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The Fama-French Five-Factor Model and Norwegian Stock Returns

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Preface

This thesis concludes my Master of Science in Financial Economics at the Norwegian University of Science and Technology. I would like to give a sincere thank you to my supervisor Snorre Lindset for guiding me through the process of writing this thesis.

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Abstract

The Fama-French five-factor model has established itself as a good model to explain US stock returns. The model consists of five common risk factors related to the overall market, company size, book-to-market equity, operating profitability and investment. This thesis investigates the model's ability to describe Norwegian stock returns in the time period of 1999 to 2017. In doing this, I employ the Fama-Macbeth two-stage econometric model and GRS tests. The analysis shows that the five-factor model fails to explain all variation in expected returns of companies listed on the Oslo Stock Exchange. In particular, the model struggles to describe stock returns of companies with low market capitalization. Nevertheless, the analysis documents interesting characteristics of Norwegian shares. The risk factors associated with the market and firm size have high explanatory power in explaining stock returns. Contrary to US findings, where shares characterized by low investment activity outperform those who invest aggressively, I document a significant positive relationship between investment activity and stock returns. The value premium associated with the book-to-market factor often targeted by investors in financial markets across the globe is seemingly absent on the OSE, and the factor's ability to explain returns is poor.

Sammendrag

Eugene Fama og Kenneth French sin fem-faktor-modell er en god modell for å beskrive amerikanske selskapers avkastning. Modellen består av fem risikofaktorer tilknyttet markedet, selskapers markedsverdi, selskapers forhold mellom bokført verdi og markedsverdi av egenkapitalen, operasjonell lønnsomhet og investering. Denne oppgaven har til hensikt å undersøke modellens evne til å forklare norske selskapers avkastning i perioden 1999 til 2017. Gjennom en Fama-Macbeth to-stegs økonometrisk modell og GRS-tester viser jeg at modellen mislykkes i å forklare all variasjon i selskapers avkastning. Modellen sliter spesielt med å fange opp utviklingen i avkastning for selskaper med lav markedsverdi. Modellens ufullkommenhet tilsidesatt, så dokumenterer analysen flere interessante funn og karakteristikker ved norske selskaper. Risikofaktorene tilknyttet markedet og selskapers markedsstørrelse viser seg å være svært gode indikatorer for å forklare virksomheters avkastning. I motsetning til amerikanske aksjer, der selskaper kjennetegnet ved lav investeringsaktivitet presterer bedre enn selskaper som investerer mye, viser jeg at norske aksjer med aggressiv investeringsaktivitet signifikant kaster bedre av seg enn virksomheter som investerer lite. Den mye omtalte verdipremien tilknyttet selskapers forhold mellom bokført verdi og markedsverdi av egenkapitalen viser seg å være fraværende på Oslo børs, og faktorens evne til å beskrive historisk avkastning er svak.

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1 Introduction

Ever since the introduction of the capital asset pricing model (CAPM) by Sharpe (1964), Lintner (1965) and Mossin (1966) in the mid-sixties, the field of asset pricing has experienced rapid growth and been of high interest to both researchers and practitioners. The CAPM is a theoretical single-factor model, stating that there is a positive linear relationship between expected stock returns and an overall market risk factor. Despite being ground-breaking work, Black et al. (1972) and many others have shown that the CAPM fails empirically. Motivated by shortcomings of the CAPM, a lot of research has been done trying to find other risk factors that alongside the market factor can explain stock returns. Among the most important contributions is the Fama-French three-factor model (Fama and French, 1993). Eugene Fama and Kenneth French identified two additional risk factors related to firm size and book-to-market equity that improved on the CAPM's ability to explain US stock returns. In 2015, the authors augmented two additional risk factors related to company profitability and investment to their latest model known as the Fama-French five-factor model (FF5) (Fama and French, 2015).

The literature on modern asset pricing is heavily influenced by findings in the US stock market. As an attempt to investigate the validity of the five-factor model in other financial markets, Fama and French (2017) tested the FF5 in a European, Japanese and Asia-Pacific market with mixed results. For the Norwegian stock market, Næs et al. (2009) tried to describe Norwegian stock returns from 1980 to 2006 with the Fama-French three-factor model and different macroeconomic variables. They found that risk factors related to the overall market and company size provide a reasonable fit for the cross-section of Norwegian shares.

This thesis expands on the work done by Næs et al. (2009) and seeks to evaluate the validity of the five-factor model in the Norwegian stock market. In doing this, I address the following research question:

> *How does the Fama-French five-factor model perform in explaining stock returns on the Oslo Stock Exchange?*

In answering this question, all factors of the five-factor model will be examined and three sub-questions addressed:

1. Is there a linear relationship between returns and the proposed risk factors?
2. Do the factors award risk premiums?
3. Which factor combinations perform best in explaining stock returns?

The sub-questions serve as good indicators to measure the model's appropriateness for Norwegian stocks.

The format of this paper is as follows. In section 2, I draw the line between a theoretical two-period consumption model and a linear factor pricing model. I also give a very brief summary of the literature on modern asset pricing. In section 3, I establish central definitions and tools needed to empirically test and evaluate the Fama-French five-factor model. Section 4 addresses all decisions made when cleaning and adjusting the data. In section 5 I present and discuss results, and section 6 concludes.

2 Theory and literature review

Theoretical asset pricing models center on the idea that investors prefer to hold assets that perform well in bad economic times. We value shares that make us less poor during economic turmoil over stocks that make us more rich when we are already well-off. In this section, I show how a central asset pricing equation derived from a theoretical two-period consumption model defines good empirical risk factors.

2.1 Consumption and asset prices

The value of an asset is the sum of all expected discounted future cash flows. An asset can be any instrument that promises a pay-off, be it a stock, an obligation or a bet. Common for all assets is that their future cash flows are uncertain and paid on different points in time. Following Cochrane (2009), to put a price on what an investor is willing to pay for an asset, we can look to a simple two-period consumption-based model and derive a basic pricing equation. Investing in an asset is simply trading consumption today for consumption tomorrow, and as a starting point we formalize this trade-off by describing the utility an investor receives over current and future levels of consumption

$$U(c_t, c_{t+1}) = u(c_t) + \beta E_t\big[u(c_{t+1})\big]. \tag{2.1}$$

Equation (2.1) states that total utility received is the sum of the utility of today's consumption c_t and expected utility of future consumption c_{t+1}, discounted by the subjective discount factor β. The utility function $u(\cdot)$ is assumed increasing, reflecting a constant desire for more, and concave, reflecting diminishing marginal value of each additional unit of consumption. The subjective discount factor captures both the investor's impatience and aversion to risk, quantifying compensation for delayed consumption and uncertainty regarding the actual level of consumption.

Let us define x_{t+1} [1] as the future pay-off an investor receives after paying the price p_t

[1]For a stock, expected future pay-offs are expected future stock price plus any dividend: $x_{t+1} = p_{t+1} + d_{t+1}$.

today. If ξ denotes the amount he chooses to buy and e his consumption level had he not bought the asset, the investor is faced with the following maximization problem:

$$\begin{aligned} \max_{\xi} \quad & u(c_t) + E_t\big[\beta u(c_{t+1})\big] \quad \text{s.t.}, \\ c_t &= e_t - p_t\xi, \\ c_{t+1} &= e_{t+1} + x_{t+1}\xi. \end{aligned} \tag{2.2}$$

Substituting the constraints and setting the derivative of the object function w.r.t. ξ equal to zero, we end up with the first order condition for optimal consumption:

$$p_t u'(c_t) = E_t\big[\beta u'(c_{t+1})x_{t+1}\big]. \tag{2.3}$$

An investor will buy or sell the asset until the marginal loss of reducing today's consumption $p_t u'(c_t)$ equals the increase in expected future discounted marginal utility of consumption $E_t\big[\beta u'(c_{t+1})x_{t+1}\big]$. In other words; the investor chooses a portfolio of consumption where the marginal cost of reduced consumption today equals the marginal benefit of increased consumption tomorrow. Rearranging equation (2.3), we end up with *the* central asset pricing formula:

$$p_t = E_t\left[\beta \frac{u'(c_{t+1})}{u'(c_t)} x_{t+1}\right]. \tag{2.4}$$

It states that given an investor's choice of present and future consumption and his impatience and appetite for risk β, he is willing to pay price p_t for an asset with uncertain pay-off x_{t+1}. In the literature, this relationship is commonly referred to as

$$\begin{aligned} p_t &= E_t(m_{t+1}x_{t+1}), \\ \text{where} \quad m_{t+1} &= \beta \frac{u'(c_{t+1})}{u'(c_t)}. \end{aligned} \tag{2.5}$$

Assets are worth their expected discounted future pay-off. Both m_{t+1} and x_{t+1} are stochastic variables, unknown as of today. m_{t+1} is the marginal utility of wealth and often called the stochastic discount factor (SDF). It is the rate at which an investor is willing to substitute consumption tomorrow $(t+1)$ for consumption today (t). Generally, m_{t+1}

should be interpreted as a countercyclical variable that is large in bad times and small in good times. During recessions, consumption level is low and the marginal benefit of an extra unit of consumption is high, causing m_{t+1} to decrease in value and the price p_t we are willing to pay to postpone consumption is small. This mechanism explains why investors value assets that perform well in bad times (countercyclical or low-beta assets) over assets that perform well in good times (cyclical or high-beta assets). A friend in need is a friend indeed.

Having established a general framework for how assets are priced, we now move to a special case of (2.5) more suited for factor pricing models. By setting $p_t = 1$ and defining asset i's pay-off ($x_{i,t+1}$) as the gross return R_i, we can rewrite the pricing equation as:

$$1 = E_t(m_{t+1} R_i). \tag{2.6}$$

Using the definition of covariance[2] and rearranging, we end up with

$$E(R_i) = R_F - R_F\, cov(m, R_i), \tag{2.7}$$

where $R_F = \frac{1}{E(m)}$ is the gross risk-free rate of return that we require of a riskless asset whose promised pay-off x_{t+1} is always paid. Equation (2.7) states that an asset's expected return is the risk-free rate plus a risk adjustment. Cyclical assets that barely or negatively covary with marginal utility of wealth m must promise higher expected returns to induce an investor to hold them. Countercyclical assets that covary positively with m (e.g. insurance), on the other hand, can attract investors despite having low or negative expected returns.

Equation (2.7) can be written as

$$E(R_i) = R_F + \left(\frac{cov(R_i, m)}{var(m)}\right)\left(-\frac{var(m)}{E(m)}\right), \tag{2.8}$$

[2] $E(mR_i) = cov(m, R_i) + E(m)E(R_i)$.

which we can reformulate further, letting $\beta_{i,m} = \frac{cov(R_i,m)}{var(m)}$ and $\lambda_m = -\frac{var(m)}{E(m)}$, and express as:

$$E(R_i) = R_F + \beta_{i,m}\lambda_m. \tag{2.9}$$

Here, $\beta_{i,m}$ is the regression coefficient of the return R_i on the stochastic discount factor m and can be interpreted as the quantity of risk. The λ_m is often interpreted as the price of risk and depends on the volatility of the discount factor.

2.2 Factor models

Many scholars, among them Campbell and Cochrane (2000), have shown that despite being a complete answer to most asset pricing questions, the consumption-based model has yet to succeed well empirically in explaining stock returns. Cochrane (2009) chapter 6 and 9 show that the pricing model of (2.9) is equivalent to a linear multi-factor model for the discount factor m:

$$m_{t+1} = a + b' f_{t+1}. \tag{2.10}$$

Here, a is a free parameter and b' is a vector containing multiple regression coefficients of returns r regressed on risk factors f. An important question within the literature of asset pricing is which variables we should use as factors. Equation (2.5) and (2.10) imply that a successful factor pricing model uses variables that proxy well for the stochastic discount factor or marginal utility of wealth m_{t+1}:

$$\beta \frac{u'(c_{t+1})}{u'(c_t)} \approx a + b' f_{t+1}. \tag{2.11}$$

Factors that successfully captures investors' preference for holding assets that perform well in bad economic times over assets that covary highly with consumption (good approximations of m_{t+1}) satisfy (2.11).

