Øyvind Blindheim

Optimal positioning considering fish welfare for Havfarm 2

Master's thesis in Marine Technology
Supervisor: Dong Trong Nguyen
June 2019

Master's thesis

NTNU
Norwegian University of Science and Technology
Faculty of Engineering
Department of Marine Technology

Illustration of Havfarm 2

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Science and Technology

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Summary

This master thesis look into a possible water quality problem for the salmon in the hindmost cage of Havfarm 2. Havfarm 2 is an unconventional aquaculture plant, designed for dynamic positioning, that is currently being designed by NSK Ship Design for Nordlaks. Havfarm 2 is one of Nordlaks' projects with outstanding innovation and substantial investments, answering to The Norwegian Directorate of fisheries arrangement to solve challenges of the Norwegian aquaculture industry.

At first, a digital twin of Havfarm 2 is developed. This is done in Simulink and Matlab, where the hydrodynamic analysis that are done in Wamit, are included using Marine Systems Simulator. It is also developed a simulation platform for the digital twin in Simulink, including environmental conditions and measurement simulations. As it is the water quality in the hindmost cage that is investigated, a simplified model for the water quality is also developed.

A control system, customized for the operational conditions in this thesis, is developed. The control system includes a state estimator, simplified thrust allocation and several control alternatives to ensure high quality water to the fish, while minimizing the fuel consumption. The different control alternatives that was tested are the following:

1. Using a PID controller to regulate the water quality by controlling force from one thruster.
2. Using dynamic programming to find an optimal heading based on the current, and a PID controller to regulate the heading.
3. Using a 3 degree of freedom (surge, sway and yaw) nonlinear model predictive control to find optimal control input for each time step.
4. Using a 3 degree of freedom (surge, sway and yaw) nonlinear model predictive control to find optimal position and heading for each time step. A PID controller is used to follow this reference.

The control alternative 3 was found to be the best alternative in this thesis, but there are limitations, and still much work to be done before realizing such a control system. From the results in this thesis, it is argued that optimal control is both possible and maybe the preferable approach to solve a possible water quality problem.

Sammendrag

Denne masteroppgaven tar for seg et mulig problem med vannkvaliteten for laksen i det bakerste buret i Havfarm 2. Havfarm 2 er et ukonvensjonelt akvakulturanlegg, designet for dynamisk posisjonering, som for øyeblikket designes av NSK Ship Design for Nordlaks. Havfarm 2 er et av Nordlaks sine prosjekter bestående av enestående innovasjon og betydelige investeringer, og svarer til Fiskeridirektoratets tilsagn om utviklingstillatelser for å løse utfordringene i norsk akvakultur.

Først utvikles en digital tvilling av Havfarm 2. Dette gjøres i Simulink og Matlab, hvor den hydrodynamiske analysen som er gjort i Wamit, er inkludert ved hjelp av Marine Systems Simulator. Det er også utviklet en simuleringsplattform for den digitale tvillingen i Simulink, inkludert miljøforhold og målesimuleringer. Ettersom det er vannkvaliteten i det bakerste buret som undersøkes, utvikles også en forenklet modell for vannkvaliteten.

Et reguleringssystem, tilpasset driftsfilosofien i denne oppgaven, er utviklet. Reguleringssystemet inkluderer en observer, forenklet kraftfordeling til fremdriftsenhetene og flere reguleringsalternativer for å sikre vann av høy kvalitet til fisken, samtidig som drivstofforbruket minimeres. De forskjellige reguleringsalternativene som ble testet, er følgende:

1. Bruk av en PID-regulator for å regulere vannkvaliteten ved å styre kraft fra en fremdriftsenhet.
2. Bruk av dynamisk programmering for å finne en optimal retning, basert på strømmen, og en PID-regulator for å regulere retningen på Havfarm 2.
3. Bruke en ikke-lineær modellprediktiv regulering i tre frihetsgrader (jag, svai og gir) for å finne optimale krefter for hvert tidssteg.
4. Bruke en ikke-lineær modellprediktiv regulering i tre frihetsgrader (jag, svai og gir) for å finne optimal posisjon og retning for hvert tidssteg. En PID-regulator brukes deretter til å følge denne referansen.

Alternativ nummer 3 ble det beste alternativet i denne oppgaven, men det er begrensninger og fortsatt mye arbeid som må gjøres før man kan realisere et slikt reguleringssystem. Av resultatene i denne oppgaven er det hevdet at optimal regulering både er mulig og kanskje den foretrukne måten å løse et mulig vannkvalitetsproblem i Havfarm 2.

NTNU Trondheim
Norwegian University of Science and Technology
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MSC THESIS DESCRIPTION SHEET

Name of the candidate: Øyvind Blindheim

Field of study: Marine control engineering

Thesis title (Norwegian): Optimal posisjonering med tanke på fiskevelferd for Havfarm 2

Thesis title (English): Optimal positioning considering fish welfare for Havfarm 2

Background

The Norwegian aquaculture has had enormous growth the last 50 years, but has now reached a natural limit, as the protected areas and fjords are “full” of aquaculture plants. An objective is to move the production to less protected areas. The Havfarm project is an initiative from Nordlaks to move the production to more exposed areas. Havfarm 2, which is currently being designed by NSK Ship Design, is a large aquaculture plant, for salmon farming, that shall use DP for a low precision station-keeping. The vessel shall have an unconventional hull shape, which makes this vessel dissimilar from other DP-vessels. In operational conditions, Havfarm 2 shall use a gravity anchor for fuel saving. As this is a project that has never been done before, there are many cases that needs to be investigated before realization. One case to be studied in this master thesis is the station-keeping when the weather is nice and the current speed is low. This can make the water quality in the hindmost cage bad. Therefore, there might be a need to change the heading of Havfarm 2 such that there are inflow of fresh water to the hindmost cages from the sides. To be able to study this case, simulations should be used. To be able to do this, the following work description shall be accomplished.

Work description

1. Perform a background and literature review to provide information and relevant references on:
   - Havfarm 2
   - Marine Systems Simulator (MSS)
   - Digital Twin
   - Anchoring systems
   - Optimal Control
     - Model predictive control (MPC)

Write a list with abbreviations and definitions of terms, explaining relevant concepts related to the literature study and project assignment.

2. Develop an anchoring characteristic for the anchoring line on Havfarm 2, and implement a look-up table in the simulator.

3. Develop a digital twin of Havfarm 2 by using the approach found in the Project Thesis, due to updated concept, geometry and data.

4. Investigate the coherence between fish health, water quality and water flow speed, and develop a model for this that fit in to the simulator.

5. Develop a control system for Havfarm 2 in operational condition in calm weather conditions which minimizes fuel consumption, but still ensures fish welfare by providing water of high quality.

6. Run simulations to verify the complex system.

7. Evaluate the remaining time to investigate other cases.

Specifications

The scope of work may prove to be larger than initially anticipated. By the approval from the supervisor, described topics may be deleted or reduced in extent without consequences with regard to grading.

The candidate shall present personal contribution to the resolution of problems within the scope of work. Theories and conclusions should be based on mathematical derivations and logic reasoning identifying the various steps in the deduction.

The report shall be organized in a logical structure to give a clear exposition of background, results, assessments, and conclusions. The text should be brief and to the point, with a clear language. Rigorous mathematical deductions and illustrating figures are preferred over lengthy textual descriptions. The report shall have font size 11 pts., and it is not expected to be longer than 60-80 A4 pages, from introduction to conclusion, unless otherwise agreed upon. It shall be written in English (preferably US) and contain the following elements: Title page, summary, thesis specification, list of symbols and acronyms, table of contents, introduction with objective, background, and scope and delimitations, main body with problem formulations, derivations/developments and results, conclusions with recommendations for further work, references, and optional appendices. All figures, tables, and equations shall be numerated. The original contribution of the candidate and material taken from other sources shall be clearly identified. Work from other sources shall be properly acknowledged using quotations and a Harvard citation style (e.g. *natbib* Latex package). The work is expected to be conducted in an honest and ethical manner, without any sort of plagiarism and misconduct. Such practice is taken very seriously by the university and will have consequences. NTNU can use the results freely in research and teaching by proper referencing, unless otherwise agreed upon.

The thesis shall be submitted with electronic copy to the main supervisor, and otherwise according to NTNU procedures. The final revised version of this thesis description must be included. Computer code, pictures, videos, data series, and a PDF version of the report shall be enclosed electronically with all submitted versions.

Start date: 15.01.19 **Due date:** 11.06.19

Supervisor: Dong Trong Nguyen
Co-advisor(s): Håkon Ådnanes & Sebastien Gros

Trondheim, ___________________

____
Dong Trong Nguyen
Supervisor

Preface

This thesis is my final act in the Integrated Master Program in Marine Technology. It has been a fluctuating travel where I have met, and passed many obstacles, hopefully stronger and better informed than before.

I work part-time for NSK Ship Design, which is the company that design Havfarm 2 for Nordlaks. As Havfarm 2 shall be designed in regards to dynamic positioning, it correspond well to the specialisation, Marine Cybernetics. It is a privilege to write my master thesis in collaboration with NSK Ship Design. NSK, more specific, Håkon Ådnanes, and their subcontractors have helped me define the objective of the thesis, and provided necessary information to accomplish the master thesis. For this, I am grateful.

I would like to thank my supervisor Dong Trong Nguyen for our frequently guidance meetings and supervisory.

This master thesis includes optimization, which I did not know much about before. I have searched for guidance in optimization when I was stuck. I am thankful to Sebastien Gros, professor at NTNU, who guided me in the optimization problem.

During these five years of the integrated master program, my parents and the rest of my family and friends have supported me in any way. For this, I am grateful as well.

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Nomenclature

AF_w	Frontal projected area of the vessel, over the water surface. 29
AL_w	Lateral projected area of the vessel, over the water surface. 29
B_V	Linear Viscous damping matrix. 20
H_s	Significant wave height, defined as the mean of the 13 highest waves. 25
Loa	Length-over-all. 29
T_n	Natural period. 39
V_{rw}	Relative wind speed. 29
Θ	Attitude, Euler angles. 14
β	Direction of propagating waves. 25
$\boldsymbol{A}_w$	Design matrix describing the sea state. 33
$\boldsymbol{B_T}$	Extended thrust configuration matrix. 40
$\boldsymbol{C}_w$	Design matrix describing the sea state. 33
$\boldsymbol{C}_A$	Added mass coriolis-centripetal matrix. 16
$\boldsymbol{C}_{RB}$	Rigid body mass coriolis-centripetal matrix. 16
$\boldsymbol{D}$	Damping matrix. 16, 20
$\boldsymbol{G}$	Restoring force matrix. 16, 24
$\boldsymbol{I}_g$	Inertia matrix about Center og gravity. 17
$\boldsymbol{J}_\Theta$	Transformation matrix. 15
$\boldsymbol{K}_D$	Derivative gain matrix. 39
$\boldsymbol{K}_I$	Integral gain matrix. 39
$\boldsymbol{K}_P$	Proportional gain matrix. 39
$\boldsymbol{K}_{1,2,\ldots,7}$	Observer gains. 34
$\boldsymbol{M}$	Mass matrix. 16, 17, 19, 34
$\boldsymbol{M}_A$	Added mass inertia matrix. 16, 19, 20
$\boldsymbol{M}_{RB}$	Rigid body mass inertia matrix. 16, 17, 19, 20
$\boldsymbol{Q}$	Process noise covariance matrix. 32, 33
$\boldsymbol{R}$	Radius of gyration. 17
$\boldsymbol{R}$	Rotation matrix. 15
$\boldsymbol{S}$	Skew symmetric matrix, according to Fossen (2011). 17, 20
$\boldsymbol{T}_b$	Design matrix for bias time constants. 33
$\boldsymbol{T}_c$	Design matrix for current time constants. 33
$\boldsymbol{T}_\Theta$	Angular velocity rotation matrix. 15
$\boldsymbol{\eta}$	Position vector in NED frame. 15, 16, 32, 39
$\boldsymbol{\mu}$	Fluid Memory effect vector. 16, 21, 24
$\boldsymbol{\nu}$	Velocity vector in body frame. 14, 15

$\boldsymbol{\nu}_c$	Current velocity vector in body. 16, 30
$\boldsymbol{\nu}_c$	NED frame current velocity vector. 33
$\boldsymbol{\nu}_r$	Relative velocity vector in body. 16, 31, 32
$\boldsymbol{\tau}$	Vector of forces in body. 15, 16, 40
$\boldsymbol{\tau}_{wave}$	Vector of wave forces. 25
$\boldsymbol{\tau}_{wind}$	Vector of wind forces. 28, 29
$\boldsymbol{\xi}$	Wave state vector. 33
$\boldsymbol{b}$	Bias. 33
$\boldsymbol{f}_b^b$	Body fixed force. 15
$\boldsymbol{g}(\boldsymbol{\eta})$	Restoring force. 24
$\boldsymbol{g}_0$	Gravity force on dynamic ballast tanks. 16
$\boldsymbol{m}_b^b$	Body fixed moment. 15
$\boldsymbol{p}$	Latitude position. 14
$\boldsymbol{r}_g$	Vector from CO to CG. 17
γ_{rw}	Relative angle of attack for the wind. 29
λ_i	Relative damping ratio of the wave spectrum. 34
ω	Wave frequency. 25
ω_0	Wave spectra peak frequency. 25
ω_{0i}	Natural frequency in each DOF. 34
ω_{ci}	The cut-off frequency in each DOF. 34
ϕ	Euler roll angle. 14
ψ	Euler yaw angle. 14
θ	Euler pitch angle. 14
ζ	Relative damping ratio. 34, 39
sH	Horizontal distance to centroid of AL_w. 29
sL	Vertical distance to centroid of AL_w. 29
μ	Latitude position. 14
l	Longitude position. 14
K	Moment about x-axis. 14
M	Moment about y-axis. 14
N	Moment about z-axis. 14
X	Force in x-direction. 14
Y	Force in y-direction. 14
Z	Force in y-direction. 14
p	Angular velocity around x-axis. 14
q	Angular velocity around y-axis. 14
r	Angular velocity around z-axis. 14

u Velocity in x-direction. 14, 20

v Velocity in y-direction. 14

w Velocity in z-direction. 14

x Position on x-axis. 14

y Position on y-axis. 14

z Position on z-axis. 14

Acronyms

AD Algorithmic Differentiation. 8

CAS Computer Algebra Systems. 8

CB Center of buoyancy. 24

CFD Computational Fluid Dynamics. 64

CG Center of gravity. 17, 24

CO Center of origin. 17

DOF Degree of freedom. vii, 13, 14, 25, 28, 29, 32–35, 38, 40, 61

DP Dynamic Positioning. 2, 3, 5, 10, 11, 31, 61–63

ECEF Earth-centered Earth-fixed. 14, 15

ES Extremum Seeking. 7

ESC Extremum Seeking Control. 6

FDI Frequency-Domain Identification. 4, 24

GNC Guidance, Navigation and Control. 4, 25, 33, 39

GPS Global Positioning System. 14, 15

ITTC International Towing Tank Conference. 25

MAB Maximum allowed biomass. 2

MPC Model Predictive Control. 6–8, 10, 11, 35–39, 47, 55, 59–62, 65

MSS Marine systems simulator. 3, 4, 9, 10, 17, 21, 24, 25, 28, 33, 39, 58

NED North-East-Down. 14–16, 28, 30, 33, 34

NSK Nordnorsk skipskonsult. 1, 3, 17, 29

NTNU Norwegian university of Science and Technology. 4

PDE Partial differential equation. 5

PID proportional–integral–derivative. 10, 11, 35, 37–39, 49, 51, 55, 56, 59, 60

PM	Position Mooring. 5, 6
RAO	Response amplitude operator. vii, 4, 25, 28
TAPM	Thruster-Assisted Position Mooring. 5
WD	Wave drift. vii, 25, 28
WF	Wave frequency. 25, 61

1 Introduction

1.1 Background

In the last 50 years, the Norwegian aquaculture have had enormous growth. Statistisk sentralbyrå (2018) An illustration of this is shown in Figure 1.

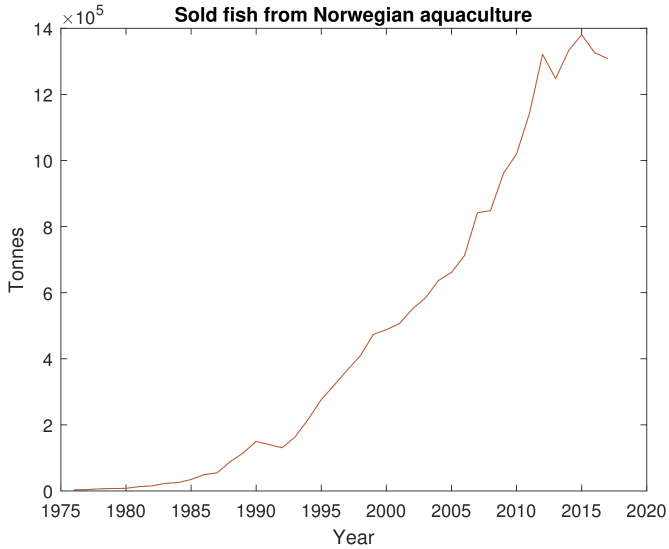


Figure 1: Illustration of sold aquaculture fish. Data collected from Statistisk sentralbyrå (2018)

This growth has now reached a natural maximum limit as the fjords of Norway are filled with aquaculture plants. The traditional plants cannot be exposed to hefty weather, and need to be located in areas without huge environmental loads. There are some challenges with this traditional way of doing aquaculture. An example is that if the plant is located at a shallow site with low current velocities, the faeces from the plant will pile up under the plant. This is not good for the biological life. To prevent more damage to the environment, there was not allowed more fish farming in the fjords.

The Norwegian Directorate of fisheries introduced a temporary arrangement including special authorizations to projects that involve outstanding innovation and substantial investments. These projects should have innovation that could contribute to solve the challenges the industry had regarding the environmental view and the lack of proper farming area. Fiskeridirektoratet (n.d.)

In 2015 Nordlaks and Nordnorsk skipskonsult (NSK) Ship Design started project "Havfarm." Originally the "Havfarm" project consisted of three different concepts. A stationary Havfarm, a Portable Havfarm and a Dynamic Havfarm. The Havfarm project is a part of Nordlaks solution to a sustainable evolution of the fish farming industry. Havfarm should be able to farm fish in areas where this is not possible with traditional plants. In 2017 was Nordlaks pledged with 21 permissions to do fish farming in the stationary Havfarm, Havfarm 1, and the dynamic Havfarm, from now referred to as Havfarm 2. Nordlaks (n.d.).

Figure 2: Illustration of Havfarm 2

1.2 Literature Review

1.2.1 Havfarm 2

Project Havfarm 2 was pledged with eight permissions from the Norwegian Directorate of fisheries. Havfarm 2 is designed as an offshore structure, with propulsion as a ship, but in principal concern a fish farming plant with a Maximum allowed biomass (MAB) of 10 000 tonnes. As mentioned, Havfarm 2 should be a dynamic plant. In other words, the plant should be equipped with thrusters and Dynamic Positioning (DP) system such that it can maintain position by its own propulsion. Havfarm 2 is planned to operate in two different zones. Mainly in an exposed zone, which can be divided in to several anchor points, and a safe zone. The operational philosophy is that Havfarm 2 shall be located in an exposed location when the weather conditions allow, and shall seek shelter in a location less exposed to waves in heavy weather. The transit between the two locations will be done using own propulsion thrusters. Havfarm 2 should also be equipped with one or more anchors. These are for fuel saving purposes only.

Havfarm 2 is a new type aquaculture plant, which is completely new way of doing aquaculture. Therefore, there will be many aspects that does not match today's aquaculture regulations. To be able to use Havfarm 2, exemptions from current regulations, regulatory clarifications, or the ultimate consequence of developing new regulations to account for Havfarm 2 is needed. Some examples of this are listed below.

- Havfarm 2 should not be anchored at a specific site, as traditional plants, but should be able to utilize a larger area, and multiple areas. This makes the normal site-clearance not good enough.
- There are regulations, NYTEK, Nærings- og fiskeridepartementet (n.d.), requiring mooring analysis, to achieve facility certificate, which is needed to farm fish. Havfarm 2 will not base the station keeping on the mooring system, and will therefore not get a facility certificate pursuant to current regulation.
- The requirements to the station keeping DP-system of Havfarm 2 need to be different than to the DP-systems to for example to offshore drilling vessels. As drilling vessels have to strictly maintain the position during a failure to be able to shut down the operation, there are no need to strictly maintain the position of Havfarm during a failure. Loss of position for Havfarm 2 does not pose an immediate severe consequence like a drilling vessel does. The

crucial objective for Havfarm 2 during a potential failure is to get the propulsion system up and running before the plant run aground.

- The DP system on Havfarm 2 must also account for the swim speed of the salmon. The relative velocity to the water in the cages must be below a given value. This applies both in an ultimate limit state, that is the maximum swim speed for a short amount of time, and a fatigue limit state, that is the maximum swimming speed during a given time period.

Another aspect in the DP system is to account for the welfare of the fish. The salmon needs a given amount of dissolved oxygen in the water. As the salmon utilizes the oxygen, the water quality in the hindmost cage can become poor. The DP system needs to counter this by either moving or change the heading of the vessel, allowing more high quality water to flow though the hindmost cage. An illustration of the water quality problem is shown in Figure 3. In the figure, the water flow are arriving from the top of the figure. If the vessel is stationary and headed along the water flow, there are many nets, structure, utilization of oxygen, pollution of water that reduce the water quality. In the figure, the dark blue water is high quality water, but the light blue are water of low quality. It is illustrated that if the vessel is stationary and headed orthogonal to the water flow, there are high quality water. The negative point of view is that is requires much energy to counteract the drag forces in this condition. By changing the heading a little bit (Seen to the right) the water quality can remain high, and the forces on the vessel remain acceptable.

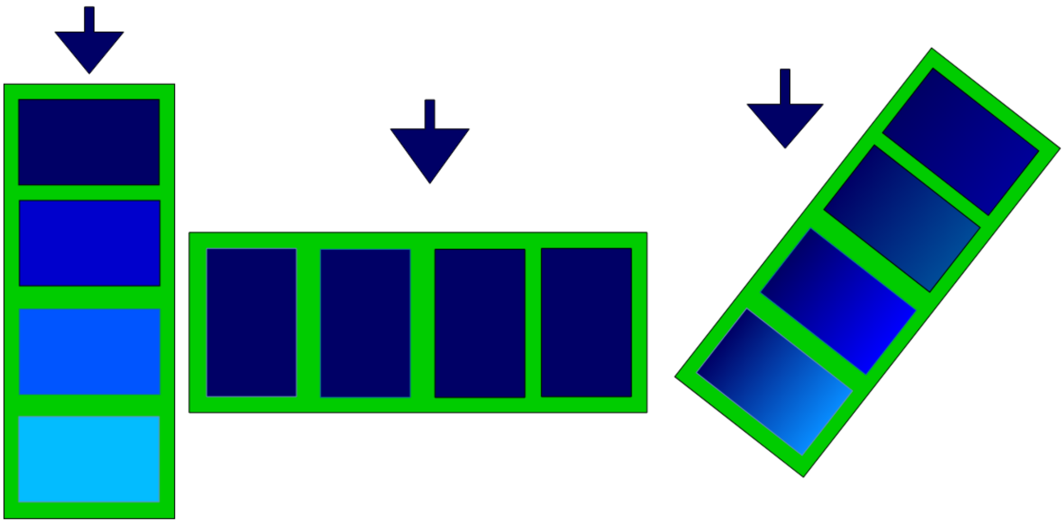


Figure 3: Illustration the water quality reduction on Havfarm 2

This is the case that is studied in this thesis. To ensure water of high quality while minimizing the fuel consumption.

The design of Havfarm 2 is still at an early stage. This means that many of the variables and data presented in this thesis may be estimates and early engineering assumptions. The data from Havfarm 2 used in this section and this thesis is, if not contradicted, collected from NSKs internal documents.

1.2.2 MSS

Marine systems simulator (MSS) is a Matlab/Simulink library for marine systems. It contains several models and tool boxes useful in design of marine control systems. In Perez et al. (2006) MSS is described as *"an environment which provides the necessary resources for rapid implementation of mathematical models of marine systems with focus on control system design."* The development of MSS started at Norwegian university of Science and Technology (NTNU), and have continued to been developed with the help of other groups.

The MSS toolbox is divided in to three main tool boxes. This is the Guidance, Navigation and Control (GNC) Toolbox Fossen & Perez (n.d.) , which *"contains basic libraries and system examples for guidance, navigation and control (GNC)"*, the HYDRO Toolbox Fossen & Perez (n.d.) , which *"reads output data files generated by hydrodynamic programs and processes the data for use in Matlab/Simulink"*. The last toolbox that is included in MSS is the Frequency-Domain Identification (FDI) Toolbox Perez & Fossen (2009). *"This is a stand-alone tool box for identification of radiation-force models and fluid memory effects of marine structures (marine craft and wave energy converters)."* Perez & Fossen (n.d.)

In this Thesis, the HYDRO Toolbox will be used. First the geometry of the hull and the loading condition (the distribution of the mass) must be analyzed in Wamit, Lee & Newman (n.d.). Wamit provides a lot of calculated data like: restoring coefficients, added-mass and damping coefficients, exciting forces transfer functions force-RAOs, body-motion transfer functions, motion-RAOs, local hydrodynamic pressure, mean-drift wave force and moment, etc. MSS Hydro can read output data from WAMIT and save all relevant data to a data structure, which can further be used in Matlab/Simulink. Further the MSS Hydro provide a Simulink library that can be used for time-domain simulations in different sea states. Perez et al. (2006)

In addition to MSS HYDRO, the MSS GNC Toolbox will be used in this thesis. This Toolbox is the core component in MSS. It contains examples of Guidance Navigation and Control, a Simulink library including predefined models for Guidance, environment, utilities, control, ship models and navigation, and several Matlab files that are used in the Simulink models, and are useful in modelling in Simulink. Perez et al. (2006)

The FDI Toolbox will also be used. This is a tool to identify radiation-force models and fluid memory effects of marine structures. As Wamit analyzes is done in frequency domain, the forces and motions in the frequency domain is found by the use of FDI Toolbox.

1.2.3 Digital Twin

Digital twin was introduced in 2002 as a "*Conceptual Ideal for PLM."* (Product Lifecycle Management) in Grieves & Vickers (2017). In Grieves & Vickers (2017) it is stated, "It is based on the idea that a digital informational construct about a physical system could be created as an entity on its own. This digital information would be a "twin" of the information that was embedded within the physical system itself and be linked with that physical system through the entire lifecycle of the system."

Since that time, digital twins have been used more and more. For starter, the US Air Force and many other car makers have used digital twin in design phase of production. (Smogeli (2017))

A digital twin is according to Øyvind Smogeli, Smogeli (2017), *A digital representation of a physical object, asset or system: a ship, a car, a wind turbine, a power grid, a pipeline, or a piece of equipment such as a thruster or an engine.* In a wider aspect, a digital twin is a mathematical representation of the real world. This digital twin have many benefits. It can be used as a simulation model, which is exposed to external and internal forces, and calculates the response. In the real world, it may be too complex, to costly or even impossible to model the object in a helpful scale. In Glaessgen & Stargel (2012) digital twins of aerospace vehicles, is described. It is stated that *Since extreme thermal, mechanical, and acoustical loading's may be impossible to reproduce in a laboratory at anything more than the component scale, the identification and quantification of limit states via computational simulation is needed.*

According to Glaessgen & Stargel (2012) digital twins will be used in design phase of more and more objects in the future. In an early phase of designing, loads and scenarios can be simulated, and the object can be optimized based on these results.

Digital twins of ships and other objects cannot only be used in the design phase. It will be implemented in control systems that live can predict what will happen if the ship is subjected to e.g. thrust. In Smogeli (2017) it is stated that *Using digital twins for control system software throughout the life cycle will help to avoid cost, prevent risk earlier, reduce the time for new systems to enter service in the field, improve systems interoperability, and enable superior system performance.*

1.2.4 Anchoring and positioning mooring systems

Havfarm 2 shall be designed as a DP vessel, which is defined in The Maritime Safety Committee (2013) as *a unit or a vessel which automatically maintains its position (fixed location or predetermined track) exclusively by means of thruster force.* In operational conditions, Havfarm 2 shall, in addition to the DP system, use a gravity anchor for fuel saving purposes. This anchor is also included is this thesis.

Using both anchor and thrusters for station keeping is nothing new, and is generally called: Thruster-Assisted Position Mooring (TAPM) or only Position Mooring (PM).(Berntsen (2008)) In the commonly used PM systems in offshore industry, the mooring system is quite complex and should maintain the position alone in moderate weather conditions. The thrusters in PM may be needed during severe weather, or more active, by providing constant for damping the surge, sway and yaw motions and for keeping the desired headings. (Berntsen et al. (2009))

General PM have been used in the industry since the end of the 1980's. The experiences is that PM is a cost-effective alternative to permanent installations in offshore oil production. The control theory of PM is according to Aamo & Fossen (2001) based on control theory from DP. The traditional DP systems was based on Optimal control theory and Kalman filtering, and was developed and extended from the early 1970's. In later years, nonlinear control theory based on integrator backstepping was developed. Aamo & Fossen (2001).

Aamo & Fossen (2001) developed a new finite element model for the mooring cable suspended in water. This was done by developing a Partial differential equation (PDE) for the mooring cable dynamics with negligible bending and torsional stiffness, applying Hooke's and use gravity, hydrostatic and hydrodynamic forces as the external forces. Thereafter discretization of the mooring line in to finite elements. The cable configuration of the cable can be seen in Figure 4.

