Phasor Estimation based on Modified Recursive Prony

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***Abstract*— Synchronized Measurement Technology represented by Phasor Measurement Units (PMUs) is a sophisticated tool for power system monitoring and contributes to a more reliable and secure operation of the power systems. Phasor estimation is the key of PMUs, providing amplitude and phase of voltage/current of the power network buses. In this paper, the performance of a recursive Prony algorithm for the phasor estimation is examined in steady state and under dynamic conditions. It is shown that the properties of the recursive algorithm have opposite impacts on the accuracy and the speed of the estimation, under these two conditions. To reach a good compromise between these two opposing properties, a modified recursive Prony, based on a time varying λ, is proposed in this paper.**

I. Introduction

With increasing penetration of renewable energy sources, the generation connected through the power electronic-based devices are significantly changing the dynamics of the power systems. This is affecting the requirements of the control, protection and monitoring. Utilization of the modern sensors and communication technologies appears to be the critical technological enabler for addressing these challenges. The synchronized Measurement Technology represented by the PMU seems to be the solution contributing to a more reliable, economical and secure operation of power systems [1], [2]. The phasor estimation is a main element of PMUs. Phase angles of voltage phasors of the power network buses have always been important to the power system engineers. As many of the planning and operational considerations in a power network are directly concerned with the flow of the real power, measuring the angle differences across the transmission lines is important. Although performance requirements (such as accuracy and settling time) are presented in recent IEEE standard for synchrophasor (C37.118 [3]), a specific algorithm is not proposed there. Therefore, phasor estimation procedures have recently attracted significant attention [4], [5]. Among others, in [6] a new method based on an adaptive band-pass filter is proposed to estimate phasors. An integrated phasor and frequency estimation using a fast recursive GaussNewton algorithm is proposed in [7] and a method based on modified Fourier transform for eliminating Direct Current (DC) offset is presented in [8]. Reference [9] introduces an angle-shifted energy operator to extract the amplitude. In [10], an approach for estimating the phasor parameters based on a recursive wavelet transform is introduced. In [11], the Prony algorithm is used to estimate the phasors with exponential amplitude and linear phase.

Prony analysis is one of the most common measurement-based identification tools to estimate oscillatory modes. The Prony algorithm approximates the main signal by exponentially damped sinusoidal signals. This algorithm is able to determine the values of frequency, damping factor, amplitude and phase of the main signal. By this algorithm, frequency and damping factor parameters are calculated in the first step and consequently the phasor is obtained in the second step [12], [13]. Despite the promising performance of the Prony algorithm, the accuracy is reduced under noisy conditions. To address this problem, multi-channel Prony is proposed in the literature [14]-[16]. The centralized multi-channel Prony is a basic solution, which is constructed based on measured data from different channels of one or several PMUs. However, due to the size of the matrices in the generalized multi-channel, different strategies may be used to make it more efficient. One solution for reducing the computational burden is a recursive pattern as proposed in [17]-[19]. In [17] the recursive approach is proposed to extract the modal information. A modified recursive Prony is also proposed in [18] to estimate the monotonous trend of an oscillating system. In addition, an oscillation detection method based on the recursive Prony is proposed in [19] to automatically detect ring-down data. Generally, the Prony is a three-stage algorithm consisting of two stages with least square (LS) minimization process. The LS algorithm is a widely used method for the phasor and mode estimation [20]-[22] while it must be recomputed for every data window in online applications, which makes it time consuming. To solve this, the recursive-least-squares (RLS) is used as a more efficient estimation method. Since the Prony is a least square-based algorithm, it can recursively be solved. With fast convergence and good robustness, the recursive Prony fits to real time processes.

According to the IEEE Standard for synchrophasors, the phasor estimation algorithms must be examined in steady state and under dynamic conditions. In this paper, the performance of the recursive Prony under both conditions is examined. Accordingly, a new parameter named, "*Forgetting Factor* (λ)", is used in the phasor estimation process giving exponentially less weight to the older error samples. The impact of λ on the performance of the recursive Prony is analyzed in steady state and under dynamic conditions. Based on this analysis, a time varying λ ($TV\lambda$) is implemented as a trade-off between the performance requirements under steady state

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and dynamic conditions.

