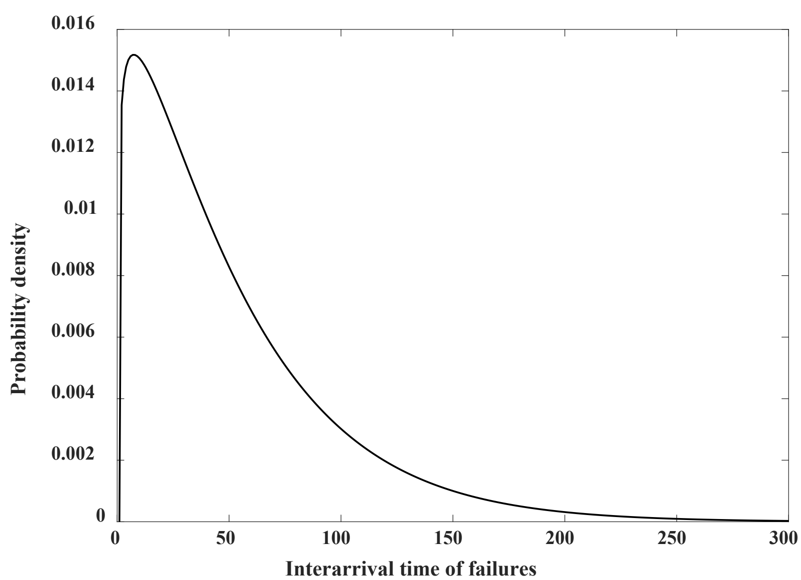


Figure 7. Obtained Weibull distribution of failure inter-arrival times considering a MR assumption.

3.3.2. Hierarchical Bayesian Model (HBM)

In order to estimate the likelihood function and posterior probability of Weibull parameters, α and β, the recorded failures were entered into the HBM. Similar to the HPP application, using MCMC simulations, three chains from separate points were assigned with 1000 burn-in iterations, followed by 300,000 iterations at each chain in order to ensure the convergence of the simulation and accurately predict the posterior probabilities of the parameters of interest. Figure 8 shows the predicted posterior distribution of α and β. The dynamic trace of Weibull parameters is depicted in Figure 9 confirming the convergence. A summary of the estimated marginal posterior distributions for α, β as well as their corresponding MTTFs are listed in Table 2.

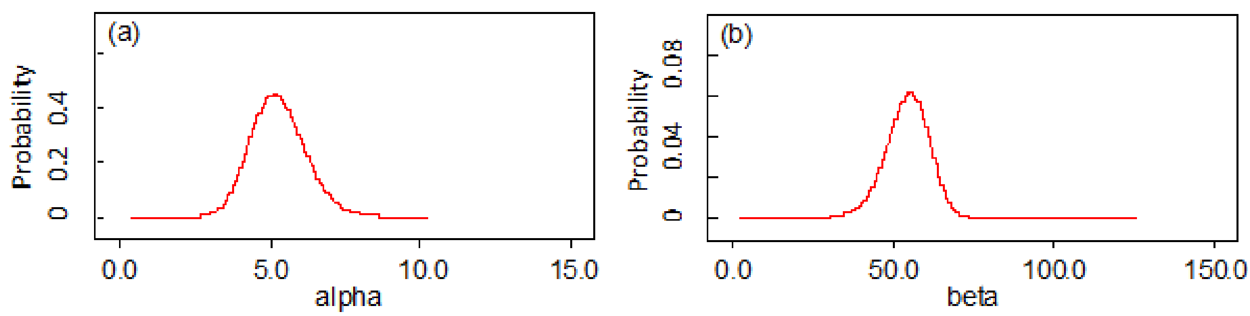


Figure 8. Posterior distributions of Weibull (a) shape parameter (b) and scale parameter.

