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# A Novel Algorithm for Estimating Purchase Incentive of The Public Based on Mobile Cloud Computing

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**ABSTRACT** Mobile Cloud Computing provides applications of cloud computing technology on mobile internet. In the era of 5G networks, Mobile Cloud Computing offers the possibility of heavy computational in parallel through different mobile terminals. This also offers us to use Mobile Cloud Computing to analyze fluctuations in user buying incentives over the course of a year. In various business scenarios, the purchase incentive of the public is often considered as being controlled by multiple factors and changes dramatically during holiday seasons. This paper proposed an evolutionary algorithm for monitoring purchase incentive of the public which can be distributed in Mobile Cloud Computing. In this work, we demonstrate that the public's shopping behavior of a commodity is a consequence of a collective behavior propagating through a social network and can be modeled as a simple ODEs model. Then, based on this new model, we develop an extended Differential Evolution algorithm that is deployed on mobile terminals and combined with 5G network computing to estimate public incentives from historical sales datasets. Results suggest that this method can successfully monitor the public's purchase incentive. Additionally, the overall purchase incentive changes dramatically during holiday seasons but fluctuates with similar patterns every year.

**INDEX TERMS** Mobile Cloud Computing, Internet of Thing, data analysis.

## I. INTRODUCTION

Mobile Cloud Computing (MCC) brings rich computational resources to cloud computing providers, network operators, and mobile users [1], [2]. These computational resources make MCC a great potential for distributed computing and have been used to solve a variety of learning and optimization problems. As an Evolutionary algorithm requires a heavy computation burden, it will consume tremendous time for a single processing unit. Here, an evolutionary algorithm is suitable to be decomposed into sub-tasks, which can be distributed in mobile devices within a cloud. We applied this method to estimate the public's purchase incentive from historical sales data. First, we need to derive a mathematical

model to describe the fluctuation of public's purchase incentive under the seasonal influence. Then, based on this model, we propose an extended differential evolution algorithm to estimate the public's purchase incentive from historical sales data.

Many large-scale real-life networks, such as mobile network, traffic system and [3], vehicular social networks [4], [5], Internet of thing [6], can be described as a complex network in which nodes represent organizations or entities, while links stand for interactions among the nodes [7], [8]. In many social, biological and communication networks, each node of networks may assume several states and can transit from one state to another, such as the susceptible-

infected-recovered model [9], [10], the susceptible-infected-susceptable model [11], the growth of membership-based website [12], complex network of language evolution [13], [14], the spreading of social behaviour [15], [16] and etc. We claim this important class of network as transition networks. Then, we can derive a general model describing the dynamic properties of a transition network and develop a simulation algorithm for studying the network evolutionary behavior [17]. Our research in transition networks provided convenient access to analyze collective human behavior. For instance, the business proceeding of the growth of the user population in term of a connected community, or a network of the growth of the user and prospective users can be modeled by a simple transition network.

In this work, we focus on developing a user growth model, based on two fundamental behaviors of decision making and construction of a networked community, that can universally describe the growth of the user amount of a product or service, such as mobile service. A user growth network is a transition network in which each node assume a state  $P$  or  $U$ , corresponding to it being a prospective user or user. Links represent a community of users and prospective users who may assert mutual influence on one another. Then, nodes can transit from one state  $P$  to state  $U$  as time elapses obeying two transition laws: the self-transition law and the peer-influenced transition law. These two fundamental laws stand for two basic types of human decision behaviors, namely, the personal choice and word of mouth [18], [19]. Then, the user growth in a transition network can be modeled as stochastic processes. Based on the mean-field method, we rigorously derive a universal growth equation that describes the user growth profile in general uncorrelated network [20], [21]. Furthermore, if the network is homogenous, the dynamics of users population obeys a simple first-order ODE:  $dx/dt = (c_1(t)Nx - c_1(t)(1 + \delta(x))x^2) + (c_2(t)N - c_2(t)x)$ , where  $x(t)$  represents the expectation number of users,  $N$  is the total users,  $c_1(t)$  is a stochastic rate of transition that determines how likely a prospective user would transit to a user by word-of-mouth influence, while another stochastic rate of transition  $c_2(t)$ , corresponding to self transition of a prospective user to an user following pure personal choice;  $\delta(x)$  is a residual factor. In the real world, user growth (product sales) speed always fluctuates dramatically in a short time. We consider these phenomenon is contributed to the fluctuation of the combined incentive for the prospective users  $C = c_1x + c_2$  in a short time, namely,  $c_1$  and  $c_2$  are not constants but time-varying, which brings  $C(t) = c_1(t)x + c_2(t)$ . The combined incentive is influenced by many factors, such as promotion, advertising, cut off, holiday effect, shopping festival and etc. Here, we will show the combined incentive  $C(t)$  and leftover market size  $\Delta N = N - x$  influences user growth (sales) speed. Then, the inverse technique is required to estimate the fluctuation of the combined incentive, which is always formulated as an optimization problem. This problem is actually a constraint nonlinear programming problem, which requires heavy computation source to solve it. Then, we

develop an extended differential evolution algorithm, which can be distributed on Moblie Cloud Computing platform.

