

# A Feedback Linearization Control of Container Cranes: Varying Rope Length

Hahn Park, Dongkyoung Chwa, and Keum-Shik Hong\*

**Abstract:** In this paper, a nonlinear anti-sway controller for container cranes with load hoisting is investigated. The considered container crane involves a planar motion in conjunction with a hoisting motion. The control inputs are two (trolley and hoisting forces), whereas the variables to be controlled are three (trolley position, hoisting rope length, and sway angle). A novel feedback linearization control law provides a simultaneous trolley-position regulation, sway suppression, and load hoisting control. The performance of the closed loop system is shown to be satisfactory in the presence of disturbances at the payload and rope length variations. The advantage of the proposed control law lies in the full incorporation of the nonlinear dynamics by partial feedback linearization. The uniform asymptotic stability of the closed-loop system is assured irrespective of variations of the rope length. Simulation and experimental results are compared and discussed.

**Keywords:** Crane control, Lyapunov function, modeling, nonlinear control, partial feedback linearization.

## 1. INTRODUCTION

During the past two decades, the endeavor to enhance the handling efficiency of container cranes has pulled vigorous research in two directions: a new mechanism design with which the cycle time in transferring loads can be reduced [4,5,18] and the quick suppression of the sway of the loads despite faster trolley traveling [1-3,6-17,20-31]. The productivity of a container ship is directly related to how quickly containers are loaded or unloaded from the ship to/from container-carrying vehicles. The container crane dynamics are simpler than the overhead crane dynamics. But, the area of container crane control still draws attention from many researchers because the overall control performance is not satisfactory. Two main obstacles in achieving good control performance are that the container cranes

operate outdoor confronting winds, and the structure becomes larger and larger, possibly resulting in structural vibrations.

Early works on crane control in some optimizing sense can be traced back to [2,6,13-15,25,29]. More recent papers include [1,3,8-12,16,17,21-24,26-28,30,31]. In contrast to the speed control of [2,13-15,25,29], the torque control method applies control forces/torques in such a way that the dynamics of the controlled system meet a given reference signal. The torque control method is more attractive from the aspects of accuracy and energy saving.

Recently, Lee [23] proposed an anti-swing control of overhead cranes with changing rope length, in which the trolley reference trajectory was modified to achieve a sufficient damping of the load sway and the asymptotic stability of the zero solutions of the

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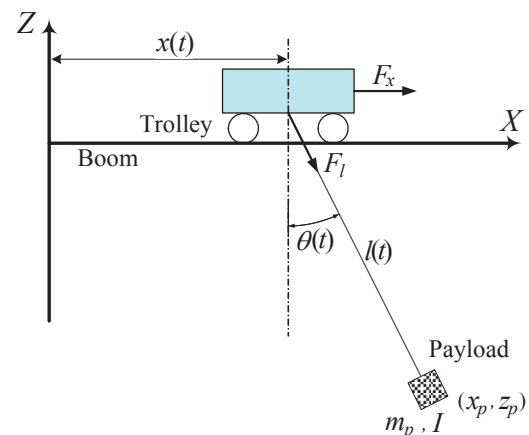


Fig. 1. Sway of the load caused by trolley movement.

position and rope length tracking errors was proved. Also, Chung and Hauser [7] and Fang *et al.* [11] have addressed Lyapunov-function-based controls in order to incorporate the full nonlinear dynamics of crane systems.

The crane is naturally an under-actuated mechanical system, in which the number of actuators is less than the degree of freedom of the system. For a container crane, the degree of freedom is three (i.e., trolley position, rope length, and sway angle; see Fig. 1), but the number of actuators is two (i.e., trolley and hoist motors). For a given target position of the load, the trolley should travel as fast as it can. On the other hand, such a fast trolley movement should not result in any residual sway of the load at the completion of the transference.

In this paper, an energy-based nonlinear control design for container cranes is investigated, where the nonlinearity of the plant is fully incorporated into control law design when the energy function is differentiated along the plant dynamics. Unlike the controller of Lee [23], the proposed anti-sway controller, which guarantees an accurate control of the trolley position and rope length, is designed in such a way that the trolley reference trajectory needs not to be modified to achieve the sufficient damping of the load sway. Thus, there is no notch in the trolley velocity. Also, the asymptotic stability of the zero solutions of the trolley position and rope length tracking errors is guaranteed not only in the case of constant rope length but also in the case of varying rope length. That is, the uniform asymptotic stability of the closed-loop system can be guaranteed by a properly selected energy function. Furthermore, the parameters in the controller can be chosen in such a way that the transient performance (i.e., the rise time to a set position of the trolley or to a desired rope length) can be adjusted in a predictable way.

The contributions of this paper are the following. An energy-based control law with improved rise time, sway suppression, and load hoist control capability for container cranes is proposed. The uniform asymptotic stability of the closed-loop system is assured. Finally, the developed algorithm is verified through simulations and experiments.

