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# A multi-layer power grid model for distribution systems (MV and LV): Where and what size to invest in Battery Storage? 

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## Sammendrag

Implementering av sol og vindkraft krever en radikal omlegging av dagens kraftsystem. Det er derfor viktig å finne løsninger som kan utnytte mulighetene den nye produksjonsmiksen gir og samtidig ivareta sikker og effektiv drift. En mulig løsning er å installere energilagringsenheter, som f.eks. batterier, i distribusjonsnettet.

Det er mange utfordringer med å finne en optimal plassering og dimensjonering av batterier i et distribusjonsnett, blant annet fordi det å framstille kraftnettverket matematisk gir en matematisk ulineæritet, videre er det ofte mangel på gode data, det er vanskelig prediktere utviklingen av batteripriser osv. Det finnes imidlertid flere måter å løse batteri optimeringsproblemet. I denne masteroppgaven har jeg valgt å bruke en mixed integer programming (MIP) energi basert modell. I dette tilfelle innebærer det å bruke energibalanse samtidig som en henter statiske data om kraftnettverket fra et stillbilde av en power flow analyse. Målet med masteroppgaven er å minimere de totale strømkostnadene samtidig som en dekker energi behovet til hus og industri i bestemte distribusjonsnett. Mer presist ønsker jeg å undersøke en type distribusjonsnett som ikke har blitt analysert tidligere - en kombinasjon av lav- og mellomspenningsnett.

Overordnet viser resultatene at det er $ø$ konomisk fordelaktig å installere batterier i distribusjonsnett som kombinerer lav- og mellomspenning, og at flesteparten av batteriene plasseres nærme noder med høy energiettersørsel. Den optimale batteriplasseringen i lavog mellomspenning er imidlertidig svært følsom når prisforskjellen økes litt mellom batterier som bare kan installeres i lavspenningsnettet og batterier som bare kan installeres i mellomspenningsnettet. Videre viser funnene at den totale batterikapasiteten reduseres når andelen av fornybare energiressurses $\varnothing$ ker. Batterikapasiteten synes først og fremst å bli påvirket av mengden av fornybar energiproduksjonen, mens plasseringen av batteriene påvirkes mest av forskjellen på batterikostnadene. Når det gjelder forskjellige nettverkstopologi (Loop og Radialt) viser imidlertid resultatene at de har mindre effekt på størrelsen og plasseringen av batteri.

## Summary

The deployment and adoption of solar and wind power is leading to a radical reorganisation of the current power system. It is therefore important to find solutions that can exploit the opportunities of the production-mix and at the same time ensure a safe and efficient power system. One possible solution is to install energy storage devices, such as batteries, in the distribution network.

Battery storage costs are projected to decrease dramatically within the next decade. Batteries will likely become affordable and have a widespread use for different power system services. Batteries provide flexibility to maintain a stable supply-demand balance. Related research has analysed the role of battery in distribution grids, but have not consider coordination between different power system layers. Raising the questions: where and what size to invest in battery storage? Should batteries be placed in the low-voltage or medium-voltage grid (or both)? To address these questions, this thesis has developed a new model that specifically represents both medium voltage and low voltage grids. It is a multi-layer grid designed for battery investments by considering short term operational decisions. The multi-layer grid model is a mixed integer program that minimises the total cost of electricity while supplying the energy demand of houses and industry in various distribution networks. The thesis analyses and investigates a distribution network problem that has not been analysed previously - a combination of low and medium voltage network.

Overall, the results reveal that it will be economic beneficial to install batteries in grids combining medium and low voltage, and most batteries close to the highest energy demand. The optimum battery placement, in the different grids, is very sensitive to cost difference between the batteries in the low and medium voltage grid. Furthermore, the findings show that the total battery capacity decrease when the proportion of renewable energy increase. The battery capacity seems to be primarily affected by the amount of renewable energy production, while the placement of the batteries is most effected by the difference in costs between low and medium voltage batteries. When it comes to different network topology (Loop and Radial), the results show that they have less effect on the battery placement and size decisions.

## Preface

This thesis concludes my masters degree at the Department of Industrial Economics and Technology Management of (IØT) at the Norwegian University of Science and Technology (NTNU).

The problem description was addressed in autumn 2018, in close collaboration with Senior researcher Pedro Crespo del Granado (IØT) and Professor Ruud Egging (IØT). However, research is not always carried out according to plan. Along the way, we discovered some challenges using the solution method, linearised power flow, in my problem description. Among other tings, this led to the need to adjust the problem description and method in my thesis. Although it is frustrating when things don't work as planned, in retrospect it has been a very educational experience.

The responsibility for the master is mine, but it would not have have been possible without many people inputs, discussions and motivation. I would like to thank my supervisor Ruud Egging for your wise comments, inputs and ideas. A special thanks to my supervisor Pedro Crespo del Granado for your always good mood, that you always are available and supportive and not least, your ability to share your deep insights. Alexander Luisi and Matthias Resch, you deserve a huge thank you for the interest you have shown in my thesis, and the effort you have made reading and commenting my drafts. I would also like to thank Kjersti Berg for interesting discussions to improve my modelling assumptions. Moreover, through Kjersti, Matthias and Pedro, I had the opportunity to share insights on battery developments in the SINTEF project: IntegER - Integration of energy storage in the distribution grid.

Last but not least, I would like to thank my friends and family for all your love and support.

Trondheim, 17 juni 2019

Erik Næss Guldbrandsøy

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\section*{| Chapter |
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## Introduction

There is a scientific consensus that we substantially need to increase renewable energy sources (RES) in our power systems if we are going to achieve the ambitious climate goals (EU, 2018). However, there are many political, economical and technological obstacles that need to be solved before RES represent the majority of the energy production.

A major challenge of any power system is to assure a stable supply-demand balance (Moslehi et al., 2010). Including RES in the existing power system has the challenge to improve the control of the unstable productions of wind and solar power. Especially in medium- and low-voltage grids as both face the challenge of integrating and managing large shares of decentralised RES. Local wind farms and solar rooftops along with an increasing number of prosumers are forcing grid owners and operators to rethink traditional investment in and management of power grids. In this regard, most grid planning research and methods tend to focus on a single layer of the power grid while aggregating and simplifying the other layers, i.e. low-, medium-, and high-voltage grids. In order to have a more complete and comprehensive understanding of coordinating flexibility and investments among power grid layers, the interactions and synergies between them should be represented and modelled. In the energy systems modelling literature, the integration of RES affects the full infrastructure and value chain of the power system, hence some papers suggest linking models and layers of the power system (Crespo del Granado et al., 2018). To this end, a multi-layer grid model that represents both the medium- and the low-voltage grids would provide a general approach for analysing trade offs on investing in flexible energy resources versus other grid investments to integrate more decentralised RES. A promising technology to complement the deployment of decentralised RES is the adoption of battery storage. However, investment in batteries compared to other options remains an open question. To some extend, this thesis, through developing a new multilayer model, analysis the opportunities and challenges in investment in batteries as well as their potential role on shaping the adoption of RES in low and medium voltage grids.

### 1.0.1 The battery revolution

It is important to develop technologies that assure a secure and reliable supply-demand balance as the energy productions from RES most likely will increase dramatically in the future. In addition, the development of cheaper and more efficient solar and wind technology is likely to result in many of today's consumers also being able to become renewable energy producers. In other words, future energy systems are likely to be more complex than those we have today.

One way to handle the some of this complexity is by installing electricity storage in the grid. One type of energy storage that has had a large decrease in cost is battery. An increase in interest, technological development and more financial support is a strong contributor to cheaper batteries and installation. According to Bloomberg New Energy Finance, renewable energy as wind and solar combined with battery storage generate an average cost which is lower than the average costs of new coal and gas-fired generators Parkinson (2018).

The investment cost of batteries has for a long time been assumed to be to expansive, but relatively new research and cost projections proves otherwise. Figure 1.1 illustrates a examination of Li-ion batteries for electrical vehicles (EV) (?). The authors (?) also claimed that the Li-ion battery cost in US was \$ 1000 per kWh in 2007 and in 2014 the cost was reduced to $\$ 350$ per kWh . If this reduction continues the cost will be $\$$ by the next decade.


Figure 1.1: Projected costs for Li-ion batteries. Figure extracted from (Nykvist and Nilsson, 2015)
The reduction of batteries costs in the EV market also applies for stationary batteries.

According to International Renewable Energy Agency (IRENA) (IRENA, 2017) the battery costs for stationary purpose drop by $66 \%$. IRENA also estimates an increase of the Li-ion batteries lifetime in 2030.

The cost reduction of battery from early 2000 until today has been very steep as mentioned, and the predictions estimate a further reduction in costs. The predictions vary from different sources, but the cost development so far, and with a high possibility of further reduction, gives plenty of motivation to investigate how Li-ion batteries can be implemented into the power market and their role in distribution grids.

### 1.0.2 Thesis objectives

Increased complexity will, among other things, make it more demanding to avoid congestion, voltage imbalance and low energy quality. Many of these challenges have already been addressed and several studies have been conducted (Das et al., 2018). Several have highlighted battery technology as a possible solution, but have assumed that it will not pay off financially. In this thesis we will look into these assumptions. More precisely, the main focus is to investigate whether battery storage can be cost-effective, and what is possibly the most cost-effective location in multi-layer grids with different amount of renewable energy productions.

## Research questions

To guide the investigation the following questions are devised as steps or orientation points. The overall question is:
What is a cost-effective battery investment strategy in combining low voltage (LV) and medium voltage (MV) grids? Where should batteries be located and what should the capacity be?

1. When we use different topology of grids?
2. When we vary the quantity energy produced from RES?
3. When we increase the difference in cost between MV and LV batteries?

Regarding the overall question it is expected that installing batteries in a multi-layer grid will reduce the total cost by taking arbitrage from the energy market price. There are three different ways battery can exploit the arbitrage in the energy market; Import more energy and store it when import price is high, and discharge the battery when prices are high. Store energy from surplus form RES instead of exporting for a low price or/and Reduce losses. The batteries can be installed at a given cost, the question will be if the investment cost is lower than the battery's exploitation of the arbitrage in the market. If there is possibility to exploit the market with installing batteries the next questions will be, what is the optimal placement and sizing of the batteries?

The most important with the follow-up questions is to investigate whether, and in what way, the most cost-effective battery strategy is affected by changes in the grid, grid topology, quantity of RES and battery prices.

### 1.0.3 Thesis structure

This thesis is paper-based and includes different chapters with different contributes to the topic. The structure of the thesis:

Chapter 1, Introduction, present the background and motivation for this thesis.
Chapter 2, Literature Review, gives and overview of related studies.
Chapter 3, Problem description, explains the problem scoop, objective, restrictions etc.
Chapter 4, Test Model, present a small problem of the master thesis and solves, to control that the components which should be used in master problem works.

Chapter 5, Where and what size to invest in Battery Storage in a reduced IEEE 13-Bus System, In this chapter the master thesis problem is present and formulated mathematically.

Chapter 6, Data, discussion and results from the reduced IEEE 13-bus system, presents the data, results and discussion for the master thesis problem.

Chapter 7, Computational study, present computational performance of the thesis model
Chapter 8, Conclusion and Recommendations for Further Work, present the most important findings and recommendations for further work.

## Chapter

## Literature Review

Integration of RES (renewable energy sources) in existing power system is not new. Neither is the use of battery storage, nor analyses of cost-effective strategies (Resch et al., 2017, 2019). In this chapter, we will look more closely into the part of the knowledge field that is most relevant for this thesis. More precisely, it review the literature about challenges that arises with RES in power systems and possible solutions, types of energy storage, the complexity of power grid and mathematical techniques to find optimal sizing and placement of batteries. Finally in the chapter is the proposed approach for this thesis.

Many power system have already installed RES in their grids (Birol, 2017). A further increase of RES, with poor control and uncertainty, will make the grid operation and planning more complex and challenging. More specific, the technical concerns will be issues related to power quality and power stability (Nazaripouya, 2017). The term power quality includes frequency and voltage fluctuation, while power stability is about power fluctuations, bi-directional flow, uncertainty and variation in power flow (Bayindir et al., 2016) (Anees, 2012). There are different approaches and combinations to address these challenges. One is that the operators, TSO and DSO, who are responsible for keeping a secure and stable energy supply to consumers upgrade the network with line enhancements (Anees, 2012). Another approach is that they can try to solve the challenges with storage devices, a third is to install automation and integrated smart grid, such as Distributed Energy Resources (DER) generation, and/or demand management described in (Tuballa and Abundo, 2016). Several point out the advantaged of installing storage devices in the power grid because it both gives control over the power quality and reduce the stability issues. This means giving the power system flexibility with balancing the generation and demand (Aneke and Wang, 2016). Energy storage can also increase the utilisation of the RES installed, and decrease the total energy cost for the RES-investor (Granado et al., 2016; Resch et al., 2019).

## Energy Storage technologies

Energy storage technologies are often divided in four types; electrohemica, mechanical, electrica and therma (ygado et al., 2018). Each of them are based on different storage technologies. However, some technologies have very similar characteristics, and the optimal sitting and sizing solution may be valid for several types of energy storage. On the other hand, different characteristics can have an impact on the allocation of storage (Wogrin and Gayme, 2015). It is therefore important to specify the type of storage technology that are used. In this thesis, the energy storage will be Li-ion battery, which is in the category electrohemica. This storage is chosen because of its high density, good cycle life and high charge/discharge efficiency. It also have some challenges as high production cost, sensitive to temperature and intolerance to deep discharges (Janko, 2014). However, there are energy analyses of lithium battery in real-time networks that indicate that storage allocation and sizing is not severely limited by this energy storage technology (Sbordone et al., 2015)

## Optimal sizing and sitting of batteries

Increasing complexity in the power grid, as a result of more RES, also makes it more demanding to find the optimal sitting and sizing of batteries due to the highly dimensional and non-convex problem. To generally solve highly dimensional and non-convex problem there are many mathematical techniques as analytic, classic, artificial and other miscellaneous techniques (Prakash and Khatod, 2016). Using the techniques to analyze a single power grid is in itself time-consuming. When optimizing the sitting and sizing of energy storage in a complex network, it needs to be simulated multiple times and it radically increase the solution time. In the literature there are mainly four techniques for solving optimal sitting and sizing problem; analytical methods, mathematical programming, exhaustive search and heuristics (Zidar et al., 2016).

