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Revolving flow of a fluid-particle suspension with suction

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Abstract The three-dimensional revolving flow of a particle-fluid suspension above a plane surface is considered. The flow represents an extension of the classical Bödewadt flow to a two-fluid problem. The governing equations for the two phases are coupled through an interaction force with the particle relaxation time τ as a free parameter. By means of a similarity transformation, the coupled set of non-linear ODEs becomes a two-point boundary value problem. The numerical results show that the radial inward particle velocity increases whereas the circumferential velocity decreases by shortening τ, thereby strengthening the spiralling particle motion. These predictions are consistent with the so-called tea-cup effect, i.e. accumulation of tea leaves at the centre of the cup. On the contrary, the revolving fluid motion is reduced as a result of the particle-fluid interactions.

1. Introduction

The steadily revolving flow of a viscous fluid above a planar surface is commonly known as Bödewadt flow; see Bödewadt [1]. The fluid motion well above the surface is characterised by a uniform angular velocity, which is reduced through a viscous boundary layer in order for the fluid to adhere to the no-slip condition at the solid surface. The reduction of the circumferential velocity component in the vicinity of the surface reduces the radially directed centripetal acceleration (or centrifugal force) such that the prevailing radial pressure gradient induces an inward fluid motion. In order to assure mass conservation, this inward fluid motion gives in turn rise to an axial upward flow. Such a spiralling flow exists near the planar surface, although more complex variations of the velocity field have been reported further away, but yet before the uniformly rotating flow conditions are reached. The oscillatory nature of the three velocity components reported by Bödewadt [1] has been subject to criticism, but this criticism was deem unjustified by Zandbergen and Dijkstra [2]. It is interesting to notice that these oscillations are damped and even suppressed in presence of a magnetic field (King and Lewellen [3]), partial slip (Sahoo, Abbasbandy and Poncet [4]), stretching surface (Turkyilmazolgu [5]) or suction (Nath and Venkatachala [6]). With a sufficiently high suction velocity through the planar surface, the axial flow is directed in the downward direction rather than upward, as is the case in the classical Bödewadt flow. In view of its fundamental importance as a prototype swirling flow the Bödewadt flow has received renewed focus in recent years. The inviscid instability of the Bödewadt boundary layer was examined by MacKerrell [7] whereas Sahoo [8,9] and Sahoo and Poncet [10] demonstrated that also

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such revolving flows of a non-Newtonian Reiner-Rivlin fluid admit exact similarity solutions.

The swirling flow induced by a steadily rotating disk was first described by von Kármán [11]. The von Kármán flow is essentially a reversed Bödewadt flow, albeit without the oscillatory features that characterize the latter. Zung [12] studied a von Kármán flow of a fluid-particle suspension and his analysis was subsequently extended by Sankara and Sarma [13] to include surface suction and further explored by Allaham and Peddieson [14].

Studies of suspensions of small particles in a continuous medium (either gas or liquid) are of fundamental interest in fluid mechanics and yet with numerous applications like, for instance, aerosol clouds and erosion protection. Additional applications were recently pointed out by Turkyilmazolgu [15]. A daily life example is the characteristic flow-induced sedimentation of tea leaves in a flat-bottomed cup of tea, as discussed by Einstein [16] and illustrated in Fig. 1.

The aim of the present study is to adopt a similar approach as that advocated by Zung [12] to Bödewadt flow of a fluid-particle suspension. After first having shown that the governing two-phase flow equations admit similarity solutions, numerical solutions of the coupled set of non-linear ordinary differential equations will show how the particle phase is revolving along with the fluid and also how the presence of particles will affect the swirling motion of the fluid phase.

2. Mathematical model equations

Let us consider the steadily revolving flow of a fluid-particle suspension above a planar surface. In cylindrical polar coordinates (r, θ, z) the governing mass conservation and momentum equations for the fluid and particle phases become:

$$\frac{\partial}{\partial r}(ru) + \frac{\partial}{\partial z}(rw) = 0, \tag{1}$$

$$\frac{\partial}{\partial r}(r\rho_p u_p) + \frac{\partial}{\partial z}(r\rho_p w_p) = 0, \tag{2}$$

$$u\frac{\partial u}{\partial r} - \frac{v^2}{r} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left[\frac{\partial^2 u}{\partial r^2} + \frac{\partial}{\partial r}\left(\frac{u}{r}\right) + \frac{\partial^2 u}{\partial z^2}\right] + \frac{F_r}{\rho}, \tag{3}$$

