Magnus Aas Hagebø

Enabling Flexiramp Capability in Unit Commitment Formulation

Master's thesis in MTENERG
Supervisor: Hossein Farahmand
June 2019

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Norwegian University of Science and Technology
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Summary

Following the climate challenges, large investments have been done in renewable energy sources like wind and solar power. Every indication points to that this trend will continue as the cost of these technologies continue to decrease and become competitive to conventional power production. These intermittent sources introduce more uncertainty in the power system, as they are highly dependent on the weather conditions. Power systems needs to adapt to be reliable under these uncertain conditions by including more flexibility.

This thesis focuses on the flexibility resource of fast ramping units. They can be called upon to respond quickly to deviance in weather forecasts. *Flexiramp* products and markets have been introduced in the US as a mean to pay generating units that can provide this flexibility to the power system. This payment also acts as an incentive to invest more in flexible units.

The aim of this thesis is to investigate and evaluate different approaches of including flexiramp in the Unit Commitment formulations used for electricity market clearing. This is done through implementation of existing formulations and running case studies on a system of five generators.

The main contribution of the student is a new formulation of the Unit Commitment formulation that includes flexiramp markets and requirements in a stochastic approach.

Proposals of future work are included in the Conclusion. These are based on experiences gained from working with the implementations of the models.

All implementations of the different models, done in Python/Pyomo, are included and submitted along with the thesis.

Sammendrag

Som følge av klimautfordringene har det de siste årene blitt gjort store investeringer i fornybare energikilder som sol- og vindkraft. Alle indikasjoner tyder på at denne utviklingen kommer til å fortsette ettersom investeringskostnadene for disse teknologiene fortsetter å synke og bli konkurransedyktige sammenlignet med konvensjonell kraftproduksjon. Disse skiftende og periodiske energikildene introduserer mer usikkerhet i driften av kraftsystemet. Dette fordi disse kildene avhenger i aller høyeste grad av været. Kraftsystemet må tilpasse seg for å kunne fortsette å være pålitelige under disse usikre forholdene gjennom å inkludere mer fleksibilitet.

Denne masteroppgaven fokuserer på fleksibilitetsressursen som finnes i generatorer som kan endre produksjonsnivået hurtig. Disse generatorene kan kalles på for å raskt respondere på avvik i værprognoser som påvirker kraftproduksjonen. *Flexiramp*-produkter og markeder er blitt introdusert i USA for å betale generatorer for å tilby denne fleksibiliteten. Dette gir også et insentiv til videre investering i raske produksjonsenheter.

Målet for denne oppgaven er å undersøke og evaluere forskjellige måter man kan inkludere flexiramp i Unit Commitment-formuleringer som blir brukt for å optimere kraftmarkedet. Dette blir gjort gjennom å implementere eksisterende formuleringer og simulere casestudier på et system med fem generatorer.

Studentens hovedbidrag er en ny Unit Commitment-formulering som inkluderer flexiramp og har en stokastisk tilnærming til problemet.

Forslag til videre arbeid innen emnet foreslås i Konklusjonskapittelet. Disse forslagene er basert på erfaringer gjort under arbeidet med de implementerte modellene.

Alle implementasjonene, gjort i Python/Pyomo, av de forskjellige modellene er inkludert og innsendt sammen med denne oppgaven.

Preface

This master thesis marks a conclusion of my Master of Science (M.Sc) degree in Energy and Environmental Engineering with the department of Electric Power Engineering at the Norwegian University of Science and Technology (NTNU). The thesis is written on the field of Energy Planning and Power Markets, under the supervision of Associate Professor Hossein Farahmand, Postdoctoral Fellow Jamshid Aghaei and PhD Candidate Kasper Emil Thorvaldsen. The thesis was completed in the spring semester of 2019.

I would like to express my gratitude to you for providing me with guidance and answers whenever needed. I am thankful for the many discussions that have motivated me to learn more on the topic and work hard on producing models and results. You have helped me gain new knowledge of new programming languages, research topics and methods.

Trondheim, 12 June 2019
Magnus Aas Hagebø

Acknowledgements

My deepest possible gratitude goes to my classmates at the Energy and environmental engineering program. You have kept me motivated through all five years of study, and especially the last semester when we all have written our theses. Special thanks to my friends which whom I shared an office with for the last year. The coffee breaks were always a highlight in the work day.

Thank you to Victoria, for using her design skills to help me make a nice figure for this thesis.

Also, a huge thank you to Kasper who has always been available to help me out with learning Python and Pyomo, as well as helping me discussing results and looking at thesis drafts for me.

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Abbreviations

CAISO	=	California Independent System Operator
CTUC	=	Continuous Time Unit Commitment
DR	=	Demand Response
ES	=	Energy Storage
KKT	=	Karush-Kuhn-Tucher
LP	=	Liner Program
MILP	=	Mixed Integer Linear Program
MINLP	=	Mixed Integer Non-Linear Program
MISO	=	Midcontinent Independent System Operator
pdf	=	Probability density function
PEV	=	Plug-in Electrical Vehicles
PV	=	Photovoltaics
RES	=	Renewable Energy Source
T	=	Tons
UC	=	Unit Commitment

Chapter 1

Introduction

1.1 Brief problem description

In the US, ramp products have been introduced in the market to give an incentive to invest in fast ramping generating units. Midcontinent Independent System Operator (MISO) calls this product *Ramp Capability*, while California Independent System Operator (CAISO) refers to it as *Flexiramp*. This thesis will use the latter term, consistent with CAISO, when refering to ramping products. Up-flexiramp (down-flexiramp) can be defined as

> [...] the designation of capacity in time interval *t* to meet higher (lower) than expected net energy demands in subsequent intervals, at which time that capacity can be optimally dispatched to meet those energy demands [41].

Flexiramp product differs from conventional reserve products like spinning and operational reserves mainly in two ways. First, such conventional reserves are usually designated to be used in the occurrence of a predetermined worst case scenario such as an outage of a line or a power plant. By this difference, flexiramp will be called upon to produce energy much more often than conventional reserve capacity. Secondly, flexiramp products is reserved and paid for in one time interval before it may be called for. This is to avoid the occurrence of an opportunity cost by holding back low cost capacity in one interval to provide ramp capability for later intervals.
MISO reports many benefits of a flexiramp product, both in market operation and system reliability [29]. Among other things, they list

- High cost resources needs to be used less,
- Reduced need for system operators to take actions that differs from the optimal market solution, providing increased consistency of market results,
- Transparent pricing and incentives for the supply of flexiramp,

as benefits to market operation. For the system reliability, these are some of the listed benefits:

- Enhanced incentives to invest in resources that provide ramping flexibility,
- Avoided and/or reduced cost of reserve shortages,
- Reduced need for operator intervention in routine real-time market operations, freeing time to focus on other issues.

Energy markets in Europe lack these flexiramp markets. The market operator in the Nordics, Nord Pool, consider ramping constraints on HVDC cables in their day-ahead energy market clearing [36] but not ramping of individual generators. Entso-E's Market Committee has put together a dedicated group on working with Renewable Energy Sources (RES) and Market design. They are currently working on getting more insight into future technical difficulties to be faced, such as ramping. Entso-E is the European Network of Transmission System Operators, and consists of TSO's from 36 countries accross Europe. They aim to further liberalize electricity and gas markets, and improve cooperation between member countries to improve system reliability.

1.2 Motivation

In 2015, 195 of the countries in the world agreed to reduce the global emissions in the Paris agreement. In the first half of December 2018, United Nations held a climate conference in Poland called COP 24. Representatives from about 200 countries attended, and discussed how the objectives from the Paris Agreement should be achieved through concrete regulations [2]. In other words the climate change problems are still a big concern.

These problems demands a change in how different sectors operates, so that green house gas emissions are reduced.

The electricity generation sector is still a large contributor to these emissions. Coal and gas are used extensively to cover the electricity demand as illustrated in **Fig. 1.1**.

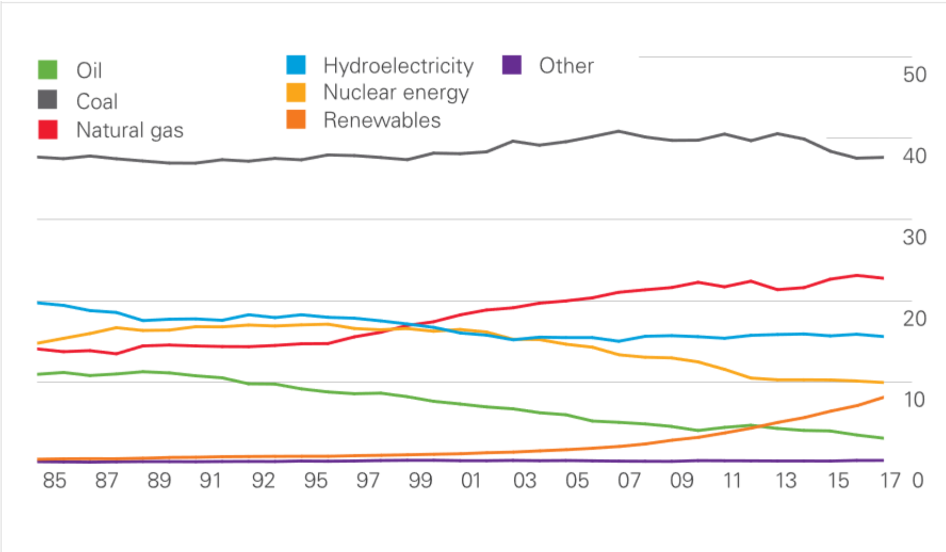

Figure 1.1: Share of global electricity generation by fuel (percentage) from 1985-2017 [1]

A positive trend to note from the graph is the increase in renewable sources. In this case, the two major contributors to the increased renewable penetration are wind and solar power. This trend is believed to continue, as the cost of these technologies have been decreasing rapidly for years. The International Renewable Energy Agency claims in their online article "Renewable Power Generation Costs in 2017" that the cost of renewable generation is becoming more and more competitive with regards to conventional production [6].
Solar and wind electricity production are intermittent resources. A large share of unpredictable energy production result in challenges for the power system operator both in planning and operation. It is worth to mention that an increase in distributed energy resources such as grid connected photovoltaics (PV) and prosumers is observed [3]. A high penetration of distributed generation involves new technical problems, like voltage rising in

the distribution grid.

On a larger scale, i.e. a large wind power plant, the intermittent nature of the wind resource rise challenges in the operation because of the uncertainty. One of the major concerns of the power system operator is to retain the system frequency at nominal value. To ensure this, electric generation must meet the load at all times. So what happens if the output of the wind power plant is much less than scheduled? The needed electricity must be generated by other means. Hence, *flexibility* in the power system is needed. There are many different types of flexibility that can be utilized, but bottom line is that either generation, load or both must be regulated so that the total load and generation is equal.
Taking a look at the wind power plant example, other generating units must be ready to start producing if the wind can't deliver as promised. One can also look at the other way around; if the wind power plant is able to produce much more than scheduled, more expensive units should ramp down their production so that the total cost of system operation is reduced. This deviation from scheduled output can happen because the scheduling is based on forecasts, which may not be accurate. This is due to the, to some extent, random behaviour of the weather. This is were flexiramp comes in handy, as a mean to optimally decide what generators to ramp up or down.

1.3 Research question

This thesis will look at different ways to model a flexiramp product in conventional Unit Commitment approaches, used to schedule and dispatch electrical power systems. The standard approach has been a deterministic model, but recent studies [41] have showed that stochastic modeling yields greater benefits. These studies are reproduced. Also a new formulation will be introduced. This new formulation will incorporate the stochastic approach while at the same time include flexiramp market clearing. This is done to maintain the transparent pricing of flexiramp products as well as the benefits gained from stochastic modeling.

1.4 Scope

The scope of this thesis is limited to shedding a light on how flexiramp products are currently being handled in Unit Commitment formulations, investigating the mechanisms in existing proposed models for flexiramp and evaluating the results. This is done through an implementation of the formulations presented in Chapter 3, in Python. In this thesis, a new formulation will also be proposed, and the performance of it's implementation will be assessed and compared to the two other existing models. The results evaluated will be generator outputs, flexiramp products, social welfare for the system and generator costs.

1.5 Limitations

The simulations of the implementations of the formulations are based on a very limited set of generators and only a few time steps. The results are therefore limited to only 1 hour of system operation, split into four 15 minute intervals. The formulations themselves are limited to only consider generator behaviour and limitations as constraints. Grid constraints and possible contingencies are not included. There are also no regular reserve capacity included.

1.6 Process

The process of working on this thesis has went to quite drastic changes along the way. Starting off, the first few months were spent on researching flexiramp, learning GAMS and begin implementing a model in that framework. GAMS is a programming language specialized for solving optimization problems. Challenges were encountered when building up the models in GAMS, and with limited resources to aid with help, a choice was taken of switching to programming in Python. The next months were then spent learning Python, building up the models in Python, develop cases for studying and gaining and analyzing results. The switching of programming language were a time consuming process, limiting the time available to extend the models and analyze more results.

1.7 Structure of Thesis

This thesis is structured in 6 chapters. This first chapter have served as an introduction to the topic, as well as explaining the scope and limitations of the work done in the following chapters.
Chapter 2 takes on and presents the relevant background theory that builds the foundation on which the problem formulations given in Chapter 3 are built upon. It also includes a literature survey to put this work in a scientific context.
The mentioned Chapter 3 introduces three different Unit Commitment models. One deterministic, one stochastic and one stochastic model with flexiramp included. Here is also included an explanation of how these models have been implemented for testing.
The next chapter, called *Case-Study*, describes the test system and the six different cases used to examine the performance of the three problem formulations.
Chapter 5 presents the outcome and results of the case studies carried out. Results will be provided along with a discussion for easier readability.
The last chapter aims to conclude the results and discussion from the previous chapter, and also discuss further work to be done.

Chapter 2

Background

This chapter will introduce the reader to the theory that is a preliminary to the models presented in later chapters. Previous scientific work in the field will also be presented as a literature survey.

2.1 Theory

Much of the theory presented in this section is taken from unpublished material produced in a project aiming to prepare for this thesis.

The European power markets are cleared as an optimization problem[15]. Therefore, this theory section aims to introduce the reader to optimization and optimization in power systems. Optimization is a branch in mathematics that deals with goal function to be minimized or maximized under certain constraints through designated algorithms.

2.1.1 Optimization

When deciding which generating units in a power system that should be producing at a point in time, there is one thing that is prominent: The supply must always match the total demand, and this should be done with the lowest possible total cost. To ensure this, optimization tools needs to be utilized. The theory presented in this section is based on Appendix B in [30].

The general mathematical formulation of an optimization problem is:

$$
\begin{aligned}
\underset{x}{\text{Minimize}} \quad & f(x) \\
\text{subject to} \quad & h(x) = 0, \\
& g(x) \leq 0.
\end{aligned}
\tag{2.1}
$$

In Equation set (2.1) the function $f(x)$ is called the goal function, which in this case is to be minimized. Usually it is either a cost function to be minimized or a function describing the profit to be maximized. Functions $h(x)$ and $g(x)$ describes the constraints. They are an equality and an inequality constraint, respectively. All the constraints in an optimization problem together define a feasibility region. This is illustrated in Figure 2.1. The variable x is called the decision variable. This is the variable that is controllable, and the whole point of optimization is finding the x that grants the best result.

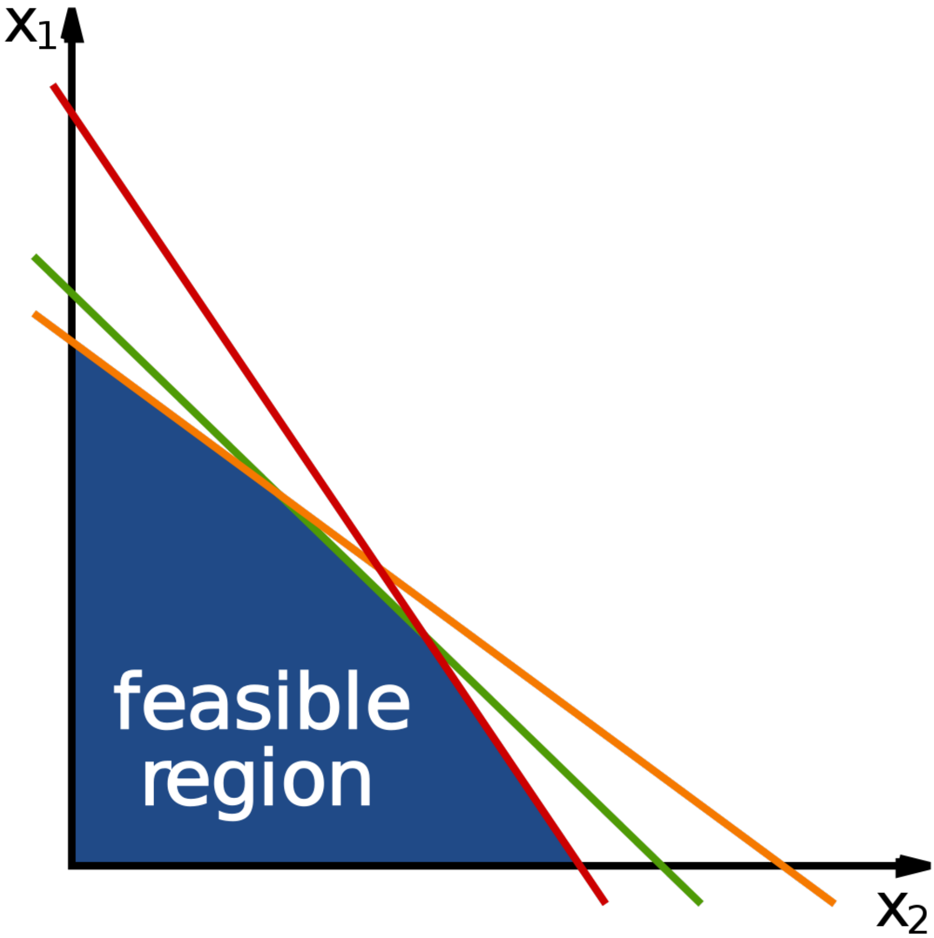

Figure 2.1: The feasibility region of a optimization problem. Taken from [4].

