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Performance analysis of redundant safety-instrumented systems subject to degradation and external demands

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ABSTRACT

Safety-instrumented systems (SISs) play a vital role in preventing hazardous events in the offshore facilities. Many of existing performance analysis of SISs are based on the constant failure rate assumption, which is however doubtful when it is applied to actuator sub-systems or mechanical final elements of a SIS. These mechanical SIS components can become vulnerable with time and with upcoming demands given the past exposures to shocks/demands. In this paper, we analyze SIS reliability and unavailability by considering that a failure occurs when total degradation of a SIS component, including continuous degradation and increments caused by random demands, exceeds to a predefined critical threshold. The dependency of two components in a redundant structure of mechanical actuators caused by random demands is also taken into account in the analysis. Approximation formulas for reliability and unavailability of the redundant SIS sub-system under a degradation process are developed. Finally, a numerical example is conducted to illustrate effects of degradation parameters on SIS performance.

1. Introduction

Safety instrumented systems (SISs), which generally consist of sensor-, logic solver- and actuator-subsystems, are widely used to prevent the occurrences of hazardous events or mitigate their consequences (Rausand, 2014). These systems are designed to perform some specific safety-instrumented functions (SIFs) to protect the equipment under control (EUC) in different industries (Rausand and Arnljot, 2004).

In terms of reliability assessment of SISs, a considerable amount of literature is available. Almost all reliability assessments of SISs are based on an assumption that the failure rates of the components within the systems are constant, such as (Guo and Yang, 2008; Liu and Rausand, 2011; Catelani et al., 2011; Jin and Rausand, 2014), even in (IEC 61511, 2010) and (IEC 61511, 2003). It means that all components or SIS channels are as-good-as-new when they are functioning, and their failures follow the exponential distribution. However, in practices many mechanical actuators of SISs become more vulnerable along with time (Zio, 2016), because they chronically expose to some failure mechanisms, such as corrosion, wear, fatigue (Rafiee et al., 2014, 2017). The actual lifetimes of actuators are determined not only by their reliability, but by the operating conditions (Nakagawa, 2007), and the assumption of constant failure rate is thus questionable. For such cases, researchers have identified that the failure rates of these items are non-constant, and they have chosen the Weibull distribution in reflecting the failure process (Rogova et al., 2017; Wu et al., 2018).

Redundant structures are often used in SISs to improve the system availability and so to enhance safety, e.g., two shutdown valves are installed in parallel to stop flow when the downstream pressure is too high. When one of them cannot be activated, the process, namely EUC, is still safe if the other valve works. Such kind of configuration is called as 1-out-of-2 (1oo2), where channels/units are also assumed identical with a same constant failure rate in most of the existing studies (Jin and Rausand, 2014; Chebila and Innal, 2015; Mechri et al., 2015; Innal et al., 2016). Actually, mechanical components in a 1oo2 configuration expose to the same environment and stand demands simultaneously, so that it is reasonable to suppose that their times-to-failure can be relevant and dependent.

In case the degradation in mechanical components is unavoidable, the performance information about system and evolving environment (Zhou et al., 2008) is helpful for the reliability assessment. Deterioration of the mechanical actuators in a SIS are not only due to chronic mechanisms, e.g. wear and material fatigue (Lai and Chen, 2016), but also from the external shocks, namely demands for SIS actuation (Nawaz, 2008). For example, in a high integrity pressure protection system (HIPPS), the required function of the actuator, valves, is to close

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the flow in the pipeline when the pressure beyond the specialization. Occasional high pressures cause unprecedented stresses on the valve, and so the effects of such demands on degradation of the valves, especially on those with serious damages, may not be neglected.

Two degradation processes should be therefore considered in assessing the performance of mechanical SIS actuators: (1) continuous aging degradation, and (2) additional damages by the randomly occurring demands. It is also natural to assume that when the overall degradation of such components arrives at a predefined level, they can not be activated as expected when a new demand comes.

Degradation challenges the common assumption of as-good-as-new after each proof-test in SIS reliability assessment (see (IEC 61511, 2010) and (IEC 61511, 2003). In general, the reliability of a system decreases as the degradation processes develop (Zio, 2016). Once the degradation reaches a specific level, the component will fail. The so-called specific level for SIS actuators is referring to a certain performance requirement, such as closing time and maximum leakage rate in closed position (Hauge et al., 2016).

There has been considerable amount of published literature that analyzes the reliability of single component experiencing either degradation or random shocks (Kharoufeh and Cox, 2005; Tang et al., 2014; Rafiee et al., 2017; Liu et al., 2017; Zhang et al., 2017; Xu et al., 2018). Models used in these researches can be divided into several categories: statistical models of time to failure (e.g. (Gebraeel et al., 2009),), stochastic models (Ye and Xie, 2015; Ye et al., 2015; Chen et al., 2015) and multi-state models (Li and Pham, 2005a, 2005b; Lin et al., 2015; Song et al., 2018). Stochastic processes are very effective in modeling time dependent degradation with taking dynamic operating conditions included (Singpurwalla, 1995). Klutke and Yang (2002) have derived an availability model for an inspected system subject to shock and graceful degradation (Deloux et al., 2009). have considered both continuous degradations and shocks in the calculation of system reliability, and propose a predictive maintenance policy as a response (Bocchetti et al., 2009). have considered wear degradation and thermal cracking in their competing risk model for in a marine Diesel engine considering (Mercier et al., 2013). have used a Poisson process and a gamma process to model the cracks of passive components within electric power plant. The homogeneous gamma process has been in fact widely used to model gradual degradation phenomena, such as fatigue crack growth (Lawless and Crowder, 2004), thinning due to corrosion (Kallen and van Noortwijk, 2005), corroded steel gates (Frangopol et al., 2004), sealing performance of O-rings (Sun et al., 2018).

However, new research is motivated by the fact that the existing results in degradation analysis, even those for redundant systems, cannot be simply applied to a SIS due to its operational characteristics, e.g.

- For components in the redundant structure of a SIS, they are expose to same environment and same demands. The damage sizes of the two components caused by a random demand can be assumed be similar or same, and the degradation processes of two components are thus correlated.
- The components in a SIS are simultaneously tested and maintained in most cases, and such an operational approach weakens the assumption of independence of the two components.
- Failures and degradations are always hidden until periodical tests. For the valves in a HIPPS as an example, they are mainly in a dormant state in the normal operation, meaning that the performance can not be estimated by visual inspection or diagnostic tests (Rausand, 2014).
- SISs are evaluated with different measures when they are operated in different modes, and the frequency of demands to activate SISs is key to decide what measure can be used. Although more demands obviously can accelerate degradation, it is necessary to value the effects of demands in consideration of measure adaptability

The average probability of failures on demand (PFD_{avg}) is a widely acknowledged measure to quantify the reliability of a low-demand SIF. In the current literature, all units are as-good-as-new as long as they are functioning at the proof-tests, so the PFD_{avg} is totally same in each test interval. It is not at all realistic for SISs with degradations. Given that no failure is revealed in a proof test, it only means that the unit is functioning, but not as-good-as-new. It is natural to suppose that the PFD_{avg} increases in step in different test intervals.

The objective of this paper is to deal with the challenges of a SIS to degradation analysis, and propose a degradation-based unavailability analysis model for a 1oo2 SIS. The specific objectives include:

- Investigating the combined effects of continuous degradation and random demands on the reliability and availability of a SIS with hidden failures;
- Developing new algorithms for calculating time-dependent PFD_{avg} in different test intervals.
- Providing guidance on decision-making for proof tests of SISs, to ensure compliance and cost-effective operation.

The remainder of this article is organized as follows. Section 2 describes the SIS operation as a stochastic degradation process, with random demand damages and demands arrivals. Section 3 discusses the reliability modeling and PFD_{avg} calculation of a 1oo2 SIS. In section 4, a numerical example is presented to demonstrate our models and sensitivity analysis is also included. Finally, Section 5 presents conclusions.

2. Definitions and assumptions

2.1. Notation

The notions used in formulating the reliability in this paper are now listed.

