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Abstract

The article presents initial ideas towards a network-based approach for sea state estimation used for maritime applications. In principle, all available means, ranging from in situ buoys, fleet of ships to remote sensing by satellite and aircraft, could be considered, emphasising that the single means, and groups among, are supposed to act simultaneously. This study focuses on just \textit{one} of the means; the use of ships as sailing wave buoys. The article introduces the \textit{wave buoy analogy}, i.e. ship-as-a-wave-buoy, and it makes a proposal on how to impose (different) weights to the single ship-specific wave spectrum estimates obtained from multiple ships. Moreover, the work includes a discussion about the importance to associate a measure to reflect the (un)certainty of the wave spectrum estimate. The article presents a numerical case study, where multiple ships act simultaneously as 'wave spectrum estimators'. The case study relies on numerical motion simulations, as appropriate full-scale data is not available. In the analysis, it is shown that the use of simultaneous data from multiple ships leads to more accurate wave spectrum estimations.

Keywords	Sea state estimation; wave buoy analogy; multiple ships; RAO-based weighting; uncertainty measure

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Kgs. Lyngby, 27-07-2018

Hereby, I hope to have my work reviewed for possible publication in Applied Ocean Research. The article is entitled "Sea state estimation using multiple ships simultaneously as sailing wave buoys" and it is a joint work with two co-authors, Dr. Astrid H. Brodtkorb and Prof. Asgeir J. Sørensen.

Highlights:

- Simultaneous use of multiple ships as sailing wave buoys
- Weight functions derived from vessels' filtering characteristics
- Weight-averaged wave spectrum estimate
- Associated uncertainty measure derived from the frequency-wise variation among the ship-specific wave spectrum estimates
- Improved wave spectrum estimation

Best regards,

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Sea state estimation using multiple ships simultaneously as sailing wave buoys

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Abstract

The article presents initial ideas towards a network-based approach for sea state estimation used for maritime applications. In principle, all available means, ranging from in situ buoys, fleet of ships to remote sensing by satellite and aircraft, could be considered, emphasising that the single means, and groups among, are supposed to act simultaneously. This study focuses on just *one* of the means; the use of ships as sailing wave buoys. The article introduces the *wave buoy analogy*, i.e. ship-as-a-wave-buoy, and it makes a proposal on how to impose (different) weights to the single ship-specific wave spectrum estimates obtained from multiple ships. Moreover, the work includes a discussion about the importance to associate a measure to reflect the (un)certainty of the wave spectrum estimate. The article presents a numerical case study, where multiple ships act simultaneously as 'wave spectrum estimators'. The case study relies on numerical motion simulations, as appropriate full-scale data is not available. In the analysis, it is shown that the use of simultaneous data from multiple ships leads to more accurate wave spectrum estimations.

Keywords:
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1. Introduction

Safe and efficient operation of marine vessels is key, independent on the actual context; for instance, navigation in rough weather and severe seas, inspection and maintenance of offshore windfarms, and drill operations for oil and gas exploration. The most important (compromising) factor in this regard is the wave system experienced during the operation. It is therefore essential to possess knowledge about the onsite wave system, or generally the *sea state*, since predictions and analyses of wave-induced loads and responses otherwise cannot be made. And, it should be clear that with the future's *autonomous ship* it becomes even more important to have reliable estimates, in real-time, of the onsite sea state; realising that direct involvement and gut feeling by the ship master - normally securing safe and efficient marine operations - may have been taken out of the loop. Altogether, it could be said that sea state estimation is one of the fundamental components towards maximized operability and risk management of ships and ocean structures.

Typical means for sea state estimation (SSE) include classical wave buoys, remote sensing by aircraft and satellite, marine wave radar measurements, and ship-as-a-wave-buoy; all of which are usually applied on an individual basis and herein referred to as *observation platforms*. In the general context of marine operations (navigating vessels, crane operations, etc.) there is no the-one-and-only means for SSE, as the individual means all have their own pros and cons, depending on the very context. For example, the classical wave buoy is probably the most reliable means if concern is about obtaining accurate estimates and wave statistics at a specific geographic location which, conversely, is of no value if the sea state at a ship's ocean route needs to be monitored.

It is not the purpose of this paper to go into any discussion about which means for SSE should be introduced depending on the specific context. Rather the paper takes the view to consider the individual means as complementary and, consequently, may be used all together in a single network of 'wave recorders'. In this sense, as a conceptual idea, the *measuring of waves* would ultimately be a matter of fusing together simultaneous data and/or sea state estimates from *multiple* observation platforms. The motivation is to allow for improved SSE on both a local on-site position and on a more global geographic scale, enabling also forecasting of a future sea state to be expected ahead of time at a given geographical site.

Although several means should act simultaneously the present work initiates the aforementioned larger conceptual study by having its focus exclusively on one of the means; *the wave buoy analogy* where a ship is used as a sailing wave buoy. The analogy builds on the fact that ships resemble classical wave buoys, and, hence, response recordings from ships can be processed to facilitate estimation of the onsite wave spectrum. The particular approach for SSE has no news in itself [1–15], but the idea to consider multiple ships simultaneously has not been studied before.

The remaining part of the article consists of five sections. The main principle of the wave buoy analogy, and the reason to have particular focus on (multiple) ships as sailing wave buoys are briefly presented in the following section, Section 2. There is available a number of mathematical procedures upon which the wave buoy analogy can be formulated, and Section 3 presents one of the methods that can be applied to process wave-induced motion data from a ship. In the application of multiple ships for SSE it becomes natural as well as critical to introduce a weighting of the individual ship-specific estimates for what reason Section 4 makes a proposal on how to possibly do this. Section 5 includes a numerical case study used to illustrate the potential improvements that can be expected, when multiple ships are applied simultaneously as sailing wave buoys. Finally, concluding remarks are given in Section 6.

2. Multiple ships as sailing wave buoys

Most of today's ships, or more generally marine vessels, are heavily instrumented with sensors to record, for instance, wave-induced acceleration levels at various places on the vessel, the rolling angle of the ship, hull girder stresses amidships, etc., see Figure 1. In this sense, ships resemble classical wave buoys, and the response recordings from ships can be processed to facilitate estimation of the on-site sea state [17]; making the analogy to wave buoys by relating the measurements and the sea state through a mathematical model. However, in general, three aspects make the estimation problem more complicated: (1) Ships have more complex geometrical forms compared to wave buoys, for what reason theoretical calculations of the ship motion dynamics in terms of the wave-to-motion transfer functions (RAOs) may have larger uncertainty associated than corresponding calculations for wave buoys. Uncertainties in RAOs may also be imposed because of uncertain operational conditions. (2) Ships generally advance relative to the progressing waves. Consequently, a ship encounters the waves at a different period (equivalently, frequency) than does an observer being fixed relative to the inertial frame of reference (i.e., the 'absolute domain'). Physically, this phenomenon is described by the Doppler Shift [18–20] that must be strictly introduced, as theoretical calculations of the wave-induced responses of the ship otherwise cannot be compared with corresponding measurements that are obtained in the 'encounter-domain'. The mathematical solution is to directly establish the governing equations in absolute domain, e.g. [1, 2, 6, 13], *or* to solve the problem in encounter-domain and, subsequently, convert the solution to absolute domain [4, 19]. (3) A ship acts as a (wave) filter, making the ship less responsive to high-frequency waves. The filtering characteristic depends on ship size (length, breadth, draught) relative to wave length; but various studies address this particular problem [14, 21, 22].

The use of ships as 'sailing wave buoys' is believed to be key for a successful development of a networked-based approach for sea state estimation using simultaneous data from multiple observation platforms; simply due to the fact that quantity is a quality of its own. This is illustrated in Figure 2 that presents the potential amount of data which is available in a single snapshot in time, if data from all vessels in (all parts of) the oceans contributes.

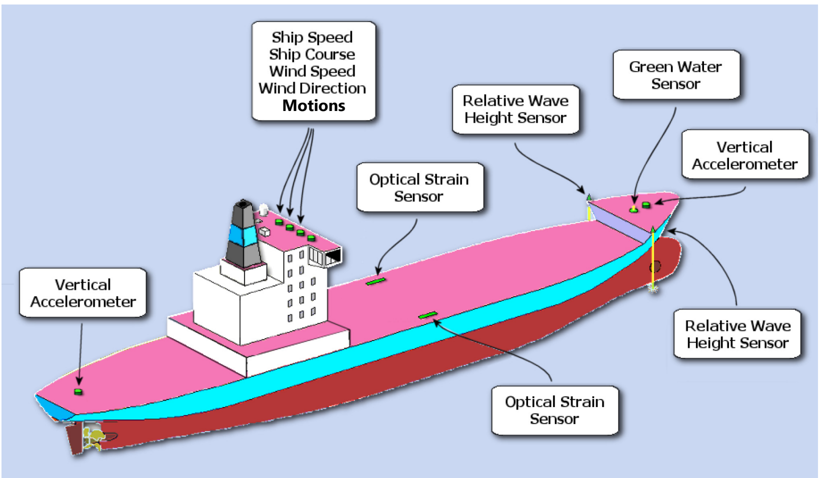


Figure 1: Illustration of (conceptual) sensor arrangement on a vessel [16].

As a related note, the use and analysis of data from the Automatic Identification System (AIS) should in itself be highly interesting, and a comprehensive survey in this regard has been given by [23]. AIS data is, indeed, a reason why navigating ships are foreseen to have a vital role in the present context (cf. Fig. 2).

