

Leak Detection in Pipelines by the use of State and Parameter Estimation

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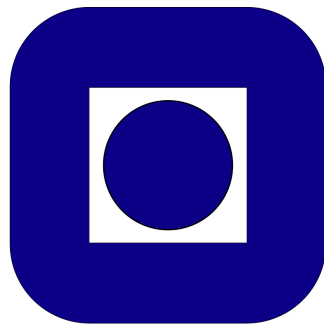
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Master Thesis

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“A small leak will sink a great ship”
Benjamin Franklin

Contents

Problem Description	1
Abstract	3
1 Introduction and Motivation	5
1.1 The State of The Worlds Oil Pipelines	5
1.2 The World's Water Distribution System	5
1.3 Scope and Emphasis	6
1.4 Outline of Thesis	7
2 Methods of Leak Detection	8
2.1 The Main Categories	8
2.1.1 Hardware Based Methods	8
2.1.2 Software Based Methods	10
2.1.3 Comparison	12
3 Pipe Flow and Leak Equations	13
3.1 Simple Hydraulic Transmission Line	13
3.2 Discretized Model	14
3.3 Hydraulic Transmission Line with a Branch	15
4 Modeling of The Water Transmission Line	17
4.1 Modeling the Pipeline	17
4.2 State Space Representation	17
4.3 Observability Analysis	19
4.4 Initial Conditions	19
4.5 Introducing the Leak	20
4.6 Process and Measurement Noise	21
5 The Kalman Filter	23
5.1 Breakdown of The Kalman Filter	23
5.2 Implementing The Kalman Filter	24
6 Results	25
6.1 No Leakage	25
6.2 Leakage at Known Position	29

6.3	Leakage at Unknown Position	32
6.3.1	Leak in Main Branch	32
6.3.2	Leak in Branch A	36
6.3.3	Leak in Branch B	40
6.3.4	Time Varying Boundary Conditions	40
6.4	Larger Model	41
6.4.1	Zero Leak	41
6.4.2	One Percent Leak	44
6.4.3	Five Percent Leak	46
7	Discussion, Suggestions and Future Work	48
7.1	Discussion	48
7.2	Future work	49
8	Conclusion	50
	Bibliography	51
	Nomenclature	53

Problem Description

The subject originates from the requirement of monitoring pipelines carrying oil and gas. Statoil operates several pipelines, for instance the one between Kollsnes and Mongstad. A system for supervision of these pipelines is imposed by the authorities. This system must incorporate a thorough model, enabling, among others, Mongstad to predict the production rate and detect leakages. Other factors that can change is the friction factor for the pipe and errors on the measuring equipment (pressure/temperature/density/rate, everything measured in and/or out of the pipe). With a good model one can estimate states like pressure, rate and temperature in the middle of the pipe and with that identify leakages. It is possible to buy commercial programs for this, but it is desirable to find alternative models and estimation routines. The project assignment is based on earlier submitted projects and theses where methods for detecting leaks in a pipeline with flow from A to B. This project wish to expand the range of uses to include networks of pipes, for instance in water or gas supply to households. We start with the simplest network of all: one inlet, one branch and two outlets. If time allows, we will look on more advanced networks. Objective:

1. Outline earlier work, and motives for the assignment.
2. Expand the mathematical description done in earlier work to include branching.
3. Consider to what degree earlier estimation strategies can be included in branching. Would it require measurements inside the network, or is it adequate with measurements in the inlet and outlet?
4. Demonstrate performance through simulation with leakages in different places and of different rates.
5. If time allows: Repeat 2-4 with a more complicated network
6. Write a report with your finds and make suggestions for further work.

Abstract

A model based estimation process is implemented in simulation of a transmission line for the purpose of leak detection. The model has one inlet, one branch and two outlets. The objective of this thesis is aimed at determining, through simulation results, the effectiveness of the Kalman Filter for leak detection in a pipeline network.

Water distribution systems often contain a large amount of unknown losses. The magnitude of the water loss range from 10 to over 50 percent of the total water pumped. Water leaks and losses lead to an unnecessary increase in energy used to pump water to users, which means higher costs for the providers and thus the customers. Our increasing population and the necessity and limited availability of water will, and are already causing problems, so it is obvious that more control efforts need to be implemented on these systems. Moreover, leaks that are not considered to be major faults tend to go unnoticed, which in total add up to a lot of water loss.

The work done in this thesis may also be relevant for the petroleum industry, as transportation of oil and gas also face many of the same problems as the water distribution industry. One should also note that the environmental impact of leaks in oil pipes are bigger than those in water pipes.

The leak detection process used in this thesis is model based. A model of a transmission line with a branch and two outlets is derived from two basic partial differential equations describing mass and momentum balances for a hydraulic transmission line. Simulations are run, and when the system is in its steady state, a leak is introduced. Flow and pressure at all three extremes are used as inputs to the Kalman Filter. The likelihood of two leaks occurring simultaneously is low, so only one leak will be expected at a time.

Localization of the leaks are not studied, only the detection and the magnitude. It is shown that a Kalman Filter can be used as a detection method in this system to a certain degree. The changes caused by the pressure change is noticeable when a leak occurs in a large scale system, even if the leak is very small. The speed of the Kalman Filter proved to be a problem area, as some of the estimated states used a long time to reach its final value. More satisfying results may be found through more rigorous testing, improvements in the model itself, and changes to the tuning parameters in the Kalman Filter.

1 Introduction and Motivation

1.1 The State of The Worlds Oil Pipelines

As the petroleum industry constantly evolves, the need for fault detection and isolation (FDI) in pipelines becomes more and more important, both to save the environment of possible harm, as well as prevention of economic loss. Thankfully, many venues (e.g. petroleum industry) are required to include some sort of FDI.

Pipelines break all the time. Exxon's Pegasus pipeline can carry more than 90,000 barrels per day of crude oil from Pakota, Illinois, to Nederland, Texas. At the end of March 2013, Pegasus had a leak and spilled between 3,500 - 5,000 barrels of crude oil into neighborhood streets and lawns in Mayflower, Arkansas. Within that same week, Shell had three major oil spills. The last one was in Texas where they estimated that 700 barrels of oil had been lost, amounting to almost 30,000 gallons of crude oil. With the correct model and measurements it may be possible to detect faults like these so that the pipeline can be closed off before it causes too much damage.

There seems to be some drawbacks with leak detection based on pressure and flow monitoring though. A leak changes the hydraulics of the pipeline and therefore changes the pressure and flow readings after some time. To detect the changes (and the leak) within a satisfying time limit, you need it to be in a steady state. Gas is compressible and therefore its use with gas may be limited [8]. It is a complex problem, and to find a solution one have to start with the easier one-phase systems, like water.

1.2 The World's Water Distribution System

Nothing living nor human made would exist without water. Everything from the economy to the environment to the health of all animals are dependent on water. Keeping the water distribution systems healthy is vital for our survival. It is an important and difficult job, and is therefore the main focus of this thesis.

Water distribution systems are, more often than not, a buried infrastructure, which makes it very difficult to determine immediate problems. Leaks over long periods

of time causes many problems, including the obvious loss of water, the chemicals used to treat it, possible contamination of the (drinking) water inside the pipe, an increase in energy usage, demand shortages leading to avoidable capacity expansions and the possibility of environmental damage [5, 9].

Advances in instrumentation for more effective leak detection and localization have been significant since the 1990's, especially in metering and logging for flow, pressure and leak noise. Despite these advances in leak detection, most water distribution systems worldwide continue to experience high level of water losses [15]. Lambert [15] tries to make a meaningful standard approach to Benchmarking and reporting of management performance. They include Non-Revenue Water, Water Losses, Apparent Losses and Real Losses, all of which has its own subgroups. Real Losses include the losses due to leaks which may be recovered by well directed leakage management activities. At the moment, the magnitude of Real Losses range from under 5 percent (Singapore) to over 40 percent (Malta) of total system input volume.

As water distribution systems age, they experience problems of deteriorating infrastructure, water loss, and service disruption [9]. A better system for fault detection can easily lead to a reduction in leakage, which again could lead to fewer situations where people experience water shortages. Approximately 9 to 30 percent of the total water pumped in Europe is unaccounted for, while some places in North America experience losses up to 50 percent [16].

Large leaks are easily detectable, not only because they will have a big impact on both pressure and flow, but because of the large amounts of water that collects on the surface above the leak. It is obvious that a better fault detection system is required to detect smaller leaks that does not show themselves as easily. Many attempts to better the situation with fault monitoring and detection has been made with some success. It is, however, clear that the problem is complex and poses many challenges. These challenges was the motivation for this thesis.

1.3 Scope and Emphasis

The scope for this thesis will be the design of an estimator for the detection of a leak in a system consisting of a pipe with one inlet, a branch, and two outlets. The estimator used is the well known Kalman Filter. A simple Luenberger type observer was tested in the previous project leading up to this thesis. A small analysis of some other methods of leak detection is included in chapter 2.

1.4 Outline of Thesis

Chapter 2 will present some of the earlier work done in leak detection, the interested reader is encouraged to read more of the referenced material. Chapter 3 and 4 will be dedicated to the theory behind the model, the leak and the estimator, and the method used. Chapter 5 gives a brief introduction to the Kalman Filter, and chapter 6 contains all simulation results. Discussion of the results and suggestions for further work are found in chapter 7, while the conclusion is presented in chapter 8.

2 Methods of Leak Detection

There are numerous methods and studies on leak detection. One could say that there are three main groups of methods, one hardware based, one based on software, and one that relies on biological methods. Each method has its advantages and disadvantages. Leak sensitivity, location estimate capability, operational change, availability, false alarm rate, maintenance requirement and cost are important attributes when comparing the different methods. Some existing methodologies offer acceptable performance, but few, if any, satisfy all conditions. False alarms often have a high rate, which is undesirable because they generate extra work for operational personnel, they reduce the confidence operators have in the system, and actual leaks may be overlooked.

2.1 The Main Categories

Different methods offer different advantages, but they all have some drawbacks, which will be mentioned in sec. 2.1.3. The main methods can be classified into three categories

- Hardware based methods: Typical hardware based detection methods include acoustic sensors, gas detectors, negative pressure detectors, and/or infrared thermography.
- Software based methods: Different software with varying complexity and reliability are used. Examples include flow/pressure change detection and mass/volume balance, model based systems and pressure point analysis.
- Biological methods: Maybe the most time consuming method. Experienced personnel (with dogs) walk along the pipes and looks for obvious damage, smell and sound.

The following sections will include description of the first two methods, as the last one is self-explanatory.

2.1.1 Hardware Based Methods

Fault detection and monitoring based on hardware often means that one would have to install equipment all along a pipeline. Turner and Zhang[19, 21] are the main sources for information used in the following descriptions, other sources will be cited.

Temperature Profile

The outside of the pipeline is monitored. In many situations a pipeline leak will cause a change in temperature in the environment surrounding the pipe. Infrared thermography can be used from moving vehicles, helicopters or postable systems and is able to cover up to hundreds of miles of pipeline per day. This is not a very cheap or fast way to detect leaks, so the development of advanced wide area temperature sensors has made the temperature profile technique more practical. Two major technologies of this type are the multi sensor electrical cable and the Optical Time Domain Reflectometry (OTDR) using fibre optic cable. These type of sensors are typically attached to the pipeline in a “piggy back” fashion, following the whole pipeline. If the radius of the pipeline is large, several of these cables can be attached around the pipe.

Infrared thermographic pipeline testing is a different kind of sensor that detect change in temperature. A radiometer scan areas along the pipeline and detects change in temperature. Fluids, such as water, may form a plume near a leak, the fluid has a thermal conductance different from the dry soil or backfill which will result in different surface temperature patterns above the leak location. The data collected by the scanner is displayed as pictures with areas of differing temperatures where different temperatures have different colors. [13]

Acoustic Emission Detectors

As liquids escape a leaking pipe an acoustic signal that is different from the normal environment is created. Acoustic sensors attached to the outside of the pipeline detect this change, which is analyzed. The sound is used as a means to locate this leak. Acoustic sensors detect and discriminate leak sounds from other sounds generated by normal operational changes. Usually, many sensors have to be placed along the pipeline to counter the limitation of the detection range.

Negative Pressure Waves

At the moment a non-catastrophic rupture in a pipeline occurs, a rarefaction wave is produced in the pipeline contents. This wave, which travels both up- and down-stream, can be detected and analyzed. The technique is based on the theory behind the characteristics of pressure waves traveling over long distances at the speed of sound guided by the pipeline walls [8]. Sensors are usually placed at each end of a pipeline segment to help discriminate between noise and externally caused pressure drops.

Gas Detectors

If the product inside a pipeline is highly volatile, a vapour monitoring system can be used to detect the level of hydrocarbon vapour in the pipeline surroundings.

2.1.2 Software Based Methods

Methods based on software relies on information gathered about flow, pressure and temperature at certain points in a pipeline. Here are some of the methods currently in use.

Conservation of Mass

Mass into, and mass out of the system is measured. If the difference between the two is above a set threshold, a leak alarm will go off. This pure balancing method is, however, not suitable for smaller leaks because of the possible noise signals, drifting measurements and the dynamic changes of both flows (\dot{m}_{in} and \dot{m}_{out}). Improved versions of this method exists. For instance, one could determine the low frequency components of the flows by discrete-time low-pass filtering. More information regarding this method can be found in [12]. The localization of a leak is not possible with this simple method.

Pressure Change

Pressure sensors are almost always installed at the extremes of pipelines. When the system has reached steady state, a limit is set such that if the pressure falls below this limit (with some wiggle room), there's likely a leak within the pipe [8]. A more advanced way to look at it is to regard a long pipeline as a low pass filter with respect to pressure disturbances. A leak is one possibility if a high rate of change of pressure can't be identified through approximations and restrictions, or no pipeline operation that could cause this pressure change is identified.[19]

Change in flow

A predefined figure is used as a model for possibilities of flow changes. If the rate of change of the flow is higher than a predefined figure within a specific time period, there is most likely a leak.

Model Based Systems

Model based systems seems to be the cheapest, but possibly also a very good method for leak detection. The common denominator for all model based systems is that

the pipe flow is described mathematically. Leaks are detected when discrepancies between calculated and measured values differ. The usual equations used to model fluid flow in pipes are

- Conservation of mass
- Conservation of momentum
- Conservation of energy
- Equations of state for the fluid

These are all differential equations which are solved by a variety of computational techniques. Alternative methods currently in use in commercial software packages include

- Finite difference
- Finite element
- Method of characteristics
- Frequency response/spatial discretisation

The model based method requires flow, pressure and/or temperature measurements at the inlet and outlet of a pipeline. If possible, measurements at several points along the pipeline is ideal.

Studies on model based leak detection

Billmann and Isermann [2] develops a nonlinear adaptive state observer and a special correlation technique based on pressure and flow measurements at the pipeline inlet and outlet. The correlation technique is a sensitive decision algorithm for leak detection. The pipeline model itself is based on a hyperbolic partial differential equation system.

Lesyshen [16] develops a transient model of a transmission line using two partial differential equations of continuity and momentum that describe pipe flow. Method of Characteristics is used as a finite difference solution of the two PDEs. Measurements at the pipe extremes are used as inputs to the Extended Kalman Filter that is used for the estimation process. The filter model places two artificial leakage states within the system. The estimates of these “fictitious” leakage states are then used to locate the actual position and magnitude of leakage within the transmission line. One leak can be located.

Aamo et al. [1] designs a leak detection system consisting of an adaptive Luenberger-type observer based on a set of two coupled one dimensional first order nonlinear hyperbolic partial differential equations governing the flow dynamics. As with the previous mentioned methods, only measurements at the extremes are available (flow

velocity and pressure). Time varying boundary conditions found through the model and observer characteristics are applied to guarantee fast convergence. The observer design is performed for the continuum model. Heuristic update laws for adaptation of the friction coefficient and leak parameters are also given.

Hauge [10] presents an adaptive Luenberger-type observer which uses measurements of velocity and temperature at the inlet, and pressure at the outlet. Output injection in the form of boundary conditions is also used in this thesis to achieve fast convergence of the observer error. OLGA, a computational fluid dynamics simulator is used to govern the one-phase fluid flow of the observer which makes it possible to incorporate temperature dynamics which had not been possible before.

In Hodne [11] a set of boundary conditions is derived from a two-fluid model. The conditions are estimated numerically and they try to mimic the behaviour of output injection by using a linearized version of the model. OLGA is used as a model to produce an observer. Furthermore, a set of adaptation laws for estimating parameters in a two-phase leak model is derived. The two-fluid model is made hyperbolic by adding a virtual force term, which is used in the boundary conditions.

Ming and Wei-qiang [17] uses a wavelet algorithm to detect pressure waves caused by the transition from the state of no-leak to leak, and the registered time instances is used to compute the position of the leak.

Rahiman et al. [18] presents a nonlinear observer design based on the circle criterion for a nonlinear model of a pipeline system, which is used for the leak detection.

2.1.3 Comparison

Hardware based leak detection systems are expensive. To install this kind of hardware along pipelines that expand over hundreds of miles is expensive regardless of where the pipe is situated or what elements it runs through. It also adds more equipment that needs service and repairs. The software based systems usually only need flow, pressure and maybe temperature measurements at the inlet and outlet. If the distribution system is very large with many branches, measurement equipment inside the pipe at certain points is necessary, but it will still be cheaper than the hardware based detection methods.

3 Pipe Flow and Leak Equations

The theory used in the thesis is shown in this chapter. All assumptions will also be presented.