$N(t)$	number of demands arrived by time t
λ_{de}	arrival rate of random demands
L	performance threshold for failure in terms of a certain degradation
$X(t)$	aging degradation of a component
y_i	damage by the i-th random demand on a component
$Y(t)$	cumulative damage of demands on the component by t
τ	function test interval
$F_Z(z, t)$	the probability of total degradation less than z at time t
$G(X, t)$	cumulative density function of $X(t)$ at time t
$f_{y_i}^k$	probability density function of the sum of k independent and identically distributed (i.i.d.) y_i variables
$Z(t)$	overall degradation of the component

2.2. Redundancy and testing of SISs

SISs are designed to protect EUC given a specific safety integrity level (SIL). IEC 61508 specifies four levels for SIL, with SIL1 being the least reliable and SIL4 being the most reliable. To fulfill the performance requirements for a certain SIL, a SIS in the low-demand mode must have an average probability of failure on demand (PFD_{avg}) in the corresponding interval, as illustrated in Table 1.

Table 1
SILs for low-demand SISs.

SIL	PFD_{avg}
SIL 4	10^{-5} to 10^{-4}
SIL 3	10^{-4} to 10^{-3}
SIL 2	10^{-3} to 10^{-2}
SIL 1	10^{-2} to 10^{-1}

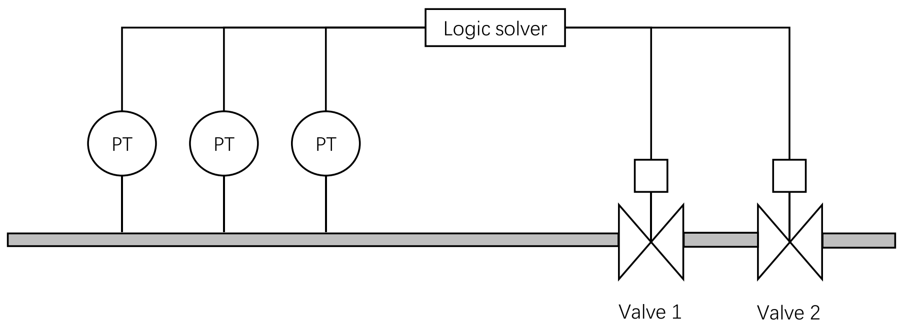

Fig. 1. Example of a HIPPS.

We can take HIPPS as an example of SISs, which architecture is shown in Fig. 1. As mentioned above, the two valves in this SIS are installed in series with a 1oo2 voting configuration to meet the requirement of (IEC 61511, 2010).

The fundamental tasks for the HIPPS is to control high pressure to keep the EUC under acceptable risk level. In general, mechanical systems are designed with safety margins to meet the specified performance requirement (Marszal and Mitchell, 2004). The performance criteria for the HIPPS, e.g. leakage rate and closing time, should be a target value with deviation (Rausand, 2014). In theory, the designed leakage rate should be 0 kg/s, but there is an acceptable deviation based on practical consideration, like 1 kg/s (Nawaz, 2008). Also the performance criteria is different under specific working scenarios. Like, in the offshore plants, acceptable leakage rates generally set higher than for an onshore installation, the main reason for this is due to lower human risk exposure in offshore plants (Nawaz, 2008).

If leakage rate is lower than this acceptable deviation, the performance of valve is acceptable, and it can be stated that the valve is functioning. Too much internal leakage also can weaken control, and even can cause a failure in control of pressure. If the actual leakage rate is higher than the acceptable, the valve is not effective any longer for risk control (Nawaz, 2008). The valve will be in a failed state. The failure mode is "Leakage (through the valve) in closed position (LCP)".

This failure mode is mainly caused by corrosion and erosion on the gate or the seat (Rausand, 2014). The failure mode is dangerous undetected failure and only can be revealed by proof tests or demands.

The possible failure causes could be:

- Normal wear due to corrosive medium. Since a valve is installed to control the pressure, the contact of its gate sealing area with erosive medium can not be avoided. The erosion of the gate sealing area is a progressive, which provides larger flow paths for leaking oil.
- Random demands beyond the specification. The intention of a shutdown valve is to shut-off the liquid flow in case an emergency that leads to a hazardous situation. Operating in higher pressure can result in the misalignment between the gate and the seat of the valve (Technical Note 101). The misalignment of a valve seat can accelerate the existing wear process.

Once high pressure occurs in a pipeline, the stresses on the 2 valves in Fig. 1 will be same or similar. The high pressure could cause a same damage on the two valves simultaneously. Considering the coupling factor, reliability analysis of 1oo2 configuration could not consider two valves separately.

2.3. Assumptions in modeling

In this paper, the aforementioned two processes of the 1oo2 actuator subsystem are regarded as stochastic processes.

For the LCP failure mode of valves, three factors are of interests: acceptable deviation, frequency of closing operations and the effects of high pressure, which will be quantified in the following analysis. First, the acceptable deviation will be the failure threshold L. The valve will be activated when a hazard or demand occurs, so the frequency of closing operation could be linked with a demand rate $\lambda_{\rm de}$ given that the occurrences of demands are modeled as a homogeneous Poisson process. Moreover, high pressure/demands can cause non-negative damage to valve and accelerate the degradation, and such side-effects are modeled by a gamma distribution since it is fairly flexible and positively-skewed distributed with the convenient mathematical properties.

The total degradation process of an actuator includes continuous deterioration and abrupt damages due to random demands as shown in Fig. 2. The occurrence times of random demands $t_1, t_2, \cdots$. are following Poisson process with parameter λ_{de}. Each demand could accelerate the degradation at some extent immediately, as $y_1, y_2, \cdots$. When the total degradation arrives at the failure threshold L, the valve will fail.

The following assumptions and considerations should be mentioned before the performance analysis of the actuators:

1. The actuator starts working at time $t = 0$ and it is subject to a continuous degradation process. In this paper, we assume that the degradation with aging $\{X(t); X(0) = 0, t \geq 0\}$ is a homogeneous Gamma process with the shape parameter $\alpha > 0$ and the scale parameters $\beta > 0$ (Van Noortwijk, 2009). For the period from s to t, $s < t$, the new degradation $X(t) - X(s)$ follows a Gamma density and probability density function (PDF)

$$
\begin{aligned}
&X(t) - X(s) \sim \Gamma(\alpha(t-s), \beta) = f_{X(t)-X(s)}(x) \\
&= \frac{\beta^{\alpha(t-s)}}{\Gamma(\alpha(t-s), 0)} x^{\alpha(t-s)-1} e^{-\beta x}, \ \alpha, \beta > 0
\end{aligned} \tag{1}
$$

The cumulative density function (CDF) of $X(t)$ for $T > 0$ (Wang et al., 2015) is

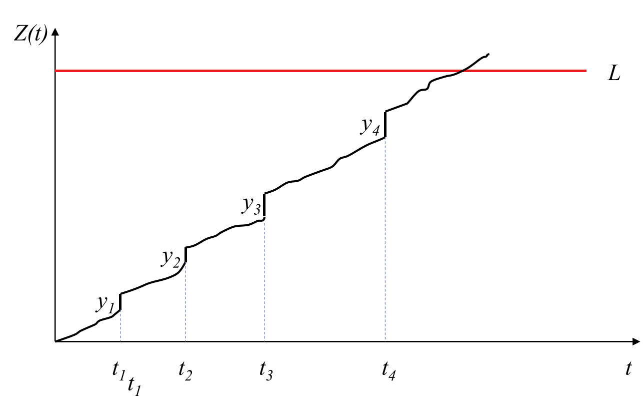

Fig. 2. The degradation behavior of one unit based on the two processes.

$$F_{X(t)}(x) = \Pr\{X(t) \le x\} = \int_0^x f_{X(t)}(z)dz = \frac{\gamma(\alpha t, x\beta)}{\Gamma(\alpha t)} \tag{2}$$

where Γ denotes the upper incomplete Gamma function defined as

$$\Gamma(\alpha) = \int_0^\infty z^{\alpha-1}e^{-z}dz,\ \alpha > 0 \tag{3}$$

γ denotes the lower incomplete Gamma function defined as

$$\gamma(\alpha, x) = \int_0^x z^{\alpha-1}e^{-z}dz,\ x \ge 0,\ \alpha > 0 \tag{4}$$

Then, the mean and variance of $X(t)$ are $\alpha t/\beta$ and $\alpha t/\beta^2$, respectively.

