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Calculating Longitudinal Wheel Slip and Tire Parameters Using GPS Velocity

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ABSTRACT

While tire parameters are quite important to both current vehicle control systems and proposed future systems, these parameters are subject to considerable variability and are difficult to estimate while driving due to the unavailability of absolute vehicle velocity. This paper details a method of generating longitudinal tire force-slip curves using absolute velocity information from the Global Positioning System (GPS). By combining GPS measurements with measured wheel speeds, the effective tire radius and longitudinal stiffness of the tires can be identified using a simple least-squares regression technique. Preliminary results demonstrate the feasibility of the technique, show that the effective radius can be identified with considerable precision and suggest that the identified longitudinal stiffness exhibits noticeable sensitivity to changes in inflation pressure.

INTRODUCTION

The longitudinal forces that produce acceleration and braking on ground vehicles with pneumatic tires arise due to deformation and sliding in the tire contact patch. While the actual motions that take place in the contact patch are somewhat complex, the force generation can generally be described with sufficient accuracy in terms of wheel slip – a measure of the difference between the rotational speed of the wheel and the translational velocity of the wheel center. The standard SAE definition of wheel slip is

$$S = -\frac{(V - R_e w)}{V} \tag{1}$$

where V is the longitudinal speed of the wheel center, w is the angular speed of the tire and R_e is the effective tire radius. The effective radius is defined to be the radius of the tire when rolling with no external torque applied about the spin axis. Since the tire flattens in the contact patch, this value lies somewhere between the tire's undeformed radius and static loaded radius.

A number of different tire models for predicting tire longitudinal force in terms of wheel slip have been derived from empirical data. Such models generally relate the longitudinal force on a tire to the wheel slip for given values of normal force, road surface conditions, tire characteristics, and other factors (such as camber angle). Figure 1 demonstrates the general shape of such a curve generated from the commonly-used "Magic Formula" tire model [2]. While models vary, several of the traits shown in Figure 1 are common to various mathematical models and empirical test data. First, the relation between force and slip is roughly linear at low values of slip below the point at which significant sliding occurs in the contact patch. In this region, force can be approximated as proportional to slip using an effective longitudinal stiffness of the tire, C_x:

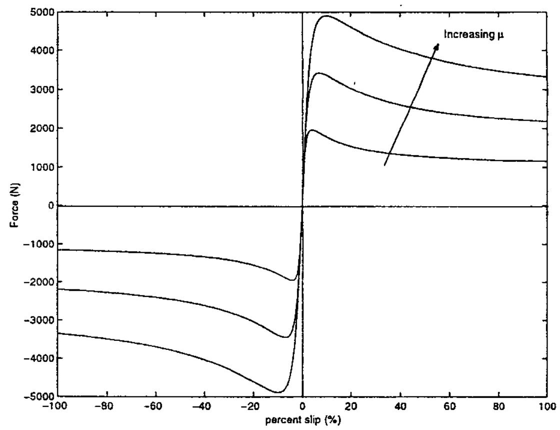

Figure 1 – General Shape of Force versus Slip Curve

$$F_x = C_x \cdot S \tag{2}$$

The stiffness depends on the foundation stiffness of the tire and the length of the contact patch between the tire and the road [8]. As a result, this value depends strongly upon tire construction and inflation pressure.

Beyond this linear region, the additional force generated per unit slip begins to decrease and ultimately reaches a peak, after which tire force decreases and braking behavior becomes unstable. The peak force at which this occurs depends strongly upon the road surface and is often approximated by scaling by a peak friction value, μ, as shown in Figure 1. Some experimental research has suggested that the longitudinal stiffness may also depend on road surface condition and this peak friction value [10]. While consistent with many mathematical representations of force versus slip curves, such dependence violates the traditional brush model's physical description of tire force generation [8].