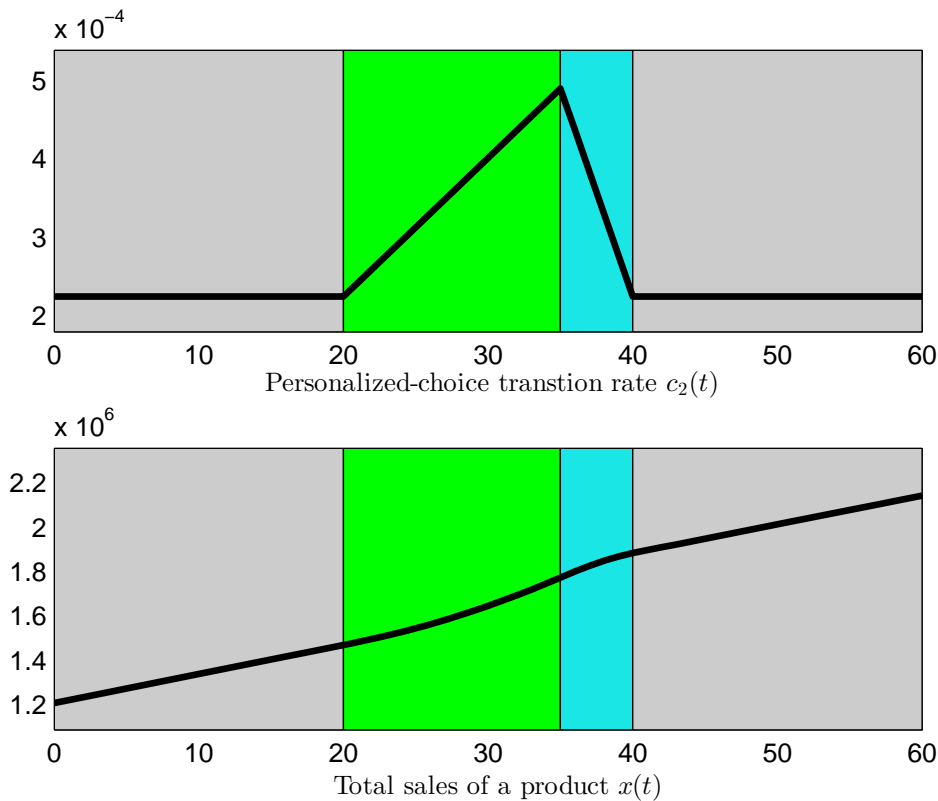
In this working, real sales data of video game market, including three seventh generation video game consoles (XBOX360, PlayStation 3, Nintendo Wii) and two handled video game consoles (Nintendo DS, PlayStation Portable) during 2005 to 2014 is utilized to analyze the dynamics of  $C(t)$  over years [22]. A video game console is a device that output a video game signal to display a video game. Inverse method is applied to find  $c_1(t)$ ,  $c_2(t)$  and the leftover market size  $\Delta N$  from the historical sales data. Then, the relative effectiveness of customer, service and promotional efforts can be studied. Results show that the combined incentive  $C(t)$  fluctuation obeys similar patterns every year:  $C(t)$  always keeps in a low level (a magnitude of  $10^{-4}$ ) from the beginning of Feb to the middle of Oct. This period is the low season of video game console market; In holiday season, from middle Oct to Christmas,  $C(t)$  increases dramatically to a magnitude of  $10^{-3}$ ; Then, after Christmas,  $C(t)$  drops quickly to a low level. We show the speed of sales is rough  $\Delta x \approx C \times \Delta N \times \Delta t$ . For example  $\Delta t = 7\text{days}$ ,  $\Delta x(t)$  means the weekly sales. This formula shows that the speed of sales, such as weekly sales of a console, in proportion to the combined incentive and the leftover market size. Hence, an attractive product with a larger  $C(t)$  always enjoys larger weekly sales. However, there are other important factors, leftover market size  $\Delta N$ , influence the speed of sales. For example, if there are two product with similar  $C(t)$ , the one with larger leftover market size has larger weekly sales. The analysis of video game console sales data supports the previous conclusion. For example, in the year 2012, we find the combined incentive of Nintendo Wii  $C^W$  is larger than  $C^X$  (Xbox360) and  $C^P$  (PlayStation 3). However, the leftover market size of Wii is  $\Delta N^W \approx 8\text{millions}$ , which is smaller than  $\Delta N^X \approx 30\text{millions}$  and  $\Delta N^P \approx 36\text{millions}$ . Hence, the weekly sales of Wii are worse than Xbox360 and PlayStation 3 in the year 2012. The result matches our intuitive feeling of selling a product.

In the next section, preliminaries of the user growth model, console sales data and holiday effect are given. In section III, a summary of the results is presented. Finally, the conclusion is given in section IV.

## II. PRELIMINARIES

### A. USER GROWTH MODEL

We define a network  $G = (V, E)$  of prospective users and users of a service or product, where  $E$  and  $V$  represent the sets of edges and nodes, respectively. Suppose there are  $N$  nodes, which denotes individuals and can assume one of two possible states:  $P$  and  $U$ . A link between two nodes means that two individuals are mutual acquaintances, e.g., being families, relatives, colleagues, etc. A prospective user ( $P$ ) can transit into a user according to two rules: (1) Word-of-mouth: a prospective user can be positively informed about a product or service by other users who are acquaintances of the prospective user. Then, the prospective user may adopt



**FIGURE 1.** Personalized-choice rate  $c_2(t)$  influences the user(sales) growth: the gray part is the low season, the green part is the “holiday season” period (peak season) and the cyan part is the “after holiday season” part (low season).

the service or product and transit into a user; (2) Personal choice: in real life, prospective users are often informed about a product or service through broadcasting, such as sales promotions, advertisements, and even personal research, and the decision to adopt a service or product is a purely personal choice. Combining the two rules, the conceptual description of the user growth profile in a user growth network is given by two transition channels

$$\begin{aligned} T_1 : (P - U) &\xrightarrow{c_1(t)} (U - U), & \text{Word - of - mouth,} \\ T_2 : (P) &\xrightarrow{c_2(t)} (U). & \text{Personal - choice,} \end{aligned} \quad (1)$$

where  $c_\mu$  ( $\mu = 1, 2$ ) is the transition rate,  $c_\mu \Delta t$  is the probability that a prospective transition link of transition channel  $T_\mu$  at time  $t$  will react in the next infinitesimal time interval  $(t, t + \Delta t)$ . “-” represents there is a link.  $(P - U)$  means a node in state  $P$  connects with another individual in state  $U$ . Here  $c_\mu(t)$  is influenced by holiday season effect and varying with time.