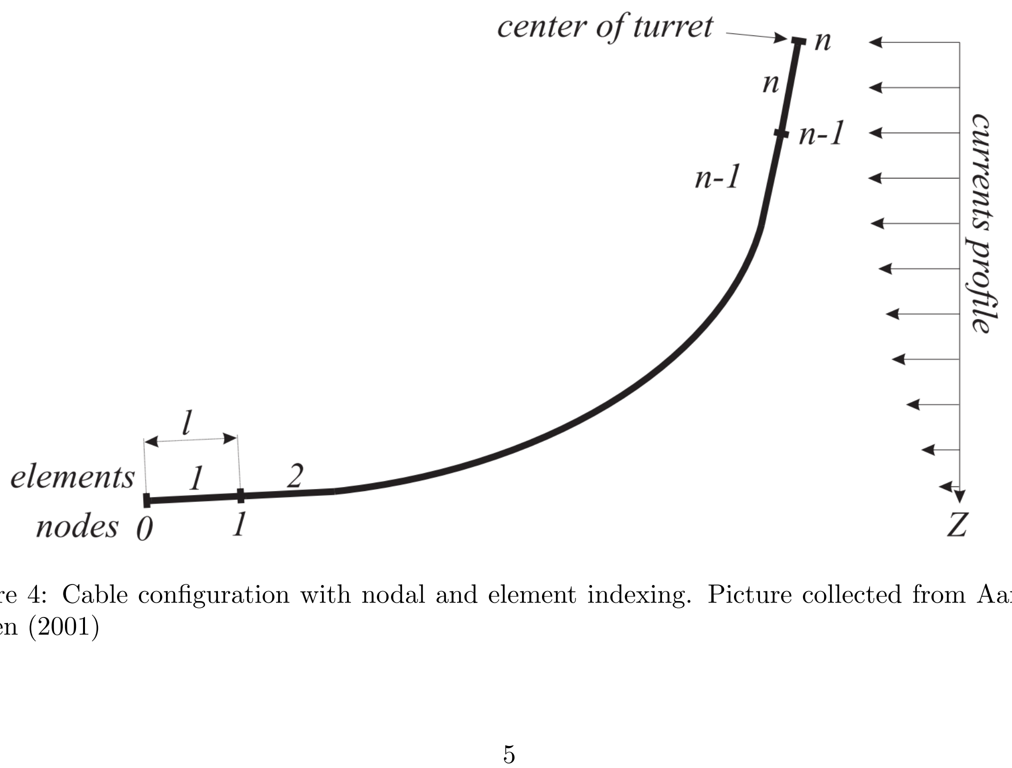


Figure 4: Cable configuration with nodal and element indexing. Picture collected from Aamo & Fossen (2001)

Common for existing PM systems is that the position of the vessel should be restricted within predefined geographical areas or safety regions. These areas are often quite small, drilling and pipe laying as an example, and the PM system can use a lot of fuel for strict positioning or preventing mooring line breakage like in Berntsen et al. (2009).

1.2.5 Optimal control

Optimization is nothing new, neither something special. Freedivers regulate the intensity of the swimming to optimize for the largest distance. Investors minimize the risk and maximize the rate of return. Engineers adjust the variables to optimize the design. In addition, nature optimizes. Every physical system tend to a state of minimum energy.

Mathematical optimization has developed from the work of Euler and Lagrange in the 18th century. Even as mathematical optimization was used before 1940, most of the modern optimization theory and practice has evolved from the development of linear programming in the 1940 until today. As a modern language relates "programming" to computer software, "programming" in the 1940s includes connotations of algorithm design and analysis in the development of optimization. Nocedal & Wright (2006)

The common mathematical optimization problems include three main components. This is the *Objective function*, which is a function that should be maximized or minimized, *decision variables*, which are mostly real variables included in the program, and *Constraints*, which are the limitations in space for the variables. The constraints are often divided in to *equality constraints* and *inequality constraints*.Foss & Heirung (2016)

When either the constraints or derivatives of the constraints are nonlinear, a nonlinear program appears. As the problem becomes more nonlinear and with more variables the solution can be harder to find. An example can be that we want to find the Global minimum, i.e. the minimum solution there is, but end up with an infeasible problem because the optimizer found a local infeasible minimum. An example of this is shown in Figure 5. When using a Rotation matrix to transform between different reference frames, the problem becomes nonlinear.

There are many methods to solve optimization problem, and the two main candidates for solving the nonlinear optimization problem in this thesis was the Extremum Seeking Control (ESC), and the Model Predictive Control (MPC). These are described short in the following paragraphs.

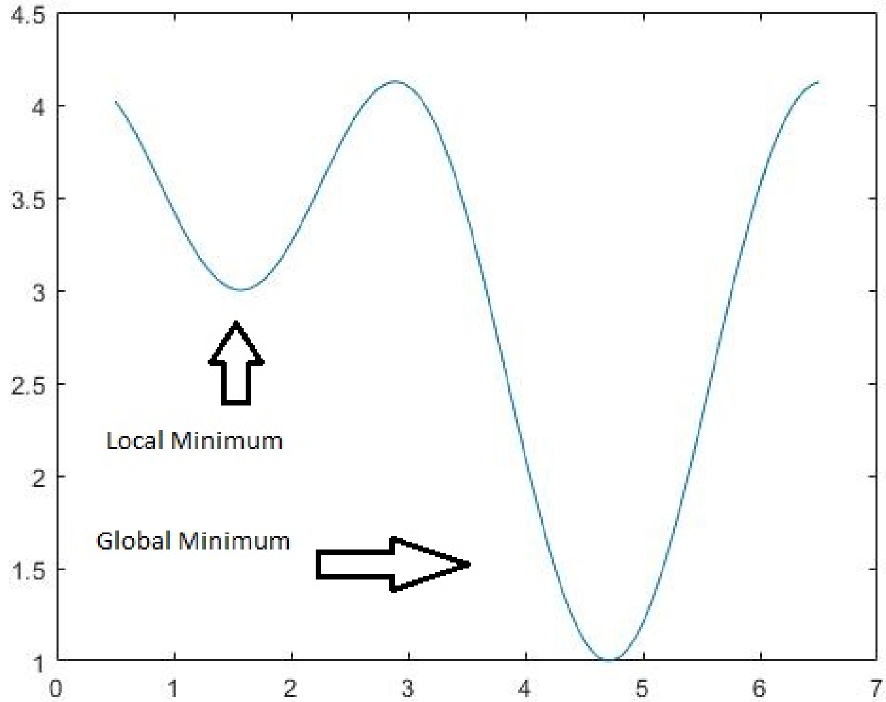


Figure 5: Visualisation of local and global minimum in one dimension

Extremum Seeking Control
Extremum Seeking (ES) is one of the most used methods for real time non-model-based optimization. Even as ES was invented in 1922, it was first heavily used in the industry in the the millennium change. As a real time non-model-based optimization, the idea of the most common Extremum Seeking approach is that perturbation signals are added for the purpose of estimating the gradient of the map which should be optimized Krstic (2014). ES is an adaptive control that tunes the variables in an objective function resulting in the system reacting a local minimum. This method is also a non-model-based method. Breu & Fossen (2010). The extremum seeking algorithm search for the local minimum by calculation the gradient of the objective. The way this is done is to by exciting the plant with a sinusoidal probing signal and analyse the result. Breu & Fossen (2010)

MPC Model Predictive Control, abbreviated MPC is a form of optimal control based on a model. In general MPC is an open loop optimization problem, which is being solved online at every time step for a finite time horizon. The initialization to the problem is the state the system at the current time, and the control input applied to the system is the first control input in the time horizon created by the optimization. At each time step, the optimization is being executed, and the first control input is being used.

Early proposals of MPC included linear programming for linear systems, using hard constraints. This was done by Propoi in 1963. The first application of MPC in process control is being described in Richalet et al. (1977). Even as there were existing forms for optimal control in the 1970s, for example linear quadratic control, this was not used much in the industry. The reason for this may be that it could not handle constraints, nonlinearities and uncertainty well enough (Mayne et al. (n.d.)). A form of MPC called IDCOM (identification and command), was proposed in Richalet et al. (1977) and Richalet et al. (n.d.). This used a finite horizon pulse response and a quadratic cost function. It also included constraints for the input and the output. The MPC has since then evolved to include hard, soft and ranked constraints, temporary violation of constraints, nonlinear dynamics and nonlinear constraints. Now the MPC is heavily used in the marine industry. The reason for this is that it accounts for natural constraints like in thrust for a ship very well. It is also an advantage for unconstrained nonlinear plants, where offline computation of the control law usually requires the dynamics to process a special structure. Mayne et al. (n.d.)

Both linear, quadratic and nonlinear MPC are described well in Foss & Heirung (2016). An example of a nonlinear MPC problem as an extension from linear MPC is shown below:

$$\min\ f(z) = \sum_{t=0}^{N-1} \frac{1}{2} x_{t+1}^T Q_{t+1}\ x_{t+1} + d_{x\ t+1} x_{t+1} + \frac{1}{2} u_t^T R_t u_t + d_{u\ t} u_t + \frac{1}{2} \Delta u_t^T R_{\Delta t} u_t$$

$$\begin{aligned} &\text{Subject to} \\ x_{t+1} &= g(x_t, u_t) \\ x_0, u_{-1} &= given \\ x^{low} &\le x_t \le x^{high} \\ u^{low} &\le u_t \le u^{high} \\ -\Delta u^{high} &\le \Delta u_t \le \Delta u^{high} \end{aligned} \tag{1}$$

$$\begin{aligned} &\text{Where} \\ Q_t &\succeq 0 \\ R_t &\succeq 0 \\ R_{\Delta t} &\succeq 0 \end{aligned}$$

Where the first line is the objective, to minimize the weighted combinations of the states and the control input in both linear and quadratic form. The second line is the propagation, expressed as an equality constraint, where the next states are a nonlinear function of the current states and the control input. Further, the initial conditions is expressed as an equality constraint. The inequality constraints is the lower and upper values for the states and the control inputs. This example also includes lower and upper bound for the derivatives of the control input. At least it is important that the weighting matrices are positive definite.

The algorithm for solving an MPC problem is shown in Foss & Heirung (2016), and given below:

Algorithm for Nonlinear MPC with state feedback
for t = 0, 1, 2, ... **do**
Get the current state x_t
Solve the optimization problem on the prediction horizon
from t to $t + N$ with x_t as the initial condition
Apply the first control move u_t from the solution above.
end for

CasADi

CasADi is an open-source software tool for numerical optimization in general and optimal control. CasADi stand for Computer Algebra Systems (CAS), and Algorithmic Differentiation (AD). In addition to AD, the CasADi tool support integration of ordinary differential equations and differential-algebraic equations. CasADi does also include sensitivity analysis, nonlinear programming and interfaces to other numerical tools. The focus of the CasADi tool is now gradient base numerical optimization, especially on optimal control. There are several layouts of the nonlinear programming, and *Opti stack* is a collection of CasADi helper classes that can solve programs where both the constraints and the objective function are nonlinear, and combination of nonlinearities. An example of this is given in Equation (2). The main characteristics for Opti Stack, or the factor which differs Opti Stack from other layouts is according to, Joel A. E. Andersson et al. (n.d.), the following:

- Allows natural syntax for constraints
- Indexing/bookkeeping of decision variables is hidden
- Closer mapping of numerical data-type to the host language.

The solver that is used is the *Ipopt*, a library for large-scale nonlinear optimization. Ipopt is released as open source code under the Eclipse Public License (EPL), and can be found in Wächter & Biegler (2006) and `https://github.com/coin-or/Ipopt`

$$
\begin{aligned}
\underset{x,y}{\text{minimize}} \quad & (y-x^2)^2 \\
\text{subject to} \quad & x^2+y^2=1 \\
& x+y \geq 1,
\end{aligned}
\tag{2}
$$

1.3 Objective and scope

1.3.1 Objective

The objective for this master thesis is to investigate the situation in operational conditions for Havfarm 2, when the weather conditions are calm. There may be a problem that the salmon in the hindmost cage of Havfarm 2 could suffer from low water flow through the cages, and the fish in the upstream cages utilize the oxygen in the water. The main objective is to develop an algorithm that ensures water of high quality to the fish living in the net pens of Havfarm 2. This should be done by merging optimization theory and control theory in order to minimize fuel consumption while ensure sufficient water flow to the salmon.

To be able to obtain the main objective, several objectives are also a part of this master thesis. This is to develop a digital twin for Havfarm 2. That is, in control system theory, a mathematical model that is as close as possible to the real system, also referred to as a "Process plant model".

Another objective to lay the foundation for the main objective is to develop a simulator for the optimization and the digital twin. This simulator should contain several subsystems, which mostly exists for traditional DP vessel, but must be customized in order to be functional for the simulation of Havfarm 2.

When the foundation is laid, simulations will be done in order to investigate and compare different approaches.

1.3.2 Scope

Work description The work done in this project thesis is listed below:

1. A small background and literature review of Havfarm 2, MSS, Digital Twin, Anchoring and Position mooring systems and Optimal control.
2. Collecting and modifying a 3D panel model of Havfarm 2 in Rhino which is used in the hydrodynamic analysis program: Wamit.
3. Collecting the mass matrix for Havfarm 2 that is needed in order to run the hydrodynamic analysis.
4. Development of input files to Wamit, and analyze the hydrodynamics of Havfarm 2 in Wamit.
5. Read and sort the output data from Wamit, using the MSS function *wamit2vessel*.
6. Development of equation of motions for Havfarm 2, model the digital twin corresponding to this equation. This is done in the following steps:

- Use MSS Simulink library and use a DP model, and modify this with the following steps.
- Use the infinite frequency added mass, linear viscous damping, restoring matrix from Wamit.
- Develop the fluid memory effects by transforming the frequency domain hydrodynamics to state space by the use of MSS function *vessel2ss*.
- Use the rigid body mass and added mass to model the Coriolis and centripetal matrices.
- Develop a quadratic drag model by the knowledge of drag coefficients.
- Develop the effect of wind on the vessel by using the MSS function *blendermann94*.
- Develop the effect of the waves on the vessel by using transfer functions from Wamit and MSS wave loads procedures
- Develop mooring characteristics according to the mooring theory in Faltinsen (n.d.)

7. Development of the environmental conditions is done in the following steps:
   - Developing a constant current model.
   - Developing a varying wind model by using wind spectrum created in MSS.
   - Developing a varying irregular wave model by using wave spectrum created in MSS.
8. Development of a simple measurement model
9. Developing a model for the water quality based on relative water velocities in different directions.
10. Developing a state estimator customized for Havfarm 2 and its needs.
11. Developing a simple thrust allocation and add simple dynamics to the thrusters.
12. Learning the CasADi program, and developing Nonlinear MPC optimization problem in Matlab using the *Opti Stack* collection. Implement this by use a sufficient control plant model in the optimization problem.
13. Implement the optimization problem in to the simulator by creating either reference points for the proportional–integral–derivative (PID) controller or direct control inputs for the thrust allocation for each time step.
14. Simulation and comparing of different control approaches to determine which is the best for this problem.

1.4 Main contributions

There are many aspects regarding the DP system of Havfarm 2. Example of this is the relocating between different zones, anchoring and the raising of the anchor, station keeping in hefty weather conditions with and without the help of mooring and the positioning in calm conditions. This master thesis will focus on the dynamic positioning in calm conditions, including an anchor line. The challenge in this operational conditions may be to ensure that the fish have access to sufficient water. The reason for this problem is that in calm weather conditions, the water flow in the hindmost cage is being reduced by multiple nets, and the quality is being reduced by fish utilizing the oxygen and produce faeces which are not transported away quickly enough by the water flow. As this is not a common problem, there are not developed sufficient solutions to this in the industry. The contribution of this master thesis is to propose an approach to solve this problem.

As Havfarm 2 is neither a traditional DP-vessel nor a traditional aquaculture plant, there are not done much modelling of this plants. Modelling of a digital twin of Havfarm 2 as a combination of an aquaculture plant and a DP-vessel is also a contribution.

In the marine control system industry, it is natural to develop a digital twin of the vessel, a process plant model, to do early stage cybernetics engineering. The main contribution is the digital twin developments of Havfarm 2, which as compared to offshore vessels has unconventional hull shape, different operational philosophy, hydrodynamics and adjustment for biological welfare. In addition to this digital twin, it is built a simulation platform where the digital twin should operate. This simulation platform includes environmental conditions like wind, waves and current, measurement simulation, state estimator, control system, thrust allocation and thruster dynamics. These "known" subsystems are customized to account for Havfarm 2's differences from traditional DP vessels.

To solve the water quality problem, different approaches are compared. The main approach uses optimal control theory by minimizing the fuel consumption while ensure enough water quality for the fish. This is done by developing a model for the water quality, and use this model in an optimization problem, more specific a nonlinear MPC problem. This optimizer includes a simplified model of the vessel(control plant model,) and necessary variables. It uses Direct multiple shooting to find the optimal solution. The optimal solution is either transformed to waypoints in a reference model which shall be followed by a PID-controller or the direct control input as thrust. The other approach is to use a PID controller to regulate the thrust from one thruster based on the water quality.

Limitations This master thesis includes modelling of Havfarm 2, an optimization algorithm and a simulation platform. In the development of this, many simplifications have been made. There are also many limitations that had to be included in the evolution of the thesis. Some of them are listed below

- The design of Havfarm 2 is not fulfilled, and there are some data used to develop this model that is not reliable. Even though the digital twin in this thesis might not be as the final version of the Havfarm 2, it should give sufficient dynamic behaviours.
- In the operational conditions of Havfarm 2, that this model is developed for, there will be used an anchor to reduce the consumption of fuel. As it is not yet fully decided whether to use chain, wire, nylon or a combination of these as the mooring line, the anchor characteristics is pretty estimated.
- Havfarm 2, the DP-system and the optimization tool in this thesis is developed for calm weather, low velocities and not strict DP systems. This will make the simulation model less suitable for rough weather and higher velocities.
- In the development of the digital twin, there are also some simplifications. Especially the quadratic drag model and the wind model are very simplified models.
- The simple thrust allocation does not account for forbidden sectors, neither for using only the wanted thrusters.
- The optimization algorithm need much further work to give satisfying results.

1.5 Organization of project

In Section 1, Introduction, the background of the project thesis is presented. A short literature review is also presented. The objective and scope of the project thesis is also a part of the introduction. Section 2, Methods, is divided in to modelling of the digital twin, 2.1, and the control system design, 2.2. The modelling section shows the kinematics, kinetics and development of the digital twin. The control system design section take care of both the state estimator, thrust allocation and the DP system including controller and optimization tool. The simulation results and the discussion of these simulation results is presented in respectively Section 3 and 4. A proposition for some further work is mentioned in Section 6.

2 Methods

2.1 Modelling of Digital Twin

When modelling marine vessels, it is common to choose between two different regimes. The regimes are maneuvering and seakeeping. When using maneuvering modelling, the vessel is assumed to be in calm water and without waves. In seakeeping the assumption is that the vessel keeps a constant course and speed. Neither of theses regimes are optimal for the modelling of Havfarm 2. The seakeeping model is considered the best option. The reason for this is that the speed of the vessel is relatively low, which fits the seakeeping model best. Knowing this, several aspects of the maneuvering regime is included in the modelling of Havfarm 2 as well.

The main particular of Havfarm 2 is shown in Table 1.

Table 1: Main particular of Havfarm 2

Unit	**Value**
Displacement	67 300 *tonnes*
Length	304.2 m
Breadth	63 m
Depth moulded	43 m
Total fish cage volume	400 000 m^3

A sketch of Havfarm 2 is shown in Figure 6.

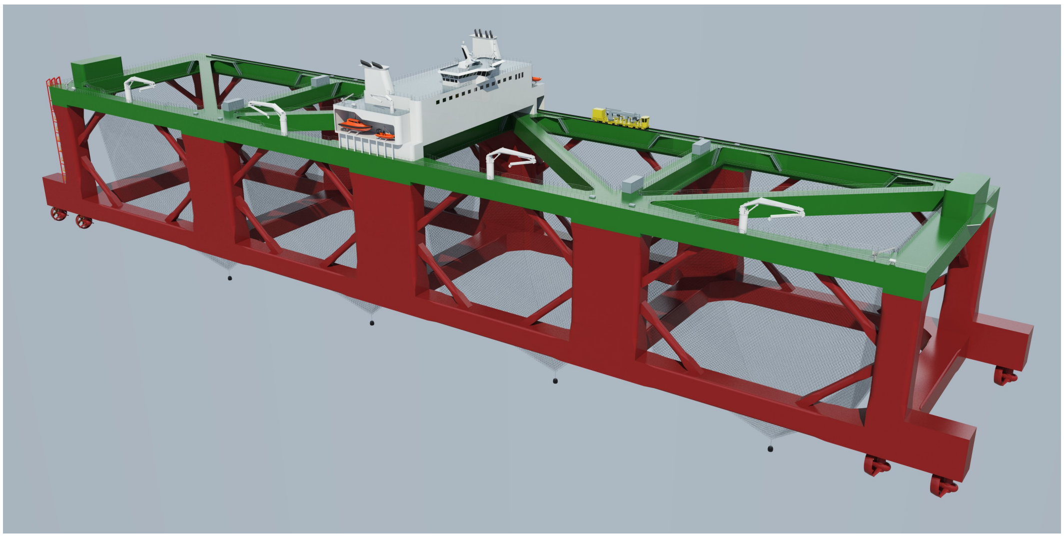


Figure 6: Sketch of Havfarm 2

Many of the hydrodynamic data that is used in this thesis are calculated in the hydrodynamic analysis program Wamit. The input to this program are several text files describing the problem that shall be solved. Included in these are the the rigid body mass matrix (See Section 2.1.3) and a *.GDF file that is a text file representing a 3 dimensional panel model. This meshed panel model includes only the parts that are submerged in the operational condition in this thesis. The panel model is shown in Figure 7.

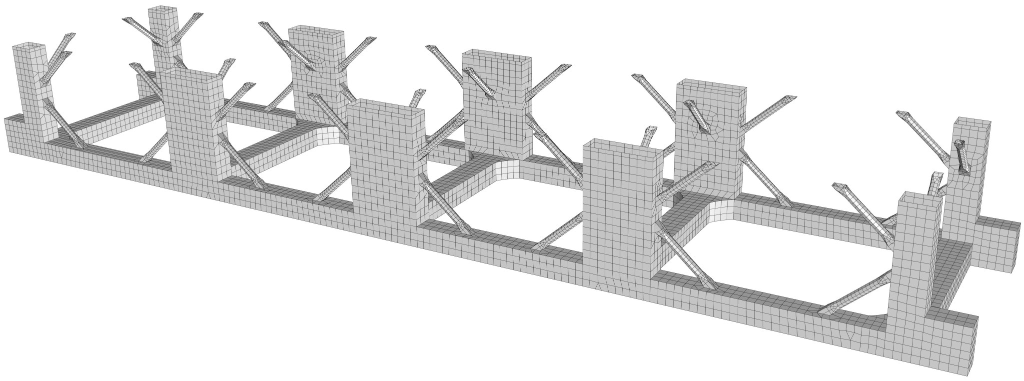


Figure 7: Meshed panel model of Havfarm 2 as input to Wamit

2.1.1 Kinematics

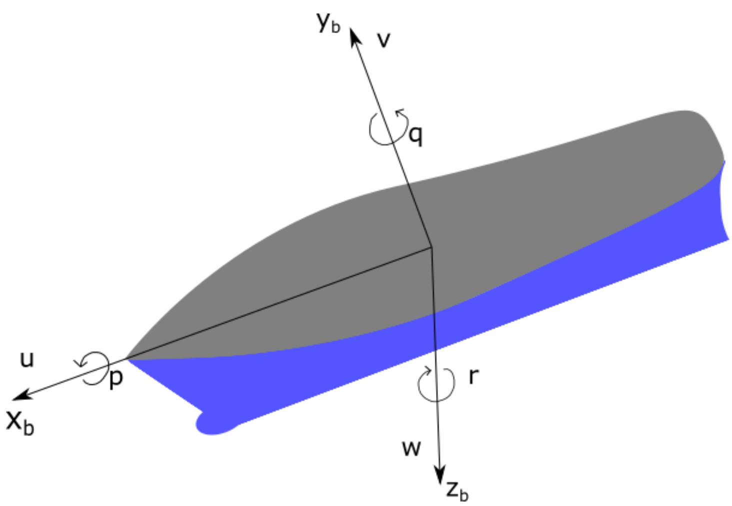


Figure 8: 6 DOF

A vehicle can be modelled in six degree of freedom (6 DOF). These are shown in Figure 8. These are position in the x, y, and z axes and the rotation around the same axis. The motions in this 6 DOF system are *surge, sway, heave, roll, pitch* and *yaw.*

An overview of this 6 DOF system are given in Table 2.

Table 2: Notation of SNAME (1950) for marine vessels, Fossen (2011)

DOF		Forces and moments	Linear and ang. velocities	Positions and Euler angles
1	motions in the x-direction (surge)	X	u	x
2	motions in the y-direction (sway)	Y	v	y
3	motions in the z-direction (heave)	Z	w	z
4	rotation about the x-axis (roll,heel)	K	p	ϕ
5	rotation about the y-axis (pitch,trim)	M	q	θ
6	rotation about the z-axis (yaw)	N	r	ψ

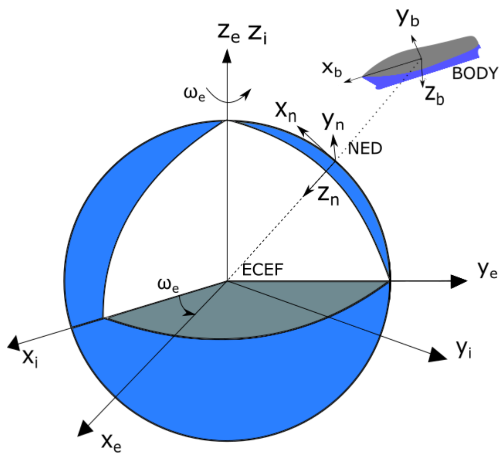


Figure 9: Reference frames

Reference frames In the modelling of marine vessels it is needed to use more than one reference frame. In this thesis, some different reference frames are used. A short description of these are following.

ECEF, {e}. This is the Earth-centered Earth-fixed reference frame. The origin of this frame is fixed to the center of the earth. The axis are directed as shown in subscript: e in Figure 9. Common Global Positioning System (GPS) coordinates are shown in this frame, as longitude and latitude respectively l and μ. Regarding that this is not a desired reference frame to model the vessel in, the GPS coordinates needs to be transformed in to another reference frame. The Earth-centered Earth-fixed (ECEF) position and respectively Longitude and latitude can be written in vectorial form as follows:

$$\boldsymbol{p}^e_{b/e} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \Theta_{en} = \begin{bmatrix} l \\ \mu \end{bmatrix} \tag{3}$$

NED {n}. This is an abbreviation for North-East-Down coordinate system. This system are used in everyday life. In this system the x-axis is pointing to the north, y-axis is pointing to the east and the z-axis is pointing to down, towards the origin of the earth. Regarding that Havfarm 2 shall be located in a local area, a "flat earth approximation" is used for this coordinate system. When this "flat earth approximation" is used, this reference frame can be assumed inertial such that Newton's law allies Fossen (2011). The North-East-Down (NED) position, $\boldsymbol{p}^n$, and the attitude (Euler angles), Θ_{nb}, are given in vectorial form as follows.

$$\boldsymbol{p}^n_{b/n} = \begin{bmatrix} N \\ E \\ D \end{bmatrix} \quad \Theta_{nb} = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} \tag{4}$$

BODY, {b}. The body-fixed reference frame is a coordinate system that have it's origin midships in the water line. The coordinate system is moving with the vessel. The x-axis is directed from aft to fore on the vessel. The y-axis is directed towards starboard on the vessel, while the z-axis are directed from top to the bottom of the vessel. Figure 8 shows this coordinate system. Fossen (2011). The body fixed linear velocity, $\boldsymbol{\nu}$, and the body fixed angular velocity, are given as follows:

$$\boldsymbol{v}^b_{b/n} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \quad \boldsymbol{\omega}^b_{b/n} = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{5}$$

The Body fixed force, $\boldsymbol{f}_b^b$, and body fixed moment, $\boldsymbol{m}_b^b$ are written in vectorial form as follows:

$$\boldsymbol{f}_b^b = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad \boldsymbol{m}_b^b = \begin{bmatrix} K \\ M \\ N \end{bmatrix} \tag{6}$$

To further compress the vectorial form $\boldsymbol{\eta}$, $\boldsymbol{\nu}$ and $\boldsymbol{\tau}$ are introduced.

$$\boldsymbol{\eta} = \begin{bmatrix} \boldsymbol{p}_{b/n}^n \\ \Theta_{en} \end{bmatrix} \quad \boldsymbol{\nu} = \begin{bmatrix} \boldsymbol{v}_{b/n}^b \\ \boldsymbol{\omega}_{b/n}^b \end{bmatrix} \quad \boldsymbol{\tau} = \begin{bmatrix} \boldsymbol{f}_b^b \\ \boldsymbol{m}_b^b \end{bmatrix} \tag{7}$$

Transformation between reference frames The transformation between reference frames is necessary to be able to use input data from GPS and other measurement data. A rotation Matrix, $\boldsymbol{R}$ is used as follows:

$$\mathbf{x}^{to} = \boldsymbol{R}_{from}^{to}\ \mathbf{x}^{from} \tag{8}$$

When getting longitude and latitude from GPS signals the rotation matrix from ECEF to NED is used. This is a function of the longitude and latitude, and is used as follows:

$$\boldsymbol{R_n^e(\Theta_{en})} = \begin{bmatrix} -cos(l)sin(\mu) & -sin(l) & -cos(l)cos(\mu) \\ -sin(l)sin(\mu) & cos(l) & -sin(l)cos(\mu) \\ cos(\mu) & 0 & -sin(\mu) \end{bmatrix} \tag{9}$$

When transforming from NED to BODY, the Euler angles are used. This rotation matrix are given as follows:

$$\boldsymbol{R_n^b(\Theta_{bn})} = \begin{bmatrix} c\psi c\theta & -s\psi c\phi + c\psi s\theta s\phi & s\psi s\phi + c\psi c\phi s\theta \\ s\psi c\theta & c\psi c\phi + s\phi s\theta s\psi & -c\psi s\phi + s\theta s\psi c\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix} \tag{10}$$

Where s is an abbreviation for *sine* and c is an abbreviation for *cosine*.

The transformation matrices have properties that satisfies

$$\boldsymbol{R}\boldsymbol{R}^T = \boldsymbol{R}^T\boldsymbol{R} = \boldsymbol{I} \qquad det(\boldsymbol{R}) = 1 \qquad \boldsymbol{R}^{-1} = \boldsymbol{R}^T \tag{11}$$

The relationship between angular velocity in BODY and NED frame is given in the same way, with a transformation matrix $\boldsymbol{T}_\Theta$. This transformation matrix is given as follows:

$$\boldsymbol{T_\Theta(\Theta_{nb})} = \begin{bmatrix} 1 & s\phi t\theta & c\phi t\theta \\ 0 & c\phi & -s\phi \\ 0 & s\phi/c\theta & c\phi/c\theta \end{bmatrix} \tag{12}$$

Where s stands for *sine*, c stands for *cosine* and t stands for *tangens*.