II. Recursive Prony Algorithm

An algorithm for the recursive Prony is presented in this section. The Prony analysis is a technique to model a signal with sum of damped complex exponentials as:

$$y(t) = \sum_{k=1}^{k=L/2} A_k e^{\alpha_k t} cos(\omega_k t + \phi_k) \quad (1)$$

where A_k , α_k, ϕ_k and ω_k are the amplitude, damping coefficient, phase and angular velocity of the k^{th} component. L represents the system order. Using the Eulers theorem, a cosine function can be represented as:

$$y(t) = 0.5 \sum_{k=1}^{k=L} A_k e^{(\alpha_k + j\omega_k)nT + j\phi_k} \quad (2)$$

where T is the sampling period. Considering $L = 2$, (2) is reduced to:

$$y(t) = 0.5A_1 e^{j\phi_1} e^{(\alpha_1 + j\omega_1)nT} + 0.5A_2 e^{j\phi_2} e^{(\alpha_2 + j\omega_2)nT} \quad (3)$$

where $Z_1 = e^{(\alpha_1 + j\omega_1)nT}$ and $Z_2 = e^{(\alpha_2 + j\omega_2)nT}$ as conjugated poles ($Z_2 = Z_1{}^*$), so:

$$y(nT) = 0.5hZ_1^n + 0.5h^* Z_1^{*n} \quad (4)$$

where $h = A_1 e^{j\phi_1}$ is the phasor of $y(t)$. To extract the phasor by Prony, three stages are anticipated. In the first stage, the linear prediction (Auto regressive) is solved. For N samples of the measured signal, we have:

$$\begin{pmatrix} y[0] \\ y[1] \\ y[2] \\ \vdots \\ y[N-1] \end{pmatrix} = \begin{pmatrix} y[-1] & y[-2] \\ y[0] & y[-1] \\ y[1] & y[0] \\ \vdots & \vdots \\ y[N-2] & y[N-3] \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \quad (5)$$

$$\boldsymbol{Y} = \boldsymbol{Q}\, \boldsymbol{a}$$

The estimate of the vector $\boldsymbol{a}$, including the frequency, is obtained by the recursive least square (RLS). The RLS needs no matrix inversion and is therefore computationally efficient. To solve (5) by a recursive solution, we have to consider one row of the matrix $\boldsymbol{Q}$ as:

$$\boldsymbol{D}(n) = [y(n-1) \quad y(n-2)] \quad (6)$$

At the first step of the recursive solution in the first stage of Prony, the difference between the estimated sample ($\hat{y}(n)$) and the new measured sample ($y(n)$) is:

$$\alpha_1(n) = y(n) - \hat{y}(n) = y(n) - \boldsymbol{D}(n)\boldsymbol{a}(n-1) \quad (7)$$

In the next step, the gain of the first stage (k_1) is calculated to determine the required change, when a new sample is acquired. The gain is calculated as:

$$\boldsymbol{k_1}(n) = \frac{\lambda^{-1}\boldsymbol{P_1}(n-1)\boldsymbol{D}^T(n)}{1 + \lambda^{-1}\boldsymbol{D}(n)^T \boldsymbol{P_1}(n-1)\boldsymbol{D}^T(n)} \quad (8)$$

where T is the transpose operator. According to the calculated gain, the new estimated vector $\boldsymbol{a}$ and its error covariance P_1 are obtained as:

$$\boldsymbol{a}(n) = \boldsymbol{a}(n-1) + \boldsymbol{k_1}(n)\alpha_1(n) \quad (9)$$

$$\boldsymbol{P_1}(n) = \lambda^{-1}\boldsymbol{P_1}(n-1)(1 - \boldsymbol{k_1}(n)\boldsymbol{D}(n)) \quad (10)$$

where λ is *forgetting factor*, which is a scalar in the (0 1] range. Setting $\lambda = 1$ corresponds to no forgetting. In this case just constant coefficients can be estimated. Setting $\lambda < 1$ implies that the past measurements are less significant and can be ignored. Therefore, by setting $\lambda < 1$, an estimate of time-varying coefficient is possible.