The paper is structured as follows. In Section 2, a container crane model is introduced. In Section 3, the design of a Lyapunov-function-based control law is discussed and the uniform asymptotic stability of the closed-loop system is proved. In Section 4, simulations and experimental results are provided. In Section 5, conclusions are drawn.

## 2. SYSTEM DYNAMICS

For the successful sway suppression and hoist control of a suspended load, it is important to know

what part of the crane dynamics should be included in the control law design process and what part can be neglected. Fig. 1 shows the swing motion of the load caused by trolley movement, in which  $X$  is the trolley moving direction,  $Z$  is the vertical direction,  $\theta(t)$  is the sway angle of the load,  $x(t)$  is the displacement of the trolley,  $l(t)$  is the hoist rope length, and  $F_x$  and  $F_l$  are the control forces applied to the trolley in the  $X$ -direction and to the payload in the  $l$ -direction, respectively.

The following assumptions are made: i) The payload and trolley are connected by a massless rigid rod, that is, a pendulum motion of the load is considered; ii) the trolley mass and the position of the trolley are known; iii) all frictional elements in the trolley and hoist motions can be neglected; iv) the rod elongation is negligible. The above assumptions give a three d.o.f. crane model, yielding the generalized coordinate vector  $q(t) \in R^3$ , as follows:

$$q(t) = [x(t) \quad l(t) \quad \theta(t)]^T. \quad (1)$$

Let the coordinate of the payload be  $(x_p, z_p)$ . Then,  $x_p$  and  $z_p$  are given by

$$x_p = x + l \sin \theta, \quad z_p = -l \cos \theta. \quad (2)$$

Using (2), the kinetic energy  $T$  and the potential energy  $V$  are given as follows:

$$T = \frac{1}{2}(m_t + m_p)\dot{x}^2 + \frac{1}{2}(m_p + m_l)\dot{l}^2 + \frac{1}{2}m_p(l\dot{\theta})^2 + m_p\dot{x}(l \cos \theta \dot{\theta} + \sin \theta \dot{l}) + \frac{1}{2}I\dot{\theta}^2, \quad (3)$$

$$V = -m_p g l \cos \theta, \quad (4)$$

where  $m_p$  is the payload mass,  $m_t$  and  $m_l$  are the equivalent mass of the trolley and hoist systems, respectively,  $I$  is the mass moment of inertia of the payload, and  $g$  is the gravitational acceleration. Finally, the equations of motion using the Lagrange's equation are derived as follows:

$$(m_t + m_p)\ddot{x} + m_p \sin \theta \ddot{l} + m_p l \cos \theta \ddot{\theta} + 2m_p \cos \theta \dot{l} \dot{\theta} - m_p l \sin \theta \dot{\theta}^2 = F_x, \quad (5)$$

$$m_p \sin \theta \ddot{x} + (m_p + m_l)\ddot{l} - m_p l \dot{\theta}^2 - m_p g \cos \theta = F_l, \quad (6)$$

$$m_p l \cos \theta \ddot{x} + (m_p l^2 + I)\ddot{\theta} + 2m_p l \dot{\theta} \dot{x} + m_p g l \sin \theta = 0. \quad (7)$$

The dynamic equations (5)-(7) can be rewritten as

$$M(q)\ddot{q} + V_m(q, \dot{q})\dot{q} + G(q) = u, \quad (8)$$

where

$$M = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & 0 \\ m_{31} & 0 & m_{33} \end{bmatrix}, \quad V_m = \begin{bmatrix} 0 & V_{m12} & V_{m13} \\ 0 & 0 & V_{m23} \\ 0 & V_{m32} & V_{m33} \end{bmatrix},$$

$$G = \begin{bmatrix} 0 & -m_p g \cos \theta & m_p g l \sin \theta \end{bmatrix}^T,$$

$$u = \begin{bmatrix} F_x & F_l & 0 \end{bmatrix}^T,$$

$$m_{11} = m_p + m_t, \quad m_{12} = m_p \sin \theta, \quad m_{13} = m_p l \cos \theta,$$

$$m_{21} = m_p \sin \theta, \quad m_{22} = m_p + m_l,$$

$$m_{31} = m_p l \cos \theta, \quad m_{33} = m_p l^2 + I,$$

$$V_{m12} = m_p \dot{\theta} \cos \theta, \quad V_{m13} = -m_p l \sin \theta \dot{\theta} + m_p \cos \theta \dot{l},$$

$$V_{m23} = -m_p l \dot{\theta}, \quad V_{m32} = m_p l \dot{\theta}, \quad \text{and}$$

$$V_{m33} = m_p l \dot{l}.$$

It is remarked that the inertia matrix  $M(q)$  and the centripetal term  $V_m(q, \dot{q})$  satisfy the skew-symmetric relationship. Therefore the following equation holds.

$$\xi^T \left( \frac{1}{2} \dot{M}(q) - V_m(q, \dot{q}) \right) \xi = 0, \quad (9)$$

where  $\dot{M}(q)$  is the time derivative of  $M(q)$ , and then the following inequalities hold.

$$k_1 \|\xi\|^2 \leq \xi^T M(q) \xi \leq k_2 \|\xi\|^2, \quad \xi \in R^3, \quad (10)$$

where  $k_1, k_2$  are positive constants.

