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Title page

'Walking a graph': Developing graph sense using motion sensor technology

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Abstract

The teaching and learning of functions have generally been considered a problematic area of school mathematics. Also in Norway, functions have been identified as a difficult topic for many students. In this paper we address some of the difficulties that may arise when introducing the concept of time-distance graphs and discuss misconceptions that can be avoided with our approach. The paper reports on a case study done within the international project FaSMEd (Formative assessment in science and mathematics education). Two groups of grade 6 students were introduced to time-distance graphs for the first time. This happened at an earlier stage than normally dictated by the national curriculum. We show how students indicate they are learning mathematics and developing graph sense through a couple of activities utilizing motion sensor technology. We will see how students express connections with translating movement into language about graphs and how they translate graphs into physical movement.

Introduction

The function concept is generally regarded as a difficult concept for students to grasp (Dreyfus & Eisenberg, 1982; Sajka, 2003; Sierpinska, 1992). In Norway, national and international tests have shown that mathematical functions are a difficult area. In the TIMSS and TIMSS Advanced tests, algebra and calculus are identified as highly problematic areas for Norwegian students (Bergem, Kaarstein, & Nilsen, 2016; Grønmo, Hole, & Onstad, 2016).

Indeed, it did after all take humanity several thousand years of mathematical activity until functions were introduced in the 17th century; and even then it took another 200 years to create a solid foundation for functions within mathematics. It is therefore maybe not surprising that students struggle with functions, as functions can be perceived as more abstract than e.g. numbers or geometry. How to meet this challenge has been addressed in contrasting ways. With the “New math” movement in the 1960-70s, it was believed that school mathematics should resemble research mathematics, and attempts were made to introduce functions during the first years of primary school. Eicholz, Martin, Brumfiel and Shanks (1963) did exactly that. This American textbook was translated into several languages, including Norwegian. The reforms were, for many reasons, deemed to be unsuccessful. The pendulum turned with the “back to basics” movement, and functions were pushed up to secondary school, e.g., in Norway. Consequently, most research done on student understanding of functions and graphs[2] is conducted with secondary school students, although notable exceptions exist, e.g. Nemirovsky, Tierney and Wright (1998) and Robutti (2009).

[2] In this paper the term “graph” is used exclusively to denote the graph of a function; and not as the term is used in e.g. discrete mathematics where graph theory is an important topic.

The research reported in this paper was conducted as part of the EU FaSMEd project, which brought together seven European countries and South Africa, researching on the use of formative assessment and technology in mathematics and science education. Part of each country's work was in the form of case study interventions in ordinary classrooms in close cooperation with schoolteachers. In this paper we focus on a particular set of lessons concerning functions and graphs, and how to use one particular type of digital technology to increase student activity and engagement and enhance learning.

Theoretical background

Dreyfus and Eisenberg (1982) pointed out that the function concept is not a single concept by itself but has several aspects and sub-concepts associated with it. Examples are domain, range, variable, growth, slope, rate of change. Further, a function can be represented in several different ways, e.g., graph, table, formula, and verbally, which also contributes to its complexity. DeMarois and Tall (1999) connected the complexity of the function concept to the learning of functions, claiming that "for many students, the complexity of the function concept is such that the making of direct links between all the different representations is a difficult long-term task" (p. 264). Sajka (2003) said that even if considerable time and effort have been spent on functions in the didactic process, it still remains a difficult concept. In her in-depth analysis of a student she found three sources for the difficulties the student experienced: the ambiguities intrinsic to the mathematical notation, the restricted contexts and limited choice of tasks occurring in school mathematics, but also the student's (mis)-interpretation of tasks.

According to Duval (2006) we can only gain access to mathematical objects by semiotic representations. Janvier (1984) distinguished between four representations of functions, viz. situations, graphs, tables and formulae, and made clear the importance of working with transitions between these. Duval (2006) also stressed that "What matters is not representations but their transformation" (p. 107). That is, when learning about a mathematical concept, what students deal with is a representation of the object, and the main difficulty is to change between different representations of the same object. Duval (2006) distinguished between conversions and treatments. Here, treatments take place between the same registers (e.g., changing $y = 2x$ from one particular scale to another), while conversions take place between registers (e.g., reading a table and interpreting the data to a situation from the numbers in it). The latter seems to be far more difficult, while the former is the most common in school. In this way, the Janvier table helps teachers to focus on the change of registers rather than just algebraic manipulation. In our study we investigated the transitions between verbal situations and graphs.

Formulae	Modelling	Fitting	Curve fitting	
Graphs	Sketching	Plotting		Sketching

Tables	Measuring		Reading off	Computing
Situations		Reading	Interpretation	Parameter recognition
To↑ From→	Situations	Tables	Graphs	Formulae

Table 1: The Janvier table

Sierpinska (1992) raised the question what it actually means to understand functions. She identified the multitude of epistemological obstacles students face when trying to grasp the function concept. These are both of a general nature, related to attitudes, beliefs and the philosophy of mathematics (or view on mathematics), to mathematical methodology and technical knowledge, and also schemes of thinking; but also more specifically related to functions and associated terms. Student attitudes towards mathematics, views on what mathematics is and beliefs in what it means to do mathematics, are instrumental in the way students approach the learning of mathematics and problem solving (Sierpinska, 1992, p. 28). For instance, building an understanding of functions involves realising that there are changes everywhere in our surrounding world, and that these changes can be captured mathematically. A way to gain insight into students' attitudes, beliefs and views on mathematics is interviewing students using Q-sort methodology, outlined in e.g. (Watts & Stenner, 2005). Q-sorting is not done to test the content knowledge of the participants nor impose any meanings onto them. Instead it is up to the participants to decide what they find meaningful, and what does and does not have value and significance from the participants perspective (Watts & Stenner, 2005, p. 74).

Arzarello and Robutti (2004) wrote that according to current research some of the most severe difficulties students have in getting to understand functions are related to understanding graphs, and time-dependent graphs in particular. Arzarello, Pezzi and Robutti (2007) focused on difficulties related to the interpretations of graphs, and in particular time-distance and time-velocity graphs. They found that the literature has identified two common misinterpretations; viz. the graph-as-picture interpretation and the slope-height confusion. Hadjidemetriou and Williams (2002) identified several misconceptions, which in addition to the graph-as-picture and slope-height confusion included misreading the scale, students' tendency to reverse the x- and y-coordinates, and inappropriate use of prototypes (e.g., students expecting 'nice' graphs like y = x).

There are several ways to working with graphs in school mathematics, also involving technology. Many research projects have used dynamic software, allowing students to manipulate parameters and studying the covariation of variables. One such project is the SimCalc project. They investigated integration of technology, curriculum, and teacher professional development aimed at improving mathematics instruction in grades 7 and 8 in the USA. Roschelle, Shechtman, Tatar, Hegedus, Hopkins, Empson, Knudsen and Gallagher (2010) describe how students using SimCalc software could control the motions of animated characters by building and editing linear or piecewise linear mathematical functions in either graphical or algebraic forms (p. 839). In a similar manner, Sinclair and Armstrong (2010) studied how stories and graphs are related and how students work with this connection using dynamic geometry software. In their geometrical approach they investigated how having students move 'a character' on screen and seeing how this relates to a time-distance graph.