A final remark about the use of ships as wave buoys should introduce the idea to make sea state estimation entirely based on measurements data and associated analytics, and simply "discarding" the need for transfer functions in real(-time) *in-service* scenarios. In this context, it is understood that vast amounts of wave-induced response data may be generated and/or measured in conditions where the *generating wave system* is exactly known; for example as is the situation in simulation studies of time history recordings, cf. Section 5. Subsequently, the generated (or measured) response recordings could be analysed and cast through machine learning procedures with the purpose to train algorithms to find optimum wave spectrum estimates under given conditions. The learning procedures may follow different approaches from more simple analytical models to deep learning algorithms for image recognition.

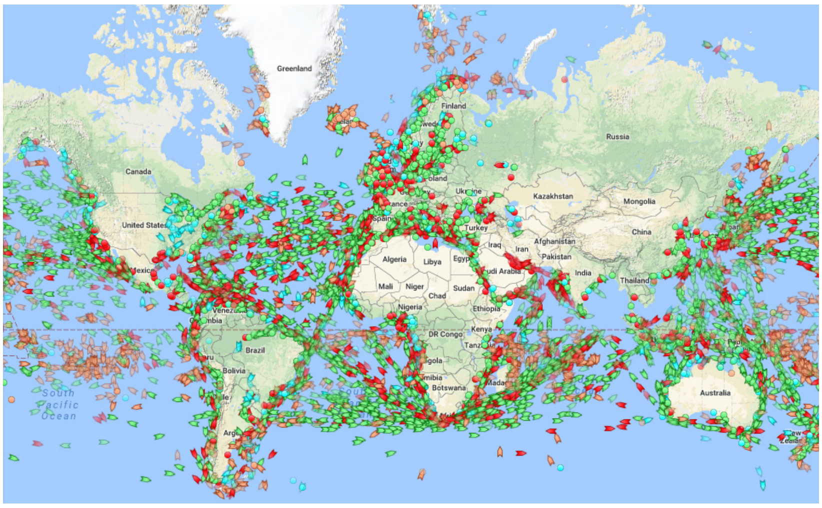


Figure 2: A snapshot of geographic vessel positions based on data from AIS (Automatic Identification System). With permission by MarineTraffic (www.marinetraffic.com/).

2.1. Communication and sharing of data

The ambition to make sea state estimation using simultaneous measurements from multiple ships and other heterogeneous observation platforms, cf. Figure 3, definitely requires levels of inter-connectivity which are higher than today's standard. Even by having focus exclusively on 'ships as sailing wave buoys' it will be necessary to ensure ultra-fast and reliable communication and sharing of data at sea and in-between the single ships and, possibly, a central processing unit. At sea and in open ocean areas, data and information may generally be communicated by, for instance, VHF radio (simply by word of mouth), Marine Broadband Radio (MBR) or by mobile units via satellites. In this sense, communication channels for data sharing among multiple ships are already in place. On the other hand, it is not given which data to be sharing. Presently, transmission speed is restricting the sharing - in real-time - of entire data sets; for example, it is not feasible to live-share the full set of wave-induced response recordings from an in-service operating vessel. Hence, it is matter of deciding which kind of data to be sharing. As a likely scenario, real-time data sharing may initially be focused on just characteristic wave parameters such as significant wave height, wave period, and wave direction. However, any further discussion about data communication and sharing is beyond this paper's

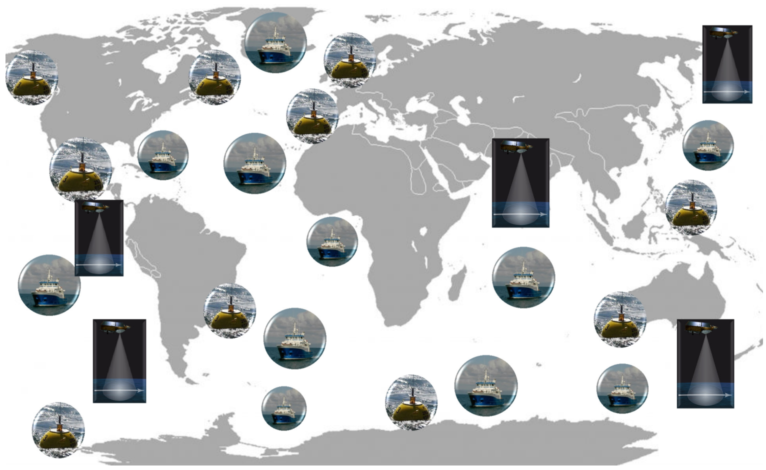


Figure 3: A huge number of heterogeneous wave observation platforms may be interconnected to complement each other for improved sea state estimation, both on the local scale and on a more global scale.

scope. In this article, it will therefore be assumed that all necessary data, including communication and sharing, is momentarily available on ad-hoc basis, *as needed.*

3. Ship motion-based wave spectrum estimation using a spectral-residual calculation

In the past, several successful studies on the *wave buoy analogy* have been conducted, and it has been concluded [17] that different procedures can be applied. Nonetheless, the underlying computational/mathematical methods have been found to suffer in many cases of being too slow or inefficient (although being fairly accurate). Recently, however, a novel procedure has been developed by Brodtkorb et al. [5] and Nielsen et al. [4], where the former study considers station-kept, dynamically positioned ships exclusively, while the latter [4] focuses on ships with a non-zero forward speed, and to some extent represents a generalisation of the former. Regardless of forward speed, the procedure relies on a brute-force, residual calculation formulated in the frequency domain through spectral analysis and the particular solution-strategy makes the specific procedure computationally very efficient with computational times in the order of a few seconds. The case study, Section 5, introduces wave spectrum estimation based on the work by Nielsen et al. [4], and the following contains a brief summary of the estimation procedure's theoretical formulation.

It is a general characteristic of the wave buoy analogy that it relates the *measured signals* of wave-induced responses with the exciting *unknown* wave spectrum through a mathematical model that couples the two parts using *theoretically calculated* transfer functions. Thus, in a short-crested, stationary seaway, the governing equation system reads

$$R_{ij}(\omega_e) = \int H_i(\omega_e, \mu+\beta)\overline{H_j(\omega_e, \mu+\beta)}E(\omega_e, \mu)d\mu \tag{1}$$

for a set (i, j) of responses taken among, say, the heave (z), roll (ϕ), and pitch (θ) motions, i.e. $i, j = \{z, \phi, \theta\}$. The complex-valued transfer functions, $H_i(\omega_e, \mu+\beta)$ and $H_j(\omega_e, \mu+\beta)$, yield the theoretical relationship between the i-th and the j-th components of the response spectra $R_{ij}(\omega_e)$ and the directional wave spectrum $E(\omega_e, \mu)$ for wave heading $(\mu + \beta)$ where μ is the direction of the single waves relative to mean wave heading β. The encounter frequency is ω_e. Note, the bar indicates complex conjugate. It should be understood that the left-hand side of Eq. (1) is estimated by the measured data while the right-hand side is obtained through the theoretical calculations. As noted μ represents the direction of the single waves, and, hence, by integration over all directions, the mean (absolute) wave direction ϑ can be derived by taking into account the compass-course of the vessel. The encountered frequency is related to the absolute frequency ω_0 through the Doppler Shift,

$$\omega_e = \omega_0 - \omega_0^2 \psi, \ \psi = \frac{U}{g} \cos(\mu + \beta) \tag{2}$$

where g is the acceleration of gravity and U is vessel speed. The Doppler Shift imposes an elementary physical problem but, in practice, this *shift* implies that the wave estimation problem is rather "delicate" to solve for ships with advance speed; making note that, in the end, the *absolute* wave energy spectrum rather than the *encountered* wave energy spectrum must be estimated. Several textbooks introduce the complications involved because of the Doppler Shift, e.g. [24–26], but giving no solution on how to deal with the problem in practice, when a wave spectrum (or a response spectrum) shall be transformed from encounter frequency-domain to absolute frequency-domain; a process that theoretically cannot be uniquely solved as it relates one single encounter frequency to three absolute frequencies in following seas. On the other hand, interrelated studies by Nielsen [19, 20] outline explicitly an elaborate and practical solution strategy for this problem.

The main (mathematical) task of the wave estimation problem is to solve Eq. (1) for the unknown wave spectrum. The actual solution follows from an iterative approach, schematically written as;

$$\tilde{R}_{ij}(\omega_e) = R_{ij}(\omega_e) - \hat{R}_{ij}(\omega_e) \tag{3a}$$

$$\hat{S}_{ij}(\omega_e) = \hat{S}_{ij}(\omega_e) + h_{ij}\tilde{R}_{ij}(\omega_e) \tag{3b}$$

$$\hat{R}_{ij}(\omega_e) = \hat{S}_{ij}(\omega_e) \int H_i(\omega_e, \mu + \beta)\overline{H_j(\omega_e, \mu + \beta)}\varphi(\mu)d\mu \tag{3c}$$

performed for any pair (i, j) of response signals for the entire set of considered encounter frequencies. The directional wave spectrum $E(\omega_e, \mu)$ is taken as the product between a point wave spectrum $S_{ij}(\omega_e)$ and a spreading function $\varphi(\mu)$, e.g. [24]. The iteration is initiated by setting the *estimated* response spectrum $\hat{R}_{ij}(\omega_e)$ and the *estimate* of the wave spectrum $\hat{S}_{ij}(\omega_e)$ equal to zero; emphasising that $\hat{S}_{ij}(\omega_e)$ is computed for each combination of signals. As seen, the iteration builds on a residual calculation (Eq. 3a), where $\hat{R}_{ij}(\omega_e)$ is subtracted from the measured response spectrum $R_{ij}(\omega_e)$ and, hence, forming the residual $\tilde{R}_{ij}(\omega_e)$. Subsequently, the residual facilitates an update (Eq. 3b) of the wave spectrum estimate, using a prescribed step size $h_{ij} > 0$, and the wave spectrum is in turn used to estimate the response spectrum (Eq. 3c). This iteration is continued until a threshold is reached for the residual $|\tilde{R}_{ij}| \leq \epsilon$, for $\epsilon > 0$.

