Stochastic subspace identification of modal parameters during ice-structure interaction

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Authors' contributions

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Statement (if applicable):
T.S.N. performed the simulations, identifications, and data analysis and drafted the manuscript.
H.H. conducted the simulations with the ice model.
Ã˜.W.P. performed the data analysis.

Stochastic subspace identification of modal parameters during ice-structure interaction

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Summary

Identifying the modal parameters of structures located in ice-infested waters may be challenging due to the interaction between the ice and structure. In this study, both simulated data from a state-of-the-art ice-structure interaction model and measured data of ice-structure interaction were both used in conjunction with a covariance-driven stochastic subspace identification method to identify the modal parameters and their corresponding variances. The variances can be used to assign confidence to the identified eigenfrequencies, and effectively eliminate the eigenfrequencies with large variances. This enables a comparison between the identified eigenfrequencies for different ice conditions.

Simulated data were used to assess the accuracy of the identified modal parameters during ice-structure interactions, and they were further used to guide the choice of parameters for the subspace identification when applied to measured data. The measured data consisted of 150 recordings of ice actions against the Norströmsgrund lighthouse in the Northern Baltic Sea. The results were sorted into groups defined by the observed ice conditions and governing ice failure mechanisms during the ice-structure interaction. The identified eigenfrequencies varied within each individual group and between the groups. Based on identified modal parameters, we suggested which eigenmodes play an active role in the interaction processes at the ice-structure interface and discussed the possible sources of errors.

1. Introduction

The understanding of ice-structure interaction originates from observations and measurements of various ice

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2   features interacting with structures such as lighthouses, bridge piers and offshore structures. The monitoring

3   of structural responses on platforms in Cook Inlet, Alaska began more than 50 years ago (1). It was soon

4   discovered that ice forces varied depending on the ice conditions, and therefore, the measurements of the

5   structural response were often supplemented with measurements of physical and mechanical properties of the

6   ice (2). Despite the measurement complexity and cost, several full-scale monitoring campaigns in (sub-)Arctic

7   areas were conducted in the past, many of which involved lighthouse structures (3, 4). Drifting ice against a

8   structure may result in various modes of ice-structure interaction. Some of these are violent with ice forces and

9   structural responses that represent a threat against the structural integrity, whereas other modes pose no

10   concern at all. These modes depend on both the structure and the ice. Structures with sloped walls at the ice-

11   structure interface typically promote a flexural type of ice failure, while vertically sided structures typically

12   promote crushing failure. Thus, sloped-walled structures are subject to significantly lower ice forces than

13   vertically sided structures, but they often incur a higher construction expense (5).

14   Ice forces on offshore structures have been debated for decades. Design engineers still find it challenging to

15   determine consistent load estimates pertaining to the expected ice conditions. A study by Timco and

16   Croasdale (6) demonstrated the difficulties in the year 2006 when they invited international experts to

17   calculate the ice forces under some selected scenarios of a structure interacting with first-year level ice, first-

18   year ridge and multi-year ice floes. Considerable scatter was presented for the different cases, up to a factor of

19   11 between the lowest and highest load estimates for a conical structure subject to level ice. Lighthouses are no

20   exception in terms of the challenges faced in the design of offshore structures in ice-infested waters. After

21   revisiting 69 lighthouses in the St. Lawrence Waterway, Danys (7) found that many old lighthouses with

22   design pressures in the range of 0.55-0.76 MPa were damaged, whereas structures with design pressures in the

23   range 2.0-2.8 MPa were undamaged. A few lighthouses in the Baltic Sea, which had design loads in the latter

24   range, were also damaged due to ice actions (5). Many of the lighthouses were built with vertical walls at the

25   ice-structure interface, allowing a greater risk for structural damage. In particular, narrow structures with a

26   low aspect ratio (diameter of the structure at the waterline divided by the ice thickness) were noted to be

27   vulnerable to damage, due to excessively low design pressures suggested by the standards (5, 8).

28   Consequently, recent guidelines for the design of Arctic offshore structures were modified to account for the

29   high ice pressures at low aspect ratios.

30   To this end, sensors and measurement techniques were developed to handle the harsh Arctic environment as

31   lighthouses and oil platforms were instrumented. The aim was to mitigate ice forces on offshore structures. It

32   was soon discovered that the structural integrity was threatened not only by the high ice forces, but also the

33   severe ice-induced vibrations (IIV) that followed from the dynamic ice actions (1). IIV represents a threat in

34   the form of low-cycle fatigue and illness of people exposed to the vibrations, and in a few cases, it was the

35   primary cause of structural damage (1, 4, 9). Although phenomenological models exist to predict IIV, i.e., as

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36   reported in (10, 11), the origin of the vibrations is still under debate (12), and the ice conditions in which IIV

37   occurs are still not entirely known. Single events of IIV can also pose a threat to the structural integrity. The

38   most critical and famous IIV episode occurred in the Beaufort sea in 1986, when the 90 by 90m wide oil

39   platform Molikpaq encountered a multi-year ice floe (13), resulting in liquefaction of the soil foundation. As

40   platforms have been deployed in ice-choked waters in regions such as Cook Inlet, Alaska, USA and Bohai Sea,

41   China, for decades, their remaining lifetimes decrease, making structural health monitoring (SHM)

42   increasingly relevant.

43   On the massive Confederation Bridge, monitoring programmes were employed to assessing ice loads and

44   SHM (14-16). Simulated damage in conjunction with response measurements under different ambient

45   loadings that included ice loads were used to evaluate the possibility for damage detection. On that particular

46   structure, ice actions were considered as part of the environmental variability of modal parameters. The

47   Confederation bridge is a massive structure, and smaller structures may experience a higher influence of ice

48   actions on the modal parameters. A limited number of studies have addressed the changes in modal

49   parameters caused by different types of ice conditions and ice failure mechanisms; however, some

50   experimental (17, 18) and simulation studies (19) have indicated that added mass and damping may occur. It

51   is, thus, unknown to what extent the size of the structure relative to the severity of the ice conditions causes

52   system changes, e.g., changes in the eigenfrequency, damping and mode shapes, or the introduction of

53   nonlinearities. It is also unknown for which ice conditions the true modal properties can be identified and

54   when the underlying assumptions of the applied algorithms are violated the most.

55   Even though many arctic offshore structures are located in vulnerable areas where a structural failure may

56   have fatal consequences for the environment, it is still unknown whether vibration-based SHM is feasible

57   during the ice-covered months. This is relevant not only for hydrocarbon exploitation, but also for advancing

58   the development of offshore wind power in cold climates. The null hypothesis is that the feasibility depends

59   on the severity of the ice-actions relative to the size of the structure, which was partly exploited in (20). An

60   automatic routine was applied to identify eigenfrequencies for data records considering various modes of ice-

61   structure interaction occurring on the Norströmsgrund lighthouse. The identified frequencies were spread

62   and only few time records rendered similar identified frequencies. The origin of both the bias errors and

63   variance errors in the estimated modal parameters when using the covariance-driven reference-based

64   stochastic subspace identification routine (SSI-cov/ref) was addressed (21). It was shown how the bias error

65   could partly be removed, whereas the variance error could only be estimated. A computationally efficient

66   implementation was presented in (22), which is exploited in this study as a means to investigate the variability

67   in the identified modal parameters for different ice conditions. The remaining paper is structured as follows:

68   section 2 explains some fundamentals of ice-structure interaction and ice forces on vertically-sided structures;

69   section 3 presents a summary of the SSI-cov/ref algorithm used to identify the modal parameters; section 4
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presents the identified modal parameters and their uncertainties for simulated cases of ice-structure interaction; section 5 presents the identified modal parameters and the corresponding uncertainties for 150 recordings of ice-structure interaction on the Norströmsgrund lighthouse; and finally, the concluding remarks are presented in section 6.

2. Modes of ice-structure interaction

Several types of ice-structure interactions against vertically sided structures are considered in this study, and they are illustrated in Figure 1. The readers are recommended to refer to (23) for an overview of the mechanics of ice-structure interaction and (24) for descriptions of the observed failure types against the Norströmsgrund lighthouse. Continuous brittle crushing (Figure 1a) is governed by the non-simultaneous occurrence of so-called high-pressure zones across the ice-structure interface. The interaction process ongoing at the ice-structure interface also involves the occurrence of many modifications of the ice material, such as recrystallization and microcrack developments (23, 25). The bending type of flexural failure is often initiated by the development of a circumferential crack followed by radial cracks (Figure 1b). Splitting failures (Figure 1c) are usually observed when the interacting ice sheet has a low lateral confinement. The buckling type (Figure 1d) of flexural failure is governed by a build-up of curvature in the ice sheet. Winds and waves as well as ice management can generate fields of broken ice (Figure 1e) that cause small impacts from floes of various sizes onto the walls of the structure. The floes split and pass around the structure, while the wind and wave actions contribute significantly to the total force. The last interaction type considered in this study is creep, in which the ice floe rests against the structure. For simplicity, no distinction is made between limit force, when the driving forces are too low to generate any of the failure mechanisms mentioned above, and limit-stress creep deformation. Among these modes, ice crushing and creep exert the highest forces on the structure. The frequency contents of the ice forces vary substantially not only between the individual failure types but also within the same type of failure with different environmental parameters, see e.g., (20, 26). As ice forces cannot be described as Gaussian white noise, the following sections describe the assessment of the influence of violating the stochastic white-noise input assumption for the SSI-cov/ref.
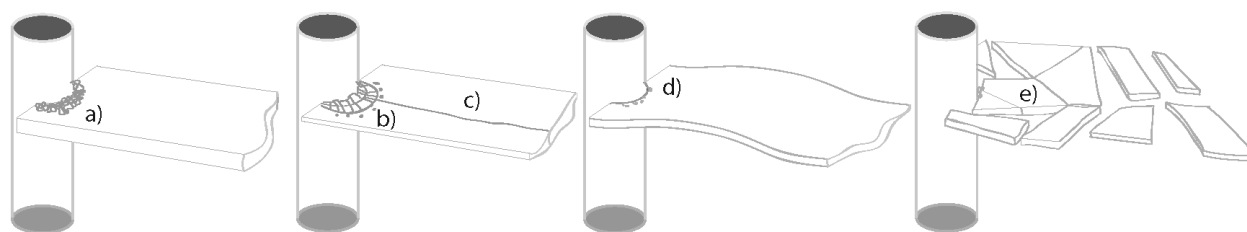
Figure 1. Types of failure modes of ice-structure interaction (from Nord et al. (20)).

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3. Estimation of modal parameters and their uncertainty

(a) System model

Consider the linear time-invariant system described by a discrete time state-space model:

$$x_{k+1} = Ax_k + Bu_k \tag{1}$$

$$y_k = Cx_k + Du_k + e_k \tag{2}$$

where $x_k \in \mathbb{R}^n$ is the state vector, $u_k \in \mathbb{R}^m$ is the input vector, $y_k \in \mathbb{R}^r$ is the measurement vector and $A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}, C \in \mathbb{R}^{r \times n}$ and $D \in \mathbb{R}^{r \times m}$ are the system matrices. Here, n is the model order and r is the number of sensors. The measurements y_k are corrupted with the measurement error e_k which is modelled as a white noise random process. The white noise random process is also used to model the input u_k, which is unknown for many practical applications, and the state-space model is thus reduced to

$$x_{k+1} = Ax_k + w_k \tag{3}$$

$$y_k = Cx_k + v_k \tag{4}$$

where $w_k = Bu_k$ and $v_k = Du_k + e_k$ are the process and output noise, respectively.

We aim to identify matrices A and C from which the modal frequencies, damping and mode shapes can be obtained. The eigenvalues and eigenvectors of the system in Eqs. (3) and (4) become

$$(A - \lambda_i I)\phi_i = 0 \tag{5}$$

$$\varphi_i = C\phi_i \tag{6}$$

from which the continuous time eigenvalues μ, eigenfrequencies f_i and damping coefficients ξ_i (in % of critical) can be obtained as follows:

$$\mu_i = \frac{\ln \lambda_i}{T}, \qquad f_i = \frac{|\mu_i|}{2\pi}, \qquad \xi_i = -100\frac{\Re(\mu_i)}{|\mu_i|} \tag{7}$$

where T is the sampling period.