2. The actuator mainly stays in a dormant state. Demands occur following a homogeneous Poisson process with rate λ_{de}. Let $N(t)$ denote the number of all demands that arrived by time t. The probability of exactly n demands occurring in the time interval $[0, t)$ is

$$\Pr(N(t) = n) = \frac{e^{-\lambda_{de}t}(\lambda_{de}t)^n}{n!},\ n = 0,1,\cdots, \tag{5}$$

3. It is assumed that the damage y_i, for $i = 1,2, \cdots, N(t)$, on the actuator caused by the i-th demand is non-negative, independent and gamma distributed with parameters (ξ_i, ρ). The cumulative damage due to demands by time t, $Y(t)$, can be given as

$$Y(t) = \begin{cases} \sum_{i=1}^{N(t)} y_i, & \text{if } N(t) > 0 \\ 0, & \text{if } N(t) = 0 \end{cases} \tag{6}$$

Meanwhile, all demands y_i are assumed to have the same scale parameter ρ, then

$$\sum_{i=1}^{N(t)} y_i \sim Gamma\left(\sum_{i=1}^{N(t)} \xi_i, \rho\right) \tag{7}$$

4. All demands will cause the same damage size on two valves simultaneously.
5. A failure occurs when the total degradation reaches a certain critical threshold L. The failure to work of the system means that both of the two components have degraded to the failure threshold L.
6. The system is regularly proof-tested after a certain period τ ($\tau > 0$). Proof-tests are non-destructive and non-damage to the actuators. During a proof-test, the only information we can collect about the system status is whether it is functioning or not.
7. Common cause failures (CCFs) in such a 1oo2 configuration are excluded, with the purpose to illustrate the effects of degradation on a redundant architecture apparently.

As mentioned before, PFD_{avg} is a widely used unavailability measure of a SIF. To describe the system performance clearly, algorithms and approximation formulas for the reliability of one unit and that of a 1oo2 configuration will be derived at first, and then such formulas will be the basis of PFD_{avg} calculation.

3. Reliability and unavailability analysis

3.1. Unit reliability analysis

According to assumptions in 2.3, the total degradation of one unit, $Z(t)$, is the sum of degradation due to aging process and the instantaneous damages due to random demands. The overall degradation of unit is expressed as $Z(t) = X(t) + Y(t)$.

Considering the demands following a Poisson process, the probability that total degradation at time t is less than z, $F_Z(z, t)$, can be derived as

$$\begin{aligned} F_Z(z, t)(t) &= \Pr(Z(t) < z) \\ &= \sum_{i=0}^{\infty} \Pr(X(t) + Y(t) < z | N(t) = i)\Pr(N(t) = i) \end{aligned} \tag{8}$$

Furthermore, a convolution integral can be used in (8). We set $G(X, t)$ as the cumulative density function of $X(t)$ at t, $f_{y_i}^k$ as the probability density function of the sum of k independent and identically distributed (i.i.d.) Y_i variables, then $F_Z(z, t)$ can be derived as:

$$F_Z(z, t)(t) = \sum_{i=0}^{\infty}\left(\int_0^z G(z-u, t) f_{y_i}^k(u)du\right)\frac{e^{-\lambda_{de}t}(\lambda_{de}t)^i}{i!} \tag{9}$$

3.2. System reliability analysis

The actuator subsystem is still functioning even when one of the two units has failed, so that the reliability of such a 1oo2 configuration by time t is the probability that total degradation of at least one component is less than the threshold level ($Z(t) < L$). The survivor function of the 1oo2 configuration is,

$$R(t) = P\{[Z_1(t) < L] \cup [Z_2(t) < L]\} \tag{10}$$

Given that one demand can result in same damage of the two units, the times-to-failure of two components are dependent (Song et al., 2014). Most existing papers only considered the dependence due to same number of demands $N(t)$. Since the two components are exposing to same damage each time. It is reasonable to consider the dependency due to impact of each demand. We need to compute it by finding the marginal distribution, $f_Y(y)$. Based on the law of total probability, we then integrate the marginal distribution to derive the system reliability, as shown in 11.

As assumed, demands are independent from the aging degradation process, then the reliability of a 1oo2 configuration is given

$$\begin{aligned} R(t) &= P\{[Z_1(t) < L] \cup [Z_2(t) < L]\} \\ &= \int_0^L [1 - \textstyle\prod_{i=1}^2 (1 - P(Z_i(t) < L | Y = y, N(t))] f_Y(y)dy \\ &= \textstyle\sum_{k=0}^{\infty} \int_0^L [1 - \prod_{i=1}^2 (1 - P(X_i(t) + Y_i(t) < L | S = y, N(t) = k)] f_Y(y) \\ &\quad dy \cdot P[N(t) = k] \\ &= \textstyle\sum_{k=0}^{\infty} \int_0^L [1 - \prod_{i=1}^2 (1 - P(X_i(t) < L - y)] f_Y(y)dy \cdot P[N(t) = k] \\ &= \textstyle\sum_{k=0}^{\infty} \int_0^L \left[1 - \prod_{i=1}^2 \left(1 - \frac{\gamma(\alpha t, (L-y)\beta)}{\Gamma(\alpha t)}\right)\right] \frac{\rho^{k\xi} \cdot y^{k\xi-1} \cdot e^{-\rho y}}{\Gamma(k\xi)} dy \cdot \frac{e^{-\lambda_{de}t}(\lambda_{de}t)^k}{k!} \\ &= \textstyle\sum_{k=0}^{\infty} \int_0^L \left[1 - \left(1 - \frac{\gamma(\alpha t, (L-y)\beta)}{\Gamma(\alpha t)}\right)^2\right] \frac{\rho^{k\xi} \cdot y^{k\xi-1} \cdot e^{-\rho y}}{\Gamma(k\xi)} dy \cdot \frac{e^{-\lambda_{de}t}(\lambda_{de}t)^k}{k!} \\ &= \left[1 - \left(1 - \frac{\gamma(\alpha t, L\beta)}{\Gamma(\alpha t)}\right)^2\right] \cdot e^{-\lambda_{de}t} + \\ &\quad \textstyle\sum_{k=1}^{\infty} \int_0^L \left[1 - \left(1 - \frac{\gamma(\alpha t, (L-y)\beta)}{\Gamma(\alpha t)}\right)^2\right] \frac{\rho^{k\xi} \cdot y^{k\xi-1} \cdot e^{-\rho y}}{\Gamma(k\xi)} dy \cdot \frac{e^{-\lambda_{de}t}(\lambda_{de}t)^k}{k!} \end{aligned} \tag{11}$$

The process for general Koo$N$ architecture is same as 1oo2 in this paper. The only consideration is to replace the survivor function in Eq. (10). Here, we take 1oo2 as a typical configuration to illustrate the tendency of $R(t)$ and PFD_{avg}.

3.3. Calculating PFD

In the existing studies, components in SISs are as-good-as-new after each proof-test, and therefore PFD_{avg} within each proof-test interval is completely same. When degradation is in consideration, the situation becomes different, namely PFD_{avg} in a proof-test interval is dependent on that in the previous one.

