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Modelling, simulation and control of macro economic systems.

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Submission date: June 2009
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Problem Description

Build a macroeconomic model based on monetary stocks and flows described by differential equations. Include household and firm sector, a private bank sector, and a government and central bank. With this model, find debt crisis mechanisms for different system variants including today's Basel-type regime, a 100% reserve system, and a system with negative interest rate, as proposed by Silvio Gesell.

Assignment given: 12. January 2009
Supervisor: Trond Andresen, ITK

Abstract

The assignment in this report is motivated by the current financial crisis, which equally, or maybe even more accurately should be called a debt crisis.

The research problem to be solved in this report is therefore to assess four conceptual different macroeconomic systems, including today's Basel-type banking system, to find advantages and disadvantages in these systems compared to each other. Advantages and disadvantages are found in terms of how debt crisis dynamics are compared to economic growth dynamics around what is found to be a sound value of debt to GDP. The two first conceptual different macroeconomic systems to be assessed is the Basel-type banking regime we have today and a 100% reserve system. These two systems are then assessed one more time, but with negative interest rate on deposits and zero borrowing rate, as proposed by Silvio Gesell. The assessment is done for these systems in terms of gross domestic product (GDP), debt burden on households and firms, and bank net lending to government net spending. The models are built in Mathwork's Simulink for simulation, and are made from differential equations for money flows and aggregation in, and in between the different economic units.

This report is built step by step, starting with the theoretical smallest economic unit of a household or firm. All these basic kinds of household and firm units are then aggregated into a sector with the same essential dynamics as the units it is made of. The dynamics for this sector of households and firms can be compared with the dynamics of a generic tank with fluid inflow, outflow and aggregate of fluid.

The model is then expanded to include a private bank sector subject to the Basel rule which enforces a certain maximum limit of bank capital to asset ratio in order to make the private banks robust against insolvency. A simulation of this system is then done to see how the system evolves through time without government control. The motivation for government control is thereby made because debt grows faster, and to a not durable ratio compared to money deposits for households and firms.

To complete the general macroeconomic model a Central Bank (CB) with government control action is included to fix the total debt to GDP ratio to a certain level, which is found from historical levels of debt to GDP ratio in Australia and the U.S. to be about 60%. Government control action to fix the debt to GDP ratio is government net deficit spending. After the inclusion of a government with a Central Bank (CB) the rest of the complete model now consists of total debt and total bank deposits of households and firms, and a private bank sector. The private bank sector is represented with a bank net lending controller and bank assets and liabilities that is the total debt and deposits of households and firms, respectively.

The results from simulations are GDP, debt service (DS) to GDP ratio and bank net lending to government net spending for the four conseptual different

economic systems. These results essentially shows that in order to increase economic growth in a Basel-type system and 100% reserve system with normal positive interest rate on deposits, the DS/GDP ratio also needs to be increased thus debt crisis and liquidity trap could occur. The two systems enforced with negative interest rate as proposed by Silvio Gesell has the inversed relationship between economic growth GDP and the DS/GDP ratio compared to the systems with "normal" interest rates. This means that for Gesell interest rates, increased GDP growth leads to decreased DS/GDP ratio, thus the two Gesell systems are robust against debt crisis and not prone to the liquidity trap.

Therefore the conclusion advocates for the Gesell-Basel-type regime system because it can increase economic growth measured by GDP side-by-side with a decrease in DS/GDP ratio. This is regarded as a good property compared to today's Basel-type regime, and the 100% reserve system which needs to increase DS/GDP ratio in order to increase economic growth, according to the models in thin report.

The reasons for introducing this system, or at least work on this idea more thoroughly should be interesting and taken into consideration for anyone interested in conceptually changing today's macro economic system to avoid debt crisis, and at the same time have a high macroeconomic growth (GDP growth). Furthermore, this suggested system should also be considered because the main generic macro economic elements for a country are incorporated in our model in a reasonable way.

Foreword

This master thesis was written during the time-period from the beginning of 2009 until the summer of 2009, under the teaching supervision of Amanuensis Trond Andresen, at the Norwegian University of Science and Technology. The master thesis is also the last part of the Master of Science degree at the Norwegian University of Science and Technology, from Department of Engineering Cybernetics.

I would like to thank my supervisor Trond Andresen for giving me the interesting insight into the field of macro economics that i needed, and for ongoing feedback and evaluation during this period. I would also like to thank my flatmates for keeping my mood up, and my caring girlfriend Michaela for great support, understanding and love.

Trondheim 05.15.2009

Stian Bu Solgård

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Chapter 1

Introduction

The title of the assignment in this report is:

> “ Modelling, simulation and control of macro economic systems.”

with the following task:

> “Build a macroeconomic model based on monetary stocks and flows described by differential equations. Include household and firm sector, a private bank sector, and a government and central bank. With this model, find debt crisis mechanisms for different system variants including today’s Basel-type regime, a 100% reserve system, and a system with negative interest rate, as proposed by Silvio Gesell.”

To solve this, four conseptually different systems have been assessed in terms of gross domestic product (GDP), debt service on households and firms (DS) to GDP ratio, and to what extent the government contributes to money creation compared to the tribute from the private bank sector. The conseptually different systems to be assessed are today’s Basel-type regime, a 100% reserve system and these two systems induced with negative interest rate on bank deposits and zero borrowing rate as proposed by Silvio Gesell (1920)[25]. All the different macroeconomic models are simulated around the same value for debt/GDP ratio found from historical data to be around 60%. This ratio is thought to be a neutral debt/GDP ratio in terms of a neutral confidence[1], to see how the systems behave dynamically in this range.

The most relevant previous findings in this area of dynamic conceptual macroeconomic systems are found from papers written by Trond Andresen [7], [5], [2], [4] and [6]. The basis for the models presented in this report is found from Andresen(2009)[7].

Motivation for this assignment is the global financial crisis of 2008-2009 that started out as a subprime mortgage crisis in the U.S. and led to a decrease in liquidity[1], and as advocated by Andrese(2008), the financial crisis is really a

[1] Neutral confidence means the confidence has no effect on the rest of the economy, so when neutral confidence is assumed to be set constant by the government, confidence can be excluded from the model. See Appendix A for further explanation

debt crisis[3]. This debt crisis could be solved by a radical change in the macroeconomic structure, and such conceptually different structures will be developed and simulated here to see how their debt burden dynamics are. Furthermore we will see if it would be advantageous for the macro economy of a generic country to change to other systems than the Basel-type regime we have today.

This study is worthwhile because the Basel-type regime we have today turned out to have disadvantages because of the debt crisis, thus the objective is to find results to back up a proposal to introduce a conseptually different system if such a system can be found to be better than the one we have today.

1.1 Target group

The target group for this report is anyone familiar with basic differential equations, and who are interested in macroeconomics.

1.2 Personal motivation

The subject has been to investigate, develop and assess different economic systems on the basis of differential equations and knowledge of control engineering.

I find macroeconomics interesting and i saw this master thesis assignment as an opertunity to use my background in the field of engineering cybernetics in order to understand the financial crisis of 2008-2009, and to find out what could be done to make a better macroeconomic system.

1.3 Research method

The different macroeconomic systems are developed by differential equations which are connected in Mathwork's Simulink block diagrams for simulation. The choices for parameter values for the models are explained, and algebraic solutions to the simulation results are developed in parallel to substantiate the findings.

Results from this research of different economic systems are then compared to each other to find a conclusion.

1.4 Structure of the report

Apart from Foreword, Abstract and Introduction, the chapters in this report is structured as follows:

Foundation. This chapter explains how the model is built in terms of how differetial equations should be understood for economic units and sectors.

A model of private banks, households and firms. This chapter introduces a Bank net lending controller to lend money to the household and firm sector. The first simulations are done here to see how the debt burden dynamics evolves over time.

A government is included. A government control action is included to stabilize the debt burden. Simulations for a Basel-type bank regime and a 100% reserve system are done without government interference, and with government interference for different debt burden ratios.

Silvio Gesell's proposal to negative interest rates is a chapter where negative interest rate on deposits and zero interest rate on loans are introduced. Simulations and algebraic solutions to the Basel-type bank regime and 100% reserve system with these interest rates are done here.

