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Sea State Estimation Using Quadratic Discriminant Analysis and Partial Least Squares Regression

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Abstract: This paper proposes non-model based sea state estimation methods for a dynamically positioned vessel. Sea state estimation entails finding the wave direction, significant wave height and peak wave period and is done based on sensor data of the vessel response. Sea state estimation is of importance because it assists the on board decision system and provides weather information for the relevant geographical position. In this paper, the methods for sea state estimation are based on machine learning algorithms, rather than the vessel transfer function. The models are trained and tested using simulated time series of response data, and yield promising results.

Keywords: Sea state estimation.

1. INTRODUCTION

Information about the sea state is necessary for decision making, securing safe marine operations. When using weather information from wave buoys, the weather for a specific position is found by interpolation between the positions of the wave buoys. On board sea state estimation may provide a more accurate sea state than information from wave buoys as it provides information in real time and for the specific position the vessel is in (Nielsen, 2017).

Nearly all marine operations have weather requirements. For example drilling operations, lifting operations and tandem operations all have strict limitations on significant wave heights. Input on real time sea state information for the exact geographical position allows for a better knowledge base for decision-making and potentially cost-savings related to waiting on weather or aborting an ongoing operation. As stated in Nielsen (2017), sea state estimation covers a wide range of purposes. Sea state estimation can be used for operational profiles, i.e. whether the ship operates in the conditions it was designed for, fuel performance evaluations, research on added resistance and accident investigations. The sea state is also of interest for autonomous ships, where the control system needs as much information as possible about the vessel surroundings and operational environment. Additionally, the sea state is an important input to the on board decision support system, as it for example can be used when detecting the occurrence of parametric roll (Galeazzi et al., 2015). Parametric roll is a phenomena which can cause serious damage, and the sea state may be a crucial input when deciding on measures to avoid parametric roll.

Previous work within the field involve model based calculation both in the time and frequency domain, for ships with forward speed and in Dynamic Positioning (DP). Most of the present day methods can be characterized as the so-called wave buoy analogy. The wave buoy analogy involves using a mathematical model to relate vessel response measurement data to the sea state. The common ground for many present model based methods is that they rely on some knowledge of the vessel's transfer function. The transfer function represents how waves are transferred into the vessel responses. Transfer functions are calculated using potential wave theory and sometimes Computational Fluid Dynamics (CFD) based on Navier-Stoke's equations and other nonlinear effects. Transfer functions can be difficult to estimate exactly, and if nonlinear effects are not accounted for, they will be inaccurate in severe waves.

Studies on sea state estimation for DP are extensive, and early studies include the use of the Maximum Likelihood Method by Waals et al. (2002). The method consists of calculating the Cross Power Spectral Densities (CPSD) using response data and minimizing the difference between this calculated spectrum and a theoretically found spectrum based on the phase difference and transfer functions. Further studies were done by Tannuri et al. (2003), who used a parametric estimation method. The directional spectrum was here represented by a 10-parameter function, capable of representing various sea states including doubly-peaked spectra. Similarly to Waals et al. (2002), the method further minimizes the difference between the measured and estimated response to yield the estimated sea state. Pascoal and Guedes Soares (2008) propose a non-parametric method which consists of an error minimization between CPSD based on sensor data and CPSD estimated using

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the transfer function, where a smoothing term yielding a smooth spectrum is also included.

The main scientific contribution in this paper is the development of non-model based methods for sea state estimation using machine learning methods. As the vessel transfer functions are not always available, the methods presented in this paper are independent of the transfer function and thus require less of detailed vessel information. A computationally efficient and accurate method for distinguishing between port and starboard waves, as well as head and following waves when relevant, is applied. The machine learning methods do not rely on transfer functions, and simply rely on estimating the sea state based on a combination of parameters calculated using the vessel response in all its Degrees of Freedom (DOFs).

The vessel used in the case study in this paper is NTNU's research vessel, R/V Gunnerus. A simulation model in Simulink was used to generate the data needed for the sea state estimation research.