Nemirovsky (2003) conjectured that "mathematical abstractions grow to a large extent out of bodily activities having the potential to refer to things and events as well as to be self-referential" (p. 106). He also pointed out that thinking and understanding are perceptuo-motor activities (p. 108).

Finally, combining relevant software and bodily motion has been addressed. Arzarello and Robutti (2004) performed an experiment with motion sensors and graphic-symbolic calculators with 14-15-year-old students. The students would produce and interpret graphs and number tables describing motions. The aim of the study was to analyse the cognitive processes of students when they were involved in constructing knowledge of mathematical objects. A result of their research was the claim that students can grasp mathematical concepts through meaningful sensory-motor experiences if they are encouraged to communicate and have the necessary support (p. 308). Arzarello, Pezzi and Robutti (2007) pointed out that teachers can use new technology to design experiences for students "where graphs can be presented in a dynamical and genetic way" (p. 135). This contrasts with the static way that functions and graphs are traditionally presented in textbooks, which may serve as a hindrance to developing meaningful understanding of the dynamism inherent in e.g. time-distance functions. Robutti (2009) conducted research on time-distance graphs with kindergarten children using motion sensors and calculators, finding that even very young children were able to make connections between the movements they made in front of the sensor and the graph sketched by a calculator.

Robutti (2006) introduced the term *graph sense* as a parallel to number sense (Greeno, 1991) and symbol sense (Arcavi, 2005). Graph sense comprises various competencies

> (…) intended not only as the ability to represent data on a graph or to read graphs, but to decipher the variety of information contained in a graph, at a global and local level; to represent graphs as functions; to distinguish between discrete and continuous representations; to keep in mind the scale factors. (Robutti, 2006, p.117)

In other words, raph sense concerns understanding of features of graphs, abilities to analyse those features, and the awareness of the links between a phenomenon and the graphical representation of it.

To sum up, literature identifies many problematic areas for students in the learning of functions. An important difficulty that has to be overcome is making transitions and understand the relations between representations of functions. Also, several common misconceptions have been identified, not least concerning time-distance graphs. There is evidence that both use of technology and use of bodily motions can help students make sense of graphs and our hypothesis is that letting students themselves experience the time and distance can prevent the obstacles with time-distance graphs. As we have seen, much research has been focused on the cognitive processes of students when they are working with graphs. The affective aspects have been less explored. Developing student understanding of graphs is dependent on developing positive attitudes towards mathematics that involves graphs from the changing world of surroundings, and student beliefs on what graphs are and how they are connected to mathematics, as well as student views on what mathematics in general is, e.g. if it is solely a school subject or if it connects to the real world (cf. Sierpinska, 1992). In order to gain insight into the relations between student understanding of graphs and their attitudes, beliefs and views, it is necessary to expose them to situations where they work with graphs, and give them opportunities to comment on their own experiences. Our research question is thus "How can motion sensor technology help primary school students develop their beliefs, attitudes and views on graphs and make sense of the relations between movement and graphs?"

In this paper we report on work done in primary school with students aged around 11 where we investigated time-distance graphs made by student movement and echo sound technology. We have chosen 11-year old students, knowing that they will not have learned about functions before. Hence, our hope is that student experience with time-distance graphs and making sense of related concepts like slope and scale can avoid some of the difficulties identified in the literature, and help students develop graph sense.

Methods

We first outline the context in which the walking-a-graph activity took place.

In the FaSMEd project we worked with six teachers from three different schools. Our case study was carried out in a grade 6 class in a primary school in Norway. The number of students in the school is close to 600 and the number of teachers around 35. The teacher participating in our study was one of these six teachers The participating teacher had background from general teacher education, with specialization in mathematics and history. At the time of the case study sessions, he had been working as a teacher for 7 years, the last three years at the school where the research was conducted. He had mainly been teaching mathematics, and also some science. During his participation in the FaSMEd project he was teaching the same students, starting in grade 5 and continuing with the same group of students in grade 6. In total there were 31 students in his class, 15 girls and 16 boys.

The topic of the case study classroom sessions was time-distance graphs. Several technological tools and different computer programs had been introduced to the project teachers at FaSMEd meetings at our university. This included the student response systems Socrative and Kahoot, and software for dynamical exploration of mathematics like GeoGebra and Desmos and also the Pasco Echo Sound Systems. Material on time-distance graphs from the FaSMEd collection of tasks (Fasmed, 2018) had also been introduced. Planning of the experimental lessons started at one of these meetings.

Working with mathematical graphs connecting situations and graphical representations is usually not done in primary schools in Norway. According to the national curriculum, students are to use tables and bar charts in connection with statistics, but otherwise functions and graphs is not a specified learning goal for students until after grade 10, which means that the topic will be introduced sometime during grades 8, 9 or 10. This would therefore be the first time this teacher had worked with students in primary school on time-distance graphs. Because of this, he first wanted to test the lesson on a group of students that he considered high achieving and with an interest in mathematics. Subsequently the lesson was repeated with a group of students considered lower achievers. The sequence of lessons are outlined in Figure 1.

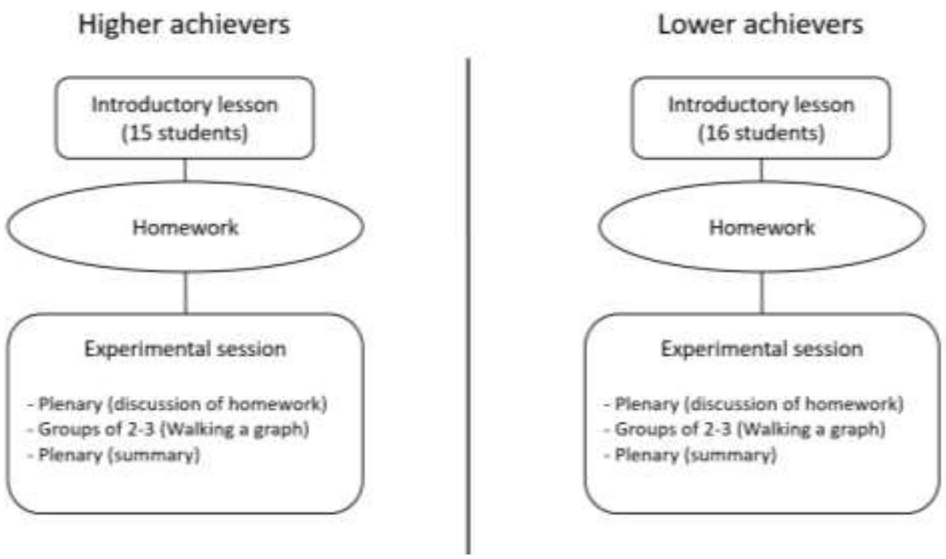


Figure 1. Outline of the lessons

The teacher would give one lesson introducing students to graphs and the connection between graphs and real-life situations. In this introductory lesson students were introduced to graphs displaying a quasi-real-life situation and how to interpret such graphs. The tasks were provided by the FaSMEd project, originally developed by the Shell Centre[3] in Nottingham, UK. The texts were translated into Norwegian by the Norwegian FaSMEd team.