Discussion chapter is needed to sum up the results and discuss them because discussions and results in the former chapters are so scattered.

Conclusion chapter is the last chapter which concludes on basis of the findings in this report, and on the basis of what the intention of this report is. This chapter also discusses if there is some parts of the model that could flawed, and suggests where further research could go.

Appendix A and B is a long development of an algebraic solution, and a discussion of how confidence could be included, how it affects the economy, and why it is not needed in our model.

Chapter 2

Foundation

To make a macroeconomic model, we first need to go into detail about how the smallest elements in the final model actually works and how they should be interpreted. Most of the knowledge presented in this chapter is found in Andresen's paper "The Macroeconomy as a Network of Money-Flow Transfer Functions"(1999)[5]. The model is made up by economic units that can store money for themselves, and receive and send money from and to other economic units. These units are bank deposits of the household and firm sectors, and the aggregate of all private bank loans which appears as assets to the aggregate of all banks. It should also be noted that the government and bank sector units are also allowed to create and destroy money[1], but we will come back to this later when these units are introduced. First we will develop the most basic elements and concepts of the model, namely how to interpret the generic unit with pipes and reservoir, and why we choose to model dynamics in continuous time.

2.1 A generic approach to the economic unit

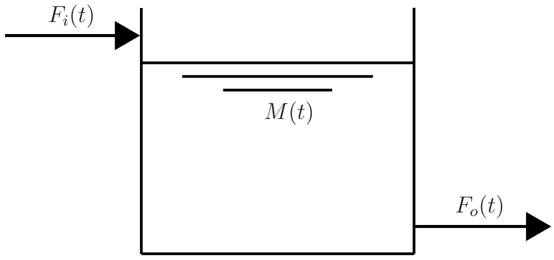

Figure 2.1: Generic economic unit

In our terms, an economic unit can be made of a household, a bank, a firm, a government, or in general any institution with money inflow, outflow and

[1]Units that can create and destroy money are the Bank sector, and the government. The Bank creates money by lending out as much as it is allowed to by keeping a certain capital/asset ratio. The government can both create money, by government deficit spending, and destroy money, by budget surplus. We will come back to how this is done later.

money stock. To understand this unit better we compare it to a generic model that is a vessel or reservoir with a varying level of fluid, which in comparison to the economic unit is money stock $M(t)$. Figure 2.1 shows money inflow $F_i(t)$, outflow $F_o(t)$, and money stock $M(t)$ in the generic unit concept. This level of fluid, $M(t)$, in the generic model depends on the fluid inflow and outflow, which is equivalent to the economic unit where the money stock of course varies with money inflow, $F_i(t)$, and outflow, $F_o(t)$.

Inflow, $F_i(t)$

To make this a bit clearer, imagine a reservoir, the generic model, with only fluid inflow and no outflow: if so, the level of fluid would rise proportional to the fluid inflow. In this same way the money stock in the economic unit would rise proportional to the income as long as there is no spending. Because the dynamics of both models are the same their rate of fluid level or rate of money stock can be modelled by the same equation, namely equation (2.1) which describes how money stock $M(t)$ varies in terms of money inflow and outflow.

$$\dot{M}(t) = F_i(t) - F_o(t) \tag{2.1}$$

We assume the unit, both the generic and the economic, to have no influence on its inflow. To explain this we can first imagine that the inflow to the generic reservoir unit is uncontrollable by the unit inherently from its construction. It is constructed like an open reservoir with no control of its inflow, but big enough to assume it will never get full and spill over. Thus a realization of the reservoir would have to be indefinitely big, but lets just image that a full reservoir will never be an issue. The economic unit which in this case is of no control over its inflow, is somewhat easier to explain because the rate of money flowing in will always be at the mercy of its sources of income, and a bank account can never full and spill over. However, the economic unit *could* refuse to accept money inflow, but we assume this situation rare enough not to be accounted for, because all households and firms are assumed to normally accept every positive amount of income. Both the economic and the generic units have now therefore no control over their inflows.

Outflow, $F_o(t)$

The dynamics of outflow $F_o(t)$ on the other hand, from both the generic and economic units, are decided by the level of fluid and the size of the money stock, respectively. With a fixed valve at the outflow from the generic reservoir unit, no friction, fixed fluid properties and fixed outlet pressure, the outflow now solely depends on the level of fluid in the reservoir which creates the pressure behind the valve. This relationship between level of fluid and reservoir outflow is assumed to be proportional.

In the generic reservoir unit, it is easy to understand that as the level rises, thus the pressure against the valve rises, hence an increase in the outflow due to higher pressure. In the economic unit however, such dynamics can be explained by what is considered here to be normal behaviour among households and firms, which is that their spending depends on the size of their money stock. If an economic unit has a big money stock it will spend more, and if the money stock is small it is likely to spend less. Equation (2.2) describes this dynamic relationship with money outflow $F_o(t)$ as a function of money stock, $M(t)$.

$$F_o(t) = \frac{1}{\tau}M(t) \tag{2.2}$$

The time lag τ, as it is called in systems theory lingo, can be interpreted as the speed of which the unit adjusts its spending to its (changed) money stock. The sort of delay τ represents in practice would be the time a person uses to decide what to spend money on before spending it after his money stock has increased. In the same way, if suddenly there is no more inflow, the outflow would decrease due to decreased money stock. This adjustment of spending, or outflow as a function of money stock is depicted in Figure (2.2) and (fig:downstep-out).

For the generic reservoir unit, it is easy to understand that a smaller τ, that is a wider valve opening, will increase the outflow, and a bigger τ, that is a smaller valve opening will decrease the outflow. The size of the "valve" for the economic unit is interpreted here to be depended on the confidence of the economic unit. It will spend more in times of optimism, hence a small time lag τ, and save more for security reasons in times of depression, hence a bigger τ. How confidence can change over time bacause of i.e. a change in consumer debt service to gross domestic product, causing a varying time lang τ, is explained in Appendix A.[2]

The economic unit

The economic unit can now be described by the change in money aggregated in the unit found from equations (2.1) and (2.2).

$$\dot{M}(t) = F_i(t) - \frac{1}{\tau}M(t) \tag{2.3}$$

To summarize, this equation (2.3) describes how money aggregated in the unit changes with time, by money income $F_i(t)$, and expenditure of money $F_o(t) = \frac{1}{\tau}M(t)$, leaving the unit as a function of money stock $M(t)$. To see how the dynamics of the economic unit are, we will now simulate the unit for two different scenarios with time lag $\tau = 2$. The first scenario is when the economic unit starts with zero money stock $M(t = 0) = 0$, and zero income, $F(t = 0) = 0$. Income to the economic unit $F_i(t)$ is then increased to one and held constant, and the equation for its reaction in terms of outflow $F_o(t)$, called the step response, becomes

$$F_o(t) = 1 - e^{-\frac{t}{\tau}} \tag{2.4}$$

This step response from a sudden unity income to economic unit can be seen in Figure 2.2. The second scenario to be simulated is for an economic unit that has reached a stationary value of money aggregation M_0, from i.e. a long time constant input of one, and then gets its income cut to zero. The reaction in outflow $F_o(t)$ can then be expressed like this

$$F_o(t) = \frac{M_0}{\tau}e^{-\frac{t}{\tau}} \tag{2.5}$$

[2]Confidence is not included in out model for simulation purposes however because development of confidence is thought by the author to be highly speculative. In addition to this, confidence will not be included because it will no effect our model when government control actions sets a certain debt to GDP ratio to enforce neutral confidence.

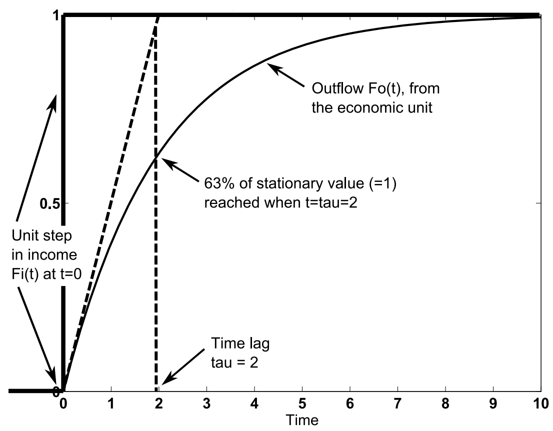

Figure 2.2: Outflow reaction $F_o(t)$, to a step in income from zero to one at $t = 1$.