When compressing the rotation matrix and the transformation matrix in vectorial form, the transformation matrix, $\boldsymbol{J}_\Theta$, is obtained.

$$\boldsymbol{J_\Theta(\eta)} = \begin{bmatrix} \boldsymbol{R}_b^n(\Theta_{nb}) & 0_{3\times3} \\ 0_{3\times3} & \boldsymbol{T}_\Theta(\Theta_{nb}) \end{bmatrix} \tag{13}$$

From this result the 6 DOF kinematic equation is obtained. In Equation (14), ν is the 6 DOF velocity and angular velocity in body frame. $\boldsymbol{\eta}$ is the 6 DOF position in NED frame.

$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}_{\Theta}(\boldsymbol{\eta})\boldsymbol{\nu} \tag{14}$$

2.1.2 Kinetics

In order to model a vessel, Newton's second law is used. This law applies in an inertial frame, and as stated before, the NED frame is assumed to be inertial. By transferring newtons second law to a marine vessel in the NED frame, the result becomes as shown in Equation (15).

$$F = ma \qquad \Longrightarrow \qquad \boldsymbol{M}(\boldsymbol{\eta})\ddot{\boldsymbol{\eta}} = \boldsymbol{\tau} \tag{15}$$

Where $\boldsymbol{M}$ is the mass matrix. Transferring Equation (15) into the rigid body $\{b\}$ frame, a Coriolis term, $\boldsymbol{C}_{RB}$ is added. The reason for this is that the BODY frame is not inertial, and a compensation for a moving and rotation reference frame is needed. This term is dependent on the velocity vector in the BODY frame. Newton's second law in $\{b\}$ is given in Equation (16.)

$$\boldsymbol{M}_{RB}\dot{\boldsymbol{\nu}} + \boldsymbol{C}_{RB}(\boldsymbol{\nu})\boldsymbol{\nu} = \boldsymbol{\tau} \tag{16}$$

Where $\boldsymbol{M}_{RB}$ is the rigid body mass matrix, $\boldsymbol{C}_{RB}$ is the rigid body coriolis matrix. The force $\boldsymbol{\tau}$ can be divided in to τ, $\tau_{mooring}$, τ_{wind} and τ_{wave}, where τ is the propulsion force, $\tau_{mooring}$ is the mooring forces. In addition the effect of the added mass, $\boldsymbol{M}_A$, and the added Coriolis term, $\boldsymbol{C}_A$, need to be added. The rigid body mass and the rigid body Coriolis term is related to respectively the acceleration and the velocity, but the added mass and added Coriolis term is related to respectively the relative acceleration and the relative velocity. The relative velocity and acceleration is denoted as $\boldsymbol{\nu}_r$ and $\dot{\boldsymbol{\nu}}_r$ where $\boldsymbol{\nu}_r$ is as follows:

$$\boldsymbol{\nu}_r = \boldsymbol{\nu} - \boldsymbol{\nu}_c \tag{17}$$

where $\boldsymbol{\nu}_c$ is the current velocity vector. A damping matrix, $\boldsymbol{D}(\boldsymbol{\nu}_r)$, that is related to $\boldsymbol{\nu}_r$ is added to account for the hydrodynamic effects like linear damping and viscous damping effects. In addition to this terms the fluid memory effects, $\boldsymbol{\mu}$, the restoring forces, $\boldsymbol{G}\ \boldsymbol{\eta}$, and the gravity force on dynamic ballast tanks, $\boldsymbol{g}_0$, are added. The result of the equation of motion in the $\{b\}$-frame are given below.

$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}_{\Theta}(\boldsymbol{\eta})\boldsymbol{\nu} \tag{18}$$

$$\boldsymbol{M}_{RB}\dot{\boldsymbol{\nu}} + \boldsymbol{C}_{RB}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{M}_A\dot{\boldsymbol{\nu}}_r + \boldsymbol{C}_A(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \boldsymbol{D}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \boldsymbol{\mu} + \boldsymbol{G}\boldsymbol{\eta} + \boldsymbol{g}_0 = \boldsymbol{\tau} + \boldsymbol{\tau}_{mooring} + \boldsymbol{\tau}_{wave} + \boldsymbol{\tau}_{wind} \tag{19}$$

To make this equation a bit easier, following simplifications is made.

- The Rigid body mass matrix, $\boldsymbol{M}_{RB}$, is independent of the current acceleration. I.e. $\boldsymbol{M}_{RB}\ \dot{\boldsymbol{\nu}}_c = 0$. Therefore the rigid body mass, $\boldsymbol{M}_{RB}$, and the added mass, $\boldsymbol{M}_A$, can be summed according to following equation:

  $$\boldsymbol{M} = \boldsymbol{M}_{RB} + \boldsymbol{M}_A \tag{20}$$

  In addition the acceleration of the current is assumed to zero. Therefore the relative acceleration is switched with the body frame acceleration.

- The ballast tanks in operational conditions is assumed constant, so that $\boldsymbol{g}_0$ is assumed zero

The remaining equation of motion to model is as follows:

$$\begin{aligned} \dot{\boldsymbol{\eta}} &= \boldsymbol{J}_{\Theta}(\boldsymbol{\eta})\boldsymbol{\nu} \\ \boldsymbol{M}\dot{\boldsymbol{\nu}} + \boldsymbol{C}_{RB}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{C}_A(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \boldsymbol{D}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \boldsymbol{\mu} + \boldsymbol{G}\boldsymbol{\eta} &= \boldsymbol{\tau} + \boldsymbol{\tau}_{mooring} + \boldsymbol{\tau}_{wave} + \boldsymbol{\tau}_{wind} \end{aligned} \tag{21}$$

2.1.3 Mass Matrix

As mentioned the mass matrix, $\boldsymbol{M}$, is a sum of rigid body mass matrix and the added mass matrix. To find the rigid body mass matrix, the inertia matrix about center of gravity, $\boldsymbol{I}_g$, is needed. $\boldsymbol{I}_g$ is assumed diagonal, where $I_x = R_{44}$, $I_y = R_{55}$ and $I_z = R_{66}$. $\boldsymbol{R}$ is the Radius of gyration in the different axis. $\boldsymbol{R}$ is found by comparing Havfarm to a semi-submersible, and using following relation.

$$\begin{aligned} R_{44} &= 0.43 \cdot B \\ R_{55} &= 0.3 \cdot Lpp \\ R_{66} &= 0.32 \cdot Lpp \end{aligned} \tag{22}$$

The vessel is modelled about Center of origin (CO), and not Center of gravity (CG). The location of CG with respect to CO is called $\boldsymbol{r}_g$, and calculated from the estimated CG from stability analysis from NSK Ship Design. To account for modelling about CO, the skew symmetric matrix, $\boldsymbol{S}(\boldsymbol{r}_g)$ is used. From this, the rigid body mass matrix, $\boldsymbol{M}_{RB}$ is found as following:

$$\boldsymbol{M}_{RB} = \begin{bmatrix} m\boldsymbol{I}_{3\times3} & -m\boldsymbol{S}(\boldsymbol{r}_g) \\ m\boldsymbol{S}(\boldsymbol{r}_g) & \boldsymbol{I}_g \end{bmatrix} \tag{23}$$

The mass used in Equation (23) is calculated from submerged body in the geometry file. As the Havfarm 2 project proceed, a later estimation of the rigid body mass matrix was available. The previous estimated rigid body mass matrix is compared to the estimated mass matrix from NSK Ship Design, and is found to be very similar. Anyway the latest version of the rigid body mass matrix was used as an input to Wamit.

The second part of the mass matrix is the added mass matrix. The added mass matrix is frequency dependent, and is calculated in Wamit and found by using the MSS Matlab function "*wamit2vessel*". The result of the frequency-dependent added mass analysis from Wamit is shown in Figure 10 and 11.

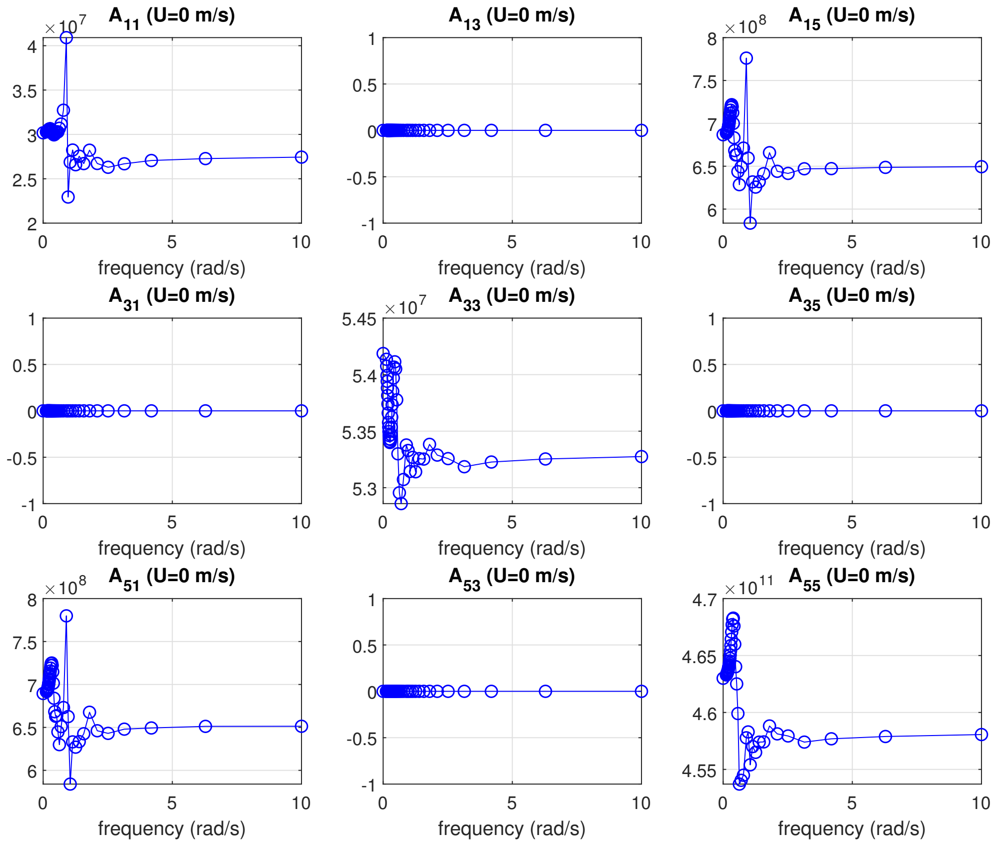


Figure 10: Frequency dependent added mass in surge, heave and pitch

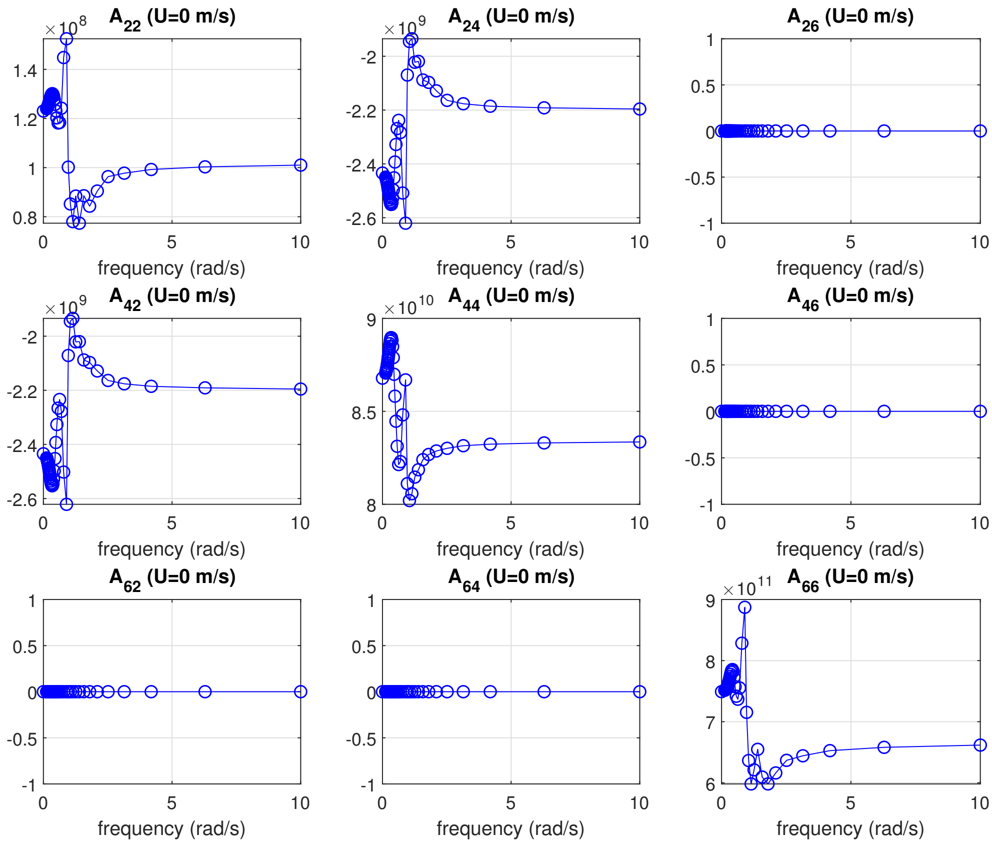


Figure 11: Frequency dependent added mass in sway, roll and yaw

As the mass matrix , $\boldsymbol{M}$, should be constant, the infinite frequency added mass is used. . The resulting mass matrix, $\boldsymbol{M}$ is calculated from the rigid body mass matrix, $\boldsymbol{M}_{RB}$ and the infinite frequency added mass matrix, $\boldsymbol{M}_A(\infty)$ as follows:

$$\boldsymbol{M} = \boldsymbol{M}_{RB} + \boldsymbol{M}_A(\infty) \tag{24}$$

2.1.4 Coriolis-Centripetal matrices

The force from the Coriolis-Centripetal matrices, are computed from the appurtenant mass matrices and the velocity vector. The added Coriolis-Centripetal matrix, uses the infinity frequency added mass matrix, and the body fixed relative velocity to the water. The seakeeping rigid body Coriolis-Centripetal matrix is found by using the rigid body mass matrix, and the body fixed velocity. The approach for this calculations is collected from Fossen (2011).

$$\begin{aligned}
\boldsymbol{M} &= \boldsymbol{M}^T = \begin{bmatrix} \boldsymbol{M}_{11} & \boldsymbol{M}_{12} \\ \boldsymbol{M}_{21} & \boldsymbol{M}_{22} \end{bmatrix} > 0 \\
\boldsymbol{\nu}_1 &= \begin{bmatrix} u \\ v \\ w \end{bmatrix} \\
\boldsymbol{\nu}_2 &= \begin{bmatrix} p \\ q \\ r \end{bmatrix}
\end{aligned} \tag{25}$$

By using this, and the fact that $\boldsymbol{S}$ is the cross product skew symmetric matrix, the Coriolis-Centripetal matrix is found as follows:

$$\boldsymbol{C}(\boldsymbol{\nu}) = \begin{bmatrix} \boldsymbol{0}_{3\times3} & -\boldsymbol{S}(\boldsymbol{M}_{11}\boldsymbol{\nu}_1 + \boldsymbol{M}_{12}\boldsymbol{\nu}_2) \\ -\boldsymbol{S}(\boldsymbol{M}_{11}\boldsymbol{\nu}_1 + \boldsymbol{M}_{12}\boldsymbol{\nu}_2) & -\boldsymbol{S}(\boldsymbol{M}_{21}\boldsymbol{\nu}_1 + \boldsymbol{M}_{22}\boldsymbol{\nu}_2) \end{bmatrix} \tag{26}$$

As these terms are functions of the relative velocity and the velocity, they need to be calculated online.

When performing open loop simulation exposed to wind, the velocities become quite high, and the forces from the Coriolis-Centripetal matrices was very big. Therefore a maneuvering-approach to model the matrices was used instead Fossen & Perez (n.d.). This is done by using the following equations:

$$\begin{aligned} \boldsymbol{C}_A &= u\boldsymbol{M}_A(\infty)\boldsymbol{L} \\ \boldsymbol{C}_{RB} &= u\boldsymbol{M}_{RB}\boldsymbol{L} \end{aligned} \tag{27}$$

Where u is the surge velocity, L is a selection matrix. $\boldsymbol{M}_{RB}$ and $\boldsymbol{M}_A(\infty)$ is respectively the rigid body mass matrix and the infinite frequency added mass matrix.

2.1.5 Damping Matrix

The damping term, $\boldsymbol{D}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r$, in Equation (21), consists of viscous damping and drag forces. The viscous damping term that is used in this model is the built-in term from "MSS Hydro 6 DOF DP Model" that uses B_V from the Matlab function: "Wamit2vessel". The drag model in "MSS Hydro 6 DOF DP Model" assumed a compact and solid hull. In other words, it could not be used for Havfarm 2. To calculate the drag forces, the following drag equation was used. Faltinsen (n.d.)

$$dF = \frac{\rho}{2} C_D D \; dz \; |u|u \tag{28}$$

Where dF is the force on a strip length of dz. ρ is the density of the water, Cd is the drag coefficient. This is assumed approximately 1 for cylinders, and 2 for squares. D is the dimensional length, in cylinders this is the diameter.

This was used on all submerged part of Havfarm 2. For simplicity, the geometry was divided in to columns, transverse pontoons and longitudinal pontoons, as shown in Figure 13a . The drag force in surge was computed by assuming the columns as circular cylinders, where the diameter is the total width of the column, the length as the total draft of the submerged body, thereafter the drag force was multiplied with the number of the columns. The transverse pontoons has a square geometry, the drag coefficient is therefore different, but the same approach was used. The projected area in surge can be seen in Figure 13b. The contributors to the sway drag force was the longitudinal pontoons and the columns. The columns was assumed to be a cylinder with diameter equal to the length of the columns. The force from the longitudinal pontoons used the same approach as mentioned above. The projected area in sway can be seen in Figure 12.

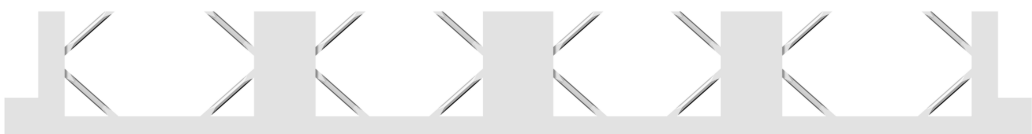
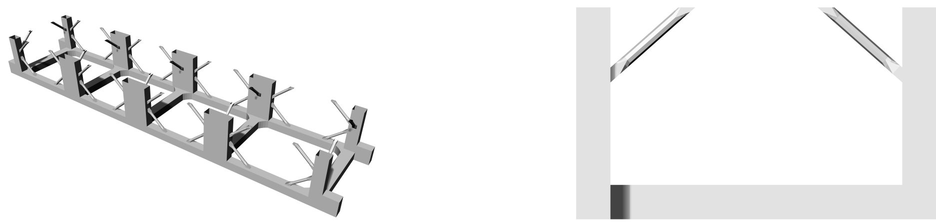


Figure 12: Projected area in sway



(a) Perspective

(b) Projected area i surge

Figure 13: Overview of components that result in drag forces

The approach used in the previous paragraph assumes that there are no reduced flow behind the objects. This is a very conservative assumption. In addition, the projected areas where both the transverse pontoons and the columns used, both contributions is added. This is also a conservative assumption. The reason for this conservative decisions are that the net for the fish cages is not included, neither a possible net for flotsam in the bow.

2.1.6 Fluid memory effect

In the modelling of Havfarm 2, the fluid memory effect, $\boldsymbol{\mu}$, is included. This is a dissipative force Fossen (2011). This term represents the fact that a change in fluid momentum due to the motion of the hull at a particular time instant affects the motion of the vessel at all subsequent times. In other words, the motions of the vessel will create waves on the free surface that, in principle, will persist at all subsequent time. These waves will affect the forces and respectively the motions of the vessel. The development of the fluid memory effects are in seakeeping coordinates, that are perturbations around the equilibrium position of the vessel.

At first, the hydrodynamic forces in an ideal fluid in seakeeping coordinates, are related to the frequency-dependent added mass (Figure 10 and 11) and potential damping. The frequency dependent damping matrix, is as the added mass matrix analyzed in Wamit, and found from the MSS function *wamit2vessel*. The result of the damping matrix can be seen in Figure 14 and 15.

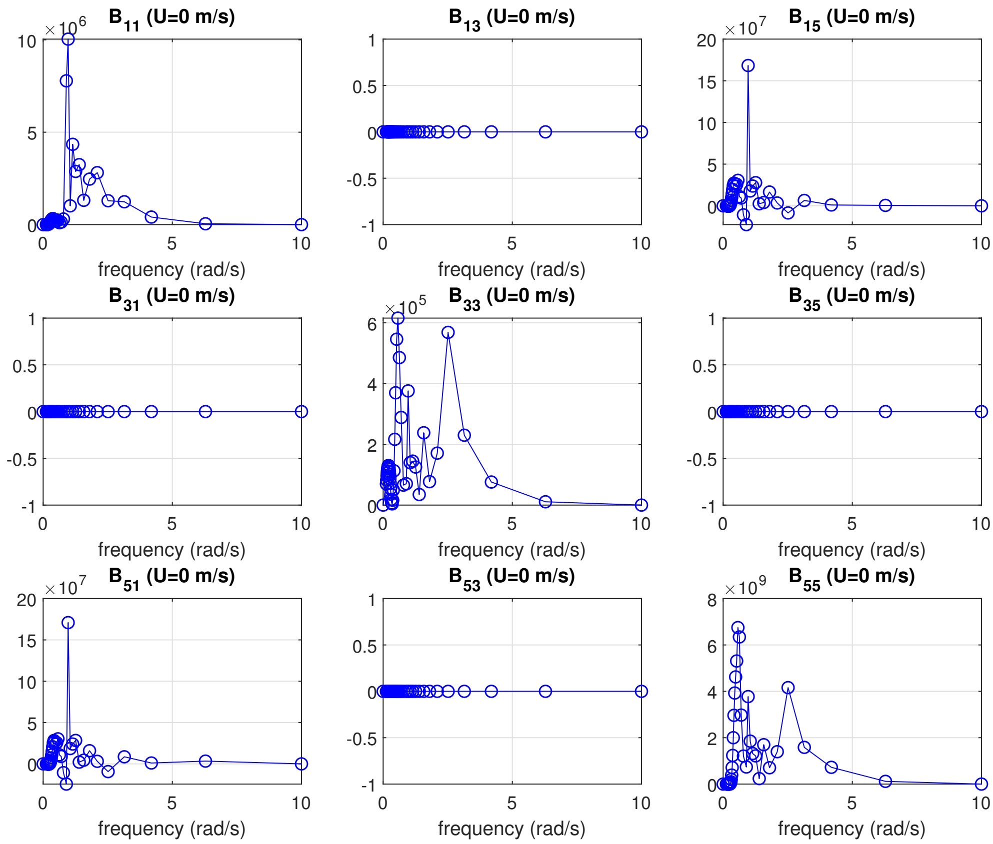


Figure 14: Frequency dependent potential damping in surge, heave and pitch

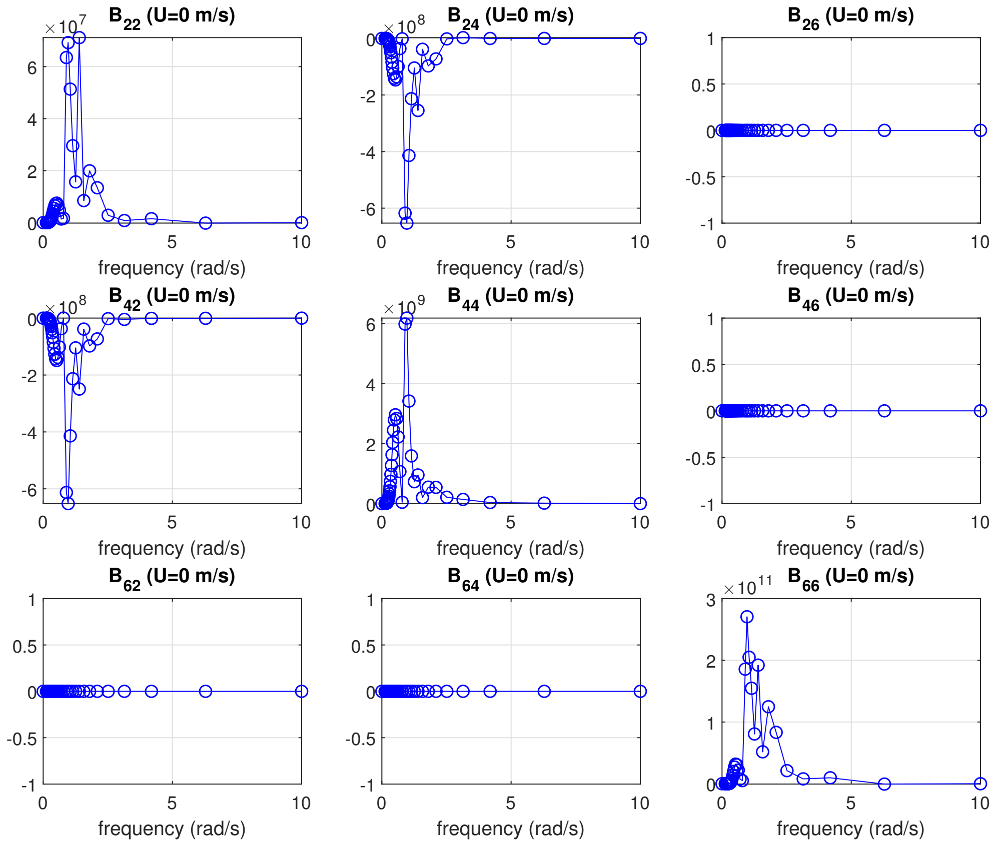


Figure 15: Frequency dependent potential damping in sway, roll and yaw

The hydrodynamic forces in seakeeping coordinates are calculated as follow :

$$\boldsymbol{\tau}_{hyd} = -\boldsymbol{A}(\infty)\ddot{\boldsymbol{\xi}} - \int_0^t \bar{\boldsymbol{K}}(t-\tau)\dot{\boldsymbol{\xi}}(\tau)d\tau \tag{29}$$

Where

$$\bar{\boldsymbol{K}}(t) = \frac{2}{\pi}\int_0^\infty \boldsymbol{B}(\omega)cos(\omega t)d\omega \tag{30}$$

is a matrix of retardation functions, and ξ is the seakeeping coordinate vector.

The convolution integral from Cummins Equation (29), is the fluid memory effects in time-domain and seakeeping coordinates. Perez & Fossen (2007). To find the time-domain body coordinate fluid memory effects, the following procedure is used.

$$\begin{aligned}\dot{\boldsymbol{\xi}} &= \delta\dot{\boldsymbol{\eta}} \\ &= \boldsymbol{J}_\Theta(\delta\boldsymbol{\eta})\delta\boldsymbol{\nu} \\ &\approx \boldsymbol{J}_\Theta(\delta\boldsymbol{\eta})(\boldsymbol{\nu} + U(\boldsymbol{L}\delta\boldsymbol{\eta} - \boldsymbol{e}_1)) \\ &\approx \boldsymbol{J}_\Theta(\delta\boldsymbol{\eta})(\boldsymbol{\nu} - U\boldsymbol{e}_1)\end{aligned} \tag{31}$$

Where
$\delta\dot{\boldsymbol{\eta}}$ is the perturbation coordinates
$\boldsymbol{J}_\Theta(\delta\boldsymbol{\eta})$ is the transformation matrix between body coordinates and seakeeping coordinates
$\boldsymbol{e}_1$ is a column vector of zeros, where only the first element are 1.

This result in the following equation for the fluid memory effects:

$$\boldsymbol{\mu} = \int_0^t \boldsymbol{K}(t-\tau)(\boldsymbol{\nu}(\tau) - U\boldsymbol{e}_1)d\tau \tag{32}$$

Where $\bar{\boldsymbol{K}}$ is replaced with $\boldsymbol{K}$ due to numerical oscillations at high frequencies from following equation:

$$\boldsymbol{K}(t) = \frac{2}{\pi}\int_0^\infty [\boldsymbol{B}_{total}(\omega) - \boldsymbol{B}_{total}(\infty)]cos(\omega t)d\omega \tag{33}$$

To implement this in the digital twin, several steps had to be made.

At first the Wamit analysis had to include enough periods that should be analyzed. The Wamit output data are read through the MSS Hydro function: *wamit2vessel*, and stored in a struct. To transform these frequency domain analyses to a state space model, the MSS FDI function: *vessel2ss* is used to do the mapping from $\delta\boldsymbol{\nu}$ to $\boldsymbol{\mu}$. This is done by a transfer function, $\boldsymbol{H}(s)$.

$$\boldsymbol{\mu} = \boldsymbol{H}(s)\delta\boldsymbol{\nu} \tag{34}$$

Where

$$\boldsymbol{H}(s) = \boldsymbol{C}_r(s\boldsymbol{I} - \boldsymbol{A}_r)^{-1}\boldsymbol{B}_r \tag{35}$$

Creates the state space model:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}_r\boldsymbol{x} + \boldsymbol{B}_r\delta\boldsymbol{\nu} \\ \boldsymbol{\mu} &= \boldsymbol{C}_r\boldsymbol{x} \end{aligned} \tag{36}$$

Where the states in $\boldsymbol{x}$, according to Fossen (2011), reflect the fact that once the marine craft changes the momentum of the fluid, this will affect the forces in the future. In other words, the radiation forces at a particular time depend on the history of the velocity of the marine craft up to the present time. The dimension of $\boldsymbol{x}$ and the matrices $\boldsymbol{A}_r$, $\boldsymbol{B}_r$ and $\boldsymbol{C}_r$ depend on the order of the identified transfer functions, which is manually chosen so that the frequency dependent added mass and damping from the Wamit analyses fit to the chosen curves.

During the development of the fluid memory effects, there was several problems. At first there was not enough freqencies in the wamit analyses, so that the low-frequency behaviour of the vessel was not mapped. Another problem was that the $\boldsymbol{A}_r$ matrices was unstable, which made the complete process plant unstable. Several approaches to solve the problem was done. According to tutorials for MSS, **?**, the eigenvalues of $\boldsymbol{A}_r$ should be calculated, and change the sign of the real part of the eigenvalues that was in the right part if the imaginary half plane, and thereafter reconstruct the matrix from the new denominator. There was plenty of scepticism for this approach, and even though the matrices became stable, the approach was not used. By adding several more frequencies in the wamit analysis and many trials and errors on the choosing of the order of the transfer function, finally all the 18 state space models for the fluid memory effects was stable.