Since tire force generation can be described in terms of wheel slip, slip is a critical parameter in control algorithms for vehicle control systems such as anti-lock brake systems (ABS) and electronic stability control (ESP) [14]. While many ABS algorithms rely primarily on the deceleration of the wheel [3], some estimate of slip is necessary to avoid lock-up on low friction surfaces. Although the definition of wheel slip in Equation 1 is quite simple, calculating slip on a vehicle is complicated by the lack of accurate measurements of either the radius or the absolute vehicle velocity. While an average radius value can usually be assumed without producing much error, some form of observer must be employed to estimate the vehicle speed [11]. Other systems determine the vehicle's absolute velocity by comparing the front and rear wheel speeds (assuming the car is two-wheel drive) [10]. Recent work has demonstrated that velocity measurements derived from the Global Positioning System (GPS) can be used to provide an absolute velocity for calculating wheel slip [4]. This avoids the drift problems inherent in observers based upon wheel speed measurement.

The use of GPS velocity information has an even greater benefit beyond the generation of an accurate slip measurement. By comparing the wheel slip to estimates of the forces acting on the vehicle, the tire force versus slip characteristics can be obtained. These, in turn, can be used to feed model-based controllers for ABS or ESP systems or more advanced driver assistance systems for lanekeeping or collision avoidance [9]. They could also be used to provide more accurate observers for periods of time when GPS information is not available. Several researchers [6,7,10,12,13] have also suggested that by fitting the low slip region of the force-slip curve to a parameterized model - ranging in complexity from the form of Equation 2 to dynamic friction models - the peak friction point can be determined. This application represents a further use for the information that can be generated from GPS-based slip measurement, although preliminary results achieved with the system demonstrate some care in interpretation is necessary for friction detection.

This paper demonstrates how tire force-slip curves – and in particular the linear region of these curves – can be determined using GPS velocity measurements and wheel speed sensors. The GPS velocity measurement is differenced to obtain absolute vehicle acceleration, which is multiplied by the vehicle mass to calculate the longitudinal force on the tires. The accuracy of the GPS data enables the estimation of the effective tire radius and longitudinal stiffness of the tires, thus completely specifying the linear part of the force-slip curves. Some preliminary tests at different pressures indicate that these values exhibit some strong dependence on tire pressure, raising a cautionary note about inferring peak friction from tire behavior at low levels of slip.

METHOD

The equation of motion of a vehicle in the longitudinal direction is:

$$F_{xf} + F_{xr} - F_{rr} - F_d - Mg\sin 6 = Ma_x \tag{3}$$

where F_{xf} and F_{xr} are the longitudinal forces (driving or braking) on the front and rear axles, respectively, F_{rr} is rolling resistance, F_d is drag and θ is the grade angle.

As a first approximation, rolling resistance, drag and grade are neglected and mass is assumed known. The equation of motion then becomes:

$$F_{xf} + F_{xr} = Ma_x \tag{4}$$

This is admittedly a simplification and better results could be obtained by including estimates of the road grade, mass and other road loads obtained online. Grade can be estimated from the GPS receiver by examining the ratio of vertical velocity to horizontal velocity after which the mass and road loads can be estimated if a value of engine torque is available [1].

The remaining step is to apportion the acceleration between the front and rear axles and – if desired – the left and right tires. With a four wheel drive vehicle, curves for each wheel can be derived during periods of acceleration alone by assuming some split of torque across the four wheels. For vehicles with only two driven wheels, the force-slip characteristics of the driving wheels can be identified while accelerating, but the undriven wheels can only be identified during periods of braking. This, in turn, requires knowledge of the brake distribution among the different wheels. The experimental results and the derivation presented here assume that the vehicle has two driven wheels, the curves are identified under acceleration alone and an effective longitudinal stiffness for the two wheels combined is desired.