A decision x is called *feasible* if all the constraints are satisfied. The decision is *optimal* if the goal function is at it's maximum/minimum, depending on the wanted outcome. Appendix C.1. shows an example of a linear model, the famous Farmer's Problem, with thorough explanations. An implementation of the model is attached as a digital file to this thesis.

Dual Values

Every constraint in a linear program has a associated dual value. The dual is defined mathematically as,

$$min\{\pi^T b | \pi^T A \geq c^T, \pi \quad unrestricted\}. \tag{2.2}$$

Where variables π are the dual variables, c defines the goal function and $A\&b$ defines a constraint. The dual value reflects how much the goal function value will change per unit increase of the right hand side of the constraint. The dual value is also referred to as the shadow price. This is because the value reflects how much you should be willing to pay for one additional unit of the resource associated with the constraint. By this logic, the dual prices for un-binded constraints are zero, because you still have available unused units of that perticular resource. Dual values are non-zero when the constraint is binding, i.e. the problem is limited by the scarcity of that resource.

Lagrange Method

If functions $f(x)$, $g(x)$ and $h(x)$ from Equation set (2.1) are all linear, the problem is called a linear programming problem. This is the simplest instance of an optimization problem and many commercial solvers are available to solve very large problems. The most famous solving method is probably the Simplex Method. Replacing $f(x)$, $g(x)$ and $h(x)$ in the general formulation with linear functions, the linear programming formulation is obtained:

$$\begin{aligned} \underset{x}{\text{Minimize}} \quad & \mathbf{c}^T \mathbf{x} \\ \text{subject to} \quad & A_E \mathbf{x} = \mathbf{b_E}, \\ & A_I \mathbf{x} \geq \mathbf{b_I}. \end{aligned} \tag{2.3}$$

The matrices and vectors seen in Equation set (2.3) represents the following:

- $\mathbf{c}$ is the cost coefficient of the decision variable $\mathbf{x}$
- A_E and $\mathbf{b_E}$ defines the equality constraints. The dimensions of these must match.

- A_I and $\mathbf{b_I}$ defines the inequality constraints. The dimensions of these must match.

If the functions in the general formulation from Equation set (2.1) are not linear, different approaches and algorithms can be used to find the optimal solution. A common approach is defining the Lagrangian function as follows:

$$\mathcal{L}(x, \lambda, \mu) = f(x) + \lambda^T h(x) + \mu^T g(x). \tag{2.4}$$

To find the optimal solution of the problem the Karush-Kuhn-Tucher (KKT) Conditions are both necessary and sufficient under certain assumptions. The KKT Conditions are formulated:

$$\nabla_x f(x) + \lambda^T \nabla_x h(x) + \mu^T \nabla_x g(x) = 0, \tag{2.5a}$$

$$h(x) = 0, \tag{2.5b}$$

$$g(x) \leq 0, \tag{2.5c}$$

$$\mu \geq 0, \tag{2.5d}$$

$$\mu^T g(x) = 0, \tag{2.5e}$$

Considering Equation set (2.1), the mentioned assumptions for the KKT Conditions are:

- $f(x)$ and $g(x)$ must be continuously differentiable and convex

- $h(x)$ must be affine, i.e. they can be expressed as a linear combination of the components of x and some constant.

- The constraint classifications must hold

The assumptions will not be discussed in detail here. However, they are assure that the problem is feasible and that the optimal solution are possible to attain through convergence. This way they make up the basis for how many solvers find the optimal solution of a non-linear problem.

2.1.2 Unit Commitment

General formulation

The type of optimization of interest in this thesis is the Unit Commitment (UC) problem for a power system. The main goal of UC is to make sure all electric demand is supplied at all times with the lowest possible total cost. It is designed usually to represent operations of a centrally dispatched power system. The objective function of such a problem is dependent on the portfolio of generation available. A general UC formulation for a system might look like this:

$$\begin{aligned} \underset{x}{\text{Minimize}} \quad & \mathbf{c}^T\mathbf{x} \\ \text{subject to} \quad & \sum \mathbf{x} - D = 0, \end{aligned} \tag{2.6}$$

Where

- $\mathbf{x}$ is the vector containing all the outputs from each power plant in MW,
- $\mathbf{c}^T$ is the vector describing the cost of production in \$/MW,
- D is the *Net Load*, which represent the total demand, plus losses in the system with renewable production subtracted.

This is a very simple formulation with only one constraint. The constraint ensures that the sum of all electric power generation equals the total demand minus the non-dispatchable renewable power produced. It is formulated this way because we have no control over output from renewable production or the actual demand. These uncertain parameters must be modelled properly to ensure a good result. The output of the generators, however, are controllable. Therefore, the vector $\mathbf{x}$ is a *control variable*. It is also important to point out that UC can be utilized both in planning and operation of a power system.

Expansion of the general UC

The simple formulation in Equation set (2.6) can be improved by adding a better cost description and more constraints. The adjustments of the general formulation proposed further in this sub-section are based on the lectures of Jordan Kern from the University of North Carolina[24]. The main costs of generation are the start-up cost, the fixed costs of keeping the plant online and the variable costs that generally increase as a function of the amount of production. To deal with these costs, a new set of decision variables and parameters are proposed.

Table 2.1: Variables introduced to expand the general UC formulation. Variables in *italic* text are decision variables.

Variable	**Type**	**Unit**	**Description**
START	Binary	-	Describes the turning on of a plant
ON	Binary	-	Plant online/offline
GEN	Continuous	[MW]	Amount of output from generator
a	Parameter	[$]	Start-up cost
b	Parameter	[$/h]	Fixed cost of generation
c	Parameter	[$/MWh]	Variable cost of generation
MAX	Parameter	[MW]	Maximum output of generator
MIN	Parameter	[MW]	Minimum output of generator
Ramp rate	Parameter	[MW]	Maximum hourly rate of change in power production
R	Parameter	[MW]	Amount of reserves provided by generator

With the variables and parameters from **Table 2.1** an improved goal function can be introduced. This will take into account the different costs occurring in each hour. For a portfolio of generators, i, in several hours, t, the function to minimize becomes,

$$\min_{START,ON,GEN} \sum_{t=1}^{T} \sum_{i=1}^{I} START_{i,t} \cdot a_i + ON_{i,t} \cdot b_i + GEN_{i,t} \cdot c_i \tag{2.7}$$

Equation (2.7) is a goal function that helps decide the outputs in each hour that ensures the lowest total cost for the whole planning period. When talking about variable cost of generation, this is usually mainly fuel costs. Labor and maintenance are so small in comparison that these costs can be neglected[11].

Controlling On/Off-status of units

To control the binary decision variables $START_{i,t}$ and $ON_{i,t}$ some smart constraints must be introduced in the UC formulation.

$$GEN_{i,t} \leq MAX_i \cdot ON_{i,t} \qquad \forall i, t \tag{2.8}$$

$$START_{i,t} \geq -ON_{i,t-1} + ON_{i,t} \qquad \forall i, t \tag{2.9}$$

Equation (2.8) ensures that whenever a generator is producing, i.e. $GEN > 0$, it's characterized as turned on, i.e. $ON = 1$. This way, the costs associated with having the plant online are 'activated' in the goal function (2.7). The variable MAX_i characterizes the physical limit of operation of generator i, as implied in Table 2.1. So this constraint also ensures that generation doesn't exceed the maximum generation of the unit. The second constraint, shown in Equation (2.9), makes sure the start-up costs are activated in the hour in which the generator goes from offline to online. How this constraint works can be explained with help from **Table 2.2**.

Table 2.2: All possible solutions of right hand side of Equation (2.9)

	$ON_t = 1$	$ON_t = 0$
$ON_{t-1} = 1$	0	-1
$ON_{t-1} = 0$	1	0

From this it is seen that there is only one case where $START_t$ is forced to take the value 1. This happens only when the generator was offline in the previous hour and is now turned online. It can be argued that the inequality in Equation (2.9) doesn't hinder the $START_t$-variable in being set to 1 for the other the possible combinations. However, this is not problematic as the lowest total cost occurs when $START_t = 0$. This way, $START_t$ needs to be forced to 1, and if not, it will always be 0.
Likewise, a shut-down cost can be modeled and included in the same way as the start-up cost. This can be seen in [7]. However, there is a common approach to neglect the shut-down cost associated with operating a generating unit, in the UC formulation. Within the CAISO system, market participants must submit The Generator Resource Data Template in order to add or change specific operating parameters to be included in the Master

File [12]. There is no shut-down cost included in this template. Hence in operation of this market, shut-down costs are neglected.

Operating limits

A generating unit usually have a lower operating limit, as well as a maximum one. This lower limit is the minimum generation the generator can deliver while preserving the synchronization with the power grid.

$$GEN_{i,t} \geq MIN_i \cdot ON_{i,t} \qquad \forall i, t \tag{2.10}$$

Equation (2.10) force $GEN_{i,t}$ to be greater than or equal to the minimum output, if $ON_{i,t} = 1$.

An important characteristic of a generating unit is the ramp rate. This can be defined as the maximum hourly rate of change in power production. This characteristic is crucial to model correctly in order to obtain a realistic dispatch of generators. If the ramp rate is not specified explicitly, it will be infinite by default when solving the optimization problem. Different energy resources and power plants have a large variance in ramp rates.

$$GEN_{i,t} - GEN_{i,t-1} \leq RampRate_i \tag{2.11}$$

$$GEN_{i,t-1} - GEN_{i,t} \leq RampRate_i \tag{2.12}$$

Equations (2.11) and (2.12) ensures that generator j cannot change it's output power between time step t an $t-1$ faster than the specified $RampRate_j$. The ramp rate is a physical limitation on the generator, so the value will be dependent on the size of the time step considered in the specific model. The ramp rate needs to be modified so that the value used in the modeling corresponds to the actual ramping the generator is able to do in one interval.

It is important that the constraints are formulated in such a way that the model mimics realistic generator behaviour.

2.1.3 Stochastic programming

From a deterministic model to a more realistic one

The models presented so far in this chapter are deterministic. In the farmer's problem, refer to Appendix C.1, the crop yield is known to the farmer beforehand, and the net load in the UC formulation is also considered to be a known value. However, the reality is more complex. The unpredictable weather will in reality influence both the farmer's crop yield and also the net load of the power system. Wind and solar illumination will directly affect the production of renewable power, and will also affect the temperatures which in turn affects our power consumption. In order to make an optimal decision regarding an unpredictable future, the models must account for this uncertainty. This leads to more complex models and hence more computational power are needed to find the optimal solution. One way to take into account the uncertainty in a optimization model is to introduce Stochastic Programming. This approach takes into account all the different realizations of an uncertain parameter in the system, and the respective probabilities of the realizations. In Appendix C.2., a modified version of the Farmer's Problem is found. To better understand how stochastic models can be made based on different scenarios, it is a recommended read.

2.1.4 Different ways to handle uncertainty in optimization

The model of the Stochastic Farmer's Problem presented in Appendix C.2 is just one of many different approaches to handle uncertain parameters in optimization problems. This sub-section will present the most common approaches and discuss the pros and cons.

Stochastic Programming

Let's go back to the basics of optimization. Considering Equation set (2.3), let us say that A_I, $\mathbf{b_I}$ and $\mathbf{c}^T$ are all dependent on of a stochastic variable λ and it's realization λ_ω. As a decision maker, you must decide $\mathbf{x}$ before the actual realization of λ_ω. Stochastic programming provides a tool to help decide $\mathbf{x}$ so that feasibility is kept for almost all plausible realizations of the random vector. The solution algorithm for solving this linear stochastic program includes descretization of λ if it is continuous. Random vector λ is then modeled as a set Ω of plausible outcomes or scenarios ω, where each $\omega \in \Omega$ has an associated probability of occurence π_ω such that $\sum_{\omega \in \Omega} \pi_\omega = 1$. [30] These ω's and π's are found through a probability density function (pdf). This pdf has to be chosen appropriately so that it describes the phenomenon in question accurately enough.

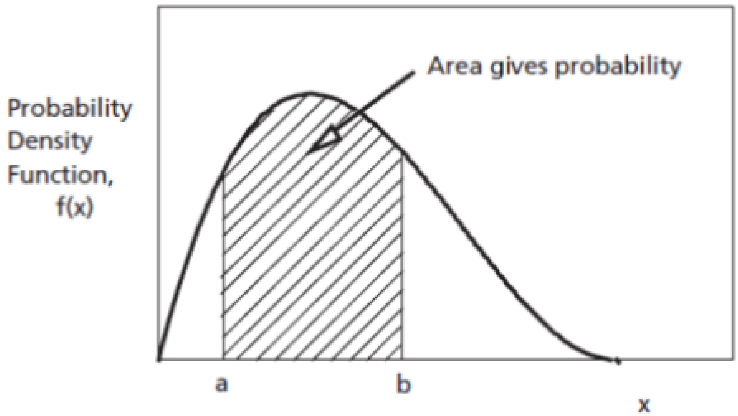

Figure 2.2: An example Probability Density Function. Taken from [5].

Figure 2.2 shows how the probability of x being between a and b can be calculated from the area under the graph. Hence,

$$P(a < x < b) = \int_a^b pdf(x)dx \tag{2.13}$$

describes the probability of x being realized as a value between a and b. This is as mentioned used to make scenarios to put into the optimization problem. In theory, it is possible to make numerous scenarios to cover all possible outcomes. However, this would require a lot of computation time, so instead it is possible to make a set of scenarios to describe the most likely outcomes.

Robust Optimization

Another way of dealing with the uncertainty in optimization problems is using Robust Optimization. Where Stochastic Programming utilizes a probability density function and scenarios to cover the most plausible realizations of the random variable, Robust Optimization uses *uncertainty sets* instead. These uncertainty sets are used to characterize the possible outcomes of the random variables. The key to Robust Optimization is to ensure feasibility for *all* realizations described in the uncertainty set. This way, the solution is only optimal for the worst case. Hence, it is a conservative approach.

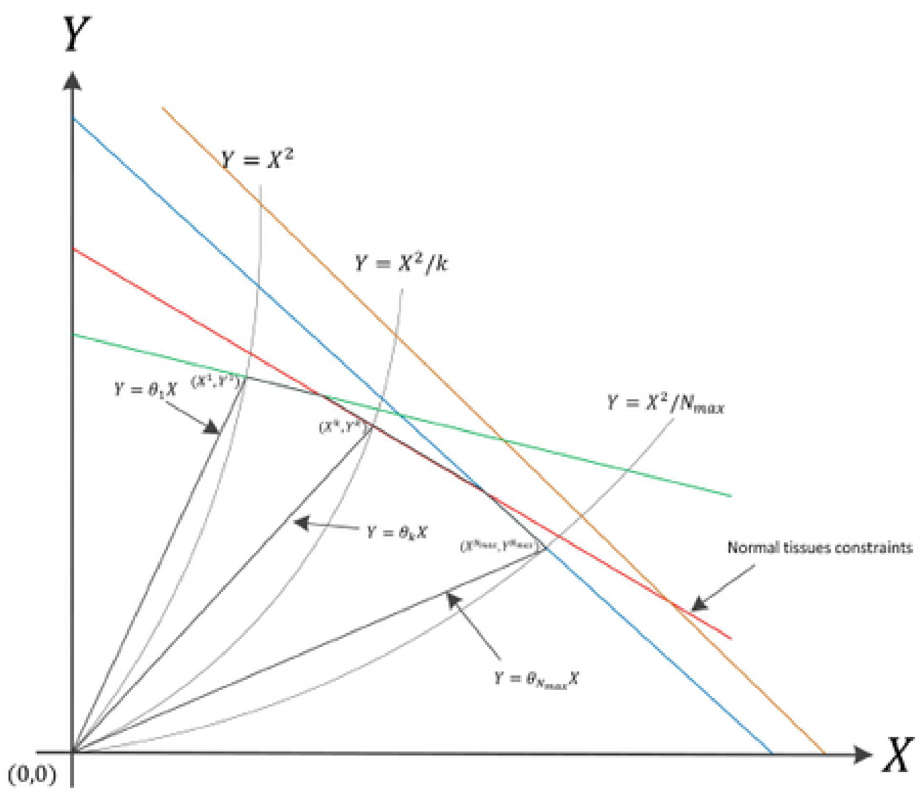

Figure 2.3: Figure showing conceptual Robust Optimization. Taken from [8].