3.1 Simple Hydraulic Transmission Line

A differential control volume, $A dx$, is used for the derivation of the hydraulic transmission line model. $A[m^2]$ is the cross section area and $x[m]$ is the spatial coordinate along the volume. The volume contains a compressible fluid with density $\rho(x, t)[\frac{kg}{m^3}]$, and the mass balance can then be found by looking at the average velocity for the fluid over the cross section. The mass flow $w[\frac{kg}{s}]$ is

$$w(x, t) = \int_A \rho v dA = \rho \bar{v} A \quad (3.1)$$

where \bar{v} is the average velocity. The mass balance for this one-dimensional pipe is

$$\frac{\delta \rho}{\delta t} = -\frac{1}{A} \frac{\delta w}{\delta x} \quad (3.2)$$

which is a differential equation describing the dynamics of the density in the volume. To change the variables to pressure, p , this equation is considered: $dp = \frac{\beta}{A} d\rho$. $\beta[Pa]$ is the bulk modulus for the substance in the pipe. The result takes this form

$$\frac{\delta p}{\delta t} = -\frac{\beta}{A} \frac{\delta q}{\delta x} \quad (3.3)$$

The momentum balance for the same control volume is considered when finding an equation for the flow rate, q

$$\frac{\delta w}{\delta t} = -A \frac{\delta p}{\delta x} - A \frac{\delta}{\delta x} \int_A \rho v^2 dA - F + A \rho g \cos(\alpha) \quad (3.4)$$

where α is the angle between positive flow direction and gravity. F is the friction force acting on the volume. Assuming the change $\frac{\delta}{\delta x} \int_A \rho v^2 dA$ is negligible, and that the density, ρ , is treated as a constant like before we then get the equation for the volumetric flow rate

$$\frac{\delta q}{\delta t} = -\frac{A}{\rho_0} \frac{\delta p}{\delta x} - \frac{F}{\rho_0} + A g \cos(\alpha(x)) \quad (3.5)$$

Moreover, we assume that this is a one dimensional flow and that the friction is linear. The simplified equation is

$$\frac{\delta q}{\delta t} = -\frac{A}{\rho_0} \frac{\delta p}{\delta x} - f q \quad (3.6)$$

The Darcy-Weisbach equation is used on a pipe with known cross section, flow, flow element (water) and pipe roughness coefficient to find the friction factor f . The cross section, flow and pipe roughness is of course the same used throughout the thesis, unless specified otherwise.

Equations 3.3 and 3.6 are the differential equations used for the modeled transmission line used in this thesis. More on these equations can be found in Egeland and Gravdahl [6].

3.2 Discretized Model

To use the transmission line model found in the previous section, it has to be discretized in both time and space. To do this, a simple finite volumes method is applied to the differential equations describing the average pressures and flows of each control volume. The pipeline of length L was divided into N segments of length l in a staggered grid (visualized in figures Fig. 3.1 and Fig. 3.2¹). Equation 3.3 becomes

$$\dot{p}_1 = \frac{\beta}{Al}(q_{in} - q_1) \quad (3.7)$$

$$\dot{p}_2 = \frac{\beta}{Al}(q_1 - q_2) \quad (3.8)$$

⋮

$$\dot{p}_N = \frac{\beta}{Al}(q_{N-1} - q_N) \quad (3.9)$$

and equation 3.6 becomes

$$\dot{q}_1 = \frac{A}{l\rho}(p_1 - p_2) - f q_1 \quad (3.10)$$

$$\dot{q}_2 = \frac{A}{l\rho}(p_2 - p_3) - f q_2 \quad (3.11)$$

¹Notice how these two figures are connected, and can be viewed as a single figure.

$$\vdots$$

$$\dot{q}_N = \frac{A}{l\rho}(p_N - p_{out}) - fq_N \quad (3.12)$$

The boundary conditions for the model are the external output and some of the available measurements, namely flow in and pressure at the end of the pipe

$$q(0, t) = q_{in}(t) \quad (3.13)$$

$$p(L, t) = p_{out}(t) \quad (3.14)$$

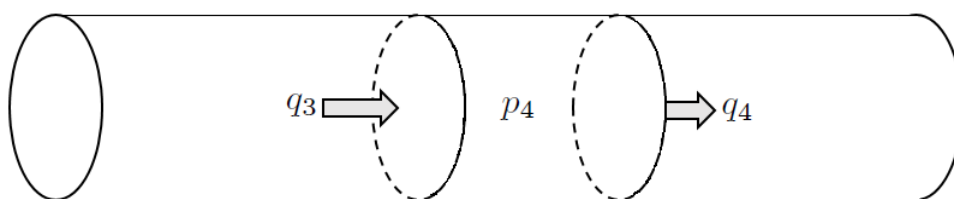


Figure 3.1: Part of staggered grid with centered p .

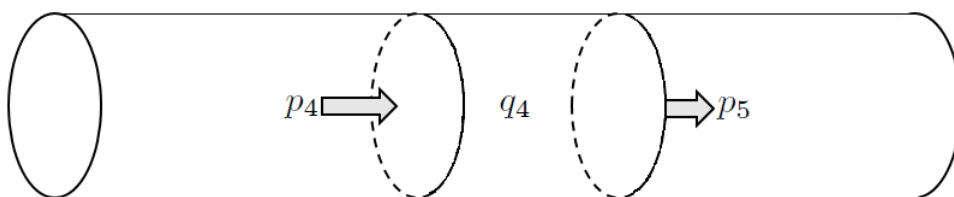


Figure 3.2: Part of staggered grid with centered q .

3.3 Hydraulic Transmission Line with a Branch

The difference between the modeling of a simple pipeline, and a pipeline with a branch and two outlets, is the updated boundary conditions and the modeling of the branch itself. Throughout the report, the single original pipe will be called the *main* pipe/branch, while the branches will be called branch *A* and *B*. The updated boundary conditions are the following

$$q(0, t) = q_{in}(t) \quad (3.15)$$

$$p(L_{main} + L_A, t) = p_{A,out}(t) \quad (3.16)$$

$$p(L_{main} + L_B, t) = p_{B,out}(t) \quad (3.17)$$

where $p(L_{main} + L_A, t)$ is the pressure at the outlet of pipe A , and $p(L_{main} + L_B, t)$ is the pressure at the outlet of pipe B .

In the original pipe, the branch itself was been the boundary condition for pressure, now it works as the common pressure at the start of both branches. Using equation 3.3, the discretized version of the branch becomes

$$\dot{p}_{Branch} = \frac{\beta}{Al}(q_{main,N} - q_{A,1} - q_{B,1}) \quad (3.18)$$

where $q_{main,N}$ is the flow coming into the branch, and q_{A1} and q_{B1} are the flows going out of the branch and into branch A and B respectively. Branch A and B uses the same differential equations as the *main* pipe, but with their own length and number of segments. The discrete equations are shown below to avoid any misunderstandings.

$$\dot{p}_{A,1} = \dot{p}_{B,1} = \dot{p}_{branch} \quad (3.19)$$

$$\dot{p}_{A,2} = \frac{\beta}{Al}(q_{A,1} - q_{A,2}) \quad (3.20)$$

⋮

$$\dot{p}_{AN} = \frac{\beta}{Al}(q_{A,N-1} - q_{A,N}) \quad (3.21)$$

$$\dot{q}_{A,1} = \frac{A}{l\rho}(p_{branch} - p_{A,2}) - f q_{A,1} \quad (3.22)$$

$$\dot{q}_{A,2} = \frac{A}{l\rho}(p_{A,2} - p_{A,3}) - f q_2 \quad (3.23)$$

⋮

$$\dot{q}_{A,N} = \frac{A}{l\rho}(p_{A,N} - p_{A,out}) - f q_{A,N} \quad (3.24)$$

The same equations are used for branch B . For simplicity, the cross section of the pipe is set to be constant, even at the branch.

4 Modeling of The Water Transmission Line

4.1 Modeling the Pipeline

The main model is supposed to imitate a small part of a water distribution system where one of the branches exits to an end user (e.g. a house). There is a constant flow out of the water reservoir, and the water always exits through both the branches. All pipes in the main model have a cross section of $0.0129m^2$, and a flow in, q_{in} , of $10l/s$. The cross section is chosen from [7], which makes it easy to choose the friction factor. The *main* pipe is 10 segments long, where each segment is 1 meter. The *branches* are 4 and 6 segments long, each segment being 1 meter. No changes in elevation occur, neither does any curves/turns. The model configuration is shown below in Fig. 4.1¹.

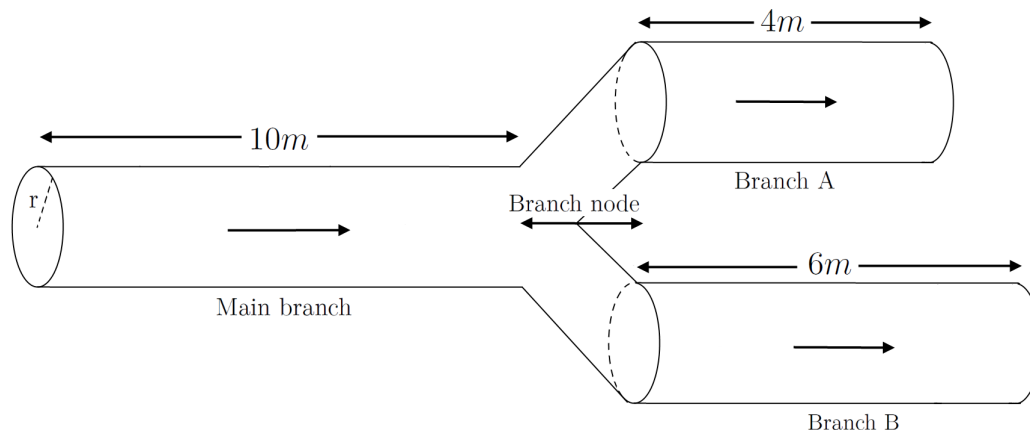


Figure 4.1: The visualized model.

4.2 State Space Representation

To get a working Matlab model of the system, the discretized equations in sec. 3.2 and sec. 3.3 are used to create a *state space* representation, $\dot{x} = Ax + Bu$ $y =$

¹Note that the *branch node*, in effect, is as big as the nodes in the main branch.

$Cx + Du$. “...the *state space* representation is a mathematical model of a physical system as a set of input, output and state variables related by first-order differential equations”². The interdependability of the two main equations describing flow and pressure populate the $A - matrix$ while the external forces/boundary conditions (i.e. flow into the system, and pressure at the ends) populate the $u - matrix$. The $B - matrix$ connects the boundary conditions in u to the correct states. *State space* representation is considered known material, but readers interested in more regarding this topic are urged to take a look at Chen[4].

As mentioned earlier, the pipeline is modelled as a staggered grid. The $x - vector$, having state for every node, is populated like this

$$\dot{x} = \begin{bmatrix} p_{main,1} \\ p_{main,2} \\ \vdots \\ p_{main,N} \\ q_{main,1} \\ q_{main,2} \\ \vdots \\ q_{main,N} \\ p_{branch} \\ p_{A,2} \\ p_{A,3} \\ \vdots \\ p_{A,M} \\ q_{A,1} \\ q_{A,2} \\ \vdots \\ q_{A,M} \\ p_{B,2} \\ p_{B,3} \\ \vdots \\ p_{B,R} \\ q_{B,1} \\ q_{B,2} \\ \vdots \\ q_{B,R} \end{bmatrix} \quad (4.1)$$

where N , M , R are the number of segments the *main* pipe, branch A , and branch B are divided into, respectively. More specifically, this is how the nodes are saved in \dot{x}

²First sentence on the Wikipedia page about State space representation

- 1 – 10: p_{main}
- 11 – 20: q_{main}
- 21: p_{branch}
- 22 – 24: p_A
- 25 – 28: q_A
- 29 – 33: p_B
- 34 – 39: q_B

4.3 Observability Analysis

Measurements are needed for us to be able to estimate unknown states. A measurement of flow and pressure at the pipeline extrema seems to be the standard in pipeline flow problems, it's easy to set up, and affordable. In this thesis, the first node describing pressure, and the first node describing flow is used as measurements at the input. The last node describing flow, and the last node describing pressure in branch A are used as measurements at the end of branch A , and similarly, in branch B . In total, 6, measurements are used.

A system is said to be completely observable if all unknown states can be determined from measurements of the output signal in finite time. To find out if a state space system with n states is observable, the rank of the observability matrix is considered:

$$\mathcal{O} = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} \quad (4.2)$$

where C and A are the matrices in the state space representation $\dot{x} = Ax + Bu$, $y = Cx + Du$ of rank n . If the row rank of this matrix is n , the system is observable. For more information regarding observability readers are directed to [14].

With all six measurements at the extrema available, the system in this thesis is completely observable.

4.4 Initial Conditions

We assume that the system is in a steady state with zero leakage from the start unless specified otherwise. Natural reflections of pressure waves inside the pipeline

makes the graphs messy and difficult to read if startup is included. Especially with low friction and no initial conditions. Additionally, with the low friction used, it would take a long time for the system to reach steady state. To have the system start out in its steady state, the state space representation with $\dot{x} = 0$ is used.

$$\dot{x} = Ax + Bu \mid \dot{x} = 0 \quad (4.3)$$

$$x = -A^{-1}Bu \quad (4.4)$$

A plot of the steady state pipeline characteristics is shown in Fig. 4.2 below. Longer branches are used in these plots, but that has no impact on the behaviour. The pressure drops linearly through the pipelines, as is expected from the linear model. The flow in the *main* pipe is 750 l/s, and it is split into the two branching pipelines. The pipelines all have the same friction factor, and considering the length of the two branches and the theory behind the “path of least resistance” it is logical that $\frac{450}{750} * 100\% = 60\%$ of the flow would go into branch A.

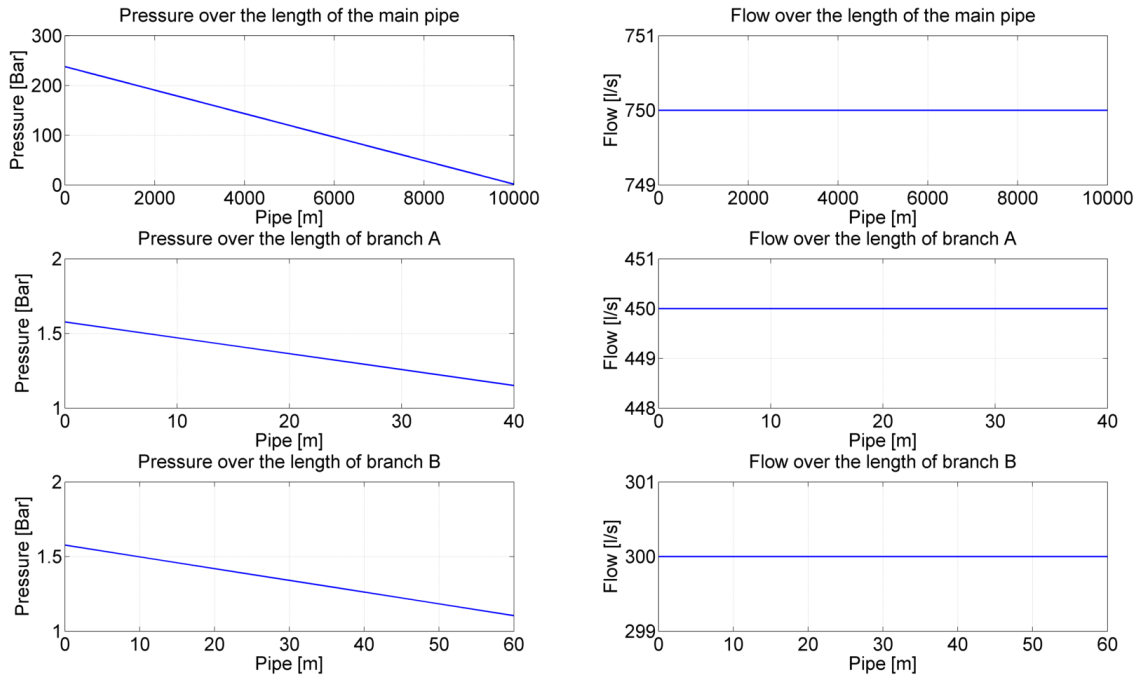


Figure 4.2: Flow and pressure in the pipeline with a branch and two outlets.

4.5 Introducing the Leak

A leak, q_{leak} , is added to the model (3.3) and (3.6). It is assumed to be a constant, point leak that occurs at $t = t_{leak}$ in one of the pressure nodes

$$\dot{p}_n = \frac{\beta}{Al}(q_{n-1} - q_n - q_{leak}) \quad (4.5)$$

q_{leak} is selected as

$$q_{leak}(x, t) = w_l H(t - t_{leak}) \quad (4.6)$$

w_l is the magnitude, and x_l is the position of the leak. H denotes the Heaviside step function. If not specified, nothing about outside pressure, the size of the hole where the leak occurs or q_{in} are considered. The leak is added as an input in the state space representation, so u and B is slightly changed.

A leak of magnitude $3l/s$ is applied to the system with $q_{in} = 10l/s$. A total of $7l/s$ should exit the two branches, which is shown in Fig. 4.3.

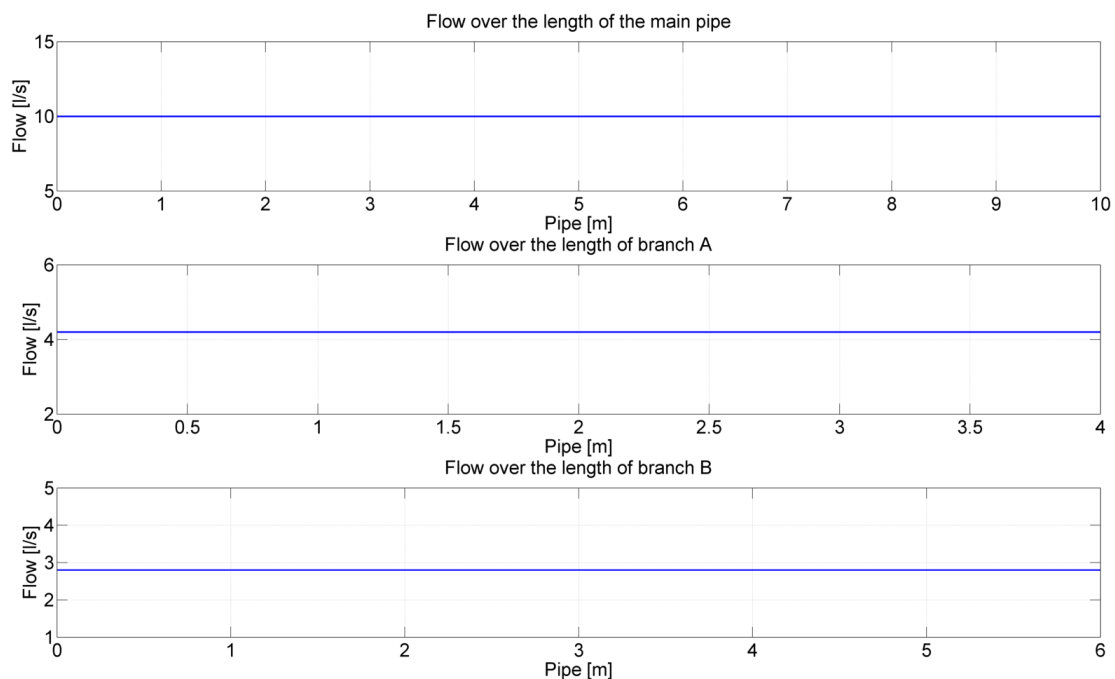


Figure 4.3: Flow and pressure in the pipeline with a branch and two outlets.