[3] Interpreting Distance-Time Graphs. © 2012 MARS, Shell Centre, University of Nottingham

In Norway, students are usually given homework to elaborate or train on topics introduced in school. As homework after the introductory lesson, the students had worked with two tasks. The first task was about a girl walking along a road from home to the bus stop. A graph was given, and the students were supposed to give details about her walking path (Figure 2). The second task was about hoisting a flag (Figure 3). A story was given: "It is the 17th of May[4] and you will take part in hoisting the national flag at school before setting for the town centre to join the parade." Four different graphs were given, and three questions for the students: "a) Explain in your own words the meaning of each of the graphs; b) Which of the graphs describes the situation most realistically. Please explain why you think so; c) Which of the graphs describe the situation least realistically. Please explain why you think so." (Figure 3). During the plenary at the start of the lesson, the teacher asked each student to describe his/her thinking about the first task, what kind of information could be read from the graph, e.g., what it means that the graph is rising or falling or horizontal.

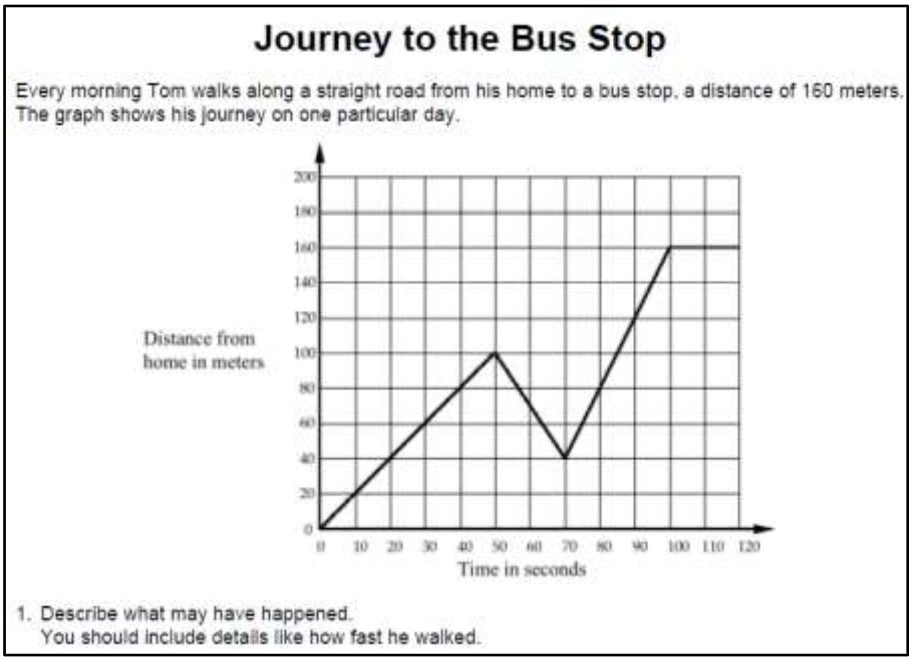
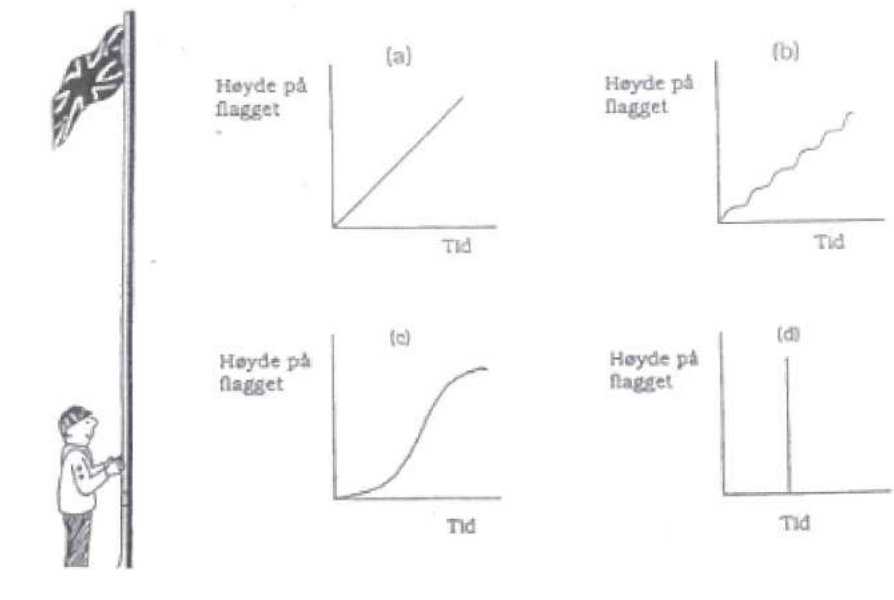


Figure 2. Homework task 1 (English version)

[4] 17th of May is the Norwegian Constitution Day



Historien som følger disse grafene er:
"Det er 17.mai, og du har fått i oppdrag å være med på å heise flagget om morgenen før dere drar til byen for å gå i tog."

a) Forklar med ord hva hver enkelt av grafene betyr.
b) Hvilken av grafene viser situasjonen mest realistisk? Forklar hvorfor.
c) Hvilken av grafene er minst realistisk? Forklar hvorfor.

Figure 3. Homework task 2: Flag hoisting (Norwegian version)

After the plenary discussion of the homework tasks, students were divided into groups of 2 or 3 for the walk-a-graph activity.

The digital tools used in our study are data loggers. These are mainly used in science, but we claim that inclusion of these types of tools may be beneficial also in the mathematics classroom and may contribute to more student-centred practices. According to Newton (2000), “data-logging methods involve the use of electronic devices to sense, measure and record physical parameters in experimental settings.” (p. 1247). Measurements and results of the logging can be displayed on a computer screen, either subsequently or simultaneously.

The digital technology used was two echo sounder devices developed by Pasco[5]. This technology was chosen as the entry level for using it was not too high in the sense that it was easy to use, and therefore the teacher considered it appropriate for grade 6 students. It allowed students to walk back and forth in relation to a logging device, such that a graph was immediately drawn on the computer screen indicating the distance to the device during a set time lapse of e.g. ten seconds.

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[5] http://www.pasco.com

When students walked in front of the echo device, the computer would give a live display of the graph depicting their walk in a time-distance coordinate system.

