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Valuation and Risk Management of Paid-up Policies

Optimizing Investment and Buffer Strategies

Master's thesis in Industrial Economics and Technology Management
Supervisor: Verena Hagspiel & Michiel Janssen
June 2019

Master's thesis

NTNU
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1 Abstract

Decreasing interest rates have significantly reduced the profitability and increased the risks on pension products offering a guaranteed annual return. In Norway, one such product is paid-up policies and account for more than 300 billion NOK of the total pension savings. Through a complex profit-sharing system, the insurer has the option to set aside reserves in two buffers, or share profit with the customers when returns are sufficiently high. The system gives the insurer numerous options to manage paid-up policies. However, the increased risk has led to the insurers reducing their share of equity in the portfolio, resulting in policyholders rarely receiving profit above the guaranteed rate. Over time, their policies therefore lose value due to inflation and salary increases.

We derive the market value of paid-up policies given different investment and buffer strategies using a combination of the Black-Scholes and Hull-White One Factor Model. In addition, we analyze the impact of these strategies on both the risk and capital requirements under Solvency II. Finally, we analyze how the insurer can manage the risks using a separate hedge portfolio.

Our results give valuable insight in the management of paid-up policies. We find that insurance companies minimize the risk and market value of the liabilities by making investments in bonds only. However, almost the same market value and capital requirements can be achieved using a CPPI investment strategy where the equity weight is adjusted according to the size of the buffers. This strategy leads to more frequent profit-sharing, which makes the policies more valuable for the customers.

2 Sammendrag

Fallende markedsrenter har ført til en markant reduksjon i lønnsomheten og økt risikoen på pensjonsprodukter med en garantert, årlig avkastning. I Norge er fripoliser et eksempel på et slikt produkt, og utgjør mer enn 300 milliarder norske kroner av de samlede pensjonsforpliktelsene. Gjennom et komplekst profittdelingssystem har forsikreren mulighet til å avsette reserver i to buffere eller dele profitt med kundene dersom avkastningen er tilstrekkelig høy. Systemet gir forsikreren mange muligheter i forvaltningen av fripolisene. Den økte risikoen har imidlertid ført til at livselskapene har redusert aksjeandelen i kollektivportføljen, slik at kundene sjelden får overskudd over garantert rente. Over tid reduseres derfor verdien av fripolisene grunnet inflasjon og lønnsøkninger.

Vi bestemmer markedsverdien av fripoliser gitt ulike investerings- og bufferstrategier, ved å bruke en kombinasjon av Black-Scholes og Hull-White enfaktormodell. I tillegg analyserer vi hvordan de samme strategiene påvirker risikoen og kapitalkravene livselskapene må oppfylle gjennom Solvens II-direktivet. Avslutningsvis analyser vi hvordan livselskapene kan redusere risiko ved å bruke en egen sikringsportfølje.

Resultatene våre gir verdifull innsikt i forvaltningen av fripoliser. Vi finner at livselskaper maksimerer markedsverdien og minimerer risikoen ved kun å investere i obligasjoner. Omtrent den samme markedsverdien og de samme kapitalkravene kan imidlertid oppnås gjennom en CPPI-investeringsstrategi hvor aksjeandelen i porteføljen justeres i henhold til bufferstørrelsen. Denne strategien fører til mer profittdeling, noe som gjør fripolisene mer attraktive for kundene.

3 Preface

This master's thesis within Financial Engineering is written as a part of our Master of Science at the Norwegian University of Science and Technology, Department of Industrial Economics and Technology Management. We first want to express our sincere gratitude to our academic supervisor, Associate Professor Verena Hagspiel, for thorough feedback and rich discussions throughout the process. The work could also not have been done without the guidance of our industry partner Michiel Janssen in EY. Thank you for your patience in answering all our questions, even if it was Friday night. Lastly, we wish to thank Investment Manager in Nordea Liv, Nina Fiskaaen, for giving valuable insight into the insurance business and providing us with relevant data.

Erlend Gjerdevik Sørtveit & Bjarte Knutsen Espedokken

Trondheim, June 10th 2019

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4 Introduction

Due to decreasing interest rates during the latest ten years, pension products offering a guaranteed rate of return to customers have become less valuable for insurance companies. In Norway, one of these products is called paid-up policies and constitutes the dominant pension product in the market. The policies account for more than 300 billion NOK, equivalent to approximately 50% of all pension liabilities.

Paid-up policies give the policyholders at least a guaranteed annual rate of return on their pension savings. An insurance company is responsible for fulfilling the guaranteed rate of the policy and invests assets backing the policies in the financial markets. If the yearly book return is sufficiently high, the insurer has the option to set aside reserves in different buffers. In addition, the policies are part of a complex profit-sharing system, where returns can be shared between policyholders and the insurer. The investment opportunities and the profit-sharing system give the insurer various investment and buffer strategies to fulfill the guaranteed rate.

As of 2016, the insurers also have to comply with capital requirements of the Solvency II directive. The regulatory framework requires insurance companies to hold a level of capital such that they are able to ensure coverage for the liabilities in one year with a 99.5% probability. The choices an insurer makes regarding investments and buffers affect the capital requirements, and increase the importance of an effective risk management of paid-up policies. This was illustrated in 2017, when the Norwegian insurance company Silver failed to comply with the capital requirements. After financial authorities intervened, Silver eventually ended up bankrupt while another insurance company acquired all of Silver's paid-up policies.

The goal of this thesis is to gain valuable insight on how we can use investment and buffer strategies to improve insurers' management of paid-up policies. We want to analyze how these strategies affect the market value of the policies, both from the insurer's and policyholders' perspective. The objective here is to maximize the market value for the insurer. As insurers must comply with capital requirements, we also want to analyze how different strategies influence these requirements, and look at how insurers can handle risk effectively. In these analyses, we focus on minimizing the capital requirements.

In order to quantify how specific decisions regarding buffer and investment strategies affect the market value and risk associated with paid-up policies, we run simulations. Risk-neutral simulations are needed to determine the market value of the policies, while the risk associated can only be derived through a combination of risk-neutral and real-world simulations. Using a combination of the Black-Scholes and Hull-White One Factor Model to simulate the evolution in stock prices and interest rates, we estimate the market value of paid-up policies in the Norwegian pension market, from the insurer's and policyholders' perspective. The model also determines the one-year risk associated with the paid-up policies. In the risk management of the policies, we examine if a hedge portfolio, consisting of derivatives, which hedge against considerable drops in the stock market or interest rate, can be beneficial for the insurer.

From an international perspective, many countries have products that offer a guaranteed rate of return. In countries like Germany, the Netherlands and the UK there exist pension schemes guaranteeing that the pensions will keep pace with specified indexes. In the UK, for instance, they use the Consumer Prices Index (CPI) to determine the pace on the pensions. The goal is to protect the pension payments against inflation. This stands in contrast to the paid-up policies in Norway, as these offer a constant guaranteed rate of return. In addition, the insurance companies that offer the policies in Norway have to cope with the specific, Norwegian profit-sharing system.

The differences between the pension products, and the regulations within the countries they are offered, make it hard to develop a universal model that can be used for valuing and analyzing the risk associated. However, several studies attempt to value specific products offering a guaranteed rate of return.

Hipp (1996), for example, applies Monte Carlo simulations to price and evaluate the risk of options for guaranteed index-linked life insurance. This is a product where the policyholder receives a return equal to the return of a reference portfolio, or at least a guaranteed minimum payment. Using the Black-Scholes formula for the evolution in the stock price, and a constant interest rate, Hipp (1996) is able to value the guarantee.

Guaranteed index-linked life insurance is in some sense similar to Norwegian paid-up policies, as it also gives the policyholder at least a guaranteed payment. However, the analysis of Hipp (1996) does not include aspects like profit-sharing, buffers and investment strategies, which are important parts of the Norwegian policies. In addition, Hipp (1996) considers products which pays a guaranteed amount at maturity, and where there are no annual rate which has to be reached. This differs from the paid-up policies in Norway, where the insurer has to reach a guaranteed rate of return each year.

Hipp's model has been extended by Aase and Persson (1996) and Miltersen and Persson (1999) to also account for interest rates that evolve randomly over time and include a so-called multi-period guarantee. This means that the contract specifies a binding guarantee for each subperiod. The Norwegian paid-up policies have the same dynamics, where the insurance companies have to fulfill an annual guarantee.

Aase and Persson (1996) also extend Hipp's model to include a participating policy, where the policyholder is entitled to a share of the surplus if the realized interest rate during the insurance period is above the assumed interest rate. This is similar to the profit-sharing system used for paid-up policies. However, as the model does not take into account any buffers, it cannot be fully used to value the policies in Norway. In addition, the model only considers bonds, while insurance companies also have the option to invest in other asset classes.

Miltersen and Persson (1999) values multi-period guarantee with different underlying return processes. They derive one valuation formula for a stock process with deterministic interest rate, and one where the guarantee is on a short-term interest rate return process. However, the model does not include an option to invest in a portfolio consisting of both stocks and bonds simultaneously. This is necessary to analyze paid-up policies, as insurance companies normally invest in both of these asset classes. In addition, the model does not include any profit-sharing system.

Also Bakken, Lindset, and Olson (2006) develops a model to price multi-period rate of return guarantees. The model works for stochastic interest rates, and the evolution in the price of the underlying assets (stocks and bonds) are modeled using one single differential equation. This implies that the assets are invested in stocks and bonds simultaneously. However, the bond and stock weight in the portfolio cannot be adjusted, which means that the model would not make it possible to compare different investment strategies. Also, Bakken et al.'s (2006) model does not include a profit-sharing system.

Other studies, like Pennacchi (1999), attempt to value products for which the pension benefits paid to customers are linked to the performance in the equity market. Pennacchi (1999) values the minimum pension benefit guarantee for customers with a defined contribution pension plan. In this pension scheme, the benefit to customers has no guaranteed payment as the benefit is dependent on the returns of the pension assets invested in the market. However, in countries such as Chile and Argentina, the government still guarantees a minimum floor of the benefits, which is similar to the guaranteed benefits offered on paid-up policies in Norway. Pennacchi (1999) derives the risk-neutral expected value of these government guaranteed payments by Monte Carlo simulations in his valuation model. The model developed takes into account stochastic pension contributions and a stochastic rate of return on the pension assets. However, the pension assets only evolve according to one geometric brownian motion as in Miltersen and Persson (1999), which was argued to be a shortcoming above. In addition, the short-term real interest rate is derived using the Vasicek (1977) model, which includes a randomly evolving interest rate with mean reversion. This short-rate model is, however, not set to fit the current term structure of interest rates. As the current market interest rates significantly influence the risk and return of paid-up policies, Pennacchi's (1999) model is unfit to value paid-up policies. Also, the model does not include a profit-sharing system.

The models mentioned above do not take into account all aspects needed to make a fair valuation of paid-up policies in Norway, and cannot be used for extensive risk analysis. To the best of our knowledge, academic studies which analyze Norwegian paid-up policies are scarce. The only one we are aware of is Høivik and Sæter (2011). They develop a model used for evaluating the risk associated with paid-up policies, and uses it to compare different investment strategies. However, their model can only be used to look at a paid-up policies in a one-year horizon. Their model will therefore not give a full picture of the risk associated with paid-up policies, as a risk analysis also has to take into account that the policies last for several years.

Høivik and Sæter (2011) make multiple simplifications in their model. For instance, the risk-free rate is assumed to stay constant, and one of the two main buffers is not considered. As the interest rate in reality changes over time and a paid-up policy lasts for several years, an annual model is not sufficient to fully analyze the dynamics of paid-up policies. In order to derive accurate analysis related to the Norwegian paid-up policies, we therefore need to develop a model which takes into account aspects like buffers and the profit-sharing system. The model also has to take into account that a contract lasts for several years, and both the interest rate and the stock price therefore have to be simulated. In order to avoid arbitrage opportunities, the simulated interest rate should also fit the current term structure of interest rates. All these aspects are included in the model we develop.

The results from our model indicate that the highest market value of paid-up policies for the insurer is achieved when all assets are invested in bonds. This investment strategy also gives the lowest capital requirements. However, a CPPI strategy where the equity weight is adjusted based on the current buffer size does not decrease the market value and increase the capital requirements considerably. In addition, a CPPI strategy leads to more profit distributed to customers compared to an investment strategy where all assets are invested in bonds. The model also suggests that building large buffers instead of sharing profit with the customers is always favorable for the insurer. Furthermore, we find that a *hedge portfolio* can be used to significantly reduce the capital requirements.

The reminders of this thesis are outlined as follows. In Section 5 we describe the Norwegian pension system and paid-up policies in detail. We then move on to Section 6 where the current discussion on paid-up policies is described. The capital requirements within the Solvency II directive are presented in Section 7. In Section 8 we introduce the model used to analyze the market value and risk of the policies. Section 9 explains how the model has been calibrated to current market data. The results are presented in Section 10. Section 11 concludes and gives some potential directions for further research.

5 Paid-Up Policies

5.1 The Norwegian Pension System

As illustrated in Figure 5.1, the Norwegian Pension System consists of three pillars (NAV, 2018a):

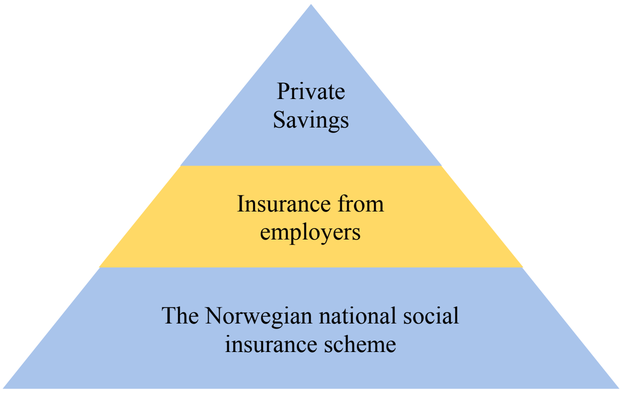

Figure 5.1: The Norwegian pension system consists of three pillars: Pension from the Norwegian national social insurance scheme, insurance from employers and private savings.

The total size of the pension received for a retiree in Norway is dependent on payments from the National Insurance Social Insurance Scheme ("Folketrygden"), the insurance from employers and private savings. A paid-up policy originates from pension contributions from employers, as highlighted in Figure 5.1.

5.1.1 The Norwegian National Social Insurance Scheme

The pension from the Norwegian National Social Insurance Scheme forms the basis of the Norwegian Pension System. All citizens are entitled to a certain pension in the years of retirement. The pension is determined by the citizen's historical salary, how long it has worked in Norway and marital status. However, the state also sets a minimum level of pension, calculated from a base amount ("G"). As of May 2018, the base amount was 96,883 NOK (1G) which gives at least a minimum annual pension payment of 153,514 NOK (NAV, 2018b).

5.1.2 Insurance from Employers

Employers also save pension for their employers during years of employment. There are two main types of occupational pension schemes in Norway, namely defined benefit and defined contribution.

Defined benefit ("Ytelsespensjon"): In this pension scheme, the pension amount received as a retiree is certain (Finansportalen, n.d.-b). The defined benefit is added on top of the amount an employee receives through the public pension system. The sum of these two equals a percentage of the salary at the time of retirement, known as the pension benefit. A typical pension benefit lies between 50-70% of the salary at time of retirement and depends on the employer's pension plan. The employer therefore saves pension during employment. The amount received as retired is fixed. This means the employer must set aside an amount during the years of employment, which is unpredictable for the employer as the salary at the time of retirement is uncertain.

When an employee retires or changes its employer, the pension amount saved up during employment will be converted to a paid-up policy, which is the focus of this thesis. At that point, an insurer takes over the responsibility of the paid-up policy and the employer has no longer any engagement in the policy. This product will be described in detail in the next section. We illustrate the connection between the defined benefit pension scheme and paid-up policies using an example. Imagine the employee retiring with the following parameters:

- Age: 67 (retirement age)
- Gross income: 800,000 NOK
- Defined benefit of 65%

In this case, the person is entitled to a yearly contribution of 520,000 NOK (65% of the salary) in pension. These guaranteed future cash flows constitute the pension product called *paid-up policy*. The paid-up policy consists of contributions from the state and the employer, saved up during the years of employment. An insurer takes on the responsibility to manage the paid-up policy and offer a guaranteed rate of return.

Defined contribution ("Innskuddspensjon"): The employer saves a fixed percentage of the employee's salary each month, decided by how large the salary is (NHO, n.d.). The defined contribution is added on top of the amount an employee receives through the public pension system. It is saved on an account, where the employee can decide a risk profile for investing the funds itself. For instance, the employee can have 100% in stocks or 20% in stocks and 80% in bonds. The risk profile is typically decided on the basis of how many years remain before retirement. The total return on the chosen investment profile will decide how much the policyholder receives at retirement.

Employers are allowed to change their employee's pension plan from defined benefit to defined contribution. When this is done, the amount saved up in the defined benefit plan will be converted to a paid-up policy. Most of the companies in Norway have in the latest years made the change, and started offering their employees defined contribution instead of defined benefit (Finans Norge, 2017). The effect is that the total value of paid-up policies increases. However, as companies stop offering the defined benefit scheme, the number of paid-up policies will decrease in the future. This will be discussed more in the next section.

5.2 About the Paid-up Policies

A paid-up policy is a pension insurance from a current or previous employment relationship (Storebrand, 2018a). After issuing the policy, the insurance company is responsible for giving the policyholder a fixed annual rate of return on her pension savings. This guaranteed rate is set when the policy is issued, and the insurer has the possibility to earn profit through a complex profit-sharing system in years where the return is above the guaranteed rate.

Historically, paid-up policies have been issued when an employee has resigned from a corporation which used defined benefit pension schemes. In addition, employees also get paid-up policies when employers terminate this type of pension plan, as discussed above.

The total value of the Norwegian paid-up policies was 308.8 billion NOK by October 2018 (Finans Norge, 2018b), and is still increasing. In about five years the value is expected to reach 490 billion NOK before it starts decreasing (Hippe & Lillevold, 2018). The major part of all paid-up policies is without investment choice, where an insurer manages the policy and the policyholder is guaranteed a specific annual rate of return. After September

1st, 2014, policyholders got the option to switch to a new policy product where they select an individual risk profile instead of receiving the guaranteed rate (Regjeringen, 2018). This new option was introduced to help insurance companies face new capital requirements, known as Solvency II (Finansdepartementet, n.d.). In short, Solvency II requires insurance companies to hold a level of capital such that they can face the 0.5% worst case scenario and still be able to pay all claims to policyholders. This will be further discussed in Section 7. As the guaranteed rate associated with paid-up policies increases the capital requirements for insurers, they benefit from clients switching to policies with investment choice. Then, the obligation for the insurer to fulfill a guaranteed rate disappears and the capital requirements are relaxed as the risk is transferred from the insurer to the policyholder.

Five years after the new type of paid-up policies was introduced, only 5% of the total value of all policies have been moved (Finans Norge, 2018a). In this thesis we will therefore solely focus on the paid-up policies without investment choice.

5.3 Collective Portfolio and Returns

All policyholders' pension savings become part of a collective portfolio managed by the insurance company. We will refer to this portfolio as the *policy portfolio*. As mentioned above, the insurer has to give the policyholders a guaranteed rate of return on their pension savings. In order to be able to pay this guarantee, the assets are invested in the financial markets.

To illustrate how the *policy portfolio* is used to back the insurance companies' liabilities to the policyholders, we consider a hypothetical insurer only offering paid-up policies. An illustration of the balance sheet for such an insurer is shown in Figure 5.2.

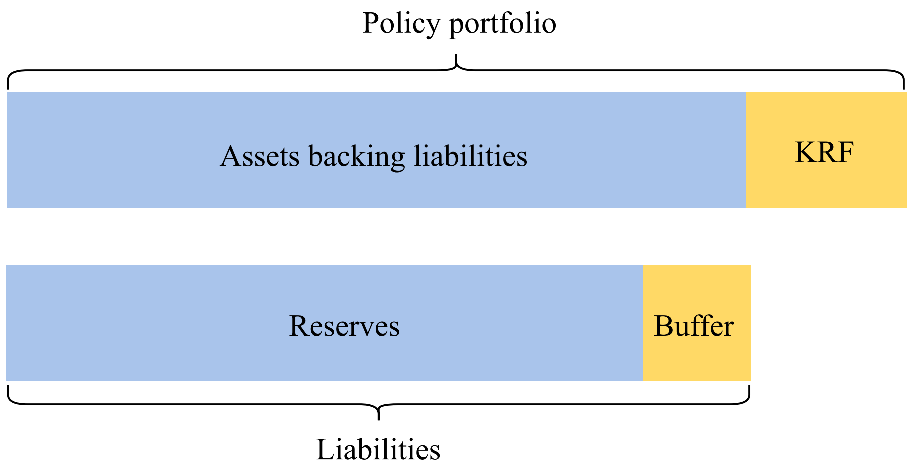

Figure 5.2: The insurer uses the *policy portfolio* to back the liabilities to the policyholders. The share of the portfolio that exceeds the liabilities becomes a part of a buffer known as the KRF.

As the figure demonstrates, the size of the *policy portfolio* has to be large enough to back the insurance company's liabilities to the policyholders. In other words, the value of the portfolio must be larger or equal to the sum of the reserves and a buffer known as the Additional Reserves ("Tilleggsavsetning/TA"). This buffer is hereby named TA and will be described in detail in Section 5.4. If the value of the portfolio exceeds the total liabilities, the additional value is part of the Value Adjustment Fund ("Kursreguleringsfond/KRF"), hereby named KRF.

To be able to discuss the options the insurer have for covering the guaranteed rate to the policyholders, we have to distinguish between unrealized return and book return. The book return is used to cover the guaranteed rate and constitutes the basis of the profit-sharing system used for paid-up policies, which is further examined in Section 5.4.

In years where the insurer has achieved a positive unrealized return, he has to determine the book return, and decide how much of the unrealized return to realize. After the guaranteed rate has been covered from the book return, the remaining, unrealized return automatically becomes part of the KRF. This fund can be realized in years where the return is insufficiently high, and serves therefore as a buffer for the insurer. If the KRF is large enough, the buffer enables him to fulfill the guaranteed rate in years of low return, as well. An illustration of the unrealized return and the book return is given in Figure 5.3.

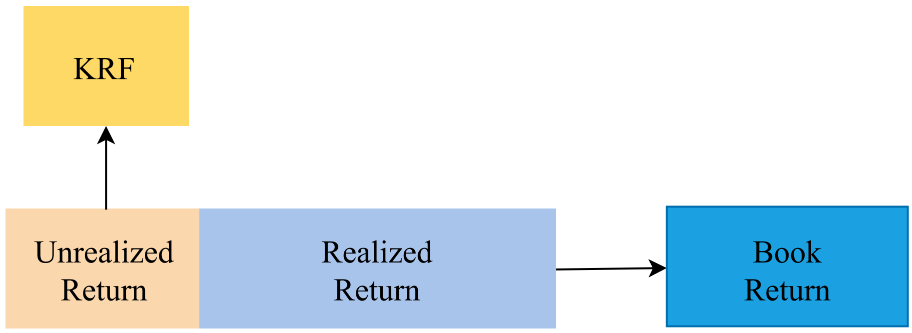

Figure 5.3: The unrealized return of the *policy portfolio* is the KRF, while the yearly realized return becomes the book return.

In the next subsection we discuss how the book return is used in the profit-sharing system, and what options the insurer has in years of both low and high return.

5.4 Profit-Sharing System

The insurance companies issuing paid-up policies have three sources of income from the product:

1. Administration charge
2. Risk result
3. Profit-sharing between policyholders and the insurer

Each year the insurer requires an administration charge from the paid-up policies. The size of the fee is fixed and dependent on the value of the policy (Finansportalen, n.d.-a). The risk result is determined by comparing the forecasted life expectancy of the population to the reserves the insurance companies set aside based on a base mortality. The risk result is positive when the actual mortality is lower than what was expected, which can be put in a risk equalization fund. The insurance companies can also benefit from a profit-sharing system in years where returns are sufficiently high, as the additional profit can be shared between the policyholders and the insurer.

In this thesis we only consider the profit-sharing system as a source of income. This follows from our main goal being to find optimal investment and buffer strategies, and see how options can be used to hedge the market risk associated the paid-up policies. In this discussion, the fixed administration charge and the risk result from longevity risk are not relevant. These income sources are therefore left out of the scope of our thesis.

The insurance companies managing paid-up policies are obliged to pay the policyholders an annual payment after they retire, known as the pension benefit. In order to fulfill the benefits, the companies have to set aside reserves. The reserves are increased by at least the guaranteed rate each year, depending on the annual book return. If the annual book return is higher than the guaranteed rate, the insurer has the option to transfer the additional profit into an additional reserve buffer, known as the TA. This buffer is assigned to the individual policyholder, and is therefore also a part of the insurer's liabilities.

The value of the funds transferred to the TA is equal to a certain percentage of the policyholder's reserves. The same percentage has to be transferred to all policyholders. However, the insurer can use a higher percentage for policyholders that have a TA less than 3.5% of the reserves. The TA can maximally reach 12% of the reserves for each policyholder (Regjeringen, 2018). The purpose of the TA is that it can be used to cover the guaranteed rate in years where the book return is too low. This will be described shortly.

In years where the book return on the assets is insufficiently high to cover the guaranteed rate, the insurer has to inject cash flows into the *policy portfolio*.

The possible scenarios for the profit-sharing system are:

- The book return is equal the guaranteed rate.
- The book return is higher than the guaranteed rate.
- The book return is non-negative, but lower than the guaranteed rate.
- The book return is negative.

In the following we present the options the insurer has in each of these scenarios. We refer to the choices the insurer makes concerning the buffers as the *buffer strategy*.

5.4.1 Book Return is Equal the Guaranteed Rate

The insurer must transfer all the returns to the reserves to fulfill the obligation of the guaranteed rate.

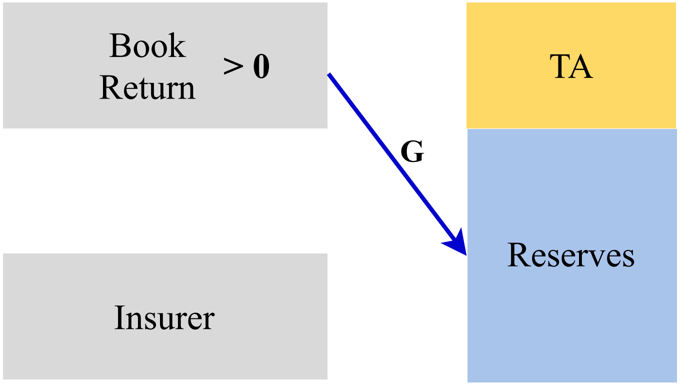

Figure 5.4: Book return is transferred directly to the reserves (blue line).

5.4.2 Book Return is Higher than the Guaranteed Rate

The insurer must transfer an amount equal the guaranteed rate to the reserves. The insurer has three options for the return exceeding the guaranteed rate:

1. Add the additional profit to the TA.
2. Share the additional profit between the policyholder and himself. In this case, the insurer keeps 20%, while the policyholder gets 80% of the additional profit (Regjeringen, 2018).
3. Combining the two options above.

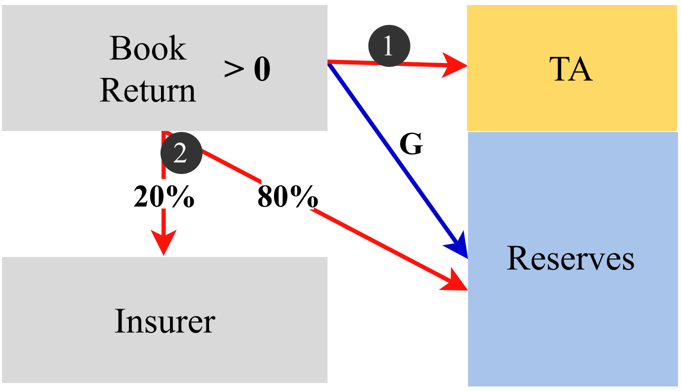

Figure 5.5: A share of the book return equal to the guaranteed rate is first transferred to the reserves (blue line). The additional profit can then be put in the TA (red line 1) or distributed between the insurer and the policyholders (red line 2).

The profit the customer receives from profit-sharing becomes part of his future pension payments. However, the customer does not receive a guaranteed rate on this part of the reserves.

5.4.3 Book Return is Non-negative, but Lower than the Guaranteed Rate

The book return is first transferred to the reserves. If the TA is large enough, the remaining part up to the guaranteed rate can be taken from this buffer (1). Otherwise, the insurer must compensate the deficit itself, by injecting cash flows into the portfolio (2).

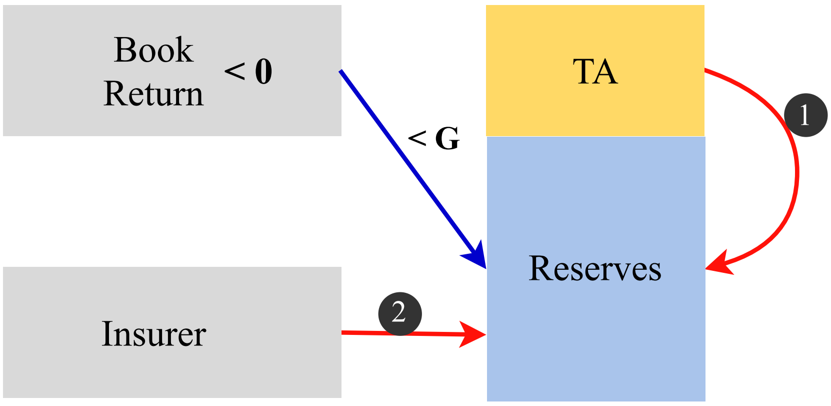

Figure 5.6: Book return is first transferred directly to the reserves (blue line). The residual can then be taken from the TA (green line 1) or compensated by the insurer (green line 2).

5.4.4 Book Return is Negative

The TA cannot be used to cover negative returns. Therefore, in this scenario, the insurer must compensate the negative part of the book return, shown by the blue line in Figure 5.7. The positive part to reach the guaranteed rate, however, can either be transferred from the TA (1) or be compensated by the insurer (2).

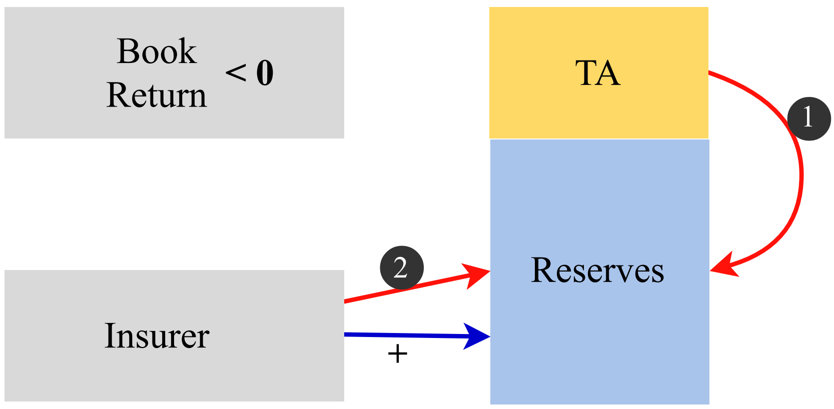

Figure 5.7: The insurer must cover the negative part of the book return (blue line). The residual can be transferred from the TA (red line 1) or compensated by the insurer (red line 2).

5.5 Investment Strategy

In addition to selecting a strategy on how to distribute the book return, the insurance companies have to decide how they want to invest the assets in the *policy portfolio*. There are three main types of investment strategies used by insurance companies (Actecan, 2011).

1. **Buy and Hold:** Buying a set of assets and keeping them regardless of what happens in the financial markets.
2. **Constant Mix:** The insurance company rebalances its portfolio such that the weight of the different asset classes stays constant.
3. **Constant proportion portfolio investment (CPPI):** The exposure to the risky assets is adjusted based on the previous return.

The buy and hold strategy can be considered the most passive one. Here, the insurance company's return on the portfolio is fully dependent on the initially bought assets.

The constant mix strategy performs worse than the Buy and Hold strategy when the return in the stock market is low (Actecan, 2011). This comes from the fact that the insurer will buy new stocks when the stock market falls, and the value of the portfolio will therefore continue to decrease at the same speed. If the stock markets perform well, insurance companies that use the constant mix strategy have to sell stocks to keep the asset mix constant. This will give a lower return than if a Buy and Hold strategy is selected.

Insurance companies using the CPPI strategy reduce the equity weight in the portfolio in years of low return. The reason is that doing this decreases the probability of becoming insolvent if the return continues to be low. In years of high return, the insurance company is able to take on higher risk and therefore invests more in risky assets.

The selected investment strategy will significantly influence the risk and return of the paid-up policies. Actecan (2011) suggests that insurance companies use a strategy which gives the possibility to perform a dynamic adjustment of the risk exposure, like the CPPI strategy. As the insurance companies have to inject capital from their own equity if the value of the liabilities is higher than the value of the portfolio, it makes sense to reduce the risk exposure when the KRF is low.

In our model, the insurance company has the opportunity to select one of these strategies. As the implementation of the first two are relatively straightforward, we will only look into the implementation of the CPPI strategy.

Using this investment strategy, the amount invested in risky assets (in our case stocks) should equal

$$E = m(P - floor) \tag{5.1}$$

where P is the total value of the assets in the portfolio, and $floor$ is the value the portfolio should not drop below. For paid-up policies, it makes sense to set the floor equal to the value of the liabilities. m is a constant multiplier which reflects the insurance companies' risk preference.

As the portfolio is shared between all policyholders, the risk profile has to be equal for all contracts. This means that we cannot have a dynamic asset mix for individual contracts, and thereby not adjust the risk exposure based on the lifetime of the contract.

5.6 Bond Characterization

As we will see in detail in Section 9, insurers often invest a large part of the assets in bonds. The bonds can be characterized as either held-to-maturity (HTM) or available-for-sale (AFS). The characterization has a major impact on how returns have to be reported in the financial statements and thus the described book return. In this section, we therefore describe the difference between the two bond characterizations, and how it affects the book return.