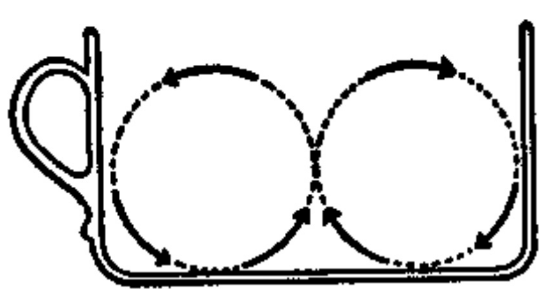

Fig. 1 Sketch of the secondary fluid motion in the vertical plane through the symmetry axis of a flat-bottomed cup of tea. The secondary motion arises due to a revolving fluid motion in the horizontal plane, caused for instance by stirring by a spoon, and sweeps the tea leaves towards the center of the cup.

$$u\frac{\partial v}{\partial r} + \frac{uv}{r} + w\frac{\partial v}{\partial z} = \nu\left[\frac{\partial^2 v}{\partial r^2} + \frac{\partial}{\partial r}\left(\frac{v}{r}\right) + \frac{\partial^2 v}{\partial z^2}\right] + \frac{F_\theta}{\rho}, \tag{4}$$

$$u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right] + \frac{F_z}{\rho}, \tag{5}$$

$$u_p\frac{\partial u_p}{\partial r} - \frac{v_p^2}{r} + w_p\frac{\partial u_p}{\partial z} = -\frac{1}{\rho_p}\frac{\partial p}{\partial r} - \frac{F_r}{\rho_p}, \tag{6}$$

$$u_p\frac{\partial v_p}{\partial r} + \frac{u_p v_p}{r} + w_p\frac{\partial v_p}{\partial z} = -\frac{F_\theta}{\rho_p}, \tag{7}$$

$$u_p\frac{\partial w_p}{\partial r} + w_p\frac{\partial w_p}{\partial z} = -\frac{1}{\rho_p}\frac{\partial p}{\partial z} - \frac{F_z}{\rho_p}, \tag{8}$$

where (u, v, w) and (u_p, v_p, w_p) are the velocity components of the fluid and particle phases in the radial, circumferential and axial directions, respectively. Here, we have assumed rotational symmetry about the vertical z-axis, i.e. $\partial/\partial\theta = 0$. The kinematic viscosity of the fluid is ν and the densities of the fluid and particle phases are ρ and ρ_p. The above set of governing equations is the same as that considered by Zung [12] and Sankara and Sarma [13] for swirling von Kármán flow of a fluid-particle suspension above a steadily rotating disk, except that the a priori unknown pressure p was assigned only to the fluid phase. In the present study, however, the pressure gradients are shared between the two phases in proportion to their density ratio. This alternative formulation was suggested by Allaham and Peddeison [14] but not adopted in their subsequent calculations. In the present problem, however, it is essential to include pressure gradient terms also in the particle-phase equations of motion. Indeed, a radial pressure gradient is required to balance the centripetal acceleration in the far field in Eq. (6).

Particle-fluid interactions are accounted for by means of the following force components

$$F_r = \rho_p(u_p - u)/\tau, \tag{9}$$

$$F_\theta = \rho_p(v_p - v)/\tau, \tag{10}$$

$$F_z = \rho_p(w_p - w)/\tau, \tag{11}$$

included in the fluid phase equations and their negative counterparts in the particle phase equations. These expressions represent force per volume and are based on the assumption of a linear drag law, i.e. Stokes drag, where τ

$$\tau = m/6\pi\mu a, \tag{12}$$

is the relaxation time of a single spherical particle with mass $m$and radius a immersed in a fluid with dynamic viscosity $\mu = \rho\nu$. The viscous fluid phase sticks to the permeable planar surface at $z = 0$ and attains a state of solid body rotation with angular velocity Ω high above the surface:

$$\left.\begin{array}{llll} u = 0, & v = 0, & w = -\sqrt{\nu\Omega}A & at \quad z = 0 \\ u = 0, & v = r\Omega, & p = \frac{1}{2}\rho r^2\Omega^2 & as \quad z \to \infty \end{array}\right\}, \tag{13}$$

where $A \geqslant 0$ is a dimensionless suction velocity. The inviscid particle phase can be assumed to follow the motion of the fluid phase far above the solid surface, i.e.