The iteration scheme (Eqs. 3a-3c) is as mentioned applied for each signal-combination (i, j) but, although not mentioned herein, it is a fact that the iteration is, on top, conducted for a discrete set of mean wave headings specified on the half circle (0-180 deg.), followed by an additional calculation to select between waves approaching on starboard or port side. Altogether, the solutions obtained from Eqs. (3a-3c) form a *set* of initial estimates of the encounter-wave energy spectrum. Thus, in order to obtain the final wave spectrum estimate $S(\omega_0)$, valid in absolute domain, the initial brute-force solutions, $\hat{S}_{ij}(\omega_e)$, must be post-processed. The details of this process are given in [4], leaving it here to address the basics only. The post-process consists of two steps: (a) Firstly, a corresponding match of the optimum wave direction and an associated set of wave spectra (composed by all signal-combinations) is selected through a minimisation of a metric based on an energy-variation between the individual wave spectra. (b) Secondly, for the particular wave direction, the mean wave spectrum of the set of wave spectra is calculated and, in sequence, this spectrum is transformed to absolute domain. The transformation to absolute domain is a non-trivial (and non-unique!) problem to solve, and efforts have been spent setting up a practical and cost-effective approach to convert the encounter-wave spectrum to a corresponding absolute-spectrum. This particular "task" is a study in itself, and specific algorithms are outlined in the works [19, 20]. The actual algorithms will not be given further attention herein, and it suffices to note that, in the numerical case study (Section 5), spectrum transformation is conducted by the algorithm outlined in [19].

In a following section, a case study, or numerical example, is considered. Basically, the case study is used to illustrate how sea state estimation can be done - and improved

- by using *multiple* ships as sailing wave buoys. Before this, however, it is essential to discuss how to introduce weight factors, based on the wave-to-motion transfer functions of the individual ships, and, furthermore, how to associate (un)certainty to the actual wave spectrum estimate.

4. Weighting of the ship-specific estimates and (un)certainty measure

This section discusses two relevant aspects of wave spectrum estimation using multiple ships. **(i)** Weighting of the estimate by the single ship: Any vessel's capability to act as a wave buoy depends on the vessel's general behavior in waves, i.e. the wave-induced ship-motion dynamics, which in turn depend on the vessel's filtering properties, often described in terms of the wave-to-motion transfer functions. For instance, if the the group of vessels consists of similar ships with almost identical filtering characteristics, one possible way to weight the single ship-specific estimates would be to associate equal weights to the estimates from the ships. **(ii)** Association of an uncertainty measure to the weight-averaged wave spectrum estimate.

4.1. Conditions

In the following discussions and contemplations, the conditions (or assumptions) are the following:

- A total multiple of K ships are considered, and index $k = \{1, 2, ..., K\}$ is used to characterise a specific ship. Hence, there will be totally K wave spectrum estimates $S_k(\omega_0)$ that, collectively, can be used to derive one (final) weight-averaged wave spectrum $S^0(\omega_0)$ from.
- The ship-specific wave spectrum estimate $S_k(\omega_0)$ obtained from the single ship is derived by the wave buoy analogy using the 'brute-force spectral' approach described in Section 3, which is a summary of [4].
- Weighting of the individual wave spectrum estimates is made solely on the basis of the single sets of transfer functions of the group of ships considered. Hereby is understood that other "weighting metrics", for instance based on the relative distances between the individual vessels, are not included.

- For any ship, the considered set of motion components are heave, roll, and pitch, i.e. $i = \{z, \phi, \theta\}$, cf. Section 3. Reference to a given ship's set of transfer functions is according to $H_{i,k}(...)$.
- Angular [rad/s] and time-wise frequency [Hz] are used interchangeably but it is *absolute* frequency being considered no matter the choice. The angular frequency ω and time-wise frequency f are related by $\omega = 2\pi f$.

4.2. Weighting of the single wave spectrum estimates

The central point in the following is to establish a frequency-dependent *and* ship-specific weight function which multiplies on the wave spectrum estimate from the single ship to give the particular estimate greater or smaller (frequency-dependent) trust than corresponding estimates from the other considered vessels. The level of trust in a frequency region is dependent on the particular vessel's filtering characteristic, where any of the six degrees-of-freedom (DOF) motion components, in principle, can be utilised. In the following, the heave and pitch filtering characteristics are considered.

The behaviour of a ship as a (linear) filter means that the given ship "filters away certain wave components"; that is, the ship does not respond to the particular wave components which conversely implies that the same ship cannot be used to estimate exactly those wave components being filtered away. It is a ship's wave-to-motion transfer functions that determine how waves are *transferred* into motions of the ship, since the set of transfer function contains the *filtering characteristic* of the ship. Consequently, it makes sense to introduce a weighting of the single ship-specific wave spectrum estimate based on the given ship's transfer functions. Note, the transfer function $H_{i,k}(...)$ is complex-valued or, equivalently, represented by a corresponding pair of modulus (i.e. magnitude) and phase, see also Section 3. In the following, only modulus $|H_{i,k}(...)|$ is considered although this is not necessarily stated.

The sets of transfer functions $|H_{i,k}(f|\beta)|$ of heave and pitch for three arbitrarily selected vessels* are shown in Figure 4 as function of frequency. In the figure, the individual subplots show the modulus at different encounter angles β; where the thinner

*The vessels are exactly the same as those used in the case study in Section 5.

(coloured) lines represent specific encounter angles between 0 deg. (following sea) and 180 deg. (head sea). The actual variation with encounter angle is of less importance, since the 'mean curve', obtained as the average of all single curves for a given frequency, is used in the further contemplations. The 'mean curve' is simply calculated by,

$$\overline{H}_i(f) = \underset{\text{All } \beta}{\text{mean}} \big[|H_{i,k}(f|\beta)| \big] \tag{4}$$

and the curve is shown as the bold, blue line in the individual sub-plots of Figure 4.

In terms of weighting, the given values of the transfer functions' mean curves should *not* be used, but rather should the weighting be based on normalised values, since vessels of different size generally may have (very) different transfer function magnitudes, depending on the considered motion component as seen for the modulus of pitch from the three right-hand side plots in Figure 4. The normalised modulus of any motion component is denoted by $\alpha_i(f)$, which applies to a specific frequency f, and normalisation is made with respect to the maximum value of the (averaged) modulus,

$$\alpha_{i,k}(f) = \frac{\overline{H}_{i,k}(f)}{\max \overline{H}_{i,k}(f)} \tag{5}$$

Figure 5 shows the normalised modula for heave (left) and pitch (right) for the three respective vessels.

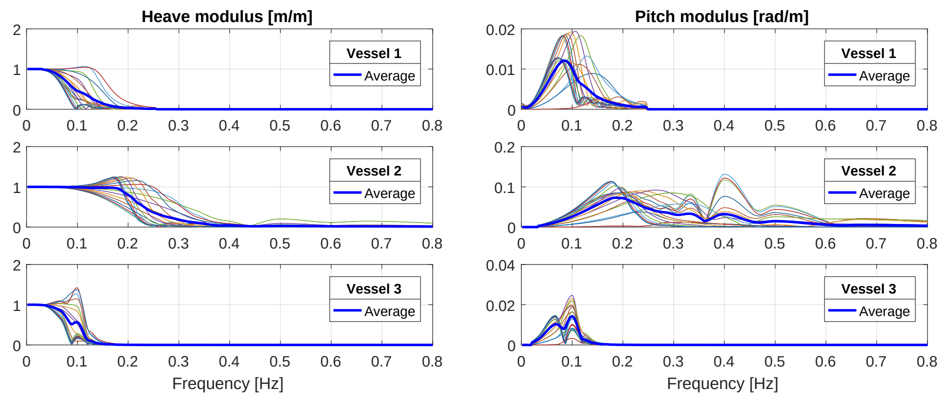


Figure 4: The filtering characteristics in terms of transfer function magnitudes of three size-wise different vessels; heave is the left- and pitch the right-hand side plots, where each plot shows the results for different wave-encounter angles. Note the difference in scales on the y-axes on pitch magnitudes.

To arrive at *one* frequency-dependent weight function ϱ applicable to a given motion component *and* to a given ship, the specific normalised modulus of the particular ship is weighted with consideration to the normalised modula of all considered vessels. Hence, the weight function is defined by,

$$\varrho_{i,k}(f) = \frac{\alpha_{i,k}(f)}{\sum_k \alpha_{i,k}(f)}, \qquad k = 1, 2, ..., K \tag{6}$$

where it is noted that ϱ by definition takes values between 0 and 1; it is 0 if the particular ship, denoted by $k = k_0$, has no response at the given frequency f, i.e. $\overline{H}_{i,k=k_0}(f) = 0$, while it is 1 at any frequency if $\overline{H}_{i,k=k_0}(f) > 0$ and $\overline{H}_{i,k\neq k_0}(f) \equiv 0$. The weight functions for heave and pitch are shown in Figure 6. For instance, it is seen that the pitch-based weighting factor for vessel 2 takes a value approximately equal $\varrho_{\theta,2}(0.10) = 0.17$ at the frequency $f = 0.10Hz$. This value is obtained as $\frac{0.37}{0.85+0.37+1.00} \approx 0.17$ with reference to the plot of the normalised pitch modula (right-hand side plot in Figure 5).