(b) Stochastic subspace identification (SSI) algorithm

In the following two sections, the covariance-driven SSI-cov/ref and the data-driven Unweighted Principal Component (UPC) SSI algorithm are briefly introduced; but for thorough explanations, see (27). Let r be the number of sensors, r_0 be the number of reference sensors, and p and q be the parameters chosen such that

6

$pr \geq qr_0 \geq n$, where n is the model order. The algorithm utilises the output data to build a subspace matrix

$H_{p+1,q} \in \mathbb{R}^{(p+1)r \times qr_0}$ which (for a sufficient number of samples) may be decomposed as

$$H_{p+1,q} = O_{p+1}Z_q, \tag{8}$$

where $O_{p+1} = \begin{bmatrix} C^T & (CA)^T & \cdots & (CA^p)^T \end{bmatrix}^T$ is the observability matrix and matrix Z_q depends on the chosen

subspace identification algorithm. The observability matrix O_{p+1} is constructed from a singular value

decomposition (SVD) of the subspace matrix $H_{p+1,q}$, which is further truncated at a user-defined model order

n:

$$H_{p+1,q} = \begin{bmatrix} U_1 & U_0 \end{bmatrix} \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_0^T \end{bmatrix}, \tag{9}$$

$$O_{p+1} = U_1 \Sigma_1^{1/2}. \tag{10}$$

The C matrix can be directly extracted from the first block of r rows of the observability matrix O_{p+1}, while

the A matrix can be obtained from a least squares solution of

$$O_{p+1}^{\uparrow} A = O_{p+1}^{\downarrow}, \tag{11}$$

where

$$O_{p+1}^{\uparrow} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{p-1} \end{bmatrix}, \qquad O_{p+1}^{\downarrow} = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^p \end{bmatrix}.$$

(c) Estimate of the subspace matrix

"Future" and "past" output data matrices are assembled from a total number of $N + p + q$ samples as follows:

$$Y^+ = \begin{bmatrix} Y_{q+1} & Y_{q+2} & \vdots & Y_{N+q} \\ Y_{q+2} & Y_{q+3} & \vdots & Y_{N+q+1} \\ \vdots & \vdots & \vdots & \vdots \\ Y_{q+p+1} & Y_{q+p+2} & \vdots & Y_{N+q+p} \end{bmatrix}, \qquad Y^- = \begin{bmatrix} Y_q^{(\text{ref})} & Y_{q+1}^{(\text{ref})} & \vdots & Y_{N+q-1}^{(\text{ref})} \\ Y_{q-1}^{(\text{ref})} & Y_q^{(\text{ref})} & \vdots & Y_{N+q-2}^{(\text{ref})} \\ \vdots & \vdots & \vdots & \vdots \\ Y_1^{(\text{ref})} & Y_2^{(\text{ref})} & \vdots & Y_N^{(ref)} \end{bmatrix} \tag{12}$$

where $Y_k^{(\text{ref})} \in \mathbb{R}^{r_0}$ contains the reference sensor data. These data matrices are further normalised with respect

to their numbers of columns, such that

$$\tilde{Y}^+ = \frac{1}{\sqrt{N}}Y^+, \qquad \tilde{Y}^- = \frac{1}{\sqrt{N}}Y^-. \tag{13}$$

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146 From the data matrices, the covariance-driven subspace matrix is defined:

$$\tilde{H}^{cov} = \tilde{Y}^{+}(\tilde{Y}^{-})^{T} \ , \tag{14}$$

148 while in the data-driven case, the subspace matrix is defined:

$$\tilde{H}^{dat} = \tilde{Y}^{+}(\tilde{Y}^{-})^{T}(\tilde{Y}^{-}(\tilde{Y}^{-})^{T})^{\dagger}\tilde{Y}^{-} \ . \tag{15}$$

150 Alternatively, the data-driven subspace matrix can be estimated from the thin LQ decomposition:

$$\begin{bmatrix} \tilde{Y}^{-} \\ \tilde{Y}^{+} \end{bmatrix} = \begin{bmatrix} R_{11} & 0 \\ R_{21} & R_{22} \end{bmatrix}\begin{bmatrix} Q_{1} \\ Q_{2} \end{bmatrix} \tag{16}$$

152 from which $\tilde{H}^{dat} = R_{21}Q_{1}$, where $R_{21} \in \mathbb{R}^{(p+1)r \times qr_0}$ and $Q_{1} \in \mathbb{R}^{qr_0 \times N}$. Note that because of the orthogonal

153 properties of Q_1, the observability matrix is estimated directly from R_{21} in the implementation.

(d) Covariance estimate of the subspace matrix

155 Statistical uncertainty in the identified modal parameters may occur due to e.g. non-white excitation of the

156 system, a finite number of data samples, non-stationarity etc., which leads to variance errors in the modal

157 parameters. In what follows, the covariance computation of the modal parameters is based upon an

158 underlying covariance-driven SSI algorithm and propagation of first-order perturbations from the data to the

159 modal parameters (21). Full-scale validations of the method were presented in (28), while our study applies a

160 computationally efficient implementation (22). Reviews of existing methods for covariance estimation of

161 identified modal parameters are presented in (28) and (29).

162 In the covariance-driven SSI, the covariance of the subspace matrix were calculated by first splitting $\tilde{Y}^{+}$ and

163 $\tilde{Y}^{-}$ into n_b blocks

$$\tilde{Y}^{+} = \frac{\sqrt{N}}{\sqrt{N_b}}\begin{bmatrix} \tilde{Y}_1^{+} & \dots & \tilde{Y}_{n_b}^{+} \end{bmatrix}, \qquad \tilde{Y}^{-} = \frac{\sqrt{N}}{\sqrt{N_b}}\begin{bmatrix} \tilde{Y}_1^{-} & \dots & \tilde{Y}_{n_b}^{-} \end{bmatrix}, \tag{17}$$

165 where for simplicity, $N_b \cdot n_b = N$; subsequently, the blocks were normalised according to the block length.

166 From each data block in Eq. (17), the subspace matrix estimate $\tilde{H}_j^{cov}$ is built with

$$\tilde{H}_j^{cov} = \tilde{Y}_j^{+}(\tilde{Y}_j^{-})^{T} \tag{18}$$

168 , and its covariance estimate $\hat{\Sigma}_{\tilde{H}^{cov}}$ reads as

$$\hat{\Sigma}_{\tilde{H}^{cov}} = \frac{N}{n_b(n_b-1)}\sum_{j=1}^{n_b}(vec(\tilde{H}_j^{cov}) - vec(\tilde{H}^{cov}))(vec(\tilde{H}_j^{cov}) - vec(\tilde{H}^{cov}))^{T}. \tag{19}$$

(e) Covariance estimate of the modal parameters

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171 The principle of the method is to propagate the covariance of the subspace matrix, Σ_H, to the modal

172 parameters through first-order perturbations. First-order perturbations of the subspace matrix H are first

173 propagated to the system matrices A and C as follows:

$$\begin{bmatrix} vec(\Delta A) \\ vec(\Delta C) \end{bmatrix} = J_{AC,H} vec(\Delta H) \tag{20}$$

174

175 where $J_{AC,H}$ is the sensitivity matrix according to the definitions given in (21, 28) and (30). The covariance of

176 the vectorised system matrices, $\Sigma_{A,C}$, can be defined as,

$$\Sigma_{A,C} \overset{def}{=} \mathrm{cov}\left(\begin{bmatrix} vec(\Delta A) \\ vec(\Delta C) \end{bmatrix}\right) = J_{AC,H}\Sigma_H J_{AC,H}^T, \tag{21}$$

177

178 in which the covariance of the subspace matrix, Σ_H, can be estimated by dividing the subspace matrix into

179 blocks as described in the previous section and in (22). The perturbations of the modal parameters can be

180 described as functions of the vectorised system matrices as follows:

$$\Delta f_i = J_{f_i,A} vec(\Delta A), \quad \Delta \xi_i = J_{\xi_i,A} vec(\Delta A), \quad \Delta \varphi_i = J_{\varphi_i,AC}\begin{bmatrix} vec(\Delta A) \\ vec(\Delta C) \end{bmatrix} \tag{22}$$

181

182 where the sensitivities $J_{f_i,A}$, $J_{\xi_i,A}$, and $J_{\varphi_i,AC}$ are defined per mode i. Finally, the covariances of the modal

183 parameters are obtained as:

$$\mathrm{cov}\left(\begin{bmatrix} f_i \\ \xi_i \end{bmatrix}, \begin{bmatrix} f_j \\ \xi_j \end{bmatrix}\right) = \begin{bmatrix} J_{f_i,A} & 0_{1,rn} \\ J_{\xi_i,A} & 0_{1,rn} \end{bmatrix} \Sigma_{AC} \begin{bmatrix} J_{f_j,A} & 0_{1,rn} \\ J_{\xi_j,A} & 0_{1,rn} \end{bmatrix}^T$$

184

$$\mathrm{cov}\left(\begin{bmatrix} \Re(\varphi_i) \\ \Im(\varphi_i) \end{bmatrix}, \begin{bmatrix} \Re(\varphi_j) \\ \Im(\varphi_j) \end{bmatrix}\right) = \begin{bmatrix} \Re(J_{\varphi_i,A,C}) \\ \Im(J_{\varphi_i,A,C}) \end{bmatrix} \Sigma_{AC} \begin{bmatrix} \Re(J_{\varphi_j,A,C}) \\ \Im(J_{\varphi_j,A,C}) \end{bmatrix}^T. \tag{23}$$

185

186 In this study, the implementation is performed in accordance with algorithm 4 presented in (22) which yields

187 fast computations when multiple model orders are considered.

188

4. Modal parameter identification using simulated data of dynamic ice-structure

190 ## interactions

191 Since ice forces have been measured by means of both inverse techniques and installations of force panels at

192 the ice-structure interface, some knowledge exists concerning their appearance in time and frequency

193 domains. The continuous brittle crushing process, which is one of the interaction types decisive for design

194 loads, is described as a random process (31). Nonsimultaneous contact across the interface between the

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195 structure that changes in space and time causes local pressures to vary across the interface (32). This means

196 that the force varies randomly superimposed to a mean level. The structure, in some cases, slows down the

197 drifting ice floe(s), thereby influences the driving force(s); therefore, the mean level is seldom constant. The

198 slow-varying processes of ice forces are also influenced by uneven ice thickness, stochastic variation of ice

199 strength, and delayed elastic and viscous properties of the ice, in addition to the intermittent occurrences of

200 other failure processes; all of these may contribute to a non-white forcing onto the structure.

201 In this study, a phenomenological ice-structure interaction model (19, 33) is used to investigate the effect of

202 variation in ice conditions on the identified modal parameters and their uncertainties. A MATLAB

203 implementation of the model, including a single-degree-of-freedom structural representation in the modal

204 domain, is available from Mendeley data (34). The ice model is coupled to the modal characteristics of the

205 Norströmsgrund lighthouse, extracted from the finite element software Abaqus CAE.

206 The Norströmsgrund lighthouse (Figure 2) is a gravity-based concrete structure located in the Gulf of Bothnia,

207 Sweden. The lighthouse was constructed in 1971 and designed to withstand ice loads of 2.2 MN/m. Below the

208 mean water level, the main structure and foundation caisson are filled with sand. Eight concrete bulkheads

209 stiffen the foundation frame, and a 0.7 m thick concrete foundation plate rests on a top layer of cement-

210 grouted crushed stones and a lower layer of morainic soil. The wall thickness varies between 0.2 m at the top

211 and 1.4 m at the mean water level. Service personnel visiting Norströmsgrund late in the winter of 1972

212 observed heavy vibrations due to ice actions. In 1973, thin cracks were detected in the most strained area of

213 the superstructure walls close to the ground floor. Despite that the cracks pervaded through the wall and

214 moisture intrusion was observed, it was concluded that the structural integrity remained intact (9).

215 The finite element model, modelled with quadrilateral finite membrane-strain shell elements with reduced

216 integration (S4R) is displayed in Figure 2 and Figure 3. The structural dimensions and material properties are

217 obtained from (35) and an elaborate description of the model can be found in (26). Because the structure is

218 near axisymmetric, structural modes occur in pairs with almost identical frequencies. Mode shape 1 (and 2) is

219 governed by tilt of both the foundation caisson and tower substructure (tower below + 16.5 m) while the

220 superstructure deflects in the tilt direction. For mode shape 3 (4), both the tower caisson and tower

221 substructure displaces in the opposite direction to the superstructure deflection. Mode shape 5 (6) has the

222 same tilt of both the caisson and tower substructure as for mode shape 1, whereas the superstructure deflects

223 in opposite direction to the substructure tilt direction. In mode shape 7 (8), the foundation caisson tilts in the

224 opposite direction to the tower substructure.

