

## Suppression of Two-Dimensional Load-Sway in Rotary Crane Control Using Only Horizontal Boom Motion\*

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### Abstract

Horizontal motion of booms in rotary cranes typically generates undesirable two-dimensional load-sway; therefore, crane operators must be highly skilled to control the crane's motion. To reduce the burden on human operators, automatic control systems that can simultaneously control the boom's position while suppressing unwanted load-sway have been widely investigated. In most existing control schemes, both horizontal and vertical boom motion must be used to suppress load-sway. However, it would be less energy intensive and indeed safer if a control scheme could be developed that only utilized horizontal boom motion, i.e. without the need for any vertical motion. In this paper, we present a nonlinear controller design that enables both boom positioning and load-sway suppression using only horizontal boom motion. Numerical simulations and experimental results demonstrate the effectiveness of the proposed method.

**Key words :** Rotary Crane, Motion Control, Anti-Sway Control, Nonlinear Control

### 1. Introduction

Rotary cranes have many applications, such as bearing load at construction sites and lifting automotive tracks. Because one-dimensional horizontal motion of booms on such cranes typically generates undesirable two-dimensional load-sway, crane operators must be highly skilled to control the crane's motion. Rotary crane motion consists of two distinct modes of operation: 1) Load lifting and lowering by varying the rope length, and 2) vertical and horizontal boom motion.

Because the first mode of operation does not cause load-sway, it seems easier for human operators to control this motion. For the second mode of operation, however, anti-sway control is required, which is much more difficult.

To reduce the burden on human operators, automatic control systems have been widely investigated. Various types of control schemes have been developed for crane control, including optimal control<sup>(1)-(4)</sup>, gain scheduling control<sup>(5)-(7)</sup>, sliding mode control<sup>(8)-(10)</sup>, adaptive control<sup>(11)</sup>, back-stepping control<sup>(12)</sup>, input-shaping control<sup>(13)</sup>, disturbance observer based control<sup>(14)</sup>, and nonlinear control based on the Lyapunov stability theorem<sup>(15)-(17)</sup>.

In most existing control schemes, both horizontal and vertical boom motion must be used to suppress load-sway. However, it would be less energy intensive and also safer if a control scheme could be developed that only utilized horizontal boom motion, i.e. without the need for any vertical motion that must overcome gravity.

There are a few studies in the literature on controller designs for load-sway suppression that utilize only horizontal boom motion. Open-loop control without requiring load-sway information has been proposed, in which a trajectory for horizontal boom motion is generated that can suppress load-sway<sup>(18)-(20)</sup>. However, the performance of an open-loop control approach may deteriorate even for a small disturbance. Another approach is via two-mode

switching control<sup>(21)</sup>: in the first control mode, the controller reduces load-sway in the uncontrollable direction around the desired position of the boom. In the second control mode, exact boom positioning and controllable load-sway suppression are achieved simultaneously around the desired position of the boom. However, this approach also suffers from poor performance when a small disturbance increases the uncontrollable load-sway around the desired position of the boom.

In this paper, we propose a nonlinear controller design that can suppress the 2-D load-sway utilizing only horizontal boom motion. Because the control system should be robust with respect to varying parameters such as load mass and joint friction, we first present a controller design based on a disturbance observer<sup>(22)</sup> and partial linearization. This design provides a simple dynamical model of rotary crane motion that includes a centrifugal force term responsible for two-dimensional load-sway. Next, an anti-sway controller design based on the Lyapunov stability theory is presented. This controller provides tracking control of the boom along an arbitrary trajectory while suppressing the two-dimensional load-sway. Simulation and experimental results demonstrate the effectiveness of the proposed method.

The paper is organized as follows: In Section 2, we present a simple dynamic model of a rotary crane. In Section 3, we describe a nonlinear controller developed by us to achieve boom positioning and anti-sway control of the load. The effectiveness of the proposed controller is demonstrated by simulation and experimental results in Section 4. Concluding remarks are given in Section 5.

## 2. Rotary Crane Dynamics

A schematic model of a rotary crane is shown in Fig. 1.  $\theta_1$  and  $\theta_2$  are load-sway angles in the plane of vertical boom motion and the tangential direction of horizontal boom motion, respectively.  $\theta_3$  and  $\theta_4$  are vertical and horizontal angles of the boom, respectively.  $l$  and  $L$  denote the length of the rope and boom, respectively, and  $(x, y, z)$  denotes the 3-D position of the load.

We assume that rotary crane dynamics has the following characteristics:

- The load can be considered as a point mass, and the torsion of the rope can be neglected.
- The angle and angular velocity ( $\theta_4$  and  $\dot{\theta}_4$ ) of horizontal boom motion, and the sway angles and their angular velocities ( $\theta_i$  and  $\dot{\theta}_i$ ,  $i=1,2$ ) are measurable.
- The sway angle  $\theta_i$  has a small magnitude, so that the approximations  $\sin \theta_i \simeq \theta_i$  and  $\cos \theta_i \simeq 1$ , ( $i=1,2$ ) are reasonable, and  $\dot{\theta}_i$  ( $i=1,2$ ) are bounded.