Figure 2.3 can be used to explain conceptually how the feasibility region and optimal solution are affected by the Robust approach. Consider an optimization problem with two inequality constraints. These are dependent on realization of a random vector. Our uncertainty set contains the two

plausible outcomes of the random vector. Then let's say constraint one is described by the blue and the yellow line in Figure 2.3, depending on the two outcomes. The green and the red line describes the other constraint. Looking at the different feasibility regions the different combinations of lines make up, it can be seen that the size vary. Realization of green and yellow line results in the largest area, while realization of red and blue lines results in the smallest area. The idea is then to make sure the output decision variables are feasible for all possible realizations described in the uncertainty set. Hence, the decision will be the optimal of the smallest, and inner most, feasible region.
An important part of Robust optimization is choosing the right definitions of the uncertainty sets. If they are chosen badly, the result from the optimization may become too conservative or too risky. Hence, a solution which grants a much lower profit than expected or even a non-feasible one.

Summing up

Historically, Stochastic Programming has been used the most when dealing with uncertain parameters. There are mainly two weaknesses with this approach:

- It is very difficult to correctly develop scenarios, through distribution functions, to describe the uncertainty of bids from rival market participants.

- Generally, the number of scenarios needed to describe every plausible outcome is very large. This makes the optimization problems big and difficult to manage and solve.

With large-scale problems, the computation time needed to find a solution increase a lot. This is problematic. The big advantage with Robust Optimization is that it leads to smaller problems to solve and hence, shorter computation times. The most obvious drawback of the Robust approach is the conservative final solution. It is only optimal if the worst possible realization of the uncertain parameters occurs. This means that at most times, the system optimized will run at less than optimal, which means a lower profit.

To minimize the drawbacks of Stochastic Programming and Robust Optimization it is possible to do a *Hybrid* of the two. There are different ways of combining the two, and it can be done in the way that is most fitting for the actual problem to be solved. In an article published July, 2018 in *Frontiers*, the authors propose a hybrid model of a unit commitment formulation[19]. Their formulation aims to preserve the reliability from Robust Optimization and the cost-efficiency of Stochastic programming.

2.1.5 Markov Decision Process

When using stochastic programming to model the uncertain reality, there are many different approaches that can be taken when considering how to construct the scenarios. A basis of a probability density function has been mentioned. In this thesis however, one approach is particularly interesting as it works well with the Unit Commitment problem. The approach in question is called a *Markov decision process* [10, 9, 22]. In this process, one aim to find optimal decisions at discrete points in time. The model considers the state of the system in time-step t, then how random parameters influences the next state in the next time-step $t + 1$. An important characteristic of a Markovian structure is that actions and outcomes are only dependent on the current state. This fits well with the time-stepwise decision making in UC. Another key aspect of this approach is defining an expedient action space and state space. As the typical way of solving such a model is through backward recursion resulting in an optimal solution for each state at each stage, a large state space will result in large computation times.

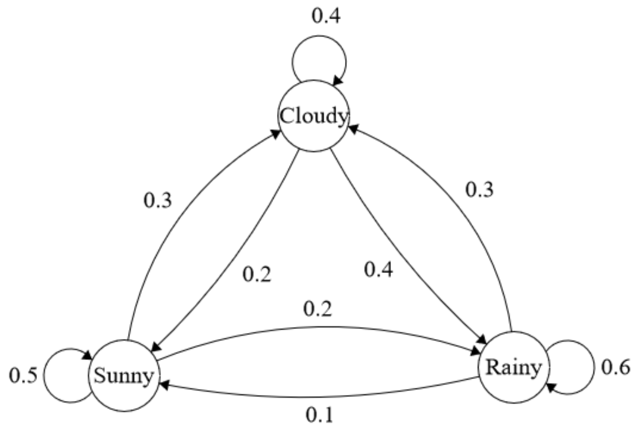

Figure 2.4: A Markov process

To elaborate the nature of a Markov process, **Fig. 2.4** will be used as a basis for an explanation. The three circles in the figure are called *nodes*. They represent the state of the system. In this case, the system describes the weather for a given day. It can be either sunny, rainy or cloudy. The arrows between nodes are called *edges*. The edges describes how the weather can change from one day to the next, given the current state (weather). The numbers on each edge describes the probability of that edge. For example, according to this model there is a 50% chance that there will be sunny tomorrow if it is sunny today. It is important that all edges going out from a node must sum up to 1, i.e. it must be 100% certain that we end up in one of the three possible states in the next day.
The Markov Property is emphasized in [16]. This property is a formal requirement for any Markov Chain. The Markov Property is that the future only depends on the immediate past. For this simple weather-example, it means that the weather tomorrow only depends on the weather today. This holds as the weather yesterday does not influence the probability of the weather tomorrow in any way.

2.2 Literature survey

A thorough literature review on the topics unit commitment and flexibility has been carried out in the preliminary work for this thesis. In order to discover research gaps, a *taxonomy table* is provided in **Table. 2.3**. The different research papers have been classified by five criteria.

- Flexibilty Sources (FS)
- Uncertainty Modelling Method (UMM)
- Type of Uncertainty (TU)
- Type of Formulation (TF)
  - Linear or non-linear
- Horizon (H)
  - Operation or planning

Table 2.3: Taxonomy table. List of abbreviations is provided in the first pages of this report.

	FS	**UMM**	**TU**	**TF**	**H**
[28]	Thermal generators	Stochastic (Normal)	Wind Power	MILP	Planning
[35]	Flexiramp	Bernstein/ Hermite	Load	MILP	Planning
[41]	Flexiramp/ DR	Stochastic (Markov)	Load/ Renew.	Stochastic MINLP	Operation
[27]	PEVs	Stochastic (Markov)	PEV use	MILP	Operation
[25]	Flexible loads(aggr.)	Stochastic (Markov)	Load	MILP	Operation
[26]	Generators and ES	-	Net Load	MILP	Operation
[34]	-	-	Load	CTUC	Planning
[23]	Pumped Hydro	Robust	Wind Power	MILP	Planning
Gaps	**Flexiramp**	**Robust**	**Wind Power**	**CTUC**	**Planning**

Ramping products and flexibility resources are topics of great interest in the scientific community. It is highlighted in [32] how ramp products are cheaper and more reliable than other available means of providing flexibility to a power system. Studies have been carried out to discuss market design [31], implementation ([39],[40],[42]) and analyzing the impact [43] on ramping products. These implemented models are based on robust approach, simulation-based optimization, stochastic and deterministic approaches. Most of them are of the MILP type. With investments in solar and wind power still on a high level, the continued work in this field is highly interesting.

Table. 2.3 shows that the most common approach to deal with uncertainty in the research papers investigated are Stochastic programming. With this as background, the aim of the work in this thesis have been to implement the proposed models in [41], as well as expanding the stochastic model presented there to include a flexiramp product.

Chapter 3

Problem Formulation

This chapter will present three different models. Each model has a different approach as to how the system is optimally dispatched with regards to flexibility and reliability. The first two models, the *Deterministic Model* and the *Stochastic Model*, are presented by Benjamin F. Hobbs and Beibei Wang in [41]. The paper is published in *IEEE Transactions On Power Systems* and are considered a well renowned source. The last model, the *Stochastic Model with Flexiramp*, is an extension of the Deterministic Model. This thesis aims to highlight the differences between the models, and compare their performance through a case-study presented in the next chapter.

3.1 Deterministic Model

3.1.1 Formulation

The mathematical formulation of the deterministic model is shown in Appendix B.2. As the California Independent System Operator (CAISO) operates their system on a combined real-time unit commitment, dispatch and settlement using 15-minutes intervals [13], which is deterministic, it is interesting to see how a deterministic model copes with the extra complexity of a flexiramp market. This model has some simplifications compared to the current standard of CAISO, i.e. dispatch adjustments every 5 minutes are not considered in this model. It is also assumed that the unit commitment decisions are made in one time-step prior to actual dispatch. The initial commitment status of long-start units are assumed to be known and pre-determined. These are considered boundary conditions for the model.

The goal function (6.1a) shows that the system aims to maximize the social surplus. This is done by maximizing the area below the demand curve, minus the total generation cost. This area is shown in **Fig. 3.1**. It is important to note that the equation includes a decision variable squared, d_t^2, which makes the problem non-linear. This complicates the problem and limits the possible solvers. The whole sum is multiplied by 0.25 because the cost parameters are given at a per-hour basis, while the time steps are of 15 minutes.

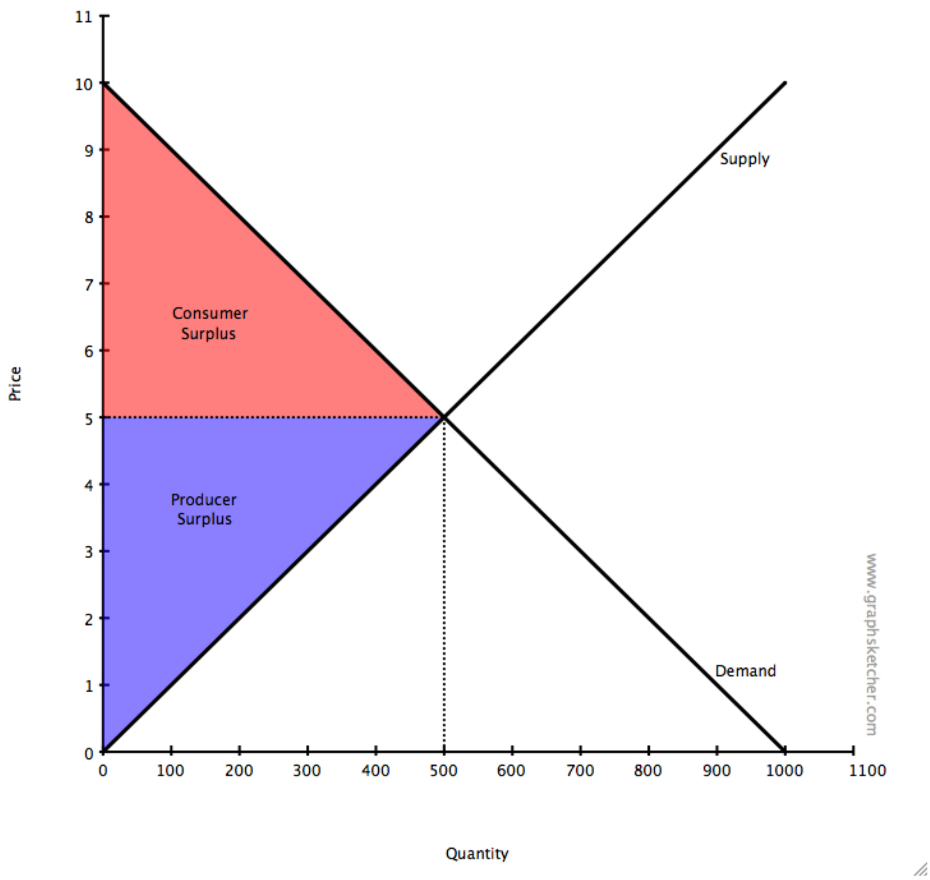

Figure 3.1: The social welfare (total surplus) is the sum of the consumer surplus and producer surplus [44]

The first three constraints, (6.1b), (6.1c) & (6.1d), are market clearing constraints. They assure that enough power is produced to match the net load in each time step, as well as enough up- and down-flexiramp to meet the predetermined requirement.

A common variable for all three formulations is the variable determining the feasible generation, $\overline{g_{it}}$, putting a cap on how much a generator can dispatch at maximum next time period. The variable acts as an auxiliary decision variable for the actual generation g_{it}. Constraints limits $\overline{g_{it}}$ based on the commitment status of the generator. For instance ($v_{it} = 0$ and $v_{i(t+1)} = 1$), meaning that the machine is being turned on, or ($v_{it} = v_{i(t+1)} = 1$), meaning the generator are turned on in subsequent intervals. The commitment status in the previous interval is important as the ramp limit might be lower than the minimum output capacity. In this case the ramp limit must be relaxed. In constraint (6.1i) this is taken care of by restricting $\overline{g_{it}}$ to fit the appropriate situation. When a machine is starting up, (6.1i) becomes $\overline{g_{it}} \leq g_{i(t-1)} + SURR_i$, and when it was turned on in the previous interval it becomes $\overline{g_{it}} \leq g_{i(t-1)} + RR_i$.

The further constraints in the problem formulation are made to assure that the generator behaves like it would in real life. For example, a generating unit should not be able to generate more than the production capacity. In fact, the sum of generation and up-flexiramp cannot exceed this limit. This is ensured in (6.1g). Without this type of constraint a unit would possibly be able to operate at it's maximum while still getting paid for upward flexiramp services for the next interval, even though it can't ramp up any further. The other constraints also consider physical limitations of each unit like ramp rate, start-up- and shut-down ramp rates, and start-up costs. It is worth noting that the *Non-Negativity* constraints does not apply to the up- and down-flexiramp. These are allowed to obtain a negative value. This will be discussed further in Chapter 5.

3.1.2 Implementation

In this subsection, the methodology of implementing the formulation is described. This deterministic formulation was implemented in Python with the use of the otimization framework Pyomo. The script, called ”deterministic.py”, with the impelemtation is attached as a digital file to this thesis.

Input data, including information about the generators, net load, flexiramp requirement and initial values for generation and commitment status were put into the model through excel-files, with the use of Pandas (Appendix A.4). From there, the input data are stored in Python-Dictionaries. The next lines of code constructs a concrete Pyomo model, and sets, parameters and variables are defined. The input data is used to initialize these elements of the model. The Non-Negativity constraints are handled when the variables are defined as within "pyo.NonNegativeReals", which makes the variable a real, positive number. Further, the goal function is defined along with all the constraints. At the end of the script, the model is solved and output saved to a new excel-file.

Implementation in Pyomo required splitting all constraints on the form

$$lowerLimit \leq variable \leq upperLimit$$

into two separate constraints. This complicated the implemented model somewhat compared to the mathematical formulation.

The output of the model is the commitment status, the generation, and provided flexiramp of each unit for every time interval. Also prices for energy and flexiramp products are found as the dual values of constraints (6.1b), (6.1c) & (6.1d). As this problem formulation is a Mixed-Integer type, the dual values are not given straight forward from the solver. To find the market prices, λ_t, μ_t^u & μ_t^d, the whole problem has to first be solved once, giving the on/off status, v_{it}, for all units in all time intervals. Then this solution can be used as input for re-solving the model, only now the commitment status is modelled as fixed parameters. Hence, the problem are then solved as a Non-linear program with no binary variables. This is done in the attached script "deterministic_duals.py"

3.2 Stochastic Model

3.2.1 Formulation

The mathematical formulation of the Stochastic Model is given in Appendix B.3. It differs from the deterministic model in quite a few ways.

The most prominent difference is the lack of a flexiramp product and market clearing. The model is meant to serve as a benchmark for comparison with the deterministic model. This is because CAISO operates with a deterministic model, but stochastic approaches are considered more accurate and efficient. Hence, the deterministic model with the intruduction of flexiramp must be compared to what is considered the best modeling. All equations of this formulation now includes variables with the subscript s. This denotes the scenario. Scenario construction and structure will be elaborated at the end of this subsection.

Equations

The goal function has been adjusted to handle the different scenarios by summing over all scenarios as well as all time steps. The parameter PR_{ts}, which describes the probability of each scenario in each time step is also included. This way, the resulting objective function value becomes an *expected value* of social welfare. Another difference is that DF_t has been swapped with D_{ts}, as we are now looking at the realization of net load in different scenarios rather than the forecasted net load.

As mentioned, this formulation has only got one market clearing constraint, equation (6.2b). This constraint ensures that net load is met by production at all times, in all realizations of net load. The dual value gives the energy price in each situation and each time step. The further constraints are the same as for the deterministic model, except all constraints concerning up- and down-flexiramp are removed. New Non-Anticipativity Constraints are added, (6.2i)-(6.2m). These constraints sees to it so that decisions are made based on the information known at that point in time, as this is very important in all stochastic models [37]. Specifically, two net load scenarios s and s' with the same history of net loads must have the same decisions up to that point in time. As before, a start-up cost is inflicted (6.2h) when the commitment status of a machine changes from 0 to 1 from one time to the next.