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Figure 2. Illustration, picture and finite element model of the Norströmsgrund lighthouse, from Nord et al. (26).
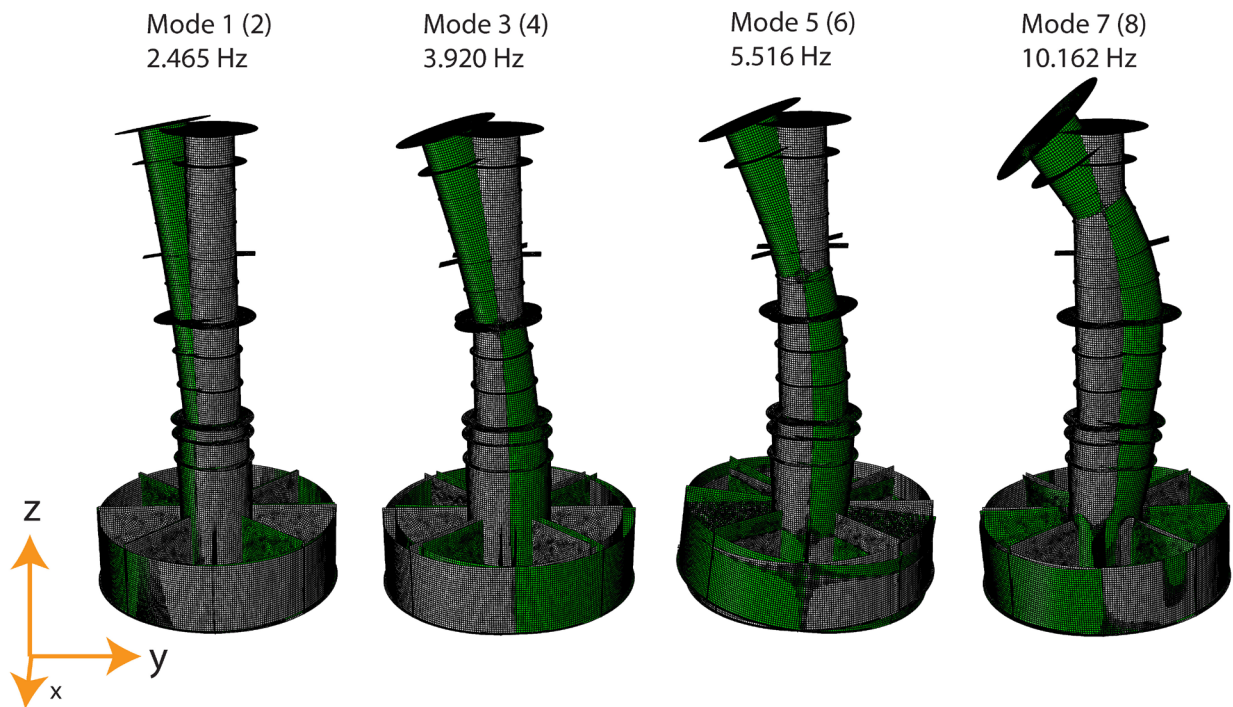

Figure 3. Mode shapes and corresponding natural frequencies of the finite element model (From Nord et al. (26)).

The forces are treated as concentrated loads acting on a linear time-invariant structure that is represented by selected vibration eigenmodes:

$$\ddot{\mathbf{z}}(t) + \mathbf{\Gamma}\dot{\mathbf{z}}(t) + \mathbf{\Omega}^2\mathbf{z}(t) = \mathbf{\Phi}^{\mathrm{T}}\mathbf{S}_{\mathrm{p}}\mathbf{p}(\mathbf{z}, \dot{\mathbf{z}}, t) \tag{24}$$

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236   Here, $\mathbf{z}(t) \in \mathbb{R}^{n_m}$ is the vector of modal coordinates, and $n_m = 24$ is the number of eigenmodes used to

237   assemble the model. The ice force, $\mathbf{p}(\mathbf{z}, \dot{\mathbf{z}}, t) \in \mathbb{R}^{n_p}$, is specified to act on the desired location "*Force N-S*" in

238   Figure 2 through the force influence matrix $\mathbf{S}_p \in \mathbb{R}^{n_{DOF} \times n_p}$, where n_p is the number of force time histories and

239   n_{DOF} is the number of degrees of freedom.

240   $\mathbf{\Gamma} \in \mathbb{R}^{n_m \times n_m}$ is the diagonal damping matrix populated on the diagonal with the terms $2\xi_j \omega_j$, where ω_j and

241   ξ_j represent the natural frequency in radians per second and damping ratio corresponding to eigenmode j,

242   respectively. $\mathbf{\Omega} \in \mathbb{R}^{n_m \times n_m}$ is a diagonal matrix containing the natural frequencies ω_j, and $\mathbf{\Phi} \in \mathbb{R}^{n_{DOF} \times n_m}$ is a matrix

243   collecting the mass-normalised mode shapes. Rayleigh damping is assigned to the model with 2% in modes 1

244   and 2, increasing up to 20% in mode 24. The lowest four eigenfrequencies and corresponding damping ratios

245   of the finite element model are given in Table 1, with the corresponding mode shapes displayed in Figure 3.

246   The axisymmetry of the model leads to mode pairs in the orthogonal directions, with each pair assigned

247   identical damping values.

248   Table 1. Modal properties of simulation model.

	Mode 1 (2)	Mode 3 (4)	Mode 5 (6)	Mode 7 (8)
f_i [Hz]	2.465	3.920	5.516	10.162
ξ_i [%]	2.000	2.005	2.367	3.642

249

250   Two time series, each of 600s, were simulated using the ice model. The mean ice thickness and ice velocity

251   were assumed to be 1 m and 0.1 ms^{-1}, respectively. Time-dependent parameters in the ice model were used to

252   generate time series with different ice conditions, herein categorised as *slow-varying* and *fast-varying* ice

253   conditions (Figure 4). Though the ice model is in essence non-linear and dependent on the structural motion,

254   the non-linearity is considered insignificant at the high ice velocities considered in this work. Under these

255   conditions, the time between contact and failure of a single ice element is short and consequently the motion

256   of the structure is insignificant compared to the deformation of the ice element. At low ice drift velocities

257   strongly non-linear interactions commonly referred to as intermittent crushing and frequency lock-in, become

258   prevalent and the presented approach is no longer applicable.

259

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260
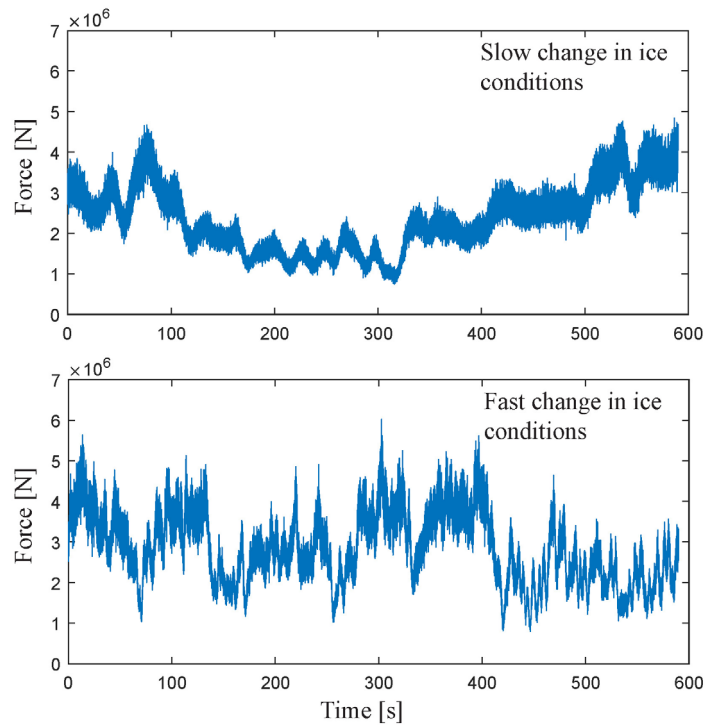
261 Figure 4. Simulated force time histories during interaction with slow-varying (top figure) and fast-varying

262 (bottom) ice conditions.

263

264 Four measurements of accelerations extracted at locations specified in Figure 2 were used for the subspace

265 identification (SSI-cov/ref), and throughout the paper the number of reference sensors is set equal to the total

266 number of sensors $r = r_0 = 4$. This involved the same number of sensors with the same locations as installed

267 on the lighthouse in the winters of years 2001 to 2003.

268 For comparison, a white noise force with an amplitude $0.3294 \cdot 10^6$ N was applied to the same model and

269 with accelerations extracted at the same locations as for the ice-structure interaction simulations. In the

270 simulations, the system matrices A, B, C, D in Eqs. (1) and (2) were assembled from the same modally

271 reduced order finite element model as described above, with forces acting in the same nodes and the sensors

272 located at the same nodes. Section 2.1 in (26) describes in depth how the modal properties of the structure are

273 used to assemble the system matrices A, B, C, D.

274 Recordings of 59000 sample points were added 5 % Gaussian white noise. The sampling frequency was 100

275 Hz, the number of blockrows was set as 100 and the number of blocks for the covariance of the subspace

276 matrix, n_b, was set as 60.

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13

277 Tolerance deviances to frequency, damping and MAC values, as well as the normalised standard deviation of

278 the frequency, $\dfrac{\hat{\sigma}_{\omega_i}}{\omega_i}$ were added into the stabilization diagram. A pole at order n was considered stable if the

279 deviances in frequency, damping, MAC, and normalised standard deviation of the frequency between a pole

280 at order n and $n-1$ were less than or equal to 0.01, 0.05, 0.95 and 0.05, respectively. A routine was used to

281 pick eigenmodes automatically from the stabilization diagrams. The poles were first sorted with increasing

282 corresponding absolute values, and a user-defined frequency slack value, S_f, defined the range from which

283 the poles were collected. From the eigenvectors of the poles in that range, MAC values were calculated

284 between all eigenvectors, and a reference eigenmode was selected as the pole that rendered the highest sum of

285 MAC values. The MAC values between the reference eigenmode and the eigenvectors of the remaining poles

286 in that frequency range (defined by S_f) must lie within a user-defined MAC-slack, S_{MAC} to be further

287 considered. The third acceptance criterion checked whether the poles that fulfilled the MAC-slack also

288 fulfilled a damping slack, S_ζ. Finally, the selected eigenmode contained the mean values of the frequency,

289 damping, mode shape and their corresponding variances. S_f, S_ζ and S_{MAC} were chosen to be 0.02, 0.3 and

290 0.5, respectively.

291 The stabilization diagrams in Figure 5 show the first four identified eigenmodes, with the square root of the

292 identified variance (standard deviation), $\pm\hat{\sigma}_\omega$, added to each pole in the diagram. Both the slow and fast-

293 varying ice conditions rendered clear columns of stable poles, and the differences were noticeable mainly in

294 the variance estimates. Table 2 presents the automatically selected eigenmodes' frequency, damping and

295 corresponding standard deviations alongside with the absolute error between the identified and model

296 frequencies. The results obtained from the time series of ice-structure interaction were compared with the

297 results obtained from the time series with Gaussian white noise as the input. As expected, the Gaussian white

298 noise input yielded more accurate frequency estimates than both the ice-structure interaction scenarios, while

299 the fast-varying ice conditions appeared to render damping as accurate as in the case of the Gaussian white

300 noise.

301

302 Table 2. Identified modal parameters (using SSI–cov/ref), standard deviations and absolute errors.

Identified eigenmodes	White noise $f_i \pm \hat{\sigma}_{f_i}(\text{Hz})/(error\ (\text{Hz}))$	Slow-varying ice conditions $f_i \pm \hat{\sigma}_{f_i}(\text{Hz})/(error\ (\text{Hz}))$	Fast-varying ice conditions $f_i \pm \hat{\sigma}_{f_i}(\text{Hz})/(error\ (\text{Hz}))$

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	White noise	Slow-varying ice conditions	Fast-varying ice conditions
Mode 1	2.465 ± 0.004/ (0.000)	2.454 ± 0.005/(0.011)	2.462 ± 0.005/(0.003)
Mode 2	3.923 ± 0.010/(0.003)	3.925 ± 0.027/(0.005)	3.909 ± 0.021/(0.011)
Mode 3	5.517 ± 0.051/ (0.001)	5.527 ± 0.057/(0.011)	5.505 ± 0.068/(0.011)
Mode 4	10.148 ± 0.134/(0.014)	10.173 ± 0.121/(0.011)	10.140 ± 0.072/(0.022)
	White noise $\xi_i \pm \hat{\sigma}_{\xi_i}(\%) / (error\ \%)$	Slow-varying ice conditions $\xi_i \pm \hat{\sigma}_{\xi_i}(\%) / (error\ \%)$	Fast-varying ice conditions $\xi_i \pm \hat{\sigma}_{\xi_i}(\%) / (error\ \%)$
Mode 1	2.149 ± 0.193/ (0.149)	1.627 ± 0.215/ (0.373)	1.804 ± 0.211/ (0.196)
Mode 2	1.793 ± 0.266/ (0.212)	2.239 ± 0.653/ (0.391)	2.070 ± 0.619/ (0.065)
Mode 3	2.623 ± 0.726/ (0.256)	2.092 ± 0.837/ (0.275)	2.188 ± 0.867/ (0.178)
Mode 4	3.840 ± 0.961/ (0.198)	3.438 ± 0.975/ (0.204)	3.566 ± 0.635/ (0.075)

303

304

305 Figure 5. Stabilization diagrams for a) slow-varying ice conditions b) fast-varying ice conditions.

306

307 Since simulations with the ice model were computationally expensive, it was not feasible to check whether the

308 point estimate of variances of the identified frequency and damping coincided with the sample statistics for

309 multiple simulations. Instead, sample statistics were generated using Monte Carlo simulations as follows:

310 Continuous brittle crushing ice forces, $p_{i,m}(t)$, were generated by realization of a typical ice force spectrum

311 from the Norströmsgrund lighthouse, $\Lambda_{pi}(\omega)$, which was discretised using $\Delta\omega = 0.01$ rad s^{-1}:

$$312 \qquad p_{i,m}(t) = \sqrt{2\Delta\omega}\,\text{Re}\left(\sum_{k=1}^{N}\sqrt{\Lambda_{pi}(\omega_k)}\,e^{i(\omega_k t + \alpha_{lk})}\right), \qquad (25)$$

313 where α_{lk} is the phase angle modelled as uniformly random between 0 and 2π.