5.6.1 HTM-bonds

As the name indicates, held-to-maturity bonds are purchased with the intention of holding the bond until it matures. Such investments should be reported at amortized costs, which means that the difference between the face value and the price will be amortized over the holding period (Forskrift om årsregnskap for livsforsikringsforetak, n.d.).

For HTM-bonds paying a yearly coupon, the coupon will become a part of the book return. The main advantage of characterizing bonds as held-to-maturity, is that one achieves predictability as the return of the bonds are unaffected by a changing market interest rate. On the downside, the HTM-bonds are not suitable if the owner needs to liquidate assets shortly, as they are intended to be held to maturity.

5.6.2 AFS-bonds

An AFS-bond is purchased with the intention of selling it before it matures. The bond should be reported at fair value, i.e. its market value, meaning that the current market value is compared to the price of the bond at issuance (Forskrift om årsregnskap for livsforsikringsforetak, n.d.).

The inverse relationship between bond price and interest rate causes bonds to lose value if the interest rate increases. For AFS-bonds, this unrealized loss will reduce the value of the KRF. If the KRF reaches a level below zero (i.e. the portfolio is smaller than the insurer's total liabilities), the insurer has to inject capital to make sure that the KRF is non-negative. For coupon bonds, the coupon will be unaffected by the fluctuations in the market interest rate, meaning that they will still be realized and become a part of the book return.

5.6.3 Comparison between HTM-bonds and AFS-bonds

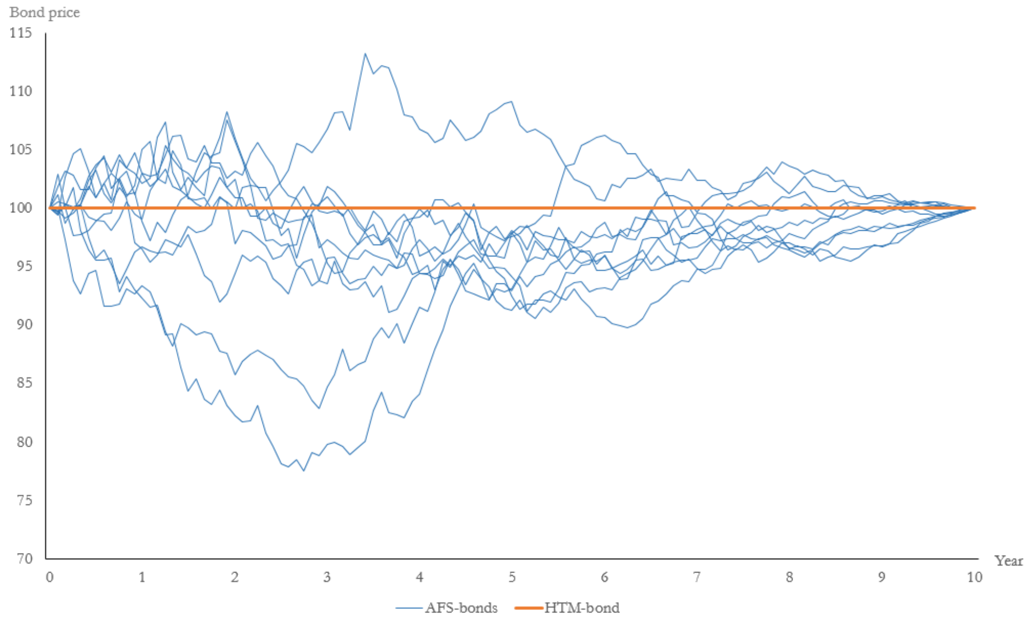

Figure 5.8: The blue lines illustrate ten different scenarios for the development in bond price for 10-year AFS-bonds paying annual coupons of 4.3% with face value of 100. The orange line shows the development in the bond price for a HTM-bond with the same parameters. The vertical axis shows the different market values of the bonds and the horizontal axis shows the years since issuance of the bonds.

In the figure, we see ten different scenarios in the development of the bond price for a 10-year AFS-bond paying annual coupons of 4.3%, illustrated by the blue lines. The orange line shows the market value each year for a HTM-bond with the same parameters as the AFS-bond. All bonds are issued at a price of 100 and have a face value of 100, meaning that they trade at par. The figure demonstrates the uncertain returns for the owner of an AFS-bond. Each of the blue lines is a possible path in the development of the bond price as the market interest rate changes. The interest rates used in the example is provided by the European We see that each line starts in the same point, 100, which is the price at issuance. After 10 years, all lines end up in the same point at 100, as this is the face value of the bonds. However, the paths from issuance price to face value are all different. Since the AFS-bonds must be reported in the financial income statement at its market values, the yearly return for the owner will be uncertain as a result of the uncertain path. The returns from a corresponding HTM-bond, meanwhile, will be amortized over the holding period, which gives certain returns. This is illustrated with the orange line, which is linear in development. On the upside, AFS-bonds are suitable for investors who may need to liquidate assets shortly and are therefore more flexible than HTM-bonds.

The bond characterization in our model is further discussed in Section 9.1.

5.7 Trade-offs for the Insurance Companies

The fact that the insurers have a wide range of alternatives regarding the management of paid-up policies, makes it hard to select a strategy that gives the highest possible market value, and keeps the risk sufficiently low at the same time.

We now state four important decisions the insurers have to make:

- How large should the buffers be before the insurer starts profit-sharing the return above the guaranteed rate?
- In what kind of assets should the insurer invest?
- When should the insurer prioritize to build TA or KRF?
- Should the insurer invest in derivatives in order to hedge against drops in the stock and bond prices?

The decisions the insurance companies make regarding all of these questions will affect the market value of the product, as well as the risk the companies are exposed to.

However, it is not trivial to select a strategy. Due to the complexity of the paid-up policies, we cannot find a closed form solution. To quantify how specific decisions affect the market value and risk we therefore run simulations. How we do so will be thoroughly described in Section 8.

6 Current Discussion on Paid-up Policies

Paid-up policies have historically constituted an attractive product for insurance companies. The main reason is that interest rates have historically been higher than the guaranteed rate offered on paid-up policies. Figure 6.1 plots the average guaranteed rate on paid-up policies (light blue line) with the Norway Government Bond 10Y rate (purple line) from 2000 to 2015. As there is virtually no chance the Norwegian government will default on this bond, the 10Y rate could be used as a proxy for the risk-free rate in the Norwegian financial market.

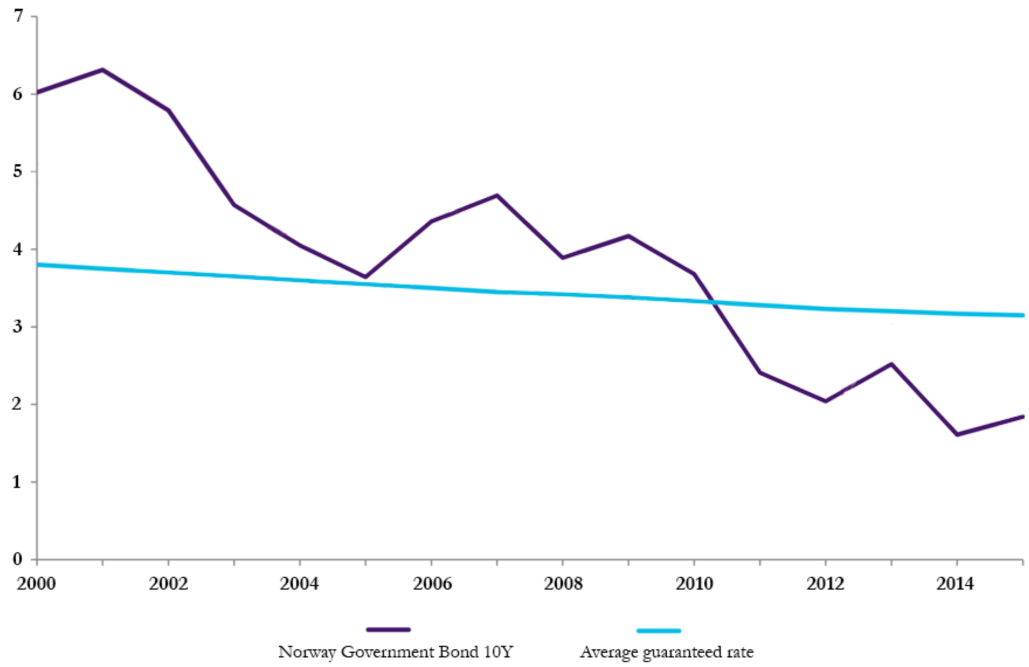

Figure 6.1: The Norway Government Bond 10Y rate (purple line) plotted with the average guaranteed rate among paid-up policies (light blue line) (The Financial Supervisory Autority of Norway, 2015).

Figure 6.1 shows that the risk-free rate (10Y rate) was above the guaranteed rate promised to policyholders until 2010. This meant that insurers could easily fulfill the guaranteed rate to policyholders. Additionally, as insurers got 20% of the profit above the guaranteed rate, insurers often earned profit from the paid-up policies. Hence, paid-up policies were historically a profitable product for the insurers associated with low risk.

When the risk-free rate dropped below the average guaranteed rate among paid-up policies in 2010, the insurance companies were forced to get a higher return on the savings than the risk-free rate in order to fulfill the guarantee. The drop in interest rates has made the paid-up policies less attractive in terms of profit-sharing and more challenging in terms of delivering the guaranteed rate to the policyholders, as insurers need to take more risk in order to fulfill the guaranteed rate.

Considering today's interest rates, it would be currently favourable for insurance companies that their customers switched to paid-up policies *with* investment choice. All policyholders have this opportunity, as described in Section 5.2. By changing, policyholders loose the promised annual return from the insurance company. In return, customers decide themselves how to invest the policy by choosing a desired risk profile, which could give higher or lower returns than the guarantee from an insurer. The insurers would in this case get rid of the obligation to fulfill the guaranteed rate. Insurance companies in Norway have therefore encouraged customers to switch. In doing so, all insurers are also obliged to inform about aspects which possibly makes switching to paid-up policies with investment choice not favorable for the customers. The information should be correct and accurate, a principle called the duty of disclosure.

One of the biggest insurance companies in Norway, Storebrand, has encouraged their holders of paid-up polices without investment choice to switch to the policies with investment choice. However, the Norwegian Financial Supervisory Authority had serious doubt whether Storebrand acted in accordance with the duty of disclosure, which led to public critism of the company in 2016. The authorities concluded the insurance company informed its policyholders of this option to switch, but the estimated return expectations for policies with investment choice were set too high (Finanstilsynet, 2016). This suggests that the company informed their customers inaccurately. "If the company's forecasts [...] are based on unreasonably high return assumptions for the individual customer, this will make the option to convert look better than what the reality is" the authorities write, and conclude that "there is considerable doubt that the company has complied with the duty of disclosure"[1].

Overall, only 5% of policyholders has switched to paid-up policies with investment choice since the option was introduced in 2014 (Finans Norge, 2018a). Consequently, insurance companies still manage the significantly largest part of paid-up policies, and must deliver

[1]The quotes are translated from Norwegian.

a guaranteed rate to policyholders. This shows the importance of an effective buffer and investment strategy, as well as methods to manage risk.

Furthermore, the low market interest rates have also made the policies less valuable for the policyholders since profit-sharing has been less likely during the latest years. The result is that policyholders rarely achieve a return above the guaranteed rate. Hippe and Lillevold (2018) argue that over time, paid-up policies lose value due to inflation and salary increases. Consequently, the Norwegian Ministry of Finance hired a commission in April 2017 to look into the matter. Their goal was to evaluate today's regulations concerning paid-up policies and propose possible changes which would be favourable for the customers (Regjeringen, 2018). In detail, the commission examined several changes to today's regulations, to see how they can stimulate the insurance companies to take more risk in order to achieve higher returns.

In September 2018, the Ministry of Finance published the commission's suggestions, which supports the following two changes to the regulations (Arbeidsgruppe, 2018).

1. Merge the TA and KRF into one buffer which can be used to cover negative return, and assign it to individual contracts. The insurance companies should also be allowed to prioritize profit-sharing for some customers, and building buffers for others.

2. Let the insurers give policyholders a financial compensation if they switch to paid-up policies with investment choice.

Based on the report, the Ministry of Finance plans to determine if there should be changes to the regulations for paid-up policies. Until then, today's regulations stand and insurers should use optimal buffer and investment strategies to manage the policies.

In April 2019, the Norwegian Pensioners' Association published a report discussing how retirees get payments from the TA (Pensjonistforbundet, 2019). As mentioned in Section 5.4, each customer has its own TA and the maximum level is 12% of the customer's reserves, stated by law. When the reserves are decreased in the years of payout, the maximum allowed size of the TA will also decrease. Therefore, the TA has to be reduced through payments to customers such that it does not exceed its maximum limit. In the report, the Norwegian Pensioners' Association argue that with today's regulations, only

15% of the TA has been paid out to the customers when they are 85 years old. If the policyholder passes away this old, the insurance company gets the amount that is left in the buffer. The association now requires new regulations which give the policyholders payments from the TA earlier.

All of the mentioned discussions might eventually lead to changes in the regulations for paid-up policies, which may have considerate impact on how insurance companies will manage the policies in the future.

7 Solvency II Capital Requirements

As of January 2016, the new regulatory framework Solvency II was introduced for insurance companies within the EU (Finanstilsynet, 2017b). The purpose of the directive is to establish a common regulatory framework to protect insurers' policyholders through sufficient capital requirements and risk management (European Parliament and of the Council, 2009). Norway agreed to follow the same standards, which means Norwegian insurance companies are obliged to comply with the directive. Solvency II consists of three pillars, as illustrated in Figure 7.1.

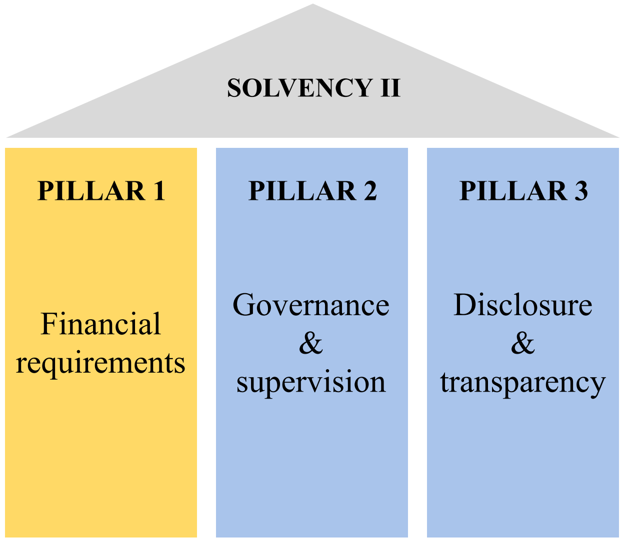

Figure 7.1: The Solvency II directive consists of three pillars: Financial requirements, Governance & supervision and Disclosure & transparency.

Pillar 1 includes requirements of the valuation of all assets and liabilities in the balance sheet and sets quantitative thresholds for required capital through the Solvency Capital Requirement (SCR) and Minimum Capital Requirement (MCR). These requirements will be described in more detail below. Pillar 2 covers governance and supervision through effective risk management systems. It includes details for conduct of supervisory reviews and interventions. Pillar 3 deals with reporting and disclosure of companies' capital adequacy and risks to the public authorities, with the goal of improving the industry's risk discipline. In this thesis we focus on optimizing the buffer and investment strategy related to the assets and liabilities of an insurer. This is related to the risks covered in Pillar 1. Compliance with Pillar 2 and Pillar 3 are considered out of the scope of this thesis.

We first describe the terminology in Pillar 1 and then explain the capital requirements. Figure 7.2 presents an overview of the assets and liabilities of an insurer in Pillar 1.

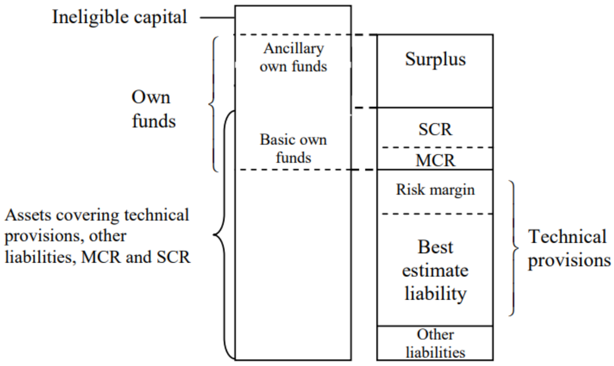

Figure 7.2: Illustration of an insurer's balance sheet in Solvency II: The assets are given on the left-hand side while the liabilities are given on the right-hand side. The capital requirements (MCR and SCR) are added on top of the liabilities, denoted as "technical provisions".

On the right hand side, we see the liabilities of an insurer ("Technical provisions"), divided into risk margin and best estimate liability. These will be described below. On top of the liabilities, the insurer must hold additional capital at least equal to the capital requirements in SCR and MCR. On the left hand side, we see the insurer's asset. These are divided into assets covering the liabilities and the capital requirements, plus the insurer's own funds. The own funds are again divided into "basic own funds" (funds the insurer holds) and "ancillary own funds" (funds the insurer can collect in special circumstances).

The main purpose of Solvency II is to make sure that insurers are sufficiently solvent to face years of large losses. Therefore, the directive sets capital requirements for insurers. The Solvency Capital Requirement (SCR) is one of these requirements. As Figure 7.2 illustrates, it is defined as a quantity threshold for the amount of funds an insurer must hold in addition to the liabilities in order to be solvent (European Parliament and of the Council, 2009). More detailed, SCR requires that insurers hold a minimum level of assets such that the assets exceed the liabilities in 99.5 % of all possible scenarios, with a time horizon of one year. The insurer must therefore hold assets A_t^* at time t at least equal to

$$A_t^* = \min\left\{ A_t \,\middle|\, \mathbb{P}_t\left(\frac{A_{t+1}}{L_{t+1}} < 1\right) \leq 0.005 \right\} \tag{7.1}$$

where A_t is the value of the assets at time t, L_t the value of the liabilities at time t and $\mathbb{P}_t$ is the probability measure at time t. This means that an insurer should be able to ensure a coverage for the liabilities in 99.5% of all possible scenarios, i.e. have. Supposing the assets achieve a return of r_t during year t, the SCR can be expressed as

$$SCR_t = \Big(Q_{0.995,t}\big(A_{t+1} - L_{t+1}\big) - \big(A_t - L_t\big)\Big)e^{-r_t} \tag{7.2}$$

where $(A_t - L_t)$ denotes the insurer's own funds at time t, and $Q_{0.995,t}$ denotes the 99.5% quantile of the distribution of the insurer's own funds at time $t+1$ conditional on the information available at time t.

In order to be able to determine the SCR under Solvency II, we need to derive the value of the assets at time $t+1$ (A_{t+1}) and the liabilities of the insurer at time $t+1$ (L_{t+1}). The directive states that assets should be valued at their market value, i.e. the amount at which the assets could be exchanged in an arm's length transaction between two willing parties. If market prices are not readily available, mark-to-model techniques can be used to value the assets. The liabilities, however, have no observed market price. According to the directive (2009), the value of the liabilities must be approximated by two separate calculations: The best estimate liability (BEL) and a risk margin (RM). The liabilities are then equal to the sum of the BEL and RM, as shown in Figure 7.2.

The BEL is the present value of all expected future cash flows, discounted with term dependent rates provided by the European Insurance and Occupational Pensions Authority (EIOPA). The goal is that the insurer makes assumptions on the cash flow projections that best reflect the characteristics of the underlying insurance portfolio. The projections should therefore take into account all relevant data, both internal and external. The actuarial and statistical methodologies used to estimate the cash flows and calculate the BEL are subject to approval (European Parliament and of the Council, 2009).

The RM is an additional margin intended to increase the liabilities to the amount that would have to be paid to another insurance company in order for them to take over an insurer's liabilities. It can be seen as a risk premium an insurance company would demand if they took over another insurance company's liabilities. In short, the RM for an insurer is calculated by projecting its SCR in respect of non-hedgeable risks and then multiplied with a Cost-of-Capital rate provided by Solvency II. This amount is then discounted with the term dependent rates to derive the RM.

At this point, we have explained how assets and liabilities are valued in Solvency II. We then have the balance sheet of an insurer, which is the basis of the SCR calculation. The SCR is derived as the VaR of a 99.5% confidence interval of the variation over one year of the insurer's basic own funds, given in Figure 7.3.

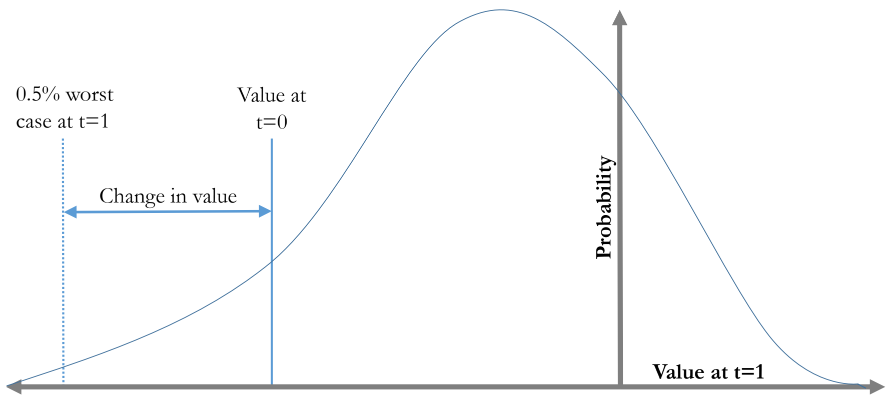

Figure 7.3: The probability distribution for the value of the policies at t=1, given a value at t=0. Insurance companies have to be able to face the 0.5% worst case scenarios. This implies the change in value marked with an arrow.

The figure illustrates how the SCR is derived. The distribution of the insurer's funds at time $t = 1$ is given in the figure, where the probability of a possible fund value is given along the vertical axis. The SCR is derived such that the insurer is able to face the change in value of his own funds from time $t = 0$ to time $t = 1$ of the 0.5% worst case.

To obtain the distribution of the insurer's own funds at $t = 1$, one of these approaches can now be used (European Parliament and of the Council, 2009):

1. Use a standard formula (SF) provided by EIOPA.
2. Use an internal, firm-specific model, subject to supervisory approval.
3. Use a partial internal model, i.e. a combination of 1. and 2.

The standard formula aims to capture the risk that an average European insurance company is exposed to. The SF takes into account market risk, non-life underwriting risk, life-underwriting risk, health underwriting risk, counterparty default risk, intangible asset risk and operational risk. The general principle of the SF is to apply a set of instantaneous shocks on the insurer's balance sheet (assets and liabilities) related to these risks. From the individual shocks, one can calculate the net impact that the shocks have on the balance sheet at the valuation date. The total instantaneous shock is then derived by applying a correlation matrix (also provided by EIOPA) between the individual shocks. The 99.5% VaR of the variation in the insurer's basic own funds is then the SCR. However, the SF may not appropriately reflect the risk profile for all insurers. Hooghwerff, van der Kamp, and Clarke (2017) point out that some assumptions in the SR may be unrealistic for insurance companies. One example is that the stress scenario for the decrease in the interest rate term structure does not allow for negative interest rates. As we will discuss in Section 8.1.2, this is unrealistic given today's interest rate term structure.

EIOPA, therefore, encourages insurance companies to develop their own internal model that better reflects the risk profile of an insurance company. A proposed internal model must effectively pass a number of tests, including a "use test". The use test examines if the model is widely used within the company and plays a significant role in the decision-making process and the solvency capital assessment. An internal model must also still generate an SCR from the 99.5% coverage of all future claims. The expected benefits of an internal model are argued to be improved risk sensitivity of SCR related to an insurer's specific risk profile, better alignment of capital requirements with economic capital and innovation in risk management methodology (Sharma, 2008). However, due to the cost of implementing an internal model, the majority of European insurers uses the SF (Hooghwerff et al., 2017).

In our analysis concerning risk associated with paid-up policies, we will use the SCR as the measure of risk. As we consider a hypothetical insurer only offering paid-up policies, the insurer's cash flows are only related to the paid-up policies. This means that the insurer's own funds in our case is equal to the market value of the paid-up polices from the insurer's perspective. As our model estimates the value of the insurer's liabilities (i.e. the paid-up policies) at $t = 0$ (L_t) and $t = 1$ (L_{t+1}), we can straightforwardly determine the SCR by a 1-year VaR calculation as

$$SCR_t = \Big(Q_{0.995,t}(L_{t+1}) - L_t\Big)e^{-r_t} \tag{7.3}$$

where $Q_{0.995,t}$ denotes the distribution of the market values for the insurer at time $t+1$ conditional on the information available at time t. We therefore do not need to use the approach of a Standard Formula in Solvency II. Our model will be discussed in detail in Section 8. Our calculation of the SCR only takes into account market risk. We are not able to consider any other of the mentioned risks, but a SCR comparison of reported risk modules between ten of the largest European insurance companies in 2018 showed that market risk was the risk module that had the considerably largest impact on the calculation of the SCR (Solvency II Wire Data, 2018).

Another important capital requirement under Solvency II is the Minimum Capital Requirement (MCR). The MCR is defined as a portion of the SCR. It is defined as at least 25%, but at most 45% of a company's SCR. If the market value of an insurance company's assets falls below the MCR the insurer is exposed to a level of risk considered unacceptable. Consequently, supervisory intervention from the authorities will follow inevitably (Vandenabeele, 2014). The Norwegian insurance company Silver is an example of a company that struggled to comply with the the SCR and MCR after the introduction of Solvency II in 2016. The Financial Authorities claimed that Silver's asset value was 3.5 billions NOK short of the SCR and 2.3 billion NOK short of the MCR (Finanstilsynet, 2017a). The company had in essence promised larger payments to their customers than their assets could cover (Lorentzen & Molnes, 2017). After several postponements of the deadline for Silver to comply with the requirements, financial authorities intervened and froze all the assets of the company. Thousands of customers could not access or move their pension savings to other companies for about eight months. Silver eventually ended up bankrupt. Another Norwegian insurance company, Storebrand, then took over the liabilities and converted the paid-up policies without consense from the customers to paid-up policies *with* investment choice (described in Section 5.2). Consequently, the customers lost the guaranteed rate associated with paid-up policies. The Consumer Council of Norway estimates that Silver's customers lost up to 12% of the market value of their paid-up policies after the policies were converted (The Norwegian Consumer Council, 2018).

The Silver-case illustrates the importance of such a directive for insurance companies. However, the directive is far from perfect. Critics of the directive claim that it works against its purpose because the capital requirements focus only on the one-year risk. This means the insurers' assets must comply with the requirements on a one-year basis, while the liabilities are typically of a long-term nature, meaning the payouts occur far ahead. If insurers, for example, make long-term investments, Solvency II treats them as if they are short-term traders and bases the risk measurement on short-term risks (Insurance Europe, 2017). This means that the calculation of the capital requirement uses the market interest rate (provided by EIOPA) to find an estimate of the liabilities, while in fact the liabilities increase with a fixed, guaranteed rate. The effect is that the capital requirements do not reflect the true risks the insurers face. As deputy director general of Insurance Europe, Olav Jones, points out: "Important improvements are needed to ensure that the framework works as intended, justifies the huge cost and effort involved in developing, implementing and operating it, and to avoid unnecessarily disincentivising insurers from making much needed long-term investments in the European economy" (Insurance Europe, 2017).

8 Model

In this chapter we present the model that we use for the analysis of paid-up policies. The focus of our analysis is on determining the effect of different investment and buffer strategies on the market value and risk associated with paid-up policies. By developing a valuation model for paid-up policies, we analyze how the choices regarding the investment and buffer strategy impact the market value of the policies. The different alternatives of buffer and investment strategies were discussed in Section 5.4.

Furthermore, we want to analyze how a hedge portfolio held by an insurer affects the capital requirements and the risk associated with the policies. By modelling a separate hedge portfolio, we are able to determine if the risk and capital requirements are affected by a hedge which can protect the insurer against large losses in the stock or bond market. In order to analyze these aspects we need to develop a model that determines the market value of paid-up policies and the associated risk. We explain in two steps how we 1) derive the market value and 2) determine the risk based on the capital requirements an insurer has to comply with, and determine how a hedge portfolio affects this risk.

First, we explain how we derive the value of paid-up policies. We want to determine the market value from both the insurer's and policyholder's perspective in order to be able to analyze the effect of different investment and buffer strategies for both parties. Looking at both parties can give valuable insights. Insurers are exposed to the largest part of the risk as they are obliged to deliver the guaranteed rate of return. Finding optimal strategies can help insurers getting a return above the guaranteed rate with less risk associated. Policyholders are the owners of the product and want insurers to deliver high returns on their pension savings. A poor investment and buffer strategy might decrease the customer satisfaction, and lead to bad publicity compared to competitors.

Recall that a paid-up policy promises given pension payments in the future when the policyholder retires. The market value of such a product is the sum of all discounted cash flows. To derive the market value for the insurer and the policyholder, we therefore need to keep track of the cash flows to both parties. If the cash flows in the future would be deterministic, we could value a policy easily by discounting all future cash flows at time t (CF_t) with an appropriate discount rate (r_t), as given by

$$MV = \sum_{t=1}^{T} CF_t \cdot e^{-r_t t} \tag{8.1}$$

where T is the length of the contract period, i.e. the lifetime of a policyholder.

However, the cash flows to a policyholder are in reality not deterministic. Recall that an insurer manages paid-up policies and promises a guaranteed rate of return to policyholders. The insurer holds an asset portfolio backing the liabilities from the paid-up policies that largely consists of stocks and bonds. We refer to this reference portfolio as *policy portfolio*. The cash flows to a policyholder will thus depend on the return of the *policy portfolio* in addition to the choices the insurer makes regarding the buffers, as explained in Section 5.4.

As the insurer pays pensions payments to customers until they pass away, the initial value of the portfolio is derived taking risk of mortality into account. How we account for mortality risk is discussed in detail in Section 8.2.

Since the *policy portfolio* largely consists of stocks and bonds, the cash flows depend on stock and bond prices, which are stochastic. In order to determine the cash flows and make a valuation of the paid-up policies, we therefore need risk-neutral scenarios of the future evolution of the stock (S_t) and bond prices (B_t). This comes from the fact that the market value of a future cash flow (CF_t) can be calculated by the expected value of the cash flow under a risk-neutral measure, $\mathbb{Q}$, discounted by the risk-free rate.

$$MV = \mathbb{E}^{\mathbb{Q}}\Big[CF_t e^{-r_t t}\Big] \tag{8.2}$$

Risk-neutral scenarios are used to simulate the development in S_t and B_t. The value of the *policy portfolio* is then found from the stock and bond return. In Section 9.1, we discuss why we only include these two assets in the portfolio. The details of how stock and bond prices are simulated are discussed in Section 8.1. Potential paths of risk-neutral scenarios are illustrated in Figure 8.1.

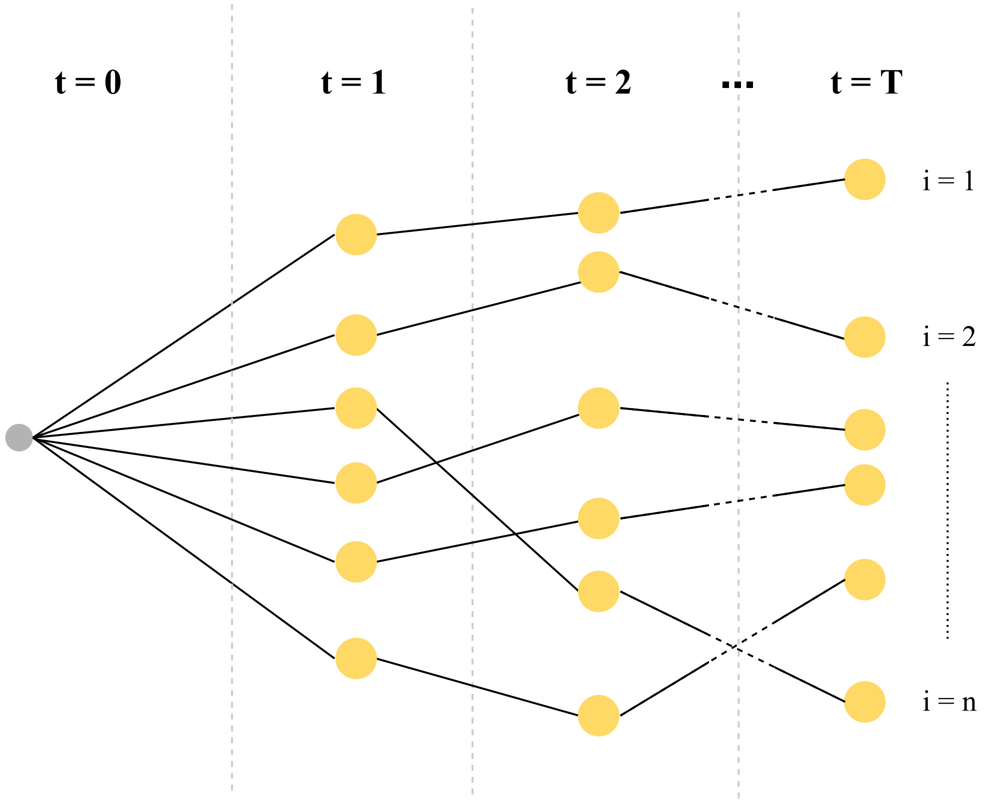

Figure 8.1: Each node represents a possible risk-neutral scenario of the value of the *policy portfolio* for the contract period $t = 0$ to $t = T$ of a paid-up policy. The cash flows in each node are discounted to determine the market value of the paid-up policy at $t = 0$.

The figure shows risk-neutral scenarios for the development of the *policy portfolio* in the contract period from time t to T. A node in the figure represents a possible state of the portfolio value at time t. For each simulated path i, the cash flows to the policyholder and the insurer are derived applying a chosen buffer strategy. How this is done is explained in Section 8.3.

As mentioned in Section 5.4, the insurer can have both negative and positive cash flows related to the paid-up policies he manages. The negative cash flows occur in years when he has to inject cash flows into the portfolio, while positive cash flows come from profit-sharing. The policyholders' cash flows will always be positive. The market value of a paid-up policy is derived after simulating multiple risk-neutral paths for the development of S_t and B_t, applying a chosen buffer strategy in each scenario. The buffer strategy might change dynamically over time, depending on the chosen overall strategy.