The equations of motion for a rotary crane are as follows:

$$l(1 + \theta_1^2)\ddot{\theta}_1 + l\theta_1\theta_2\ddot{\theta}_2 - l\theta_2\ddot{\theta}_4 + l\theta_1\dot{\theta}_1^2 + l\theta_1\dot{\theta}_2^2 - 2l\dot{\theta}_2\dot{\theta}_4 - (l\theta_1 + L \sin \theta_3)\ddot{\theta}_4 + g\theta_1 = 0 \quad (1)$$

$$l(1 + \theta_2^2)\ddot{\theta}_2 + l\theta_1\theta_2\ddot{\theta}_1 - l\theta_2\dot{\theta}_4^2 + l\theta_2\dot{\theta}_1^2 + l\theta_2\dot{\theta}_2^2 + 2l\dot{\theta}_1\dot{\theta}_4 + (l\theta_1 + L \sin \theta_3)\ddot{\theta}_4 + g\theta_2 = 0 \quad (2)$$

$$J\ddot{\theta}_4 + d = Ku \quad (3)$$

where  $g$ ,  $J$ ,  $d$ ,  $K$ , and  $u$  denote gravitational acceleration, inertia for horizontal boom motion, a nonlinear force term including disturbance, plant gain, and command voltage from the computer, respectively. Equation (3) assumes that the inertia for horizontal boom motion  $J$  is constant, and the time-variant effect is included in the disturbance  $d$ .

To compensate for the effect of the disturbance, the following disturbance observer<sup>(22)</sup> is applied to Eq. (3):

$$u = \frac{J}{K} \left\{ \dot{v} + \left( \frac{K}{J}u - s\dot{\theta}_4 \right) \frac{\omega}{s + \omega} \right\} \quad (4)$$

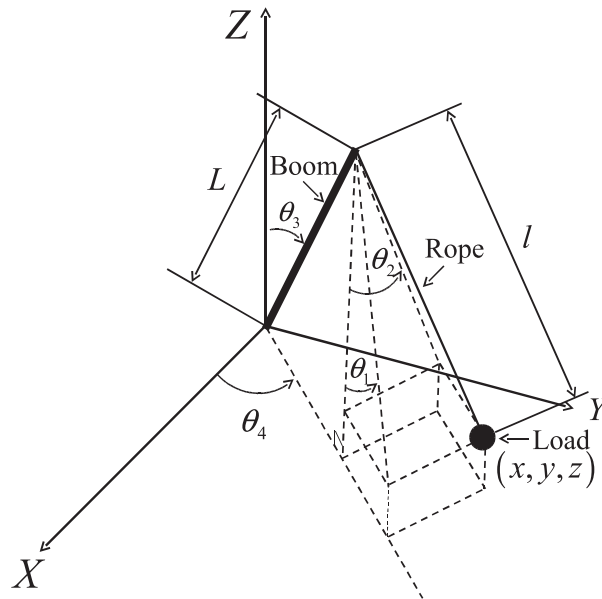


Fig. 1 Schematic model of rotary crane

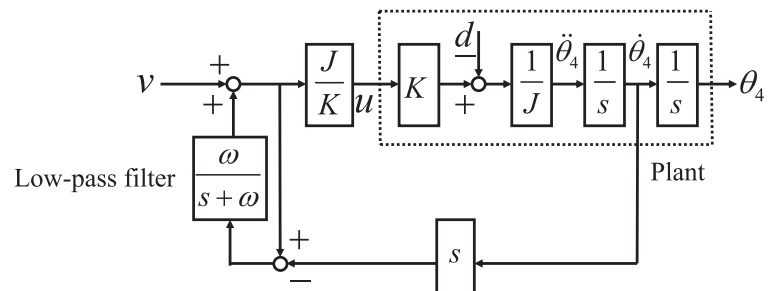


Fig. 2 Control system with disturbance observer

The block diagram of this control system is shown in Fig. 2, where  $s$ ,  $v$ , and  $\omega$  denote a differential operator, new virtual control input calculated by feedback controller, and cut-off angular frequency of the low-pass filter, respectively. The dynamics of the system in the low-frequency range is as follows:

$$\ddot{\theta}_4 \approx v \quad (5)$$

Because the centrifugal force caused by horizontal boom motion and gravitational force affect the two-dimensional load-sway significantly, to derive the control law, we linearize Eqs. (1) and (2) except for the centrifugal and gravitational force terms by assuming that  $\theta_i$  and  $\dot{\theta}_i$  ( $i=1,2$ ) are small such that  $\theta_i\theta_j \approx 0$ ,  $\theta_i\dot{\theta}_j \approx 0$  ( $j=1,2$ ),  $\dot{\theta}_i\dot{\theta}_4 \approx 0$ , and  $\theta_i\ddot{\theta}_4 \approx 0$  are satisfied.

$$\ddot{\theta}_1 = -\frac{g}{l}\theta_1 + \frac{L \sin \theta_3}{l}\dot{\theta}_4^2 \quad (6)$$

$$\ddot{\theta}_2 = -\frac{g}{l}\theta_2 - \frac{L \sin \theta_3}{l}\ddot{\theta}_4 \quad (7)$$

Combining Eqs. (5), (6), and (7), we arrive at a simple dynamical model of rotary crane motion as follows:

$$\ddot{\theta}_1 = -a\theta_1 + b\dot{\theta}_4^2 \quad (8)$$

$$\ddot{\theta}_2 = -a\theta_2 - bv \quad (9)$$

where  $a = g/l$ , and  $b = L \sin \theta_3/l$ .