15

314   The ice forces were first obtained with a sampling frequency of 1660 Hz and further resampled down to 100

315   Hz (Figure 6). The simulated acceleration time series was thereafter generated by using Eqs. (1) and (2) by

316   applying the ice force $p_{i,m}(t)$ onto the ice-action point (cf. Figure 2). In the simulations, the system matrices

317   A, B, C, D were assembled from the same modally reduced order finite element model as described

318   previously. In total 250 acceleration time series were simulated using ice forces.
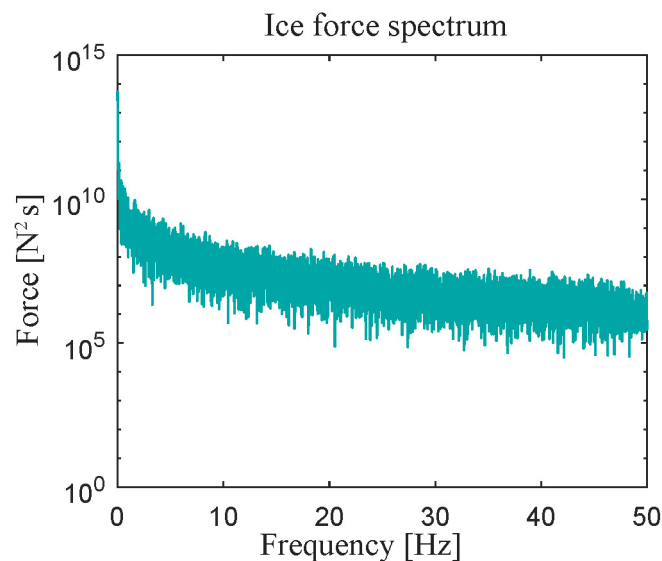
319
320   Figure 6. Ice force spectrum used for monte-carlo simulations.

321   Five percent Gaussian white measurement noise was added to the four acceleration signals, and all the

322   settings for the SSI-cov/ref were identical to those in the abovementioned examples. For each identification,

323   the first automatically identified eigenmodes in the frequency range 0-12 Hz with their corresponding

324   damping and variances were stored. For the sake of further illustrations and result presentation, the standard

325   deviations were calculated from the variances. 250 simulated time series with Gaussian white noise input with

326   an amplitude of $0.3294 \cdot 10^6$ N was used for comparison. The collection of identified frequencies during the

327   ice-structure interactions shows presence of spurious modes, spread out from the true eigenfrequency, most

328   with larger standard deviations (Figure 7). For simulations of ice-structure interactions, the sample standard

329   deviations for the identified frequencies and damping (σ_{f_i} and σ_{ξ_i}) did not correspond to the mean values of

330   the identified standard deviations ($\hat{\bar{\sigma}}_{f_i}$ and $\hat{\bar{\sigma}}_{\xi_i}$, Table 3), a result which is significantly influenced by four

331   spurious frequencies below 2 Hz (Figure 7a). In the case of the simulations with Gaussian white noise, the

332   sample standard deviations corresponded well with the identified standard deviations.

333

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334 Table 3. Natural frequencies and damping ratios of the first identified eigenmode. μ_{f_i} and μ_{ξ_i} : mean values

335 over 250 simulations. $\hat{\bar{\sigma}}_{f_i}$ and $\hat{\bar{\sigma}}_{\xi_i}$: mean of the estimated standard deviations. σ_{f_i} and σ_{ξ_i} : sample standard

336 deviation.

Mode	μ_{f_i} (Hz)	$\hat{\bar{\sigma}}_{f_i}$	σ_{f_i}	μ_{ξ_i} (%)	$\hat{\bar{\sigma}}_{\xi_i}$	σ_{ξ_i}
Mode 1 (ice-structure interaction)	2.459	$1.153 \cdot 10^{-2}$	$1.202 \cdot 10^{-1}$	1.530	$8.096 \cdot 10^{-1}$	$4.288 \cdot 10^{-1}$
Mode 1 (white noise input)	2.465	$4.999 \cdot 10^{-3}$	$4.857 \cdot 10^{-3}$	2.047	$2.152 \cdot 10^{-1}$	$1.977 \cdot 10^{-1}$
Mode 1 (ice-structure interaction) Short time series	2.433	$2.317 \cdot 10^{-2}$	$5.887 \cdot 10^{-2}$	1.859	1.864	$4.035 \cdot 10^{-1}$
Mode 1 (white noise input) Short time series	2.464	$1.598 \cdot 10^{-2}$	$4.577 \cdot 10^{-3}$	2.004	$7.992 \cdot 10^{-1}$	$1.977 \cdot 10^{-1}$

337

17

a) Ice-structure interaction   b) Gaussian white noise input

338

339 Figure 7. Identified frequencies and their uncertainties for each simulation with a) ice-structure interaction,

340 and b) Gaussian white noise input. Each simulation contained *N=59000 sample points.*

341 Until now, the simulated time series had a long duration. In the following section, we describe the analysis of

342 the effects of having a short duration of time series and of resampling down to 30 Hz. The short duration

343 resembles the nature of rapidly changing ice conditions, which vary by site. Because we aim to assess the

344 identification of modal parameters for different ice conditions, the duration constraint is inevitable. Figure 7

345 shows that the variances of the identified frequencies for spurious modes were larger than those for the true

346 eigenmodes; therefore, a tolerance value of the normalised standard deviation that could eliminate spurious

347 modes and retain true eigenmodes was attempted to be determined. Each simulation had 17800 sample points

348 and 5 % noise was added to each acceleration signal. The number of blockrows, number of blocks for the

349 calculation of the variance of the subspace matrix, n_b, and tolerance value for the stabilization criterion, $\dfrac{\hat{\sigma}_{\omega_i}}{\omega_i}$,

350 were changed to 30, 20 and 0.03, respectively. All other settings remained as described above. Figure 8

351 displays the identified eigenfrequencies and their estimated standard deviations for both simulations with ice-

352 structure interaction and Gaussian white noise input. It was noted that some spurious modes remained in the

353 identifications, some of them with standard deviations in frequency nearly as low as the standard deviations

354 of true eigenfrequencies. Therefore, reducing the tolerance value, $\dfrac{\hat{\sigma}_{\omega_i}}{\omega_i}$, further suppressed the correctly

355 identified eigenfrequencies alongside the spurious frequencies. The eigenfrequencies were reasonably well

356 identified during ice-structure interaction, whereas the ensemble statistics in Table 3 (bottom two rows) show

357 that neither the identified damping nor the mean of the estimated standard deviations, $\hat{\bar{\sigma}}_{f_i}$ and $\hat{\bar{\sigma}}_{\xi_i}$ matched

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358 with the sample standard deviations σ_{f_i} and σ_{ξ_i}. These discrepancies may be explained by both the violation

359 of stationary white noise excitation and the chosen slack values for the automatic mode selection routine.

360
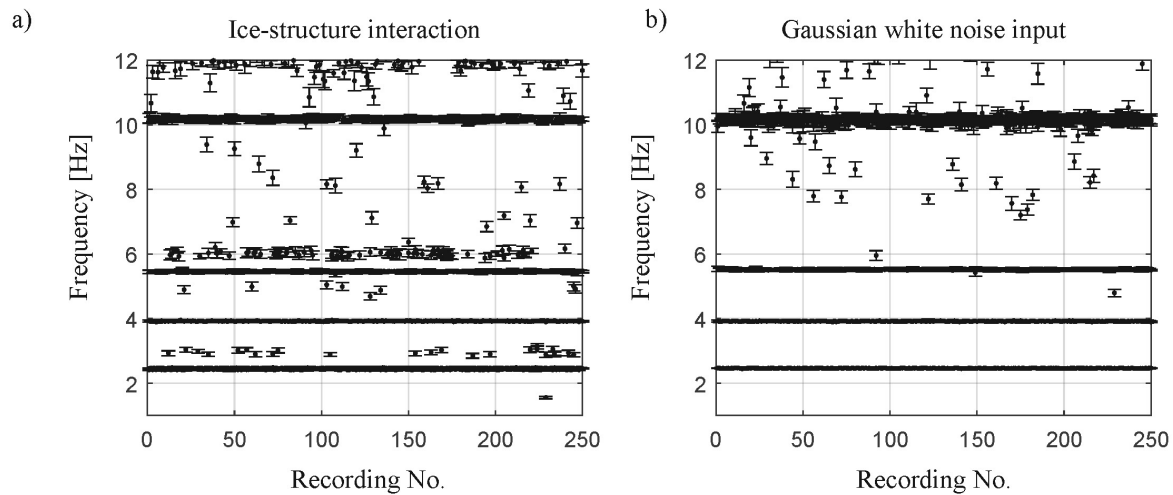
362 Figure 8. Identified frequencies and their uncertainties for each simulation with a) ice-structure interaction,

363 and b) Gaussian white noise input. Each simulation contained *N=17800 sample points.*

364

5. Modal parameter identification using measured data of ice-structure interactions

(a) Measurements of ice-structure interaction at Norströmsgrund lighthouse

367 At the Norströmsgrund lighthouse (Figure 9), the structural responses, ice forces, ice thicknesses, air

368 temperatures, wind speeds, wind directions and ice conditions during the winter seasons from 1999 to 2003

369 were monitored in the measurement projects LOLEIF (LOw LEvel Ice Forces) (36) and STRICE (STRuctures in

370 ICE) (24). Nine panels were installed at the mean water level to measure the ice forces (36), covering the outer

371 perimeter from 0 (North) to 162 degrees. Four acceleration channels (Shaevitz SB) and four inclinometer

372 channels (Schaevitz DC inclinometer series and Applied Geomechanics biaxial Model 716-2A) measured the

373 structural accelerations in the north-south and east-west directions, and the tilts about the same directions,

374 respectively. The accelerometers were located close to the ice-action point at an elevation of +16.5 m and close

375 to the top at an elevation of +37.1 m. A summary of the STRICE project and earlier measurement campaigns

376 can be found in Bjerkås (37).

19
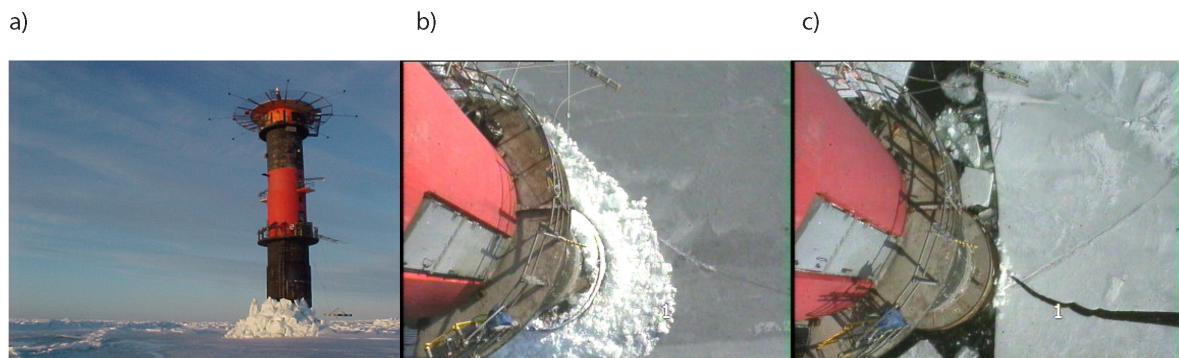
377

378 Figure 9. The Norströmsgrund lighthouse during ice action: a) rubble formation after ice interaction (Photo by

379 courtesy of Basile Bonnemaire); b) crushing ice failure and c) splitting ice failure.

380

(b) Data selection

382 Extensive efforts were devoted to the selection of data for this study, as several criteria were required to be

383 fulfilled; a similar selection was used in (20), although minor modifications were adopted in the present study.