2.1.7 Restoring forces

The restoring forces for surface vessels will depend on metacentric height, the location of CG and Center of buoyancy (CB) and the shape and size of the water plane area. This forces works as a spring in the system. The restoring force, $\boldsymbol{g}(\boldsymbol{\eta})$, is the nonlinear restoring forces from the vessel. This have a linear approximation that works well for small roll and pitch angles, small heave motions, on box shaped vessels. By using these assumptions the following approximation can be done for Havfarm 2.

$$\boldsymbol{g}(\boldsymbol{\eta}) = \boldsymbol{G}\boldsymbol{\eta} \tag{37}$$

Where $\boldsymbol{G}$ is the restoring force matrix. In this project, the restoring force matrix is obtained from the Wamit analyzes, and the MSS Hydro function: *wamit2vessel*.

2.1.8 Propulsion

The propulsion force in Equation (21) is the force from the thrusters. This is modelled in section 2.2.9, thrust allocation and thruster dynamics.

2.1.9 Wave forces

$\boldsymbol{\tau}_{wave}$ in Equation (21) is the forces from the waves on the plant. To model the waves, the International Towing Tank Conference (ITTC)-spectrum is used. This is an approach to determine the constants, A and B, in a Bretschneider spectrum, which is a Pierson-Moskowitz spectrum ITTC (2002):

$$S(f) = \frac{A}{f^5} exp\left(\frac{-B}{f^4}\right) \tag{38}$$

Where S is the spectral density, and f is the frequency. A Simulink block, *Waves*, from the MSS GNC library utilizes the significant wave height, H_s, the peak frequency, ω_0, and the mean direction to create the wave spectrum and the individual waves.

The wave forces are divided into Wave frequency (WF) forces and WD forces. The WF forces are zero mean oscillating forces from first order wave theory. The WD forces are second order waves, which are not zero mean. These are divided in to sum/difference frequencies and a mean WD force, which is accounted for in this project thesis. For each of the two wave cases and for each, DOF, wave direction, β, and wave frequency, ω, a normalized force RAO is used as follows Fossen (2011):

$$F^l_{wave1}(\omega_k, \beta_i) = \left|\frac{\tilde{\tau}^l_{wave1}(\omega_k, \beta_i)}{\rho g A_k}\right| e^{j\angle \tilde{\tau}^l_{wave1}(\omega_k, \beta_i)} \tag{39}$$

$$F^l_{wave2}(\omega_k, \beta_i) = \left|\frac{\tilde{\tau}^l_{wave2}(\omega_k, \beta_i)}{\rho g A_k^2}\right| e^{j\angle \tilde{\tau}^l_{wave2}(\omega_k, \beta_i)} \tag{40}$$

Where Equation (39) is the force RAO for WF, and Equation (40) is the force RAO for WD. In the two equations, l, k and i are respectively the DOFs, frequencies, and directions. j is the imaginary number and A_k is the wave amplitude. To calculate the amplitude and the phase of the force RAOs, imaginary part and the real part is defined as respectively Im and Re. The calculation for first order is given in Equation (41) and for second order in Equation (42)

$$\begin{aligned} \left|F^l_{wave1}(\omega_k, \beta_i)\right| &= \sqrt{Im_{wave1}\{l\}(k,i)^2 + Re_{wave1}\{l\}(k,i)^2} \\ \angle F^l_{wave1}(\omega_k, \beta_i) &= atan2(Im_{wave1}\{l\}(k,i), Re_{wave1}\{l\}(k,i)) \end{aligned} \tag{41}$$

$$\begin{aligned} \left|F^l_{wave2}(\omega_k, \beta_i)\right| &= Re_{wave2}\{l\}(k,i) \\ \angle F^l_{wave1}(\omega_k, \beta_i) &= 0 \end{aligned} \tag{42}$$

The MSS Hydro function *wamit2vessel*, calculates the phases and amplitudes for the wave frequency, these are shown in Figure 16 to 18, and the amplitudes of of the RAO in wave drift, these are shown in Figure 19.

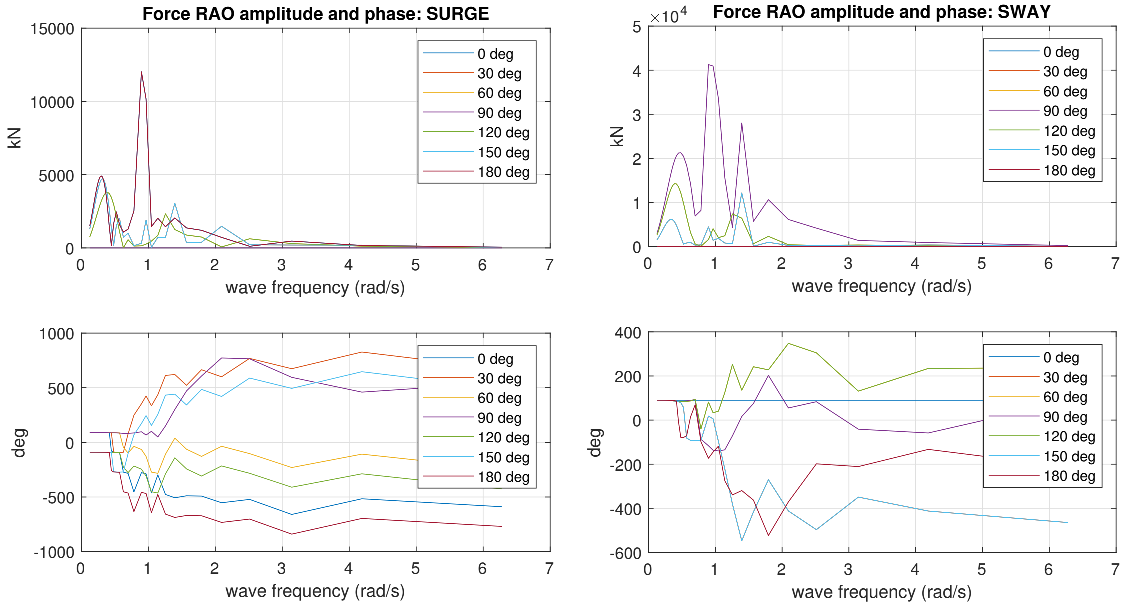


Figure 16: Force RAOs

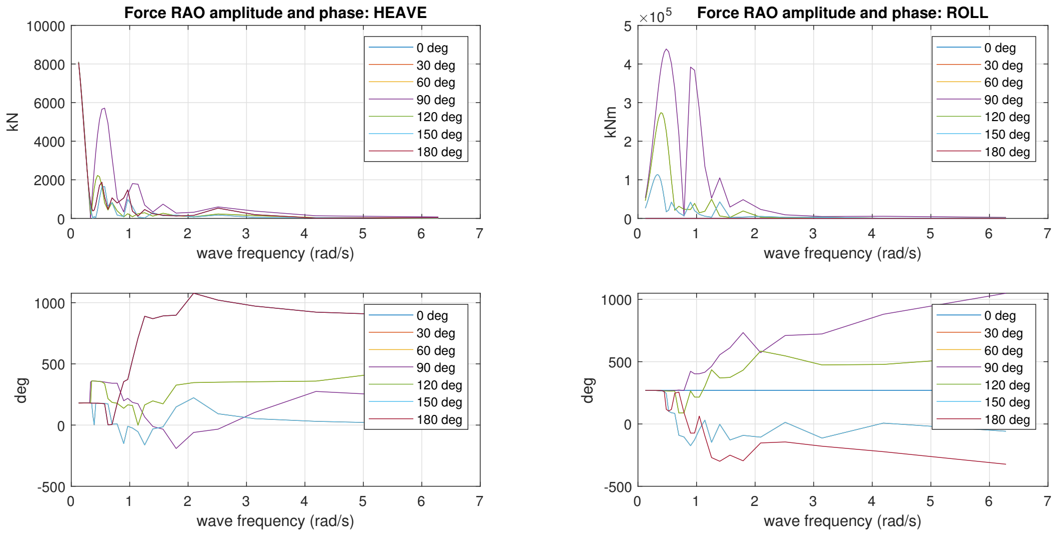


Figure 17: Force RAOs

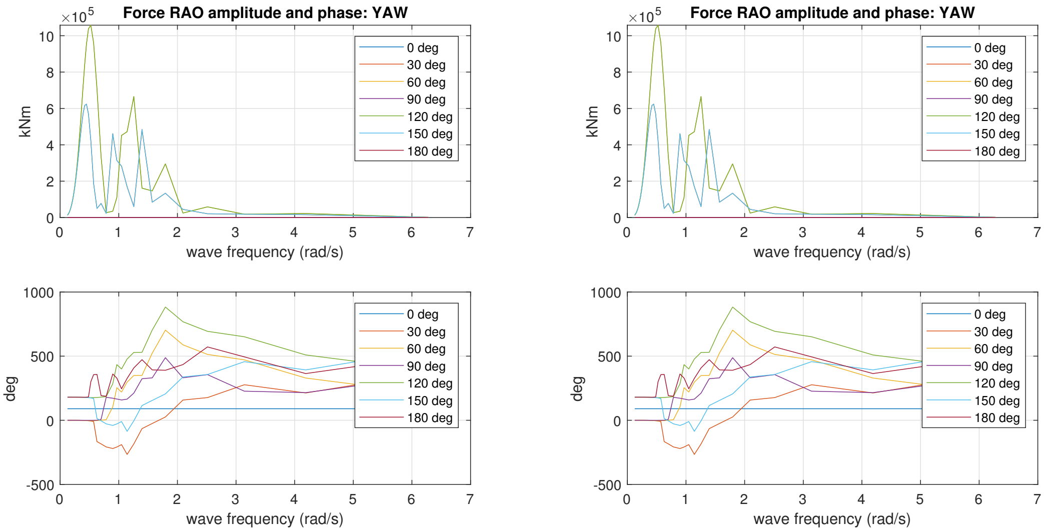


Figure 18: Force RAOs

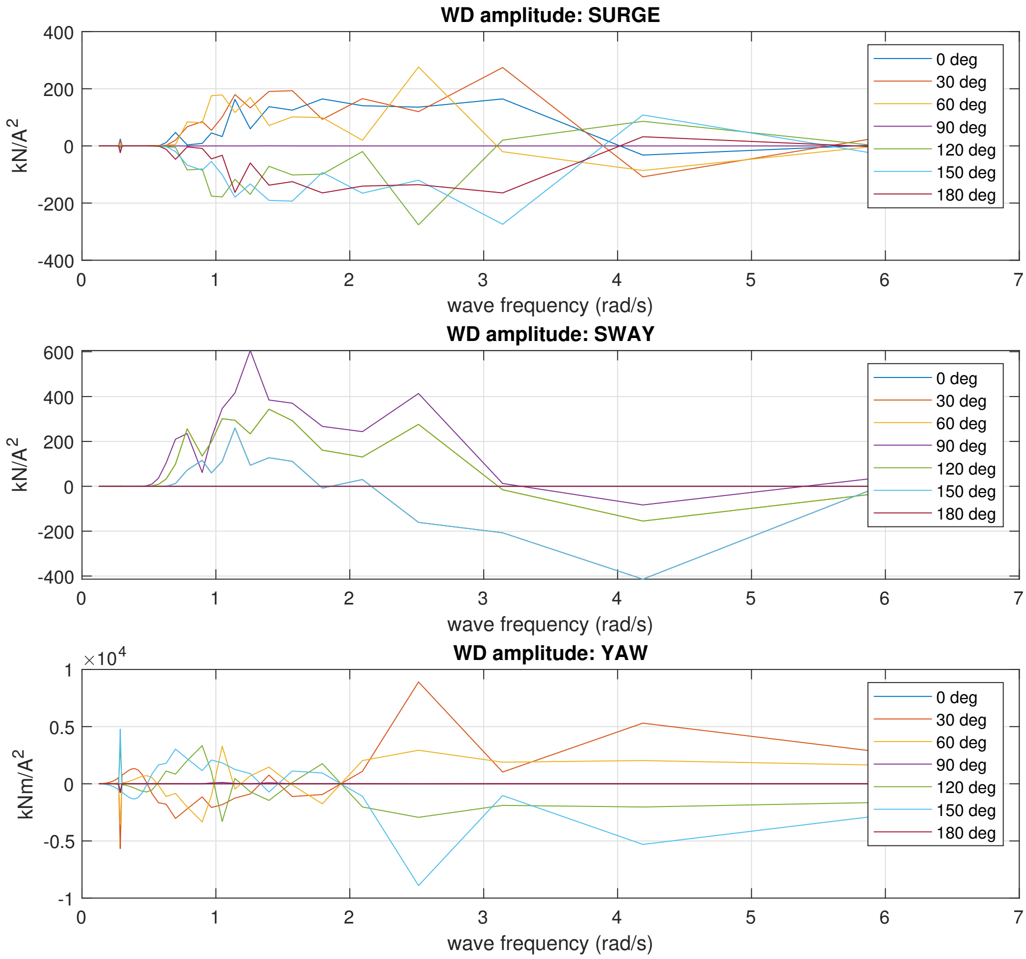


Figure 19: WD force RAOs

These are being used to find the forces on the vessel from the waves. In the Wamit analysis, there was done analysis each ten degree. To compensate for this, there are used a interpolation function between two directions for each wave.

In the implementing of the wave forces, there was some difficulties. The wave forces and therefore the vessel in the open loop analysis had some strange behaviour. When applying waves with mean direction north, the vessel moved uncontrolled south. By inspecting the wave drift amplitudes, the amplitudes between 180 degrees and 360 degrees had wild points and incorrect signs in some cases. This was fixed by mirroring the Wave drift amplitude in surge with respect to the x,z-axis, as the amplitudes for direction 0-180 degrees looked fine.

2.1.10 Wind forces

Wind is added to this digital twin. The mean wind velocity ten meters above the sea surface and the mean wind direction is specified. This is used by the MSS Simulink *Wind*-block to develop a Harris wind spectrum. To calculate the wind forces on the vessel, the time varying wind in the NED-frame is rotated to the body frame, and the MSS Matlab function *blendermann94* is used. This function returns the wind force/moment vector in 3 DOF, $\boldsymbol{\tau}_{wind}$, and the optionally wind coefficients for merchant ships using the formulas of Isherwood (1972). In this project the *blendermann* function is used as follows:

$$\boldsymbol{\tau}_w = blendermann94(\gamma_{rw}, V_{rw}, AF_w, AL_w, sH, sL, Loa, vessel_{no}) \tag{43}$$

Where
$\boldsymbol{\tau}_{wind}$ is the 3 DOF vector of wind forces
γ_{rw} is the relative angle of attack for the wind, defined as $\gamma_{rw} = -atan2(v_{rw}, u_{rw})$
V_{rw} is the relative wind speed
AF_w is the frontal projected area of the vessel, over the water surface
AL_w is the lateral projected area of the vessel, over the water surface
sH is the horizontal distance to centroid of AL_w
sL is the vertical distance to centroid of AL_w
Loa is the overall length of the vessel
$vessel_{no}$ is which database-vessel Havfarm 2 should be compared to. By comparing the wind forces from Havfarm 1, to the forces from the different vessels, and regarding that the superstructure of Havfarm 2 should be in the center of the vessel, $vessel_{no}$ is chosen to be 8, which is a ferry. The rest of the data is estimated, or collected from NSKs internal documents.

2.1.11 Anchoring

The anchor model that is developed for this digital twin is based on one mooring line, and a gravity anchor or a drag anchor. As the final anchor system is not yet decided, a simplified model is used. The approach for the mooring characteristics is found in Faltinsen (n.d.), and is as follows:

$$a = \frac{T_H}{w} \tag{44}$$

$$D = l - h\left(1 + 2\frac{a}{h}\right)^{\frac{1}{2}} + a\ cosh^{-1}\left(1 + \frac{h}{a}\right) \tag{45}$$

Where T_H is a vector containing the horizontal component of the tension at the waterplane.
w is the weight per unit length of line in water.
D is a vector containing the horizontal distance from the anchor point.
l is the total length of the mooring line (Including the part that lies on the bottom)
h is the depth, assuming constant depth.

Table 3: Input used to develop anchor characteristics.

Unit	**Value**
Total length	$950m$
Length of line on bottom	$100m$
Total mass of line	$150000m$
Depth	$600m$

Using the values if Table 3, the anchor characteristics is as shown in Figure 20.

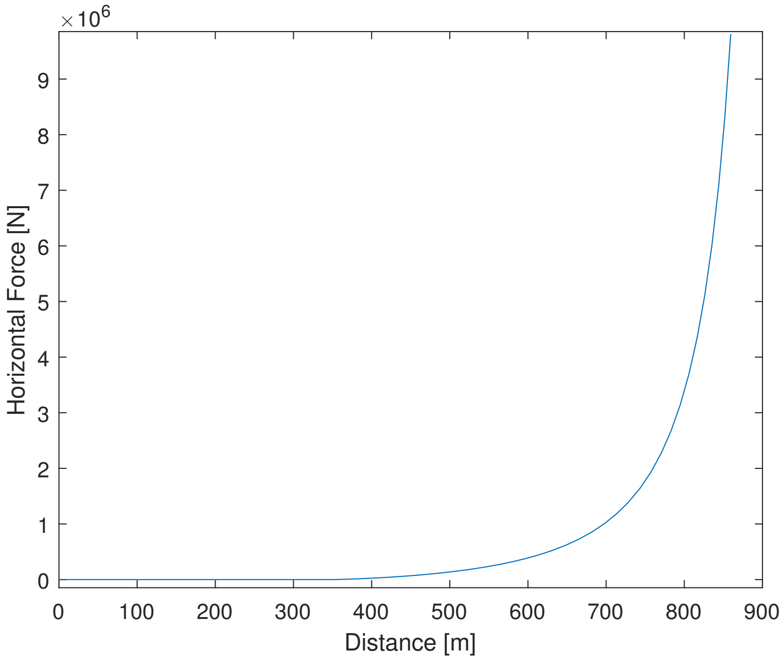


Figure 20: Anchor characteristics

In the simulation model, the estimated north and east position from the observer is used to calculate the horizontal distance from anchor point, D, by the sum of squares. This is used in a lookup table to find the horizontal force on the vessel, T_H from the anchor system. Based on the north and east position, compared to the anchor point, and the heading, ψ, the resulting forces on the vessel is found according to Equations (46), (47), (48) and (49). Note that this is a simplified model. In this model, the distance from the anchor point, D, is calculated from the center of the vessel. To correct this, the distance should have been calculated from the front of the vessel where the mooing line shall be mounted. The real distance could have been calculated by the use of the heading. Knowing that the anchor and mooring line are still very early in the design phase, the model was left as follows for simplicity.

$$T_N = \frac{-N\ T_H(D)}{D} \tag{46}$$

$$T_E = \frac{-E\ T_H(D)}{D} \tag{47}$$

$$\begin{bmatrix} T_X \\ T_Y \\ 0 \end{bmatrix} = R_b^n(\psi)^T \begin{bmatrix} T_N \\ T_E \\ 0 \end{bmatrix} \tag{48}$$

$$T_\psi = T_Y\ \frac{L}{2} \tag{49}$$

2.1.12 Current

The way that the current is implemented in the digital twin, and in Equation (21) is by using the relative velocity in body frame for the hydrodynamic terms. The relative velocity is shown in Equation (17). To develop the current velocity, $\boldsymbol{\nu}_c$, a 2D model of the current is used. The direction of the current and the speed is initialized in NED-frame, and rotated online to the body frame to be able to use Equation (17).

2.1.13 Water quality

There will probably be many sensors on Havfarm 2 in order to monitor the water quality. This may be sensors measuring the relative velocity, and sensors measuring the oxygen saturation. It is not common to measure the relative water velocity on DP-vessels, when the velocities is as small as is it here. One of the reason for this is that the thrusters disturb the measurement with wakes and currents. At Havfarm 2 it would likely be possible to use these sensors because the thrusters are so deep in the water. Traditionally it is difficult to measure somewhere it is not disturbed by the hull, but because of the unconventional hull shape of Havfarm 2 this should be possible. By measuring the oxygen saturation at different locations on the vessel, the minimum water quality can be calculated. By using this, we must always keep the lowest quality over a given value.

The given quality value, probably given in oxygen saturation and/or relative water velocity, is not focused in this master thesis. This is the reason that it is modelled in a simplified manner. As the model is used in a control plant model in the optimization, it is important that it is correct in real life. It is probably difficult to model this good enough offline, it must probably be used an adaptive model that can change both first and second order flow/quality reduction.

A model of the water quality to the fish is a compound of the relative current inside the fish cages and the general water quality based on faeces and oxygen saturation. The modelling is done by using the current relative to the vessel, $\boldsymbol{\nu}_r$, from the digital twin. As the current is assumed two dimensional, the first value in the vector, $\boldsymbol{\nu}_r$, is parallel to the x-axis of the vessel, and the second value is is the relative current parallel to the y-axis of the vessel. In this model, there are three factors that reduce the local relative current velocity or the quality of it.

- The flow reduction from the nets. When the water flow is transported through the nets, there are drag, and the relative velocity of the water will decrease. This will happen for both the fish cage nets and for eventually protection nets in the front. If the heading of the vessel is assumed towards the current, the relative current in the cages is assumed reduced by a factor 0.86 for each of the net walls. This comes from Equation (50), (51) and (52), which is collected from respectively formula 199, 211 and 208 in Løland (1991) with solidity ratio of 0.21 which is the assumed solidity ratio of a net wall in Havfarm 2.

$$C_d(Sn) = 0.04 + (-0.04 + 0.33Sn + 6.54Sn^2 - 4.88Sn^3)cos(\alpha) \tag{50}$$

$$r = 1 - 0.46C_d \tag{51}$$

$$U_i = U_\infty r^i \tag{52}$$

- There will also be reduction in the flow velocity because of the columns. Especially if the relative current velocity is not orthogonal to the x-axis or the y-axis of the vessel. As this is a bit harder to model, and are different for each cage, it is preliminary only modelled as a general reduction factor of 0.8.

- The quality of the water or maybe more exact, the oxygen saturation is important for the fish. As water (coming from the first cage) reach the hindmost cage, the fish in the previous cages has utilized much of the oxygen in the water. This is amongst other factors depending on numbers of fish in the cages, size of the fish, temperature, feeding cycle, and of course the velocity of the water through the cages. The most correct approach to model this would be to use a higher order of quality/flow reduction. The reason for this is that in addition to slow relative water brings slower quality water to the fish, the slower the water flows through the previous case, the more oxygen is being utilized and more faeces is added to the water. It should be possible to make an explicit formula based many factors, but as this value is assumed to be given as a measurement of the oxygen saturation, it is modelled in a more simplified manner. The water that flows along the vessel is assumed an additional reduction factor of 0.95 for the flow for each cage the flow comes through.

The result from this model is given in Equation (53)

$$\begin{aligned} X_{Quality} &= |u_{rel}|\ r_{net}^7\ r_{protect}\ r_{columns}\ r_{quality}^3 \\ Y_{Quality} &= |v_{rel}|\ r_{net}\ r_{columns} \end{aligned} \tag{53}$$

The $X_{Quality}$ and the $Y_{Quality}$ are later summed to be only one water quality variable.

$$Q = X_{Quality} + Y_{Quality} \tag{54}$$

As the relative water velocities in Equation (53) can oscillating with a high frequency, the resulting "real" quality, Q, will also be oscillating. To prevent too much oscillation on this signal a first order low pass filter with a time constant of ten seconds is added in the model. This is shown in the equation below:

$$\frac{Q}{Q_{earlier}} = \frac{1}{10s + 1} \tag{55}$$

2.1.14 Measurements

As the output from the process plant model is the actual position, $\boldsymbol{\eta}$, and the relative water velocity $\boldsymbol{\nu}_r$ it would be incorrect to assume that the measurements of the position and the relative water velocity is perfect. To compensate for the imperfect measurements, measurement noise is added. This is done by adding band limited white noise of different noise power to the different states separately. The variance of the different signals could also be measured to later be used in a state estimator, which uses the variance, which is not the case in this master thesis.

The measurements used in this thesis is the position in 3 DOF, $\boldsymbol{\eta}$, velocity relative to the water, $\boldsymbol{\nu}_r$, and the water quality, $\boldsymbol{Q}$. These measurements are quite realistic, as the position is measured by amongst other a GPS and a gyroscope, the relative velocity can be measured multiple spots by example a Doppler velocity logger, and merged to estimate the real relative velocity. The water quality or the dissolved oxygen saturation can be measured by a dissolved oxygen meter in different locations, and estimate the minimum quality.

2.2 Control Design

2.2.1 Observer / State Estimator

An observer or a state estimator is an important tool. This have multiple tasks. One of them is to distinguish the high frequency wave motions and the low frequency motions of the vessel. It is preferable to only use the low frequency motions in the control system. This is to prevent the thrusters from trying to compensate for the high frequency wave motions. Another job for the state estimator is to reconstruct unmeasured states from the measured states and a model. The observer is also a tool to estimate the real states based on the measurement for the last time step. This is by applying the control input to the dynamics of the system. In this way the observer predict the states of the system before the measurements are available.

In this model, the measurements is the two dimensional velocity relative to the water, the north and east position, the heading and the water quality. These measurements, the control input and the control plant model are used to estimate the position and heading, velocities, current and the water quality.

The observer was implemented by combining a known nonlinear Passive observer from MSS and develop observer for the parts that is not accounted for in the MSS nonlinear passive observer. The result is a Luenberger-like observer. The nonlinear passive observer is collected directly from the MSS GNC library, and tuned partly according to Fossen (2011). The observer is also modified to account for the velocity relative to the water and the water quality for the fish. The observer model is given in Equation (56)

$$
\begin{aligned}
\dot{\boldsymbol{\xi}} &= \boldsymbol{A}_w \boldsymbol{\xi} \\
\dot{\boldsymbol{\eta}} &= \boldsymbol{R}(\psi)\boldsymbol{\nu} \\
\dot{\boldsymbol{b}} &= -\boldsymbol{T}_b^{-1}\boldsymbol{b} \\
\boldsymbol{M}\dot{\boldsymbol{\nu}} &= -\boldsymbol{D}\boldsymbol{\nu}_r + \boldsymbol{R}^T(\psi)\boldsymbol{b} + \boldsymbol{\tau} \\
\dot{\boldsymbol{\nu}}_c &= -\boldsymbol{T}_c^{-1}\boldsymbol{\nu}_c \\
\dot{Q} &= K_7\left(Q - \boldsymbol{r}\boldsymbol{\nu}_r\right) \\
\boldsymbol{y}_1 &= \boldsymbol{\eta} + \boldsymbol{C}_w \boldsymbol{\xi} \\
\boldsymbol{y}_2 &= \boldsymbol{\nu}_r \\
\boldsymbol{y}_3 &= Q
\end{aligned}
\tag{56}
$$

Where $\boldsymbol{\xi}$ is the 3 DOF wave state vector, which is high frequency motions and shall be distinguished from the low frequency vessel motions. $\boldsymbol{A}_w$ and $\boldsymbol{C}_w$ are design matrices describing the sea state. $\boldsymbol{T}_b$ is the design matrix for bias time constants, describing the slowly dynamics of the bias, $\boldsymbol{b}$, which accounts for the non-modelled vessel dynamics. This could be non-modelled geometry, high order motions, wind forces and wave drift motions. The bias is modelled in the NED frame because the forces from wind + waves changes less in NED than in body frame. $\boldsymbol{\nu}_c$ is the 2 dimensional NED frame current and $\boldsymbol{T}_c$ is the design matrix for the current time constants. $\boldsymbol{Q}$ is the quality of the water, modelled and based on the vessels velocity relative to the water.

The development of the observer is done by using the following observer equations:

$$\begin{aligned}
\dot{\hat{\boldsymbol{\xi}}} &= \boldsymbol{A}_w\hat{\boldsymbol{\xi}} + \boldsymbol{K}_1(\boldsymbol{\omega_o})\tilde{\boldsymbol{y}}_1 \\
\dot{\hat{\boldsymbol{\eta}}} &= \boldsymbol{R}(\psi)\hat{\boldsymbol{\nu}} + \boldsymbol{K}_2\tilde{\boldsymbol{y}}_1 \\
\dot{\hat{\boldsymbol{b}}} &= -\boldsymbol{T}^{-1}\hat{\boldsymbol{b}} + \boldsymbol{K}_3\tilde{\boldsymbol{y}}_1 \\
\boldsymbol{M}\dot{\hat{\boldsymbol{\nu}}} &= -\boldsymbol{D}\hat{\boldsymbol{\nu}}_r + \boldsymbol{R}^T(\psi)\hat{\boldsymbol{b}} + \boldsymbol{\tau} + \boldsymbol{R}^T(\psi)\boldsymbol{K}_4\tilde{\boldsymbol{y}}_1 \\
\dot{\hat{\boldsymbol{\nu}}}_c &= -\boldsymbol{T}_c^{-1}\hat{\boldsymbol{\nu}}_c + \boldsymbol{R}(\psi)\boldsymbol{K}_5\tilde{\boldsymbol{y}}_2 \\
\dot{\hat{Q}} &= K_7\left(\hat{Q} - \boldsymbol{r}\boldsymbol{\nu}_r\right) + K_6\tilde{\boldsymbol{y}}_3 \\
\hat{\boldsymbol{y}}_1 &= \hat{\boldsymbol{\eta}} + \boldsymbol{C}_w\hat{\boldsymbol{\xi}} \\
\hat{\boldsymbol{y}}_2 &= \hat{\boldsymbol{\nu}_r} \\
\hat{\boldsymbol{y}}_3 &= \hat{Q}
\end{aligned} \tag{57}$$

Where the hat is the estimated state, the tilde indicates the error i.e. $\tilde{y} = y_{measured} - \hat{y}$. The derivative of the sea state, $\dot{\hat{\boldsymbol{\xi}}}$, is based on the sea state, and corrected for the measurement of the position. The derivative of the NED frame position, $\dot{\hat{\boldsymbol{\eta}}}$, is based on the rotated estimated body frame velocity, and corrected by the position measurement. The estimated bias propagation is based on the previous bias and this is also corrected by the position measurement. The dynamics of the vessel is used to calculate the estimated body frame acceleration,$\dot{\hat{\boldsymbol{\nu}}}$, in addition to correction based on the position measurements. The derivative of the current is based on the estimated current, and the measurement of the velocity relative to the water. $\tilde{\boldsymbol{y}}_2 = \boldsymbol{\nu}_{r\ measurement} - \hat{\boldsymbol{\nu}}_r$ is the derived error in body frame relative velocity, which is rotated to NED frame to influence the propagation of the current velocity. The propagation of the quality is based both on the water quality model, the estimated quality and the measurements. The $\boldsymbol{K}_{1,2,\ldots,7}$ matrices are respectively the Wave estimator, position estimator, bias, velocity, current, quality model and quality measurement observer gains. The tuning of the gain matrices is done partly according to Fossen (2011), using the following procedure:

$$\begin{aligned}
K_{1i}(\omega_{0i}) &= -2(\zeta_{ni} - \lambda_i)\frac{\omega_{ci}}{\omega_{0i}} \\
K_{1(i+3)}(\omega_{0i}) &= 2\omega_{0i}(\zeta_{ni} - \lambda_i) \\
K_{2i} &= \omega_{ci} \\
\boldsymbol{K}_3 &= 0.1\boldsymbol{K}_4 \\
\boldsymbol{K}_4 &= \boldsymbol{M} \cdot diag\{0.1, 0.1, 0.01\} \\
\boldsymbol{K}_5 &= diag\{0.5, 0.5\} \\
K_6 &= 1 \\
K_7 &= 0.5
\end{aligned} \tag{58}$$

Where
ζ_{0i} is the relative damping ratio in each DOF
λ_i, is the relative damping ratio of the wave spectrum
ω_{0i} is the natural frequency in each DOF
ω_{ci} is the cut off frequency in each DOF
$\boldsymbol{M}$ is the 3×3 mass matrix

2.2.2 Optimization/Controller

The main objective in this thesis is to ensure enough water flow to the salmon in operational conditions, while minimizing the fuel consumption. This is done in different approaches. One is by using MPC like optimization. This MPC like optimization, which is from now referred to as only MPC, differs from the normal MPC optimization described in section 1.2.5. The difference is that in this thesis not only the control input from the MPC optimization is used, but also the optimal position and velocities as references for the controller to follow. There are more differences. The MPC optimization in section 1.2.5 runs the optimization every time step (In this thesis, 0.1 seconds) and apply only the first control input or reference. As the optimisation solver requires much computational power, the optimization cannot be executed every time step in this thesis. Instead, the reference/control action are obtained from the first five seconds from the MPC. In addition, interpolation is used to calculate the missing values due to uncalculated MPC. This is acceptable since the Havfarm dynamics is much slower than 0.1 second. When applying this optimization in the real life, it would be advantageous to run the optimization for each time step of the positioning system.