4.6 Process and Measurement Noise

Zero mean noise was added to both the input (q_{in}) and the measurements in the model. The Kalman Filter (chapter 5), which is the estimator used in this thesis, is made with the presumption that noise is present. The noise added to the input signal, q_{in} , is to model the small fluctuations one would get from a water pump, in addition to natural fluctuations in a system such as this. The measurement noise is added to simulate the uncertainty or error that is associated with head measurement. The system with noise is illustrated in Fig. 4.4.

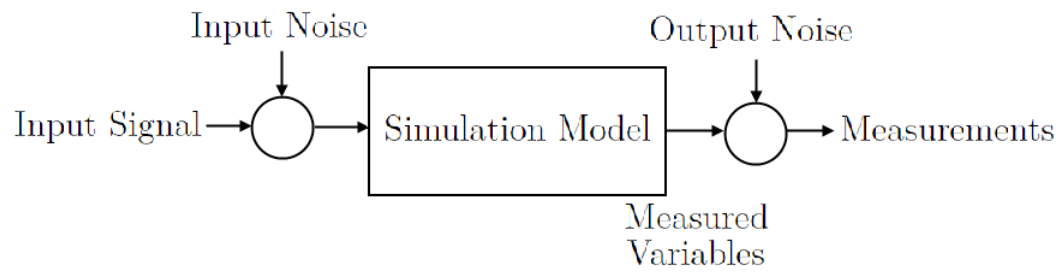


Figure 4.4: Complete model with noise added to input and measurements.

5 The Kalman Filter

The Kalman Filter, a linear quadratic estimation process, is an algorithm that uses a series of noisy measurements observed over time to produce estimates of unknown variables. It is a recursive solution to the “discrete-data linear filtering problem”. The filter is a predictor-corrector algorithm, where the filter minimizes the estimated error covariance in a linear stochastic system. Stochastic systems are systems with random noises in the process itself and in the measurements. Process noise is associated with the system and its states, while measurement noise comes from sensors and the instrumentation used to monitor the process. The Kalman Filter works in a way that makes it capable of handling systems with a high degree of data uncertainty; which makes it a good candidate for leak detection in pipelines.

The complete dissertation of The Kalman Filter is outside the scope of this thesis, but the main elements are mentioned below. More information regarding the Kalman Filter can be found in numerous books and articles (e.g. [16, 3]).

5.1 Breakdown of The Kalman Filter

There are two phases of the Kalman Filter, the prediction and the updated prediction of the estimated values. They are often referred to as the *a priori* and *a posteriori* state estimates.

Predict

Predicted (a priori) state estimate

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{F}_k \hat{\mathbf{x}}_{k-1|k-1} + \mathbf{B}_k \mathbf{u}_{k-1} \quad (5.1)$$

Predicted (a priori) estimate covariance

$$\mathbf{P}_{k|k-1} = \mathbf{F}_k \mathbf{P}_{k-1|k-1} \mathbf{F}_k^T + \mathbf{Q}_k \quad (5.2)$$

Update

Innovation or measurement residual

$$\tilde{\mathbf{y}}_k = \mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_{k|k-1} \quad (5.3)$$

Innovation (or residual) covariance

$$\mathbf{S}_k = \mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^T + \mathbf{R}_k \quad (5.4)$$

Optimal Kalman gain

$$\mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k^T \mathbf{S}_k^{-1} \quad (5.5)$$

Updated (a posteriori) state estimate

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \tilde{\mathbf{y}}_k \quad (5.6)$$

Updated (a posteriori) estimate covariance

$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k|k-1} \quad (5.7)$$

$\hat{\mathbf{x}}_{n|m}$ is the *a priori* estimate of \mathbf{x} at time n given observations up to, and including time m . $\hat{\mathbf{x}}_{k|k}$ is the *a posteriori* state estimate at time k given observations up to and including time k . $\mathbf{P}_{k|k}$ represents the *a posteriori* error covariance matrix which is a measure of the estimated accuracy of the state estimate. The way the filter equations are presented means that no history (except the previous timestep) of any kind are needed to compute the estimate for the current state. \mathbf{Q} and \mathbf{R} represent the noise covariances of process noise and observation noise respectively, and are defined based on facts, observations and/or assumptions. Measurement noise is often easily defined since the precision of the measurement device or sensor are specified in its manual or the like. The process noise on the other hand is often difficult to characterize, therefore, \mathbf{Q} may serve as a tuning parameter (along with \mathbf{R} if it's difficult to characterize observation noise aswell). Satisfying results can be found through trial and error.

5.2 Implementing The Kalman Filter

The task of implementing the Kalman Filter is straightforward. The state space matrices are used in a state space block in MatLab's Simulink, y is taken from the state space block and used in the equations above (5.1-5.7). Noise is added to the simulation as shown in Fig. 4.4. With the assumptions made, the noise and its variance is known, which means that \mathbf{Q} and \mathbf{R} are easily defined. Different values of \mathbf{Q} and \mathbf{R} are tried out to see how much effect this has on the estimation process though.

The leak, with respect to the Kalman Filter, is a state to be estimated. It's assumed that the leak appears at a known location, namely the pressure node in the branch. The state vector, x , the *A-matrix*, and the *B-matrix* is changed within the filter since the leak is modelled as an input in the model itself.

6 Results

All results found, and some notes regarding these results will be presented in this chapter, while a discussion and the conclusion will be saved for chapter 7 and chapter 8. The first section is about the simple observation of the system in its steady state. Each subsequent section will deal with an incrementation until we end at this model and its Kalman Filters limit. On a side note, time will always describe one of the axes in the plots since the time it takes for the Kalman Filter to reach a good estimate is the interesting factor. Considering the tens of nodes describing the pipelines, focus will be given the average plots, and the plots showing some kind of special behaviour. The majority of plots follows the small scale model described earlier. The last part of this chapter include a bigger, more realistic model to show how the Kalman Filter and the model itself behaves on a bigger scale. The simulation of this bigger and more realistic model is extremely slow with the computer power available throughout this thesis, therefore only a small amount of tests were done to see if everything learned from the small scale model also applied to the big one.

6.1 No Leakage

In sec. 4.3 it was mentioned that the system is completely observable. Which means that the Kalman Filter should be able to estimate all states with a low error. First, in Fig. 6.1 we can see that the Kalman Filter was able to identify that the system did not have any leaks.

The estimation of flow and pressure is shown in Fig. 6.2-Fig. 6.7. Two nodes of each branch is used, one node being the basis for pressure, and one node being the basis for flow. The plots that are not shown have both smaller and bigger transients before they converge to the correct value, but they always converge before, or right after the 100 second mark.

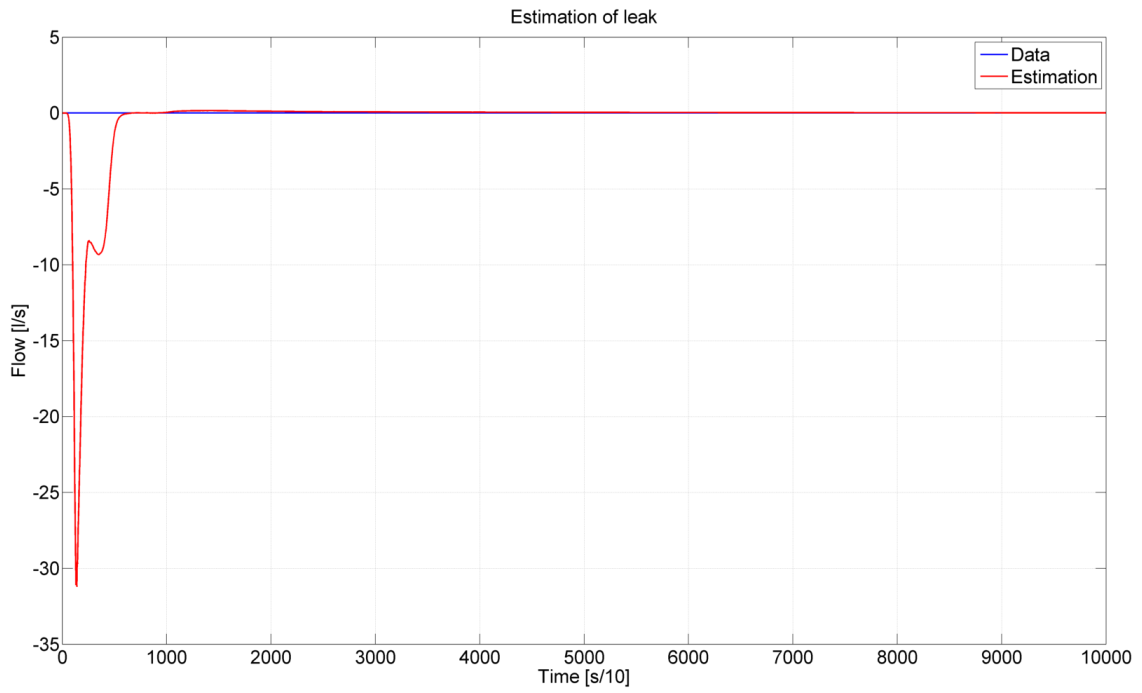


Figure 6.1: Estimation of leakage with $0l/s$ leak added.

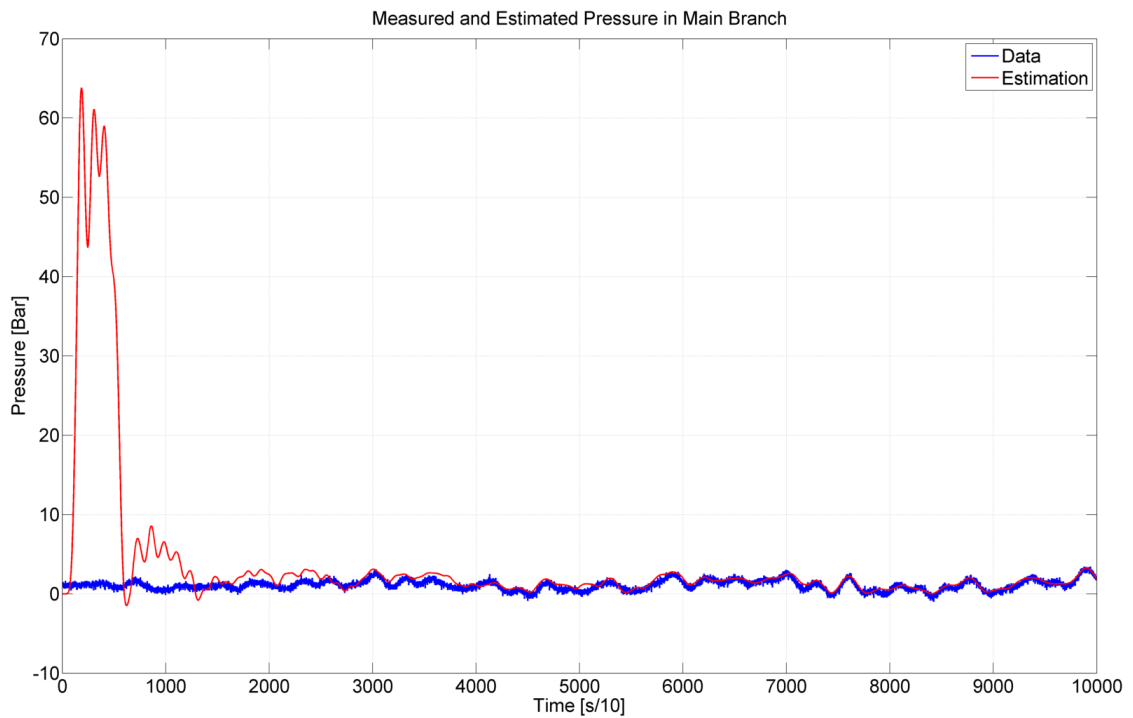


Figure 6.2: Measured and estimated pressure in an average node in the main branch. Zero leakage.

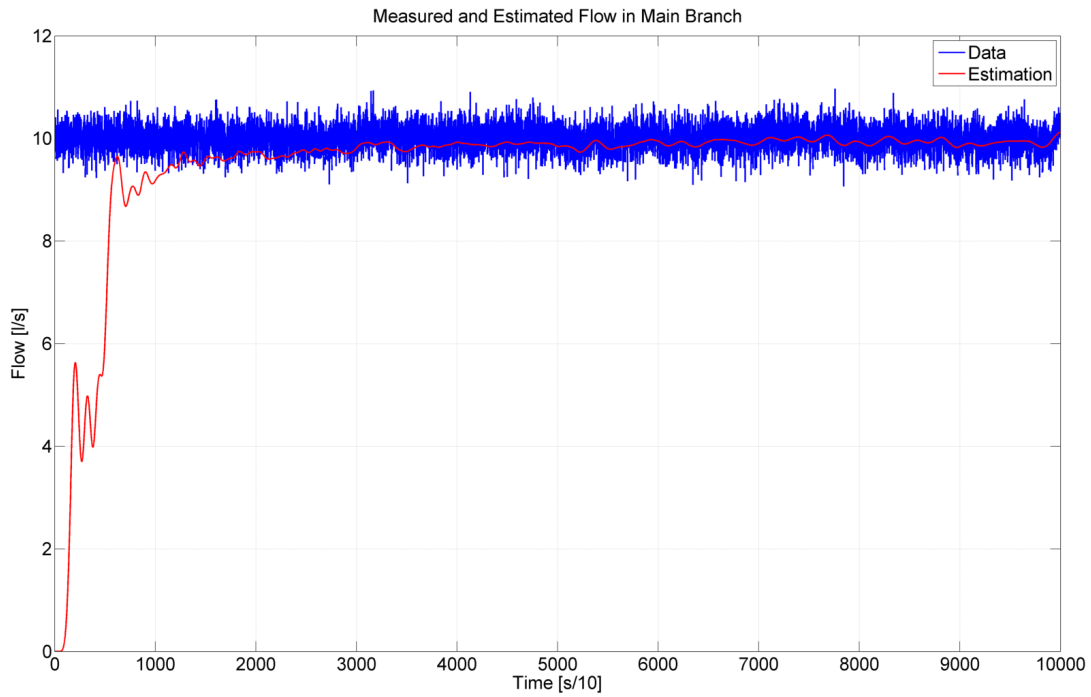


Figure 6.3: Measured and estimated flow in an average node in the main branch. Zero leakage.

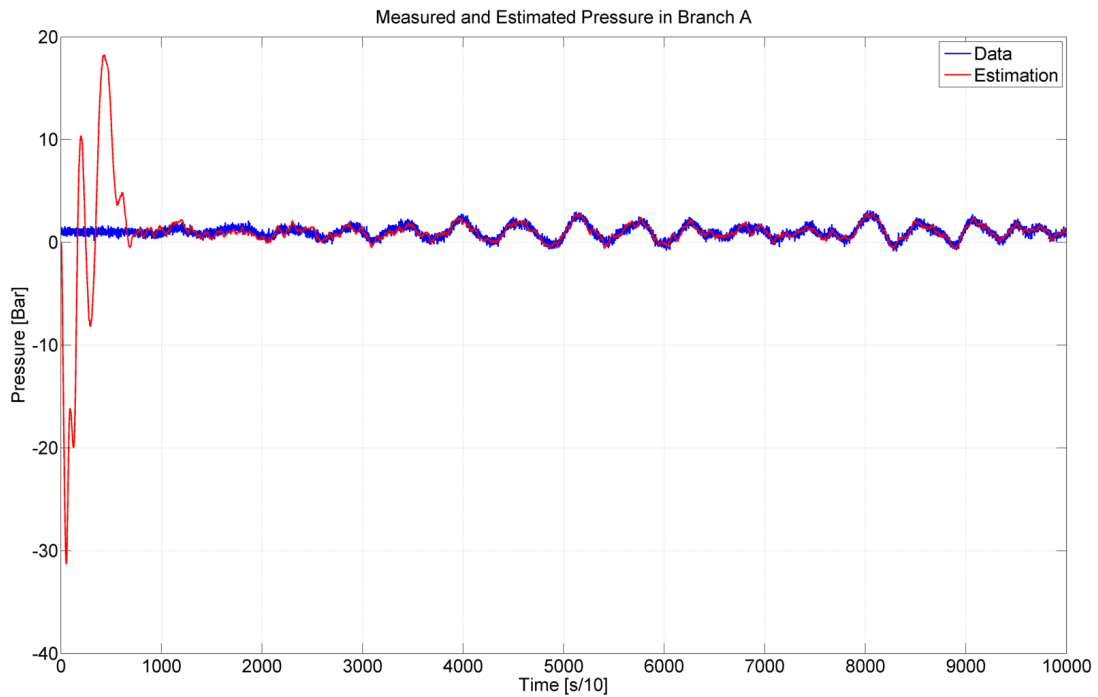


Figure 6.4: Measured and estimated pressure in an average node in branch A. Zero leakage.

