restart

#Initialize linear algebra
$with(LinearAlgebra)$:
$with(VectorCalculus)$:
$with(ArrayTools)$:
$with(CodeGeneration)$:

#Import expressions from other worksheets:

read "../Equations_of_motion/eom.m";
read "diff_eq_phi.m";

$DDphi\_sol := f$:

#Inserting for virtual holonomic constraint:

$q1sub := \mathrm{theta}(t)$:
$q2sub := \mathrm{phi}(\mathrm{theta}(t))$:

$eom := M.Vector([DDq1, DDq2]) + Cc.Vector([Dq1, Dq2]) + G$:

$$aby := collect\left(subs(q1 = q1sub, q2 = q2sub, Dq1 = diff(q1sub, t), Dq2 = diff(q2sub, t), DDq1 = diff(q1sub, t, t), DDq2 = diff(q2sub, t, t), eom), \left[\frac{d^2}{dt^2}\,\theta(t), \left(\frac{d}{dt}\,\theta(t)\right)^2\right]\right):$$

$aby1 := aby(1)$;
$aby2 := aby(2)$;

$$aby1 := collect\left(aby1, \left[\frac{d^2}{dt^2}\,\theta(t), \left(\frac{d}{dt}\,\theta(t)\right)^2\right]\right):$$
$$aby2 := collect\left(aby2, \left[\frac{d^2}{dt^2}\,\theta(t), \left(\frac{d}{dt}\,\theta(t)\right)^2\right]\right):$$

$$alpha1 := coeff\left(aby1, \frac{d^2}{dt^2}\,\theta(t)\right);$$
$$alpha2 := coeff\left(aby2, \frac{d^2}{dt^2}\,\theta(t)\right);$$
$$gamma1 := subs\left(\left(\frac{d}{dt}\,\theta(t)\right)^2 = 0, \frac{d^2}{dt^2}\,\theta(t) = 0, aby1\right);$$
$$gamma2 := subs\left(\left(\frac{d}{dt}\,\theta(t)\right)^2 = 0, \frac{d^2}{dt^2}\,\theta(t) = 0, aby2\right);$$

$$beta1 := subs\left(\frac{d^2}{dt^2}\,\theta(t) = 0, \left(\frac{d}{dt}\,\theta(t)\right)^2 = 1, aby1\right) - gamma1;$$

$\beta 2 := subs\left(\frac{d^2}{dt^2}\theta(t)=0, \left(\frac{d}{dt}\theta(t)\right)^2=1, aby2\right)-\gamma 2;$

#Inserting for error coordinates:

$q1sub := \mathrm{theta}(t) :$
$q2sub := y(t) + \mathrm{phi}(\mathrm{theta}(t)) :$

$eom\_y := subs(q1 = q1sub, q2 = q2sub, Dq1 = diff(q1sub, t), Dq2 = diff(q2sub, t), DDq1 = diff(q1sub, t, t), DDq2 = diff(q2sub, t, t), eom);$
$temp := solve\left(\{eom\_y[1], eom\_y[2]\}, \left\{\frac{d^2}{dt^2}\theta(t), \frac{d^2}{dt^2}y(t)\right\}\right) :$
$DDtheta\_sol := temp[1];$
$DDy\_sol := collect\left(temp[2], \left[D^{(2)}(\phi)(\theta(t))\right]\right);$

#Inserting phi"=f0:

$DDtheta\_sol\_DDphi\_sub := subs\left(D^{(2)}(\phi)(\theta(t)) = DDphi\_sol, DDtheta\_sol\right);$
$DDy\_sol\_DDphi\_sub := subs\left(D^{(2)}(\phi)(\theta(t)) = DDphi\_sol, DDy\_sol\right);$

#Simplifying aby with weights:

$mu1 := p3 :$
$mu2 := p2 \cdot \cos(\mathrm{theta}(t) - \mathrm{phi}(\mathrm{theta}(t))) :$

$sub\_out\_time := (f) \rightarrow subs(diff(\mathrm{theta}(t), t, t) = DDtheta, diff(\mathrm{theta}(t), t) = Dtheta, \mathrm{theta}(t) = \mathrm{theta}, diff(y(t), t, t) = DDy, diff(y(t), t) = Dy, y(t) = y, f) :$
$sub\_out\_error := (f) \rightarrow subs(diff(y(t), t) = 0, y(t) = 0, Dy = 0, y = 0, f) :$