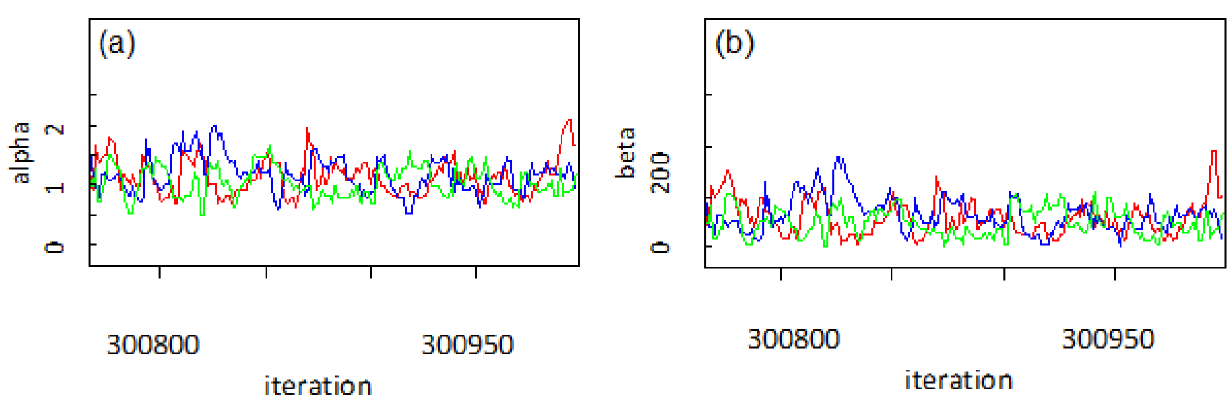


Figure 9. Dynamic trace of Weibull shape parameters (a) and scale parameter (b) in MCMC simulation.

4. Comparison, results and discussion

4.1. *comparison*

According to the presented models, a range of comparisons are drawn to illustrate the deviation of uncertainty quantifications throughout the characterized failure functions. To this end, Cumulative Distribution Functions (CDFs) of each failure modelling approach were developed, as illustrated in Figure 10. The estimated MTTF for each approach is also shown in their CDFs in this Figure. Table 2 summarizes the details of obtained results. An assumption-based approach covers the comparison between the two assumptions made regarding the distribution of inter-arrival times of failure events during the studied operation i.e. those with the iid assumption, represented by a HPP and those without this assumption which are modelled as a NHPP. The comparison of HBM and MLE reveals a difference in the estimated posterior distribution of parameters, which is attributed to the impact of correlation between the observed data. The results of these analyses are discussed in more details in the following sections.

4.1.1. Assumption-based comparison

The MTTF values, estimated by the MLE approach, are 40.67 and 50.87 days for PR and MR, respectively. The significant difference between the obtained MTTF is due to the fact that the PR assumption neglects the dependency amongst failure interarrival while MR accounts for this. As shown in Figure 10, a similar difference level is observable between the MTTF values estimated by using a HBM approach. The MTTF of HPP and NHPP are found to be 41.15 and 55.57 days, respectively. These results confirm that discounting the time dependency of failure events may lead to between 25% to 35% difference in MTTF, regardless of the modelling approach (HBM or MLE).

4.1.2. *Parameter estimator-based comparison*

Regarding the HBM, to allow the results to be compared with MLE, independent and diffuse priories were adopted for α and β. A gamma distribution prior was used for both shape and scale parameters, as suggested by Kelly and Smith (2009).

Based on PR, the MLE yields a failure rate of $\lambda_{MLE} = 0.0246$ with a 95% confidence interval of (0.0137,0.0385) while the posterior mean of this parameter is estimated by HBM as $\lambda_{HBM} = 0.0243$, having a 95% confidence interval of (0.0044,0.0631). In the light of estimated value for parameter of interest, lambda, given HPP, it is interpreted that the source to source variability of data carrying out by MCMC simulation in the HBM is less reflected. Subsequently, the MLE and HBM of MTTF are estimated at 40.67 and 41.15, respectively, suggesting a minimal difference (see Figure 10).

For a MR assumption, HBM yields a posterior mean for the shape parameter $\alpha = 1.107$ with a 95% credible interval of (0.605, 1.757) while the MLE model resulted in $\alpha = 1.106$, highlighting a good agreement between the employed approaches. Bayesian inference β, yields a posterior mean of 57.71. However, the MLE, which disregards the correlation between observed data, results in a smaller shape parameter of 52.83 (8% deviation). Finally, this deviation in the posterior mean of β in HBM and MLE is appeared again in the MTTF (see Table 2).

Table 2. Summary of statistical analyses results of different failure modelling approaches.