### 3. Controller Design and Stability Analysis

Two control objectives are considered in this study. The first objective is to make the horizontal boom angle follow a given desired trajectory  $r$ , and the second objective is to suppress load-sway. We assume that the first and second derivatives of  $r$ ,  $\dot{r}$  and  $\ddot{r}$ , are available. To design a control law, we consider the following Lyapunov function candidate:

$$V = \frac{1}{2}K_e s_e^2 + \frac{1}{2}K_{\theta_1} s_1^2 + \frac{1}{2}K_{\theta_2} s_2^2 \quad (10)$$

$$\begin{aligned} e &= r - \theta_4, & s_1 &= \dot{\theta}_1 + \lambda_1 \theta_1, \\ s_e &= \dot{e} + \lambda e, & s_2 &= \dot{\theta}_2 + \lambda_2 \theta_2 \end{aligned}$$

where  $K_e$ ,  $K_{\theta_1}$ ,  $K_{\theta_2}$ ,  $\lambda$ ,  $\lambda_1$ , and  $\lambda_2$  are positive constants. The time derivative of Eq. (10) is given by:

$$\begin{aligned} \dot{V} &= K_e s_e (\ddot{r} + \lambda \dot{e}) + K_{\theta_1} s_1 (-a\theta_1 + \lambda_1 \dot{\theta}_1) \\ &\quad + K_{\theta_2} s_2 (-a\theta_2 + \lambda_2 \dot{\theta}_2) + K_{\theta_1} b s_1 \dot{\theta}_4^2 \\ &\quad - (K_e s_e + K_{\theta_2} b s_2) v \end{aligned} \quad (11)$$

Considering Eqs. (8), (9) and (11) to apply the Lyapunov stability theorem, we propose the following control law:

$$v = \begin{cases} K_1 p + \frac{w}{p} + \frac{K_2 K_{\theta_1} b |s_1 \dot{\theta}_4^2|}{p} & \text{if } |p| \geq \epsilon \\ K_1 p + \frac{w}{\epsilon} + \frac{K_2 K_{\theta_1} b |s_1 \dot{\theta}_4^2|}{\epsilon} & \text{otherwise} \end{cases} \quad (12)$$

$$\begin{aligned} p &= K_e s_e + K_{\theta_2} b s_2, \\ w &= K_e s_e (\ddot{r} + \lambda \dot{e}) + K_{\theta_1} s_1 (-a\theta_1 + \lambda_1 \dot{\theta}_1) \\ &\quad + K_{\theta_2} s_2 (-a\theta_2 + \lambda_2 \dot{\theta}_2) \end{aligned}$$

where  $K_1$  and  $K_2$  are positive constants, and  $\epsilon$  is a small positive constant.

Next we verify the validity of the proposed control law to achieve the two objectives. If  $|p| \geq \epsilon$ , then Eqs. (11) and (12) yield

$$\dot{V} = -K_1 p^2 + K_{\theta_1} b (s_1 \dot{\theta}_4^2 - K_2 |s_1 \dot{\theta}_4^2|) \quad (13)$$

If  $K_2 > 1$ , then  $\dot{V}$  is negative definite (assuming  $\dot{\theta}_4 \neq 0$ ). If  $|p| < \epsilon$ , then Eqs. (11) and (12) yield

$$\dot{V} = -K_1 p^2 + w \left(1 - \frac{p}{\epsilon}\right) + K_{\theta_1} b \left(s_1 \dot{\theta}_4^2 - \frac{p}{\epsilon} K_2 |s_1 \dot{\theta}_4^2|\right) \quad (14)$$

We have the following relation from Eq. (14):

$$\dot{V} \leq -\alpha V + \beta \quad (15)$$

$$\begin{aligned} \alpha &= 2K_1 \cdot \min\{K_e, K_{\theta_2} b^2\}, \\ \beta &= -2K_1 K_e K_{\theta_2} b s_e s_2 + \frac{1}{2} \alpha K_{\theta_1} s_1^2 + w \left(1 - \frac{p}{\epsilon}\right) \\ &\quad + K_{\theta_1} b \left(s_1 \dot{\theta}_4^2 - \frac{p}{\epsilon} K_2 |s_1 \dot{\theta}_4^2|\right) \end{aligned} \quad (16)$$

Because  $\theta_i$  and  $\dot{\theta}_i$  ( $i=1,2$ ) are bounded from the previous assumptions and  $|p| < \epsilon$ , the second term of the right hand of Eq. (15) is bounded. Hence,  $V$  is bounded and the proposed control law may provide good control performance by adjusting controller parameters.

In addition, this variable structure controller may provide the robustness to the ignored nonlinear effect in the model Eqs. (8) and (9).