Scenario Tree

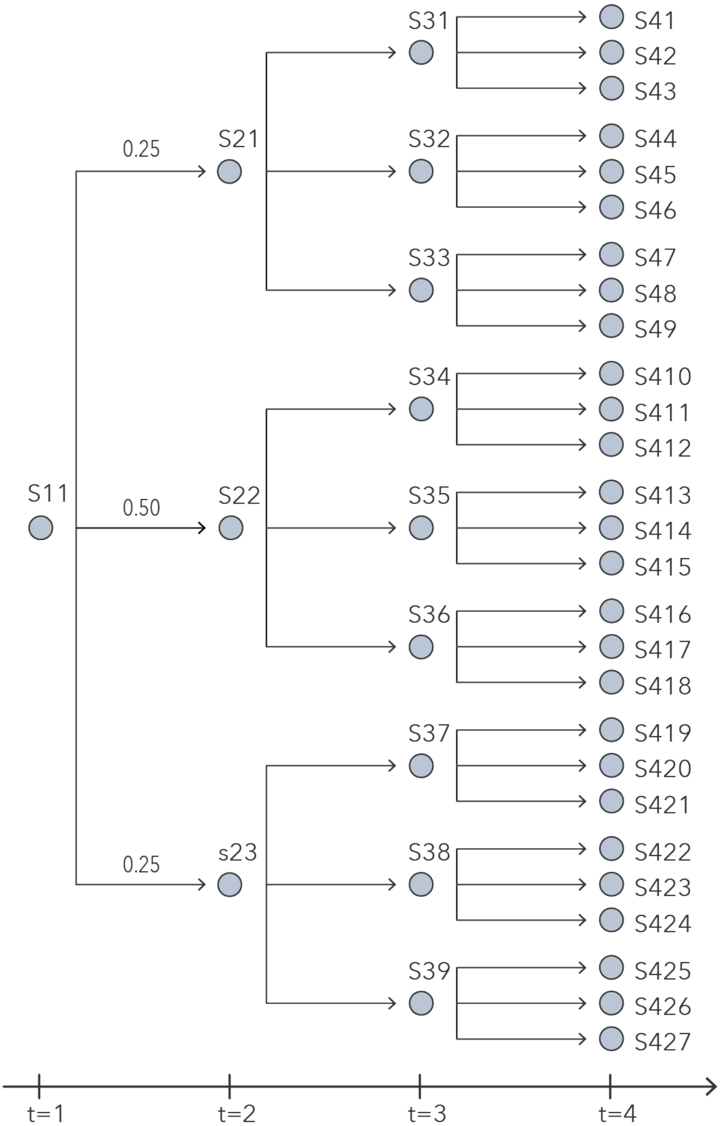

Figure 3.2: Scenario tree describing the 3 possible outcomes in each node and their probability.

Fig. 3.2 shows the Markovian stucture of the scenarios. In time $t = 1$ there is only one node, this is the boundary condition or the starting point. Then there are three different scenarios that can occur in $t = 2$, and three

more for each of these nodes in $t = 3$ and so on. The scenarios describe how the net load might change in the future. A more thorough description of the three different outcomes in each node are given in Chapter 4. The Markov Property can be explained by looking at one of the nodes in $t = 3$. In that given state, all three possible outcomes in the next time step $t = 4$ is dependent only on the system state in that given node. Going further back in time does not give any more information as to how the state will change in $t = 4$. The way we got to that exact state in $t = 3$ does not matter as it does not change the probability of the three outcomes in the next time step. Just like the weather example in Chapter 2, it does not matter if it was rainy yesterday, and what the net load was in time $t = 2$ is equally irrelevant to the current system state.
The four time steps given here are for the purpose of testing the model for these four time steps. The probability tree could go on further, but the downside to that is the rapidly expanding number of scenarios which complicates the solving process. With this current approach, it would be possible to solve on a so called rolling horizon. Where you always look four time-steps ahead, but use the current realized system state as the boundary condition and solve again for the next four time steps.

3.2.2 Implementation

The attached file ”stochastic_hobbs.py” shows the actual implementation of this Stochastic Model. This sub-section will explain it’s components and how it works.

Input data

Just as in the Deterministic Model, input data is fed into the model through an excel-file. The excel-file now contains a scenario description of the possible net load scenarios and their respective probabilities, as well as the generator data and initial system state. The net load of the different scenarios and their probabilities are pre-calculated in the excel-file, according to the Case Study description in the next chapter. So no manipulation of the numbers are needed later in the Python-script.

Pyomo model

The approach in building the model in the Pyomo enviroment are pretty much the same as for the previous model. However, a few changes are made. Now there is a set with scenarios that are defined and used to iterate all the constraints. This set includes both the four time steps and the scenarios connected to each time step. Also, a new parameter is included, called s'. This parameter is included to do the job of the Non-Anticipativity Constraints, by keeping track of what scenario happened in the previous time step.

Output

With such a system of scenarios and generators, the number of output variables are becoming very large. One output variable per node per generator. This is a lot of data to handle and analyze, so the implementation focus on the scenarios that equals the net load used in the deterministic model to get a common ground of variables to look into. The overall expected social surplus, as well as the actual social surplus for the scenarios in question are calculated. Finding the dual value of the energy market clearing constraint, i.e. the energy price, becomes slightly more complicated for this model as v_{its} are 3-dimentional (unit, time and scenario). It is dealt with in "stochastic_hobbs_duals.py" by running the whole model once, then storing the output data of all 200 optimal v_{its} and use it as input in the model as a fixed parameter before solving again. Same as for the previous model, this is done to get rid of the binary variables that makes extracting dual values impossible.

3.3 Stochastic Model with Flexiramp

3.3.1 Formulation

This model is as previously mentioned an extension of the Deterministic Model presented at the beginning of this chapter. It's mathematical formulation can be found in Appendix B.4. The background for this formulation is to make some kind of a hybrid between the stochastic and deterministic formulations. The constraints concerning the flexiramp products are kept, unlike in the Stochastic Model. This was done so that the flexiramp markets could be examined in a stochastic process.

The formulation is a hybrid of the two other formulations in the following way: The constraints are from the Deterministic Model, while the scenarios are the same as for the Stochastic Model. This way, the constraints had to be adapted to hold for the different scenarios described previously. The most prominent change that has been done to the constraints is introducing the indices s and s'. The s describes the different scenarios, while s' keeps track of what scenario and decisions were taken in the previous time step. One of the things to keep in mind when working with a stochastic model is that the future is uncertain. One can not take any decision based on knowledge of the future, unlike in a deterministic model. Hence, the constraints including the index $(t+1)$ had to be shifted so that the decision making is done based on previous and current states of the system. This was done by simply shifting the whole constraint by $t = -1$, so that index t becomes $(t-1)$ and $(t+1)$ becomes t. The variables from the previous time step, subscript $(t-1)$ utilize the index s' to keep track of the state of the system at that time. This makes the Markovian Property hold for this formulation as well. An example showing this is Eq. (6.31).

The output of this model is decision variables for every generating unit, in all four time steps and for every scenario. The goal function, still maximizing social welfare, is weighted by the parameter PR_{ts}, which describes the probability of the occurrence of scenario s in time t. This means that the value of the goal function will be the *expected value* for the system, same as for the Stochastic Model. The actual social welfare will depend on the realization of the uncertainties. A direct comparison between the Deterministic Model and the Stochastic Model with Flexiramp can be done by comparing the deterministic solution to the scenario that corresponds to the "deterministic scenario". The main difference between these to processes are at what time information is acquired. The Deterministic Model will expectantly preform better as it will utilize information "from the future". The stochastic model will behave more conservative, as it holds little information of what the future will bring. It is expected that the faster and more expensive units must be online more often, in case of the occurrence of a scenario that demands a large ramping.

3.3.2 Implementation

The implementation of this model is found in the attached file ”stochastic.py”. It is implemented in the same manner as the two previous models, in the Pyomo environment in Python.

Chapter 4

Case-study

In this chapter the cases to study is presented. The cases are the same as in *Wang, Hobbs (2016)* [41], for comparison of their results with the results from the implementation of their models made in this thesis, as well as the new proposed Stochastic Model with Flexiramp. All these models including scenario trees are described in the previous chapter.

4.1 Generator data

Table. 4.1 describes the five generators used for the case-studies. The generator data includes capacities, ramp rates and different costs. Generator 1 is a cheap base load unit that provides 300 MW of power, with no flexibility. Generators 2-4 are increasingly costly to operate and has an increasingly larger start-up cost. These three generators have slightly different operating limits, but have the same ramp rate. Generator 4 has a larger start-up and shut-down ramp rate than Generator 2 and 3, but at the same time it is the most expensive unit to start up.

Table 4.1: Data for generators in case study. [41] Units are as given in the Nomenclature in Appendix B.1.

Gen i	$\overline{Cap}$	RR	OC	$\underline{Cap}$	SURR	SDRR	HC	NLC
G1	300	0	0	300	0	0	0	0
G2	150	40	20	50	60	60	300	300
G3	200	40	40	50	60	60	600	300
G4	150	40	60	50	100	100	900	300
G5	100	100	200	10	60	60	0	0

4.2 Rising and descending net load

The two different base cases to investigate is a case with an increase in the net load and one with a decrease in the net load. In the rising net load case, we consider an increase of $DNL_t = 40MW$ from interval t to $t+1$, starting at net load $DF_0 = 500MW$. For the case with decreasing net load, the starting point is $DF_0 = 700MW$, and the net load decrease (or increase when considering the negative sign) with $DNL_t = 40MW$. For the stochastic models, DNL_t will represent the *expected* change in net load between intervals. Four intervals of 15 minutes are considered in all cases.

Table 4.2 Gives the starting point for the two different base cases. This includes the initial commitment status of the generators as well as the power output. This can also be considered the boundary condition of the models. These are required in order to get the models to converge to a optimal solution.

Table 4.2: Initial data for the case studies. [41]

	Initial g_{i0} [MW]		Initial v_{i0}		Initial v_{i1}	
	Rising net load	Descending net load	Rising net load	Descending net l.	Rising net load	Descending net load
G1	300	300	1	1	1	1
G2	100	150	1	1	1	1
G3	100	130	1	1	1	1
G4	0	120	0	1	0	1
G5	0	0	0	0	0	0

It is obviously important that these initial values are feasible and within the limits of the generator operation. For example, if a unit is producing the commitment status must be set to 1, i.e. the unit must be turned on. The generator output must also be between the maximum and minimum capacity.

4.2.1 Flexiramp requirement

Within the two base cases, there are three different cases of flexiramp requirement. The basis flexiramp requirement is

$$FRup_{t-1} = DNL_t + ERR_t/2$$

and

$$FRdn_{t-1} = -(DNL_t - ERR_t/2).$$

For all cases the range of error in net load change forecast, $ERR = 100MW$. This means that for the rising net load case, up-flexiramp requirement will be 90 MW while down-flexiramp requirement will be 10 MW. For the descending net load case, up-flexiramp requirement is 10 MW and down-flexiramp requirement is 90 MW.

The two other flexiramp cases to consider are a 20 % increase and a 20% decrease of the base requirement. These flexiramp-cases will not influence the result of running the Stochastic Model, as this model does not include a flexiramp product.

To sum up, there are six cases to analyse:

- **Case 1**: Rising net load with base flexiramp requirement.
- **Case 2**: Rising net load with 20% less flexiramp requirement.
- **Case 3**: Rising net load with 20% more flexiramp requirement.
- **Case 4**: Descending net load with base flexiramp requirement.
- **Case 5**: Descending net load with 20% less flexiramp requirement.
- **Case 6**: Descending net load with 20% more flexiramp requirement.

4.3 Scenario tree for Stochastic Models

The scenario tree shown in **Fig. 3.2** previously is a general representation of how the models are built on a Markov chain basis. For the case-study described on the previous pages, the *Stochastic Model* and *Stochastic Model with Flexiramp* will have three specific possible realizations of the forecasted net load, DF_{t+1}. These three possible realizations are described below.

1. *Underforecast*: The net load are realized as higher than expected from the forecast, $DF_{t+1} = DF_t + DNL_t + ERR_t/2$. This outcome has a probability of 25%.

2. *Correct forecast*: The net load are realized as expected from the forecast, $DF_{t+1} = DF_t + DNL_t$. This occurs with a probability of 50%.

3. *Overforecast*: The net load are realized as lower than expected from the forecast, $DF_{t+1} = DF_t + DNL_t - ERR_t/2$. This outcome has a probability of 25%.

These possible outcomes applies to every node in the probability tree. Note that the *Overforecast* and *Underforecast* scenarios corresponds to the basis flexiramp requirement. Hence, it is seen that the flexiramp product is designed to cope with the possibility of an error in the forecast.

Chapter 5

Results & Discussion

Out of the wast amount of results the case-study produced, some results will be presented in this chapter while the rest will be made available through attached files. The results chosen will be used to validate the models, highlight market operation differences and change in social welfare. This chapter has four sections, one for each Unit Commitment model plus one section summing up. In the first section, the results Deterministic model will be presented along with a discussion of how the constraints influence the optimal solution. The next section will compare the deterministic model results with the output of the Stochastic model. Then, the results from the modified Stochastic Model with Flexiramp will be presented and discussed. Finally, the economic aspects of the different models will be compared in the last section.
Most emphasis will be laid on Case 1 & 4 from Chapter 4. Also Case 3 will be discussed to observe how a change in flexiramp requirement influence the optimal solution. To follow the discussion along easier, it should be kept in mind that generators 1 through 5 are increasingly more expensive to operate.

5.1 Deterministic Model

Solving the optimization problem with five generators for four time steps gives a lot of results to analyze. The main decision variables considered are the power output, g_{it}, up- and down-flexiramp, ur_{it} and dr_{it}, and on/off status, v_{it}. These variables were at the first stage of the analysis used to validate the model and the optimal solution, to see to that the generator behave as it should within it's operating limits, etc. Dual values for the three market clearing constraints are the other variables central to the analysis.

5.1.1 Case 1: Rising net load

Model Behaviour

To validate the results in terms of basic generator operation limits, the generator outputs are compared to operating limits of the generators (refer to **Table 4.1**) and on/off status of the generators.

Table 5.1: Case 1: Optimal commitment of generating units (v_{it}). Deterministic model.

	t=1	t=2	t=3	t=4
G1	1	1	1	1
G2	1	1	1	1
G3	1	1	1	1
G4	0	1	1	1
G5	0	0	0	0

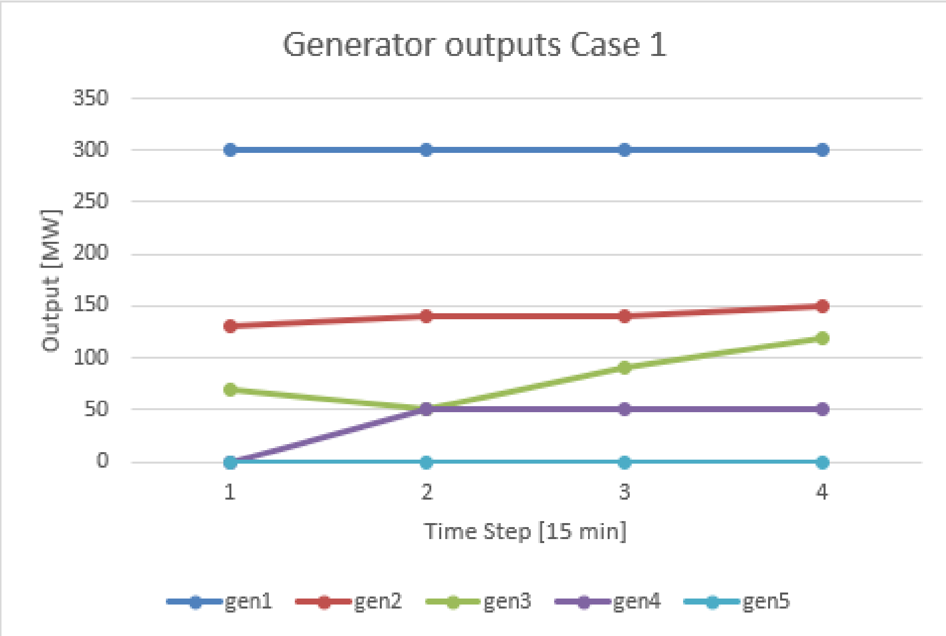

Figure 5.1: Case 1: Generator output (g_{it}) in all time steps. Deterministic model.

Comparing the generator outputs in **Fig. 5.1** with the commitment status shown in **Tab. 5.1** it is quickly seen that generators that are turned off does not have any output. Remember that Case 1 considers a rising net load, starting at 500 MW, then increasing with 40 MW each time step. It is seen that generator 4 is turned on between $t = 1$ and $t = 2$, and starts producing power to meet this increased demand.

The generator outputs are well within the physical limits and capacities as well. Generator 1 operates at a constant 300 MW output, while generators 2-4 operates above the minimum output of 50 MW at all times, when turned on. These generating units also operates below their maximum capacities. The ramp rates are also not violated, as the maximum change in output from t to $t + 1$ is observed as 40 MW for generator 3 between $t = 2$ and $t = 3$.

The aggregated output of the generators sums up to the net load in all time steps. As it should according to the Energy Market Clearing constraint (6.1b).

Table 5.2: Case 1: Up- and Down-Flexiramp provided by each unit. Deterministic model.

	Up-Flexiramp ur_{it} [MW]		
	t=1	t=2	t=3
G1	0	0	0
G2	-40	10	10
G3	40	40	40
G4	100	40	40
G5	0	0	0
	Down-Flexiramp dr_{it} [MW]		
	t=1	t=2	t=3
G1	0	0	0
G2	40	40	40
G3	20	-30	10
G4	-50	0	-40
G5	0	0	0

Table 5.2 shows some interesting incidents, for example the occurrence of negative values. These will be discussed shortly, but first the basics. Generator 1 has only one operating point, which is at 300 MW. This generator can therefore not provide any ramping, neither up or down. Looking at generator 2 in time step $t = 2$ and $t = 3$, it operates at 140 MW output, which is close to it's upper limit of operation at 150 MW. Hence, it is seen that it can only provide 10 MW of up-flexiramp in these periods, because it can't provide more than the maximum capacity.