Assumption	Distribution	Estimation method	Parameter value	MTTF	SD*
Perfect repair	Exponential	MLE	λ= 0.0246	40.666	1.6538e+03
Perfect repair	Exponential	HBM	λ= 0.0243	41.152	1.6935e+03

Minimal repair	Weibull	MLE	α= 1.106 β= 52.83	50.872	2.1179e+03
Minimal repair	Weibull	HBM	α= 1.107 β= 57.71	55.569	2.5270e+03

*Standard Deviation

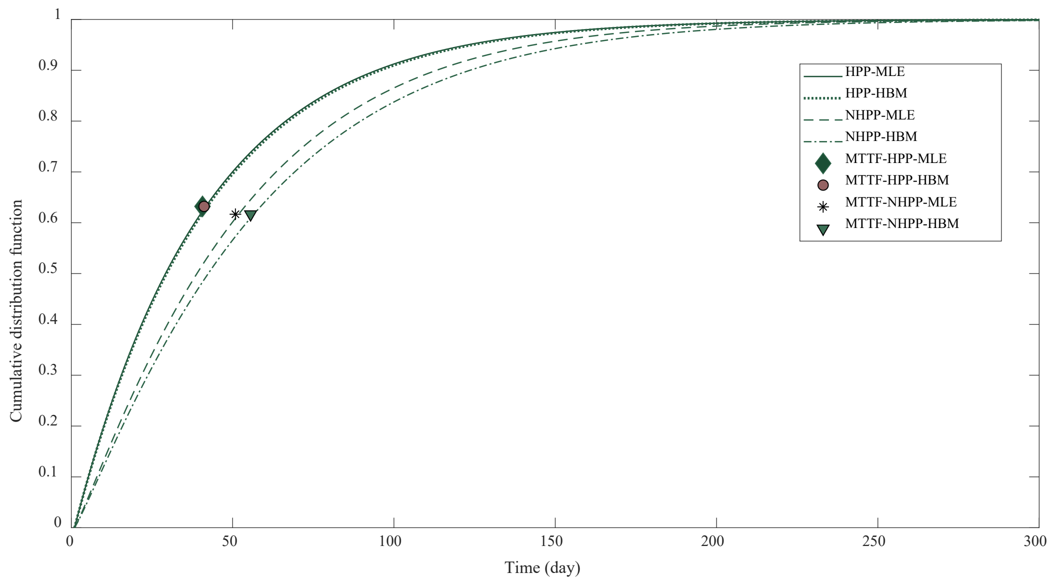


Figure 10. Cumulative distribution function and corresponding MTTF values for different repair categories estimated by MLE and HBM methods.

4.2.Discussion: the unbiased and minimal variance category of failure modelling approaches

The results listed in Table 2 reveal that the value of the Weibull shape parameter, estimated using both parameter estimation approaches (HBM, MLE), are higher than 1, confirming that the number of failure events are dependent upon time. This is in contrary to the PR assumption, where the failure rate is assumed to be constant with time. That is, a MR assumption is appearing to be more credible for the failure modelling of NGRMS. In order to categorized the efficient approach among the presented models given MR assumption, the probability plot for Weibull distribution was developed, as depicted in Figure 11. As shown in the figure, the ML of shape and scale parameters of Weibull distribution include higher uncertainty than their estimation through the HBM approach. Therefore, the HB model given an MR assumption seems to be the most reliable approach amongst the reviewed methods. That is, the Bayesian method can efficiently take advantage of the available data to predict the parameters of failure model hence providing an opportunity for improvements of asset management plans.