This paper is organized as follows: Firstly in Section 2 the theory on wave modeling and vessel response is covered. Further, in Section 3 the theory behind the methods used for sea state estimation is described, followed by Section 4 including a description of how the sea state estimation algorithm combines these methods to produce results. Simulation results for estimation of wave direction, significant wave height and peak wave period are presented in Section 5. Lastly, the paper is concluded in Section 6.

2. THEORY

A description of the way waves are modeled in the simulations as well as the basis for the algorithm's ability to distinguish between port/starboard and head/following waves are covered in this section.

2.1 Modelling Waves

The standardized wave spectrum called the JONSWAP spectrum is used to model waves in the simulation model. The JONSWAP spectrum, or the Joint North Sea Wave Project spectrum, can be parametrized by the equation

$$S(\omega) = \alpha \frac{g^2}{\omega^5} \exp \frac{-5}{4} \left(\frac{\omega_p}{\omega} \right)^4 \gamma^{exp[-0.5(\frac{\omega - \omega_p}{\sigma \omega_p})^2]} \qquad (1)$$

where g is the gravitational constant, ω is the wave frequency, ω_p is the peak wave frequency, γ is a non-dimensional peak shape parameter, σ is the spectral width parameter and α is a parameter determining the shape of the spectrum in the high frequency range (Myrhaug, 2014). The JONSWAP spectrum can be directly related to significant wave height by

$$H_s = 4\sqrt{m_o} \qquad m_0 = \int_0^\infty S(\omega) d\omega \qquad (2)$$

where m_0 is the first spectral moment of the spectrum. The JONSWAP spectrum is a singly-peaked spectrum for fully developed sea, and simulations are done with long-crested waves.

Table 1 (Price and Bishop, 1974) shows realistic combinations of significant wave heights and peak wave periods. These sea states are used to generate a dataset with response data and the associated sea state.

Sea State	Description	$\mathbf{H_s}$ [m]	$\mathbf{T_p}$ [s]
0	Calm (glassy)	0	-
1	Calm (rippled)	0 - 0.1	4.87 - 5.66
2	Smooth (wavelets)	0.1 - 0.5	5.66 - 6.76
3	Slight	0.5 - 1.25	6.76 - 7.95
4	Moderate	1.25 - 2.5	7.95 - 9.24
5	Rough	2.5 - 4.0	9.24 - 10.47
6	Very rough	4.0 - 6.0	10.47 - 11.86
7	High	6.0 - 9.0	11.86 - 13.66
8	Very high	9.0 - 14.0	13.66 - 16.11
>8	Phenomenal	>14.0	>16.11

Table 1. Table of realistic combinations of significant wave heights and peak periods (Price and Bishop, 1974).

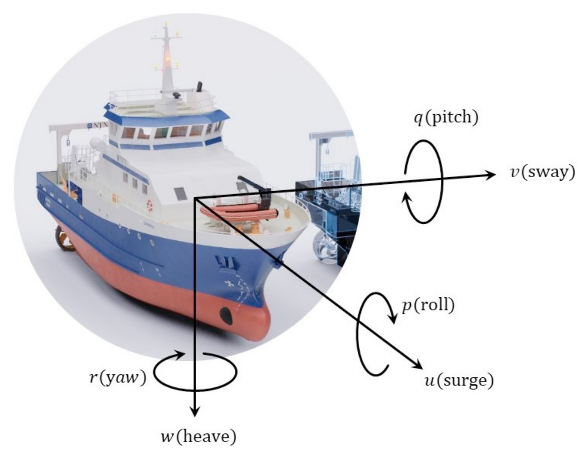

Fig. 1. R/V Gunnerus and its DOFs.

2.2 Differentiating Between Port and Starboard Waves

This section is based on information from Brodtkorb et al. (2018). Most vessels, including R/V Gunnerus, are port/starboard symmetric. This means that the response of the vessel is in practice the same for equal angles port and starboard of the center line. It therefore makes little sense to compare the energy in the sea state to distinguish between port and starboard waves.