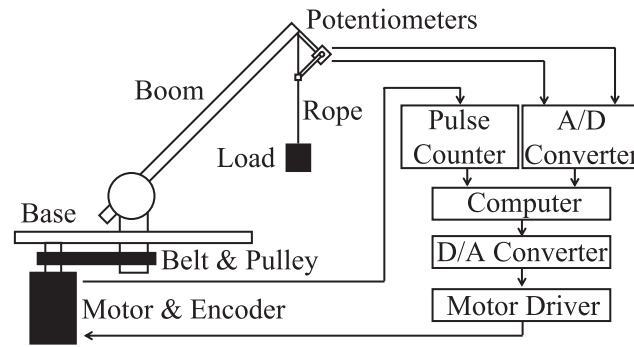


Fig. 3 Experimental system

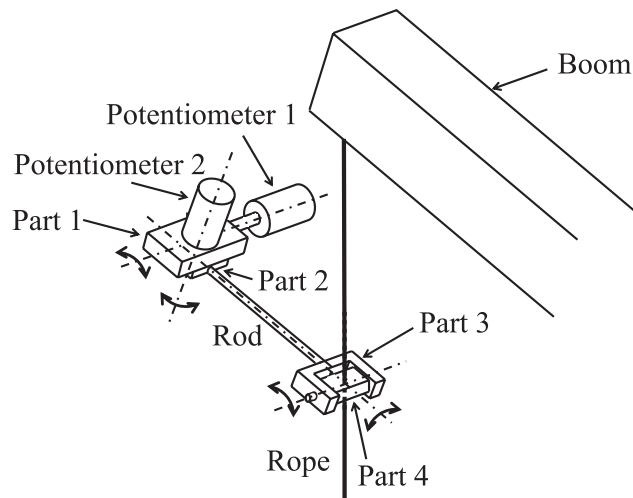


Fig. 4 Sensor system for measuring load-sway

#### 4. Simulation and Experimental Results

The experimental system is shown in Fig. 3. A DC servo motor drives the boom and base. The horizontal boom angle ( $\theta_4$ ) is measured using a rotary encoder, whose angular measurement resolution is  $1.8 \times 10^{-3}$ [deg].

The load-sway angles ( $\theta_1$  and  $\theta_2$ ) are measured by potentiometers 1 and 2 in Fig. 4. Potentiometer 1 is fixed to the boom. Potentiometer 2 is fixed to part 1, which rotates around the rotational axis of the potentiometer 1. Part 2 rotates around the rod and the rotational axis of the potentiometer 2. Part 3 rotates around the rod, and part 4 rotates around part 3. Part 4 slides smoothly along the rope. The angular measurement resolution is  $6.4 \times 10^{-2}$ [deg].  $\theta_1$  and  $\theta_2$  can be calculated from the output voltage values of potentiometers 1 and 2.

To demonstrate the effectiveness of the disturbance observer and the proposed controller, we conducted a simulation and an experiment with the parameters shown in Table 1. The following cycloid curve is employed for a desired angular trajectory of the boom:

$$r = (\theta_f - \theta_0) \left\{ \frac{t}{t_s} - \frac{1}{2\pi} \sin \left( 2\pi \frac{t}{t_s} \right) \right\} + \theta_0 \quad (17)$$

where  $\theta_f$  is the final angle,  $\theta_0$  is the initial angle, and  $t_s$  is the settling time, respectively. We set  $\theta_0 = 0$ ,  $\theta_f = 45$ [deg], and  $t_s = 3$ [s].  $r$  is set as  $r = \theta_f$  for  $t \in (t_s, t_f]$ , and  $t_f = 10$ [s]. The cycloid curve provides zero acceleration at initial and terminal points, and is widely used in industrial applications.

The controller parameters were assigned as shown in Table 2 by a trial and error manner. The value of  $\epsilon$  can be used to adjust the trade-off between the chattering in the command voltage and control performance.

Table 1 Parameters of rotary crane

$m[\text{kg}]$	0.24	$K[\text{N}\cdot\text{m}/\text{V}]$	2.06	$L[\text{m}]$	0.65
$l[\text{m}]$	0.25	$g[\text{m}/\text{s}^2]$	9.81	$\theta_3[\text{deg}]$	45.0

Table 2 Proposed controller gains

$K_e[\text{kg}\cdot\text{m}^2]$	1.00	$K_{\theta_1}[\text{kg}\cdot\text{m}^2]$	1.00	$K_{\theta_2}[\text{kg}\cdot\text{m}^2]$	0.30
$K_1[1/(\text{kg}\cdot\text{m}^2\cdot\text{s})]$	50.0	$K_2[1/\text{rad}]$	1.20	$\epsilon[\text{kg}\cdot\text{m}^2\cdot(\text{rad}/\text{s})]$	0.005
$\lambda[1/\text{s}]$	0.01	$\lambda_1[1/\text{s}]$	3.00	$\lambda_2[1/\text{s}]$	3.00