In order to separate the unactuated dynamics ( $\theta$ -dynamics) from the actuated dynamics ( $(x, l)$ -dynamics), (7) is rewritten as

$$\ddot{\theta} = \frac{1}{m_p l^2 + I} (-m_p l \cos \theta \ddot{x} - 2m_p l \dot{l} \dot{\theta} - m_p g l \sin \theta). \quad (11)$$

It can be seen from (11) that a change of the rope length leads to a change of the sway dynamics. This coupling effect has to be effectively controlled. Using (11), (5) and (6) can be rewritten as

$$\begin{aligned} & (m_p + m_t - m_p^2 l^2 \cos^2 \theta / (m_p l^2 + I)) \ddot{x} + m_p \sin \theta \ddot{l} \\ & = 2m_p \cos \theta \dot{l} \dot{\theta} (m_p l^2 / (m_p l^2 + I) - 1) \end{aligned} \quad (12)$$

$$+ m_p l \sin \theta (\dot{\theta}^2 + m_p g l \cos \theta / (m_p l^2 + I)) + F_x,$$

$$m_p \sin \theta \ddot{x} + (m_p + m_l) \ddot{l} = m_p l \dot{\theta}^2 + m_p g \cos \theta + F_l. \quad (13)$$

Now, we introduce a new vector  $r = [x \ l]^T$  for the purpose of partial feedback linearization and the tracking of both trolley and hoist dynamics at the same time (see Section 3). Then, the coordinate vector

can be written as  $q = [x \ l \ \theta]^T = [r^T \ \theta]^T$ . And, by rearranging the terms in (12)-(13), the  $r$ -dynamics become

$$\ddot{r} = P(F + \bar{W}) = PF + W, \quad (14)$$

where

$$\begin{aligned} P &= \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = [M']^{-1} \\ &= \frac{1}{\det(M')} \begin{bmatrix} m_p + m_l & -m_p \sin \theta \\ -m_p \sin \theta & m_p + m_t - \frac{m_p^2 l^2 \cos^2 \theta}{m_p l^2 + I} \end{bmatrix}, \end{aligned}$$

$M'$  is the inertia matrix of  $(x, l)$ -dynamics,

$$\begin{aligned} \det(M') &= (m_p + m_t - m_p^2 l^2 \cos^2 \theta / (m_p l^2 + I)) \\ &\quad \times (m_p + m_l) - m_p^2 \sin^2 \theta, \end{aligned}$$

$$F = \begin{bmatrix} F_x & F_l \end{bmatrix}^T,$$

$$W = \begin{bmatrix} w_1 & w_2 \end{bmatrix}^T = P\bar{W}, \quad \text{and}$$

$$\bar{W} = \begin{bmatrix} 2m_p \cos \theta \dot{l} \dot{\theta} (m_p l^2 / (m_p l^2 + I) - 1) \\ + m_p l \sin \theta (\dot{\theta}^2 + m_p g l \cos \theta / (m_p l^2 + I)) \\ m_p l \dot{\theta}^2 + m_p g \cos \theta \end{bmatrix}.$$

### 3. CONTROL LAW DESIGN

In this section, a nonlinear control law for suppressing the sway angle of the suspended load and controlling the trolley position and the hoisting rope length is derived. The novelty of this law lies in improving the transient performance and also guaranteeing the asymptotic stability of the trolley position and hoisting rope length tracking errors, irrespective of the variations of the rope length. The information on the sway angle, sway angular velocity, trolley displacement and velocity, hoisting rope length and its time rate of change is assumed to be known. Let the tracking errors of the trolley position and the hoisting rope length be defined by

$$e = r - r_d = [x_e \ l_e]^T, \quad (15)$$

where  $r_d = [x_d \ l_d]^T$ ,  $x_e = x - x_d$ ,  $l_e = l - l_d$ , and  $x_d$  and  $l_d$  are the desired trolley position and hoisting rope length, respectively.

To improve the transient performance in positioning and load hoisting control, and sway suppression, the following new control law is proposed:

$$F = P^{-1} \cdot \left( \ddot{r}_d - K_p e - K_d \dot{e} - W + \begin{bmatrix} f \\ 0 \end{bmatrix} \right), \quad (16)$$

where  $K_p = \bar{K}^2$ ,  $K_d = 2\bar{K}$ ,  $\bar{K} = \text{diag}(k_x, k_l)$ ,

$$f = \bar{f} - \ddot{x}_d + (k_\theta \dot{\theta} / a - 2\dot{l}\dot{\theta}) / \cos \theta, \quad (17)$$

$$a = \frac{m_p l}{m_p l^2 + I}, \quad (18)$$

$$\bar{f} = \left| 2k_x \dot{x}_e + k_x^2 x_e + (c^* - c) \sin \theta / (a \cos \theta) \right| \cdot \kappa(k_\theta \theta + 2\dot{\theta}), \quad (19)$$