The two most common approaches are mathematical programming and heuristics. And within analyze and calculation of power grid the most used tools is a mathematical programming called power flow (PF) or optimal power flow (OPF). These tools reconstruct very precisely the complex grid by mathematical equations. PF and OPF includes for instance the technical requirements of the power system as voltage, load, reactive power (Boroojeni et al., 2016). At the same time is it possible to include decision variables, and these can be used to find the best placement and size of the batteries. One challenge with using OPF is that the reconstruction of the grid is so detailed that that it becomes highly dimensional and non-convex and makes it difficult to ensure convergence (Lavaei and Low, 2012). To establish convergence with use of OPF the complexity has to be reduced. There are different ways to reduce the complexity and they have different magnitude of simplification. The two main simplifications techniques are relaxation of the OPF, network model linearzation, or using heuristic. In table 2.1 there is listed different articles using relaxation of OPF to find optimal size and placement, and in table 2.2 there is different approaches finding sitting and sizing using heuristic.

| Title of article | Author | Method | Focus | Challenge with method | Conclusion |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Optimal sizing and placement of distribution grid connected battery systems through an SOCP optimal power flow algorithm | Etta Grover-Silva, Robin Girard, George Kariniotakis | Second order cone program OPF to guarantee algorithmic performance | Techno-economic analysis of benefits in comparison to investment cost of batteries in medium voltage distribution feeder | -The model is very sensitive to investment cost of the batteries <br> - Not considered other solution as upgrading infrastructure and active load instead of energy storage | The total cost is reduced when batteries are installed in the grid compared with no battery in the grid. |
| Optimal Sizing and Placement of Distributed Storage in Low Voltage Network (2) | Philipp <br> Fortenbacher, <br> Martin Zellner, <br> Goran Andersson | Novel Optimal power flow method; based on a linearized multiperiod optimal power flow method, forward backward sweep optimal power flow (FBS-OPF) | Cost-optimal placement and sizing of battery. More specific: which storage configuration (centralized or distributed) is more viable under the assumption that the LV-network with high photovoltaic is connected to the power market. | - Solution deviates slightly from the optimum <br> - Expanding to bigger networks with a larger number of storages will make the problem too complex due to the storage coupling | - Distributed storage / decentralized storage are preferable. <br> -The results from this article also indicates that the economic impact of curtailing PV energy is higher than the saved network losses from the energy market transactions. |
| Optimal placement and sizing of battery storage to increase the PV hosting capacity of low voltage grids <br> (3) | Vasileios Poulios, Evangelos Vrettos, Florian Kienzle, Florian Kienzle, Hansruedi Luternauer, Göran Andersson | Optimal Power Flow (OPF) algorithms: simplifications of AC OPF and simplifications of DC OPF without neglecting crucial operational data such as voltages | Cost-optimal placement and sizing of battery storage as a PV integration measure in distribution grids with use of both AC OPF and DC OPF | -With the approximation the solution and computation time is very high using AC OPF <br> - Use DC OPF also because this approach is less time consuming, but missing crucial operational data <br> -The mix of both these methods can lead to incorrect solutions | The results of this article show that a DSO would have to bear high costs if the battery should be the exclusive solution to PV penetration challenges |

Figure 2.1: Articles using relaxation of OPF
(1) Grover-Silva et al. (2018), (2) Fortenbacher et al. (2016), (3) Poulios et al. (2015)

| Title of article | Author | Method | Focus | Challenge with method | Conclusion |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Optimal siting \& Sizing of Battery Energy Storage System In Active Distribution Network | Zhong Qing, <br> Yu Nanhua, <br> Zhang <br> Xiaoping, You <br> Yi, Liu Dong | Particle swarm optimization (PSO) with weight factors | Multi-objective optimal sitting and sizing of battery storage, concerning peak shaving capacity, voltage quality and active power adjustment capacity | - The problem is complex and the expanding of the problem will massivly increase the complexity <br> - The weight factor for the different objectives is difficult to decide | Battery energy storage system can give benefits regardless of peak shaving, voltage quality, providing active power adjustment capacity |
| Optimal Location and Sizing of Distributed Storage Systems in Active Distribution Networks | M. Nick, M. Hohmann, R. Cherkaoui, M. Paolone | A two-stage iterative procedure. First stage is a genetic algorithm, second stage evaluates the fitness of the solution by solving a daily AC optimal power flow | Optimal siting and sizing of distributed storage systems. In particular, the paper proposes voltage support of storage systems to the grid, the network losses and the cost of the energy-flow towards the external grid. | - The problem consist of a time period of one day of summer and winter <br> - The problem is kept nonlinear and non-convex which will increase the problem exponential when size increases | Results shows that energy storage have improved all the terms taken into account within the multi-objective function, namely the network voltages, the network losses as well as the cost of imported energy from the external grid. <br> Distributed Storage System contributions to the energy balance and support the grids ancillary services. |
| Optimal placement and sizing of the storage supporting transmission and distribution networks | Mahdi <br> Motalleb, Ehsan Reihani, Reza Ghorbani | A not categorized heuristic method with multi-purpose algorithm, while transmission Storage part used Complex-Valued Neural Networks and Time Domain Power Flow | Optimize the location(s) and size of the Battery Energy Storage System (BESS) regarding minimize cost, while it decrease fluctuations from the RES and controlling the frequency and voltage. | - The non-linear and nonconvex part of the problem is solved with Newton-Raphson method which is linearize the problem and can create a dual gap <br> - Only using load for one day which makes the model less realistic | The results from this article shows that the in the BESS some nodes are more voltage sensitive to change in RES and those are the nodes where the battery should be placed |

Figure 2.2: Articles using heuristic
(1) Zhong Qing et al. (2013), (2) Nick et al. (2013), (3) Motalleb et al. (2016)

The articles mention in figure 2.1 use convex approximations as FBS-OPF and SOCP
relaxation to simply the OPF problem. These approaches will make the problem convex and possible to solve in a polynomial time. One weakness with the approximation is that it provides a dual optimal solution, and the duality gap can be non-zero. There will be a non-zero duality gap if the difference between the actual optimal solution and the approximated. In article (Farivar and Low, 2013) results shows that the duality gaps can be up to $30 \%$ from the primal (actual) optimal solution when using convex approximation of AC OPF in a IEEE test system. The relaxation of the OPF gets more and more incorrect, duality gap increases, when the losses in the lines gets higher as in MV and LV due to the non-linearity of the losses, R/X ratio is large. In the last article about convex approximation there are used a combination of a simplified AC and DC OPF. The AC OPF will find optimal placement and sizing with voltage and other technical restrictions, but it is very time consuming. While the DC OPF is less time consuming, but neglect crucial operating data as voltage, reactive power etc. So they use a combination of these to find a optimal sizing and sitting of battery. This strategy can work, but they still do simplifications that cause uncertainty of it is the primal optimal solution.

In figure 2.2 the battery placement problem has been solved with a heuristic method of different types PF. It is a method to reduce the complexity of OP, but can't ensure global optimal solution. Even when simplifying the problem using this method it is still often very complex and the resolution time often very high. Together article 1 and 2 in figure 2.2 conclude that if the problem gets expanded in size and/or time horizon the complexity will increase a lot. In the second article, the authors used heuristic method on a OPF, simplified with Newton-Raphson method. This makes it possible to investigate larger problems, but there will be inaccuracy. So heuristic methods and simplifications of OPF have weaknesses due to uncertainty in optimal solution and/or high complexity, but they include the technical requirements. These requirement are crucial to make sure that the power grid is operating inside its physical restrictions. As mention earlier in this chapter the installing of new RES, as wind and solar, gives new technical problems for the grid and one solution is to install batteries. The articles mention here concludes that batteries can solve the technical problems, but for a high cost. There are also concluded that decentralised battery are economical preferred compared with centralised batteries.

In this thesis, finding the best placement and size of the battery is based on an energy balance model formulation. However, energy-based methods generally do not include losses or other technical operational data. This might bring some limitation on deciding placement of batteries, but are appropriate for deciding optimal size as seen in article (Harsha and Dahleh, 2011) and (Pedro Crespo Del Granado, 2014). Trying to make the energy based model suitable for technical restrictions the grid is build up with technical restrictions of a snap shot of a PF analysis. So the losses, line restrictions etc are included in this thesis, but it does not include the voltage and reactive power compared to other works (Nazaripouya et al., 2015). Also, given the complexity in modelling a multi-layer grid, considering detail technical specification will make the problem intractable, difficult to solve and analyze.

### 2.1 Proposed approach

Despite many challenges related to analysing power systems, it is important to emphasize that the various approaches used to optimize sitting and sizing of energy storage have contributed with important insights. Most papers have investigate the optimal size and placing in one layer grid - LV, MV or HV (see the article sin figures 2.1 and 2.2) and some have look at best placement in HV and MV-grids.

Based on this literature, I have not found related work analysing sitting and sizing of batteries in a multi-layer grid consisting of LV and MV. It is therefore interesting to investigate further what answers such an analysis will provide. More precise the thesis will try to provide insight in the placing and sizing of batteries when minimising the cost for DSO and end-user with different network attributes. The battery sizing and sitting problems will be minimized by using energy balance with technical information gathered from a snap shot PF analysis. This method is used because is less complex and support the possibility to do analyses on larger grids and for a long time period.


## Problem description

In the light of the introduction and literature review, this thesis aims to evaluate the economical value of battery sitting and sizing in a multi-layer grid, combining LV and MV, with the following research questions mention in introduction:

What is a cost-effective battery investment strategy in combining low voltage (LV) and medium voltage (MV) grids? Where should batteries be located and what should the capacity be?

1. When we use different topology of grids?
2. When we vary the quantity energy produced from RES?
3. When we increase the difference in cost between MV and LV batteries?

To address these questions, the main objective of this thesis is to make a model which minimise the total cost of multi-layer grids while the energy demand within the grid is supplied. The energy demand in the MV and LV distribution system can be supplied by three different sources; importing from the HV-grid, energy production from RES, discharging battery or a combination of these, see figure 3.1.

The model has two decisions to consider when minimising the total cost: invest in battery and/or upgrade transformer capacity, see figure 3.2. This is an cost-efficiency model and both decisions will be considered to minimise the total cost. Deciding to install batteries in one specific node will make it possible to store energy in it, while deciding to upgrade the transformer capacity will make it possible to transport more energy between the MV and LV-grid, see 3.3. The decision variables for investing in batteries are integer, while the decision variables for the transformers are binary, which makes it a Mix Integer Problem (MIP).


Figure 3.1: The energy demand-supply balance
If the model decides to not invest there are no more decisions to be made, and the model will only cover the demand with RES and imported energy. If only the transformer should be upgraded, the model has to pay an investment cost and determine which transformer to upgrade. If investing in batteries the model will have to pay a investment cost for each battery and decide how many, where to place them and when to charge and discharge. Both decisions are taken under deterministic circumstances.


Figure 3.2: Decisions to be made in the problem

The decisions variables will invest in batteries or/and transformers for two reason; to reduce total cost or/and forced by the technical restrictions when trying to cover the energy demand. The binary transformer decision variables will take value 1 if there is more to save from transporting a larger amount of energy between the MV and LV grid. Or if the
capacity on the transformers is to low for supplying some energy demand. The integer battery decisions variables take a integer value if there is technical restrictions in the grid that gives problem to supply the demand, or if it is possible to take larger arbitrage from the energy marked then the cost of the batteries. The battery can arbitrage the energy market by exploiting the dynamic importing cost, saving the surplus from RES and reducing the losses in the lines.

## Objective function

The objective is to minimise the total cost when supplying energy demand in LV-nodes for all time periods. The objective will consist of an operational cost and one-time investment cost. The operational cost will consist of the total cost for imported energy subtracted by the revenue for exporting energy during the two week period. The one-time investment costs will include the integer decision variables for building MV and LV-batteries, and the binary decision variables for upgrading the transformer. It is one one-time cost attached to each of the three different decision variables, but all these one-time costs are different.

## Available information

In this problem we are given information about the energy demand that has to be supplied, and the energy production from RES, for each time period. Additionally we are given the price for importing and exporting energy per kWh for all time periods. The importing price is like the energy production and load dynamic throughout all the time periods, while the exporting price is very low and static. The energy production from RES is limited by the amount decided, while the amount of energy imported has no-limitation. The one time cost for investing in one transformer, MV and LV-batteries is given and it is static.

## Construction of grid and battery

Each MV-node is connected to either HV-node, MV-nodes, LV-nodes or a combination. There can be transported energy between all nodes which are connected. If two MV-nodes are connected there will be a MV-line between them, see figure 3.3. This line will, in this problem, have two characteristics: maximum capacity and losses. The capacity is equal for all the MV-lines while the losses are different in the MV-lines due to length. The capacity of the MV-line will restrict the amount of energy that can be transported from one MV-node to another, while the losses will remove some the amount of energy transported in that line.

There will also be a connection between MV-node and LV-nodes, again see figure 3.3. Between this connection there will also be a MV/LV- transformer with capacity and loss. As mention earlier, all the energy demand is in the LV-nodes, therefore the capacity for each MV/LV-transformer is set to be higher than the maximum load in the LV-nodes connected to the transformer. The HV/MV transformer will have the same capacity as the MV-lines. The losses in all the transformers, HV/MV and MV/LV, will be equal and static.


Figure 3.3: Multi-layer grid description

There will be two types of batteries, batteries installed in MV-grid and batteries installed in LV-grid. Both these types will have a maximum capacity and minimum capacity. Each type will be given a maximum discharge and charge rate, which decide the maximum amount of energy going in and out of the battery in one time period. The last attribute for the batteries is the efficiency. The difference between the two types will be that the each battery installed in MV will have have higher amount of all the attributes, except the efficiency which will be the same for both.

## ${ }_{c}$ Cmane 4

## Test Model

This chapter will present a mathematical model, data, analyses and results of a small problem called the "Test Problem". The main purpose of this Test Problem is to see if the modelling of the grid and all the components are working. The objective of the Test Problem is the same as the objective in the master problem, minimising the total cost while meeting the energy demand. The Test Model will be further explained in the next section.

### 4.1 Description of the Test Model

The Test Model is illustrated in figure 4.1. The model consists of 8 nodes. 1 slack node, 2 medium-voltage nodes and 5 low-voltage nodes. Node 1 is a slack bus, which controls the energy, Energy is imported from the HV-node when the multi-grid lack energy and exported when it is a surplus of energy in the grid. The blue nodes, node 2 and 3, are the MV-nodes, while the green nodes 2.1, 2.2, 3.1, 3.2 and 3.3 are the LV-nodes.

As said before, the objective is to minimise the cost when covering the dynamic energy demand (load) in the LV-nodes in each time period. The duration of one time period is 30 minutes. To cover the energy demand in each period, the energy can be bought from the HV-grid, energy production in RES, or/and discharged energy from installed batteries. The amount of energy bought from the HV-grid is unlimited but there is a cost for each kWh imported, but no fixed investment cost. The RES production is placed as seen in figure 4.1, and it will be considered as a sunk cost, with no investment or maintain cost for RES. The alternative is to build batteries in the MV or LV-nodes, except node 1. The size and placement of the batteries will be decided by minimising the total cost. The batteries can supply energy by discharging if they are bought. Dynamic energy prices can give incentives to arbitrage the energy market, but installing batteries comes at a fixed investment cost. The model can decide to build batteries in the system, and it will do that if there is reduction of cost. It will find the optimal size and placement of the batteries.

As seen in figure 4.1 there will be installed RES in node $3,2.1,3.1,3.2$, and 3.3. There
will be solar panels in all nodes, except node 3.1 where there will be a wind park. Between the MV-grids and the LV-grids there is a transformer with an opportunity for an upgrade.