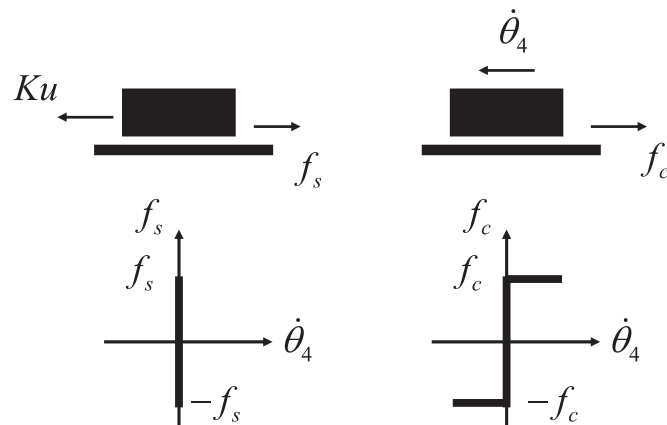


Fig. 5 Friction model

In this system, the disturbance  $d$  in Eq. (3) includes not only nonlinear force terms but also the following friction term:

$$f = C\dot{\theta}_4 + f_n \quad (18)$$

where  $C$  denotes a viscous friction coefficient, and  $f_n$  denotes a static friction term or a Coulomb friction term. The profile of  $f_n$  is shown in Fig. 5, and can be represented as follows:

$$f_n = \begin{cases} \text{sgn}\{Ku\} \cdot \min\{|Ku|, f_s\} & (\dot{\theta}_4 = 0) \\ \text{sgn}\{\dot{\theta}_4\} f_c & (\dot{\theta}_4 \neq 0) \end{cases} \quad (19)$$

where  $f_s$  and  $f_c$  denote the magnitude of the static and Coulomb friction, respectively. Their numerical values are given in Table 3.

The cut-off angular frequency for the disturbance observer is an important parameter for adjusting the trade-off between the stability and the disturbance rejection of the control system. We assigned it as  $\omega = 40$  [rad/s] by a trial and error manner. All angular velocities are calculated by taking the backward difference between successive positional measurements.

The effectiveness of the disturbance observer for overcoming the influence of the frictional disturbance is shown in Figs. 6(a) and (b). The results without the disturbance observer, as shown in Fig. 6(b), are also based on the proposed anti-sway control law. Simulation results well coincide with experimental ones in these figures. Although both experimental and

Table 3 Friction parameters

$C$ [N·m/(rad/s)]	1.39
$f_s$ [N·m]	3.71
$f_c$ [N·m]	3.58

Table 4 PD controller gains

$K_p$ [1/s <sup>2</sup> ]	50.0
$K_v$ [1/s]	10.0

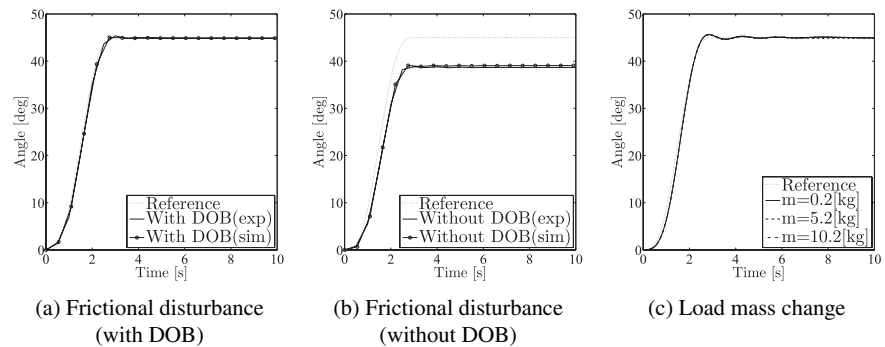


Fig. 6 Effectiveness of disturbance observer (DOB) for horizontal boom motion ( $\theta_4$ )

simulation results track the reference trajectory in Fig. 6(a), there remains significant tracking error in Fig. 6(b). These results confirm that the frictional disturbance can be compensated by using the disturbance observer.

We conducted a simulation based on the proposed method for verifying the effect of changes in load mass without the frictional term in Eq. (18). The results of this simulation are given in Fig. 6(c), which confirm that the proposed disturbance observer is indeed robust with respect to changes in load mass. The dynamics of rotary crane is described by Eqs. (1)-(3). Equations (1) and (2) do not include any load mass parameter because the oscillation property (natural frequency) of the rope-load system is determined mainly from the rope length. Hence, we need to consider only Eq. (3) for the robustness to the load mass change. Since the load mass change effect can be included in the disturbance term  $d$  in Eq. (3), we can compensate for the effect by using the disturbance observer.

To verify the effectiveness of the proposed anti-sway controller, we conducted simulation and experiment by using a conventional feedback controller without considering load-sway suppression. These results were obtained by using PD control with the same disturbance observer as used in our proposed method. The PD controller is as follows:

$$v = K_p e + K_v \dot{e} \quad (20)$$

where  $K_p$  and  $K_v$  are control gains, and were assigned as in Table 4 by a trial and error manner.

The results for the PD controller in terms of horizontal boom angle, command voltage, and load-sway angles are shown in Fig. 7, respectively. In Fig. 7(c), the experimental result is considerably different from the simulation result because of the effect of friction between the rope and the top of the boom. Friction gradually reduced the load oscillation in the plane of the vertical boom motion.

The results for our proposed controller in terms of horizontal boom angle, command voltage, and load-sway angles are shown in Fig. 8, respectively. The performance of load-sway suppression shown in Figs. 8(c) and (d) is much better than that displayed in Figs. 7(c) and (d), while similar steady-state error magnitude is shown in Figs. 7(a) and 8(a).