For each simulation path i there might be cash flows to and from the insurer in each year

t ($CF_{i,t}^{Ins}$). Extending Equation (8.2) to account for annual cash flows during the contract period, the market value equals the average value of the discounted net cash flows of all simulations to the insurer, as in

$$MV_t^{Ins} = \frac{\sum_{i=1}^{n} \sum_{t=1}^{T} CF_{i,t}^{Ins}(S_t, B_t, BS) \cdot e^{-r_t^* t}}{n} \tag{8.3}$$

where n is the number of simulated paths, T is the length of the contract period while BS refers to the chosen buffer strategy, which we discuss in Section 8.3. How to derive the discount rate r_t^* will be discussed in detail in Section 8.4.

The policyholder receives cash flows ($CF_{i,t}^{Pol}$) at least equal to the guaranteed promised payments. The market value for the policyholder is also calculated by taking the average value of the discounted net cash flows of all simulations.

$$MV_t^{Pol} = \frac{\sum_{i=1}^{n} \sum_{t=1}^{T} CF_{i,t}^{Pol}(S_t, B_t, BS) \cdot e^{-r_t^* t}}{n} \tag{8.4}$$

In the model, we have also accounted for the fact that the insurer has several different customer groups depending on the time to expiration of the policy, i.e. when the policyholder passes away. Each customer group has a different guaranteed rate, as their policies were issued in different years. How we account for the dynamics of several customer groups will be discussed further in Section 8.5.

We now have explained how we value paid-up policies. However, an insurer must also manage the risk associated with the policies. This comes from the fact that there is a trade-off between selecting investment and buffer strategies that give the highest market value for the policyholder and balancing the associated risk. For instance, an investment strategy with only equity in the *policy portfolio* can give a high market value to the customer as profit-sharing will be likely, but the same strategy will have a huge associated risk.

In order to analyze how insurers can manage risk more effectively, we also include a *hedge portfolio* which the insurer holds. It is kept separately from the *policy portfolio*, and consists of different types of options. The goal of the *hedge portfolio* is to make sure that its return is negatively correlated to the *policy portfolio*, such that it hedges the negative movements of S_t and B_t. At the expiration date of the options, the value of the hedge portfolio increases by the option payoffs. However, due to the option price, the value of the *hedge portfolio* will decrease in years of high return, as in this case, the options bought to hedge risk have no value at maturity.

As presented in Section 7, the insurer must act in accordance with the Solvency Capital Requirement. The one-year risk is the basis of this capital requirement, and therefore, it has to be modelled. We could also model the risk each year of the contract period, but this would result in a considerable increase in the computational complexity as the number of simulations needed increases exponentially. Given the fact that the Solvency II focuses on the one year risk, we follow the directive and only focus on the one-year risks.

In order to be able to measure the one-year risk, we have to use a combination of real-world scenarios and risk-neutral scenarios. Real-world scenarios also simulate the development in S_t and B_t, but the difference from the risk-neutral scenarios is that real-world scenarios take into account a risk-premium when simulating stock prices. The bond prices are simulated without any risk premium, as we disregard default risk of the bonds. The risk premium will be discussed further in Section 9.6.

The one-year risk is determined as follows. Figure 8.2 illustrates the procedure. By simulating m real-world scenarios from $t = 0$ to $t = 1$, we have m different scenarios of the value of the *policy portfolio* and *hedge portfolio* at $t = 1$. These are represented by the blue nodes in Figure 8.2. For each real-world scenario at $t = 1$, we need to calculate the market value at the given node by simulating n risk-neutral scenarios from each real-world scenario at $t = 1$ to the end of the contract period. The risk-neutral scenarios are represented by the yellow nodes in Figure 8.2. Consequently, we derive m different market values at $t = 1$. The different market values then constitute the basis of the risk calculation by the Value-at-Risk (VaR) measure, discussed below.

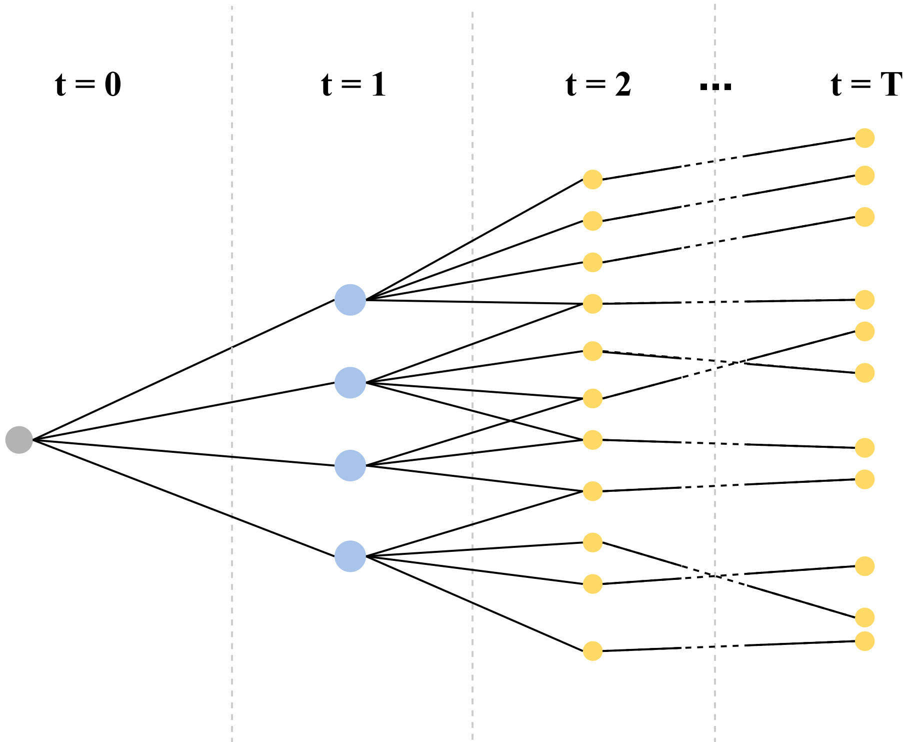

Figure 8.2: Each blue node represents a possible real-world scenario in the value of the *policy portfolio* and *hedge portfolio* from $t = 0$ to $t = 1$. From $t = 1$ to $t = T$, the market value is derived using risk-neutral scenarios (yellow nodes).

As we discussed in Section 7, the Solvency II-directive requires the insurance companies to hold a certain amount of funds in order to be solvent. The minimum level of the insurer's funds must be equal to the 99.5% VaR of the change in market value of the insurer's liabilities from $t = 0$ to $t = 1$. To quantify the risk, we measure the Value-at-Risk based on the derived m market values for the insurer ($MV_1^{InsPolicy}$), the value of the *hedge portfolio* at $t = 1$ (MV_1^{Hedge}) and the net cash flows at $t = 1$ (CF_1^{Ins}). The value of the *hedge portfolio* is given by the sum of the payoffs from the options minus the price of the options bought. The pricing of the options and deriving the payoffs are described in more detail in Section 8.7. The total market value for the insurer at $t = 1$ is thus equal to

$$MV_1^{Ins} = MV_1^{InsPolicy} + MV_1^{Hedge} + CF_1^{Ins} \tag{8.5}$$

The VaR is then calculated using the following equation

$$VaR_p = Percentile(MV_1^{Ins}, p) \tag{8.6}$$

where p denotes the level of confidence. The VaR is chosen as the measure of risk because it is easily understood, widely used in the insurance business and the Solvency Capital Requirement is based on the VaR.

We have now described the modelling approach we use to value paid-up policies and estimate the associated risk. Figure 8.3 illustrates the simulation structure of the overall model, where we use a contract length of 54 years ($T = 54$). This choice will be further discussed in Section 9.3. The figure illustrates the real-world scenarios for the development of the *policy* and *hedge portfolio* as the blue nodes and the risk-neutral scenarios as the yellow nodes. All scenarios model the development in S_t and B_t. In order to reduce the computational run time of the simulations, we use control variates in all simulations. This is discussed further in Section 8.6.

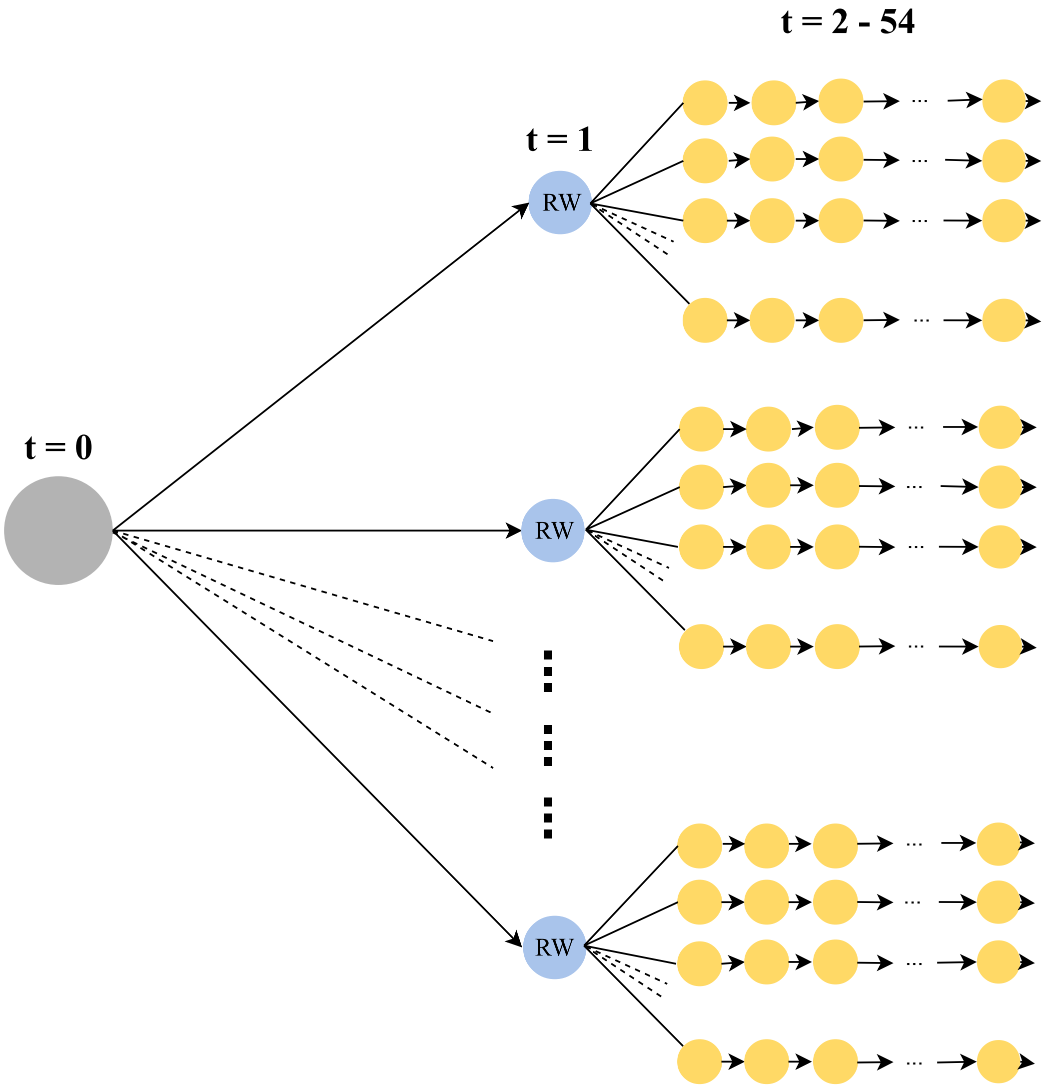

Figure 8.3: The blue nodes at time t=1 represents different real-world scenarios for the development in the *policy* and *hedge portfolio*. For each of these scenarios, we calculate the market value for the insurer and policyholder from the generated risk-neutral scenarios (yellow nodes).

The rest of the chapter is structured as follows. Section 8.1 presents how we model stock and bond prices, using the Black-Scholes and the Hull-White One-Factor Model. Section 8.2 discusses how the initial value of the *policy portfolio* is derived, taking into account mortality risk. Section 8.3 describes the buffer strategy applied in each scenario, which is a decisive part of the Norwegian profit-sharing system. Section 8.4 discusses how the discount rate of the cash flows from the simulations is determined. How we include multiple contracts and customer groups in our model is discussed in Section 8.5. The use of control variates is described in Section 8.6. Finally, Section 8.7 describes the *hedge* *portfolio* and the option types included in this portfolio.

8.1 Simulating Stock and Bond Returns

This section presents in detail how we simulate stock prices (S_t) and bond prices (B_t). As discussed above, we need models for S_t and B_t for both risk-neutral and real-world scenarios. S_t can be simulated using the well-known Black-Scholes model, which is explained below. In order to simulate B_t, we first need to simulate the short-term interest rate, the so-called *short rate*. In Section 8.1.2 we compare possible short-rate models that can be used to do this simulation and end up with the Hull-White One-Factor model as the best fit. This short-rate model includes the desired characteristics of the development of the interest rate such as mean-reversion and that it fits the current term-structure of interest rates, i.e. the model is calibrated to today's market interest rates. Bond prices (B_t) are then derived from the simulated short rates.

8.1.1 Stocks

The Black-Scholes procedure uses risk-neutral scenarios of the stock price in their valuation of options. The assumptions underlying this framework are given by Black-Scholes (1973):

- The short-term interest rate is known and is constant through time.
- The stock price follows a random walk in the continuous time with a variance rate proportional to the square of the stock price. Thus, the distribution of possible stock prices at the end of any finite interval is lognormal. The variance rate of the return on the stock is constant.
- The stock pays no dividends or other distributions.
- The option is "European", that is, it can only be exercised at maturity.
- There are no transaction costs in buying or selling the stock or the option.
- It is possible to borrow any fraction of the price of a security to buy it or to hold it, at the short-term interest rate.
- There are no penalties to short selling. A seller who does not own a security will simply accept the price of the security from a buyer, and will agree to settle with

the buyer on some future date by paying him an amount equal to the price of the security on that date.

Under these Black-Scholes assumptions, the stock price S_t evolves as

$$dS_t = \mu S_t dt + \sigma_S S_t dz_t \tag{8.7}$$

μ denotes the expected rate of return from the stock and σ_S the volatility of the stock price. dz_t is a Wiener process.

When the stock price evolves according to Equation (8.7), it follows the Markov property, i.e. the current stock price is only dependent on the previous period. Furthermore, the stock price has a lognormal property. This means that $\ln S_T$ has a mean value equal to $\ln S_0 + \left(\mu - \frac{\sigma_S^2}{2}\right)T$ and a variance equal to $\sigma_S^2 T$. In other words

$$\ln S_T \sim \phi\left[\ln S_0 + \left(\mu - \frac{\sigma_S^2}{2}\right)T, \sigma_S^2 T\right] \tag{8.8}$$

By multiplying the standard deviation, σ_S, and adding the mean, we can convert the standard normal random variable, Z, into one with an arbitrary mean or variance. In addition, in the Black-Scholes world, investors are risk-neutral. The expected return μ is therefore the risk-free rate, r_f. Considering the stock price following Equation (8.7) it can now be simulated as

$$S_T = S_0 \cdot e^{(r_f - \frac{1}{2}\sigma_S^2)T + \sigma_S \sqrt{T} Z} \tag{8.9}$$

Equation (8.9) is used for simulating risk-neutral stock prices. How to derive the risk-free interest rate r_f is discussed in the next section. The volatilty σ_S is discussed in Section 9.1. The simulation of the stock price in real-world scenarios takes into account a risk premium, which will be discussed in Section 9.6.

8.1.2 Interest Rate and Bonds

In order to determine bond prices, we need to simulate interest rates. This has to be done as bond returns constitute a major part of the total return from the policy portfolio.

We start by defining the distinction between short rates, forward rates and the instantaneous forward rate, which all found basis of the interest rate simulation. Following Hull (2012) we define the short rate, r_t, as the interest spot rate that applies to an infinitesimally short period of time at time t. It will be simulated by short-rate models, which is the topic of this section.

To understand short-rate models, we need to look at the forward rate, denoted $f_0(t,T)$. This rate is defined as the rate at which you can contract at $t = 0$ to borrow or lend money starting at period t, to be paid back at period T. The determination of the forward rate is based on the non-arbitrage principle, such that

$$e^{R_t t} e^{f_0(t,T)(T-t)} = e^{R_T T} \tag{8.10}$$

where R_t denotes the t-year spot rate at time $t = 0$. Solving Equation (8.10) for $f_0(t,T)$ and rearranging, we observe that the forward rate is equal to.

$$f_0(t,T) = \frac{R_T T - R_t t}{T-t} = R_T + (R_T - R_t)\frac{t}{T-t} \tag{8.11}$$

In the limit $(T-t) \downarrow 0$, we get the *instantaneous* forward rate $f_0(t)$. This is the forward rate at time 0 that is applicable to an infinitely small time period from t. The instantaneous forward rate is derived using the following formula (Veronesi, 2010)

$$f_0(t) = R_t + t \cdot \frac{\partial R_t}{\partial t} \tag{8.12}$$

where R_t denotes the current spot rate for maturity t. This means that the instantaneous forward rate then depends on the initial spot rate plus a term that depends on the slope of the estimated spot rate curve.

Using the chain rule, Equation (8.12) for the instantaneous forward rate can also be written as

$$f_0(t) = -\frac{\partial \ln(P(0,t))}{\partial t} \tag{8.13}$$

where $P(0,t)$ is the the price of a zero-coupon bond at time 0 with maturity t, such that $P(0,t) = e^{-R_t t}$. We will use both formulations of $f_0(t)$ when we later show how bond prices are simulated in our model. Appendix A.5 shows how we go from Equation (8.12) to Equation (8.13).

The future evolution in the short rate can be described using short-rate models. The simulation of the short rate can be done by martingale models where we assume a standard differential equation (SDE) for the short rate r_t under the martingale measure $\mathbb{Q}$ (Boshuizen, van der Vaart, van Zanten, Banachewicz, & Zareba, 2006). These short rates can then be used to value a zero-coupon bond maturing at time T by applying the following formula:

$$P(t,T) = \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^T r_s ds}\right] \tag{8.14}$$

A number of short-rate models has been suggested. One of them is the Vasicek's model, where the risk-neutral process for r_t is given by

$$dr_t = a(b - r_t)dt + \sigma_B dz \tag{8.15}$$

Here a, b and σ_B are constants, and dz is a Wiener process. a represents a parameter for mean reversion. Even though the model can be used to obtain an expression for the price of a zero-coupon bond, it suffers from especially one important drawback; the model is not set to fit the current term structure of interest rates. This is important since interest rates not following the current term structure would lead to arbitrage opportunities.

Thomas Ho and Sang Bin Lee solved this when they presented the first no-arbitrage model in 1986. Such models make sure that the simulated short rate fits today's term structure of interest rate. The Ho-Lee Model is given by

$$dr_t = \theta(t)dt + \sigma_B dz \tag{8.16}$$

Here, $\theta(t)$ defines the average direction in which r_t moves at time t. dz is still a Wiener process.

However, even though this model makes sure that the output fits the observed term structure, it does not contain any mean reversion. Due to the randomness in the second term on the right hand side of Equation (8.16), excluding the mean reversion might be problematic if we face a very high or very low interest rate. Since $\mathbb{E}(dz) = 0$, we can in such a case expect that the future evolution in interest rate will be parallel, but too high or too low, as it does not revert to the curve implied by θ.

Hull and White (1990) proposed a no-arbitrage model that allows to overcome both drawbacks of the Vasicek's model and the Ho-Lee Model by combining them. The result is a model which contains mean reversion and fits the current term structure of interest rate. Hull and White (1990) propose to model the dynamics of the short rate by the following PDE:

$$dr_t = [\theta(t) - ar_t]dt + \sigma_B dz \tag{8.17}$$

Equation (8.17) implies that at time t, the short rate reverts to $\theta(t)/a$ at rate a, where $\theta(t)$ reflects the current term structure of interest rates. a, σ_B and $\theta(t)$ have to be calibrated using market data, such that they are consistent with interest rate derivatives.

In the Hull-White model, interest rates are assumed to be normally distributed, which implies that they can also take negative values. This has historically been argued to be one of the drawbacks of the model. However, as the interest rate has dropped below zero in many countries during the last years, this can now be considered an advantage.

In order to get an explicit formula for r_t, we start by defining $G(t, r_t) = e^{at}r_t$, and apply Ito's lemma on $dG(t, r_t)$. The derivations in this subsection are based on Boshuizen et al. (2012) and van Buul (2010).

$$\begin{aligned} dG(t, r_t) &= \frac{\partial G}{\partial r_t}dr_t + \underbrace{\frac{1}{2}\frac{\partial^2 G}{\partial r_t^2}(dr_t)^2}_{=0} + \frac{\partial G}{\partial t} \\ &= ae^{at}r_t dt + e^{at}\Big((\theta(t) - ar_t)dt + \sigma_B dz\Big) \\ &= \theta(t)e^{at}dt + \sigma_B e^{at}dz \end{aligned} \tag{8.18}$$

By integrating Equation (8.18) and moving e^{at} from $G(t, r_t)$ over to the other side, we obtain a formula for r_t

$$r_t = e^{-at}r_0 + e^{-at}\int_0^t \theta(u)e^{au}du + \sigma_B e^{-at}\int_0^t e^{au}dz \tag{8.19}$$

In order to construct the Hull-White interest rate model, we apply this equation and the formula for the price of a zero-coupon bond given by Equation (8.14).

This lets us calculate the distribution of $\int_t^T r_s ds$, which is needed to find a price for a zero-coupon bond, using Equation (8.14).

$$\int_t^T r_s ds = B(t,T)r_t + \int_t^T B(u,T)\theta(u)du + \sigma_r \int_t^T B(u,T)dz \tag{8.20}$$

where

$$B(t,T) \stackrel{\text{def}}{=} \frac{1 - e^{-a(T-t)}}{a}. \tag{8.21}$$

The proof can be found in Appendix A.1.

As dz is normally distributed with mean 0 and variance 1, and the first two terms of Equation (8.20) are discrete, we can conclude that $\int_t^T r_s ds$ is normally distributed with the following values for the mean and variance.

$$\int_t^T r_s ds \sim \mathcal{N}\Big(B(t,T)r_t + \int_t^T B(u,t)\theta(u)du, \sigma_B^2 \int_t^T B^2(u,T)\Big) \tag{8.22}$$

For a normally distributed variable X with a mean μ and variance σ_B^2 it holds that $\mathbb{E}[e^X] = e^{\mu + 0.5\sigma_B^2}$. The proof can be found in Appendix A.2. Inserting the mean and variance from Equation (8.22) into this equation, gives us the following price for a zero-coupon bond.

$$P(t,T) = e^{A(t,T) - B(t,T)r_t} \tag{8.23}$$

where

$$A(t,T) = \int_t^T \Big(0.5\sigma_B^2 B^2(u,T) - \theta(u)B(u,T)\Big)du \tag{8.24}$$

Inserting Equation (8.23) into Equation (8.13), we obtain the following equation for the instantaneous forward rate

$$f_0(t) = \frac{\partial B(0,t)}{\partial t} r_t - \frac{\partial A(0,t)}{\partial t} \tag{8.25}$$

The theoretical forward prices given by Equation (8.25) have to be matched to the ones observed in the market. We therefore insert $A(0,t)$ from Equation 8.24 and $B(0,t)$ from Equation 8.21 into Equation (8.25). After differentiating and integrating, we end up with the following formula for $\theta(t)$.

$$\theta(t) = af_0(t) + \frac{\partial f_0(t)}{\partial t} + \sigma_r^2 B(0,t)\Big(e^{-at} + \frac{1}{2} aB(0,t)\Big) \tag{8.26}$$

The proof of Equation (8.26) can be found in Appendix A.3.

To find a formula for r_t, we insert Equation (8.26) into Equation (8.19) and get

$$\begin{aligned}
\alpha_t &= f_0(t) + \frac{\sigma_B^2}{2} B^2(0,t) \\
\beta_{t+\Delta t} &= e^{-a\Delta t}\beta_t + \sqrt{\frac{1}{2}\sigma_B^2 B(0, 2\Delta t)} Z_t \\
r_t &= \alpha_t + \beta_t
\end{aligned} \tag{8.27}$$

The derivation of the formula is given in Appendix A.4. In order to calculate α_t, we need values for the observed, instantaneous forward rates $f_0(t)$. These rates can be derived from the observed spot rates in the market. In this thesis we rely on current interest data provided by the European Insurance and Occupational Pensions Authority (EIOPA). However, these spot rates are discrete and therefore we need to estimate a continuous zero-curve. This can be done by doing a regression on the spot rates using polynomials, and then apply Equation (8.12). The more polynomials one includes, the closer one gets to the actual curve. In this thesis we have used the ninth order polynomials, which gives a R-squared value of 0.9999. The results of this calibration can be found in Section 9.2.

We are then only left with deriving bond prices for bonds with different maturities. By using the derived value for $\theta(t)$ from Equation (8.26) in Equation (8.23) and Equation (8.24), we find the price of a zero-coupon bond at time t with time to maturity T.

$$P(t,T) = \frac{P(0,T)}{P(0,t)} \exp\left(B(t,T)f(0,t) - \frac{\sigma_B^2}{4a}B^2(t,T)(1-e^{-2at}) - B(t,T)r_t\right) \tag{8.28}$$

Setting Equation (8.28) equal to the bond price given by $P(t,T) = e^{-r_{t,T}(T-t)}$, we can derive the following

$$r_{t,T} = \frac{\ln P(t,T)}{t-T} \tag{8.29}$$

$r_{t,T}$ denotes the interest rate at time t for a zero-coupon bond maturing at time T. This is the rate used in our model.

8.1.3 Hull-White-Black-Scholes Model

We now have the Black-Scholes model to simulate stock prices S_t and the Hull-White One-Factor Model to simulate the short rate r_t and the bond prices $P(t,T)$. Combining these two models, we are able to simulate the evolution in stock prices and interest rates. In the following, we call this combined model the Hull-White-Black-Scholes Model. The model can be used for valuation of financial products simultaneously affected by equity and interest rates, such as paid-up policies.

The interest rates simulated using the Hull-White model are used as an input when calculating the evolution in stock prices using Equation (8.9). The Wiener process from Black-Scholes, denoted $dz_1(t)$, is correlated with the Wiener process from Hull-White, $dz_3(t)$ by a factor ρ such that

$$dz_1(t) = \rho dz_2(t) + \sqrt{1-\rho^2}dz_3(t) \tag{8.30}$$

where $dz_1(t)$ and $dz_2(t)$ are independent.

From the Hull-White model, we know that the short rate, r_t, is expected to change over time. To find the evolution in the stock price, we put the model in Equation (8.9) into discrete time steps Δt. This gives

$$S_{t+\Delta t} = S_t \cdot e^{(r_t - \frac{1}{2}\sigma^2)\Delta t + \sigma_S \sqrt{\Delta t} Z} \tag{8.31}$$

As we discuss later in Section 8.3.1, the dividend (δ) becomes a part of the book return each year. In order to separate it from the unrealized equity return, we therefore handle the dividend manually, and do not include dividend yield in Equation (8.31).

8.2 The Initial Value of the Reserves

In order to value paid-up policies by simulating the value of the *policy portfolio* (i.e. development of S_t and B_t), we need an initial value of the portfolio. The *policy portfolio* consists of reserves from promised guaranteed payments and payments originating from profit-sharing. The reserves are therefore a liability for the insurer, as it is the future promised payments from the insurer to the policyholder. When a paid-up policy is issued, the initial value of the reserves is calculated by discounting these future payments. Since the insurer promises a certain guaranteed rate to the policyholder at the time of issuance, this guaranteed rate is used as the discount rate.

The length of a contract, i.e. the lifetime of a policyholder, will also decide the initial value of the reserves. We therefore take into account risk of mortality for a policyholder and rely on the mortality tables provided by Statistics Norway and the Financial Supervisory Authority of Norway from 2013 (Finanstilsynet, 2013; SSB, 2019). The mortality tables can be found in Appendix A.6.

In the model, we assume all customers retire at the age 67, as this is the retirement age stated by law in Norway (Regjeringen, 2017). The mortality tables are then used to calculate the risk of mortality for customer aged in the interval between 67 and 100. No customers are assumed to reach an age higher than 100 years old. Furthermore, the risk of mortality for a customer is high in the interval [90, 100] years old. To save run time, the payments to customers in this age group is gathered up in one payment, i.e. when a policyholder becomes 90 years old, she receives her last payments as one payment. As these last expected payments are low, this simplification is not expected to have an impact on the end result.

As we have customer groups with different guaranteed rates and length of the policy before it expires, the initial values of the reserves will be different for all customer groups. This is discussed further in Section 8.5.

8.3 Buffer Strategy

So far, we have described how we simulate the interest rate and stock price which is used to determine the value of the *policy portfolio*. Based on each year's return from the simulated stock price and interest rate, the insurer must choose how to distribute the returns. The possible cases were illustrated in Section 5.4. The choices the insurer makes are referred to as the buffer strategy. This strategy is applied each year in each scenario, both real-world and risk-neutral. This section describes in detail what the buffer strategy implies.

Figure 8.4 illustrates the setup of the model and when the buffer strategy is applied.

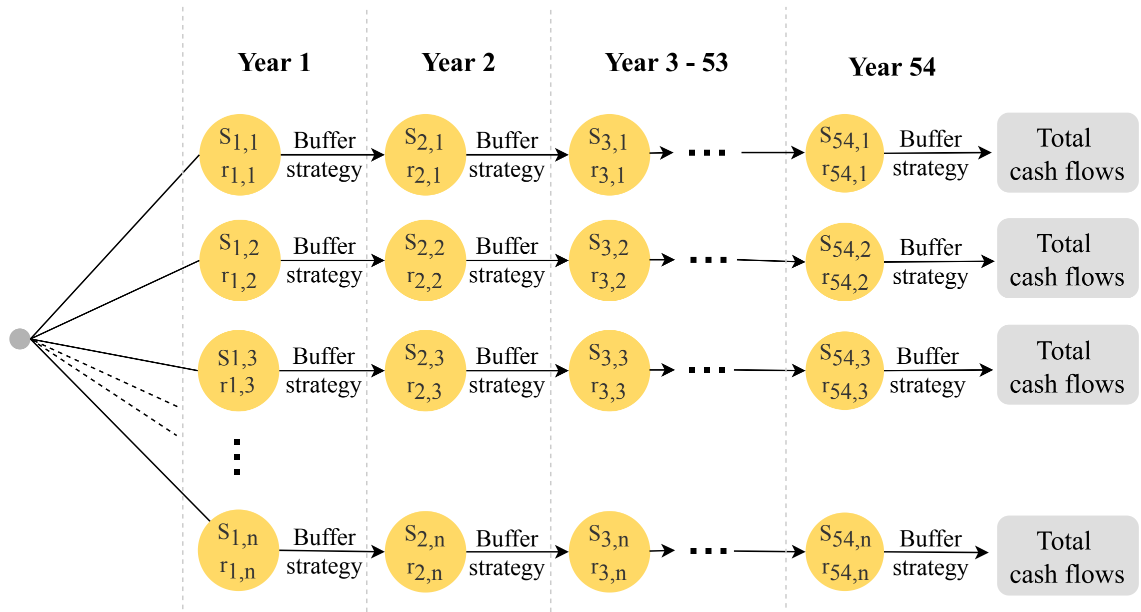

Figure 8.4: The model is run for a selected number of different scenarios of evolution in the stock price and short rates. $S_{i,j}$ and $r_{i,j}$ represent the stock price and short rate in year i for scenario j. For each of these scenarios, the chosen buffer strategy is applied yearly. Based on the last year's market and book return, the insurer decides how much to put in the TA or share between himself and the policyholder through the profit-sharing system.

Each node in Figure 8.4 represents a scenario j in year i where the stock price $S_{i,j}$ and interest rate $r_{i,j}$ are simulated. Each year, we calculate the total return based on how the stock price and interest rate have evolved since last year. The insurer uses his buffer strategy to decide the book return and distribute profit. The buffer strategy is carried out for all years in each scenario and is illustrated as "Buffer strategy" in the figure.

The rules related to the TA and profit-sharing differ before and after the policyholder retires. In order to explain the buffer strategy, we will therefore in the following discuss these two periods separately. The periods will be referred to as the build-up period (before retirement) and payout period (after retirement). The general buffer strategy which is carried out in both periods is discussed in Section 8.3.1. Section 8.3.2 then describes the special needed steps in the buffer strategy for the payout period of a policyholder.

8.3.1 General Buffer Strategy for All Years

Recall that the yearly book return founds the basis of the buffer strategy. The first step of the buffer strategy is therefore to decide the yearly book return. This is the part of the yearly unrealized return that is realized. No matter what buffer strategy the insurer selects, parts of the annual return will always be realized. We refer to this as the *Initial book return*.

It is given by the following equation.

$$\begin{aligned} \textit{Initial book return} \ &= \textit{Coupons from bonds} \ + \ \textit{Dividends} \ + \\ &\quad \textit{Realized return from rebalancing} \end{aligned} \tag{8.32}$$

The initial book return equals the sum of three parts. First, coupon bonds pay a constant annual coupon, and these cash flows become a part of the realized return. Second, dividends from stocks are realized yearly and increase the realized return. Third, if the portfolio has been rebalanced the last year (i.e. bonds and/or stocks have been sold), a part of the unrealized returns in the KRF has been realized and becomes a part of the realized return as well.

We now look at how the insurer covers the guaranteed rate to the policyholders, and the options he has for book returns exceeding the guaranteed rate. We divide his strategy into two stages. The first stage checks if the insurer has to cover negative return by injecting capital into the portfolio. The second stage carries out the buffer strategy using the KRF, TA and profit-sharing.

- **Stage 1: Determine if the book return plus the market value of the portfolio is less than the liabilities**.

The insurer must inject capital into the portfolio if the market value of the portfolio plus the initial book return (see Equation (8.32)) is less than his liabilities. Figure 8.5 illustrates how this is determined.