We assume that the network is homogeneous [21] and uncorrelated network [20], [23]. For transition channel  $T_\mu$ , all prospective transition links have the same transition rate  $c_\mu(t)$ . Therefore, at time  $t$ ,  $c_\mu(t) \Delta t$  stands for the probability that a prospective transition link will make a transition in the

next infinitesimal time interval  $(t, t + \Delta t)$ . We assume the number of users in a network  $G(V, E)$  is  $X$ . It is easily known that  $X$  is a positive integer. Let  $P(X(t) = m)$  represents the probability that there are  $m$  users at time  $t$ , and  $P(X(t + \Delta t) = n | X(t) = m)$  denotes the transition probability that there is  $n$  users at  $t + \Delta t$ , conditioned upon having  $m$  users at time  $t$ . For brevity, we define  $P_m(t) \triangleq P(X(t) = m)$  and  $P_{n,m}(t, \Delta t) \triangleq P(X(t + \Delta t) = n | X(t) = m)$ . Therefore, we have

$$\begin{aligned} P_n(t + \Delta t) &= \sum_{m=1}^{n-1} P_m(t) P_{n,m}(t, \Delta t) \\ &+ P_n(t) \left[ 1 - \sum_{m=n+1}^{\infty} P_{m,n}(t, \Delta t) \right]. \end{aligned} \quad (2)$$

Then, we assume that at most one prospective user will adopt a service or product at infinitesimal time interval  $\Delta t$ . Hence,  $P_{n,n-1}(t, \Delta t) \neq 0$  for  $n = 2, 3, \dots, N$ ; and  $P_{n,m}(t, \Delta t) = 0$  otherwise. Thus, we have  $P_n(t + \Delta t) = P_{n-1}(t) P_{n,n-1}(t, \Delta t) + P_n(t) (1 - P_{n+1,n}(t, \Delta t))$ , and the transition probability  $P_{n,n-1}$  is  $P_{n,n-1}(t, \Delta t) = h_1(n-1) c_1(t) \Delta t + h_2(n-1) c_2(t) \Delta t$ , where the number of prospective transition links  $h_\mu(n-1)$  stands for the number of distinct prospective transition links with  $n-1$  users for transition channel  $T_\mu$  ( $\mu = 1, 2$ ). Obviously, we have  $h_2(n-1) =$

$(N - n + 1)$ . For  $h_1(n - 1)$ , we assume that links  $(U - P)$  are homogeneously distributed in the network. Then, the number of links  $(U - P)$  is proportional to  $(n - 1)(N - (n - 1))$ , i.e.,  $h_1(n - 1) = \alpha(n - 1)(N - (n - 1))$ , where  $0 < \alpha \leq 1$  is a constant. Then, we have  $P_{n,n-1} = (\alpha c_1(t))(n - 1)(N - (n - 1))\Delta t + c_2(t)(N - n + 1)\Delta t$ . For brevity, we define  $c_1(t) \triangleq \alpha c_1(t)$ . The transition probability can be simplified as  $P_{n,n-1} = c_1(t)(n - 1)(N - n + 1)\Delta t + c_2(t)(N - n + 1)\Delta t$ , and the probability of having  $n$  users at time  $t + \Delta t$  is

$$P_n(t + \Delta t) = P_{n-1}(t)[(n - 1)(N - n + 1)c_1(t)\Delta t + (N - n + 1)c_2(t)\Delta t] + P_n(t)[1 - n(N - n)c_1(t)\Delta t - (N - n)c_2(t)\Delta t]. \quad (3)$$

In the following step, we attempt to find an ordinary differential equation model involving the mean number of users based on the stochastic process represented by (3).  $E[X(t + \Delta t)]$  denotes the expectation and has the form  $E[X(t + \Delta t)] = \sum_{n=1}^{\infty} nP_n(t + \Delta t)$ . Then, we have

$$E[X(t + \Delta t)] = \sum_{n=1}^{\infty} nP(X(t) = n) + \sum_{n=1}^{\infty} n[P(X(t) = n - 1) \times (n - 1)(N - n + 1) - P(X(t) = n) \times n(N - n)]c_1(t)\Delta t \quad (4a)$$

$$+ \sum_{n=1}^{\infty} n[P(X(t) = n - 1) \times (N - n + 1) - P(X(t) = n) \times (N - n)]c_2(t)\Delta t \quad (4b)$$

where  $E[X^2(t)] = \sum_{n=1}^{\infty} n^2 P(X(t) = n)$  and  $\text{var}[X(t)] = E[X^2(t)] - (E[X(t)])^2$  is the variance of  $X(t)$ . Then, Eq. (4a) is

$$\sum_{n=1}^{\infty} n[P(X(t) = n - 1) \times (n - 1)(N - n + 1) - P(X(t) = n) \times n(N - n)]c_1(t)\Delta t = c_1(t)\Delta t(N \times E[X(t)] - (E[X(t)])^2 \times (1 + \frac{\text{var}[X(t)]}{(E[X(t)]^2)})) \quad (5)$$

Similarly, from Eq.(4b), we have

$$\sum_{n=1}^{\infty} n[P(X(t) = n - 1) \times (N - n + 1) - P(X(t) = n) \times (N - n)]c_2(t)\Delta t = (N - E[X(t)]) \times c_2(t)\Delta t \quad (6)$$

Finally, combine Eq. (5) and (6), we have

$$E[X(t + \Delta t)] = E[X(t)] + (N - E[X(t)]) \times c_2(t)\Delta t + (N \times E[X(t)]c_1(t)\Delta t - (E[X(t)]^2 \times (1 + \frac{\text{var}[X(t)]}{(E[X(t)]^2)})) \times c_1(t)\Delta t \quad (7)$$



FIGURE 2. Photos of these consoles, including Xbox one, Playstation 4, and Nintendo Switch.

From (7) and taking the limit as  $\Delta t \rightarrow 0$ , we achieve the simple ODEs model as

$$\frac{dE[X(t)]}{dt} = Nc_1(t)E[X(t)] - c_1(t) \left(1 + \frac{\text{var}[X(t)]}{(E[X(t)]^2)}\right) (E[X(t)])^2 + c_2(t)N - c_2(t)E[X(t)] \quad (8)$$

Defining  $x(t) \triangleq E[X(t)]$  as the number of users at time  $t$ , equation (8) can be formulated as

$$\frac{dx}{dt} = (c_1(t)Nx - c_2(t)(1 + \delta(x))x^2) + (c_2(t)N - c_2(t)x), \quad (9)$$

where  $\delta(x) = \text{var}[X(t)]/E[X(t)]^2$ . Moreover, for large-scale networks,  $\delta(x) \ll 1$  generally holds, leading to

$$\dot{x}(t) \approx \underbrace{(c_1(t)x + c_2(t))}_{\text{Purchase incentive}} \times \underbrace{(N - x)}_{\text{Market size}}. \quad (10)$$

Here, the factor  $(N - x)$  clearly refers to the effective market size, and  $(c_1(t)x + c_2(t))$  is the combined incentive for the prospective users to make a purchase decision which consists of a peer-influence term  $c_1(t)x$  and a personal-choice term  $c_2(t)$ .