In the second stage of the Prony algorithm, roots of the polynomial $F(z)$ are extracted as:

$$F(z) = (z - Z_1)(z - Z_1^*) = a_0 z^2 + a_1 z + a_2 \quad (11)$$

In the third (last) stage, the phasor equations are solved based on the estimated roots (Z_1, Z_1^*). Presume the $y(t)$ is sampled at N samples:

$$\begin{pmatrix} y[0] \\ \vdots \\ y[n] \\ \vdots \\ y[N-1] \end{pmatrix} = 0.5 \begin{pmatrix} 1 & 1 \\ \vdots & \vdots \\ Z_1^n & Z_1{}^{-n} \\ \vdots & \vdots \\ Z_1^{(N-1)} & Z_1{}^{-(N-1)} \end{pmatrix} \begin{pmatrix} h \\ h^* \end{pmatrix} \quad (12)$$

$$\boldsymbol{Y} = \boldsymbol{J}\, \boldsymbol{H}$$

The phasor is estimated using the recursive least square (RLS) by creating vector $\boldsymbol{C}$ as one row of matrix $\boldsymbol{J}$:

$$\boldsymbol{C}(n) = [0.5Z_1^n \quad 0.5Z_1{}^{-n}] \quad (13)$$

The difference between the estimated sample $\hat{y}(n)$ and the new measured sample $y(n)$ is:

$$\alpha_2(n) = y(n) - \hat{y}(n) = y(n) - \boldsymbol{C}(n)\boldsymbol{H}(n-1) \quad (14)$$

The gain of recursive solution for the third stage of the Prony is calculated as:

$$\boldsymbol{k_2}(n) = \frac{\lambda^{-1}\boldsymbol{P_2}(n-1)\boldsymbol{C}^T(n)}{1 + \lambda^{-1}\boldsymbol{C}(n)^T \boldsymbol{P_2}(n-1)\boldsymbol{C}^T(n)} \quad (15)$$

According to the calculated gain, the new estimate and the covariance of error are:

$$\boldsymbol{H}(n) = \boldsymbol{H}(n-1) + \boldsymbol{k_2}(n)\alpha_2(n) \quad (16)$$

$$\boldsymbol{P_2}(n) = \lambda^{-1}\boldsymbol{P_2}(n-1)(1 - \boldsymbol{k_2}(n)\boldsymbol{C}(n)) \quad (17)$$

Finally, the phasor is calculated by the recursive Prony. According to the explained formulation, the recursive Prony introduces a new parameter entitled *forgetting factor (λ)*. The *forgetting factor* is an important tuning parameter to improve the estimation process (accuracy and speed). In the next section, this parameter will be examined and a new strategy for defining it is proposed.

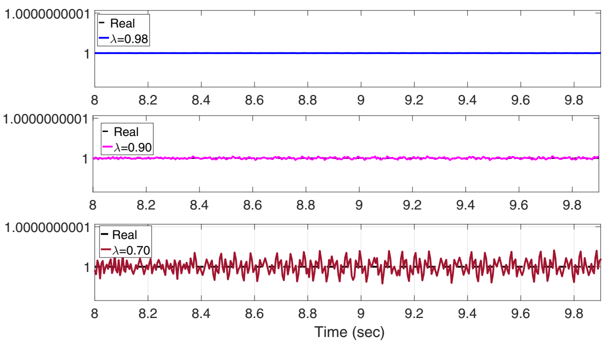

Fig. 1. Performance of recursive Prony with three different values of λ during steady state