384 The sampling frequency had a minimum value of 30 Hz, video footage was available to define the type of

385 failure, and the ice failure was governed by one of the aforementioned failure modes for a minimum of 10

386 minutes. The individual data files contained time series of various lengths, and they were selected by

387 operators to capture specific types of interactions. Often, one data file had several events of interaction that

388 fulfilled the criteria above, and each of these events is hereon referred to as a recording. In total, 150

389 recordings with lengths of 10 minutes were selected and further used in this study; of these, the number of

390 recordings pertaining to continuous crushing, flexural, splitting, ice floe, and creep failures were 79, 30, 14, 8

391 and 19, respectively. Examples of acceleration signals during continuous crushing and flexural failures are

392 displayed in Figure 10. For these examples, the continuous crushing excites the modes with lower

393 eigenfrequencies stronger than the flexural failure does (Figures 10 b and d). An important disadvantage with

394 the STRICE dataset was that data were stored with different sampling frequencies depending on the level of

395 dynamic response of the lighthouse. If the vibrations were significant, the original data was resampled down

396 and stored at 100 Hz, whereas if less dynamic ice action was present, data was resampled and stored at 30 Hz.

397 For the purpose of this study, regardless of which sampling frequency the data were stored at, all

398 accelerations were resampled to 30 Hz.

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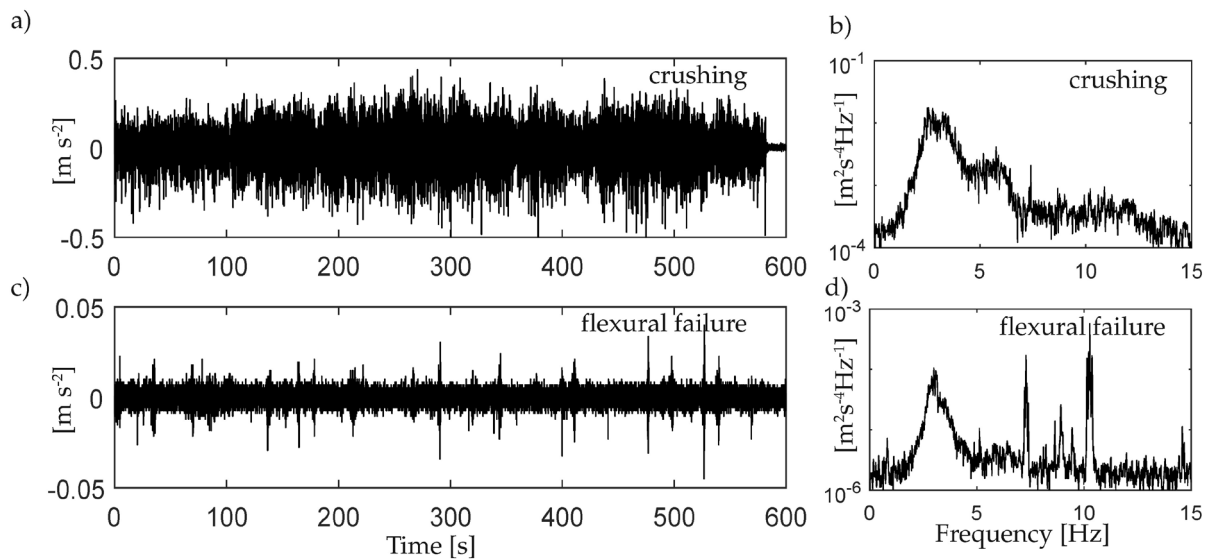
400   Figure 10. Accelerations measured during: crushing failure a) and b), and flexural failure c) and d).

401

(c)   Modal parameters and their uncertainty

403   The measured data were used in conjunction with the automatic routine for identifying the modal parameters

404   and their uncertainties. The automatic routine involved recordings of 17800 sample points, sampling

405   frequency of 30 Hz and the same input values as for the simulated examples in Figure 8. The identified

406   eigenfrequencies and corresponding standard deviations are shown in Figure 11a. A collection of points forms

407   horizontal branches, indicating the eigenfrequencies at approximately 2.9, 5.0, 7.5 and 10.2 Hz. A scatter of

408   points with somewhat higher standard deviations is displayed between these horizontal lines. Assuming that

409   these are not true eigenfrequencies of the lighthouse, another attempt is made to eliminate these by lowering

410   the tolerance value $\dfrac{\hat{\sigma}_{\omega_i}}{\omega_i}$ for the stabilization criterion, now set as 0.01. The horizontal branches are more

411   pronounced as much of the scatter is successfully eliminated via the tolerance value (Figure 11b), and another

412   branch becomes apparent at approximately 12.4 Hz.

413

21

a) $\dfrac{\hat{\sigma}_{\omega_i}}{\omega_i} < 0.03$ b) $\dfrac{\hat{\sigma}_{\omega_i}}{\omega_i} < 0.01$
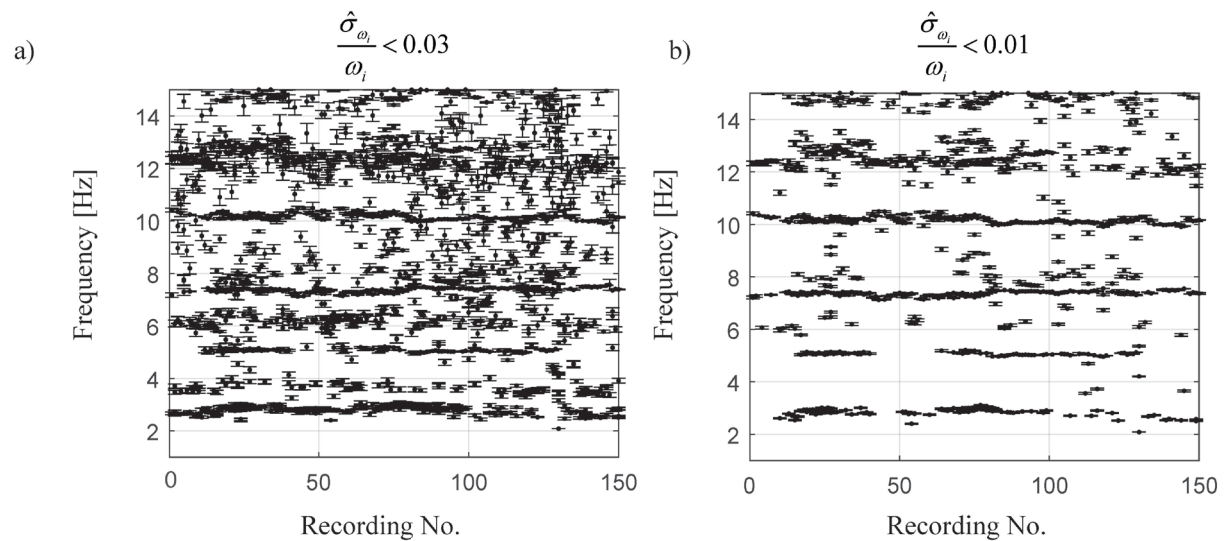
414

415 Figure 11. Identified frequencies and their uncertainties for each recording with two different tolerance values

416 for the stabilization criterion: a) tolerance criterion $\dfrac{\hat{\sigma}_{\omega_i}}{\omega_i} < 0.03$, and b) tolerance criterion $\dfrac{\hat{\sigma}_{\omega_i}}{\omega_i} < 0.01$.

417 Figure 12 displays the identified damping obtained from the poles used to extract the eigenfrequencies in

418 Figure 11b. Large variations can be observed between the identified damping at frequencies corresponding to

419 eigenmodes 1 and 5 (approximately 2.9 Hz and 12.4Hz, respectively) and the damping identified for the other

420 eigenmodes (at 5.0, 7.5 and 10.2 Hz, respectively). The eigenmodes 1 and 5 have significantly higher damping

421 than the other eigenmodes. A possible explanation for this observation may be that eigenmodes 1 and 5 play a

422 more active role in the interaction between the ice and structure than the other eigenmodes do. Similar scatters

423 can commonly be observed for wind-sensitive bridges, for which it is well established that the modal damping

424 varies with the wind velocity (38). High damping was also estimated during laboratory model-scale ice-

425 structure interaction, where damping as high as 13 % was identified during ice crushing (18). Another

426 hypothesis considers the data quality, which may be inadequate for damping estimates for low excitation

427 amplitudes. However, these hypotheses require verification with higher quality data.

428

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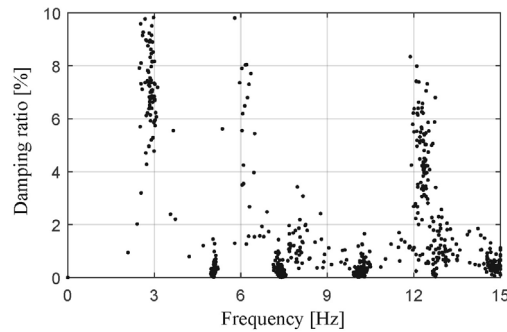
Figure 12. Identified damping for all recordings (tolerance criterion $\dfrac{\hat{\sigma}_{\omega_i}}{\omega_i} < 0.01$)

430

431 Since the recordings were categorised according to the governing interaction mode, it was possible to

432 investigate differences between the identified quantities for different interaction modes. The two interaction

433 modes, crushing and flexural failure, excited the most frequencies (Figure 13). When the ice rested against the

434 structure (creep, Figure 13), higher eigenmodes at 5.0, 7.5 and 10.2 Hz were excited for most recordings,

435 whereas no eigenfrequencies were identified at around the first eigenmode at 2.9 Hz, and only a few were

436 identified in the vicinity of 12.4 Hz. These observations support the hypothesis that the eigenmodes at 2.9 Hz

437 and 12.4 Hz participate in the interaction process at the ice-structure interface, thereby causing higher

438 damping. It is also suggested that eigenmodes 2, 3 and 4 are less affected by both the interaction process at the

439 ice-structure interface and the support provided from the ice resting against the structure.

440

441
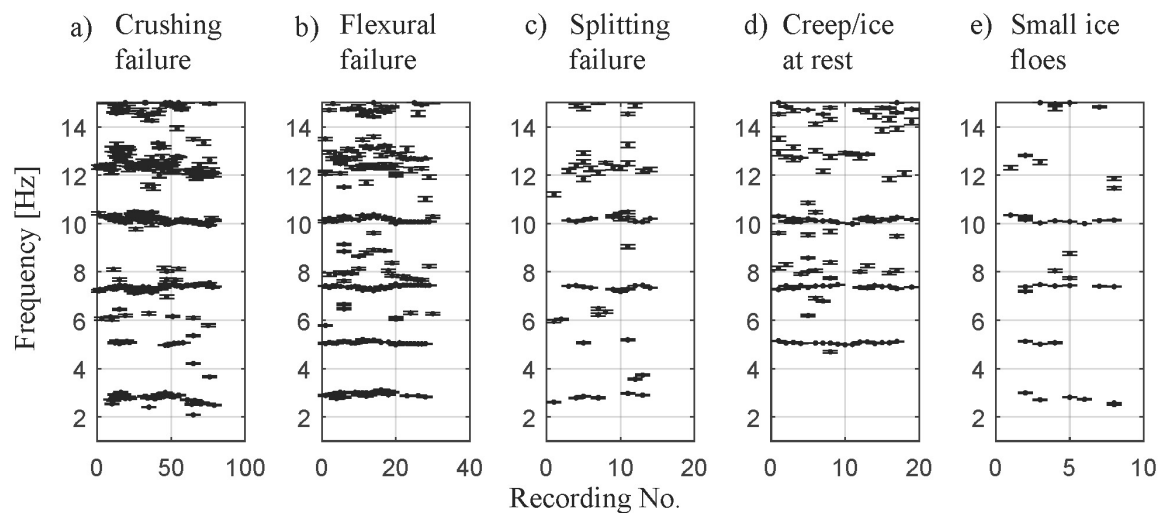
442 Figure 13. Frequencies identified from the automatically selected poles for different regimes of ice-structure

443 interaction: a) crushing failure; b) flexural failure c) splitting failure; d) creep and e) floe ice.

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444   The horizontal branches formed by frequencies in Figures 11 and 13 indicate the eigenfrequencies. These

445 branches barely resemble straight lines. Laboratory experiments of ice-induced vibrations show that the

446 frequency content of the structural response could be dominated by different modes of the structure

447 depending on the ice velocity (39). In full-scale, the ice velocity near a structure may change in a matter of

448 seconds, thereby influencing the governing failure mode and thus the structural response (40). The observed

449 variability in the results may also be explained by the differences in the ice thickness, ice-drift direction,

450 mechanical properties of the ice, air temperature, ice rubble accumulation at the structure, or other met-ocean

451 conditions that in turn cause differences in the structural behaviour.