In Figure 1 the DOFs in which the vessel can move, as well as the axes, are shown. The roll motions are anti-symmetric about the x-axis, meaning that when roll is negative on starboard side, it is positive on the port side and vice versa. On the contrary, the heave motions are symmetric about the x-axis. The cross-spectrum of roll and heave motions can therefore be used to estimate whether waves are incoming from port or starboard. In complex analysis, the imaginary part is an indication of the phase of the cross-spectra. It is therefore the imaginary part of the cross-spectra of heave and roll that can indicate if waves are incoming from port or starboard.

The following rule can be used to distinguish between port and starboard waves:

$$\hat{\beta} \in \begin{cases} [0°, 180°], & \text{if } \Gamma_{z\phi} \geq 0 \text{ (port)} \\ (-180°, 0°), & \text{if } \Gamma_{z\phi} < 0 \text{ (starboard)} \end{cases} \qquad (3)$$

where

$$\Gamma_{z\phi} = \int_{\omega=0}^{\omega_N} \text{Im}(R_{z\phi}(\omega)) d\omega \qquad (4)$$

and $\hat{\beta}$ is the wave direction estimate. $\text{Im}(R_{z\phi}(\omega))$ is the imaginary part of the cross-spectra between the heave and roll motion and ω_N is the highest frequency.

2.3 Differentiating Between Head and Following Waves

This section is also based on information from Brodtkorb et al. (2018). Although R/V Gunnerus is not fore/aft symmetric, the response from head and following directions can be similar and difficult for the algorithm to distinguish. Correction of the initially estimated wave direction is therefore included. In a similar manner as for port/starboard waves, the imaginary part of cross-spectra can be used to distinguish between head and following sea. In this case, it is the heave and pitch cross spectrum that is relevant, as the pitch motion is anti-symmetric about the y-axis. Corrections for head and following wave directions are then made according to

$$\hat{\beta} \in \begin{cases} (0°, 90°), & \text{if } \Gamma_{z\theta} < 0 \text{ (following)} \\ [90°, 180°], & \text{if } \Gamma_{z\theta} > 0 \text{ (head)} \end{cases} \tag{5}$$

where

$$\Gamma_{z\theta} = \int_{\omega=0}^{\omega_N} \text{Im}(R_{z\theta}(\omega)) d\omega \tag{6}$$

$\text{Im}(R_{z\theta}(\omega))$ is the imaginary part of the heave and pitch cross spectrum and ω_N is the highest frequency.

3. METHODOLOGY

This section includes a description of how the raw data is processed as well as theory on the methods used for sea state estimation. The methods include Quadratic Discriminant Analysis (QDA) used for estimating the wave direction, and Partial Least Squares Regression (PLSR) used for estimating the significant wave height and peak wave period.

3.1 Preprocessing of Raw Data

To obtain comparable values for the different sea states, the fast Fourier transform was used. The time series of the vessel response in all DOFs for each sea state were transformed to the frequency domain using the Fourier transformation. To obtain a single value for each DOF for the particular sea state, the frequency domain response was integrated over the frequency range. The results are then one response value for surge, sway, heave, roll, pitch and yaw for every sea state. These values, as well as the wave direction, significant wave height and peak wave period were then recorded and used as training data to make models to estimate the sea state.

3.2 Quadratic Discriminant Analysis

QDA is the method used for estimating the wave direction. Estimation of the wave direction is here considered a classification problem. Classification is a commonly used machine learning method, and involves classifying elements based on input data. The model is trained based on a training dataset with classes, which are the chosen wave directions. The output can therefore only be as precise as the training data. In practice this means that since the training data covers 18 different directions, the trained

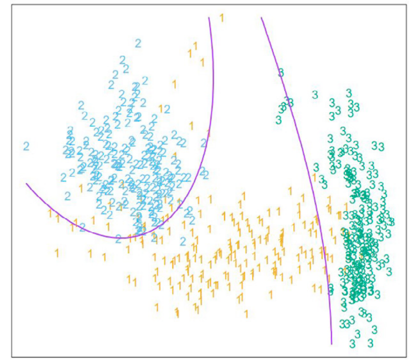

Fig. 2. Quadratic Discriminant Analysis (Hastie et al., 2001).

model can only output these 18 wave directions since these are the possible classes, i.e. the wave direction is discrete.