Assuming that the tires are in the linear region of the force versus slip curve, the longitudinal equation is given by:

$$F_x = F_{xf} + F_{xr} = C_x \cdot S = C_x \cdot \frac{(wR_e - V)}{V} \tag{5}$$

Combining this with Equation 4 gives simply:

$$C_x \cdot \frac{(wR_e - V)}{V} = Ma_x \tag{6}$$

Acceleration is found by differencing the GPS velocity measurements in post-processing according to:

$$a(k) = [v(k+1) - v(k-1)]/2T \tag{7}$$

where T is 0.1 s, the time interval between consecutive GPS data points.

Therefore,

$$a(k) = \frac{1}{M} C_x \left[\frac{R_e w(k) - V(k)}{V(k)} \right] \tag{8}$$

Defining a speed ratio P as,

$$P = \frac{w(k)}{V(k)} \tag{9}$$

This can be rearranged to form

$$a(k) = \frac{C_x}{M} [R_e \cdot P(k) - 1] \tag{10}$$

or, alternately,

$$a(k) = \left[\frac{C_x \cdot R_e}{M} \right] \cdot P(k) - \left[\frac{C_x}{M} \right] \tag{11}$$

which clearly represents a linear relationship between the acceleration and the speed ratio.

Both acceleration, *a(k)*, and speed ratio, *P(k)*, can be determined from GPS and wheel speed sensor readings. From these measurements, a simple least-squares algorithm can be used to determine the slope and intercept of the line:

$$a(k) = mP(k) + c \tag{12}$$

Once the slope, *m*, and intercept, *c*, of the line have been determined, the longitudinal tire stiffness can be easily calculated from the intercept and vehicle mass:

$$C_x = -c \cdot M \tag{13}$$

Knowing the mass and the stiffness, the effective radius follows from:

$$R_e = \frac{m \cdot M}{C_x} \tag{14}$$

The effective radius can then be used to present the data in terms of the SAE standard definition of wheel slip given in Equation 1 without having to guess at the radius.

EXPERIMENTAL SYSTEM

The sensor system used in this study consists of a NovAtel Millenium GPS receiver and an A-dat high-resolution wheel speed sensor. The GPS receiver provides absolute velocity information at 10 Hz with a 1σ noise level of less than 3cm/s observed in experiments [4]. The receiver is connected to a magnet-mounted antenna placed on the centerline of the car's roof, as close as possible to the car's center of gravity. The velocity data is averaged internally by the receiver over each 100 ms, giving each velocity point a theoretical latency of 50 ms.

Delays in GPS computation and data transfer increase this latency further. To align the GPS information with the wheel speed sensor data, the receiver outputs a short-duration (20 μs) pulse at the start of each integer second. This pulse is not long enough to reliably capture with the base sampling rate of 1kHz on the data acquisition computer, so it is broadened using a 1-shot circuit before being read into the A/D board and stored in the data set. By comparing the start of the GPS second and the arrival of the GPS data point, the latency of each pulse is determined. This latency was measured to be, on average, 95 ms [5], making the total latency of the GPS data roughly 150 ms. Since the data used in this study was post-processed, the latency provided no problems after synchronizing the data streams. Should real-time generation of slip be required, the velocity measurement can be combined with a longitudinal accelerometer to produce an unbiased measurement with higher update rate and no latency (as was demonstrated for lateral dynamics in [5]).

An externally mounted high-resolution wheel speed sensor measures the angular speed of the drive wheel. It outputs 1000 digital pulses per revolution to a counter circuit, which is read directly into the single-board computer. The digital signal, sampled at 1 kHz, is run through a 300-point moving-average filter, making the signal more continuous and giving each data point an average latency of 150 ms, closely matching the GPS receiver's total latency. This sensor has a much greater resolution than production wheel speed sensors used in ABS or ESP systems and was chosen for ease of interfacing. As demonstrated in the experimental results, the low frequency GPS velocity measurement is the limiting factor in setting the accuracy of the slip measurement, not the wheel speed. Hence, this resolution could be greatly reduced without altering the basic character of the results presented in this paper.