There is an extensive literature that tries, both theoretically and empirically, to find state variables and factors that proxy well for marginal utility of wealth. The Capital Asset Pricing Model by Sharpe (1964), Lintner (1965) and Mossin (1966) was the first model to formalize a relationship between asset returns and risk factors. Merton et al. (1973) extended the CAPM to a multi-factor model called ICAPM (Intertemporal Capital Asset Pricing Model) addressing some of the CAPM shortcomings. Some years later, Ross (1976) came up with the Arbitrage Pricing Theory (APT), which is another multi-factor model that has greatly influenced the literature.

Systematic patterns in stock returns that violate the CAPM are what we call anomalies. One of these anomalies is related to the CAPM failing to explain returns of portfolios based on firm size (Reinganum, 1981). Portfolios consisting of small-valued companies were found to systematically outperform portfolios of large stocks. The value anomaly is another well-documented irregularity of the CAPM. Rosenberg et al. (1985) showed that "cheap" stocks with a low ratio of book equity over market value (book-to-market) on average have higher returns than their "expensive" counterparts with high book-to-market ratios. In 1993, Fama and French presented the three-factor model that addressed and successfully managed to explain the size and value anomaly. In more recent times, two more factors related to profitability and investment have been augmented by the same authors to combat new anomalies. In the five-factor model, the investment factor is motivated by Novy-Marx (2013), who identified a strong positive relationship between profitable firms and average return. The firm-specific factor related to investment was discovered by Aharoni et al. (2013), who documented a negative relationship between investment activity and expected returns.

3 Methodology

We need a solid methodological foundation to examine how the Fama-French five-factor model performs in explaining Norwegian stock returns. In this section, I formally provide central definitions and the framework for the five-factor model. The last part establishes important tools needed to empirically evaluate the model.

3.1 Central definitions

3.1.1 Monthly returns

Asset pricing revolves around rates of return. An asset's monthly rate of return is the percentage change in price from one month to the next

$$r_{it} = \frac{Adjusted\ price_{it}}{Adjusted\ price_{i,t-1}} - 1. \tag{3.1}$$

$Adjusted\, price_{it}$ is company i's market price adjusted for corporate events such as stock splits, dividends and share repurchases.

3.1.2 Equally- and value-weighted portfolios

Empirical asset pricing often deals with portfolios of stocks rather than individual shares when explaining stock returns. In the literature, two main portfolio versions are used. The first is the value-weighted (VW) portfolio where all stocks are weighted according to their market capitalization

$$\begin{aligned} r_{pt}^{VW} &= \sum_{i}^{N} r_{it} \times w_{it}, \\ w_{it} &= \frac{Market\ capitalization_{it}}{Market\ capitalization_{pt}}. \end{aligned} \tag{3.2}$$

The return of a value-weighted portfolio p for month t is the sum of the weighted return of all portfolio assets N. As a result, high valued stocks affect the portfolio's return greatly, whereas smaller stocks are of less importance. The equally-weighted (EW) portfolio, on the other hand, gives every stock the same weight regardless of their market capitalization

$$r_{pt}^{EW} = \sum_{i}^{N} r_{it} \times \frac{1}{N}, \tag{3.3}$$

The Norwegian stock market is dominated by very few high-valued companies. On the 31.12.2018, three companies alone (Equinor, Telenor and DNB) stood for about 54.5% of the total value of the OBX[3]. We often seek to form portfolios that proxy well for the market as a whole. In this context, the value-weighted portfolio is preferred to the equally-weighted, and it will be our main portfolio of choice going forward.

3.1.3 Factor definitions

To investigate whether firm-specific characteristics related to size, book-to-market, profitability and investment proxy well for marginal utility of wealth, I first have to formally define the risk factors. The factor mimicking the underlying risk in return related to size is measured in terms of a firm's market capitalization. A firm's market cap is simply the price of all its outstanding shares times the current market price per share

$$\textit{Market capitalization}_t = \textit{Price}_t \times \textit{Shares outstanding}_t. \tag{3.4}$$

The value factor, book-to-market, is calculated as a firm's book value of equity relative to its market cap

$$B/M_t = \frac{\textit{Book value of equity}_{t-1}}{\textit{Market capitalization}_{t-1}}. \tag{3.5}$$

Book-to-market is the inverse of the common price-to-book (P/B) ratio, and tells us the dollar value received in book equity per dollar spent buying stocks. Value stocks, typically risky stocks with slim growth potential, trade at high book-to-market ratios, whereas growth stocks, generally well performing stocks that have high growth potential,

[3]OBX is a value-weighted index of the 25 most liquid stocks listed on the OSE.

have low book-to-market ratios.

The factor proxying for operating profitability is defined as a firm's operating profit (total revenue minus total operating expenses) net its interest expenses over the book value of equity

$$OP_t = \frac{Operating\ profit_{t-1} - Interest\ expense_{t-1}}{Book\ value\ of\ equity_{t-1}}. \tag{3.6}$$

Companies that generate high profits relative to their book equity are defined as robust, whereas weak companies are unprofitable relative to their book equity. The factor proxying for a firm's investment activity is simply defined as the percentage change in total book assets from one fiscal year to the next

$$Inv_t = \frac{Total\ assets_{t-1}}{Total\ assets_{t-2}} - 1. \tag{3.7}$$

I name companies that experience a high percentage change in total assets aggressive, whereas firms with low or negative change are called conservative. The time subscripts in all factor equations refer to years, where t is this year and $t-1$ is last year.

3.1.4 The explanatory returns

To test if the proposed factors defined above can explain stock returns on the Oslo Stock Exchange (OSE), we need a way to incorporate them as explanatory variables in empirical models. The factor mimicking portfolio is a highly useful tool for doing this. Roll and Srivastava (2018) define a mimicking portfolio as a "tradable fund engineered to closely copy the factor sensitivities of an individual asset, a fund, or a non-tradable variable such as a macro-economic quantity". We form mimicking portfolios for all factors of the FF5 by taking long (buying) and short (selling) positions in stocks motivated by CAPM failures. For instance, to proxy for the underlying risk related to size, we form a portfolio by buying low-valued stocks and selling stocks of high market value. All portfolios are backward-looking, meaning that they are implementable trading strategies based on information available to all investors at the time they were created. As Norwegian companies are required by law (Finansdepartementet, 2018) to first publish last fiscal year's annual report by 01.07 (six months into the next FY), we first form and rebalance portfolios at the end of June each year.

At the end of each June, we allocate all equities on the OSE into two size groups, small and big, using the median market capitalization given by (3.4) as a breakpoint. Stocks are also independently allocated to three book-to-market, operating profitability and investment groups using the 30th and 70th percentiles of B/M, OP and Inv defined by (3.5-3.7) as breakpoints. The factor proxying for risk related to size, SMB, is then the average monthly return of all *small* portfolios minus all *big* portfolios. The value factor, HML, is the average monthly return of all *high* portfolios minus all *low* portfolios. The factor mimicking risk related to profitability, RMW, is the average monthly return of all *robust* minus all *weak* portfolios and the investment factor, CMA, is the average monthly return of all *conservative* minus *aggressive* portfolios. The market factor, $R_M - R_F$, is the value-weighted monthly average return of a portfolio consisting of all stocks on the OSE (the market portfolio) less the monthly risk-free rate.

3.1.5 The returns to be explained

Having formalized a way to incorporate the explanatory factors empirically, I now introduce the explained variable. Blume (1970) argues that betas (an asset's exposures to risk factors) are estimated with error that we can diversify away by pooling stocks in portfolios. Hence, instead of testing the FF5 model on single stocks, we define so-called test portfolios. Following Ødegaard (2019b), a portfolio is well diversified when it consists of ten or more stocks.

We systematically construct test portfolios based on firm-specific characteristics of the FF5, enabling us to look for patterns in coefficients as portfolio characteristics, and thereby the dependent variable, change. In this thesis, we evaluate models for test portfolios formed by simple and double sorts. For the simple sorted portfolios, all stocks on the OSE are sorted in ascending order based on their market capitalization, book-to-market ratio, operating profitability and investment (defined by 3.4-3.7) individually. Decile breakpoints are calculated and used to split the equities into 10 portfolios per factor. As for the mimicking portfolios, portfolios are sorted and rebalanced annually at the end of June.

Factors tend to be correlated (e.g. Fama and French (2015) find a strong correlation in returns between companies with high book-to-market and low investment activity), possibly distorting our analysis. To deal with correlation among the factors, we try to isolate firm-specific characteristics by using an independent double sorting technique. For the double sorted portfolios, we allocate all equities on the OSE to three even size groups using the 33.33th and 66.66th percentile of market capitalization as breakpoints. Stocks are then independently allocated to four groups using the 25th, 50th and 75th percentile of book-to-market ratio breakpoints as a second sort variable. The intersection of the two independent sorts leaves us with twelve value-weighted Size-B/M portfolios. Sorting on operating profitability and investment as a second variable instead, we form twelve Size-OP and Size-Inv portfolios in the same manner as Size-B/M.

3.2 Testing tools

3.2.1 Fama-Macbeth analysis

There are many ways we can proceed to empirically test asset pricing models. Traditional methods follow the two-step procedure of Fama and MacBeth (1973). The first step involves running OLS time-series regressions for the monthly excess return (monthly return less the risk-free rate) of each test portfolio i on all factors f_j

$$r_{it} - r_{Ft} = \alpha_i + \sum_{j}^{J} \beta_i^j f_{jt} + \epsilon_{it}, \quad t = 1, ..., T. \tag{3.8}$$

We want to estimate the sensitivity of each test portfolio's excess return to movements in the factors. These estimated coefficients are the β_i^js above. The r_{it} is the net return ($r_{it} = R_{it} - 1$) of test portfolio i in time period t, and r_{Ft} is the net risk-free return. All factor models center on this time-series regression, and for the Fama-French five-factor model we rewrite (3.8) as

$$r_{it} - r_{Ft} = \alpha_i + b_i(r_{Mt} - r_{Ft}) + s_i SMB_t + h_i HML_t + rw_i RMW_t + c_i CMA_t + \epsilon_{it}. \quad (3.9)$$

Here, $r_{Mt} - r_{Ft}$ is the excess return on a value-weighted market portfolio, SMB_t is the return on a diversified portfolio of small stocks minus the return of a portfolio of big stocks, HML_t is the difference between the returns on diversified high and low book-to-market portfolios, RMW_t the difference between the returns on portfolios with robust and weak operating profitability, CMW_t the difference between the returns of portfolios with conservative and aggressive investment behaviour and ϵ_{it} is a zero-mean residual. The coefficients b_i, s_i, h_i, rw_i and c_i are the respective factor exposure (also commonly referred to as factor loadings or betas) to the five risk factors.

Interpreting the results from the time-series regression above, we are mainly interested in the model intercepts α_i and estimated factor exposures. If the factors capture all variation in expected returns, the regression intercept α_i equals zero for all securities and portfolios i. A correct model should be consistent with this result[4]. The estimated betas ($\hat{b_i}$, $\hat{s}_i$, $\hat{h_i}$, $r\hat{w}_i$ and $\hat{c}_i$) represent how the excess returns of the test portfolio i move in relationship to the respective factor f. The sign, magnitude and consistency across different test portfolios i will be of interest. If, for example, size is a true factor and model (3.9) is run for test portfolios formed from a simple sort on size, we should expect its factor exposures s_i to be positive for small portfolios and negative for large portfolios. To judge how different models (factor combinations) succeed in explaining the test portfolios' variation in excess return, we look to the R^2 goodness-of-fit measure. The average absolute value of the intercepts we get by running (3.8) on different test portfolios for different factor combinations also gives an idea of how well a model performs in explaining stock returns.