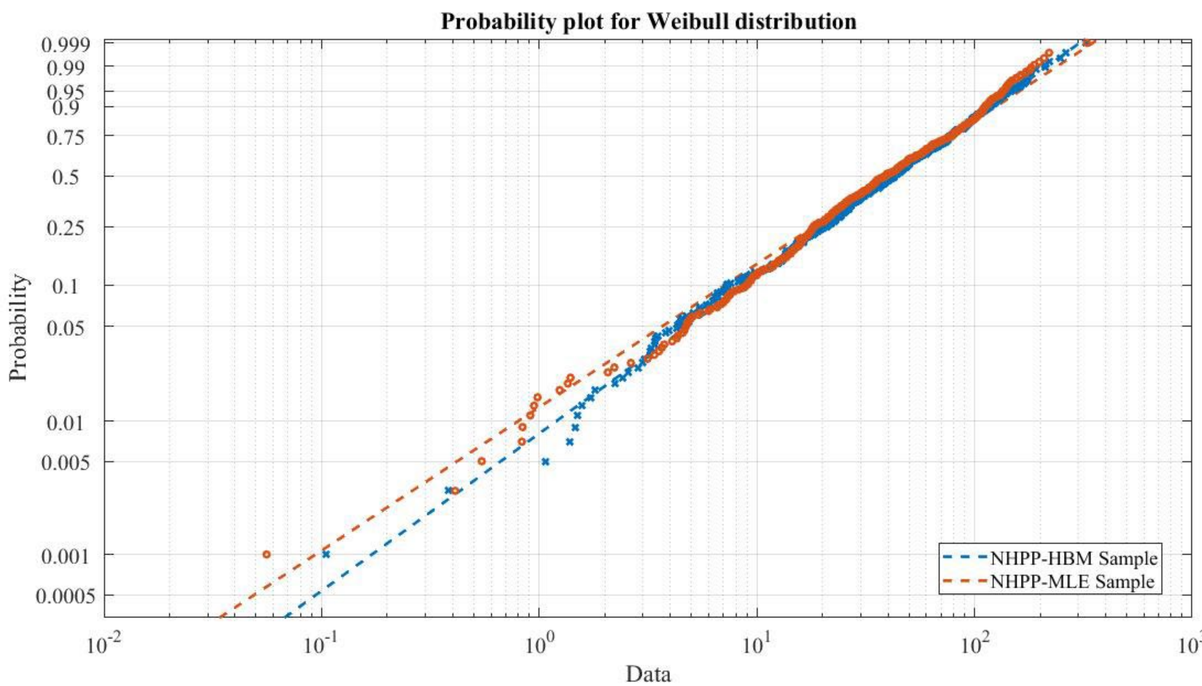


Figure 11. Weibull probability plot for time-dependent failure modelling approaches.

5. Conclusion

A major challenge in failure modelling of repairable systems is choosing applicable tools and making valid assumptions. This will also help in reducing the uncertainty associated with the obtained results. The differences between application of two mostly utilized assumptions in failure modelling, MR and PR, have been addressed in this paper. This was carried out in a case study of natural gas regulation and measurement plant by MLE and Bayesian inference method. The final results highlighted that relaxing the renewal process assumption (constant failure rate) and taking the time dependency between the observed failure times into account, results in a more precision of failure modelling where the shape parameter value of Weibull distribution in both parameter estimation approaches (HBM, MLE) are higher than 1, confirming that the number of failure events are dependent upon time. On the other hand, HBM is able to model the correlation between the failure data through an MCMC simulation, leading to less uncertainty in MTTF calculations. This is approved through the developed probability plot for Weibull distribution where the estimated shape and scale parameters of HB model has better precisions than ML estimation. The results also suggest that a minimal repair assumption for an HBM failure analysis estimates longer MTTF which avoid the conduct of premature maintenance or compromise operational safety. As a further investigation, it is recommended to model generalized perfect repair assumption with hierarchical Bayesian inference.

Reference

Abaei, M. M., E. Arzaghi, R. Abbassi, V. Garaniya and S. Chai (2018). "A novel approach to safety analysis of floating structures experiencing storm." Ocean Engineering **150**: 397-403.

Abaei, M. M., E. Arzaghi, R. Abbassi, V. Garaniya, M. Javanmardi and S. Chai (2018). "Dynamic reliability assessment of ship grounding using Bayesian Inference." Ocean Engineering **159**: 47-55.

Antonov, A. and V. Chepurko (2017). "Statistical analysis of data on failures of the nuclear plant equipment in conditions of a non-homogeneous flow of events. Part 2." Nuclear Energy and Technology **3**(3): 206-210.

Arzaghi, E., M. M. Abaei, R. Abbassi, V. Garaniya, J. Binns, C. Chin and F. Khan (2018). "A hierarchical Bayesian approach to modelling fate and transport of oil released from subsea pipelines." Process Safety and Environmental Protection **118**: 307-315.

Arzaghi, E., M. M. Abaei, R. Abbassi, V. Garaniya, C. Chin and F. Khan (2017). "Risk-based maintenance planning of subsea pipelines through fatigue crack growth monitoring." Engineering Failure Analysis **79**: 928-939.