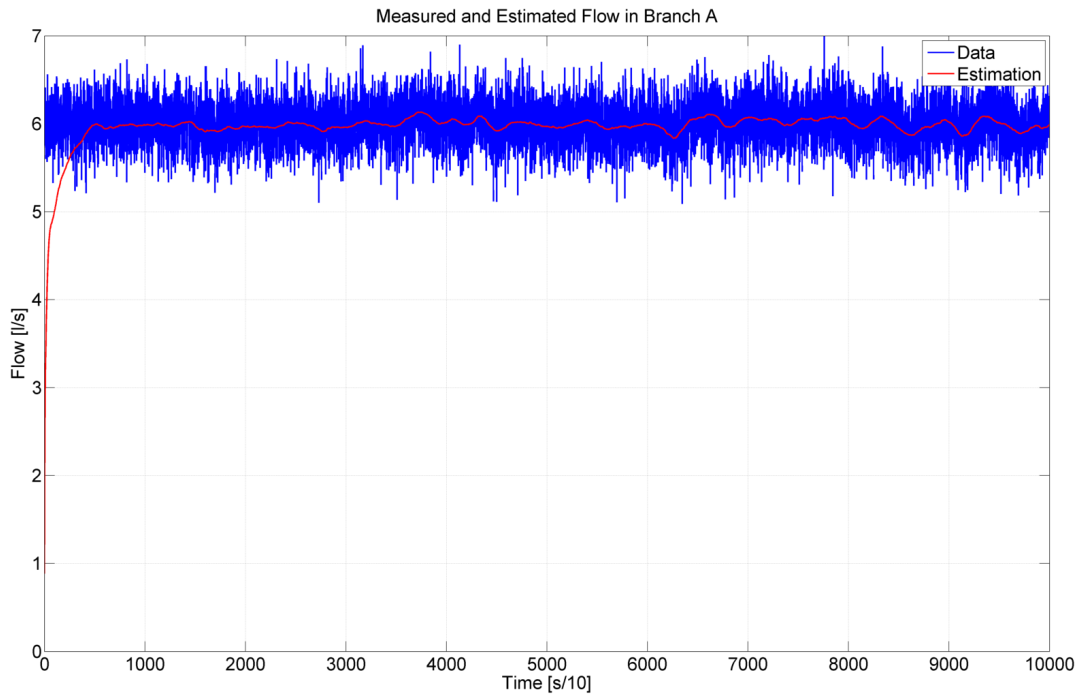


Figure 6.5: Measured and estimated flow in an average node in branch A. Zero leakage.

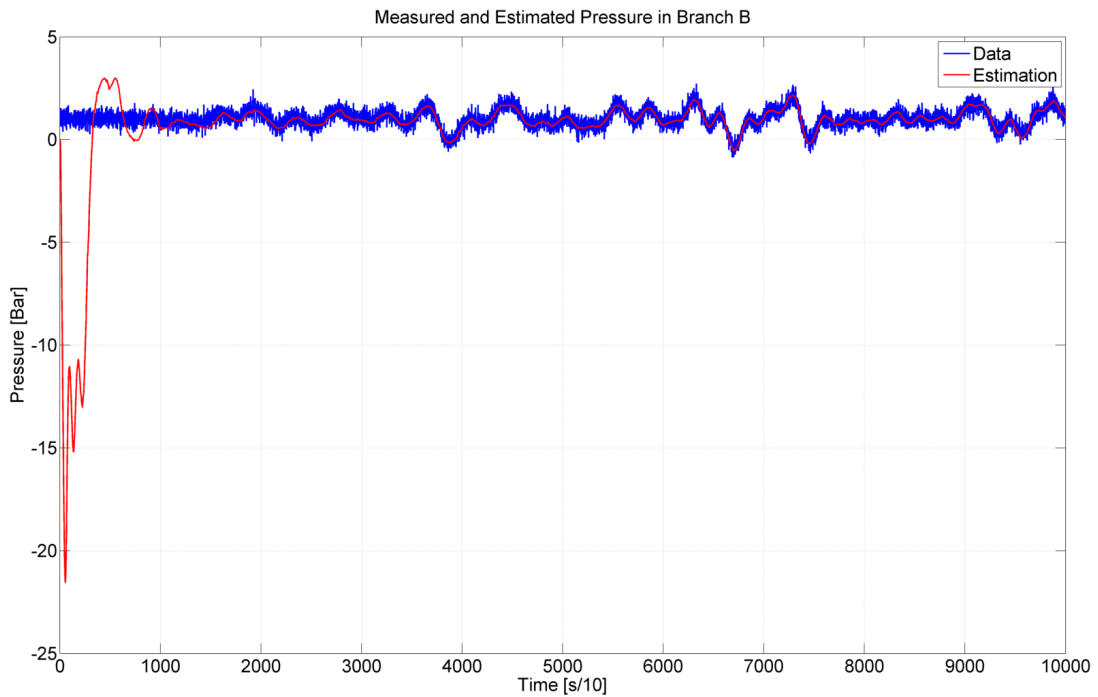


Figure 6.6: Measured and estimated pressure in an average node in branch B. Zero leakage.

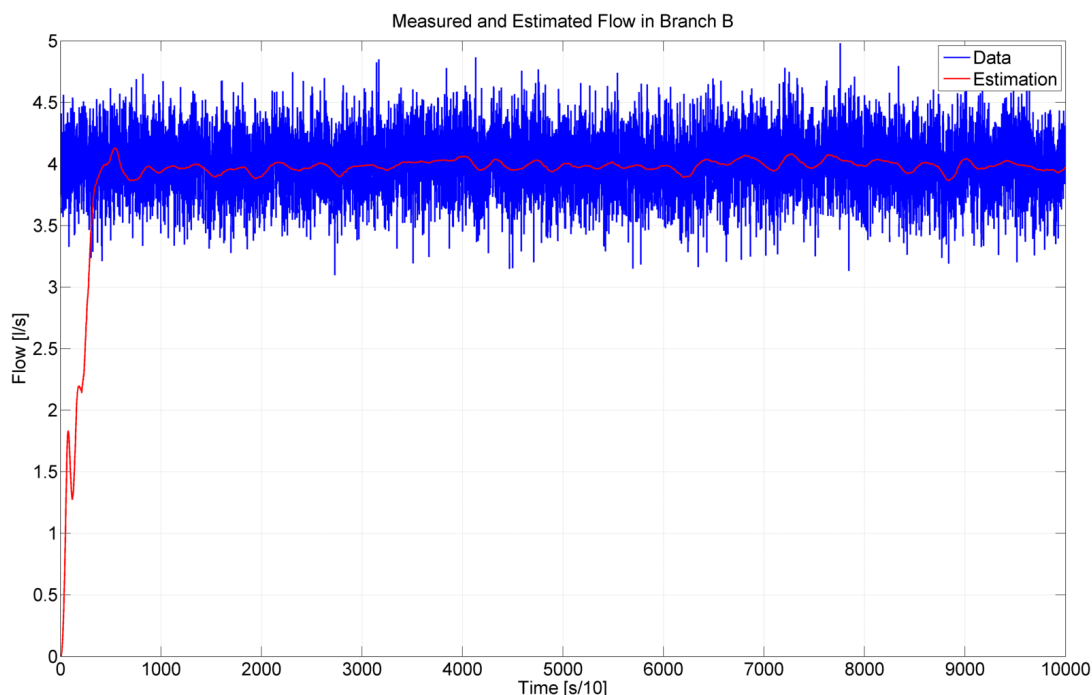


Figure 6.7: Measured and estimated flow in an average node in branch B. Zero leakage.

Without leakage, the system is completely observable, without any unknown factors. Nothing unexpected happened with the estimation. The time it took for the estimation to converge towards the real values is approximately 100s.

6.2 Leakage at Known Position

A leak is applied to the pressure node in the branch itself. It is applied after a set amount of time to let the estimated values reach a stationary value before something new happens. This is to see if the Kalman Filter noticeably reacts to the change. Applying the leakage causes jumps in pressure and changes the flow. With the low friction factor initially used, this causes the system to oscillate for a long time before it reaches steady state. To counter this, the friction factor is increased, which dampens the system, which in turn leads to an increase in pressure to be able to drive the flow.

Restrictions in computer power made it very time consuming to simulate the results, so it is stopped at 10000s. This is long enough to see that the estimated values either converges towards, or reaches the actual values. As mentioned earlier, the Kalman Filter is designed to know where the leak is.

The interesting plots is shown in the figures below (Fig. 6.8-Fig. 6.12).

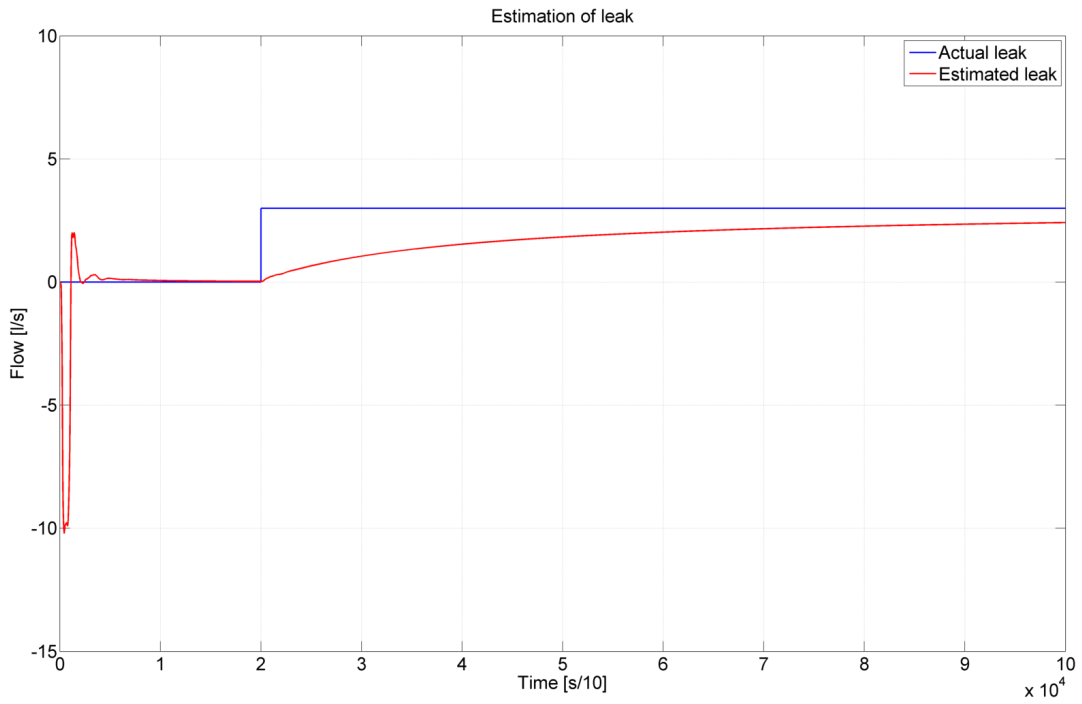


Figure 6.8: The applied and estimated magnitude of the leak.

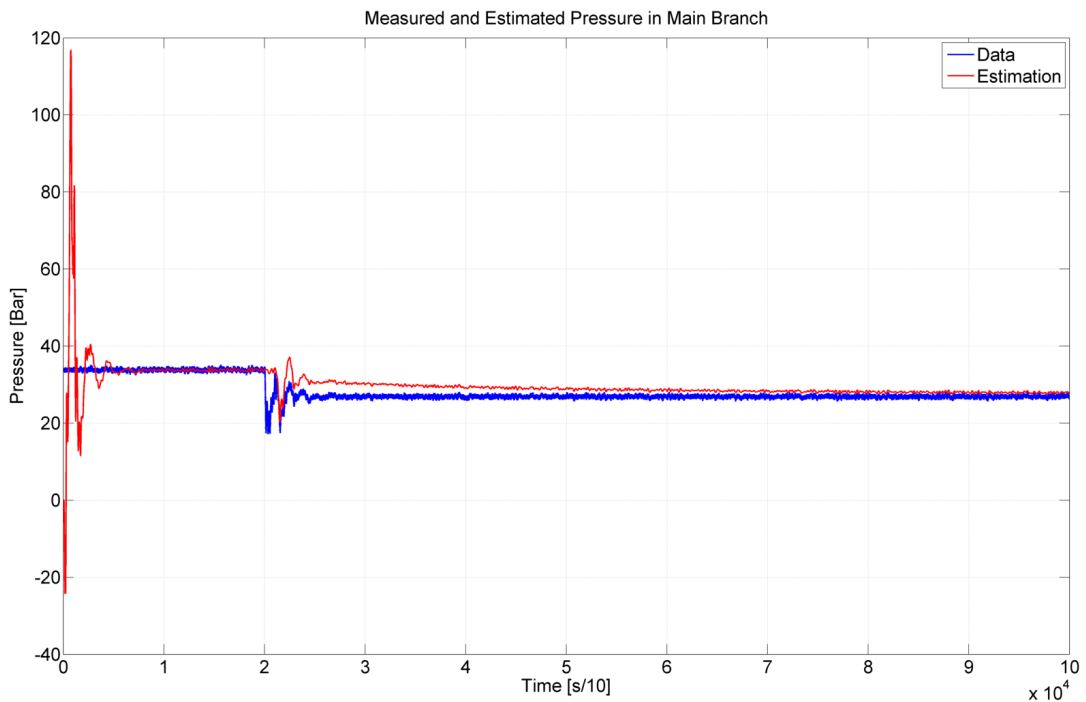


Figure 6.9: Measured and estimated pressure in the main branch, close to the branch itself where the leak is located.

6.2 Leakage at Known Position

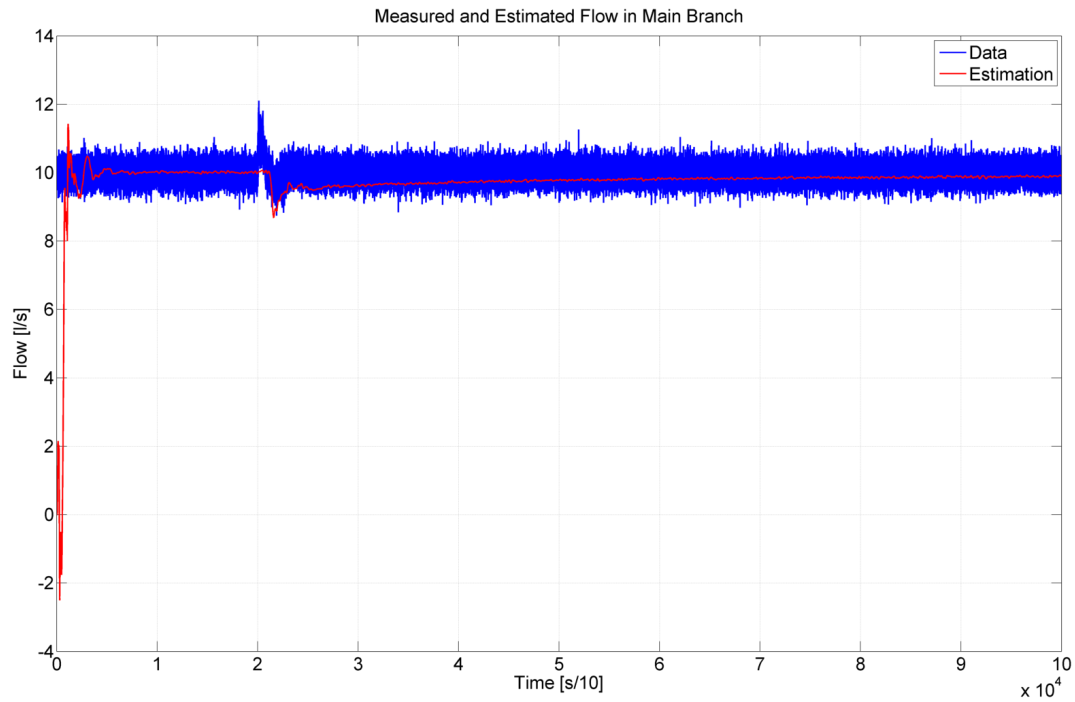


Figure 6.10: Measured and estimated flow in the main branch, close to the branch itself where the leak is located.

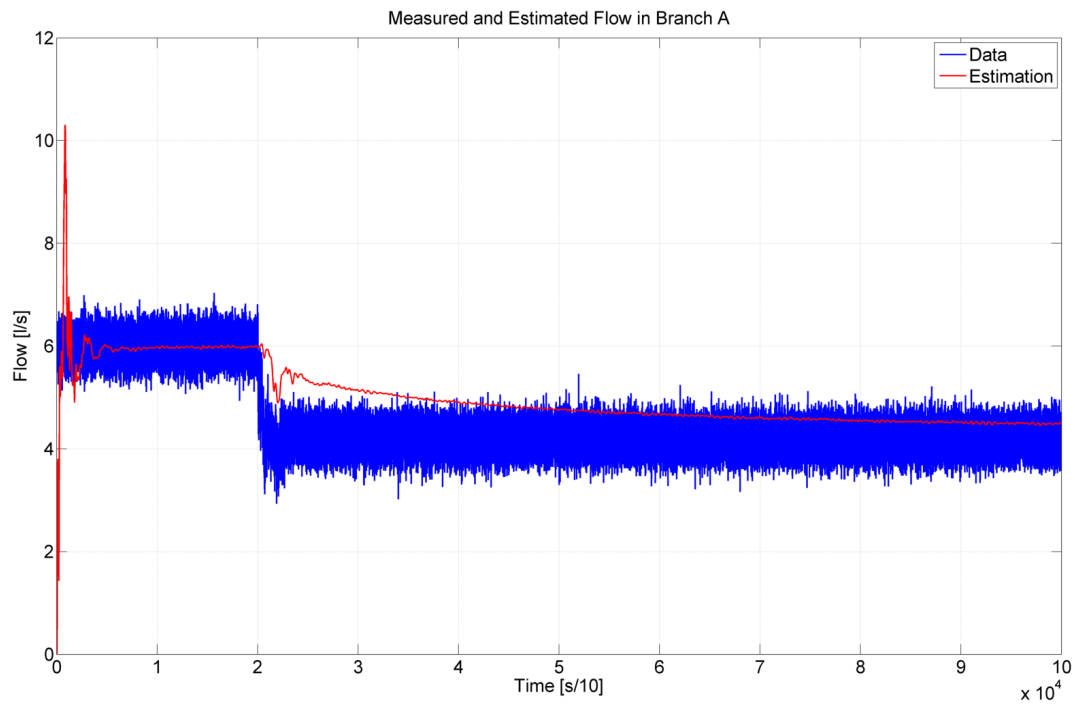


Figure 6.11: Measured and estimated flow in branch A, close to the branch itself where the leak is located.