452   The numerical simulations were limited to a specific mode of interaction, namely continuous brittle crushing.

453 In the full-scale measurements, it is important to note that even if a failure mode governed the process for

454 each recording, other failure mechanisms were often involved in the process. For instance, some local crushing

455 often took place during interactions governed by flexural failure, and even for short time sequences, the

456 interactions were entirely governed by crushing failure. This led to a non-stationary force and response,

457 possibly contributing to the observed scatter in the experimental results. Presently models for predicting

458 dynamic response in such mixed-mode scenarios are limited to the combination of ice crushing and buckling

459 (41).

460   Even though the structure is almost symmetric, there may be differences in the modes depending on the

461 direction of excitation. The recordings ranging from 23-30 in Figure 13b display identified frequencies that

462 resemble straight lines. For all these recordings, the interaction was governed by flexural failures, the ice-drift

463 direction was from south to south-southwest and the air temperature was warmer than -2 $^{\circ}$C.

464   Attempts were made to test whether the data were inaccurate at low vibration amplitudes by sorting out

465 the recorded events with the highest standard deviations of the acceleration; however no clear indication was

466 found. Similar results were also obtained using the UPC algorithm instead of the SSI-cov/ref. It is, however, a

467 fact that this is an imperfect axisymmetric problem with asymmetric mass distribution at the waterline caused

468 by the heavy force panels and varying mass distribution due to ice rubble. Further, an unknown extent of

469 nonlinearity is involved and the system identification is handled with only a few sensors. An attempt was also

470 made to investigate whether the first eigenmode had preferred mode shape alignments in the horizontal

471 plane. Only vague indications of the preferred directions were found, but the results highly depend on the

472 slack values for the automated selection of modes, and such analysis lies beyond the scope of this study. Due

473 to the few sensor locations, the identified mode shapes and their variances were also not considered in this

474 study. By using a hierarchy clustering method as described in (42), similar results as shown in Figure 11b were

475 obtained when low importance was assigned to the weights for the MAC-values and clusters with few poles

476 were discarded. It must also be noted that the identified eigenfrequencies indicate that the FE model contained

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477 modelling errors. Updating the model would benefit numerical simulations in the future, but it does not

478 influence the conclusions of this study.

479   Nord et al. (40) studied 61 events of frequency lock-in vibrations of the Norströmsgrund lighthouse. This is

480 a mode of vibrations caused by crushing failure under certain conditions; lock-in vibrations have been a topic

481 of research during the past 50 years (43) and still represent a popular research topic. The vibrations' signature

482 is an amplified periodic response near one of the eigenfrequencies of the structure. For the 61 frequency lock-

483 in events analysed, the governing frequency of vibrations was 2.3 Hz, more than 20% lower than the

484 suggested eigenfrequency found in this study at 2.9 Hz, which is somewhat more than that reported for

485 channel markers in the Baltic Sea (44).

486

6. Conclusions

488 Simulated and measured recordings of a structure subjected to ice actions were used to assess the possibility

489 to identify consistent modal parameters during ice-structure interaction and consistent system changes with

490 observed ice conditions.

491 The combined use of ice-structure interaction simulations and identification of variances alongside modal

492 parameters rendered insight into how the identified modal parameters were influenced by the presence of ice.

493 The simulations guided the choice of tolerance values for the stabilization criterion, in turn rendering some

494 consistent estimates of eigenfrequencies for different ice conditions with measured data.

495 Even though the eigenfrequency often varied between each data recording, some consistency was noted

496 between the ice failure mode and identified frequencies. The lowest eigenfrequency at approximately 2.9 Hz

497 was identified in all interaction modes except in the case of creep, i.e., when the ice was resting against the

498 structure. This eigenmode, as well as another mode at 12.4 Hz, exhibited significantly higher damping than

499 the other eigenmodes, and these eigenmodes were mostly identified during crushing and flexural ice failures.

500 It is therefore suggested that these modes were influenced by the interaction process at the ice-structure

501 interface. Since eigenmodes at 5.0, 7.5 and 10.2 Hz were identified also during creep and had for all cases

502 significantly lower damping, it is suggested that these higher modes were less influenced by the interaction

503 process. For the sake of SHM, identifying eigenmodes insensitive to the interaction process significantly

504 reduces the environmental variability and may turn out useful in selecting damage-sensitive features.

505 The variability in the results may be explained by the violation of the underlying assumptions used to derive

506 the applied identification routine, the structural complexity and limited sensor data of uncertain quality.

507

Additional Information

509

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513

Authors' Contributions

515

516 T.S.N. performed the simulations, identifications, and data analysis, and drafted the manuscript.
517 H.H. conducted the simulations with the ice model.
518 Ø.W.P. performed the data analysis.

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536 References

1.   Blenkarn KA, editor Measurment and analysis of ice forces on Cook Inlet structure. Offshore Technology Conference; 1970; Houston, TX.
2.   Peyton HR. Sea Ice Strength: University of Alaska; 1967.
3.   Frederking R, Haynes FD, Hodgson TP, Sayed M. Static and dynamic ice loads on the Yamachiche lightpier, 1984-1986. 8th International Symposium on Ice, IAHR Iowa, USA 1986. p. 115-26.
4.   Määttänen M. Experiences of ice forces against a steel lighthouse mounted on the seabed, and proposed constructional refinements. Port and Ocean Engineering under Arctic conditions (POAC); Fairbanks, Alaska 1975. p. 857-67.
5.   Bjerkås M, Nord TS. Ice action on Swedish lighthouses Revisited. The 23rd IAHR International symposium on ice; Ann Arbor, Michigan, USA 2016.
6.   Timco G, Croasdale KR. How well can we predict ice loads? IAHR; Sapporo, Japan 2006. p. 167-74.
7.   Danys JV. Ice forces on old and new offshore lighthouses in the St. Lawrence Waterway. Port and Ocean Engineering under Arctic Conditions (POAC); St. Johns, Newfoundland, Cananda 1977. p. 115-38.
8.   Mättänen M, Kärnä T. ISO 19906 ice crushing load design extension for narrow structures. Proceedings of the 21st International Conference on
Port and Ocean Engineering under Arctic Conditions (POAC); Montréal, Canada 2011.
9.   Bjørk B. Ice-induced Vibration of Fixed Offshore Structures. Part 2: Experience with Baltic Lighthouses: Ship Research Institute of Norway, Information Department; 1981.

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10.   Hendrikse H, Metrikine A. Interpretation and prediction of ice induced vibrations based on contact area variation. International Journal of Solids and Structures. 2015;75–76:336-48.

11.   Määttänen M. On conditions for the rise of self-excited ice induced autonomous oscillations in slender marine pile structures [PhD]: UNIVERSITY OF OULU; 1978.

12.   Sodhi DS. Ice-induced vibration of structures. Proceedings of the 9th IAHR International Symposium on Ice; Sapporo, Japan 1988. p. 625-57.

13.   Jefferies MG, Wright WH. Dynamic response of "Molikpaq" to ice-structure interaction. Proc 7th International conference on Offshore Mechanics and Arctic Engineering (OMAE 88); Houston, Texas, United States 1988. p. 201-20.

14.   Londoño NA. Use of vibration data for structural health monitoring of Bridges. Ottawa: Carleton University; 2006.

15.   Brown TG, Tibbo JS, Tripathi D, Obert K, Shrestha N. Extreme ice load events on the Confederation Bridge. Cold Regions Science and Technology. 2010;60(1):1-14.

16.   Shrestha N, Brown TG. 20 years of monitoring of ice action on the Confederation Bridge piers. Cold Regions Science and Technology. 2018;151:208-36.

17.   Hendrikse H, Metrikine A, Evers K-U. A method to measure the added mass and added damping in dynamic ice-structure interaction. "Ice Research for a Sustainable Environment"(IAHR) Dalian, China Paper no 094 2012.

18.   Singh SK, Timco GW, Frederking R, Jordaan IJ. Tests of ice crushing on a flexible structure. OMAE Houston TX. USA 1990. p. 89-94.

19.   Hendrikse H. Ice-induced vibrations of vertically sided offshore structures: Delft University of Technology; 2017.

20.   Nord TS, Kvåle KA, Petersen ØW, Bjerkås M, Lourens E-M. Operational modal analysis on a lighthouse structure subjected to ice actions. Procedia Engineering. 2017;199:1014-9.

21.   Reynders E, Pintelon R, De Roeck G. Uncertainty bounds on modal parameters obtained from stochastic subspace identification. Mechanical Systems and Signal Processing. 2008;22(4):948-69.

22.   Döhler M, Mevel L. Efficient multi-order uncertainty computation for stochastic subspace identification. Mechanical Systems and Signal Processing. 2013;38(2):346-66.

23.   Jordaan IJ. Mechanics of ice–structure interaction. Engineering Fracture Mechanics. 2001;68(17–18):pp.1923-60.

24.   Kärnä T, Jochmann P. Field observations on failure modes. Port and Ocean Engineering under Arctic Conditions; Trondheim, Norway 2003. p. 839-49.

25.   Wells J, Jordaan I, Derradji-Aouat A, Taylor R. Small-scale laboratory experiments on the indentation failure of polycrystalline ice in compression: Main results and pressure distribution. Cold Regions Science and Technology. 2011;65(3):pp. 314-25.

26.   Nord TS, Øiseth O, Lourens E-M. Ice force identification on the Norströmsgrund lighthouse. Computers & Structures. 2016;169(Supplement C):24-39.

27.   Peeters B, De Roeck G. Reference-based stochastic subspace identification for output-only modal analysis. Mechanical Systems and Signal Processing. 1999;13(6):855-78.

28.   Reynders E, Maes K, Lombaert G, De Roeck G. Uncertainty quantification in operational modal analysis with stochastic subspace identification: Validation and applications. Mechanical Systems and Signal Processing. 2016;66-67:13-30.

29.   Mellinger P, Döhler M, Mevel L. Variance estimation of modal parameters from output-only and input/output subspace-based system identification. Journal of Sound and Vibration. 2016;379:1-27.

30.   Döhler M, Lam X-B, Mevel L. Uncertainty quantification for modal parameters from stochastic subspace identification on multi-setup measurements. Mechanical Systems and Signal Processing. 2013;36(2):562-81.

31.   ISO. ISO/FDIS 19906. 2010. p. 188.

32.   Sodhi D, Haehnel R. Crushing Ice Forces on Structures. Journal of Cold Regions Engineering. 2003;17(4):153-70.

33.   Hendrikse H, Nord TS. Dynamic response of an offshore structure interacting with an ice floe failing in crushing. Marine Structures. 2019;65:271-90.

Phil. Trans. R. Soc. A.

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Submitted to Phil. Trans. R. Soc. A - Issue

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34.   Hendrikse H. Model for simulation of dynamic ice-structure interaction for vertically sided offshore structures - SDOF MATLAB implementation. Mendeley Data v2, https://data.mendeley.com/datasets/582m8565dj/2, 2019.

35.   Heinonen J, Kärnä T, Luo C. Dynamic Behavior of the Nordströmsgrund Lighthouse, STRICE-REPORT. VTT Technical Research Centre of Finland and Luleå University of Technology; 2003.

36.   Jochmann P, Schwarz J. Ice force measurements at lighthouse Norströmsgrund- winter 1999, LOLEIF Report No. 5, MAS3-CT-97-0098. Hamburgische shiffbau-versuchsantalt GmbH; 1999.

37.   Bjerkås M. Ice action on offshore structures [PhD]: NTNU, ISBN 82-471-7756-0; 2006.

38.   Hong AL, Ubertini F, Betti R. Wind Analysis of a Suspension Bridge: Identification and Finite-Element Model Simulation. Journal of Structural Engineering. 2011;137(1):133-42.

39.   Nord TS, Øiseth O, Lourens E, Määttanen M, Høyland KV. Laboratory experiments to study ice-induced vibrations of scaled model structures during their interactions with level ice at different ice velocities. Cold Regions Science and Technology. 2015(119):1-15.

40.   Nord TS, Samard ija I, Hendrikse H, Bjerkås M, Høyland KV, Li H. Ice-induced vibrations of the Norströmsgrund lighthouse. Cold Regions Science and Technology. 2018;155:237-51.

41.   Hendrikse H, Metrikine A. Ice-induced vibrations and ice buckling. Cold Regions Science and Technology. 2016;131:129-41.

42.   Magalhães F, Cunha Á, Caetano E. Online automatic identification of the modal parameters of a long span arch bridge. Mechanical Systems and Signal Processing. 2009;23(2):316-29.

43.   Määttänen M. Ice induced frequency lock-in vibrations - Coverging towards concensus. Proceedings of the 23rd International Conference on Port and Ocean Engineering under Arctic Conditions; Trondheim, Norway 2015.