Consider a 1oo2 configuration and let T denote the time to failure of the actuator subsystem. The failure probability by t is $F(t) = Pr\{[Z_1(t) > L] \cap [Z_2(t) > L] | T \le t\}$, and the instantaneous unavailability of the SIS subsystem within the first proof test interval, $\text{PFD}_1(t)$, is

$$\mathrm{PFD}_1(t) = \Pr([Z_1(t) > L] \cap [Z_2(t) > L] \text{ by } t) = F(t) = 1 - \Pr([Z_1(t) < L] \cup [Z_2(t) < L]) = 1 - R(t) \quad (12)$$

The average value of $\mathrm{PFD}_1(t)$ in the first proof test interval$(0,\tau)$ can be obtained then

$$\mathrm{PFD}_{\mathrm{avg}} = \frac{1}{\tau}\int_0^\tau \mathrm{PFD}_1(\mathrm{t})dt = 1 - \frac{1}{\tau}\int_0^\tau R(t)dt \quad (13)$$

Using the survivor function of the system $\mathrm{R}(t)$ in (11), we can get

$$\mathrm{PFD}_{\mathrm{avg}} = 1 - \frac{1}{\tau}\int_0^\tau R(t)dt = 1 - \frac{1}{\tau}\int_0^\tau \left\{\left[1 - \left(1 - \frac{\gamma(\alpha t, L\beta)}{\Gamma(\alpha t)}\right)^2\right] \cdot e^{-\lambda_{de}t} + \sum_{k=1}^{\infty}\int_0^L \left[1 - \left(1 - \frac{\gamma(\alpha t, (L-y)\beta)}{\Gamma(\alpha t)}\right)^2\right] \frac{\rho^{k\xi} \cdot y^{k\xi-1} \cdot e^{-\rho y}}{\Gamma(k\xi)}dy \cdot \frac{e^{-\lambda_{de}t}(\lambda_{de}t)^k}{k!}\right\}dt \quad (14)$$

A proof-test will be executed at time τ. If the subsystem is functioning at τ with unknown degradation level, $\mathrm{PFD}_2(t)$ becomes the conditional probability of failure with $t > \tau$ given functioning by τ

$$\mathrm{PFD}_2(t) = \Pr[T < t | T > \tau, t > \tau] = 1 - \Pr[T < t | T > \tau, t > \tau] = 1 - \frac{\Pr[T > t \cap T > \tau, t > \tau]}{\Pr[T > \tau]} = 1 - \frac{R(t)}{R(\tau)} \quad (15)$$

The $\mathrm{PFD}_{\mathrm{avg}}$ in the second test interval$(\tau,2\tau)$ is then:

$$\mathrm{PFD}_{\mathrm{avg}} = \frac{1}{\tau}\int_\tau^{2\tau} PFD_2(t)dt = \frac{1}{\tau}\int_\tau^{2\tau}\left[1 - \frac{R(t)}{R(\tau)}\right]dt = 1 - \frac{1}{\tau}\int_\tau^{2\tau}\frac{R(t)}{R(\tau)}dt \quad (16)$$

Similarly, if the subsystem is functioning in the i-th proof-test interval of $((i-1)\tau, i\tau)$, the $\mathrm{PFD}_i(t)$ can be calculated as:

$$\mathrm{PFD}_i(t) = \Pr[T < t | T > (i-1)\tau, t > (i-1)\tau] = 1 - \Pr[T > t | T > (i-1)\tau, t > (i-1)\tau] = 1 - \frac{\Pr[T > t \cap T > (i-1)\tau, t > (i-1)\tau]}{\Pr[T > (i-1)\tau]} = 1 - \frac{R(t)}{R((i-1)\tau)} \quad (17)$$

In the i-th proof-test interval $((i-1)\tau, i\tau)$, $\mathrm{PFD}_{\mathrm{avg}}$ can be calculated as:

$$\mathrm{PFD}_{\mathrm{avg}} = \frac{1}{\tau}\int_{(i-1)\tau}^{i\tau}\mathrm{PFD}_i(t)dt = 1 - \frac{1}{\tau}\int_{(i-1)\tau}^{i\tau}\frac{R(t)}{R((i-1)\tau)}dt \quad (18)$$

Based on the results above, it is still difficult to generate a straightforward expression of $\mathrm{PFD(t)}$ and $\mathrm{PFD}_{\mathrm{avg}}$. Therefore, in the rest of this paper, a numerical example is chosen to manifest differences between the proposed method and the existing ones.

4. Case studies

In this section, an example is given to illustrate the function of the proposed algorithm. We will compare the results based on the method in this paper and those from the widely used formulas for $\mathrm{PFD}_{\mathrm{avg}}$. We will also perform sensitivity analysis for the effects of parameters on $R(t)$ and $\mathrm{PFD}_{\mathrm{avg}}$. The three following variables will be evaluated: failure threshold L, demand rate λ_{de}, shape parameter ξ of demand damage.

4.1. Reference values from simplified formulas

In the simplified formulas in (Rausand and Arnljot, 2004), the subsystem is assumed as-good-as-new after each proof test. The units in a 1oo2 configuration have the same failure rate λ, and they are tested at the same time with an interval τ. The approximation formulas for the $\mathrm{PFD}_{\mathrm{avg}}$ of this 1oo2 configuration is

$$\mathrm{PFD}_{\mathrm{avg}}^{(1oo2)} \approx \frac{(\lambda\tau)^2}{3} \quad (19)$$

Table 2
Parameter values.

Parameter	Value
L	0.00125 (Tanner and Dugger, 2003)
λ_{de}	$2.5 \times 10^{-5} h^{-1}$
α	1.02×10^{-4}
β	1.2×10^4
ξ	4.0
ρ	4×10^4
τ	$8760h$

*h means hour.

The SIL requirement for a 1oo2 valve actuator subsystem in the IEC standard is SIL3 (IEC 61511, 2010). Following the corresponding values of $\mathrm{PFD}_{\mathrm{avg}}$ listed in Table 1, the upper and lower limits of $\mathrm{PFD}_{\mathrm{avg}}$ for SIL3 is 10^{-4} and 10^{-3}. Using Eq. (19), we can get the constant failure rate λ with $\tau = 8760$ is 2×10^{-6} and 6.25×10^{-6}, the maximum and minimum mean time to failure (MTTF) is 5×10^5 and 1.6×10^5, respectively. In other words, if the design of actuator can follow the requirement of SIL3, the maximum acceptable failure rate of each unit in the 1oo2 configuration is 6.25×10^{-6}.

The failure rate of LCP for valve is obtained from (Rausand, 2013) as 2.7×10^{-6}. These three failure rates will be used as reference values to validate the proposed degradation model.

The parameters of aging degradation and random demands are provided in Table 2, and then the two processes are simulated in Matlab R2018a.

To investigate the effect of damage caused by random demands on system, we compare two degradation modes: degradation only with the aging process, and degradation as the combination the aging and random demands.

Based on Eq. (11), under the combined effects of aging and demands, reliability of the 1oo2 configuration decreases along with time as plotted in Fig. 3.

In Fig. 3, it is easy to notice that $R(t)$ of the 1oo2 configuration only with the continuous process is overlapping with that subject to two processes by around 0.5×10^5. When the time in consideration is longer, random demands gradually have more obvious effects on degradation, with the reflection in Fig. 3 that $R(t)$ only with the continuous process is higher. The difference between two curves reflects the accumulating effect caused by random demands. With time going on, the effect of random demands is more obvious. If only the aging process is considered, reliability of the SIS will be overestimated and risk of EUC will be underestimated.

$\mathrm{PFD}_{\mathrm{avg}}$ values of the SIS in the two degradation modes are shown in Fig. 4.

It is easily noticed that there are much difference on $\mathrm{PFD}_{\mathrm{avg}}$ for the two degradation modes. For the degradation mode with two processes, the $\mathrm{PFD}_{\mathrm{avg}}$ of this 1oo2 configuration is not in the range of SIL3 anymore after 7τ. But if only considering the aging processes, this system can still meet the required SIL3 in the test interval $[9\tau,10\tau)$. Considering the safety requirement of EUC, the combined degradation processes could make the reliability and $\mathrm{PFD}_{\mathrm{avg}}$ more stricter than only aging process.

After the valves installation, their reliability and availability should be assessed through periodic and diagnostic tests. In order to meet the required SIL, it is necessary to maintain an accurate record not only operating time and proof test results but also the previous operation history. Considering the harsh operating environment, valves that report only on installation time may not be sufficient for assessing the status.