$$\begin{aligned} &u_p = u = 0, \quad v_p = v = r\Omega, \\ &w_p = w, \qquad \rho_p = \rho, \qquad \textit{for } z \to \infty. \end{aligned} \tag{14}$$

3. Similarity transformation and resulting ODEs

The two-phase flow that arises above the planar surface can be characterized by the length scale $\sqrt{\nu/\Omega}$, the time scale Ω^{-1} and the velocity scale $\sqrt{\nu\Omega}$. We can therefore introduce the same dimensionless similarity variables used already by von Kármán [11] and Bödewadt [1]:

$$\eta = z\sqrt{\Omega/\nu}. \tag{15}$$

The same similarity transformations are used for the fluid phase velocities and pressure:

$$\begin{aligned} &u(r,z) = r\Omega F(\eta), \\ &v(r,z) = r\Omega G(\eta), \\ &w(r,z) = \sqrt{\nu\Omega} H(\eta), \\ &p(r,z) = \rho\left(-\nu\Omega P(\eta) + \tfrac{1}{2} r^2\Omega^2\right). \end{aligned} \tag{16}$$

The latter pressure transformation was not required in Bödewadt's original approach which was based on the boundary layer approximations which imply that the pressure is constant all across the boundary layer. The variables characterizing the particle phase can be recast into dimensionless forms by means of the transformation used for von Kármán flow by Sankara and Sarma [13]:

$$\begin{aligned} &u_p(r,z) = r\Omega F_p(\eta), \\ &v_p(r,z) = r\Omega G_p(\eta), \\ &w_p(r,z) = \sqrt{\nu\Omega} H_p(\eta), \\ &\rho_p(r,z) = \rho Q(\eta), \end{aligned} \tag{17}$$

where Q is the ratio between the densities of the two phases. The governing set of partial differential Eqs. (1)–(8) transforms into a set of ordinary differential equations:

$$2F + H' = 0, \tag{18}$$

$$Q'H_p + QH_p' + 2QF_p = 0, \tag{19}$$

$$F'' - HF' - F^2 + G^2 + \beta Q(F_p - F) = 1, \tag{20}$$

$$G'' - HG' - 2FG + \beta Q(G_p - G) = 0, \tag{21}$$

$$P' = -2FH + 2F' - \beta Q(H_p - H), \tag{22}$$

$$F_p'H_p + F_p^2 - G_p^2 + \beta(F_p - F) = -1/Q, \tag{23}$$

$$G_p'H_p + 2F_pG_p + \beta(G_p - G) = 0, \tag{24}$$

$$QH_p'H_p + \beta Q(H_p - H) = P' \tag{25}$$

where the prime denotes differentiation with respect to the similarity variable η. The accompanying boundary conditions (13) and (14) transform into:

$$F(\eta) = 0, \quad G(\eta) = 0, \quad H(\eta) = -A, \qquad \textit{at} \quad \eta = 0,$$

$$\left.\begin{aligned} &F(\eta) = 0, \quad G(\eta) = 1, \qquad P(\eta) = 1, \\ &F_p(\eta) = 0, \quad G_p(\eta) = 1, \quad H_p(\eta) = H(\eta), \\ &Q(\eta) = 1. \end{aligned}\right\} \textit{as} \quad \eta \to \infty. \tag{26}$$

The resulting two-fluid flow problem now depends only on two dimensionless parameters, namely the suction parameter A and the interaction parameter $\beta = 1/\Omega\tau$ where τ was introduced in Eq. (12). The ratio between a particle time scale and a fluid time scale, e.g. τ/Ω^{-1}, is often referred to as a Stokes number. The single-phase Bödewadt flow with suction is recovered in the particular case when the interaction parameter $\beta = 0$.