Considering generator 4 in the first time step $t = 1$. The generator is turned off, but provides flexiramp products because it can, and will, be turned on in the next time step. It provides 100 MW in up-flexiramp and -50 MW of down-flexiramp. The first value is because of the start-up ramp rate, while the latter value comes as a result of that when the unit is committed, it has to at least produce 50 MW. This effect is governed by (6.1l) and (6.1o). By inserting $v_{4,1} = 0$ and $v_{4,2} = 1$ the effects are highlighted. First for the up-flexiramp:

$$-40{\cdot}1-100{\cdot}[0-1]-150{\cdot}(1-0) \leq ur_{4,1} \leq 40{\cdot}0+100{\cdot}[1-0]+150{\cdot}[1-1]$$

$$-90 \leq ur_{4,1} \leq 100$$

Together with (6.1n) that further limits $ur_{4,1}$ downwards:

$$50 \leq ur_{4,1} \leq 150$$

In sum, the up-flexiramp must be between 50 MW and 100 MW for generator 4 in $t = 1$, the table shows that it provides the maximum of 100 MW. As for the down-flexiramp, inserting the same values for the commitment status and generator data in (6.1o) gives:

$$-1 \cdot 150 \leq dr_{4,1} \leq 0 \cdot 150 - 1 \cdot 50$$

$$-150 \leq dr_{4,1} \leq -50$$

which clearly governs the down-flexiramp provided to be maximally -50 MW.
Similar observations can be done for the other flexiramp products provided.

Market Impact

In **Table 5.3** below, the market prices for energy and flexiramp products can be seen for each time step. These are found, as described in Chapter 3, as the dual values to constraints (6.1b), (6.1c) & (6.1d).

Table 5.3: Case 1: Dual values & social welfare for the deterministic model.

	t=1	t=2	t=3	t=4
Energy Price, λ_t [$/MW]	5	8.05	11.02	10
Up-Flexiramp, μ_t^u [$/MW]	0	3.05	6.02	
Down-Flexiramp, μ_t^d [$/MW]	5	0	0	
Social welfare	$15,798,612.50			

The increase in energy price can be seen intuitively as **Fig. 5.1** shows that the more expensive units, generator 3 & 4, gradually increase their output. More expensive production leads to higher prices. The small drop in energy price from $t = 3$ to $t = 4$ can be explained by the lack of any flexiramp requirement in the last time step, $t = 4$. This grants the generators "more freedom" to operate at a more optimal point, which reduce the stress on the Energy market clearing constraint. This trend can be seen for all results across all models and cases. The energy price in $t = 4$ are always lesser than or equal to the price in $t = 3$.

Let's consider time step by time step when analyzing the flexiramp prices. In $t = 1$, the up-flexiramp price is 0. This is because generator 4 is turning on and can provide all needed up-flexiramp alone, all the while generator 2 & 3 are able to provide more if needed. As there is no cost associated with providing flexiramp in it self, only scarcity will trigger price incentives. Taking a closer look at the down-flexiramp in this first time step reveals why there is a price of 5 $/MW for downward flexiramp. As discussed in the previous section concerning model behaviour, generator 4 is forced to provide -50 MW of down-flexiramp. As the requirement is 10 MW, generator 2 & 3 must together provide 60 MW to compensate for generator 4. Remember that generator 1 can't provide ramping. It is seen from **Table 5.2** that generator 2 provides 40 MW of down-flexiramp, which is equal to the maximum ramp rate of the generator. Generator 3 is operating at 70 MW in $t = 1$, which means it can only ramp down with 20 MW in the next interval, as the minimum output of the unit is 50 MW. Hence, if the flexiramp requirement increased a little more, a new optimal solution must be found to stay within operating and flexiramp limits. This new solution would have a lesser social surplus due to the more constrained problem.

For the next time steps, $t = 2$ and $t = 3$, the output of the generators increases which moves the operating point of the units away from the minimum output limitation. With the small down-flexiramp requirement and practically no limitations in providing it, the price goes down to 0. The opposite is true for the up-flexiramp product. The requirement is 90 MW for all time steps, and generators 3 & 4 provides their maximum ramp rates of 40 MW each in order to meet the requirement. Generator 2 however are already operating at 140 MW, which is only 10 MW under it's maximum capacity of 150 MW. Hence, it can only provide 10 MW of up-flexiramp. Since all units are at it's limit with regards to providing up-flexiramp, the price is larger than 0 by the same argument of scarcity as before.
On another note, we can observe how the flexiramp requirement limits the optimal solution in this case. A pure economical dispatch to maximize social welfare would mean utilizing the cheapest generators to it's maximum. Generator 2 is the cheapest, but is forced to operate 10 MW under it's maximum capacity in order for the system to meet the flexiramp requirements. This is emphasized by observing how generator 2 ramps up to it's maximum of 150 MW in $t = 4$ when there is no flexiramp requirement.

5.1.2 Case 3: Increased flexiramp requirement

Generator dispatch and flexiramp

In Case 3 there were an increase in flexiramp requirement of 20%, up to 108 MW up-flexiramp and 12 MW of down-flexiramp. The basis was, as before, a rising net load starting from 500 MW. **Fig. 5.2** below shows how the generator output had to change from case 1 in order to meet the new flexiramp requirement.

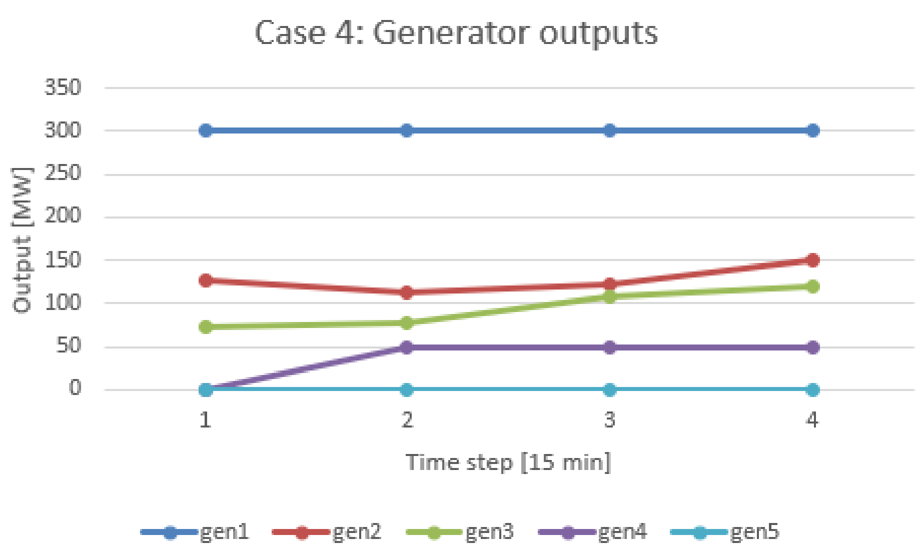

Figure 5.2: Case 3: Generator output (g_{it}) in all time steps. Deterministic model.

Following the same line of arguments as in the previous section on Case 1, the generator output seems to follow the same governing mechanisms as before. The cheapest generator, number 2, must now ramp down it's production in order to meet the up-flexiramp requirement by shifting further away from it's maximum operating point. This can be seen from **Table 5.4** as well, where generator 2 provides 28 MW in both $t = 2$ and $t = 3$.

Table 5.4: Case 3: Up- and Down-Flexiramp provided by each unit, deterministic model.

	Up-Flexiramp ur_{it} [MW]		
	t=1	t=2	t=3
G1	0	0	0
G2	-16	28	28
G3	40	40	40
G4	100	40	40
G5	0	0	0
	Down-Flexiramp dr_{it} [MW]		
	t=1	t=2	t=3
G1	0	0	0
G2	40	12	12
G3	22	0	0
G4	-50	0	0
G5	0	0	0

The table above shows many similarities to the results from Case 1 (**Table 5.2**). The distribution of flexiramp provided among the units are pretty much the same. The increase in flexiramp requirement is covered by generator 2 alone, both for up and down. The results in case 3 are somewhat less ”messy” as negative values for flexiramp occurs less often.

Market impact

As the optimization problem now is stricter constrained, the social welfare is expected to decrease compared to Case 1. In fact, the decrease is only $315, but keep in mind that this case study only considers 1 hour of operation. Extrapolating and assuming the same social welfare is lost every hour, the total loss of welfare for a whole year will be $2.76 million.

Table 5.5: Case 3: Dual values & social welfare for the deterministic model.

	t=1	t=2	t=3	t=4
Energy Price, λ_t [$/MW]	10	10	10	10
Up-Flexiramp, μ_t^u [$/MW]	5	0	0	
Down-Flexiramp, μ_t^d [$/MW]	0	0	0	
Social welfare	$15,798,297,50			

Looking at **Table 5.5**, the prices differ in fact quite a lot from Case 1. As discussed, the generators are now only really limited in their ability to provide up-flexiramp in $t = 1$, when generator 2 hasn't ramped down it's production yet. This is what triggers the price in this first time step, while the price for up-flexiramp is 0 for the other times as generator 2 now can provide more for "free".
Down-flexiramp is never a scarcity, hence, the price is always zero. It may seem counter intuitive that even though the requirement for down-flexiramp is increased, the scarcity is less. This is because when the requirement changed, the whole optimization problem change, and as shown in **Fig 5.2** the generator dispatch are all different than before. This affects the amount of flexiramp the units are able to provide, which in turn affects the prices more than the flexiramp requirement itself in this case.

5.1.3 Case 4: Descending net load

In Case 4, the net load was descending with 40 MW between intervals, starting from 700 MW. In this case, the flexiramp requirement is oposite as in Case 1. Down-flexiramp requirement is 90 MW and up-flexiramp is 10 MW.
As seen in **Fig. 5.3**, the generator dispatch is quite different than from the rising net load cases. Naturally, the biggest difference is the initial conditions and generally descending output, but also now generator 5 is included in operation.

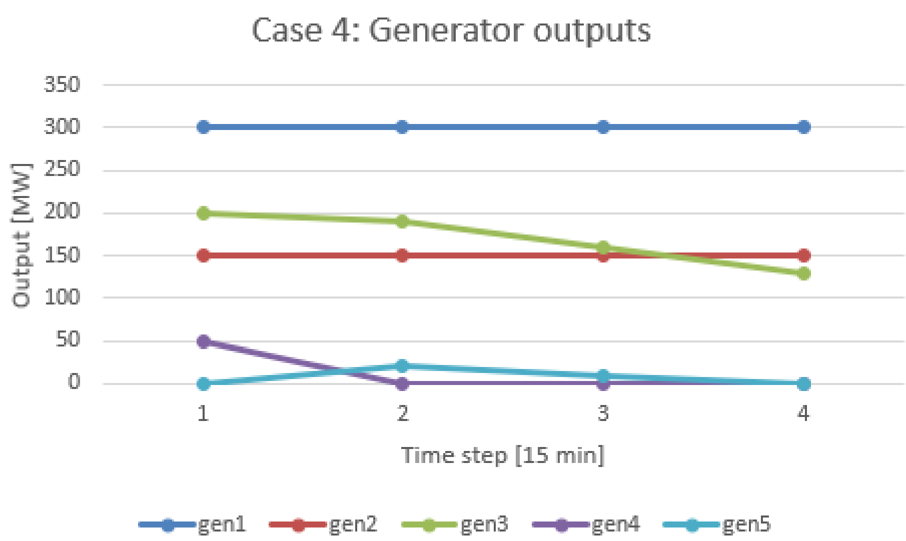

Figure 5.3: Case 4: Generator output (g_{it}) in all time steps. Deterministic model.

Naturally, the most expensive generators start ramping down first in order to minimize the fuel cost. Generator 4 is turned off immediately between $t = 1$ and $t = 2$, while generator 3 ramps down with 30 MW each time step from there and out.

So why is generator 5 turned on, when it is actually the most expensive generator to operate? Some simple calculations can be carried out to explain that phenomenon. Running generator 4 on it's minimum output of 50 MW for all four time intervals would cost \$3000. As generator 5 has \$0 start-up cost, running the dispatch shown in the above figure cost the system \$1500, ie half the fuel cost of generator 4.

Table 5.6: Case 4: Up- and Down-Flexiramp provided by each unit, deterministic model.

	Up-Flexiramp ur_{it} [MW]		
	t=1	t=2	t=3
G1	0	0	0
G2	0	0	0
G3	-10	10	20
G4	-40	0	0
G5	60	0	-10
	Down-Flexiramp dr_{it} [MW]		
	t=1	t=2	t=3
G1	0	0	0
G2	40	40	40
G3	40	40	40
G4	20	0	0
G5	-10	10	10

For this case, the flexiramp provided by each generator in each time step are pretty intuitive at the first glimpse. In $t = 1$, generator 5 is turned on and generator 4 is turned off. Hence, generator 5 will provide 60 MW, equal to the start-up ramp rate of the unit, to compensate for the ramping down of generator 4. At the same time, generator five provides -10 MW of Down-flexiramp as it must ramp up to at least 10 MW in the next time step. In fact, a weakness with the model needs to be pointed out here. As generator 4 is shut down from 50 MW to 0 MW between the two first time periods, it should provide -50 MW as it cannot ramp down only 40 MW. Also for the down-flexiramp generator 4 promises to ramp down 20 MW, when it in reality ramps down 50 MW. In the other time steps, the flexiramp provided does not violate any logic. It is suspected that there might be a fault in the implementation where input data is handled. In the implementation, input data from **Table 4.2** are considered boundary conditions, ie g_{it} are allowed to change in $t = 1$. This seems to make the model fail in defining the correct flexiramp bounds. Fixing the values in $t = 1$ removes this problem, but also removes the energy price in $t = 1$ as a result. For this thesis, the market impact is of interest so that is why the results shown are kept. The flaw is not considered to not impact the system to a significant extent.

Market impact

The market prices for flexiramp products shown in **Table 5.7** shows that the mentioned fault does not have any impact on generator revenues as the price is 0.

Table 5.7: Case 4: Dual values & social welfare for the deterministic model.

	t=1	t=2	t=3	t=4
Energy Price, λ_t [$/MW]	12.62	10	10	10
Up-Flexiramp, μ_t^u [$/MW]	0	0	0	
Down-Flexiramp, μ_t^d [$/MW]	0	40	40	
Social welfare	$20,599,767.50			

Not surprisingly, the up-flexiramp price is 0 at all times. This is because of the combination of a small requirement and that not all generators are operating close to their maximum capacity. The most interesting to discuss about these results is the down-flexiramp price in $t = 2$ and $t = 3$ that suddenly rockets. Generator 1 are as always not able to provide any flexiramp, while generator 4 is turned off so it can only provide up-flexiramp. Generator 2 and 3 provide its maximum down-flexiramp product equal to their ramp rates. In $t = 2$ generator 5 can only provide 10 MW of down-flexiramp, because it operates 10 MW over it's minimum output. While in $t = 3$ unit 3 can only provide the required 10 MW by shutting down. Hence, in order to provide any more down-flexiramp the whole system must be redispatched to keep the expensive generator 4 running at a even higher point than its minimum. Again, the scarcity of flexiramp-products is what drives the price up. The high price is because of the need to operate the expensive generator 4 at a higher level than optimal in order to provide any more than the required down-flexiramp.
The social welfare is a lot higher than the previous cases only because it is more energy involved in the system because of the differing starting points.

5.2 Stochastic Model

The Stochastic Model does not include any flexiramp, neither products nor requirement. Hence, only Case 1 and Case 4 are evaluated. The other cases does not give different results as the only difference there is the flexiramp requirement. The results from this model includes optimal values for all scenarios, but only the results for the ”mean” scenarios are shown, i.e. where the forecast is correct. These results are though linked with the under- and overforecast scenarios through the constrain formulation. These specific results are chosen because they are the only ones directly comparable to the deterministic model results.

5.2.1 Case 1: Rising net load

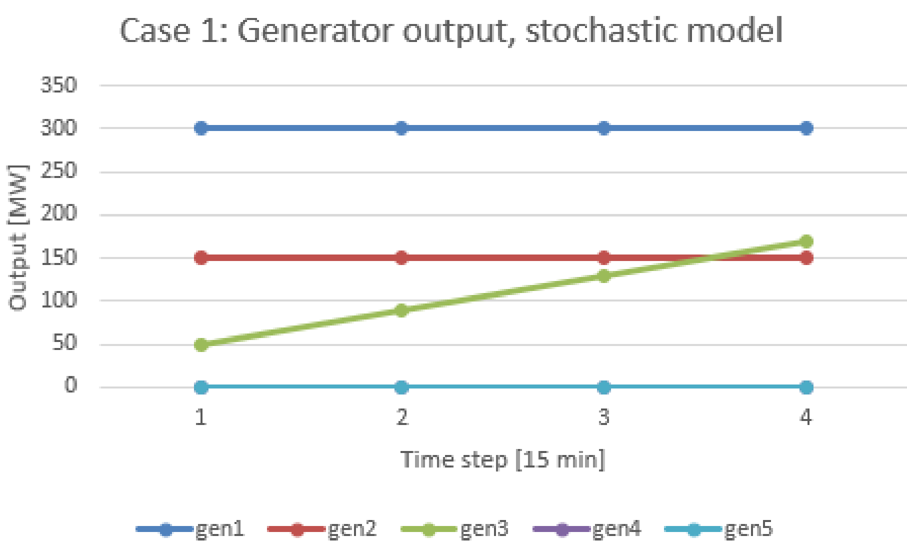

Figure 5.4: Case 1: Generator output (g_{it}) in all time steps. Stochastic model.

