REAL-TIME OPTIMIZATION OF OIL PRODUCTION USING MODIFIER ADAPTATION

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Preface

This report is a result of a project thesis in Cybernetics and robotics at the Norwegian University of Science and Technology. Mature oil fields that have passed plateau production often have a complex bottleneck structure, which makes it hard to decide which well to produce from at any time. Furthermore uncertainty of reservoir characteristics and equipment capacities makes model-based optimization challenging. The motivation of this thesis is to investigate whether the modifier adaptation approach to real-time optimization can be a solution to the mentioned challenges.

The thesis is done with the support from my teaching supervisor Lars Imsland. I am grateful for his help and optimism during this work, and would like to thank him for it.

Summary

The report starts with an introduction of the problem, where the challenges are described. In the following section the Modifier adaptation approach in real-time optimization is described. Further a static optimization problem in a two-well system is presented. In this chapter the details around the system, which is being investigated, is modelled and described in detail. Moreover the static optimization problem is solved in an ideal situation. In other words the problem is solved in a situation where the mentioned challenges do not take place. This section is followed by the results and discussion of the obtained solution. Finally in chapter 4 the modifier adaptation approach for the two-well system is presented. To put it differently in this chapter the modifier adaptation approach, which is presented in chapter 2, is applied to the two-well system. Different from chapter 3, in this chapter the uncertainty of reservoir characteristics is taken into account.

The techniques used in this thesis is based on the methods proposed by (Marchetti et al. 2016). To the writers best knowledge, applying similar methods to handle model uncertainty in oil production optimization is reported in the literature once, by (Matias et al. 2018). On the other hand this was done on a completely different oil production facility structure.

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Abbreviations

GOR	=	Gas-oil ratio
MA	=	Modifier Adaptation
RTO	=	Real-time Optimization
FDA	=	Finite-difference approximation

Chapter 1

Introduction

As oil and gas reserves are getting more expensive and hard to explore it is important that we ensure best utilization of the resources. These projects are quite complex, thus there are many decisions which have to be taken carefully. Development of these fields involve multibillion investments with huge expecting returns. Therefore, the industry has been forced to come up with innovative strategies to obtain optimal operation of the production. To optimize there are many factors that have to be taken into consideration as in example increased water depths, environmental conditions, reserve structure and the ratio between gas and oil.

Since the oil is the most valuable product in the reserves the optimal production in oil fields involves maximizing oil production. When producing oil there will also be produced gas and water, but the amount of the different fluids varies from well to well. The production facilities have gas handling capacities, which is the maximum amount of gas the field can handle. Therefore, in oil production the ratio between gas and oil in each well, gas-oil ratio (GOR), is an important factor when deciding how to produce the oil. In other words it is essential that the oil

production is from the wells with lowest GOR at all times, for optimal production with respect to the gas handling capacity constraint.

At the moment there exist simulators which can predict the GOR for different wells. Combining these simulators with model-based optimization, as for example Real-time Optimization(RTO), is a powerful approach to optimize large scale production problems. However, if there are disturbances, sudden changes in GOR or reservoir pressures between the RTO iterations the GOR simulators will not be accurate enough. Hence the optimal operation calculated by the RTO will not be the actual optimal operation point for the plant. This problem, called plant-model mismatch, is researched a lot by the process engineering community and several RTO variants have arose. In this thesis I will investigate if this plant-model mismatch problem can be solved by a RTO method called Modifier Adaptation(MA).

Chapter 2

Modifier adaptation approach in real-time optimization

2.1 Real-time optimization

As mentioned in chapter 1 RTO, real-time optimization, arose to cope with difficulties associated with plant-model mismatch. RTO methods are typically structured with three main steps. First of all the process optimization contain a model of the process, also known as process modeling, followed by a numerical optimization of the obtained process model. Then an application of the optimal inputs, obtained from the numerical optimization, on the plant. If the models were a perfect representation of the plants it would be this easy, but unfortunately in the real world it is more challenging. The model-based optimal inputs are indeed optimal for the model, but as long as the model is not a perfect representation the inputs often are sub-optimal for the plant. However, RTO has showed its power to converge to optimal inputs, even when there exists mismatches between the plant and the model.

2.2   Modifier adaptation approach

In section 2.1 the challenge of making an accurate model of the plant was introduced. These inaccuracies, called plant-model mismatches, are mainly caused by one of the following reasons or a combination of multiple of them:

- Parametric uncertainty - when the model parameters do not correspond to the real process

- Structural plant-model mismatch - when there is mismatch in the structure of model, in example due to simplified/neglected dynamics or unknown characteristics of the process.

- Process disturbances

When choosing RTO-method there are some important properties that have to be considered. Guaranteed plant optimality upon convergence is one of the desired properties. In addition to that fast convergence and feasible-side convergence are two ideal properties too. In fact the latter two properties often oppose each other. As an illustration fast convergence often require large steps, while feasible-side convergence often call for small steps. Hence to satisfy these two requirements there must be a compromise between large and small step sizes. The key characteristic of modifier adaptation is that it satisfies the first-mentioned property. In other words it guarantees convergence to plant optimum even if there exist structural plant-model mismatches.

Modifier adaptation is a RTO method that uses the process measurements to improve cost and constraint functions. In general it uses correction terms for the cost and constraint function to update the plant model, instead of estimating the plant parameters, which is a more common strategy. More precisely it estimates

the plant gradients, from the measurements, which are used as gradient correction term in the model to modify both cost and constraint functions in the optimization problem. The use of gradients is justified by the necessary conditions of optimality that include constraints with sensitivity conditions. By enforcing the plant's and model's necessary conditions of optimality to match, the modified model will be a likely candidate to solve the plant optimization problem.