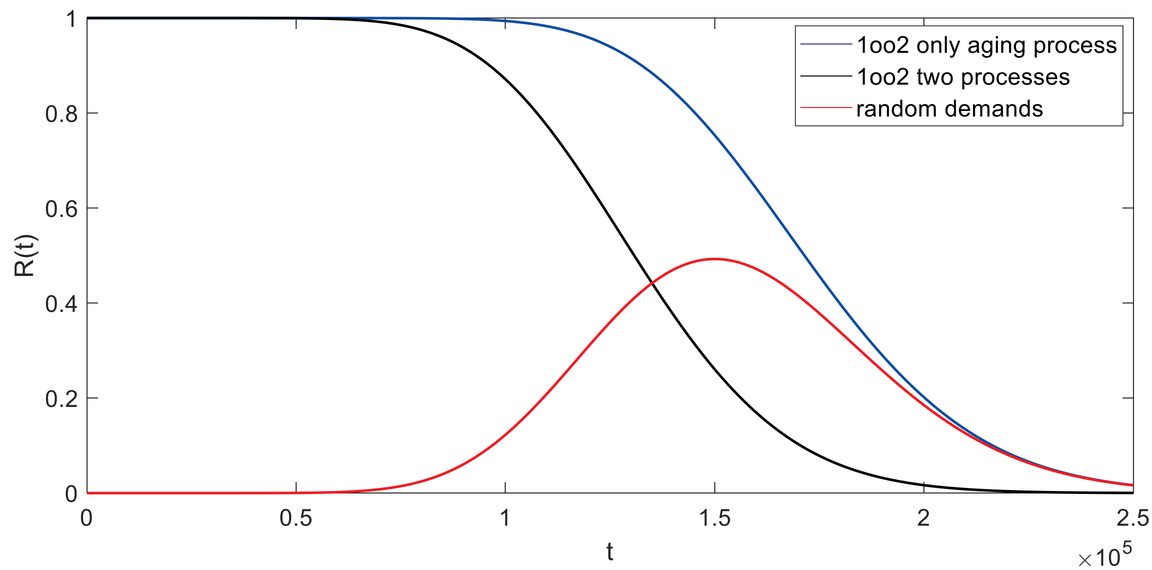

Fig. 3. Reliability of the 1oo2 configuration subject to aging degradation and random demands.

4.2. Sensitivity analysis of parameters

To investigate the effect of degradation process on the PFD_{avg}, several parameters will be discussed: i.e. threshold L, demand rate λ_{de}, and shape parameter ξ. Here, these three parameters are used to describe the working conditions of the 1oo2 configuration.

4.2.1. Effects of thresholds

Taking the LCP failure mode as an example, the maximum allowable leakage rate of a valve can act as a threshold for determining whether a failure occurs. The reflection of leakage rate on the valve is the depth of erosion and corrosion.

In practices, the maximum allowable depth is determined by several aspects. First of all, during the design stage, it is the property of material. Designers should choose more stable material for valves installed harsh working condition. Secondly, it is related with the leakage rate requirement. Therefore, the working condition should be considered during the selection and installation of the valve.

In this paper, the maximum value of depth under each specific scenario is assumed as the failure threshold L. Different threshold values are given under a constant demand rate $\lambda_{de} = 2.5 \times 10^{-5}$ per hour, and their effects on the reliability are shown in Fig. 5.

It can be found that $R(t)$ is not sensitive to L until t reaches a certain value around 0.5×10^5, meaning that the maximum depth values have slight effect on the system reliability at the beginning. The system stays with high reliability by this time, with no consideration about manufacturing error or failures, because the 1oo2 configuration has just experienced slight aging degradation and seldom demands have come. Along with longer time, the reliability decreases dramatically. By increasing thresholds L shifts from 0.00115 to 0.00155, namely releasing the requirement for acceptable leakage rate, $R(t)$ shifts to the right. Such a shift is from the loosing definition on the system functioning.

As seen in Fig. 5, the reliability profiles (solid lines) based on the proposed degradation model are totally different with those having constant failure rates (dashed lines). The hypothesis of a constant failure rate provides easier mathematical models to assess the performance of actuators. At the early stage, the method based on the constant failure rate (the dashed solid line) underestimates the reliability of system. The underestimation can bring unnecessary costs in SIS design and over protection in some degree. While more focus should be put on

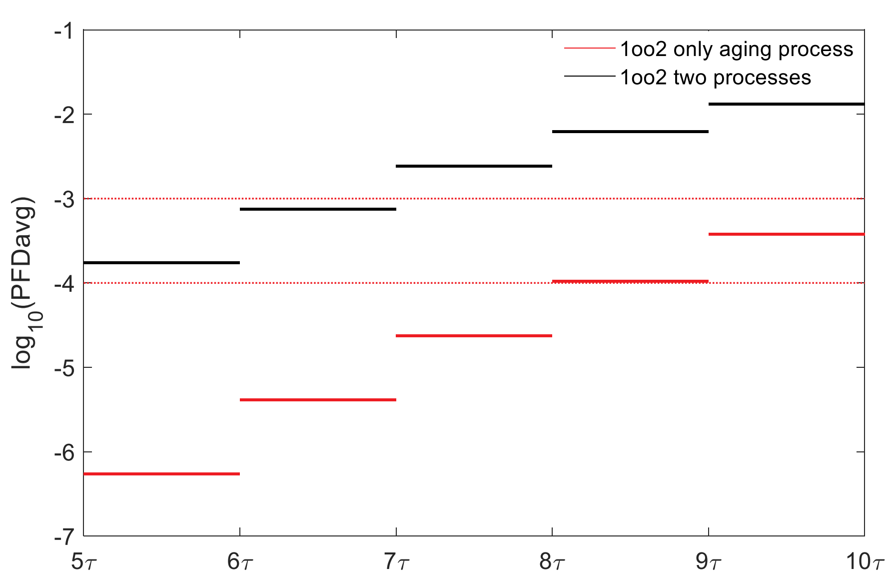

Fig. 4. PFD_{avg} of the 1oo2 configuration subject to aging degradation and random demands.

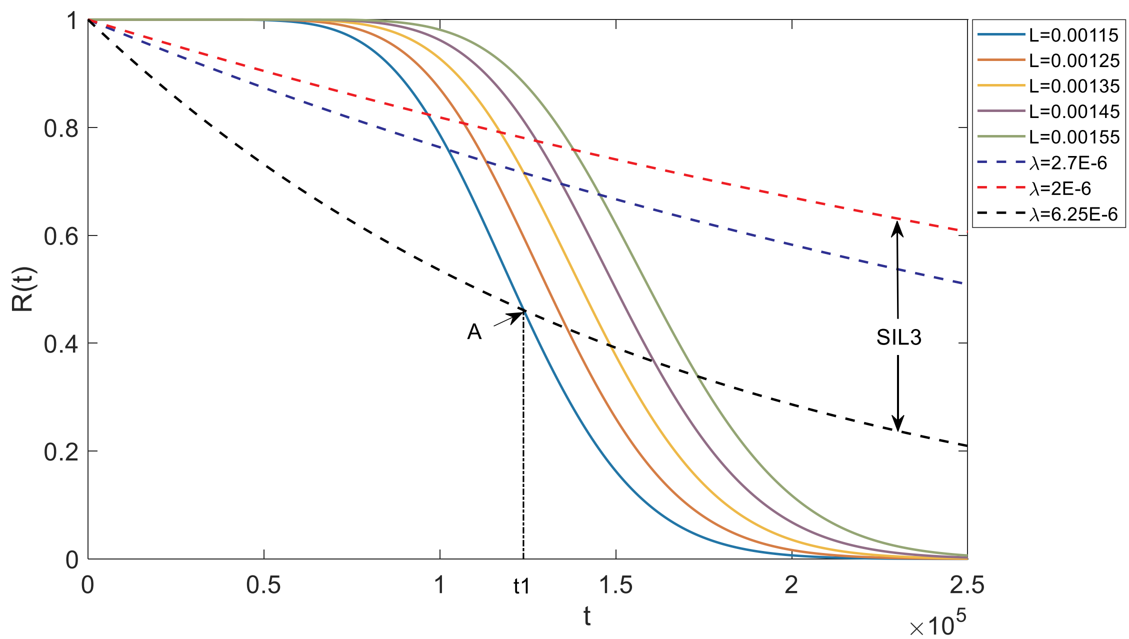

Fig. 5. Sensitivities of $R(t)$ on threshold L.

the period after a specific time point, e.g. the intersections of the dashed blue lines with the solid ones in Fig. 5, where the constant rate-based method overestimates the system reliability. In such contexts, even the SIS can be degraded at a high SIL, it actually cannot provide sufficient protection on EUC.