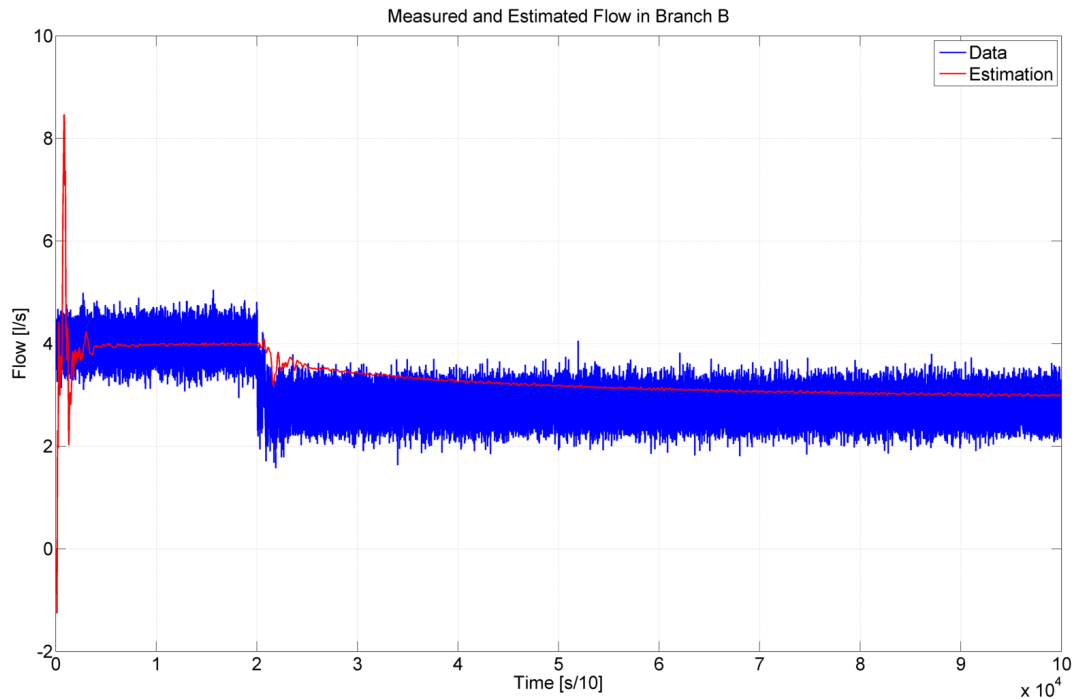


Figure 6.12: Measured and estimated flow in branch B, close to the branch itself where the leak is located.

The leak is applied at $t = 2000s$, after which the estimated leak converges slowly towards a value very close to the leak, $3l/s$. Regarding the plots not shown: the closer we get to the position of the leak, the slower the Kalman Filter is to estimate the true values. Only the two nodes closest to the branch (in all three pipes going out of said branch) where the leak is, shows this behaviour. At node three and out, in all directions, the estimated value reaches the true value within a short amount of time (this applies to both flow and pressure).

6.3 Leakage at Unknown Position

This test is to check the limits of the Kalman Filter. To see how it reacts to a leak that is in a different location than where it is supposed to be. Three tests will be executed, a leak in the main branch, branch A, and branch B. The Kalman Filter, like before, is designed to only expect a leak in the branch.

6.3.1 Leak in Main Branch

For this test, the $3l/s$ leak is applied to pressure node number 5 in the main pipe. The figures below shows the estimation of flow in the pipes.

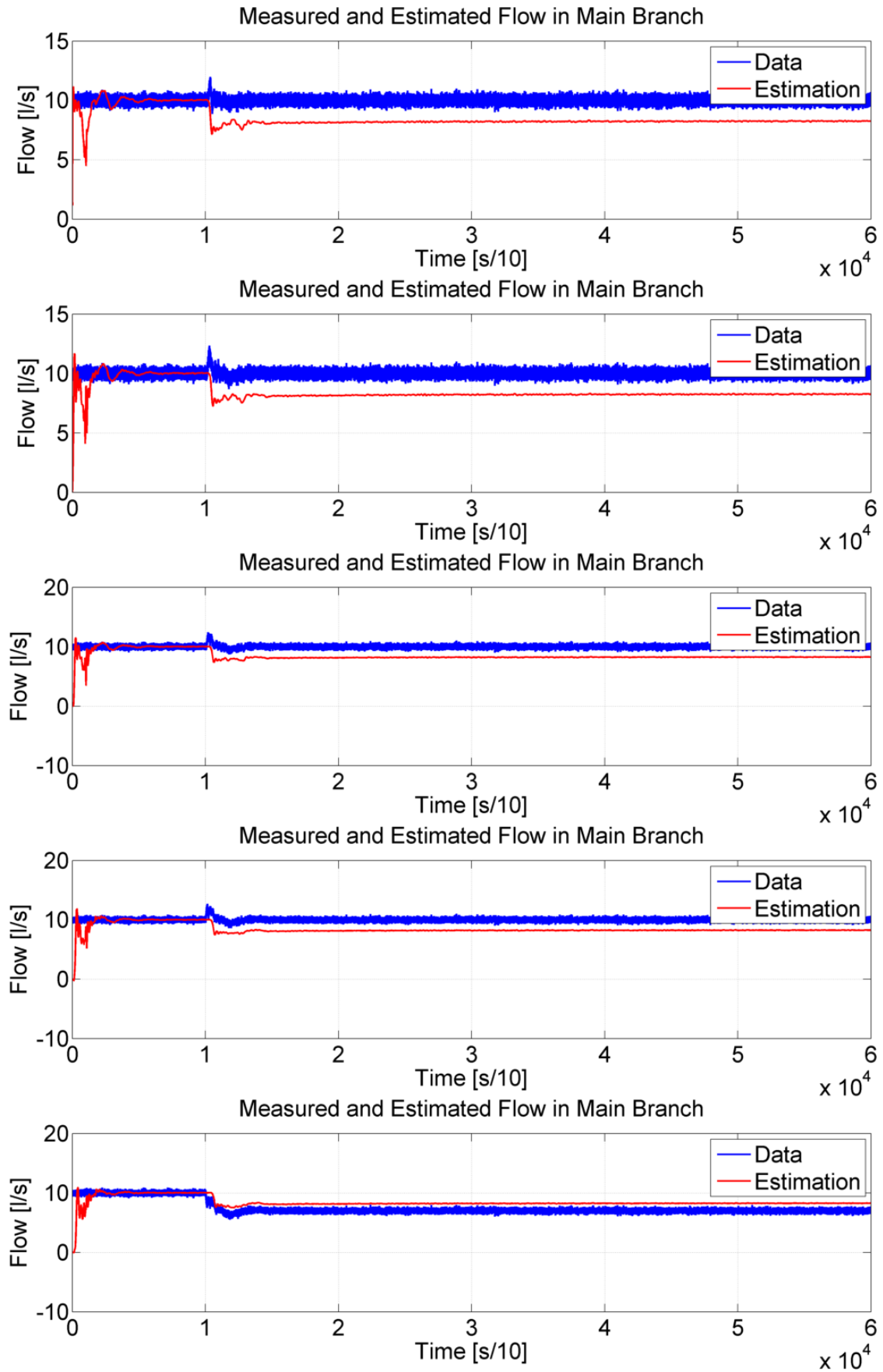
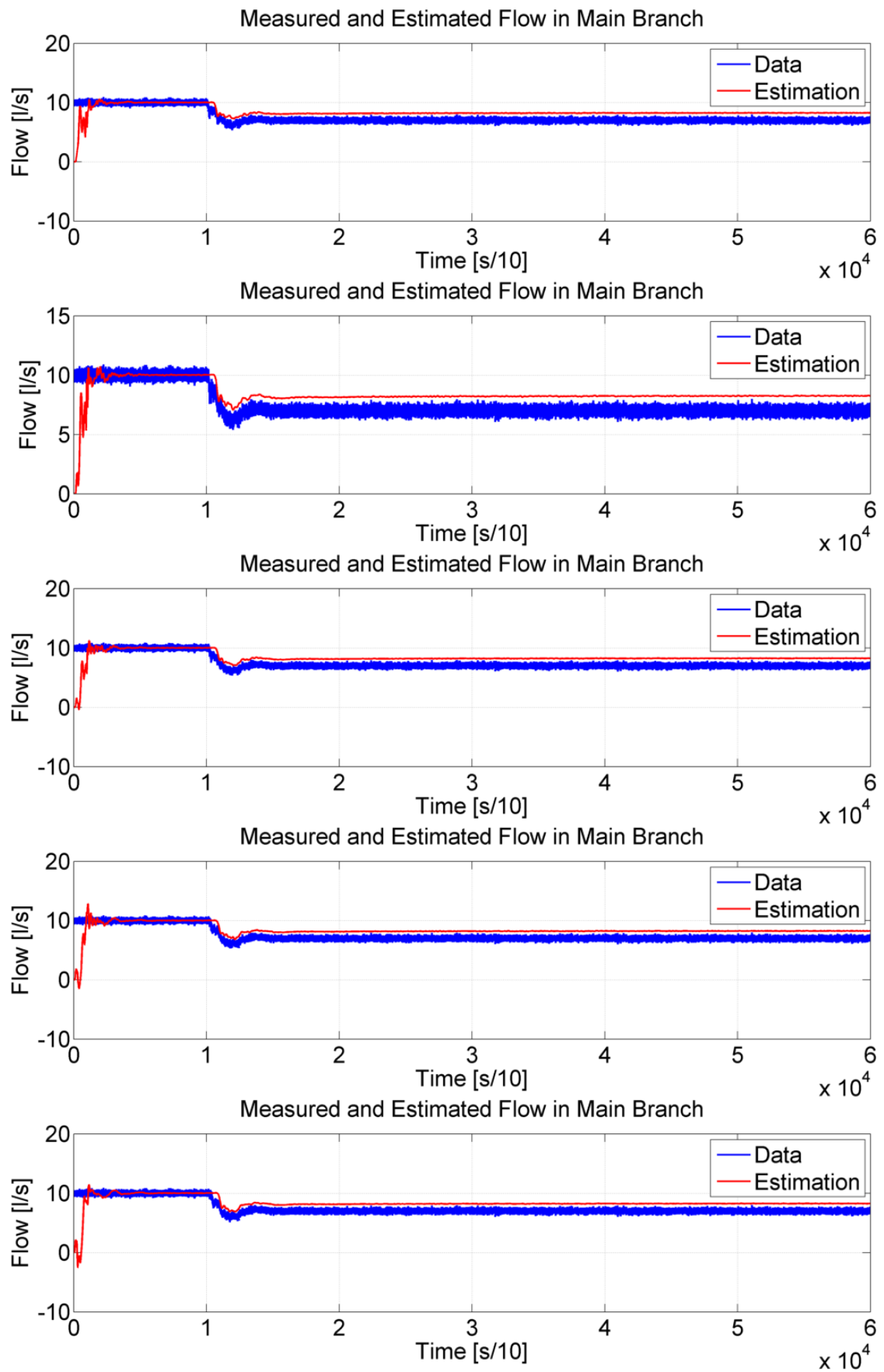


Figure 6.13: Measured and estimated flow in nodes 1 – 5 in main branch. 33



34 **Figure 6.14:** Measured and estimated flow in nodes 6 – 10 in main branch.

6.3 Leakage at Unknown Position

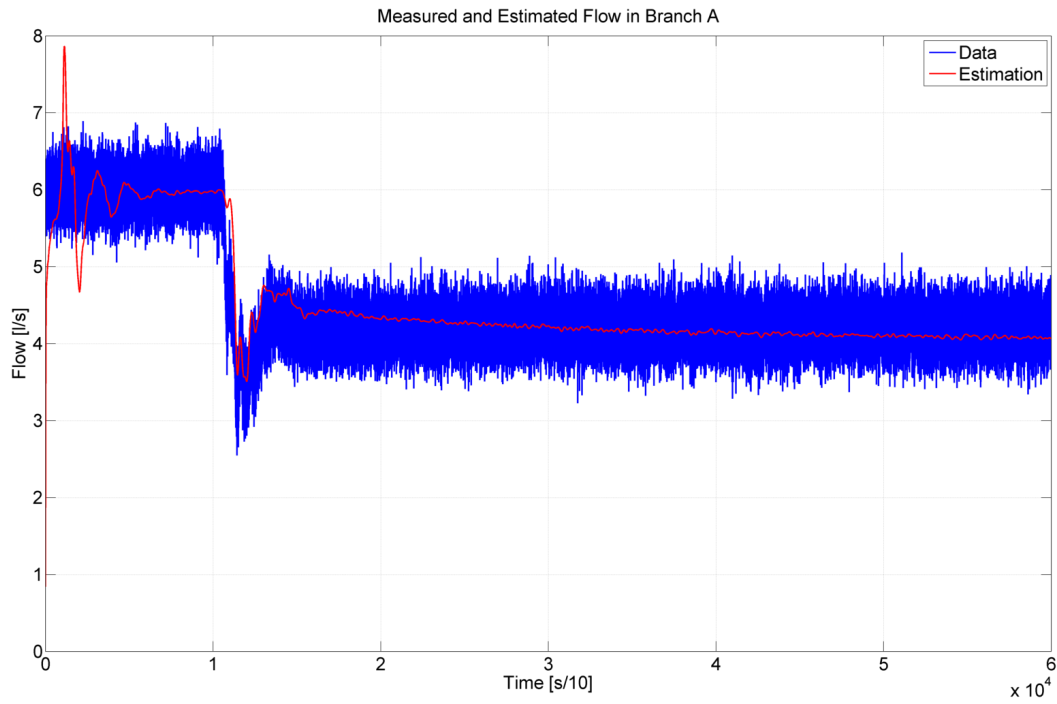


Figure 6.15: Measured and estimated flow in branch A with leak in main pipe.

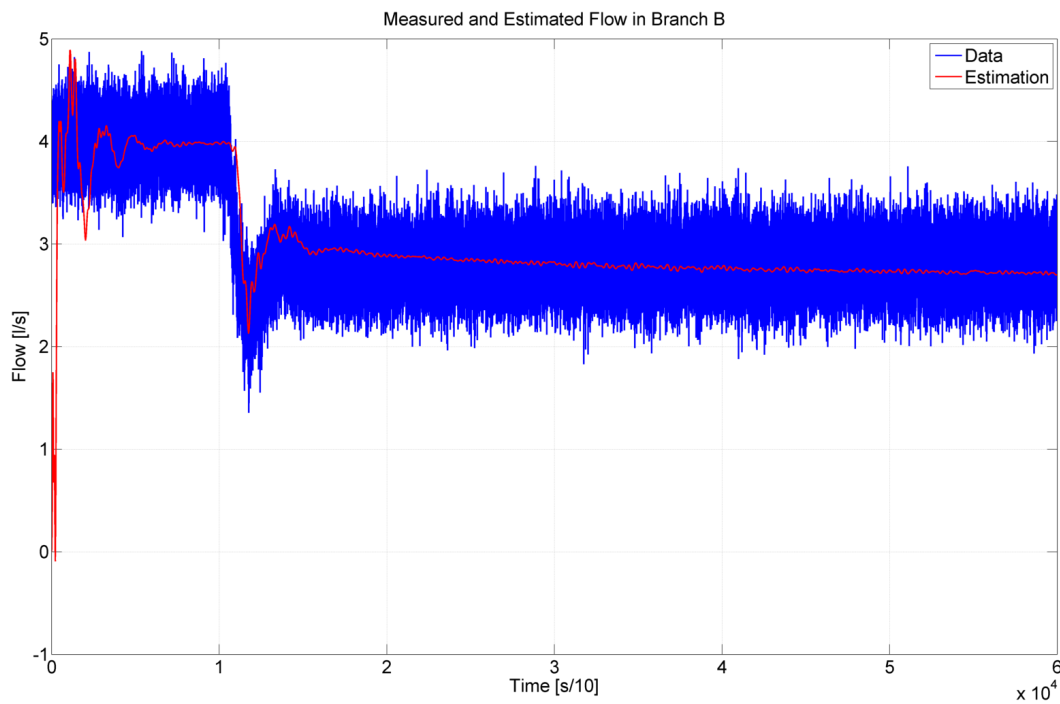


Figure 6.16: Measured and estimated flow in branch B with leak in main pipe.

The plots showing flow in the main pipe (where the leak occurs) reveals that the

Kalman Filter is not able to estimate the flow if a leak occurs at a different position than it should have. The estimation has a constant deviation, though it seems like this deviation is smaller in the nodes *after* and *including* the leak. With the numbers used in this test, the deviation in the nodes *before* the leak is approximately 1.8 l/s (18.0% deviation), while it is approximately 1.2 l/s (17.1% deviation) in the nodes at and *after* the leak.

The plots showing estimation of flow in branch A and B display some of the same characteristics as Fig. 6.11 and Fig. 6.12. The estimation is still slow, but it is also apparent that the estimations will settle at values which gives a small deviation compared to the true value. All nodes in the branches showed the exact same behaviour.

The leak estimation settles at 1.5 l/s , while the true leak is 3 l/s . Like with the other estimations, a constant deviation occurs.