The optimization/Controller in this thesis is done using four different control alternatives.

- **Alt. 1** - A traditional PID controller is tuned to control the heading of the vessel based on the water quality. This approach uses only the thrusters in one of the aft corners of the vessel.
- **Alt. 2** - Using the 1 DOF (yaw) MPC optimizer reference as dynamic programming. And use the last reference as a steady state heading. A 1 DOF PID controller is used to calculate the desired thrust.
- **Alt. 3** - Using a 3 DOF (surge, sway and yaw) nonlinear model predictive control to find optimal control input for each time step.
- **Alt. 4** - Using a 3 DOF (surge, sway and yaw) nonlinear model predictive control to find optimal position and heading for each time step. A PID controller is used to follow this reference.

2.2.3 MPC Optimization

Common for Alt. 3 and 4 is that the MPC-optimizer use the online values from the state estimator. This is the position/heading, velocities and the current. In addition, the vessel parameters, e.g. mass, damping, etc. are used. The Model Predictive Control Optimizer runs first when initializing the simulation. Thereafter the program runs every five seconds. Every time the optimizer runs, a complete plan for the vessels position, velocity and control input is developed for every 2 second in the next 200 seconds. This is respectively the time if the control intervals, dt, and the time of the MPC horizon, T, of the optimization.

The 3 DOF nonlinear MPC optimization problem is set up as follows: At first all the variables in the MPC are declared. This is the states,$x = [N\ E\ \psi\ u\ v\ r]^T$, the control input, $\tau = [X\ Y\ N]^T$, the quality, Q, and a slack variable, s.

The objective of the MPC function is to minimize the fuel consumption. The mathematical objective is given below.

$$\text{Minimize} \quad \int_0^T a\ X(t)^2 + b\ Y(t)^2 + c\ \left(\frac{2}{L}N(t)\right)^2 + d\ s(t)\ dt \tag{59}$$

Table 4: Weighting factors for MPC function

Weighting factor	**Value**
a	$1e-2$
b	$1e-2$
c	$1e-3$
d	$1e3$

Where the letters, a, b, c, d, are the weighting factors. These are tuned to get the most satisfying results, and the values are given in Table 4. The slack variable, s, is a variable that is included to make the optimization problem more robust. The first revision of the MPC problem used a water quality constraint that was brought into action after a given time. This was because the problem could be infeasible because the system could not provide sufficient water quality in a short time. When the water quality constraint was brought into action after a given time, the system got much oscillation and would be unstable. This instability was fixed by introducing the slack variable, s, defined as follows.

$$\begin{aligned} -\infty \leq Q_{min} - Q - s \leq 0 \\ s \geq 0 \end{aligned} \tag{60}$$

Equation (60) shows that if the water quality not can be sufficient for each time step, the program can still fulfil the task. The slack variable in Equation (59) are weighed heavily, which makes the solver wanting the water quality constraint to be fulfilled.

The next step in the MPC function is to set boundaries on the variables. There are different boundaries on the different types of variable. All the boundary conditions are applied as constraints.

- The first value in every state, x, must be equal to the states of the vessel, i.e. the online values that comes from the state estimator.
- Lower and upper boundary on the position of the vessel. These are defined so that the vessel shall not transport too large distances.
- To make the simulations comparable the heading is forced to be between -180 and 180 degrees. This is to prevent that the solver seeks solutions that including rotate the vessel.
- There are also applied lower and upper boundaries on the rotation velocity, r
- Boundary conditions are also applied to the control input. These are based on an early capability plot. This prevents the forces to be larger than the capability of the vessel.
- There are applied a limitation for rapid change in yaw moment. This is to prevent oscillations, and is implemented by defining the maximum change per time step of $\pm 5[MNm]$

The propagation of the vessel are also applied as constraints. This done by using Runge Kutta 4 discrete integration, and is shown in Equation (61). Zlatev et al. (2014)

$$\begin{aligned} k1 &= f(\boldsymbol{x}(k), \boldsymbol{\tau}, \boldsymbol{\nu_c}, \boldsymbol{M}, \boldsymbol{D}) \\ k2 &= f(\boldsymbol{x}(k) + \frac{dt}{2}k1, \boldsymbol{\tau}, \boldsymbol{\nu_c}, \boldsymbol{M}, \boldsymbol{D}) \\ k3 &= f(\boldsymbol{x}(k) + \frac{dt}{2}k2, \boldsymbol{\tau}, \boldsymbol{\nu_c}, \boldsymbol{M}, \boldsymbol{D}) \\ k4 &= f(\boldsymbol{x}(k) + dt \cdot k3, \boldsymbol{\tau}, \boldsymbol{\nu_c}, \boldsymbol{M}, \boldsymbol{D}) \\ \boldsymbol{x}(k+1) &= \boldsymbol{x}(k) + \frac{dt}{6}(k1 + 2 \cdot k2 + 2 \cdot k3 + k4) \end{aligned} \tag{61}$$

Where f is the derivative function of the states, $\dot{x}$. These are calculated based on the control plant model, shown in Equation (62).

$$\begin{aligned} \dot{\boldsymbol{\eta}} &= \boldsymbol{R}(\boldsymbol{\eta})\boldsymbol{\nu} \\ \dot{\boldsymbol{\nu}} &= \boldsymbol{M}^{-1}\left(-\boldsymbol{D}\boldsymbol{\nu}_r + \boldsymbol{\tau} + \boldsymbol{\tau}_{mooring}(\boldsymbol{\eta})\right) \\ f(x, ...) &= \dot{x} = [\dot{\boldsymbol{\eta}}, \dot{\boldsymbol{\nu}}]^T \end{aligned} \tag{62}$$

To make the solver run faster and to initialize the solver close to the global minimum, initial values are provided for each time step for the states and the control inputs. This is done by propose to aim for a heading of 90 degrees, and no transnational movement. This approach works only for one current condition/direction. To generalize the initial values a reference frame in line with the current could be made to use this as a reference for the heading.

The output from the MPC optimization is the vectors for each state and control input containing values for each control interval in the MPC time horizon. Processing of the output data is done by creating a zero order hold function for each time step in Simulink. This is done by linear interpolation between each control interval for each time step. Every time the optimization is being executed, the processing of the output data resets.

While developing this nonlinear model predictive control optimization, many obstacles was faced. Some of them had very simply ways to work around, and some required deeper investigation. Examples of this is given.

- The control input that should be used in MPC optimization had to be scaled to prevent the solver from operating with large numbers, and to prevent large numbers being multiplied with small numbers.
- $\tau_{mooring}$ in Equation (62) should be calculated online. In Simulink and Matlab generally, an interpolation function is used to linearly interpolate between point in the mooring/anchor characteristics. When using CasADi Opti Stack in Matlab, the interpolation functions is not supported. At first, a power curve fitting function was used to imitate the mooring characteristics. This approach resulted in too large numbers being multiplied with too small numbers. The final approach was to manually extrapolate the mooring characteristics for distances between 0 and 350 meters to give a polynomial curve fitting function enough points at low values. The degree of the final polynomial curve fitting approach was eight.
- Small changes in the objective function resulted in large deviations the output. It has been problematic when changing the MPC code, and need to tune the objective function according to new changes.

2.2.4 Control alternative 1 - PID

This is the simplest of all four alternatives. To obtain at least the minimum water quality, a PID controller that shall transform water quality error in to thrust is developed. This approach is because an increased heading (relative to upstream) also increases the water quality. Knowing this, an increased sway force from the hindmost thrusters will create a yaw moment and will increase the heading. Only one of the hindmost thrusters are being used. The reason for this is to prevent counteracting the mooring force as much as possible. Knowing that it may be preferable to only use one or two thrusters, this approach skip the thrust allocation and give desired thrust on only F4y (See Figure 21). All the other modelled thrusters are put to zero.

This approach use the estimated water quality from the state estimator and calculate the error relatively the quality reference, $\tilde{Q}$. The quality error is used in the proportional and the integral term in the controller, where the proportional term is saturated to be positive. This means that if the quality is higher than the reference quality the proportional gain will not counteract the control system. The reasoning behind this is that there is no problem if the quality is higher than the minimum quality. The integral term is summed with the proportional term according to Equation (64). The other term in Equation (64) is the derivative term, which use the derivative of the quality. The derivative could be collected from the calculations in the state estimator, but is in this case only estimated by a transfer function that calculate the numerical running derivative of the estimated water quality. The transfer function for this operation is given in Equation (63).

$$\frac{\dot{Q}}{Q} = \frac{s}{10s+1} \tag{63}$$

If this should have been implemented in reality, it would be preferable to use the derivative of the estimated quality from the calculations in the state estimator, but as this works, it is not done in this thesis.

At the end, all three contributions are summed and saturated to be positive. To compare the simulation from this approach to the others, the negative value for F4y (See Figure 21) is used to create a positive yaw moment.

$$u_8 = -sat_0^{\infty}\Big\{sat_0^{\infty}\{K_p^Q\tilde{Q}\} - K_d^Q\dot{Q} + K_i^Q\int_0^t \tilde{Q}dt\Big\} \tag{64}$$

The different gains in Equation (64) are tuned, and given in Table 5. Even as the integrator gain is definitely the smallest, this term definitely accounts for most of the control action. The reason for this is that the integral of the error can grow over a long time, as the dynamics of the vessel are slow.

Table 5: Quality PID controller gains

Gain	**Value**
K_p^Q	$1e7$
K_d^Q	$1e7$
K_i^Q	$4.5e4$

2.2.5 Control alternative 2 - 1 DOF Optimization

The control alternative 2 use the MPC like setup as described in Section 2.2.3, but differs in especially two ways. Control alternative 2 do not have an online MPC optimization, it only runs the MPC algorithm once, and follows this as a reference. This approach can be called Dynamic Programming, Rush D. Robinett & Hurtado (2005). The optimal position i.e. the last heading in the MPC output is used as a steady state reference heading in a PID controller. The other main difference is that the PID controller used is only a 1 DOF controller that regulate the heading (Yaw).

The MPC like optimization that is being executed only once, i.e. dynamic programming is set up as follows:

$$\begin{aligned} &\text{Minimize} && \int_0^T N(t)^2 dt \\ &\text{Subject to} && \begin{cases} lb \leq x \leq ub \\ lb \leq \tau \leq ub \\ lb \leq \nu_r(x, \nu_c) \leq ub \\ Q(x, \nu_c, r_{(x,y)}) \geq Q_{min} \\ x(k+1) = x(k) + f(x(k), \tau(k), \nu_c)dt \end{cases} \end{aligned} \tag{65}$$

Where $x = [\psi, r]$ is the 1 DOF states. N is the Yaw force. The last state in the optimization contains the optimal heading which is used in the following 1 DOF PID controller.

$$\tau_3 = -\tau_{moor}^3 + K_p^{3,3}\tilde{\psi} - K_d^{3,3}\dot{\psi} + K_i^{3,3}\int_0^t \tilde{\psi}dt \tag{66}$$

Where the first term in Equation (66) is a term that counteract the mooring forces in Yaw. The mooring forces is estimated inside the PID controller based on the position and heading of the vessel in the same manner as described in Section 2.1.11. The controller gains in Equation (66) is described in Section 2.2.8.

Note that this control alternative may work in quite the same manner as control alternative 1, but the dynamic programming optimization need to be executed for every sea state. If the environmental loads changes during weather/current change, another heading may be the optimal one.

Control alternative 2 will not ensure high enough water quality before the system reach a steady state.

2.2.6 Control alternative 3 - 3 DOF Control input optimization

Control alternative 3 is the alternative that is most similar to the traditional MPC algorithm. This alternative use the same 3 DOF problem formulation as in Section 2.2.3. To be able to run the online MPC optimization, a Matlab function block is added in Simulink. This block prepares the MPC algorithm and feeds the online information to the MPC Algorithm. This is the six states, $x = [x, y, \psi, u, v, r]^T$, the estimated current, ν_c, the mass matrix, M, damping matrix, D, the water flow reduction factors, $r_{(x,y)}$, the minimum quality, Q_{min} and information concerning the set up of the problem. CasADi Opti Stack is used to build the optimization problem in Matlab. This contains quite understandable functions to be used. An overview and explanation of the MPC functions are given in Section 2.2.3.

The MPC optimization is being executed every 5 seconds, and produce a 3 DOF optimal control input for the next 200 seconds, with a step size of 2 seconds. The output data is sent to a post processor that always use the most resent values, and interpolate between every 2 seconds to get the simulation time step of 0.1 seconds.

2.2.7 Control alternative 4 - 3 Dof Reference optimization

Control alternative 4 is the same MPC formulation as used in Control alternative 3, but this approach use the optimal states as a reference, and not the control input. The output, $x = [x, y, \psi, u, v, r]^T$, is used as input to the traditional PID controller, described in Section 2.2.8.

2.2.8 Controller

A simple controller is implemented to be able to run proper simulations. A nonlinear PID set-point controller was used to control the system. This was collected from MSS GNC library, but modified to do trajectory tracking.

The control law is as follows:

$$\boldsymbol{\tau} = -\boldsymbol{K}_p \boldsymbol{R}(\psi)\tilde{\boldsymbol{\eta}} - \boldsymbol{K}_d \tilde{\boldsymbol{\nu}} - \boldsymbol{K}_i \int_0^t \boldsymbol{R}(\psi)\tilde{\boldsymbol{\eta}} dt \tag{67}$$

Where $\tilde{\boldsymbol{\eta}} = \boldsymbol{\eta}$ - $\boldsymbol{\eta}_c$. $\boldsymbol{\eta}_c$ is the commanded position. $\boldsymbol{K}_P$, $\boldsymbol{K}_D$ and $\boldsymbol{K}_I$ are respectively the proportional, derivative and integral gain matrices. To tune these matrices, the relative damping ratio, ζ, is set to 0.7 and the natural period, T_n, is set to 100s.

From this, the gain matrices is found from following:

$$\begin{aligned} \omega_n &= \frac{2\pi}{T_n} \\ \boldsymbol{K}_d &= 2\zeta\omega_n \boldsymbol{M} \\ \boldsymbol{K}_p &= \omega_n^2 \boldsymbol{M} \\ \boldsymbol{K}_i &= \frac{\omega_n}{10}\boldsymbol{K}_p \end{aligned} \tag{68}$$

2.2.9 Thrust allocation and thruster dynamics

A simple thrust allocation is also a part of the simulation model. The thrust allocation uses the desired thrust in three DOFs directly from the optimization tool or the controller to distribute the thrust on the different thrusters.

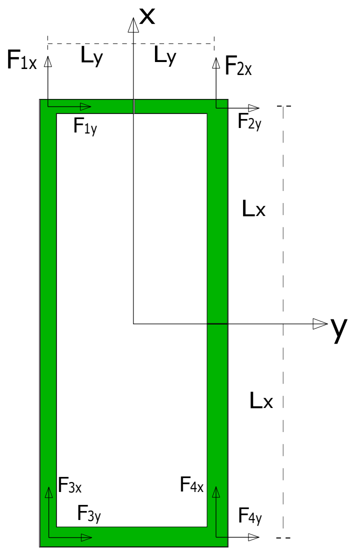


Figure 21: Visualization of the modelled thruster arrangement

At the current moment, Havfarm 2 will use eight thrusters. There will be thruster blisters as elongations of the longitudinal pontoons, where the thrusters will be mounted. Two thrusters in each "corner" of the vessel. These thrusters will of course have restricted sectors, based on preventing flushing and suction of water in the fish nets. Also preventing a thruster in another thrusters wake. This is not modelled in this thesis. When developing the thruster arrangement in this thesis, there are only four thrusters, one in each corner. The thrusters will be able to rotate, to model this, each thruster are decomposed in to force in x-direction and force in y-direction, and modelled as two different thrusters. A figure showing the thruster arrangement is shown in 21. The thrust allocation is done by mapping which influence the thrusters make in the body 3 DOF forces. This is done in the following equation:

$$\boldsymbol{\tau} = \boldsymbol{B_T}\boldsymbol{u}^c \tag{69}$$

Where $\boldsymbol{\tau}$ is the 3 DOF force vector in body coordinates. $\boldsymbol{B_T}$ is the extended thrust configuration matrix, which map the desired thrust to each thruster, shown in Equation (70).

$$\boldsymbol{B_T} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ L_y & L_x & -L_y & L_x & L_y & -L_x & -L_y & -L_x \end{bmatrix} \tag{70}$$

$\boldsymbol{u}^c = [F_{1x}\ F_{1y}\ F_{2x}\ F_{2y}\ F_{3x}\ F_{3y}\ F_{4x}\ F_{4y}]^T$ is the commanded force for the corresponding thruster.

To find $\boldsymbol{u}^c$, a "Moore–Penrose inverse," also called a pseudo-inverse is used. This method minimizes the norm of $\boldsymbol{u}^c$. The solution of the thrust allocation is shown below:

$$\begin{aligned} \boldsymbol{u}^c &= \boldsymbol{B_T}^{\dagger}\boldsymbol{\tau} \\ &= \boldsymbol{B_T}^T(\boldsymbol{B_T}\boldsymbol{B_T}^T)^{-1}\boldsymbol{\tau} \end{aligned} \tag{71}$$

When applying the forces on Havfarm 2, thruster dynamics are added. This accounts for the delay in the thrusters. The thruster dynamics are modelled as follows:

$$\dot{u} = -\frac{1}{T_u}(u - u^c) \tag{72}$$

Where u is the force resulting force from each decomposed thruster, u^c is the commanded thrust and T_u is the time constant for the thrust. A 6×8 $\boldsymbol{B_T}$-matrix is used to do the mapping to 6 DOFs in the same way as in Equation (69), to apply to the digital twin. In Equation (73), L_z is also included. This is the distance from the thrusters to the water line.

$$
\boldsymbol{B_T}(6\times 8) = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_z & 0 & L_z & 0 & L_z & 0 & L_z \\ L_z & 0 & L_z & 0 & L_z & 0 & L_z & 0 \\ L_y & L_x & -L_y & L_x & L_y & -L_x & -L_y & -L_x \end{bmatrix} \tag{73}
$$

The pure pseudo-inverse thrust allocation problem is not a recommended approach. This approach does not have saturation limits for the thrusters. Neither does the thrust allocation account for the dynamics of the thrusters. When the thrust allocation should be implemented to Havfarm 2, there must be implemented optimization functions, with manually weighting matrices. An example of this is the Matlab function: *Quadprog*, which use quadratic programming to optimize $\boldsymbol{u}^c$

3 Results

3.1 Observer

The observer/state estimator that is used is described in Section 2.2.1. This observer gets noisy measurements of position and heading, relative water velocity and the water quality from the last time step. Based on the measurements, the control plant model and the forces from the thrusters, the states in current time step are estimated. The estimated states are in all simulations positions and heading, velocities, water quality and the North-East current. The presentation of the observer result are given in this Section. As the observer copes well with the position and heading, only 50 seconds are simulated. The reason for this is that the results should be able to see. The velocities, current and quality needs a bit longer to initialize, and are simulated in 300 seconds.

The observer is tested in environmental loads which are given in Table 6. The environmental loads have both high and low frequencies, the observer needs to cope with.

Table 6: Environmental conditions for testing of the observer. All directions are given with degrees from north.

Environmental condition	**Value**
Average wind velocity	15 $[m/s]$
Average wind direction	-60 $[deg]$
Significant wave height	2 $[m]$
Wave peak period	7 $[s]$
Average wave direction	180 $[deg]$
Current velocity	0.316 $[m/s]$
Current direction	198 $[deg]$

The fist simulation was simulated for 50 seconds. The results can be seen in Figure 22. It can be seen that both the estimated position and heading are quite good already in the beginning. Even with considerable amount of noise, the maximum error in the NED-plane are 0.4 meters. The maximum heading error is 0.1 degrees.

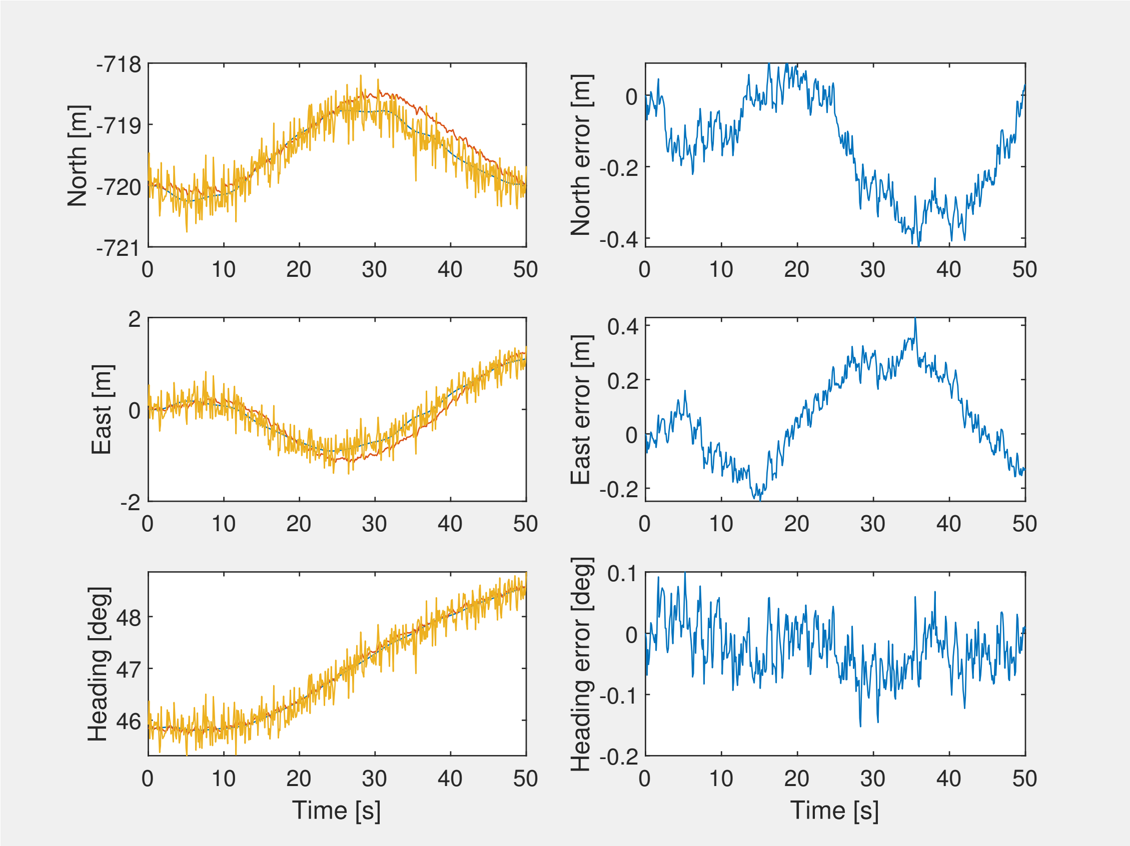


Figure 22: Position and heading estimation from observer. To the left are the blue line the actual position, the red line are the estimated position and the yellow are the measurement.

The second simulation was simulated for 300 seconds. The results can be seen in Figure 23 and 24. Figure 23, which estimate the velocities, use around 100 seconds to stabilize the estimation. The reason for this is that the velocities is not measured. Maximum error in surge and sway are respectively 0.1 and 0.2 meters per second. Maximum error in yaw velocity is 0.04 deg. per second. It can be seen in Figure 23 that the real velocities is very oscillating, but the estimated states are less oscillating. The reason for the oscillating velocities is the high frequency wave motion.

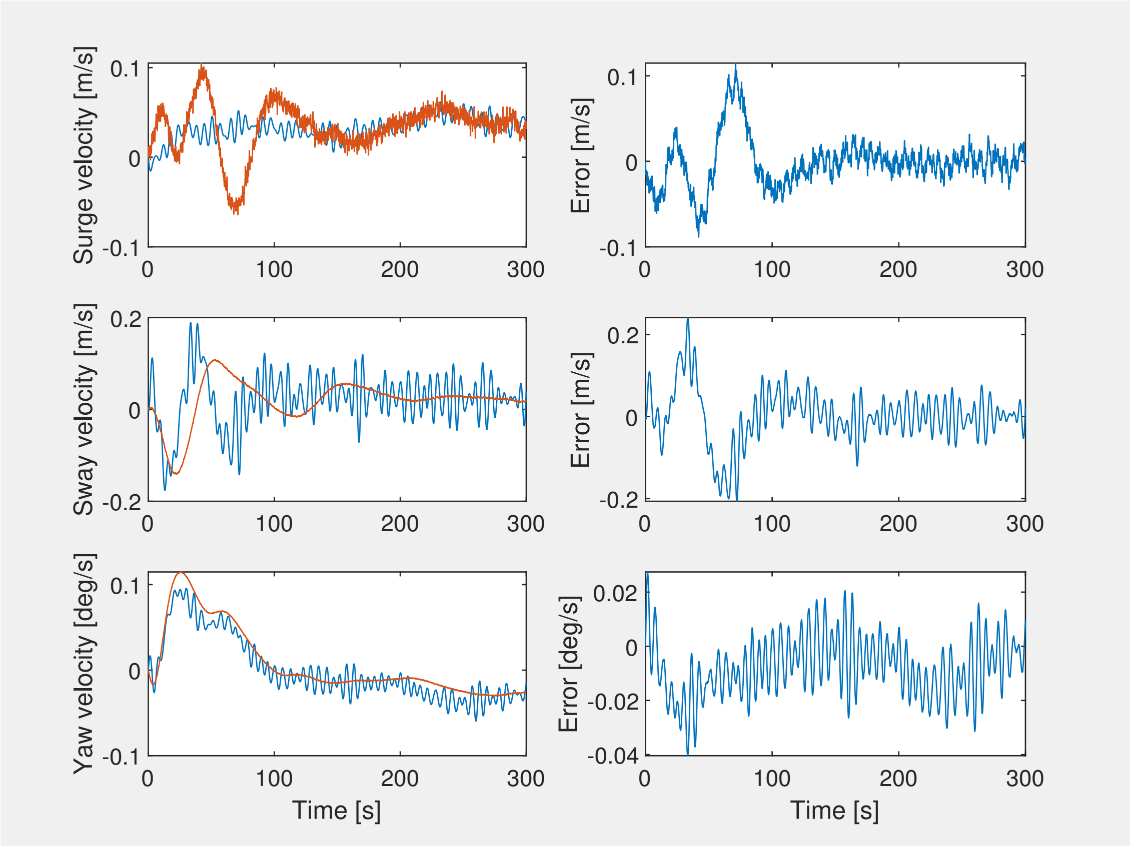


Figure 23: Velocity estimation from observer. To the left are the blue line the actual velocity and the red line are the estimated velocity.

In Figure 24 shows the results from estimating the water quality and the north-east current. The quality estimation are on point. Even as the real quality have a lot high frequency oscillations, the estimated quality have a bit less oscillations. The north and east current use some time to stabilize the estimates. This is as expected, because of the non-measured current.