Remarks: With the set of weight functions $\varrho_{i,k}(f)$ available, a few remarks should be noteworthy. **(1)** As already indicated, the hypothesis is that just *one* of the weight functions should be used to arrive at a weight-averaged (overall) wave spectrum estimate in a given operational scenario. Which one of the weight functions, the heave- *or* the pitch-based, to use may be chosen subjectively or by some objective criterion; although it is beyond the scope of the present paper to make a study of such an objective criterion. Herein, it is therefore a matter of selecting the one or the other, but, in either case, the

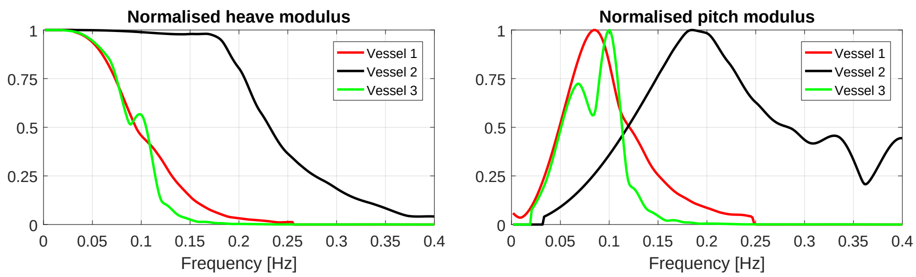


Figure 5: The normalised modula (of 'mean curves') of heave (left-hand side) and pitch (right-hand side). Normalisation is made with respect to the maximum value of the 'mean curves', cf. Figure 4

weight-averaged wave spectrum estimate is obtained from,

$$S_i^0(\omega_0) = \sum_k \varrho_{i,k}(\omega_0) \cdot S_k(\omega_0) \tag{7}$$

emphasising that index i refers to the specific motion component upon which the weighting function is established and index k refers to a given ship. As an example, if the pitch-based weight function ($i = \theta$) from Figure 6 is used to compute an average wave spectrum estimate in some operational scenario, it will be found that the averaged estimate is primarily based on the estimates from Vessels 1 and 3 at the lower frequency region [0.02-0.10] Hz. On the contrary, for frequencies larger than about 0.20 Hz, the averaged wave spectrum estimate will be (almost) entirely based on the wave spectrum estimate from Vessel 2; in the intermediate region [0.10-0.15] Hz, the wave spectrum estimates from all three vessels are weighted somewhat equally. Note, with little need to mention, it should be realised that the sum of weight factors at any given frequency always add to 1, cf. Eq. (6), independently on the considered motion component. **(2)** It was decided to calculate the given ship-specific weight functions on the basis of the normalised modula obtained from the 'mean curves' of the transfer functions' magnitudes, cf. Figures 4 and 5. Obviously, this is an approximation and it may be argued that the ship-specific weight functions should not only be frequency-dependent, but should also be dependent on the encounter angle. However, as this would require knowledge about which exact encounter angle to use it is decided to *not* make the weight functions (explicitly) dependent on encounter angle. Note, the wave buoy analogy does actually offer an *estimate* of the "exact" encounter angle experienced during operational scenar-

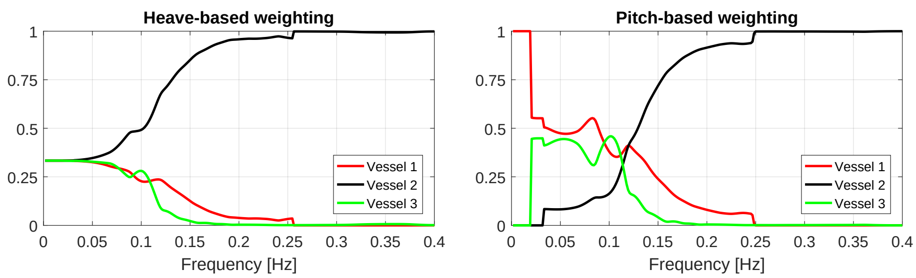


Figure 6: Ship-specific heave and pitch-based weighting functions derived from the modula of corresponding transfer functions.

ios, see Section 3, but the point here is that the estimate is an uncertain *estimate* and, together with all other uncertain parameters and operational conditions in realistic in-service scenarios, it should not be critical to derive the set of weight functions from just (frequency-dependent) 'mean curves' of the transfer functions' magnitudes.

4.2.1. Estimation of wave heading.

As noted, the single ship-specific wave spectrum estimate is associated with an estimate β_k of the mean *wave heading*; or, equivalently, the mean *wave direction* ϑ_k can be obtained if β_k is combined with vessel no. k's compass course. As multiple ships are considered simultaneously, it makes sense to introduce a weighting of the individual wave direction estimates ϑ_k, $k = 1, 2, ..., K$; for instance, using a similar filtering-based approach like the one introduced for the wave *spectrum*, cf. Eq. (7). For the wave *direction*, however, the weighting is not frequency dependent but, instead, it is suggested to calculate ship-specific weight *factors* $\Lambda_{i,k}$ to be multiplied on the single wave direction estimates associated to the given ships. Herein, the weight factors are defined by,

$$\lambda_{i,k} = \int_0^\infty |\varrho_{i,k}(\omega_0)|^2 S^0(\omega_0) d\omega_0 \tag{8}$$

$$\Lambda_{i,k} = \frac{\lambda_{i,k}}{\sum_k \lambda_{i,k}} \tag{9}$$

repeating that indices i and k refer to a particular motion component and a specific ship, respectively. It is noted that $\lambda_{i,k}$ resembles the 0th order spectral moment; but emphasising that the previously defined weight functions (Eq. 6) are used rather than the modula of the actual transfer functions. In the end, the weight-averaged wave direction estimate is obtained with due account to all considered vessels

$$\tilde{\vartheta}_i = \sum_k \Lambda_{i,k} \cdot \vartheta_k \tag{10}$$

using the tilde to refer to the weight-averaged result, and pointing out that the index i on the wave direction estimate $\tilde{\vartheta}_i$ indicates that some sort of criterion should/could be introduced to select the motion component upon which the weight factor $\Lambda_{i,k}$ should be derived from; see also remark (1) above. In the case study below, both heave and pitch will, for illustrative purposes, be used to establish ship-specific weight functions $\varrho_{i,k}(\omega_0)$ and factors $\Lambda_{i,k}$ from.

4.3. (Un)certainty measure of wave spectrum estimate

Qualitatively, the frequency-wise variation, or deviation, between the individual ship-specific wave spectrum estimates $S_k(\omega_0)$ reflects a level of (un)certainty. Thus, as a *hypothesis*, it is assumed that (almost) equal spectral density estimates at any frequency indicate a high level of certainty at the given frequency, while larger deviations between the spectral density estimates reflect larger uncertainty at the given frequency. Herein, it is understood that the calculation of deviations between the ship-specific estimates must be made before any sort of weighting is introduced. A quantification of the preceding is obtained by the frequency-dependent measure $\Delta \equiv \Delta(\omega_0)$ defined by,

$$\Delta^2 = \sum_{k=1}^{K-1} \sum_{l=k+1}^{K} \left[|S_k(\omega_0) - S_l(\omega_0)|^2 \right] \tag{11}$$

repeating that index k refers to a particular ship, and noting that $\Delta(\omega_0)$ should be calculated for a discrete set of frequencies. It is noted that for $K = 3$, Eq. (11) reads $|S_1(\omega_0) - S_2(\omega_0)|^2 + |S_1(\omega_0) - S_3(\omega_0)|^2 + |S_2(\omega_0) - S_3(\omega_0)|^2$. Moreover, the measure resembles basically that of a frequency-wise 'standard deviation'.

Now, the matter of subject is to calculate the "accumulated", or *total*, (un)certainty ψ by considering the whole range of (discrete) frequencies. This is achieved by making the frequency-integrated version of Eq. (11), and relate it to the energy level of the (estimated) sea state. The energy level of a sea state is generally represented by the 0-th order spectral moment, obtained as the area under the wave spectrum that characterises the sea state. In this case, the sea state is represented by the weight-averaged (overall) wave spectrum estimate $S^0(\omega_0)$, see Subsection 4.2 and Eq. (7). Hence, the (un)certainty measure is calculated as

$$\psi = \frac{\frac{1}{K} \displaystyle\int_0^\infty \Delta(\omega_0) d\omega_0}{\displaystyle\int_0^\infty S^0(\omega_0) d\omega_0} \tag{12}$$

taking into account that totally K ships are considered. It should be clear that, in the one direction ψ is bounded by 0. Thus, $\psi = 0$ is representing perfect agreement between all of the K ship-specific wave spectrum estimates considered; that is, the spectra are truly identical. On the other hand, ψ is unbounded in the opposite direction, and the larger the number of ψ, the larger deviation between the single estimates which in turn represents increasing degree of uncertainty. In this sense, the (un)certainty measure ψ is not an absolute measure but should instead be considered relatively, understood in the way that the measure indicates increasing or decreasing levels of (un)certainty rather than absolute levels. And, with little reason to mention, the measure does not indicate if a wave spectrum estimate is good (or bad), equivalently how close the estimate is to the truth[†]. Per definition, ψ is simply a measure that expresses how the single ship-specific estimates deviate from each other. Thus, $\psi = 0$ does not necessarily indicate an accurate (overall) estimate, as it could be the result of an extreme case where all ship-specific estimates were identical but (completely) off the truth. However, the measure of ψ should - with reasonable assumptions and as-good-as-possible data - be a measure that can be used to infer about the (relative) *likeliness* that a particular wave spectrum estimate is good in given operational conditions.