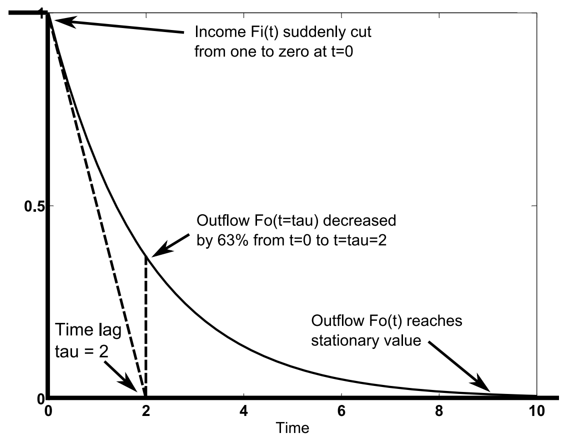

Figure 2.3: Outflow reaction $F_o(t)$, to stationary income cut from one to zero.

The graph produced of outflow reaction $F_o(t)$, by simulation of such a scenario with income $F_i(t)$, cut from constant one to zero is depicted in Figure 2.3.

2.2 Pipes and flows

Economic units in the model to be made are interconnected by "pipes" with money flowing in these pipes between the units. The denomination of the money flow is [currency unit/ time unit] (=[$/y])[3] which can be compared to the generic model where the denomination of flow would be [volume unit/ time unit]. Because money will always belong to a unit or institution, there is a difference from the reservoir analogy with the "pipes" that connects the different economic units where the flows are. The difference is that the "pipes"

[3] $ is the denomination of the currency unit used, but does not have anything to do with American dollars except dollars are also a currency unit. y means years, as we will look at how our models evolves over time spans of years, hence y as the denomination for time.

between the economic units never actually contain any money because money always belongs to someone and thus, the "pipes" does not represent any delay or aggregate of money in the system. The reservoir anlalogy of the "pipes" in the economic model can therefore be understood as frictionless, short and thin enough to contain nothing. This is in fact actually often how reservoir systems with flows of liquid are modelled, by neglecting the contained volume, friction and delay imposed by the pipes.

2.3 Dynamics and continuous time

A macro economy evolves through time with different economic units interacting with each other by money flows. The system is thus a dynamic system, and so is the model to be made of it in this report, namely on the basis of differential equations. As Andresen[5] points out, dynamics are conceptually and mathematically far more complicated than comparrative statics, but they are still used in this paper to make a closer approach to the real world. To make the model dynamic, algebraic equations corresponding to intersecting schedules widely used in economics are substituted with differential equations. While it is relatively easy to find graphical or algebraic solutions in a static framework, this is much harder to do with differential equations without computer-implemented solution software. We also desire to use block diagrams in our model to place our differential equations in modules, or blocks, to make it easier to understand the model as a whole and how the units interfere with each other. Therefore, to implement and analyse our dynamic model in a block diagram with implemented functionality to run simulations, computer software Mathwork's Matlab[20] with Simulink is used because it meets these requirements.

While real world economic transactions is sent in discrete packets, we will consider the flow to be continuous. This is reasonable for the time span years which is used throughout our simulations. Another reason that makes it reasonable to assume money flow to be continuous in the model is because one unit in the model really is an aggregate of all units of this kind in the macro economy. This means, for example, that the household unit in our model really is an aggregate of all households in one country, the bank unit is an aggregate of all banks, and the firm unit is an aggregate of all firms. How this aggregation of units is done is explained in the next section, on the basis of findings from Andresen(1999)[5] Therefore, if we assume there is a million households in the aggregate household unit, and they all make their individual monthly instalment payments at different points of time during a month, this descrete money flow of debt service to the aggregate of all banks will be very close to a continuous flow.

2.4 An aggregate of economic units

The economic units in the model will represent sectors in which specific economic units like a household in the household sector, or a bank in the bank sector, is aggregated. Such aggregates that make up a sector could be the sector of a country, an area of countries, or even the whole world. In our model however, these aggregates are assumed to be aggregates from within a country. What defines a sector is that it exists of units of the same type, but they can have different sizes of money stock, money inflow and money outflow. The outflow from each unit in the sector is split between going into other units in the sector and leaving the sector, and these parts of the outflow from each unit are denoted ρ and $(1-\rho)$ respectively, where $0 < \rho \leq 1$. The system of units inside a sector

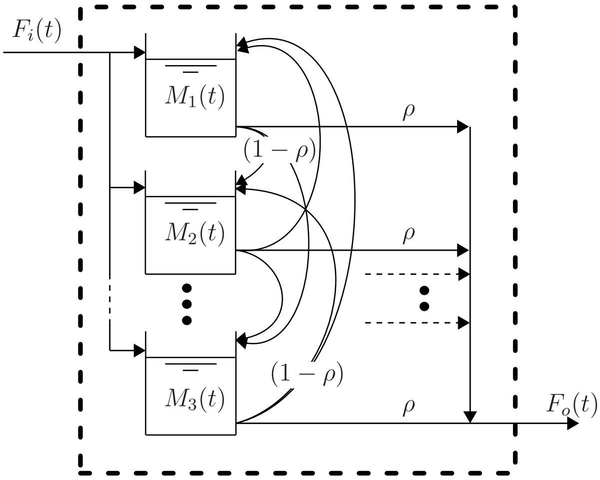

Figure 2.4: Sector of aggregated units

is illustrated in Figure 2.4. Under the assumptions that ρ and the time lag τ is common for the whole sector, Andresen[5] proves the aggregate sector to have the same first order dynamics as the units it consists of, namely

$$h_a(s) = \frac{1}{1+T_a s}, \text{ where } T_a = \frac{\tau}{\rho} \geq \tau \tag{2.6}$$

To give some insight to the dynamics of this sector model we can look at a system with a small ρ, that is, where a significant part of the unit outflow $(1-\rho)$ goes back to other units in the sector and only a small remaining part of the outflow ρ from each unit flows out of the sector. The resulting aggregate time lag T_a for such a sector will be bigger and money will stay in the sector for a longer period, thus making the sector input-output dynamics slower. A sector with such behaviour could be a sector of firms where often a relatively big amount of the income is used for purchasing goods and services from other firms which would lead to a big $(1-\rho)$, thus a smaller part of the turnover would be used to cover employer salaries and thereby leave the firm sector. Households on the other hand, are assumed to use most of their spending in other sectors, thus the ρ of households will be close to unity which, according to equation (2.6), gives a T_a close to τ. This means that the sector of households nearly has the same time lag as the individual units it consists of because their input-output workings in the sector are close to parallel. As the time lag T_a of the household sector is close to the individual time lag of households, τ, as opposed to the firm sector, the time lag of the household sector is also assumed to be much smaller.

2.5 Block diagrams

The models will be made of differential equations, and presented as block diagrams to show how different sectors of the macro economy are interconnected. These diagrams will be illustrated as they appear in a Simulink diagram because this is nearly the same as the traditional block diagram scheme that can be found in Reguleringsteknikk(2004)[15] and because there is an one to one mapping between them.

The reader is assumed to be familiar with both traditional block diagrams and blocks in Simulink, both for differntial equations, so this will not be explained further. As an example of such mapping, Figure 2.5 shows the mapping of an integration block between the traditional block diagram and the integrator block in Simulink.

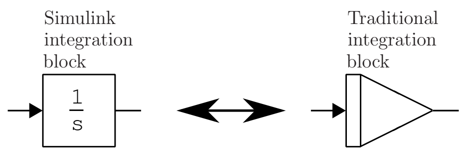

Figure 2.5: Mapping of an integration block between traditional standards and Simulink

Chapter 3

A model of private banks, households and firms.

After establishing the foundation for the model in the previous chapter, we will now start building it. The first model we will build is a model of the private bank sector, which from here will be called the Bank with capital B, meaning the sector of all commercial banks. The Bank will lend money to the household and firm sector, thus creates both debt and deposits. This model is basically the same model as developed by Andresen(2009)[7], but it will still be explained here because it is of great importance to understand it before moving on to an extended model, and because we assume this to be new to the reader. If the presentation in this chapter should appear vague, or cause confusion, Andresens paper "Basel Rules, endogenous Money Growth, Financial Accumulation and Debt Crisis"(2009)[7] should be read to get the proper understanding.