Regarding that the plant gradients cannot be measured directly, implementing MA in real situations can be difficult. Especially obtaining reliable estimates of the gradients from noisy measurements can be quite challenging. Estimating gradients can be done by dynamic perturbation methods, that uses transient data, or steady-state perturbation methods, which is simpler since they only use stationary data.

Finite difference approximation is the simplest steady-state method to estimate the gradients. First each input is perturbed around the operating point at the current step. Secondly the corresponding gradient elements get measured when the process reaches steady state. Finite difference approximation is sufficient for processes without noise and with few inputs. However, most realistic processes have noise and the method can lead to constraint violation when it operates near a constraint.

A more robust alternative to finite difference approximation is a quadratic approximation. The quadratic model obtains a local quadratic approximation of the cost and constrain functions, using the current and past operating points. Further the model calculates the plant gradient. In view of that higher order of approximation captures more precise information of the process it consequently decreases the in-

fluence of the noise. Hence this is a more accurate alternative to finite-difference approximation, but is indeed more complex.

Chapter 3

Static optimization in a two-well system

In this section a simple static optimization problem for oil production in a two-well system with gas capacity is presented. Figure 3.1 shows a sketch of the system. As can be seen from the figure the system consists of three submodels. One for reservoir inflow, a model for pressure drop through vertical pipe and a final one for the flow across a valve. The three submodels are described and modeled in the following three sections.
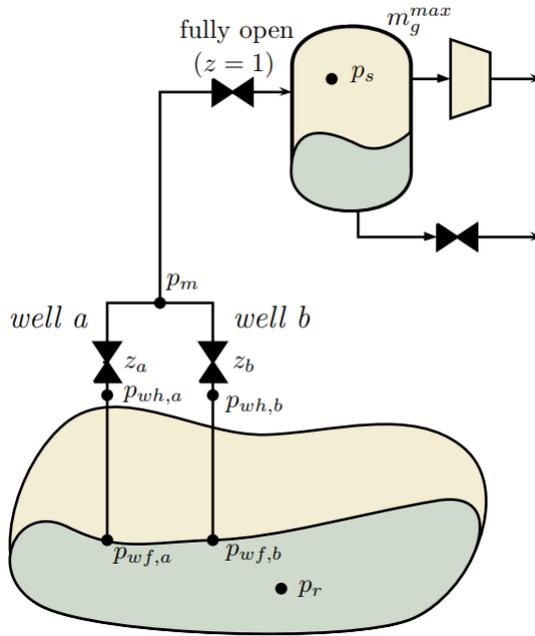
Figure 3.1: Static two-well system

3.1 Reservoir inflow

The reservoir inflow are assumed to follow Fetkowichs(1973) quadratic deliverability equation as following:

$$\dot{m}_o = k_o(p_r^2 - p_{wf}^2) \tag{3.1}$$

$$\dot{m}_w = k_w(p_r^2 - p_{wf}^2) \tag{3.2}$$

where $\dot{m}_o$ denotes the mass flow of the oil production and $\dot{m}_w$ is the mass flow of water from production. k_o and k_w are the flow coefficients for oil and water, respectively. p_r is the reservoir pressure and p_{wf} denotes the pressure in the pipe orifice in the reservoir. Combining the oil mass flow with GOR, gas oil ratio, the mass flow for gas production, $\dot{m}_g$, is obtained as following:

$$\dot{m}_g = GOR \cdot \dot{m}_o \tag{3.3}$$

3.2 Pressure drop across valve and through pipe

3.2.1 Modeling one phase pseudo fluid

Before modeling pressure drops, the three phased fluid has to be modeled. The fluid from the reservoir consists of three parts, specifically oil, water and gas. To simplify, the multiphased fluid will be approximated to one-phased pseudo fluid. Accordingly no mixing volumes is assumed. Furthermore oil and water are incompressible and the gas is assumed to follow the ideal gas law, described by the following equation:

$$\rho_g^{ig} = \frac{pM_g}{RT} \tag{3.4}$$

Where ρ_g^{ig} is the density, p is the pressure, M_g molar weight of gas, R is the ideal gas constant and T is the Temperature. Further the one phased pseudofluid is approimated by its volumetric average:

$$\rho_{mix}(p) = \frac{\dot{m}_o + \dot{m}_g + \dot{m}_w}{\frac{\dot{m}_o}{\rho_o} + \frac{\dot{m}_w}{\rho_w} + \frac{\dot{m}_g}{10^5 \rho_g^{ig}(p)}} \quad (3.5)$$

where ρ_o and ρ_w are the densities of the oil and water, respectively. The 10^5 term is used to scale up, since bar is used as unit for the pressure and Pascal is the standard unit in equation 3.4.

3.2.2 Pressure drop across valve

The mass flow across a valve is given by the following valve equation:

$$\dot{m}_o + \dot{m}_w + \dot{m}_g = f(z)C_dA\sqrt{\rho_{avg}(p_2 - p_1)} \quad (3.6)$$

where f(z) is describing the valve characteristics, with z between 0, when fully closed, and 1, when fully open. C_d is the valve constant, A is the cross section area of the pipe and the pressure on each side is denoted by p_1 and p_2. Hence the pressure drop across the valve is given by $\Delta p = p_2 - p_1$. From equation 3.5 it can be observed that the density for the one-phase fluid is dependent of the pressure. Therefore, ρ_{avg} in equation 3.6 is approximated by the average of the density on each side of the valve, given by the following equation:

$$\rho_{avg} = \frac{1}{2}(\rho_{mix}(p_1) + \rho_{mix}(p_2)) \quad (3.7)$$

Figure 3.2: Pressure drop across valve

Regarding that the manifold pressure, p_m, is set by the designer the pressure on the upper side, p_1 of the valve can easily be calculated. Further the pressure on the downside of the valve, p_2, will be an equation with respect to the massflows and f(z). In this model the valve characteristics are assumed to be linear as following:

$$f(z) = z \qquad\qquad , z \in [0, 1] \qquad\qquad (3.8)$$

Combining equations 3.6, 3.7 and 3.8 with some manipulations the following second order equation for p_2 is obtained:

$$p_2^2(\rho_{mix,1}\beta + \dot{m}_{tot}) + p_2(\rho_{mix,1}\alpha - p_1(\rho_{mix,1}\beta + \dot{m}_{tot}) - \frac{2 \cdot 10^6}{\gamma}\dot{m}_{tot}^2\beta)$$

$$-(p_1\rho_{mix,1}\alpha + \frac{2 \cdot 10^6}{\gamma}\dot{m}_{tot}^2\alpha) = 0$$

$$(3.9)$$

$$\alpha = \frac{RT\dot{m}_g}{10^5 \cdot M_g} \tag{3.10}$$

$$\beta = \frac{\dot{m}_o}{\rho_o} + \frac{\dot{m}_w}{\rho_w} \tag{3.11}$$

$$\dot{m}_{tot} = \dot{m}_o + \dot{m}_w + \dot{m}_g \tag{3.12}$$

$$\rho_{mix,1} = \frac{\dot{m}_{tot}}{\frac{\alpha}{p_1} + \beta} \tag{3.13}$$

$$\gamma = (f(z)C_dA)^2 \tag{3.14}$$

As can be seen the valve pressure drop is obtained by solving equation 3.9.

3.2.3 Pressure drop through vertical pipe

To estimate the pressure drop through a vertical pipe the stationary mechanical energy balance is used. No slip between phases and no friction are assumed. In addition to that work and kinetic energy are neglected. Hence the mechanical energy balance is as following:

$$dp = \rho_{mix}gdh \tag{3.15}$$

where g is the gravitation acceleration. Combining 3.5, 3.10, 3.11 and 3.12 the following equation is obtained:

$$dp = \frac{\dot{m}_{tot}}{\frac{\alpha}{p} + \beta}gdh \tag{3.16}$$

Integrating 3.16 from (p_1, h_1) to (p_2, h_2) the relation becomes

$$\beta(p_2 - p_1) + \alpha ln(\frac{p_2}{p_1}) = \dot{m}_{tot}g(h_2 - h_1) \tag{3.17}$$

As can be observed equation 3.17 cannot be solved exactly for p_2, due to the logarithm. For this reason a serial expansion of the natural logarithm is used:

$$ln(\frac{p_2}{p_1}) = ln(1 + \frac{p_2 - p_1}{p_1}) \approx \frac{p_2 - p_1}{p_1} \tag{3.18}$$

Combining 3.17 and 3.18, and using $\Delta h = h_2 - h_1$, the pressure p_2 can be expressed as

$$p_2 = p_1 + \frac{\dot{m}_{tot}gp_1\Delta h}{\alpha + \beta p_1} \tag{3.19}$$
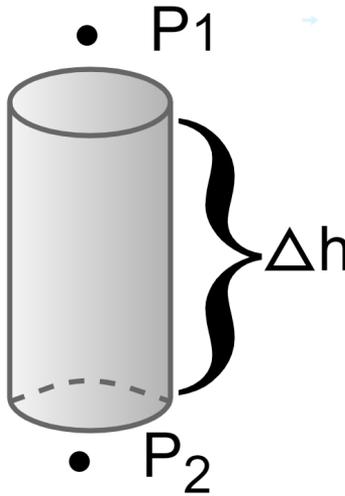
Figure 3.3: Pressure drop across vertical pipe

3.3   Formulating the optimization problem

As the dynamics of the system is modeled the optimization problem can be formulated. First of all the decision variables have to be determined. Further the

objective function and constraints have to be formulated.

In this problem it is desired to maximize the oil production with respect to gas capacity constraints. Therefore, the massflow of oil, $\dot{m}_o$, and gas, $\dot{m}_g$, have to be described by the decision variables. Since the massflows are dependent of the well inflow pressure, p_{wf}, and the valve pressure,p_{wh}, these will also be described by the decision variables. From equation 3.6 it can be observed that the massflow is dependent of the valve opening function $f(z) = z$. Therefore, the problem has one degree of freedom for each well, $f(z) = z$. Accordingly the valve openings for the the well will be one of the decision variables. Hence there is one decision variable,z , for each well. Regarding that the production is from two wells the optimization problem will in total consist of two decision variables. Furthermore it is assumed that there is no limit on how much water, from the production, the system can handle.

Regarding that the objective is to maximize oil production the objective function will be so simple as: maximize $\dot{m}_{o,1} + \dot{m}_{o,2}$

Prior to formulating the problem there are some restriction that have to be considered. Equation 3.1 and 3.3 are two of the constraints that have to be respected. In addition to that the system has a gas capacity constraint, from here will be denoted as $\dot{m}_{g,max}$, which set an upper limit on how much gas that can be produced.

Finally the optimization problem can be formulated:

$$x^T = \begin{bmatrix} x_1 & x_2 \end{bmatrix} = \begin{bmatrix} z_1 & z_2 \end{bmatrix}$$

where the subscripts in z denotes which well the variables belong to. Before formulating the problem, the massflows and other variables will be presented with respect to x_1 and x_2.