It is noteworthy that, while the proposed (un)certainty measure realises only because the exact same sea state is estimated by *multiple* ships, it would be useful and relevant to propose a measure if/when wave spectrum estimation is made from just a single ship. Efforts in this direction do exist [27], but in the present study, uncertainty is considered/computed entirely by the calculation of ψ in Eq. (12).

5. Case study: Multiple ships as sailing wave buoys

It may require further technical developments, notably related to communication and sharing of data, before *sea state estimation using multiple ships as sailing wave buoys* becomes practically feasible. Nonetheless, the potentials and steps towards such a network-based approach have been outlined in the preceding sections. Hence, it will be the task in the following to numerically evaluate the proposed ideas, and the present section includes a case study based on numerically *simulated* data. The case study builds on an operational scenario where three ships are used simultaneously as sailing wave buoys by use of the wave buoy analogy, see Section 3. The actual ships are selected somewhat arbitrarily albeit they are chosen in such a way that they reflect different

[†]Note, the exact truth itself is never known although estimates from other means (buoys, satellites, ...) may be available.

filtering characteristics to better illustrate the discussions in Section 4. The ships will be presented in the following, but their pertinent data has already been shown in Figure 4 presenting the modula of the sets of transfer functions.

The use of the spectral-residual approach, Section 3, as the mathematical model for the wave buoy analogy, requires a *stability analysis* [28] to find suitable values for the iteration gains $h_{ij,k}$, cf. Eq. (3b). The values of the gains are important to ensure a reasonable compromise between computational speed and accurate (or mathematically stable) results. However, this specific exercise and the related analyses are beyond the scope of the present study, and, thus, it is left to say that the gains $h_{ij,k}$ take "reasonable" values, but they have not been tuned accordingly [28]. It is noteworthy, too, that there are other mathematical models of the wave buoy analogy available; an account has been given by [17]. The prime argument to use the spectral-residual approach, cf. Section 3, in favour of the other available models, is the fact that it has high(er) computational efficiency, and, at the same time, yields reasonable results [4, 5, 28–30].

5.1. Simulated motion measurements of three ships

Wave-induced motion measurements have been simulated for three vessels, all operating in a given confined ocean area. The physical distances between the ships are at all times assumed to be no more than what allows for statistically identical sea states, including wave direction, at the vessels' positions. The three vessels are taken as an advancing medium-sized Ro-Ro vessel ('Vessel 1'), an advancing smaller research vessel ('Vessel 2'), and a station-kept FPSO ('Vessel 3'), respectively. The specific vessels are characterised by main dimensions as listed in Table 1. The wave-to-motion transfer functions of the Ro-Ro ship and the research vessel (R/V) have been calculated by linear strip theory, while a linear potential theory-based panel code has been used for the FPSO. The motion transfer functions have been computed for all six degrees of freedom, considering the whole range of wave-encounter angles (0 - 360 deg.), spaced at 10 deg., and the sets of cut-off frequencies are reflected by the plots of the modula of the transfer functions, see Figure 4. Note, onwards the three vessels are referred to by their type-specific names, i.e. Ro-Ro, R/V, and FPSO.

The motion responses are simulated using standard procedure considering short-crested waves and assuming the wave elevation as a Gaussian process, see e.g. [4].

Table 1: Main dimensions of the three vessels, a Ro-Ro ship, a research vessel (R/V), and an FPSO, used as sailing wave buoys.

Vessel	Length [m]	Breadth [m]	Draught [m]	Block coef. [-]
Ro-Ro	232.0	33.0	6.1	0.61
R/V	28.9	9.6	2.7	0.56
FPSO	200.0	44.0	12.0	0.79

Realisations are made for the motion components of heave, roll, and pitch, respectively, for all three vessels.

The operating scenario is illustrated in Figure 7, where it is important to note that the indicated irregular wave system approaches from the same mean direction and is represented by a stationary, and identical, sea state in the whole operational area; that is, the sea state does not change temporally, during the given measurement period, say, in

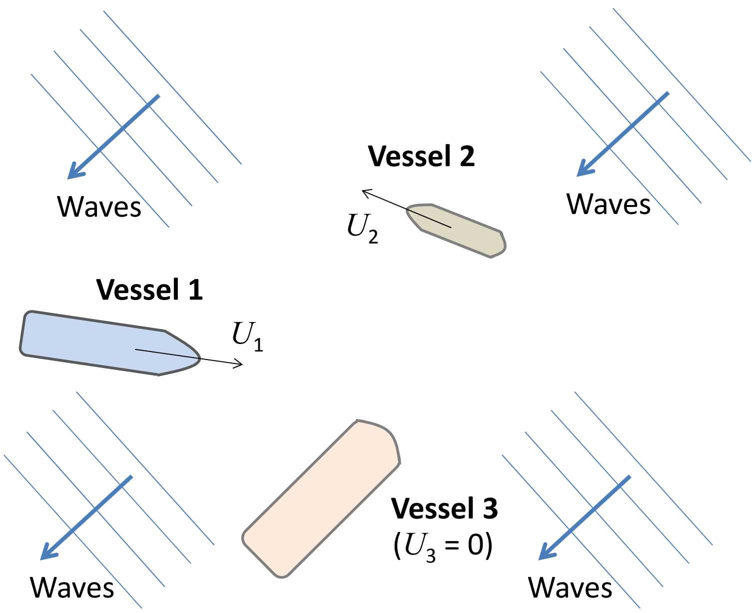


Figure 7: Illustration of the operating scenario where three vessels are used simultaneously for sea state estimation. The sea state and the mean wave direction are the same in the whole operational domain and, likewise, it is assumed that the sea state and wave direction do not change in the considered period of time, in the order of half an hour to an hour.

the order of 30 minutes, and remains the same at all the spatial positions of the vessels in any considered operational scenario. The combination of sea state and operational parameters (vessel speeds and course/heading) of *multiple* ships leads to a large number of possible operational scenarios. This study has its focus on just a few ones, where it has been of little priority to look at any special operational scenario(s); for whatever reason that could be. The case study, the first of its kind, serve exclusively to introduce and illustrate the use of multiple ships simultaneously as sailing wave buoys. The results to be presented apply to totally eight sub-scenarios, distinguished by different sea states and/or different operational parameters. Thus, four main scenarios are defined according to sea state variations, A, B, C, and D, while for each wave scenario two combinations of the operational parameters are considered, 1 and 2. The sub-scenarios are presented in Table 2. It is seen that sea states A and B represent wave conditions described by a unimodal wave spectrum, here taken as a Bretschneider spectrum overlaid with a spreading function, e.g. [24], to introduce short-crested waves. Sea states C and D characterise wave conditions observed during the existence of wind waves and swell at the same time, and approaching from the same direction. Specifically, the respective sea states C and D are defined by the summation of two Bretschneider spectra, again overlaid with a spreading function to resemble directional, short-crested waves. The theoretically known 'generating wave conditions' are used to simulate sets of motion recordings of the ships for given (true) wave headings β_1, β_2, and β_3, and vessel speeds, U_1, U_2, and U_3, respectively. The wave heading, equivalently the encounter angle, is defined as the angle between the mean direction ϑ of the propagating waves and the individual ship's centerline, and $\beta = 180$ deg. is head sea. The operating conditions and the characteristic sea state parameters of the sub-scenarios are listed in Table 2, where specifications are given for all ships and their respective location; and mentioning again that the wave system at any ship's location is simulated by the same *generating* sea state.

Based on the specified sea state and a set of operational parameters, cf. Table 2, the 'initial outcome' of a given sub-scenario consists of three sets of simulated (stochastic) motion time history recordings of {heave, roll, pitch}; one set for each vessel. Specifically, the time histories are generated for a given 30-minutes period, and, consequently, the simulated "measurements" represent one single 30-minutes realisation out of infinitely many

Table 2: Sub-scenarios: Operational conditions and generating sea state at the vessels' point of operation. The (true) sea state is assumed to be the same over the whole ocean area in which the three vessels operate.

	Speed [kts]			Heading [deg.]			Swell		Wind sea	
	U_1	U_2	U_3	β_1	β_2	β_3	H_s [m]	T_p [s]	H_s [m]	T_p [s]
A1	11	10	0	240	030	180	N/A	N/A	3.0	8.0
A2	16	10	0	240	070	180	N/A	N/A	3.0	8.0
B1	11	10	0	240	030	180	N/A	N/A	3.0	12.0
B2	16	10	0	240	070	180	N/A	N/A	3.0	12.0
C1	11	10	0	240	030	180	3.0	15.0	3.0	8.0
C2	16	10	0	240	070	180	3.0	15.0	3.0	8.0
D1	11	10	0	240	030	180	3.0	17.0	3.0	10.0
D2	16	10	0	240	070	180	3.0	17.0	3.0	10.0

comprising the whole ensemble. Strictly speaking, it is therefore necessary to consider *several* of these time history realisations in order to properly account for the stochastic nature of ocean waves and, hence, allowing a comprehensive and valid statistical analysis of the 'post-processed outcome', i.e. the estimation of sea states. In the present context, it suffices to note that the following analyses will be based on (statistical) evaluations made from sets of 20 time history realisations of {heave, roll, pitch} for the Ro-Ro ship, the research vessel (R/V), and the FPSO, respectively.

5.2. Results and discussions

The following bullets summarise a few noteworthy points about the results to be presented:

- Wave spectrum estimation is made using simultaneous data from three ships, i.e. $K = 3$. For any given ship, a certain ship-specific estimate is obtained by the brute-force spectral procedure reviewed in Section 3.