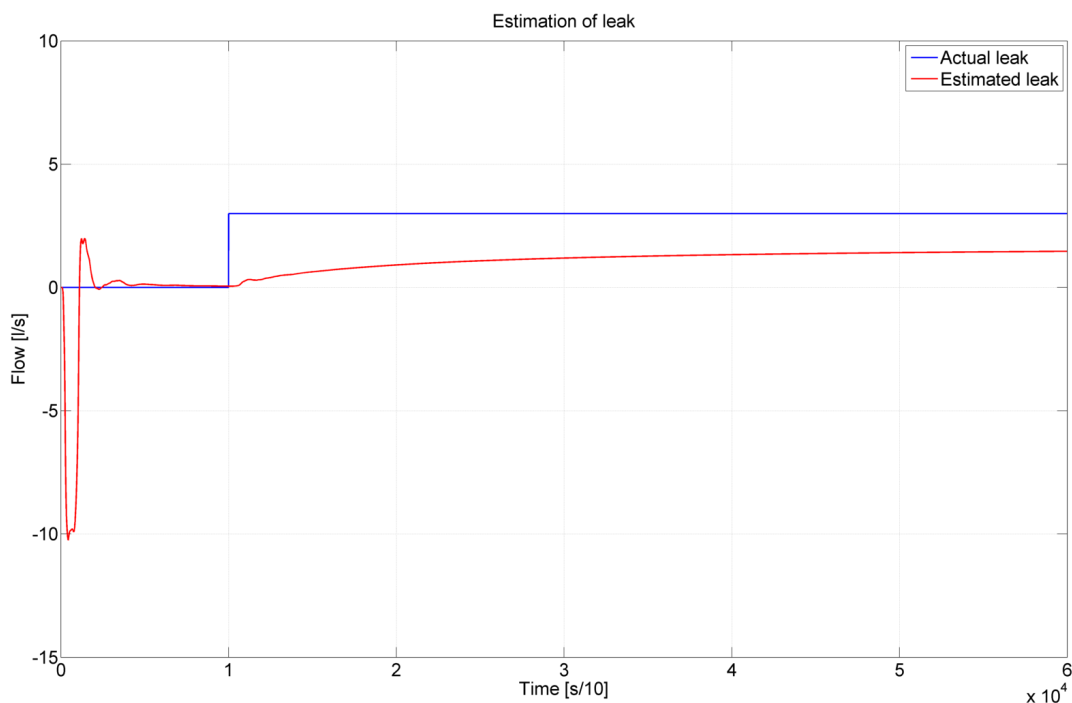


Figure 6.17: Actual and estimated leak.

6.3.2 Leak in Branch A

The leak is applied to pressure node number 3 in branch A (which has 4 pressure nodes including the node in the branch). Some of the problems that occurred in the last section also occur here. There is a deviation in the estimated flow in the main branch and in branch B. In branch A, where the leak is applied, we have the exact same characteristics as in Fig. 6.13 and Fig. 6.14: the nodes ahead of the leak

have a bigger estimation deviation than the nodes at, and after the leak. The most interesting result, is that the leak itself is estimated with a close-to-zero error, even better than the estimation where the leak is at the supposed position with respect to the Kalman Filter.

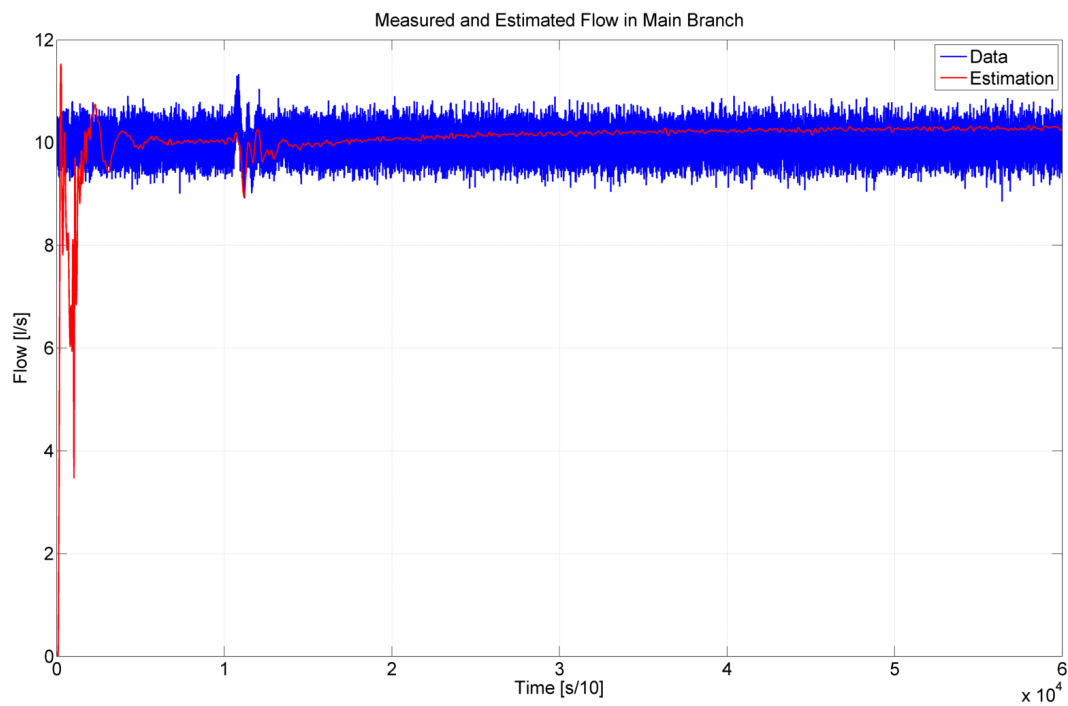


Figure 6.18: Measured and estimated flow in the main branch with leak in branch A.

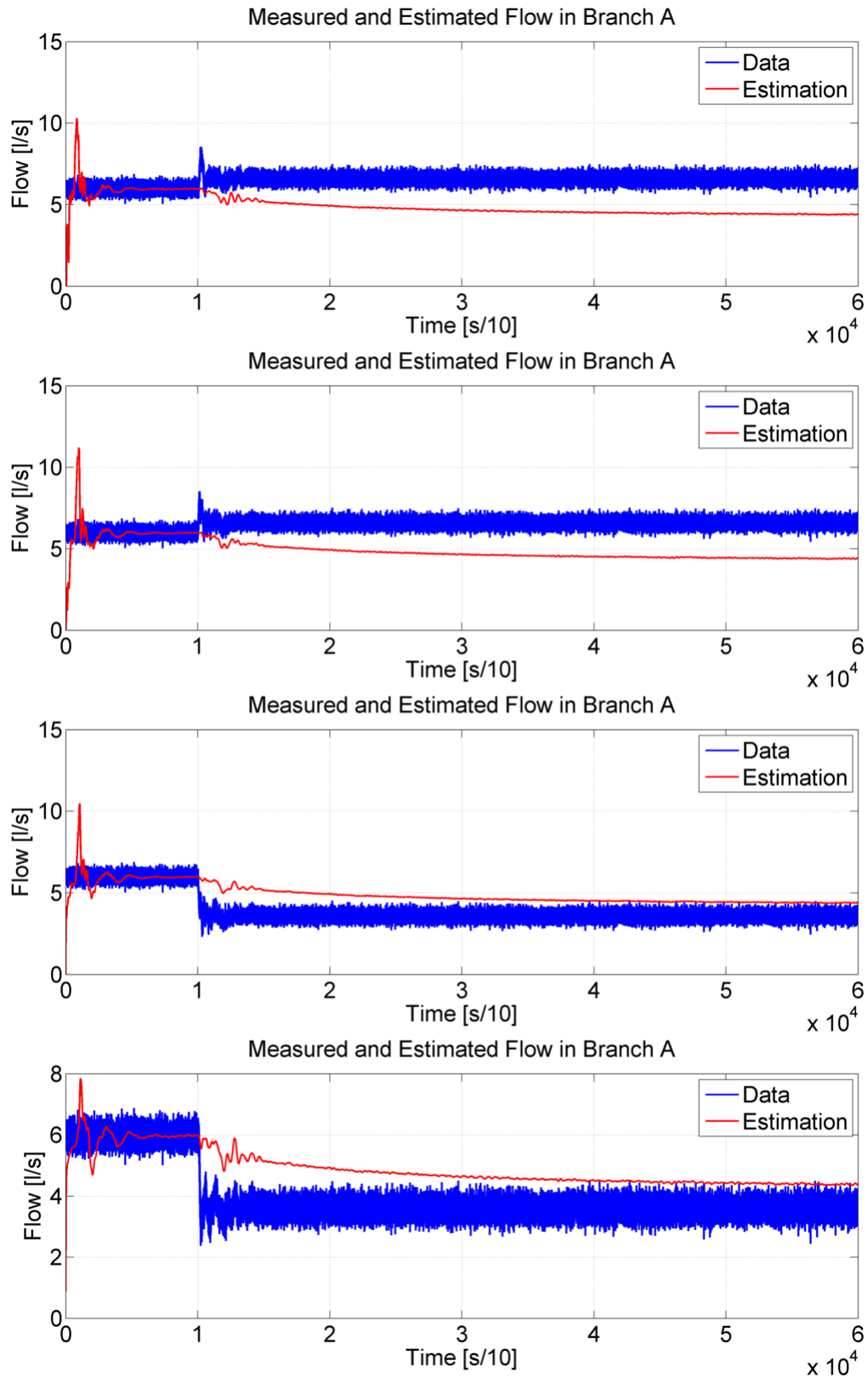


Figure 6.19: Measured and estimated flow in branch A with leak in branch A.

6.3 Leakage at Unknown Position

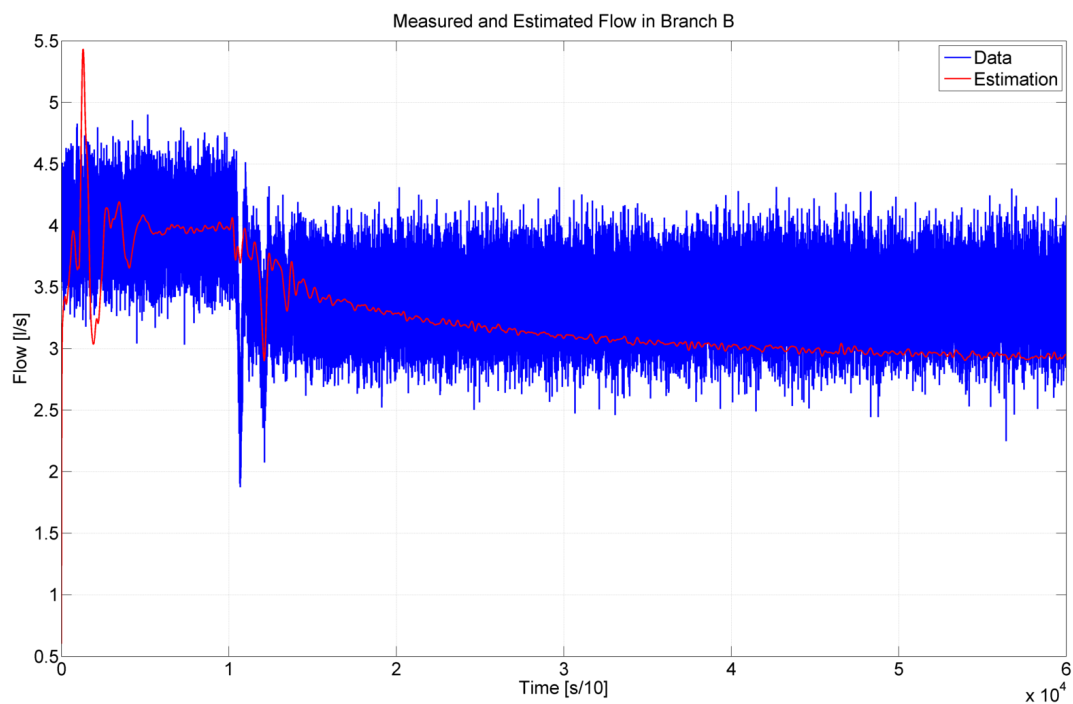


Figure 6.20: Measured and estimated flow in branch B with leak in branch A.

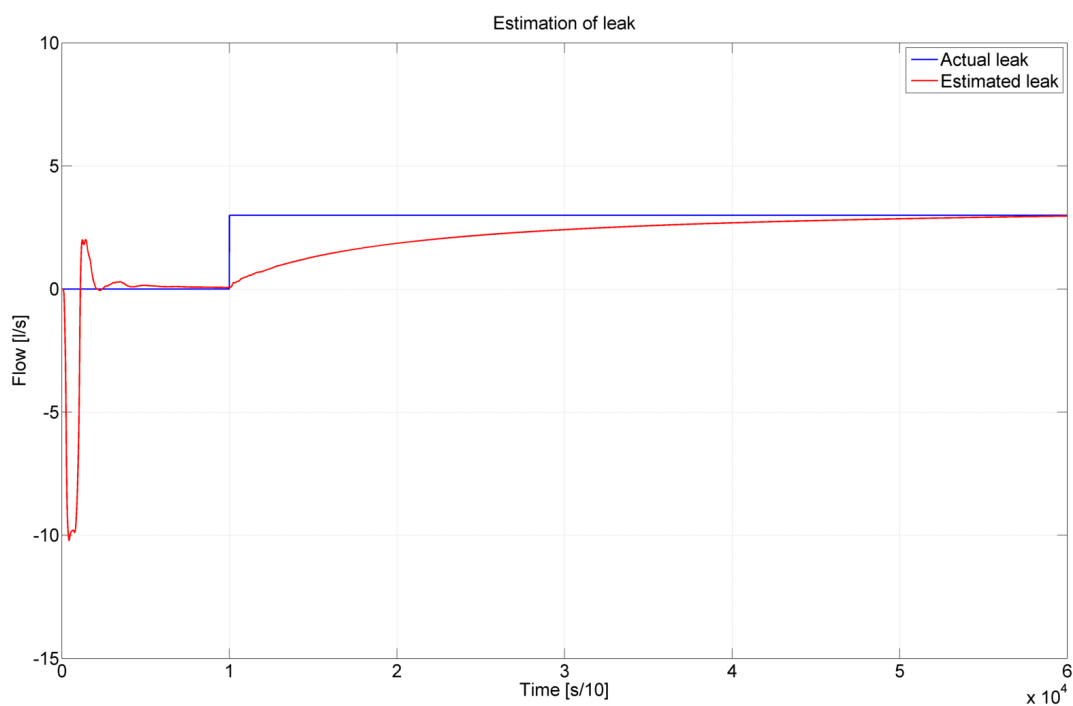


Figure 6.21: Actual and estimated leak with the leak being in branch A.

6.3.3 Leak in Branch B

The results here are similar to the ones in sec. 6.3.2. The difference is that the leak estimation does not reach the actual value of the leak.

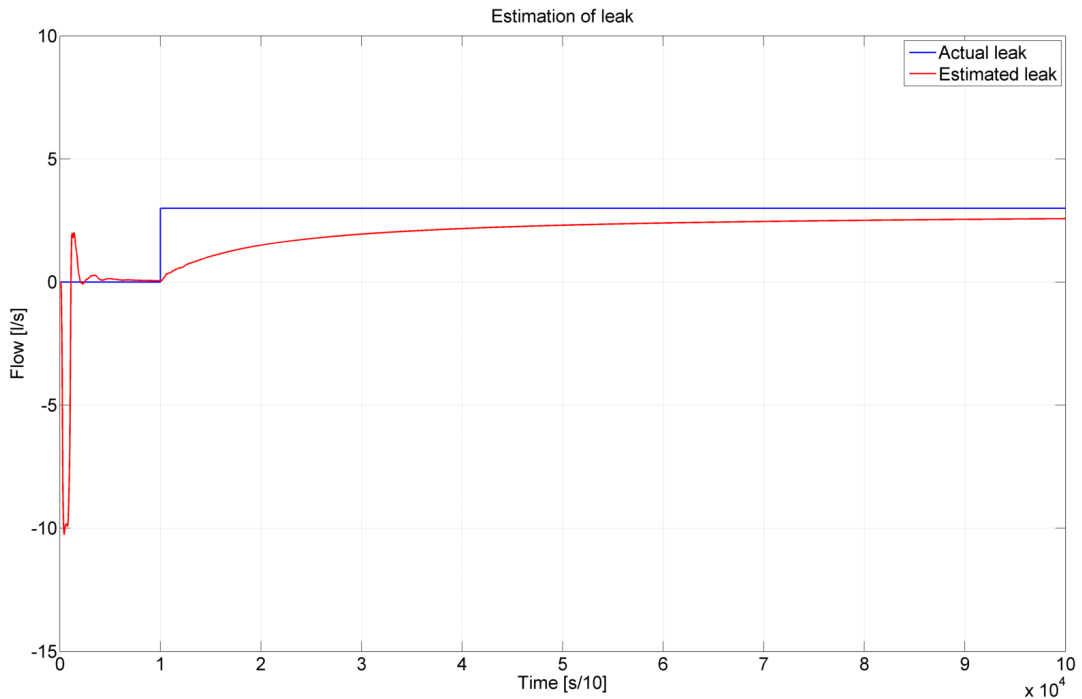


Figure 6.22: Actual and estimated leak with the leak being in branch B.

6.3.4 Time Varying Boundary Conditions

To see how the Kalman Filter reacts to change in boundary conditions, a time varying flow was applied to the system along with the leak. The flow into the system is a sine wave with a bias of 10, an amplitude of 5 and a frequency of 0.01. The leak is set to be 10% of the applied flow. As in sec. 6.2, all state estimations are satisfactory with close-to-zero error. The leak estimation becomes an average. See Fig. 6.23. With a lower frequency sine wave, the estimation tries to follow the leak, but lags behind.