where  $k_x$ ,  $k_l$ ,  $k_\theta$ ,  $c^*$ , and  $c$  are positive constants. Here,  $c^*$  satisfies  $c^* > c := m_p g l / (m_p l^2 + I)$  and  $\kappa(x)$  is defined as

$$\kappa(x) = \begin{cases} \text{sgn}(x) & \text{if } |x| \geq \gamma(t) \\ \sin\left(\frac{x}{\gamma(t)} \cdot \frac{\pi}{2}\right) & \text{if } |x| < \gamma(t), \end{cases} \quad (20a)$$

where  $\text{sgn}(x)$  is defined such that  $\text{sgn}(x) = 1$  for  $x > 0$ ,  $\text{sgn}(x) = -1$  for  $x < 0$ , and  $\text{sgn}(x) = 0$  for  $x = 0$ ,  $\gamma(t) > 0$  is given by

$$\gamma(t) = \begin{cases} \frac{1}{\alpha + \beta e^{\eta t}} & \text{if } 0 \leq t \leq T_f \\ \frac{1}{\alpha + \beta e^{\eta T_f}} & \text{if } t > T_f, \end{cases} \quad (20b)$$

and  $\alpha$ ,  $\beta$ ,  $\eta$ ,  $T_f$  are positive constants such that  $0 < \gamma(t) \leq \varepsilon < 1$  for a positive constant  $\varepsilon$  and  $\kappa(x)$  converges to  $\text{sgn}(x)$ .

**Theorem 1:** Consider the plant (11) and (14) with  $|\theta(0)| < \pi/2$  and  $l(0) > 0$ , and the control law given by (16)-(19). Then, the tracking errors of the trolley position and hoisting rope length ( $x_e$  and  $l_e$ ) and the swing angle  $\theta$  converge to zero asymptotically.

**Proof:** By substituting (16) into (14) and rearranging the terms, the position and load hoisting error dynamics become

$$\ddot{e} + K_d \dot{e} + K_p e = \begin{bmatrix} f \\ 0 \end{bmatrix}. \quad (21)$$

Rewriting of (21) and the  $\theta$ -dynamics in (11) become

$$\ddot{x}_e + 2k_x \dot{x}_e + k_x^2 x_e = f, \quad (22)$$

$$\ddot{l}_e + 2k_l \dot{l}_e + k_l^2 l_e = 0, \quad (23)$$

$$(m_p l^2 + I) \ddot{\theta} + m_p l \cos \theta \ddot{x} + 2m_p l \dot{l} \dot{\theta} + m_p g l \sin \theta = 0. \quad (24)$$

First, a positive definite function for the  $x$ - and  $l$ -dynamics is considered as follows:

$$V_1 = (\dot{e} + \bar{K}e)^T (\dot{e} + \bar{K}e) / 2. \quad (25)$$

Then, the time derivative of (25) using (22)-(23) becomes

$$\begin{aligned} \dot{V}_1 &= (\dot{e} + \bar{K}e)^T (\ddot{e} + \bar{K}\dot{e}) \\ &= (\dot{e} + \bar{K}e)^T \left( -\bar{K}(\dot{e} + \bar{K}e) + \begin{bmatrix} f \\ 0 \end{bmatrix} \right) \\ &= -(\dot{e} + \bar{K}e)^T \bar{K}(\dot{e} + \bar{K}e) + (\dot{x}_e + k_x x_e) f \\ &= -k_x (\dot{x}_e + k_x x_e)^2 - k_l (\dot{l}_e + k_l l_e)^2 \\ &\quad + (\dot{x}_e + k_x x_e) f. \end{aligned} \quad (26)$$

Also, using (22) and (17), the sway dynamics (24) are expressed as

$$\dot{\theta}_1 = \theta_2, \quad (27a)$$

$$\begin{aligned} \dot{\theta}_2 &= -c \sin \theta_1 - a \cos \theta_1 (\bar{f} - 2k_x \dot{x}_e - k_x^2 x_e) \\ &\quad - k_\theta \theta_2, \end{aligned} \quad (27b)$$

where  $\theta_1 = \theta$ ,  $\theta_2 = \dot{\theta}$ , and  $c = m_p g l / (m_p l^2 + I)$ .

A Lyapunov function candidate for system (27a,b) (i.e., a positive definite function for the sway dynamics) is considered as follows:

$$V_2 = 1/2 \cdot \theta_2^2 + 1/2 \cdot (k_\theta \theta_1 + \theta_2)^2 + 2c^* (1 - \cos \theta_1). \quad (28)$$