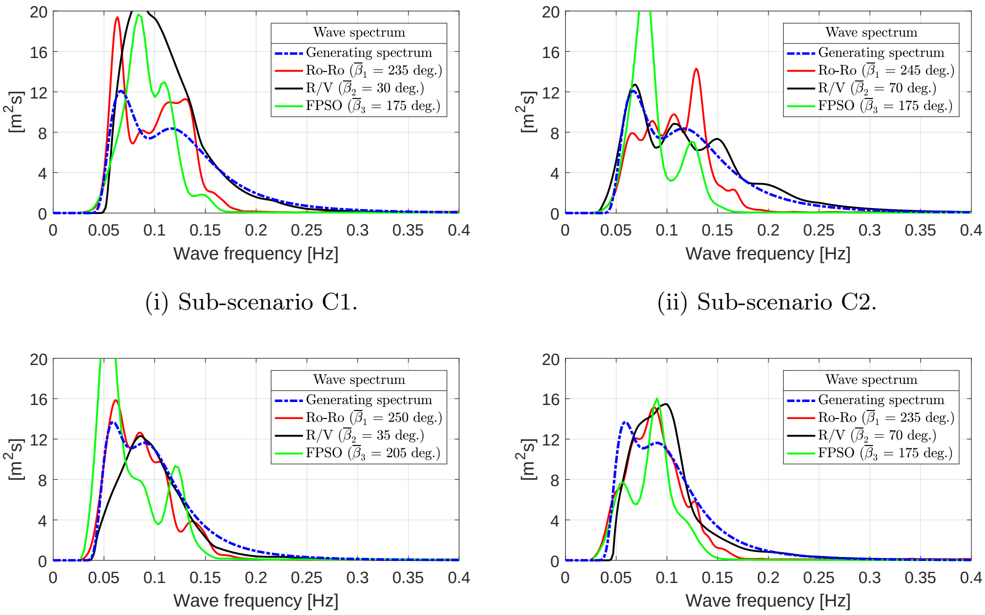
- Sets of results are presented for three different weight functions, cf. Eqs. (6) and (8); one set based on an equal weighting of the individual ships, that is, $\varrho_{i,k}(\omega_0) \equiv \Lambda_{i,k} = \frac{1}{3}$, and two other sets based on the heave and the pitch motions, respectively; cf. Figure 6.

- The *true* ship-specific encounter wave-angle, or wave heading, cf. Table 2, has been subtracted from the single (ship-specific) *estimate*. The resulting value is "defined" as the wave direction ϑ_k. Consequently, ϑ_k will be zero for a perfect wave heading estimate; emphasising that this is so for all the considered ships.
- Each operational sub-scenario, cf. Figure 7 and Table 2, is evaluated statistically by the analysis/processing of totally twenty sets of realisations of simulated motion measurements.

5.2.1. Plots of wave spectra

Examples of ship-specific wave spectrum estimates are seen in Figure 8, where arbitrarily selected outcomes of the four sub-scenarios C1-D2 are presented in sub-plots (i)-(iv). In the figure, any sub-plot includes the actual spectrum *estimates* and the corresponding true *generating* wave spectrum; emphasizing, however, that the latter spectrum is not necessarily identical to the true *realised* wave spectrum as would/could be derived from the encountered wave elevation record (and subsequently transformed to absolute domain). In principle, the comparison should thus be relative to the (true) realised wave spectrum, but since the comparison subsequently follows from a statistical evaluation by considering several (i.e. twenty) realisations, it is for the present illustrative purpose acceptable to compare the individual ship-specific estimates with the *true generating* wave spectrum.

A detailed inspection of the spectrum estimates in Figure 8 reveals that that the two larger vessels, as expected, cannot be used to infer about the higher-frequency components of the wave system, explained because of the vessels' filtering effect. Furthermore, it is seen that the single estimates of all three ships do not match exactly with respect to the number of spectral peaks and the locations of the peaks. However, the main observation from the sub-plots in Figure 8 is that the individual spectrum estimates, by and large, agree on a 'fair level' with the generating spectrum, and the same applies for the estimates of the (mean) wave heading. Notably, it is observed that, collectively, the *set* of spectrum estimates "captures" the exact, true spectral density on the the whole range of frequencies; a finding applicable to any of the sub-scenarios (C1, C2, D1, and D2), and also to all other remaining sub-scenarios although not shown. In this sense, the indication is that it should indeed be useful to *simultaneously* consider the multiple spectrum estimates (sub-scenario by sub-scenario) by introducing a weight-averaged estimate like discussed in Section 4. Thus, the ship-specific wave spectrum estimates can be weighted according to the approach outlined previously, and, if so, the outcomes appear from Figure 9. The shown examples correspond to the same operational sub-scenarios considered in Figure 8. The sub-plots' legends refer to different weighting functions, where "equal" means that the three individual ship-specific wave spectrum estimates (from Figure 8) are weighted equally with $\varrho_{i,k} \equiv \frac{1}{3}$ in Eq. (7). Legends "heave" and "pitch" refer to weighting functions derived from the normalised modula of heave and pitch, respectively; cf. Figure 6. Furthermore, the legends include the weight-averaged wave direction estimate $\tilde{\vartheta}$ applicable to the particular (weight-averaged) spectrum. It is



(i) Sub-scenario C1.

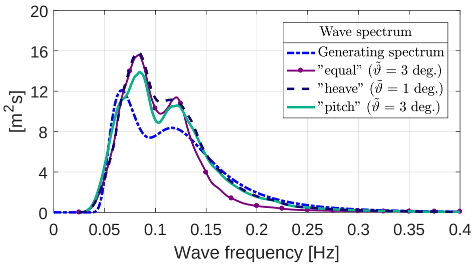
(ii) Sub-scenario C2.

(iii) Sub-scenario D1.

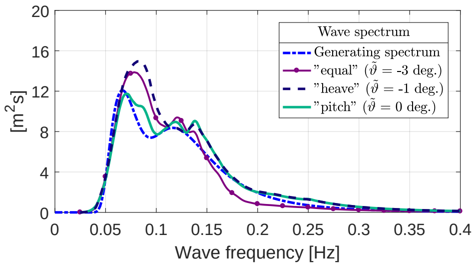
(iv) Sub-scenario D2.

Figure 8: Examples of ship-specific wave spectrum estimates obtained by processing the sets of motion time history recordings from the single vessels. The input-wave spectrum is shown together with the estimates, and the estimated wave heading is included in the subplots' legend. Note, the associated wave *heading* estimates are included in the plots' legend.

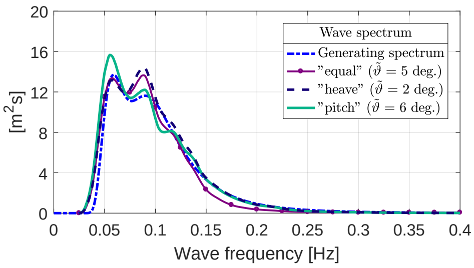
evident from Figure 9 that the weighted spectrum estimates ("equal", "heave", "pitch") indeed compare well with the true generating (Bretschneider-type-of) spectrum for all the sub-scenarios; and the agreement is improved considerably compared to the individual ship-specific estimates (cf. Figure 8). It is observed that the two (true) peaks are estimated at almost consistent frequencies and also with the spectral values of the peaks in reasonable agreement. Another specific observation from the four sub-plots in Figure 9 is noteworthy; the capability of the weight-averaged estimates to capture the higher-frequency spectral wave components. This is especially true for the heave- and pitch-based results, where the estimate by the (smaller) research vessel is given the most weight (cf. Figure 6). Generally, the mentioned observations apply to all remaining realisations of the four considered operational sub-scenarios and also to all the other sub-scenarios (cf. Table 2) and their respective realisations. It is, however, not practically



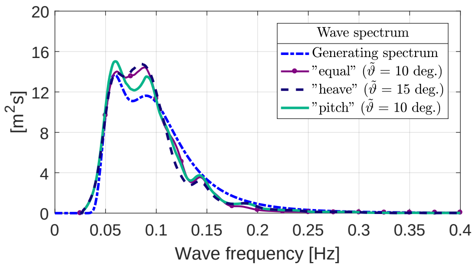
(i) Sub-scenario C1.



(ii) Sub-scenario C2.



(iii) Sub-scenario D1.



(iv) Sub-scenario D2.