$$\dot{m}_{o,i} = K_{o,i}(P_r{}^2 - \mathbf{P^2_{wf_i}}(x_i)) \qquad\qquad i = [1,2]$$

$$\dot{m}_{w,i} = K_{w,i}(P_r{}^2 - \mathbf{P^2_{wf_i}}(x_i))) \qquad\qquad i = [1,2]$$

$$\dot{m}_{g,i} = GOR_i K_{o,i}(P_r{}^2 - \mathbf{P^2_{wf_i}}(x_i))) \qquad\qquad i = [1,2]$$

$$\dot{m}_{tot,i} = \dot{m}_{o,i} + \dot{m}_{w,i} + \dot{m}_{g,i} \qquad\qquad i = [1,2]$$

$$\alpha_i = \frac{RT\dot{m}_{g,i}}{10^5 \cdot M_g} \qquad\qquad i = [1,2]$$

$$\beta_i = \frac{\dot{m}_{o,i}}{\rho_o} + \frac{\dot{m}_{w,i}}{\rho_w} \qquad\qquad i = [1,2]$$

$$\rho_{avg,i} = \frac{1}{2}(\frac{\dot{m}_{tot,i}}{\frac{\alpha_i}{p_m} + \beta_i} + \frac{\dot{m}_{tot,i}}{\frac{\alpha_i}{P_{wh,i}} + \beta_i}) \qquad\qquad i = [1,2]$$

Where $\mathbf{P^2_{wf_i}}(x_i)$ denotes the inflow pressure as a function of the valve opening, x_i. Ultimately the final optimization problem can be formulated:

$$maximize \quad K_{o_1}(P_r{}^2 - \mathbf{P^2_{wf_1}}(x_1)) + K_{o_2}(P_r{}^2 - \mathbf{P^2_{wf_2}}(x_2)) \qquad (3.20)$$

$$s.t.$$

$$\mathbf{P_{wh_1}} = p_m + \frac{10^6 \cdot \dot{m}_{tot,1}^2}{\rho_{\mathbf{avg,1}} \cdot (x_1 C_d A)^2}$$

$$\mathbf{P_{wf_1}} = \mathbf{P_{wh_1}} + \frac{10^{-5} \cdot \dot{m}_{tot,2} \cdot gh \cdot \mathbf{P_{wh_1}}}{\alpha_1 + \beta_1 \cdot \mathbf{P_{wh_1}}}$$

$$\mathbf{P_{wh_2}} = p_m + \frac{10^6 \cdot \dot{m}_{tot,2}^2}{\rho_{\mathbf{avg,2}} \cdot (x_2 C_d A)^2}$$

$$\mathbf{P_{wf_2}} = \mathbf{P_{wh_2}} + \frac{10^{-5} \cdot \dot{m}_{tot,2} \cdot gh \cdot \mathbf{P_{wh_2}}}{\alpha_2 + \beta_2 \cdot \mathbf{P_{wh_2}}}$$

$$\dot{m}_{g,1} + \dot{m}_{g,2} \leq \dot{m}_{g,max}$$

3.3.1   Problem set-up

In the problem formulation, from previous section, it can be observed that the oil production is dependent on various constants. Among others it is dependent on the GOR and the manifold pressure, P_m, which is given by the separator pressure, P_s, together with the facility specifications. To illustrate which impact these values have I solved the static optimization problem for various GOR and P_s. For simplicity only the values for well 2 was changed, while keeping the values for well 1 constant. The values for the other constants, determined by the facility characteristics, can be found in section 5.2. The optimization problem was solved using the "fmincon"-function from the optimization toolbox in MATLAB, which uses the "interior-point" - method.

3.4   Results and discussion

As we can see from figure 3.4 the oil production is low for relative high GOR and high separator pressure. High GOR means that the production of gas is relatively high. Hence the oil production will reduce, since the production facility only can
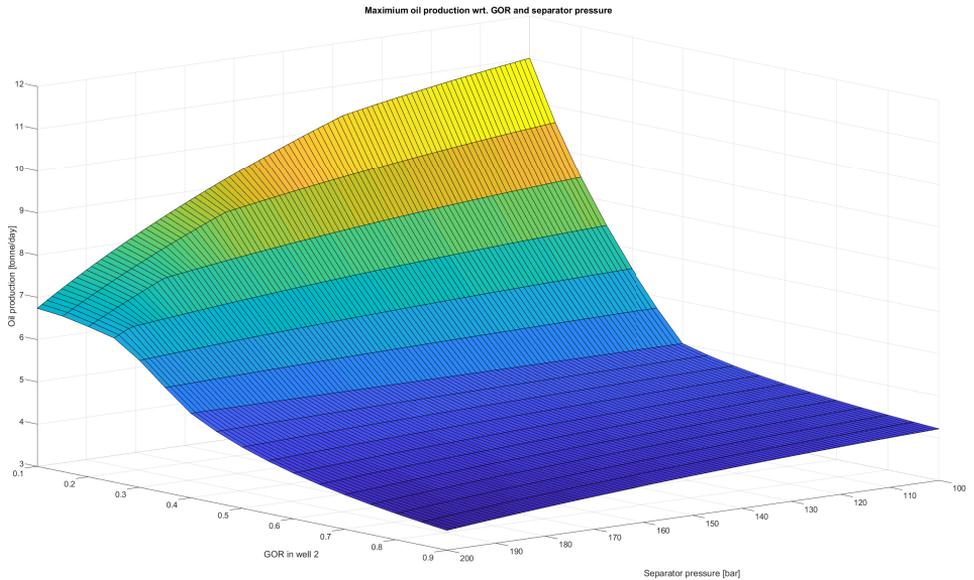
Figure 3.4: Produced oil for varying GOR and separator pressures

handle a certain amount of gas. On the other hand the results shows that the oil production also will reduce for high separator pressures. This also makes sense, since the pressure difference between the separator and the reservoir will reduce. Consequently the production flow will also reduce, which means that less oil will be produced. As we can observe from figure 3.4 the gas-oil ratio makes the biggest impact on production. When GOR is high the oil production is relatively low even for low separator pressures. Contrary when the GOR is low the oil production is relatively high even for high separator pressures . However, the oil production is significantly higher when both GOR and the separatory pressure is low.