4. Numerical integration technique

Our primary interest is in the flow field. The pressure gradient P' can therefore be eliminated from the axial components of the fluid and particle equations to give

$$H_p' = \left(2F' - 2\beta Q(H_p - H) - 2FH\right)/H_pQ. \tag{27}$$

We have used the bvp4c MATLAB solver, which gives very good results for the non-linear ODEs with multipoint BVPs. This finite-difference code utilizes the 3-stage Lobatto IIIa formula, that is a collocation formula and the collocation polynomial provides a C1-continuous solution that is fourth-order accurate uniformly in [a,b]. For multipoint BVPs, the solution is C1-continuous within each region, but continuity is not automatically imposed at the interfaces. Mesh selection and error control are based on the residual of the continuous solution. Analytical condensation is used when the system of algebraic equations is formed; see Shampine et al. [17]. The coupled set of non-linear ODEs are integrated for $A = 2.0$ and some different values of β.

For the particular parameter value $\beta = 0$, the two-point boundary value problem defined in Section 3 simplifies since the particle phase decouples from the fluid phase. In our recent paper [18], results for this single-phase flow compared excellently with earlier results provided by Nath and Venkatachala [6] for some different values of the dimensionless suction velocity $A = 0, 1$, and 2.

For non-zero values of the interaction parameter β, the fluid phase momentum equations are coupled to the particle phase momentum equations through interaction force terms. Although our numerical integration approach worked perfectly well for $\beta = 0$, we were unable to obtain converged numerical solutions for $\beta > 0$ in absence of suction ($A = 0$). We first computed some sample solutions for $A = 3$ (Rahman and Andersson [19]). In the present paper we instead consider $A = 2$ after first having validated the computational accuracy by comparisons with the results obtained by Nath and Venkatachala [6].

Allaham and Peddieson [14] mentioned that numerical solutions of particulate von Kármán flow driven by an impermeable disk did not exist for some parameter combinations, but also that no such restrictions were found when suction was imposed. It is well known that Bödewadt flow over an impermeable surface, i.e. $A = 0$, is more complex than the corresponding Kármán flow. The three velocity components exhibit an oscillatory behaviour and the Bödewadt boundary layer is substantially thicker than the von Kármán boundary layer. In presence of suction, however, the Bödewadt boundary layer becomes substantially thinner and the oscillatory behaviour vanishes. For these reasons, numerical solutions are more readily obtained.

5. Results and discussions

We are primarily interested in particle-fluid interactions in the three-dimensional flow field. We therefore considered five different values of the particle-fluid interaction parameter β in the range from 0.2 to 2.0. The suction parameter A was kept constant and equal to 2.0. This particular parameter value was chosen since Nath and Venkatachala [6] considered the same albeit without a particle phase.

We can see from Fig. 2 that the radial inward flow is reduced in the presence of particles and this reduction increases with β. This is caused by the interaction force F_r which is positive since $u_p > u$ everywhere except in the near vicinity of the surface. It is noteworthy that also the particle phase flows towards the symmetry axis. Contrary to u, however, u_p does not obey no-slip at the surface. This gives rise to a change-of-sign of the slip velocity $u_p - u$ next to the surface and thereby a reversal of F_r. The thickness of the thin near-wall layer with $F_r < 0$ increases from about 0.2 to $0.4\sqrt{v/\Omega}$ as β increases from 0.2 to 2.0.

The particle velocity u_p in Fig. 3 is radially inward and increases in magnitude all the way towards the surface. This inward motion strengthens monotonically with increasing interaction parameter β, primarily due to a reduction of the circumferential particle velocity v_p. The gradual reduction of the magnitude of the centripetal acceleration in Eq. (6) is partly compensated by an increasing magnitude of the convection $u_p \partial u_p/\partial r = 1/2 \partial u_p^2/\partial r < 0$.

The interaction parameter β has an almost negligible effect on the circumferential fluid velocity v in Fig. 4 which reduces monotonically from that of solid body rotation $v = r\Omega$ to no-slip $v = 0$ at the surface. The circumferential slip velocity $v_p - v$ becomes inevitably positive in the viscous boundary layer since the particle velocity v_p is neither affected by viscous forces nor obeys no-slip. The circumferential interaction force F_θ on the fluid phase is thus positive and slightly increases v with increasing β, whereas the corresponding reaction force on the particle phase $-F_\theta < 0$ and therefore tends to enhance the deceleration of the particle motion with increasing β, as one can observe in Fig. 5.