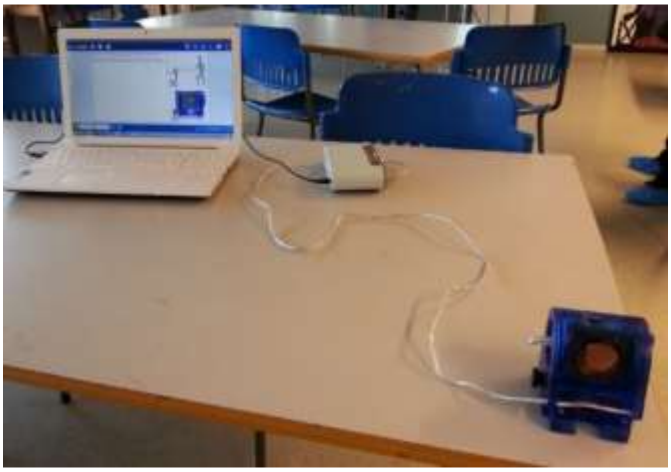

Figure 4. The echo sound logger setup

The computer was set up with an application with premade tasks that were presented to the students. Students worked in groups of two or three, where one student walked in front of the sensor while the other(s) monitored the data collection on the computer. Some tasks were taken from the software bundled with the Pasco software. Instructions for using the software and tasks were translated into Norwegian by the FaSMEd team. Some additional tasks were also added. The tasks were a mix of practical tasks, called "Walk-a-graph", and open-ended questions about interpretation of the graphs from the walks. The tasks were

a) (Warm – up exercise) Try out the software by walking randomly in front of the sensor. See how it works.
b) Walk with steady speed in constant distance from the sensor. What shape is your graph?
c) Start at the sensor and walk straight away from it with steady speed. Repeat this task twice. First walking slowly, then quickly. What is the difference between the two graphs you get?
d) Walk a graph where you try to make the letter "W".
e) Walk a graph and change the speed during your walk. How can a person who didn't observe your walk use the graph to see where your speed was highest?

Task b) asks the students to walk back and forth at constant distance from the sensor without changing their pace, which would ideally produce a horizontal graph. Task c) asks the students to start at the sensor and then walk away from it in a straight line without changing their speed. This would ideally produce a linear graph starting at the origin. Students were supposed to do this twice, at different speed, producing lines with different slopes.

All the graphs and students' written responses were saved and could be used by the teacher for assessment and feedback to the students. These data were e.g. used by the teacher at the end of the sessions to determine which student groups should present their work in plenary. The order was chosen so that each group would contribute something unique about their solutions, bringing in new aspects compared to the previous groups. Examples of such aspects could be straight or curved graphs, steepness, or horizontal parts of graphs. Students were chosen deliberately to give good examples of graphs made and how to interpret them. An example of a student response to task b) is shown in Figure 5: the student has walked away from the sensor and then walked back and forth at the same distance from it. As we see the sensor alternatively hits the student (at close to 3 meters away from the sensor) and the classroom wall (at just over 6 meters).

Figure 5. Student response to task b)

Two weeks after the graph activity, students were interviewed in a Q-sorting activity (e.g. Watts & Stenner, 2005). Five students from the group of higher achievers and five students from the group of lower achievers were selected by the teacher for Q-sorting. The students were selected on the grounds that they were considered by the teacher to be highly communicative. The Q-sorting was done to gain insight into the students' beliefs, attitudes and view on mathematics (cf. Sierpinska (1992)) and its learning and how the technology could help them in the learning of mathematics. An example would be whether they found the use of technology meaningful in the learning of mathematics in the sense that they found it fun, exciting etc. Students were presented with a set of statements printed on cards and asked to sort the cards according to whether they agreed, disagreed or were undecided about, the statement on the card, see Figure 7. The statements concerned beliefs and attitudes towards mathematics, towards the learning of mathematics, and towards use of technology in mathematics. The full set of statements is found in Appendix 1. The statements were developed in the FaSMEd project and agreed upon in an international consortium meeting. The Q-sorting activity was carried out in groups of 4 or 5 students. When working on sorting the cards, students were encouraged to discuss and reason about their choices of placement of the cards. In this way we learned not only whether the students agreed to or disagreed with the statements on the cards, but also why they agreed or disagreed, including whether they related their opinions to the walk-a-graph sessions. The Q-sorting contributed to strengthening the construct validity (Yin, 2018, p. 43), both by giving us multiple sources of evidence and by students 'revisiting' the learning experience.

Data sources collected during the experiment include a) observation sheets from two classroom sessions; b) audio recordings from two classroom sessions, from teacher pre- and post-interviews, post-lesson reflections, interviews and Q-sorting activities with students; c) video recordings from two classroom sessions; d) transcriptions of audio and video recordings; e) photos taken at classroom and Q-sorting sessions, and of student work; f) files and screenshots from the PC screens during student activity; g) teacher lesson plans for two classroom sessions (used preparatory).

In light of our research question, there were three aspects we wanted to study. The first concerns student development of graph sense, the second concerns student use of technology in mathematics, and the third concerns student attitudes, beliefs and views of mathematics. From our experiences with the lessons, the Q-sorting activity and the initial read-through of the transcripts, we could see these three aspects occurring in the data. Audio- and video transcripts and student graphs and written responses in the Pasco software (example screenshots, see Figures 5, 6) were subsequently coded according to these three themes.

Students' development of graph sense is connected to the aspects of the Janvier table and the transitions between different representations of functions. The graph sense theme also concerns emerging understanding of mathematical concepts. Examples include how students noticed important features of graphs, like slope, intersections, and linearity.

The technology theme concerns affordances and hindrances offered by the software and hardware. One of the affordances of the technology used is the ability to quickly represent real world phenomena, giving close to direct links between phenomena and representation, and thus helping students develop awareness of such links. This theme is therefore closely connected to the task of developing graph sense.

A particular aspect of students' attitudes, beliefs and views of mathematics is the ability to relate school mathematics to real life situations. The relation between a phenomenon and the graphical representations of it is at the centre of graph sense. Some of these phenomena are indeed real world situations. It is thus helpful to place students in situations which afford connections between the real world and the mathematical representations of it in meaningful ways.

As we see, the technology and real world themes are also related to student development of graph sense, and thus provides ways to answer our research question.

Whenever we found any interesting sequences, or the meaning was unclear from the transcript alone, we reverted to the original audio- and video recordings for confirmation or clarification.

Results

We first report on findings from the lesson with the walking-a-graph activity. Subsequently we report on findings from the Q-sorting. The full result of statements that students agreed or disagreed with can be found in Appendix 2. From the walking a graph activity we are mainly concerned with evidence of student understanding and developing graph sense. We looked for evidence of ability to read graphs, decipher information at global or local level and relate their movements to what was happening on the screen. The Q-sorting gives us insight into student beliefs, attitudes and possible engagement with the mathematics and the connection between mathematics and real life. Both activities give us insight into issues regarding technology from a student perspective. Taken together, the walking-a-graph activity and the Q-sorting give us insights into the emerging graph sense of students, both as described by Robutti (2006, 2009) and sensemaking that includes the affective aspects that include beliefs, attitudes and views on mathematics.

Making sense of graphs

An important constituent of graph sense is awareness of the links between a phenomenon and the graphical representation of it (Robutti, 2009). We can see evidence that students were able to connect the pace of their walking to the slope of the graph, using their everyday language to articulate their experiences. A student responding to task c) during the group session, trying to describe and explain the difference between the two graphs to the teacher, said:

Student E: It rises earlier because you walk faster.

