restart

#Initialize linear algebra
with(*LinearAlgebra*) :

#Vector of generalized coordinates

$q := Vector([q1, q2])$;
$Dq := Vector([Dq1, Dq2])$:
$DDq := Vector([DDq1, DDq2])$:

#Homogenous transformation matrices

$H01 := Matrix([[\cos(-q1), \sin(-q1), 0, \sin(-q1)\cdot b], [-\sin(-q1), \cos(-q1), 0, \cos(-q1)\cdot b], [0, 0, 1, 0], [0, 0, 0, 1]])$;
$H1h := Matrix([[1, 0, 0, 0], [0, 1, 0, a], [0, 0, 1, 0], [0, 0, 0, 1]])$;
$H02 := Matrix([[\cos(q2), -\sin(q2), 0, 1\cdot\sin(-q1) + a\cdot\sin(q2)], [\sin(q2), \cos(q2), 0, 1\cdot\cos(-q1) - a\cdot\cos(q2)], [0, 0, 1, 0], [0, 0, 0, 1]])$;

$H2e := Matrix([[1, 0, 0, 0], [0, 1, 0, -b], [0, 0, 1, 0], [0, 0, 0, 1]])$:

#Coordinate frame origins

#Center of mass first link:
$o1 := MatrixVectorMultiply(H01, Vector([0, 0, 0, 1]))$:
$o1 := combine(o1, trig)$:
$o1x := o1[1]$;
$o1y := o1[2]$;
#Center of mass hip:
$H0h := MatrixMatrixMultiply(H01, H1h)$:
$oh := MatrixVectorMultiply(H0h, Vector([0, 0, 0, 1]))$:
$oh := combine(oh, trig)$:
$ohx := oh[1]$;
$ohy := oh[2]$;

#Center of mass second link:
$o2 := MatrixVectorMultiply(H02, Vector([0, 0, 0, 1]))$:
$o2 := combine(o2, trig)$:
$o2x := o2[1]$;
$o2y := o2[2]$;
#End of swing leg:
$H0e := MatrixMatrixMultiply(H02, H2e)$:
$oe := MatrixVectorMultiply(H0e, Vector([0, 0, 0, 1]))$:
$oe := combine(oe, trig)$:
$oex := oe[1]$;
$oey := oe[2]$;

#Linear velocities of centers of mass:

#First link:
$Do1x := diff(o1x, q1)\cdot Dq1$;
$Do1y := diff(o1y, q1)\cdot Dq1$;
#Hip:

$Dohx := diff(ohx, q1) \cdot Dq1;$
$Dohy := diff(ohy, q1) \cdot Dq1;$

#Second link:
$Do2x := diff(o2x, q1) \cdot Dq1 + diff(o2x, q2) \cdot Dq2;$
$Do2y := diff(o2y, q1) \cdot Dq1 + diff(o2y, q2) \cdot Dq2;$

#Kinetic energy of system

#First link:
$K1 := \frac{m1}{2} \cdot \left(Do1x^2 + Do1y^2\right) :$
$K1 := collect(combine(K1, trig), Dq1);$

#Hip:
$Kh := \frac{mH}{2} \cdot \left(Dohx^2 + Dohy^2\right) :$
$Kh := collect(combine(Kh, trig), Dq1);$

#Second link:
$K2 := \frac{m2}{2} \cdot \left(Do2x^2 + Do2y^2\right) :$
$K2 := collect(combine(K2, trig), Dq2);$

#Total kinetic energy T(q,Dq):
$K := K1 + K2 + Kh :$
$K := collect\left(K, \{Dq1, Dq2\}\right);$
$K :$
#Simplify
$par1 := mH \cdot l^2 + b^2\, m1 + l^2\, m2 :$
$par2 := m2\, l\, a :$
$par3 := m2\, a^2 :$

$K0 := algsubs\left(m1\, b^2 + m2\, l^2 + mH\, a^2 + 2 \cdot mH\, a\, b + mH\, b^2 = par1, K\right) :$
$K0 := algsubs(par1 = p1, K0) :$
$K0 := algsubs(par2 = p2, K0) :$
$K0 := algsubs(par3 = p3, K0) :$

$K0;$

#Computing inertia matrix

$m11 := algsubs\left(2\, a\, b + b^2 + a^2 = l^2, collect(sort(diff(K0, Dq1, Dq1)), [mh, m1, m2])\right);$
$m12 := sort(diff(K0, Dq1, Dq2));$
$m21 := m12;$
$m22 := sort(diff(K0, Dq2, Dq2));$
$M := Matrix([[m11, m12], [m21, m22]]);$

#Potential energy of system

#First link:

$P1 := m1 \cdot g \cdot o1y;$

#Hip:

$Ph := mH \cdot g \cdot ohy;$

#Second link:

$P2 := m2 \cdot g \cdot o2y;$

#Total potential energy:

$P := P1 + P2 + Ph :$

$P := combine(P, trig) :$

$P := collect(P, [\sin, \cos]) :$

$P + const;$

#Simplify

$par4 := (m1 \cdot b + m2 \cdot l + mH \cdot l) \cdot g;$

$par5 := m2\, a \cdot g;$

$P0 := algsubs((m1 \cdot b + m2 \cdot l + mH \cdot a + mH \cdot b) \cdot g = par4, P);$

$P0 := algsubs(\text{par4} = \text{p4}, P0) :$

$P0 := algsubs(par5 = p5, P0) :$

$P0;$

#Lagrangian of the dynamical system:

$L := K0 - P0;$

#First link:

$L1 := diff(L, Dq1) :$

$G1 := diff(L, q1) :$

$F1 := diff(L1, q1) \cdot Dq1 + diff(L1, Dq1) \cdot DDq1 + diff(L1, q2) \cdot Dq2 + diff(L1, Dq2) \cdot DDq2 - G1 :$

$F1 := collect(F1, \{Dq1, Dq2, DDq1, DDq2, \});$

#Second link:

$L2 := diff(L, Dq2) :$

$G2 := diff(L, q2) :$

$F2 := diff(L2, q1) \cdot Dq1 + diff(L2, Dq1) \cdot DDq1 + diff(L2, q2) \cdot Dq2 + diff(L2, Dq2) \cdot DDq2 - G2 :$

$F2 := collect(F2, \{Dq1, Dq2, DDq1, DDq2\});$

#Coriolis and centrifugal torque matrix C(q,Dq):

$F := F1 :$

$$c11 := collect\left(simplify\left(diff(F, Dq1, Dq1) \cdot \left(\frac{Dq1}{2}\right) + diff(F, Dq1, Dq2) \cdot \left(\frac{Dq2}{2}\right)\right), [\sin, \cos]\right) :$$

$$c12 := collect\left(simplify\left(diff(F, Dq2, Dq1) \cdot \left(\frac{Dq1}{2}\right) + diff(F, Dq2, Dq2) \cdot \left(\frac{Dq2}{2}\right)\right), [\sin, \cos]\right) :$$

$F := F2 :$

$$c21 := collect\left(simplify\left(diff(F, Dq1, Dq1) \cdot \left(\frac{Dq1}{2}\right) + diff(F, Dq1, Dq2) \cdot \left(\frac{Dq2}{2}\right)\right), [\sin, \cos]\right) :$$

$$c22 := collect\left(simplify\left(diff(F, Dq2, Dq1) \cdot \left(\frac{Dq1}{2} \right) + diff(F, Dq2, Dq2) \cdot \left(\frac{Dq2}{2} \right) \right), [\sin, \cos] \right) :$$
$$Cc := Matrix([[c11, c12], [c21, c22]]);$$

#Gravitational torque vector G(q):

$$G := Vector([F1, F2]) - MatrixVectorMultiply(M, DDq) - MatrixVectorMultiply(Cc, Dq) :$$
$$G := collect(G, [DDq1, \sin(q1), mh]) :$$
$$G := algsubs(a + b = l, G, 'exact') :$$
$$G := simplify\left(algsubs\left(a^2 + 2\, a\, b + b^2 = l^2, G, 'exact' \right) \right);$$
 # Finding value of constant term in potential energy
$$P0 := int(G(1), q1 = 0..q1) + int(G(2), q2 = 0..q2);$$

#Conversion to Matlab Code:

$$with(CodeGeneration) :$$

$$Matlab(o1x, resultname = "o1x");$$
$$Matlab(o1y, resultname = "o1y");$$
$$Matlab(ohx, resultname = "ohx");$$
$$Matlab(ohy, resultname = "ohy");$$
$$Matlab(o2x, resultname = "o2x");$$
$$Matlab(o2y, resultname = "o2y");$$
$$Matlab(oex, resultname = "oex");$$
$$Matlab(oey, resultname = "oey");$$

$$Matlab(M[1, 1], resultname = "m11");$$
$$Matlab(M[1, 2], resultname = "m12");$$
$$Matlab(M[2, 1], resultname = "m21");$$
$$Matlab(M[2, 2], resultname = "m22");$$

$$Matlab(c11, resultname = "c11");$$
$$Matlab(c12, resultname = "c12");$$
$$Matlab(c21, resultname = "c21");$$
$$Matlab(c22, resultname = "c22");$$

$$Matlab(G[1], resultname = "g1");$$
$$Matlab(G[2], resultname = "g2");$$

$$Matlab(par1, resultname = "p1");$$
$$Matlab(par2, resultname = "p2");$$
$$Matlab(par3, resultname = "p3");$$
$$Matlab(par4, resultname = "p4");$$
$$Matlab(par5, resultname = "p5");$$

#Storing matrices for other worksheets:

$$save\ M, Cc, G, K0, P0, `eom.m`;$$