In the previous work, we fix the transition rate  $c_1(t)$  and  $c_2(t)$  as constants, which represent average rates over a long time period. Take Facebook as an example, the  $c_1$  and  $c_2$  are average rates over 10 years. However, in the real world, users growth (sales) speed always fluctuate dramatically in a short time period. Note that the weekly sales of video game consoles in Fig. 3 (c) and (d). One can find that there are many ‘‘spikes’’, which means the sales increase dramatically and then fall in a short time period. We consider these phenomenon is contributed to the fluctuation of  $c_1(t)$  and  $c_2(t)$  in a short time. Then the combined incentive for the prospective  $C = c_1(t)x + c_2(t)$  is also changed. Hence, the sales speed, such as weekly sales  $\Delta x_7$  is also changed

$$\Delta x_7 \approx [(c_1(t)Nx - c_1(t)(1 + \delta)x^2) + (c_2(t)N - c_2(t)x)] \times 7 \text{days}. \quad (11)$$

Fig. 1 shows an example of how  $c_2(t)$  influences sales speed. In this simple example, the first 20 days are normal days with the personalized-choice rate  $c_2(t) = 2 \times 10^{-4}$ ,

TABLE 1. Notation for DE algorithm.

$g \in Z^+$ :	The generation of the DE algorithm
$N_\alpha \in Z^+$ :	The population size
$\hat{\gamma} = \{\hat{\theta}, \hat{\tau}\} \in R^k$ :	Unknown parameters of the system
$XV_{i,g} = \{\hat{\gamma}_{i,g}, \alpha_{i,g}\}$ :	The $i^{th}$ individual vector of the population, $i = 1, 2, \dots, N_\alpha$
$XV_{best,g}$ :	The best individual producing the minimal $L(XV_{i,g})$ in the $g^{th}$ generation
$POP_g = \{XV_{1,g}, XV_{2,g}, \dots, XV_{N_\alpha,g}\}$ :	The population of generation $g$
$\delta$ :	Probability of $\alpha$ being involved in the evolution.
$F$ :	The scale factor with $F \in [0, 1]$
$CR$ :	The crossover rate with $CR \in [0, 1]$
$L(XV_{i,g})$ :	The cost function value with $i^{th}$ vector of the population $XV_{i,g}$
$MV$ :	The mutant vector after mutation

which means on average, 2 out of 10000 prospective users will adopt this product in a day. During the 20th to 35th day, there is a holiday season.  $c_2(t)$  increases every day, while after a holiday, it drops dramatically (the 35th to 40th day) and returns to a normal level (the 40th to 60th day). The number of users is shown in Fig. 3 (b) correspondingly.

### B. AN EXTENDED DIFFERENTIAL EVOLUTIONARY ALGORITHM BASED ON MCC

#### Algorithm 1 Pseudo-code for the new DE algorithm

```

Initialize generation number  $g = 0$ 
Initialize population  $POP_g = \{XV_{1,g}, XV_{2,g}, \dots, XV_{N_\alpha,g}\}$ 
for  $i = 1$  to  $N_\alpha$  do
    Random generate  $XV_{i,g} = \{\hat{\gamma}_{i,g}, \alpha_{i,g}\}$ 
end for
Main Program:
while stopping criterion is not satisfied do
     $g = g + 1$ 
    for  $i = 1$  to  $N_\alpha$  do
         $j_{rand} = rand(1, length(XV_{i,g}))$ 
        for  $j = 1$  to  $length(XV_{i,g})$  do
            if  $rand(0, 1) \leq CR$  or  $j = j_{rand}$  then
                 $TV_{i,g}[j] = MV_{i,g}[j]$ 
            else
                 $TV_{i,g}[j] = XV_{i,g}[j]$ 
            end if
        end for
        if  $L(TV_{i,g}) \leq L(XV_{i,g})$  then
             $XV_{i,g} = TV_{i,g}$ 
            if  $L(TV_{i,g}) < L(XV_{best,g})$  then
                 $XV_{best,g} = TV_{i,g}$ 
            end if
        else
             $XV_{i,g+1} = XV_{i,g}$ 
        end if
    end for
end while

```

First, we give a brief introduction to the video game console market. The electronic systems used to play video games are known as consoles, the example of these are video game

console and handled game consoles (shown in Figure 2). A home video game console is a machine designed for playing a video game on a separate television, while a handled game console is a lightweight, portable electronic device with a built-in screen, game controls, and speaker. In the last decade, the video game console market is monopolized by three companies, Nintendo, Sony, and Microsoft. A video game console was about 400 USD (seventh video game console) and 200 USD (handled video game console). So, video game consoles are not luxury. Furthermore, the number of sales are recorded accurately, which is suitable for our analysis. A total of 5 sales data of video game consoles, including 3 seventh generation video game consoles (XBOX360, PlayStation 3 and Nintendo Wii) and 2 handled video game consoles (Nintendo DS and PlayStation Portable) [22], are utilized to support the study<sup>1</sup>.