To address sub-question two, of whether the proposed risk factors are priced, we investigate if there is a positive (or negative) relationship between increased risk exposure and expected

[4]When test portfolios and factors all are excess returns, the expectation of (3.9) implies that all regression intercepts α_i are zero. See Cochrane (2009) chapter 12 for a complete take on this.

returns. If this relationship exists, we say that a factor is priced. We follow the second step of the Fama-Macbeth framework to estimate these factor risk premiums. By using the estimated factor exposure coefficients $\hat{\beta}_i^j$ from the first step regressions for each factor j and test portfolio i, cross-sectional regressions for each time period t are run, estimating the risk premium λ_j related to each factor j:

$$r_i - r_F = \lambda_0 + \sum_j^J \lambda_j \hat{\beta}_i^j + \epsilon_i. \tag{3.10}$$

Here, λ_0 is the regression intercept. For the FF5, (3.10) corresponds to

$$r_i - r_F = \lambda_0 + \lambda_b \hat{b}_i + \lambda_s \hat{s}_i + \lambda_h \hat{h}_i + \lambda_{rw} r\hat{w}_i + \lambda_c \hat{c}_i + \epsilon_i, \tag{3.11}$$

where λ_b, λ_s, λ_h, λ_{rw} and λ_c are risk premiums associated with their respective factor, and $\hat{b}$, $\hat{s}$, $\hat{h}$, $r\hat{w}$ and $\hat{c}$ are the corresponding factor loadings estimated from (3.9). As the cross-sectional regression (3.11) is run T times, the risk premium associated with risk factor f_j is the mean of all estimated premiums $\hat{\lambda}_j$:

$$\hat{\lambda}_j = \frac{1}{T} \sum_{t=1}^T \lambda_{jt}. \tag{3.12}$$

If $\hat{\lambda}_j$ is significantly different from zero, the factor is priced and should be regarded as a good proxy for marginal utility growth, satisfying the central factor pricing model relationship given by (2.11).

The fact that the second stage Fama-Macbeth regressions in (3.10) uses dependent variables β_j^i that are already estimated from the first stage regression causes a so-called generated regressors problem. Shanken (1992) argues that the estimated factor premiums $\hat{\lambda}_j$ are subject to an errors-in-variables (EIV) problem rendering them biased in small samples and suggest correcting the standard errors. The Generalized Method of Moments (GMM) approach introduced by Hansen (1982) estimates (3.8) and (3.10) simultaneously, ultimately solving the generated regressor problem. In this thesis, I chose to stick with the traditional Fama-Macbeth analysis for estimating risk premiums as my time constraint did not allow me to experiment with the GMM framework.

3.2.2 GRS test

A perfect asset pricing model explains all variation in the expected excess return of all securities and portfolios. This implies that the regression intercepts, or pricing errors α_i, from the Fama Macbeth 1. step regression (3.9), jointly equal zero. To test this, I employ a Gibbons, Ross and Shanken test (Gibbons et al., 1989). The test statistic is given by

$$\left(\frac{T}{N}\right)\left(\frac{T-N-L}{T-L-1}\right)\left[\frac{\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha}}{1+\bar{\mu}'\hat{\Omega}^{-1}\bar{\mu}}\right] \sim F(N, T-N-L). \tag{3.13}$$

Here, T is the number of months, N is the number of test portfolios, L the number of factors, $\hat{\alpha}$ is a vector containing all the N estimated alphas from, $\hat{\Sigma}$ is the estimated residual covariance matrix, $\bar{\mu}$ is a vector of all test portfolio sample means and Ω is an unbiased estimate of the test portfolios' covariance matrix. The GRS-statistic assumes that the errors ϵ from (3.9) are normal, as well as uncorrelated and homoskedastic. The null hypothesis of the GRS test states that all alphas are zero ($H_0 : \alpha_i = 0,\ \forall i$). A good model has marginal pricing errors, resulting in a small GRS-statistic that cannot reject the null. See Cochrane (2009) chapter 12 for a more complete take on the GRS-statistic. I use the R package "GRS.test" by Kim (2017) to calculate the test statistics.

4 Data

Evaluating factor pricing models is a data-centred exercise. In this section, I explain decisions made when working with the data at hand. I address all relevant steps regarding data cleaning and sample adjustments, with the intention of enabling the interested reader to completely replicate the study.

4.1 Data wrangling

Forming factor mimicking portfolios and test portfolios requires stock market and accounting data. Looking at the factor definitions in equation 3.4–3.7, we need data containing monthly observations of stock prices and market capitalization, in addition to yearly data of company book equity, total assets, total revenue, total operating expenses and interest expense. To my knowledge and given my restricted access to data, the financial database for Norwegian academic institutions TITLON is the best source of data for this analysis. TITLON offers daily stock market data back to 1980 and onward and sufficient yearly accounting data from 1998 until 2017. As the five-factor model consists of three accounting based factors, where the factor proxying for a company's investment activity is lagged, this analysis is limited to the time period of June 1999 to December 2017. A total of 223 months. Monthly risk-free rates are taken from Ødegaard's data library (Ødegaard, 2019a). The risk-free rates are forward looking, estimated from Norwegian government securities and the Norwegian InterBank Offered Rate (NIBOR).

The journey of transforming the raw TITLON data into complete manageable samples ready to be analyzed was long and challenging at times. The programming language R, and in particular the data manipulation package "dplyr" (Wickham et al., 2019), has been extensively used to solve tasks ranging from data manipulation to writing algorithms for the Fama-Macbeth analysis. TITLON provides samples of about 1,500,000 stock market data points and 203,000 accounting data points. An initial task was to convert the daily stock market sample to a time series of monthly data restricted to our time period. As the data contained non-equities such as derivatives and exchange-traded funds (ETFs), another task was to remove all entries that were not stocks. As TITLON does not differ between security types, this work had to be done manually. Many companies were listed multiple times under different ISINs (International Securities Identification Number). To correct for this, all duplicate entries were removed from the original sample. To make the data easier to work with, I merged the stock market data and accounting data into one data frame by matching observations across both data sets with company-IDs and dates. After the initial data wrangling, we are left with one single sample containing 48,804 observations and 557 unique firms over the 223 month long time period.

4.2 Factor concerns

To ensure that the factors have their intended economic meaning, certain adjustments to the data sample needs to be made.

4.2.1 Class A and B stocks

A challenge when working with TITLON data is how to approach listed companies with different stock classes. Many companies listed on the OSE issue both class A and B stocks, often referred to as common and preferred stock. Common stock is an ordinary share giving the stockholder the right to a share of a company's profits and to vote on matters regarding corporate policy. A preferred stock, however, is generally regarded as a hybrid instrument including properties of both equity and debt instruments. Although there exist no universal definition of a class B stock, typical properties include seniority in dividends and assets in the event of liquidation and nonvoting. Disregarding class definition, common and preferred stock always represent identical fundamental company characteristics. Hence, including more than one security per company in our empirical framework would be wrong, and for that reason I chose to exclude all preferred stocks.

4.2.2 Negative book value of equity

If a company's balance sheet's total debt exceeds total assets, it has a negative book value of equity. In similar fashion as Fama and French (2015), I exclude all these stocks in constructing the factors. Without the removal of negative BE stocks, both the book-to-market (B/M) and profitability (OP) factor would lose all economic meaning and intended properties. For consistency, the same stocks are also dropped when constructing factors that are not dependent on book equity. In constructing the test portfolios and the market factor, however, all stocks are included to preserve the portfolios' ability to represent the market.

4.2.3 Financial firms

As for the removal of negative BE stocks, firms that represent misleading financial information can be problematic when constructing the factors. Financial firms fall under this category, as their leverage (the proportion of assets financed with debt) is high by nature in comparison to normal firms, where high leverage is an indicator of financial distress. Following this line of reasoning and Fama and French (1992), all financial firms are excluded leaving the sample with only so-called operational firms.

4.2.4 Penny stocks

Penny stocks, also commonly referred to as micro-cap stocks, are stocks issued by low-valued firms that trade at a very low market price. These stocks are characterized by being more prone to risk than mid-cap and even small-cap stocks, as they tend to be highly volatile, illiquid and exhibit limited disclosure of information. Urbański et al. (2014) show that the inclusion of penny stocks in an empirical context contribute to inconsistent stock pricing. An ideal asset pricing model should be able to explain returns of all assets perfectly, including penny stocks. Unfortunately, a perfect model does not exist, and we instead look for a model that gives us the *best* insight in how asset returns change across securities over time. An easy way to exclude penny stocks is to filter stocks on a minimum required price and market capitalization. Ødegaard (2019b) recommends dealing with penny stocks by excluding all shares with a price below NOK 10 and a total market value falling short of NOK 1 million[5]. He further suggests requiring a stock to have a minimum of 20 trading days before being included in the sample, as a way to handle

[5] In the data, all companies except one are valued above NOK 1 million.

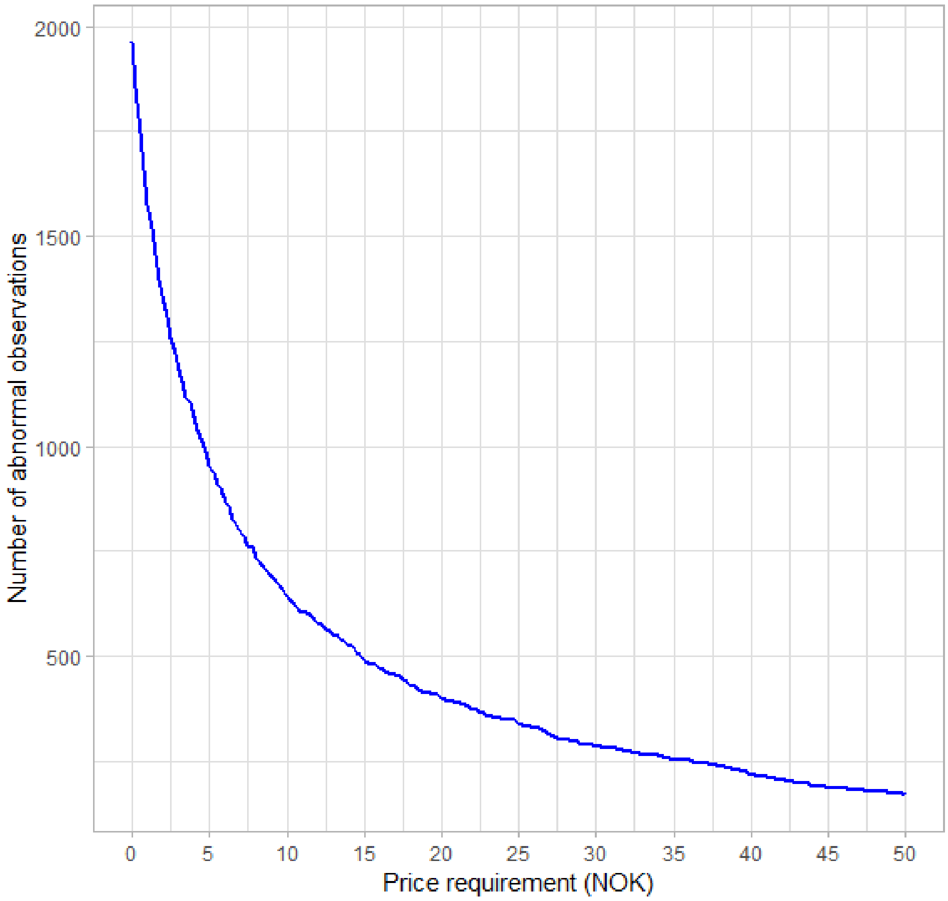

Figure 4.1: Penny stocks and abnormal monthly returns

The figure shows that penny stocks stand for a high proportion of observations having abnormal monthly returns. Abnormal returns are defined as observations with monthly returns within the 2.5th percentile (more negative than -31%) or 97.5th percentile (greater than 40%). The y-axis represents the number of abnormal observations in the sample after stocks are excluded for different price requirements, given by the x-axis.

stocks that are seldom traded. Instead of blindly following this advice, I first investigate the characteristics of penny stocks on the OSE thoroughly.

In figure 4.1, I graphically show that penny stocks stand for a high proportion of observations having abnormal monthly returns. I define an observation as abnormal if its return for a particular month is within the 2.5th percentile (more negative than -31%) or the 97.5th percentile (greater than 40%). The y-axis of the figure represents the number of abnormal observations in the sample after stocks are excluded for different price requirements, given by the x-axis. Out of the 1939 observations defined as abnormal when all in-sample stocks are included (price requirement of NOK 0), we observe a drastic fall in the number of abnormal returns as we start excluding the lowest priced stocks.