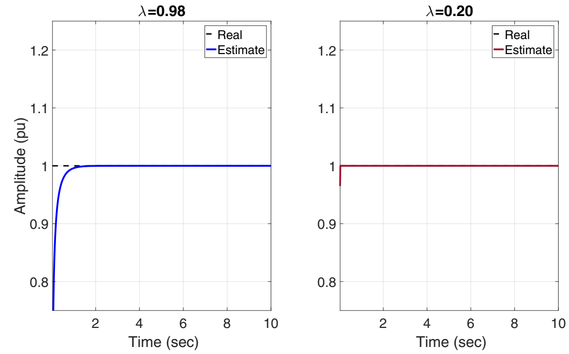

Fig. 3. Amplitude estimate by recursive Prony with two values of λ

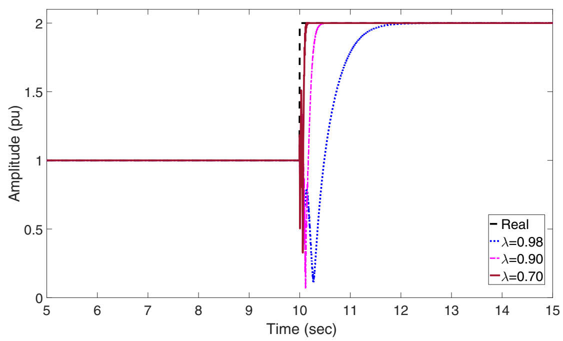

Fig. 2. Performance of recursive Prony with three different values of λ during step change in amplitude

III. Modified recursive Prony

The main attribute of the recursive Prony is being independent of the matrix inversion. The parameter (λ) can be used to improve the accuracy and time response of the phasor estimation. Consider a sinusoidal signal as $cos(2\pi f_0 t)$, where the frequency is $50Hz$. By assigning three different values to the forgetting factor ($\lambda = 0.98, 0.9, 0.7$) and conducting the recursive Prony under steady state, the results obtained are shown in Fig.1.

According to Fig.1, the highest value of λ ($\lambda = 0.98$) delivers the most accurate amplitude. This observation aligns with the definition of λ, where higher values provides more accurate estimate in steady state due to use of more samples in the estimation process. Therefore, a higher value of λ is better under steady state condition. However, the method should also perform well under dynamic conditions. Consider a sinusoidal signal where the amplitude is stepped up. This test is also run by recursive Prony assigning three values for λ ($\lambda = 0.98, 0.9, 0.7$). The results are shown in Fig.2.

According to Fig.2, the three values of λ provide different time responses. The lowest value of λ delivers fastest response and shortest settling time due to use of less samples. Hence, lower value of λ is better under dynamic conditions. Fig.1 and Fig.2 indicate contrasting trends of performance with respect to the values of λ. The accuracy of the phasor estimation during steady state increases by increasing λ and in contrast, lower values of λ improve the time response under dynamic condition. Since the objective is to provide appropriate performance under both conditions, a trade-off is proposed in this paper. By defining a time-varying λ, a better phasor estimate can be achieved under both conditions. By defining upper and lower limits for λ, this parameter is allowed to change between these values. An index of activation is proposed to trigger the variation of λ as:

$$TRIG(n) = y(n) - \boldsymbol{C}(n)\boldsymbol{H}(n-1) \tag{18}$$

If $TRIG$ is bigger than a threshold value (TH), λ is set on lower limit ($\lambda = 0.20$) and it is set on upper limit ($\lambda = 0.98$) if the $TRIG$ is smaller.

IV. Simulation Results

To analyze the performance of the modified recursive Prony (proposed method), different test cases are provided in this section.

A. Performance of the classical recursive Prony

Firstly, the accuracy of the classical recursive Prony is evaluated. The main signal is considered as:

$$y(t) = Acos(\omega_0 t + \theta_0) \tag{19}$$
$$A = 1; \omega_0 = 2\pi 50; \theta_0 = \pi/4$$

The signal presented in (19) is synthesized in MATLAB. The fundamental frequency is $50Hz$ and there are 20 samples per cycle. To show the impact of λ on the accuracy of phasor estimates, two different values of λ are analyzed: $\lambda = 0.98$ and $\lambda = 0.2$.

The amplitude and phase estimates for two different values of λ are shown in Fig.3 and Fig.4. According to the figures, the recursive Prony operates accurately with both values. However, there is a considerable delay when $\lambda = 0.98$. Two indices are utilized to quantify the accuracy and delay under these two conditions. The first one is settling time defined as the time delay to reach to 0.98% of nominal value, and the second index is Total Vector Error (TVE) defined as:

$$TVE = \frac{|X_r - X_e|}{|X_r|} \tag{20}$$

where X_r and X_e are the real and estimated values of the phasor. $X_r = Ae^{j\theta_0}$ and $X_e = |h|\angle h$ that is estimated phasor