3.1 Bank fundamentals

A bank is a government-licensed financial institution, offering loans for interest, and interest for deposits at a lower rate than the borrowing rate. This deviation from interests earned on loans given by the bank, and interests paid on deposits held by bank, is mainly how banks make money. The level of government regulation on the bank depends on the country it is situated in, and the regulation can be relatively heavy, like in China, or relatively light, like in Iceland, the United Kingdom and the United States. A bank can also be allowed to be involved with other financial activities like insurance services or ownership of non-financial companies[28] depending on the country it is situated in, but in our model the Bank is assumed not to be involved in such other financial activities.

3.1.1 Basel Rules

"The Bank for International Settlements (BIS) is an international organisation which fosters international monetary and financial cooperation and serves as a bank for central banks."[8] BIS sets requirements on banks, here called Basel rules, to aim at promoting monetary and financial stability. The most important of these Basel rules, which will be focused on here, is the minimum capital/asset ratio requirement. Banks subject to this rule are required to have their capital/asset ratio κ as described in equation (3.1), above a certain reference level κ_{ref}, assumed here to be $\kappa_{ref} = 0,08$. This is done, as Andresen[7] explains, to make banks robust against insolvency, but this also has the important side effect of endogenous money growth, by making banks capable of creating money to lend out as long as they stay above a certain κ_{ref}. Equation (3.1) explains how the current capital/asset ratio is calculated, and is from now on referred to as the BIS rule.

$$\kappa_{ref} = 0,08 \leq \kappa = \frac{A(t) + R(t) - L(t)}{A(t)} \tag{3.1}$$

3.2 Assumptions made to better understand the model

We will make the Bank model easier to develop and understand by making certain assumptions:

- The only Basel rule inforced, is the BIS rule from equation (3.1), and this is the only regulation banks need to fulfill.
- There is only one type of Bank assets, and that is money lended out to borrowers, which in our model are households and firms.
- There is only one type of Bank liability, and that is Bank deposits belonging to households and firms.
- Interest rate i_R on Central Bank (CB) reserves $R(t)$, are zero. This means CB reserves can be seen as a constant through time when government control action is not included.
- Influence from confidence in any sector is not accounted for.[1]
- The Bank lends out as much as it is allowed to according to the BIS rule (3.1), which also means the borrowers will borrow as much as they are able to.

3.3 Constants and variables

The model, as illustrated in the simulink block diagram in Figure 3.1, consists of the following constants and variables explained below. The time derivative of any variable x is from now on denoted with a dot by the convention in equation (3.2):

$$\dot{x}(t) \equiv \frac{dx}{dt} \tag{3.2}$$

The denomination of the different variables and constants is signified by brackets with the denomination inside. The denomination is built up by money, "\$",

[1]To see an explanation of how confidence could be accounted for, one should take a look at Appendix A. It is not included in our models because building such a model is thought to be highly speculative, involving too many unknown parameters.

time "y" (a year), or dimensionless: "-". Examples of denomination are Bank assets, $A(t)$: [$], any money flow: [$/y], interest rate: [1/y], or the Bank capital/asset ratio κ, which is dimensionless: [-].

3.3.1 Constants

All the values of these constants except for R was found in Andresen(2009)[7]. The value for R is set to be positive to have a reasonable initial condition for simulation, but the choise to set it to 10 was just picked as any other positive number could be picked, and 10 will be the initial value of R in all further simulations done in this report.

$\boldsymbol{R} =$ 10 [$]. It is the reserves of the Central Bank. Later on, when government action is included, this will serve as a variable.

$\boldsymbol{\kappa_{ref}} =$ 0,08 [-], and is the lowest capital/asset ratio allowed according to the BIS rule in equation (3.1).

$\boldsymbol{\lambda} =$ 0,005 [1/y], is the annual loss rate, as a fraction of $A(t)$ because of borrowers defaulting on their loans.

$\boldsymbol{i_A} =$ 0,07 [1/y], is the borrowing rate, paid to the Bank where the total of loans given to customers are the Banks assets, $A(t)$.

$\boldsymbol{i_L} =$ 0,03 [1/y], is the interest on deposits, that is the Bank liabilities $L(t)$, paid by the Bank.

$\boldsymbol{\beta} =$ 0,2 [-], is net interest income left for the bank after expenses, including wages, are paid. $0 < \beta < 1$

3.3.2 Variables

$\boldsymbol{\kappa} =$ the actual, computed capital/asset ratio [-], used to calculate the deviation from allowed minimum capital/asset ratio, κ_{ref}, to send to the controller which use this deviation to compute new Bank net lending. See equations (3.12) and (3.14).

$\boldsymbol{l - rA} =$ net lending from the Bank [$/y]. This net lending increases both the assets and liabilities of the Bank with just as much, and at the same time. This is because the money borrowed will appear at the borrowers deposits account ($= L(t)$) as soon as it appears as outstanding debt to the bank ($= A(t)$). If the borrower chooses to pay someone else with the money borrowed, this would appear as a deposit in their bank, and since the Bank is an aggregate of all banks, this would still appear as a liability to the Bank.

$\boldsymbol{A(t)} =$ assets of the Bank, [$]. This is money belonging to the Bank, lent out to borrowers, not yet paid back, and therefore the aggregate of all outstanding debt from households and firms to the Bank.

$\boldsymbol{L(t)} =$ liabilities of the Bank, [$]. It is the customer's deposits in the bank, belonging to the customers that consists households and firms.

3.4 Making the model

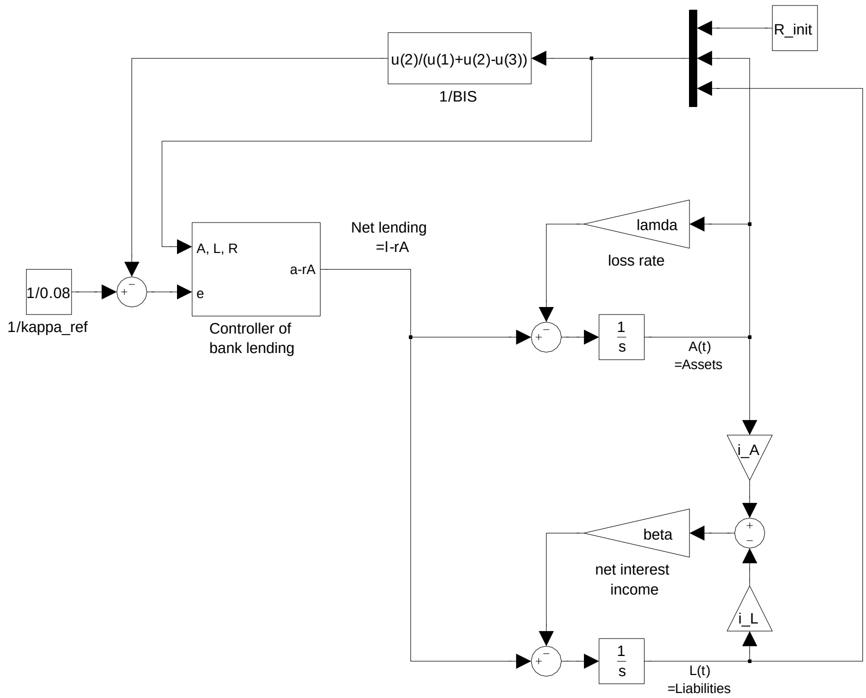

Figure 3.1: Bank model in Simulink

The main components of the model are the constants, variables and assumptions as explained above. We will now show how all these parts fits together by equations and Simulink block diagrams. The model is shown in Figure 3.1 and explained by equations in the different subsections below. Exactly how the equations are derived are explained by Andresen(2009)[7], and an abstract of his papers, with simulations, given here.

3.4.1 Assets

Bank assets are the loans given to the borrowers, where the differential equation for asset change is

$$\dot{A}(t) = l - rA - \lambda A(t) \tag{3.3}$$

The only addition to net lending, $l - rA$, affecting the asset change, is the borrowers defaulting on their loans causing loss of assets by the effect of $-\lambda A$. This is shown in figure 3.1.