Figure 4.1: Description of the Test Problem

## Notation

In this subsection the notations for the mathematical model is presented.
Table 4.1: Indexes and magnitude in the example problem

| Index | Description | Units |
| :---: | :---: | :---: |
| t | Time Period | Hours, Half-hours, Minutes |
| i | Indicates all node in set I | Each nodes in problem |
| j | Indicates all node in set I | Each nodes in problem |
| Magnitude | Description | Units |
| T | Set of Time periods | Hours,Half-hours, Minutes |
| I | Set of nodes | $1,2,3,(2.1),(2.2),(3.1),(3.2),(3.3)$ |
| L | Set of LV-nodes | $(2.1),(2.2),(3.1),(3.2),(3.3)$ |
| M | Set of MV-nodes | 2,3 |

Table 4.2: Parameters

| Parameter | Description | Units |
| :---: | :---: | :---: |
| $\eta_{C h}$ | Efficiency of Battery Charging | \% |
| $\eta_{\text {Disch }}$ | Efficiency of Battery Discharging | \% |
| Batt MaxCapacityLV $^{\text {a }}$ | Maximum Energy Capacity of battery in LV | kWh |
| Batt MaxCapacity MV $^{\text {a }}$ | Maximum Energy Capacity of battery in MV | kWh |
| $S o C_{\text {minLV }}$ | Minimum Allowed State of Charge on Battery in LV | kWh |
| $S o C_{\text {min } M V}$ | Minimum Allowed State of Charge on Battery in MV | kWh |
| Price ${ }_{\text {Import }}^{(t)}$ | Price to pay for kWh imported from the grid | Pound/kWh |
| Price ${ }_{\text {Export }}^{(t)}$ | Price earned per kWh exported to the grid | Pound/kWh |
| BattCh maxLV | Maximum Charging Rate of battery in LV | kWh |
| BattCh maxMV | Maximum Charging Rate of battery in MV | kWh |
| BattDch $\operatorname{maxLV}$ | Maximum Discharging Rate of battery in LV | kWh |
| BattDch maxMV | Maximum Discharging Rate of battery in MV | kWh |
| Line $_{\text {Limit, }, \text { ij }}$ | Maximum Line Energy Thermal Limit from node i to j | kWh |
| Line $_{\text {Limit, }, \text { i }}$ | Maximum Line Energy Thermal Limit from node j to i | kWh |
| Load ${ }_{2.1}^{(t)}$ | Load in Node 2.1 | kWh |
| Load ${ }_{2.2}^{(t)}$ | Load in node 2.2 | kWh |
| Load ${ }_{3.1}^{(t)}$ | Load in node 3.1 | kWh |
| Load ${ }_{3.2}^{(t)}$ | Load in node 3.2 | kWh |
| Load ${ }_{3.3}^{(t)}$ | Load in node 3.3 | kWh |
| $P V_{i}^{(t)}$ | Generated Energy in Solar PV System in node i | kWh |
| Transformer ${ }_{\text {Capacity }, m}$ | Capacity of the MV/LV transformer in node m | kWh |
| Cost $_{\text {BatLV }}$ | Cost of installing battery in LV | Pound |
| Cost $_{\text {BatMV }}$ | Cost of installing battery in MV | Pound |
| Cost $_{\text {TransformerUpgrade }, m}$ | Cost of upgrading the transformer | Pound |
| Loss $_{\text {MV-line }}$ | Losses in line between MV-nodes | \% |
| Loss ${ }_{\text {Line\& }}$ Trans | Losses in LV-MV line plus transformer | \% |

Table 4.3: variables in the example problem

| Variable | Description | Units |
| :---: | :---: | :---: |
| $E_{\text {Lineij }}^{(t)}$ | Energy transported from node i to node j at time t | kWh |
| $E_{\text {Lineji }}^{(t)}$ | Energy transported from node $j$ to node i at time t | kWh |
| $B a t_{D i s c h, i}^{(t)}$ | Energy discharged from the battery in node i at time t | kWh |
| $B a t_{C h, i}^{(t)}$ | Energy charged to the battery in node $i$ at time $t$ | kWh |
| $\text { Input }_{L V 2}^{(t),}$ | Energy transported from the MV to LV in node 2 at time t | kWh |
| Output ${ }_{L V 2}^{(t)}$ | Energy transported from the LV to MV in node 2 at time t | kWh |
| Input ${ }_{L V 3}^{(t)}$ | Energy transported from the MV to LV in node 3 at time t | kWh |
| Output ${ }_{L V 3}^{(t)}$ | Energy transported from the LV to MV in node 3 at time t | kWh |
| $S o C_{i}^{(t)}$ | State of charge of battery in node i at time t | kWh |
| $y_{i}$ | Integer value of amount of batteries build LV-node i | Integer |
| $v_{i}$ | Integer value of amount of batteries build MV-node i | Integer |
| $b_{i}$ | ry value 1 if upgrade transformer in MV-node i, and 0 otherwise | Binary |

### 4.1.1 Mathematical formulation

In this section, the mathematical equation used in the Test Problem is presented. First the energy balance for the nodes is introduced, then the boundaries for the lines, batteries, PVsystem, transformers, and in the end the objective function witch includes the investment cost are presented.

## Slack bus (node 1)

$$
\begin{equation*}
E_{\text {Line } 12}^{(t)}-E_{\text {Line } 21}^{(t)} \quad \forall t \in T \tag{4.1}
\end{equation*}
$$

Equation 4.1 illustrates the energy imported to and exported from the multi-layer grid. $E_{\text {Line12 }}^{(t)}$ represent the energy imported and $E_{\text {Line } 21}^{(t)}$ the energy exported.

## Energy balance for node 2

$$
\begin{align*}
& \left(\text { Output }_{L V 2}^{(t)} * \text { Loss }_{\text {Line\&Trans }}\right)+E_{\text {Line } 12}^{(t)}+\left(E_{\text {Line } 32}^{(t)} * \text { Loss }_{M V-\text { line }}\right)+ \\
& \quad \text { Bat }_{\text {Disch } 2}^{(t)}+P V_{2}^{(t)}=E_{\text {Line } 21}^{(t)}+E_{\text {Line } 23}^{(t)}+\text { Bat }_{\text {Ch2 } 2}^{(t)}+\text { Input }_{\text {LV } 2}^{(t)} \quad \forall t \in T \tag{4.2}
\end{align*}
$$

Equation 4.2 illustrates the energy balance for node 2. This is the only node that is connected to the slack bus, node 1. The connection between the HV-grid and the multilayer grid is represented by the variables $E_{\text {Line } 21}^{(t)}$ and $E_{\text {Line12 }}^{(t)}$. Node 2 is also connected to node 3 with the variables $E_{\text {Line } 32}^{(t)}$ and $E_{\text {Line } 23}^{(t)}$, respectively energy transported from node 3 to 2 and from node 2 to 3 . The variable $I n p u t_{L V 2}^{(t)}$ represent the energy transported from
the MV, node 2, to the LV-nodes. While Output ${ }_{L V 2}^{(t)}$ is the energy transported from the LVgrids to the MV. The variables $P V_{2}^{(t)}, B a t_{D i s c h 2}^{(t)}$ and $B a t_{C h 2}^{(t)}$ express the discharged and charged energy from the battery and production of energy in the PV-system in node 2.The loss between the LV and MV nodes are represented by Loss $_{\text {Line\&Trans }}$ and $\operatorname{Loss}_{M V-l i n e}$ is the loss between the MV-grids.

## Energy balance for node 3

$$
\begin{align*}
& \left(\text { Output }_{L V 3}^{(t)} * \text { Loss }_{\text {Line\&Trans }}\right)+\left(E_{\text {Line } 23}^{(t)} * \text { Loss }_{M V-\text { line }}\right) \\
& \quad+\text { Bat }_{\text {Disch } 3}^{(t)}+P V_{3}^{(t)}=E_{\text {Line } 32}^{(t)}+B a t_{\text {Ch3 } 3}^{(t)}+\text { Input }_{L V 3}^{(t)} \quad \forall t \in T \tag{4.3}
\end{align*}
$$

In equation 4.3 the energy discharge and charge from the battery and the production from the PV-system in node 3 are represented by $B a t_{D i s c h 3}^{(t)}, B a t_{C h 3}^{(t)}$ and $P V_{3}^{(t)}$. Node 3 is connected with node 2, and the energy transported between them will be $E_{\text {Line } 23}^{(t)}$ and $E_{\text {Line32. }}^{(t)}$. The energy transported to the LV-nodes connected to node 3 is called Input $t_{L V 3}^{(t)}$, and energy from these LV-nodes to node 3 is Output $_{L V 3}^{(t)}$. The loss between the LV and MV nodes are represented by $\operatorname{Loss}_{\text {Line\&Trans }}$ and $\operatorname{Loss}_{M V-l i n e}$ is the loss between the MV-grids.

## Energy balance for LV-grids connected to node 2

$$
\begin{align*}
& \left(\text { Input }_{L V 2}^{(t)} * \text { Loss }_{\text {Line\&Trans }}\right)+B a t_{\text {Disch } 2.1}^{(t)}+B a t_{\text {Disch } 2.2}^{(t)}+P V_{2.1}^{(t)}+P V_{2.2}^{(t)}= \\
& \text { Load }_{2.1}^{(t)}+\text { Load }_{2.2}^{(t)}+\text { Bat }_{\text {ch2.1 }}^{(t)}+\text { Bat }_{\text {ch2.2 }}^{(t)}+\text { Output }_{L V 2}^{(t)} \quad \forall t \in T \tag{4.4}
\end{align*}
$$

In equation 4.4 the energy balance for all the LV-grids connected to node 2 are represented. They can be seen as one node because in this model there will not be any losses between the LV-nodes 2.1 and 2.2. The energy transported between the LV-grids and MVgrid is represented by the same variable as in 4.2 Input $t_{L V 2}^{(t)}$. In the LV-nodes there can be produced energy from PV-system, $P V_{2.1}^{(t)}, P V_{2.2}^{(t)}$. In this problem $P V_{2.2}^{(t)}$ will be zero. If the model decided to build batteries in one of these LV-nodes it will be possible to charge and discharge energy with the variables $B a t_{D i s c h 2.1}^{(t)}, B a t_{D i s c h 2.2}^{(t)}, B a t_{c h 2.1}^{(t)}$ and $B a t_{c h 2.2}^{(t)}$. If there is no invested in batteries these variables will be forced to zero. In the LV-grid energy balance the loads are represented by the parameters $\operatorname{Load} d_{2.1}^{(t)}$ and $L o a d_{2.2}^{(t)}$. The loss between the LV and MV nodes are represented by Loss $_{\text {Line\& }}$ Trans and $\operatorname{Loss}_{M V-l i n e}$ is the loss between the MV-grids. The loss between the LV and MV nodes is represented by Loss Line\&Trans

## Energy balance for LV-grid nodes connected to node 3

$$
\begin{array}{r}
\left(\text { Input }_{L V 3}^{(t)} * \text { Loss }_{\text {Line\&Trans }}\right)+\text { Bat }_{\text {Disch } 3.1}^{(t)}+B a t_{\text {Disch } 3.2}^{(t)}+B a t_{\text {Disch } 3.3}^{(t)}+P V_{3.1}^{(t)}+P V_{3.2}^{(t)}+ \\
P V_{3.3}^{(t)}=\operatorname{Load}_{3.1}^{(t)}+\text { Load }_{3.2}^{(t)}+\text { Load }_{3.3}^{(t)}+B a t_{c h 3.1}^{(t)}+B a t_{c h 3.2}^{(t)}+B a t_{c h 3.2}^{(t)}+O u t p u t_{L V 3}^{(t)} \\
\forall t \in T \tag{4.5}
\end{array}
$$

In 4.5 the energy balance for the LV-grids connected to node 3 is represented. This is the same equation as 4.4 but with three LV-grids represented with 3.1, 3.2 and 3.3.

### 4.1.2 Constraints

This section will define the bounds for the components in multi-grid system first for the lines. Thereafter, characterise the batteries in LV-nodes and MV-nodes and in the end the max capacity of the transformer will be determined.

## Line

$$
\begin{array}{ll}
E_{\text {Line }, i j}^{(t)} \leq \text { Line }_{\text {Limit }, i j} & \forall i, j \in M \backslash\{i=j\} \\
E_{\text {Line }, j i}^{(t)} \leq \text { Line }_{\text {Limit }, j i} & \forall t \in T  \tag{4.7}\\
& \forall i, j \in M \backslash\{i=j\}
\end{array} \quad \forall t \in T
$$

In equations 4.6 and 4.7 the energy flow between node $i$ and $j$ is restricted by the limit of the line Line ${ }_{\text {Limit,ij }}$. The equations will ensure that it is not possible to transport more energy between the nodes than the capacity of the line.

## Battery for low-voltage grid

$$
\begin{gather*}
S o C_{i}^{(t)}=S o C_{i}^{(t-1)}-B a t_{\text {Dischi }}^{(t)} * \frac{1}{\eta_{\text {Disch }}}+B a t_{c h i}^{(t)} * \eta_{c h} \quad \forall i \in L \quad \forall t \in T  \tag{4.8}\\
S o C_{i}^{(t)} \leq B a t_{\text {MaxCapacityLV }} * y_{i} \quad \forall i \in L \quad \forall t \in T  \tag{4.9}\\
S o C_{i}^{(t)} \geq S o C_{\operatorname{minLV}} * y_{i} \quad \forall i \in L \quad \forall t \in T \tag{4.10}
\end{gather*}
$$

$$
\begin{equation*}
B a t_{c h, i}^{(t)} \leq{B a t C h_{\operatorname{maxLV}}} y_{i} \quad \forall i \in L \quad \forall t \in T \tag{4.11}
\end{equation*}
$$

$$
\begin{equation*}
\text { Bat }_{\text {Disch }, i}^{(t)} \leq \text { BatDisch }_{\operatorname{maxLV}} * y_{i} * \text { AllowBatteries }_{i} \quad \forall i \in L \quad \forall t \in T \tag{4.12}
\end{equation*}
$$

In equation 4.8 the state of charge (SoC) for LV-node battery is defined. To define the SoC in period t , one have to consider the SoC of the previous time period, $S o C_{i}^{(t-1)}$. The SoC from the previous periods will be subtracted by the the amount of energy discharged from the battery divided by the efficiency, and then added the amount of energy used to charge the battery multiplied with the charge efficiency. In the first period there will also be given an initial state of charge.

In equations 4.9 and 4.10 the maximum and minimum capacity of the battery in all the LV-grid is defined.

Connected to the SoC-equations and discharge/charge-equations there is an integer variable $y_{i}$. When $y_{i}$ takes the value 0 in node $i$ there will not be possible to save energy in that node, which indicates that there will not be a battery at that node. If it takes a value there will be possible a to store energy in that node. This integer value also exist in the objective function 4.20. The Allow Batteries ${ }_{i}$ is a value that is set to 0 when there will not allowed to build batteries in the grid, and 1 otherwise.

## Battery for medium-voltage grid

$$
\begin{array}{cl}
S o C_{i}^{(t)}=S o C_{i}^{(t-1)}-B a t_{\text {Disch } i}^{(t)} * \frac{1}{\eta_{\text {Disch }}}+B a t_{c h i}^{(t)} * \eta_{c h} & \forall i \in M \\
S o C_{i}^{(t)} \leq \text { Bat }_{\text {MaxCapacityMV }} * v_{i} & \forall t \in T \\
S o C_{i}^{(t)} \geq S o C_{\operatorname{minMV}} * v_{i} & \forall i \in M \\
B a t_{c h, i}^{(t)} \leq B a t C h_{\max M V} * v_{i} & \forall i \in M \tag{4.16}
\end{array} \quad \forall t \in T,
$$

$$
\text { Bat } t_{\text {Disch }, i}^{(t)} \leq \text { BatDisch }_{\max M V} * v_{i} * \text { AllowBatteries }_{i} \quad \forall i \in M \quad \forall t \in T
$$

The battery in the MV-nodes has the same characteristic as in LV-nodes. The only difference is that the battery can hold more energy and the discharge/charge rate is higher in the MV-nodes for each integer variable $v_{i}$.

## Transformer

$$
\begin{equation*}
\text { Input }_{L V, i}^{(t)} \leq \text { Transformer }_{\text {Capacity }, i}+M * b_{i} \quad \forall i \in M \quad \forall t \in T \tag{4.18}
\end{equation*}
$$

$$
\begin{equation*}
\text { Output }_{L V, i}^{(t)} \leq \text { Transformer }_{\text {Capacity }, i}+M * b_{i} \quad \forall i \in M \quad \forall t \in T \tag{4.19}
\end{equation*}
$$

In equation 4.18 and 4.19 the amount of energy transported through the transformer (MV/LV). Output ${ }_{L V, m}^{(t)}$ and Input $_{L V, m}^{(t)}$ has to be lower than the capacity of the current transformer. If the model wants to transfer more energy than the capacity of the transformer, it has to be upgraded with a capacity of M. $b_{m}$ is a binary variable which take the value 1 if the transformer should be upgraded.