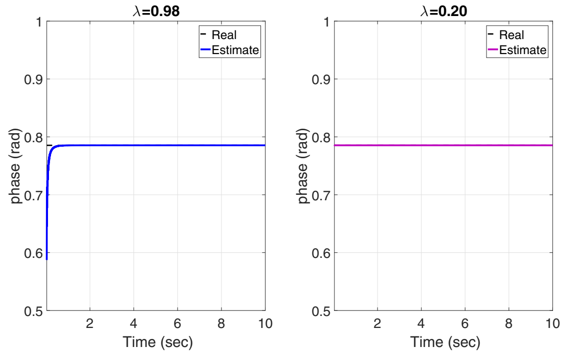

Fig. 4. Phase estimate by recursive Prony with two values of λ

TABLE I

SETTLING TIME AND TVE UNDER TWO DIFFERENT VALUES OF λ

–	$\lambda = 0.98$	$\lambda = 0.2$
Settling time (sec)	0.542	0.024
TVE (%)	3.22×10^{-4}	0.2407

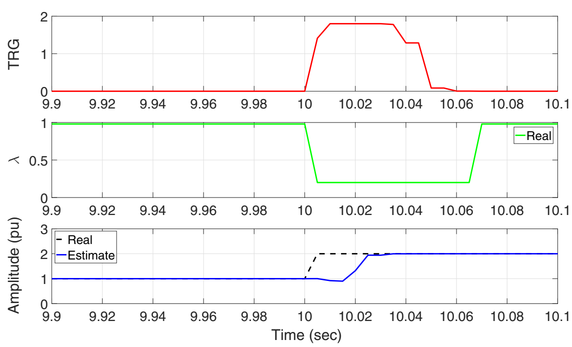

Fig. 5. Amplitude estimation during step changes by $TV\lambda$

using the recursive Prony. The two mentioned indices are calculated under two conditions and the results are tabulated in Table I. According to the table, a higher value of λ ($\lambda = 0.98$) provides a lower error while a lower λ ($\lambda = 0.2$) provides a faster time response (shorter settling time).

B. Performance of the proposed method under amplitude and phase step

A step change is considered in this section to examine the performance of the proposed method. The test case is:

$$y(t) = A[1 + k_x f_1(t)]cos(\omega_0 t + \theta_0[1 + k_a f_1(t)]) \quad (21)$$
$$A = 1; \omega_0 = 2\pi 50; \theta_0 = \pi/8$$
$$k_x = 1; k_a = 1; f_1(t) = u(t - t_0); t_0 = 10$$

where A is the amplitude of the input signal, ω_0 is the nominal power system frequency, $f_1(t)$ is a unit step function, k_x is an magnitude step size and k_a a phase step size. This case is used as transition test between two steady state conditions (the amplitude and phase are increased at $t = 10sec$ to two times of the primary values).

According to the proposed method, the strategy $TV\lambda$ can achieve a trade-off between the two conditions (steady state and dynamic). By defining an upper limit ($\lambda = 0.98$) and a lower limit ($\lambda = 0.20$) for forgetting factor, this parameter can be changed between these limits. The proposed method is activated by comparing the error index (TRG) with a threshold value ($TR = 2 \times 10^{-4}$). If the TRG is bigger than the TR,the λ is set on $\lambda = 0.20$ and it is set on $\lambda = 0.98$ if the TRG is smaller than the TR. The proposed strategy based on $TV\lambda$ is employed for the test case (21) and the results are shown in Fig.5 and Fig.6. Fig.5 shows the variation of TRG, λ and the amplitude estimate under a step change. According to the figure, before the step time ($t_0 = 10sec$), λ is set on the upper limit ($\lambda = 0.98$) to provide the most accurate estimate in steady state condition. Suddenly, a step change occurs at $t_0 = 10sec$ and the real amplitude is changed from one to two per-unit. According to the proposed strategy, the algorithm detects the sudden change of TRG and sets λ to its lower limit ($\lambda = 0.20$). This reduction in *forgetting factor* ensures the short settling time under the dynamic condition. Likewise, after reaching the steady state condition, the proposed strategy sets λ on its upper limit ($\lambda = 0.98$) again to ensure an accurate estimate in the steady state. Moreover, the same procedure can be explained for phasor estimation and the results are shown in Fig.6.