Fig. (3) (a) and (b) shows the cumulative worldwide sales of video game consoles, while Fig. (3) (c) and (d) shows the weekly total worldwide sales. Almost all developed countries in Western and even the Eastern developed countries (regions) celebrate western holidays, especially Christmas. During the Holiday season, companies, vendors will cut off, promotion and etc., to attract customers. Note that there are “spikes” in Fig. (3) (c) and (d), which are always around Christmas, we consider it is the holiday effect. Here, the console sales data can be used to analyze the combined incentive rate  $C(t)$  during Holiday season, which from Halloween (31 Oct) until New Year’s Day (1 Jan). Here, we assume the dynamics of transition rates in a year can be modeled by the following piecewise function

$$c_\mu = \begin{cases} c_{\mu,0} & (t_0 \leq t < t_1), \\ c_{\mu,0} + \alpha_{\mu,1}(t - t_1) & (t_1 \leq t < t_2), \\ c_{\mu,0} + \alpha_{\mu,1}(t_2 - t_1) + \alpha_{\mu,2}(t - t_2) & (t_2 \leq t < t_3), \\ c_{\mu,0} + \alpha_{\mu,1}(t_2 - t_1) + \alpha_{\mu,2}(t_3 - t_2) \\ \quad + \alpha_{\mu,3}(t - t_3) & (t_3 \leq t < t_4), \\ c_{\mu,0} + \alpha_{\mu,1}(t_2 - t_1) + \alpha_{\mu,2}(t_3 - t_2) + \alpha_{\mu,3}(t_4 - t_3) \\ \quad + \alpha_{\mu,4}(t - t_4) & (t_4 \leq t < t_5). \end{cases} \quad (12)$$

<sup>1</sup>We make our dataset publicly available online at [https://1drv.ms/u/s!An5r\\_ZiZKwRPh89dLnVWWIoZbs4MCA](https://1drv.ms/u/s!An5r_ZiZKwRPh89dLnVWWIoZbs4MCA)

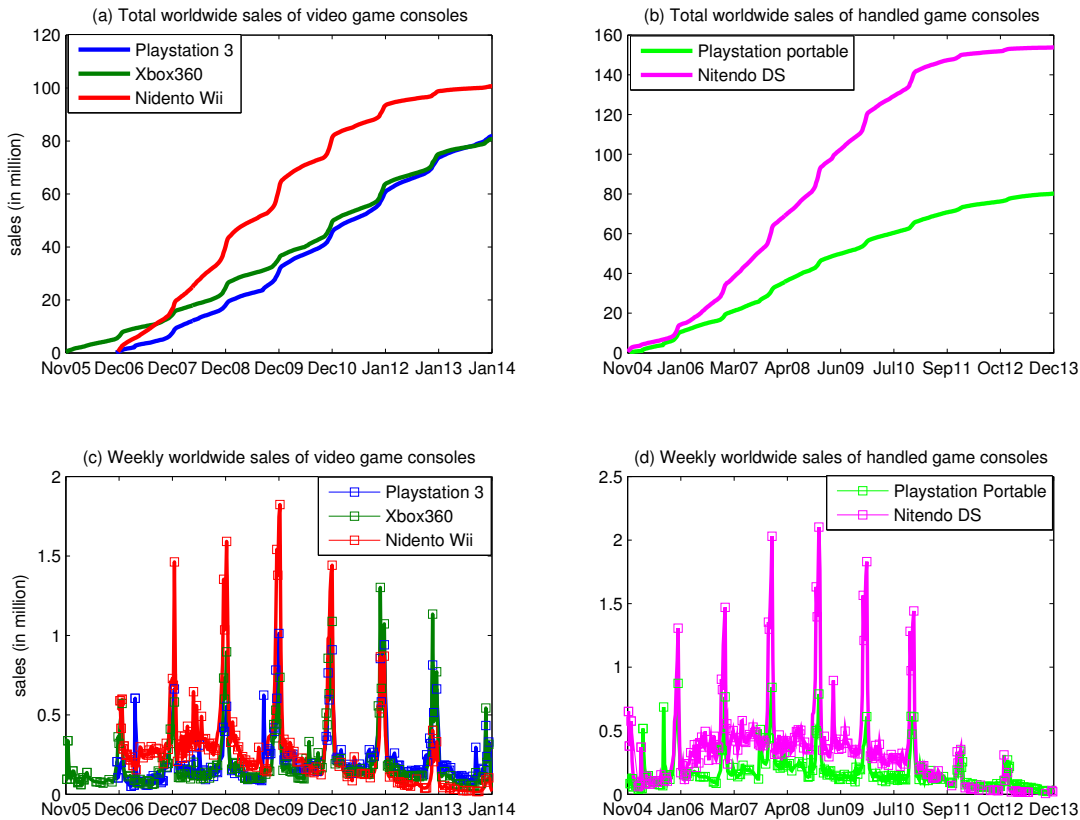


FIGURE 3. Sales of video game Consoles from 2005 to 2014.

Here,  $c_{\mu,0}$  and  $\alpha_{\mu,i}$  ( $i = 1, 2, \dots, 5$ ) are unknown constant parameters to be determined, while  $t_i$  is settled as following

- $t_1$  is one day before Halloween in middle Oct;
- $t_2$  is a day around Thanks giving day;
- $t_3$  is just after about a week of Thanks giving;
- $t_4$  is around Christmas day;
- $t_5$  is a day at the end of Jan.

The basic mathematical structure is settled. However, a set of unknown parameters  $\{c_{\mu,0}, \alpha_{\mu,i}, N\}$  has to be estimated. The parameter estimation problem is actually a constraint nonlinear programming (NLP) problem for calibrating the unknown parameters so that the estimated trajectory is in good alignment with the measured data [24], [25]. Inverse engineering method is applied to find a feasible set of parameters  $\{c_{\mu,0}^*, \alpha_{\mu,i}^*, N^*\}$ . Then, we should estimated the unknown parameters from Equation 10 and 12. Table shows 1 the notations of algorithm, while the pseudo-code is shown in 1. Note that this algorithm is suitable for parallel computing and requires rich computation resources, which can be distributed on parallel computing platforms, such as MCC.