This relates the time they have walked in front of the sensor (horizontal axis), current distance from the sensor (vertical axis) to the speed of their movement (how steep the curve is), a fundamental relationship in understanding time-distance graphs, and of course in everyday life. Students generally did not master the mathematical vocabulary yet, instead using everyday language. A pair of students, C and F, wrote "The lines appeared much lower when we walked slower". As we see, these students refer to the appearance on the screen, whether the line (graph) drawn from their walking would be above or below their previous line according to their walking speed. In these instances, it was natural for the teacher to introduce relevant vocabulary, like e.g. in this instance 'slope'. Thereafter it was possible for the teacher to use slope in his dialogue with

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students. The dialogue below concerns task c), when students were working in groups of 2 or 3 walking graphs:

Teacher: You got different slopes. Why is that?

Student D: On this I walked a bit faster

Student D and her colleague had walked twice straight away from the sensor at steady speed, first slow and then faster. The students were able to recognize which graph was made from the slow walk and which one from the faster walk. Having been introduced to the word ‘slope’, they were able to communicate with the teacher in a meaningful way. Understanding the relation between their walking speed and the slope of the graph can be seen as deciphering information contained in a graph, thus part of developing graph sense in the sense of (Robutti, 2006). In addition, it concerns learning about real-world experiences and graphs and so relates to the affective sides of making sense of graphs.

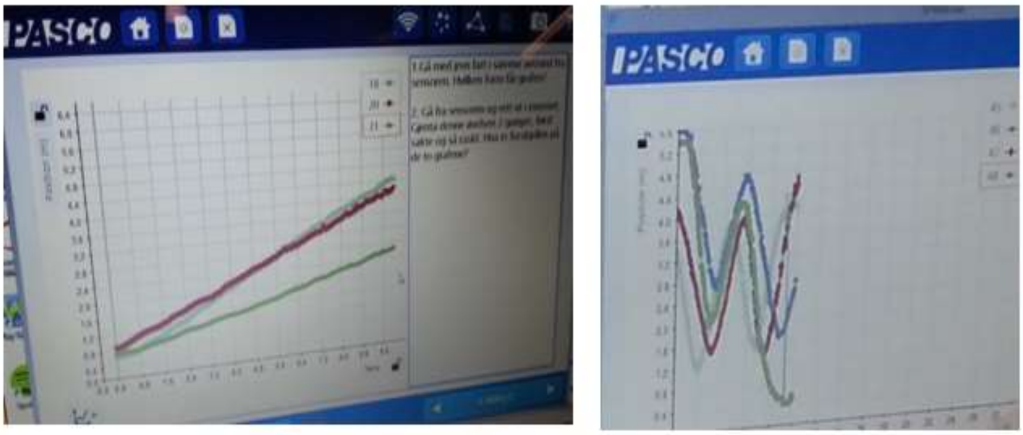

Figure 6. PC screenshots. Graphs where students were supposed to walk 1) with constant speed, and 2) to form the shape of a W.

The software made it easy to spot the difference between slopes, and all students were able to both experience the difference and comment on the various graphs made. Working with task e), students experienced that a single graph can have varying slope. Some student descriptions of the graphs made were “The faster you walk, the steeper it gets”, “If you walk slowly first and then faster the graph suddenly goes upwards”, “When I walked fast it was steep and when I walked slow it became slack”. Here we see three important aspects of graph sense (Robutti, 2006) emerging. Students are 1) reading the graphs produced on screen, 2) deciphering the inherent information about speed and connecting it to their own walking; and 3), commenting on the global properties, like the overall look of the graph, and local properties, like where the graph changes slope due to speed changes. Students also experienced the fact that a graph does not have to start at the origin. This was evident when working with task d) where they were supposed to try walking in a way that would produce the letter “W” as graph:

Student H: You have to start far away [from the sensor] because then it goes downwards and then it goes upwards and then it goes downwards.

The student argued that to get the shape of a W, the starting point of the graph has to be somewhere else than the origin. This means that the student has to start walking at some (more or less arbitrary) distance from the echo sensor, and then walk towards the sensor to make the graph decline to the right on the screen. We see here that they are experiencing a) that a graph can cross anywhere on the vertical axis, and b) the relationship between distance from the sensor and time passed. Their descriptions and discussions do not use mathematical vocabulary or concepts. Rather, they are describing what they see in everyday terms. Students’ use of natural language and their reading of the graph exemplifies that they are in the process of learning to change registers (Duval, 2006), and not only operate within the same register.

Tasks b), c) and d) concern starting with a situation and making a graph to match the situation described. This contrasts with task e) where the students were supposed to make a graph with varying speed and then explain how and why the graph made matched the walk. In this way we can see that students work with and are made aware of both directions in the transition between graphs and situations in the Janvier table (Table 1).

When the teacher asked Student D about the different slopes, the student could identify the slope as a global feature of a graph and relate it to the speed of the walk. When walking the letter W, it was necessary to also relate to local features like turning points (extremal values) and where to start on the vertical axis (the distance from the sensor, the intercept with the y-axis). Thus, students were again exposed to situations involving both local and global features, central features of graph sense (Robutti, 2009).

Real world connections

Classroom activities like these, involving bodily movement, can naturally make students connect their mathematical activity to other aspects of the real world. Having themselves produced graphs by walking in front of the sensors, affords making connections between real world situations and graphs also in other contexts. During Q-sorting, we found that students generally agreed to statements connecting mathematics to real life. E.g., students agreed to the statements “Mathematics helps us to understand our surroundings”, “Mathematics is used in everyday life”, and “Mathematics is important” (see Appendix 2). Further, they disagreed with the statements “Mathematics is only for the classroom, not for real life outside school”, “I can do without mathematics” and “Mathematics is not relevant for my future life”.

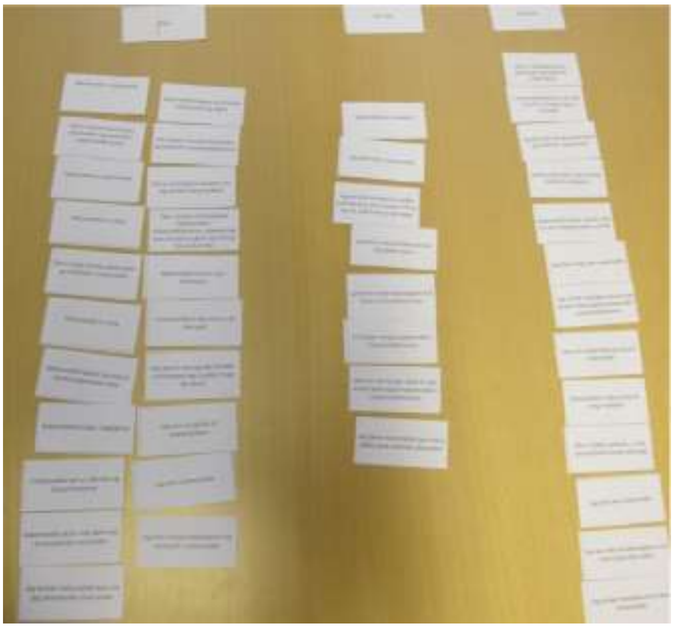

Figure 7. Result of Q-sorting in one group

The Q-sorting activities were done around two weeks after the time distance graph lesson with motion sensors. We may therefore claim that there is some evidence indicating that the lesson had made them aware of, or strengthened their awareness of, connections between mathematics in school and real world situations that can be described by mathematics or where mathematics is used. In particular, it may be easier for students to understand graphs as something that can be used when describing real world events. Students also agreed that "Mathematics is important", e.g. claiming that

Student C: We use it all the time. Everywhere. In the shop. (…) On trains. Airplanes. The bus.