The remaining parts of this section are based on theory from Hastie et al. (2001). QDA models each class as a multivariate Gaussian function as follows:

$$f_K(x) = \frac{1}{(2\pi)^{p/2}|\mathbf{\Sigma}_k|^{1/2}} \exp -\frac{1}{2}(x-\mu_k)^\top \mathbf{\Sigma}_k^{-1}(x-\mu_k) \tag{7}$$

$\mathbf{\Sigma}_k$ is the covariance matrix for class k, μ_k is the mean for class k, and p is the number of dimensions. Unlike for Linear Discriminant Analysis (LDA), the covariance matrix, $\mathbf{\Sigma}$, is not the same for all classes. By working out the log ratio:

$$\log \frac{P(G=k|X=x)}{P(G=l|X=x)} \tag{8}$$

where l and k are classes, and using

$$P(G=k|X=x) = \frac{f_k(x)\pi_k}{\Sigma_{l=1}^K f_l(x)\pi_l} \tag{9}$$

one obtains a function where it can be seen that the decision boundary where $P(G=k|X=x) = P(G=l|X=x)$ is quadratic in x. K is the number of classes, and π_l is the prior probability of class l. The discriminant function for class k, δ_k, is shown in (10).

$$\delta_k(x) = -\frac{1}{2}\log|\mathbf{\Sigma}_k| - \frac{1}{2}(x-\mu_k)^\top \mathbf{\Sigma}_k^{-1}(x-\mu_k) + \log \pi_k \tag{10}$$

The boundaries between the classes are then defined by the function $\{x : \delta_k(x) = \delta_l(x)\}$. An example of boundaries on a plot of normally distributed mixtures is shown in Figure 2.

3.3 Partial Least Squares Regression

PLSR is a multivariate regression method which performs regression in one step by using the output data directly when decomposing the input data. PLSR aims to find new variables (latent variables) in the directions of both high variance and high correlation to the output (Hastie et al., 2001).

PLSR is based on the two equations below (Esbensen, 2001)

$$X = T \cdot P^\top + E \tag{11}$$

$$Y = U \cdot Q^\top + F \tag{12}$$

where X is the predictor variables, Y is the response variables and E and F are error terms. P and Q are latent variables, meaning they are not the observed X and Y, but the new variables obtained through a mathematical model (Salkind, 2010).

Results shown in this paper are obtained using MATLAB which takes use of the SIMPLS algorithm. The first step in this algorithm is to centralize the data, i.e. give each column a mean of zero by subtracting the mean from every value. The SIMPLS algorithm uses Singular Value Decomposition (SVD), which finds the U, D and V satisfying the equation

$$X = U \cdot D \cdot V^\top \tag{13}$$

U and V are the left and right unitary matrices respectively, and D is a diagonal matrix of singular values. U and V are orthogonal matrices, where U contains the eigenvectors of the covariance matrix, $XX^\top$. V contains the eigenvectors of $X^\top X$ and D contains the non-negative square roots of the eigenvalues of $XX^\top$ (Golub, G. and Reinsch, C., 1970).