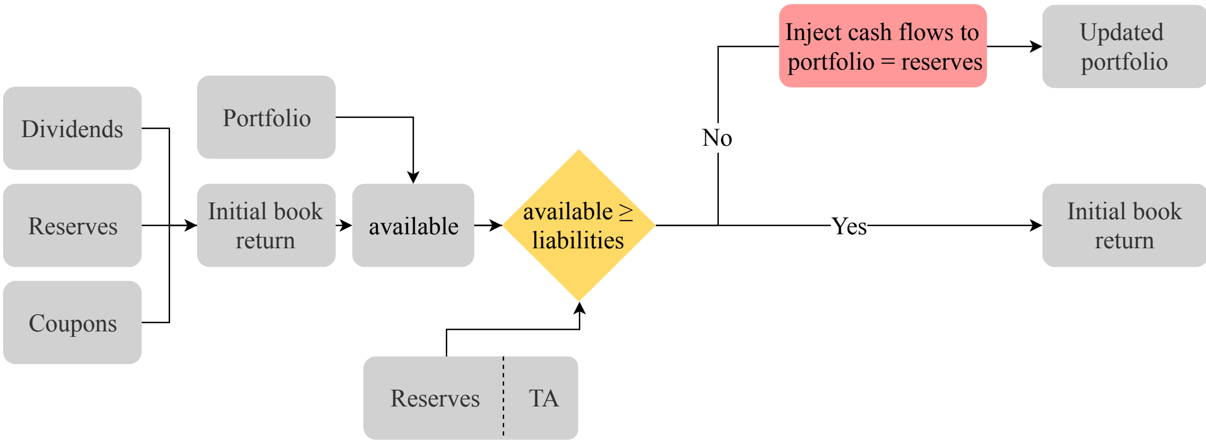

Figure 8.5: Stage 1 of the buffer strategy checks if the insurer's available funds (the *initial book return* plus the market value of the portfolio) are large enough to cover the liabilities. If the available funds are lower than the liabilities, the insurer must inject capital into the portfolio.

The value of the reserves in Figure 8.5 is still not increased by the guaranteed rate. This enables us to determine if the insurer has to inject capital into the portfolio in case of a negative return that cannot be covered by the KRF.

The output of this stage is a possibly updated portfolio which matches the insurer's liabilities. The initial book return and the KRF goes into stage 2. Notice that if the insurer has injected capital, the KRF is 0 after stage 1. This follows from the value of the portfolio exactly matching the value of the liabilities in this case.

- **Stage 2: Fulfill the guaranteed rate and allocate excess book return.** In this stage, the sum of the initial book return and the unrealized return (KRF) is either greater or equal to the guaranteed rate of return (a), or below it (b).

  (a) If the returns are above the guaranteed rate, we increase the reserves with the guaranteed rate. The book return exceeding the guaranteed rate is either shared between the policyholder and insurer or put into the TA. The insurer also determines if parts of the unrealized returns should be realized in order to increase the TA or be shared with the policyholder.

(b) If the returns are below the guaranteed rate, we check if the TA can cover the guaranteed rate. If this buffer is not sufficiently large, the insurer must again inject cash flows into the portfolio to cover the guaranteed rate.

Stage 2 is illustrated in Figure 8.6.

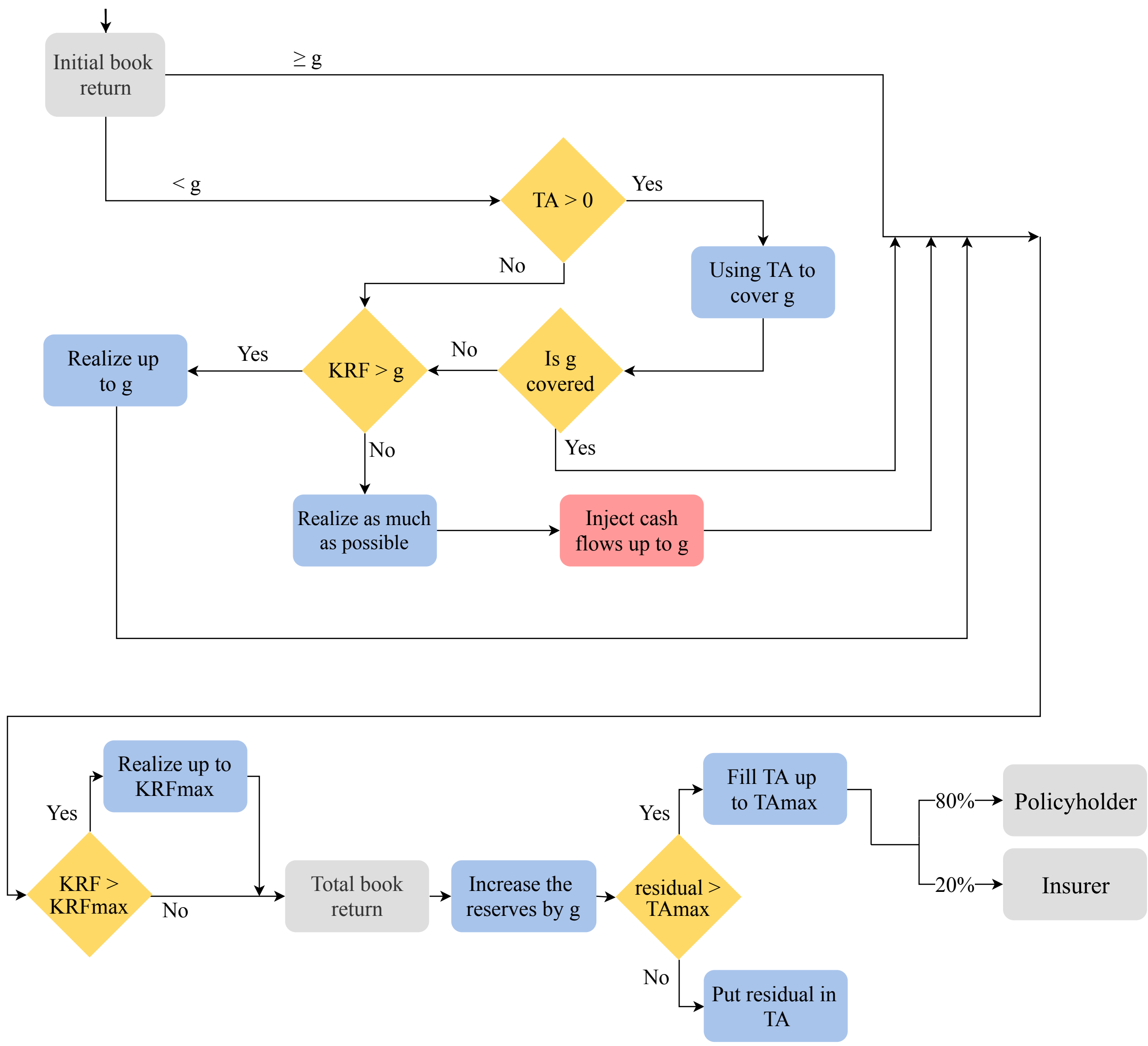

Figure 8.6: Flow chart for stage 2 of the buffer strategy. The input into the model is the initial book return coming from dividend, coupon and rebalancing, plus the KRF. In case of available returns over the guaranteed rate g, the returns can stay unrealized in the KRF, be distributed to the TA, profit-shared, or a combination of these alternatives. In case the available returns are lower than g, the KRF and the TA can be used or the insurer must inject extra capital to cover g.

In stage 2, we have decided the following priority for the buffer strategy:

1. Fill up the KRF to a certain maximum level (defined as a percentage of the reserves, denoted $KRFmax$). The maximum level can be adjusted according to the insurer's buffer preference.
2. Fill up the TA to its maximum level (defined as a percentage of the reserves, denoted $TAmax$). Also the maximum level of this buffer can be adjusted by the insurer.
3. Profit-share between policyholders and insurer.

As mentioned in Section 5.4, the KRF and TA can be used in years where the total return is not sufficiently large to cover the guaranteed rate. While the KRF can be used both to cover negative return and the guaranteed rate, the TA can only be used to cover the guaranteed rate. Due to the extra flexibility of the KRF we prioritize filling this buffer before the TA. To be able to analyze how the size of the buffers affects the market value, we define upper limits ($KRFmax$ and $TAmax$), which can be adjusted.

8.3.2 Buffer Strategy for Years of Payout

When the policyholder retires, the payout period starts. The reserves will thus be reduced each year as the insurer realizes this part of the portfolio and pays pension benefits to the policyholder. As the policyholder receives her promised payments, the TA also gets automatically reduced as the maximum level stays at 12 percent of the reserves. According to regulations, "the sum of the reserves and the TA [...] cannot be reduced in other ways than through payouts to the policyholder (Forsikringsvirksomhetsloven, n.d.)" [2]. Therefore, the client receives an additional payout whenever the TA decreases due to a reduction in the reserves.

Figure 8.7 illustrates the additional actions needed in the years of payout.

[2] Translated from Norwegian

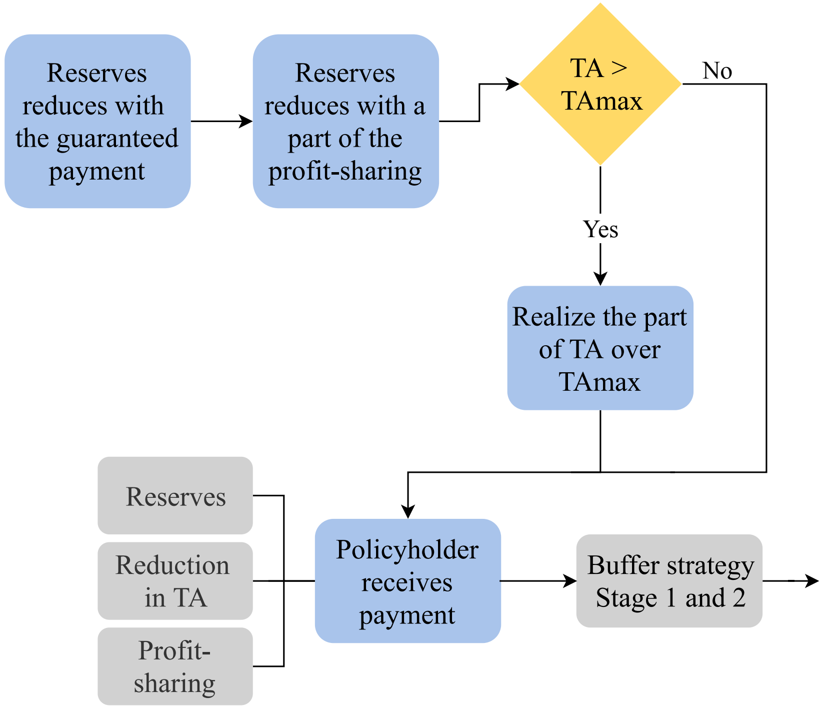

Figure 8.7: Additional buffer strategy for the years of payout. The reserves are reduced and possibly also the TA if its value exceeds the defined upper limit. The policyholder receives her yearly payment. Stage 1 and 2 of the buffer strategy of the buffer strategy is then applied.

The first action in the buffer strategy in the years of payout is to reduce the reserves with the guaranteed payment. The guaranteed payment takes into account risk of mortality as described in Section 8.2, and is therefore reduced by a certain mortality factor each year. Furthermore, the policyholder's reserves might have increased in cases of profit-sharing. If this is the case, the policyholder gets an additional payment in the payout period.

As earlier mentioned, the upper limit of the TA automatically gets reduced as it is defined as a percentage of the reserves. We therefore possibly have to reduce the buffers as well. As the TA belongs to the policyholder, the reduction is given to her.

After the reserves and possibly also the buffers are reduced, the buffer strategy from stage 1 and stage 2 in Figure 8.5 and Figure 8.6, respectively, is again carried out, since the policyholder still has the obligation to fulfill the guaranteed rate of return in the years of payout.

When all policyholders have received their guaranteed payments, there might still be some value left in the KRF. We contacted the Financial Supervisory Authority of Norway to learn if this value belongs to the policyholder or insurer. As we were told there were no clear rules, we have decided to let what's left in the KRF stay undistributed.

8.4 Determining the Discount Rate of the Cash Flows

We have now simulated stock and bond prices which decide the yearly market return of the insurer's portfolio and determined the cash flows generated to the policyholder and insurer by applying a buffer strategy. In the start of this chapter, we described how to find the market value by discounting the cash flows. However, we still need a discount rate. This is derived from the simulations of the short rate in the Hull-White-Black-Scholes model.

Under risk neutral measures, future cash flows are discounted using the bank account numéraire (BA_t) (Wüthrich, Bühlmann, & Furrer, 2010; Wüthrich, 2013). For an initial investment of one unit of currency, the future value of the bank account at any $t \geq 0$ is given by

$$BA_t = e^{\int_0^t r_s ds} \tag{8.33}$$

where r_s is the simulated short rates. In this equation we assume that we have the future spot rates under an arbitrage free measure ($\mathbb{Q}$) at every infinitely small time step. The market value (MV_0) of a future cash flow (CF_t) is then given by

$$MV_0 = \mathbb{E}^{\mathbb{Q}}\Big(CF_t e^{-\int_0^t r_s ds}\Big) \tag{8.34}$$

As calculating the change in the short-term interest rate continuously is not possible, we have to make some assumptions to be able to apply Equation (8.33) for discrete values. If we assume that the interest rate stays constant between two intervals, Equation (8.33) can be written as a sum. In this discrete setting, the present value of a future value can now be calculated using

$$MV_0 \approx \mathbb{E}^{\mathbb{Q}}\Big[CF_t e^{\left[\sum_{s=0}^{t} r_s\right]}\Big] \tag{8.35}$$

By Equation (8.35) we can value paid-up policies from the generated cash flows and the bank account numéraire as the discount rate.

8.5 Multiple Contracts

An insurer has several customers with different guaranteed rates, reserves and sizes of the TA. Recall that the guaranteed rate is set at issuance of the policy and stays constant throughout the contract period. This means that the time of issuance and age of the policyholder highly impact the characteristics of the paid-up policy. The sizes of the reserves and the TA are also dependent on the age. As a result, we segment the customers based on their age in our model.

We divide the customers in 54 groups, where each group consists of customers with the same age, size of the reserves and size of TA. Figure 8.8 illustrates how we have divided the customer groups.

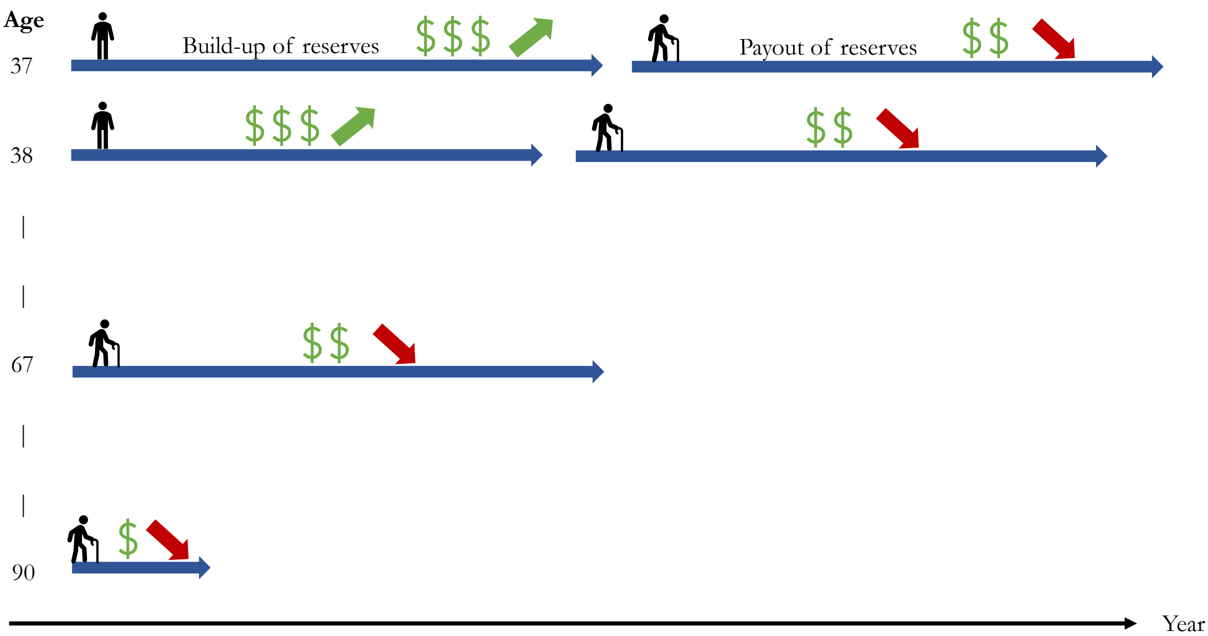

Figure 8.8: The customers are divided into 54 customer groups, where each group consists of customers with the same age and also with the same characteristics of their paid-up policies. All policyholders retire at age 67. The first group has 30 years where the reserves are built-up and 24 years where the reserves are reduced and paid out. The last group has only payment left.

The youngest group consists of 37 year old policyholders, while the policyholders within the oldest group are 90 years old. All policyholders are assumed to retire at 67, which is the retirement age stated by law in Norway (Regjeringen, 2017). With this retirement age, the first 30 groups are in a period where their reserves are being built-up, i.e. invested in the financial market by the insurer. We illustrate the build-up period with the two topmost left lines in Figure 8.8. The first group receives their first pension payment (benefit) after 30 years when they reach 67 years old. The payout period then starts and lasts 24 years, illustrated by the lines where the reserves are being paid out. The latter 24 groups have already retired and their reserves therefore reduce as they receive promised payments. The oldest group included in our model only has one payment before the paid-up policy expires, i.e. the policyholders pass away.

The reserves of all groups together pose the liabilities for the insurer. We use the mortality tables described in Section A.6 to calculate the promised payments to each customer group. As we see from Figure 8.8, the insurer manages several paid-up policies with different characteristics at the same time. The guaranteed rates, as well as the initial values of the TA and the reserves are calibrated according to market data. This is discussed in Section 9.5.

8.6 Control Variate

The model illustrated in Figure 8.3 requires multiple valuations of paid-up policies, and therefore also a huge number of simulations. In order to limit the number of simulations required to obtain accurate market values of paid-up policies, we therefore look at how the variance can be reduced.

The payments the policyholders receive and the net cash flows for the insurer are both correlated to the evolution in the stock price and interest rate. In order to reduce the variance in the estimated market value of the paid-up policies for the policyholders and insurer, the control variate method can therefore be applied.

Letting $\boldsymbol{Y} = (Y_I, Y_C)$ be a vector of random variables whose mean is to be determined through simulation. In our case, these variables are the market value of the payments to the customer (Y_C) and the market value of all cashflows to and from the insurer (Y_I).

Also, letting $\boldsymbol{Z} = (Z_S, Z_B)$ be a vector containing the control variates for stock prices (S_t) and interest rates (r_s), where

$$Z_S = \frac{\sum_1^n S_t e^{-\sum_0^t r_s}}{n} \tag{8.36}$$

$$Z_B = \frac{\sum_1^n e^{-\sum_0^t r_s}}{n} \tag{8.37}$$

n is the number of simulations.

These control variates have the known mean values $\boldsymbol{\mu} = (S_0, B_0)$, where S_0 is the initial stock price, and B_0 is the price of a zero-coupon bond, given a spot rate curve. Using the control variates Z_S and Z_B, $\bar{Y}_{CV}(\boldsymbol{\beta})$ is now a consistent and unbiased estimator for the expected values of Y_I and Y_C.

$$\bar{Y}_{CV}(\boldsymbol{\beta}) = \bar{Y} - \boldsymbol{\beta}'(\boldsymbol{Z} - \boldsymbol{\mu}) \tag{8.38}$$

We want to minimize $\text{Var}(\bar{Y}_{CV}(\boldsymbol{\beta}))$. If we assume that the covariance matrix Σ_{ZZ} is non-singular (i.e. that it has a matrix inverse), it can be shown via the second-order optimality conditions that the following holds (Glynn & Szechtman, 2002)

$$\boldsymbol{\beta}^* = \Sigma_{ZZ}^{-1}\Sigma_{YZ} = \begin{bmatrix} \text{cov}(Z_S, Z_S) & \text{cov}(Z_S, Z_B) \\ \text{cov}(Z_S, Z_B) & \text{cov}(Z_B, Z_B) \end{bmatrix}^{-1} \begin{bmatrix} \text{cov}(Z_S, Y^{(i)}) \\ \text{cov}(Z_B, Y^{(i)}) \end{bmatrix} \tag{8.39}$$

where i represents either the payments to the customer or the cash flows to and from the insurer.

Lavenberg, Moeller, and Welch (1982) derive that with two control variates, the variance of $\bar{Y}_{CV}(\boldsymbol{\beta})$ is given by

$$\text{Var}(\bar{Y}_{CV}(\boldsymbol{\beta}^*)) = \frac{n-2}{n-4}(1 - R^2)\text{Var}(\bar{Y}) \tag{8.40}$$

where R^2 is the squared multiple correlation coefficient between Y and Z defined by

$$R^2 = \frac{\Sigma_{YZ}^T \Sigma_{ZZ}^{-1} \Sigma_{YZ}}{\sigma_Y^2} \tag{8.41}$$

Using Equation (8.38) allows us to value the paid-up policies with a lower variance in the simulations. We can thus reduce the number of simulations, which is discussed in Appendix A.7.

8.7 Hedge Portfolio

The uncertainty related to future returns on stocks and bonds can result in large losses for insurance companies, as they guarantee the policyholders a certain annual return on their pension savings. To hedge against a drop in the stock price and interest rate, the insurers can trade different kind of options. The payoffs from the options will be negatively correlated to the return on the *policy portfolio*.

These investments are a part of a separate *hedge portfolio* that is not connected to the paid-up policies. The main reason why we keep the *hedge portfolio* separated from the *policy portfolio* is that options are reported differently in the financial statements than bond and stock holdings. Recall that the book return of the *policy portfolio* has to be used to cover the guaranteed rate. If the book return is higher than this rate, the additional return has to be distributed to the TA, or shared between the policyholder and insurer. Returns on stocks and bonds mainly lead to changes in the unrealized return. Payoffs from the options, however, lead to changes in the book return, and will therefore directly affect the profit-sharing. The *hedge portfolio* is therefore kept separate in order to manage the paid-up policies without being affected by the option payoffs.

We consider a case where the insurer invests in options that hedge negative movements of the *policy portfolio*. The two portfolios and their relations are illustrated in Figure 8.9.

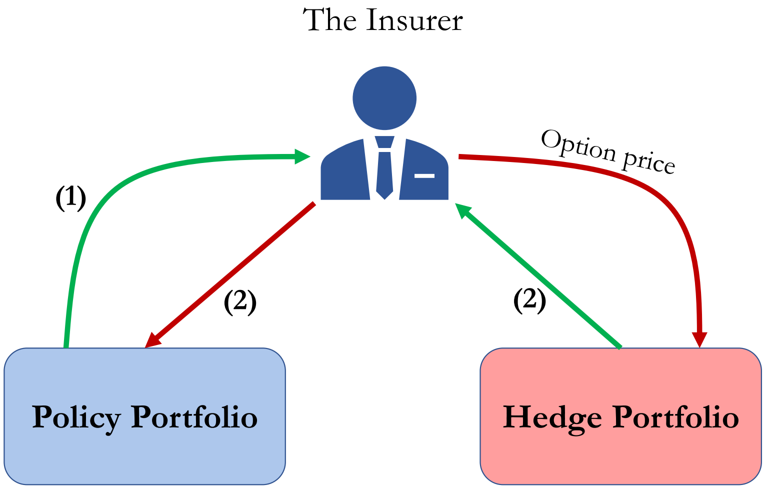

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Figure 8.9: The insurer holds two portfolios, the *policy portfolio* and the *hedge portfolio*. The red arrow to the right indicates the option premiums, which are always paid. In case (1), the total market return of the *policy portfolio* can give a payoff to the insurer if profit-sharing occurs. In case (2), the negative market return of the *policy portfolio* leads to the insurer injecting cash flows into this portfolio. However, the insurer then receives payoffs from the options in the *hedge portfolio*

When a hedge strategy is included, the insurer holds two portfolios: The *policy portfolio* and the *hedge portfolio*. The options premiums on the options bought are paid by the insurer in all cases, illustrated by the red arrow to the right in Figure 8.9. When the total book return of the *policy portfolio* is sufficiently high (1), it can lead to payoffs to the insurer if profit-sharing occurs. This is illustrated by the green arrow to the left. In this case, the insurer receives no payoffs from the *hedge portfolio* as the options are worthless at expiration date. However, if the market value of the *policy portfolio* falls below the market value of the insurer's liabilities, he must inject cash flows into the *policy portfolio* (2), represented by the red line to the left. In this case, the options from the *hedge portfolio* will give a positive payoff to the insurer, illustrated by the green line to the right in Figure 8.9.

Rabobank, which is one of the major banks in the Netherlands, has had a success with its extensive hedging policy against risks on interest rates, equity, inflation and currency on its pension fund. In 2011, for instance, the company managed to achieve a total return of 14.7%, even though the long-term interest rates were decreasing, and the equity portfolio generated a 6.5% loss (Preesman, 2012). However, their trading strategy has led to losses in years when the value of stocks and bonds have increased. This happened for instance in 2014 (Preesman, 2014).

Buying stock and zero-coupon bond options gives the insurer a hedge against fluctuations in both the stock price and interest rate. As mentioned above, the *policy portfolio* consists of stocks and coupon bonds. However, as the return on zero-coupon bonds are highly correlated to coupon bonds, we expect that they will give a sufficient hedge against decreasing interest rates. Notice that more complicated hedging strategies are possible. However, we want to keep it simple to analyze the idea of a separate hedge portfolio.

Both stock and zero-coupon bond options are priced using risk-neutral scenarios. This is done to achieve full consistency with the valuation of the paid-up policies. In the following subsections, we briefly describe the different options. Notice that the insurer also has the possibility to sell options. Including short positions reduces the costs related to the option investments, and allows to tailor the portfolio better to the insurance companies' risk preferences.

8.7.1 Hedge Portfolio Affecting the Balance Sheet

In order to understand how a hedge portfolio affects the capital requirements, we consider the balance sheet of an insurer. In Table 8.1, we have constructed the balance sheet of a hypothetical insurance company only offering paid-up policies.

Assets	**Liabilities**
Hedge portfolio	Market value of paid-up policies
Policy portfolio	**Equity**
	Own funds

Assets = Liabilities + Equity

Table 8.1: Balance sheet of a hypothetical insurance company only offering paid-up policies.

In order to comply with the capital requirements under the Solvency II directive, the insurer's own funds have to be at least equal to the SCR (see Section 7). In detail, large enough to cover the one-year risks with a 99.5% probability. The SCR is thus the difference between the market value of paid-up policies at time $t = 0$ and the 99.5% VaR of the market values at $t = 1$. If the insurer's own funds equal the SCR at time $t = 0$, he cannot increase the risk without violating the capital requirements.

The goal of the *hedge portfolio* is to reduce the SCR, and hence the amount of own funds the insurance company has to hold. If the insurance company invests in a hedge portfolio, the risk gets reduced, as the value of this portfolio will increase if the value of the *policy portfolio* decreases. This will therefore lead to a reduction in the SCR. We assume that buying options is financed by selling assets that earned a variable interest rate (assumed risk-free).

8.7.2 Stock Options

A put option gives the buyer the right to sell a stock at a pre-specified price, known as the strike price. It can therefore be used to hedge against a reduction in the stock value, as illustrated in the payout diagram in Figure 8.10.

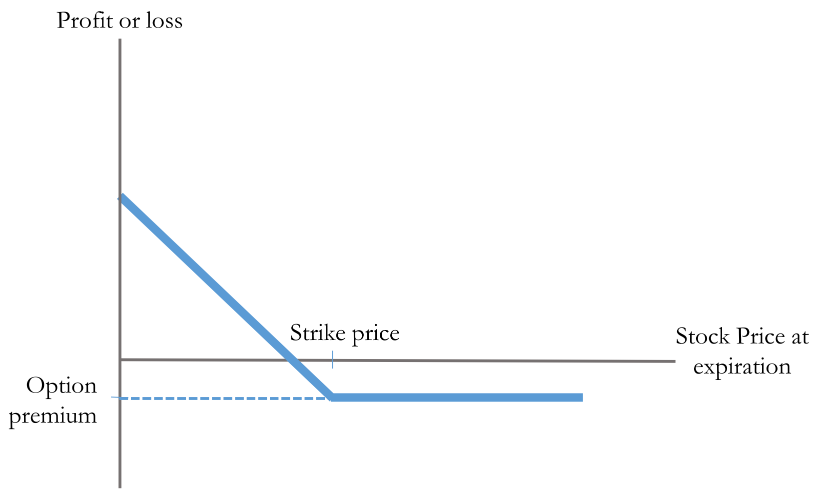


Figure 8.10: The payoff of a stock put option.

The diagram illustrates that a stock price below the strike price at maturity, leads to a payoff at maturity. If the stock price is higher than the strike price, the net payoff from the option is only the initial option premium paid. A call option gives the right to buy a stock at a certain price, and hedge against an increase in the stock prices. In the model we assume that all options are of an European type, which means that they cannot be exercised before maturity. The value of the options therefore solely depend on the stock value at maturity.

The formulas for valuing stock call (C_0^S) and put options (P_0^S), respectively, are given below.

$$C_0^S = \frac{\sum_{i=1}^{n}\left[(S_t - K)\cdot e^{-\sum_{s=0}^{t} r_s}\right]}{n} \tag{8.42}$$

$$P_0^S = \frac{\sum_{i=1}^{n}\left[(K - S_t)\cdot e^{-\sum_{s=0}^{t} r_s}\right]}{n} \tag{8.43}$$

where K is the strike price, S_t the stock price at time t, r_s the short rates and n the number of simulations.

8.7.3 Bond Options

The zero-coupon bond options work similar to the stock options, and give the option holder the right to buy or sell a bond at a pre-specified price.

There is an inverse relationship between interest rates and bond prices. This means that if interest rates decrease, the bond value will increase. If the interest rates instead increase, the bond will lose value. For an insurance company, a reduction in the interest rate will therefore be beneficial in the short-term, as the value of the bonds increases. However, in the long-term, lower interest rates mean that the probability of reaching the guaranteed rate in the coming years gets lower. To hedge against low interest rates, zero-coupon bond call options can be bought. The payoff diagram for such call options is given in Figure 8.11.

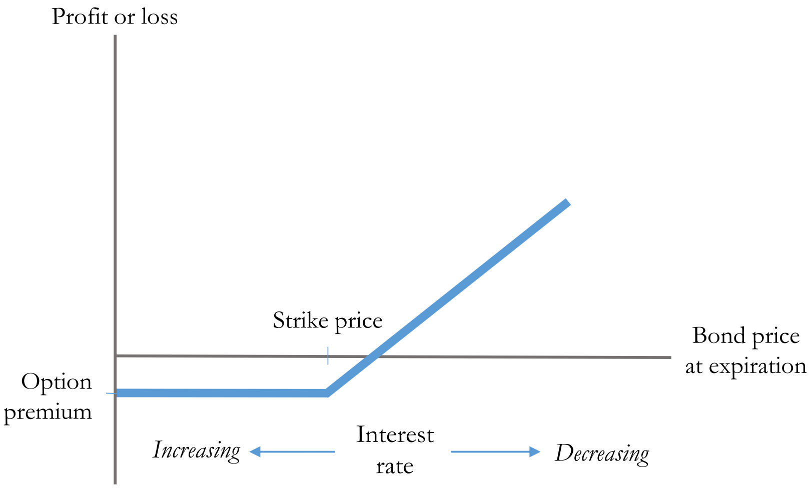


Figure 8.11: The payoff of a zero-coupon call option.

The figure illustrates that decreasing interest rates leads to higher option payoffs due to the inverse relationship between bond price and interest rate.

The prices of a call (C_0^B) and put option (P_0^B) linked to a bond with maturity T are given by the following formulas, respectively.

$$C_0^B = \frac{\sum_{i=1}^{n} \left[(P(t,T) - K) \cdot e^{-\sum_{s=0}^{t} r_s} \right]}{n} \tag{8.44}$$

$$P_0^B = \frac{\sum_{i=1}^{n} \left[(K - P(t,T)) \cdot e^{-\sum_{s=0}^{t} r_s} \right]}{n} \tag{8.45}$$

$P(t,T)$ is the value of a T-year zero coupon bond at the t. K is a chosen strike price, S_t the stock price at time t, r_s the short rates and n the number of simulations. In our model, the bond options are linked to the price of a 10-year bond. As bonds with different maturities are highly correlated in the Hull-White model, this choice will not impact the effect of the hedge portfolio.

8.7.4 Option Combinations

An investor can combine call and put options with different strike prices in order to achieve specific hedges. For example, buying a put option and selling a call option creates a so-called collar. The advantage of this option combination is that the net option premium is lower than if the insurer only buys options. However, the combination also hedge both increased and reduced asset prices. It therefore makes the net cash flows for the insurer more certain. Figure 8.12 illustrates the payoff of a collar consisting of put and call stock options.

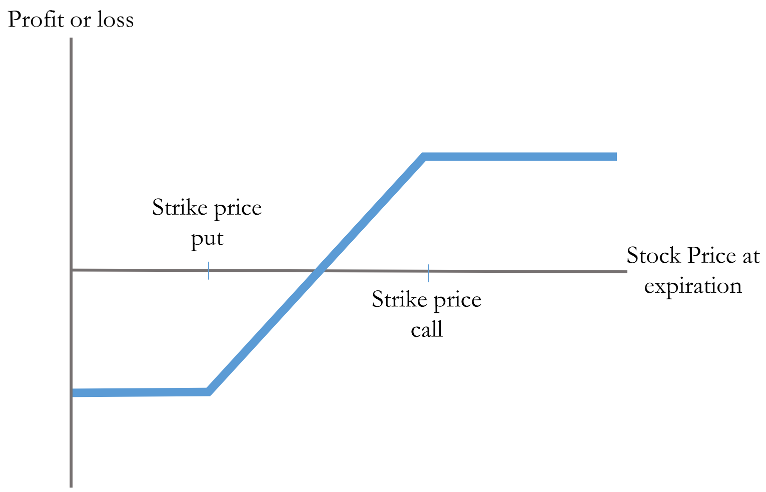


Figure 8.12: The payoff of a collar option, where a stock put option is bought and a stock call option is sold.