Then, using (27b) and (19), the time derivative of (28) is given by

$$\begin{aligned} \dot{V}_2 &= \theta_2 \dot{\theta}_2 + (k_\theta \theta_1 + \theta_2)(k_\theta \dot{\theta}_1 + \dot{\theta}_2) + 2c^* \sin \theta_1 \cdot \dot{\theta}_1 \\ &= (k_\theta \theta_1 + 2\theta_2) \dot{\theta}_2 + (k_\theta \theta_1 + \theta_2) k_\theta \theta_2 + 2c^* \sin \theta_1 \cdot \theta_2 \\ &= (k_\theta \theta_1 + \theta_2) k_\theta \theta_2 + 2c^* \sin \theta_1 \cdot \theta_2 - (k_\theta \theta_1 + 2\theta_2) c \sin \theta_1 \\ &\quad - (k_\theta \theta_1 + 2\theta_2) a \cos \theta_1 \cdot (\bar{f} - 2k_x \dot{x}_e - k_x^2 x_e) \\ &\quad - (k_\theta \theta_1 + 2\theta_2) k_\theta \theta_2 \\ &= -k_\theta c^* \theta_1 \sin \theta_1 - k_\theta \theta_2^2 + (c^* - c)(k_\theta \theta_1 + 2\theta_2) \sin \theta_1 \\ &\quad - a \cos \theta_1 (k_\theta \theta_1 + 2\theta_2) (\bar{f} - 2k_x \dot{x}_e - k_x^2 x_e) \\ &= -k_\theta c^* \theta_1 \sin \theta_1 - k_\theta \theta_2^2 + a \cos \theta_1 (k_\theta \theta_1 + 2\theta_2) \\ &\quad \times \{ 2k_x \dot{x}_e + k_x^2 x_e + (c^* - c) \sin \theta_1 / (a \cos \theta_1) - \bar{f} \} \\ &\leq -k_\theta c^* \theta_1 \sin \theta_1 - k_\theta \theta_2^2 + |a \cos \theta_1 (k_\theta \theta_1 + 2\theta_2)| \\ &\quad \times |2k_x \dot{x}_e + k_x^2 x_e + (c^* - c) \sin \theta / (a \cos \theta)| \\ &\quad \times \{ 1 - \kappa(k_\theta \theta + 2\dot{\theta}) \}. \end{aligned} \quad (29)$$



Then, (20a) and (20b) can be used to show that  $\theta$  and  $\dot{\theta}$  asymptotically converge to the set  $\{(\theta, \dot{\theta}) : |k_\theta \theta + 2\dot{\theta}| \leq \gamma(t)\}$  which becomes sufficiently small as time goes on, due to the property of  $\gamma(t)$  in (20b). Here, note that  $\theta_1 \sin \theta_1$  in (29) is always positive if  $-\pi/2 < \theta_1 < \pi/2$ . Then, we can see that as (29) becomes  $\dot{V}_2 \leq -k_\theta c^* \theta_1 \sin \theta_1 - k_\theta \theta_2^2 + \varepsilon$  for sufficiently small constant  $\varepsilon > 0$  as time goes on,  $\theta_1 \rightarrow 0$  and  $\theta_2 \rightarrow 0$  as  $t \rightarrow \infty$  accordingly. That is, as  $t \rightarrow \infty$ , the sway angle is suppressed (i.e.,  $\theta \rightarrow 0$ ) and (27a,b) is asymptotic stable.

As  $\theta, \dot{\theta} \rightarrow 0$ ,  $f$  approaches zero (see Remark 2). Then, the third term in the right hand side of (26) approaches to zero and (26) is given as follows:

$$\begin{aligned} \dot{V}_1 &= -k_x(\dot{x}_e + k_x x_e)^2 - k_l(\dot{l}_e + k_l l_e)^2 \\ &\leq -\min(k_x, k_l) \{(\dot{x}_e + k_x x_e)^2 + (\dot{l}_e + k_l l_e)^2\} \quad (30) \\ &\leq -2\min(k_x, k_l)V_1. \end{aligned}$$

Hence,  $e$  and  $\dot{e}$  in (25) tends to zero exponentially. Since  $x_e$  and  $l_e$  are considered as states and  $\theta$  as input, the unforced system (i.e.,  $f = 0$ ) is exponentially stable. With this reason, (22) and (23) is input-to-state stable. Consequently, the closed-loop system with the proposed control law is asymptotically stable ([19], p. 174).  $\square$

**Remark 1:** Whereas the anti-swing control law proposed in [23] guarantees the asymptotic stability only in the case of constant-speed hoisting zone, the proposed controller in this paper guarantees the asymptotic stability in any varying-speed hoisting zone.

**Remark 2:** The crane control problem is a regulation problem, not a tracking problem. In general, the target trolley position  $x_d$  may not be constant. But, for loading/unloading purpose,  $x_d$  can be considered as constant and hence  $\ddot{x}_d$  is zero. For this reason, it can be concluded that  $f$  converges to zero (or becomes sufficiently small) as  $\theta$  and  $\dot{\theta}$  converge to zero (or become sufficiently small). The function  $\kappa(x)$  in (19) makes the control input  $\bar{f}$  with a signum function smoother and renders the boundary layer of the swing surface to vanish, so that the stability of the system is retained. Another notable contribution is that Theorem 1 does not require  $\ddot{l}_d = \dot{l}_d = 0$ , as was done in [23], for the convergence of  $x_e$ ,  $l_e$ , and  $\theta$  to zero, and thus the result in this paper is more general.