## Objective function

$$
\begin{array}{r}
\min (10 * 52) *\left(\sum_{t}^{T}\left[P_{\text {Import }}^{(t)} * E_{\text {Line } 12}^{(t)}-P_{\text {Export }}^{(t)} * E_{\text {Line } 21}^{(t)}\right]\right)+ \\
\left(\operatorname{Cost}_{\text {BatLV }} * \sum_{i}^{L} y_{i}\right)+\left(\operatorname{Cost}_{\text {BatMV }} * \sum_{i}^{M} * v_{i}\right)+\sum_{i}^{M}\left[\text { Cost }_{\text {TransformerUpgrade }} * b_{i}\right] \tag{4.20}
\end{array}
$$

In the objective function 4.20 the energy imported $E_{\text {Line12 }}^{(t)}$ multiplied with Price ${ }_{\text {Import }}^{(t)}$ will represent the total cost, and total revenue is represented by the $E_{\text {Line } 21}^{(t)}$ multiplied the Price ${ }_{\text {Export }}^{(t)}$. The imported and exported energy is only simulated over 1 week, so to get approximate cost over 10 years this function is multiplied by 52 weeks and 10 years. The prices for import and export change throughout the week, so storing energy can give economic benefits. Storing energy in batteries comes for an investment cost Cost $_{B a t L V}$ and Cost $_{B a t M V}$. There will also be a possibility to increase the amount of energy transferred between the MV and LV-nodes, upgrade the transformer for a given investment cost. Since all the investments is optional the only reason it would build batteries or/and upgrade transformer is if the savings in drift is higher then the investment costs.

### 4.1.3 Data

In this section, the data collected for the Test Problem will be presented and accounted for. All the parameters defined and explained in the section mathematical model have to be given predetermined data. Some of these parameters has a static value and others are dynamic during time or/and different node.

The load in all the LV-grids is gathered form an European LV-Test feeder (Koirala, 2018). From this project, the energy consumption from 100 households was measured, the load was per minute, but it was summed into periods of 30 minutes which is the time period. How the load from the 100 households is divided in the Test Problem is listed below.

- $\operatorname{Load}_{2.1}$ represent households 1-50
- Load $_{2.2}$ represent households 51-100
- $\operatorname{Load}_{3.1}$ represent households (1-33)*1.2
- $\operatorname{Load}_{3.2}$ represent households (34-67)*1.2
- $\operatorname{Load}_{3.3}$ represent households (68-100)*1.2

Previous mention the loads in connected to node 2 and 3 consist of 100 households. They are divided into the different LV-nodes, these nodes are connected with no losses in the lines. In the model $\operatorname{Load}_{2.1}$ and $\operatorname{Load}_{2.2}$ are not separated and can be seen as one load, but for more possibility in the future analyses they are set apart. The total load in LV-nodes connected to MV-node 3 is 1.2 times higher. The total for the different loads was $\operatorname{Load}_{2.1}$ 441 kWh, Load $_{2.2} 401 \mathrm{kWh}$, Load $_{3.1} 303 \mathrm{kWh}$, Load $_{3.2} 263 \mathrm{kWh}$ and Load $_{3.3} 276 \mathrm{kWh}$.

The capacity for the transformer in node 2 was set to be $20 \%$ higher than the maximum load for $\operatorname{Load}_{2.1}$ and Load $_{2.2}$ in one time period, 31.5 kWh . So the transformer was set to 37.8 kWh . In node 3 the maximum load in one time period will be 37.8 kWh and the transformer was set to 45.36 kWh .

The energy production in RES is also like the load, a predetermined dynamic parameter. In this test model there will be installed solar panels in node 3, 2.1, 3.2 and 3.3. In each of these nodes the solar production will be $30 \%$ of the load. In the MV-node 3 there is not a load, so in this node the solar production is set to be $30 \%$ of the load in 2.1. Node 3.1 is the only node that produce energy from wind. In this node the total production will be $30 \%$ of load, but the shape of production each period is different. The shape is varying, and is not depended by the time of day.

The last dynamic data in this problem is the cost for importing energy form the HVgrid. The reference price data is taken from the electricity spot market in UK collected by APX Group (Group, 2019), the pound is converted NOK. See section 6.1 for more detailed information. The importing price will go up and down several times throughout the week simulated. One can see the imported cost in 4.6 , the green graph. The price for exporting
energy to the HV-grid is set to $0.2213 \mathrm{NOK} / \mathrm{kWh}$.

All the other parameters, as battery, lines etc. are static and will not be specified with numbers in this chapter, but in the master problem they will be defined. The prices for investing in batteries are changed in the different cases, and will therefore be given in the next section.

- Case 0 , no battery allowed to be build
- Case 1 , allowed to build battery
- Case 2, allowed to build battery and reduced cost for MV batteries
- Case MV-battery price, where I run the model with different prices


### 4.2 Results

### 4.2.1 Case 0

In case 0 the parameter Allow $^{\text {Batteries }}{ }_{i}$ is set to 0 , which deny the model to discharge energy form a battery. So there will no reason to build batteries. In this case the investment cost for batteries is not relevant. The results in this case represent the multi-grid without any batteries.

## Total cost

When the model is running with no battery the minimal cost for 10 years is 3915610 NOK. The total energy imported in one week is 16923.3 kWh and the exported is 151.56 kWh.

## Energy behaviour

The energy behaviour in the multi-grid shows how the energy demand is supplied and the losses.


Figure 4.2: Results from case 0

In figure 4.2 the total energy production, PV and wind, is represented in the blue bars and the yellow bars represent the imported energy. When the energy production is less the imported energy is raised, and when the production is very high the surplus of energy in the multi-grid will lead to energy exported. In this case the total losses throughout a week are $3,500.44 \mathrm{kWh}$.

### 4.2.2 Case 1

In this case the model is allowed to invest in batteries, AllowBatteries ${ }_{i}$ is set to 1 . The price for a 10 kWh battery in the LV-grid is $17,300 \mathrm{NOK}$. While the battery price for MV-battery, 15 kWh , is 23,355 NOK.

## Total cost and exported/imported energy

The total cost for the 10 years becomes $3,913,720$ NOK. The imported energy for the week is $16,774.51 \mathrm{kWh}$ and the exported energy is 129.86 kWh . The model decides to build one battery of 10 kWh for 17,300 NOK in node 3.3 , se figure 4.1.

## Energy behaviour



Figure 4.3: Energy behaviour from case 1

In figure 4.3 the total production from RES is represented by the blue bars and the imported energy is represented by the yellow bars. The loss is also illustrated by the red line diagram. In this case the loss in lines is $3,490.80 \mathrm{kWh}$. The losses due to charging and discharge of the battery is 12.63 kWh , and the total energy loss $3,503.43 \mathrm{kWh}$.


Figure 4.4: Battery behaviour in case 1

In figure 4.6 the behaviour of the battery is illustrated. SoC in the batteries depends on the production of energy in the RES and the prices of importing energy. One can see that when there is high production of energy or/and import prices are low the SoC raises, and when the prices are high and production is low the SoC falls to its minimum. The state of charge is changing between 1.5 kWh and 10 kWh , the yellow bars.

### 4.2.3 Case 2

In this case, the MV-battery cost is reduced by $25 \%$, and the reduced prices becomes 19,608 NOK/battery. This price is not realistic at present time, but for the future.

## Total cost and imported/exported energy

In this case the minimum cost over the 10 years is $3,909,530$ NOK and 3 MV -batteries are build in node 3 , see figure 4.1 . The price of the battery installed in node 3 be 58,824 NOK and have a size of 45 kWh . For one week the total imported and exported energy is respectively $16,894.21 \mathrm{kWh}$ and 97.48 kWh .

## Energy behaviour



Figure 4.5: Results from case 2

In figure 4.5 the total production from RES is represented by the blue bars and the imported energy is represented by the yellow bars. Because this case has a large battery park, the periods when the battery is discharging and charging has a bigger impact on the total energy needed. the losses are agains shown by the red graph. In this case the total loss in the lines was $3,472.50 \mathrm{kWh}$, and the loss in the battery is 28.62 kWh . This give a total loss of $3,501.13 \mathrm{kWh}$ in this case.


Figure 4.6: Battery behaviour in case 2

The SoC raises in periods with high energy production or/and low import prices and decreases in periods with high import prices or/and low production. The state of charge is changing between 6 and 45 kWh .

### 4.2.4 Case MV-battery prices

In this case do a sensitivity analysis of the MV-battery cost. The MV-battery price will be in a range fom $10,000 \mathrm{NOK} /$ battery to $20,000 \mathrm{NOK} /$ battery with an interval of 2,000 NOK/kWh. The cost of LV-battery will be kept at 14,700 NOK/battery.

| MV battery price (NOK) | Total MV batteries | Total LV batteries | Total cost (NOK) |
| :---: | :---: | :---: | :---: |
| 10,000 | 26 | 0 | $3,780,000$ |
| 12,000 | 22 | 0 | $3,825,432$ |
| 14,000 | 18 | 0 | $3,859,874$ |
| 16,000 | 9 | 0 | $3,886,214$ |
| 18,000 | 6 | 0 | $3,900,541$ |
| 20,000 | 0 | 4 | $2,905,614$ |

Table 4.4: Number of batteries in the multi-grid

As one can see, when the MV-battery price increases the total cost goes up while the number of MV-batteries goes down. When the MV-battery price is 18,000 NOK the best solution is still to install MV-batteries, while in the 20,000 NOK scenario the test model finds that the minimised cost is to build LV-batteries.

| MV battery cost (NOK) | Node 2 | Node 3 | Node 2.1 | Node 2.2 | Node 3.1 | Node 3.2 | Node 3.3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10,000 | 11 | 14 | 0 | 0 | 0 | 0 | 0 |
| 12,000 | 9 | 11 | 0 | 0 | 0 | 0 | 0 |
| 14,000 | 7 | 8 | 0 | 0 | 0 | 0 | 0 |
| 16,000 | 4 | 6 | 0 | 0 | 0 | 0 | 0 |
| 18,000 | 1 | 5 | 0 | 0 | 0 | 0 | 0 |
| 20,000 | 0 | 0 | 0 | 0 | 2 | 0 | 3 |

Table 4.5: Number and placemnt of batteries in the multi-grid

From table 4.4 the size of battery was illustrated. In table 4.5 the placement of the batteries is shown. The model always prefer to install batteries in node 3 then in node 2 . In the last scenario the test model builds 5 LV-batteries, 3 are build in node 3.1 and 2 in node 3.1.

### 4.3 Discussion

In this section, I will first talk about the different results from case 0,1 and 2 , and then discuss the results from the sensitivity analysis of MV-battery prices.

## The objective

- Case 0,3915610
- Case 1, 3913720
- case 2, 3909530

The best economical result is in case 2. This sounds reasonable when we look at the two other cases. Case 2 are given the same or better opportunities as case 0 and 1 . So the result from case 2 should be better, or as good as case 1 . This is also consistent with the results.

The total cost in cases 1 and 2 are lower than case 0 which indicates that the operational cost is reduced with the investment cost plus the difference in total cost. In case 2 the investment cost for 3 MV-batteries is 58,824 NOK, and the total cost becomes 6,080 NOK lower then case 0 . Over a 10 year period the operational cost is reduced with 64904 NOK.

## Battery size and placement

The battery prices used in case 1 is $1,730 \mathrm{NOK} / \mathrm{kWh}$ for batteries in LV-nodes, and 1,557 $\mathrm{NOK} / \mathrm{kWh}$ for the batteries in MV-grid. In case 1 the model choose to build one battery in the LV-node 3.3. This results indicate that it would rather build a battery in a LV-node for a higher NOK/kWh price than battery in a MV-node for a lower NOK/kWh. In case 2 the

MV-battery is reduced with an amount that favourites building batteries in the MV. The price reduction also increases the size of total installed battery. This results indicates that lower battery prices gives more intensives to exploit the arbitrage in the energy market.

The total battery size installed in case 1 and 2 are respectively 10 and 45 kWh . This is the optimal size based on results from the Test Problem where each battery has the size of 10 kWh in LV and 15 kWh in MV. If each battery had a smaller size optimal size could be different or it would install some battery in LV and some in MV. So reducing the size of each battery will make the model more flexible to install batteries.

## The energy behaviour

Looking at the results of the energy behaviour the case with no batteries has more transportation of energy between the MV-nodes, node 2 and node 3 . The reason for this is when there is surplus in the production in node 3 and the connected LV-grids energy is transported to node 2. In cases 1 and 2 it is possible to store some of the surplus to other periods, which reduce the amount of energy transported between the 2 MV-nodes. Because case 2 has a bigger battery in node 3, which can store more energy when there is surplus the energy transported between node 2 and 3 is lower then in case 1 .

The energy transferred between LV-nodes and the MV-nodes are exact the same in case 0 and case 2 . The reason for this is that energy in case 2 is stored in the MV-grid so the energy transported between the LV and MV-nodes will be the same. While in case 1 the energy transported into the LV-grid is increased. In some periods there will be transported energy into the batteries from node 3, increasing the input energy to LV-nodes. On the other hand the output energy from LV-nodes because of surplus of energy production in 3.1, 3.2 and 3.3 can be stored instead of transported to the MV-grid.

## Case MV-battery prices

The results from this sensitivity analysis of MV-battery prices shows that the total cost increases when the cost of the MV-batteries increases. These results seem very reasonable. When the price of MV-battery raise the total battery size decreases because it reduces amount of arbitrage in the energy market. When the price of the MV-battey gets high enough there are better to install LV-batteries. This results show that when batteries becomes cheaper there will be more intensives to store energy and reduce the total cost.

The placement of the batteries in the first 5 scenarios shows that installing batteries in MV-node 3 is better than in node 2. In the last scenario where LV-batteries are installed they are all placed in the LV-nodes connected to MV-node 3. This means that when minimising the total cost the model favourite building batteries in node 3 and LV-nodes connected to it instead of node 2 connected LV-nodes. There can be many factors that contribute to this result. Node 3 is further away from the HV-grid then node 2, there are more renewable energy production in node 3 and the LV-nodes connected to it or is it a combination of both. What factors have the biggest affect on the placement of batteries will be interesting to investigate further.

### 4.4 Conclusion

An analysis of the results proves out that the model and all the components seems to be working, which was the main purpose of this Test Problem, but there is also other interesting results from the Test Problem. From case 1 the battery price on $1,730 \$ / \mathrm{kWh}$ for LV-batteries very close to the breakeven cost when it only builds 1 battery. Also from this case it building batteries in LV-nodes compared to MV-nodes. From case 2 and case MVbattery prices, a reduction of battery investment cost increase the total storage installed and reduce the total cost. With cheaper batteries in the future, these results indicate that it would be possible to arbitrage the energy market. Regarding the placement of the batteries, all the cases which allowed battery there were always preferred to install in MV-node 3 or in the LV-nodes connected to the node. In chapter 5, with the expended model the topology of MV-grids and the amount and type of energy production will be in focus.

## Where and what size to invest in Battery Storage in a reduced IEEE 13-Bus System

In this chapter, the multi-layer grid will be constructed very similar to the IEEE 13-Bus System in (Schneider et al., 2018). The original and simplified can be seen in figure 5.1. In this chapter the grid will be reduced, node 633 and 675 are removed. Node 692 is representing a switch and Node 633 is representing loss in transformer, and since this represent MV-nodes, node 634 is placed where node 633 was.


Figure 5.1: IEEE 13-Bus system


Figure 5.2: The model with Loop topology

This reduced grids will have different shapes, see figure 5.2 and 5.3. The figure in a) is called Radial Tree and in b) Loop. The IEEE 13-Bus system which is used in this master has been analysed with power flow by others on high loads (Schneider et al., 2018). This analysis provides data used in this master thesis as losses and capacity in lines and transformers. As mention, the snap shot power flow analysis article is described as a high load. So in this master thesis the maximum energy demand in each node, as one can see in figure 5.2 and 5.3 , will be the load analysed in the snap shot. The purpose of using the IEEE 13-Bus system is to collect realistic data of losses and capacity from power flow analyses. This data is important when this model in this master uses energy balance that does not contain losses in the lines. In section 6.1, the data will be further explained. Both topology consist of 1 slack bus (HV), 10 MV buses and 21 LV nodes. In figure 5.2 the model of the Loop structure is presented, while in the figure 5.3 shows the Radial Tree connected grid. The windmill represent the wind production and PV-system solar production, and in the figures the placement of them are shown. The battery can be installed in all the nodes except node 1 and the energy demand is only in the LV-nodes. In the next sections the model will be described mathematically.