Notice that the shape of the graph is identical for a collar consisting of zero-coupon bond call and put options.

9 Calibration of Model

This section presents how the model is calibrated using available market data and internal data provided by the Norwegian insurance company Nordea Liv. Section 9.1 presents the parameters used in the simulation of stocks and bonds. Section 9.2 explains how we use market data to calibrate the Hull-White model that simulates the development of bond prices. Section 9.3 discusses the length of the simulated contract period. Section 9.4 presents the calibration of the KRF and TA, using internal data from Nordea Liv. Finally, Section 9.5 presents the calibration of the promised guaranteed rate on the paid-up policies.

9.1 Asset Classes in the Portfolio

As mentioned in Chapter 8, the assets in the *policy portfolio* are invested in stocks and bonds only. This assumption is made as in the general asset mix for insurance companies in Norway, where stocks and bonds account for approximately 90% of the total portfolio for paid-up policies (Regjeringen, 2018). The remaining part is invested in real estate, accounting for about 10% of the portfolio. Based on the annual reports from Storebrand (2018b) and Nordea Liv (2018), rental income from real estate accounts for approximately 3% of portfolio return each year. The two insurance companies manage more than 50% of paid-up policies in the Norwegian market. The implementation of real estate will be disregarded in our model, but is compensated by a higher share of the *policy portfolio* invested in stocks, which is discussed below. As the annual return of real estate is the same as the dividend yield on stocks used in our model, stocks and bonds are the sole focus. When we include a hedge portfolio in the model, we also make sure that we are not able to buy options for the part of the stocks that corresponds to real estate.

The annual reports of Storebrand and Nordea Liv also state that stocks only constitute for 7% and 9% of the portfolio, respectively. Bonds constitute the dominating asset class for both companies, where Storebrand has 37% HTM-bonds and 30% AFS-bonds, while Nordea Liv has 60% HTM-bonds and 19% AFS-bonds. As we observe, the portion of stocks is generally low while the bond characterization varies across companies. Based on the data and the fact that we do not model real estate, we choose to set the initial equity weight in our *policy portfolio* to 20%, the initial weight of HTM-bonds to 40% and the initial weight of AFS-bonds to 40%. The initial asset mix in our portfolio is illustrated in Figure 9.1.

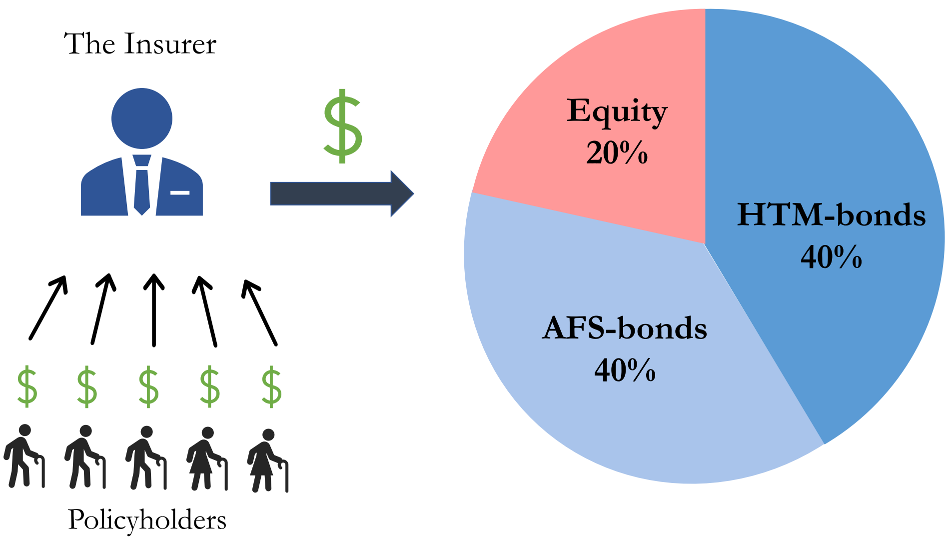


Figure 9.1: The insurer invests the policyholders' pension savings (the *policy portfolio*) in 20% stocks, 40% HTM-bonds and 40% AFS-bond in our model.

An invested portfolio which consists of only stocks and bonds also has the advantage that it is easy to adjust according to the insurer's desired risk profile. This will be helpful when determining how the market value of a policy changes according to different investment strategies.

Recall that we use the Black-Scholes model to simulate the development of the stock price. The volatility in the Black-Scholes model is based on the historical volatility of the Oslo Stock Exchange, and is set to 25%. A quarterly dividend of 0.50% is being paid out. This is based on the average historical dividend since 1990 of S&P500 of approximately 3% yearly[3]. The dividend is immediately reinvested in the stock market.

Based on historical data, the correlation between the stock prices and the interest rates is set equal to 0.249. This is used in the Hull-White-Black-Scholes model, discussed in Section 8.1.3.

[3]Data is taken from Quandl.com, which provides different financial datasets.

9.2 Calibrating Bond Prices

As mentioned in Section 8.1.2, the initial interest rates are calibrated using data provided by EIOPA. EIOPA provides yearly spot rates for zero-coupon bond maturities between 1 and 150 years. However, these spot rates are discrete while we need to calibrate the Hull-White Model using a continuous zero rate curve. To estimate a continuous zero rate curve, we therefore perform a regression analysis using the Curve Fitting Toolbox in Matlab. The spot rate curve and the corresponding instantaneous forward rate curve are illustrated in Figure 9.2. The figure illustrates that the market interest rate is expected to increase.

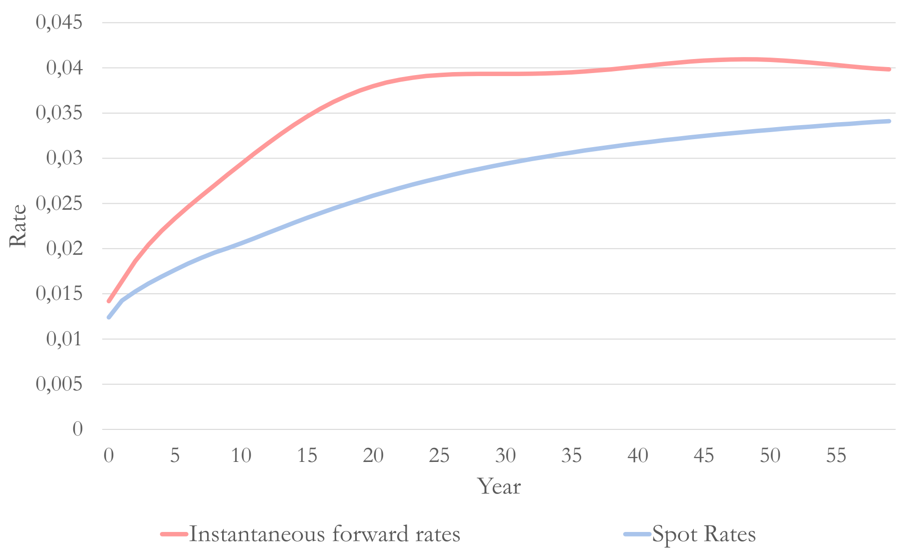


Figure 9.2: The blue line shows the regression of the zero rates provided by EIOPA. The red line shows the instantaneous forward rate curve derived from the zero rates.

Also the drift in the model, denoted α, and the volatility, denoted σ_B, have to be calibrated using market data. There are several ways to do this calibration. However, all have in common that they require prices of interest rate derivatives. As these prices are not easily available, we choose to use the parameters from a calibration done by Russo and Torri (2018). We select α and σ_B values obtained from the Jamshidian's (1989) approach [4], which is usually adopted by practitioners to calculate prices for coupon bond options. Using theoretical swaption prices Russo and Torri (2018) concludes that the drift α is equal to 0.0304, and the volatility σ is equal to 0.0085.

9.3 Length of the Contracts

The retirement age in Norway is 67 years, stated by law. In our model, we start to pay out the reserves when policyholders reach this age. We include customer groups ranging from 37 to 100 years old, where the last group consists of all customers from 90-100 years old as described in Section 8.5. By including this range of customers, we account for more than 99% of all paid-up policies under management in Nordea Liv. The youngest group which is 37 years today has 30 years where the reserves are being built up and 24 years where the reserves are being paid out as pension payments. This means that we simulate 54 years in total in our model.

9.4 Initial Size of the Buffers

The initial size of the KRF is set to 2% of the reserves. This is based on numbers from the latest annual reports from Nordea Liv (2018) and Storebrand (2018b).

The initial size of the TA is calibrated using internal data provided by Nordea Liv, where the value of the TA is aggregated based on the groups described in Section 8.5. Recall that the customers are divided by age, which gives customer groups from 37 to 90 years old. The initial size of the groups' TAs are given in Figure 9.3.

[4] The method gives a closed-form solution for pricing risk-free bonds. The interest rate is assumed to follow a mean reverting Gaussian process.

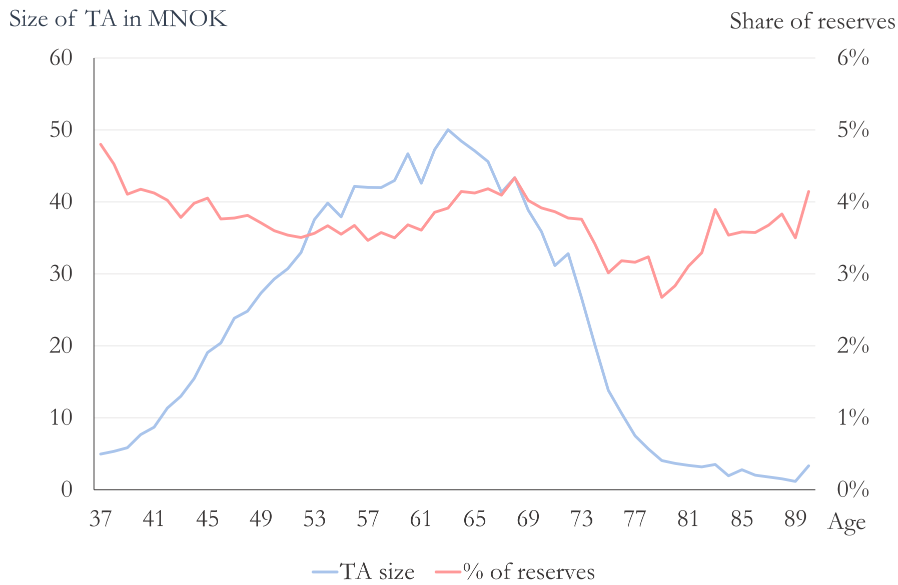


Figure 9.3: The blue line shows the size of the TA for different customer groups consisting of policyholders segmented based on their age. The red line shows the size of the TA expressed as a percentage of the group's reserves, given by the secondary vertical axis on the right.

Figure 9.3 plots the size of the TA (blue line) on the left vertical axis for each customer group on the horizontal axis. The red line expresses each group's TA as a share of the group's size of the reserves, given by the right vertical axis. Recall that the TA cannot exceed 12% of the reserves. We observe from the figure that the size of the TA is smallest for the youngest and oldest groups, and largest for the customers approximately 60 years old. This is in line with what we would expect, as customers aged 60 have experienced returns being distributed to their TA in more occasions than the younger groups. The size of the older groups' TA is lower due to the fact that some of the TA has been paid out as the reserves decrease, as discussed in Section 8.3.2. The result is that the size of the TA relative to the reserves stays more or less constant for most groups, as the red line illustrates.

9.5 Guaranteed Rate on the Paid-up Policies

The model must also be calibrated according to the actual guaranteed rates of paid-up policies. The guaranteed rate has a huge impact on the market value of a policy, as it decides the value of the reserves and the yearly annual return the insurer must strive to achieve. Nordea Liv has provided us with the guaranteed rate on their different customer groups, also segmented on their age. The guaranteed rates for the groups are illustrated below.

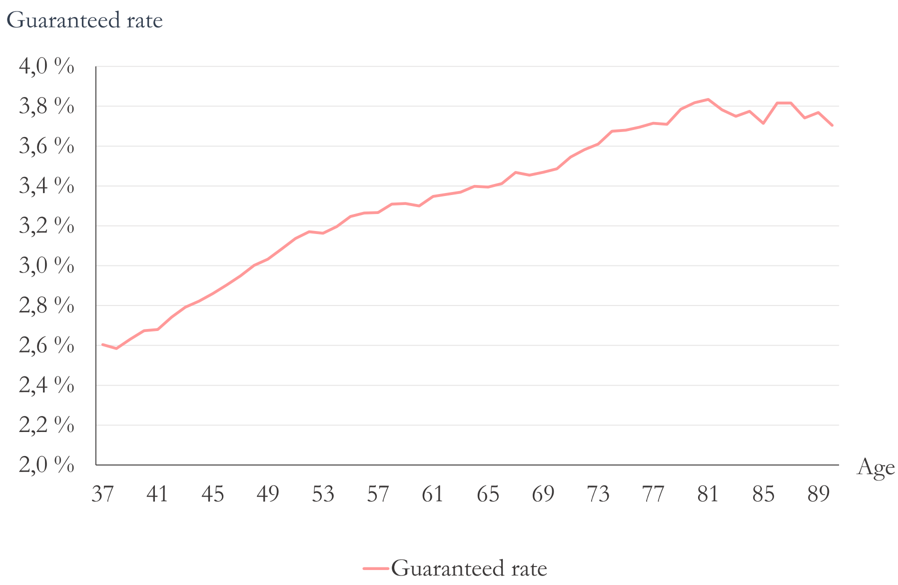


Figure 9.4: The red line shows the guaranteed rate insurers promise to policyholders, segmented on their age along the horizontal axis.

The guaranteed rate offered to policyholders ranges from 2.6% to 3.8%, where the oldest policyholders have the highest guarantee on their pension savings. This follows from the fact that when the policies of the oldest groups were issued, the general offered guaranteed rate was higher. Also recall that the guaranteed rate for a customer group stays constant through the contract period. The average guaranteed rate this insurance company promises to policyholders is 3.30%.

Our model is calibrated such that each customer group has its guaranteed rate as in Figure 9.4. With these guaranteed rates, the total initial value of the reserves, i.e. the insurer's liabilities, is equal to 33.1 billion NOK.

9.6 Risk Premium in Real-World Simulations

In order to analyze the risk associated with paid-up policies, we simulate real-world scenarios, as presented in Chapter 8. In the real-world scenarios, the simulations of the stock price in the Black-Scholes model takes into account a risk premium (r_p) above the risk-free rate (r_t) simulated in the Hull-White model. Equation (9.1) states the calculation of the stock price one time step (Δt) ahead

$$S_{t+\Delta t} = S_t \cdot e^{(r_t + r_p - \frac{1}{2}\sigma_S^2)\Delta t + \sigma\sqrt{\Delta t}Z} \tag{9.1}$$

where σ_S is the volatility of the stock price and Z denotes a standard normal random variable.

In our case, we simulate the development of the stock price using quarterly steps, i.e. $\Delta t = 0.25$. In order to decide the risk premium, we analyze the historical return of the stock indexes S&P500, the 500 largest publicly traded companies in the U.S, and OSEBX, the Oslo Stock Exchange. Since January 1983, OSEBX has delivered an average quarterly return of 3.7% and S&P500 a quarterly return of 2.7% in the same period[5]. Our results are not very sensitive to that choice, so we choose a risk-premium equal to 3%.

[5]The return has been derived from datasets retrieved from Yahoo Finance (2019a, 2019b)

10 Results

This section presents the results of the model presented in Section 8. The goal is to find an optimal investment and buffer strategy. We will consider two different definitions of what is optimal. First, we maximize the market value of the paid-up policies for the insurer. Thereafter, we minimize the capital requirements.

The results will be presented as the following. First, in Section 10.1, the dynamics of the model, i.e. how the book return, the profit-sharing and buffers change during the contract period, is presented. We then look at how different investment strategies affect the market value of paid-up policies from the insurer's and policyholder's perspective in Section 10.2. For this examination, we use risk-neutral scenarios and analyze how different asset mixes and rebalancing strategies affect the market value of the policies. We then analyze how the selection of a buffer strategy can improve the perspective of the insurer in Section 10.3. The size of the KRF and the TA, plus the use of a dynamic buffer strategy adjusted according to the interest rate, will be the topic in this section. The second main part of the results includes the one-year risk and the Solvency Capital Requirement, which was explained in Section 7. This is presented in Section 10.5. In the same section, we discuss how options can be used to hedge the risk associated with paid-up policies.

In Section 9 we discussed the selections of parameters in the simulations. In Table 10.1, some of the most important ones are repeated. These parameters will be used unless otherwise specified.

Parameter	Value
Number of RN simulations	5,000
Number of RW simulations	1,000
Number of intervals per year	4
Initial value of KRF	2%
Initial equity weight	20%
AFS-bonds	5 year: 50%, 10 year: 50%
HTM-bonds	1 year: 40%, 3 year: 20% 5 year: 20%, 10 year: 20%
Upper limit for equity weight	21%
Lower limit for equity weight	19%
Correlation stocks and bonds	0.249
Retirement age	67
Initial asset value	34.5 billions NOK

Table 10.1: Base parameter values used in the model. Risk-neutral is abbreviated RN and real-world is abbreviated RW.

10.1 General Insights

As discussed in Section 5.4, the yearly book return is the basis of the profit-sharing system. Coupons from bonds, dividends from stocks and realized returns from rebalancing the portfolio will always be a part of the book return. This was referred to as the *initial book return* in Section 8.3.1. In addition, the insurer has to decide how much to realize from the unrealized return into book return. We define the *total return* here as the yearly market return plus the initial book return. In other words, this is the yearly return the insurer achieves before making any decisions regarding buffers and profit-sharing.

Figure 10.1 gives the distribution of the total yearly return of the insurer's *policy portfolio* from 5,000 risk-neutral simulations.

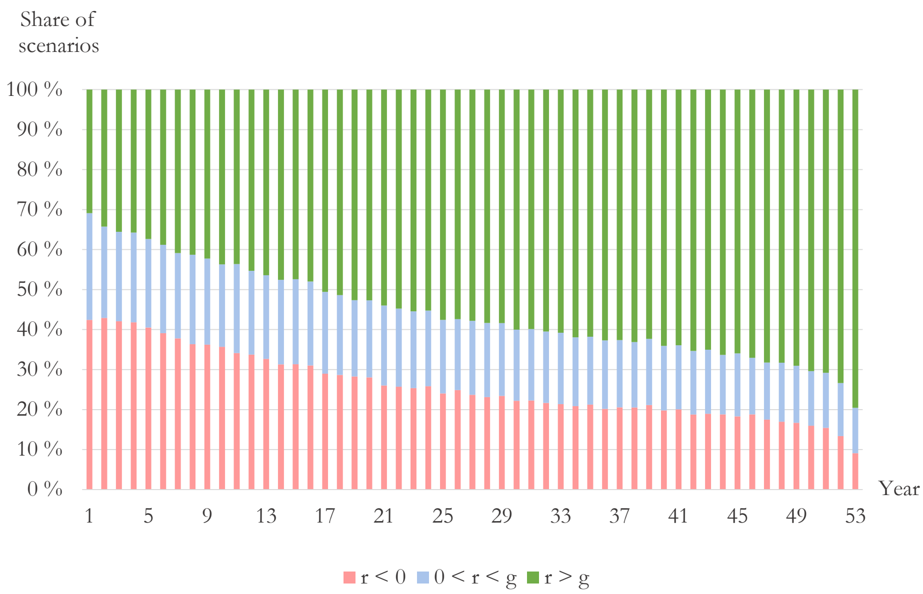


Figure 10.1: The percentage of the scenarios that has a total return below zero (red), between zero and the guaranteed rate (blue) and above the guaranteed rate (green) for each year in the contract period.

The bars in Figure 10.1 indicate the portion of scenarios with a total return below zero (red bars), between zero and the guaranteed rate (blue bars) and above the guaranteed rate (green bars) for each year. As we observe, the number of scenarios where the total return is below zero decreases through the simulation period. The cases where the total return is above the guaranteed rate increases. This means that the insurer more frequently has the option to set aside reserves in buffers or share profit with the policyholders. As mentioned in Section 9.2, market interest rates are expected to increase. Moreover, recall from Section 9.5 that each customer group has a different guaranteed rate on their paid-up policies. The guarantee stays constant through the contract period. In general, the younger customers (i.e. the groups with the longest lifetime of the policies) have a lower guaranteed rate. As a result, the average guaranteed rate on the policies decreases through the simulation period. Combining this with a higher market interest rate, the insurer achieves total return above the guaranteed rate more often in the end of the contract period.

We can also see how the market return for bonds and stocks vary in the risk-neutral simulations. Figure 10.2 plots these returns in year 1.

Figure 10.2: Stock returns (blue bars) and bond returns (red bars) for year 1 with 5,000 simulations.

The blue bars correspond to the stock returns while the red bars represent the bond returns. As expected, Figure 10.2 illustrates that bond returns (volatility of 1.8%) are clearly less volatile than stock returns (volatility of 25.0%).

During the contract period, the sizes of the two buffers (the TA and the KRF) change as the insurer distributes profit to the buffers, and use the buffers to cover insufficient returns. Recall from Section 8.3.1 that building buffers is prioritized over profit-sharing to customers, where the KRF has priority over the TA. This means that sharing profit between customers and the insurer only occurs after the buffers have reached their defined maximum limits ($KRFmax$ and $TAmax$), set by the insurer. The sizes are expressed as a percentage of the insurer's liabilities. We first present how the size of the KRF impacts the market value of the paid-up policies for the insurer in Figure 10.3. The initial asset mix consists of 20% equity and 80% bonds, and $TAmax$ is set to 12%.

Figure 10.3: The market value for the insurer of the paid-up policies for different customer groups along the horizontal axis. The red line represents the case with a $KRFmax$ of 5% and the blue line with a $KRFmax$ of 30%.

Figure 10.3 plots the market values for each customer group of paid-up policies with a $KRFmax$ of 5% (red line) and 30% (blue line). The total market value for the insurer is the sum of the market value for each group. As discussed in Section 8.5, the customers are divided into groups decided by their age in the start of the simulation period. This gives 54 customer groups, ranging from 90 years old to 37 years old along the horizontal axis. We first see that the insurer faces the largest losses for the customer groups 62 to about 53 years old in both cases. This comes from the fact that these groups are the largest in terms of the size of their reserves, and they also have a high guaranteed rate compared to younger groups. When $KRFmax$ is changed from 30% to 5%, the losses for the older groups in the left part of Figure 10.3 seems unaffected by the $KRFmax$. A large part of these customer groups is in the payout phase and has not many years until the contract expires and the KRF, therefore, has less impact. For the younger groups, however, the defined upper size of the KRF makes more impact. We see from the gap between the two lines in the right part of Figure 10.4 that the insurer takes larger losses with a small $KRFmax$ than a large $KRFmax$ for these groups. This implies that the importance of a large buffer for the insurer increases with a longer time to expiration of the contract.

We can further examine how profit-sharing to customers are affected by the size of the KRF. Figure 10.4 plots the market value of the funds above the guaranteed rate distributed to customers each year in the contract period for $KRFmax$ equal to 30% and 5%.

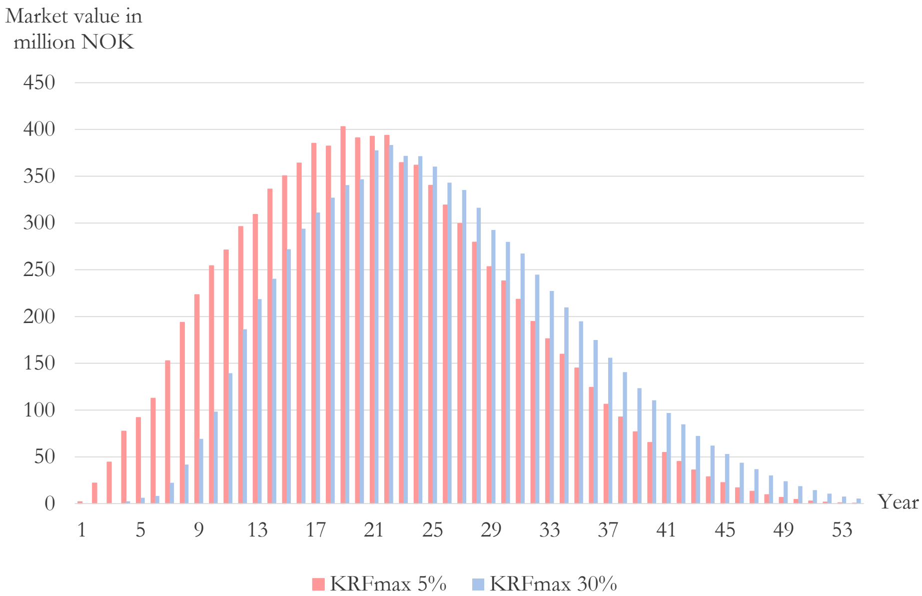


Figure 10.4: Market value each year of the profit above the guaranteed rate which is distributed to customers in case of a $KRFmax$ at 5% (red bars) and 30% (blue bars). The initial equity and bond weights are 20% and 80%, respectively.

The bars in Figure 10.4 represent the market value of the profit distributed to customers each year when $KRFmax$ is 5% (red bars) and when $KRFmax$ is 30% (blue bars). We see that more profit is distributed the first 22 years when the maximum size of the KRF is small (5%) compared to large (30%). This can be explained by the dynamics of the model, where it is more likely that the KRF will be filled to its maximum level sooner when the $KRFmax$ is 5% compared to 30%. Additional profit is then distributed to customers when the KRF (and the TA) have reached their maximum limits. After a while, it is also likely that the KRF can reach its maximum limit in the case of $KRFmax = 30\%$, which increases the profit-sharing to customers, seen from the blue bars increasing after about five years. Towards the end, we see that the blue bars are higher than the red bars. This means that more profit is distributed in the case of a large buffer compared to a small buffer, which can be surprising. However, recall that the KRF is a common buffer the insurer holds. When policyholders pass away, the funds will be kept in the KRF. The effect is that the insurer builds a large KRF in the first part of the simulation period, which the younger customer groups benefit from later. Even so, more profit is distributed in total when the maximum level of the KRF is kept low compared to large. Further analysis on the buffers follows in Section 10.3.

10.2 Investment Strategy

We now focus on the first part of finding an optimal investment and buffer strategy, namely maximizing the market value of the paid-up policies for the insurer. We start our analysis by examining how different asset mixes impact the market value of paid-up policies. In order to analyze how the market values at $t = 0$ for the insurer and policyholders depend on the asset mix, we vary the time to maturity of the bonds and the initial equity weight in the *policy portfolio*. How the time to maturity of the bonds affects the market value is first examined in Section 10.2.1. Furthermore, we want to find which of the three mentioned rebalancing strategies in Section 5.5 that delivers the best results. We therefore then analyze how the equity weight in the portfolio changes through the contract period without any rebalancing (the buy and hold strategy) in Section 10.2.2. Section 10.2.3 then examines the same case applying the constant mix strategy. Here, we analyze how the market value is affected by a varying initial asset mix using this rebalancing strategy. The effect of a Constant Proportion Portfolio Investment (CPPI) rebalancing is examined in Section 10.2.4.

10.2.1 Time to Maturity of Bonds

The insurer has a variety of different options when it comes to what time to maturity he wants on the bonds he buy. All bonds are traded at par, i.e. the price equals the face value of the bonds, and pays an annual, constant coupon. The equity weight is set to 20%. Figure 10.5 plots the market value given an investment strategy where all bonds have a time to maturity of either 1, 3, 5 or 10 years.

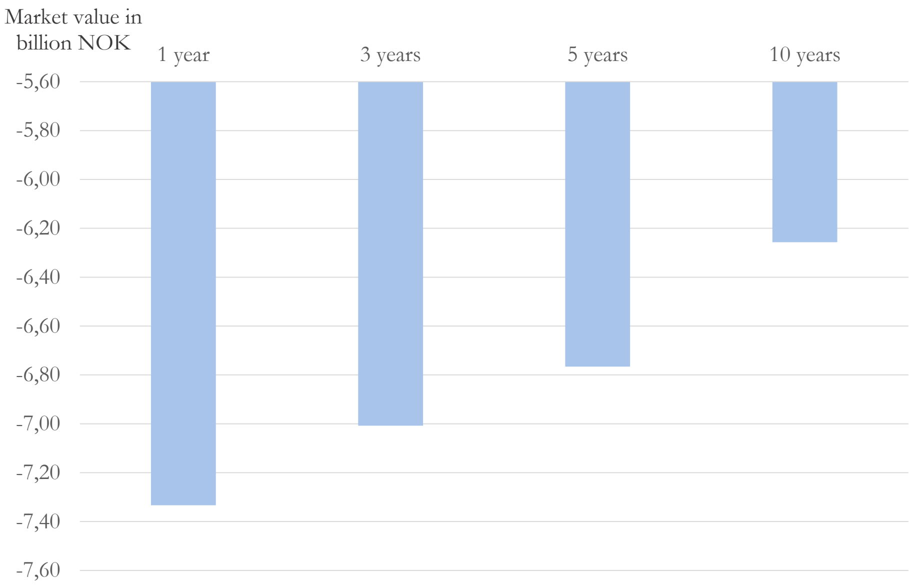


Figure 10.5: The market value for the insurer given bonds with different time to maturity.

We observe that bonds with higher time to maturity seem to give the insurer the highest market value. To find out why this is the case, we plot the average simulated interest rate for different bond maturities in Figure 10.6.

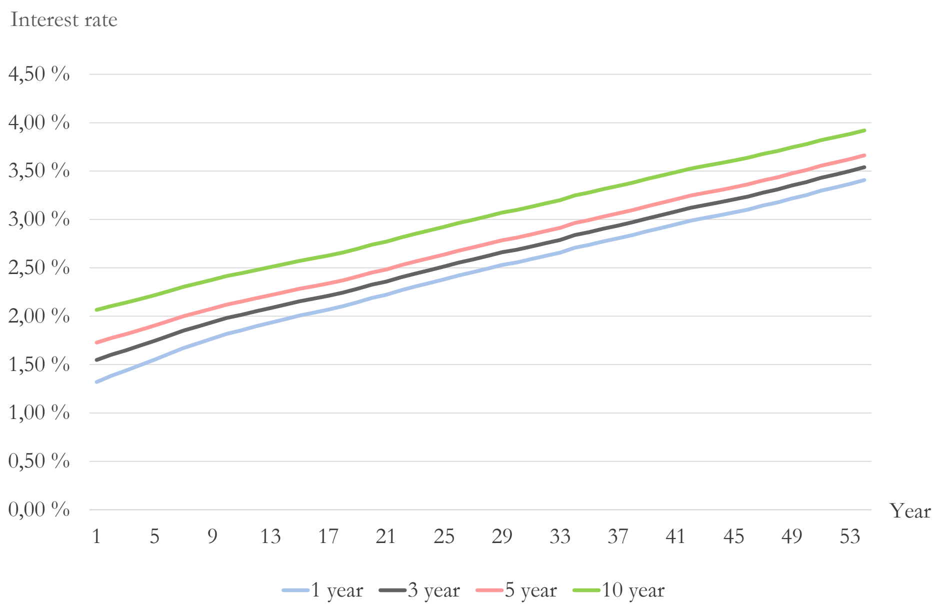


Figure 10.6: The market value for the insurer given bonds with different time to maturity.

The blue line corresponds to a bond with a time to maturity of 1 year, the black to a 3 year-bond, the red to a 5-year bond while the green corresponds to a 10-year bond. The average interest rate is given along the vertical axis, while the year into the contract period increases along the horizontal axis.

As we see from Figure 10.6, the 10-year bonds give on average a higher coupon than bonds with a shorter time to maturity. This can explain why the market value increases as we increase the time to maturity on the bonds in the *policy portfolio*. However, there is a trade-off for the insurer between ensuring a fixed coupon in a long-term perspective with long-term bonds or investing in short-term bonds that can benefit from increased interest rates. The first alternative benefits if the interest rate decreases or stays constant, while the latter is better if the interest rate increases. Our results indicate that the insurer on average benefits from investing in long-term bonds.

There are, however, reasons why insurance companies do not only include bonds with a long time to maturity in their *policy portfolio*. As insurance companies have to make regular payments to the customers, they have to realize parts of the *policy portfolio*. After a discussion with Nordea Liv, it has been made clear that bonds with maturity above 3 years are less liquid. Therefore, they have to include bonds with a short time to maturity as well, even though they lead to a lower market value. In the following, the initial time to maturity of the bonds will be as stated in Table 10.1 (AFS-bonds: 5 year: 50% and 10 year: 50%, HTM-bonds: 1 year: 40%, 3 year: 20% 5 year: 20%, 10 year: 20%).

10.2.2 Buy and Hold Strategy

We now focus on the share of equity in the *policy portfolio*. We first see how the equity weight in the portfolio develops with the buy and hold strategy, i.e. when there is no rebalancing of the portfolio throughout the entire contract period. To illustrate how the equity weight may vary over the years, we have selected eight simulations that are plotted as a function of the year into the contract period in Figure 10.7.