TABLE 2. Number of consoles and games in different areas

	France	Germany	Japan	UK	USA	Global
Consoles	11	11	21	11	11	11

### III. EXPERIMENTAL RESULTS

We utilize real-life data of sales of 21 consoles in different areas, including e.g., France, Germany, Japan, UK, USA, and the whole world, as an illustration, examples show the availability of the proposed model (9). Note that this mode only requires the historical data about the sales of a video game console to estimate the purchase incentive. Table 2 shows the number of consoles sold in each areas. Please note that our data is incomplete and does not include all game console sales information. For instance, we only have global sales data for 11 consoles, while the total number of consoles is far larger than 11. The Root Mean Square Percentage Error (RMSPE) is adopted as a criterion measuring the difference between the sales predicted by the proposed model and true

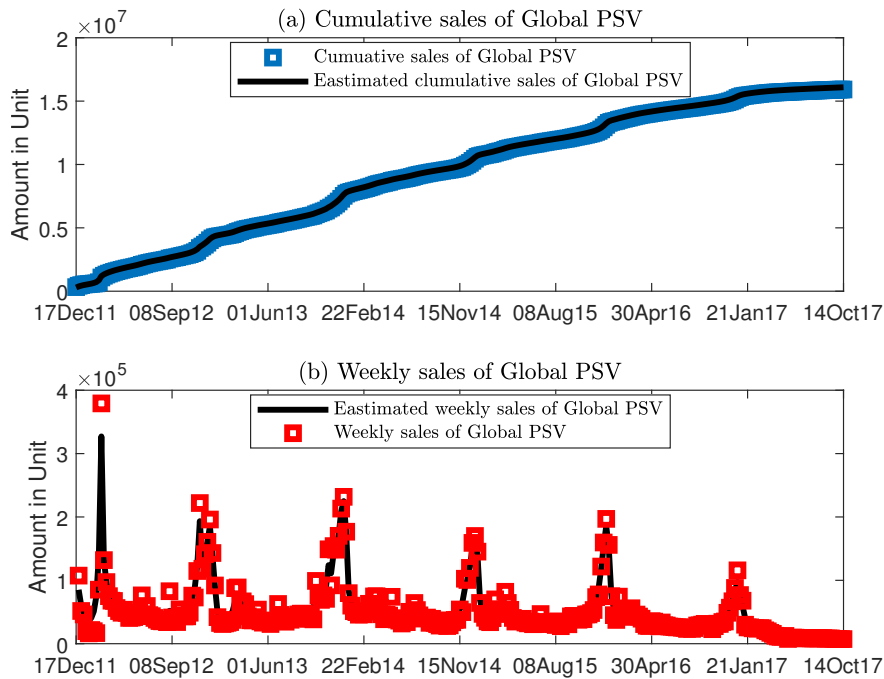


FIGURE 4. (a) Cumulative global sales of Playstation Portable;(b) Weekly global sales of Playstation Portable.

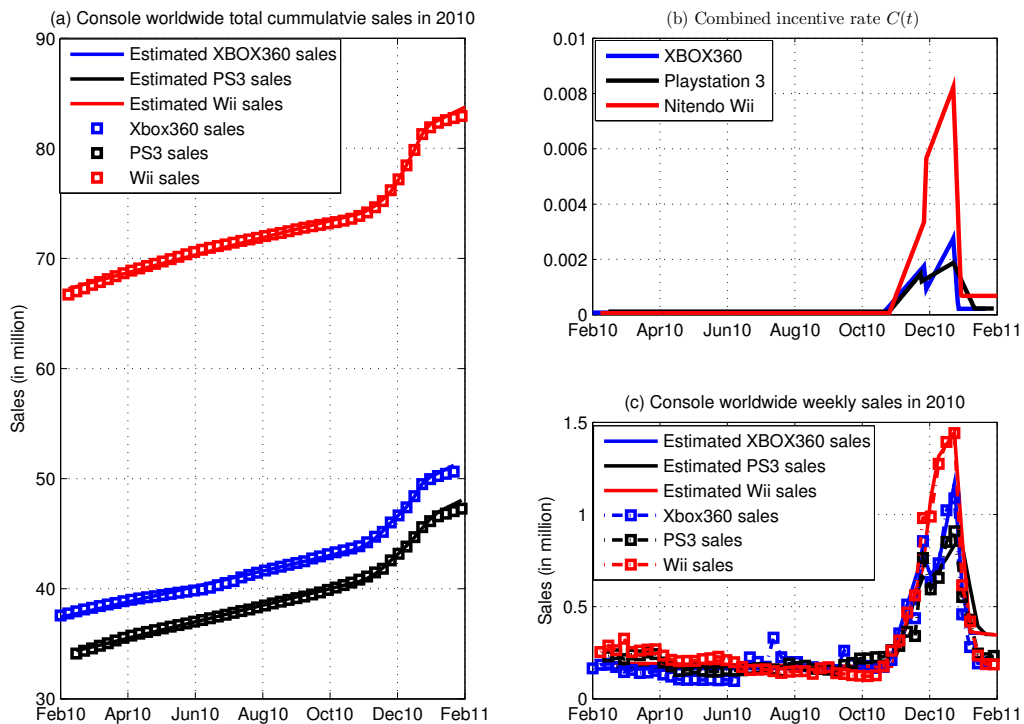


FIGURE 5. (a) Sales of world wide cumulative video game Consoles in 2010;(b) Personalized-choice transition rate  $c_2(t)$ ;(c) Weekly sales of video game console in 2010.

sales. The RMSPE is defined as

$$\text{RMSPE} = \sqrt{\frac{1}{N} \sum_{i=1}^N \left( \frac{\hat{x}(t_i) - x(t_i)}{x(t_i)} \right)^2} \times 100\%, \quad (13)$$

where  $\hat{x}(t_i)$  is the estimated sales at time  $t_i$ . Figure 4 shows one representative example, the fitting results of cumulative