As mentioned above, **Fig. 5.4** shows the generator dispatch for the scenario with an increase in net load of 40 MW each time step, same as the deterministic model. These results require only a short discussion. The generators are prioritized in the order of cheapest to most expensive. The two cheapest generators are dispatched at their maximum, while generator 3 takes care of the ramping up. The dispatch differs from the deterministic model dispatch because it is not limited by any flexiramp requirement. This way the generators are able to be dispatched in a more economic way, only limited by the generator data. The overforecast scenario that can occur in every time step can be covered by simply turning on generator 4, so the system is feasible.

Market comparison

By what is seen from the generator dispatch, the prices are expected to be lower, as the cheapest generators are used at all times and no start-up costs occur.

Table 5.8: Case 1: Energy price and social welfare comparison between stochastic and deterministic model.

	Stochastic [$]	Determ. [$]
λ_1	5	5
λ_2	2.84	8.05
λ_3	1.54	11.02
λ_4	1.54	10
Social welfare	15,892,912.11	15,798,612.50

Table 5.8 shows the prices are as expected lower at all time steps except the first one, where the prices are equal for the two models. These lower prices lowers the generator income, but also increase the consumer surplus as they get cheaper power. The energy prices for the deterministic model are quite extreme because of the flexiramp requirement. The fact that generators are forced to be dispatched in a less economical way impacts the energy market in a significant way in this case.

The social welfare shown for the stochastic model in the previous table is the *expected* social welfare. This value considers all scenarios and weights the resulting social welfare with the probability of its occurrence. The result of stochastic optimization yields a social welfare that is expected to be $94,299.61 larger than the deterministic optimization. The actual social welfare for the excact scenario presented was $1,400 larger than the deterministic result. One could think that the deterministic optimization would yield a larger social surplus because of the benefit of being able to get information of the future, but the flexiramp requirement limits more.

5.2.2 Case 4: Descending net load

Same as for Case 1, results from the scenarios that follow the same net load trend as the deterministic model are chosen for comparison.

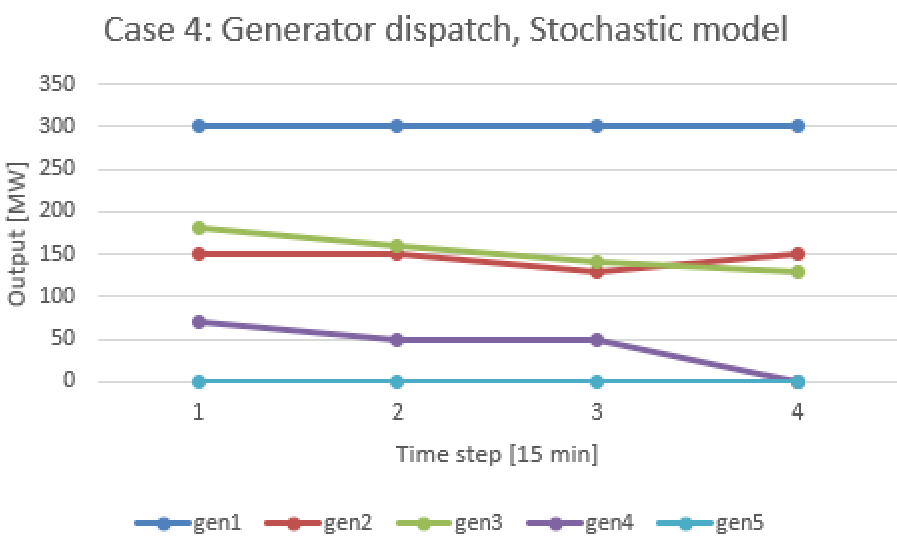

Figure 5.5: Case 4: Generator output (g_{it}) in all time steps. Stochastic model.

The generation dispatch mechanisms are quite similar as to the previous case. Generally, the most expensive units are prioritized to ramp down first. However, the generator limitations complicates the dispatch a little. Generator 4 can only ramp down to 50 MW from $t = 1$ to $t = 2$, because that is the minimum operating capacity. The other 20 MW decrease in net load are met by ramping down generator 3. Simply turning the expensive generator 4 off in the first time step would not be possible. That would ramp the production down with 70 MW, and the other generators are only able to ramp up 20 MW to compensate. Thus, the production would be 10 MW lower than the net load in the next time step. From $t = 2$ to $t = 3$ both generator 3 and the cheaper generator 2 ramps down. The reason for this ramping down of generator 2 is to be able to ramp it back up when generator 4 is turned off in the last time step.

Market comparison

Table 5.9: Case 4: Energy price and social welfare comparison between stochastic and deterministic model.

	Stochastic [$]	Determ. [$]
λ_1	15	12.62
λ_2	5.95	10
λ_3	1.25	10
λ_4	1.25	10
Social welfare	20,693,044.14	20,599,287.50

Table 5.9 shows that the energy is generally cheaper than with the deterministic model for this case also. The exception is in $t = 1$, where the stochastic model gives a slightly higher price. This is due to the operation of generator 4 at a slightly higher point compared to the deterministic solution. The reasons behind the otherwise lower prices are the same as for Case 1.

The expected social surplus is $93,756.64 higher than for the deterministic model in this case, while the actual social surplus for the presented scenarios are only $350 dollar larger. The reasons for this increased social welfare is the more relaxed problem now that flexiramp is not included.

5.3 Stochastic model with Flexiramp

Same as for the Stochastic Model, only results for the scenarios that corresponds to the results from the Deterministic Model are considered. This model does include flexiramp, with the same requirement as the deterministic model, so comparison on flexiramp products and prices will be shown in this section. Results from Case 1 will be presented here, while the other results from Case 3 and 4 can be found in Appendix D. This decision were made in order to make this section a less tedious read. The results are similar to the results already presented, and the driving mechanisms behind the results are explained earlier. A more executive look at the economical impacts of the different model are instead shown and discussed in the last section of this chapter.

5.3.1 Case 1: Rising net load

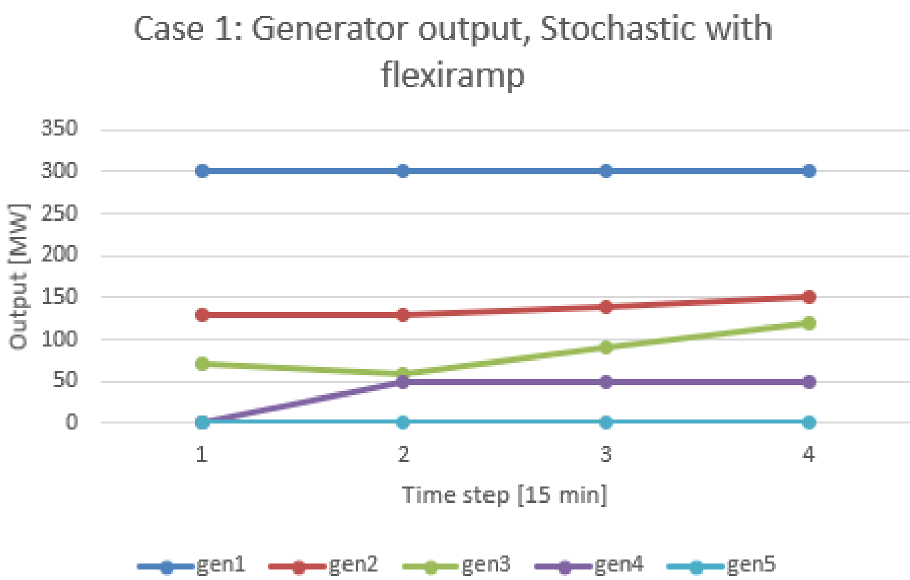

Figure 5.6: Case 1: Generator output (g_{it}) in all time steps. Stochastic model with Flexiramp.

Due to the flexiramp requirements, the generator dispatch seen in **Fig. 5.6** is very similar to the dispatch seen in the deterministic dispatch from **Fig. 5.1**. Where the Stochastic Model was able to meet the increase in net load solely by ramping up generator 3, this model needs generator 4 to help provide the up-flexiramp required.

Table 5.10: Case 1: Up- and Down-Flexiramp provided by each unit, stochastic model with flexiramp.

	Up-Flexiramp ur_{it} [MW]		
	t=1	t=2	t=3
G1	0	0	0
G2	-20	10	10
G3	30	40	40
G4	100	40	40
G5	0	0	0
	Down-Flexiramp dr_{it} [MW]		
	t=1	t=2	t=3
G1	0	0	0
G2	40	10	10
G3	20	0	0
G4	-50	0	0
G5	0	0	0

The up-flexiramp provided by each machine, shown in **Table 5.10**, is almost excactly the same as for the deterministic model (**Table 5.2**). Note that the sum of up-flexiramp provided in $t = 1$ now is 110 MW, even though the requirement is only 90 MW. This is however not problematic as the flexiramp market clearing constraints requires the sum of flexiramp provided to be greater than or equal to the requirement.
The down-flexiramp provided are less complex than for the deterministic solution as there are no negative values after $t = 1$, where the start-up of generator 4 occurs. This is a better solution to handle in a market sense, because less generators are involved in the settlement.

Energy Market

As this model is a hybrid between the two other models, it is expected that the prices and social welfare will be somewhere in between the results from the two others.

Table 5.11: Case 1: Energy price and social welfare comparison all models.

	Stoch+Flex [$]	Stochastic [$]	Determ. [$]
λ_1	5.6	5	5
λ_2	2.5	2.84	8.05
λ_3	2.1	1.54	11.02
λ_4	1.25	1.54	10
Social welfare	15,891,914.82	15,892,912.11	15,798,612.50

The results from **Table 5.11** are more or less as expected. The energy prices for the stochastic model with flexiramp are however much closer to the stochastic model than the deterministic one.

Flexiramp Markets

Table 5.12: Case 1: Up-Flexiramp price comparison between deterministic and stochastic model with flexiramp.

	t=1	t=2	t=3
Stoch+Flex [$]	0	0	0.8
Determ. [$]	0	3.05	6.02

Table 5.13: Case 1: Down-Flexiramp price comparison between deterministic and stochastic model with flexiramp.

	t=1	t=2	t=3
Stoch+Flex [$]	2.65	0	0
Determ. [$]	5	0	0

For both models, there is scarcity of flexiramp that drives the price up above 0. By observing the prices, the stochastic approach makes the system more flexible in it self, so there are less scarcity of flexiramp resources. Both **Table 5.12 and 5.13** show this tendency.

5.4 Economic comparison of the models

Some key points taken away from the comparison and analysis of the results in this chapter will be discussed in this section. Also a more general overview of how generator dispatch and prices influence the market participants will be shown.

Key take-aways

The introduction of flexiramp prices affects energy prices greatly. Some of these issues have been adressed by CAISO when designing the market [14]. The deterministic model gives energy prices that are up to 7 times higher than prices from the stochastic model. This affects all market participants. This is because more expensive units are committed to provide the required flexibility.

The results show occarrances of seemingly unnecessary negative flexiramp values. Usually, the TSO pays for ancillary services needed for grid operation, so payment for flexiramp goes from the TSO to the generator. However, with negative values of flexiramp it seems like generators at some times generators must pay the TSO for providing negative flexiramp.
Both these issues occur rarer with the stochastic model with flexiramp.

On the use of dual values as prices

During the process of analyzing the results of the different models in this section, a weakness in the implementations have been discovered. The dual values used as energy prices does not appear as they should. If we take a closer look at the energy prices for the Stochastic Model in Case 1, seen in **Table 5.8**, there is something that makes little sense. As the net load ramps up, the energy price goes down. This is counter intuitive as more expensive units have to ramp up their production to meet this increased load. This should be reflected in increased prices. The dual values of the energy market clearing constraint should reflect the cost of demanding 1 MW more from the generators. In this case, this translates to 1 MW more produced from generator 3, which would cost $15 for 15 minutes.
When calculating the generator revenue using the given dual prices, the generators will run with a economic loss. This is not behaviour that should

be seen in this case. Hence, the results are considered faulty.

This realization brings doubts to the correctness of the flexiramp prices. The previous discussion in this chapter of the mechanisms of scarcity that triggers the price signals still stands, but the actual prices are maybe not correct.

One reason revealed from further investigation into the implementations is the way the dual values are gained. By using the pre-solved commitment status of the machines as a fixed parameter for re-solving the problem, generators not committed will not be reflected in the dual values. To further elaborate, a generating unit defined to be turned off cannot provide any flexiramp when re-solving. This factor is not evaluated when the dual values are calculated. This is considered an implementation weakness that must be further investigated in future work.

Economic influence of the different models

As in [41], the stochastic model will be used as benchmark for economic comparison. This is because stochastic optimization gives the best results. Social welfare is compared, and for the stochastic models it is the *expected* social surplus that is evaluated. The Stochastic Model is denoted as SUC, the Deterministic Model as DT and the Stochastic Model with Flexiramp as SFUC.

Table 5.14: Comparison of social welfare. Rising net load case

	SUC	DT (change)	SFUC (change)
Social welfare [10^3\$]	1589.3	-94.3	-1.0
Generator cost [10^3\$]	8	+1.4	+1.45

The new Stochastic Model with flexiramp performs much better than the deterministic model in terms of social welfare, as shown in **Table 5.14**. With regards to generator costs, the two models performs approximately equally bad compared to the benchmark. This is due to the use of more expensive machines in order to provide the required flexiramp.

Table 5.15: Comparison of social welfare. Descending net load case

	SUC	DT (change)	SFUC (change)
Social welfare [$10^3$$]	2069.3	-93.8	-0.5
Generator cost [$10^3$$]	12.6	+0.175	+0.4

The descending net load case shows slightly better results for both models than for the rising net load case. Note that the generator costs are higher for the SFUC than for the DT, even though the SFUC can show a much larger social welfare. This is because the generator costs are calculated from the actual generator dispatch in the scenarios in question. The social surplus on the other hand, is for the SFUC calculated as an expected value across all possible scenarios.

Chapter 6

Conclusion

The increased penetration of intermittent renewable energy sources like wind and solar power are a challenge to grid operation and system reliability. Flexibility is a needed resource that is not properly valued in the energy markets of Europe today. Among the many flexibility resources available, like industrial batteries, demand response, etc, utilizing the flexibility in fast ramping generators are per now the most convenient and cheap resource. Flexiramp markets gives an incentive to invest in fast-ramping generators, to help secure system reliability and optimal electricity market operation.

This thesis has focused on investigating different approaches on modelling flexiramp products in a Unit Commitment formulation. Validity of the model results have been tested and economical differences in the different approaches. The main findings include the fact that flexiramp requirements trigger a loss in social welfare because more expensive generators must operate in order to provide the required flexibility. This loss is triggered both by higher generator cost and higher prices that reduce the consumer surplus.

Performance of the Stochastic Model with Flexiramp

The new formulation proposed in this thesis, refered to as a Stochastic Model with Flexiramp, has proven to perform better than the Deterministic Model, both in terms of a higher social welfare in all cases and also a more optimal flexiramp designation in terms of less generators involved. The big advantage of the new formulation compared to the existing Stochastic Model is the enabling of flexiramp markets. This retains the transparacy in pricing and keeps the incentives for investments in supply of flexiramp, like in the Deterministic Model. However, the Stochastic Model still gives the largest social welfare for all cases as well as the lowest generator costs. The introduction of flexiramp has shown to introduce a market inefficiency in terms of a more constrained optimization problem. However, if compared to the consequences of lacking flexibility, the inefficiency is small. Constant lack of flexibility due to a lack of investment incentive would be increasingly problematic for a grid operator as renewables make up a larger part of power production.

The analysis of results pointed out a flaw in the *implementation* of the models. Dual values gained from solving the model with the available solver seems to be wrong. This was not noticed at first, as the values seemed to be in a plausible magnitude at first glance. An alternative way of retrieving energy prices will be discussed in the next section on Future Work.

The formulations are presented in a general manner, with T time steps and I generating units. However, the implementation is done on the basis of the case studies. Looking at five units for four time steps. The limited system and limited planning horizon are only able to show the concepts of including flexiramp, but how it will perform on a realistic power system is not investigated in this implementation.

On stochastic vs deterministic market optimization

Today's standard is using deterministic Unit Commitment formulations based on net load forecasts. Scenario based stochastic models can show to better results than today's standard. Especially when coping with a high degree of uncertainty. This degree of uncertainty is expected to increase along with the increased penetration of renewable energy sources. Changing to a Stochastic model will increase the complexity of the model and require more computational power than the current deterministic models.

6.1 Future Work

There are many interesting research questions to investigate further in the topic of flexiramp enabling in the Unit Commitment problem. Especially how the implementation can be done in a European context. The formulations presented in this thesis are limited to only include generator behavior. Grid operation constraints are not included.

Adapting to European Markets

The European electricity markets operates with zonal prices. Generator bids and consumer demand are aggregated within distinct zones. Transfer capacity between the zones can sometimes be a bottleneck which trigger different prices in each zone. In order to adapt the formulations to cope with this, transfer capacities between zones must be included in the constraints.

Further, the generator bids into the day-ahead market are based on the marginal cost of operation of the generating units, whereas these models are based on the three costs start-up cost, no-load cost and operational cost. A proposal for future work is to change the goal function from maximizing social welfare into minimizing total system cost, based on the marginal cost of the generators. This could possibly solve the encountered problem of gaining dual values that correctly reflects the energy pricing. With this approach, energy price could be found by looking at the marginal cost of the *marginal unit*, ie the most expensive units who's bid get accepted in the clearing of the market.