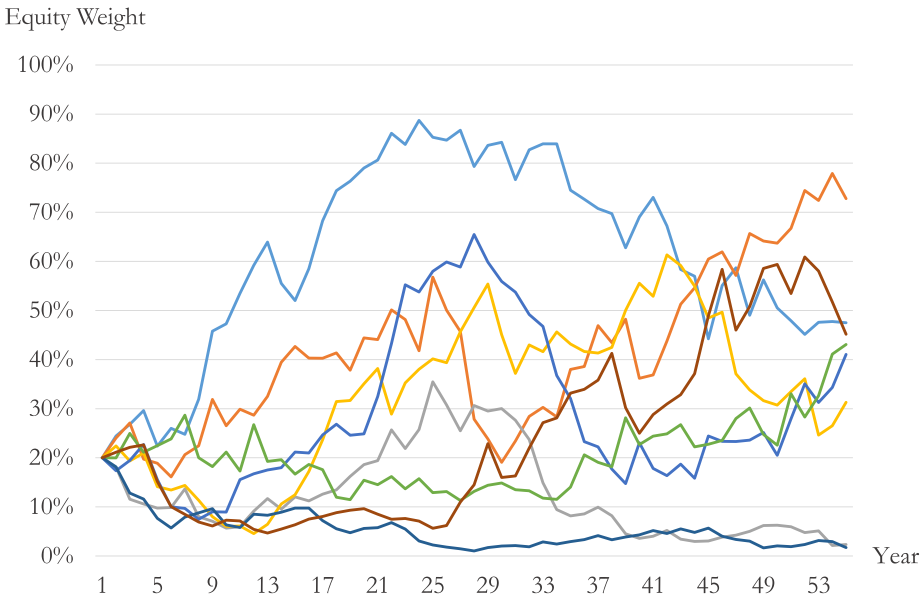


Figure 10.7: Each line represents the development over the years of the equity weight in the portfolio for one of eight potential paths, where the buy and hold strategy is used.

The intial equity weight is set to 20% and the bonds account for 80% of the *policy portfolio*, equally divided between HTM-bonds and AFS-bonds. The horizontal axis shows the year into the contract period, while the vertical axis shows the weight of equity in the portfolio at a given point in time into the contract period. We observe from Figure

10.7 that the share of stocks in the portfolio may vary considerably from simulation to simulation through the period. The top, light blue line, for example, shows an equity weight of almost 90% at one point, while another is down to almost 0%. The great variations follow from the fact that no rebalancing is done throughout the period. After the insurer starts with an initial weight of stocks of 20%, the weight will vary with the return of stocks and bonds without any interference by the insurer. If the stock returns outperform the bond returns, the weight of equity in the portfolio will increase. The inverse happens if the stock market falls.

It is not realistic that an insurance company managing paid-up policies applies the buy and hold strategy. As Figure 10.7 illustrates, the equity weight might be both 90% at some point and 2% at another even if we started with just 20% equity in the initial asset mix. For an insurance company dependent on delivering an annual guaranteed rate to its policyholders, the buy and hold strategy will lead to unpredictable returns as the share of stocks in the portfolio is out of the insurer's control. This can also result in unattainable capital requirements, which insurers must comply with in order to stay in business. The buy and hold strategy is, therefore, considered not suitable as an investment strategy.

10.2.3 Constant Mix

A more suitable investment strategy for insurers managing paid-up policies, might be the constant mix rebalancing strategy. As explained in Section 5.5, this strategy implies that the insurer rebalances the portfolio such that the weight of the different asset classes stays constant over the given time period. For the constant mix strategy we have defined the limits for the allowed equity shares in the portfolio by

$$\text{Equity Weight} = \text{Initial Equity Weight} \pm 1\% \tag{10.1}$$

With the given limits, the share of stocks in the portfolio will be kept constant within $\pm$ 1%, and the other asset classes are kept constant accordingly.

The development of the equity weights for eight simulations are plotted through the contract period in Figure 10.8.

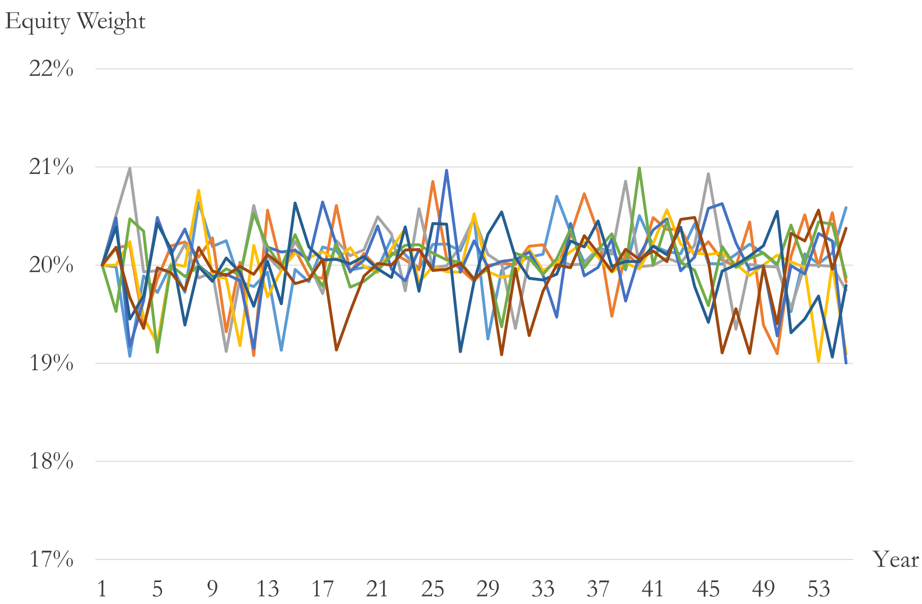
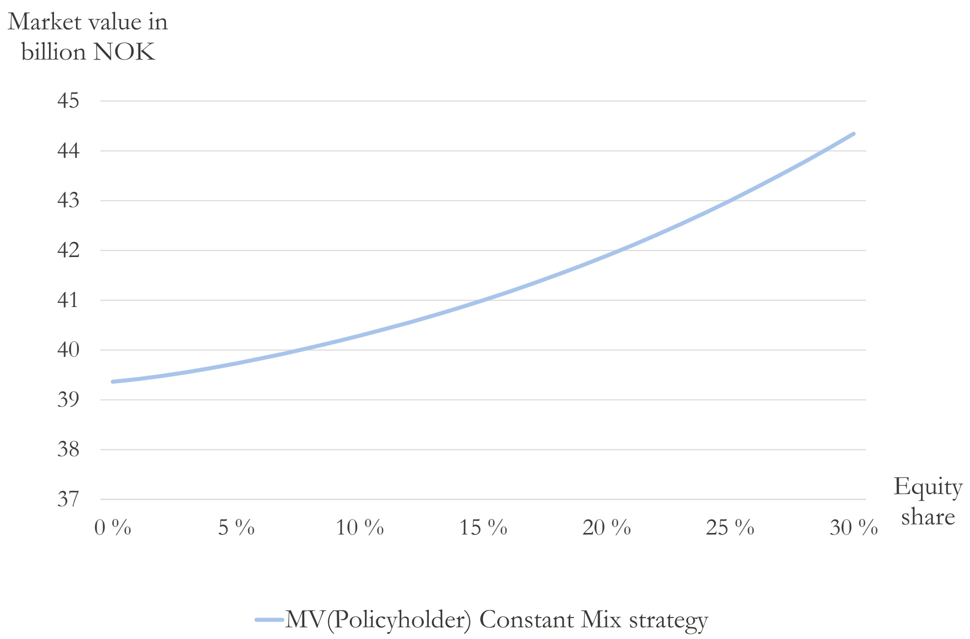


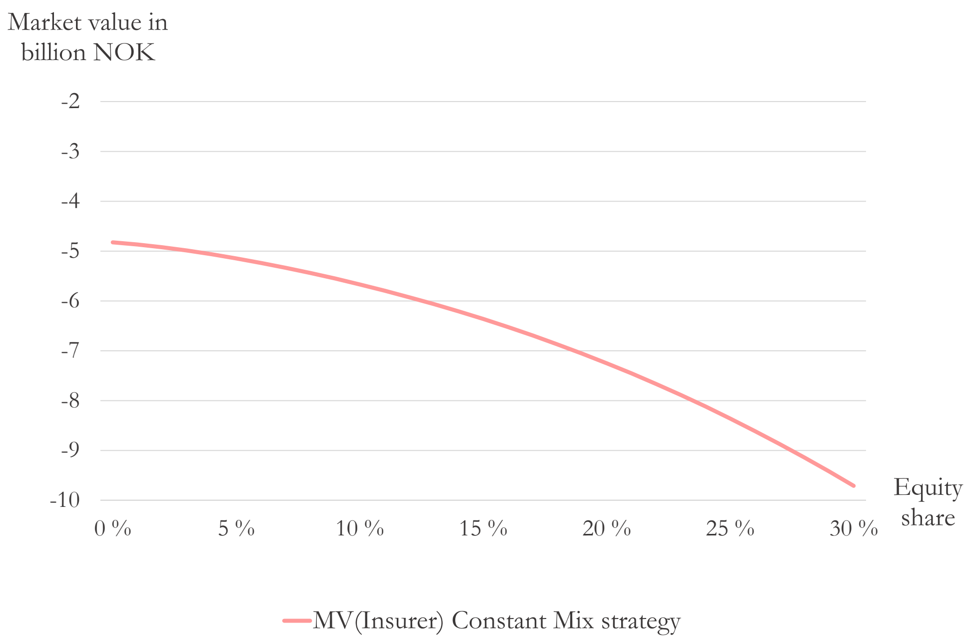
Figure 10.8: Each line represents the weight of equity in the portfolio after rebalancing for a simulation using the constant mix strategy.

The initial equity weight is as before set to 20%. As we see from the figure, all simulations start at 20% equity weight in year 1. Notice that the left axis, showing the equity weight in the portfolio, now only ranges from 17% to 22%. We see that the share of stocks in the portfolio stays within the defined lower limit (19%) and upper limit (21%) through the entire period. Compared to the situation where the insurer used a buy and hold strategy, the insurer is now able to control the risk of the portfolio as the equity weight stays almost constant. The returns are, of course, still uncertain, but the amount of assets allocated to stocks is under the insurer's control with this strategy. Consequently, the insurer can better adjust his desired risk profile with the constant mix strategy by allocating the assets to a desired initial asset mix.

We now analyze how the initial asset mix affects the market value of the paid-up policies, using the constant mix rebalancing strategy. Therefore, the initial equity weight is varied from 0% to 30%, while the rest of the assets in the portfolio is allocated equally to HTM-bonds and AFS-bonds. The result is illustrated in Figure 10.9.



(a) The blue line presents the market values at $t = 0$ for the policyholders with a constant mix strategy as a function of the equity share in the *policy portfolio*.



(b) The red line presents the market values at $t = 0$ for the insurer with a constant mix strategy as a function of the equity share in the *policy portfolio*.

Figure 10.9: The figures plot the initial equity share in the *policy portfolio* with the corresponding market value of the paid-up policies from (a) the policyholders' perspectives and (b) the insurer's perspective. The market values are given by NOK in billion on the vertical axis. The maximum size of the KRF and the TA is set to 12%.

Figure 10.9 plots the market values at $t = 0$ of the paid-up policies for the policyholders (a) and the insurer (b) for different equity weights in the portfolio. Both buffers, the KRF and the TA, are set to have a maximum size of 12% of the reserves in this case. The market values are given in billion NOK and plotted on the vertical axis. Recall that the initial value of the portfolio is equal to 34.5 billion NOK.

As we observe in Figure 10.9a, the market value for the policyholders increases with an increasing initial equity weight. This is expected, as the policyholders get 80% of the profit above the guaranteed rate through the profit-sharing system. They thus benefit from the insurers having a larger part of equity in their portfolios, as returns above the guaranteed rate and profit-sharing would be more likely with more stocks. The insurer, however, takes a larger risk with more equity. This comes from the fact that he has to cover all the downside in case of negative book returns, while he only gets 20% of the upside in case of book returns above the guaranteed rate. Therefore, the insurer will lose market value from an increasing equity share. This is apparent in Figure 10.9b, which shows that the market value declines with an increasing equity share. Also notice how the graphs are a mirror image of each other, i.e. the sum of the customer's and insurer's market value is constant across all combinations of assets in the *policy portfolio*. This follows from the fact that the paid-up policies can be considered a closed system, where the returns are shared between the policyholder and the insurer. The only injection of capital from the outside comes from the insurer in years of return below the guaranteed rate. But since these cash flows go to the policyholder, the net value is equal to zero.

10.2.4 CPPI

Another possible rebalancing strategy is the CPPI, which was explained in Section 5.5. Recall that this strategy is meant to help the insurer adjust the risk profile of the portfolio based on the value of the KRF.

Figure 10.10 illustrates the dynamics of the KRF (red line) and the equity weight of the portfolio (blue line) for one single simulation during the first 30 years of the contract period.

Figure 10.10: The red line represents the development in the KRF, while the blue line corresponds to the equity weight of the portfolio, plotted as a function of the year.

The curves show how the equity weight is determined by the size of the KRF, as the insurer increases the amount of stocks in the portfolio when the KRF is high. Notice that we have set a floor on the equity weight equal to 3%.

We then use the CPPI strategy to analyze how this strategy impacts the market value of paid-up policies. Recall from Section 5.5 that the CPPI coefficient is the m in

$$E = m(P - floor) \tag{10.2}$$

where E is the equity weight, P is the total value of the portfolio and $floor$ is the value the portfolio should not drop below. A higher coefficient m corresponds to a higher risk profile. This implies that the lower the coefficient is, the lower is the insurer's willingness to take on risk. The simulations are run with an equal amount invested in HTM and AFS-bonds, and with maximum values of the TA and KRF equal to 12% and 30% respectively. The floor on the equity weight in the CPPI strategy is set to 0.

Figure 10.11 shows how the market value at $t = 0$ changes for different CPPI coefficients.

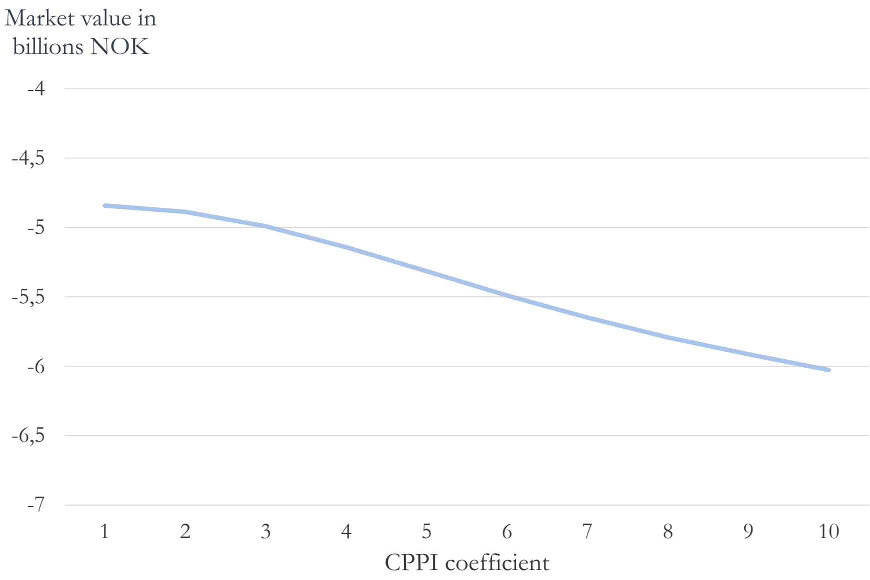


Figure 10.11: The market value for the insurer, given a CPPI investment strategy with different CPPI coefficients (m).

When we compare the values in Figure 10.11 to the results from Section 10.2.3, we observe that a CPPI strategy does not give a higher market value than when we use a strategy where the equity weight is permanently kept low. Notice that on the left hand side, the graph in Figure 10.11 converges towards the market value we get when the entire *policy portfolio* is invested in bonds (see Figure 10.9b for this result). This can be explained by the fact that we have set the floor in the CPPI strategy equal to the liabilities. If we then select a coefficient of 1, for instance, we only invest an amount equal to the KRF in stocks. As the figure illustrates, the market value for the insurer decreases as we increase the CPPI coefficient.

As we see from the results above, the CPPI strategy does not significantly reduce the market value for the insurer compared to the case where all assets are invested in bonds. For instance, CPPI coefficients of 1, 2 or 3 give -4.84, -4.89 and -4.99 billion NOK in market value for the insurer, while 100% bonds gives a market value of -4.83 billion NOK. Using a CPPI strategy could also benefit the policyholders through more frequent profit-sharing. Figure 10.12 plots the market value of the profit distributed to policyholder each year given different investment strategies.

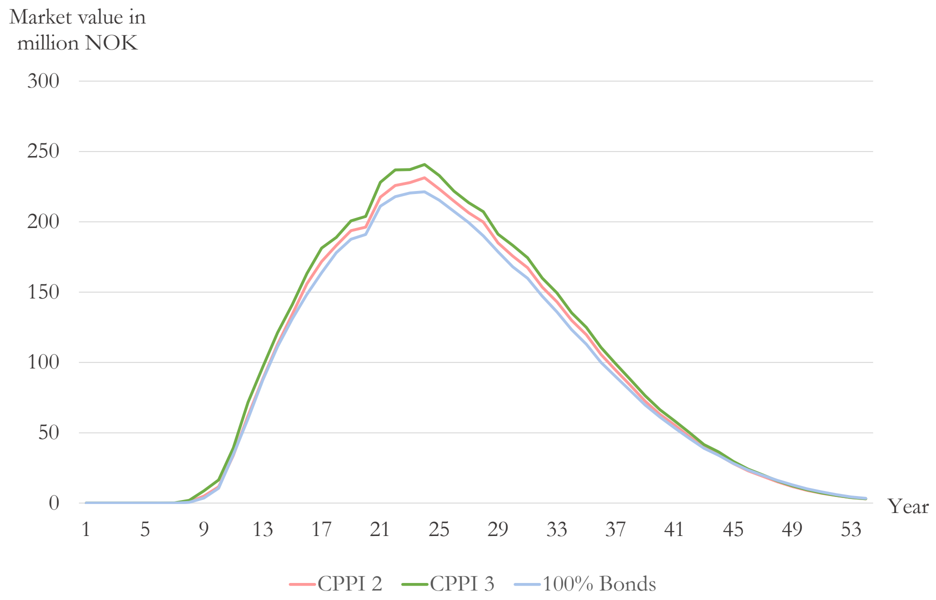


Figure 10.12: The market value of the profit distributed to policyholders each year, using a CPPI strategy with coefficient $m = 2$ (red line), $m = 3$ (green line) and 100% of the assets invested in bonds (blue line).

The red line and green line represent investments with the CPPI strategy, with coefficients (m) of 2 and 3, respectively. The blue line corresponds to the profit distributed when the insurer has invested 100% in bonds. Figure 10.12 illustrates that when the insurer uses a CPPI strategy, more profit is distributed to policyholders. We see this from the red and green line laying above the blue line. More precisely, using the CPPI strategy with coefficients of 2 and 3 increases the value of the profit distributed by 3.6% and 8.6% compared to the case with only bonds. Our results therefore suggest that the CPPI strategy increases the pension benefit to policyholders, as more profit is distributed.

10.2.5 Discussion

The results illustrate that the insurance companies' market value of paid-up policies will be higher if they keep a low risk profile on the *policy portfolio*, i.e. prioritize investing in bonds. This can easily be explained by the fact that the insurers have to get a certain annual return on the assets in order to prevent losses. At the same time, the insurers only get 20% of a return above the guaranteed rate if they select profit-sharing instead of building buffers. This percentage does not compensate for the additional risk related to large stock investments.

From the analysis above, we find that independent of what rebalancing-strategy the insurer applies, he gets the highest market value when everything is invested in bonds. The profit-sharing system and an asymmetric distribution of risk between policyholders and insurers is one main reason for explaining this. Investing in stocks may lead to a return above the guaranteed rate, even when the interest rates are low. However, stocks are also more risky. The question is if it is better to invest in bonds that are expected to give a return below the guaranteed rate, or invest a portion in equity which can give sufficient returns for the insurer, but potentially also larger losses. Our analysis suggests that the first alternative gives the highest market value.

However, it is not trivial for an insurance company to invest all assets in bonds. Even though this strategy would maximize the market value from the insurer's perspective, the policyholders would rarely receive any additional return above the guaranteed rate. This could lead to lower customer satisfaction. When the general asset mix for insurance companies in Norway is about 10% equity, it could lead to bad publicity for an insurer if the equity exposure is reduced to zero. As a consequence, customers might switch to other insurance companies that are more willing to take on risk in order to deliver higher returns to customers.

We have also seen that a CPPI strategy does not increase the market value for the insurer, compared to a Constant-Mix strategy where the assets in the *policy portfolio* are invested in bonds only. However, even though a CPPI strategy does not seem to be favorable for the insurer, it could be an alternative that keeps the customer satisfaction higher, as it can lead to more frequent profit-sharing to customers. Having a higher equity weight when the KRF is high, shows the customers a willingness to take on risk that is beneficial for them. At the same time, the additional stock investments do not reduce the market value for the insurer dramatically. Our analysis indicates that the insurer can expect a reduction in the market value of 0.39% and 1.32% if he uses a CPPI strategy with a coefficient equal to 1 and 2, respectively. The market value for the customer will increase correspondingly.

10.3 Buffer Strategy

We have now seen that the insurer minimizes the liabilities related to paid-up policies by keeping a very low risk profile on the *policy portfolio*. Another important aspect of the paid-up policies, which also has important implications on the market value, is the selection of the buffer strategy. Recall from Section 5.4 that insurance companies have the possibility to use the two buffers called KRF and TA to cover the guaranteed rate in years where the return on the *policy portfolio* is not sufficiently high. We now continue the analysis of paid-up policies by focusing on the size and the dynamics of these two buffers.

Section 10.3.1 first examines how the maximum allowed sizes of the buffers affect the market value of the paid-up policies. We then examine the use of a dynamic strategy for the TA, where the size of the buffer changes with the current interest rate in the market, in Section 10.4.

10.3.1 Comparing Sizes of the Buffers

We start the analysis of the buffers by examining how the market value changes with a varying maximum size of the buffers. Figure 10.13 presents how the market value at $t = 0$ is affected by the maximum level of the KRF ($KRFmax$) and the TA ($TAmax$) when the equity weight is set equal to 20%. The $KRFmax$ ranges from 2% to 30% of the value of the reserves, while the $TAmax$ ranges between 5% and 12%. The points on the graphs highlight the optimal value of the $TAmax$ for a given value of $KRFmax$.

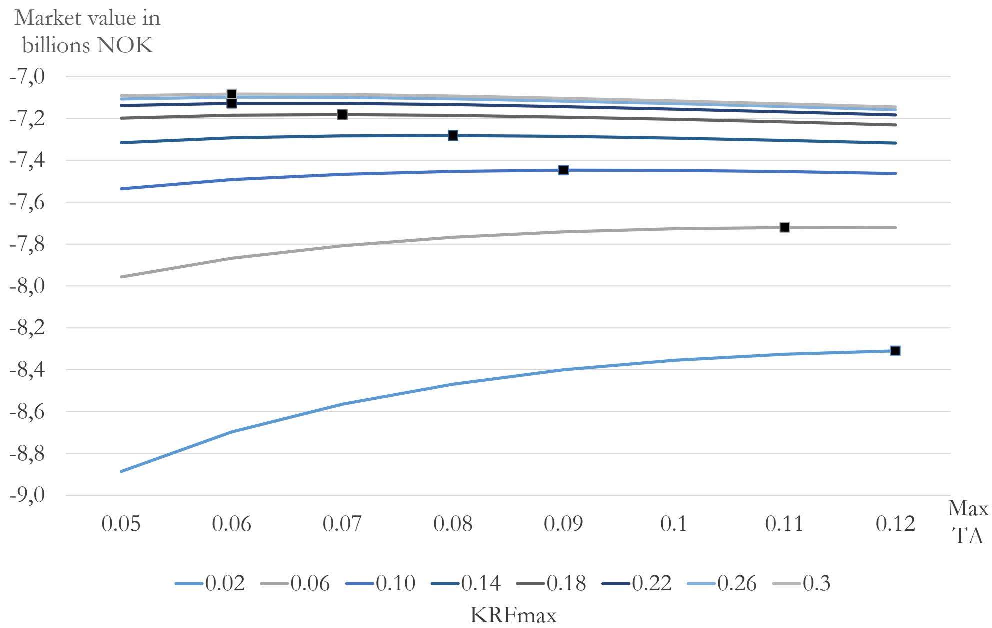


Figure 10.13: The insurer's market value at $t = 0$ for different values of $TAmax$. The equity weight is set to 20%. Each line represents a given value of $KRFmax$. The highest market value on each line is marked with a black square.

We observe that for small maximum levels of the KRF (the lower line in Figure 10.13), increasing the maximum level of the TA increases the market value considerably. This suggests the maximum size of the TA should be set high when the size of the KRF is low. However, if we select a higher $KRFmax$ (the upper line in Figure 10.13), a high $TAmax$ decreases the market value. In other words, instead of increasing the TA, the insurer is better off sharing the profit between himself and the policyholders in case of a high KRF prioritization. This indicates that the KRF serves as a sufficient buffer for the insurer. Therefore, instead of distributing the additional profit to the TA, the insurer should prioritize profit-sharing and get 20% of the additional profit.

The results can also be illustrated in a three dimensional plot like in Figure 10.14.

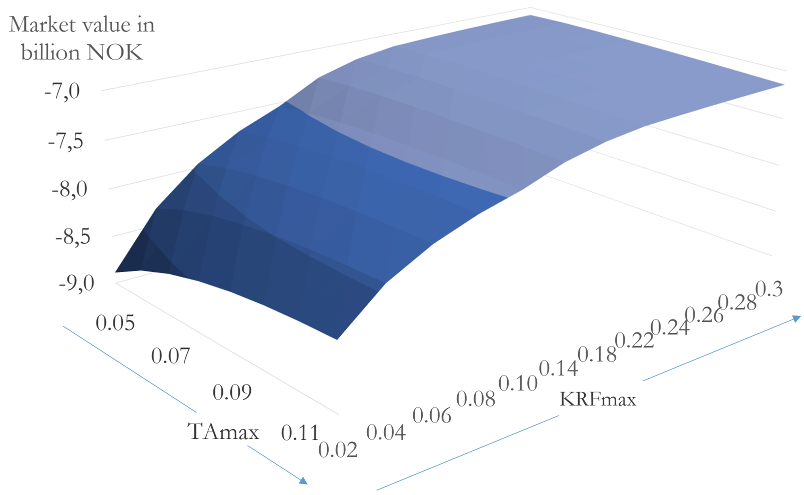


Figure 10.14: Market value for the insurer given different combinations of KRFmax and $TAmax$. The equity percentage is set equal to 20% with constant mix rebalancing strategy.

The graph illustrates more clearly that the importance of the KRF seems to be significantly higher than of the TA. This can be seen from the graph increasing faster on the axis corresponding to the $KRFmax$ than on the axis corresponding to the $TAmax$. For all maximum levels of the TA, having a $KRFmax$ equal to 30% instead of 2% increases the market value by an amount between 14% and 20%. The almost constant market value for combinations of $TAmax$ and $KRFmax$ on the right hand side of the graph indicates that the TA seems to be less important as long as building KRF is prioritized. Also, for small values of the KRF, it is beneficial for the insurer to fill the TA before he shares profit.

We now change the investment strategy to investing in bonds only. As mentioned in Section 10.2.5, a low risk profile of the portfolio is beneficial for the insurer. Figure 10.15 plots the market value at $t = 0$ for different levels of $KRFmax$ and $TAmax$ when all assets are invested in bonds.

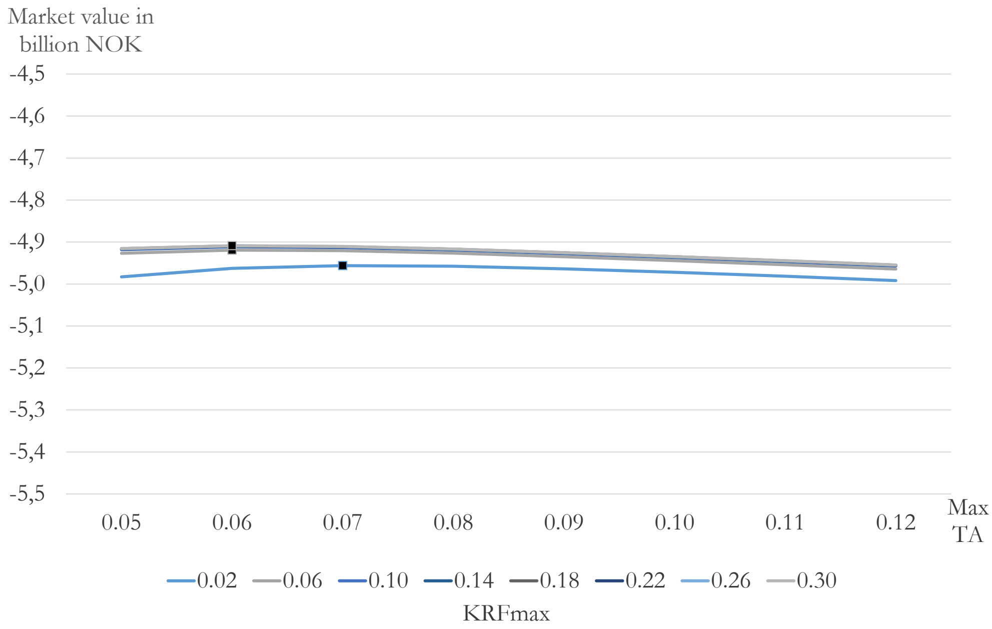


Figure 10.15: The insurer's market value at $t = 0$ for different values of $TAmax$. The equity weight is set to 0%. Each line represents a given value of $KRFmax$. The highest market value on each line is marked with a black square.

The figure shows that the importance of building buffers decreases when assets are invested in bonds only. This can be seen from the lines in Figure 10.15 laying significantly closer to each other than the lines in Figure 10.13, which means that the maximum sizes of the buffers have less impact on the market value. In the case with only bond investments, it is still favorable to increase the size of the KRF, but the difference between having a small and large $KRFmax$ is considerably smaller than for the case where 20% of the assets are being invested in equity. This can be explained by the lower volatility related to bond investments leading to a reduction in the need for buffers. As we see from Figure 10.2 in Section 10.1, bonds rarely give very low returns compared to the returns for stocks. This results in a reduction in the use of buffers.

A high KRF prioritization now results in a market value that is between 0.5% and 1.5% higher than with a low KRF prioritization. However, due to the lower risk profile, building TA instead of sharing profit is now not favorable, even for small values of $KRFmax$. This comes from the fact that the TA belongs to the policyholder. As in the case where 20%

of the assets in the portfolio are invested in equity, we observe that the market value for the insurer increases if the additional profit is shared with the customer instead of being distrubuted to the TA.

10.4 Dynamic TA

In years where the risk-free rate is high, we expect that the need for buffers decreases. One way to set a $TAmax$ value that gets gradually reduced as the size of the risk-free rate increases, is by dividing an arbitrary coefficient by the interest rate. We also saw earlier that the optimal size of the TA seems to depend upon the maximum size of the KRF and the equity weight of the portfolio. We will therefore now look at a dynamic buffer strategy, where the maximum size of the TA changes based on the current risk-free rate. The maximum size of the TA will for the analysis in this section be given by.

$$TAmax = \min\Big(l/|r_t|, 0.12\Big) \tag{10.3}$$

where l is a constant coefficient that should be optimized and r_t is the current interest short rate. Since the interest rate rarely becomes considerably below zero, we take the absolute value of r_t. This ensures a high value of $TAmax$ when the interest rate drops below zero. Recall that the TA can only be reduced through payments to the customer. We therefore never set $TAmax$ below the current level of the TA, as this would have led to unnecessary customer payments for the insurer.

To see the effect of using a strategy where $TAmax$ is adjusted dynamically, we start by looking at a case where $KRFmax$ is set to 10%. In this case, the TA will be more frequently used, as the KRF more often reaches its maximum limit. Figure 10.16 shows the market value for the insurer given different values of l.

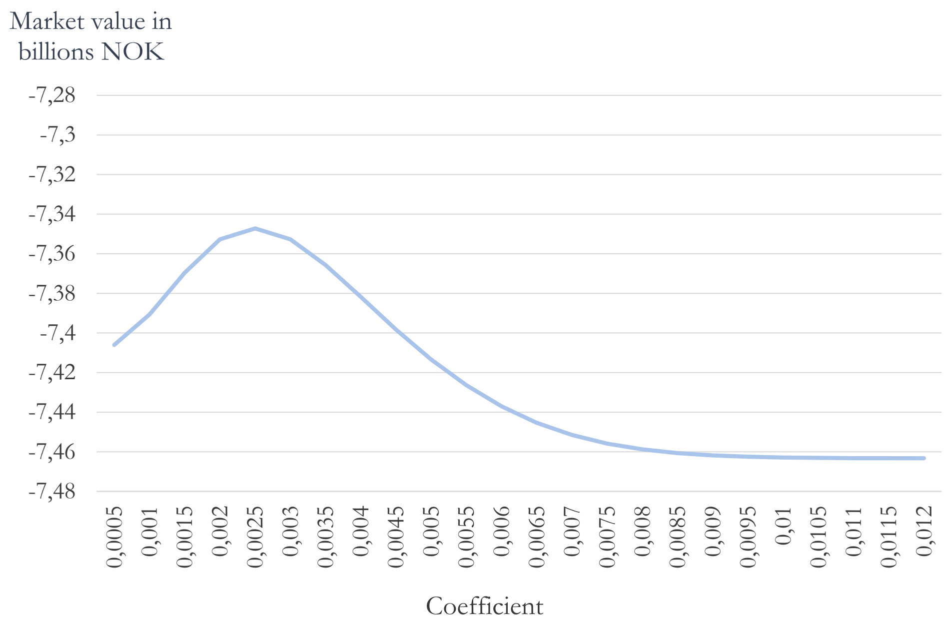


Figure 10.16: The market value having a dynamic $TAmax$, given different values of the l coefficient.

To compare these results to the results from the simulations where $TAmax$ is constant, we have set the minimum level of the $TAmax$ equal to 5%. From Figure 10.16, we observe the highest market value for the insurer is achieved with a coefficient l of 0.0025 with a dynamic TA strategy. Using the dynamic strategy, the market value increases from -7.41 billion NOK to -7.35 billion NOK, compared to a case where $TAmax$ constantly equals 12%. This corresponds to an increase in the market value of 1.45%. The graph in Figure 10.16 converges towards the market value we get when we set $TAmax$ equal to 12%.