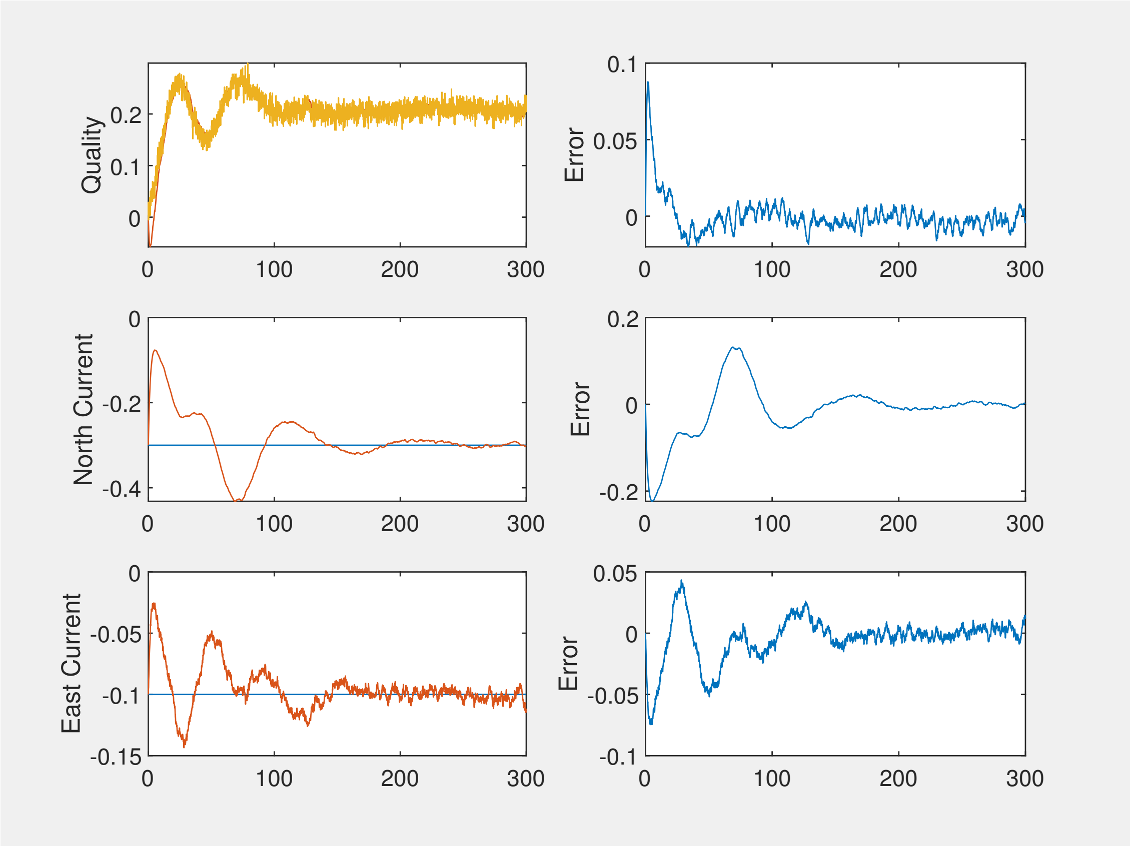


Figure 24: Water quality and current estimation from observer. In the quality plot, are the blue line the actual position, the red line are the estimated position and the yellow are the measurement. In the plots of the current, the blue are the input current to the system, and the red are the estimated current.

3.2 Station keeping

To validate the digital twin's co-operation with the control system design including state estimator, controller and thrust allocation a station keeping simulation is executed. This is done without the anchor, but including environmental loads. The included environmental conditions are explained in Section 2.1.9, 2.1.10 and 2.1.12, and the values are given in Table 7.

Table 7: Environmental conditions for Station keeping results. All directions are given with degrees from north.

Environmental condition	**Value**
Average wind velocity	15 $[m/s]$
Average wind direction	-60 $[deg]$
Significant wave height	2 $[m]$
Wave peak period	7 $[s]$
Average wave direction	180 $[deg]$
Current velocity	0.316 $[m/s]$
Current direction	198 $[deg]$

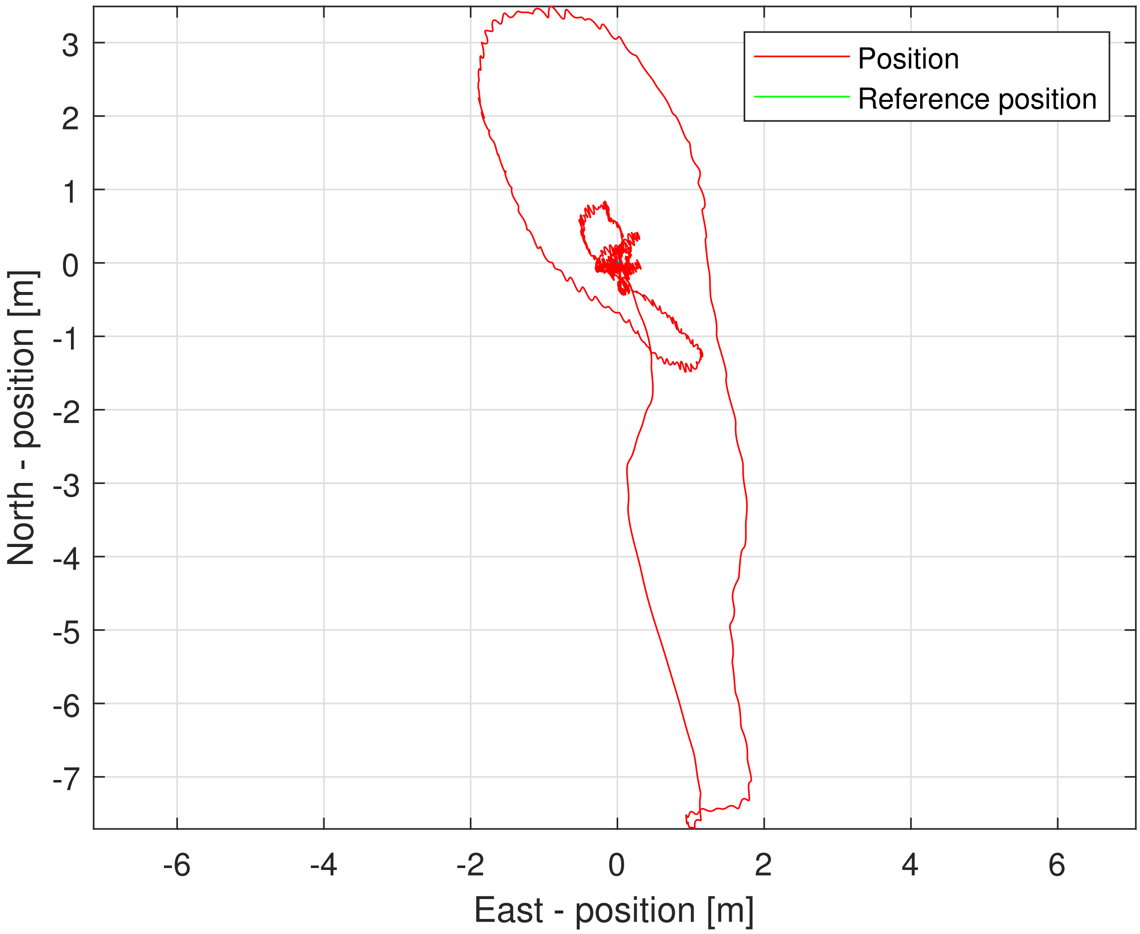


Figure 25: 2 dimensional plot of the trajectory of Havfarm 2

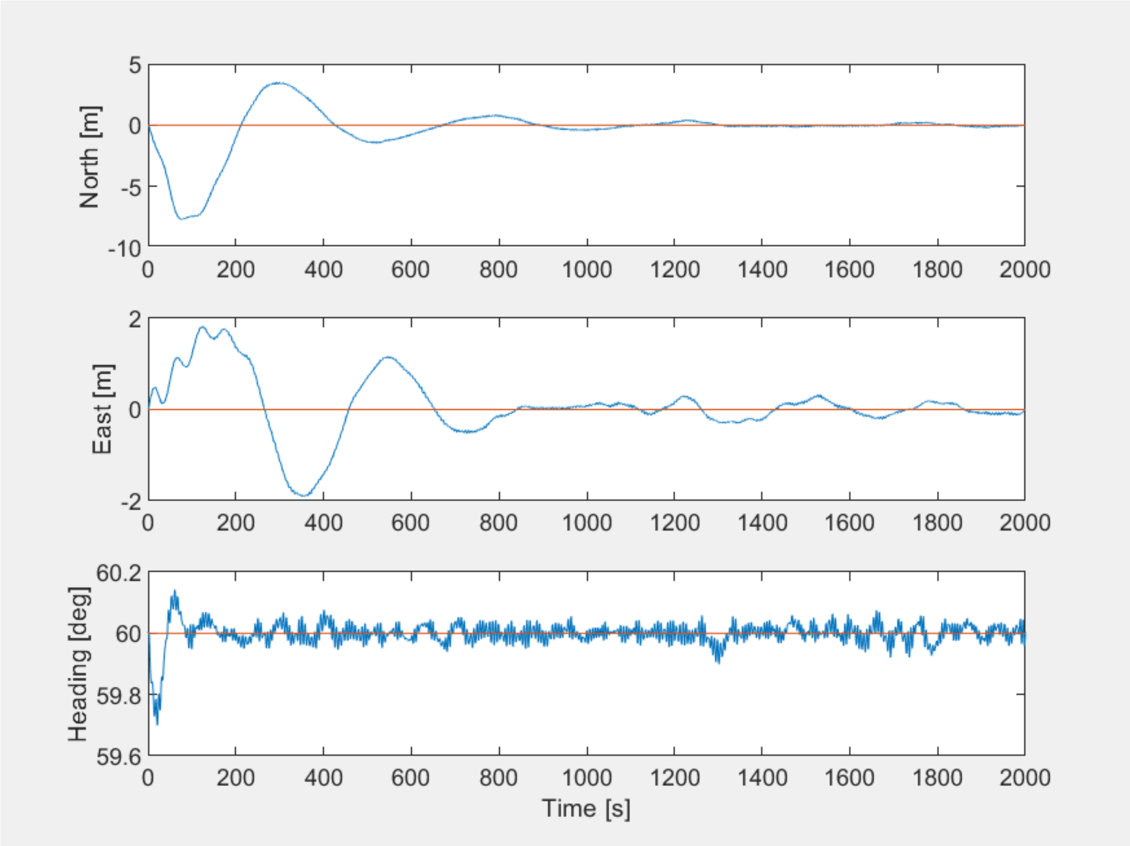


Figure 26: 3 DOF plot showing the desired position (red) and the actual position (blue)

3.3 Water quality simulations

When executing and comparing the different control approaches for ensuring sufficient water quality there are some limitations in the results. As it takes very long time to simulate using the approaches including online MPC all the simulations are initialized as close as possible to a steady state. This is to prevent using a large amount of time to see the systems stabilize. All the water quality simulations are executed exposed to the same environmental loads. These are quite low, because of the operational conditions of this master thesis. The environmental conditions are given in Table 8.

Table 8: Environmental conditions for water quality simulations. All directions are given with degrees from north.

Environmental condition	**Value**
Average wind velocity	$5\ [m/s]$
Average wind direction	$-60\ [deg]$
Significant wave height	$1\ [m]$
Wave peak period	$7\ [s]$
Average wave direction	$180\ [deg]$
Current velocity	$0.3\ [m/s]$
Current direction	$180\ [deg]$

There are many ways to present the result from these simulations. As the objective of these simulations is to get sufficient water to the fish while minimizing the fuel consumption, the comparison of these results is given in this section.

To compare the fuel consumption, a model i used. The model is for each of the eight thrusters modelled in Section 2.2.9. The fuel consumption model is given in Equation (74), where the t is the time, i is to count for all the thrusters and u is the thrust from the individual thruster. Note that for each second in Equation (74) there are 10 time steps that are summed.

$$\boldsymbol{E} = \sum_{t=1000}^{2000} \sum_{i=1}^{8} u_i(t)^2 \tag{74}$$

The result from the fuel consumption model is shown in Figure 27. The result are also given in Table 9. Note that there were simulated 2000 seconds in every approach, but the comparison of fuel consumption use only values for the 1000 last seconds. The reason for this is that especially alt. 3 and alt. 4 use some time to stabilize the system.

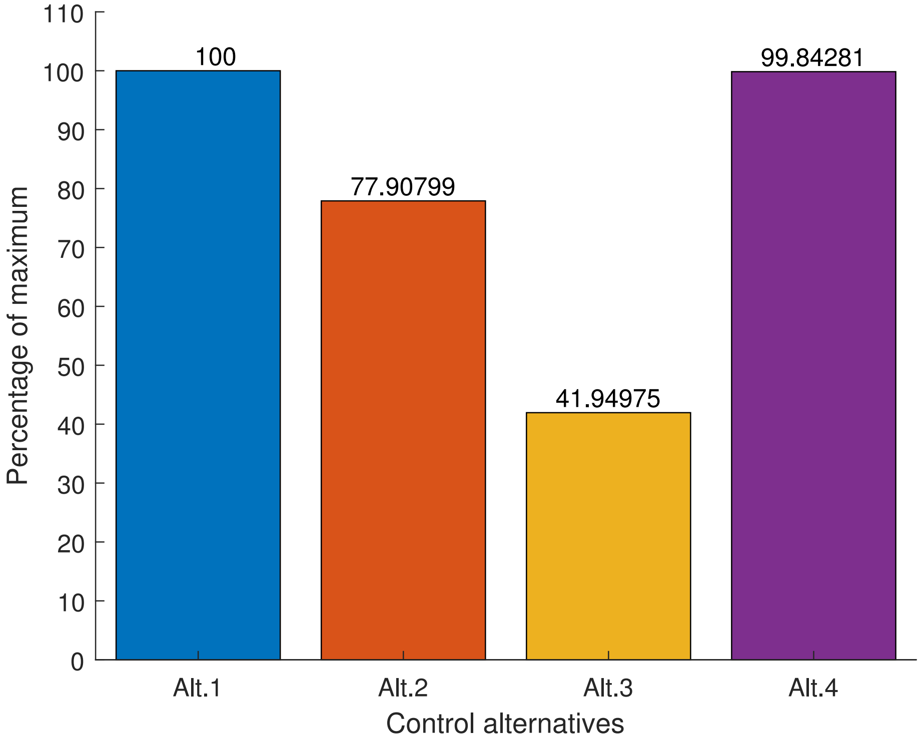


Figure 27: Comparison of fuel consumption. The comparison is normalized with the control Alt. 1.

Figure 27 shows the relative fuel consumption compared to the other control alternatives. It can be seen that control alternative 1 have the highest fuel consumption, with control alternative 4 consuming almost as much. Control alternative 2 use approximately 78 % of the fuel that was used by control alternative 1 and 4. The best alternative according to the results shown in Figure 27 is control alternative 4, consuming only 42% of the fuel used by alternative 1.

The water quality are also compared. Presentation of this is done in time series for each alternative, and shown in Figure 28.

Table 9: Comparison of fuel consumption. 100% is the alternative with most fuel consumption.

Alternative	Value
Alternative 1	100%
Alternative 2	77.9%
Alternative 3	41.9%
Alternative 4	99.8%

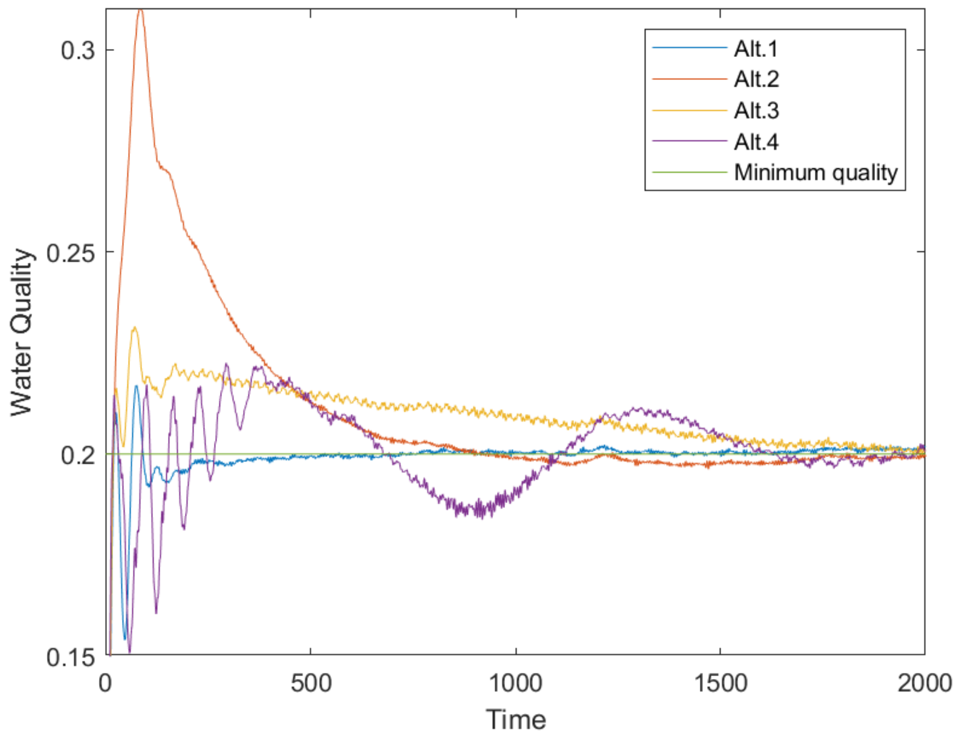


Figure 28: Comparison of water quality.

In Figure 28 it can be seen that all the control alternative approaches the limit of minimum quality. There are quite a lot oscillations in the beginning. Especially in alternative 4 and some in alternative 1. Alternative 2 overshoots pretty much in the beginning. Alternative 3 overshoots also a bit in the beginning, but decreases towards the minimum quality limit.

3.3.1 Control alternative 1

The control alternative 1 that use a traditional PID controller start 650m south and 300m east of the anchor point, with a heading of 28 degrees. In the start of the simulation, the proportional and the derivative term are the dominating (See Equation 64). When time passes, the integral term becomes the dominating term. Note that according to Equation (64) the PID controller will only induce a control action if the quality are too low. To compensate for this, the minimum water quality as input to the PID controller is 0.202.

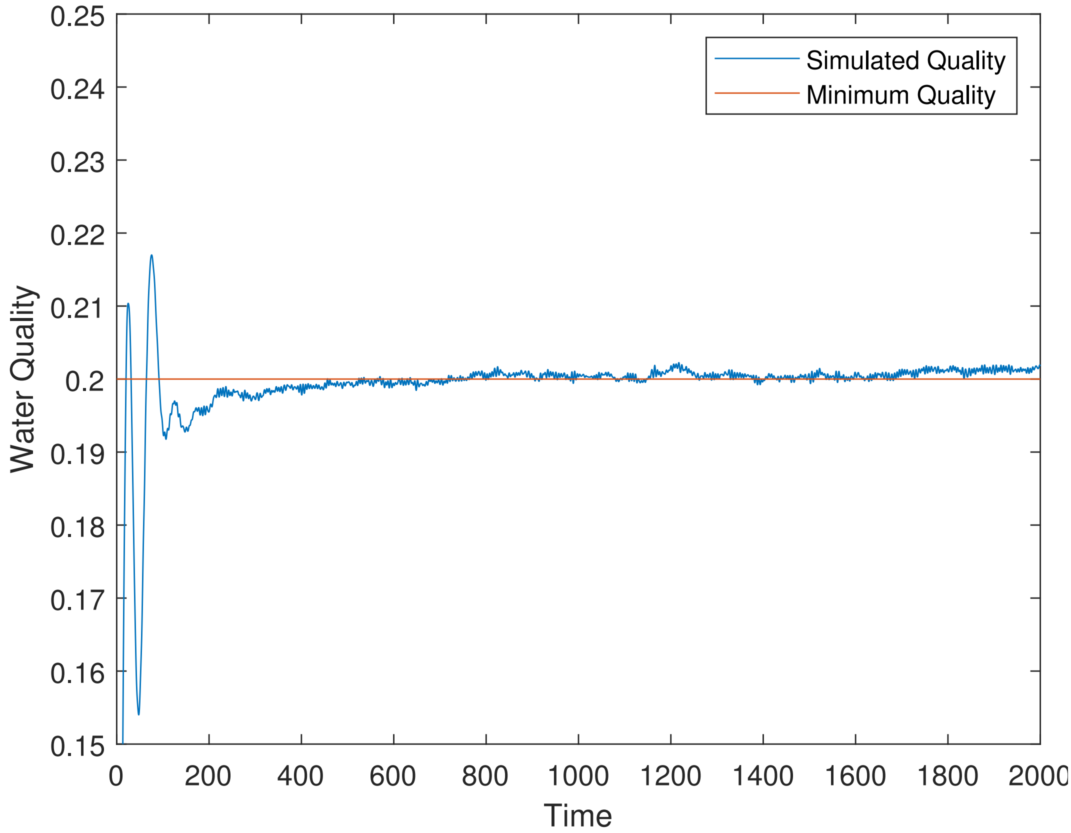


Figure 29: Water quality for control alternative 1

In Figure 29 it can be seen that control alternative 1 manage to obtain the quality constraint for almost the whole simulation. The exceptions are in the initialization where the deviations are large, but over a small period, and around 1000 seconds where the deviation are negotiable. In the end of the simulation, the quality overshoot a bit. The reason for this may be the integral term that is built up over a time. Note that this is not negative for the fish, but it could use more fuel than necessary.

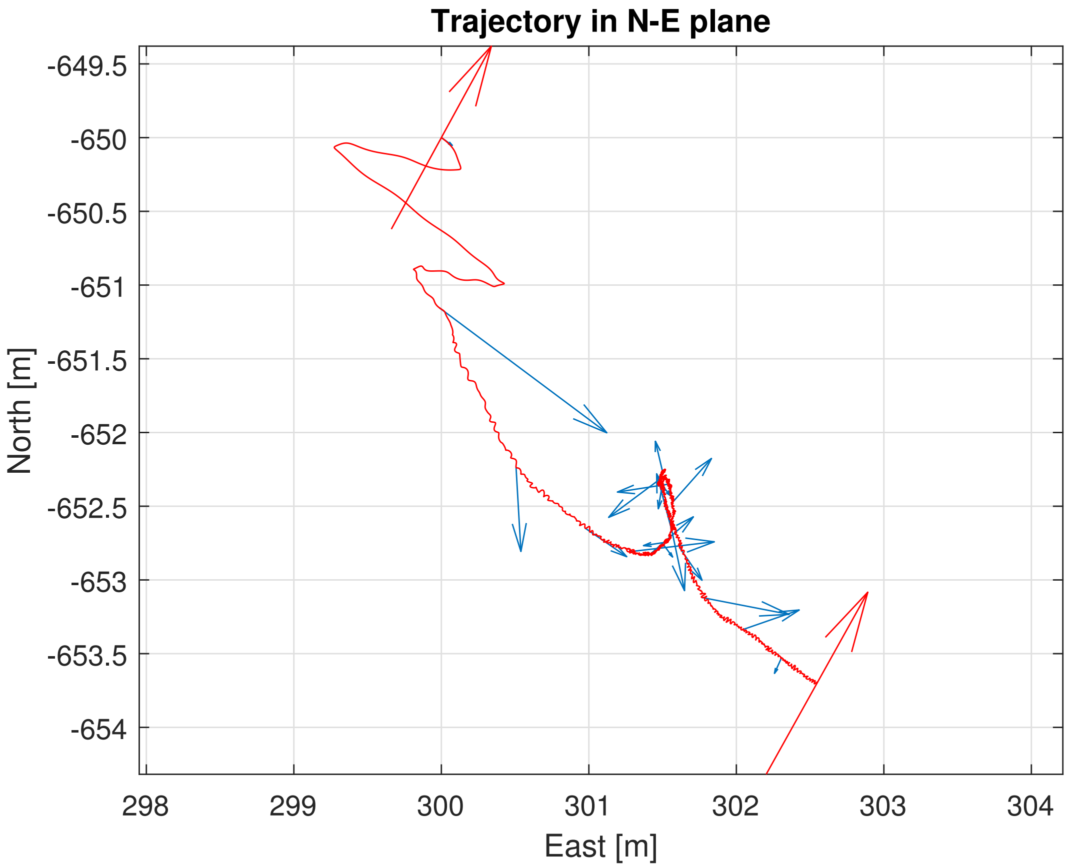


Figure 30: 2D trajectory of Havfarm 2 in control alternative 1. The blue arrows are the velocity vectors, and the red arrows are the heading vectors.

Figure 30 shows that Havfarm 2 stays at almost the exactly same position during the 2000 seconds. The transnational distance are maximum 12 meters from the starting position.

3.3.2 Control alternative 2

The control alternative 2 use a MPC optimization function once, which creates the optimal heading for the vessel to have, when the current is constant. The PID controller described in Section 2.2.8 is used in 1 DOF (Yaw) to obtain this optimal position. There are no applied force in surge nor sway. In the PID-controller there are also added a feed forward term that shall equalize the yaw moment from the mooring line.

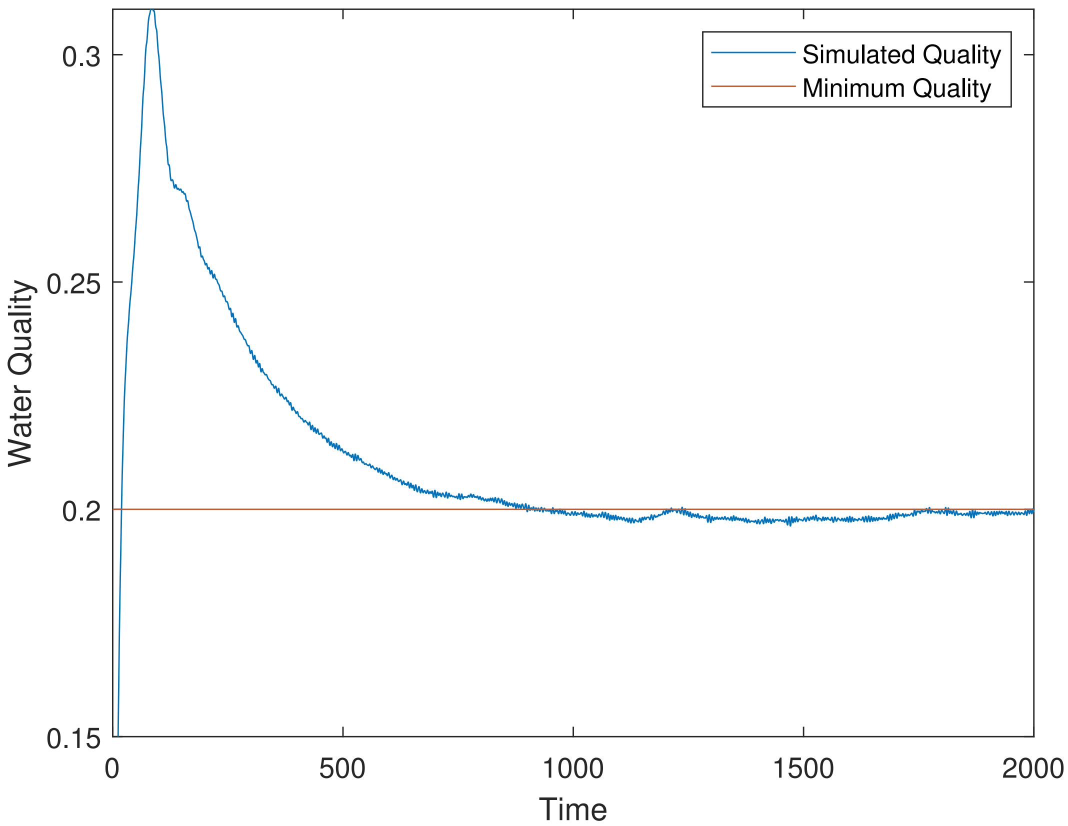


Figure 31: Water quality for control alternative 2

Figure 31 shows that the control approach use approximately 1000 seconds to reach a steady state. After this, the quality lies negligible under the minimum quality.

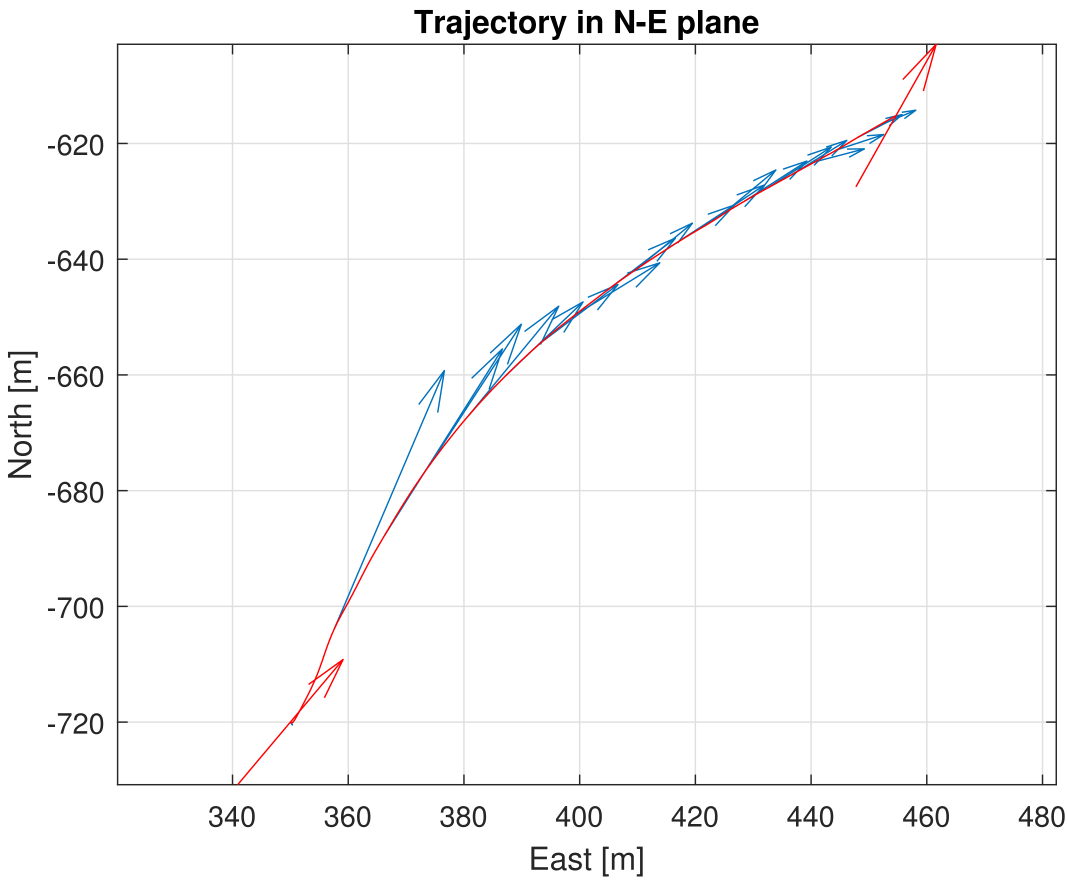


Figure 32: 2D trajectory of Havfarm 2 in control alternative 2. The blue arrows are the velocity vectors, and the red arrows are the heading vectors.

Figure 32 shows the trajectory of Havfarm 2 using control alternative 2. It can be seen that the transnational velocity(blue arrows) in the beginning is quite high, which is reflected in the quality (See Figure 31)

3.3.3 Control alternative 3

Control alternative 3 is the approach that has the least fuel consumption. Nevertheless it can be seen in Figure 33 that the control approach obtain constraint of the minimum quality. Note that in this simulation, the minimum quality are put to 0.22 that are not obtained. In other words, the problem can be solved, but need to be tuned a bit more.