The fluid phase flows axially towards the surface, i.e. $w < 0$, as can be seen in Fig. 6. This opposite flow direction compared

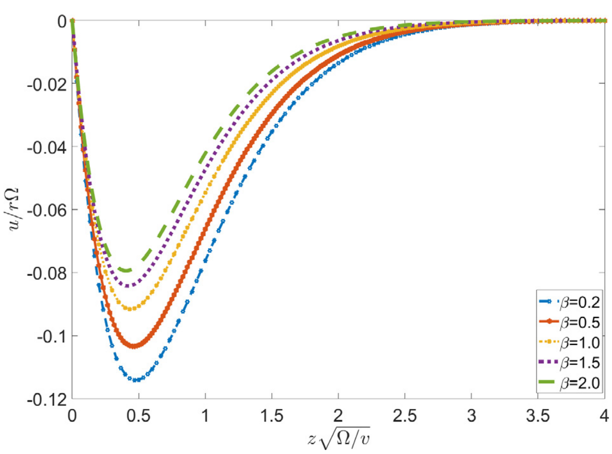

Fig. 2 Radial fluid velocity component $F = u/r\Omega$ for some different values of the interaction parameter β.

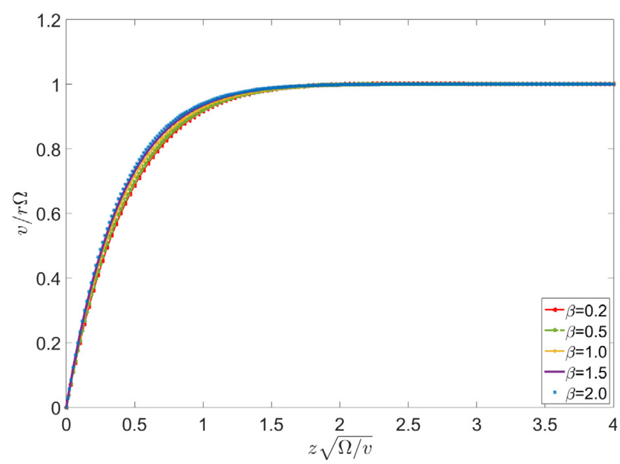

Fig. 4 Circumferential fluid velocity component $G = v/r\Omega$ for some different values of the interaction parameter β.

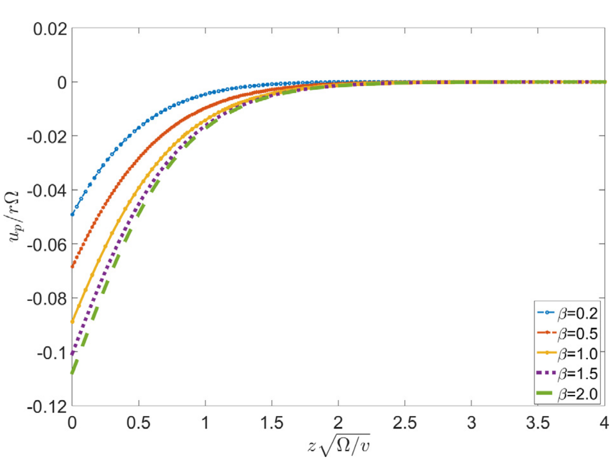

Fig. 3 Radial particle velocity component $F_p = u_p/r\Omega$ for some different values of the interaction parameter β.

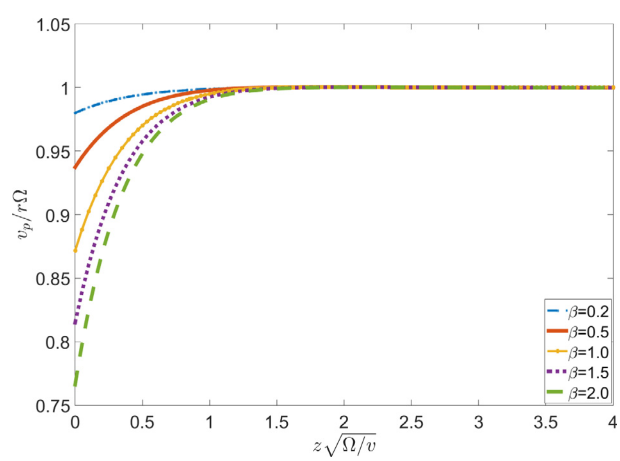

Fig. 5 Circumferential particle velocity component $G_p = v_p/r\Omega$ for some different values of the interaction parameter β.