The two-well static optimization problem has two degrees of freedom. One from each of the two valve openings. The lower flow pressure, P_{wf} which is explicitly in the oil production equation, is dependent on the valve openings. In fact the

more open the valve is the less will the lower flow pressure be. The following figure show the contour lines of the oil production with respect to the lower flow pressures in both wells.
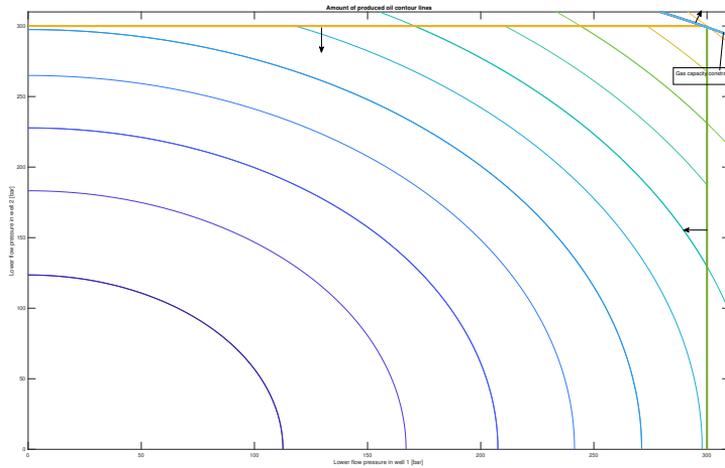
Figure 3.5: Oil produciton contour lines w.r.t. lower flow pressures

Not surprisingly it can be observed from figure 3.5 the lower P_{wf_1} and P_{wf_2} are the more is the oil production. This can also be verified from the objective function in equation 3.20. There are several reasons for why we cannot drive the lower flow pressures towards zero. One of them is simply the physics of the system, which we cannot do anything about. On the other hand the other constraint, which is interesting when solving this problem, is the gas-capacity constraint, which is visualized by the blue line in figure 3.5. Figure 3.6 shows a closer look of this constraint.
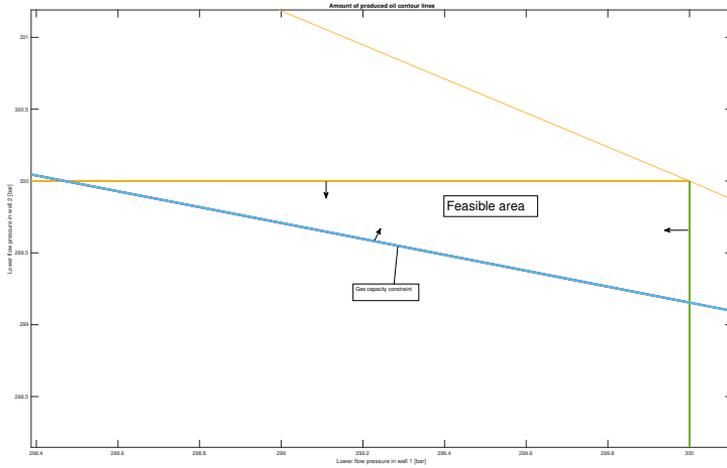
Figure 3.6: Oil produciton contour lines w.r.t. lower flow pressures

As can be seen from figure 3.6, the constraint pushes the lower flow pressures up. In other words the higher gas capacity there is the lower will the blue line parallel shift. In addition to that the gas-oil ratios will decide the slope of the constraint. The steepness is determined by the ratio between the GORs in both wells. For instance in this case it can be observed that the GOR in well 1 is higher than i well 2, since the slope is higher than 1, with respect to P_{wf_1}.

Chapter 4

MA-RTO of a two-well system to handle uncertain parameters

As mentioned in the introduction GOR is a very important parameter when solving the optimization problem. GOR can be modeled as a function of reservoir pressures and lower flow pressure. Thus GOR uncertainty can be a result of disturbances in measurements of the pressures. Disturbances in the process, sudden changes in GOR or reservoir pressures will also cause wrong GOR in the model, which can result in a suboptimal optimum for the plant when solving the optimization problem. As described earlier this uncertainty is called plant-model mismatch. In this section we will try to solve this problem using the Modifier Adaptation approach, which is described in section 2.

The Modifier Adaptation approach which has been used in this section is described in detail in "Marchetti et al. 2016". First of all two models have to be developed in order to study the plant-model mismatch. One model for the plant is needed, which will be updated by the measurements, referred as plant. Further one model

for the optimization layer, using the modifier adaptation framework, referred as the model. Moreover by applying zeroth and first order modifiers in the cost- and constraint functions the model-based optimization is supposed to reach the plant optimum.

First the zeroth order correction modifier is added as the difference between the plant and model values in each iteration. In other words the difference between the most recent plant measurement and model values are used in the optimization. Similarly the difference between the plant gradients and the gradients of the model are applied as the first order modifiers.

4.1 Formulating the optimization problem

In this thesis the focus is to study the plant-model mismatch. Therefore, it is assumed that both cost functions and constraint functions are measured without noise. Hence the plant-model mismatch only comes from the parametric uncertainty of the gas-oil ratios.