To demonstrate the effectiveness of the proposed method for different desired horizontal boom angle, we conducted simulations and experiments by setting  $\theta_f = 25$ [deg] and  $\theta_f =$

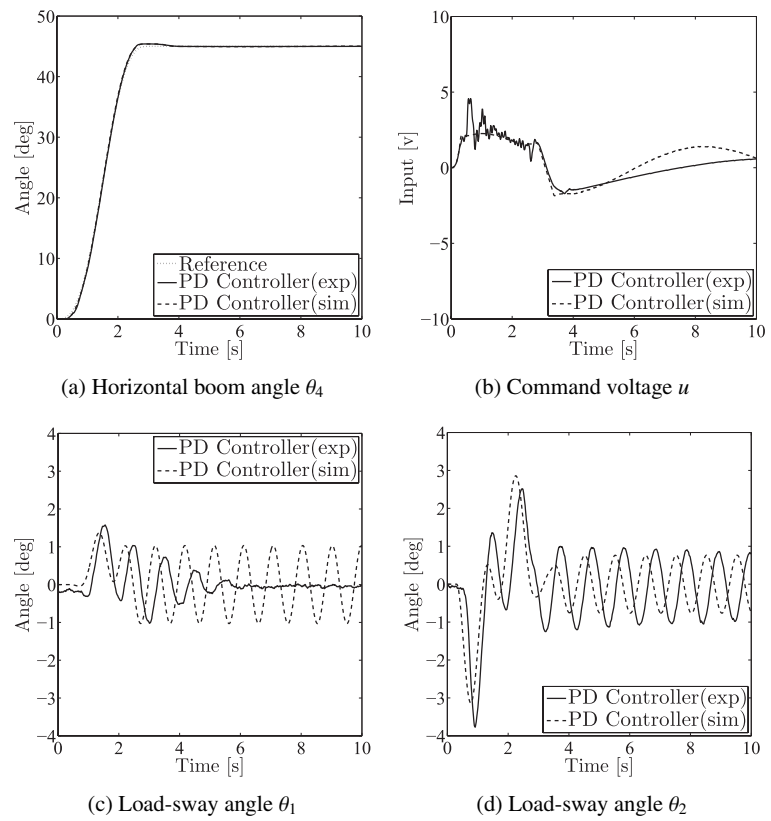


Fig. 7 PD control without anti-sway function

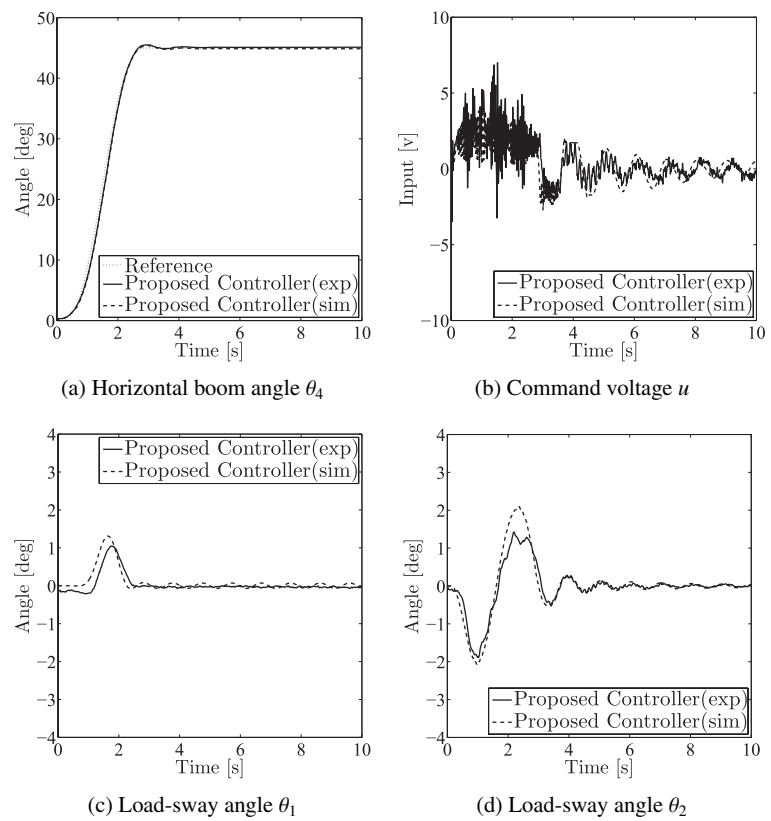


Fig. 8 Proposed control ( $\theta_f=45^\circ$ )