SVD is done on the matrix S, which is the product $X^\top Y$. The left singular vector from the SVD, U_a is multiplied by X to obtain t_a.

$$t_a = XU_a \tag{14}$$

This is an iterative process, so $a = 1, 2, ..., A$ where A is the number of latent variables, and hence also the number of iterations. t_a is then normalized by $t_{a,norm} = \frac{t_a}{\sqrt{t_a^\top t_a}}$, and used in the following calculations (Alin and Ali, 2012).

$$p_a = X^\top t_{a,norm} \tag{15}$$

$$q_a = \frac{D \cdot V}{t_{a,norm}} \tag{16}$$

Here p_a are the X-loadings, i.e. the coefficients transforming the X-variables to the new latent variables, and q_a are the Y-loadings, i.e. the coefficients needed to map the latent variables to the output, for iteration a. Both the X-loadings and Y-loadings are then orthonormalized through the modified Gram-Schmidt process. The iteration is then accounted for by deflating the S matrix as shown in (17).

$$S_{a+1} = S_a - v_a(v_a^\top S_a) \tag{17}$$

In the above equation the X-loadings have been orthonormalized and are denoted v_a.

When having obtained both X and Y-loadings, regression coefficients can be obtained which give a linear combination of the input variables mapping them to the output variables.

4. SEA STATE ESTIMATION ALGORITHM

The approach for estimating the sea state is based on a few steps. Firstly, the QDA model is trained to find the wave direction. The response of the vessel varies with varying wave directions, so PLSR is done for each of the wave directions. In practice, this means that each wave direction has an associated set of regression coefficients that can be used to estimate the significant wave height and peak wave period. QDA is chosen for estimating wave direction, as a linear method is not applicable as vessel motions will not vary linearly with incoming wave direction.

The wave direction is first found using the trained model. Based on the output from this model, the regression coefficients for the estimated wave direction are chosen and used to estimate the significant wave height and peak wave period. Figure 3 outlines the process.

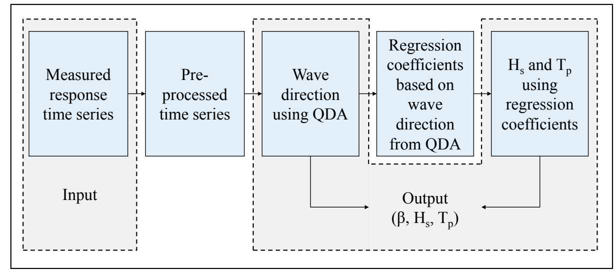

Fig. 3. Flow chart of sea state estimation algorithm.

Sea State	β [deg]	H_s [m]	T_p [s]
1	-163	2.0	8.3
2	161	2.7	9.4
3	-4	3.4	9.4
4	144	4.0	11.7
5	-140	4.7	11.3
6	-40	5.4	11.2
7	-35	6.1	12.1
8	-133	6.8	13.4
9	165	7.4	13.0
10	-159	8.1	12.5
11	-53	8.8	12.8
12	-175	9.5	14.6
13	-119	10.2	13.8
14	84	10.8	14.2
15	-18	11.5	14.0
16	-74	12.2	14.1
17	-112	12.9	14.2

Table 2. Sea states used to demonstrate sea state estimation results.

5. RESULTS AND DISCUSSION

This section presents sea state estimates using the methods described. The sea states in the results shown are made by deciding on a set of significant wave heights and randomly generating a peak wave period based on the ranges in Table 1, as well as a random wave direction. This means that the sea states will not be exactly the same as in the training data, which demonstrates the algorithm's performance on sea states other than the specific ones in the training data.

5.1 Simulation Results

The sea states used to demonstrate the algorithm's performance are shown in Table 2. Results for estimation of the wave direction are shown in Figures 4 and 5.

Results show that estimation of wave direction is done quite accurately. It is clear that for two of the sea states the algorithm unsuccessfully distinguishes between port and starboard waves. These sea states have incoming wave direction of $-163°$ and $-4°$, thus close to head and following sea respectively. The likely reason for the wrong estimation of the wave direction is that for these two sea states the roll motions are low and the heave-roll cross-spectra therefore carries limited information. Figure 5 shows the absolute value of the deviation in the wave direction.