Asher, H. and H. Feingold (1984). Repairable system reliability: Modeling, Inference, Misconceptions and their cause, Marcel Dekker, New York.

BahooToroody, A., M. M. Abaei, E. Arzaghi, F. BahooToroody, F. De Carlo and R. Abbassi (2019). "Multi-level optimization of maintenance plan for natural gas system exposed to deterioration process." Journal of hazardous materials **362**: 412-423.

BahooToroody, A., M. M. Abaei, F. BahooToroody, F. De Carlo, R. Abbassi and S. Khalaj (2019). "A condition monitoring based signal filtering approach for dynamic time dependent safety assessment of natural gas distribution process." Process Safety and Environmental Protection **123**: 335-343.

Barabadi, A., J. Barabady and T. Markeset (2014). "Application of reliability models with covariates in spare part prediction and optimization–a case study." Reliability Engineering & System Safety **123**: 1-7.

Behmanesh, I., B. Moaveni, G. Lombaert and C. Papadimitriou (2015). "Hierarchical Bayesian model updating for structural identification." Mechanical Systems and Signal Processing **64**: 360-376.

Chitsazan, N., A. A. Nadiri and F. T.-C. Tsai (2015). "Prediction and structural uncertainty analyses of artificial neural networks using hierarchical Bayesian model averaging." Journal of Hydrology **528**: 52-62.

Crow, L. H. (1975). Reliability analysis for complex, repairable systems, ARMY MATERIEL SYSTEMS ANALYSIS ACTIVITYABERDEEN PROVING GROUND MD.

Dar, M. A., D. Gowsami and A. Chaturvedi (2015). "Testing Effort Dependent Software Reliability Growth Model with Dynamic Faults for Debugging Process." International Journal of Computer Applications **113**(11).

El-Gheriani, M., F. Khan, D. Chen and R. Abbassi (2017). "Major accident modelling using spare data." Process Safety and Environmental Protection **106**: 52-59.

Gonzalez-Bustamante, J., J. Sala, L. López-González, J. Míguez and I. Flores (2007). "Modelling and dynamic simulation of processes with 'MATLAB'. An application of a natural gas installation in a power plant." Energy **32**(7): 1271-1282.

Hajati, T., C. Langenbruch and S. Shapiro (2015). "A statistical model for seismic hazard assessment of hydraulic-fracturing-induced seismicity." Geophysical Research Letters **42**(24): 10,601-610,606.

Huang, N. E., Z. Shen, S. R. Long, M. C. Wu, H. H. Shih, Q. Zheng, N.-C. Yen, C. C. Tung and H. H. Liu (1998). The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis. Proceedings of the Royal Society of London A: mathematical, physical and engineering sciences, The Royal Society.

Kelly, D. L. and C. L. Smith (2009). "Bayesian inference in probabilistic risk assessment—the current state of the art." Reliability Engineering & System Safety **94**(2): 628-643.

Kumar, N. and S. Chakraborti (2015). "Improved phase I control charts for monitoring times between events." Quality and Reliability Engineering International **31**(4): 659-668.

Leoni, L., A. BahooToroody, F. De Carlo and N. Paltrinieri (2018). "Developing a risk-based maintenance model for a Natural Gas Regulating and Metering Station using Bayesian Network." Journal of Loss Prevention in the Process Industries.

Li, B. and M. D. Pandey (2017). "An advanced statistical method to analyze condition monitoring data collected from nuclear plant systems." Nuclear Engineering and Design **323**: 133-141.

Li, Q., F. Guo, S. G. Klauer and B. G. Simons-Morton (2017). "Evaluation of risk change-point for novice teenage drivers." Accident Analysis & Prevention **108**: 139-146.

Louit, D. M., R. Pascual and A. K. Jardine (2009). "A practical procedure for the selection of time-to-failure models based on the assessment of trends in maintenance data." Reliability Engineering & System Safety **94**(10): 1618-1628.

Mahadevan, S. and R. Rebba (2005). "Validation of reliability computational models using Bayes networks." Reliability Engineering & System Safety **87**(2): 223-232.

Majeske, K. D. (2007). "A non-homogeneous Poisson process predictive model for automobile warranty claims." Reliability Engineering & System Safety **92**(2): 243-251.