Finally the modified problem formulation becomes:

$$
x_{k+1}^* = \underset{\mathbf{x}}{argmax} \quad K_{o_1}(P_r{}^2 - \mathbf{P^2_{wf_1}}(x_1)) + K_{o_2}(P_r{}^2 - \mathbf{P^2_{wf_2}}(x_2)) + \lambda_k^J \begin{bmatrix} x_1 & - & x_1^k \\ x_2 & - & x_2^k \end{bmatrix}
$$

$$(4.1)$$

s.t.

$$P_{wh_1} = p_m + \frac{10^6 \cdot \dot{m}_{tot,1}^2}{\rho_{\mathbf{avg}}(P_{wh_1}) \cdot (x_1 C_d A)^2} + (\mathbf{C_{p,1}}(x_1^k) - \mathbf{C_1}(x_1^k)) + \lambda_k^{C_1} \begin{bmatrix} x_1 & - & x_1^k \\ x_2 & - & x_2^k \end{bmatrix}$$

$$P_{wf_1} = P_{wh_1} + \frac{10^{-5} \cdot \dot{m}_{tot,2} \cdot gh \cdot P_{wh_1}}{\alpha_1 + \beta_1 \cdot P_{wh_1}}$$

$$+ (\mathbf{C_{p,2}}(x_1^k) - \mathbf{C_2}(x_1^k)) + \lambda_k^{C_2} \begin{bmatrix} x_1 & - & x_1^k \\ x_2 & - & x_2^k \end{bmatrix}$$

$$P_{wh_2} = p_m + \frac{10^6 \cdot \dot{m}_{tot,2}^2}{\rho_{\mathbf{avg}}(P_{wh_2} \cdot (x_2 C_d A)^2} + (\mathbf{C_{p,3}}(x_2^k) - \mathbf{C_3}(x_2^k)) + \lambda_k^{C_3} \begin{bmatrix} x_1 & - & x_1^k \\ x_2 & - & x_2^k \end{bmatrix}$$

$$P_{wf_2} = P_{wh_2} + \frac{10^{-5} \cdot \dot{m}_{tot,2} \cdot gh \cdot P_{wh_2}}{\alpha_2 + \beta_2 \cdot P_{wh_2}}$$

$$+ (\mathbf{C_{p,4}}(x_2^k) - \mathbf{C_4}(x_2^k)) + \lambda_k^{C_4} \begin{bmatrix} x_1 & - & x_1^k \\ x_2 & - & x_2^k \end{bmatrix}$$

$$\dot{m}_{g,1} + \dot{m}_{g,2} + (\mathbf{C_{p,5}}(x_1^k, x_2^k) - \mathbf{C_5}(x_1^k, x_2^k)) + \lambda_k^{C_1} \begin{bmatrix} x_1 & - & x_1^k \\ x_2 & - & x_2^k \end{bmatrix} \leq \dot{m}_{g,max}$$

where the modifiers are defined as following:

$$\lambda_k^J = (1 - K^J)\lambda_{k-1}^J + K^J(\widehat{\nabla \mathbf{J}_{\mathbf{p,k}}} - \widehat{\nabla \mathbf{J}_{\mathbf{k}}})$$

$$\lambda_k^{C_i} = (1 - K^{C_i})\lambda_{k-1}^{C_i} + K^{C_i}(\widehat{\nabla \mathbf{C}_{\mathbf{i,p,k}}} - \widehat{\nabla \mathbf{C}_{\mathbf{i,k}}}) \qquad i = [1, 2, 3, 4, 5]$$

$\mathbf{C_{p,i}}(x^k)$ is the constraint measurements from the plant model. Thus the plant constraints $\mathbf{C_{p,i}}(x^k)$ for i=1 to i=5 are the plant measurements of P_{wh_1}, P_{wf_1}, P_{wh_2}, P_{wf_2} and $\dot{m}_{g,1} + \dot{m}_{g,2}$, respectively. Similarly $\mathbf{C_i}(x^k)$ are the model values for the same constraints. Moreover using these values the derivatives of the model constraints, $\widehat{\nabla \mathbf{C}_{\mathbf{i,k}}}$, and plant constraint gradients, $\widehat{\nabla \mathbf{C}_{\mathbf{i,p,k}}}$, are estimated. In addition to that the plant cost gradient, $\widehat{\nabla \mathbf{J}_{\mathbf{i,p,k}}}$, is also estimated from the measurements.

4.1.1 Gradient estimation using FDA with past RTO points

As mentioned in the previous section the plant gradients have to be estimated. The biggest challenge when estimating these are noise, due to the fact the the gradients cannot be measured directly. In "Marchetti et al. 2016" multiple methods are presented. One of these are finite-difference approximation using past RTO points. Initialization of this technique requires number of operating points equal to the number of inputs, in the problem, plus one in order to estimate the gradients. Hence it will require 3 operating points in this case, regarding that this problem has 2 inputs. One can obtain these points by perturbating each input around the current operating point two times. Thus the following matrices can be constructed

$$U_k = \begin{bmatrix} x_k^1 - x_{k-1}^1 & x_k^1 - x_{k-2}^1 \\ x_k^2 - x_{k-1}^2 & x_k^2 - x_{k-2}^2 \end{bmatrix}$$

$$\delta \widetilde{J}_{p,k}^{\ T} = \begin{bmatrix} \widetilde{J}_{p,k} - \widetilde{J}_{p,k-1} & \widetilde{J}_{p,k} - \widetilde{J}_{p,k-2} \end{bmatrix}$$

$$\delta \widetilde{C}_{p,i,k}^{\ T} = \begin{bmatrix} \widetilde{C}_{p,i,k} - \widetilde{C}_{p,i,k-1} & \widetilde{C}_{p,i,k} - \widetilde{C}_{p,i,k-2} \end{bmatrix}$$

Finally the plant gradients become

$$\widehat{\nabla \mathbf{J}_{\mathbf{p,k}}} = \delta \widetilde{J}_{p,k}^{\ T} (U_k)^{-1}$$

$$\widehat{\nabla \mathbf{C}_{\mathbf{p,i,k}}} = \delta \widetilde{C}_{p,i,k}^{\ T} (U_k)^{-1} \qquad\qquad i = [1, 2, 3, 4, 5]$$

where k indicates which time instant the values belong to.

4.2 Problem set-up

Now as the modifier adaptation approach for this two-well static optimization problem is described, the algorithm can be presented. The optimization problem

was solved using the "fmincon"-function from the optimization toolbox in MAT-LAB, which uses the "interior-point" - method. For the interested readers, the MATLAB-code can be found in section 5.1.