When I remove all stocks below NOK 5, the number of abnormal observations is reduced to 953, a 51% decrease. The curve flattens out after about NOK 10, confirming the belief that penny stocks have more extreme returns than medium or highly priced stocks. To get an idea of how what a normal stock price on the OSE is, I can report that the mean and median price of all stocks on the OSE from 1999 to 2017 were about NOK 51 and NOK 22, respectively.

To further investigate how penny stocks affect important variables, table 4.1 presents descriptive statistics for monthly return, market capitalization and number of listed companies for different price requirements. Each column represents different price thresholds, where *Original sample* is the original TITLON data (after initial data cleaning and removal of class B stocks and financial firms) and $p > 10$ a sample only containing stocks priced above NOK 10. Systematically removing low-priced stocks increases the monthly average return from 0.94% to 1.69%, while also cutting its standard deviation in half (35.22% to 15.14%). This confirms that, on average, micro-cap stocks are more volatile (higher standard deviation) and perform worse (lower average return). Regarding market capitalization, we read from the table that the average observation's market value increases as cheap stocks are excluded, confirming my belief that low-priced stocks also on average have lower market values. Looking at sample size, we observe a reduction from the original 39,766 to 25,876 observations when all stocks below NOK 10 are excluded. The average number of listed companies is reduced from about 177 to 115. Figure 4.2 shows how the number of firms listed on the OSE changes during the time period, highlighting how sensitive the number of companies listed is to different price requirements.

We are looking for a sample suitable for statistical modeling, but we also want to keep as many data points as possible to maintain the data's representativeness of the cross section of firms on the OSE. Following this line of argumentation and insight from table 4.1, I choose to be less restrictive than Ødegaard (2019b) and require all stocks to have a price above NOK 2 and more than 20 consecutive trading days to be included in the sample. After excluding the cheapest stocks, we are still left with a sample characterized by high average monthly return (1.47%) and standard deviation (35.7%). Comparing our return data to sample statistics of other European stock markets (Corhay and Rad, 1994), we

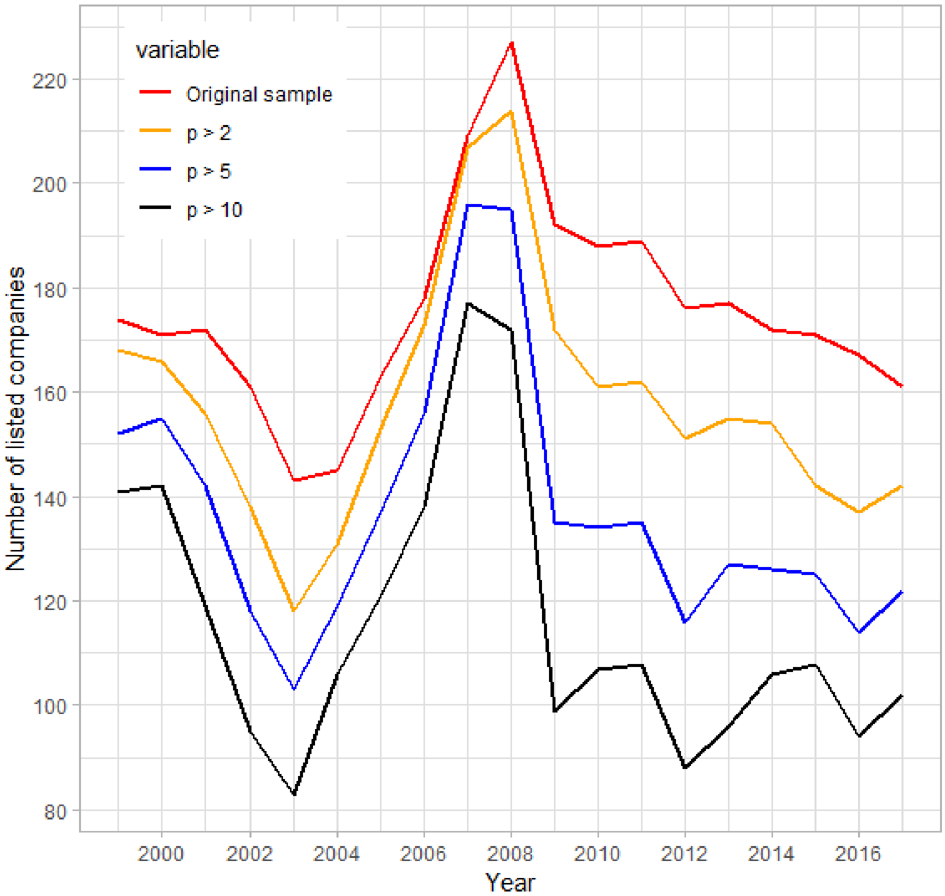

Figure 4.2: Number of listed companies 1999-2017

The figure plots the number of listed companies at the end of each year from 1999 to 2017 for samples formed by different price requirements. Class B stocks and financial firms are removed from the data prior to the filtering on price.

realize that the third (skewness) and fourth (kurtosis) moment of the return distribution seem very high. TITLON has its flaws as a data base with some very questionable data points[6] that have a big impact on the distribution of returns. To deal with extreme values I use a method called winsorizing. By removing all observations that fall within the 0.1th and 99.9th percentile of returns, I drop an additional 69 observations (winsorization of 99.8%) and reduce the mean, standard deviation, skewness and kurtosis to a level that compares better to other return series. We end up with the sample *Winsorized p>2* in the last column of table 4.1 and are now ready to examine the Fama-French five-factor model's ability to describe Norwegian stock returns.

[6]Although TITLON adjusts prices for dividends and corporate events, there are some companies that seemingly have extreme price fluctuations that simply must be due to wrong data.

Table 4.1: Penny stocks' effect on important variables

The table presents descriptive statistics for central variables for TITLON stock market data from June 1999 to December 2017 based on different price requirements. All columns represent different samples where stocks are included based on a minimum required price. *Original sample* is the sample where all stocks are included. In the last column, *Winsorized* $p > 2$, I winsorize the data removing all observations that fall within the 0.1th and 99.9th percentile of returns, in addition to including all stocks priced above NOK 2. Class B stocks and financial firms are removed from the data prior to the filtering on price.

	Original sample	$p > 2$	$p > 5$	$p > 10$	*Winsorized* $p > 2$
N observations	39,766	35,527	30,684	25,876	35,458
Monthly return					
Mean(%)	0.94	1.47	1.51	1.69	1.15
Std. dev(%)	35.22	35.70	16.19	15.14	15.35
Median(%)	0.00	0.00	0.00	0.00	0.00
Skewness	116	125	4.3	4.7	1.32
Kurtosis	19011	20172	77	94	10
Market cap					
Mean(mil.)	6313	7036	8053	9365	7049
Std. dev(mil.)	26472	27919	29913	32398	27945
Median(mil.)	799	991	1265	1568	994
Skewness	9.0	8.5	7.9	7.2	8.48
Kurtosis	98	88	76	65	87.9
Companies listed					
Mean	176.6	158.2	137.2	115.2	157.9
Std. dev	20.72	24.46	25.92	27.31	23.49
Median	172.2	155	134	107	155
Min	144	118	103	83	118
Max	227	214	196	177	214

5 Results

5.1 Test portfolios and their expected returns

5.1.1 Simple sort

To get an overview of how the test portfolios of the five-factor model behave, we want to examine how their average returns change with portfolio characteristics. We start off by looking at portfolios formed by simple sorts. Table 5.1 and 5.2 present summary statistics for average monthly percent excess return for simple sorted value-weighted portfolios in the time period June 1999 to December 2017. At the end of each June, all stocks on the OSE are sorted in ascending order based on their market capitalization, book-to-market ratio, operating profitability and investment (defined by 3.4-3.7) individually. Decile breakpoints are calculated and used to split the equities into 10 portfolios per factor. For the size portfolios in table 5.1 Panel A, portfolios 1 is then a portfolio of the 10% least valued and portfolio 10 of the 10% most valued companies. All panels display summary statistics for the excess monthly return and the number of stocks of each portfolio. We are curious about which direction average returns move as characteristics of the portfolios change. To address the significance of these movements, I perform simple t-tests of whether the difference between the average excess return for the five lowest and highest portfolios are different from zero. The test results are displayed at the bottom of each panel.

For portfolios formed on size, we observe a clear relationship between market capitalization and average return. The smallest portfolio has an average monthly excess return of 3.52%, clearly outperforming the largest portfolio which only gave a return of 1.23%. The extreme portfolios do not only differ greatly, but we also observe a distinctive pattern of falling returns with size. The pattern is tested and confirmed formally, where we see that the 50% least valued shares significantly outperform the 50% highest value by 0.91% on average per month with a high t-value of 4.41. The well-documented size anomaly seems to be present on the OSE, confirming previous findings by Næs et al. (2009).

Table 5.1: Average monthly percent excess return for value-weighted test portfolios sorted on size and book-to-market

The table presents summary statistics for the excess monthly return (in percent) of portfolios formed by a simple sort on size (Panel A) and book-to-market (Panel B) from June 1999 to December 2017, 223 months.
At the end of each June, all stocks on the OSE are sorted on their market capitalization and book-to-market ratio. Decile breakpoints are calculated and used to split the equities into 10 portfolios. For size, portfolio 1 (Small) and 10 (Big) are then the portfolios with the 10% least and highest valued firms, respectively. For book-to-market, portfolio 1 (Low) and 10 (High) are the portfolios with the 10% lowest and highest book-to-market ratios. The table also presents each portfolio's number of stocks over the time period.
The last part of the table shows the t-value and the corresponding p-value from a test of whether the difference between the average excess return for the five smallest (highest) and largest (lowest) portfolios are different from zero. A small p-value indicates a significant difference in returns between the small (high) and large (low) portfolios.

Panel A:

Size

	Excess return					Number of stocks		
Portf.	Mean	std.dev	min	median	max	min	median	max
1 S	3.52	8.89	−17.30	2.35	34.61	8	14	23
2	2.59	7.49	−17.35	2.47	31.09	10	14	21
3	2.17	7.62	−20.28	1.42	31.82	9	15	25
4	2.64	8.18	−17.18	1.54	43.13	9	14	22
5	2.08	7.33	−21.47	2.06	26.99	9	14	23
6	1.92	6.92	−22.38	1.98	23.02	10	14	22
7	2.03	6.96	−16.86	1.42	27.57	8	14	22
8	1.73	6.59	−23.30	1.63	22.47	9	14	22
9	1.56	6.96	−25.84	1.50	21.68	12	15	21
10 B	1.23	5.71	−23.61	1.50	15.97	10	15	21

Small	Large			t-test	
Mean(1-5)	Mean(6-10)	Diff	std.dev	diff=0	p-value
2.60	1.69	0.91	6.84	4.41	0.011

Panel B:

Book-to-market

	Excess return					Number of stocks		
Portf.	Mean	std.dev	min	median	max	min	median	max
1 L	1.56	8.61	−26.79	1.65	24.93	9	13	19
2	1.64	8.30	−40.85	1.53	36.15	10	13	19
3	1.50	8.21	−34.57	1.67	35.30	8	13	19
4	0.63	8.06	−34.53	0.80	27.21	8	13	18
5	1.45	5.88	−21.22	1.32	22.02	8	13	18
6	1.36	6.45	−18.98	1.78	23.33	9	14	18
7	0.95	7.71	−29.68	1.30	17.82	9	13	19
8	1.59	8.09	−27.21	2.09	26.17	9	14	18
9	0.65	8.18	−21.95	0.68	28.86	8	13	20
10 H	1.65	9.81	−21.13	1.00	48.30	8	12	18

High	Low			t-test	
Mean(6-10)	Mean(1-5)	Diff	std.dev	diff=0	p-value
1.24	1.36	-0.12	8.13	-0.49	0.623

At first glance, the value premium often targeted by money managers in the US (Fama and French, 2015) is nowhere to be found on the OSE when examining simple sorted portfolios on book-to-market. As we go from the low (1) to the high (10) portfolio, we do not observe any clear pattern in the movement of excess returns. The portfolio containing stocks with the 50% highest book-to-market ratio provided monthly returns of 1.24% and was outperformed by its counterpart, which had an average return of 1.36%. The difference is, however, not significant with a small t-value of 0.49. Results for equally-weighted portfolios on size and book-to-market sorts can be found in the appendix (A0.1).