Most students know mathematics can be connected to shopping. Student C's statement also indicates that s/he is able to see the notions of time, distance and speed experienced in the walk-a-graph activity as relevant to real life travelling, like on a bus or train. Since we interviewed the students during Q-sorting, we do not only get knowledge about what they tend to agree to, but also their reasoning and explanation for doing so. As in the case with Student C, s/he argues for the importance of mathematics by relating to real-world situations that, apart from the shop example, were part of the discussions in the walk-a-graph sessions.

It seems that these groups of students were able to see mathematics as relevant for themselves and for real world situations, hence displaying a wider view on mathematics and belief of what mathematics is (cf. Sierpinska,1992). These are important for the development of graph sense, both in the sense of Robutti (2006, 2009) and in a wider sense that includes affective aspects as important in mathematics learning. During the echo sound activity, the students had to relate what they were doing physically, i.e. the way they were walking in front of the sensor to the graph the software would display on the PC screen.

The echo sound activity made this lesson stand out from ordinary mathematics lessons. The tasks were considered different to normal mathematics exercises on two accounts. First, students were not used to doing mathematics tasks using computers. Second, in the class room they usually have to compute things, whereas in these lessons

> Student B: There were word problems and we had to do things.

Another student paid particular attention to the bodily movement in the walk-a-graph lesson:

> Student A: It was very different (…) In maths lessons we never move, we sit at our seat; except sometimes we go out to do measurements, but that is always during summer.

In other words, students described the walk-a-graph lessons as something different to traditional mathematics lessons, and thus closer to real world experiences. This indicates that students may have altered their beliefs of what a mathematics lesson constitutes, and possibly also their view on mathematics itself.

From our observations we see that students are in the process of making sense of graphs, both by connecting their own movements in front of the sensor to the graphs being made on the computer screen, and in connecting to everyday life. In our view, students sensemaking consists of both developing graph sense as described by Robutti (2006, 2009) and also an awareness of how mathematical graphs is a possible and useful way of describing the world. Making sense of graphs thus include both affective as well as cognitive aspects.

Technological aspects

We experienced two main types of technological issues concerned with the walking a graph activity. The first relates to *the artefact itself*, i.e. the hardware-software configuration. The technical setup with echo sounders was unfamiliar to most of the didacticians, as well as for the teacher and the students. In addition, the software had several minor bugs, some causing unpredicted results and stops in the activity. A student said that "Nothing is happening here", upon which the didactician investigated the matter and figured out that a minor alteration in the software setup had caused the measurements to not be recorded. The setup and inner workings of the echo sounder hardware and software was left to the didacticians (and the teacher), and not the students. The students were supposed to focus their attention on the mathematics, not the technology itself.

The second issue relates to the students' *understanding* of the hardware-software setup. Students did at times find it difficult to see the direct connection with their moving and the graph that appeared on screen, e.g. how to make sure that the graph started exactly where they wanted it to start.

Teacher: It stops automatically after ten seconds, so you will measure for ten seconds, so you must, in a way, be ready to begin walking when you… so maybe one can walk and one can press the start button, for instance.

Student: Is it possible with a longer time span? To adjust it?

It is possible to adjust the time span of the measurement in this software, but it requires some tweaking which would disrupt the activity. As well as the time span of the recording, there were limitations on the distance the sensor could measure.

Student: It said, I stopped at the end, because it could probably not see me.

The student realized that he had moved too far away (i.e. out of range) from the sensor. Both types of issues experienced relate to the setup of the echo sounder, and are thus avoidable. One could be addressed by the students by assuring that they are within the range of the sensor; whereas the first issue described needed tending to by the instructor. In light of these two issues we note that there was indeed some time invested in dealing with technicalities before we could consider the tool as being transparent and so allowing for the focus to turn to graph sense development.

The teacher asked a student working with task e) how you can find from the graph *where* you walked faster, pointing at a screen with several graphs from different walks, the graphs being in different colours:

Teacher: If I did not see you walking just now, but only looked at that [the screen].

Student G: They are all in different colours.

The student is commenting on the colouring of the graphs made by the software to distinguish the graphs, while the teacher expects the student to reason about the mathematical properties of the graphs on the screen:

Teacher: Yes, but what if we only consider one measurement.

Student G: Yes, if you remove them after each walk you will only see the one you just walked.

Teacher: Yes, but now you changed speed. You walked both fast and slow.

Student G: Yes.

Teacher: How can I see, if I didn't see you walking, how I can determine from the graph where you walked faster?

Student G: You can see, because, first it is quite slanted, and then it goes straight up.

In the software, all graphs produced are kept on the screen until they are removed or hidden by the students. The student first responded regarding all the graphs displayed on the screen whereas the teacher wanted to focus the attention on one particular walk. Deciphering the variety of information contained in a graph is an aspect of graph sense. In the transcripts above, the teacher used the features of the technology to facilitate the deciphering for the students. Student G showed that s/he has the idea of slope in mind without having acquired the mathematical vocabulary to explicate it.

According to both the students and the teacher, the technology provided affordances for learning about time-distance graphs. After trying to form the shape of a W by walking, students were eager to try more. Also, in the interviews, students said the tasks in this lesson were more challenging than the mathematics tasks they normally work with, e.g., in that they had to explain how they did things. Being challenging is not really a bad thing, and students said they found the sessions to have been great fun and exciting. They claimed that they had learned a lot about graphs. During Q-sorting, students who were agreeing to the statement "I can better understand when I use the technology tools in our mathematics lessons" said they agreed to that statement referring to learning about graphs:

Student E: I learned a lot about graphs and how they change with the computers

A statement from a student saying that s/he learned a lot, cannot be taken as definite proof that learning took place, but it does indicate an openness and eagerness from the student to learning and also a developing understanding of what graphs are about, and thus an emerging graph sense.

The walk-a-graph activity prompted student communication and discussion. Students agreed to this in the interviews, and also in the post-lesson interview the teacher found that students who normally keep quiet were engaged in discussion.

Teacher: In particular, some of the girls in the last group, they were talking, usually they are very quiet. Now they talked, without me having to point at them, prompting them; now they gave their opinion (…) And I was positively surprised at how easy it was for them, to listen to each other's arguments.

When the lesson was repeated with students that were considered to be lower achievers, the setup remained unchanged, making it more relevant for comparison. We found, and also articulated by the teacher, that it was difficult to distinguish between the first session with higher achievers and the second session with lower achievers.

Teacher: It was indeed very similar (…) maybe these needed a little bit more time. And I would be tougher, push the others more. (..) But I think they were clever, they were good at cooperating, learning from each other. (…) It shows that if you have open and good tasks, you have a lot of differentiation included.