4.2.1 The algorithm

1. Choose an initial point $[x_0^1 \quad x_0^2]$

2. Perturbate x(0) two times to get enough data points for initialization.

3. Obtain the gradients with FDA using the recent three data points.

4. Calculate the current Modifiers: $\lambda_k^J, \lambda_k^{C_i}$

5. Solve the static optimization problem in 4.1 and find the next optimal input, z_{k+1}.

6. Apply the new input , z_{k+1}, and measure the outputs.

7. Return to point 3.

From the algorithm one can observe that the pertrubation is only used in the finite-difference approximation in order to estimate the initial gradient. Further it is solving a static optimization problem in every iteration. Accordingly it is applying the new valve input after each iteration and solving the problem with the most recent data points. Hence the modifiers are also updated after each input, using the new gradient estimations from the finite-difference approximation.

In this example the actual GORs of the plant were 0.6 and 0.4 for well 1 and well 2, respectively. On the other hand the modelled GORs were 0.45 and 0.4 for well 1 and well 2, respectively. In other words the modelled GOR for well 2 is correct,

but for well 1 is too low. Hence the model is underestimating the produced gas in well 1.

4.3   Results and discussion
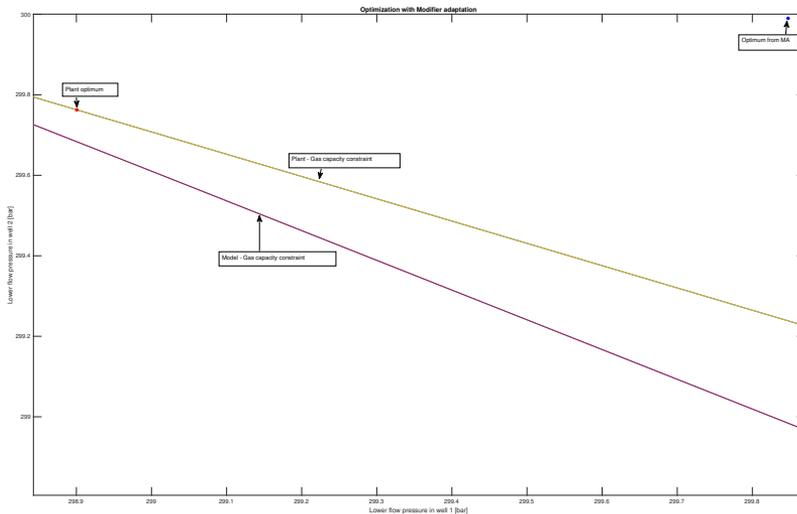
Figure 4.1: Solving the optimal production in a two-well system with Modifier Adaptation method

The result using the MA approach is presented in figure 4.1. The plant-model mismatch can be observed from the gas capacity constraints. As the model is undervaluing the GOR in well 1, the constraint is more steep and shifted more to the left compared to the actual gas constraint of the plant. Important to realize is that the area between the plant's- and model's constraint is infeasible for the plant. In fact all points below the plant's gas capacity constraint are infeasible for the plant. Moreover the red point indicates the optimal data-point for the plant. To emphasize this point represent the point where the oil production is maximized and is the point where the MA algorithm should converge to, if it works as desired. Ob-

viously, from the figure, this is not the case. The MA approach converged to the blue point, which is quite far from the plant optimum. By all means this is at least a feasible solution, which would not be the case if the optimization problem were solved only based on the model.

Unfortunately I did not get the desired results using the MA approach. To investigate this several things were inspected. In a complex and huge problem as this many things can go south. First of all these methods rely on many approximations and simplifications. In addition to that a good initial point is important. Hence the problem can be that the initial points, which were examined, were not sufficient. The initial perturbation were done to obtain an approximation of the gradient, but if they were not independent enough they would not span the whole feasible area of the plant. Thus a solution can be to explore more of the area before exploiting, but exploring too much would be inefficient regarding time, also known as the explore/exploit-dilemma.

Another important key point is the estimation method which were used to estimate the gradient. As described in section 4.1.1 the estimation method which were used was FDA with past RTO points. First thing to remember is that this is the simplest steady-state method to estimate the gradients. With this in mind if the deviation between the plant and the FDA is too big, the FDA representation of the plant will not be sufficient. Thus obtaining reliable estimates can be a challenge. There are more robust and precise, but more complex, alternatives to FDA. In example quadratic approximation, which also uses past RTO-points, could be used for a more precise estimation. Of course even higher order estimators can be used, but the increasing complexity must also be taken in mind.

4.4 Conclusion

The motivation for this project was to investigate whether the modifier adaptation approach to real-time optimization could be a solution to cope with parameter uncertainty in a static oil production system. As discussed in the section above, I was not able to solve the problem with the modifier adaptation approach to real-time optimization. However, the result were at least better than the model optimum without MA, as the model optimum was not feasible. After all we discussed things that can be improved, which may improve the solution. In fact this problem may also be too complex to solve with this method. To sum up there are several things that could have been done differently in order to get a better result, but in view of the big infeasible area of the model the method at least drew the solution to a feasible point.

Future work

All things considered this project was very educational and interesting. Especially applying theory I have learned in previous courses on an actual problem was educational. There were lot of unexpected challenges that showed up during the work, and things I took for granted was not that trivial in reality. In example model simplifications and making the algorithm work were challenging at times. All these challenges made me undoubtly more motivated to work further with this problem. Obviously I did not get the desired results, but as mentioned in section 4.3 there were several things that could had been done differently. Thus in my future work I will investigate if other techniques, that has been discussed in section 4.3, will give the desired solution. Moreover I want to investigate if I can use some machine

learning techniques, on all the measured data, to give an accurate estimate of the plant GOR.