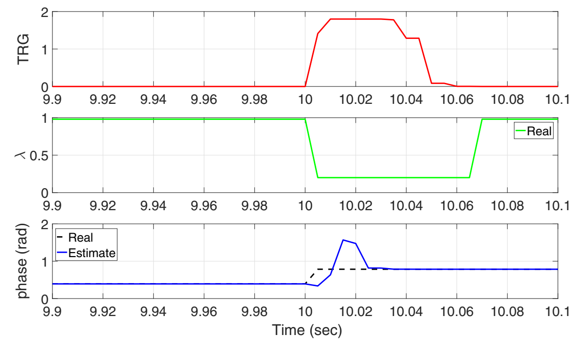

Fig. 6. Phase estimation during step changes by $TV\lambda$

C. Performance of the proposed method in a three-machine power system

In order to examine the proposed method in a real power system, a three-machine power system shown in Fig.7 is modelled and simulated using SIMULINK [23]. Fundamental frequency is $60Hz$ and there are 512 samples per cycle. A three-phase fault is simulated at 90% of the line connecting bus 5 and bus 1. The fault occurs at $t = 1s$ and is cleared after $0.03sec$. This event causes a power swing and is observed by a distance relay installed at the transmission line L_5.

The main signal is in steady state condition during $0s < t < 1s$ and a power swing is started after $t = 1.03s$. Variation of α, variation of TRG and the estimated amplitude are shown in Fig.8. According to this figure, before $t_0 = 10sec$, λ is set on its upper limit ($\lambda = 0.98$) to provide highest level of estimation accuracy in steady state. However, λ is set on its

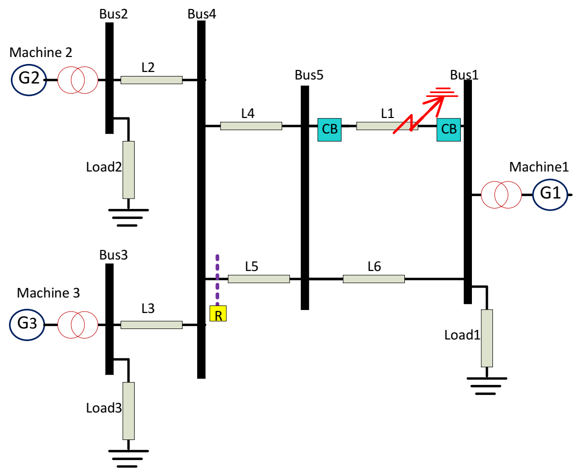

Fig. 7. Three-machine power system

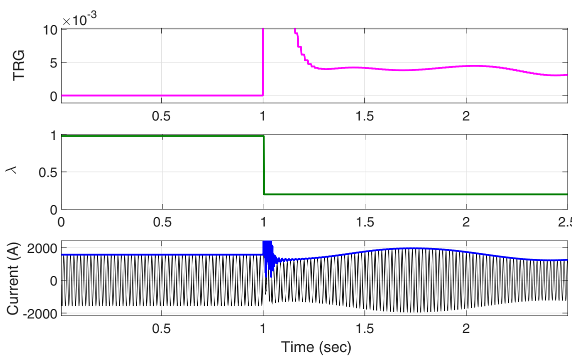

Fig. 8. Amplitude estimation by proposed method

lower limit ($\lambda = 0.20$) to deliver appropriate performance during the power swing.

V. Conclusion

An adaptive recursive Prony phasor estimation method is examined under steady state and dynamic conditions. First, the performance of the classical recursive Prony is examined. Next, the adaptive recursive Prony based on a time varying *forgetting factor* ($TV\lambda$) is proposed and tested. The results indicate that higher values of λ provides more accurate estimation at the expense of a longer delay under dynamic conditions. In contrast, lower values of λ provides faster response at the expense of accuracy of estimation. A good compromise is achieved with the proposed time-varying λ. According to the obtained results from the modified recursive Prony, this method can satisfy both steady state and dynamic objective function performance requirements simultaneously.

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