44.   Kärnä T. Steady-state vibrations of offshore structures. Hydrotechnical Construction. 1994;28(8):446-53.

Tables

Table 1. Modal properties of simulation model.

	Mode 1 (2)	Mode 3 (4)	Mode 5 (6)	Mode 7 (8)
f_i [Hz]	2.465	3.920	5.516	10.162
ξ_i [%]	2.000	2.005	2.367	3.642

Table 4. Identified modal parameters, their uncertainties and percentage errors.

Identified eigenmodes	White noise $f_i \pm \hat{\sigma}_{f_i}(\text{Hz})/(\textit{error}\ (\text{Hz}))$	Slow-varying ice conditions $f_i \pm \hat{\sigma}_{f_i}(\text{Hz})/(\textit{error}\ (\text{Hz}))$	Fast-varying ice conditions $f_i \pm \hat{\sigma}_{f_i}(\text{Hz})/(\textit{error}\ (\text{Hz}))$
Mode 1	2.465 ± 0.004/ (0.000)	2.454 ± 0.005/(0.011)	2.462 ± 0.005/(0.003)
Mode 2	3.923 ± 0.010/(0.003)	3.925 ± 0.027/(0.005)	3.909 ± 0.021/(0.011)
Mode 3	5.517 ± 0.051/ (0.001)	5.527 ± 0.057/(0.011)	5.505 ± 0.068/(0.011)

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	White noise	Slow-varying ice conditions	Fast-varying ice conditions
Mode 4	$10.148 \pm 0.134/(0.014)$	$10.173 \pm 0.121/(0.011)$	$10.140 \pm 0.072/(0.022)$
	$\xi_i \pm \hat{\sigma}_{\xi_i}(\%) / (error\ \%)$	$\xi_i \pm \hat{\sigma}_{\xi_i}(\%) / (error\ \%)$	$\xi_i \pm \hat{\sigma}_{\xi_i}(\%) / (error\ \%)$
Mode 1	$2.149 \pm 0.193/ (0.149)$	$1.627 \pm 0.215/ (0.373)$	$1.804 \pm 0.211/ (0.196)$
Mode 2	$1.793 \pm 0.266/ (0.212)$	$2.239 \pm 0.653/ (0.391)$	$2.070 \pm 0.619/ (0.065)$
Mode 3	$2.623 \pm 0.726/ (0.256)$	$2.092 \pm 0.837/ (0.275)$	$2.188 \pm 0.867/ (0.178)$
Mode 4	$3.840 \pm 0.961/ (0.198)$	$3.438 \pm 0.975/ (0.204)$	$3.566 \pm 0.635/ (0.075)$

Table 5. Natural frequencies and damping ratios of the first identified mode. μ_{f_i} and μ_{ξ_i}: mean values over 250 simulations. $\hat{\bar{\sigma}}_{f_i}^2$ and $\hat{\bar{\sigma}}_{\xi_i}^2$: mean of the estimated variances. $\sigma_{f_i}^2$ and $\sigma_{\xi_i}^2$: empirical sample variances.

Mode	μ_{f_i} (Hz)	$\hat{\bar{\sigma}}_{f_i}$	σ_{f_i}	μ_{ξ_i} (%)	$\hat{\bar{\sigma}}_{\xi_i}$	σ_{ξ_i}
Mode 1 (ice-structure interaction)	2.459	$1.153 \cdot 10^{-2}$	$1.202 \cdot 10^{-1}$	1.530	$8.096 \cdot 10^{-1}$	$4.288 \cdot 10^{-1}$
Mode 1 (white noise input)	2.465	$4.999 \cdot 10^{-3}$	$4.857 \cdot 10^{-3}$	2.047	$2.152 \cdot 10^{-1}$	$1.977 \cdot 10^{-1}$
Mode 1 (ice-structure interaction) Short time series	2.433	$2.317 \cdot 10^{-2}$	$5.887 \cdot 10^{-2}$	1.859	1.864	$4.035 \cdot 10^{-1}$
Mode 1 (white noise input) Short time series	2.464	$1.598 \cdot 10^{-2}$	$4.577 \cdot 10^{-3}$	2.004	$7.992 \cdot 10^{-1}$	$1.977 \cdot 10^{-1}$

Figure and table captions

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a)

b)

c)

d)

e)

Mode 1 (2)
2.465 Hz

Mode 3 (4)
3.920 Hz

Mode 5 (6)
5.516 Hz

Mode 7 (8)
10.162 Hz
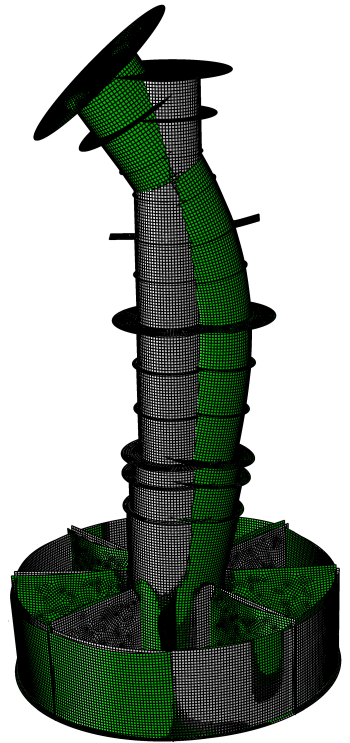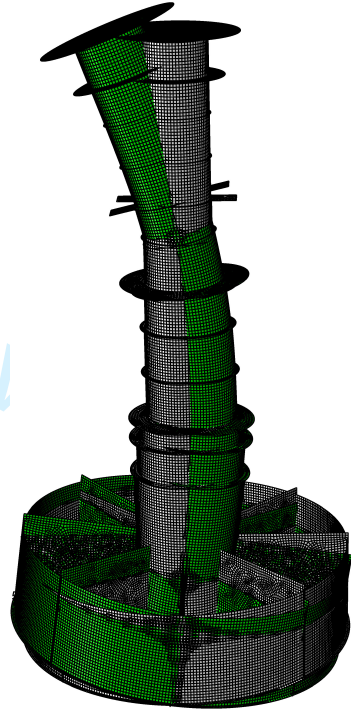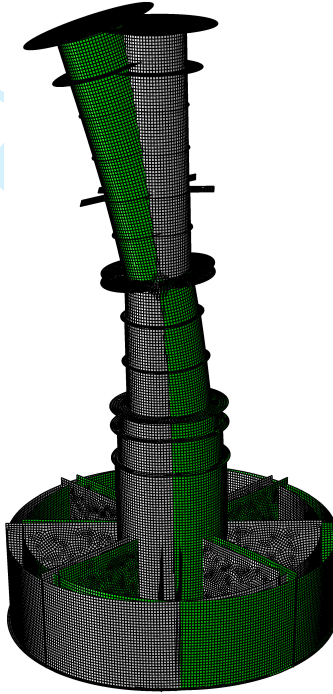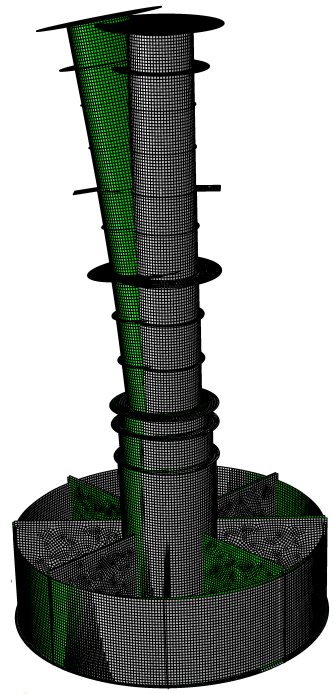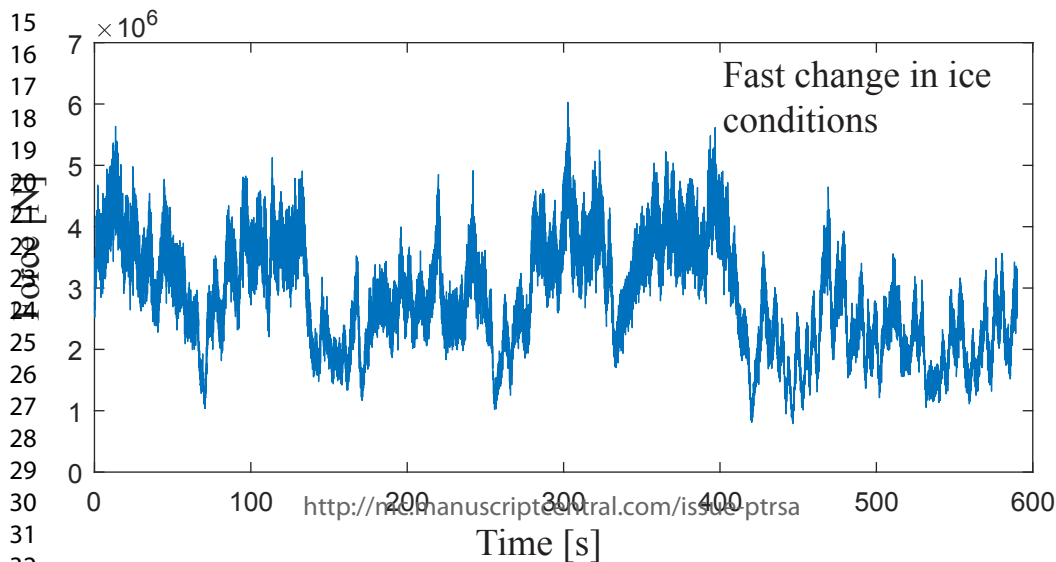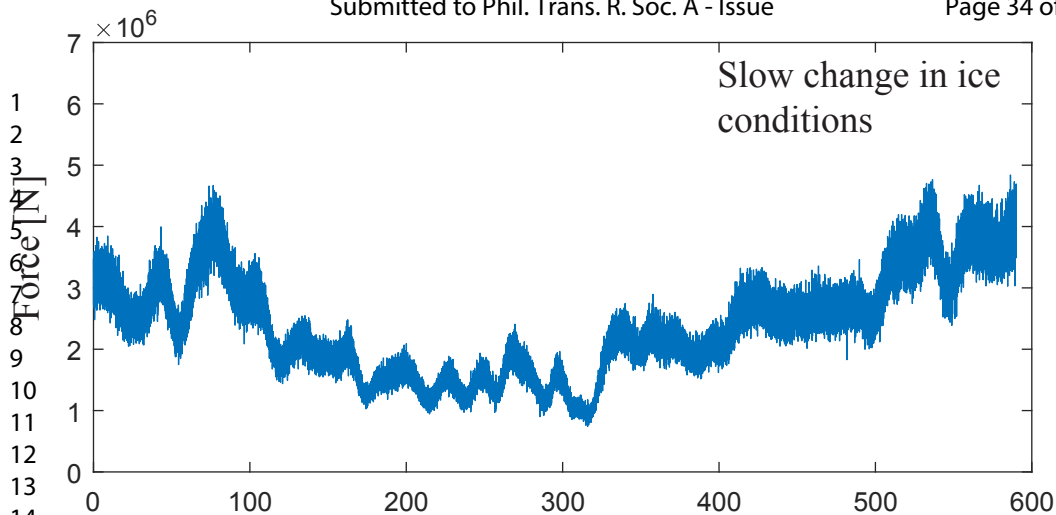
a) Criteria: $\Delta\omega/\omega$ < 1% and $\Delta\xi/\xi$ < 5% MAC > 95% and σ_ω/ω < 0.05
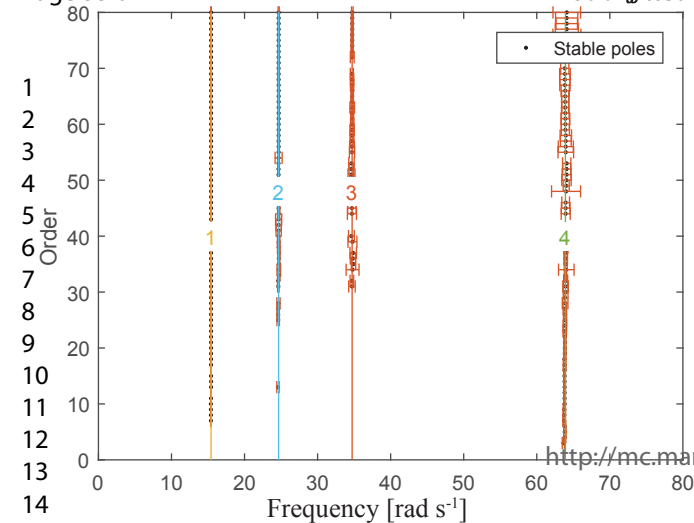
b) Criteria: $\Delta\omega/\omega$ < 1% and $\Delta\xi/\xi$ < 5% MAC > 95% and σ_ω/ω < 0.05