We can use MTTF to denote the system reliability and compare the results with different values of L and the constant failure rate.

$$\mathrm{MTTF} = E(T) = \int_0^{\infty} tf(t)dt \approx \int_0^{\infty} R(t)dt \tag{20}$$

Based on the configuration, we can firstly determine the minimum MTTF of a subsystem for SIL3 is 1.60×10^5. Then, it can be found in the table that when $L = 0.00155$, MTTF is within the range of SIL3. For the other settings of L, the subsystem only can comply with SIL2. Meanwhile, the higher threshold also means a higher tolerance for SIS which has a longer MTTF. From Table. 3, we can see that the integrity of SIS is partly dependent on how much the EUC can tolerate the rate of leakage. If the EUC is sensitive to leakage, we need to be more conservative in grading its SIS.

Meanwhile, the degradation analysis provide an opportunity to estimate the overhaul time for the SIS. Normally, if after testing we discovered an anomaly, we can schedule intervention, such as lubrication; On the contrary, if the valves is functional after each proof test, theoretically, we will not act on the valves. But when the reliability of valves has decreased considerably so it is risky to meet the safety requirement for possible demands. That is, the valves should be arranged with an overhaul time even if it has not given any symptoms of having a problem. When the calculated reliability under degradation cannot satisfy the requirement of SIL3, shown as point A. The overhaul time can be settled as time t_1 in Fig. 5. Similarly, considering different acceptable threshold, the overhaul time can be adjusted.

To illustrate the effects of threshold values on PFD_{avg}, $\log_{10}$-scale on the y-axis is then adopted since it can present more details when the value of PFD_{avg} is rather small. PFD_{avg} is calculated for every interval $[(i-1)\tau, i\tau)$ based on the proposed formula (18). A numerical comparison of PFD_{avg} under different thresholds is shown in Fig. 6.

The system reliability $R(t)$ of different thresholds before $0.5 \times 10^5$is overlapping, that is, the PFD_{avg} is easily affected by the calculation accuracy given the property of gamma function. Hereby, PFD_{avg} during the test intervals $[5\tau,6\tau)$, $[6\tau,7\tau)$, $[7\tau,8\tau)$,$[8\tau,9\tau)$,$[9\tau,10\tau)$ are analyzed respectively. To compare with the results of reference value, PFD_{avg} calculated based on assumptions of constant failure rate ($\lambda = 2.7 \times 10^{-6}$) and as-good-as-new after proof-tests ($\tau = 8760$), is drawn in red dashed line in Fig. 6.

Generally speaking, the PFD_{avg} is decreasing with the threshold in the same test interval, e.g. SIL4 for $L = 0.00155$ in interval $[6\tau,7\tau)$, but SIL2 for $L = 0.00115$, with the same assumption that the valve is functioning at 6τ during the proof test. The PFD_{avg} for $L = 0.00115$ is almost 100 times higher for $L = 0.00155$. It means that the EUC with lower threshold of leakage is more risky. Meanwhile, in these test intervals, the increment of PFD_{avg} between two consecutive thresholds keeps more or less the same value in each test interval. It means that under the same operating environment (demand rate), the PFD_{avg} increments of the two consecutive thresholds are proportional to the difference between the thresholds, which is proved by the constant difference of MTTF between two consecutive thresholds in Table 3.

During these test intervals, for the same threshold L, PFD_{avg} is also increasing with time, e.g. SIL of L= 0.00155 from qualifying SIL4 in $[5\tau,6\tau)$ is released to SIL2 in $[9\tau,10\tau)$. Such a change manifests that the probability of the system failing to demand is increasing even it is functioning at each proof test. Namely, the assumption of as-good-as-new after each proof test is too optimistic for PFD_{avg}. The valves are activated during the proof test, it only means that valves are functioning but unnecessary to be a total new state. Consequently, the periodic test policy is questionable and becoming insufficient to meet SIL requirement with time going by. This finding could be used as a rough guideline for proof test plans. For example, the PFD_{avg} when $L = 0.00125$ in interval $[6\tau,7\tau)$ is within SIL3, but for the latter interval $[7\tau,8\tau)$, the PFD_{avg} jumps to SIL2. In practical applications, the test intervals should be updated and shortened after 7τ rather than to keep $\tau = 8760$.

4.2.2. *Effects of demand rates*

Given the characteristics of low demand systems, they are required to be activated when a hazardous event occur. λ_{de} could be an indicator to describe the working condition of HIPPS.

In this subsection, we fix the failure threshold $L = 0.00125$, and observe PFD_{avg} of the 1oo2 configuration when the demand rate λ_{de} is set as different values as shown in Fig. 7.

PFD_{avg} acts as an effective measure for the low-demand system. The

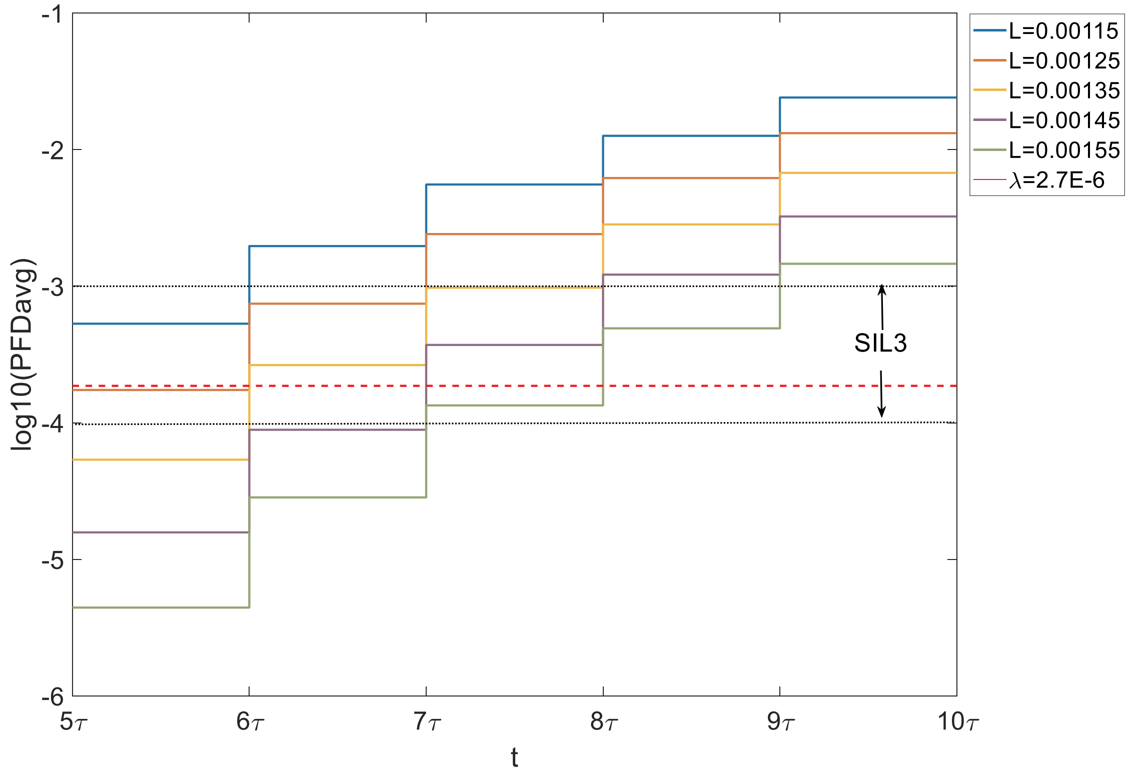

Fig. 6. Effects of threshold L on.PFD_{avg}.

Table 3
Estimated MTTF under different L.