In Panel A of table 5.2 we investigate the anomaly related to operating profitability. Albeit less systematic than for size, we observe a slight increase in average returns as we move from weak to robust portfolios. The 50% weakest stocks had an average return of 1.09%, insignificantly beaten (t-stat of 1.22) by the 50% most robust stocks who had a monthly average excess return of 1.43%. Interestingly, we also observe that weak stocks have higher standard deviations of return than robust stocks. By now, we have only compared series of returns unadjusted for risk, but by looking at the large differences in the portfolio risk (measured by standard deviation), we could have seen a more significant difference in returns had we adjusted for the extra risk involved in holding stocks with poor profitability.

Contrary to findings in the US that firms with conservative investment activity deliver higher expected returns than those who invest aggressively, the results from Panel B table 5.2 tell a different story. As we move from portfolios with low investment activity (conservative) to high (aggressive), we see a pattern of increasing average monthly returns. The 50% most aggressive stocks delivered an average return of 1.66%, whereas the conservative counterpart only gave 1.08% per month. The 0.58% average per month difference is significant with a t-statistic of 2.44. We should not be too surprised to see results that deviate from US findings. For example, in the Japanese stock market aggressive stocks outperformed conservative stocks by an average monthly return of -0.75% over the same time period as ours (French, 2019). Results for equally-weighted portfolios on profitability and investment sorts can be found in the appendix (A0.2).

Table 5.2: Average monthly percent excess return for value-weighted test portfolios sorted on operating profitability and investment

The table presents summary statistics for the excess monthly return (in percent) of portfolios formed by a simple sort on operating profitability (Panel A) and investment (Panel B) from June 1999 to December 2017, 223 months.
At the end of each June, all stocks on the OSE are sorted on their operating profitability and investment level. Decile breakpoints are calculated and used to split the equities into 10 portfolios. For operating profitability, portfolio 1 (Weak) and 10 (Robust) are then the portfolios with the 10% least and most profitable firms, respectively. For investment, portfolio 1 (Conservative) and 10 (Aggressive) are the portfolios with the 10% lowest and highest investment. The table also presents each portfolio's number of stocks over the time period. The last part of the table shows the t-value and the corresponding p-value from a test of whether the difference between the average excess return for the five most robust (conservative) and weak (aggressive) portfolios are different from zero. A small p-value indicates a significant difference in returns between the robust (conservative) and weak (aggressive) portfolios.

Panel A:

Operating profitability

	Excess return					Number of stocks		
Portf.	Mean	std.dev	min	median	max	min	median	max
1 W	1.23	11.45	−28.89	−0.47	41.35	7	12	19
2	1.18	13.09	−37.59	1.37	59.00	9	13	19
3	1.09	10.54	−30.13	1.09	55.81	9	13	20
4	1.23	8.24	−27.33	0.99	34.82	8	13	19
5	0.70	8.17	−28.64	0.49	27.30	9	13	18
6	1.34	8.88	−40.84	0.89	35.77	9	14	18
7	1.46	7.55	−37.22	1.96	24.86	10	13	18
8	1.54	6.89	−32.52	2.06	25.93	10	14	18
9	1.58	6.94	−32.45	1.00	25.02	10	14	18
10 R	1.20	6.09	−19.69	1.10	19.21	10	13	17

Robust Mean(6-10)	Weak Mean(1-5)	Diff	std.dev	t-test diff=0	p-value
1.43	1.09	0.34	9.33	1.22	0.224

Panel B:

Investment

	Excess return					Number of stocks		
Portf.	Mean	std.dev	min	median	max	min	median	max
1 C	1.73	9.65	−25.67	0.45	40.68	7	12	18
2	0.70	9.24	−24.46	0.01	35.63	7	12	18
3	0.77	7.53	−23.35	0.51	20.03	9	13	17
4	0.94	6.70	−24.67	0.70	21.90	9	12	18
5	1.24	6.99	−30.67	1.07	20.47	10	12	17
6	1.49	6.93	−31.17	1.42	30.62	9	13	17
7	1.47	6.99	−18.63	1.47	33.45	9	13	18
8	0.94	6.89	−24.45	1.38	20.29	10	13	17
9	1.60	8.03	−38.83	1.33	29.20	9	13	17
10 A	2.79	9.75	−32.33	2.15	44.49	7	12	17

Conservative Mean(1-5)	Aggressive Mean(6-10)	Diff	std.dev	t-test diff=0	p-value
1.08	1.66	-0.58	7.89	-2.44	0.015

5.1.2 Double sort

I now disentangle the factors and investigate how the sets of double sorted test portfolios behave. Table 5.3 presents the average monthly percent excess return for value-weighted test portfolios formed by double sorts on size, book-to-market, operating profitability and investment. The intersection of the two independent sorts on size and B/M, OP and Inv leaves us with the twelve value-weighted Size-B/M portfolios in Panel A, Size-OP in Panel B and Size-Inv in Panel C. In all panels, the three rows represent different size levels and each column represents different B/M, OP and Inv levels. Inspecting table 5.3, we look for systematic patterns in average excess return as portfolio characteristics change both vertically and horizontally.

The strong size effect observed from simple sorted portfolios on market cap still stands out after having controlled for book-to-market, operating profitability and investment. All panels display a systematic increase in average excess return as we move upward from big to small portfolios. After isolating book-to-market from size in table 5.3 Panel A, we still see no clear relationship between average return and the book-to-market ratio. Moving from low to high B/M, we observe a slightly increasing trend for small and medium sized portfolios. For the big portfolio, however, there is a clear declining relationship between B/M and average returns. The value factor still seems absent on the OSE. For operating profitability in panel B, we observe a more clear trend. After controlling for size, average returns are almost systematically increasing with profitability moving from left to right. Interestingly, the portfolio containing the largest and least profitable companies is the only in-sample portfolio to deliver negative average return. It is also the rarest group of stocks with an average portfolio size of only four shares, well below the advice of Ødegaard (2019b). We should therefore not overstate this result. Panel C strengthens previous results that, in contrast to US findings, there is a positive relationship between investment activity and average returns for Norwegian stocks. Moving from low to high, we see that average returns increase systematically with investment. Results for equally-weighted double sorted portfolios can be found in appendix (A0.3).

Table 5.3: Average monthly percent excess return for value-weighted test portfolios formed by double sorts

The table presents the average monthly percent excess return for test portfolios formed by double sorts on size, book-to-market, operating profitability and investment from June 1999 to December 2017, 223 months.
At the end of each June, all equities on the OSE are allocated to three *size* groups (Small, 2, Big) using market capitalization as breakpoints. Stocks are then independently allocated to four groups (Low, 2, 3, High) using book-to-market ratio as breakpoints. The intersection of the two independent sorts leaves us with the twelve value-weighted Size-B/M portfolios in Panel A. Size-OP and Size-Inv in Panel B and C are formed in the same manner as Size-B/M, now sorting on operating profitability and investment as a second sort variable. The right side of the table shows the average number of stocks in each portfolio over the time period.

	Average excess return				Average number of stocks			
	Low	2	3	High	Low	2	3	High
Panel A: Size-B/M portfolios								
Small	2.16	2.56	2.13	2.88	9	10	10	13
2	1.75	2.02	2.25	1.58	12	10	11	10
Big	1.54	1.26	1.12	1.06	11	12	11	9
Panel B: Size-OP portfolios								
Small	2.18	2.21	2.87	3.29	16	10	9	7
2	1.30	1.91	1.49	2.49	10	11	10	11
Big	-0.20	0.50	1.32	1.30	4	11	14	16
Panel C: Size-Inv portfolios								
Small	1.79	2.20	2.70	3.52	13	9	8	9
2	1.37	1.37	1.49	2.21	9	11	8	11
Big	0.90	0.96	1.37	1.81	7	11	14	10

5.2 Fama-Macbeth regressions

5.2.1 The factors

Table 5.4 presents summary statistics for the monthly percent return for all factor mimicking portfolios. The market factor, being the monthly return of a value-weighted market portfolio less the risk-free rate, has a mean of 1.42% and is, as expected, highly significant with a t-statistic of 3.80. The mimicking portfolio proxying for risk related to company size, SMB, is in line with hitherto results significant with a t-statistic of 3.54. Its mean of 0.67% tells us that a value-weighted portfolio of the 50% least valued stocks on average outperforms a portfolio of the 50% highest valued firms by 0.67% percentage points per month. The book-to-market portfolio, HML, is positive but insignificant. The same goes for the mimicking portfolio for profitability, RMW, which is slightly positive but also insignificant. We see that the factor proxying for investment activity, CMA, has a significant (t-statistic of 2.29) negative mean of -0.73% per month.

5.2.2 First stage Fama-Macbeth regressions

We now turn to the estimation of the five-factor model. We run time-series regressions following the first step Fama-Macbeth specification of equation (3.9) using double sorted portfolios on size, book-to-market, operating profitability and investment as dependent variables. The regression outputs for the three different sets of variables, Size-B/M, Size-OP and Size-Inv portfolios, are displayed in table 5.5, 5.6 and 5.7 respectively. To answer sub-question one, of whether there is a linear relationship between returns and the proposed factors, the estimated factor exposures (b, s, h, rw and c) provide important insight. We pay close attention to the coefficients' sign, magnitude, t-statistic and direction of movement as portfolio characteristics change. We also look at each model's intercept α, who we interpret as pricing errors. Intercepts significantly different from zero indicate a bad model.

Table 5.4: Summary statistics for monthly factor returns in percent

The table presents summary statistics for the monthly percent excess return for all factors from June 1999 to December 2017, 223 months. The t-statistic and the corresponding p-value comes from a test of whether the average factor return is different from zero. $r_M - r_F$ is the value-weighted monthly average return for the market portfolio minus the monthly risk-free rate. At the end of each June, all equities on the OSE are allocated to two size groups using the median market capitalization as a breakpoint. Stocks are then independently allocated to three book-to-market (B/M), operating profitability (OP) and investment (Inv) groups using the 30th and 70th percentiles of B/M, OP and Inv as breakpoints. SMB is the average monthly return of all small portfolios minus all big portfolios, HML the average monthly return of all high portfolios minus all low portfolios, RMW the average monthly return of all robust minus all weak portfolios and CMA the average monthly return of all conservative minus aggressive portfolios.

	$r_M - r_F$	*SMB*	*HML*	*RMW*	*CMA*
Mean	1.42	0.67	0.34	0.21	-0.73
Std.dev	5.59	4.8	6.23	6.52	5.04
t-statistic	3.80	3.54	0.46	0.91	-2.29
p-value	0.000	0.000	0.647	0.362	0.023

Starting off by looking at the intercepts α for all sorts in table 5.5-5.7, we immediately conclude that the Fama-French five-factor model fails to explain all variation in expected returns in the Norwegian stock market. The average absolute value of the twelve intercepts is 0.41% for Size-B/M, 0.65% for Size-OP and 0.39% for the Size-Inv portfolios. In other words, on average the model fails to explain about half a percent of each month's excess return. For Size-B/M and Size-Inv portfolios, the model performs relatively well in explaining returns for big portfolios judged by the intercepts. The alphas center on values around zero, indicating good performance. Overall, there seems to be a trend towards better explanatory results for portfolios consisting of large cap stocks. The factor exposures and significance of the market factor ($r_M - r_F$) on big portfolios seem to strengthen this belief. For the market factor, we observe highly significant exposures for all portfolios ranging from about 0.8 to 1.2. Large portfolios have factor exposures close to 1. Comparing t-statistics, we realize that there is less uncertainty related to the estimated coefficients of the big portfolios compared to the small. Looking at the R^2-values, the large portfolios on average capture between 74% and 78% of the variance in expected returns, whereas the small portfolios only explain between 45% and 52%. The fact that the FF5 model performs best in explaining large portfolios is closely connected to the fact that both the market factor and dependent portfolios are value-weighted. A few high-valued stocks greatly dictate how the market factor changes over time, and these same stocks are only assigned to the big (66.66th percentile) test portfolios.