3.4.2 Liabilities

The Liabilities in this model is the total of customer deposits, being deposits from all households and firms within the economy. To derive this result, we first consider the deposits of households and firms to be two distinct economic agents, each of these being the total aggregate of its kind, represented as sectors as explained in the previous chapter. Both transfer functions in (3.4) and (3.5) have the same essential dynamics as can be found from Andresen[2] This means that each unit to look the same as the other, except from their time lags T_F and T_H. The sectors are assumed to be interconnected as in Figure 3.2, and the explanation is as follows: Members of the households in the household sector work in the firm sector to earn wages which they consume by buying products and services from the same firm sector they work in and get their wages from. The transfer function of the household sector is

$$H_H(s) = \frac{1}{1 + T_H s} \tag{3.4}$$

with wages W as an input, and consumption C as an output, as depicted in Figure 3.2. The transfer function of the firm sector looks the same as for the household sector

$$H_F(s) = \frac{1}{1 + T_F s} \tag{3.5}$$

except for the time lag T_F instead of T_H, with consumption C as an input from the household sector, and wages W as output to pay the workers in the household sector from the firm sector. Together, these transfer functions make a Simulink block diagram as depicted in Figure 3.2. If we now assume we have a common

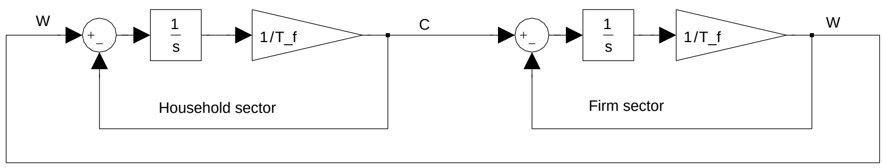

Figure 3.2: Simulink block diagram the of household and firm sectors, and how they are interconnected.

sector representing deposits of both households and firms, the aggregate time lag of the transfer function of the new aggregate sector is assumed to become simply

$$T = T_H + T_F \tag{3.6}$$

This new time lag T leads to a much simpler transfer function

$$H_{H+F}(s) = \frac{1}{Ts}, \text{ where } T = T_H + T_F \tag{3.7}$$

This is easy to see if we first melt the two sectors in Figure 3.2 in the Simulink block diagram, but keep everything else the same. Figure 3.2 now becomes Figure 3.3, and we can see the simple result in Figure 3.4, done by block diagram manipulation[15]. To do this, we simply have to recognize that the same money flow that is subtracted at the input of the integrator block in Figure 3.4 is also added at the same place, and therefore the money flows cancel each other out.

A value is found for T.

The circulation inertia T used to calculate GDP from $L(t)$ is assumed to be 0.7, meaning the mean time lag (in years) for entrance and departure of money in

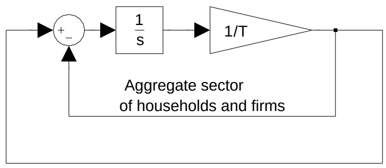

Figure 3.3: The household sector and firm sector melted together

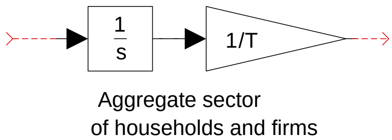

Figure 3.4: The simplified aggregate sector of households and firms

$L(t)$. This number can not be backed up statistically, but it can be found by discussions of the mean time lag for households and firms, T_H and T_F respectively.[2] Mean time lag for households T_H, is assumed to be around one month, one tenth of year that is, as households are assumed to spend almost their whole monthly salary before they receive a new one. Because there is little money flow directly between households, this does not contribute to increase the mean time lag of the household sector. In the firm sector however, the mean time lag T_F, is assumed to be largely influenced by internal money flows as just a fraction of average firm expenses goes to payment of wages, and the rest goes to other firms, keeping money in the firm sector. The circulation inertia (=mean time lag) of the firm sector T_F, is therefore assumed to be a bit more than half a year, 0,6 years that is, which is 7,2 months. According to equation (3.7), total circulation inertia T now becomes

$$T = T_H + T_F = 0,1 + 0,6 = 0,7[y] \tag{3.8}$$

We can now calculate the gross domestic product which, according to the Penguin Reference Dictionary of Economics[11], is

> "A measure of the total flow of goods and services produced by the economy over a specific time period, normally a year or a quarter. ... Note that all intermediate goods are excluded, and only goods used for final consumption or investment goods or changes in stocks are included. This is because the values of intermediate goods are implicitly included in the prices of final goods. The word 'gross' means that no deduction for the value of expenditure on capital goods for replacement purposes is made. Because income arising from investments and possessions owned abroad is not included, only the value of the flow of goods and services produced in the country

[2] The same argument for finding T_H and T_F is made by Fitje(2008)[13]

is estimated; hence, the word 'domestic' to distinguish it from the gross national income."

From this, the flow from dividing liabilities $L(t)$ (=total deposits) with the circulation inertia T, is thought to be a good estimate of GDP.

$$GDP = \frac{1}{T_H + T_F} L(t) = \frac{1}{T} L(t) \tag{3.9}$$

Further more, GDP is thought to be the right word for this flow instead if GNP because GNP also includes income arising from investments and possessions owned abroad which is not a part of our model.

The error from combining the household and firms sectors.

The assumption done in equation (3.6) only leads to a small dynamic error between the original subsystem, with two distinct sectors, and the aggregated sector of these two, as can be seen in Figure 3.7. To demonstrate the error we

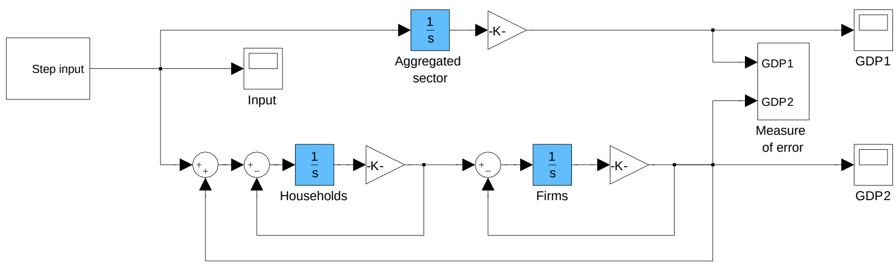

Figure 3.5: Simulink model to simulate and find dynamic error between original model with distinct sectors of households and firms, and the new simplified aggregate unit.

get from the simplification, we simulate the block diagram in Figure 3.5 over 50 years, with step inputs as shown in Figure 3.6. The error that arises from this simplification, as described in equation (3.10)

$$e(t) = \frac{GDP_{aggr} - GDP_{H+F}}{0.5(GDP_{aggr} + GDP_{H+F})} \tag{3.10}$$

and demonstrated in Figure 3.7, is divided by average GDP from the two to become an error measured as a fraction of average GDP. Initial total liabilities in the simulation of error is \$100, which shows that for the simulations done in this thesis, with liabilities usually above \$100, the simplification done in (3.6) only leads to a small, insignificant error. The simplified model in Figure 3.4, done by the assumption in equation (3.6) is found to be satisfying because we will only look at the system over a time horizon of 50 years or more, and because the error is very small when compared to aggregated deposits.

Dynamics of liabilities

Liabilities is now the united aggregates of deposits in the Bank from households and firms, and the equation for liability change is

$$\dot{L}(t) = l - rA - \beta(i_A A(t) - i_L L(t)) \tag{3.11}$$

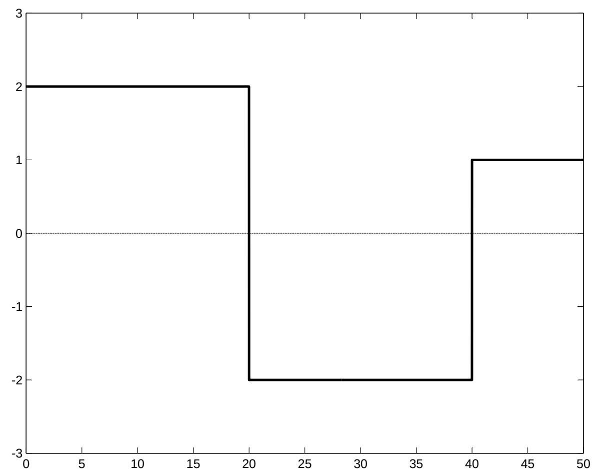

Figure 3.6: Input to system of distinct households and firms, and to simplified system.