Selecting a dynamic TA with a coefficient of 0.0025 implies that the interest rate has to reach a level of 2% before the $TAmax$ gets reduced. This is illustrated in Figure 10.17, where $TAmax$ for different interest rates r_t is plotted with the coefficient l equal to 0.0025.

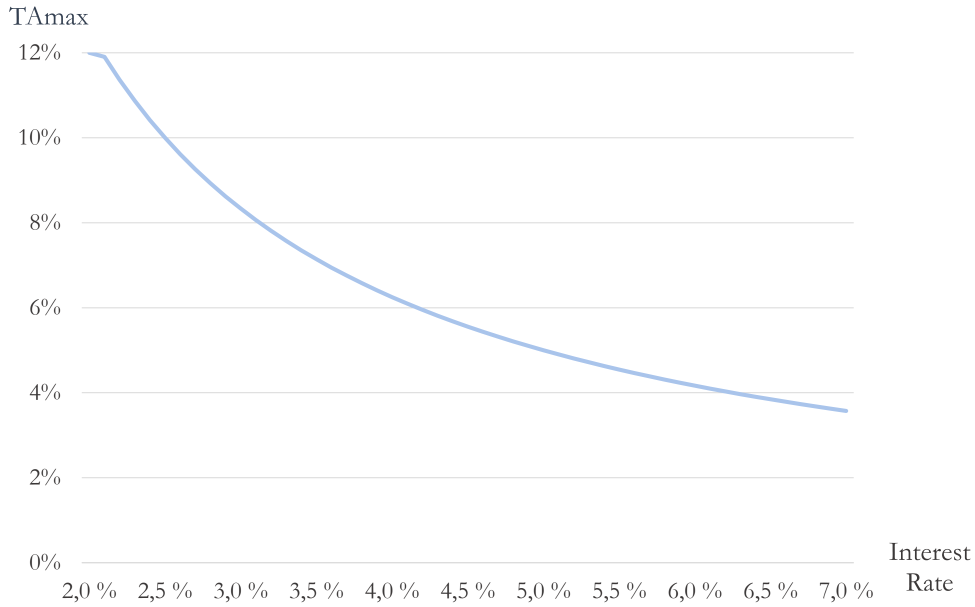


Figure 10.17: $TAmax$ for different interest rates r_t, given a coefficient l of 0.0025 in the dynamic $TAmax$ equation.

Recall that the $KRFmax$ is set to 10% in these simulations, i.e. below the level we have shown gives the highest market value for the insurer. However, in the cases where the *initial book return* from coupons, dividend and rebalancing is higher than the guaranteed rate, the insurer gets forced to select between building TA or sharing profit with the policyholders (see Section 8.3.1 for explanation of the *initial book return*). Our results indicate that this decision should be based on the size of the current interest rates.

If we run the same simulations, but now invest in bonds only, a dynamic TA strategy improves the market value for the insurer by 1.51%. The optimal coefficient is now equal to $l = 0.0015$. The lower risk related to bond investments reduces the need for buffers. Therefore, the maximum level of the TA gets gradually reduced already when the risk-free rate has reached 1.25%.

As we have illustrated above, the need for a high TA decreases if we increase $KRFmax$. If we now run the same simulations, but set $KRFmax$ equal to 30% instead of 10%, we observe that the dynamic TA strategy has limited effect, as it is beneficial for the insurer to permanently keep the buffer low. This comes from the fact that the KRF is a sufficient buffer in this case.

The importance of the KRF compared to the TA can be explained by the additional flexibility of the KRF buffer. The reason for this is that the KRF can be used to cover negative return, and also be realized and lead to profit-sharing for the insurer. Recall that the TA belongs to the customer, and return added to this buffer can only be used to cover the guaranteed rate and not to compensate for negative return.

The only way to reduce the size of the buffer is to make an additional payment to the customers. This happens in the years of payout, as according to the regulations the size of the buffer can never exceed 12% of the reserves. Figure 10.18 presents how much of the TA that is actually used to cover the guaranteed rate for each customer group in the cases of a maximum size of 5% and 12%. The value of the amount paid out from the TA to customer groups in the case where the reserves are reduced are also plotted in the same figure for sizes of 5% and 12%. All values are discounted using the bank account to time $t = 0$. The bank account was discussed in Section 8.4.

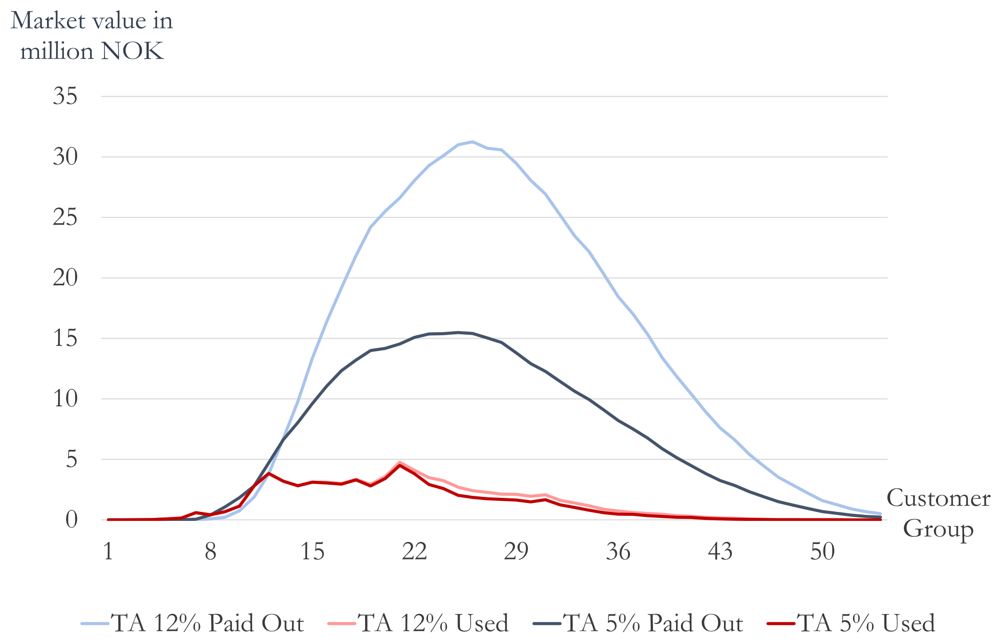


Figure 10.18: The market value of the amount paid out from the TA to customers in the case where the maximum size is 12% (light blue line) and 5% (dark blue line). The value of the amount used in the TA to cover the guaranteed rate plotted for a TA size of 12% (light red line) and 5% (dark red line). The maximum size of the KRF is set to 30% in these simulations.

The vertical axis shows the market value of the TA that is used to fulfill the guaranteed rate (red lines) and the amount of the TA that is paid out to the customers (blue lines). The values are given for each customer group. If we look at how much of the TA that is actually used to cover the guaranteed rate (red lines), we observe that for a high $TAmax$ (12%), the value almost stays constant compared to the case where the $TAmax$ is 5%. These results indicate that the effect of a larger TA to cover the guaranteed rate seems to be limited. However, the additional amount transferred to the TA with a high $TAmax$ gets paid out to the policyholders when the customer group's reserves decrease (blue lines). This implies that the insurer instead of building a TA should have shared the profit above the guaranteed rate.

The use of a dynamic strategy for the KRF buffer has also been tested, but this strategy did not make a considerable impact on the market value.

10.4.1 Implications for the Suggested Regulations

Recall from Section 6 that a commission hired by the Norwegian Ministry of Finance suggested to merge the KRF and TA into one buffer and let this buffer cover negative returns. As our results indicate that the TA seems to have limited effect when the KRF is prioritized, we expect that merging the two buffers will not make a large difference in the market value for neither the customer nor the insurer. However, it will probably make the policies easier to manage for the insurance companies and easier to understand for the customers if there would be only one buffer.

Also, as a part of this suggested change in the regulations, the commission suggested to let the insurance companies differentiate between the customers in how they build buffers. This would imply that the insurers could increase the buffers for some customers, while they share the profit with others. This alternative is not possible given today's regulations. The commission argues that with these suggested changes in the regulations, the insurance companies would share profit with more customers. The results from our simulations, however, indicate that the insurance companies are almost always better off building the KRF instead of sharing profit.

The suggested change of letting insurers differentiate their buffer strategies between customer groups could still make a difference if we use a dynamic TA strategy, as discussed above. With today's regulations regarding the TA, insurers must distribute an equal share of profit to each customer group's TA. This makes it impossible to have a dynamic TA strategy based on the time to expiration for each customer group's paid-up policies. With the suggested regulations, it could, for example, be beneficial for the insurer to increase the TA early in the contract period for one customer group, while he shares the profit with a customer group consisting of older customers.

The commission also concludes that the maximum size of the new buffer could be about 19%, which was the sum of the KRF value in 2016 and the maximum value of the TA (12%). Our results indicate that if the maximum size of this new buffer is limited, it will benefit the customers, and make the paid-up policies even less profitable for the insurance companies. This comes from the fact that the insurers get forced to profit-share return when they reach the maximum limit of the buffer.

10.5 Risk Management of Paid-up Policies

By now, we have discussed how the insurers should select an investment and buffer strategy to maximize the market value of paid-up policies from the insurer's perspective. However, the decisions the insurers make regarding investments and buffers will also affect the risk associated with the policies. Determining the risk and capital requirements given different strategies will be the focus of this section. The objective of this analysis is to minimize the capital requirements for the insurer.

For the risk analysis, we use a combination of real-world and risk-neutral scenarios. Real-world scenarios are run from time $t = 0$ to time $t = 1$, then risk-neutral scenarios are run from time $t = 1$ to $t = 54$, as illustrated in Figure 8.3 in Section 8. With this setup, we obtain a distribution of market values at time $t = 1$, which can be used to derive the capital requirements in Solvency II. The distributions of market values at time $t = 1$ obtained with different asset mixes and buffer strategies will be presented in this section. Recall from Section 7 that the SCR at $t = 0$ is calculated as the difference between the market value of the paid-up policies at $t = 0$ (L_0) and the 99.5% VaR of the obtained market values at $t = 1$ ($Q_{0.995,t}(L_1)$), given by

$$SCR_0 = \Big(Q_{0.995,0}(L_1) - L_0\Big)e^{-r_0} \tag{10.4}$$

where r_0 is the discount rate. The SCR presented for different investment and hedge strategies is thus how much the market value of the insurer's liabilities can change from $t = 0$ to $t = 1$ in the 0.5% worst case.

In Section 10.5.1, we analyze at how investing in stocks versus bonds affects the capital requirements for the insurance companies. We then add a *hedge portfolio* to the analysis in Section 10.6, to study how buying derivatives linked to stocks and the interest rate can reduce the capital requirements.

10.5.1 Capital Requirements with Constant Mix Strategy

The results from the previous sections have highlighted the importance of keeping the KRF buffer as large as possible. Using the TA as a buffer seems to have limited effect as long as increasing the KRF is prioritized. For now, we therefore leave the maximum size of the TA equal to 5% and the maximum size of the KRF equal to 30%.

The analysis on the asset mix suggested that the insurer's market value was highest with all assets invested in bonds. However, as discussed in Section 10.2.5, the insurer will usually include some equity in the portfolio in order to avoid negative publicity and keep policyholders satisfied. In the following, we will therefore analyze the risk associated with paid-up policies when also equity is included in the *policy portfolio*.

The simulations are run with 20% equity, 10% equity and with only bond investments using a constant mix rebalancing strategy. The distribution for the insurer's market value at $t = 1$ is plotted in Figure 10.19. The blue line corresponds to the case with only bonds, the yellow line to an initial asset mix of 10% equity while the red line represents the simulations where 20% of the portfolio is invested in equity.

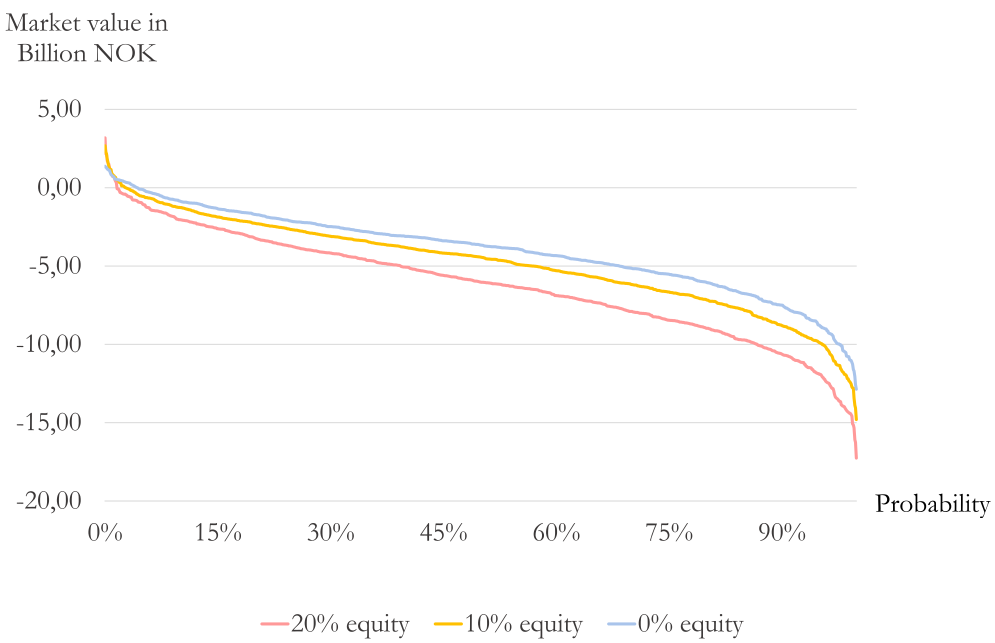


Figure 10.19: The distribution of market values at $t = 1$ for the insurer given an investment strategy with 0% stocks (blue line), 10% stocks (yellow line) and 20% stocks (red line).

Assume that we are at the point where the market value is -5,00 billion NOK on the red line in Figure 10.19. The horizontal axis then tells that there is a probability of approximately 40% that the insurer will achieve a market value of -5,00 NOK or better with an equity share of 20% in the *policy portfolio*. The figure illustrates the additional risk the insurer faces if he invests in stocks. In the upper left corner, we observe that the absolute best cases for the investment strategy with 20% stock investments are slightly better than for the case with 10% stocks and clearly better than with only bonds. These results are expected, as a larger part of assets invested in stocks would give higher returns in the best cases. In the right part of the figure, i.e. the worst scenarios for the insurer, we observe that the market values are worse the higher the equity share in the portfolio. Especially the 99.5% value in the distributions, which is the basis of the the Solvency Capital Requirement, is clearly worse for the case with 20% stocks compared to the case with bonds only. Overall, the bond investments (blue line) give a higher average market value at $t = 1$ and a 99.5% VaR that is 26% lower than the case with 20% equity. This investment strategey gives a SCR equal to 6.44 billion NOK, which is the minimum amount the Solvency II directive requires insurance companies to hold.

10.5.2 Capital Requirements with CPPI

As discussed in Section 10.2.4, the market value for the insurer and customers stays almost constant if the insurer uses a CPPI investment strategy compared to a constant mix strategy where all assets are invested in bonds. We will now look at how a CPPI strategy affects the capital requirements for the insurer. The simulations are run with a CPPI coefficient (m) of 1, 2 and 3 (see Section 5.5 for the explanation of m). Figure 10.20 plots the distribution of the market values at $t = 1$ for the three cases.

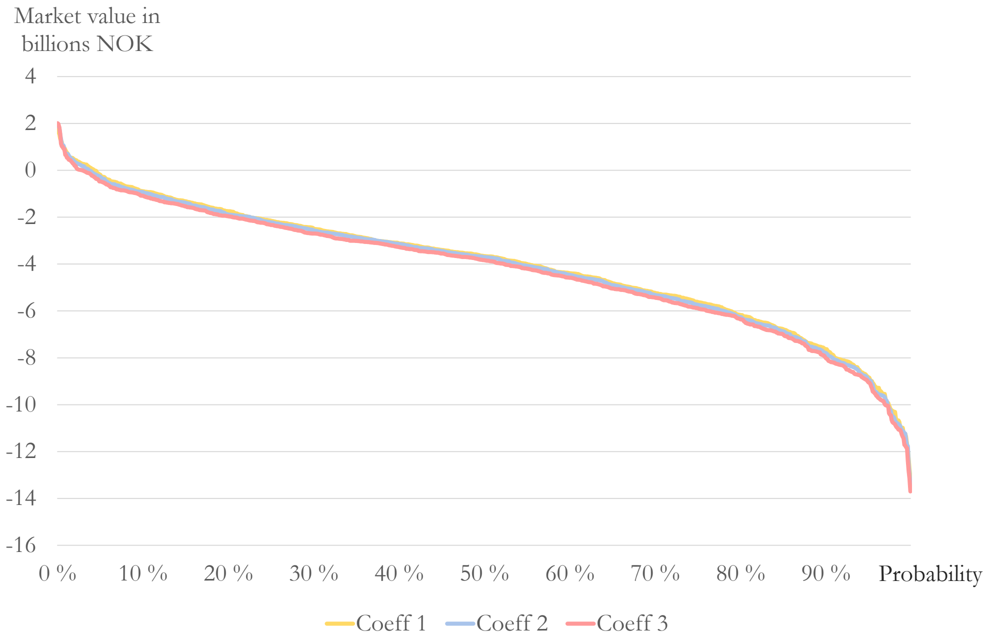


Figure 10.20: The distribution of market values at $t = 1$ given different choices of coefficients (m) in the CPPI strategy.

The yellow line corresponds to a CPPI coefficient of 1, the blue lines gives the distribution for a CPPI coefficient of 2, while the red line is the case with a coefficient of 3. We see that all lines have the same shape and that they are very close, which indicates that the distribution of market values do not change significantly with an increasing CPPI coefficient from 1 to 3. Compared to an investment strategy where all assets are invested in bonds as the blue line in Figure 10.19, the CPPI strategy leads to almost the same capital requirements (6.51 billion NOK for the CPPI against 6.44 billion NOK for 100% bonds). This can be seen from the right part of the figure, where the curves have the same shape as the case with 100% bond investments. A CPPI coefficient of 2 or 3 increases the capital requirements by 1.1% and 4.8% respectively, compared to the case when we use a CPPI coefficient equal to 1.

This indicates that a CPPI strategy where the equity weight is adjusted based on the current KRF does not considerably increase the risk the insurer faces. As we only increase the equity weight to a level sufficiently covered by the KRF, we make sure that the probability of the insurer having to inject capital into the portfolio remains low.

10.5.3 Comparing Market Value and Capital Requirements

Figure 10.21 illustrates the market value at $t = 0$ and the capital requirements for different investment strategies. The graph indicates that investing in only bonds gives the insurer the highest market value, and at the same time the lowest capital requirements. We also observe how the CPPI strategy gives almost the same market value and capital requirements as the case where the asset mix is 100% bonds. Investing in either 10% or 20% equity gives a significantly worse market value, while the capital requirements also increase.

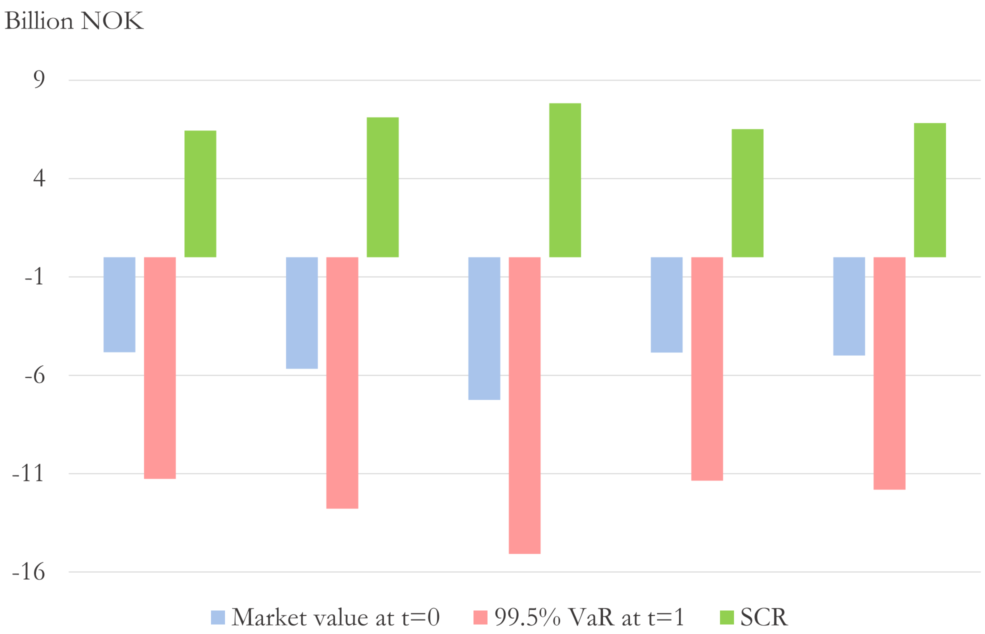


Figure 10.21: The market value at $t = 0$ (blue bars), the 99.5% VaR of the market values at $t = 1$ (red bars) and the Solvency Capital Requirement (green bars) calculated as the difference between the two, given different investment strategies.

10.6 Hedging Risk

We have now seen the distribution of market values at $t = 1$ of paid-up policies given different investment strategies. To illustrate the variations, we plot the market values at

$t = 1$ against the stock return and bond return at $t = 1$. The scatter plots are given in Figure 10.22 and Figure 10.23, respectively.

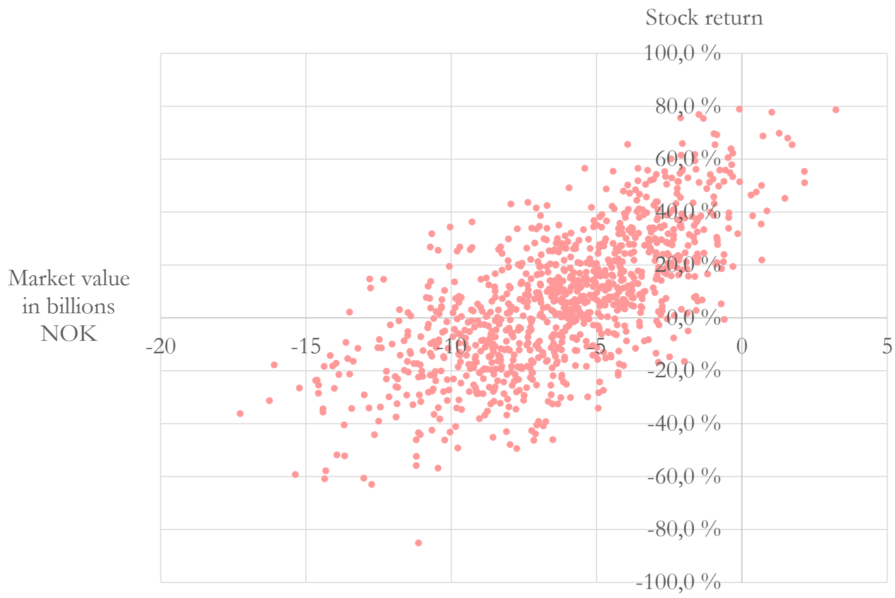


Figure 10.22: The insurer's market values at $t = 1$ for different stock returns during the first year.

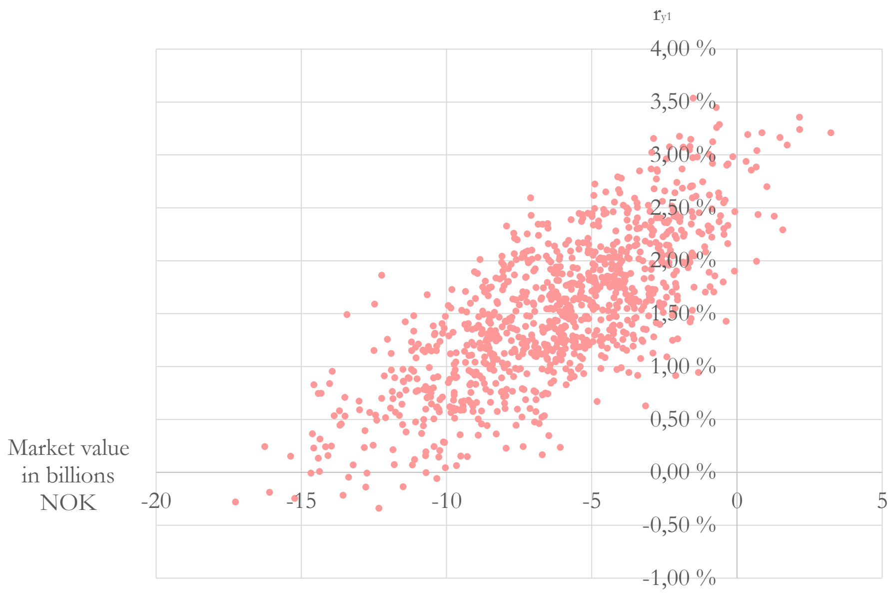


Figure 10.23: The insurer's market values at $t = 1$ for different interest short rates after one year.

The market value at $t = 1$ is given by the horizontal axes, while the stock and bond returns are given by the vertical axes. The market value of the paid-up policies varies considerably with the returns, as shown in Figure 10.22 and Figure 10.23. As discussed in Section 8.7.1, this will influence the balance sheet for the insurance company, as the variation is used to determine the capital requirements within the Solvency II directive.

From the figures, we observe that the market value for the insurer is highly dependent on the stock return and the evolution in the interest rate during the first year. The relations between the market value at $t = 1$ and the interest rate and stock return seem to be linear. The correlation between the market value at $t = 1$ and the stock return is equal to 0.69, while the correlation between the market value and the interest short rate in year 1 is equal to 0.75.

Due to the fact that the market value of paid-up policies is highly correlated to the stock and bond return, we are able to hedge it by investing in derivatives linked to stocks and interest rate. We therefore include a *hedge portfolio* in the calculations.

As discussed in Section 8.7, the goal of the *hedge portfolio* is to make the change in the value of this portfolio from $t = 0$ to $t = 1$ negatively correlated with the change in the market value of the *policy portfolio* during the same time period. This implies that we want the *hedge portfolio* to hedge potential negative market returns in the *policy portfolio*.

For now, the *hedge portfolio* only consists of call options for zero-coupon bonds and put options for stocks. This gives us a payout in scenarios of low stock return, and in scenarios where the interest rate decreases, as discussed in Section 8.7.

We now want to analyze the effect the derivatives have on the market values for the insurer. To see how the strike price affects the hedge, we will compare simulations with different strike prices K of the options. In one case, the strike price for the stock put option is set to 10% below the initial value of stocks in the *policy portfolio* and to 10% above the initial value of bonds in the *policy portfolio* for the zero-coupon bond call options. We then have a hedge for both stocks and bonds. In the other case, the strike prices are set to 5% above and below for the stock put options and bond call options, respectively.

As we have seen, the insurer has to fulfill the guaranteed rate of return each year. To make sure that he does not face considerate losses in case of very low returns during the next year, we set the time to maturity of the options to one year. In these simulations, the *policy portfolio* consists of 20% equity, and we buy options for the entire portfolio, i.e. the notional of the options equals the corresponding initial value of bonds and stocks in the *policy portfolio*. The distributions of market values at $t = 1$ are given in Figure 10.24.

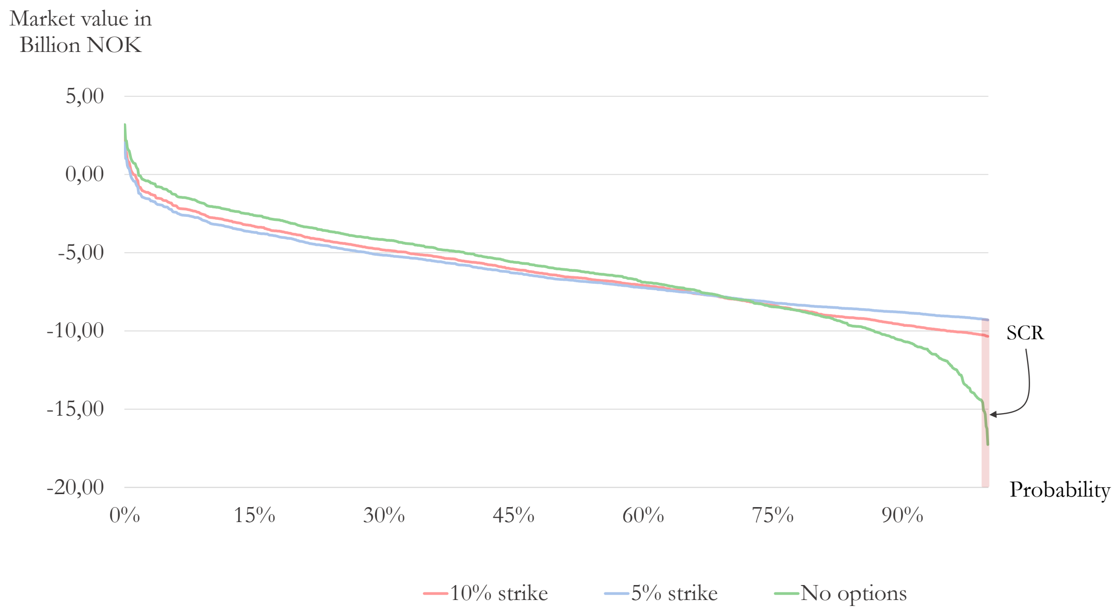


Figure 10.24: The distribution of market values for the insurer given the case with no options (green line), the case with options with a strike price ±5% (blue line) and ±10% (red line) of the initial investments in stocks and bonds. The SCR is derived as the difference between the market value at $t = 0$ and the 99.5% worst market value at $t = 1$, red shaded in the figure.

The green line in Figure 10.24 gives the distribution of the market value at $t = 1$ if no options are bought. The red and blue line show the market values if the *hedge portfolio* consists of options that protect against a fall of more than 10% and 5% of the initial value of the *policy portfolio*, respectively. The best cases for the insurer is found to the left in the figure, while the worst cases are to the right.

We observe that in the case with no options (green line), the market value will be higher than when we include a hedge portfolio with a 70% probability. This comes from the fact the insurer has no financing costs of options in this case, which we see from the figure where the green line is placed a bit over the red and blue line in these cases. However, the worst scenarios to the right give values considerably below the alternatives with options. This implies a reduction in the 99.5% VaR, i.e. the value used in the calculation of the Solvency Capital Requirement. The SCR is 7.84 billion NOK for the case with no options. Including the options with the 10% strike (red line) improves the SCR by 62% to 3.01 billion NOK. With a strike price of 5% (blue line) from the initial value, the SCR is further improved to 2.1 billion NOK, 74% better than the case with no options.

We also see that the case with 5% strike gives better values in the right part of the figure, but worse values in the left part (the best cases). This can be explained by the option prices. With a strike price of the options just 5% below the initial asset value, the options will be more expensive than the case with a strike price 10% below the initial asset value. However, this additional price for options having a strike price closer to the notional value is repaid in the worst cases, where the payoffs are higher. In general, when the insurer buys options in the *hedge portfolio*, the market value is likely to be lower. However, in the worst cases, the market values are considerably higher.

10.6.1 Collar Option

One of the main drawbacks of buying option to hedge is that the size of the total option premium might be very high. The total price of call options for zero-coupon bonds and put options for stocks that hedge against a drop in the *policy portfolio* of 5%, is about 1.2 billion NOK. To reduce the total option premium, we can sell stock call options and zero-coupon bond put options, in addition to the options we buy. This combination creates a so-called collar, as explained in Section 8.7.4. For the options we sell, we select a strike price of 2% above the initial value in stocks and 8% below the initial value in bonds. For the options we buy, we select a strike price of 5% above and below the initial value in stocks and bonds, respectively. This makes the *hedge portfolio* approximately self-financed, i.e. the price of the options bought equals the price of the options sold. The option prices are given in the table below.

	Stock options		Bond options	
Type	Put	Call	Put	Call
Strike (% of initial value)	0.95	1.02	0.92	1.05
Price (in millions)	-536.6	553.0	569.1	-536.6

The results from the collar along with the results from Figure 10.24 are plotted in Figure 10.25.

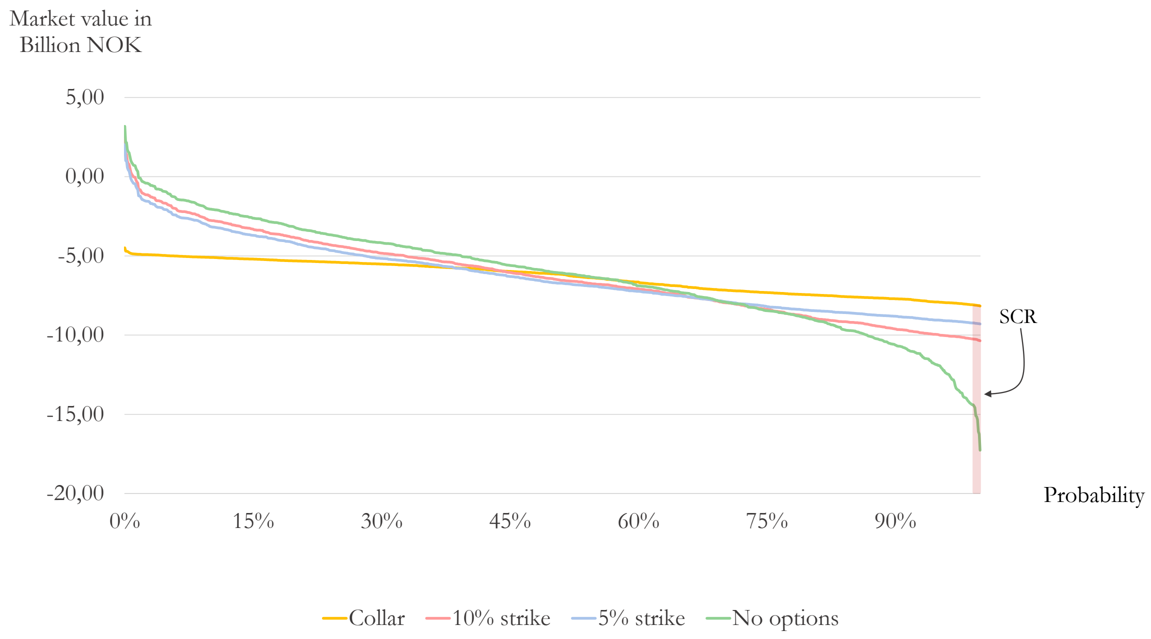


Figure 10.25: The distributions of market values at $t = 1$ for the insurer given an investment strategy with 20% stocks. The green line corresponds to a situation without a hedge portfolio. The red and blue line show the value when we also include a separate hedge portfolio consisting of options with strike equal to 90% and 95% of the initial portfolio value, respectively. The yellow line represents a self-financed option portfolio with put and call options (collar). The SCR is derived as the difference between the market value at $t = 0$ and the 99.5% worst market value at $t = 1$, red shaded in the figure.