Figure 5.3: The model with Radial Tree topology

### 5.1 Notation

In this subsection the notations for the mathematical model is presented.
Table 5.1: Indexes and magnitude

| Index | Description | Units |
| :---: | :---: | :---: |
| t | Time Period | Hours, Half-hours, Minutes |
| i | Indicates all node in set I | All nodes in problem |
| j | Indicates all node in set I | All nodes in problem |
| Magnitude | Description | Units |
| T | Set of Time periods | Hours,Half-hours, Minutes |
| I | All nodes | All nodes defined below from HV/MV and LV |
| HV/MV | All HV-MV-nodes | $1,2,3,4,5,6,7,8,9,10,11$ |
| MV | All HV-MV-nodes | $2,3,4,5,6,7,8,9,10,11$ |
| LV | All LV-nodes | $(1.1), \ldots,(11.3)$ |
| LV3 | Set of LV-nodes to 3 | $(3.1),(3.2),(3.3)$ |
| LV4 | Set of LV-nodes to 4 | $(4.1),(4.2)$ |
| LV5 | Set of LV-nodes to 5 | $(5.1),(5.2),(5.3)$ |
| LV6 | Set of LV-nodes to 6 | $(6.1),(6.2),(6.3),(6.4),(6.5)$ |
| LV8 | Set of LV-nodes to 8 | $(8.1),(8.2),(8.3)$ |
| LV9 | Set of LV-nodes to 9 | $(9.1),(9.2)$ |
| LV11 | Set of LV-nodes to 11 | $(11.1),(11.2),(11.3)$ |

Table 5.2: Parameters

| Parameter | Description | Units |
| :---: | :---: | :---: |
| $\eta_{C h}$ | Efficiency of Battery Charging | \% |
| $\eta_{\text {Disch }}$ | Efficiency of Battery Discharging | \% |
| Batt MaxCapacityLV $^{\text {a }}$ | Maximum Energy Capacity of LV-battery | kWh |
| Batt $_{\text {MaxCapacity }}$ V | Maximum Energy Capacity of MV-battery | kWh |
| $S o C_{\text {minLV }}$ | Minimum Allowed State of Charge of LV-Battery | kWh |
| $S o C_{\text {min MV }}$ | Minimum Allowed State of Charge of MV-battery | kWh |
| Price ${ }_{\text {Import }}^{(t)}$ | Price paid per kWh imported from the grid in period $t$ | Pound/kWh |
| Price ${ }_{\text {Export }}^{(t)}$ | Price earned per kWh exported to the grid in period $t$ | Pound/kWh |
| BattCh maxLV | Maximum Charging Rate of LV-battery | kWh |
| BattCh maxMV | Maximum Charging Rate of MV-battery | kWh |
| BattDch $\operatorname{maxLV}^{\text {m }}$ | Maximum Discharging Rate of LV-battery | kWh |
| BattDch maxMV | Maximum Discharging Rate of MV-battery | kWh |
| $\operatorname{Load}_{i}^{(t)}$ | Load $i$ in LV-set in period $t$ | kWh |
| $P V_{L V, i}^{(t)}$ | Generated solar energy in node $i$ in LV-set in period $t$ | kWh |
| $P V_{M V, i}^{(t)}$ | Generated solar energy in node $i$ in MV-set in period $t$ | kWh |
| Wind ${ }_{L V, i}^{(t)}$ | Generated wind energy in node $i$ in LV-set in period $t$ | kWh |
| Wind ${ }_{M V, i}^{(t)}$ | Generated wind energy in node $i$ in MV-set in period $t$ | kWh |
| Transformer Capacity,$i^{\text {r }}$ | Capacity of transformer $i$ in set MV | kWh |
| Cost $_{\text {BatLV }}$ | Cost of installing LV-battery | Pound |
| Cost $_{\text {BatMV }}$ | Cost of installing MV-battery | Pound |
| Cost $_{\text {TransformerUpgrade, } i}$ | Cost of upgrading the transformer in node $i$ | Pound |
| Loss ${ }_{\text {Line\&Trans }}$ | Losses in transformer and MV-LV line | \% |
| ZeroBatteries | Set to 0 when no batteries should be build, 1 otherwise | Binary |
| GridConnection $_{i, j}$ | Binary matrix that connecting nodes $i j$ | Binary |
| LineCapacity ${ }_{i, j}$ | Matrix that gives the maximum allowed energy through line $i j$ | kWh |
| LineLossesMV $V_{i, j}$ | Matrix that gives the losses in MV-line $i j$ | \% |

Table 5.3: variables

| Variable | Description | Units |
| :---: | :---: | :---: |
| $E_{L i n e i j}^{(t)}$ | Energy transported from node $i$ to node $j$ at time $t$ | kWh |
| $E_{L i n e j i}^{(t)}$ | Energy transported from node $j$ to node $i$ at time $t$ | kWh |
| $B a t_{D i s c h, i}^{(t)}$ | Energy discharged from the battery in node $i$ at time $t$ | kWh |
| $B a t_{C h, i}^{(t)}$ | Energy charged to the battery in node $i$ at time $t$ | kWh |
| ${I n p u t_{L V, i}^{(t)}}_{O u t p u t_{L V, i}^{(t)}}$ | Energy transported from node MV-node $i$ to LV-nodes at time $t$ | kWh |
| $S_{0} C_{i}^{(t)}$ | Energy transported from the LV-nodes to MV-node $i$ at time $t$ | kWh |
| $y_{i}$ | Integer variable, choose amount of battery in LV-node $i, 0$ otherwise | integer |
| $v_{i}$ | Integer variable, choose amount of battery in MV-node $i, 0$ otherwise | integer |
| $b_{i}$ | Takes value 1 if transformer is upgrade in node $i, 0$ otherwise | binary |

### 5.2 Mathematical model

In this section the mathematical model of the reduced IEEE 13-Bus System is presented. There will be repetition of some components from chapter 4 . First will the energy balances be presented, and afterwards the boundaries on grid and batteries and the objective function.

### 5.2.1 Energy balance

## MV-nodes

$$
\begin{array}{r}
{\left[\text { Output }_{L V, i}^{(t)} * \text { Loss }_{\text {Line\&Trans }}\right]+\sum_{j}^{H V / M V}\left[E_{\text {Line }, j, i}^{(t)} * \operatorname{Loss}_{\text {line }, j, i}\right]+\text { Bat }_{\text {Disch }, i}^{(t)}+} \\
P V_{i}^{(t)}+\text { Wind }_{i}^{(t)}=\sum_{j}^{H V / M V}\left[E_{\text {Line }, i, j}^{(t)}\right]+\text { Bat }_{C h, i}^{(t)}+\text { Input }_{L V, i}^{(t)} \\
\forall i \in 3,4,5,6,8,9,11 \quad \forall t \in T \tag{5.1}
\end{array}
$$

$$
\begin{align*}
& \sum_{j}^{H V / M V}\left[E_{\text {Line }, j, i}^{(t)} * \operatorname{Loss}_{\text {line }, j, i}\right]+B a t_{\text {Disch }, i}^{(t)}+P V_{i}^{(t)}+\text { Wind }_{i}^{(t)}= \\
& \sum_{j}^{H V / M V}\left[E_{\text {Line }, i, j}^{(t)}\right]+B a t_{C h, i}^{(t)} \quad \forall i \in 2,7,10 \quad \forall t \in T
\end{align*}
$$

$$
\begin{align*}
& \text { Output }_{L V, i}^{(t)}, E_{\text {Line }, j, i}^{(t)}, B a t_{D i s c h, i}^{(t)}, B a t_{C h, i}^{(t)}, E_{L i n e, i, j}^{(t)}, \text { Input }_{L V, i}^{(t)} \geq 0 \\
& \forall i \in M V \quad \forall t \in T \tag{5.3}
\end{align*}
$$

In equation 5.1 the energy balance for all MV-nodes connected to LV-nodes in all time periods are illustrated. Output $t_{L V, i}^{(t)}$ and $I n p u t_{L V, i}^{(t)}$ represent respectively the amount of energy going in and out from the LV-nodes connected to MV-node $i$. In 5.2 the energy balance for MV-nodes not connected to LV-nodes. $E_{\text {Line }, j, i}^{(t)}$ and $E_{\text {Line }, i, j}^{(t)}$ represent the income and outgoing energy from other MV-nodes. While $B a t_{\text {DischMV,i, }}^{(t)}, P V_{i}^{(t)}$, Wind $_{i}^{(t)}$ and $B a t_{C h M V, i}^{(t)}$ represent the energy production, charge and discharge of battery and the energy supply production inside the node.

## LV-nodes

In this subsection I will describe the energy balance for the LV-nodes. The LV-nodes connected to each MV-node will not be separated by any losses. Which means that basically the load, energy production and battery placement in LV-nodes can be considered as one connection point.

$$
\begin{align*}
& {\left[\text { Input }_{L V, i}^{(t)} * \text { Loss }_{\text {Line\&Trans }}\right]+} \sum_{j}^{L V i}\left[\text { Bat }_{\text {Disch }, j}^{(t)}+P V_{j}^{(t)}+\text { Wind }_{j}^{(t)}\right]= \\
& \sum_{j}^{L V i}\left[\text { Load }_{j}^{(t)}+B a t_{c h, j}^{(t)}\right]+\text { Output }_{L V, i}^{(t)} \\
& i \in 3,4,5,6,8,9,11 \quad \forall t \in T \tag{5.4}
\end{align*}
$$

$$
\begin{equation*}
\text { Output }_{L V, i}^{(t)}, \text { Bat }_{D i s c h, j}^{(t)}, \text { Bat }_{c h, j}^{(t)}, \text { Input }_{L V, i}^{(t)} \geq 0 \quad \forall i \in 3,4,5,6,8,9,11 \quad \forall t \in T \tag{5.5}
\end{equation*}
$$

In equation 5.4 the energy balance for the LV-grids is presented. Input $_{L V, i}^{(t)}$ and Output $t_{L V, i}^{(t)}$ represent the energy going in and out of the LV-nodes. These two variables is the only way energy can be transported from or to the LV-nodes. While $B a t_{D i s c h L V, j}^{(t)}$, $P V_{j}^{(t)}$, Wind $_{j}^{(t)}$, Load $_{j}^{(t)}, B a t_{c h L V, j}^{(t)}$ represent the produced or consumed energy in LVnodes, $L V i$ represent the LV nodes connected to MV-node $i$. There will be one equation like 5.4 for each MV-node $i$ for all time periods.

### 5.2.2 Grid and battery characteristic features and boundaries

## Line



$$
\begin{equation*}
E_{\text {Line }, j, i}^{(t)} \geq 0 \quad \forall i, j \in H V / M V \backslash\{i=j\} \quad \forall t \in T \tag{5.7}
\end{equation*}
$$

In equations 4.7 the energy flow between node $i$ and $j$ is restricted by the matrix LineCapacity $_{i, j}$ and matrix GridConnection $_{i, j}$. The line capacity matrix sets the value of the maximum energy that can be transported between node $i$ and $j$, while the binary matrix GridConnection ${ }_{i, j}$ takes value 1 for the nodes $i$ and $j$ that should be connected, and all other nodes connections will be given the value 0 , and there can not be transported energy in these lines. See appendix for the matrix GridConnection $_{i, j}, 8.1$ is for radial tree topology and 8.2 is for Loop topology.

## Transformer

$$
\begin{gather*}
\text { Input }_{L V, i}^{(t)} \leq \text { Transformer }_{\text {Capacity }, i}+M * b_{i} \quad \forall i \in M V \quad \forall t \in T  \tag{5.8}\\
\text { Output }_{L V, i}^{(t)} \leq \text { Transformer }_{\text {Capacity }, i}+M * b_{i} \quad \forall i \in M V \quad \forall t \in T \\
\text { Input }_{L V, i}^{(t)}, \text { Output }_{L V, i}^{(t)} \geq 0 \quad \forall i \in M V \quad \forall t \in T
\end{gather*}
$$

In equation 5.8 and 5.9 the amount of energy transported through the transformer (MV/LV). Input ${ }_{L V, i}^{(t)}$. If there is necessary to upgrade the capacity of the transformer, it will be upgraded with a given amount of $\mathrm{M} . b_{i}$ is a binary decision variable which take the value 1 if the transformer should be upgraded, and 0 otherwise.

## Battery for low-voltage grid

$$
\begin{align*}
& S o C_{i}^{(1)}=S o C_{m i n L V} * y_{i}-B a t_{D i s c h i}^{(1)} * \frac{1}{\eta_{D i s c h}}+B a t_{C h, i}^{(1)} * \eta_{c h} \quad \forall i \in L V \quad(5.1  \tag{5.11}\\
& S o C_{i}^{(t)}=S o C_{i}^{(t-1)}-B a t_{D i s c h i}^{(t)} * \frac{1}{\eta_{D i s c h}}+B a t_{C h, i}^{(t)} * \eta_{c h} \quad \forall i \in L V \quad \forall t \in T \tag{5.12}
\end{align*}
$$

$$
\begin{equation*}
S o C_{i}^{(t)} \leq B a t_{\text {MaxCapacityLV }} * y_{i} \quad \forall i \in L V \quad \forall t \in T \tag{5.13}
\end{equation*}
$$

$$
\begin{array}{cc}
S o C_{i}^{(t)} \geq S o C_{\operatorname{minLV}} * y_{i} & \forall i \in L V
\end{array} \quad \forall t \in T
$$

$$
\begin{equation*}
\text { Bat }_{\text {Disch }, i}^{(t)} \leq \text { BatDisch }_{\operatorname{maxLV}} * y_{i} * \text { AllowBatteries }_{i} \quad \forall i \in L V \quad \forall t \in T \tag{5.16}
\end{equation*}
$$

$$
\begin{equation*}
S o C_{i}^{(t)}, B a t_{C h, i}^{(t)}, B a t_{D i s c h, i}^{(t)} \geq 0 \quad \forall i \in L V \quad \forall t \in T \tag{5.17}
\end{equation*}
$$

In equation 5.12 the state of charge (SoC) for LV-node battery is defined. To define the SoC in period t , one have to consider the SoC of the previous time period, $S o C_{i}^{(t-1)}$. The SoC form the previous periods will be subtracted by the the amount of energy discharged from the battery divided by the discharge efficiency. Then the amount of energy used to charge the battery multiplied with the charge efficiency is added. In the first period there will also be given an initial state of charge see equation 5.18

In equations 5.13 and 5.14 the maximum and minimum capacity of one battery in all the LV-grid is defined. This capacity will raise with the amount of battery installed in the LV-node $i$. The number will be decided by the integer decision variable $y_{i}$, so if the is decided to build none batteries in LV-node $i$ the variable will take value 0 . This integer value also exist in the objective function 5.25. To add the investment cost for the LV-battery.

In the equations 5.15 and 5.16 decide the rate of discharge and charge rate in each period. In 5.16 there is included a parameter called AllowBatteries $_{i}$, this is set to 0 , when the model are forced to not build any batteries. If there is not possible to discharge the battery, no batteries are added to the system.