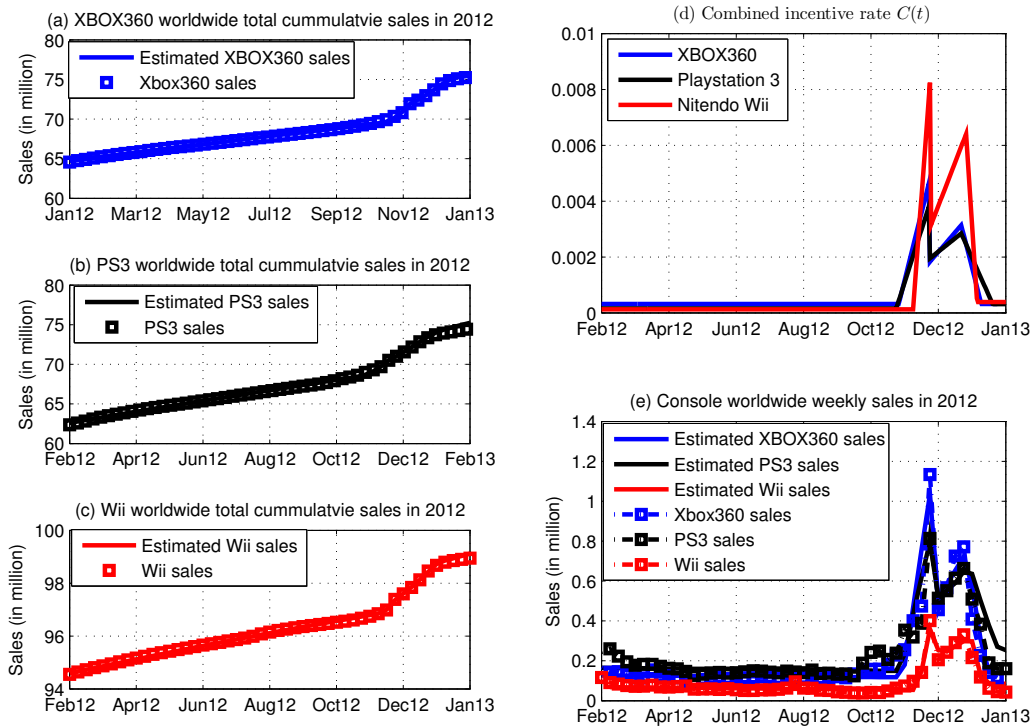


FIGURE 6. (a), (b) and (c) Sales of world wide cumulative video game Consoles in 2012;(d) Personalized-choice transition rate  $c_2(t)$ ; (e) Weekly sales of video game console in 2012.

and weekly global sales of PlayStation Portable (PSP). The square lines represent the historical sales, while the solid lines represent the sales generated by the model. The estimated sales generated by the model can capture all the historical sales very accurately. We also apply this model to analyze video game sales. The RMSPE is  $1.34 \pm 1.02\%$  for all consoles.

Here, for conciseness, we show the result of Xbox360, PS3 in the year 2010 and 2012, Wii in the year 2008, 2010 and 2012. The result of other game consoles is similar and we will not present them here. Experimental evaluation reveals that the estimated trajectories can capture the dynamics of these sales datasets accurately, shown in Fig. (5),(6) and(7). We also give the combined incentive rate  $C(t)$  and fitting results of weekly sales. Square represents the historical sales data, while the solid line represents an estimated sales number by our model. The experimental result supports our view, during the holiday season,  $C(t)$  will increase dramatically. After the holiday season,  $C(t)$  will return to normal level.

There are other interesting phenomenons. In the year 2010, Wii sold much better than PS3 and Xbox360 (Fig. (5)). Nintendo sold out almost 1.5 million units of Wii in a week around Christmas (Fig. (5) (c)), while Xbox360 and PS3 only were sold out 1 million. Fig. (5) (b) shows  $C(t)$  of these consoles. In the year 2010, combined incentive rate of Wii

$C^W(t)$  was always larger than  $C^X(t)$  and  $C^P(t)$  during the whole year. For instance, around Christmas,  $C^W \approx 0.008/\text{day}$ , which means, in one day, 8 out of 1000 potential Wii users purchased Wii, while  $C^X, C^P \approx 0.002/\text{day}$ . Hence, the weekly sales of Wii  $\Delta x^W$  larger than  $\Delta x^P$  and  $\Delta x^X$  seems reasonable. However, in the year 2012,  $C^W(t)$  was still larger than  $C^X(t)$  and  $C^P(t)$  (Fig. (6) (d)). But, weekly sales of Wii  $\Delta x^W$  was smaller than  $\Delta x^P$  and  $\Delta x^X$ . For Wii, the largest weekly sales was about 0.4 million in the year 2012, while about 1 million for Xbox360 and PS3, respectively. Furthermore, take a look at sales of Wii in the year 2008, 2010, and 2012 (Fig. (7)). Experimental results shows that  $C^W(2012) > C^W(2008)$ . However, the sales speed of Wii in the year 2012 was much slower than in the year 2008. How this phenomenon happens?

From the model, we have

$$\begin{aligned} \dot{x}(t) &= c_1(Nx - c_1(1 + \delta)x^2) + c_2(N - x) \\ &= (c_1x + c_2)(N - x) - \delta c_1x. \end{aligned} \quad (14)$$

For the video game console market case, there are always tens of millions users. Hence,  $\delta(x) \ll 1$  generally holds leading to  $\dot{x}(t) \approx (c_1x + c_2) \times (N - x)$ . Then, Eq. (14) can be simplified as

$$\begin{aligned} \dot{x}(t) &\approx (c_1x + c_2)(N - x) = \Delta N \times C, \\ \Rightarrow \Delta x(t) &\approx \Delta N \times C \times \Delta t. \end{aligned} \quad (15)$$