**Remark 3:** We used the feedback of the sway angle

and the sway angular rate to reduce the payload swing; see (17). In case that only the sway angle is measured, the sway angular rate can be estimated by implementing an appropriate observer; see [21]. It is remarked that the inclusion of the sway angular rate term makes the control system robust to the change of payload weight and the existence of an initial swing angle.

#### 4. SIMULATION AND EXPERIMENT

Now, computer simulations and experimental results of the proposed feedback linearization control law in Section 3 are discussed. The system parameters of the pilot crane in Fig. 2 are

$$\begin{aligned} m_p &= 0.73\text{kg}, \quad m_t = 1.06\text{kg}, \quad m_l = 0.5\text{kg}, \text{ and} \\ I &= 0.005 \text{ kgm}^2. \end{aligned} \quad (31)$$

For comparison purpose, the same values are used in both simulation and experiment.

The 3-D crane used in experiment is an InTeCo 3DCrane. As shown in Fig. 2, it consists of a trolley, a girder and a payload hanging at the end of a string whose top end is hinged by a pulley, which is again mounted on the trolley. The trolley is capable of rectilinear motions along the girder in the  $X$ -direction. The 3DCrane is driven by three DC motors. It has five encoders to measure five variables; the trolley and girder displacements, the string length, and the two swing angles of the payload. All five encoders are physically identical. The trolley and hoist motors are driven by a power interface board, which amplify the signals from PC to DC motors and transmit the pulse signals of the encoder after converting them to 16-bit digital signals. The PC communicates with the power interface board via an internally equipped RT-DAC

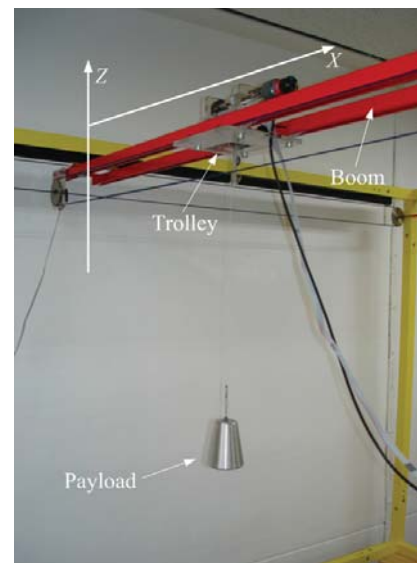


Fig. 2. The 3D pilot crane used in experiment.

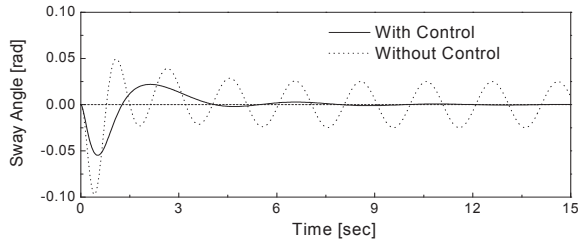


Fig. 3. Comparison of the sway angles with and without control, that is,  $f=0$  in (16).

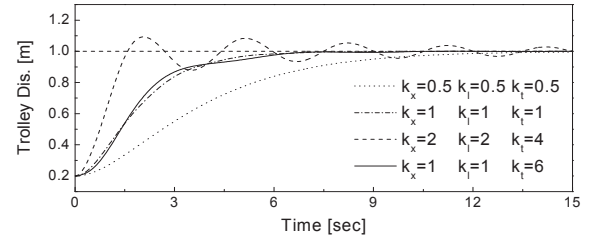
multi-purpose digital I/O board. Since we are focusing on controlling the sway motion during loading and unloading, we consider only the trolley motion, not the girder motion.

The sampling time was 0.01 sec. The initial displacement of the trolley and the initial length of the rope were set to 0.2 m, respectively, whereas their target values were set to 1.0 m, respectively. Fig. 3 compares the sway angles of the load with (i.e.,  $k_x=1$ ,  $k_l=1$ ,  $k_\theta=6$ ,  $c^*=14$ ,  $\alpha=0.5$ ,  $\beta=0.5$ ,  $\eta=0.5$ ,  $T_f=10$ ) and without control, that is,  $f$  in (16) was set to zero. It is seen that, without control, almost the same magnitude of the sway motion remains after the trolley reaches its target position.

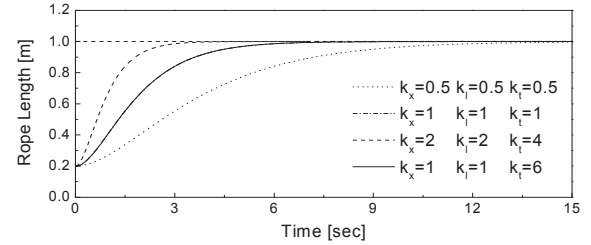
Fig. 4 compares the control performance of four different sets of control gains. As seen in Fig. 4(a), the transference time of the trolley reduces as the gain  $k_x$  increases (i.e., if the trolley positioning problem is seen as a regulation problem, the rise time to the trolley target position improves as  $k_x$  increases). On the other hand, as seen in Fig. 4(b), the rope length control improves as the gain  $k_l$  increases.