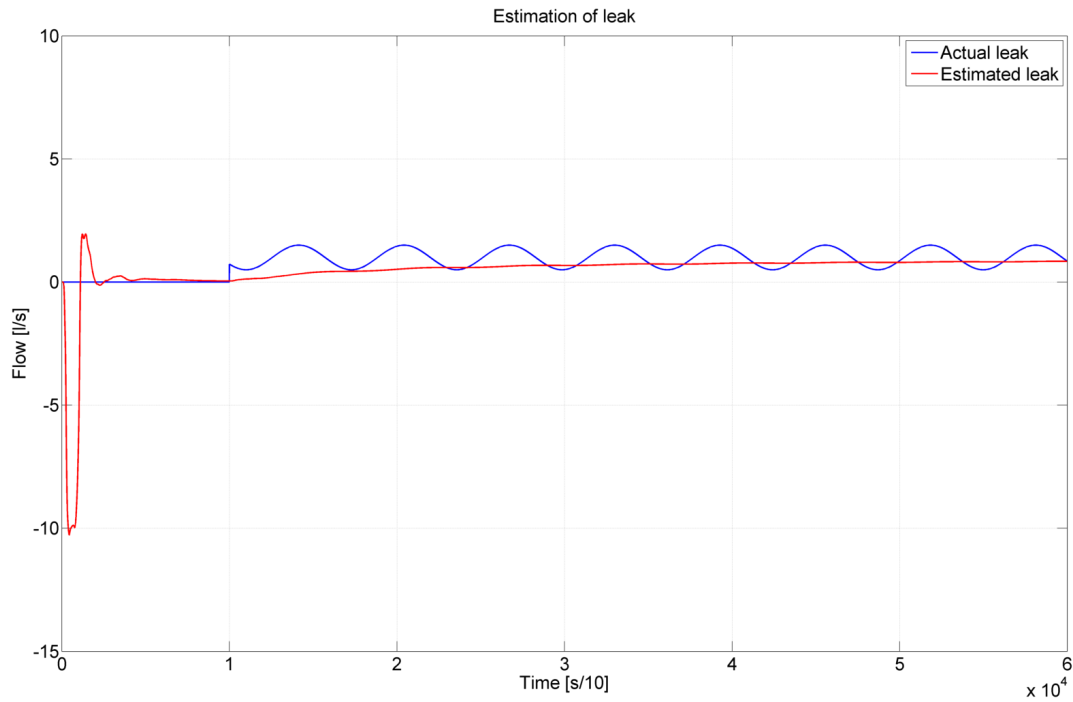


Figure 6.23: Actual and estimated leak with time varying q_{in} and leak.

6.4 Larger Model

First, let's start with the specifications:

- Main branch: 1000m divided into 50 nodes
- Branch A: 40m divided into 4 nodes
- Branch B: 60m divided into 6 nodes
- $q_{in} = 750l/s$
- $A = 0.785m^2$

6.4.1 Zero Leak

As with the smaller model, plots of the estimations with no leak are shown first. Fig. 6.24 shows that the closer we get to the measurements at the extrema, the faster the estimation starts. The 2nd plot in the figure shows a long period where the estimation does not do anything, while there is a short delay in the third plot.

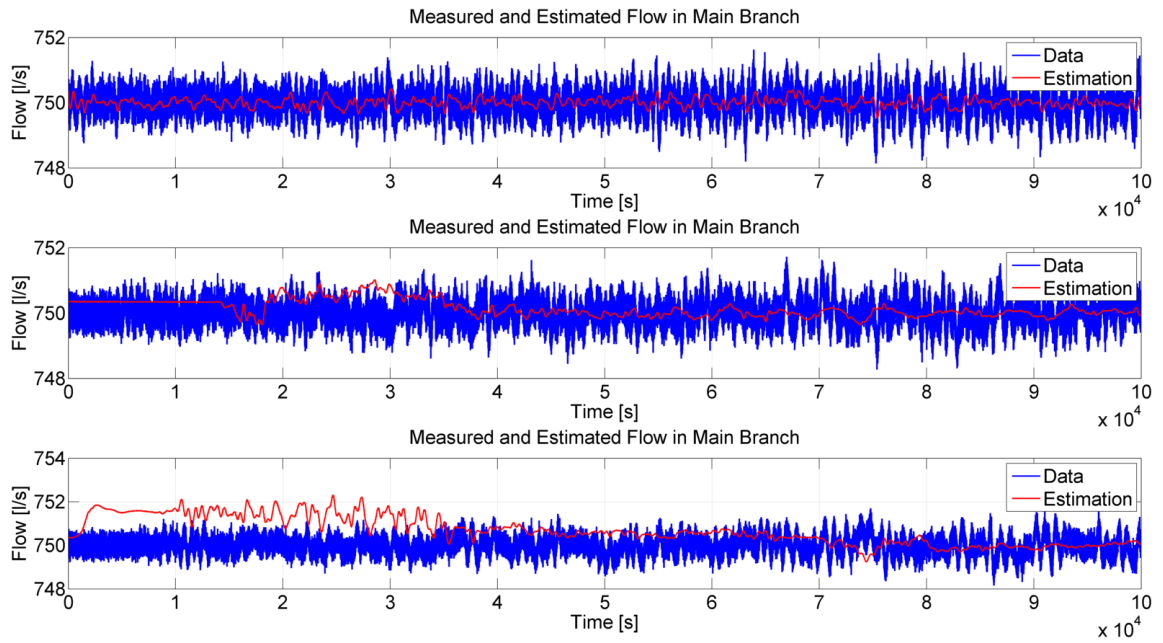


Figure 6.24: Measured and estimated flow in the first, center and last node of the main branch.

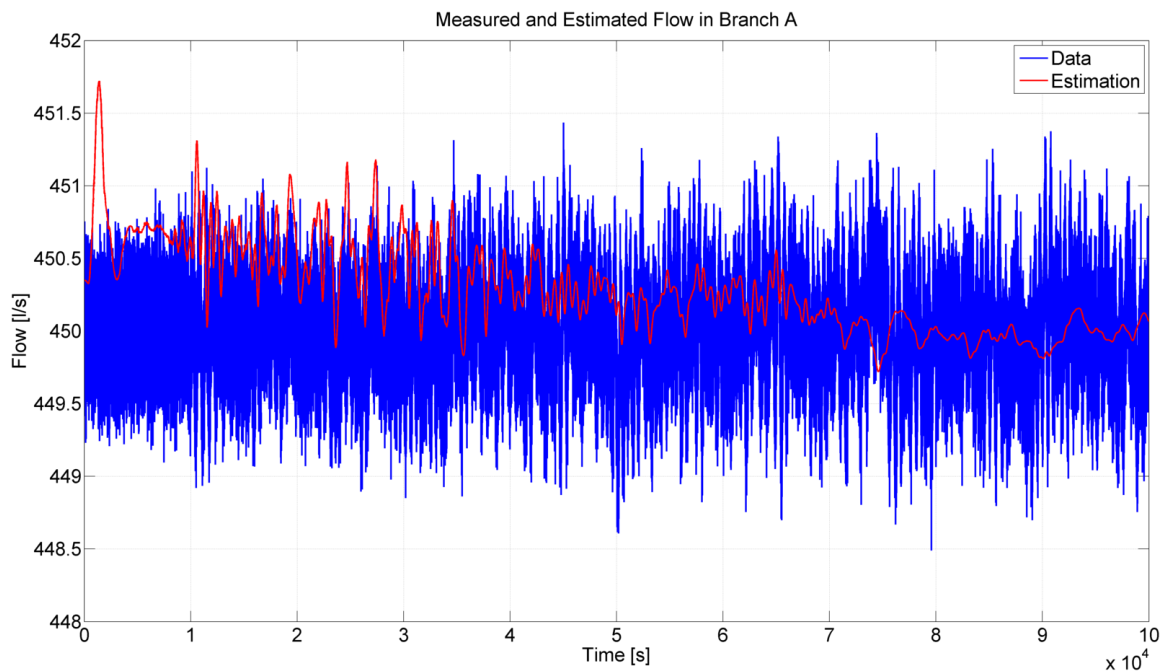


Figure 6.25: Measured and estimated flow in branch A.

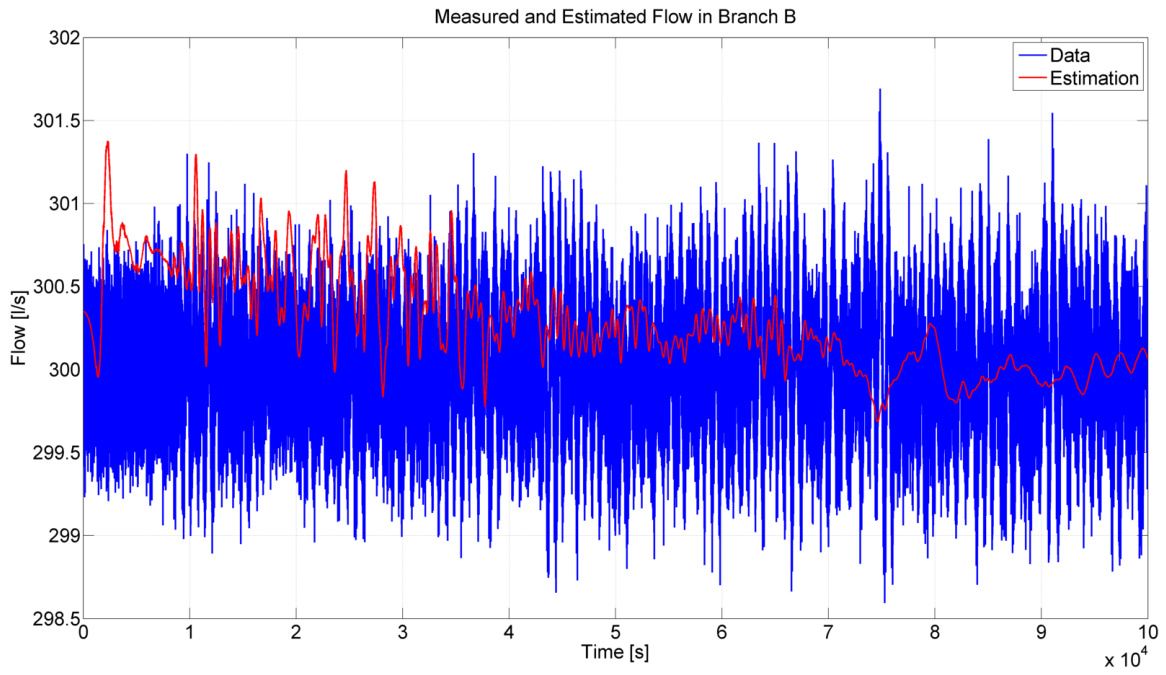


Figure 6.26: Measured and estimated flow in branch B.

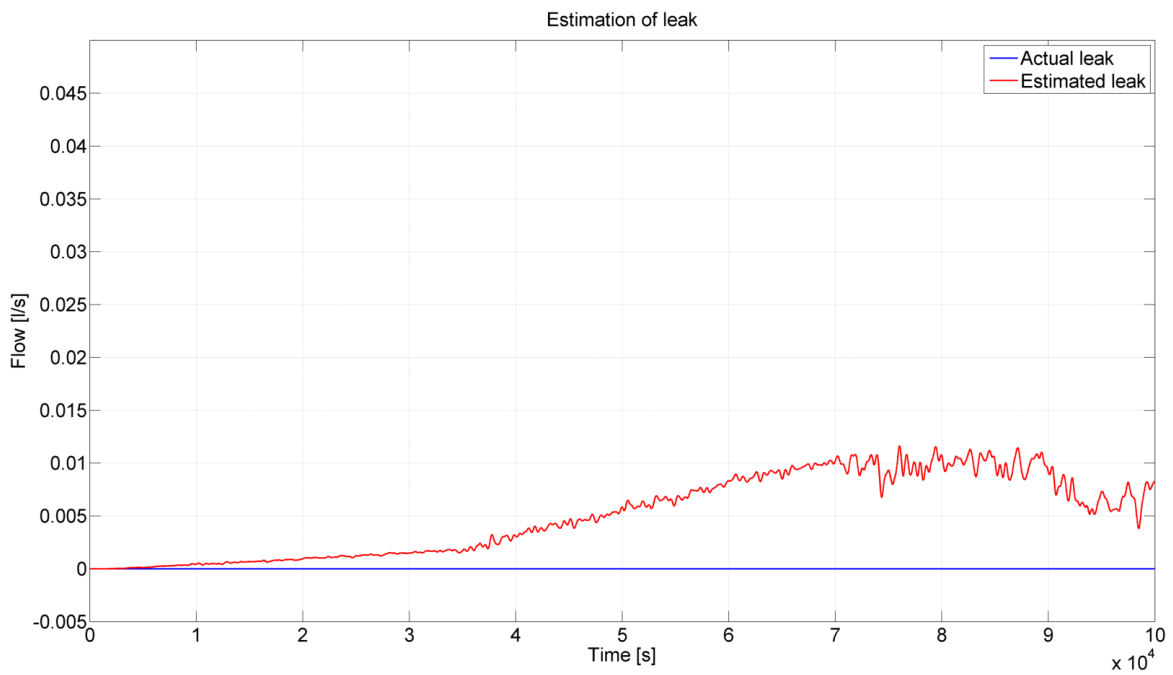


Figure 6.27: Measured and estimated flow in branch B.

Figures Fig. 6.25-Fig. 6.27 shows that, with the much higher q_{in} and the longer pipes, we have slower estimations, and thus more uncertainty.

6.4.2 One Percent Leak

A one percent leak was added to the branch node (where the leak is expected). As expected, the response is much slower as it takes some time for the pressure waves caused by the leak to reach the extrema where the sensors are located. Fig. 6.28 shows that, even at such a small leak, the Kalman Filter reacts within a reasonable amount of time to the change. It does, however, take too long to reach the final estimated value. Fig. 6.29 shows the estimation of the first, middle and last node in the main branch. The estimation of the last node in the main branch is fast to react to the leak, but as with the leak estimation, it's slow to reach the final estimated value. Fig. 6.30-Fig. 6.31 displays the estimation of flow in branch A and B, more precisely in the node closest to the leak, and the node at the exit. The same problem as with the other estimations arises here.

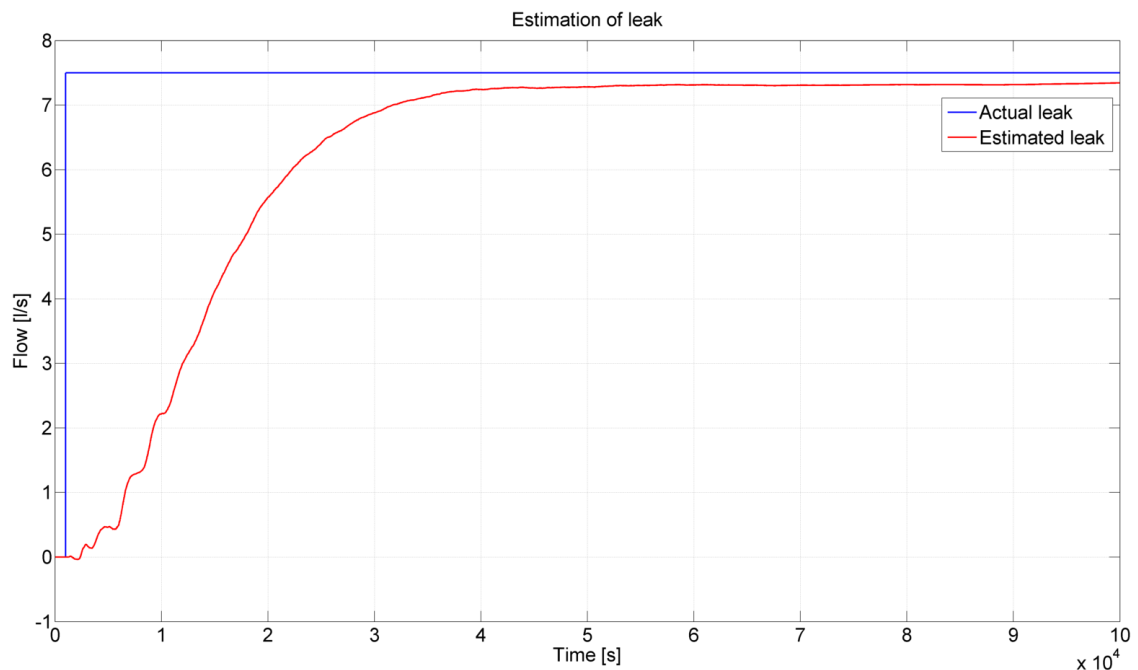


Figure 6.28: Measured and estimated leak.

6.4 Larger Model

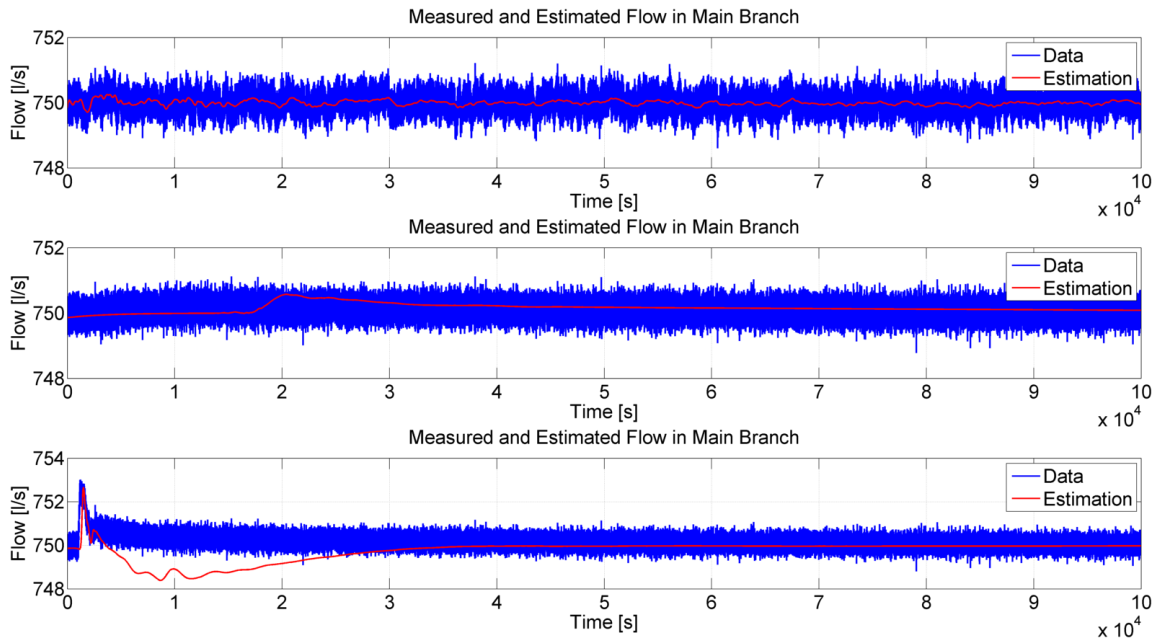


Figure 6.29: Measured and estimated flow in main branch. The node at the entrance to the pipe, the middle node and the node closest to the leak are shown.