In addition to net lending affecting liability change the same way it affects asset change in (3.3), there is also the effect of Bank income surplus from borrowing rate and interest rate on deposits after paying wages and other expenditures. This Bank income surplus appears in the form of reduced liabilities because this is where the payment of the Bank income surplus has to come from, namely from the deposits of households and firms. This part of the model is shown in the lower right part of Figure 3.1.

3.4.3 Bank net lending controller

The controller takes the error between required minimum capital/asset ratio κ_{ref}, inversed and actual capital/asset ratio κ, inversed,

$$e(t) = \frac{1}{\kappa_{ref}} - \frac{1}{\kappa}, \text{ where } \frac{1}{\kappa} = \frac{A}{A + R - L} \tag{3.12}$$

as an input to make the measured κ as close to the reference κ_{ref} as possible. The Bank is thereby lending out as much as it is allowed to, thus earning as much as possible. The reason κ_{ref} and κ are inverted is to make sure the controller works properly because the initial value in our simulation for assets is zero and the denominator of measured capital/asset ratio, κ, is assets. Not using the inversed of κ and κ_{ref} would therefore result in an infinite κ which would disturb the simulation. Output from the controller is Bank net lending, $l - rA$ defined by equation (3.13). The controller input, and A, $L(t)$ and R used to make κ, should not be seen as flows of money into the controller, but merely as measures of money flows. The point is that actual money and actual money flows only exist from the output of the controller, $l - rA$, to $\dot{A}(t)$, and $\dot{L}(t)$, and between $A(t)$ and $L(t)$, and not through the controller because that is merely measures and calculations as it is in any other control system.

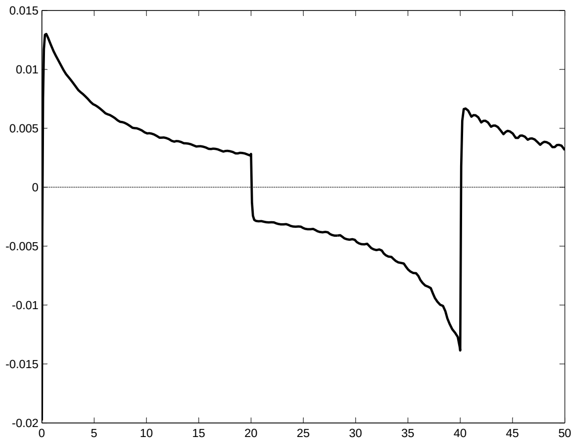

Figure 3.7: Error between simplified aggregate system of liabilities and the original system, as a fraction of average GDP between the two systems.

Inner workings

The controller is shown as a subsystem in Figure 3.1, called "Controller of Bank lending", and its inner workings is shown in Figure 3.8. By taking the measured

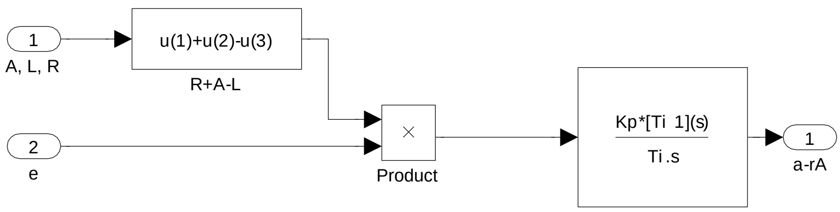

Figure 3.8: PI controller

error between the inverted reference κ_{ref}, and the inverted actual measure of κ, the controller is made to lend out as much as allowed according to the BIS rule, to make the Bank earn as much money as possible. This is done by multiplying the error with $R+A-L$, which is the capital of the bank, and send the result to a Proportional-Integral (PI) controller to calculate the appropriate net lending. The reason for multiplying the error with $R+A-L$ is because the amplitude of the control action needed to bring κ to κ_{ref} is proportional to the Banks capital, $R+A-L$, because the control action acts on both assets and liabilities just as much. Another, more intuitive reason for multiplying the error $R+A-L$, will become clear when we simulate a 100% reserve system. The proportional part (P) of the PI controller compensates for the present error, and the integral part (I) makes sure to remove long time deviation, that is, error offset. The appropriate parameters K_p and T_i in the PI controller equation (3.13)

$$l - rA = K_p \hat{e}(t) + \frac{1}{T_i} \int_0^t \hat{e}(\tau)\, d\tau \tag{3.13}$$

where

$$\hat{e}(t) = (R + A - L)e(t) \tag{3.14}$$

were found by a trial and error process to be $K_p = 10$, and $T_i = 1$.

Windup integrator due to saturated output

The Bank is assumed to be unable of having a negative net lending, which means that the Bank will always lend out at least as fast as debt from households and firms are paid back. This is reasonable to believe for normal conditions. To enforce this rule; a saturation block is set before the Bank controller output, as can be seen in Figure 3.9. This saturation makes sure the controller output

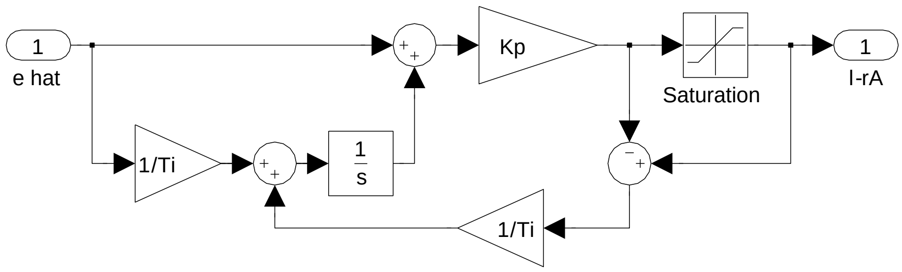

Figure 3.9: Bank net lending PI controller with saturated output and anti-windup functionality.

is limited from zero to positive infinity, meaning Bank net lending can only be positive. With such saturation after a controller with an integrator term, this leads to an integrator windup when a negative error, here $\hat{e}(t)$, is integrated, and at the same time, if the output is saturated, like it is our case. This could lead to serious overshoot or oscillations. To prevent this, anti-windup functionality is included, as can be seen in Figure 3.9. This was found from Åstrøm[22], section 6.5. The implementation can be understood intuitively because the difference between the output before and after the saturation is added in front of the integrator block. If the saturation is not active because the controller output is within the physical limits of the system, the difference before and after the saturation will be zero and the anti-windup functionality is not affecting the integrator, which sounds reasonable. However, if the controller output is outside the physical limits of the system and the saturation is active, the difference before and after the saturation is added before integrator, reducing its windup, which also sounds reasonable.

3.5 Simulation

We will now simulate the system with two different ground rule scenarios for Bank lending. The first scenario we will simulate is the Basel-type bank regime, which is made to make banks able to lend out as much money as possible, and at the same time make the banks robust against insolvency. This regime also have the side effect of endogenous money growth, with the rate of money growth inversely proportional to the capital/asset ratio, as shown by Andresen[7]. To show how the side effect of endogenous money growth could be counteracted, the second simulation is done of a 100% reserve system scenario, were banks can only lend more money to borrowers if the central bank increases its reserves. In this case $\kappa = 1$ which means that bank assets have to mirror the central bank reserves, R. Both simulations are done with the same parameter values and time range (50 years), the only difference is the value for capital/assets ratio reference, κ_{ref}, which is $\kappa_{ref} = 0.08$ for the Basel-type bank regime, and $\kappa_{ref} = 1$ for the 100% bank regime.

3.5.1 A Basel-type bank regime

As mentioned in the previous section, a Basel-type bank regime has the side effect of endogenous money growth. To show this we simulate the model with these parameter values:

Parameter	Value	Denomination
A_{init}	0,00	[$]
L_{init}	0,00	[$]
R_{init}	10,00	[$]
i_A	0,07	[-/y]
i_L	0,03	[-/y]
i_R	0,00	[-/y]
κ	0,08	[-]
β	0,20	[-]
λ	0,005	[-/y]
T	0,70	[y]
$K_{p,bank}$	10,00	[-]
$T_{i,bank}$	1,00	[-]

Table 3.1: Parameters for Simulink simulation of a Basel-type bank regime without government control

With these parameter values, the Bank will start with an infinite measured capital/asset ratio because the denominator of κ, initial Bank assets A_{init}, is zero, and it is in the Banks interest to lend out money until assets grow to a point where κ in decreased to $\kappa \approx \kappa_{ref} = 0,08$. This point is when the Bank will earn as much money as possible (from the differense between interest on loans and interest on deposits) and still be in accordance with the basel rule from equation (3.12).