Incorporate ramping between time steps in models

The net load in the implementations shown in this thesis are modelled as constant between the time steps of 15 minutes. For example in Case 1, the net load is 500 MW in $t = 1$ for 15 minutes, then at an instant it jumps up to 540 MW in $t = 2$. This does not accurately represent the reality of how a real power system demand ramps up over time. Future work could include modeling the net load as linear between time steps, and investigate how the system responds to this change in terms of generator dispatch and total cost.

Improve implementation by shortening time steps

The formulations opens up for shorter time steps as it is based on a general t for time step. The implementations can be modified to evaluate shorter time steps of for example 5 minutes or 1 minute. Many interesting observations could be done from such a study. Less flexiramp would be required per time intervall. Would this limit the impact on the social welfare and generator cost?
The formulations could also be adapted to continuous time problems by utilizing Bernstein polynomials.

Expand the proposed formulation

This thesis has proposed a new formulation. This formulation has it's limitations and shortcomings when it comes to grid operation and system planning. A proposal for future work would be adding new constraints to both the formulation and the implementation of the model. These constraints could include transfer capacities between different bidding zones. If different bidding zones are considered in the model to better fit with European markets, it would also be interesting to investigate zonal flexiramp requirement compared to a system wide requirement.

Develop new case-studies

The case studies performed in this thesis are limited both in system size and in time horizon. Future work could include working with the attached implementations, adjusting them for a new proposal of case studies. These new case studies should include more generators and more time steps to simulate a more realistic system in order to observe the performance of the models on such a system.

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A Applied Software

A.1 Python

Python [17] is a free-to-use and open-source programming language. It is increasingly popular and has a large online community providing help and guidance to users. Python includes a large standard library as well as thousands of available third-party packages and modules to solve specific problems. This makes it a very handy environment for implementing software solutions. This thesis utilizes some of these third-party packages for implementing the optimization models. These packages are mentioned below.

A.2 Pyomo

To develop both the deterministic and the stochastic model presented in this thesis, the open-source software package *Pyomo* [20, 21] was used. Pyomo is a Python-based optimization framework for formulating, solving and analyzing optimization problems. Pyomo is capable of handling a vast selection of different optimization problem types.

One of Pyomo's strengths is that it is based on Python. This gives the user access to the rich libraries that comes with such a high-level programming language. Pyomo let's the user choose between two different modelling styles. The abstract modelling enables the user to define the model without any input data, just mathematical symbols and formulation of the problem. Then instances with specific input data can be constructed within the abstract model to be solved. The concrete modelling lets the user initialize the model with input data directly.

Pyomo also provides the possibility to utilize many different available solvers, both commercial and open-source. For the models presented in this thesis, the concrete modelling approach were used, and *Gurobi* [18] was used as solver.

A.3 Gurobi

To be able to solve the problem type referred to as Mixed-integer nonlinear programming in the models, Gurobi was used. Gurobi is a robust and reliable solver, proven to be able to handle millions of decision variables as well as being able to solve all major optimization problem types.

A.4 GLPK

The solver used for the Farmer's Problem implementations in Pyomo was *GLPK* [38]. Both the stochastic and the deterministic model used this solver. GLPK is developed and published by the GNU Project. It is intended for solving large-scale Linear Programs, Mixed-Integer Programs and other related problems. For the work in this thesis, it was the Linear Program solver that was used. The popularity of GLPK can be explained by the fact that it is both open-source and free to use.

A.5 Pandas

Pandas [33] is a Python based data analysis tool. It is open-sourced and free to use. Pandas provides high-performance and easy to use data stuctures. For the work in this thesis, it was mainly used to extract data from excel-files to Python scripts.

B Problem Formulations

B.1 Nomenclature

This nomenclature is the same as in [41]. *Indices and Set*

i	Index for generating unit, i = 1.I.
s	Index for scenario, s = 1.S
s’	Index for previous scenario in Markov chain.
t	Index for time interval, t = 1.T
S’(s,t)	Set of scenarios that are indistinguishable from scenario *s* for time intervals 1 to *t*.

Parameters

$\overline{Cap_i}$	Capacity, unit *i* [MW]
$\underline{Cap_i}$	Minimum output when committed, unit *i* [MW]
D_{ts}	Realized reference net load, scenario *s* in interval *t*. This is net of variable renewable generation, but does not include the effect of demand response.
DF_t	Forecast value of reference net load, interval *t* [MW], for which the deterministic unit commitment model schedules generation and demand response.
DNL_t	Forecast change in net load [MW] from interval *t* to *t + 1*, equal to $DF_{t+1} - DF_t$.
ERR_t	Net load forecast error range [MW] in interval t.
$FRdn_t$	Down-Flexiramp requirement, interval *t* [MW/interval].

$FRup_t$	Up-Flexiramp requirement, interval *t* [MW/interval].
HC_i	Start-up cost, unit *i* [$].
NLC_i	No-load cost, unit *i* [$].
OC_i	Variable operating cost, generating unit *i* [$/MWh].
PR_0	Reference price for demand curve [$/MWh].
PR_{ts}	Probability of occurrence of scenario *s* in interval *t*.
RR_i	Ramping limit, unit *i* [MW/interval].
Slope	Slope of demand curve [($/MWh)/MW].
$SDRR_i$	Shut-Down ramp limit, unit *i* [MW/interval].
$SURR_i$	Start-Up ramp limit, unit *i* [MW/interval].

Decision Variables

c_{its}	Start-up cost, unit *i* in *t,s*.
d_{ts}	Net load [MW], in *t,s*. This equals the reference consumption of power (which excludes demand response) minus the sum of variable renewable generation and load reductions from demand response.
dr_{its}	Down-flexiramp [MW] provided by unit *i* in *t,s*.
g_{its}	Generation [MW] unit *i*, in *t,s*.
$\overline{g_{its}}$	Maximum feasible generation, unit *i*, in *t,s*.
ur_{its}	Up-flexiramp [MW] provided by unit *i*, in *t,s*.
v_{its}	Binary on/off variable, unit *i*, in *t,s*.
λ_{ts}	Energy price in *t,s* (dual) [$/MWh].
μ_t^u	Up-flexiramp price in *t* (dual) [$/MWh].
μ_t^d	Down-flexiramp price in *t* (dual) [$/MWh].

B.2 Deterministic Model

$$\begin{aligned}\text{Max} \quad & \sum_{t=1}^{T} 0.25 * \Bigg[\Big((PR_0 - \textit{slope} * DF_t) * d_t + \textit{slope} * d_t^2/2 \Big) \\ & - \left(\sum_{i=1}^{I} OC_i * g_{it} + \sum_{i=1}^{I} NLC_i * v_{it} + \sum_{i=1}^{I} c_{it} \right) \Bigg] \end{aligned} \tag{6.1a}$$

Subject to: *Energy Market Clearing:*

$$\sum_{i=1}^{I} g_{it} - d_t = 0 \qquad \forall t : \lambda_t \tag{6.1b}$$

Up- and Down-Flexiramp Market Clearing:

$$\sum_{i=1}^{I} ur_{it} \geq FRup_t \qquad t = 1, ..., T-1 : \mu_t^u \tag{6.1c}$$

$$\sum_{i=1}^{I} dr_{it} \geq FRdn_t \qquad t = 1, ..., T-1 : \mu_t^d \tag{6.1d}$$

Generation Bounds, Accounting for Start-up/Shut-down:

$$\underline{Cap_i} * v_{it} \leq g_{it} \leq \overline{g_{it}} \qquad \forall t, i \tag{6.1e}$$

Generation + Flexiramp Bounds, Accounting for Start-up/Shut-down:

$$\begin{aligned} &\underline{Cap_i} * \left[v_{i(t+1)} + v_{it} - 1\right] \leq -dr_{it} + g_{it} \leq \overline{g_{i(t+1)}} \\ &+ \overline{Cap_i} * \left[1 - v_{i(t+1)}\right] \qquad \forall t, i \end{aligned} \tag{6.1f}$$

$$\begin{aligned} &\underline{Cap_i} * \left[v_{i(t+1)} + v_{it} - 1\right] \leq ur_{it} + g_{it} \leq \overline{g_{i(t+1)}} \\ &+ \overline{Cap_i} * \left[1 - v_{i(t+1)}\right] \qquad \forall t, i \end{aligned} \tag{6.1g}$$

Definition of Maximum Feasible Generation:

$$\overline{g_{it}} \leq v_{it} * \overline{Cap_i} \qquad \forall t, i \tag{6.1h}$$

$$\begin{aligned} &\overline{g_{it}} \leq g_{i(t-1)} + RR_i * v_{i(t-1)} + SURR_i * \left(v_{it} - v_{i(t-1)}\right) \\ &+ \overline{Cap_i} * (1 - v_{it}) \qquad \forall t, i \end{aligned} \tag{6.1i}$$

$$\overline{g_{it}} \leq SDRR_i * \left(v_{it} - v_{i(t+1)}\right) + \overline{Cap_i} * v_{i(t+1)} \qquad \forall t, i \tag{6.1j}$$

Ramp Limits on Forecast Generation:

$$\begin{aligned} g_{i(t-1)} - g_{it} \leq RR_i * v_{it} + &SDRR_i * \left(v_{i(t-1)} - v_{it}\right) \\ &+ \overline{Cap_i} * \left(1 - v_{i(t-1)}\right) \qquad \forall t, i \end{aligned} \tag{6.1k}$$

Bounds on Flexiramp:

$$\begin{aligned} -RR_i * v_{i(t+1)} - SDRR_i * \left[v_{it} - v_{i(t+1)}\right] \\ -\overline{Cap_i} * (1 - v_{it}) \leq ur_{it} \\ \leq RR_i * v_{it} + SURR_i * \left[v_{i(t+1)} - v_{it}\right] \\ + \overline{Cap_i} * \left[1 - v_{i(t+1)}\right] \qquad \forall t, i \end{aligned} \tag{6.1l}$$

$$\begin{aligned} -RR_i * v_{it} - SURR_i * \left[v_{i(t+1)} - v_{it}\right] \\ -\overline{Cap_i} * \left[1 - v_{i(t+1)}\right] \leq dr_{it} \\ \leq RR_i * v_{i(t+1)} + SDRR_i * \left[v_{it} - v_{i(t+1)}\right] \\ + \overline{Cap_i} * (1 - v_{it}) \qquad \forall t, i \end{aligned} \tag{6.1m}$$

$$\begin{aligned} -v_{it} * \overline{Cap_i} + v_{i(t+1)} * \underline{Cap_i} \\ \leq ur_{it} \leq v_{i(t+1)} * \overline{Cap_i} \qquad \forall t, i \end{aligned} \tag{6.1n}$$

$$\begin{aligned} -v_{i(t+1)} * \overline{Cap_i} \leq dr_{it} \leq v_{it} * \overline{Cap_i} \\ - v_{i(t+1)} * \underline{Cap_i} \qquad \forall t, i \end{aligned} \tag{6.1o}$$

Definition of Start-Up Cost:

$$c_{it} \geq HC_i * \left(v_{it} - v_{i(t-1)}\right) \qquad \forall t, i \tag{6.1p}$$

Non-Negativity:

$$g_{it}, \overline{g_{it}}, c_{it}, d_t \geq 0, \quad v_{it} \in \{0, 1\}, \quad \forall t, i \tag{6.1q}$$

B.3 Stochastic Model

$$\begin{aligned} \text{Max} \quad & \sum_{t=1}^{T} \sum_{s=1}^{S} PR_{ts} * 0.25 * \Bigg[\left((PR_0 - \textit{slope} * D_{ts}) * d_{ts} + \textit{slope} * d_{ts}^2 / 2 \right) \\ & - \left(\sum_{i=1}^{I} OC_i * g_{its} + \sum_{i=1}^{I} NLC_i * v_{its} + \sum_{i=1}^{I} c_{its} \right) \Bigg] \end{aligned} \tag{6.2a}$$

Subject to: *Energy Market Clearing:*

$$\sum_{i=1}^{I} g_{its} - d_{ts} = 0 \qquad \forall t, s : \lambda_{ts} \tag{6.2b}$$

Generation Bounds, Accounting for Start-up/Shut-down:

$$\underline{Cap_i} * v_{its} \leq g_{its} \leq \overline{g_{its}} \qquad \forall i, t, s \tag{6.2c}$$

Definition of Maximum Feasible Generation:

$$\overline{g_{its}} \leq v_{its} * \overline{Cap_i} \qquad \forall i, t, s \tag{6.2d}$$

$$\begin{aligned} \overline{g_{its}} \leq g_{i(t-1)s} + RR_i * v_{i(t-1)s} + SURR_i * \left(v_{its} - v_{i(t-1)s}\right) \\ + \overline{Cap_i} * (1 - v_{its}) \qquad \forall i, s, t \end{aligned} \tag{6.2e}$$

$$\overline{g_{its}} \leq SDRR_i * \left(v_{its} - v_{i(t+1)s}\right) + \overline{Cap_i} * v_{i(t+1)s} \qquad \forall i, s, t \tag{6.2f}$$

Generation Ramp Limits:

$$\begin{aligned} g_{i(t-1)s} - g_{its} \leq RR_i * v_{its} + SDRR_i * \left(v_{i(t-1)s} - v_{its}\right) \\ + \overline{Cap_i} * \left(1 - v_{i(t-1)s}\right) \qquad \forall i, s, t \end{aligned} \tag{6.2g}$$

Definition of Start-Up Cost:

$$c_{its} \geq HC_i * \left(v_{its} - v_{i(t-1)s}\right) \qquad \forall i, s, t \tag{6.2h}$$

Non-Anticipativity Constraints:

$$g_{its} = g_{its'} \qquad \forall t, i, s, s' \in S'(s,t) \tag{6.2i}$$

$$\overline{g_{its}} = \overline{g_{its'}} \qquad \forall t, i, s, s' \in S'(s,t) \tag{6.2j}$$

$$c_{its} = c_{its'} \qquad \forall t, i, s, s' \in S'(s,t) \tag{6.2k}$$

$$v_{its} = v_{its'} \qquad \forall t, i, s, s' \in S'(s,t) \tag{6.2l}$$

$$d_{its} = d_{its'} \qquad \forall t, i, s, s' \in S'(s,t) \tag{6.2m}$$

Non-Negativity:

$$g_{its}, \overline{g_{its}}, c_{its}, d_{ts} \geq 0, \quad v_{its} \in \{0, 1\}, \quad \forall i, t, s \tag{6.2n}$$

B.4 Stochastic Model with Flexiramp

$$\text{Max} \quad \sum_{t=1}^{T}\sum_{s=1}^{S} PR_{ts} * 0.25 * \Bigg[\Big((PR_0 - \textit{slope} * D_{ts}) * d_{ts} + \textit{slope} * d_{ts}^2/2 \Big) - \left(\sum_{i=1}^{I} OC_i * g_{its} + \sum_{i=1}^{I} NLC_i * v_{its} + \sum_{i=1}^{I} c_{its} \right) \Bigg] \tag{6.3a}$$

Subject to: *Energy Market Clearing:*

$$\sum_{i=1}^{I} g_{its} - d_{ts} = 0 \qquad \forall t, s : \lambda_{ts} \tag{6.3b}$$

Up- and Down-Flexiramp Market Clearing:

$$\sum_{i=1}^{I} ur_{its} \geq FRup_t \qquad \forall s, t = 1, ..., T-1 : \mu_{ts}^{u} \tag{6.3c}$$

$$\sum_{i=1}^{I} dr_{its} \geq FRdn_t \qquad \forall s, t = 1, ..., T-1 : \mu_{ts}^{d} \tag{6.3d}$$

Generation Bounds, Accounting for Start-up/Shut-down:

$$\underline{Cap_i} * v_{its} \leq g_{its} \leq \overline{g_{its}} \qquad \forall i, t, s \tag{6.3e}$$

Generation + Flexiramp Bounds, Accounting for Start-up/Shut-down:

$$\underline{Cap_i} * \big[v_{its)} + v_{i(t-1)s'} - 1 \big] \leq -dr_{i(t-1)s'} + g_{i(t-1)s'} \leq \overline{g_{its}} + \overline{Cap_i} * [1 - v_{its}] \qquad \forall i, s, t = 2, ..., T \tag{6.3f}$$

$$\underline{Cap_i} * \big[v_{its} + v_{i(t-1)s'} - 1 \big] \leq ur_{i(t-1)s'} + g_{i(t-1)s'} \leq \overline{g_{its}} + \overline{Cap_i} * [1 - v_{its}] \qquad \forall i, s, t = 2, ..., T \tag{6.3g}$$

Definition of Maximum Feasible Generation:

$$\overline{g_{its}} \leq v_{its} * \overline{Cap_i} \qquad \forall i, t, s \tag{6.3h}$$

$$\overline{g_{its}} \leq g_{i(t-1)s'} + RR_i * v_{i(t-1)s'} + SURR_i * \big(v_{its} - v_{i(t-1)s'} \big) + \overline{Cap_i} * (1 - v_{its}) \qquad \forall i, s, t = 2, ..., T \tag{6.3i}$$

$$\overline{g_{i(t-1)s'}} \leq SDRR_i * \left(v_{i(t-1)s'} - v_{its}\right) + \overline{Cap_i} * v_{its} \qquad \forall i, s, t = 2, ..., T \tag{6.3j}$$