Parameter	MTTF
L = 0.00115	1.23×10^5
L = 0.00125	1.32×10^5
L = 0.00135	1.42×10^5
L = 0.00145	1.52×10^5
L = 0.00155	1.61×10^5
Maximum MTTF for SIL3	1.60×10^5

boundary of high-and low-demands can be approximated as once per year when the proof test frequency is also once per year (Liu, 2014), so the maximum $\lambda_{de} = 1 \times 10^{-4}$ is chosen in this paper.

It is not so hard to imagine that the system reliability decreases when it is operating with higher arrival rates of random demands. The overall tendency is similar as in Fig. 5. No further discussion about system reliability here.

The effects of demand rate λ_{de} on PFD_{avg} are shown in Fig. 7. Similarly to threshold L, in each test interval, the PFD_{avg} is increasing with the demand rate λ_{de}. When the demand rate increase 10 times from $\lambda_{de} = 2.5 \times 10^{-6}$ to $\lambda_{de} = 2.5 \times 10^{-5}$, the PFD_{avg} increases almost 100 times in [5τ,6τ). It is more than SIL4 under $\lambda_{de} = 2.5 \times 10^{-6}$, while, up to SIL1 under $\lambda_{de} = 1 \times 10^{-4}$ which is far from the required SIL3. It means the valves are less reliable when they are installed in higher demand rate circumstance. In a higher demand rate working condition, the 1oo2 configuration is easier to get the damage from random demands. The accumulated damage increase the overall degradation which make the valves are more fragile for the upcoming demands.

Under the same λ_{de}, the PFD_{avg} of 1oo2 configuration is increasing with time. In order to meet SIL3, the test interval $\tau = 8760$ is enough until 7τ under the demand rate $\lambda_{de} = 2.5 \times 10^{-5}$. From [7$\tau$,8$\tau$) on, it is out of the range of SIL3 but in SIL2 or higher instead. To meet the performance requirement, the proof test interval should be shorter than $\tau = 8760$ after 7τ in this example.

Compared to the threshold L, the demand rate has a more obvious effect on PFD_{avg}. When the valves are installed in a higher demand context, the SIL could beyond the safety requirement even in the early stage. These effects should attract the attentions of maintenance crews. More stricter proof tests and maintenance should be arranged for higher demand rate operating environment. After each demand, therefore, the basic visual check or simple maintenance should be followed to ensure safety. Similar to threshold L, λ_{de} is worth being taken into account when determining the overhaul time of the SIS. When demand rate is higher, it suffers more damages from demands, which requires earlier services.

4.2.3. Effects of the shape parameter of demands

As another key parameter of the working condition, the demand damage size on system should be discussed in this section. This parameter could be linked with the pressure in EUC. As assumed in 2.3, the size of damage by each random demand follows a gamma distribution with parameters (ξ_i, ρ), while the shape parameter ξ is the contributor for damage size under the same scale parameter ρ. Since the sum of k damages also follows gamma distribution with parameters ($\sum_{i=1}^{k} \xi_i$, ρ), the shape parameter can be estimated as $k\xi$ when assume these demands have same shape parameter ξ. In the sensitivity analysis, different shape parameter values are given under a constant demand rate $\lambda_{de} = 2.5 \times 10^{-5}$ per hour and threshold $L = 0.00125$.

The effects of shape parameter ξ on PFD_{avg} is shown in Fig. 8. For each of ξ, PFD_{avg} increases with time. Meanwhile, PFD_{avg} has a positive relationship with shape parameter ξ of demand. With the higher value of shape parameter of demand ξ, PFD_{avg} increases in same test interval, e.g. it is following SIL4 for $\xi = 2$, and only following SIL3 for $\xi = 4$ in [5τ,6τ). This phenomena means that the average unavailability increases with higher average damage size under same demand rate. If the HIPPS is installed in the severe pressure condition, it is becoming more risky for the upcoming demands. The possible solution is to choose the higher tolerance equipment for more severe working

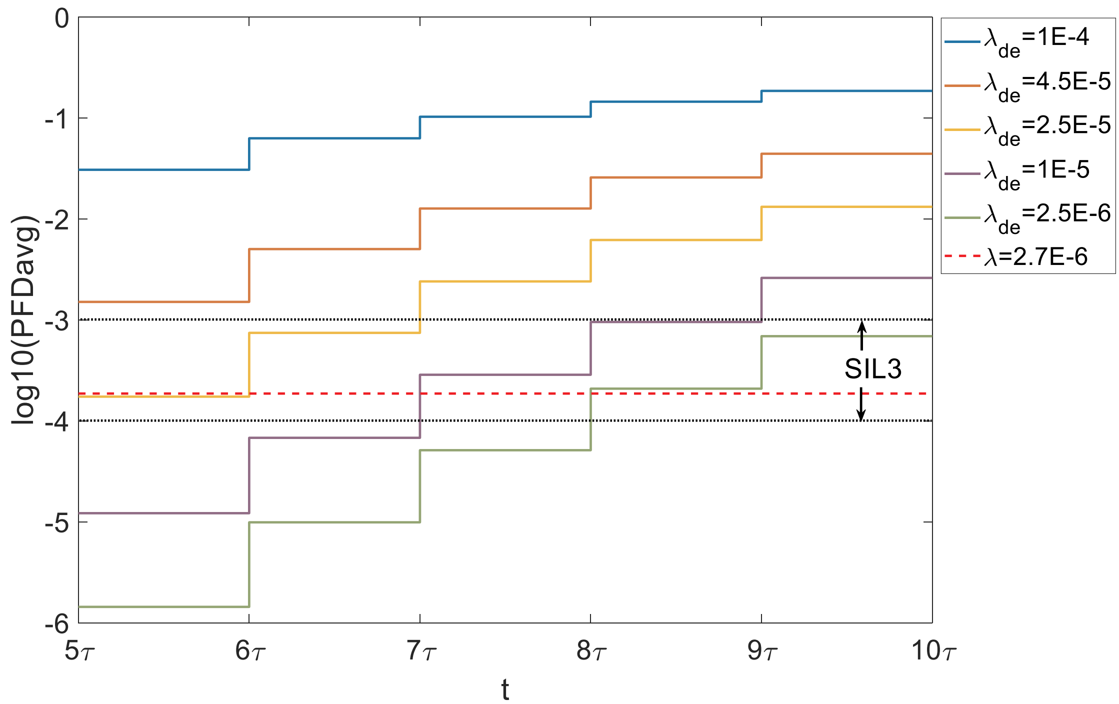

Fig. 7. Effects of demand rates λ_{de} on PFD_{avg}.

condition. Another way is to execute preventive maintenance after demands.

4.3. Updating the test intervals

Having considered the degradation, it is interesting to consider the length of test intervals. Given that degradation has been found influential on the decision-making for testing strategies, the most constraint is the SIL level to be followed. Normally, the EUC system will shutdown for the proof test of SISs. The shutdown and re-operation of EUC will cause an economic loss. In order to avoid unnecessary loss, the minimum proof test frequency should be settled. Here, we are going to discuss the first 6 test interval under different threshold L to get the different time dates.