$alpha0 := sub\_out\_time(simplify(mu1 \cdot alpha1 + mu2 \cdot alpha2));$
$beta0 := sub\_out\_time\left(simplify\left(\mu 1 \cdot beta1 + mu2 \cdot beta2\right)\right);$
$gamma0 := sub\_out\_time(simplify(mu1 \cdot gamma1 + mu2 \cdot gamma2));$

 # Verify that equation is 0 on orbit (y=0,Dy = 0)
$DDtheta\_sol\_on\_orbit := simplify(sub\_out\_time(sub\_out\_error(rhs(DDtheta\_sol\_DDphi\_sub))));$
$verification = simplify\left(DDtheta\_sol\_on\_orbit \cdot alpha0 + beta0 \cdot Dtheta^2 + gamma0\right);$

#Defining right-hand side of perturbated aby equation:

$g := sub\_out\_time\left(collect\left(alpha0 \cdot rhs(DDtheta\_sol\_DDphi\_sub) + beta0 \cdot diff(\mathrm{theta}(t), t)^2 + gamma0, \left[diff(\mathrm{theta}(t), t, t), diff(\mathrm{theta}(t), t)^2\right]\right)\right);$
$g0 := sub\_out\_error(g) :$

$verification = simplify(sub\_out\_error(g));$

$$g\_I := simplify\left(\frac{-diff(g0, \text{theta}) \cdot DDtheta + diff(g0, Dtheta) \cdot Dtheta}{2 \cdot (DDtheta^2 + Dtheta^2)}\right);$$

$g\_y := simplify(sub\_out\_error(diff(g, y)));$

$g\_Dy := sub\_out\_error(diff(g, Dy));$

#Creating linearized system matrix for transverse coordinates:

$h := collect(sub\_out\_time(rhs(DDy\_sol\_DDphi\_sub)), [Dtheta^2]);$

$h0 := sub\_out\_error(h) :$

$$h\_I := \frac{-diff(h0, \text{theta}) \cdot DDtheta + diff(h0, Dtheta) \cdot Dtheta}{2 \cdot (DDtheta^2 + Dtheta^2)};$$

$h\_y := sub\_out\_error(diff(h, y)) :$

$h\_y := collect(numer(factor(h\_y)), \{diff, Dtheta^2, DDtheta, \sin, \cos\}) / denom(factor(h\_y));$

$h\_Dy := sub\_out\_error(diff(h, Dy));$

$A := Matrix(3, 3, 'fill = 0') :$

$$\text{mu} := \frac{2 \cdot Dtheta}{alpha0} :$$

$A_{1,1} := \text{mu} \cdot (g\_I - beta0) :$

$A_{1,2} := \text{mu} \cdot g\_y :$

$A_{1,3} := \text{mu} \cdot g\_Dy :$

$A_{2,3} := 1 :$

$A_{3,1} := h\_I :$

$A_{3,2} := h\_y :$

$A_{3,3} := h\_Dy :$

#Creating Matlab code:

$sub\_out\_theta := (f) \rightarrow subs(\mathrm{D}^{(3)}(\phi)(\theta) = DDDphi, \mathrm{D}^{(2)}(\phi)(\theta) = DDphi, \mathrm{D}(\phi)(\theta) = Dphi,$
$\text{phi}(\text{theta}) = \text{phi}, f) :$

$Matlab(sub\_out\_theta(alpha0), resultname = 'alpha0');$

$Matlab(sub\_out\_theta(beta0), resultname = 'beta0');$

$Matlab(sub\_out\_theta(gamma0), resultname = 'gamma0');$

$A0 := sub\_out\_theta(A) :$

$$Matlab\left(A0_{1,\,1}, resultname='a11'\right) :$$
$$Matlab\left(A0_{1,\,2}, resultname='a12'\right) :$$
$$Matlab\left(A0_{1,\,3}, resultname='a13'\right) :$$
$$Matlab\left(A0_{2,\,1}, resultname='a21'\right) :$$
$$Matlab\left(A0_{2,\,2}, resultname='a22'\right) :$$
$$Matlab\left(A0_{2,\,3}, resultname='a23'\right) :$$
$$Matlab\left(A0_{3,\,1}, resultname='a31'\right) :$$
$$Matlab\left(A0_{3,\,2}, resultname='a32'\right) :$$
$$Matlab\left(A0_{3,\,3}, resultname='a33'\right) :$$