## Battery for medium-voltage grid

$$
\begin{gather*}
S o C_{i}^{(1)}=S o C_{m i n M V} * v_{i}-B a t_{D i s c h i}^{(1)} * \frac{1}{\eta_{D i s c h}}+B a t_{C h, i}^{(1)} * \eta_{c h} \quad \forall i \in M V \quad \text { (5.18 }  \tag{5.18}\\
S o C_{i}^{(t)}=S o C_{i}^{(t-1)}-B a t_{D i s c h i}^{(t)} * \frac{1}{\eta_{D i s c h}}+B a t_{C h, i}^{(t)} * \eta_{c h} \quad \forall i \in M V \quad \forall t \in T \tag{5.19}
\end{gather*}
$$

$$
\begin{equation*}
S o C_{i}^{(t)} \leq \text { Bat }_{\text {MaxCapacityMV }} * v_{i} \quad \forall i \in M V \quad \forall t \in T \tag{5.20}
\end{equation*}
$$

$$
\begin{equation*}
S o C_{i}^{(t)} \geq S o C_{\min M V} * v_{i} \quad \forall i \in M V \quad \forall t \in T \tag{5.21}
\end{equation*}
$$

$$
B a t_{C h, i}^{(t)} \leq B a t C h_{\max M V} * v_{i} \quad \forall i \in M V \quad \forall t \in T
$$

$$
\begin{equation*}
\text { Bat }_{\text {Disch }, i}^{(t)} \leq \text { BatDisch }_{\operatorname{maxMV}} * v_{i} * \text { AllowBatteries }_{i} \quad \forall i \in M V \quad \forall t \in T \tag{5.23}
\end{equation*}
$$

$$
\begin{equation*}
S o C_{i}^{(t)}, B a t_{C h, i}^{(t)}, B a t_{D i s c h, i}^{(t)} \geq 0 \quad \forall i \in M V \quad \forall t \in T \tag{5.24}
\end{equation*}
$$

The battery in the MV-nodes has the same characteristic as in LV-nodes. The only difference is that each MV-battery the $B a t_{\text {MaxCapacityMV }}, S o C_{\operatorname{minMV}}, B a t C h_{\operatorname{maxMV}}$ and BatDisch $\operatorname{maxMV}$ are higher. The integer variable $v_{i}$ decides the amount of battery installed in the MV-node $i$.

### 5.2.3 Objective function

$$
\begin{gather*}
\min (10 * 26) *\left(\sum_{t}^{T}\left[P_{\text {Import }}^{(t)} * E_{\text {Line12 }}^{(t)}-P_{\text {Export }}^{(t)} * E_{\text {Line } 21}^{(t)}\right]\right)+ \\
\left(\text { Cost }_{\text {BatLV }} * \sum_{i}^{L V} y_{i}\right)+\left(\operatorname{Cost}_{\text {BatMV }} * \sum_{i}^{M V} * v_{i}\right)+\sum_{i}^{M V}\left[\text { Cost }_{\text {TransformerUpgrade }} * b_{i}\right]  \tag{5.25}\\
E_{\text {Line12 }}^{(t)}, E_{\text {Line } 21}^{(t)} \geq 0 \tag{5.26}
\end{gather*}
$$

In the objective function 5.25 the energy imported from node 1 to node $2, E_{\text {Line12 }}^{(t)}$, multiplied with Price ${ }_{\text {Import }}^{(t)}$ will represent the total cost, and total revenue is represented by the $E_{\text {Line } 21}^{(t)}$ times the Price ${ }_{\text {Export }}^{(t)}$. The imported and exported energy is simulated for 2 weeks, so to get approximate cost over 10 years this function is multiplied with 26 weeks and 10 years. Each battery investment cost is represented by $\operatorname{Cost}_{\text {BatLV }}$ and $C o s t ~_{\text {BatMV }}$, and the integer decisions variables $y_{i}$ and $v_{i}$ represent the total battery investment. There will also be possible to increase the amount of energy transferred between the MV and LV-nodes by upgrade the transformer for a given investment cost. Since all the investment cost is optional the only reason it would build batteries or/and upgrade transformer if it is economical beneficial or forced by the energy balance.

## Chapter

## Data, discussion and results from the reduced IEEE 13-bus system

In this chapter, I will present the data which will be used in the mathematical model, then the results from the different scenarios are shown, and finally, there will be discussions about these results

### 6.1 Data

In this section, the data used will be presented. There will be given data about the load, energy production, importing prices, transformers and lines and batteries.

The price of importing energy is collected from UK spot price and the shape and magnitude from this data is used. From the collected energy load and production data, only shapes were important. As mention the in chapter 5, the topology is collected from a IEEE power system 13-Bus Feeder (Schneider et al., 2018), by using the a snap shot power flow analysis for the 13 -Bus system the network structure and characteristic were set. In this snap shot power flow analysis there are given loads, see figures 5.2 and 5.3. The paper (Schneider et al., 2018), where the power flow analysis was done, claimed that this analysis was done at a high load. So the loads described in the figures is in this master thesis the maximum load in one period during the simulation. The production magnitude of production of energy will be a percentage of the load.

The shape for the loads are collected from real measurements of households in UK, it is taken from the database of the Low Carbon London project ${ }^{1}$. Which was a project collecting energy consumption from 5567 households. The load profiles for industry building were taken from (Carpinelli et al., 2014). Both loads are collected from one summer and one winter week. From the Low Carbon London project the loads are divided into three

[^1]different categories, adversity, comfortable and affluent. In this master thesis this categorising of household loads will also be kept, see in figures 5.2 5.3. The categorising is based on the economical income in the households. As mention, in this model the only interest is the shape of the loads, and the shapes are very similar. So sorting the energy households between these categories is more for keeping the data as collected from Low Carbon London project. Even when the shapes of adversity, comfortable and affluent are very similar

The energy production in the multi-layer grid is placed where there are solar panels and wind turbine signs, see figures 5.2 5.3. The solar production mainly depends on global horizontal irradiation and temperature, while wind production mostly depends on the wind speed and the height of installation. The shapes of the wind and solar production was obtained from the paper (Lth et al., 2018) ${ }^{2}$. The magnitude of production throughout the two weeks will be change for different scenarios, 15,30 and $60 \%$ of the total load. In table 6.1 the total load for the two weeks are defined, the magnitude, type of production, and which grid, LV or MV, the energy is placed.

| Scenario | Total load (kWh) | Solar MV | Wind MV | Solar LV | Wind LV | Total |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $15 \%$ | $20,0038.18$ | $2.18 \%$ | $7.07 \%$ | $2.89 \%$ | $2.88 \%$ | $15 \%$ |
| $30 \%$ | $20,0038.18$ | $4.36 \%$ | $14.14 \%$ | $5.78 \%$ | $5,76 \%$ | $30 \%$ |
| $60 \%$ | $20,0038.18$ | $8.72 \%$ | $28.28 \%$ | $11.56 \%$ | $11.52 \%$ | $60 \%$ |

Table 6.1: Amount of energy production in different grids at different scenarios

The table shows that approximate $2 / 3$ of the production is in the MV-nodes and $1 / 3$ in the LV-nodes. Also the production from wind is approximate $2 / 3$ of the total production and $1 / 3$ is from solar.

The battery storage strategy depends on the price assumed in the energy market, importing and exporting price. In this model the import price is gathered from the UK electricity spot market ${ }^{3}$. The price will vary between 0.1 and $0.25 £$, where the average price is $0.14 £$. The exporting price is set to one third of the average price $0.046 £$. which means that the importing price will be dynamic between the 0.1 and $0.25 £$, while the exporting price will be constant at $0.046 £$.

The transformers in the grid will be defined by a capacity and loss. The capacity of the transformers are set to around $10 \%$ higher than the maximum load connected to it, see table 6.2. While the losses in the transformer is set to be $4 \%$ for all of them. This value contains the losses in the lines between the MV-node and LV-nodes and the transformer. The total loss in distribution grids is varying due to the lines and transformers, but the total loss in distribution grids is predicted to be from $4-8 \%$ in developed countries(WirfsBrock, 2019) (Vaillancourt, 2019).

[^2]| Node | 3 | 4 | 5 | 6 | 8 | 9 | 11 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Capacity $(\mathrm{kWh})$ | 111 | 50 | 65 | 370 | 50 | 36 | 233 |
| Max load connected LV $(\mathrm{kWh})$ | 100 | 42.5 | 57.5 | 331.25 | 42.5 | 32 | 210.75 |
| Losses $(\%)$ | 4 | 4 | 4 | 4 | 4 | 4 | 4 |

Table 6.2: Transformer characteristics

The data, capacity and loss, used for the lines is given in the figures 6.1 and 6.2. As mention, the losses in the lines are collected from the snap shot power flow analysis in IEEE 13-Bus system. The losses are depending on the length of the lines, and the length of the lines can be seen in the figures. In figure 6.1 at node 3 the characteristic for the MV/LV transformer is shown, capacity at 111 kWh and $4 \%$ loss.


Figure 6.1: Grid properties for Radial Tree structure


Figure 6.2: Grid properties for Loop structure

| Type | MV-battery | LV-battery |
| :--- | :--- | :--- |
| Max Capacity $(\mathrm{kWh})$ | 4 | 2 |
| Min SoC $(\mathrm{kWh})$ | 0.8 | 0.4 |
| Discharge Rate $(\mathrm{kWh})$ | 1.6 | 0.7 |
| Charge Rate $(\mathrm{kWh})$ | 1.6 | 0.7 |
| Discharge Efficiency $(\%)$ | 0.95 | 0.95 |
| Charge Efficiency $(\%)$ | 0.95 | 0.95 |

Table 6.3: Battery characteristics

The cost of 157 ffor Lithium-ion battery is optimistic and ment for the future, a stationary Lithium-ion battery can fall to below USD $200(157 £)$ kilowatt-hour by 2030 (IRENA, 2017). The charging and discharging are constrained by 1.6 kWh and 0.7 kWh for the different batteries, this values can are high due to other researches. The data of the battery is gathered from (Crespo Del Granado et al., 2014)(Granado et al., 2016). In this model there will also not be any degradation processes and do not consider lifetime expansion.

### 6.2 Results

In this section, the energy storage is integrated into the grid in the most cost-effective way for different scenarios. The cost-effective strategy of optimal sitting and size of energy storage changes through the scenarios. The objective in this chapter is to unwind the battery placement and sizing when attributes changes in the grid. The results will hopefully
give some insight for deploying storage in a multi-layer grid.

Before presenting the results for different scenarios. The results from the scenario, no batteries with $15 \%$ RES will be presented to show that the model represent a very realistic grid for the tree topology. In this case the total cost for 10 years was $7,101,506$ pounds and $710,150.6$ pounds each year. The total imported energy from the HV-grids was $4,994,378$ kWh per year. Dividing the total cost for one year by the imported gives an average cost for energy 0.142 pounds $/ \mathrm{kWh}$. The average kWh price do reflect the price in the grid today. The prices used for battery will be 157 and 141.3 pounds $/ \mathrm{kWh}$ these are optimistic cost, but as mention it can be a reality in 2030 . This is the prices which will be used when running all the different cases.

### 6.2.1 RES and topology analyses with lower price on MV-battery

In this section, there will be done sensitivity analyses of RES production and topology when the cost of MV-battery is lower than LV-battery. There will be 3 scenarios for both topology, RES production of 15,30 and $60 \%$. The cost of MV-battery will be 141.3 $£ / \mathrm{kWh}$ and $157 £ / \mathrm{kWh}$ for LV-batteries, a reduction of $10 \%$. The results for both topology are presented in figure 6.3 and 6.4.

| Topology | Loop |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| RES (\%) | 15 \% |  | 30 \% |  | 60 \% |  |
| Battery allowed | No | Yes | No | Yes | No | Yes |
| Total cost (£) | 7,160,880 | 6,982,000 | 5,909,172 | 5,773,614 | 3,579,784 | 3,463,951 |
| Difference in total cost with and without battery | 178,800 |  | 135,558 |  | 115,833 |  |
| Battery in MV | 0 | 413 | 0 | 335 | 0 | 257 |
| Battery in LV | 0 | 0 | 0 | 0 | 0 | 11 |
| Total capacity installed (kWh) | 0 | 1652 | 0 | 1340 | 0 | 1050 |

Figure 6.3: Results from Loop topology

| Topology | Radial tree |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| RES (\%) | 15 \% |  | 30 \% |  | 60 \% |  |
| Battery allowed | No | Yes | No | Yes | No | Yes |
| Total cost ( f ) | 7,135,891 | 6,958,425 | 5,890,397 | 5,755,841 | 3,570,683 | 3,456,131 |
| Difference in total cost with and without battery | 177,466 |  | 134,556 |  | 114,470 |  |
| Battery in MV | 0 | 413 | 0 | 334 | 0 | 255 |
| Battery in LV | 0 | 0 | 0 | 0 | 0 | 11 |
| Total capacity installed (kWh) | 0 | 1652 | 0 | 1336 | 0 | 1042 |

Figure 6.4: Result from Radial Three topology

When comparing the two topology for the three different RES scenarios presented above. One can see that the total cost is a bit lower in the Radial Tree topology in all the tree different scenarios with RES. On the other hand the amount of money saved by implementing batteries in the Loop network is higher compared to Radial Tree. The total capacity installed is a bit higher in the Loop topology when RES is 30 and $60 \%$, but the amount of capacity is very similar.

Comparing the capacity in the different scenarios for Loop and Radial Tree topology separately. It shows that higher RES installed reduce the total capacity. In the Loop topology the capacity is reduced with $18,9 \%$ (scenario $30 \%$ ) and $36,4 \%$ (scenario $60 \%$ ) from the scenario with $15 \%$ RES. In the Radial Tree topology the capacity is reduced with 19,1 \% (scenario $30 \%$ ) and $37,1 \%$ (scenario $60 \%$ ) from the scenario with $15 \%$ RES. The total cost in both topology is reduced due to more free energy produced.

The battery placement in the different RES scenarios can be seen in figure 6.5 . The amount batteries is reduce with RES as seen in the previous tables. The major of batteries installed is in node 6 and node 11, where the load is highest and around the most RES production. The battery placement in the Loop topology is similar, the capacity installed is reduced, and the highest reduction is in the nodes furthest away from the HV-node and closes to nodes with high energy demand.


Figure 6.5: Battery placement and sizing 15,30 and $60 \%$ RES in Loop grid


Figure 6.6: Battery placement and sizing 15,30 and 60 \% RES Radial Tree grid

### 6.2.2 RES analyses with equal battery prices

In this section, the cost of LV and MV-battery will now be $157 £ / \mathrm{kWh}$. There will be done analyses with RES producing 15 and $60 \%$ and in both multi-layer grid topology.

The total cost and total capacity installed when the cost are equal is presented in the figures 6.7 and 6.8. As one can see from this figures the total cost and capacity are reduced when the RES increases for both topology, this is similar to the case when the MV-battery had a lower cost.

The battery placement when the MV and LV-battery cost are equal has changed. As one can see is the amount of batteries placed in MV-nodes only $15 \%$ and 5,8 \% in the Loop topology in respectively scenario 15 and $60 \%$ RES. In the Radial Tree topology the batteries placed in MV-nodes are $16,8 \%$ and $3,3 \%$. The results prefer to place batteries in LV-nodes when the prices are equal.

| Topology | Loop |  |
| :--- | :--- | :--- |
| RES (\%) | 15 | 60 |
| Total cost (£) | $7,001,464$ | $3,475,180$ |
| Battery in MV | $61(15,9 \%)$ | $14(5,8 \%)$ |
| Battery in LV | $647(84,1 \%)$ | $449(94,2 \%)$ |
| Total capacity installed <br> $(k W h)$ | 1538 | 954 |

Figure 6.7: Results from Loop structure

| Topology | Radial tree |  |
| :--- | :--- | :--- |
| RES (\%) | 15 | 60 |
| Total cost (£) | $6,977,863$ | $3,466,912$ |
| Battery in MV | $65(16.8 \%)$ | $8(3.3 \%)$ |
| Battery in LV | $643(83.2 \%)$ | $462(96,7 \%)$ |
| Total capacity installed <br> (kWh) | 1546 | 956 |

Figure 6.8: Results from Radial Tree structure

In figures 6.9 and 6.10 the specific placement of the batteries are shown. From the view of these figures, most batteries are installed in the LV-nodes with the highest load.