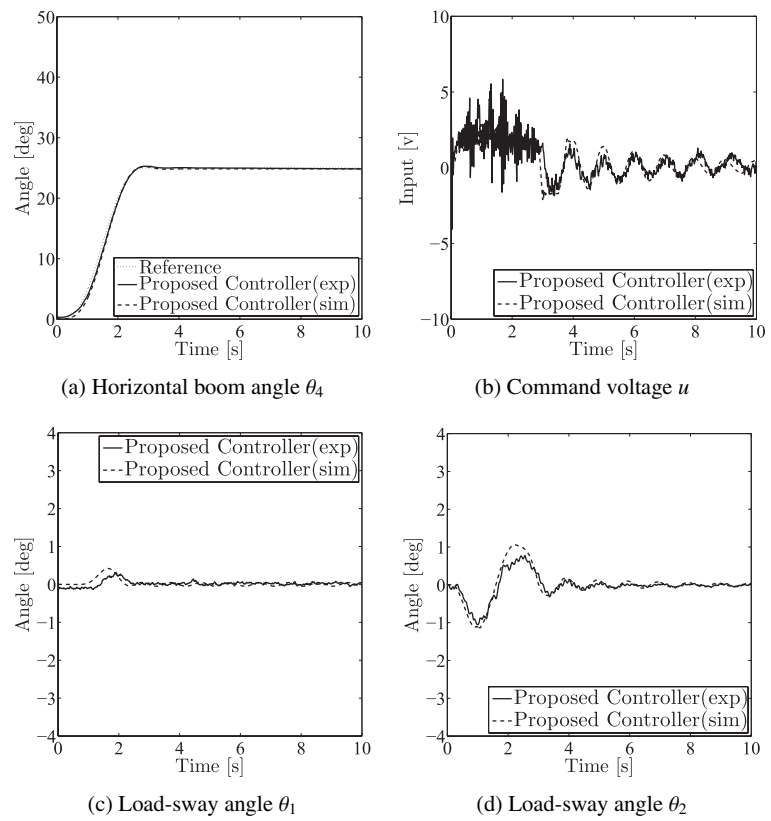


Fig. 9 Proposed control ( $\theta_f=25[\text{deg}]$ )

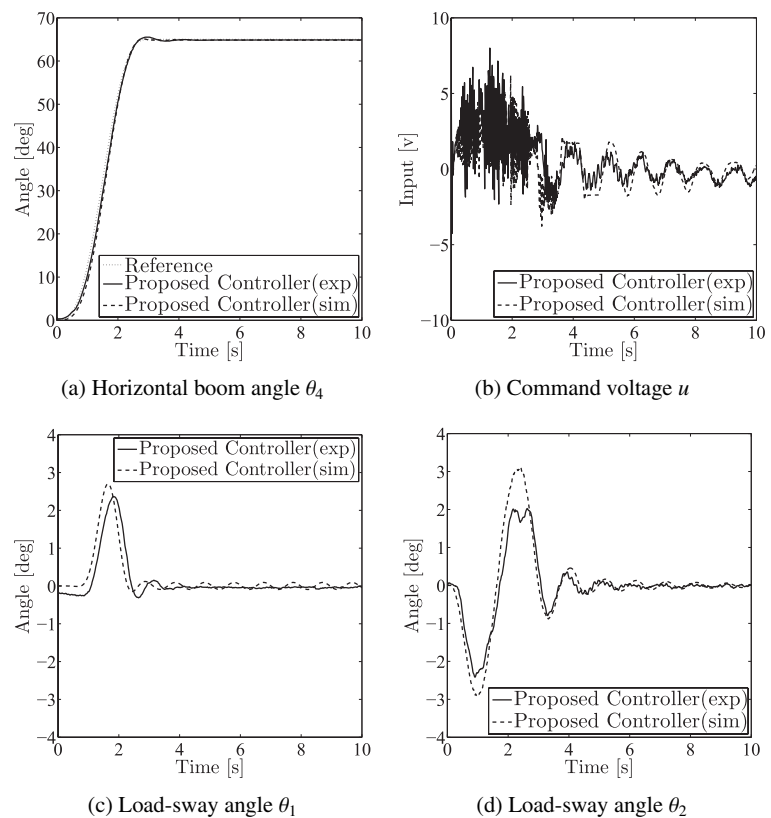


Fig. 10 Proposed control ( $\theta_f=65[\text{deg}]$ )

65[deg] with the same parameters as shown in Table 2. The results are shown in Figs. 9 and 10. We obtained almost the same results in these setups as were obtained with the condition  $\theta_f = 45[\text{deg}]$ .

These results confirm the effectiveness of the proposed method for anti-sway control of the load.

## 5. Conclusion

Our objectives were to suppress two-dimensional load-sway utilizing only horizontal boom motion, while making the horizontal boom motion track a given trajectory. To achieve them, we first derived a simple model for rotary crane dynamics using a disturbance observer. Next, we proposed an anti-sway controller based on the Lyapunov stability theory. The proposed control law may provide safer and energy efficient control of rotary cranes compared with existing approaches, because vertical boom motion is not employed for suppressing load-sway. Because the rope length affects the stability and performance of the control system, we are extending the proposed method to the varying rope length case in the future work.