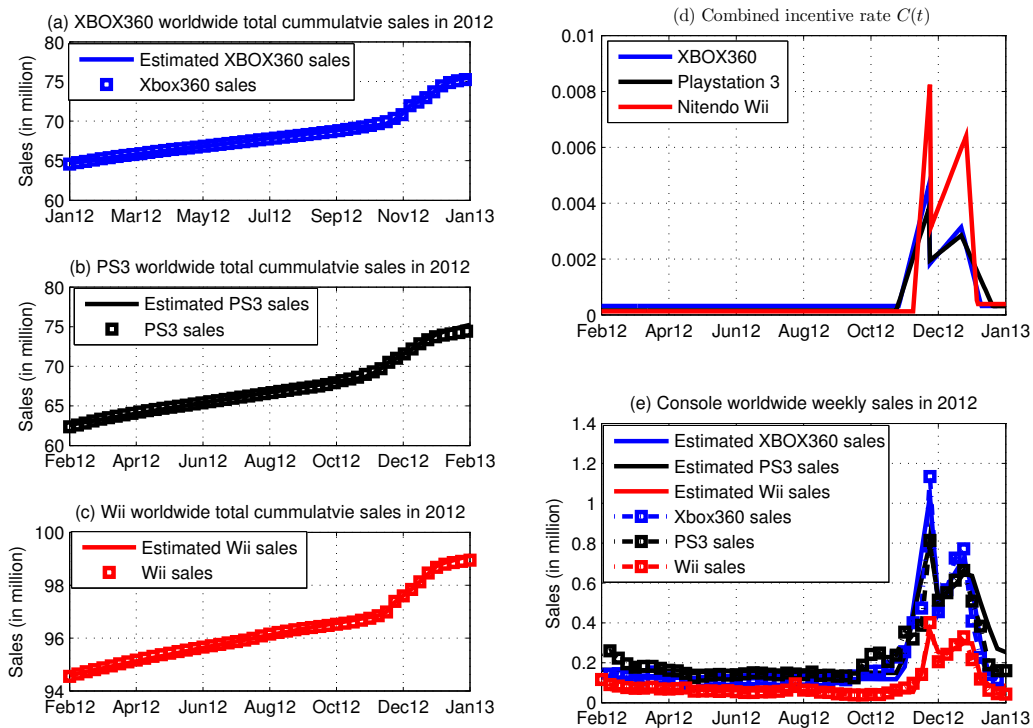


FIGURE 7. (a) Sales of world wide cumulative Wii in 2007,2008 and 2009;(b) Weekly sales of video game console in 2008,2010 and 2012; (c) Personalized-choice transition rate  $c_2(t)$ .

where  $\Delta x(t)$  is the sales number in a period  $\Delta t$ . Here,  $\Delta N = N - x$  is the number of leftover prospective users or the leftover market size.  $C = c_1x + c_2$  is the combined incentive rate of the leftover prospective users. It can be treated as sales speed. For instance, weekly sales ( $\Delta t = 7\text{day}$ ) is

$$\Delta x(t) \approx \Delta N \times C \times 7\text{days}. \quad (16)$$

Intuitively, we know that (1)if the market size is large, the sales of this product will be good; (2) if the purchaser's desire is aroused, the sales speed will rise. So, there is an intuitive conclusion, the speed of sales is highly related to the market size and the purchaser's desire, namely,

$$\text{Speed of sales} = f(\text{Market size}, \text{Purchaser's desire}) \quad (17)$$

where  $f(\cdot)$  is a function.

Now, in theory, we derive the exact mathematical relationship of sales speed, purchaser's desire and market size. The formula is the simple equation (15). In the holiday season,  $c_\mu$  increases, then,  $C$  also increases. During the holiday season, a prospective user more likely transits into a user. We know that the sales speed of a product in the holiday season will be better than the low season. However, market size  $\Delta N$  is actually the number of leftover prospective user ( $N - x$ ). For different products with similar  $C$ , the one with larger  $\Delta N$  enjoys a larger sales speed. For a product, take Wii for an

example, as the number of  $x(t)$  increases, the leftover market size decreases every year. Hence, even with the same  $C$ , the sales speed in the later year will be slow.

The theory result is supported by the real sales data of consoles. We find in year 2012, the number of total potential users of Wii are about  $N^W \approx 104$  millions,  $N^X \approx 100$  millions and  $N^P \approx 106$  millions. Hence, the number of leftover users of Wii is about  $\Delta N^W \approx 8$  millions, while the number of leftover users of Xbox360 and PS3 are  $\Delta N^X \approx 30$  millions and  $\Delta N^P \approx 36$  millions. The leftover market size of Wii is much smaller than Xbox360 and PS3. So, even  $C^W$  is larger than  $C^X$  and  $C^P$ , the weekly sales of Wii in the year 2012 in worse than PS3 and Xbox360. Similarly, the leftover market of Wii in year 2008, 2010 and 2012 are  $\Delta N^W(2008) \approx 60$ millions,  $\Delta N^W(2010) \approx 30$ millions and  $\Delta N^W(2012) \approx 8$ millions, respectively. Hence, that is why  $C^W(2012) > C^W(2008)$ , but the weekly sales of Wii in the year 2012 is worse than the year 2008 and 2010. The smaller leftover market size of Wii in 2012 brings a slow sales speed.

One of the meanings of the conclusion, maybe that, the manager should not blame the advertisement/salesman department of Nintendo in the year 2012. It is not the salesman's fault. They did their job very well, so most of the prospective users transit into users from 2007 to 2012. Actually, they did better in 2012 than in 2008 ( $C$  in 2012 is always

slightly larger than in 2008). However, the weekly sale results are worse. It is not their fault, the leftover market size in 2012 is very small. Nintendo possibly knew the lifetime of Wii almost came to an end. So, they released the eighth generation video game console Wii U in DEC/18/2012. Based on our analysis, we believe release Wii U is a good decision.

#### IV. CONCLUSION

In modern society, plenty of large scale networks exist [26], [27]. In this work, we show that the market size and the extent of the influence of different incentive factors contributing to the growth of the user population of a product and service. Furthermore, we also derive that the user growth (sales) speed of a product is rough  $\Delta x \approx C \times \Delta N \times \Delta t$ . Here, the factor  $\Delta N = N - x$  clearly refers to the effective market size, and  $C = c_1 x + c_2$  is the combined incentive for the prospective users to make a purchase decision with consist of peer-influence term  $c_1$  and a personal-choice term  $c_2$ . Historical sales data of video game consoles is applied to analyze the dynamics of the combined incentive rate over the years. Results reveal that  $C(t)$  fluctuated dramatically during the holiday season. It implies that short impulsive effect, such as holiday effect, cutting the price in short time, anniversary sales and etc., can influence  $c_\mu$  dramatically. However, assume one product, such as Wii, cut its price forever, instinctively,  $c_\mu$  will change. In this case, how business operation affects  $c_\mu$  will be an interesting and important problem. Given a set of historical data of the user growth profile, the leftover market size can be predicted. Hence, effective marketing and time strategies can thus be developed to ensure the success of the product or services in question or the timely initiation launching of new products or services. In this paper, the proposed model only has been tested in video game console sales data. In future, we will construct more datasets for evaluating this model.

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