Figure 6.9: Battery placement and capacity 15 \% RES both topology


Figure 6.10: Battery placement and capacity $60 \%$ RES both topology

### 6.2.3 Comparing the different scenarios for both topology

In this section the prices for MV-battery is increased to the same cost as LV-battery. The total cost is higher in this case compared to when the MV-battery price was lower. In table 6.4 the difference of the total cost with different battery prices.

|  | RES 15 \% L | RES 15\% R | RES 60\% L | RES 60\% R |
| :--- | :--- | :--- | :--- | :--- |
| Equal battery costs $(£)$ | $7,001,464$ | $6,977,863$ | $3,475,180$ | $3,466,912$ |
| Low MV-battery costs $(£)$ | $6,982,000$ | $6,958,425$ | $3,463,951$ | $3,456,131$ |
| Change in Total $(£)$ | 19,464 | 19,438 | 11,229 | 10,781 |

Table 6.4: Comparing the total cost ( L is for Loop and R is for radial)

Table 6.4 shows that total cost for the two topology when RES is 15 and $60 \%$ and the MV-battery cost is change between 10 \% lower and equal to the LV-battery cost. As seen the total cost saved when the cost is reduced is almost the same for both topology.

|  | Low MV-battery price |  |  | Equal battery prices |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Node | $15 \%$ RES | $60 \%$ RES | Capacity change | $15 \%$ RES | $60 \%$ RES | Capacity change |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 64 | 68 | $4(+6.3 \%)$ | 60 | 40 | $-20(-33.3 \%)$ |
| 3 | 136 | 76 | $-60(-44.1 \%)$ | 8 | 16 | $8(100 \%)$ |
| 4 | 100 | 96 | $-4(-4 \%)$ | 4 | 0 | -4 |
| 5 | 24 | 0 | -24 | 0 | 0 | 0 |
| 6 | 652 | 248 | $-404(-62 \%)$ | 0 | 0 | 0 |
| 7 | 12 | 0 | -12 | 0 | 0 | 0 |
| 8 | 64 | 52 | $-12(-18,5 \%)$ | 0 | 0 | 0 |
| 9 | 144 | 100 | $-44(-30,6 \%)$ | 84 | 0 | -84 |
| 10 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11 | 456 | 388 | $-68(15 \%)$ | 88 | 0 | -88 |
| $3.1,3.2,3.3$ | 0 | 0 | 0 | 112 | 74 | $-38(-34 \%)$ |
| $4.1,4.2$ | 0 | 0 | 0 | 80 | 62 | $-18(22.5 \%)$ |
| $5.1,5.2,5.3$ | 0 | 0 | 0 | 44 | 12 | $-32(-72.7 \%)$ |
| $6.1, \ldots, 6.5$ | 0 | 0 | 0 | 672 | 520 | $-152(-22.6 \%)$ |
| $8.1,8.2,8.3$ | 0 | 40 | 40 | 62 | 58 | $-4(-6.5 \%)$ |
| $9.1,9.2$ | 0 | 0 | 0 | 22 | 18 | $-4(18.1 \%)$ |
| $11.1,11.2$ | 0 | 0 | 0 | 302 | 154 | $-148(-49 \%)$ |
| 11.3 | 0 |  |  |  |  |  |

Table 6.5: Loop topology

|  | Low MV-battery price |  |  | Equal battery prices |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Node | $15 \%$ RES | $60 \%$ RES | Capacity change | $15 \%$ RES | $60 \%$ RES | Capacity change |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 144 | 108 | $-36(-25 \%)$ | 68 | 32 | $-36(-53 \%)$ |
| 3 | 76 | 56 | $-20(-26.3 \%)$ | 0 | 0 | 0 |
| 4 | 96 | 56 | $-40(-41.7 \%)$ | 0 | 0 | 0 |
| 5 | 4 | 8 | $4(+100 \%)$ | 0 | 0 | 0 |
| 6 | 708 | 484 | $-224(-31.6 \%)$ | 56 | 0 | -56 |
| 7 | 56 | 20 | $-36(-64.3 \%)$ | 44 | 0 | -44 |
| 8 | 56 | 40 | $-16(-28.6 \%)$ | 0 | 0 | 0 |
| 9 | 132 | 100 | $-32(-24.2 \%)$ | 92 | 0 | -92 |
| 10 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11 | 380 | 148 | $-232(-61 \%)$ | 0 | 0 | 0 |
| $3.1,3.2,3.3$ | 0 | 0 | 0 | 112 | 74 | $-38(-34 \%)$ |
| $4.1,4.2$ | 0 | 0 | 0 | 76 | 62 | $-14(-18.4 \%)$ |
| $5.1,5.2,5.3$ | 0 | 0 | 0 | 32 | 12 | $-20(-62.5 \%)$ |
| $6.1, \ldots, 6.5$ | 0 | 0 | 0 | 672 | 592 | $-80(-12 \%)$ |
| $8.1,8.2,8.3$ | 0 | 40 | 40 | 72 | 58 | $-14(-19.4 \%)$ |
| $9.1,9.2$ | 0 | 0 | 0 | 20 | 18 | $-2(-10 \%)$ |
| $11.1,11.2$ | 0 | 0 | 0 | 302 | 108 | $-194(-64.2 \%)$ |
| 11.3 | 0 |  |  |  |  |  |

Table 6.6: Radial Tree topology

In the two tables 6.5 and 6.6 represent the capacity installed in all the nodes when RES is 15 and $60 \%$, for both topology and there are difference in battery cost. Table 6.5 represent the Loop and table 6.6 the Radial Tree. When the RES increases from 15 to $60 \%$ the total capacity decreases, and in tables the capacity and percentage change are presented. The results shows that the nodes with the highest reduction of capacity are the nodes with already significant capacity and close to the highest energy demand.

### 6.3 Discussion

In this section the results will be discussed. As the results indicates the main focus is to investigate the new model I made, multi-layer grid consisting of MV and LV-grid. This have led to smaller analysis of different components in the multi-layer grid, as topology, battery prices and RES production. While the wide sensitivity analysis with high density rate has been less focused. The topics of the discussions will mainly consist of how these different scenarios affect the total cost, battery placement in LV and MV-grid and the total capacity installed.

### 6.3.1 Comparing the different topology

Before discussing the results from the different topology, it is again important to mention that the 13 -Bus system had low losses, and these losses where implemented into the master
thesis model.
The two topology are very similar, as one can see in figures 5.2 and 5.3. The change of the topology is that the line between node 2 to 6 in Radial Tree are removed in Loop, and moved to node 5 to 8 and 3 to 11 . In this section, these two topology will be discussed. More precisely, the impact they have on the total cost and placement and size of batteries when RES and battery costs vary.

## RES analyses with lower cost on MV-battery

In the case where the MV-battery has a lower cost, and the RES is changed from 15 to $60 \%$ of the total load the two topology have similar changes. The Loop topology has bit higher total cost in all RES scenarios, around $0.3 \%$ higher, and the capacity installed in scenario with 30 and $60 \%$ RES is around $0.3 \%$ higher. Which shows that the two topology do not have a big impact on the installed capacity and total cost. The small total cost difference occurs due higher losses, when energy is transported from HV-node to the nodes furthest away, the energy goes through more lines in the Loop topology, see figures 5.2 and 5.3. The installed capacity of battery is the same for both topology when RES is $15 \%$ of the total load. When the RES increases to 30 and $60 \%$ there is slightly more installed capacity in Loop topology. Slightly higher losses in the lines gives slightly more installed capacity installed when RES is 30 and $60 \%$. This can give an indication that higher losses between the nodes, especially the HV-node and the production from RES is at a certain amount, gives more incentives to install higher capacity to reduced the total cost.

From the figures 6.3 and 6.4 the difference of total cost from allowing and not allowing batteries is presented. Again, the results are very similar. The reduction of total cost when allowing batteries compared to not, in all RES scenarios, are almost the same for both topology. As mention in the problem description there are three ways to exploit the arbitrage with batteries in the energy market in my model; buying energy when the prices are low and use it when prices are high, store RES surplus and reduce losses in lines.

The placement of batteries in the two topology are also similar, but there are some difference worth mentioning. In the Radial Tree structure, see figure 6.6, the most flexible node is 6 and it is connected to the LV-nodes with the highest load. When RES is increased and the total capacity decreases in the system, node 6 is less reduced in Radial Tree than in the Loop topology. In Loop topology the node that gets less affected by the reduction of capacity is node 11 . See tables 6.6 and 6.5. The reason for this is that the HV-grid is closer to node 6 in the Radial Tree and node 11 in Loop topology, and storing energy closer to the HV-grid while keeping a high flexibility is preferred.

## RES analyses with equal battery cost

In the scenario when the battery cost for MV and LV-battery is equal and the RES increases between 15 and $60 \%$ RES, comparing the change in total cost and capacity in the two topology is very similar to when MV-battery has lower cost than LV-battery.

In this case when most of the batteries are installed in LV-nodes, both topology keeps a high amount of energy in the LV-nodes connected to MV-node 6. In both topology when the RES increases to $60 \%$ the capacity installed in the LV-nodes connected to node 6 is kept high. The reason for this is that the energy produced from the Wind park in node 10 needs to be stored close, due to reduce the losses in the lines. The load in the LV-nodes connected to node 6 will always be needing a high amount of energy so stored energy there will most likely be used inside these LV-nodes, never transported into the MV-grid again.

### 6.3.2 Comparing two different MV-battery costs to LV-battery cost

In this section, the impact of changing the MV-battery cost compared to LV-battery cost will be discussed. It is important to mention that this analysis main focus is to investigate when battery in the MV-battery decreases by 10 \% compared to LV-battery, not how the strategy changes when one change the cost of both batteries.

The initiative to have a lower cost for MV-battery compared to LV-battery is that there are more likely to have smaller batteries in the LV-grid, and smaller batteries usually have higher costs per kWh . I started to reduce the price with $10 \%$ and planned to do more, but due to the major changes in the placement I felt that this results was enough to capture the sensitivity of the battery cost.

The total cost raises in the cases when both batteries have a cost of $157 £ / \mathrm{kWh}$. This is reasonable because the cost of the MV-battery raises while all other parameters are kept the same. The total capacity installed in the multi-layer grid decreases in the case with equal batteries. The reason is that when the one-time investment cost for battery increases the model demands a higher arbitrage, especially regarding the import price, from the energy market for each battery. To be sure that the total capacity decreases, there is ran models of the Radial Tree topology with higher battery costs, see appendix 8.3.

When the MV-battery cost was $10 \%$ lower than the LV-battery, all or almost all battery were placed in MV-nodes, while in the equal cost cases the model favourites placing in the LV-nodes. See figures 6.7 and 6.8. This results indicate the total cost is lower when battery are placed closer to the loads, this is the same conclusion as in article (Fortenbacher et al., 2016) which claimed that decentralised battery was proffered.

When the cost for each kWh is equal, there will not be anything to save per kWh installed in MV. The energy demand in the multi-layer grid is in the LV-nodes, so if storing energy in the MV-nodes the energy has be transported into one of the LV-nodes to supply the demand. So if the energy eventually has to enter the LV-nodes why not just store the energy in the LV-nodes. All the surplus of RES production in the LV-nodes can if there are battery in LV be stored there instead of be transported through transformers and into MV-grid for a given loss. But installing batteries in the MV-node gives more flexibility in supplying different LV-nodes, which is the reason for the installed batteries in the MV-grid, even when the battery costs are equal.

### 6.3.3 Discuss the impact the different amount RES production have on the results

In this section, the multi-layer grid will be discussed when RES is varying from 15 and 60 \&, for both topology, the lower MV-battery and equal battery cost.

## RES analyses with lower cost on MV-battery

When the RES increases from 15 to $60 \%$ the total cost is reduced by a lot. This is very reasonable because in this master thesis the cost installation of RES is not considered, and the energy produced is free. The reduction in total cost from $15 \%$ to $60 \%$ RES is approximately $50 \%$. Which means that increasing the RES with with $45 \%$ reduced the cost in by $49,6 \%$ for both topology.

From figures $6.3,6.4$ one can see that the capacity decreases when RES increases. These results was interesting because I was expecting the capacity to increase when the total RES increased. After looking through the results and the energy movement it made sense. The total imported energy had decreased by a lot. So when the importing prices was high the energy needed to be stored until the prices was low again decreased, and the reason was that more of the load was covered by the RES production.

So in this model, when neglecting technical restrictions as voltage, reactive power etc. and minimising the total cost the capacity decreases when RES increases. One important notice from this results which needs to be commented is that articles using optimal power flow analysis to optimise battery placement and sizing, see literature review, have the opposite results. When including technical restrictions as voltage, reactive effect etc. increasing RES will lead to an increase of needing battery/energy storage in the grid.

The capacity installed closed to nodes with highest energy demand also had a higher reduction of capacity when the RES increased. The reason for this is that when the RES increases the energy demand supplied by the battery decreases. Since most of the energy was stored close to the nodes with high energy demand their capacity was also most reduced, see tables 6.6 and 6.5.

## RES analyses with equal cost on batteries

When increasing the RES when the battery costs are equal the changes in the multi-layer grid is very similar to the changes when the MV-battery cost is lower. The changes is more due to increase of total cost and decrease of total installed capacity, see figure 6.7, 6.8.

## Computational study

In this chapter, the computational performance of the model for the different scenarios will be presented and discussed. Before this the computer used to run the model will be described.

All scripts where run with HP Laptop 14-cm0xx 64-biters window 10 home, with processor AMD Ryzen 5 2500U with Radeon Vega Mobile 2.00 Ghz. The installed memory 8,00 GB RAM. The model and data is implemented into JetBrains Pycharm community edition 2018.3.4 x64. The solver used is Gurobi 8.0.1.

### 7.1 Results and discussion

In this section the main results of computational performance for most of the cases with different RES, topology and battery cost will presented. The results will be presented in table 7.1. And afterwards the different preformance results will be discussed.

## Results

In table 7.1 the performance from many of the scenarios is presented. This is a Mix Integer Problem (MIP), but due to very small size of batteries ( 2 and 4 kWh ), battery size for the model can choose almost perfect placement and size of the installed batteries, and this gives makes the MIP solution very close to the optimal solution if the model could integer decision variables the MIP very small.

| Scenario | Solution time | MIP gap | Topology | Battery Cost |
| :--- | :--- | :--- | :--- | :--- |
| No battery 15 \% RES | 0.94 s | $0 \%$ | Loop | - |
| Battery 15 \% RES | 1699.61 s | $0.0001 \%$ | Loop | MV-low |
| Battery 15 \% RES | 1025.98 s | $0,0002 \%$ | Loop | Equal |
| No battery 15 RES \% | 0.91 s | $0 \%$ | Radial | - |
| Battery 15 RES \% | 1072.58 s | $0 \%$ | Radial | MV-low |
| Battery 15 RES \% | 976.06 s | 0,0003 | Radial | Equal |
| No battery 60 \% RES | 0.91 s | $0 \%$ | Loop | - |
| Battery 60 \% RES | 984.42 s | $0.0002 \%$ | Loop | MV-low |
| Battery 60 \% RES | 1169 s | 0,0002 | Loop | Equal |
| No bat 60 \% RES | 0.74 s | $0 \%$ | Radial | - |
| Battery 60 \% RES | 779.50 s | $0.0001 \%$ | Radial | MV-low |
| Battery 60 \% RES | 1021.22 s | $0.0003 \%$ | Radial | Equal |

Table 7.1: The computational results from the different cases in this master thesis

## Discussion

In this section the computational performance will be discussed, mainly the solution time and MIP gap. First the difference between giving the possibility to build batteries compared to scenarios where the model is forced to not build batteries. Thereafter, the solution time and MIP gap between the different RES, 15 and $60 \%$ and different MV-battery costs will be discussed.

The solution time when the model is forced no batteries is very close to zero, the time is between 0.74 seconds and 0.94 seconds. The MIP gap is zero, and that is reasonable because when the battery is forced to be zero there will not be a MIP problem anymore, but a binary problem. When the model is allowed to build batteries in the multi-layer grid, the model has to investigate all the possibilities to build MV-batteries and LV-batteries to get the minimal cost. The solution time increase to between 779.50 seconds and 1699.61 seconds.