When we look at factor exposures related to the size factor, SMB, there is a clear systematic trend of falling factor exposure with size for all three sorts. Both the magnitude and significance of coefficients decrease with size, further confirming our previous findings that company size affects expected returns in the Norwegian stock market. For book-to-market factor, HML, we see a slightly positive and significant relationship between B/M exposure and average excess return for the Size-B/M large cap portfolios. This vague trend is, however, non-existent when we control for profitability and investment in table 5.6 and 5.7. The factor proxying for operating profitability, RMW, is generally weak with small and insignificant factor exposures. For the Size-OP portfolios in table 5.6, we see that big portfolios have a positive and significant connection between the profitability exposure and average returns. Controlling for book-to-market and investment, also this effect disappears. The same pattern applies to the investment factor, CMA, for the Size-Inv portfolios in table 5.7. The CMA factor exposures are decreasing with investment, supporting our previous findings that aggressive portfolios outperform conservative portfolios. When we control for book-to-market and operating profitability, however, also this link seemingly vanishes. There is a clear linear relationship between expected returns and risk factors associated with the overall market and firm size. This relationship looks to be non-existing for the value factor, and barely present for the factors related to profitability and investment.

Table 5.5: First stage Fama-Macbeth time-series regressions using twelve double sorted value-weighted size-B/M test portfolios as dependent variables

At the end of each June, all equities on the OSE are allocated to three *size* groups (Small, 2, Big) using market capitalization as breakpoints. Stocks are then independently allocated to four groups (Low, 2, 3, High) using book-to-market ratio as breakpoints. The intersection of the two independent sorts leaves us with the twelve value-weighted Size-B/M portfolios. The dependent variables in each of the twelve regressions are the monthly excess returns of the Size-B/M portfolios. The set of independent variables are equal for all regressions and consist of the excess market return ($r_M - r_F$), the size factor (SMB), the value factor (HML), the profitability factor (RMW) and the investment factor (CMA). Intercepts (α), factor exposures (b, s, h, rw and c), t-statistics for all coefficients and adjusted coefficients of determination ($Adj.R^2$) are presented for each of the twelve regressions.

$$r_{it} - r_{Ft} = \alpha_i + b_i(r_{Mt} - r_{Ft}) + s_i SMB_t + h_i HML_t + rw_i RMW_t + c_i CMA_t + \epsilon_{it},$$

B/M →	Low	2	3	High	Low	2	3	High
Size ↓		α				$t(\alpha)$		
Small	-0.31	-0.67	0.07	1.11***	-0.56	-1.29	0.18	2.87
2	-0.90**	0.34	0.59	-0.18	-2.45	0.92	1.43	-0.45
Big	0.08	-0.06	-0.05	-0.53*	-0.29	-0.25	-0.25	-1.92
		b				$t(b)$		
Small	1.10***	1.15***	0.93***	0.72***	11.35	12.71	13.36	10.73
2	1.17***	0.82***	0.86***	0.98***	18.32	12.48	12.00	14.61
Big	1.02***	1.00***	0.95***	1.07***	21.26	25.04	26.95	22.04
		s				$t(s)$		
Small	0.91***	1.42***	0.70***	0.63***	7.62	12.69	8.08	7.58
2	0.78***	0.49***	0.31***	0.33***	9.92	6.04	3.54	3.96
Big	0.05	-0.09*	-0.15***	-0.02	0.85	-1.79	-3.41	-0.41
		h				$t(h)$		
Small	-0.04	-0.22***	0.09	0.12**	-0.41	-2.70	1.49	1.99
2	0.00	0.05	0.06	0.23***	0.02	0.82	0.99	3.81
Big	-0.27***	-0.16***	0.16***	0.64***	-6.28	-4.63	5.13	14.86
		rw				$t(rw)$		
Small	-0.21**	0.16*	0.13*	-0.07	-2.19	-1.85	1.92	-1.01
2	0.13**	-0.07	-0.11	-0.15**	2.14	-1.09	-1.51	-2.35
Big	-0.09*	-0.02	-0.01	-0.13***	-1.83	-0.51	-0.19	-2.74
		c				$t(c)$		
Small	0.04	0.06	0.17**	-0.04	0.43	0.57	2.27	-0.50
2	-0.05	0.03	-0.01**	-0.01	-0.68	0.39	-1.71	-0.12
Big	-0.04	-0.04	0.05	-0.05	-0.74	-0.91	1.26	-0.95
		$Adj.R^2$						
Small	0.52	0.57	0.52	0.48				
2	0.64	0.50	0.46	0.59				
Big	0.73	0.78	0.81	0.81				

*Note: Significant codes: p<0.01: '***', p<0.05: '**', p<0.1: '*'*

Table 5.6: First stage Fama-Macbeth time-series regressions using twelve double sorted value-weighted Size-OP test portfolios as dependent variables

At the end of each June, all equities on the OSE are allocated to three *size* groups (Small, 2, Big) using market capitalization as breakpoints. Stocks are then independently allocated to four groups (Low, 2, 3, High) using profitability (OP) as breakpoints. The intersection of the two independent sorts leaves us with the twelve value-weighted Size-OP portfolios. The dependent variables in each of the twelve regressions are the monthly excess returns of the Size-OP portfolios. The set of independent variables are equal for all regressions and consist of the excess market return ($r_M - r_F$), the size factor (SMB), the value factor (HML), the profitability factor (RMW) and the investment factor (CMA). Intercepts (α), factor exposures (b, s, h, rw and c), t-statistics for all coefficients and adjusted coefficients of determination ($Adj.R^2$) are presented for each of the twelve regressions.

$$r_{it} - r_{Ft} = \alpha_i + b_i(r_{Mt} - r_{Ft}) + s_i SMB_t + h_i HML_t + rw_i RMW_t + c_i CMA_t + \epsilon_{it},$$

OP →	Low	2	3	High	Low	2	3	High
Size ↓		α				$t(\alpha)$		
Small	-0.42	-0.54	0.72*	1.13***	-0.81	-1.07	1.72	2.82
2	-1.75***	0.13	0.04	0.81**	-3.32	0.33	0.12	2.23
Big	-1.24*	-0.59**	0.08	-0.29**	-1.87	-2.05	0.27	-2.10
		b				$t(b)$		
Small	1.20***	1.04***	0.78***	0.85***	13.41	11.88	10.72	12.16
2	1.44***	0.95***	0.74***	0.76***	15.77	13.58	12.88	12.02
Big	1.11***	1.03***	0.98***	1.01***	9.69	20.49	19.36	41.93
		s				$t(s)$		
Small	0.86***	1.36***	0.80***	0.75***	7.79	12.50	8.91	8.69
2	0.84***	0.36***	0.33***	0.51***	7.40	4.12	4.69	6.56
Big	-0.04	-0.16**	-0.12*	0.05*	-0.30	-2.51	-1.95	1.75
		h				$t(h)$		
Small	0.07	-0.17**	0.15**	0.01	0.90	-2.17	2.35	0.11
2	0.09	0.06	0.04	0.06	1.14	1.04	0.86	1.02
Big	-0.03	0.14***	0.21***	-0.10***	-0.29	3.08	4.73	-4.86
		rw				$t(rw)$		
Small	-0.10	0.02	0.09	0.19***	-1.14	0.29	1.29	2.75
2	-0.03	-0.19***	-0.10*	0.13**	-0.31	-2.70	-1.77	2.14
Big	-1.06***	-0.32***	-0.02	0.21***	-9.44	-6.51	-0.32	9.08
		c				$t(c)$		
Small	0.07	0.32***	-0.10	-0.02	0.76	3.42	-1.31	-0.30
2	-0.05	-0.09	-0.06	0.06	-0.51	-1.22	-1.00	0.81
Big	0.09	0.12**	0.07	-0.04	0.72	2.30	1.25	-1.41
		$Adj.R^2$						
Small	0.56	0.60	0.46	0.46				
2	0.60	0.55	0.51	0.44				
Big	0.59	0.76	0.69	0.90				

*Note: Significant codes: p<0.01: '***', p<0.05: '**', p<0.1: '*'*

Table 5.7: First stage Fama-Macbeth time-series regressions using twelve double sorted value-weighted Size-Inv test portfolios as dependent variables

At the end of each June, all equities on the OSE are allocated to three *size* groups (Small, 2, Big) using market capitalization as breakpoints. Stocks are then independently allocated to four groups (Low, 2, 3, High) using investment (Inv) as breakpoints. The intersection of the two independent sorts leaves us with the twelve value-weighted Size-Inv portfolios. The dependent variables in each of the twelve regressions are the monthly excess returns of the Size-Inv portfolios. The set of independent variables are equal for all regressions and consist of the excess market return ($r_M - r_F$), the size factor (SMB), the value factor (HML), the profitability factor (RMW) and the investment factor (CMA). Intercepts (α), factor exposures (b, s, h, rw and c), t-statistics for all coefficients and adjusted coefficients of determination ($Adj.R^2$) are presented for each of the twelve regressions.

$$r_{it} - r_{Ft} = \alpha_i + b_i(r_{Mt} - r_{Ft}) + s_iSMB_t + h_iHML_t + rw_iRMW_t + c_iCMA_t + \epsilon_{it},$$

Inv →	Low	2	3	High	Low	2	3	High
Size ↓		α				$t(\alpha)$		
Small	-0.61	-0.11	0.77	0.99*	-1.15	-0.23	1.52	1.94
2	-0.70	0.32	0.72*	0.05	-1.53	0.87	1.90	0.11
Big	0.06	-0.13	0.17	-0.02	0.15	-0.55	0.84	-0.08
		b				$t(b)$		
Small	1.08***	0.89***	0.75***	1.05***	11.71	10.44	8.55	11.85
2	1.05***	0.69***	0.85***	1.09***	13.22	10.83	12.90	14.67
Big	0.96***	1.02***	0.83***	1.15***	13.83	25.20	25.54	22.70
		s				$t(s)$		
Small	0.89***	1.03***	0.67***	0.84***	7.83	9.76	6.20	7.70
2	0.48***	0.18***	0.20**	0.81***	4.90	2.33	2.44	8.85
Big	-0.05	-0.10**	-0.12***	-0.07	-0.57	-2.09	-2.71	-1.16
		h				$t(h)$		
Small	0.14*	-0.11	0.10	-0.04	1.74	-1.42	1.32	-0.45
2	0.24***	0.08	0.13**	-0.02	3.42	1.42	2.16	-0.23
Big	-0.05	0.04	-0.01	0.18***	-0.80	1.07	-0.30	4.09
		rw				$t(rw)$		
Small	0.01	0.08	0.01	-0.06	0.13	0.97	0.09	-0.66
2	-0.17**	-0.07	0.01	0.00	-2.18	-1.17	0.16	0.00
Big	-0.39***	0.02	-0.06*	-0.17***	-5.73	0.51	-1.68	-3.39
		c				$t(c)$		
Small	0.23**	0.17*	-0.10	-0.14	2.33	1.83	-1.02	-1.51
2	-0.06	0.00	-0.04	-0.05	-0.75	0.04	-0.60	-0.62
Big	0.40***	0.33***	-0.21***	-0.40***	5.34	7.71	-5.64	-7.35
		$Adj.R^2$						
Small	0.51	0.48	0.34	0.48				
2	0.55	0.41	0.47	0.57				
Big	0.64	0.79	0.77	0.77				

*Note: Significant codes: p<0.01: '***', p<0.05: '**', p<0.1: '*'*

5.2.3 Second stage Fama-Macbeth regressions

Table 5.8 presents the estimated risk premiums associated with each factor, obtained by running second stage Fama-Macbeth regressions on the three sets of test portfolios represented by panel A, B and C. A factor is priced if there is a significant linear relationship between increased factor exposure and average returns. Looking at at the table, we see that the risk premium for size is highly significant and priced at a significance level below 1%. The size premiums are positive, implying that there is positive relationship between increased exposure to the SMB factor and expected returns. For the Size-Inv portfolios, the investment factor is (surprisingly) priced at a significance level below 1%. The coefficient of the risk premium is, as we would expect from previous results, negative, implying a positive relationship between aggressive investment activity and expected returns. The market factor is also priced for Size-B/M and Size-OP portfolios. The signs of the estimated risk premiums for the market factor are, contrary to what we would expect, negative. What this tells us is that an increased exposure to the market as a whole results in reduced expected returns. As the OBX increased by a factor of about six from 1999 to 2017, the negative estimated risk premium should make us question the implications of table 5.9. The HML and RMW factors are not priced for any portfolios.