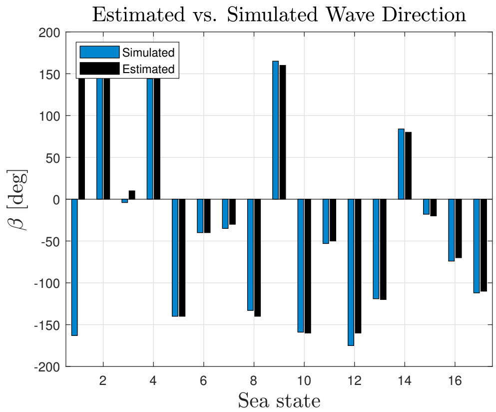

Fig. 4. Estimation of wave direction for sea states in Table 2.

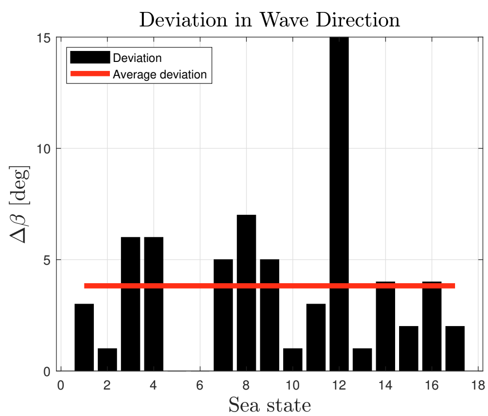

Fig. 5. Absolute deviation in wave direction for sea states in Table 2.

As expected, nearly all sea states have an error of less than 10° as the classification algorithm is trained on data for every 10th degree. The exception is sea state 12, which has a deviation of 15°.

Results showing the algorithm's performance on estimation of significant wave height are shown in Figures 6 and 7. Results show that the average deviation between simulated and estimated significant wave height is 0.7 m. Sea states 7-11 and 17 largely contribute to increasing this average with deviations up to 1.3 m. That deviations are large for higher sea states is expected as the method used is a linear method and in severe waves there are nonlinear phenomena present.

Figures 8 and 9 show results for peak wave period. The average deviation is 1.5 s, and many of the sea states are well below this average. However, especially sea state 16 largely increases the average deviation with a deviation of almost 4 seconds.

5.2 Discussion

Results demonstrated above are promising, and comparable to model-based methods. Comparing for example with results from Brodtkorb et al. (2018), the average deviation is approximately 0.25 m, which is significantly lower than average deviation presented above. However, the results in Brodtkorb et al. (2018) are for a low sea state with

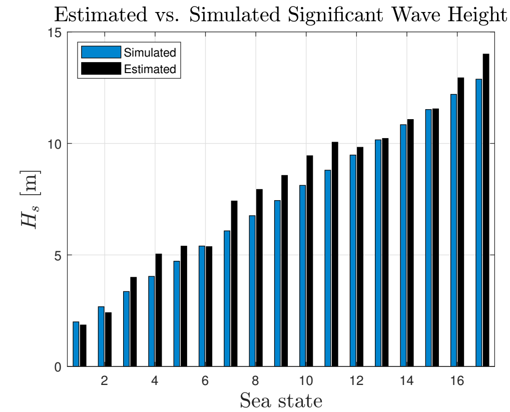

Fig. 6. Estimation of significant wave height for sea states in Table 2.

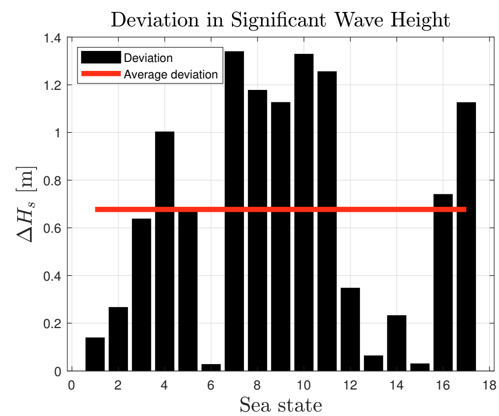

Fig. 7. Deviation in significant wave height for sea states in Table 2.