The yellow line represents the alternative with a collar. We observe that the market values at $t = 1$ becomes more certain when we use a collar strategy in the *hedge portfolio*. The collar reduces the SCR to 859 million NOK, which is a reduction of 89% compared to the case with no options. After constructing a collar, the insurer will have negative payoffs on the sold call stock options and put zero-coupon bond options. Due to the fact that these negative payoffs reduce the value in the best-case scenarios, we end up with a market value that on average is 4% below a case without options, and 2.5% below the case with options ±10% strike. Compared to a case without the *hedge portfolio*, the five percent best scenarios in the collar-case gives a market value that is 80% lower.

The drawback of a collar investment is that the market value for the insurer at $t = 1$ will never be higher than approximately -5 billion NOK. This implies that a hedge to avoid high capital requirements leads to a reduction in the market value in the best cases, as well. Without any hedge, however, the variation of possible market values at $t = 1$ is huge compared to the situation where an insurer buys a collar. Without a hedge, the insurer can achieve both much better and worse values.

The variation in possible market values is illustrated in Figure 10.26.

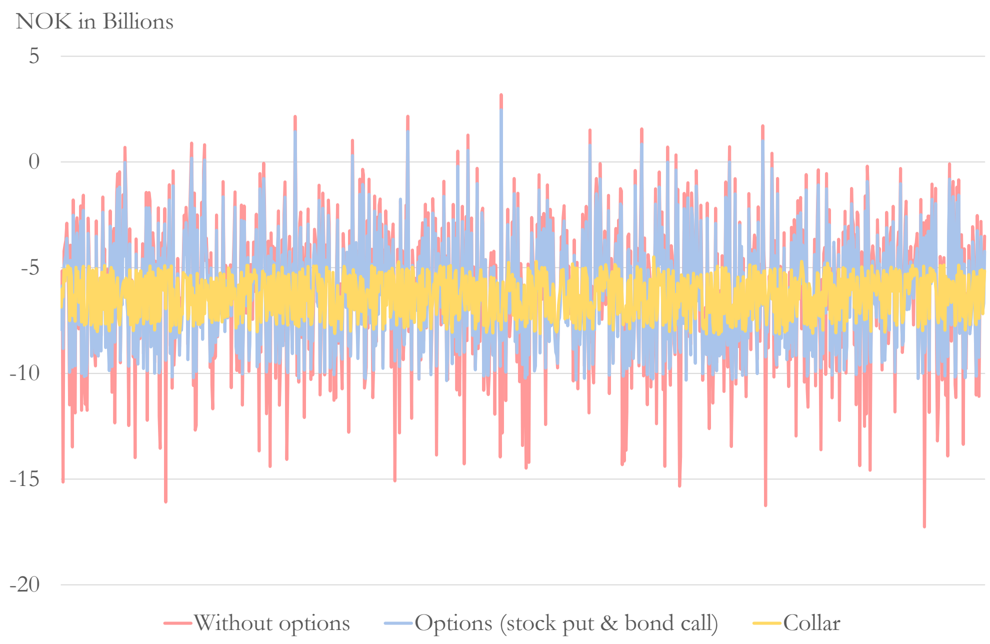


Figure 10.26: The red lines represent the market values at $t = 1$ for the insurer in all scenarios when no options are included. The blue and yellow lines represent the market values at $t = 1$ for option combinations with stock and bond options, and the collar combination, respectively.

We observe from the red lines that without a hedge portfolio, the insurer may face several scenarios where the market value at $t = 1$ ends at between -10 and -15 billions NOK. To hedge against these worst cases, he can, as mentioned above, buy stock put options and zero-coupon call options. The scenarios when we include a hedge portfolio that consists of such options (strike price $\pm 5\%$ from the initial value) are given by the blue lines. We observe that the worst cases are now considerably improved. However, due to the cost of buying options, the best cases also get reduced. The yellow lines show the results if we invest in a self-financed collar. We now observe a further reduction in the variance, and both the upside and downside get capped. However, we now never end up with scenarios that give the insurer a market value of more than -5 billion NOK.

Table 10.2 summarizes some of the key statistics from the simulations. The market value at $t = 0$ for this case is -7.25 billion NOK.

Values in billions NOK	SCR	5% best case	Average
No options	7.84	-1.04	-6.08
Options (strike = 90%)	3.01	-1.77	-6.16
Options (strike = 95%)	2.01	-2.12	-6.18
Self-financed collar	0.86	-4.99	-6.34

Table 10.2: Summary of the 99.5% VaR, the 5% best case and the average of all simulated market values at $t = 1$, given different hedging strategies. The asset mix is 20% equity and 80% bonds, which gave a market value of -7.25 billion NOK at time $t = 0$.

We see from Table 10.2 that the case with no options gives the highest Solvency Capital Requirement. This is not surprising, as there is no hedging against the worst cases that decide the SCR. However, when the insurer has no options, he earns the best average market value at $t = 1$. When the insurer invests in options, we see that the average market value gets worse. The price of the options can explain this tendency. The SCR, meanwhile, gets significantly reduced by investing in options, where the collar gives the best results regarding this capital requirement. This follows from the worst scenarios being hedged by the options.

10.6.2 Hedging Only Bond Investments

In Section 10.2.3, we observed that the best market values in $t = 0$ for the insurer was achieved with only bond investments. We now look at the risk associated with paid-up policies given this investment strategy. The distribution of the market values for the insurer at $t = 1$ given this investment strategy is plotted in Figure 10.27.

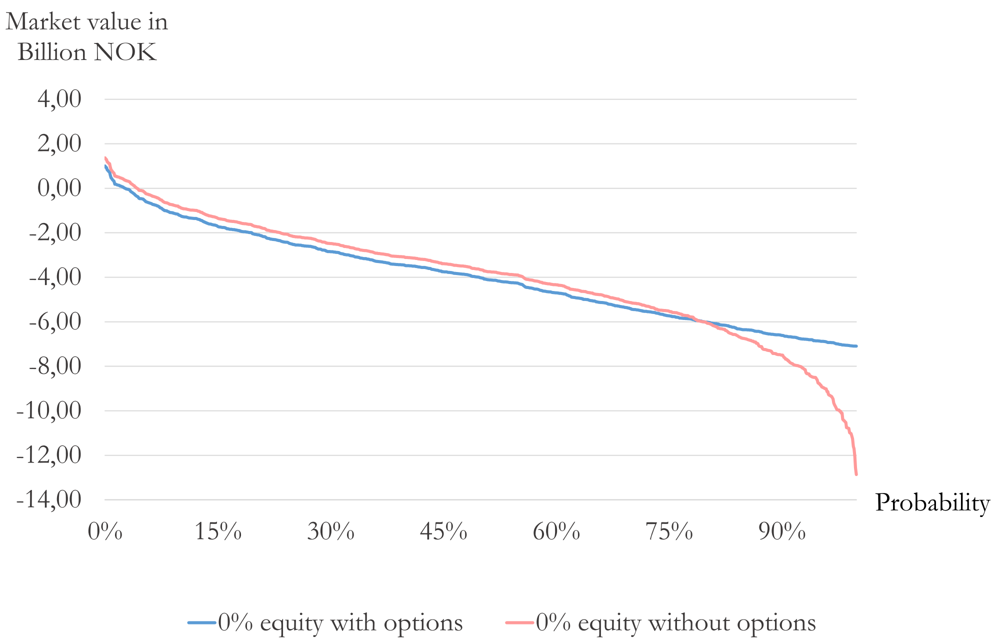


Figure 10.27: The distribution of market values at $t = 1$ when the *policy portfolio* consists of bonds, only. The red line represents the case with no options and the blue line the case where we invest in options where the strike price is 10% above of the initial investments in bonds.

The blue line illustrates the distribution of market values at $t = 1$ when no options are included, while the red line gives the distribution when the *hedge portfolio* consists of bond call options with a strike price 10% above the initial investments in bonds. We see that the graph has the same shape as in the case where 20% of the assets in the *policy portfolio* was invested in stocks. However, the variance is lower. This comes from the volatility of bonds being lower than the volatility for stocks. The SCR if no hedge portfolio is included, is now 6.44 billions in NOK.

The probability of the *hedge portfolio* leading to a higher market value at $t = 1$ than the case without any hedge is now 20%. The *hedge portfolio* still gives a considerate reduction in the capital requirement; the SCR becomes 2.26 billions, which is 65% less when derivatives are bought.

11 Conclusion

One of the most important pension savings products in the Norwegian market, with currently more than 300 billion NOK under management, is paid-up policies ("fripoliser"). The policyholders of this product are guaranteed a fixed annual return on their pension savings, which an insurance company is responsible of fulfilling. With market interest rates currently below the promised guaranteed rate, insurers have to take risk to cover the guarantee. In addition, the policies are a part of a complex profit-sharing system where returns above the promised guaranteed rate can be distributed to two buffers or shared between policyholders and the insurer. This system gives the insurer several buffer and investment strategies that significantly influence the risk and return of the paid-up policies. Insurers also have to comply with the Solvency II directive, which states the capital insurers are required to hold in order to be solvent.

In this thesis, we value the paid-up policies and derive the risk associated with the product. Our focus is to improve insurers' management of paid-up policies by searching for optimal investment and buffer strategies. We also analyze how the strategies affect the capital requirements and how insurers effectively can hedge the market risk related to the paid-up policies. In order to do this, we create a valuation model that combines the Black-Scholes and Hull-White One Factor model to simulate stock prices and interest rates, respectively. Implementing the central aspects and dynamics of the policies allows us to analyze how different investment and buffer strategies determine the market value of the policies and affect the capital requirements of the insurer. Our results indicate the following.

First, the results suggest that the assets backing the liabilities from paid-up policies should be invested in only bonds to maximize the market value for the insurer. This investment strategy is expected to give a return below the guaranteed rate with today's market interest rate. However, the possible gain of investing in equity does not seem to compensate the additional risk the insurer faces with the profit-sharing system.

Second, using a CPPI strategy where the equity weight of the portfolio is adjusted based on the current buffer size does not seem to increase the capital requirements or decrease the market value for the insurer considerably. Also, a CPPI strategy leads to more frequent profit-sharing, which benefits the policyholders.

Third, building buffers should be prioritized over profit-sharing to customers. Our analysis indicates that the most flexible buffer KRF is more valuable for the insurer than the TA buffers, which are split on customer groups. Using a dynamic buffer strategy where the size of the TA is adjusted according to the simulated market interest rate gives a slight increase in the market value for the insurer.

Fourth, investing in equity increases the capital requirement for the insurer, compared to a case where the portfolio consists of bonds only. Using options, an insurer can reduce the requirement. From the option combinations analyzed, a collar reduces the requirement the most.

The model explained in this thesis takes into account the most important aspects of paid-up policies. However, we base our analysis on current market data, and do not take into account that the number of paid-up policies is expected to increase in the future. This comes from the fact that several companies switch from defined benefit scheme to defined contribution scheme, which leads to new paid-up policies being issued. Further research should focus on modeling the future size of the reserves, based on for instance data on wage increases, turnovers, mortality and the current number of defined benefit schemes.

Also, in the risk analysis, we have only considered the one-year risk by simulating real world scenarios one time step ahead, followed by risk-neutral scenarios for each of these real world scenarios. The risk analysis could be extended to also determine the risk throughout the contract period. This can be done by doing what is known as nested simulations, where the risk is measured at each time step. This would, however, increase the computational complexity exponentially.

In order to hedge against the fluctuations in the stock and bond market, we have included a hedge portfolio. The hedge portfolio consists of protective puts against losses in the stock market and zero-coupon bond call options against decreasing interest rates. Further research could look at a more comprehensive hedge portfolio, where for instance more complex option types such as hybrid options are included. This could potentially give a sufficient hedge with lower option prices. Other hedging strategies, such as shorting equity or bonds could also be examined.

In our modelling of paid-up policies, the assets backing the liabilities of the insurer have been invested in stocks and bonds. As mentioned above, the prices of these assets are simulated using a combination of the Black-Scholes and the Hull-White One Factor Model. Insurers, however, often allocate a portion of the assets in the portfolio to real estate. To make the dynamics of the asset returns more realistic, the model could be extended to include simulations of real estate prices. Furthermore, as the bond prices have a considerate impact on the market value and risk associated with the paid-up policies, using other dynamics of the evolution in the interest rate could also be examined. The Hull-White Two Factor model is one example of a more comprehensive short-rate model. This model enables the yield curve to twist, i.e. the short rate moves in one direction while the long rate moves in another. The Hull-White One Factor model we have implemented does not include this feature.

With the mentioned CPPI strategy for the rebalancing of the assets in the portfolio, the insurer adjusts the equity weight based on the current size of the KRF. The CPPI strategy could be extended such that the equity weight gets adjusted based on other measures, such as the current size of the liabilities. This could potentially give valuable insights into how the market value for the insurer can be further improved.

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Appendix A

A.1 Proof 1

Integrating Equation (8.19) gives us

$$\int_t^T r_s ds = \int_t^T e^{-as} r_0 ds + \int_t^T \int_0^s e^{-as}\theta(u)e^{au} du ds + \sigma_r \int_t^T \int_0^s e^{-as} e^{au} dz ds \tag{A.1}$$

We evaluate the three terms on the right hand side separately, and start by looking at the first one.

$$\begin{aligned}\int_t^T e^{-as} r_0 ds &= -\frac{r_0}{a}\Big(e^{-aT} - e^{-at}\Big)\\ &= \frac{r_0}{a}\Big(e^{-at} - e^{-aT}\Big)\\ &= r_0 e^{-at}\Big(\frac{1-e^{-a(T-t)}}{a}\Big)\\ &= r_0 e^{-at} B(t,T)\end{aligned} \tag{A.2}$$

Solving the second term requires interchanging the order of integration.

$$\begin{aligned}\int_t^T \int_0^s e^{-as}\theta(u)e^{au} du ds &= \int_0^t \int_t^T e^{-as}\theta(u)e^{au} ds du + \int_t^T \int_u^T e^{-as}\theta(u)e^{au} ds du\\ &= \int_0^t \theta(u)e^{au}\int_t^T e^{-as} ds du + \int_t^T \theta(u)e^{au}\int_u^T e^{-as} ds du\\ &= \int_0^t \theta(u)e^{au}\Big[\frac{1}{a}e^{-as}\Big]_t^T du + \int_t^T \theta(u)e^{au}\Big[\frac{1}{a}e^{-as}\Big]_u^T du\\ &= e^{-at}B(t,T)\int_0^t \theta(u)e^{au} du + \int_t^T \theta(u)B(u,T) du\end{aligned} \tag{A.3}$$

The last term of (A.1) is evaluated using the same procedure by changing the order of integration.

$$\begin{aligned}\sigma_r \int_t^T \int_0^s e^{-as} e^{au} dz ds &= \sigma_r \int_0^t \int_t^T e^{-as}\theta(u)e^{au} ds dz + \sigma_r \int_t^T \int_u^T e^{-as} e^{au} ds dz\\ &= \sigma_r e^{-at} B(t,T)\int_0^t e^{au} dz + \sigma_r \int_t^T B(u,T) dz\end{aligned} \tag{A.4}$$

Summarizing (A.2), (A.3) and (A.4) into one equation leads to

$$\begin{aligned}\int_t^T r_s ds &= r_0 e^{-at} B(t,T) + e^{-at} B(t,T) \int_0^t \theta(u) e^{au} du + \int_t^T \theta(u) B(u,T) du \\ &+ \sigma_r e^{-at} B(t,T) \int_0^t e^{au} dz + \sigma_r \int_t^T B(u,T) dz \\ &= B(t,T) r_t + \int_t^T \theta(u) B(u,T) du + \sigma_r \int_t^T B(u,T) dz\end{aligned} \tag{A.5}$$

In the last line we have used the expression obtained for r_t from Equation (8.19).

A.2 Proof 2

Let $X \sim \mathcal{N}(\mu, \sigma_r^2)$. Then, $Z = \frac{X-\mu}{\sigma_r} \sim \mathcal{N}(0,1)$, and $X = \mu + \sigma_r Z$.

$$\mathbb{E}[e^X] = \mathbb{E}[e^{\mu + \sigma_r Z}] = e^{\mu} \mathbb{E}[e^{\sigma_r Z}] \tag{A.6}$$

Since Z follows a standard normal distribution, we see that

$$e^{\mu} \mathbb{E}[e^{\sigma_r Z}] = e^{\mu} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\sigma_r z} e^{-\frac{z^2}{2}} dz = e^{\mu} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\sigma_r z - \frac{z^2}{2}} dz \tag{A.7}$$

Notice that $\sigma_r z - \frac{z^2}{2}$ can be expanded, such that

$$\sigma_r z - \frac{z^2}{2} = \frac{1}{2}\sigma_r^2 - \frac{1}{2}(z - \sigma_r)^2 \tag{A.8}$$

The expected value of X can now be written as

$$\mathbb{E}[e^X] = e^{\mu} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\frac{1}{2}\sigma_r^2 - \frac{1}{2}(z-\sigma_r)^2} dz = e^{\mu + \frac{1}{2}\sigma_r^2} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(z-\sigma_r)^2} dz \tag{A.9}$$

Since $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(z-\sigma_r)^2} dz$ is the probability density of the standard normal distribution, it equals 1 by definition. Therefore $\mathbb{E}[e^X] = e^{\mu + \frac{1}{2}\sigma_r^2}$. $\square$

A.3 Proof 3

Inserting $A(0,t)$ and $B(0,t)$ into Equation (8.25), and dividing the equation into three parts to simplify the derivations.

$$f_0(t) = \underbrace{\frac{\partial B(0,t)}{\partial t} r_t}_{(1)} - \underbrace{\frac{\partial}{\partial t}\int_0^t \frac{1}{2}\sigma_r^2 B^2(u,t)du}_{(2)} + \underbrace{\frac{\partial}{\partial t}\int_0^t \theta(u)B(u,t)du}_{(3)} \tag{A.10}$$

We now solve the three parts individually.

$$\begin{aligned} (1) &= \frac{\partial B(0,t)}{\partial t} r_t \\ &= \frac{\partial}{\partial t}\Big(\frac{1-e^{-at}}{a}\Big) r_t \\ &= e^{-at} r_t \end{aligned} \tag{A.11}$$

$$\begin{aligned} (2) &= \frac{\partial}{\partial t}\Big(\int_0^t \frac{1}{2}\sigma_r^2 B^2(u,t)du\Big) \\ &= \frac{\partial}{\partial t}\Big(\int_0^t \frac{1}{2}\sigma_r^2 \frac{1-2e^{-a(t-u)}+e^{-2a(t-u)}}{a^2}du\Big) \\ &= \frac{\partial}{\partial t}\Big(\frac{\sigma_r^2}{2a^2}\int_0^t 1-2e^{-at}e^{au}+e^{-2at}e^{2au}du\Big) \\ &= \frac{\sigma_r^2}{2a^2}\frac{\partial}{\partial t}\Big[u-2e^{-at}\frac{1}{a}e^{au}+e^{-2at}e^{2au}\frac{1}{2a}\Big]_0^t \\ &= \frac{\sigma_r^2}{2a^2}\frac{\partial}{\partial t}\Big(t-\frac{2}{a}+\frac{1}{2a}+\frac{2}{a}e^{-at}-\frac{1}{2a}e^{-2at}\Big) \\ &= \frac{\sigma_r^2}{2a^2}\Big(1-2e^{-at}+e^{-2at}\Big) \\ &= \frac{\sigma_r^2}{2a^2}(1-e^{-at})^2 \end{aligned} \tag{A.12}$$

$$\begin{aligned} (3) &= \frac{\partial}{\partial t}\int_0^t \theta(u)B(u,t)du \\ &= \frac{1}{a}\frac{\partial}{\partial t}\int_0^t \theta(u)(1-e^{-at})du \\ &= \int_0^t e^{-a(t-u)}\theta(u)du \end{aligned} \tag{A.13}$$

Putting all parts together gives us

$$\begin{aligned} f_0(t) &= e^{-at}r_0 + \int_0^t e^{-a(T-u)}\theta(u)du - \frac{\sigma_r^2}{2a^2}(1-e^{-at})^2 \\ &= g(t) - h(t) \end{aligned} \tag{A.14}$$

Since g now solves $g'(t) + ag(t) = \theta$, where $g(0) = r_0$ and $h(t) = \frac{\sigma_r^2 B^2(0,t)}{2}$, the following holds

$$
\begin{aligned}
\theta(t) &= \frac{\partial f_0(t)}{\partial t} + \frac{\partial h(t)}{\partial t} + a(f_0(t) + h(t)) \\
&= \frac{\partial f_0(t)}{\partial t} + af_0(t) + \frac{\partial}{\partial t}\Big(\sigma_r^2 \frac{B^2(0,t)}{2}\Big) + a\Big(\sigma_r^2 \frac{B^2(0,t)}{2}\Big) \\
&= \frac{\partial f_0(t)}{\partial t} + af_0(t) + \frac{\sigma_r^2}{a}\Big(e^{-at} - e^{-2at}\Big) + \frac{\sigma_r^2}{2a}\Big(1 - 2e^{-at} + e^{-2at}\Big) \\
&= \frac{\partial f_0(t)}{\partial t} + af_0(t) + \frac{\sigma_r^2}{2a}\Big(1 - e^{-2at}\Big)
\end{aligned}
\tag{A.15}
$$

It can easily be shown that

$$
\theta(t) = \frac{\partial f_0(t)}{\partial t} + af_0(t) + \sigma_r^2 B(0,t)\Big(e^{-at} + \frac{1}{2}aB(0,t)\Big) \tag{A.16}
$$

A.4 Proof 4

From (8.19) we define the following α_t and β_t.

$$
\alpha_t = e^{-at}\Big(r_0 + \int_0^t \theta(u)a^{au}du\Big) \tag{A.17}
$$

$$
\beta_t = \sigma_r e^{-at}\int_0^t e^{au}dZ_u \tag{A.18}
$$

After inserting $\theta(t)$ into A.17, we get the following equation

$$
\begin{aligned}
\alpha_t = e^{-at}\Big(r_0 &+ \int_0^t af_0(u)e^{au}du + \int_0^t \frac{\partial f_0(u)}{\partial u}e^{au}du \\
&+ \sigma_r^2\int_0^t B(0,u)du + \frac{a\sigma_r^2}{2}\int_0^t B^2(0,u)e^{au}du\Big) \\
&= e^{-at}\Big(r_0\Big[e^{au}f_0^(u)\Big]_0^t + \sigma_r^2\int_0^t \frac{1-e^{au}}{a}du + \frac{a\sigma_r^2}{2}\int_0^t \big(\frac{1-e^{au}}{a}\big)^2 e^{au}du\Big) \\
&= e^{-at}r_0 + f_0(t) - e^{-at}f_0(0) + \frac{\sigma_r^2 e^{-at}}{a}\Big(\int_0^t (1 - e^{-au}du)\Big) \\
&+ e^{-at}\frac{\sigma_r^2}{2a}\int_0^t \Big(1 - 2e^{-au} + e^{-2au}\Big)e^{au}du
\end{aligned}
\tag{A.19}
$$

Since $f_0(0) = r_0$, we get the following

$$\begin{aligned}
\alpha_t &= f_0(t) + \frac{\sigma_r^2 t e^{-at}}{a} - \frac{e^{-at}\sigma_r^2}{a}\Big[-\frac{1}{a}e^{-au}\Big]_0^t \\
&+ e^{-at}\frac{\sigma_r^2}{2a}\Big(\Big[\frac{e^{au}}{a}\Big]_0^t - \Big[2u\Big]_0^t - \Big[\frac{e^{-au}}{a}\Big]_0^t\Big) \\
&= f_0(t) + \frac{\sigma_r^2 t e^{-at}}{a} + \frac{\sigma_r^2 e^{-2at}}{a^2} - \frac{\sigma_r^2 e^{-at}}{a^2} + \frac{\sigma_r^2}{2a^2} - \frac{2\sigma_r^2 t e^{-at}}{2a} - \frac{\sigma_r^2 e^{-2at}}{2a^2} \\
&= f_0(t) + \frac{\sigma_r^2}{2}\Big(\frac{e^{-2at}}{a^2} - \frac{2e^{-at}}{a^2} + \frac{1}{a^2}\Big) \\
&= f_0(t) + \frac{\sigma_r^2}{2}B^2(0,t)
\end{aligned} \tag{A.20}$$

□

In order to derive the formula for β_t we observe that the random variables in

$$e^{a(t+\Delta t)}\beta_{t+\Delta t} - e^{at}\beta_t = \sigma_r \int_t^{t+\Delta t} e^{au} dZ_u \tag{A.21}$$

are independent and normally distributed with mean 0 and variance

$$\sigma_r = \int_t^{t+\Delta} e^{2au} du = \sigma_r e^{2at}\frac{e^{2a\Delta t} - 1}{2a} \tag{A.22}$$

Dividing both sides by $e^{a(t+\Delta t)}$ we end up with Equation (8.27), which completes the proof.

□

A.5 Proof 5

We are given the following formula for the instantaneous forward rate.

$$f_0(t) = R_t + t \cdot \frac{\partial R_t}{\partial t} \tag{A.23}$$

Notice that $P(0,t) = e^{-R_t t} \implies R_t = -\ln P(0,t)t^{-1}$.

$$
\begin{aligned}
f_0(t) &= R_t + t \cdot \frac{\partial}{\partial t}\Big(- \ln\left(P(0,t)t^{-1}\right)\Big) \\
&= R_t + t \cdot \left(t^{-1} \frac{\partial\Big(- \ln\left(P(0,t)\right)\Big)}{\partial t} + \ln\left(P(0,t)\right)(t^{-2}) \right) \\
&= R_t - \frac{\partial}{\partial t} \ln(P(0,t)) + \frac{\ln\left(P(0,t)\right)}{t} \\
&= R_t - \frac{\partial}{\partial t} \ln\left(P(0,t)\right) - \ln\left(e^{-R_t t}\right)t^{-1} \\
&= -\frac{\partial}{\partial t} \ln\left(P(0,t)\right)
\end{aligned}
\tag{A.24}
$$

A.6 Mortality Tables

The formula for mortality risk is given by (Finanstilsynet, 2013; SSB, 2019)

$$
\mu_{Kol}(x,t) = \mu_{Kol}(x,2013)\left(1 + \frac{w(x)}{100}\right)^{t-2013} \tag{A.25}
$$

where $\mu_{Kol}(x, 2013)$ denotes the mortality for a person of age x in 2013, while $\mu_{Kol}(x,t)$ is the mortality for a person of age x in year $t \geq 2013$.

The decrease in the mortality, denoted by $w(x)$, is given by

$$
w(x) = \min(2.671548 - 0.172480x + 0.001485x^2, 0) \qquad \text{for men} \tag{A.26}
$$

$$
w(x) = \min(1.287968 - 0.101090x + 0.000814x^2, 0) \qquad \text{for women} \tag{A.27}
$$

$\mu_{Kol}(x, 2013)$ is defined for a policyholder at age x in 2013 as

$$
1000 \cdot \mu_{Kol}(x,2013) = 0.189948 + 0.003564 \cdot 10^{0.051x} \qquad \text{for men} \tag{A.28}
$$

$$
1000 \cdot \mu_{Kol}(x,2013) = 0.189948 + 0.003564 \cdot 10^{0.051x} \qquad \text{for women} \tag{A.29}
$$

All customers are assumed to reach the retirement age of 67. Risk of mortality then yields in the interval 67 years to 100 years old. The calculated mortality risks are compared to numbers from Statistics Norway to ensure reliability in the calculations.

A.7 Number of Simulations

Regarding the number of simulations chosen, we have to consider the trade-off between run time and accuracy. The more simulations we run, the closer the expected value gets to its actual value. However, the model gets computationally heavier.

Given a desired confidence interval, the following equation gives the number of iterations required (Hahn, 1972).

$$n = \left(\frac{Z_{1-\frac{\alpha}{2}} S}{\epsilon \overline{x}} \right)^2 \tag{A.30}$$

$Z_{1-\frac{\alpha}{2}}$ is the critical value of the normal distribution for $\alpha/2$, S is the sample standard deviation, $\overline{x}$ is the sample mean and ϵ is the desired margin of error.

We run 5,000 simulations, which gives a confidence interval of about 12% when we include the control variates. The variates reduce the variance by approximately 50 %.

As we focus on the overall trends, we use the same random variables in all runs (i.e. for different input values). By doing this we make sure that the results are comparable, as the evolution in interest rate and stock prices will be the same for all sets of input values. We have also in some cases used 1,000,000 simulations to compare the results. These tests indicate that the market value for the insurer and customer deviates with between 0% and 1% from its true value.

A.8 Martingale Testing

Martingale tests have been run to control that the discounted value of the stock and bond prices using the bank account equals the initial values. This is done in order to validate the model, and make sure that cash flows get discounted correctly.

Figure A.1 presents the results from Martingale tests where the discounted value of a bond paying 1 in t years is calculated using the bank account from Section 8.4 and the simulated quarterly interest rate. As the graph illustrates, the expected value of the bond is very similar for both discounting approaches, which means that discounting using the bank account gives us the expected market value of the bond.

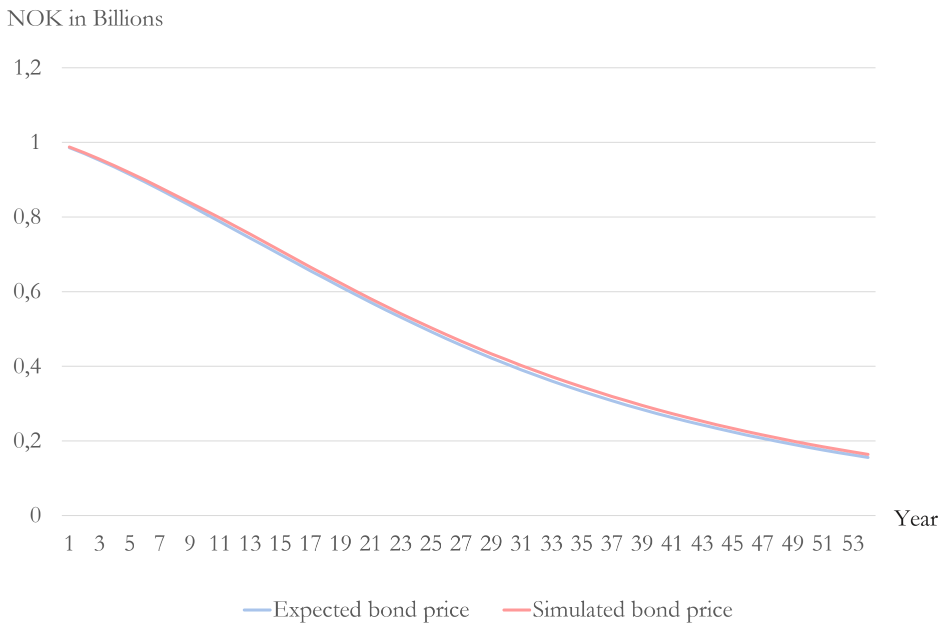

Figure A.1: Martingale testing of discounted bond prices. The blue line corresponds to bond price obtained by discounting using the interest rates provided by EIOPA. The red line represents the bond value after discounting using the bank account.

Figure A.2 shows the results from a similar test run on stock prices.

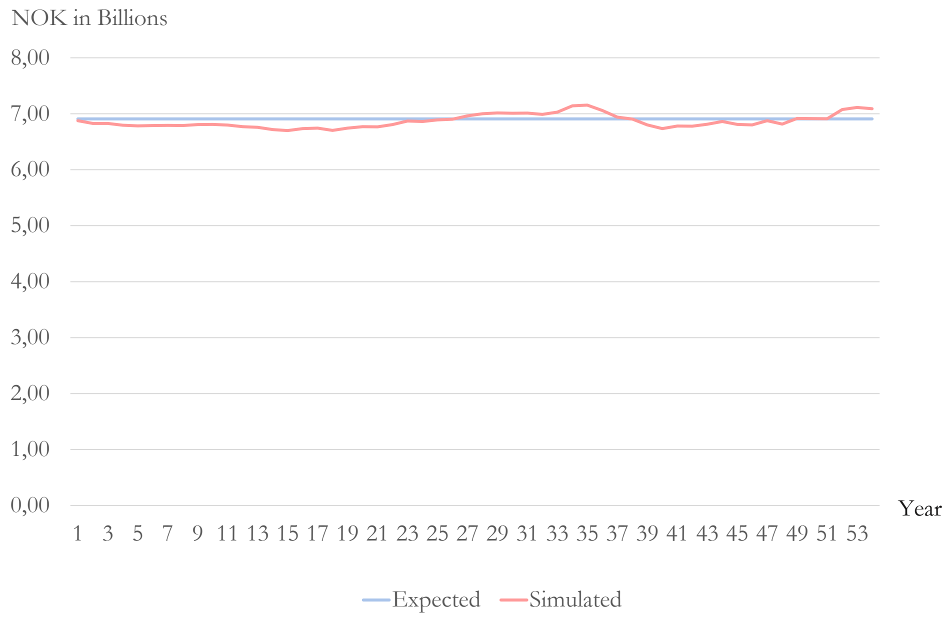

Figure A.2: Martingale testing of discounted stock prices. The blue line corresponds to the initial stock price. The red line corresponds to the discounted stock price using the bank account.

In each case, there is on average a minor deviation between the lines. The deviation might be related to discretization errors, as the stock and bond prices are only calculated for given intervals. The fact that we do not run more simulations could also be a possible explanation of the deviation.

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Paid-up Policies

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