When looking at the solution time and MIP gaps when RES is $60 \%$ of the total load, the solution time for equal battery cost has lower solution time for both topology. In the case when MV-battery cost is lower than LV-battery cost, most batteries are installed in MV-grid. When $85 \%$ is imported from HV-grid the will be flowing a lot of energy in the MV-grid, and the decision to find optimal sizing and sitting gets higher. The solution time is opposite when the RES is $60 \%$, in this cases the equal battery price has a more of a mix of MV and LV-batteries, which leads more option to check. Another interesting result is that the MIP gap is higher for most cases when the battery cost is equal, which means that the MIP optimal solution is further away from the optimal solution.

## Conclusion and Recommendations for Further Work

The topic of this thesis has not been addressed in the literature. More precisely, based on my knowledge, previous research have not explored the optimal size and location of batteries in a multi-layer grid consisting of medium voltage (MV) and low voltage (LV). In addition, the thesis has examined the optimal investment decisions of battery storage in two multi-layer grids (Loop and Radial Tree) with different topology. Furthermore, different scenarios of RES productions and battery cost have been analysed. To understand this complexity, it has been necessary to make a number of choices, such as determining solution methods, technical restrictions and data limitations. It is important to be aware of these delineates both in terms of the interpretation of the conclusions and recommendations for further work.

Overall, the results reveal that it will be economic beneficial to install batteries in grids combining MV and LV. The developed model place most of them close to the nodes with high energy demand. However, the optimum battery capacity is sensitive to the proportion of RES while the battery location is very sensitive to battery costs. An interesting insight is that different grid topology seems to have less effect on the battery storage strategy.

The analysis and findings show that two important changes in battery investment decision occur: 1) when we keep the cost equal for LV-battery and MV-battery, and 2) when there is an increase in the RES share from 15 to $60 \%$. The first is that the total battery capacity is reduced by approximately $36-39 \%$, and the other is that the vast majority of the batteries are placed in LV-grids. However, the location of the batteries changes dramatically when the cost of MV-battery is reduced with $10 \%$, then almost all batteries are placed in MV-grids. The main insight is that the cost-efficiency strategy of battery sizing is most impacted by the proportion of RES production in the network, while the placement of batteries are most influenced by cost difference between MV- and LV batteries.

### 8.1 Recommendations for Further Work

The suggestions for further work presented below is based on main shortcomings in the thesis, literature, discussions with supervisors, fellow students and teachers and other ideas.

- Optimise sizing and placement with more technical restrictions. For example, using modelling approaches such as optimal power flow that represent technical restrictions in detail ( voltage, reactive power, active power etc.). It would be important to test these restrictions in order to capture power quality issues and other battery applications.
- Forecasting algorithms and stochastic optimisation. There is uncertainty about some of the data in the thesis, for example electricity prices, load and RES production. Therefore, optimisation of battery sizing and sitting with stochastic values should be considered.
- Simulate the problem for longer time horizons. The multi-layer grid is only simulated for two representative weeks (summer and winter). These weeks operational insights are scaled up to calculate a 10 years investment analysis. This upscale neglect a lot of variation that exists in reality. It should therefore be tested whether simulation over a longer period of time affects the findings. However, this might raise computational complexity.
- Expanding with implementation of electric vehicles (EVs. There are not considered any energy storage from EVs and the high increase of energy demand in some periods. The portable storage which is in the network can give less intensives to install static batteries. In the same time there can be interesting to investigate the investment strategy when the energy demand in some periods raises by a lot. This can maybe exceed the transformer capacity and the model would be forced to do investment strategy for technical reasons and not only for economical intensives.


## Increased understanding of the model

- Increase the size of each battery. Usually, batteries intended for energy network will have a larger sizes than 2 and 4 kWh . Increasing the size will force the model to make more trade-offs if it is worth installing batteries.
- Identify break-even cost for placement of MV and LV-battery. Perform analyses with different reduction costs for MV-batteries to see how it effect the placement of batteries, and maybe find some critical break-even costs.
- Identifying break even cost for investing in batteries. In section 6.1, the data used assumes a battery investing cost project for 2030. It is important that also expenses for installing and maintenance of batteries are included in the analyses.
- Carry out analyses with different amount of RES produced in MV- and LV grid Changing the location of the RES production. In different scenarios, place a larger share of RES in MV- or LV grid.
- Do analyses with different type of RES production - wind or solar. Carry out analyses where wind is the only energy production, and others with only solar production.
- Increase the losses in the lines and do sensitivity analyses. Due to very few interesting results from the two topology, it would be interesting to increase the losses in this model. One possibility is to find larger power networks with higher losses, another might be to change the topology or add new ones.
- Similar approach containing a detailed battery model that includes degradation features. In this model the battery do not age, which means that the maximum storage, discharge and charge efficiency and minimum SoC don't change. In reality these batteries should age and make the properties worse.


## Appendix

| Scen | Node_i | Node_j | Value | Scen | Node_i | Node_j | Value |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 0 | 0 | 1 | 5 | 5 | 0 |
| 1 | 0 | 1 | 1 | 1 | 5 | 6 | 1 |
| 1 | 0 | 2 | 0 | 1 | 5 | 7 | 0 |
| 1 | 0 | 3 | 0 | 1 | 5 | 8 | 0 |
| 1 | 0 | 4 | 0 | 1 | 5 | 9 | 1 |
| 1 | 0 | 5 | 0 | 1 | 5 | 10 | 1 |
| 1 | 0 | 6 | 0 | 1 | 6 | 0 | 0 |
| 1 | 0 | 7 | 0 | 1 | 6 | 1 | 0 |
| 1 | 0 | 8 | 0 | 1 | 6 | 2 | 0 |
| 1 | 0 | 9 | 0 | 1 | 6 | 3 | 0 |
| 1 | 0 | 10 | 0 | 1 | 6 | 4 | 0 |
| 1 | 1 | 0 | 1 | 1 | 6 | 5 | 1 |
| 1 | 1 | 1 | 0 | 1 | 6 | 6 | 0 |
| 1 | 1 | 2 | 1 | 1 | 6 | 7 | 1 |
| 1 | 1 | 3 | 1 | 1 | 6 | 8 | 1 |
| 1 | 1 | 4 | 0 | 1 | 6 | 9 | 0 |
| 1 | 1 | 5 | 1 | 1 | 6 | 10 | 0 |
| 1 | 1 | 6 | 0 | 1 | 7 | 0 | 0 |
| 1 | 1 | 7 | 0 | 1 | 7 | 1 | 0 |
| 1 | 1 | 8 | 0 | 1 | 7 | 2 | 0 |
| 1 | 1 | 9 | 0 | 1 | 7 | 3 | 0 |
| 1 | 1 | 10 | 0 | 1 | 7 | 4 | 0 |
| 1 | 2 | 0 | 0 | 1 | 7 | 5 | 0 |
| 1 | 2 | 1 | 1 | 1 | 7 | 6 | 1 |
| 1 | 2 | 2 | 0 | 1 | 7 | 7 | 0 |
| 1 | 2 | 3 | 0 | 1 | 7 | 8 | 0 |
| 1 | 2 | 4 | 0 | 1 | 7 | 9 | 0 |
| 1 | 2 | 5 | 0 | 1 | 7 | 10 | 0 |
| 1 | 2 | 6 | 0 | 1 | 8 | 0 | 0 |
| 1 | 2 | 7 | 0 | 1 | 8 | 1 | 0 |
| 1 | 2 | 8 | 0 | 1 | 8 | 2 | 0 |
| 1 | 2 | 9 | 0 | 1 | 8 | 3 | 0 |
| 1 | 2 | 10 | 0 | 1 | 8 | 4 | 0 |
| 1 | 3 | 0 | 0 | 1 | 8 | 5 | 0 |
| 1 | 3 | 1 | 1 | 1 | 8 | 6 | 1 |
| 1 | 3 | 2 | 0 | 1 | 8 | 7 | 0 |
| 1 | 3 | 3 | 0 | 1 | 8 | 8 | 0 |
| 1 | 3 | 4 | 1 | 1 | 8 | 9 | 0 |
| 1 | 3 | 5 | 0 | 1 | 8 | 10 | 0 |
| 1 | 3 | 6 | 0 | 1 | 9 | 0 | 0 |
| 1 | 3 | 7 | 0 | 1 | 9 | 1 | 0 |
| 1 | 3 | 8 | 0 | 1 | 9 | 2 | 0 |
| 1 | 3 | 9 | 0 | 1 | 9 | 3 | 0 |
| 1 | 3 | 10 | 0 | 1 | 9 | 4 | 0 |
| 1 | 4 | 0 | 0 | 1 | 9 | 5 | 1 |
| 1 | 4 | 1 | 0 | 1 | 9 | 6 | 0 |
| 1 | 4 | 2 | 0 | 1 | 9 | 7 | 0 |
| 1 | 4 | 3 | 1 | 1 | 9 | 8 | 0 |
| 1 | 4 | 4 | 0 | 1 | 9 | 9 | 0 |
| 1 | 4 | 5 | 0 | 1 | 9 | 10 | 0 |
| 1 | 4 | 6 | 0 | 1 | 10 | 0 | 0 |
| 1 | 4 | 7 | 0 | 1 | 10 | 1 | 0 |
| 1 | 4 | 8 | 0 | 1 | 10 | 2 | 0 |
| 1 | 4 | 9 | 0 | 1 | 10 | 3 | 0 |
| 1 | 4 | 10 | 0 | 1 | 10 | 4 | 0 |
| 1 | 5 | 0 | 0 | 1 | 10 | 5 | 1 |
| 1 | 5 | 1 | 1 | 1 | 10 | 6 | 0 |
| 1 | 5 | 2 | 0 | 1 | 10 | 7 | 0 |
| 1 | 5 | 3 | 0 | 1 | 10 | 8 | 0 |
| 1 | 5 | 4 | 0 | 1 | 10 | 9 | 0 |

Figure 8.1: Matrix which connect radial tree topology

| Scen | Node_i | Node_j | Value | Scen | Node_i | Node_j | Value |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 0 | 0 | 1 | 5 | 5 | 0 |
| 1 | 0 | 1 | 1 | 1 | 5 | 6 | 1 |
| 1 | 0 | 2 | 0 | 1 | 5 | 7 | 0 |
| 1 | 0 | 3 | 0 | 1 | 5 | 8 | 0 |
| 1 | 0 | 4 | 0 | 1 | 5 | 9 | 1 |
| 1 | 0 | 5 | 0 | 1 | 5 | 10 | 1 |
| 1 | 0 | 6 | 0 | 1 | 6 | 0 | 0 |
| 1 | 0 | 7 | 0 | 1 | 6 | 1 | 0 |
| 1 | 0 | 8 | 0 | 1 | 6 | 2 | 0 |
| 1 | 0 | 9 | 0 | 1 | 6 | 3 | 0 |
| 1 | 0 | 10 | 0 | 1 | 6 | 4 | 0 |
| 1 | 1 | 0 | 1 | 1 | 6 | 5 | 1 |
| 1 | 1 | 1 | 0 | 1 | 6 | 6 | 0 |
| 1 | 1 | 2 | 1 | 1 | 6 | 7 | 1 |
| 1 | 1 | 3 | 1 | 1 | 6 | 8 | 1 |
| 1 | 1 | 4 | 0 | 1 | 6 | 9 | 0 |
| 1 | 1 | 5 | 0 | 1 | 6 | 10 | 0 |
| 1 | 1 | 6 | 0 | 1 | 7 | 0 | 0 |
| 1 | 1 | 7 | 0 | 1 | 7 | 1 | 0 |
| 1 | 1 | 8 | 0 | 1 | 7 | 2 | 0 |
| 1 | 1 | 9 | 0 | 1 | 7 | 3 | 0 |
| 1 | 1 | 10 | 0 | 1 | 7 | 4 | 1 |
| 1 | 2 | 0 | 0 | 1 | 7 | 5 | 0 |
| 1 | 2 | 1 | 1 | 1 | 7 | 6 | 1 |
| 1 | 2 | 2 | 0 | 1 | 7 | 7 | 0 |
| 1 | 2 | 3 | 0 | 1 | 7 | 8 | 0 |
| 1 | 2 | 4 | 0 | 1 | 7 | 9 | 0 |
| 1 | 2 | 5 | 0 | 1 | 7 | 10 | 0 |
| 1 | 2 | 6 | 0 | 1 | 8 | 0 | 0 |
| 1 | 2 | 7 | 0 | 1 | 8 | 1 | 0 |
| 1 | 2 | 8 | 0 | 1 | 8 | 2 | 0 |
| 1 | 2 | 9 | 0 | 1 | 8 | 3 | 0 |
| 1 | 2 | 10 | 1 | 1 | 8 | 4 | 0 |
| 1 | 3 | 0 | 0 | 1 | 8 | 5 | 0 |
| 1 | 3 | 1 | 1 | 1 | 8 | 6 | 1 |
| 1 | 3 | 2 | 0 | 1 | 8 | 7 | 0 |
| 1 | 3 | 3 | 0 | 1 | 8 | 8 | 0 |
| 1 | 3 | 4 | 1 | 1 | 8 | 9 | 0 |
| 1 | 3 | 5 | 0 | 1 | 8 | 10 | 0 |
| 1 | 3 | 6 | 0 | 1 | 9 | 0 | 0 |
| 1 | 3 | 7 | 0 | 1 | 9 | 1 | 0 |
| 1 | 3 | 8 | 0 | 1 | 9 | 2 | 0 |
| 1 | 3 | 9 | 0 | 1 | 9 | 3 | 0 |
| 1 | 3 | 10 | 0 | 1 | 9 | 4 | 0 |
| 1 | 4 | 0 | 0 | 1 | 9 | 5 | 1 |
| 1 | 4 | 1 | 0 | 1 | 9 | 6 | 0 |
| 1 | 4 | 2 | 0 | 1 | 9 | 7 | 0 |
| 1 | 4 | 3 | 1 | 1 | 9 | 8 | 0 |
| 1 | 4 | 4 | 0 | 1 | 9 | 9 | 0 |
| 1 | 4 | 5 | 0 | 1 | 9 | 10 | 0 |
| 1 | 4 | 6 | 0 | 1 | 10 | 0 | 0 |
| 1 | 4 | 7 | 1 | 1 | 10 | 1 | 0 |
| 1 | 4 | 8 | 0 | 1 | 10 | 2 | 1 |
| 1 | 4 | 9 | 0 | 1 | 10 | 3 | 0 |
| 1 | 4 | 10 | 0 | 1 | 10 | 4 | 0 |
| 1 | 5 | 0 | 0 | 1 | 10 | 5 | 1 |
| 1 | 5 | 1 | 0 | 1 | 10 | 6 | 0 |
| 1 | 5 | 2 | 0 | 1 | 10 | 7 | 0 |
| 1 | 5 | 3 | 0 | 1 | 10 | 8 | 0 |
| 1 | 5 | 4 | 0 | 1 | 10 | 9 | 0 |

Figure 8.2: Matrix which connect Loop topology

|  | Capacity (kWh) |  | Total cost |  |
| :--- | :--- | :--- | :--- | :--- |
|  | RES 15 \% | RES 60 \% | RES 15 \% | RES $60 \%$ |
| $300 \$$ | $956(40 ~ M V, ~$ <br> $916 ~ L V) ~$ | 472 (All in LV) | $7,090,014$ | $3,525,661$ |
| $400 \$$ | 104 (All in LV) | 218 (All in LV) | $7,137,400$ | $3,552,945$ |

Figure 8.3: Radial Tree with higher and equal battery costs for MV and LV-battery

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[^1]:    ${ }^{1}$ For further information, please refer to https://data.london.gov.uk/dataset/ smartmeter-energy-use-data-in-london-households

[^2]:    ${ }^{2}$ For further information, please refer to https://wiki.openmod-initiative.org/wiki/Data
    ${ }^{3}$ For further information, please refer to http://www.apxgroup.com/market-results/ apx-power-uk/ukpx-rpd-historical-data/