However, as  $k_x$  increases, the sway angle gets bigger, as seen in Fig. 4(c). The best sway control, among these four cases, is achieved when  $k_x=1$ ,  $k_l=1$ , and  $k_\theta=6$ . These results could have been expected because  $k_x$  and  $k_l$  were the two weighting factors on  $x_e$  and  $l_e$  in (26), respectively, and the Lyapunov function candidates  $V_1$  and  $V_2$  were designed independently. These effects can be strategically utilized by separating the loading and unloading cycle into two regions, that is, i) a quick transference region and ii) a sway suppression and rope length adjustment region.

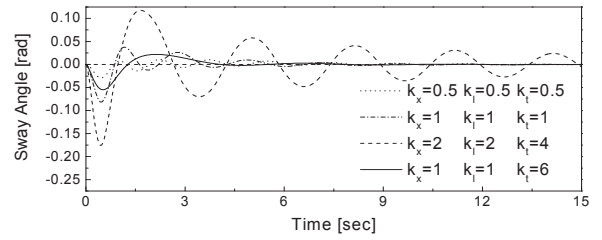
Fig. 5 illustrates the suppression of impulse disturbances to the load of -2 N at 3 sec and +3 N at 6 sec, respectively, where negative sign means against the trolley movement. It is seen that such disturbances brought up instantaneous increases of the sway angle, but the increased sway angle died out immediately because the control law was made in such a way that the time derivative of the Lyapunov function was



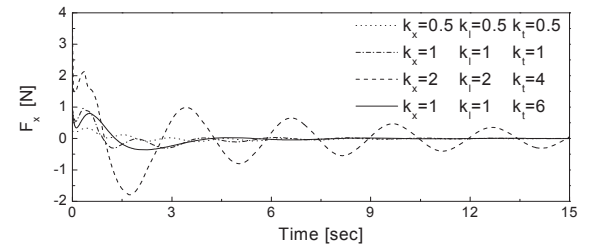
(a) Trolley position.



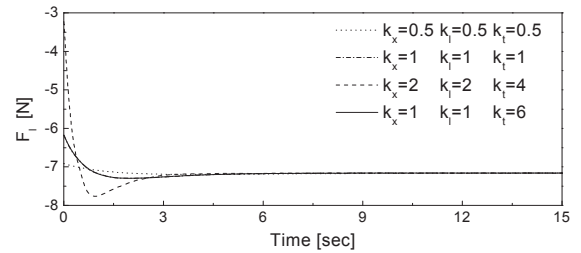
(b) Rope length.



(c) Sway angle.



(d) Control force for trolley.



(e) Control force for hoist.

Fig. 4. Control performance comparison of four sets of control gains: simulation results.

negative definite.

Fig. 6 compares the control performance of the proposed controller with that of [23], in which the convergence pattern looks similar but an improved performance of the proposed control law is demonstrated. This is because the trolley position and

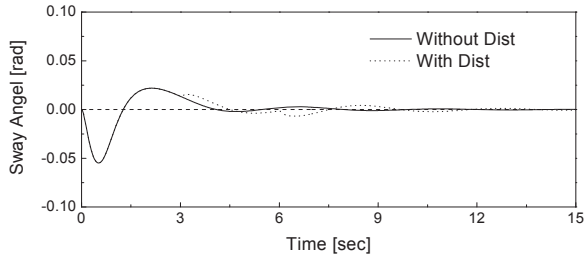
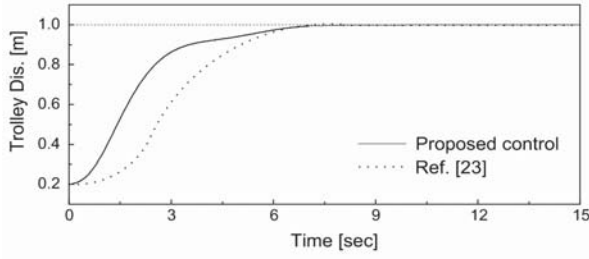
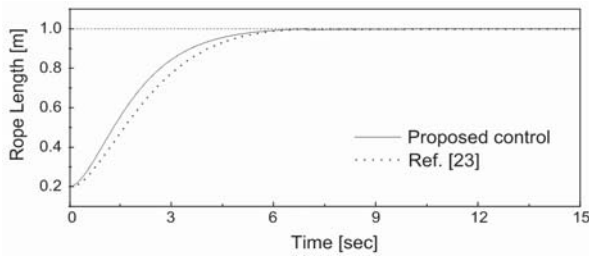


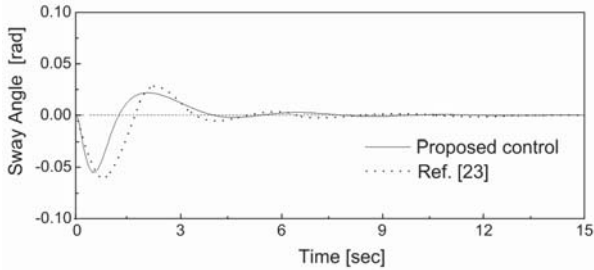
Fig. 5. Suppression of the disturbances of -2 N at 3 sec and +3 N at 6 sec, respectively ( $k_x = 1$ ,  $k_l = 1$ ,  $k_\theta = 6$ ).