Figure 9: Examples of (corresponding) weight-based mean wave spectrum estimates, cf. Figure 8. Note, the associated wave *direction* estimates are included in the plots' legend; the true value is $\vartheta_0 = 0$ deg.

feasible to make detailed visual inspections of all the realisations and their outcomes in terms of wave spectrum estimates, like presented in Figure 9. It is therefore decided to calculate a representative difference-metric which can be used to evaluate the comparison between the single weight-averaged estimates and the true generating wave spectrum from *and*, at the same time, be a measure of the relative agreement between the weighted spectrum estimates. The difference-metric ε_i is based on the root-squared error, equivalently the absolute value of the error, between the spectral components of the particular weight-averaged estimate $S_i^0(\omega_0)$, cf. Eq. (7), and the true generating spectrum $S_B(\omega_0)$,

$$\varepsilon_i = \frac{\displaystyle\int \left| S_i^0(\omega_0) - S_B(\omega_0) \right| d\omega_0}{\displaystyle\int S_B(\omega_0) d\omega_0} \tag{13}$$

where index i, as usual, refers to a weighting-based result derived from the motions heave or pitch, or from an equal weighting of the ship-specific estimates. It is noted that the integral in the numerator is calculated *after* the subtraction. The parametric

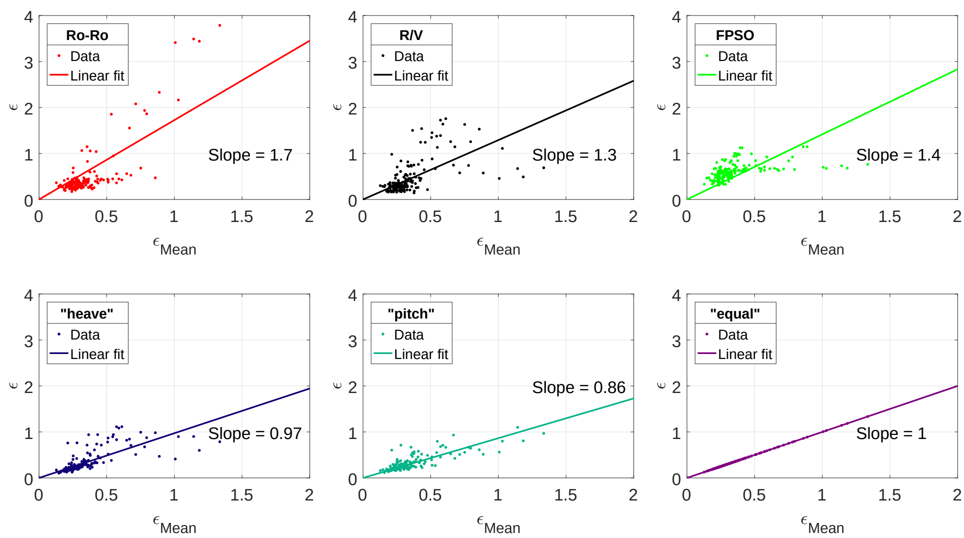


Figure 10: Evaluation of the detailed - but relative - agreement in terms of the error measure ε between spectrum estimates and the true generating spectrum for all realisations of all sub-scenarios. The base-reference is $\varepsilon_{\text{"}equal\text{"}} \equiv \varepsilon_{Mean}$. The slope of the fitted linear trendline has been included in the respective sub-plots.

formula for the generating spectrum, here taken as the summation of two Bretschneider spectra or one single spectrum, follows from any standard textbook in ocean engineering. Qualitatively, ε_i is the (normalised) "area-deficit", found as that encased by the estimated spectrum and the generating spectrum, and, hence, the metric provides a measure that enables a detailed, but relative, comparison between the different weight-based estimates. It should be noted that the metric has also been evaluated for each of the *ship-specific estimates*, which means that totally six values of the error measure is calculated for each of the realisations for any sub-scenario. If one of the measures, take $\varepsilon_{\text{"}equal\text{"}}$, is selected as base-reference, it can be readily visualised which of the six spectrum-estimates that yields the best (relative) agreement with the true generating wave spectrum by plotting ε as function of $\varepsilon_{\text{"}equal\text{"}}$. The outcome is shown in Figure 10, and it is seen that linear fitting lines are associated to the respective results; pointing out that the slope of the linear fit is a direct measure of the "average relative" agreement with the truth. Thus, it is found that among all six types of spectrum estimates (ship-specifics as well as weighted), the best agreement is achieved from the pitch-based estimate, since it has the smallest slope (= 0.86).

5.2.2. Integrated wave parameters

The difference-metric ε is useful as a comparative measure of the relative agreement among the various types of estimates to the true generating spectrum of the sub-scenarios. To supplement the comparative study between the different types of estimates, the calculation of integrated wave parameters, or sea state parameters, such as significant wave

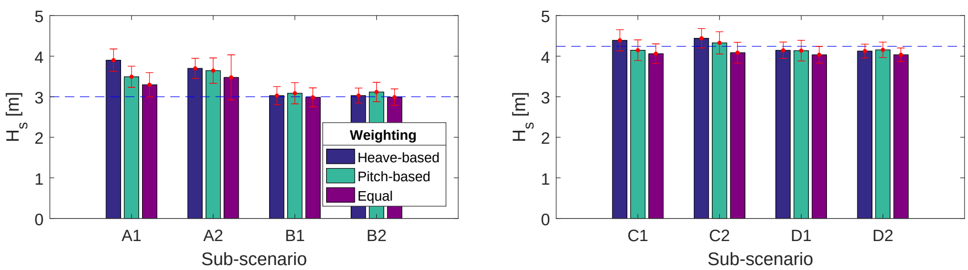


Figure 11: Statistics of estimates of the **significant wave height**. The true value is indicated by the dashed blue line.

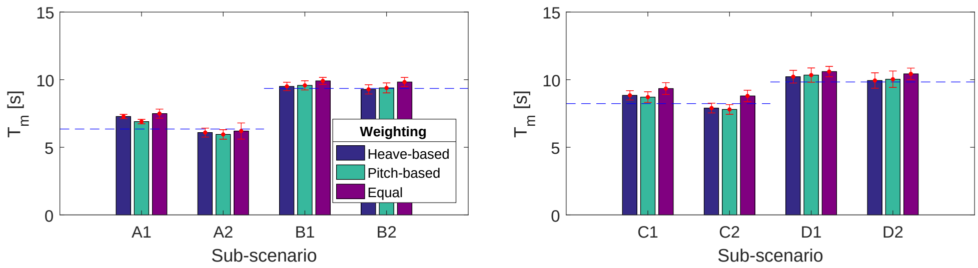


Figure 12: Statistics of estimates of the **mean wave period**. The true value is indicated by the dashed blue line.

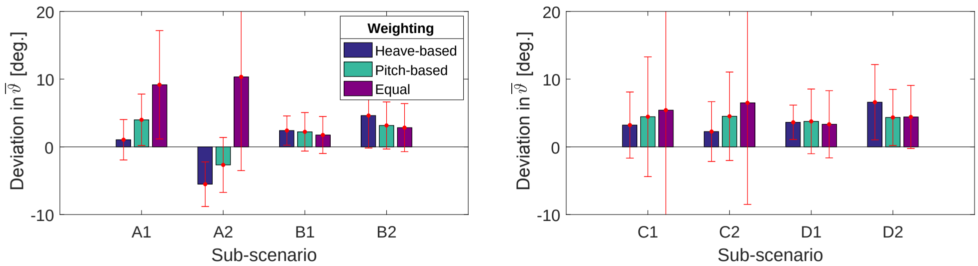


Figure 13: Statistics of estimates of the **mean wave direction**. The true value is, per definition, zero.

height H_s, mean wave period T_m, and mean wave direction ϑ can be used to provide a (more) physical inference of the comparisons. Consequently, Figures 11-13 show the statistics derived for estimates of the sea state parameters; pointing out that results are focused exclusively on the weight-averaged estimates. It is understood that the statistics have been computed from the twenty realisations (i.e. spectrum estimates) of a given sub-scenario, and, in the plots, the height of the bars indicates the mean value, while the 'error markers' indicate plus/minus the standard deviation. The statistics of H_s and T_m are based on values obtained from the spectral moments m_n of the actual wave spectrum,

$$H_{s,i} = 4\sqrt{m_{0,i}} \qquad \text{Significant wave height} \qquad (14)$$

$$T_{m,i} = 2\pi \frac{m_{0,i}}{m_{1,i}} \qquad \text{Mean wave period} \qquad (15)$$

$$m_{n,i} = \int_0^\infty \omega_0^n S_i^0(\omega_0) d\omega_e \qquad n\text{-th order spectral moment} \qquad (16)$$

The statistics of the wave direction estimate are directly obtained from the values of the estimated wave direction, which in turn is derived from the estimated wave *heading* subtracted the true heading, cf. subsection 4.2.1.

Generally, it can be seen from Figures 11-13 that the (weighted) estimates of wave parameters agree well with the true wave parameters, and, with little surprise (cf. Figure 10), it is found that the heave- and pitch-based weighted results have the best agreement (almost) consistently for the set of wave parameters. Having a special focus on the individual wave parameters the most noteworthy observations are the following: *Significant wave height*; the poorest estimates are obtained for sub-scenarios A1 and A2. This is explained because the wave systems of these sub-scenarios have most of the wave energy distributed at the higher frequencies and, hence, makes the sub-scenarios "susceptible" to the negative effects of the ships' filtering characteristics. *Mean wave period*; good estimates are found throughout. *Wave direction*; the mean values, i.e. the heights of the bars, are reasonable but large(r) standard deviations are observed, notably for some of the equally-weighted results. The reason for this is because of "outliers" obtained by the Ro-Ro and the FPSO vessels, especially in conditions where the wave system has energy distributed at the higher frequencies, which is the case for sub-scenarios A1 and A2, and C1 and C2, respectively.

5.3. Summary and additional discussions

The case study has shown that it is possible to obtain more accurate sea state estimates by considering multiple ships simultaneously in a network of 'sailing sea state estimators'. There are unsolved questions and problems to be addressed (especially with concerns about online data handling, including the sharing and communication among the *multiple* vessels), but the presented results serve, indeed, as a first step towards a proof of concept.