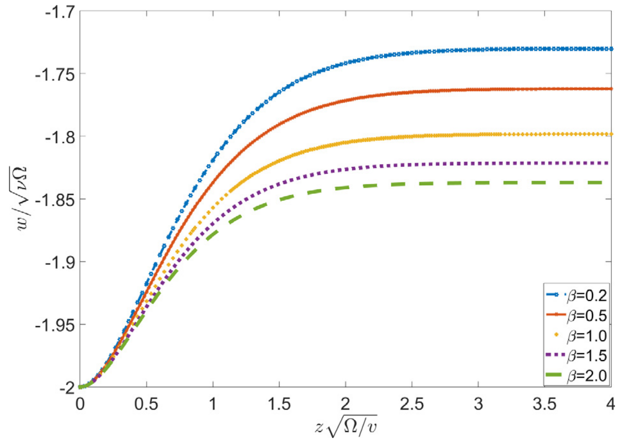

Fig. 6 Axial fluid velocity component $H = w/\sqrt{v\Omega}$ for some different values of the interaction parameter β.

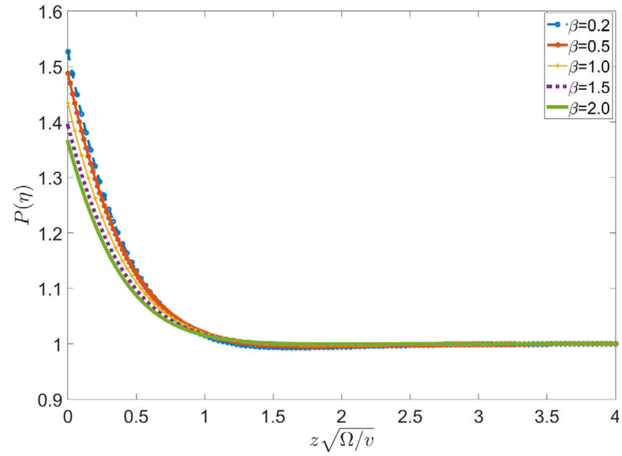

Fig. 8 Pressure P for some different values of the interaction parameter β.

to that in classical Bödewadt flow ($w > 0$) is due the surface suction. The magnitude of the downward flow can be seen to increase from that beyond $\eta = z\sqrt{\Omega/v} \approx 2$ where solid-body rotation prevails through the viscous boundary layer until $w/\sqrt{v\Omega} = -2.0$ at the surface $z = 0$. The interaction force F_z is generally positive since the slip velocity $w_p - w > 0$ (compare Figs. 6 and 7). The axial convection is $w_p \partial w_p/\partial z = 1/2\partial w_p^2/\partial z$ is partially balanced by the negative reaction force $-F_z$ in the particle Eq. (8). However, the dimensionless pressure P decreases monotonically upwards, i.e. $P' < 0$, as seen in Fig. 8. This variation gives rise to a positive pressure gradient $\partial p/\partial z$ in the axial direction, i.e. a pressure force that acts towards the surface and thus tends to support the axial motion towards the surface.

The relative density Q, i.e. the ratio between the particle and fluid densities ρ_p/ρ, is a variable quantity that has been obtained as an integral part of the numerical solution of the present two-fluid flow problem. The Q-profiles in Fig. 9 show that the particle density ρ_p is some 10% lower than the fluid density near the surface but increases to match ρ outside of the viscous boundary layer.

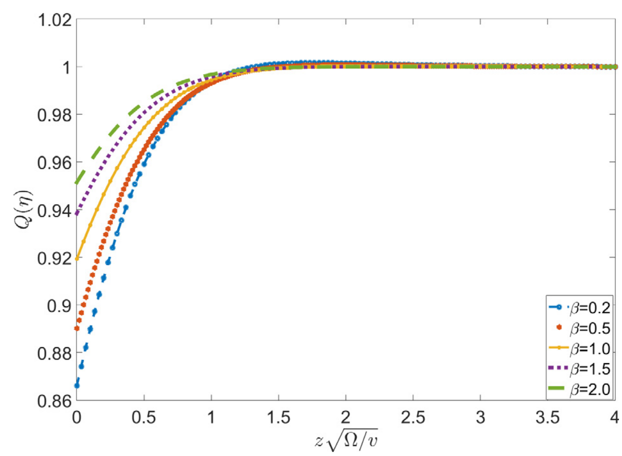

Fig. 9 Ratio of particle and fluid densities $Q = \rho_p/\rho$ for some different values of the interaction parameter β.