It was obvious that even though these type of activities with graphs are usually not done in primary school in Norway, it would not have been too difficult to incorporate it, and that this is a topic which could easily have been done at 6th grade. The teacher said:

Teacher: I think, interpreting graphs, it could have been done quite easily. (…) I think this might be more fun in primary than in lower secondary school. They still find it exciting with graphs (...) they are more curious and less biased.

In the interviews all students said that they had enjoyed taking part in the project and performing the lessons with graphs.

Student I: In my opinion everything was good (…) We learned a lot about graphs.

Even if these lessons involved unknown technologies for the students, it was obvious that they appreciated working with it, and that technical difficulties were considered minor. During Q-sorting this was further confirmed by students when they agreed to statements like “Using technology in mathematics is fun” and “When we use technology during the mathematics lesson I quickly understand if and why I am wrong”. This echoes well with the digital tools’ affordance of giving non-judgmental feedback to students. Likewise, students were disagreeing with “For me technology does not work or help” and “I do not like using technology in mathematics”. It seems that the students and the teacher were engaging in an activity that provided opportunities to develop students’ graph sense.

Discussion

To support the learning of functions, many different kinds of digital tools have been used (e.g., Roschelle et al., 2010; Sinclair & Armstrong, 2011; Tan, Hedberg, Koh, & Seah, 2006). Regardless of point of view on learning outcomes from using technological tools, it is important to realize that use of technology is more than the introduction of new tools. In a survey on mathematics teachers’ use of technology in England, Bretscher (2014) found that while ICT might contribute to change, the direction of this change was as likely to be towards “more teacher-centred practices rather than encouraging more student-centred practices” (p. 43). Our experience indicates that activities with echo sound technologies may naturally contribute to student centred practices and engagement. In the SimCalc project (Roschelle et al., 2010) a main idea was to democratize access to the mathematics of change and variation, thus making mathematics more meaningful to a broad range of students. In the FaSMEd project one of the foci was to support teachers use of technology with low achieving students (Fasmed, 2018). We found that both those that were considered high and those considered low achievers were able to engage meaningfully in the walk-a-graph activity.

Students got hands-on experience in using modern technology and using their own physical movements to create something to talk about mathematically. As we saw, this made the lessons stand out from ordinary mathematics lessons, and thus contributed to broadening students' view of mathematics and beliefs about what mathematics constitutes. Acquiring experience with new technologies can be an educational goal in itself, as digital skills is identified as one of five basic skills in the Norwegian national curriculum. In particular, echo sound technology is not common in the classroom, but it is well known in other aspects of life. In the interviews, students claimed using technology in mathematics was an important part of what they had learned, and which distinguished these lessons from ordinary lessons. In traditional data logging experiments, students might see the data collection and the data analysis as two separate entities as these are separated in time (Barton, 1997). In our experiment, the time gap between the collection of the data and the displayed graph is narrowed down to practically zero. In this respect, this activity also resembles working with dynamical graph tools, like GeoGebra or TI-Inspire. The latter software tools allow students to explore graphs by manipulating parameters within designated bounds, while walk-a-graph changes freely the look of a graph only limited by the range of the echo sound device. This is more in line with the work with motion sensors done by e.g. Arzarello and Robutti (2004) and Robutti (2009). In light of our findings, we see students being positive towards mathematics as a way of describing the world. So we see that the concept of graph sense could be extended to include students' views on graphs as a relevant description of real world phenomena. Students who agree with statements of mathematics being descriptive of the real world would probably be open to accept links between mathematical functions and everyday life.

There are, however, other gaps to be considered in addition to the time between a walk and the graph produced. There is also a "gap" between the phenomenon happening in the classroom and the corresponding picture produced on the screen. The role of task a) and task b) was to make students familiar with the constraints and support that the hardware-software setup could provide. Even though the transition from a three-dimensional walk in the classroom to a two-dimensional drawing on screen is not an obvious one, the students would relatively quickly work on noting important features of the graphs corresponding to features of their walk, that is, developing graph sense, and not too much on meddling with software issues. Hence, the tools were made more or less transparent in a reasonably short time span.

As evident from the Q-sorting data, students were able to see the relevance of mathematics in everyday life. The walk-a-graph activity provided real-life data, not only quasi real-life data as is most common in school mathematics. In addition, using their own body as moving object may have made the experience even more realistic to the students than moving a fictional character on screen as in the SimCalc project (Roschelle et al., 2010) or the approach described by Sinclair and Armstrong (2011). Data from the Q-sorting activity showed that the lessons reported on here may have had positive impact on the epistemological obstacles as highlighted by Sierpinska (1992), like attitudes and beliefs.

The walk-a-graph activity explored two aspects of working with graphs. On the one hand, students had to translate a given situation into a movement in front of the echo sound device, observe the graph being plotted on the computer screen and adapt their movement to change the graph as needed. On the other hand, students would interpret a graph plotted on the screen into what kind of movement that this would correspond to. Referring to Table 1 (Janvier, 1984), we see that what the students had engaged in was making a transition between a situation and a graph and vice versa. However, the typical sketching activity proposed by the Janvier table when working with functions usually has a different feel than in this experiment. Not only is the sketching part of the activity itself done in a kinaesthetic manner. There is also a dual aspect in that the students continually interpret the graph whilst the graph is sketched by the program on the screen. This way we can say that students work simultaneously with two cells in the Janvier table, giving further evidence that changing registers (Duval, 2006) is a difficulty that can be overcome by using appropriate teaching materials. The complexity of making direct links between representations as highlighted by DeMarois and Tall (1999) can certainly be addressed for the case of language and graphs.

The kinaesthetic part of the activity, the walking, is in itself an important aspect of the experiment. As it turned out, the designated lower achievers were able to perform well and display great enthusiasm during the session. This can be related to the way learning through movement can be an alternative approach to put students in a receptive state, ready for learning. Learning through actually moving your body is rarely an aspect of mathematics lessons, but can certainly encourage engagement as seen in this experiment, and as emphasised by Nemirovsky (2003). In our sessions students not only saw graphs and mathematics, but they got a bodily experience connected to time, distance and speed.

The walk-a-graph activity may possibly address the two main misinterpretations pointed out by Arzarello, Pezzi and Robutti (2007), the graph-as-picture interpretation and the slope-height confusion. The graph-as-picture interpretation concerns interpreting a rising graph as a hill in the landscape. This interpretation can hardly occur in the type of activity with the motion sensor. The students walk on the flat floor and thus they would not see the graph as a rising hill or valley. The slope-height confusion occurs when students interpret the value of a graph at a certain point as the slope of the graph at that point. The activity reported made it easy for the students to clearly distinguish between distance from the sensor and instantaneous speed.