3.5.2 A 100% reserve system

In a 100% reserve system, the Bank has to mirror deposits, the Bank liabilities that is, with reserves. This is done by setting the reference capital/assets ratio to one, $\kappa_{ref} = 1$. If the controller can be assumed to work perfectly, that is

$$\kappa = \kappa_{ref} = 1, \tag{3.15}$$

equation (3.12) becomes

$$\frac{1}{\kappa} = \frac{A(t)}{A(t) + R(t) - L(t)} = 1 \tag{3.16}$$

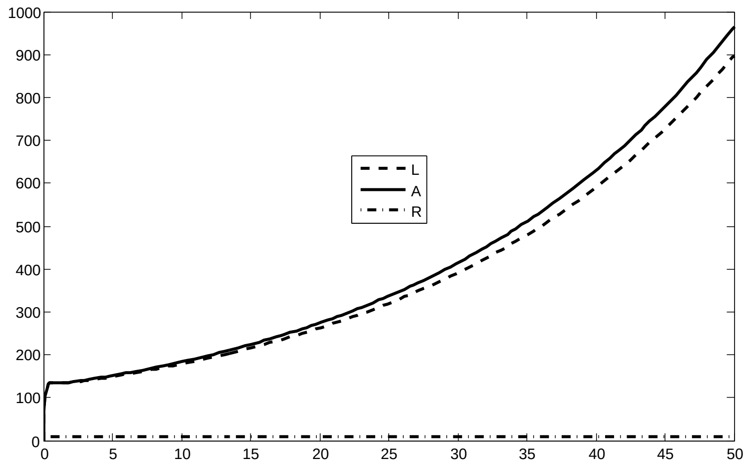

Figure 3.10: Assets, liabilities and reserves for a Basel-type bank regime.

which means that

$$L(t) = R(t). \tag{3.17}$$

An intuitive reason for multiplying the error with $R(t)+A(t)-L(t)$ inside of the controller with a 100% reserve system now becomes clear, even if the controller is not working properly, $\kappa \neq \kappa_{ref}$ that is. The error $\hat{e}(t)$, from equation (3.14), is multiplied with $R + A - L$ which gives

$$\hat{e}(t =)e(t) \cdot (R(t) + A(t) - L(t)) = R(t) - L(t). \tag{3.18}$$

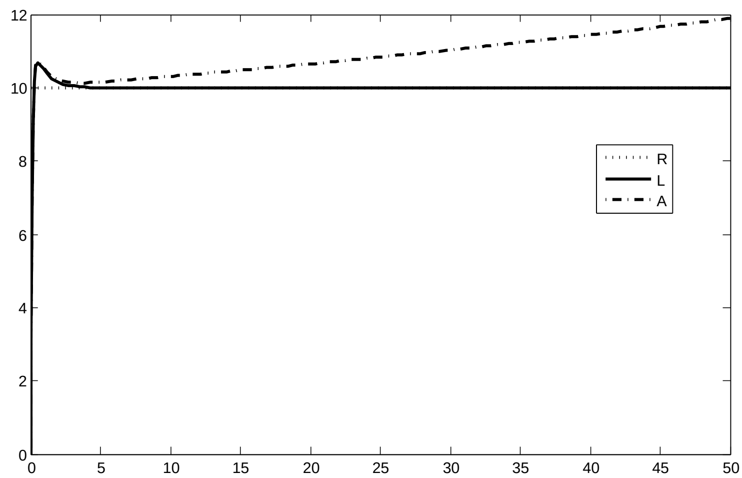

Figure 3.11: Assets, liabilities and reserves for a 100% reserves system.

3.6 Discussion

As mentioned before, the only difference between the two last subsections is the reference of capital/assets ratio κ_{ref}, which should be interpreted as the only law concerning the macro economy in this model, namely the BIS rule as explained in subsection 3.1.1. We can clearly see that both assets and liabilities grow exponentially in the Basel-type bank regime with $\kappa_{ref} = 0.08$. With the 100% reserve system on the other hand, liabilities are tied to the amount of reserves, but assets still grows however as seen in Figure 3.11, which motivates government control action to prevent debt crisis.

Chapter 4

A government is included

We have now established how the capital/assets ratio κ can be controlled and held as low as the Bank is allowed to, to make it earn as much money as possible. However, our macroeconomic system is still prone to collapse and/or debt crisis due to the problem of controlling the asset/liability ratio, directly affecting the debt service to GDP ratio (as can be seen in equation (4.7)), from here on called the DS/GDP ratio. How confidence is affected by the DS/GDP ratio, and how confidence affect the rest of the economy is discussed in Appendix A, but not included in our model[1]. The start of the current "financial crisis" is thought to be caused by a debt problem, according to Andresen[7], and many others [27],[26] and [1]. This makes the motivation for including a government controller to look at how the government can stabilize the economy by controlling the DS/GDP ratio[2] with deficit spending or taxation, and thereby prevent debt crisis.

4.1 The Central Bank (CB)

Andresen(2009)[7] explains the relationship between the government and the central bank:

> "It is assumed that banks' deposits with the CB fluctuate with government spending and taxation, and grow due to interest paid for these deposits. Government bonds are in their entirety assumed to be held by banks, and concidered to be equivalent with interest-bearing deposits at the central bank."p.3

Furthermore he exlains that any CB situated in a country with its own national currency is essentially an arm of the government. Government debt build-up with the CB through government deficit spending is therefore only an accounting and legal convention.

[1]Confidence is not included as a part of our model because to derive a model of confidence is thought by the author to be too uncertain and speculative, and it would complicate further analysis. In addition, as mentioned in Appendix A, by including a good government controller to control DS/GDP ratio, confidence could be held neutral, not affecting the rest of the economy, and it is therefore not included here. For a more thoroughly discussion on this, see Appendix A

[2]The DS/GDP ratio is interpreted as an indication of total debt burden on households and firms because debt service is paid as a fraction of GDP

4.2 Debt service and the DS/GDP ratio

To control the DS/GDP ratio to avoid debt crisis, we first need to identify DS and GDP. According to [12]: “One way to calculate GDP is to add up all expenditures on all new final goods and services.” This is the approach taken in our modelling to calculate GDP. What is not spent on services and goods is assumed to be hoarded by households and firms. As discussed in chapter 2, the time lag T expresses willingness to spend of income, and is therefore interpreted as a circulation inertia. A big time lag T therefore means spending grows slowly with a sudden grow in income, and a small time lag T means spending reacts faster to changes in income.

DS (=total debt service per year) is derived directly from assets, as can be seen in equation (4.7). To do this we first need to choose how to model debt service.

Annuity loan

An annuity loan has a fixed sum of principal plus interest to be paid each year. The equation for annually payment of interest and principal for an annuity loan is found in [23] to be

$$P = \frac{i_A A}{1 - (1 + i_A)^{-N}} \tag{4.1}$$

with parameters

$\boldsymbol{P} =$ annual payment [\$/y].

$\boldsymbol{A} =$ amount borrowed [\$].

$\boldsymbol{i_A} =$ borrowing rate [1/y].

$\boldsymbol{N} =$ payback period, [y], in years.

This type of loan could be the best approximation of the ”average” loan, but it is considered to be hard to implement in our Simulink model, and as Andresen[6] put it: ”algebraically less tractable”, due to time delays involved in the annuity dynamics.