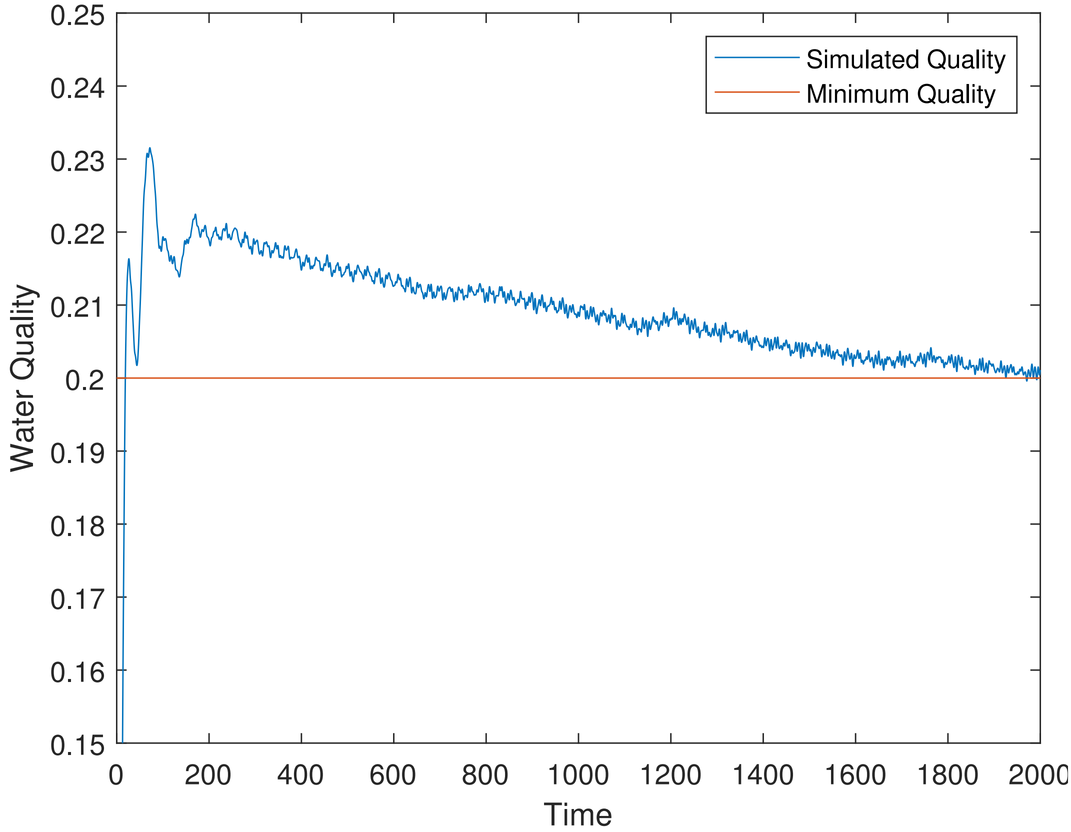


Figure 33: Water quality for control alternative 3

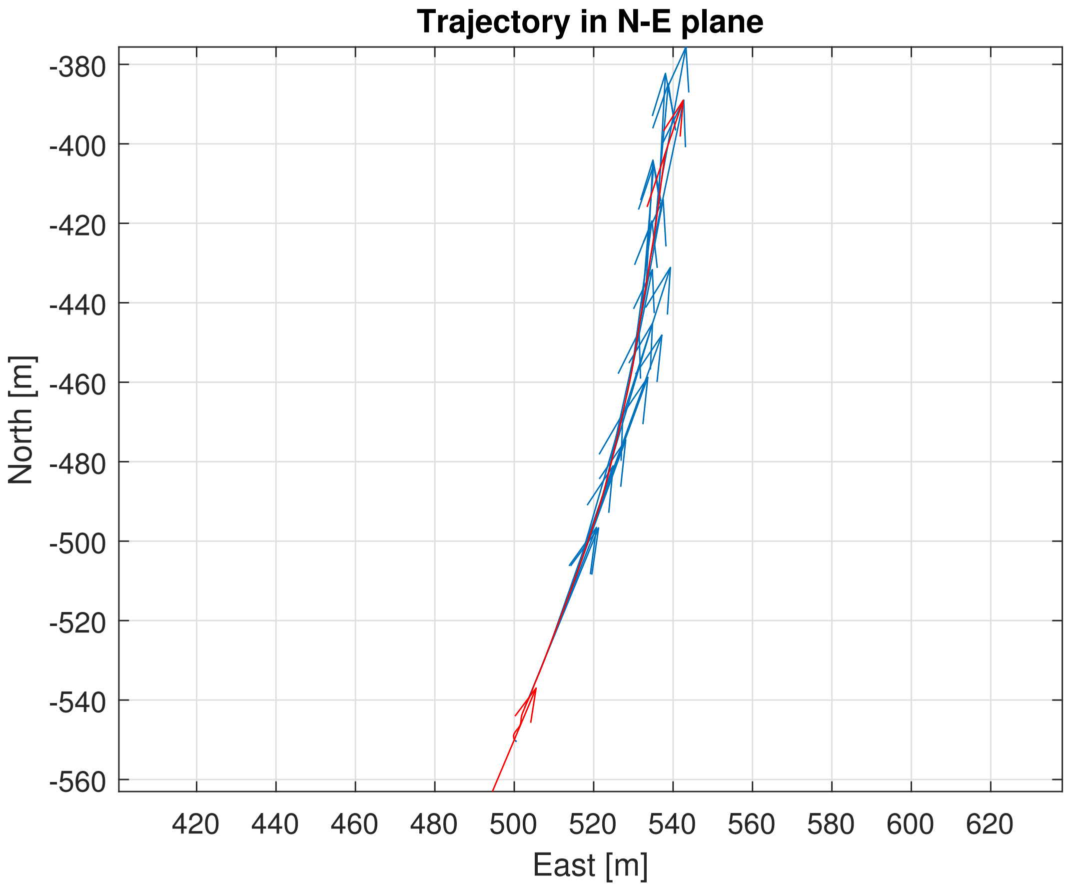


Figure 34: 2D trajectory of Havfarm 2 in control alternative 3. The blue arrows are the velocity vectors, and the red arrows are the heading vectors.

3.3.4 Control alternative 4

Control alternative 4 use the states output from the MPC optimization as reference in the traditional PID controller. The result is shown in the following figures.

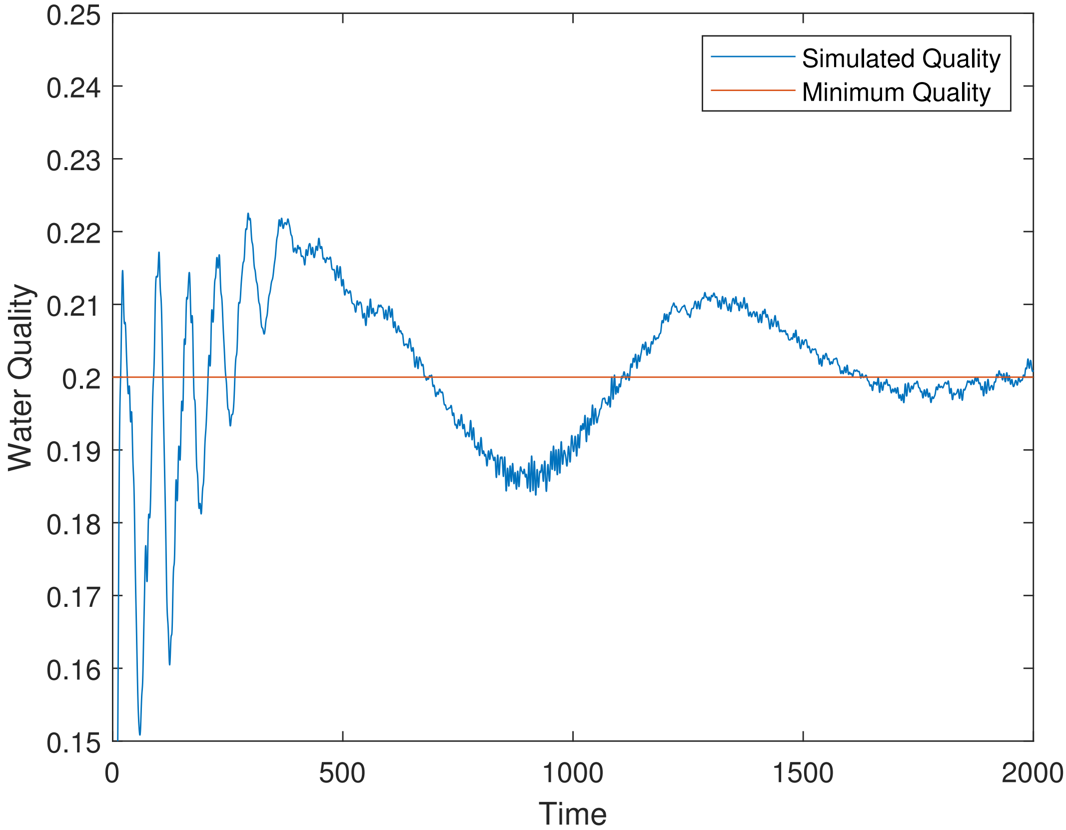


Figure 35: Water quality for control alternative 4

Figure 35 shows that control alternative 4 are oscillating quite a lot, and cannot obtain water of high enough quality. At least in the simulated 2000 seconds. The oscillations in the first 500 seconds, and clearly the Control system counteract itself. In the 1500 next seconds, there are relative low frequency damped oscillations. By the look of the low frequency oscillations, the PID controller are tuned a bit too aggressive.

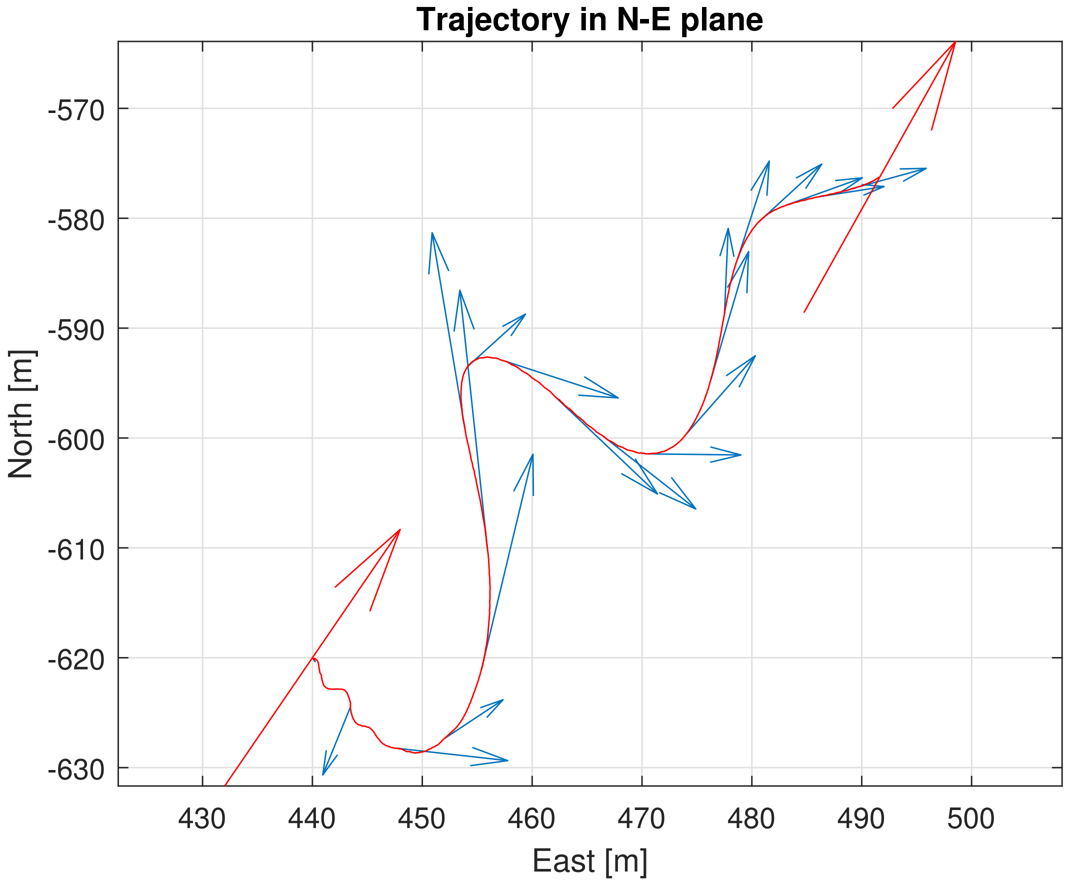


Figure 36: 2D trajectory of Havfarm 2 in control alternative 4. The blue arrows are the velocity vectors, and the red arrows are the heading vectors.

The trajectory of Havfarm 2 exposed to control alternative is shown in Figure 36. It can also be seen in the figure that there are fist relative high frequency, later a low frequency damped oscillation.

4 Discussion

In this master thesis, the objective is to develop a digital twin of Havfarm 2, develop a simulation platform for this digital twin, and design some different control approaches to solve a water quality problem that may happen in calm environmental conditions.

4.1 Digital Twin

The digital twin, a mathematical representation of Havfarm 2, is developed through hydrodynamic analysis and general modelling. A station keeping regime was first used in the modelling, but during the development, several aspects from manoeuvring modelling was used. The hydrodynamic data that was found in the analysis program *Wamit*, was translated to cooperate with the Simulink/-Matlab system. For developing the digital twin, both the hydrodynamic analysis program *Wamit*, and the MSS toolboxes was crucial. Amongst these data was the frequency dependent added mass and frequency dependent potential damping matrices. By looking at those, Figure 10, 11, 14, 15,it can be seen that most of the dynamics of the vessel are at low frequencies, i.e. high periods. This is logical as the inertia of the vessel are huge, and moves generally slowly.

There are much of the modelling of Havfarm 2 that is done accurately, but there are also part of the modelling that are estimates and early engineering assumptions. Examples of this is the anchor system, wind model and the second order drag model.

To be able to test the digital twin and the control system, a simulation platform is developed. This includes varying wind and waves and constant current. A measurement model simulating the noise in the measurement and a water quality model are developed.

When developing the environmental loads on Havfarm 2, this should be minor regarding the operational conditions that are being studied. The current are modelled constant, an assumption that only can be done in a short period of time, one sea state. This only applies for the simulation platform, as the control system, more specific the observer, are built to account for different sea states.

The measurement model are not optimal for this simulation platform as the noise are more high frequency then in the real world, where there are a more constant offset plus a minor high frequency noise. As the control system in this master thesis is a non-strict control system, it should not matter if there are an offset of 1 meter in the measurements.

Trying to obtain optimal fish welfare for the salmon is something that are seemingly not done before. There will probably be sensors aboard Havfarm 2 to measure relative water velocity and oxygen saturation in the water. At first, it was desired to not develop a water quality model, but only base the control system on the measurements of the quality. Many different approaches for the control system design was considered, but as it was desirable to use model predictive control, a model was necessary. To develop a water quality model for salmon in fish cages could be a science are for itself. Therefore only a very simplified model was developed. This model and the limit of minimum quality are not based on the salmon oxygen requirement nor anything biological. It was only developed for the testing of the control system in this thesis. If a more realistic water quality model should be developed, many factors should be regarded. Examples of these may be temperature, size and number of fish in the upstream cages that utilize oxygen and pollute the water, oxygen saturation, reduction of water flow both global and local in a fish cage. Some of these factors clearly coherence with each other and may be of higher order than one, which is used in this thesis.

4.2 Control system Design

The control system design in this thesis is based on the digital twin, the simulation platform that is developed and the calm weather conditions that can result in the water quality problem. The control system that are developed is the observer/state estimator, four different control approaches and thrust allocation. These were all developed separate, to do only its own work. It can be seen in Section 3 that the control system is working, but it could be better in many ways. If the solution should be optimal, the observer and the control approaches could be merged to one system. In this way, the estimated bias, and other slowly varying states could be used in an optimization problem.

4.2.1 Observer

The observer used in the control system is a common nonlinear passive DP wave filter plus state estimator. This was customized to account for the specialities of Havfarm 2 and the control system design. In addition to introduce new states, there are done changes to account for low velocities relative to the current. The observer works quite satisfactory for the purposes in the control system. The high frequency wave motions and noise are filtered away, and the position and heading are estimated very quickly. The velocities need some more time to stabilize, but this is expected, as there are no measurements of these. The estimation of the quality are very responsive as it weigh the measurements quite heavily. There are some high frequency oscillations in the quality estimation that should have been low-pass-filtered if it is necessary for the rest of the control system. The current in two different directions also use quite a while to be stabilized, but these are not measured either.

Other forms of observers was considered. Amongst these, a Kalman filter was developed, but as the edited nonlinear passive produced sufficient results, it was not developed further.

4.2.2 Optimization

Four different approaches was tested when trying to minimize the fuel consumption, but still ensure fish welfare. Control alternative 1 did not directly include optimization, but used a PID controller regulate the water quality. This was done by controlling the hindmost thruster on the starboard side to produce thrust. This thrust was (in this model) directed in sway direction, which also resulted in a yaw moment that turned the vessel. Even as this is the control alternative that consume most fuel when regulating the quality, it may not be a very bad approach after all. The reason for this is that the fuel consumption model favour minor force on multiple thrusters, instead of major force in one thruster. Even as this may be a good fuel consumption model, it might not be the best fuel consumption model for Havfarm 2. There are possibilities that the thrust allocation in the operational conditions studied in this thesis should only include one or two thrusters. If that will be the case, the other control alternative (That uses the existing thrust allocation) would get a higher value in the comparison of fuel consumption in Figure 27.

Common for control alternative 1 and 2 is that these are much harder to expose for restrictions. In the control in a MPC optimization as control alternative 3 and 4, the vessel can be subjected to constraints. These constraints may be geographical restrictions, maximum anchor load or maximum relative velocities. It may be preferable to use a control system that has these types of functionality.

Control alternative 2, the optimal heading reference, need to change for every sea state. In the presence of tide, the current can turn quickly during high tide and low tide. To account for this, the optimal heading algorithm need to be executed quite often based on the environmental conditions.

Both control alternative 3 and 4 used online nonlinear MPC optimization. The MPC implementation and MPC function used in this in this thesis is a relatively simple model predictive control optimization. CadADi Opti Stack formulations are quite easy to understand and use, and direct multiple shooting by the *ipopt* solver worked fine for these simulations. There are nevertheless some aspects in the nonlinear MPC optimization that must be discussed.

- When using control alternative 3 and 4 there was quite a lot oscillations in the output. When the optimization function was executed every five seconds, the desired control input was oscillating with periods of five seconds, including rapid change between every execution.

It would be desirable to run the optimization function every time step, at least every second in the control system. In these simulations, the thruster dynamics, including a time constant can solve these rapid changes or oscillations, but it is not desired to put oscillating control input into a real thruster. This could be solved by making the MPC optimization account for rapid changes in control input in constraints. The problem in itself may disappear if the observer, controller/optimizer and thrust allocation was merged in to one system.

- It is important to realize that nothing in this thesis is designed for rapid simulation. Many actions could be made to make the control system run faster. This is not emphasized at all. Firstly, the solver *ipopt* is not created for solving almost the same problem each time very fast. Secondly the MPC function has to build the problem formulation every time the function should be executed. As the problem formulation is the same every time, the function should accept new input without recompiling the problem.
- It is important to understand that the term optimizing easily can be misunderstood. It is developed a MPC function that shall minimize the fuel consumption. What is really happening is that the objective function is minimized subjected to constraints? As the objective function are weighed, in other words tuned, the MPC function acts differently. This is a basis for seeing that even as the system is optimized, it is not certain that the best solution is found. Another aspects of this is that the optimizer uses the derivative to search for a minimum, which might be a local minimum and not a global minimum (See Figure 5.)
- As mentioned, the input to control alternative 3 is a minimum quality of 0.22. Regarding this, Alternative 4 managed to obtain the minimum quality better than alternative 3, The reason for this is probably that from alternative 3, the control input is used. This has the intention of counteract the current and the mooring force. As there also are wind and waves present, there are not enough force to counteract this. Alternative 4 on the other hand, has an optimal position, heading and velocities reference output. As this is used as reference in a PID controller, the vessel use enough force to counteract every environmental term. There are several ways to round this obstacle for control alternative 3. One may be the mentioned approach to merge the optimization and the observer, where the estimated slowly varying environmental forces can be used in the optimization. Another approach could be to include an integrator term in the optimization. In a steady state situation for alternative 4, the integral term in the PID controller are the most dominating.
- In the model predictive control optimization, the variable, Q, is the water quality for every step in the MPC horizon. In other world there are created an expectation to the quality. To obtain good enough quality for the fish, the simplified model for the quality in the MPC optimizer could be adaptive. If the vessel does not obtain the planned/desired quality, the flow reduction coefficients, R_x and R_y, could be changed to plan for better quality the next optimization. Adaptive quality model in the optimization could also be an alternative to a large and complex nonlinear water quality model.

4.2.3 Thrust allocation and thruster dynamics

The thrust allocation and the thruster dynamics, described in Section 2.2.9, are very simplified. The thruster dynamics that actually could be seen a part of the digital twin are not very genuine, but rather developed for a simplified control system. As the thrust allocation may in general be the most time consuming part of a control system design, this was not attached importance to in this thesis. As the net pen stretch deeper than the thrusters on Havfarm 2, in addition to thruster places closely to each other there are forbidden sectors for the thrusters. This is not possible to model using the pseudo inverse thrust allocation used in this thesis. To be able to reduce the number of active thrusters in calm conditions, implement forbidden zones and utilize as least energy as possible, a more complex thrust allocation have to be made if this system should be realized. The thruster allocation modelled in this thesis use eight thruster where half can give thrust in surge and the other half in sway. On Havfarm 2, (See Figure 6), there are thrusters that can rotate to be able to give thrust in any desired direction, except the restricted sectors of course.

4.3 Simulation results

Section 3 shows the results from three tests done. The first test was to validate the observer. This was done with a bit heavier environmental condition than in the rest of the simulations. The observer use a control plant model and measurement of position, heading, relative water velocity in two dimensions and the water quality. It can be seen in Figure 22 that the positions and the heading estimates are very good, even as there are quite a lot measurement noise. The estimated position and heading contain a bit of high frequency noise. Even as there are small amplitudes in the remaining noise, it would have been preferable to dispose of this. Figure 23 shows the estimates of the velocities. These estimates are not as good as the position estimates. The reason for this is that there are no measurement of the velocities, these are estimated based on the position measurements, control plant model and the relative water velocity measurements. This is also the case for the current estimation in Figure 24. These are also based on the relative water velocity, control plant model and the position measurements. The water quality in the same figure are more precise, because of the measurements. From the early stage water quality model, the water quality was a direct function of the relative water velocity. As this changes much due to oscillations (Waves, fluid memory effects, wind) the "correct" water quality also have high frequency oscillations, including WF. To prevent too much oscillation in the "correct" water quality, a low pass filter is added. This is also described in the end of Section 2.1.13.

The station keeping simulation results was also done with the same environmental conditions as the observer simulation. The station keeping simulation is done without the use of an anchor, meaning it is only the DP system performing the station keeping. The vessel starts in the same point and with the same heading that are the desired ones. Even as the vessel translate around eight meters from the starting point and have some damped oscillations, the results show that the DP system is capable to keep the vessel stationary. At first the vessel are forced a bit south and to the east. As the DP system forces the vessel back to the correct position there are a bit overshoot, meaning that the vessel travels a bit too far. If the control system should been optimized for station keeping, the controller should be tuned to avoid this overshoot.

Regarding the water quality simulation in Section 3.3 there was developed four different four different control alternatives that should ensure sufficient water quality for the fish, while minimizing the fuel consumption. To be able to compare the different approaches, a model for fuel consumption was used. This model, given in Equation (74), may not be the best model for comparing the different approaches. The reason for this is described in Section 4.2.2.

It is one more factor that speaks against the presented fuel consumption comparison. By looking at the trajectory for the different control alternatives, Figure 30, 32, 34 and 36, it can be seen that the control alternative 3 and 4 have a large horizontal translation during the 2000 seconds. This means that the control alternatives did not start in the steady state solution. In addition to change the heading of the vessel, the 3 DOF nonlinear MPC finds it advantageous to use thrust to increase the relative water velocity in surge, even as the reduction factor in surge are 0.323 versus 0.817 in sway. The reason for this is that the damping in surge are much lower. Regarding that the vessel also use an anchor, this horizontal translation cannot continue forever. This means that the simulation of 2000 seconds did not show the whole picture. The behaviour was investigated by running simulations with different starting positions and headings with control alternative 3. The result of these was that it looked like the vessel never reached a steady state solution. The vessel used a positive surge force that decreased as the time passed. After a while, the surge force will counteract the mooring force. When this is the situation, the vessel tends to increase the heading to almost 90 degrees, and somehow follow the current south, but counteract the current and move very slowly. This situation are much more fuel consuming than the other control alternatives. When following the current south, situations arises where the solver not can find a feasible solution. With a bit critical view, it can be stated that control alternative 3 and 4 do not work properly. It is clearly that in this situation the MPC optimization plan for a too short period. By looking into the different simulations, it can be assumed that the problematic behaviour could have a time period of around 10 hours. If a MPC optimization like this should be realized, there are needed actions to prevent this problematic behaviour. If the MPC function should be executed every time step it would probably be a too big problem to increase the MPC horizon to a sufficient value. The solution could be to define an area where it is assumed that the steady state solution, and use this area as a constraint in the MPC optimization. There could also be other approaches to counter the problematic behaviour.

4.4 General discussion

The main objective of this master thesis, described in Section 1.3.1, was to develop a digital twin of Havfarm 2, develop a simulation platform for this digital twin and design a control system that should ensure optimal fish welfare and minimize the fuel consumption. The master thesis is built upon an assumption or possibility that without a part of the DP that ensure high quality water for the fish, the water quality would become a problem. The possibility that this situation could be a problem is not investigated in this thesis. The problem formulation was chosen in collaboration with NSK Ship Design that is designing Havfarm 2 for Nordlaks.

At the end, there are mainly three parts that are done in this master thesis. The first part was to develop a mathematical representation of Havfarm 2. By using quite a lot of time studying the modelling of Havfarm 2 and accomplishing the modelling, the digital twin was developed, and it was convenient. There were minor parts of the model that was implemented with high precision, even as there were major parts that are not modelled with the same precision. This is not incorrect, but it would be preferable to model the terms with the most influence with the highest precision. The objective was to model the Havfarm 2 as close to the reality as possible. As Havfarm 2 is not a reality yet, the model have to be updated based on updated data.

The next part of the master thesis was to develop a simulation platform for the digital twin. Including environmental conditions, measurements model and water quality model. These parts was developed for aiding the development of the control system and the validation of the digital twin.

The control system is developed by designing a customized State estimator and a simply thrust allocation. The controller/optimizer was developed for Havfarm 2 and the welfare of the fish. This control system is somewhat preliminary and only gives insight in the solving of the water quality problem, which means that there are still much work remaining before this could be realized. Nevertheless, in the process of developing this control system many obstacles are met and cared for, resulting in a convenient control system.

5 Conclusion

The main objective in this thesis is to look into and develop a control system for Havfarm 2 that shall ensure high quality water for the salmon in calm weather. To be able to to this, a digital twin of Havfarm 2 is developed. Developing the digital twin, was different from developing digital twins for conventional DP vessels, because of the special operational philosophy and unconventional hull shape. The modelling of the digital twin was done quite precise in the areas where time and data allowed it. In addition to develop the digital twin, maybe more valuable is the development of the approach used to model the digital twin, and described in this thesis. To be able to connect this digital twin to the control system, a simulation platform is also created. The subsystems in the development of both the digital twin, simulation platform and control system design are often based on existing models and approaches. When using these existing subsystems they are customized to fulfil the complex tasks that Havfarm 2 requires. The control design is developed in three separate subsystems. An observer, which estimate the states, a thrust allocation that distribute the desired thrust to the different thrusters and a controller/optimiser. In the control system design it was decided to test four different approaches for solving the water quality problem. All alternatives fulfilled the task for a while, but none can be stated as finished. The main conclusion about the master thesis is that it is possible, and probably advantageous to use optimal control to solve the water quality problem for the fish. The conclusion for the different control alternatives is that none of them are optimal, but an approach where the whole control system, including control alternative 3, a nonlinear model predictive control, is the best way to continue the developing of a control system for Havfarm 2 in calm weather.

6 Further work

This master thesis can be contemplated as a first look into the control system that could control Havfarm 2 in operational conditions in calm weather. Knowing this, there are very much further work that should be done to realize a system like this on Havfarm 2 or on any other vessel. This applies the modelling, Section 2.1, but specially the Control Design, Section 2.2.

6.1 Modelling

In the modelling part of this thesis, there are quite a lot that are preliminary in the design of Havfarm 2. Just introducing the final parameters in the developed Digital twin will increase the quality and correctness of the model.

To start with the hydrodynamic analysis in Wamit can be more precise. It can include parts that are not modelled in the 3D panel model. Examples of this may be the thrusters and the anchor equipment. The mesh size in the model can also be increased. In the Wamit analysis, the program analyse 19 different chosen frequencies. This can be increased for a more precise result in among other, the frequency dependent added mass matrix that is the basis for added Coriolis matrix, and the frequency dependent potential damping matrix.

In the damping matrix modelled in Section 2.1.5, there are done many simplifications. As the relative water velocity is constant everywhere is a quite drastic simplification. This is done because of the trusses and the fish nets are not included, which can equalize the effects. As the second order damping is a quite important factor, this model should be improved. By including the trusses, and the fish nets, and do a Computational Fluid Dynamics (CFD) analysis of the structure, a big step is done.

The wind forces in Section 2.1.10 are also done in a simplified manner. To increase the validity of the wind model, a CFD analysis should be done of the structure exposed to wind. An alternative to this can be a wind tunnel model test. The wave force model can also be more comprehensive. It can include sum frequencies and slowly varying wave forces.

When it comes to the mooring and the anchor line forces in Section 2.1.11, there are not much that are decided on Havfarm 2. As the anchor is very important for this master thesis, there was used a very simplified anchor model based on preliminary values for the chain. The forces can easily be improved by using final and correct values for the mooring line. As mentioned in Section 2.1.11, the horizontal distance of the mooring line are calculated based on the centre of the vessel. This should be improved by calculating the distance based on the actual position for the mounted anchor line.

The modelling of the water quality in Section 2.1.13 is very preliminary. It is developed without knowledge of how much oxygen there must be in the water to ensure optimal fish welfare. The quality model is in this thesis is a linear model based on the relative water flow, but the utilization of oxygen and pollution of the water clearly should be of a higher order. A more correct model based on the oxygen saturation, water pollution and relative water velocity should be developed. The minimum quality of the water in the simulation should also be based on the real welfare of the salmon.