Generally, the results of table 5.9 should be regarded with some scepticism. All factor premiums are estimated with high uncertainty and most are more than five standard deviations away from their mean. The estimated market risk premium for Size-Inv portfolios is 443(!) times its standard deviation away from the mean. I believe the factor premiums lack explanatory power as they are only estimated from twelve (3×4) observations each month. Had the cross-section of companies been larger (as in the US), we could split all stocks into groups on finer sorts resulting in more precise results. Following the discussion from section 3 about the generated regressor problem arising in the Fama-Macbeth framework, another estimation approach that corrects for this bias could also have led to more robust results.

Table 5.8: Risk premiums estimated by Fama-Macbeth second stage regressions

The table presents estimated risk premiums and their summary statistics. The risk premiums are obtained by running second stage Fama-Macbeth regression run on the three sets of double sorted portfolios Size-B/M, Size-OP and Size-Inv given by Panel A, B and C respectively. The t-statistic and the corresponding p-value comes from a test of whether the estimated risk premium is different from zero. A low p-value indicates that the risk factor is priced.

$$r_i - r_F = \lambda_0 + \lambda_b \hat{b_i} + \lambda_s \hat{s}_i + \lambda_h \hat{h_i} + \lambda_{rw} r\hat{w}_i + \lambda_c \hat{c}_i + \epsilon_i$$

	λ_b	λ_s	λ_h	λ_{rw}	λ_c
Panel A: Size-B/M portfolios					
Mean	-2.51	1.19	-0.29	-0.17	-1.34
Std.dev	13.37	5.56	7.95	19.97	26.38
t-statistic	-2.80	3.19	-0.54	-0.13	-0.76
p-value	0.006	0.002	0.590	0.899	0.448
Panel B: Size-OP portfolios					
Mean	-1.83	1.33	1.32	1.04	-0.89
Std.dev	12.46	5.55	15.3	9.82	17.46
t-statistic	-2.19	3.59	1.29	1.58	-0.76
p-value	0.030	$<$0.001	0.200	0.115	0.447
Panel C: Size-Inv portfolios					
Mean	-0.03	1.15	-1.85	0.32	-1.85
Std.dev	13.3	5.72	19.91	15.73	8.29
t-statistic	-0.03	2.99	-1.39	0.30	-3.340
p-value	0.976	0.003	0.167	0.762	$<$0.001

5.3 Comparing models

Until now, we have only looked at how the five-factor model performs in explaining stock returns. I now also examine how more parsimonious models consisting of fewer factors manage to describe Norwegian returns.

5.3.1 Factor abundance

The Fama-Macbeth analysis confirms our belief and previous empirical findings that the market and size factors are important for explaining stock returns on the OSE. The last three factors, HML, RMW and CMA, seemingly add little extra explanatory power to the already existing model of the market and size factor. We can, however, not conclude whether the HML, RMW and CMA factors are weak alone, or if the information they exhibit is already explained by other factors. In the same fashion as Fama and French (2015), we examine this more closely by looking at factor correlations and regression results from running each factor on the other four in table 5.8. Looking at the correlation matrix in Panel A, the profitability factor, RMW, stands out. It correlates rather highly with all factors, especially with SMB. Panel B investigates formally how the variation in average return of each factor is explained by the other four. In the first row, the market factor $(r_M - r_F)$ is regressed on SMB, HML, RMW and CMA. The intercept, which we interpret as the average return unexplained by exposure to the other four factors, is 2.01% per month with a high t-statistic of 5.48. The size factor, SMB, also proves itself very important with an intercept of about 1.70% and t-statistic of 5.77. The book-to-market factor, however, comes out with a low intercept of 0.42% and small t-statistic of 0.96. This result indicates, in line with the findings of Fama and French (2015), that the HML factor becomes redundant when adding factors proxying for profitability and investment. Despite proving themselves explanatory weak in the Fama-Macbeth analysis, both the RMW and CMA factor significantly leave 1.39% and 0.92% average returns unexplained when left out, respectively.

Table 5.9: Factor correlations and abundance

Panel A presents correlations between all factors and Panel B shows regression outputs from running four factors on the fifth for all factors.
$r_M - r_F$ is the value-weighted monthly average return for the market portfolio minus the monthly risk-free rate. At the end of each June, all equities on the OSE are allocated to two size groups using the median market capitalization as a breakpoint. Stocks are then independently allocated to three book-to-market (B/M), operating profitability (OP) and investment (Inv) groups using the 30th and 70th percentiles of B/M, OP and Inv as breakpoints. SMB is the average monthly return of all small portfolios minus all big portfolios, HML the average monthly return of all high portfolios minus all low portfolios, RMW the average monthly return of all robust minus all weak portfolios and CMA the average monthly return of all conservative minus aggressive portfolios.
Int. is the regression intercept.

Panel A: Correlations between different factors

	$r_M - r_F$	*SMB*	*HML*	*RMW*	*CMA*
$r_M - r_F$	1.00	-0.17	0.00	-0.27	0.03
SMB	-0.17	1.00	0.21	-0.40	0.25
HML	0.00	0.21	1.00	-0.35	0.15
RMW	-0.27	-0.40	-0.35	1.00	-0.29
CMA	0.03	0.25	0.15	-0.29	1.00

Panel B: Regressing each factor on the other four

	Int.	$r_M - r_F$	*SMB*	*HML*	*RMW*	*CMA*	$Adj.R^2$
$r_M - r_F$							
Coef.	2.01		-0.37	-0.07	-0.36	0.01	0.15
t-statistic	5.48		-4.64	-1.24	- 5.83	0.09	
SMB							
Coef.	1.70	-0.24		0.03	-0.31	0.13	0.24
t-statistic	5.77	-4.64		0.70	-6.18	2.24	
HML							
Coef.	0.42	-0.09	0.07		-0.33	0.06	0.12
t-statistic	0.96	-1.24	0.70		-4.54	0.69	
RMW							
Coef.	1.39	-0.38	-0.49	-0.27		-0.19	0.35
t-statistic	3.59	-5.83	-6.18	-4.54		-2.61	
CMA							
Coef.	-0.92	0.01	0.17	0.04	-0.16		0.09
t-statistic	-2.58	0.09	2.24	0.69	-2.61		

5.3.2 Model performance

To evaluate model performance further, I present in table 5.9 GRS statistics, average absolute value of intercepts $A|\alpha_i|$ and adjusted R^2 values for five different models. All models are run on the three sets of double sorted test portfolios Size-B/M, Size-OP and Size-Inv given by Panel A, B and C respectively. In each panel, every row represents a model and its factors. The GRS test, who under the null states that all intercepts from the Fama-Macbeth first step regression of (3.8) jointly equal zero, is easily rejected for all models and portfolios. No model perfectly captures all variation in returns. The GRS test falls in value (the models improve) as we move from the CAPM (only $r_M - r_F$) to the Fama-French five-factor model ($r_M - r_F$, SMB, RMW, CMA and HML) for all portfolios, except the Size-B/M in Panel A who slightly favours the model excluding the value factor HML. The same trend applies to the average absolute value of intercepts $A|\alpha_i|$ (average pricing error), that almost systematically decreases in value as we add factors. For all test portfolios, however, we only observe a marginal model improvement as we go from a four-factor model of $r_M - r_F$, SMB, RMW and CMA to the FF5 adding the value factor HML. In fact, the four-factor model actually outperforms the FF5 judged by $A|\alpha_i|$ and the GRS-statistic in Panel A. The value factor has proved itself very weak in explaining average returns in the Norwegian stock market, once again confirmed by the results of table 5.9. Generally, the by far biggest model improvement comes from adding the size factor SMB to the model only consisting the the market factor $(r_M - r_F)$. Further augmenting firm-specific risk factors has relatively poor improvement on model performance.

Table 5.10: Summary statistics for tests of different models

The table presents summary statistics for five different models run on the double sorted test portfolios Size-B/M, Size-OP and Size-Inv given by Panel A, B and C respectively. In each panel, every row represents a model and its factors. The GRS-statistic and the corresponding p-value comes from the test of whether the expected value of all twelve intercepts from equation (3.8) jointly equal zero. Small p-values indicate a bad model. $A|\alpha_i|$ is the average absolute value of the twelve intercepts α_i. $Adj.\ \overline{R^2}$ is the average adjusted coefficient of determination from the same twelve regressions.

	GRS	p-value	$A\|\alpha_i\|$	$Adj.\ \overline{R^2}$
Panel A: Size-B/M portfolios				
$r_M - r_F$	3.86	< 0.001	0.70	0.47
$r_M - r_F$, *SMB*	2.44	0.006	0.42	0.58
$r_M - r_F$, *SMB*, *RMW*	2.13	0.016	0.40	0.59
$r_M - r_F$, *SMB*, *RMW*, *CMA*	2.00	0.026	0.40	0.59
$r_M - r_F$, *SMB*, *RMW*, *CMA*, *HML*	2.19	0.014	0.41	0.62
Panel B: Size-OP portfolios				
$r_M - r_F$	7.45	< 0.001	1.05	0.43
$r_M - r_F$, *SMB*	5.98	< 0.001	0.95	0.55
$r_M - r_F$, *SMB*, *RMW*	4.27	< 0.001	0.71	0.58
$r_M - r_F$, *SMB*, *RMW*, *CMA*	3.83	< 0.001	0.66	0.59
$r_M - r_F$, *SMB*, *RMW*, *CMA*, *HML*	3.81	< 0.001	0.65	0.59
Panel C: Size-Inv portfolios				
$r_M - r_F$	3.83	< 0.001	0.82	0.44
$r_M - r_F$, *SMB*	2.46	0.005	0.59	0.54
$r_M - r_F$, *SMB*, *RMW*	1.73	0.063	0.53	0.54
$r_M - r_F$, *SMB*, *RMW*, *CMA*	1.19	0.29	0.39	0.56
$r_M - r_F$, *SMB*, *RMW*, *CMA*, *HML*	1.15	0.325	0.39	0.56

5.4 Robustness test

Despite using monthly data over almost 19 years, we do not know for sure if the findings are sample specific and created by chance. To reduce the uncertainty related to the results, I reproduce table 5.10 using a sample that includes all financial firms that were removed from our original test sample.

Table 5.11 is identical to table 5.10, now showing summary statistics of different models run on a sample containing 43,893 observations (of which 8,435 are financial firms). The same modifications are done to this sample as to the original one. Glancing over the table, we are met with interesting results. The inclusion of financial firms almost strictly improves the performance of every model by all measures. The variation in returns explained by the model (R^2) is higher for all sorts and models, and the pricing errors given by the average absolute value of the twelve intercepts α_i are smaller. It may be the case that removing the financial firms, despite having high leverage by nature, was wrong and that their inclusion improves on the factors' ability to explain stock returns. The more natural explanation, however, comes from the fact that the financial sector stood for about 15% of the total market value of the Norwegian stock market (Ødegaard, 2019b) in the same time period as ours, and that the inclusion of these stocks leads to value-weighted portfolio returns closer to the market development. This again strengthens the explanatory power of the market factor, and thereby the models. Also, by including financial firms, the number of unique firms listed in our sample increases from 477 to 557. This again increases the average number of stocks in each test portfolio, making the portfolios more diversified. Nevertheless, the results and implications of this thesis still stand after controlling for the exclusion of financial firms.

Investigating the effect of defining penny stocks differently, I can also report that an increased price requirement of NOK 10 only slightly worsened model performance judged by the same measurements as the one of table 5.10.