In this example, such a 1oo2 SIS needs to meet SIL3. Here, we take different thresholds L in Fig. 9 as an example. Values of the two variables are at first set as $\lambda_{de} = 2.5 \times 10^{-5}$, and $\xi = 4$ respectively. Similar to Eq. (18), we can connect reliability and average PFD in a test interval

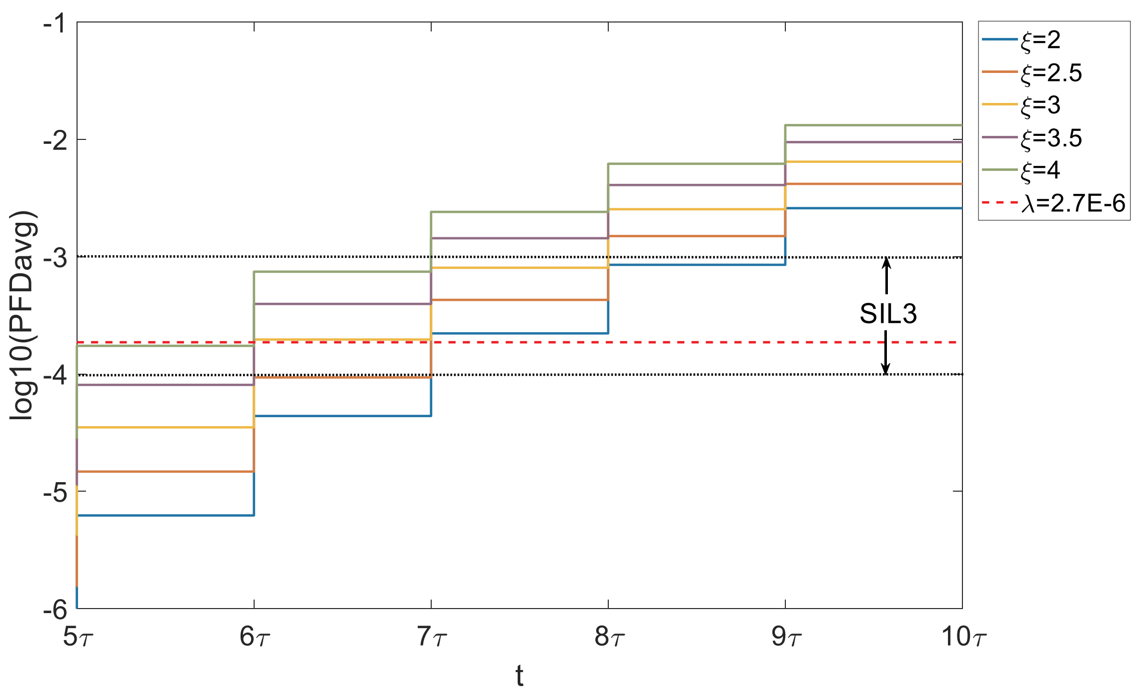

Fig. 8. Sensitivities of PFD_{avg} on shape parameter ξ of demands.

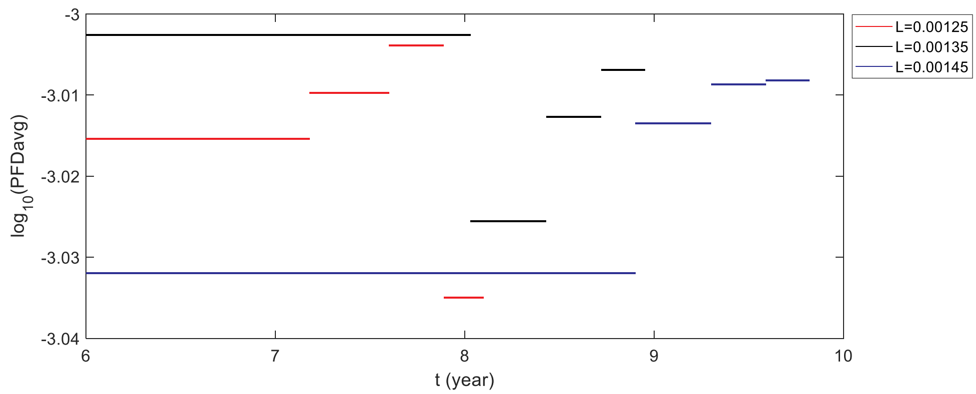

Fig. 9. Updated test interval under SIL3.

$$\mathrm{PFD}_{avg} = 1 - \frac{1}{t - t_0}\int_{t_0}^{t} \frac{R(u)}{R(t_0)}du \quad (21)$$

The idea is to calculate time t when PFD_{avg} is in the range of $[10^{-4}, 10^{-3})$ given functional at time t_0, where $R(u)$ is changing with time as in Eq. (11).

Here, we take the $\tau = 8760h$ (1 year) as time unit. In order to keep safety, 3 years is set as the maximum length of the proof-test interval (Hauge and Lundteigen, 2008). For the first test interval $[0,3\tau)$, the SIL is much higher than SIL4. As for the second interval $[3\tau, 6\tau)$, the values of $\log_{10}\mathrm{PFD}_{avg}$ under thresholds are − 3.99, − 4.53 and − 5.09, respectively, which satisfying SIL3. It means, for first two test intervals, 3-year interval is sufficient to keep the 1oo2 configuration to meet SIL3.

Considering the proposed degradation process, Eq. (21) is used to calculate the proof test time. Results of the updated test dates after the first two intervals are shown in Fig. 9. The exact values of the each test time point are shown in Table 4.

It can be found that the length of test interval is becoming shorter and shorter, decreasing from 3 to 0.2 years. The test interval is longer under the higher threshold L. For example, the 3rd test interval for $L = 0.00145$ is almost 3 years, while only around 1 year for $L = 0.00125$. Different test interval should be adopted for SISs under different working conditions.

But, it is worth mentioning that the values in Table 4 are calculated only based on the assumption of functioning at the previous proof test without considering any other factors.

In practices, the following factors should be considered in updating proof test intervals for a certain SIL requirement:

- Test quality: In order to estimate the performance accurately, the potential leakage rate should be detected perfectly in proof tests. Considering the errors of tests, the calculated PFD_{avg} with a confidence interval should be used to estimate the test interval.
- Maintenance: Since no maintenance work is considered in this example, we regard the system same no matter 0 leakage is existing or the leakage rate is close to the threshold. In practices, when the leakage rate is approaching threshold, preventive maintenance can be conducted to stop or at least slow down the degradation. After preventive maintenance, the reliability of system can be supposed to improved, the test interval should be lengthened.
- Partial tests: The length of test interval refers to the full proof test, but partial proof tests can be introduced between two full proof tests. The efficient partial test can collect the performance information which will reduce possible damages on the actuators. According to the result of partial tests, the full proof test interval could be adjusted.

Table 4
Updated test interval under different L based on SIL3.

Parameter	3rd	4th	5th	6th
$L = 0.00125$	6τ, 7.18τ	7.18τ, 7.6τ	7.6τ, 7.89τ	7.89τ, 8.1τ
$L = 0.00135$	6τ, 8.03τ	8.03τ, 8.43τ	8.43τ, 8.72τ	8.72τ, 8.95τ
$L = 0.00145$	6τ, 8.9τ	8.9τ, 9.3τ	9.5τ, 9.59τ	9.59τ, 9.82τ

*$\tau = 8760$ h = 1 year.

5. Conclusions

In order to evaluate the effects of aging and demands effects on SISs, this paper has presented a degradation-based approach for performance analysis of 1oo2 actuators of SISs. The model is developed taking account a continuous aging process and random demands on individual units. Considering the dependency of two units due to same demands, reliability algorithm for the 1oo2 subsystem has been proposed, and the approximation formulas for $R(t)$ and PFD_{avg} of the subsystem have been developed.

A numerical example is given to illustrate usefulness of the proposed models. Sensitivity analyses are conducted to examine the effects of failure threshold, demand rate and shape parameter PFD_{avg}. Based on the operational assumption at each test date, we found that the conditional PFD_{avg} is increasing with time under the assumption of functional in proof tests. PFD_{avg} is negatively related with the value of failure thresholds L and positively with demand rate λ_{de} and shape parameter ξ.

According to the results of sensitivity analysis, we propose to adjust proof test intervals based on the testing results. Flexible proof test intervals could be settled rather than keep them fixed. At the early stage of the system, the reliability of SIS is high, and so the proof test interval could be settled longer based on the unavailability acceptable criteria, to reduce operational costs. With time goes by, the length of proof test interval should be shorter to ensure safety.

This paper focuses on the calculation of $R(t)$ and PFD_{avg} of a 1oo2 SIS without considering maintenance work. One extension of the current work is to take maintenance work for restoration into consideration, since system resilience has been regarded as significant measure (Cai et al., 2018; Feng et al., 2019; Ren et al., 2019). Another extension is to study the general $KooN$ architecture in SIS. Dependency of a common number of shocks, $N(t)$ and dependency due to impact of each demand on all among components will be studied separately and reported later.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.jlp.2019.103946.

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