The data acquisition system includes a VersaLogic single-board target computer (SBC) and a notebook host computer connected via an Ethernet link. The target computer operates under the xPC target environment of MATLAB's Real-time Workshop package, providing a consistent 1ms sample time. The GPS receiver is connected to the SBC through a serial port while the wheel speed and synchronization pulses are obtained through the SBC's digital and analog inputs, respectively. Data post-processing and storage are handled exclusively by the notebook. Since the complete system is very portable, tests were performed on several passenger vehicles.

EXPERIMENTAL RESULTS

A series of short test runs (30-60 seconds) were performed in two test vehicles, maintaining normal driving speeds within a limited range on open roads with minimal grade. In keeping with the assumptions made for this study, only periods of acceleration and engine braking were used in the experimental runs, ensuring that all driving and braking forces acted through the driven wheels. Only the driven wheels were identified in this study. The data was post-processed according to the method described in the previous section to determine slip, force, longitudinal stiffness, and effective radius for each data set. Figure 2 shows a 35 second data set that represents the typical maneuver

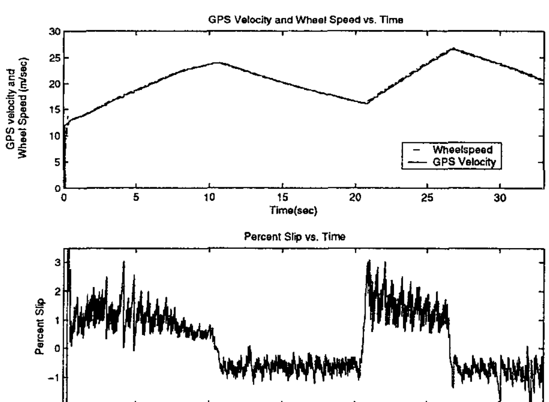

Figure 2 – Speeds and Wheel Slip During Maneuver

Clearly, the GPS velocity and wheel speed signals are close to identical when plotted in the same graph. The differences can be seen in the lower plot once these values have been converted to slip. Periods of acceleration result in positive values of slip, while periods of deceleration produce negative values. During the experiments, slip was measured to be between 3% and –3%, typical of normal driving conditions and well within the linear region of the force-slip curves.

The apparent noise in the slip curve is actually a saw-tooth pattern caused by the 10Hz update rate of the GPS velocity information. Since the wheel speed is available at a higher update rate than the GPS velocity, periods of acceleration and deceleration produce this pattern when viewed at the higher update rate. This does not alter the force-slip curve generation since the data is downsampled to match the 10Hz update rate of the GPS receiver prior to the use of the least-squares regression.

Figure 3 displays the raw acceleration and speed ratio data used in the least-squares algorithm together with the best-fit line through the data. No filtering was performed on this data other than the moving average filter for the wheel speeds and the elimination of a few outliers caused by occasional loss of GPS satellite visibility. The experimental data points clearly support the linear tire modeling assumption at low slip.

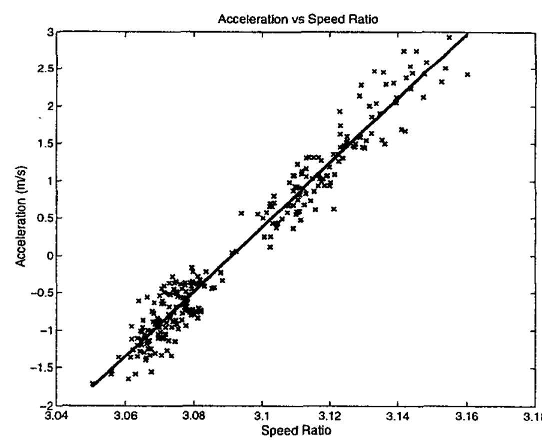

Figure 3 – Raw Acceleration Data versus Speed Ratio

After running the least-squares routine to obtain the radius, this curve can be rearranged into the more familiar units of force versus slip, presented in Figure 4.