Mishra, M., J. Martinsson, M. Rantatalo and K. Goebel (2018). "Bayesian hierarchical model-based prognostics for lithium-ion batteries." Reliability Engineering & System Safety **172**: 25-35.

Modarres, M., M. P. Kaminskiy and V. Krivtsov (2016). Reliability engineering and risk analysis: a practical guide, CRC press.

Nandi, S., H. A. Toliyat and X. Li (2005). "Condition monitoring and fault diagnosis of electrical motors—A review." IEEE transactions on energy conversion **20**(4): 719-729.

Neil, M., M. Tailor, D. Marquez, N. Fenton and P. Hearty (2008). "Modelling dependable systems using hybrid Bayesian networks." Reliability Engineering & System Safety **93**(7): 933-939.

Nielsen, J. S. and J. D. Sørensen (2017). "Bayesian estimation of remaining useful life for wind turbine blades." Energies **10**(5): 664.

Peng, W., H.-Z. Huang, Y. Li, M. J. Zuo and M. Xie (2013). "Life cycle reliability assessment of new products—A Bayesian model updating approach." Reliability Engineering & System Safety **112**: 109-119.

Peng, W., L. Shen, Y. Shen and Q. Sun (2018). "Reliability analysis of repairable systems with recurrent misuse-induced failures and normal-operation failures." Reliability Engineering & System Safety **171**: 87-98.

Pesinis, K. and K. F. Tee (2017). "Statistical model and structural reliability analysis for onshore gas transmission pipelines." Engineering Failure Analysis **82**: 1-15.

Quy, N., J. Vrijling, P. v. Gelder and R. Groenveld (2006). On the assessment of ship grounding risk in restricted channels. 8th international conference on marine sciences and technologies.

Quy, N., J. Vrijling and P. Van Gelder (2008). "Risk-and simulation-based optimization of channel depths: Entrance channel of Cam Pha Coal Port." Simulation **84**(1): 41-55.

Rebba, R. and S. Mahadevan (2008). "Computational methods for model reliability assessment." Reliability Engineering & System Safety **93**(8): 1197-1207.

Ross, S. M. (2014). Introduction to probability models, Academic press.

Sheu, S.-H., T.-H. Liu, Z.-G. Zhang and H.-N. Tsai (2018). "The generalized age maintenance policies with random working times." Reliability Engineering & System Safety **169**: 503-514.

Siu, N. O. and D. L. Kelly (1998). "Bayesian parameter estimation in probabilistic risk assessment1." Reliability Engineering & System Safety **62**(1-2): 89-116.

Slimacek, V. and B. H. Lindqvist (2016). "Nonhomogeneous Poisson process with nonparametric frailty." Reliability Engineering & System Safety **149**: 14-23.

Spiegelhalter, D., A. Thomas, N. Best and D. Lunn (2007). "OpenBUGS user manual, version 3.0. 2." MRC Biostatistics Unit, Cambridge.

Straub, D. (2009). "Stochastic modeling of deterioration processes through dynamic Bayesian networks." Journal of Engineering Mechanics **135**(10): 1089-1099.

Toroody, A. B., M. M. Abaiee, R. Gholamnia and M. J. Ketabdari (2016a). "Epistemic-based investigation of the probability of hazard scenarios using Bayesian network for the lifting operation of floating objects." Journal of Marine Science and Application **15**(3): 250-259.

Toroody, A. B., M. M. Abaiee, R. Gholamnia, M. B. Torody and N. H. Nejad (2016b). "Developing a risk-based approach for optimizing human reliability assessment in an offshore operation." Open Journal of Safety Science and Technology **6**(01): 25-32.

Wu, Z. and N. E. Huang (2004). A study of the characteristics of white noise using the empirical mode decomposition method. Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences, The Royal Society.

Wu, Z., N. E. Huang, S. R. Long and C.-K. Peng (2007). "On the trend, detrending, and variability of nonlinear and nonstationary time series." Proceedings of the National Academy of Sciences **104**(38): 14889-14894.

Yu, H., F. Khan and B. Veitch (2017). "A flexible hierarchical Bayesian modeling technique for risk analysis of major accidents." Risk analysis **37**(9): 1668-1682.