(a) Trolley position.



(b) Rope length.

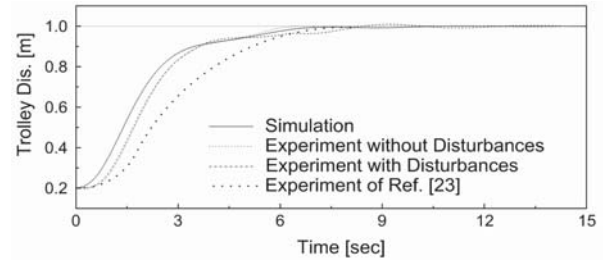


(c) Sway angle.

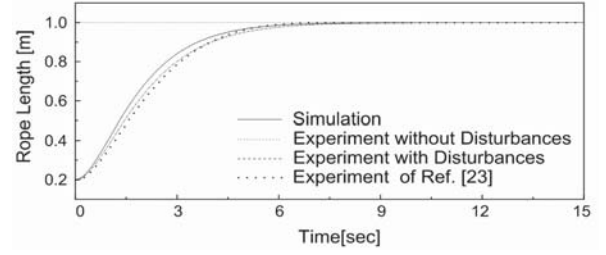
Fig. 6. Comparison of the proposed control law with the one in [23]: simulation results.

rope length errors decrease exponentially with the proposed control law.

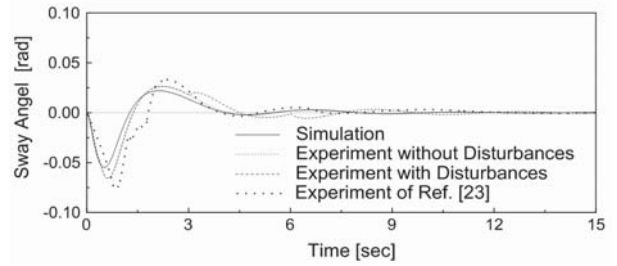
Finally, Fig. 7 verifies the correctness of the simulation results through experimentation. As seen in Fig. 7(a)-(c), the closeness between simulation and experimental results are first demonstrated. The disturbance in simulation was given in the form of an impulse, i.e., -2 N and +3 N at 3 and 6 seconds, respectively. On the other hand, that in experiment was given in the form of an impact at 3 and 6 seconds, hitting the load with a stick. The exact realization of



(a) Trolley position.



(b) Rope length.



(c) Sway angle.

Fig. 7. Comparison of simulation and experimental results.

the disturbance in experiment may not have been done. Except such an instantaneous disturbance, all other types of disturbance were hardly feasible. However, it can be said that the sway suppression capability of the proposed controller in the presence of disturbance has been demonstrated experimentally. Through both simulation and experiment, the effectiveness of the proposed controller has been firmly demonstrated.

**Remark 4:** If the rope length is  $l$ , the natural frequency of a sway motion is given by  $\omega_n = \sqrt{g/l}$ . If the length of the hoist rope in the pilot crane is reduced by  $1/\kappa$ , where  $\kappa$  is the scaling factor, it will not yield the same natural frequency because the gravitational acceleration  $g$  cannot be reduced by the factor of  $\kappa$ . Assuming that the rope lengths of the pilot and real cranes are 1 m and 40 m, respectively,  $\kappa = 40$ . With the rope length reduced by  $\kappa$ , the sway frequency increases by  $\sqrt{\kappa}$ . If following the work of [21],  $\omega t$  can be maintained constant. Hence, if the velocity profile in the real crane is  $v(t)$ , the profile in the pilot crane has to be modified to  $v(t)/\sqrt{\kappa}$ . Also, if the target error range of the load in the real

crane is  $\pm 30$  mm, then that in the pilot crane should be  $\pm 30/\kappa$  mm. However, unfortunately, since the used pilot crane can not provide torque control, that is, the exact performance of the control law (17) cannot be verified, it was used only for relative-comparison purpose.

## 5. CONCLUSIONS

In this paper, a nonlinear control law for container cranes with load hoisting using the feedback linearization technique and the decoupling strategy of  $\theta$ -dynamics from  $(x, l)$ -dynamics was investigated. Even though the feedback linearization method has a weak point in handling model uncertainty in general, it was proven effective in compensating a large nonlinearity and known variations, for instance, the rope length change in our case. Simulations and experimental results revealed that the proposed nonlinear controller could achieve the position control of the trolley and the quick swing suppression of the payload under the change of rope length. An extension of the proposed controller to a 3-D overhead crane with load hoisting can be pursued as a future work.

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