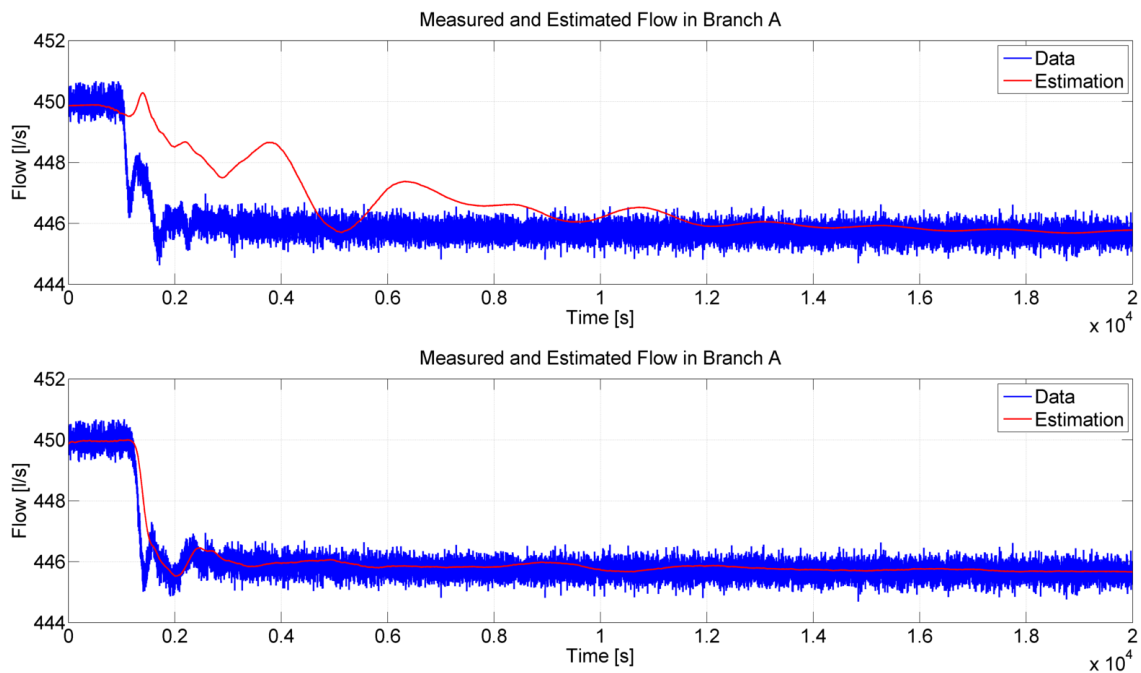


Figure 6.30: Measured and estimated flow in branch A. The node closest to the leak and the node at the exit is shown.

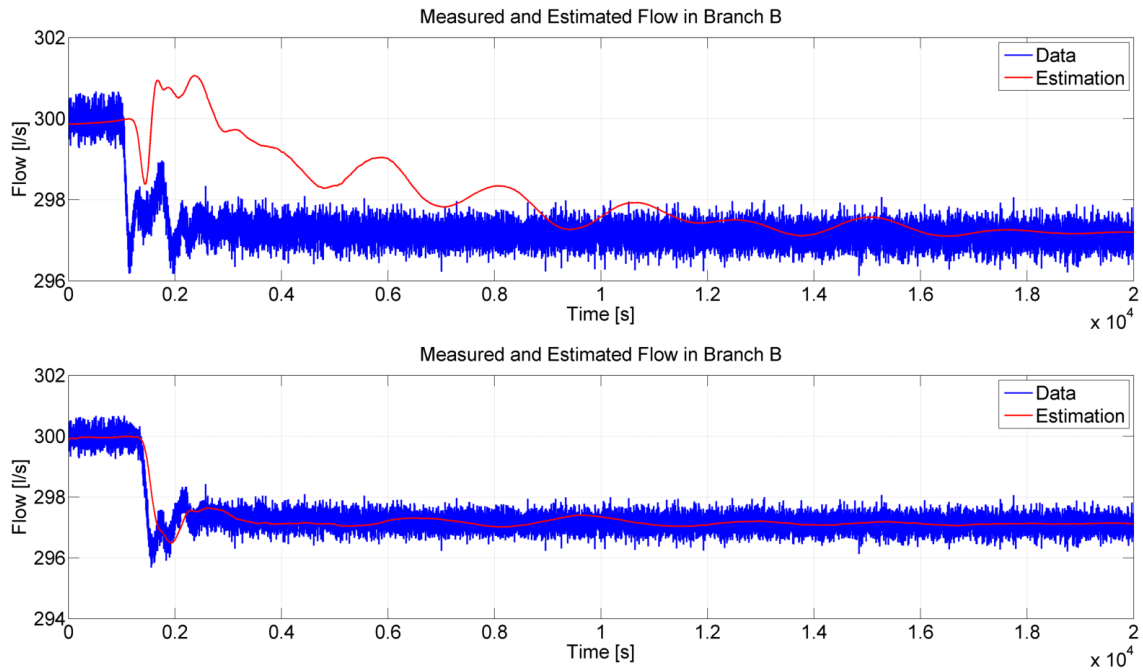


Figure 6.31: Measured and estimated flow in branch B. The node closest to the leak and the node at the exit is shown.

6.4.3 Five Percent Leak

The results found in sec. 6.4.2 are very similar to the ones found in this test. The noticeable difference is that estimation is a bit faster, and it reaches an estimate that is closer to the real value. The figure below shows the leak estimation. The other plots in this test shows the same characteristics as in sec. 6.4.2.

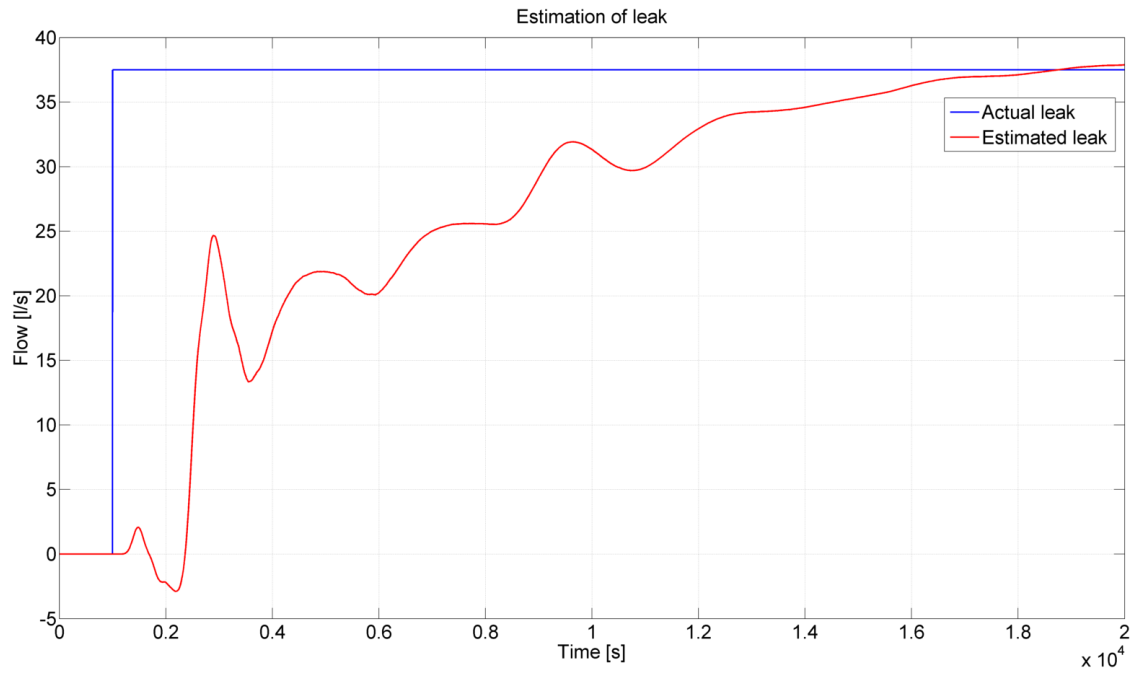


Figure 6.32: Measured and estimated leak.

7 Discussion, Suggestions and Future Work

This chapter will discuss the main results found in the previous chapter, suggestions for improvements, and future work. To simplify the discussion, some shortcuts has been made, by state estimation it is meant the estimation of all states except the leak, which will be specified as *leak estimation*.

7.1 Discussion

The state space model and the Kalman Filter made it possible to detect and estimate the states in the pipeline system to a satisfying degree when no leak was applied to the system. When a leak occurred at the known position, it was hard to find a satisfying set of tuning parameters which made the leak estimation fast enough. The state estimations on the other hand, did converge to the correct values within a small amount of time, with the exception of the two nodes closest to the leak. Attempts were made to see if any improvements were possible, but to no avail. Why the nodes closest to the leak showed this surprising behaviour remains unknown at this time. Bigger changes to the Kalman Filter could have solved this problem, suggestions are mentioned below.

The larger system with the 1000m main branch experienced an expected lag before noticing changes in the system. This is expected because of the time it takes for the changes to actually reach the sensors at the extrema. The system itself, and thus the estimation are likely slower than they would have been in a real system because of the increased friction factor used. Without a higher friction it would take a long time for the system to reach steady state, and thus increase the simulation time, which ended up being very time consuming. A possible improvement that could improve simulation time would be to use two different friction factors, one when the system is in steady state, and one at the time of the leak until the system is at steady state again. Another solution could be to change the boundary conditions, and use output injection, like in Hodne [11].

With leakage at an unknown position, we were able to detect the change, and somewhat estimate the states, but with constant deviations. The leak estimation when the leak was applied to branch A can not be seen as concluding, it was lucky

that the leak estimation was correct. The results from when the leak was applied to branch B support this claim, as these two events should yield similar results. It is interesting though, that the convergence time of the leak estimation is faster in the tests where the leak was applied to branch A and B compared to when the leak was applied to the branch itself where the Kalman Filter expected it to be. They both also reach a value closer to the true value of the leak.

Changing the tuning parameters available in the Kalman Filter changed the behaviour of the estimations, but only to a certain degree. A better, more realistic model along with more specified values within the Q and R matrices may improve the results. Also, the system has been in a steady state for a long time before the leak occurs, therefore the filter's matrices has stabilized to not expect changes. Early tests showed a better convergence rate if the leak was applied before the initial estimations had stabilized, but it was not included in the thesis as it is not likely that a leak occurs at the time the Kalman Filter is turned on. When an immediate change in flow and/or pressure is noticed, an improvement could be to reset the Kalman Filter at that moment, to see if that improves the leak estimation.

7.2 Future work

The Kalman Filter seems to work reasonable in some scenarios, but there are definitely room for improvement. Furthermore, temperature, changes in elevation and friction, and twists and turns in the pipeline has been ignored in this thesis. Adding these changes to the model, would certainly challenge the observer.

The obvious next step is to implement a way to locate the leak. A suggestion is to implement a bank of observers (Verde [20] and Lesyshen [16]) where each observer expects a leak at different locations along the pipeline, and create an algorithm that uses the estimated values to suggest where the leak is.

It would be interesting to get data from operational distribution pipelines that has experienced a leak to see the behaviour of the observer if that exact pipeline was modeled. An alternative would be to create a scaled test rig to test the model and observer in practice.

8 Conclusion

At the time, the Kalman Filter works as *leak detection*. It is easy to spot when a change happens, but the leak estimation itself is too slow. The simulations displayed results that may be seen as promising as even the smallest leaks of 1% was detectable. With improvements, it could be possible to completely identify the magnitude and position of the leaks as well.

The method used has its limitations. For it to work in a water delivery system, one would have to install sensors at all pipeline access points, that is every building connected to the pipeline. Alternatively, every branch that connects a group of buildings. How the observer would react to multiple outlets, and the changes in these outlets at different times in a realistic setting is unknown. The best use of this method would be either to implement it in pipelines transporting large amounts of liquid to few end users, or use it as a leak detection method capable of locating leakage zones.

Earlier estimation strategies has been based on a flow from A to B, but many of them could easily be adapted to the branching problem as long as the model is correct. An obvious one, is to use a different version of the Kalman Filter, for example the Extended Kalman Filter which is used by Lesyshen [16], though one would have to make a new model. Changing the boundary conditions with input injection could be a bit more difficult considering how the flow reacts to the branching, though a simplistic model would not be hard to make. As long as the branch is modeled correctly, there's no reason the earlier strategies shouldn't work here, though I wouldn't recommend using the simplest methods.

As there are no results in this thesis that makes one doubt the viability of the Kalman Filter, I would not throw away this method without further testing.

Bibliography

- [1] Ole Morten Aamo, Jørgen Salvesen, and Bjarne A Foss. Observer design using boundary injections for pipeline monitoring and leak detection. In *Proc. IFAC Symp. Adv. Control Chem. Process*, pages 2–5. Citeseer, 2006.
- [2] L Billmann and Rolf Isermann. Leak detection methods for pipelines. *Automatica*, 23(3):381–385, 1987.
- [3] Robert Grover Brown, Patrick YC Hwang, et al. *Introduction to random signals and applied Kalman filtering*, volume 1. John Wiley & Sons New York, 1992.
- [4] Chi-Tsong Chen. *Linear system theory and design*. Oxford University Press, Inc., 1998.
- [5] Andrew F Colombo and Bryan W Karney. Energy and costs of leaky pipes: Toward comprehensive picture. *Journal of Water Resources Planning and Management*, 128(6):441–450, 2002.
- [6] Olav Egeland and Jan Tommy Gravdahl. *Modeling and simulation for automatic control*, volume 76. Marine Cybernetics Trondheim, Norway, 2002.
- [7] EngineeringToolBox. Pressure loss in schedule 40 steel pipes.
- [8] Ing Gerhard Geiger. Principles of leak detection. *Fundamentals of leak detection. KROHNE oil and gas*, 2005.
- [9] Driss Halhal, Godfrey A Walters, Driss Ouazar, and DA Savic. Water network rehabilitation with structured messy genetic algorithm. *Journal of Water Resources Planning and Management*, 123(3):137–146, 1997.
- [10] Espen Hauge. *Advanced leak detection in oil and gas pipelines using a nonlinear observer and OLGA models*. PhD thesis, Norwegian University of Science and Technology, 2007.
- [11] Kjetil Hodne. *Leak Detection in Two-Phase Oil and Gas Pipelines by Parameter-and State Estimation*. PhD thesis, Norwegian University of Science and Technology, 2008.
- [12] Rolf Isermann. Process fault detection based on modeling and estimation methods - a survey. *Automatica*, 20(4):387–404, 1984.
- [13] C.N. Jackson, C.N.; Sherlock. Non-destructive testing handbook: Leak testing. *Library of Congress Cataloging-in-Publication Data*, page page 519, 2008.

- [14] R Kalman. On the general theory of control systems. *IRE Transactions on Automatic Control*, 4(3):110–110, 1959.
- [15] AO Lambert. International report: Water losses management and techniques. *Water Science & Technology: Water Supply*, 2(4):1–20, 2002.
- [16] Ryan M Lesyshen. *Water transmission line leak detection using extended kalman filtering*. PhD thesis, University of Saskatchewan, 2005.
- [17] Wu Ming and Wang Wei-qiang. Application of wavelet to detect pipeline leak point. In *Intelligent Systems Design and Applications, 2006. ISDA '06. Sixth International Conference on*, volume 2, pages 779–782. IEEE, 2006.
- [18] Wan Rahiman, Bo Li, Buzhou Wu, and Zhengtao Ding. Circle criterion based nonlinear observer design for leak detection in pipelines. In *Control and Automation, 2007. ICCA 2007. IEEE International Conference on*, pages 2993–2998. IEEE, 2007.
- [19] NC Turner. Hardware and software techniques for pipeline integrity and leak detection monitoring. *Offshore Europe*, 1991.
- [20] C Verde and N Visairo. Bank of nonlinear observers for the detection of multiple leaks in a pipeline. In *Control Applications, 2001.(CCA '01). Proceedings of the 2001 IEEE International Conference on*, pages 714–719. IEEE, 2001.
- [21] Jun Zhang. Designing a cost-effective and reliable pipeline leak-detection system. *Pipes and Pipelines International*, 42(1):20–26, 1997.

Nomenclature

α	Angle between gravity and positive flow direction [$^{\circ}$]
\bar{v}	Average velocity [m/s]
β	Bulk modulus for water [Bar]
ρ	Average density of fluid in cross-section [kg/m^3]
A	Area of a pipeline cross-section [m^2]
F	Friction force [kgm/s^2]
f	Friction factor
g	Force due to gravity [m/s^2]
H	Heaviside step function
l	Length of pipesegment [m]
L_A	Length of branch A [m]
L_B	Length of branch B [m]
L_{main}	Length of main pipe [m]
N	Number of segments the pipe is divided into
p	Pressure [Bar]
p_{branch}	Pressure in the branch [Bar]
q	Flow [m^3/s]
q_{leak}	Pipeline leak [l/s]
t	Time [s]
t_{leak}	Time where the leak occurs [s]
w	Mass flow [$\frac{kg}{s}$]

w_l	Leak magnitude [l]
x	Length in flow direction [m]
x_l	Position of the leak [m]