1 x stability
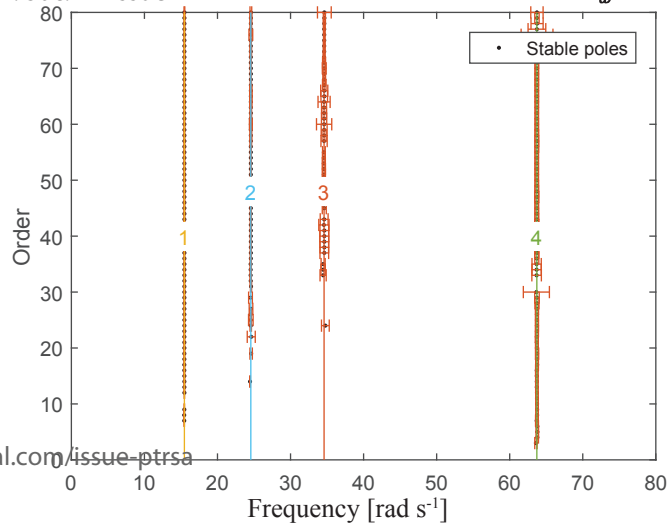
Ice force spectrum

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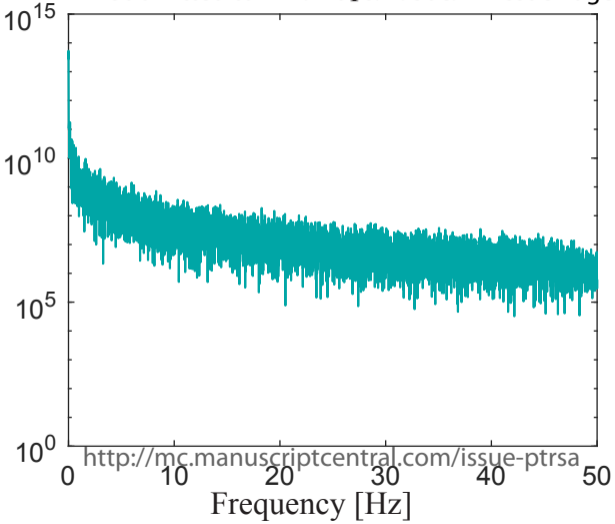
a) Ice-structure interaction
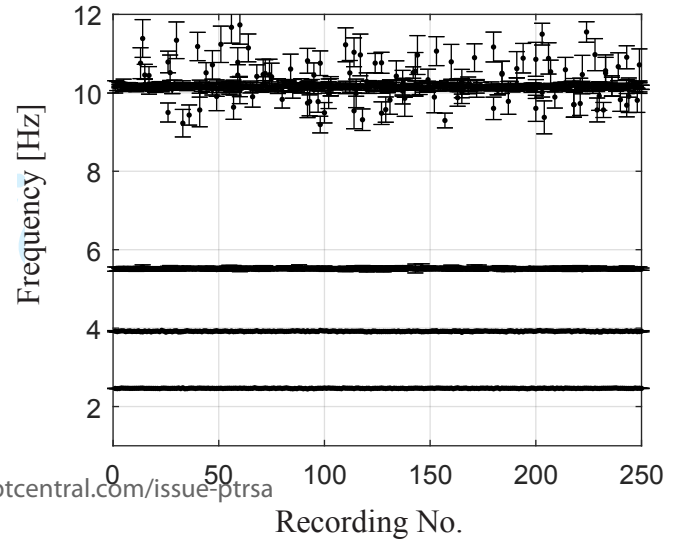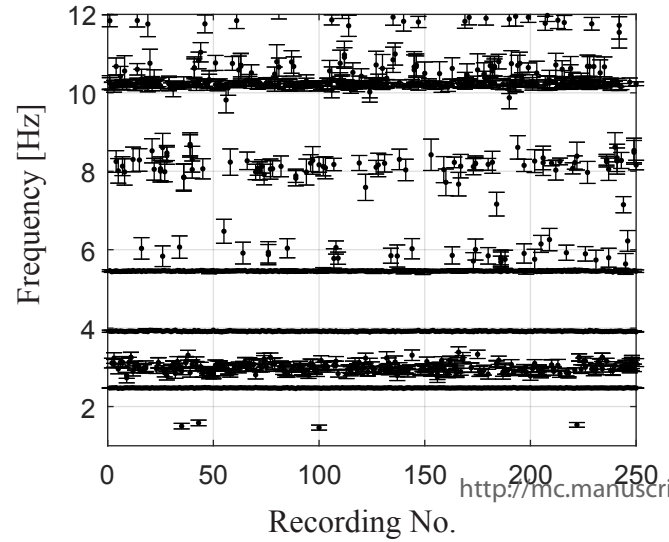
b) Gaussian white noise input

a)
b)
c)

a)

crushing

1
2
3
4
5

b)

crushing

c)

flexural failure

6
7
8
9
10
11
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13

d)

flexural failure

14
15
16

a) $\dfrac{\hat{\sigma}_{\omega_i}}{\omega_i} < 0.03$ b) $\dfrac{\hat{\sigma}_{\omega_i}}{\omega_i} < 0.01$
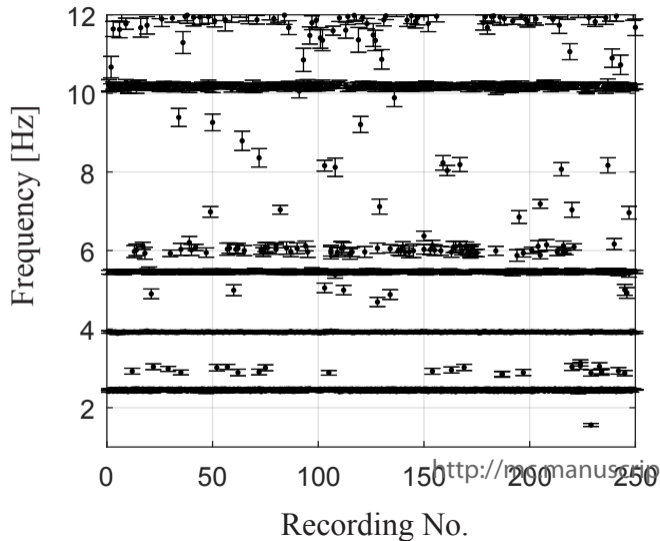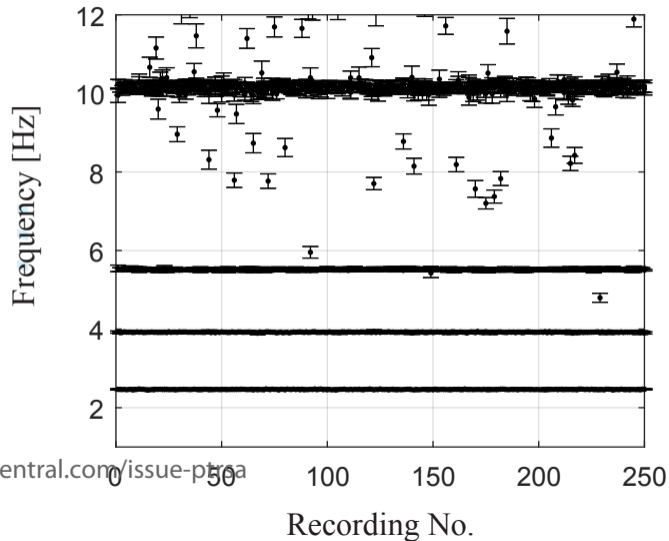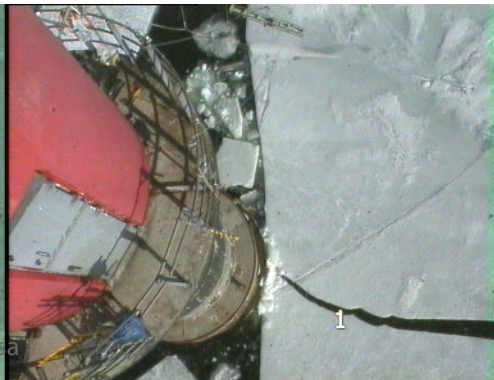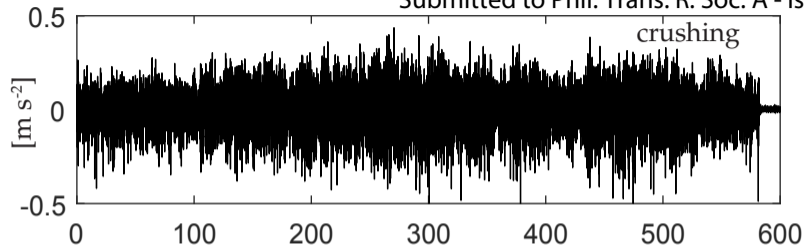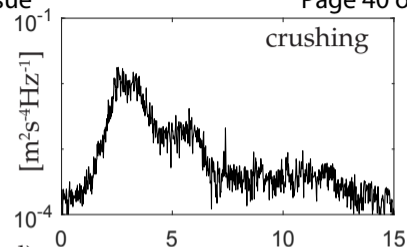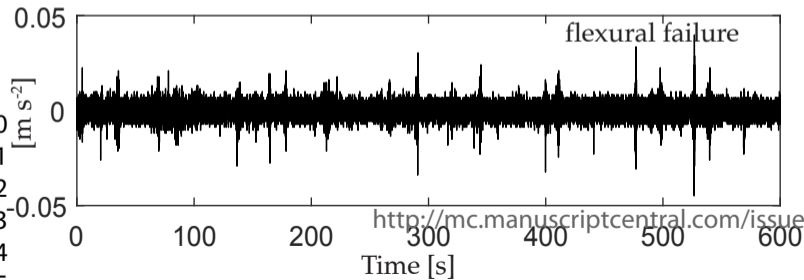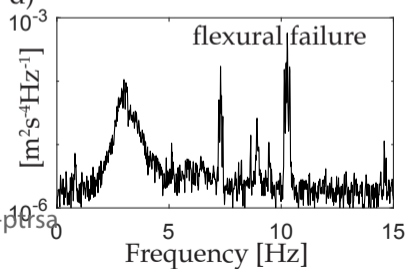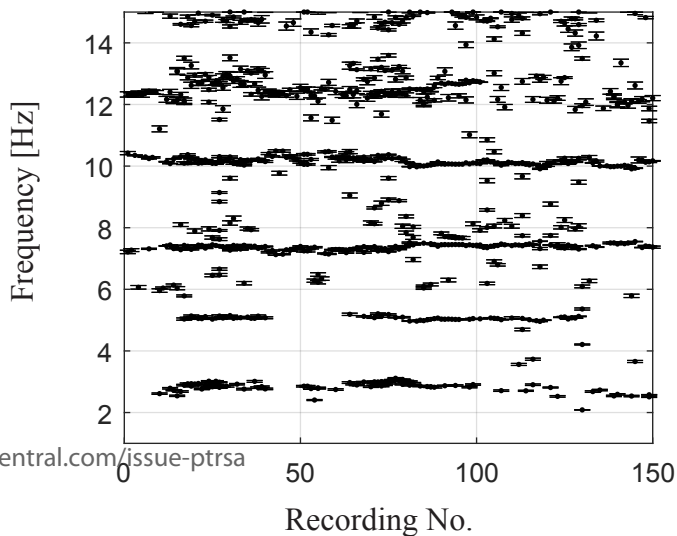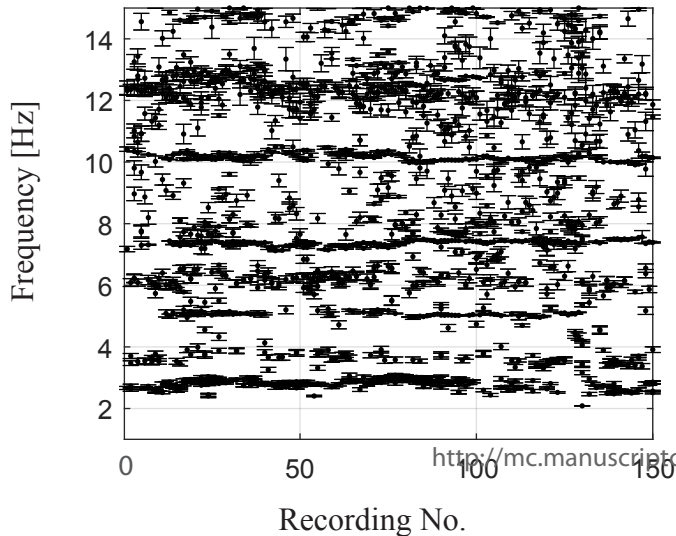
a) Crushing failure b) Flexural failure c) Splitting failure d) Creep/ice at rest e) Small ice floes

Frequency [Hz]

Recording No.