The measurements in Section 2.1.14 are also simplified. The in real life the measurement noise or measurement error does have a larger scale of error, but does not have as much high frequent noise as modelled in this thesis. Modelling of measurement noise can be improved by use a more real world like noise/error.

6.2 Control System Design

In this thesis, many aspects of a customized control system design for Havfarm 2 in operational calm conditions are investigated. The control system that is developed in this thesis consists of an observer, thrust allocation and a controller/optimizer. A more detailed discussion of the control system design and its integration into Havfarm 2 is given in Section 4.

The thrust allocation need to be done all over again, with a completely new approach. The model used in this thesis is a very simplified model. The observer in this thesis can be continued to be developed by improving the precision and decrease the oscillations. The approach that is used in this thesis could be continued, or another form of state estimator, like a Kalman Filter, could be developed. The controller/optimizer must be improved a lot. The recommended approach is to use control alternative 3, nonlinear MPC, for further development. This could be improved continuing to work on the optimizer itself, but the recommended action is to merging the different control subsystems and make them work together. The more specific answer could be to continue to develop a nonlinear 3 DOF MPC function by implementing an observer-like adaptive control that accounts for waves, wind, slowly varying forces and non-modelled terms. This should also include a thrust allocation that accounts for amongst other the desirable numbers of thrusters used and forbidden sectors. The merged control system needs to be developed in a precise and strategic manner.

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A Matlab Codes

Note that these Matlab codes are not developed for presentation. They may therefore be hard to understand.

A.1 Initialization function

```
%% INIT FILE
% This is an input file for simulation of DP on Havfarm
clc
clear all
%% Read output from Wamit
% vessel = wamit2vessel(filename,T_draught,Lpp,Boa,plot_flag)
vessel = wamit2vessel('H2_06',37,304.2,63,'1111');
% load H2_06.mat
vesselABC = vessel2ss2(vessel);
% load H2_06ABC.mat
vesselABC.Ar{1,5}(1,3)= -vesselABC.Ar{1,5}(1,3); % Prevention of positive
    eigenvalues in the fluid memory effects.
vesselABC.Ar{5,1}(1,3)= -vesselABC.Ar{5,1}(1,3);
M_A = vessel.A(:,:,1);
MRB = vessel.MRB;
M = MRB + M_A;
M_inv = inv(M);
%% Environment
%Current:
nu_c = [-0.3;0;0];
V_c = sqrt(nu_c(1)^2+nu_c(2)^2);
beta_c = (180/pi)*atan2(nu_c(2),nu_c(1));

% beta_c = 180; %degrees
% V_c = 0.3;


% Waves
Hs = 1;
Tp = 7;
w_0 = 2*pi/Tp; %rad
wave_dir = pi; %rad

% Wind
wind_velocity = 5;
wind_dir = -pi/3; %rad

%% Measurements
% Measurement noise gains
init_noise = 0.1;
measureGPS = 0.1;
measurePSI = 0.1*pi/180;
measureNUR = 0.01;
measureQ = 0.005;


%% Observer parameters
M3x3 = diag([M(1,1), M(2,2), M(6,6)]);
invM = inv(M3x3);
D_v = diag([vessel.Bv(1,1),vessel.Bv(2,2) , vessel.Bv(6,6)]);
K5 = 0.5 * diag([1,1]);
K6 = 1;
K7 = 0.5;
T_c =diag([2000,2000]);
invT_c = inv(T_c);

%% Controller
zeta = 0.7;
Tn = 100;
```

```
wn = 2*pi/Tn;

Kd = 2*zeta*wn*M3x3;
Kp = wn^2 *M3x3 *3;
Ki = wn*Kp/20 *0.5;

%% Thrust Allocation and thruster dynamics
Lx = 140;
Ly = 25;
Lz = 40;
BT = [1,0,1,0,1,0,1,0;
     0,1,0,1,0,1,0,1;
     Ly, Lx, -Ly, Lx, Ly, -Lx, -Ly, -Lx]; % mapping from forces from eight thrusters to a 3 DOF force vector

BTinv = pinv(BT); %Pseudo inverse, mapping from 3 DOF force vector to the eight thrusters

BT6 = [1,0,1,0,1,0,1,0;
     0,1,0,1,0,1,0,1;
     0,0,0,0,0,0,0,0;
     0,Lz,0,Lz,0,Lz,0,Lz;
     -Lz,0,-Lz,0,-Lz,0,-Lz,0;
     Ly, Lx, -Ly, Lx, Ly, -Lx, -Ly, -Lx]; %Mapping from eight thrusters to 6 DOF force vector

T_T = 2; %time constant Thruster
wt = 2*pi/T_T;

%% Reference Model, Used for path development and path following
% S = [0,0,0;0,0,0];%Setpoints
% [At,Vt,Pt] = initrefmod(S);

%% Water quality model
%nu_r is positive when vessel is moving forward and to starbord --> U_rel
% inside cages moves aft and port.
r_net = 0.86;
r_protect = 1;
r_columns = 0.95;
r_quality = 0.85;
nFishcages = 3;
nNet_long = 2*nFishcages -1;


R_x = r_net^nNet_long * r_protect * r_columns * r_quality^(nFishcages -1);
R_y = r_net * r_columns;

%% Development of mooring line
g = 9.81;
h = 600; %m, depth
l = 950; %total length of line
x = 100; % length that lies on the bottom
line_weigth = 150000;

% Calculations
ls = l-x; %length of line exept on bottom
w = line_weigth*g/ls; %weigth force per unit mooring line length
T_H_max = 10e5*g; %N
% T_H = linspace(1,T_H_max,100);
T_H = logspace(log10(1),log10(T_H_max),100);
a = T_H/w;
X = l-h*(1+2*a/h).^(1/2) + a.* acosh(1+h./a);

X0 = linspace(0,350,100);
T_H0 = linspace(0,1,100);
T_H2 = [T_H0 T_H];  %Needed for fit function to work
X2 = [X0 X];

y = fit(X2',T_H2','poly8'); % Used as mooring characteristic referance in CasADi
```

```
% Mooring line dynamics:
M_T = 3; %time constant Thruster
M_wt = 2*pi/M_T;

figure
plot(X,T_H);% Mooring characteristics
hold off


%% PID regulator gains
Q_Kp = 1e7;
Q_Ki = 45000;
Q_Kd = 1e7;


%% Simulation:

% init_pos = [-570;500;0;0;0;0.4]; % Good for MPC_3 Dyn programming
% init_pos = [-650;300;0;0;0;0.5]; % GOOD for PID
% init_pos = [-720;350;0;0;0;0.7]; % OK for 1dof ref
init_pos = [-550;590;0;0;0;0.4]; % OK MPC_8 ref + control + 1Dofcontroll
% init_pos = [-600;500;0;0;0;0.4]; % Try sim 04 if time.
init_pos3D = [init_pos(1);init_pos(2);init_pos(6)];
init_x = [init_pos3D;0;0;0];
Q_min = 0.2;
N = 100;
time = 200;
start = 4;
%Optimization
[N_d, E_d, psi_d, u_d,v_d,w_d,ux_d,uy_d,upsi_d,time,N] = MPC_8(init_x,nu_c,D_v,M3x3
    ,R_x,R_y,Q_min,N,time,start);
% [psi_d,w_d,nu_r_time,upsi_d,time,N] = MPC_3([0;0],nu_c,D_v,M3x3,R_x,R_y,Q_min
    ,300,300,200);
% optheading = psi_d(end);

simTime = 2000;
SampleTime = 0.1;
sim H2_08.slx
Simulink.sdi.view
```

A.2 MPC - Main function

```
function [N_d, E_d, psi_d, u_d,v_d,w_d,ux_d,uy_d,upsi_d,T,N] = MPC_9(x_last,nu_c,D,
    M,r_x,r_y,Q_min,N,T,start)
% NONLINEAR MPC FUNCTION, Revision 9.
% This is a 3 DOF Nonlinear MPC optimization function. That use CasADi Opti
% Stack and ipopt solver.

close all
opti = casadi.Opti();        % Optimization problem

% ---- decision variables ---------
X = opti.variable(6,N+1);    % Declaration of state variables
north = X(1,:);
east = X(2,:);
psi = X(3,:);
u = X(4,:);
v = X(5,:);
w = X(6,:);

Q = opti.variable(1,N+1);    % Declaration of Quality variable
s = opti.variable(1,N+1);    % Declaration of Slack variable

tau = opti.variable(3,N);    % Declaration of Control input variables
tau_x = tau(1,:);
tau_y = tau(2,:);
tau_psi = tau(3,:);


% ---- objective ---------
% minimize control input squared + slack variable
opti.minimize(sum(1e-2*tau_x.^2) + sum(1e-2*tau_y.^2) + sum(1e-3*(tau_psi*(1/150))
    .^2)+ 1e3*sum(s));



% ---- boundary conditions --------
opti.subject_to(north(1)==x_last(1));   % start at last position
opti.subject_to(north <=x_last(1)+500);
opti.subject_to(north >=x_last(1)-500);
opti.subject_to(east(1)==x_last(2));    % start at last position
opti.subject_to(east<=x_last(2)+500);
opti.subject_to(east>=x_last(2)-500);
opti.subject_to(psi(1)==x_last(3));     % start at last position
opti.subject_to(psi>=-pi);
opti.subject_to(psi<=pi);               % Force heading to be less than pi
opti.subject_to(u(1)==x_last(4));       % start at last position
opti.subject_to(v(1)==x_last(5));       % start at last position
opti.subject_to(w(1)==x_last(6));       % start at last position
opti.subject_to(w>= -pi/30);            % Prevent rapid heading change
opti.subject_to(w<= pi/30);             % Prevent rapid heading change

max_psi = 4.5*150;
opti.subject_to(tau_x >= -5.4);         % Upper and lower boundary conditions for
    control input
opti.subject_to(tau_x <= 5.4);
opti.subject_to(tau_y >= -4.5);
opti.subject_to(tau_y <= 4.5);
opti.subject_to(tau_psi >= -max_psi);
opti.subject_to(tau_psi <= max_psi);

max_change = 5; %Max Change every timestep in Yaw moment
for i = 2:N
    delta_tau_psi = tau_psi(i)-tau_psi(i-1);
    opti.subject_to(delta_tau_psi <= max_change);
    opti.subject_to(delta_tau_psi >= -max_change);
end


dt = T/N; % length of a control interval
```

```
for k=1:N % loop over control intervals
   % Runge-Kutta 4 integration
   k1 = f8(X(:,k),          tau(:,k),nu_c,M,D);
   k2 = f8(X(:,k)+dt/2*k1, tau(:,k),nu_c,M,D);
   k3 = f8(X(:,k)+dt/2*k2, tau(:,k),nu_c,M,D);
   k4 = f8(X(:,k)+dt*k3,   tau(:,k),nu_c,M,D);
   x_next = X(:,k) + dt/6*(k1+2*k2+2*k3+k4);
   opti.subject_to(X(:,k+1)==x_next); % close the gaps
end

%------ Inequality constraint------
for i = 1:N+1
    R3 = [cos(psi(i)) -sin(psi(i)) 0;
          sin(psi(i)) cos(psi(i))  0;
          0          0           1];
    nu3 = [u(i);v(i);w(i)];
    nu_r3 = nu3 - (R3')*nu_c;
    Q(i) = r_x *abs(nu_r3(1)) + r_y *abs(nu_r3(2));

    opti.subject_to(nu_r3(1) >= 0);% Prevent relative reversing
    opti.subject_to(nu_r3(1) <= 0.4);% Prevent too large velocities

    % Quality constraint including slack variable
    opti.subject_to(Q_min - Q(i)-s(i) <= 0); %
    opti.subject_to(s(i)>=0);
end

% %Alternative before slack variable was introduced
% for i = start:N+1
%     opti.subject_to(Q(i) >= Q_min);
% end




% ---- initial values for solver ---
opti.set_initial(north, ones(1,N+1)*x_last(1));
opti.set_initial(east, ones(1,N+1)*x_last(2));
optpsi = [linspace(x_last(3),pi/2,start) ones(1,N-start+1)*(pi/2.5)];
opti.set_initial(psi, optpsi);
opti.set_initial(u, ones(1,N+1)*x_last(4));
opti.set_initial(v, ones(1,N+1)*x_last(5));
optw = [ones(1,start)*0.003 zeros(1,N-start+1)];
opti.set_initial(w, optw);
opti.set_initial(tau_psi,ones(1,N)*max_psi);
opti.set_initial(tau_x,zeros(1,N));
opti.set_initial(tau_y,zeros(1,N));


% ---- solve NLP              ------
options = struct;
options.ipopt.print_level = 0;
opti.solver('ipopt', options);
sol = opti.solve();   % actual solve

%% ---- post-processing      ------


t = linspace(0,T,N+1);
tu = linspace(0,T,N);

N_d = sol.value(north)';
E_d = sol.value(east)';
psi_d= sol.value(psi)';

u_d = sol.value(u)';
v_d = sol.value(v)';
w_d = sol.value(w)';
```

```
ux_d = sol.value(tau_x)';
uy_d = sol.value(tau_y)';
upsi_d = sol.value(tau_psi)';


eta_time(:,1) = sol.value(north)';
eta_time(:,2) = sol.value(east)';
eta_time(:,3) = sol.value(psi)';

nu_time(:,1) = sol.value(u)';
nu_time(:,2) = sol.value(v)';
nu_time(:,3) = sol.value(w)';


u_time(:,1) = sol.value(tau_x)';
u_time(:,2) = sol.value(tau_y)';
u_time(:,3) = sol.value(tau_psi)';

q_time(:,1) = sol.value(Q)';

q = zeros(N+1,1);
nu_r_time = zeros(N+1,3);
for i = 1:N+1
    psi2 = eta_time(i,3);
    R2 = [cos(psi2) -sin(psi2) 0;
        sin(psi2) cos(psi2)  0;
        0         0          1];
    nu_r = nu_time(i,:)'- R2' *nu_c;


%     q(i) = r_x *abs(nu_r(1)) + r_y *abs(nu_r(2));
%     q(i) = abs(nu_r(1)) + abs(nu_r(2));

    nu_r_time(i,1) = nu_r(1);
    nu_r_time(i,2) = nu_r(2);
    nu_r_time(i,3) = nu_r(3);
end

figure
hold on
subplot(4,3,1)
plot(t,sol.value(north));
xlabel('Time [s]');
ylabel('North');

subplot(4,3,2)
plot(t,sol.value(east));
xlabel('Time [s]');
ylabel('East');

heading = sol.value(psi).*180/pi;
subplot(4,3,3)
plot(t,heading);
xlabel('Time [s]');
ylabel('Heading [deg]');

subplot(4,3,4)
plot(t,sol.value(u));
xlabel('Time [s]');
ylabel('u-velocity');

subplot(4,3,5)
plot(t,sol.value(v));
xlabel('Time [s]');
ylabel('v-velocity');

heading_vel = sol.value(w)*180/pi;
subplot(4,3,6)
plot(t,heading_vel);
```

```
xlabel('Time [s]');
ylabel('Rotation velocity [deg/s]');

subplot(4,3,7)
plot(t,nu_r_time(:,1));
xlabel('Time [s]');
ylabel('u-rel');

subplot(4,3,8)
plot(t,nu_r_time(:,2));
xlabel('Time [s]');
ylabel('v-rel');


subplot(4,3,9)
plot(t,q_time);
xlabel('Time [s]');
ylabel('Quality');

subplot(4,3,10)
plot(tu,sol.value(tau_x));
xlabel('Time [s]');
ylabel('surve force [MN]');

subplot(4,3,11)
plot(tu,sol.value(tau_y));
xlabel('Time [s]');
ylabel('sway force [MN]');


subplot(4,3,12)
plot(tu,sol.value(tau_psi));
xlabel('Time [s]');
ylabel('yaw moment [MNm]');

set(figure(1),'WindowState','maximized');
hold off
```

A.3 MPC - Propagation

```
function x_dot = f8(x_last,u,nu_c,M,D)  % Calculate the derivative of the states
    psi = x_last(3);                    % Heading
    R = [cos(psi) -sin(psi) 0;          % Rotation matrix
        sin(psi) cos(psi)   0;
        0        0          1];

    nu = x_last(4:6);                   % Body frame velocity
    nu_r = nu - R'*nu_c;                % Body frame velocity relative to the water

    %-----------Calculate mooring forces----
    length = sqrt(x_last(1)^2 + x_last(2)^2); % Distance from anchoring point
    % line_force = 3.162e-35*length^14.13; % Early stage fit function
      % There are used polyfit of of the mooring characteristics, as the
      % matlab function "interp1" are not supported when using CasADi.
        p1 = 9.013e-15;
        p2 =  -2.696e-11;
        p3 =   3.286e-08;
        p4 =  -2.096e-05;
        p5 =    0.007485;
        p6 =      -1.479;
        p7 =       148.3;
        p8 =       -6111;
        p9 =   5.707e+04;
    line_force = p1*length^8 + p2*length^7 + p3*length^6 + p4*length^5 +...
        p5*length^4 + p6*length^3 + p7*length^2 + p8*length + p9;


    North = -line_force*x_last(1)/length;
    East = -line_force*x_last(2)/length;

    NED = [North,East,0]';
    tau_moor = R'* NED;
    tau_moor(3) = 150*tau_moor(2);


    % ------ Calculate the derivatives------
    eta_dot = R*nu;
    nu_dot = inv(M)*(u*1e6 + tau_moor -D*nu_r);


x_dot = [eta_dot;nu_dot];
end
```

B Simulink setup

B.1 Simulink Overview

Overview of the main Simulink system. The yellow subsystems are parts of the control System.

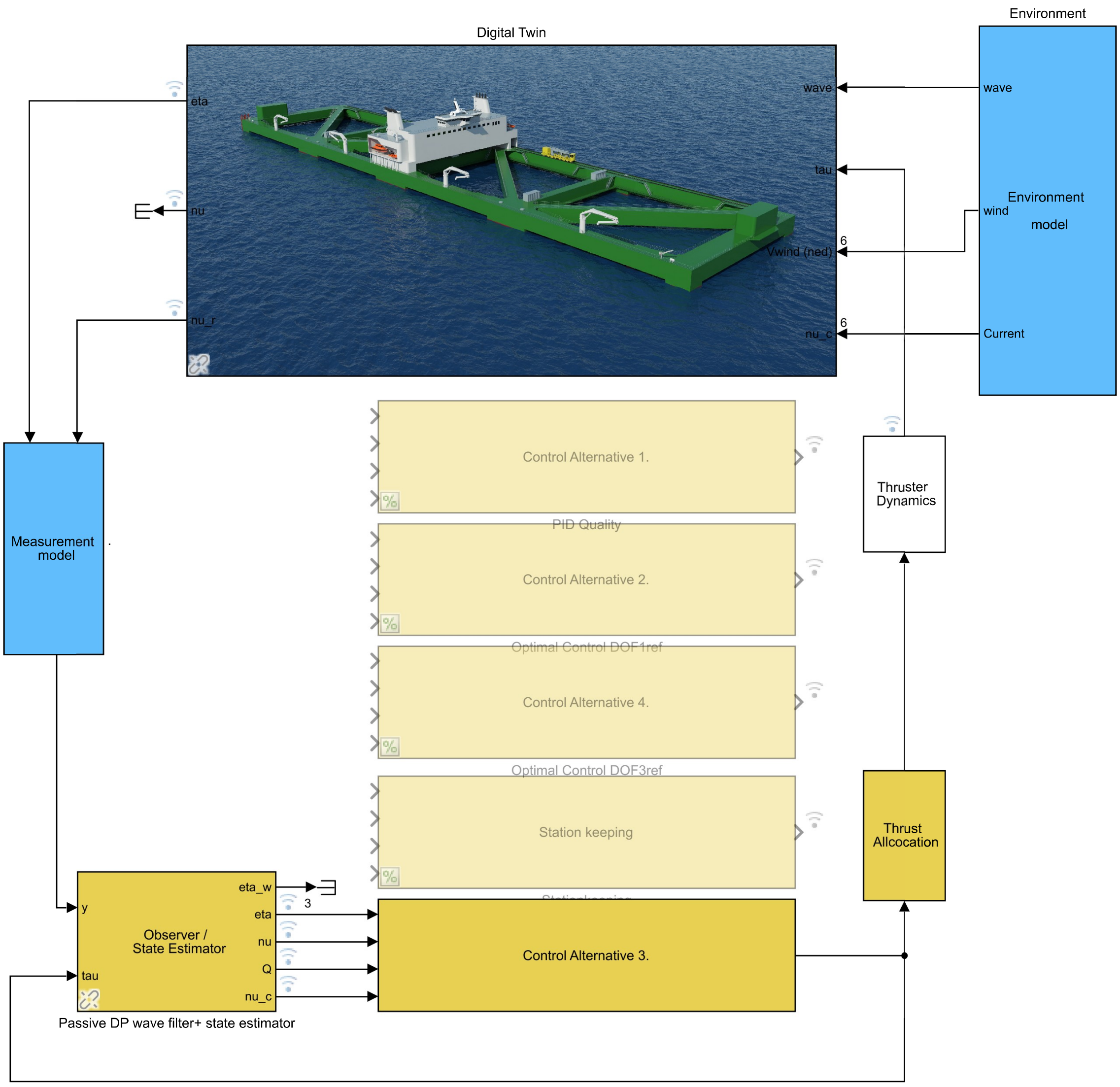

Figure 37

B.2 Digital Twin, Equation of motion

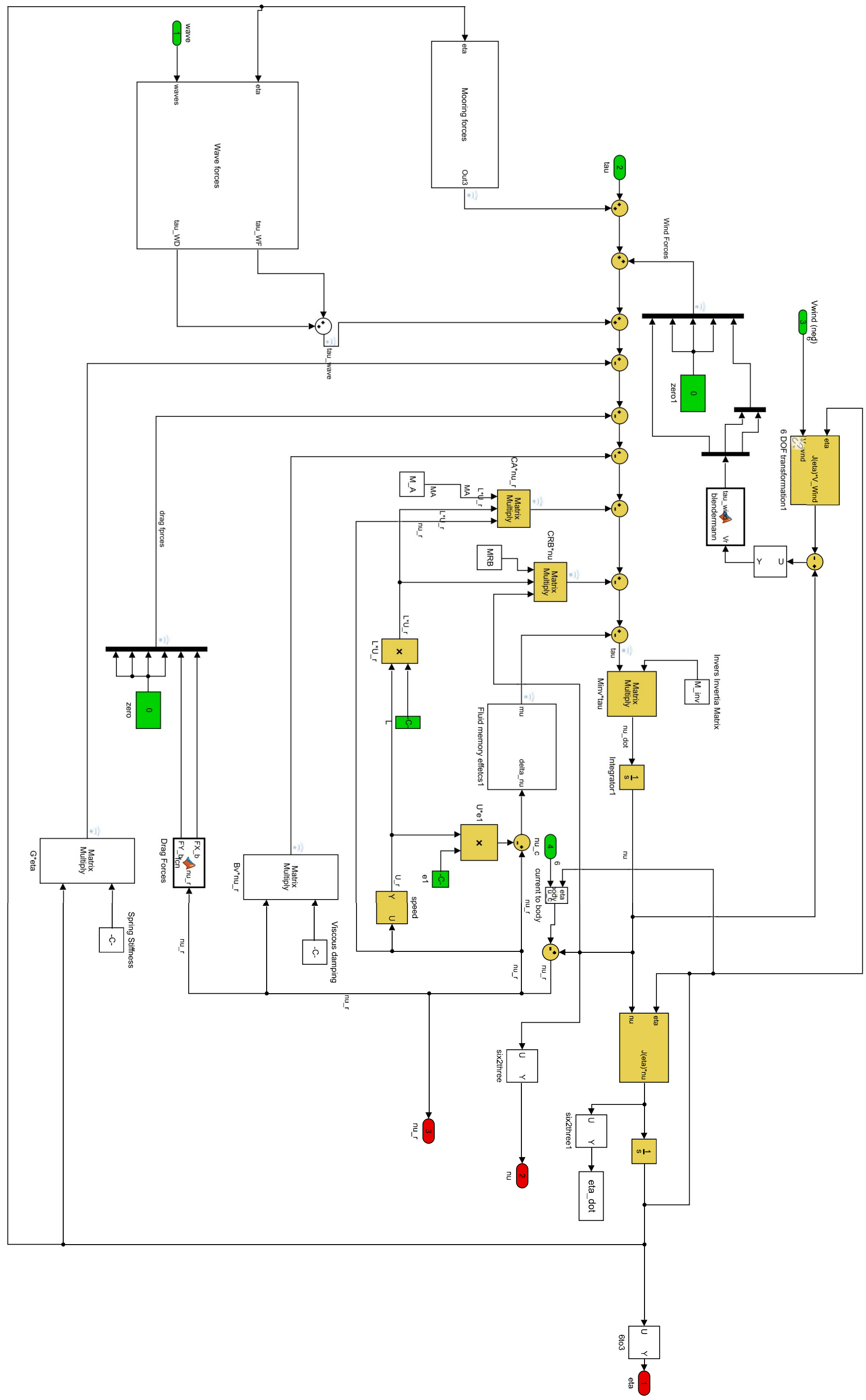

Figure 38

B.3 Observer / State Estimator

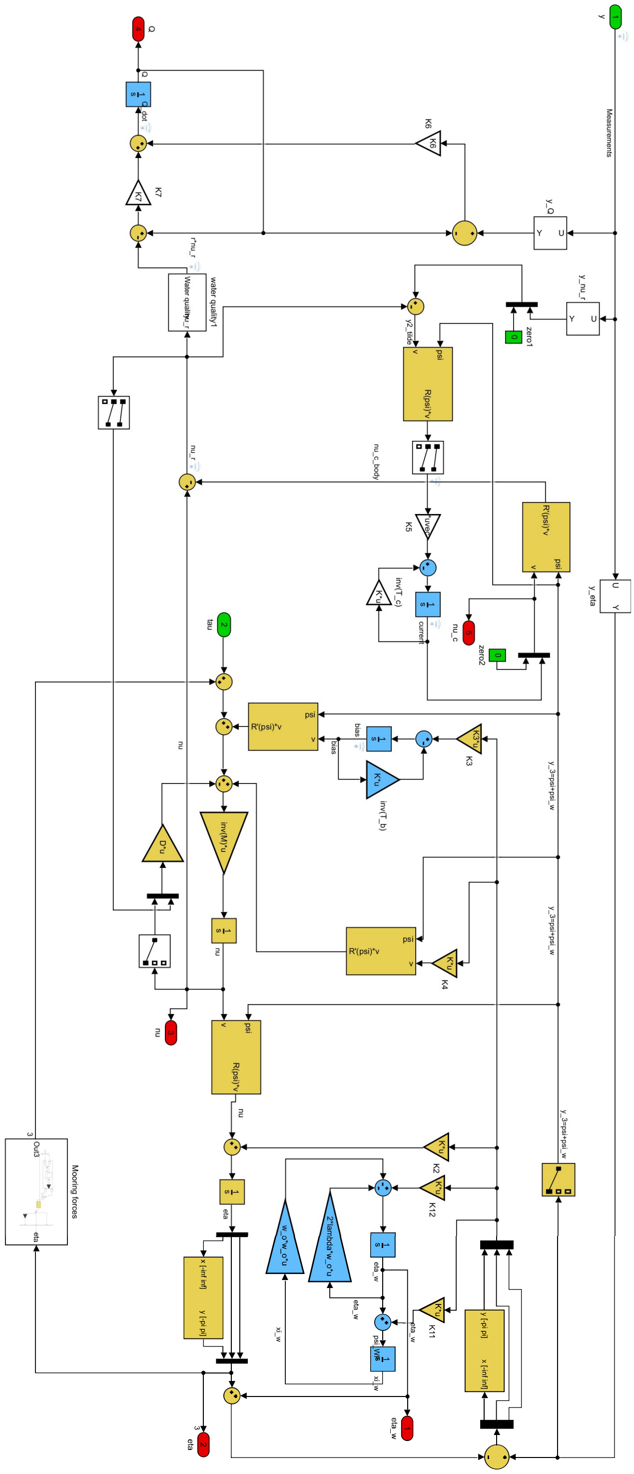

Figure 39

B.4 MPC Setup

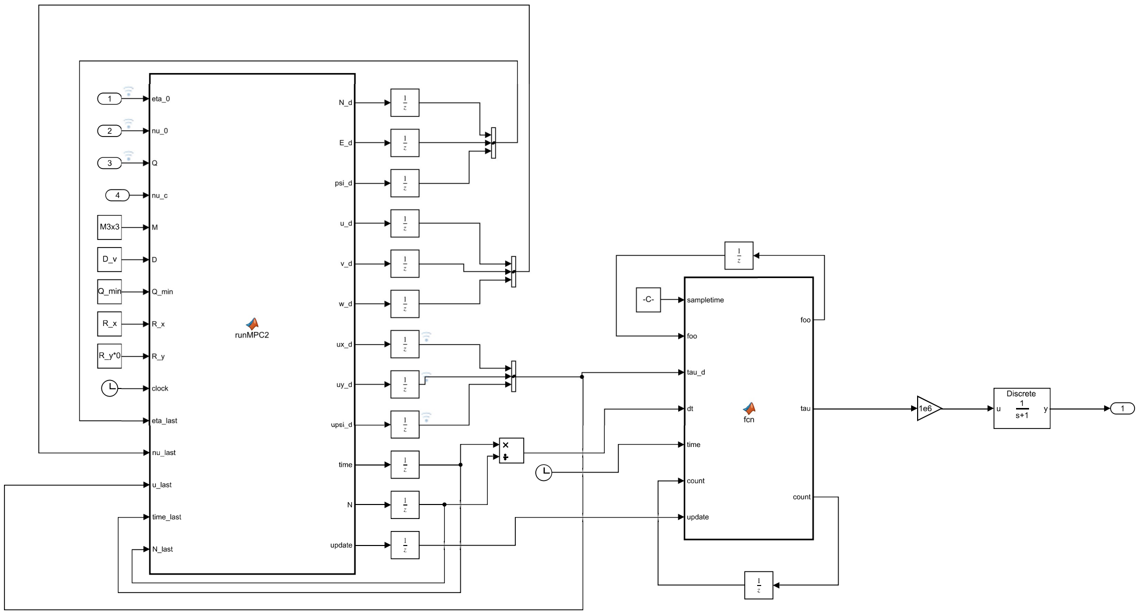

Figure 40

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