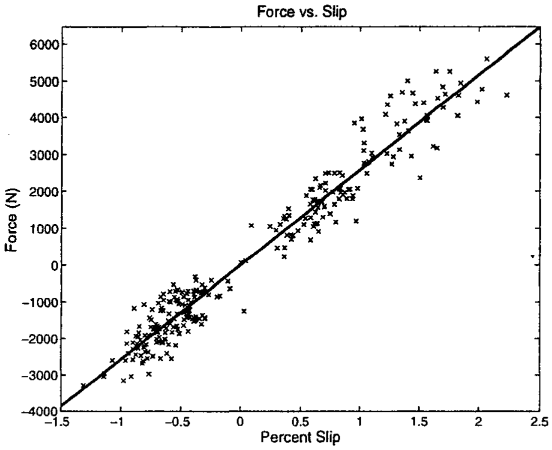

Figure 4 – Experimental Force versus Slip Curve

While identical in content to Figure 3, this plot has a bit more physical meaning since the slope of the best fit line through these points is the effective longitudinal stiffness, *Cx*, of the tires.

Effective radius was found to be between .3222m and .3246m, depending upon inflation pressure. Higher inflation pressures tended to increase the effective radius; the average value at 25psi inflation was .3229m, while the average value at 45psi was .3244m. Standard deviation of the radius obtained from multiple tests at the same inflation pressure was remarkably low, falling well below 1mm. More tests are required to statistically establish the precision of this measurement, but the precision is qualitatively quite high. Furthermore, the values obtained matched physical intuition by consistently falling between the tire's unloaded radius and static loaded radius for each tire pressure.

Stiffness values experienced more deviation than did effective radius values. A typical value for longitudinal stiffness was 1.5 x 10^6 N/m. Variations in inflation pressure had a noticeable effect upon stiffness, though in an initially counter-intuitive way: lower pressures tended to increase stiffness measurements. While more rigorous testing is necessary to define this relationship, a physical argument for this result is that the increased size of the contact patch increases the tractive force for a given level of slip. As illustrated in Figure 5, the slope of the best-fit line through the run taken at 45 psi is less than the slope of the run taken at 25 psi.

The results presented here are only preliminary and based upon a limited number of data sets. Further testing and refinement of the slip measurement is required before drawing any conclusive statements regarding the variability of tire longitudinal stiffness with road surface condition or inflation pressure.

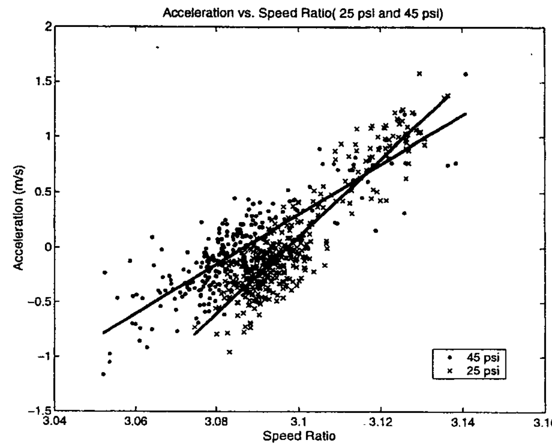

Figure 5 – Comparison at Two Inflation Pressures

CONCLUSIONS

The data shows that GPS velocity information can be combined with wheel speed information to measure tire slip and estimate longitudinal stiffness and effective radius. The data gathered are consistent with the assumption of a linear relationship between force and slip at low levels of slip as predicted by classical tire models. Radius estimation using this method exhibited considerable precision and accuracy within the difference between the undeformed and static loaded tire radii. In preliminary testing, increased inflation pressure appeared to systematically lower the longitudinal stiffness. Future work will concentrate on increasing the amount of collected data and refining data processing to establish more definitive statistical information regarding the effectiveness and sensitivity of this measurement system.

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