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## Appendix A1 Comparative Simulation with a Linear State Feedback Control with an Integrator

We conducted a simulation to compare the proposed controller with a conventional linear state feedback controller (LSFC) with an integrator. Because the problem dealt with in this paper is to suppress the residual vibration using only horizontal boom motion, the linearized dynamics around the boom desired position becomes an uncontrollable system, and the conventional LSFC cannot be used for suppressing the vibration of angle  $\theta_1$ . Hence, the LSFC for controlling only  $\theta_2$ ,  $\dot{\theta}_2$ ,  $\theta_4$ , and  $\dot{\theta}_4$  is designed (i.e.,  $\theta_1$  and  $\dot{\theta}_1$  are uncontrollable). In this design, we use the same disturbance observer in Fig. 2 to compensate for the disturbance  $d$  in Eq. (3). The linearized crane dynamics is represented as follows:

$$\begin{aligned}\dot{x} &= Ax + bv \\ y &= c^T x\end{aligned}\tag{A1}$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 0 \\ -\gamma \\ 1 \end{bmatrix}, c = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, x = \begin{bmatrix} \theta_2 \\ \theta_4 \\ \dot{\theta}_2 \\ \dot{\theta}_4 \end{bmatrix}$$

where  $\alpha = g/l$ , and  $\gamma = L \sin \theta_3/l$ .

The LSFC with an integrator is designed as in Fig. A1, where  $P$  is the linearized crane model,  $k$  is the four dimensional state feedback vector,  $K_I$  is the integrator gain,  $v$  is the virtual control input,  $r$  is the reference signal for the boom motion, and  $y$  is the control output.

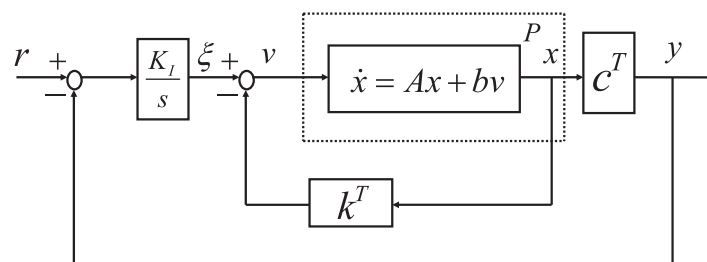


Fig. A1 State feedback control system

Table A1 Linear state feedback controller gains

$k_1[1/s^2]$	-65.9	$k_2[1/s^2]$	37.9	$k_3[1/s]$	-1.32
$k_4[1/s]$	24.3	$K_I[1/s]$	22.2	—	—

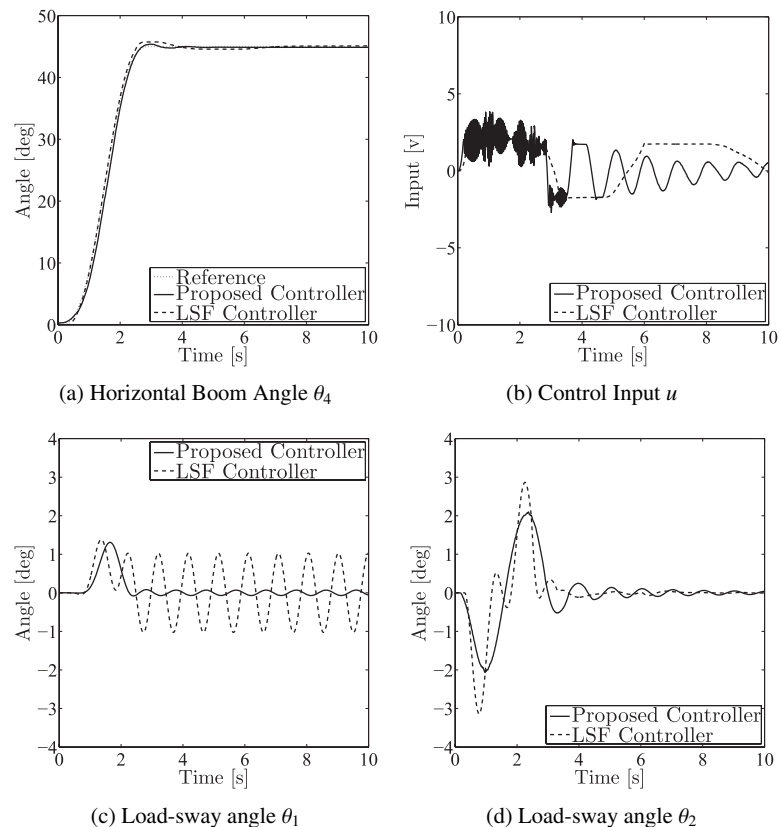


Fig. A2 Comparison results between proposed controller and LSFC (simulation)

The simulation condition is the same as that used in Fig. 8. Controller parameters were assigned as shown in Table A1 by solving an optimal control problem with the following objective function to achieve good control performance:

$$J = \int_0^\infty [\hat{x}^T(t)Q\hat{x}(t) + v^T(t)Rv(t)]dt \quad (A2)$$

where  $Q = \text{diag}\{1500, 315, 1500, 315, 50\}$ ,  $R = 1$ ,  $\hat{x} = [x^T, \xi^T]$ , and  $\xi$  is a state variable after the integrator in Fig. A1. The simulation results are shown in Fig. A2, where the control performance is compared with the proposed method. We obtained almost the same results in Figs. A2(a) and (d) between the proposed controller and the LSFC, while the performance of suppressing the load-sway angle  $\theta_1$  by using the proposed controller is much better than that with the LSFC in Fig. A2(c).

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