Bibliography

1. Nocedal, J. (2006). Numerical Optimization. New York: Springer.

2. Marchetti, A., Francois, G., Faulwasser, T., and Bonvin, D. (2016). Modifier Adaptation for Real-Time Optimization - Methods and Applications. Processes, 4(4), 55.

3. Matias, A., Roux, G., Jschke, J. (2018). Modifier adaptation for real-rime optimization of a gas lifted well network. IFAC PapersOnLine, 51(8): 31-36.

4. Urbanczyktl, C., Wattenbargert, R. (1994). Optimization of Well Rates under Gas Coning Conditions. SPE Advanced Technology Series, 2(2): 61-68.

5. Mjaavatten, A., Aasheim, R., Saelid, S., Gronning, O. (2008). A Model for Gas Coning and Rate-Dependent Gas/Oil Ratio in an Oil-Rim Reservoir. SPE, 11(5): 842-847.

6. Ferreira, T., Shukla, H., Faulwasser, T., Jones, C., Bonvin, D. (2018). Real-Time optimization of Uncertain Process Systems via Modifier Adaptation and Gaussian Processes. IEEE, 2018 ECC: 465-470.

7. Fetkovich, M. (1973). The isochronal testing of oil wells. Society of Petroleum Engineers, In Fall Meeting of the Society of Petroleum Engineers of AIME: 137142.

Chapter 5

Appendix

5.1 Optimization with Modifier Adaptation - MATLAB code

5.1.1 Static optimization

```matlab
1  function [x, fval] = opt(x0)
2  evalin('base','save myvars.mat');
3  load myvars.mat; % Loader variabler fra workspace
4
5  fun = @(x) -(Ko(1)*(Pr(1)^2 - x(1)^2) + Ko(2)*(Pr(2)^2
       - x(2)^2)  - Lamb_J_k*[x(5)-x_k(1); x(6)-x_k(2)] )
       ;
6
7  A = [];
8  b = [];
9  Aeq = [];
```

```
10  beq = [];

11

12

13  lb = [Pm(1) Pm(1) Pm(1) Pm(1) 0 0];

14  ub = [Pr(1) Pr(2) Pr(1) Pr(2) 1 1]';

15

16  nonlcon = @ineqConstraint;

17  [x,fval] = fmincon(fun,x0,A,b,Aeq,beq,lb,ub,nonlcon);

18  end
```

5.1.2 Implementation of the Modifier adaptation approach

```
1   init;

2

3   Iterations = 10;

4   x_k = [0.2018 1];

5   x_k1 = [0.3018 0.8];

6   x_k2 = [0.35 0.6];

7

8

9   GOR_real = [0.6, 0.4];

10  GOR_est = [0.45, 0.4];

11  x0 = [0 0 0 0 0 0];

12

13

14  Lamb_J_k1 = [0 0];

15  Lamb_C_k1 = [0 0;0 0;0 0;0 0;0 0];

16
```

```
17  K_J = 0.5;
18  K_C = 0.5;
19  Result = zeros(Iterations,7);
20
21  for j = 1:Iteratons
22      [Plant_cost_grad, Plant_con_grad] = PlantGradient(
            x_k2, x_k1, x_k, GOR_real);
23      [mod_cost_grad, mod_con_grad] = PlantGradient(x_k2
            , x_k1, x_k, GOR_est);
24
25      Lamb_J_k = (1-K_J)*Lamb_J_k1 + K_J*(
            Plant_cost_grad -  mod_cost_grad);
26      Lamb_C_k = (1-K_C)*Lamb_C_k1 + K_C*(Plant_con_grad
             - mod_con_grad   ) ;
27
28      digitsOld = digits(100); %Increse precision of the
             casting
29      Lamb_J_k = double(Lamb_J_k);
30      Lamb_C_k = double(Lamb_C_k);
31      digits(digitsOld); % reset to default precision
             setting
32
33      [x0(1), x0(3)] = MeasurePressure(x_k, GOR_est, 1);
34      [x0(2), x0(4)] = MeasurePressure(x_k, GOR_est, 2);
35      x0(5) = x_k(1);
36      x0(6) = x_k(2);
37
```

```
38        [x,fval] = opt(x0);
39        Result(j,1:6) = x;
40        Result(j,7) = fval;
41
42        x_k2 = x_k1;
43        x_k1 = x_k;
44        x_k = [x(5), x(6)];
45        Lamb_J_k1 = Lamb_J_k;
46        Lamb_C_k1 = Lamb_C_k;
47        x0 = x;
48        clear x;
49   end
```

5.2 Units

Table 5.1: Table

Lift-gas	Unit	
p	bar	
h	m	1000
C_d	$\sqrt{\frac{kg}{mbarday^2}}$	84600
M_g	$\frac{kg}{mol}$	16.04
R	$\frac{J}{kmol \cdot Kelvin}$	8314
T	Kelvin	373
k_{o1}	$\frac{Tonn}{bar^2}$	$6.576 \cdot 10^{-3}$
k_{g1}	$\frac{Tonn}{bar^4}$	$8.239 \cdot 10^{-7}$
k_{w1}	$\frac{Tonn}{bar^2}$	$3.344 \cdot 10^{-3}$
k_{o2}	$\frac{Tonn}{bar^2}$	$5.462 \cdot 10^{-3}$
k_{g2}	$\frac{Tonn}{bar^4}$	$5.373 \cdot 10^{-7}$
k_{w2}	$\frac{Tonn}{bar^2}$	$1.031 \cdot 10^{-2}$
ρ_o	$\frac{kg}{m^3}$	800
ρ_w	$\frac{kg}{m^3}$	1000
g	$\frac{m}{s^2}$	9.81
K^J		0.5
$K^{C_i} \quad i = [1, 2, 3, 4, 5]$		0.5