The walk-a-graph activity affords a teacher to naturally address those additional typical misconceptions mentioned by Hadjidemetriou and Williams (2002), viz. misreading the scales, reversing the x- and y-axes and use of prototypes. The scales are directly connected to the time spent walking and distance covered by the walking student. For instance, students walking to make the letter W had to consider how far away from the sensor they should start in order for the graph to start at a certain point on the y-axis, thus considering the scale on the y-axis. In the walk-a-graph activity it was evident for the students that if they came to a standstill, a horizontal graph was produced as they were standing there. This could give the teacher opportunity to help the students make sense of the correct labelling of the axes. The last misconception mentioned, the reliance on 'nice' graphs as prototypes or the belief that graphs have to pass through the origin, does not appear in this setting. Students produce their own graphs and experience different shapes and forms, and certainly not only perfect smooth or symmetrical graphs.

The type of activity exemplified in this experiment is completely devoid of focus on algorithms or procedural performance in the form of computations. Students do not know in advance how to solve the problems presented, and so focus is on developing conceptual knowledge about function graphs. From their statements we also see that they relate mathematical concepts, like slope, to real world experiences, like speed. This is similar to findings in Robutti (2009, p. 68). A well founded conceptual understanding of functions and graphs in a time-distance setting may contribute to better understanding of functions on a more general level. When students encounter functions at higher grades, their conceptual foundation will make it easier to grasp other aspects and algorithmic approaches to functions. In this way this type of activity may help overcome the problems pointed out by Sajka (2003), in particular problems related to the restricted contexts and limited choice of tasks usually found in school mathematics, and also problems connected to students' interpretation of tasks.

Student talk in the mathematics classroom is traditionally limited. As we saw, in these sessions, students engaged frequently and voluntarily in discussions. When doing so, they used the language available to them at the moment, i.e. everyday language. Engagement was high, e.g. evidenced by students wanting to improve their results several times without being ordered to do so. To introduce mathematical terminology and connect graphs to formulae and symbolic algebra, other activities are obviously necessary. The several aspects that Dreyfus and Eisenberg (1982) mentioned, like domain, range, slope etc. may in subsequent mathematics lessons more easily be connected to graphs and functions by referring to the walk-a-graph activity.

The literature suggests that teachers need support of different kinds in order to conduct teaching with new technologies in a meaningful manner. E.g., building on a large teacher survey in Singapore, Tan et al. (2006) suggest that teachers need support from laboratory technicians, data logger training, and instructional material to use data loggers effectively. In our case, none of these were present. We do however acknowledge the collaborative effort between teacher and researchers as instrumental to the success of the sessions. It is also important to stress that learning is not an automatic outcome from playing with technological tools, no matter how sophisticated the tools are. The role of the teacher is instrumental in bringing about learning, as highlighted by Clark-Wilson, Robutti and Sinclair (2014, p. 396).

Conclusion

The purpose of this paper was to illuminate how motion sensor technology can help primary school students make sense of the transition between movement and graphs. We have seen that the activities afforded students opportunities to communicate, both peer-to-peer and with the teacher, even without the teacher having to prompt them to do so. This communication offered opportunities for developing vocabulary connected with graphs, contributing to the students developing their graph sense. Students clearly expressed that the walk-a-graph session stood out from ordinary mathematics lessons, indicating that there has been a change in their beliefs about and views on what mathematics is. By having them move their bodies it was easier for them to connect mathematics to real life situations, and thus helping them to make sense of graphs.

Knowing that misconceptions are abundant and that many students do not see the relevance of functions in school mathematics, the walk-a-graph activity may address these issues successfully.

It seems that technological issues are an unavoidable part of using technological tools in the classroom. In particular this will be the case with tools that are new or originally not meant for classroom use. The affordances offered by the technology are in our opinion outweighing the technical difficulties that occur.

Through the walk-a-graph activity we gained insight into students' cognitive processes in learning about graphs. In the Q-sorting interviews we also gained more direct insight into students' beliefs, attitudes and views on mathematics. As both cognitive and affective aspects are important in the learning of mathematics, making sense of graphs has to include both aspects. Whether there is a modified approach to others that can address these two aspects together may be a goal for future research.

We do not claim that the topic of functions or graphs should necessarily be taught at an earlier stage than what is already common, but our research project has confirmed that relatively young children can perform tasks naturally leading to the formation of concepts relevant to graph sense. We can thus see motion sensor technology as a fruitful teaching aid in the introduction to functions at any stage.

We have tried to come to an understanding of what it means for students to make sense of graphs by both having students take part in an activity where they themselves produced graphs and also by letting them perform Q-sorting. These two approaches gave us insight into both cognitive and affective aspects. There might be other approaches that could help us understand how the cognitive and affective work together; this could be a goal for further research.

Note

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Appendix 1

Q-sorting statements about mathematics (English version)

Mathematics is difficult	Using technology in mathematics is difficult
Mathematics is fun	Using technology in mathematics is fun
Mathematics is important	Mathematics is exciting
Using technology in mathematics is exciting.	Mathematics is something everybody can learn
Mathematics is used in everyday life	Mathematics helps us to see/understand our surroundings
I like mathematics	I like using technology in mathematics.
I can do without mathematics	I learn/understand mathematics best when I work on my own
Mathematics is either right or wrong	I do not like mathematics
I do not like using technology in mathematics	Mathematics is not relevant for my future (life)
Mathematics means exploring and experimenting	To do mathematics means to solve many of the same tasks/exercises
I learn things quickly in mathematics	Mathematics helps us to think systematically and logically
I need the textbook to learn mathematics	Mathematics is best learnt by doing practical activities
Mathematics requires a lot of repetition	Mathematics is only for the mathematics classroom, not for real life outside.
Only gifted people understand mathematics	If I do not understand something, I work with it until I get it right

I feel that I can do/understand mathematics	To learn/understand mathematics depends on the teacher
In mathematics (lessons) there is no room for expressing one's own ideas	Mathematics is best learnt (in collaboration) with others.
I understand better if I work with friends in mathematics	Working with technologies in mathematics is useful
We use a lot of tools in our mathematics lessons	Our teacher in mathematics always uses some kind of technology for the lessons
I can better understand when I use the technology tools in our mathematics lessons	When we use technology during the mathematics lesson, I quickly understand if and why I am wrong
When we work together, it makes sense to use the technology	I feel that the teacher knows much better where we are, when s/he uses the technology tools
For me, the technology does not work, or help	I prefer to talk to the teacher, rather than find out myself with the technology

Appendix 2

Results of the Q-sorting

Both student groups agreed with the following statements: Mathematics is fun, Mathematics is important, Mathematics is used in everyday life, I like mathematics, Mathematics means exploring and experimenting, Mathematics requires a lot of repetition, Using technology in mathematics is fun, Mathematics is exciting, Mathematics is something everybody can learn, Mathematics helps us to see/understand our surroundings, Mathematics helps us to think systematically and logically, If I do not understand something, I work with it until I get it right, When we use technology during the mathematics lesson, I quickly understand if and why I am wrong

Both student groups disagreed with the following statements: I can do without mathematics, I do not like using technology in mathematics, Only gifted people understand mathematics, In mathematics (lessons) there is no room for expressing one's own ideas, I do not like mathematics, Mathematics is not relevant for my future (life), Mathematics is only for the mathematics classroom, not for real life outside, For me, the technology does not work, or help