The specific case study has been considering an arbitrarily selected group of vessels, but it should be interesting to consider other types of vessels to look at the results' sensitivity in this respect. Similarly, various weighting functions can be established; either with the purpose to be multiplied directly on the actual ship-specific wave spectrum estimates, like suggested in this study, or the weighting might also be introduced with respect to a "post-processed calculation" of integrated wave parameters. In any case, it will be relevant to associate some sort of uncertainty to the estimate(s). In the present work, one practical measure has been suggested, see Subsection 4.3, and its outcomes for the considered sub-scenarios are shown in Figure 14. Herein, the (un)certainty measure is based on the frequency-wise variation among the individual ship-specific wave spectrum estimates, cf. Eq. (12). The immediate inspection of Figure 14 reveals that large variations occur between the ship-specific estimates of sub-scenarios A1 and A2, whereas sub-scenarios B1, B2, D1, and D2 have small(er) uncertainty associated. In this sense, it is evident from the figure that the most "problematic" sub-scenarios, in regards to reliable estimates, are A1, A2, C1, and C2; which were the same scenarios that generally reflected the least good agreement with the true generating wave spectrum. It needs to be kept in mind, however, that the (un)certainty measure (Eq. 12) is a *relative number* and, as such, cannot be used to directly infer whether any particular sea state estimate is good or bad. On the other hand, it should be interesting to apply/associate the measure in larger-scale studies where different subsets of a group of *several* vessels are used in network-based wave estimation. In such case, it should be viable/feasible to select the *final* wave spectrum estimate from the *subset* of vessels associated with the smallest value of the uncertainty measure.

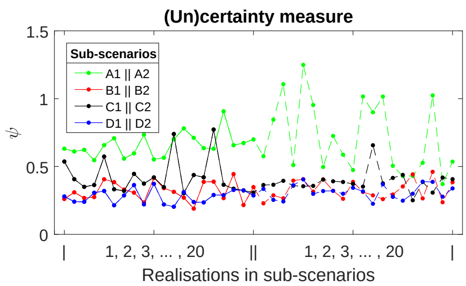


Figure 14: The measure based on Eq. (12) can be used as an indicator of the (un)certainty related to the wave spectrum estimate produced from multiple ship-specific estimates. A large number reflects large variation, i.e. uncertainty, among the particular ship-specific estimates in a given case.

6. Concluding remarks

The article has investigated the possibility to make sea state estimation using data from multiple ocean vessels simultaneously; where the vessels, for instance, could be sailing ships and station-kept offshore-operating vessels.

The use of the single vessel as a (sailing) wave buoy is not new, but the simultaneous use of a group of vessels is. Herein, it becomes a natural and critical aspect to introduce weighting of the single (ship-specific) wave spectrum estimates, and the article has discussed one practical approach for this. Thus, it is proposed to establish weight functions based on the sets of motion transfer functions of the particular vessels. The proposal does *not* include weighting based on the relative distance between the vessels, although this might be relevant (and fairly simple to do).

The possession of simultaneous wave spectrum estimates from multiple ships facilitates an associated (un)certainty measure. The article has discussed one possible measure based on the frequency-wise variation of the individual ship-specific wave spectrum estimates.

The article contains a case study focused on numerical simulations of motion measurements of three vessels operating in a given confined ocean area. In the case study it is shown that the (final) wave spectrum estimate indeed can be improved by considering data from multiple ships simultaneously. In particular, it is interesting to note the capability to estimate (also) the higher-frequency wave components, if vessels of different dimensions are considered; thus observing a "reduced" effect of the fact that ships generally act as wave filters depending on their size relative to wave length.

The realisation of the proposed network-based approach for sea state estimation requires further technical developments; especially related to data communication and sharing, since the amount of data to be handled and exchanged from and in between the single vessels will be extensive. These topics have not been addressed, but they are certainly necessary to consider in future works.

On a higher level, the proposed use of multiple ships for sea state estimation could be the first step towards *sea state estimation using a heterogenous network of (wave) observation platforms*, where observation platforms such as classical wave buoys, wave radar, satellites, and ship-as-a-wave-buoy are relevant means to consider. The reason to start by looking at multiple ships as 'sailing wave buoys' is because this means is believed to be key for a successful development of a networked-based approach for sea state estimation; simply due to the fact that quantity is a quality of its own, cf. Figure 2.

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References

[1] U. D. Nielsen, Estimations of on-site directional wave spectra from measured ship responses, Marine Structures 19 (2006) 33–69.

[2] U. D. Nielsen, Introducing two hyperparameters in Bayesian estimation of wave spectra, Probabilistic Engineering Mechanics 23 (2008) 84–94.

[3] U. D. Nielsen, D. C. Stredulinksy, Sea state estimation from an advancing ship - A comparative study using sea trial data, Applied Ocean Research 34 (2012) 33–44.

[4] U. D. Nielsen, A. H. Brodtkorb, A. J. Sørensen, A brute-force spectral approach for wave estimation using measured vessel motions, Marine Structures 60 (2018) 101–121.

[5] A. H. Brodtkorb, U. D. Nielsen, A. J. Sørensen, Sea State Estimation Using Vessel Response in Dynamic Positioning, Applied Ocean Research 70 (2018) 76–86.

[6] T. Iseki, K. Ohtsu, Bayesian estimation of directional wave spectra based on ship motions, Control Engineering Practice 8 (2000) 215–219.

[7] T. Iseki, D. Terada, Bayesian Estimation of Directional Wave Spectra for Ship Guidance Systems, International Journal of Offshore and Polar Engineering 12 (2002) 25–30.

[8] E. A. Tannuri, J. V. Sparano, A. N. Simos, J. J. D. Cruz, Estimating directional wave spectrum based on stationary ship motion measurements, Applied Ocean Research 25 (2003) 243–261.

[9] R. Pascoal, C. G. Soares, A. J. Sørensen, Ocean Wave Spectral Estimation Using Vessel Wave Frequency Motions, Journal of Offshore Mechanics and Arctic Engineering 129 (2007) 90–96.

[10] A. N. Simos, J. V. Sparano, E. A. Tannuri, V. L. F. Matos, Directional Wave Spectrum Estimation Based on a Vessel 1st Order Motions: Field Results, in: Proc. of 17th International Offshore and Polar Engineering Conference, Lisbon, Portugal, 2007.

[11] J. V. Sparano, E. A. Tannuri, A. N. Simos, V. L. F. Matos, On the Estimation of Directional Wave Spectrum Based on Stationary vessels 1st Order Motions: A New Set of Experimental Results, in: Proc. of OMAE'08, Lisbon, Portugal, 2008.

[12] R. Pascoal, C. G. Soares, Kalman filtering of vessel motions for ocean wave directional spectrum estimation, Ocean Engineering 36 (2009) 477–488.

[13] R. Pascoal, L. P. Perera, C. G. Soares, Estimation of Directional Sea Spectra from Ship Motions in Sea Trials, Ocean Engineering 132 (2017) 126–137.

[14] F. L. de Souza, E. Tannuri, P. cardozo de Mello, G. Franzini, J. Mas-Soler, A. Simos, Bayesian estimation of directional wave-spectrum using vessel movements and wave-probes: Proposal and preliminary experimental validation, in: Proceedings of 36th OMAE, Trondheim, Norway, 2017.

[15] I. A. Milne, M. Zed, Full-scale validation of the hydrodynamic motions of a ship derived from a numerical hindcast, Ocean Engineering (accepted for publication).

[16] J. Nielsen, N. Pedersen, J. Michelsen, U. Nielsen, J. Baatrup, J. Jensen, E. Petersen, SeaSense - Real-time Onboard Decision Support, in: Proc. of WMTC2006, London, UK, 2006.

[17] U. D. Nielsen, A concise account of techniques available for shipboard sea state estimation, Ocean Engineering 129 (2017) 352–362.

[18] G. Lindgren, I. Rychlik, M. Prevosto, Stochastic Doppler shift and encountered wave period distributions in Gaussian waves, Ocean Engineering 26 (1999) 507–518.

[19] U. D. Nielsen, Transformation of a wave energy spectrum from encounter to absolute domain when observing from an advancing ship, Applied Ocean Research 69 (2017) 160–172.

[20] U. D. Nielsen, Deriving the absolute wave spectrum from an encountered distribution of wave energy spectral densities, Ocean Engineering 165 (2018) 194–208.

[21] U. D. Nielsen, Response-based estimation of sea state parameters - influence of filtering, Ocean Engineering 34 (2007) 1797–1810.

[22] U. D. Nielsen, The wave buoy analogy - estimating high-frequency wave excitations, Applied Ocean Research 30 (2008) 100–106.

[23] E. Tu, G. Zhang, L. Rachmawati, E. Rajabally, G.-B. Huang, Exploiting AIS Data for Intelligent Maritime Navigation: A Comprehensive Survey, IEEE Transactions on Intelligent Transportation Systems 19 (2018) 1559–1582.

[24] R. Bhattacharyya, Dynamics of Marine Vehicles, John Wiley & Sons, 1978.

[25] R. Beck, W. Cummins, J. Dalzell, P. Mandel, W. Webster, Vol. III: Motions in Waves and Controllability, in: E. Lewis (Ed.), Principles of Naval Architecture, Second Revision, SNAME, 1989, pp. 1–188.

[26] J. Journée, W. Massie, Offshore Hydromechanics, lecture notes in course offered at TU Delft (January 2001).

[27] N. Montazeri, J. J. Jensen, U. D. Nielsen, Uncertainties in ship-based estimation of waves and responses, in: Proc. of MTS/IEEE OCEANS15, Washington, DC, USA, 2015.

[28] A. H. Brodtkorb, U. D. Nielsen, A. J. Sørensen, Online wave estimation using vessel motion measurements, in: Proc. of 11th IFAC Conf. CAMS, Opatija, Croatia, 2018.

[29] U. D. Nielsen, A. H. Brodtkorb, Ship motion-based wave estimation using a spectral residual-calculation, in: Proc. of MTS/IEEE OCEANS18, Kobe, Japan, 2018.

[30] U. D. Nielsen, A refining technique for ship motion-based sea state estimation, in: Proc. of 6th

World Maritime Technology Conference, Shanghai, China, 2018.