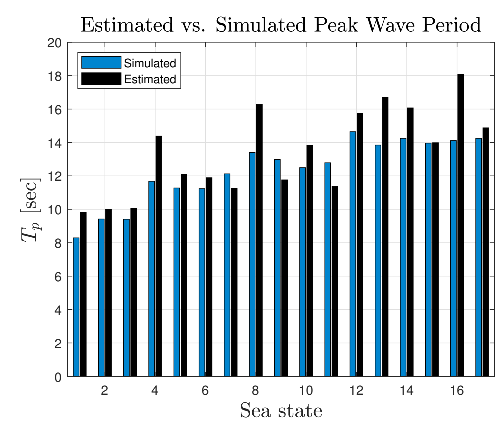

Fig. 8. Estimation of peak wave period for sea states in Table 2.

$H_s = 2$m, where the algorithm in this paper also yields results of similar size. For the same sea state, the model-based algorithm gives a deviation in peak wave period of 0.1 rad/s, which is just above the average deviation presented in this paper of 0.06 rad/s. Table 3 summarizes the comparison with Brodtkorb et al. (2018), for the sea most comparable sea state.

Nielsen (2007) demonstrates results for a vessel with forward speed, where the average deviation in significant wave height was 0.45 m using a parametric method.

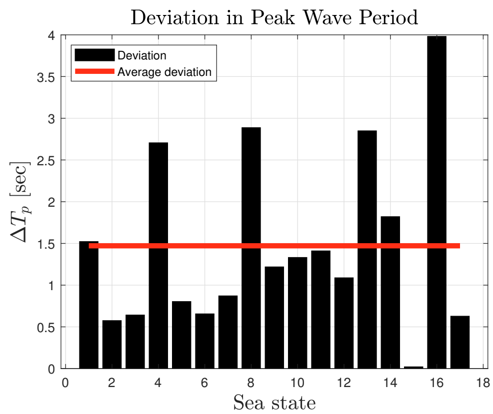

Fig. 9. Deviation in peak wave period for sea states in Table 2.

	Brodtkorb et al. (2018)		**This paper**	
	H_s [m]	T_p [s]	H_s [m]	T_p [s]
Sea State	2.0	8.3	2.0	12.0
Deviation	0.14	1.5	0.25	1.6

Table 3. Comparison between results in Brodtkorb et al. (2018) and methods presented in this paper, for a chosen comparable sea state.

Average deviation in peak wave period was shown to be 0.96 s. This again was for low sea states with $H_s \leq$ 3m. Although the algorithm presented in this paper performs at level with the model-based methods for many of the sea states, many of the estimated sea states yield much higher deviations, meaning that the robustness of the method can be questioned.

The strengths of the algorithm presented in this paper is that it requires little knowledge of the vessel, i.e. do not rely on vessel transfer function, but rather relies on a large amount of collected data which can be obtained from sensors on the vessel. However, a weakness of the method is that it can be computationally inefficient to generate datasets carrying enough information to build a model. Additionally, the results presented are for perfectly long-crested waves. In practice this is seldom the case, so simulations for different types of sea states might be necessary for the method to be applicable at sea, as the response data generated for training would likely be different in short-crested, more realistic waves. Lastly, it may not be sufficient to use simulated data to develop models, as simulation models are unable to capture all phenomena and external effects present at sea.

6. CONCLUSIONS

The sea state estimation algorithm in this paper estimated wave direction, significant wave height and peak wave period with promising results. As expected, significant wave height and peak wave period have been estimated with more accuracy for lower sea states, due to nonlinear effects in more severe waves. The estimation algorithm for wave direction showed very accurate results, and a method for efficiently distinguishing between port and starboard waves has been presented.

Interesting continuance of the work presented includes testing the algorithms on full-scale experiments. This could yield a conclusion on whether simplified simulated data for training is in fact sufficient to develop algorithms applicable at sea.

Further, changing the spectrum used in simulations and thus allowing for higher variations in sea states would be interesting. Good results with a large variety of sea states would likely yield a model which is more applicable.

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