Ramp Limits on Forecast Generation:

$$\begin{aligned} g_{i(t-1)s'} - g_{its} \leq RR_i * v_{its} &+ SDRR_i * \left(v_{i(t-1)s'} - v_{its}\right) \\ &+ \overline{Cap_i} * \left(1 - v_{i(t-1)s'}\right) \qquad \forall i, s, t = 2, ..., T \end{aligned} \tag{6.3k}$$

Bounds on Flexiramp:

$$\begin{aligned} &-RR_i * v_{its} - SDRR_i * \left[v_{i(t-1)s'} - v_{its}\right] \\ &\quad -\overline{Cap_i} * \left(1 - v_{i(t-1)s'}\right) \leq ur_{i(t-1)s'} \\ &\qquad \leq RR_i * v_{i(t-1)s'} + SURR_i * \left[v_{its} - v_{i(t-1)s'}\right] \\ &\qquad\quad + \overline{Cap_i} * [1 - v_{its}] \qquad \forall i, s, t = 2, ..., T \end{aligned} \tag{6.3l}$$

$$\begin{aligned} &-RR_i * v_{i(t-1)s'} - SURR_i * \left[v_{its} - v_{i(t-1)s'}\right] \\ &\quad -\overline{Cap_i} * [1 - v_{its}] \leq dr_{i(t-1)s'} \\ &\qquad \leq RR_i * v_{its} + SDRR_i * \left[v_{i(t-1)s'} - v_{its}\right] \\ &\qquad\quad + \overline{Cap_i} * \left(1 - v_{i(t-1)s'}\right) \qquad \forall i, s, t = 2, ..., T \end{aligned} \tag{6.3m}$$

$$\begin{aligned} &-v_{i(t-1)s'} * \overline{Cap_i} + v_{its} * \underline{Cap_i} \\ &\leq ur_{i(t-1)s'} \leq v_{its} * \overline{Cap_i} \qquad \forall i, s, t = 2, ..., T \end{aligned} \tag{6.3n}$$

$$\begin{aligned} &-v_{its} * \overline{Cap_i} \leq dr_{i(t-1)s'} \leq v_{i(t-1)s'} * \overline{Cap_i} \\ &- v_{its} * \underline{Cap_i} \qquad \forall i, s, t = 2, ..., T \end{aligned} \tag{6.3o}$$

Definition of Start-Up Cost:

$$c_{its} \geq HC_i * \left(v_{its} - v_{i(t-1)s'}\right) \qquad \forall i, s, t = 2, ..., T \tag{6.3p}$$

Non-Negativity:

$$g_{its}, \overline{g_{its}}, c_{its}, d_{ts} \geq 0, \quad v_{its} \in \{0, 1\}, \quad \forall i, t, s \tag{6.3q}$$

C Optimization Models

C.1 Farmer's Problem

As an introduction to optimization models, *The Farmer's Problem* from [10] will be used as an example. This example is a 2-stage deterministic model. That it is deterministic means that all information is known to the decision maker in advance, i.e. no uncertainty is involved at all. A 2-stage problem is a problem with decisions that needs to be taken at two different times, or stages. For this specific model a farmer needs first to decide what grains to sow, then at a later stage needs to decide how much of the crops to sell.

Consider a farmer who specializes in growing wheat, corn and sugar beets on her 500 acres of fertile soil. The farmer also owns cattle, which need to be fed both corn and wheat in order to grow. Nearby to her farm there is a market available where her products can be bought and sold. She wants to make a educated decision on how much land to devote to each plant in order to maximize her profit.
From earlier years the farmer knows that the mean yield on her land is approximately 2.5 tons (T) of wheat, 3 T of corn and 20 T of sugar beets per acre. Her cattle needs at least 200 T of wheat and 240 T of corn. In the local market, wheat and corn can be sold for \$170 and \$150 per T respectively. Purchase prices are however 40% higher due to transportation costs and the wholesaler's margins. Sugar Beets are a profitable crop, selling at 36 \$/T. However, there is a quota on the production of Beets, so that any ton of Beets over 6000 T can only be sold for 10\$/T. These key numbers

are summarized in **Table. 6.2** below together with the planting cost:

Table 6.2: Data for farmer's problem

	Wheat	Corn	Sugar Beets
Yield(T/acre)	2.5	3	20
Planting cost ($/acre)	150	230	260
Selling price ($/T)	170	150	36 under 6000 T 10 above 6000 T
Purchase price ($/T)	238	210	-
Minimum requirement (T)	200	240	-

Total available land: 500 acres

By introducing the following variables, the farmer can take a decision on how to distribute her land area:

x_1 = acres of land devoted to wheat,
x_2 = acres of land devoted to corn,
x_3 = acres of land devoted to sugar beets,
w_1 = tons of wheat sold,
w_2 = tons of corn sold,
w_3 = tons of sugar beets sold at the favourable price,
w_4 = tons of sugar beets sold at the lower price,
y_1 = tons of wheat bought,
y_2 = tons of corn bought.

The whole decision problem can now be formed as a Linear Program (LP) as follows:

$$
\begin{aligned}
\text{min} \quad & 150x_1 + 230x_2 + 260x_3 + 238y_1 + 210y_2 \\
& -170w_1 - 150w_2 - 36w_3 - 10w_4 \\
\text{subject to} \quad & x_1 + x_2 + x_3 \leq 500, \\
& 2.5x_1 + y_1 - w_1 \geq 200, \\
& 3x_2 + y_2 - w_2 \geq 240, \\
& w_3 + w_4 \geq 20x_3, \\
& w_3 \leq 6000, \\
& x_1, x_2, x_3, y_1, y_2, w_1, w_2, w_3, w_4 \geq 0.
\end{aligned}
\tag{6.4}
$$

Note that the costs are added, while the incomes are subtracted. This is in order to make this into a minimization problem, which is the most common way to solve a LP. Minimazing the negative profit is mathematically equal to maximizing the profit. The constraints are mathematical formulations of the above introductory problem text.
The set of equations in (6.4) can be solved with available LP solvers. Attached to this thesis is an implementation done in Pyomo, using the solver *glpk* [38]. The file is called ”farmers_problem.py”. The results of the solver is shown in **Table. 6.3**.

Table 6.3: Optimal solution of farmer’s problem

	Wheat	Corn	Sugar Beets
Surface (acres)	120	80	300
Yield (T)	300	240	6000
Sold (T)	100	-	6000
Bought (T)	-	-	-
Overall profit: $118,600			

The results let’s the farmer take the optimal decision of how much land to devote to each plant in order to maximize her profit while being able to meet the feeding requirement for her cattle. The solution makes intuitively sense as well. As sugar beets grants the most $ per acre up to 6000 T, the first 300 acres are designated to sugar beets. Then the next 80 acres are designated solely to meet the minimum requirement of corn, as any extra corn doesn’t give any profit when sold due to the low price. The last 120 acres are then dedicated to growing wheat. After feeding the cattle, the farmer can sell 100 T of wheat and all 6000 T of sugar beets to gain the overall profit.

C.2 Farmer’s Problem, a Stochastic approach

In the previous approach, the crop yield was as mentioned modelled as known to the farmer even before she put the seeds in the dirt. More realistically, the amount of rain and sun would have a large impact on how many tons of product she gets per acre. The easiest way to expand the model in Equation set (6.4) is to develop a *scenario representation* of the crop yield.

A simple approach would be to define three different outcomes, or scenarios, for the actual yield. A good, bad or average year would correspond to ”above”, ”below” and ”average” crops. The ”average” year would be the same as presented in **Table. 6.2**, while ”above” and ”below” will indicate an increase or decrease of 20% for the crop yield. For simplicity, the weather conditions are not considered to have an impact on the prices at the wholesaler.
The farmer’s initial approach is to run the model from (6.4), but with changed input data appropriate for the two new scenarios.

Table 6.4: Optimal solution of farmer’s problem with an above average yield (+20%)

	Wheat	Corn	Sugar Beets
Surface (acres)	183.33	66.67	250
Yield (T)	550	240	6000
Sold (T)	350	-	6000
Bought (T)	-	-	-
Overall profit: $167,667			

Table 6.5: Optimal solution of farmer’s problem with a below average yield (-20%)

	Wheat	Corn	Sugar Beets
Surface (acres)	100	25	375
Yield (T)	200	60	6000
Sold (T)	-	-	6000
Bought (T)	-	180	-
Overall profit: $59,950			

The optimal solutions for the good and bad year, shown in **Table. 6.4 & 6.5** are not very surprising and are following the same strategy as for the average year. However, this does not help the farmer much, as the range in optimal surface area designation varies with over 100 acres for the different scenarios. Remember that the decision of how many acres to sow with each plant must be taken before the farmer knows what kind of year she will face. The model must be updated to be able to find the optimal solution for any scenario. To do so, it would be preferable to consider two stages. In the first stage, decision on land assignment (x_1, x_2, x_3) is taken. Sales $(w_1, ..., w_4)$ and purchases (y_1, y_2) are decided in the next stage, i.e. after the actual crop yield is observed. For simplicity, each of the three scenarios have a probability of occurring of 1/3. The new mathematical formulation of the stochastic model now becomes:

$$
\begin{aligned}
\text{min} \quad & 150x_1 + 230x_2 + 260x_3 \\
& + \frac{1}{3}(+238y_{11} + 210y_{21} - 170w_{11} - 150w_{21} - 36w_{31} - 10w_{41}) \\
& + \frac{1}{3}(+238y_{12} + 210y_{22} - 170w_{12} - 150w_{22} - 36w_{32} - 10w_{42}) \\
& + \frac{1}{3}(+238y_{13} + 210y_{23} - 170w_{13} - 150w_{23} - 36w_{33} - 10w_{43}) \\
\text{subject to} \quad & x_1 + x_2 + x_3 \leq 500, \\
& 3x_1 + y_{11} - w_{11} \geq 200, \\
& 3.6x_2 + y_{21} - w_{21} \geq 240, \\
& w_{31} + w_{41} \geq 24x_3, \\
& w_{31} \leq 6000, \\
& 2.5x_1 + y_{12} - w_{12} \geq 200, \\
& 3x_2 + y_{22} - w_{22} \geq 240, \\
& w_{32} + w_{42} \geq 20x_3, \\
& w_{32} \leq 6000, \\
& 2x_1 + y_{13} - w_{13} \geq 200, \\
& 2.4x_2 + y_{23} - w_{23} \geq 240, \\
& w_{33} + w_{43} \geq 16x_3, \\
& w_{33} \leq 6000, \\
& x, y, w \geq 0.
\end{aligned} \tag{6.5}
$$

This way to set up the stochastic program is called the *extensive form* because all the different second-stage variables are described explicitly. A proposed implementation of this model in Pyomo is shown in the attached file ”farmers_problem_stochastic.py”. Solving gives the following results:

Table 6.6: Optimal solution of farmer's problem in the extensive form. Solution gained from running the attached script "farmers_problem_stochastic.py"

		Wheat	Corn	Sugar Beets
First stage	Area (acres)	170	80	250
s=1 Above	Yield (T)	510	288	6000
	Sales (T)	310	48	6000 (favor. price)
	Purchase (T)	-	-	-
s=2 Average	Yield (T)	425	240	5000
	Sales (T)	225	-	5000 (favor. price)
	Purchase (T)	-	-	-
s=3 Below	Yield (T)	340	192	4000
	Sales (T)	140	-	4000 (favor. price)
	Purchase (T)	-	48	-

Overall profit: $108,390

Table. 6.6 shows a unique solution with regards to land assignment. Solving the model in (6.4) with the three different inputs would not give the optimal decision. Neither would the average of the three. This shows that the farmer couldn't have made the right decision without utilizing a stochastic model like in Equation set (6.5).

D Results from Stochastic model with flexiramp

D.1 Case 3

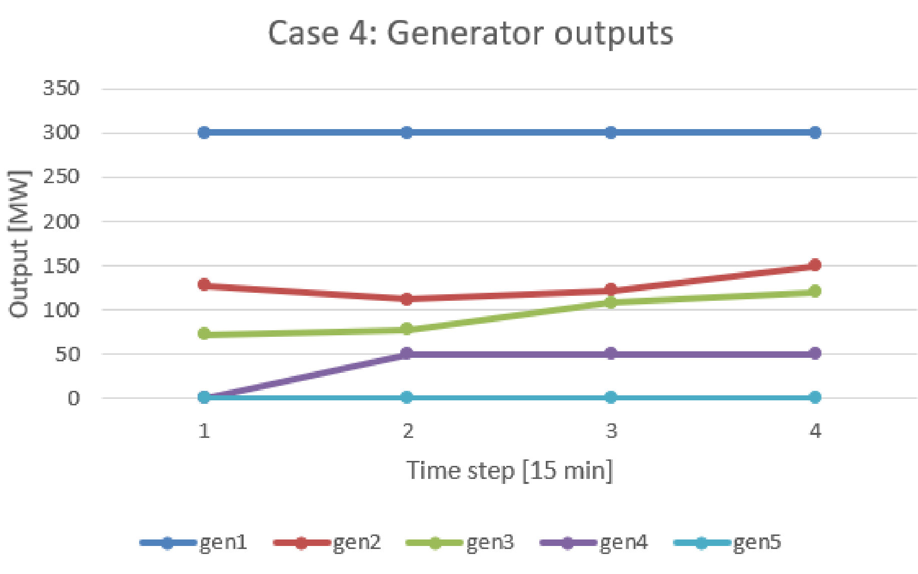

Figure 6.1: Case 3: Generator output (g_{it}) in all time steps. Stochastic model with flexiramp.

Table 6.7: Case 3: Up- and Down-Flexiramp provided by each unit. Stochastic model with flexiramp.

	Up-Flexiramp ur_{it} [MW]		
	t=1	t=2	t=3
G1	0	0	0
G2	-16	28	28
G3	40	40	40
G4	100	40	40
G5	0	0	0
	Down-Flexiramp dr_{it} [MW]		
	t=1	t=2	t=3
G1	0	0	0
G2	40	12	12
G3	22	0	0
G4	-50	0	0
G5	0	0	0

Table 6.8: Case 3: Dual values & social welfare for the stochastic model with flexiramp

	t=1	t=2	t=3	t=4
Energy Price, λ_t [$/MW]	5	2.5	2.19	1.25
Up-Flexiramp, μ_t^u [$/MW]	0	0	0.94	
Down-Flexiramp, μ_t^d [$/MW]	2.34	0	0	
Social welfare	$15,891,906.40			

D.2 Case 4

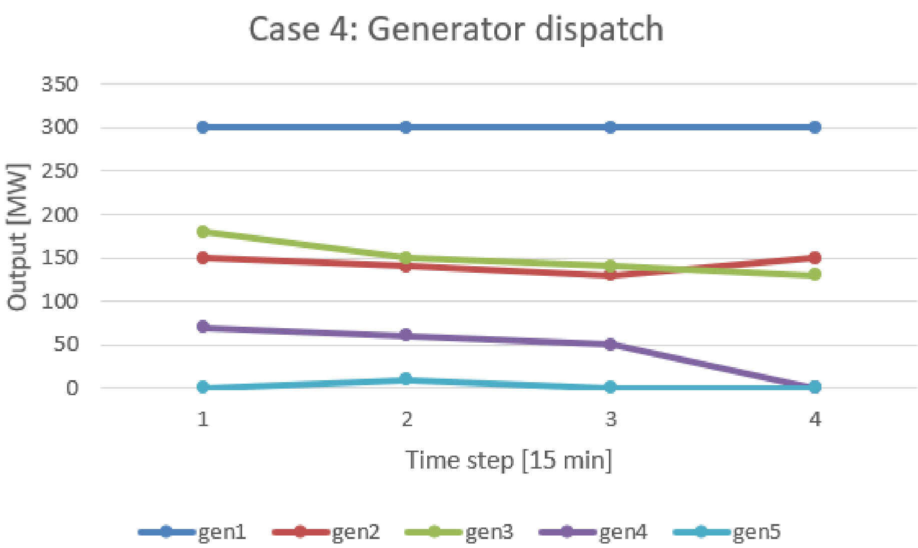

Figure 6.2: Case 4: Generator output (g_{it}) in all time steps. Stochastic model with flexiramp.

Table 6.9: Case 4: Up- and Down-Flexiramp provided by each unit. Stochastic model with flexiramp.

	Up-Flexiramp ur_{it} [MW]		
	t=1	t=2	t=3
G1	0	0	0
G2	0	10	-30
G3	-40	40	40
G4	30	0	0
G5	60	0	0
	Down-Flexiramp dr_{it} [MW]		
	t=1	t=2	t=3
G1	0	0	0
G2	40	40	40
G3	40	40	40
G4	20	10	10
G5	-10	0	0

Table 6.10: Case 4: Dual values & social welfare for the stochastic model with flexiramp.

	t=1	t=2	t=3	t=4
Energy Price, λ_t [$/MW]	12.88	2.5	1.25	1.25
Up-Flexiramp, μ_t^u [$/MW]	0	0	0	
Down-Flexiramp, μ_t^d [$/MW]	2.11	5	0	
Social welfare	$20,692,555.10			

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Enabling Flexiramp Capability in Unit Commitment Formulation

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