Table 5.11: Robustness test: Summary statistics for tests of different models with a sample including financial firms

The table presents summary statistics for five different models run on the double sorted test portfolios Size-B/M, Size-OP and Size-Inv given by Panel A, B and C respectively. In each panel, every row represents a model and its factors. The GRS-statistic and the corresponding p-value comes from the test of whether the expected value of all twelve intercepts from equation (3.8) jointly equal zero. Small p-values indicate a bad model. $A|\alpha_i|$ is the average absolute value of the twelve intercepts α_i. $Adj.\ \overline{R^2}$ is the average adjusted coefficient of determination from the same twelve regressions.

	GRS	p-value	$A\|\alpha_i\|$	$Adj.\ \overline{R^2}$
Panel A: Size-B/M portfolios				
$r_M - r_F$	3.08	< 0.001	0.58	0.50
$r_M - r_F$, *SMB*	2.24	0.011	0.39	0.62
$r_M - r_F$, *SMB*, *RMW*	1.70	0.070	0.36	0.62
$r_M - r_F$, *SMB*, *RMW*, *CMA*	1.52	0.119	0.35	0.62
$r_M - r_F$, *SMB*, *RMW*, *CMA*, *HML*	1.52	0.119	0.35	0.65
Panel B: Size-OP portfolios				
$r_M - r_F$	6.67	< 0.001	0.91	0.47
$r_M - r_F$, *SMB*	6.02	< 0.001	0.78	0.60
$r_M - r_F$, *SMB*, *RMW*	4.60	< 0.001	0.56	0.63
$r_M - r_F$, *SMB*, *RMW*, *CMA*	4.17	< 0.001	0.53	0.63
$r_M - r_F$, *SMB*, *RMW*, *CMA*, *HML*	4.16	< 0.001	0.53	0.64
Panel C: Size-Inv portfolios				
$r_M - r_F$	3.38	< 0.001	0.72	0.47
$r_M - r_F$, *SMB*	2.02	0.024	0.49	0.57
$r_M - r_F$, *SMB*, *RMW*	1.29	0.224	0.41	0.59
$r_M - r_F$, *SMB*, *RMW*, *CMA*	0.75	0.698	0.28	0.60
$r_M - r_F$, *SMB*, *RMW*, *CMA*, *HML*	0.76	0.694	0.28	0.61

6 Conclusion

This thesis investigated how the five-factor model of Fama and French performs in explaining stock returns on the Oslo Stock Exchange. Although the GRS test clearly rejected the model's ability to capture all variation in average returns, the model still has explanatory value for Norwegian stock returns. The model explained between 34% and 90% of the variance in expected returns for the three sets of test portfolios formed by double sorts. The five-factor model performed particularly well in describing returns of test portfolios of stocks with large market capitalization, capturing more of the variance in returns and delivering smaller average pricing errors.

Looking at the model's common risk factors, we can confidently declare the factors associated with the overall market and company size to have high explanatory power for Norwegian stock returns. Observing how the average return of simple and double sorted portfolios changed as the average market capitalization of the test portfolios increased, we saw a clear negative linear relationship between size and average returns. The importance of the size factor was further strengthened by the Fama-Macbeth analysis, where I documented highly significant SMB coefficients that were falling with size. The second stage regression showed that the factor was priced for all three sets of test portfolios. The value factor HML, however, proved to be poor in explaining Norwegian stock returns. The factor was neither priced nor seen to have any clear relationship with average returns. The same can be said for the profitability factor RMW. The investment factor CMA, on the other hand, provided us with some interesting results. In contrast to US findings, aggressive firms significantly outperformed conservative companies during our time period of 1999 to 2017. Its explanatory power, however, proved to be rather weak, despite being priced for one the three sets of test portfolios. Comparing the performance of the five-factor model to other factor combinations, we saw that augmenting additional factors to the parsimonious two-factor model consisting of the market and company size factor added relatively little extra explanatory power. Picking the best model was, however, not the objective of this thesis, and doing that would require more formal tests.

As already pointed out, one weakness of this study lies in the estimation of the risk premiums. An estimation technique that solved the generated regressors problem would have led to more precise estimates. A common critique of how the five-factor model is evaluated is based on the fact that test portfolios are formed on the same characteristics as the factors, causing high correlation between the explained variable and explanatory variables and thereby artificially small pricing errors and R^2-values. To control for this, further research could replicate the analysis with test portfolios formed on different characteristics as the factors themselves. Compared to research done on the US stock market, the results of this thesis are also based on a rather short time series. It would be very interesting to see how the findings of this thesis would change when analyzed on a complete time series from 1980 as Næs et al. (2009).

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Appendix

Table A0.1: Average monthly percent excess return for equally-weighted test portfolios sorted on size and book-to-market

The table presents summary statistics for the excess monthly return (in percent) of portfolios formed by a simple sort on size (Panel A) and book-to-market (Panel B) from June 1999 to December 2017, 223 months.
At the end of each June, all stocks on the OSE are sorted on their market capitalization and book-to-market ratio. Decile breakpoints are calculated and used to split the equities into 10 portfolios. For size, portfolio 1 (Small) and 10 (Big) are then the portfolios with the 10% least and highest valued firms, respectively. For book-to-market, portfolio 1 (Low) and 10 (High) are the portfolios with the 10% lowest and highest book-to-market ratios. The table also presents each portfolio's number of stocks over the time period.
The last part of the table shows the t-value and the corresponding p-value from a test of whether the difference between the average excess return for the five smallest (highest) and largest (lowest) portfolios are different from zero. A small p-value indicates a significant difference in returns between the small (high) and large (low) portfolios.

Panel A:

Size

	Excess return					Number of stocks		
Portf.	Mean	std.dev	min	median	max	min	median	max
1 S	1.59	7.30	−20.26	0.96	20.62	8	14	23
2	0.72	6.34	−18.35	0.80	19.09	10	14	21
3	0.60	7.20	−22.96	0.17	29.18	9	15	25
4	1.20	7.24	−18.90	0.76	27.30	9	14	22
5	0.46	7.08	−24.98	0.83	19.88	9	14	23
6	0.58	6.71	−25.97	0.95	18.20	10	14	22
7	0.74	6.97	−22.61	0.60	17.92	8	14	22
8	0.81	6.72	−24.97	0.74	22.94	9	14	22
9	0.76	7.12	−27.33	0.67	21.88	12	15	21
10 B	0.64	6.86	−28.78	1.34	19.06	10	15	21

Small Mean(1-5)	Big (Mean(6-10)	Diff	std.dev	t-test diff=0	p-value
0.91	0.71	0.21	5.92	1.16	0.247

Panel B:

Book-to-market

	Excess return					Number of stocks		
Portf.	Mean	std.dev	min	median	max	min	median	max
1 L	0.40	8.15	−33.88	0.13	25.36	9	13	19
2	0.82	7.60	−28.56	1.07	21.01	10	13	19
3	0.45	6.93	−25.64	0.74	22.39	8	13	19
4	0.50	6.94	−23.68	1.10	18.58	8	13	18
5	1.14	5.75	−14.52	1.30	17.68	8	13	18
6	0.70	5.78	−21.33	0.85	17.69	9	14	18
7	0.85	6.59	−26.99	0.73	24.20	9	13	19
8	1.24	6.38	−23.89	1.19	24.62	9	14	18
9	0.75	6.52	−19.71	0.46	20.25	8	13	20
10 H	1.10	8.09	−24.70	1.45	23.28	8	12	18

High Mean(6-10)	Low Mean(1-5)	Diff	std.dev	t-test diff=0	p-value
0.93	0.66	0.26	6.02	-1.46	0.144

Table A0.2: Average monthly percent excess return for equally-weighted test portfolios sorted on operating profitability and investment

The table presents summary statistics for the excess monthly return (in percent) of portfolios formed by a simple sort on operating profitability (Panel A) and investment (Panel B) from June 1999 to December 2017, 223 months.
At the end of each June, all stocks on the OSE are sorted on their operating profitability and investment level. Decile breakpoints are calculated and used to split the equities into 10 portfolios. For operating profitability, portfolio 1 (Weak) and 10 (Robust) are then the portfolios with the 10% least and most profitable firms, respectively. For investment, portfolio 1 (Conservative) and 10 (Aggressive) are the portfolios with the 10% lowest and highest investment. The table also presents each portfolio's number of stocks over the time period. The last part of the table shows the t-value and the corresponding p-value from a test of whether the difference between the average excess return for the five most robust (conservative) and weak (aggressive) portfolios are different from zero. A small p-value indicates a significant difference in returns between the robust (conservative) and weak (aggressive) portfolios.

Panel A:

Operating profitability

	Excess return					Number of stocks		
Portf.	Mean	std.dev	min	median	max	min	median	max
1 W	−0.51	9.62	−32.69	−0.96	28.85	7	12	19
2	−0.27	9.14	−28.90	−0.48	29.61	9	13	19
3	0.37	8.42	−26.34	0.85	25.45	9	13	20
4	0.58	7.26	−22.80	0.25	22.76	8	13	19
5	0.59	6.33	−24.58	0.78	18.76	9	13	18
6	0.63	6.07	−26.74	1.21	19.49	9	14	18
7	1.50	5.32	−20.96	1.63	17.33	10	13	18
8	1.79	5.55	−17.42	1.83	24.66	10	14	18
9	1.39	5.46	−26.38	1.79	17.56	10	14	18
10 R	1.80	6.01	−20.49	2.01	19.77	10	13	17

Robust	Weak			t-test	
Mean(6-10)	Mean(1-5)	Diff	std.dev	diff=0	p-value
1.42	0.15	1.27	6.17	6.88	<0.001

Panel B:

Investment

	Excess return					Number of stocks		
Portf.	Mean	std.dev	min	median	max	min	median	max
1 C	0.36	8.99	−24.64	0.27	32.72	7	12	18
2	−0.25	8.06	−26.51	−0.36	30.29	7	12	18
3	0.11	6.79	−23.85	−0.12	18.05	9	13	17
4	0.51	5.89	−24.38	0.25	18.19	9	12	18
5	0.87	5.33	−20.75	0.92	17.13	10	12	17
6	1.52	6.16	−22.96	1.61	19.26	9	13	17
7	0.77	5.65	−20.99	1.32	14.84	9	13	18
8	0.98	6.21	−22.58	0.98	15.08	10	13	17
9	1.20	6.90	−23.04	2.01	20.87	9	13	17
10 A	1.65	8.36	−30.53	1.71	24.93	7	12	17

Conservative	Aggressive			t-test	
Mean(1-5)	Mean(6-10)	Diff	std.dev	diff.=0	p-value
0.32	1.23	-0.90	5.98	-5.04	<0.001

Table A0.3: Average monthly percent excess return for equally-weighted test portfolios formed by double sorts

The table presents the average monthly percent excess return for portfolios formed by a double sort (3x4) on size, book-to-market, operating profitability and investment from June 1999 to December 2017, 223 months.
At the end of each June, all equities on the OSE are allocated to three *size* groups (Small, 2, Big) using market capitalization as breakpoints. Stocks are then independently allocated to four groups (Low, 2, 3, High) using book-to-market ratio as breakpoints. The intersection of the two independent sorts leaves us with the 12 equally-weighted Size-B/M portfolios in Panel A. Size-OP and Size-Inv in Panel B and C are formed in the same manner as Size-B/M, now sorting on operating profitability and investment as a second sort variable. The right side of the table shows average number of stocks for each portfolio over the time period.

	Average excess return				Average number of stocks			
	Low	2	3	High	Low	2	3	High
Panel A: Size-B/M portfolios								
Small	0.96	0.98	0.87	1.46	9	10	10	13
2	0.59	0.53	0.92	0.74	12	10	11	10
Big	0.48	0.97	0.78	0.5	11	12	11	9
Panel B: Size-OP portfolios								
Small	0.27	0.87	2.04	2.49	16	10	9	7
2	-0.55	0.62	0.59	1.66	10	11	10	11
Big	-1.05	0.10	0.90	1.32	4	11	14	16
Panel C: Size-Inv portfolios								
Small	0.22	1.07	1.65	1.96	13	9	8	9
2	-0.02	0.33	1.09	1.40	9	11	8	11
Big	0.15	0.58	0.99	0.90	7	11	14	10