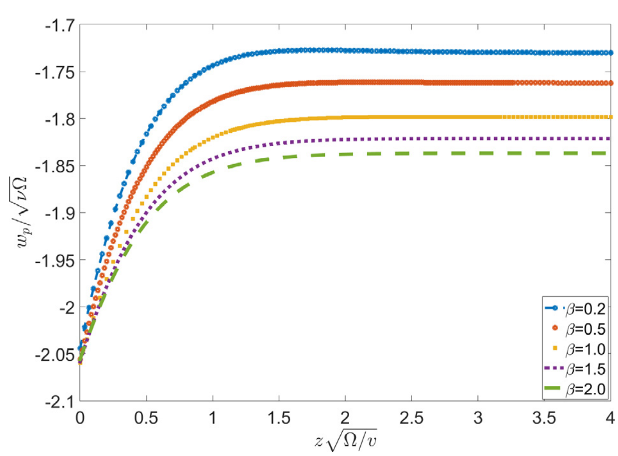

Fig. 7 Axial particle velocity component $H_p = w_p/\sqrt{v\Omega}$ for some different values of the interaction parameter β.

Let us finally look at the motion of the particle phase in the limit as $\beta \to 0$ if $\rho_p = \rho$. The horizontal velocity components become $u_p = 0$ and $v_p = r\Omega$, i.e. the radial component vanishes and the linearly increasing circumferential velocity makes the centripetal acceleration exactly balance the radial pressure force in Eq. (6).

The boundary layer characteristics obtained from the numerical solutions are given in Table 1. For the fluid phase: $-F'(0), G'(0), -H(\infty)$. For the particle phase: $-F_p(0), G_p(0), -H_p(0)$.

We learned from Table 1 that:

- The magnitude of the slope of the radial velocity $F'(0)$ decreases with increasing fluid-particle interactions.
- The slope of the circumferential fluid velocity $G'(0)$ increases with increasing β (not clearly visible in Fig. 4).
- The magnitude of the downward fluid velocity $-H(\infty)$ is slightly increased from 1.73 to 1.84 with increasing β.
- The circumferential particle velocity $G_p(0)$ decreases from 0.98 to 0.77 as β increases from 0.2 to 2.0.
- The inward radial particle velocity $-F_p(0)$ more than doubles from about 0.05 to 0.11 as β is increased.

Table 1 Flow characteristics for suction parameter $A = 2.0$.

β	$-F'(0)$	$-F_p(0)$	$G_p(0)$	$G'(0)$	$-H_p(0)$	$-H(\infty)$
0.2	0.6136	0.0492	0.9815	2.2165	2.0443	1.7301
0.5	0.5770	0.0685	0.9373	2.3038	2.0559	1.7622
1.0	0.5396	0.0889	0.8716	2.4342	2.0599	1.7983
1.5	0.5192	0.1008	0.8149	2.5453	2.0585	1.8213
2.0	0.5082	0.1078	0.7661	2.6402	2.0559	1.8370

6. Concluding remarks

In this study we adopted the mathematical model of mass and momentum transport in two-fluid systems used earlier for von Kármán flow by Zung [12] and Sankara and Sarma [13]. In the Bödewadt flow, however, it was essential to include pressure force terms in the particle phase momentum equations in order to balance the centripetal acceleration in Eq. (6) in the far field.

- The governing equations of the three-dimensional two-fluid problem have been transformed into a coupled set of ordinary differential equations by means of an exact similarity transformation.
- The resulting set of ODEs is a two parameter problem in terms of the dimensionless suction velocity A and the fluid-particle interaction parameter β.
- β is the ratio between the rotation time scale Ω^{-1} and the particle relaxation time τ. β is therefore a reciprocal Stokes number.
- The particles are spiralling inwards in the vicinity of the surface. The radial inward velocity $|u_p|$ increases and the circumferential velocity v_p decreases with increasing interaction parameter β, i.e. the spiralling increases with β. This phenomenon is commonly known as the tea cup effect: when the tea is stirred, tea leaves near the bottom move towards the centre of the cup and heap up.
- Contrary to the particle phase, the inward spiralling of the fluid phase is gradually reduced as the fluid-particle interaction parameter is increased. The circumferential flow is only modestly affected, but the surface shear stress is nevertheless increased.

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