The “Exponential loan” is considered to be similar to the serial loan

The annual payment of a serial loan is done by repayment of the same fraction of the initial borrowed amount A, plus the interest of the current remaining debt, as depicted in Figure 4.1. Thus, the formula for an annual down payment $P(k)$, each year k, for a loan starting at initial amount A, over a time period of N years, with fixed interest rate i_A for a serial loan is found to be

$$P(k) = \frac{A}{N} + A i_A \frac{N - k + 1}{N} \tag{4.2}$$

This formula is derived by the author from the knowledge that annual repayment is the same fraction of the initial borrowed amount A each year, which is what the first term in equation (4.2) means. The second term is a bit more difficult to derive, but we know it represents interest to pay from the remaining debt, in a certain year k. Thus, to derive the second term in equation (4.2), representing interest to pay from remaining debt in year k, we first look at what interest to pay the first year of down payment, for $k = 1$. We know this first year we have to pay interest for the whole initial borrowed amount, and we get

$$P(k=1) = \frac{A}{N} + Ai_A\frac{N}{N} = \frac{A}{N} + Ai_A \tag{4.3}$$

The second year ($k = 2$), we have to pay interest on the initial borrowed value A, minus repayment done the first year from $P(k = 1)$. Thus, for the second year ($k = 2$), we get

$$P(k=2) = \frac{A}{N} + Ai_A\frac{N-1}{N} \tag{4.4}$$

We also know that interest payment the last year $k = N$, is done for the last fraction of the initial loan value. The second term in $P(k)$ for the last year $k = N$, becomes

$$P(k=N) = \frac{A}{N} + Ai_A\frac{1}{N} \tag{4.5}$$

By this, we can see that equation (4.2) fulfils all requirements mentioned above.

Both the "exponential loan" type as described by Andresen[6], and the serial loan type is thought by the author to very similar because they both pay interest on the current remaining debt. The principal however, is not paid in the same manner because the "exponential loan" type pays principal as a fraction (the time period T_r) of the *current debt,* while the serial loan type pays principal as a fraction (the time period T_r) of the *initial debt.* Therefore, if we assume the average time left to repay a loan is $T_r/2 = 7,5$ years, since new loans are issued and repaid all the time meaning the average loan is halfway repaid, the initial lending value of the current debt can be assumed to be twice as big as the current debt. Thus, the annual repayment of principal in a serial loan scheme can be said to be equal to the "exponential loan" scheme described by Andresen[6], if we assume the average loan to be halfway repaid because new loans are issued and old loans down paid all the time. To make this clearer, we can say that a serial loan with a down payment period of $T_r = 15$ years can be implemented in our model by saying it is the same as an "exponential loan", but now with an average down payment period left that is half that long, namely $T_r = 15/2 = 7,5$ years.

The type of loan chosen to be used in our model

The main differnce between a serial loan (or an "exponential loan") and an annuity loan, as depicted in Figure 4.1, found from [10], is essentially that the total interest paid is higher for an annuity loan than for a serial loan. Another difference between the two types of loans is that calculated debt service for a serial loan to be implemented in Simulink is simpler, and is also easier to work with in later on analysis, and the serial loan calculation from equation (4.2) is therefore preferred. The time continuous version of this equation (4.2), meaning annual debt service DS, is equation (4.7). To get a sense of what the difference in total interest payments are between a serial loan and an annuity loan, as depicted in Figure 4.1, we calculate it. With a repayment period of 15 years ($N = 15$), a fixed interest rate of 7% ($i_A = 0.07$), and initial loan value of \$100.000 ($A = 100000$), we get total interests paid for a serial loan to

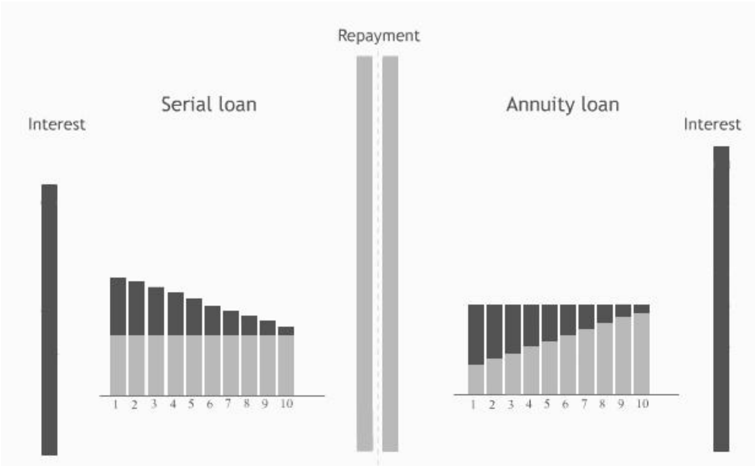

Figure 4.1: The essential differnce between an annuity loan and a serial loan

be \$56.000, while total interests paid for an annuity loan is a bit more, namely \$64.692. These results was derived using a Microsoft Excel spread sheet found in [19], but could also be found from equations (4.1) and (4.2). The differnce in interests paid in these two loan schemes, compared to the initial borrowed amount is

$$\frac{\$64.692 - \$56.000}{\$100.000} = 8.692\% \approx 8.7\% \tag{4.6}$$

which is a quite small difference. This difference can also be accounted for by simply adjusting the interest rate for the serial loan up by 1.01% to 8.08%, which gives total interests paid \$64.640, that is (almost) the same as total interests paid by the annuity loan scheme. The continuous time version of equation (4.2) for the relationship between assets and debt service for our chosen serial loan is

$$DS(t) = (\frac{1}{T_r} + i_A)A(t) \tag{4.7}$$

where $T_r = 15/2 = 7,5$ years, because we assume this to be the current average time left to repay a loan. As mentioned before, this is because of new loans being issued all the time ($T_r = 15$ years), and old loans getting repaid ($T_r = 0$ years), leaving an average repayment period of about $T_r = 7,5$ years.

Motivation for government control

Either if asset rate is higher than liability rate, which is most probable, or the opposite, it is in the author's opinion that it will hurt the economy, with one of them growing faster than the other, and that is why the government controller is included: to keep assets and liabilities in proportions to each other over time to avoid debt crisis or a bank system insolvency.

4.3 Government controller

Without a government able to have deficit spending to decrease the asset/liability ratio, or a government to tax more than it spends, this would debt crisis in both the 100% reserve system and the Basel-type bank regime system. This is because, without government control action, the asset/liability ratio is very difficult to control. Such a debt crisis can be identified by a high DS/GDP ratio. To prevent debt crisis[1], government action is included by a second controller, in addition to the bank lending controller, to make sure there exists a sound ratio between assets and liabilities. This is done indirectly by ensuring a ratio between reserves and assets

$$R(t) = \theta A(t) \tag{4.8}$$

by the use of $\gamma(t)$ (=government net spending) in the reserve rate equation

$$\dot{R}(t) = i_R R(t) + \gamma(t) \tag{4.9}$$

as proposed by Andresen[7]. In our simulations however, the interest rate on reserves i_R, is assumed to be zero to keep our simulations easier to analyze. This assumption leads to a new reserve rate equation

$$\dot{R}(t) = \gamma(t) \tag{4.10}$$

which is used as the government control action throughout our further simulations to ensure that equation (4.8) holds, and thereby counteracting a debt crisis in both the 100% reserve system and the Basel-rule regime system. If we assume this relationship to hold, as we in later simulations will find out, this leads to a new, interesting relationship. By the use of the Bank capital/assets ratio κ, from equation (3.1), with equation (4.8) inserted for $R(t)$, we get

$$\kappa = \frac{A(t) + \theta A(t) - L(t)}{A(t)} \tag{4.11}$$

thus we get the new relationship

$$L(t) = (1 + \theta - \kappa)A(t) \tag{4.12}$$

between $L(t)$, meaning total deposits, and $A(t)$, meaning total debt. It is by this established a relationship between assets and liabilities that prevents a debt crisis indirectly by ensuring equation (4.8) holds true with the government controller.

4.3.1 Inner workings

The government controller, now included in the Simulink block diagram in Figure 4.2, works by taking measured assets and liabilities as an input. Assets are multiplied with θ, and then measured reserves are subtracted to generate an error

$$e_\theta(t) = \theta A(t) - R(t) \tag{4.13}$$

[1] Changes in parameter values, and initial values of assets and liabilities in our model, could lead to the opposite, namely a liability rate higher than asset rate, eventually leading to insolvency of the banking system. However, it is in the authors opinion that the banking system should be kept alive, and stabilized. This new government controller would counteract such a scenario with liabilities growing faster than assets, as well as a debt crisis scenario by taxation of, or government deficit spending to, households and firms.

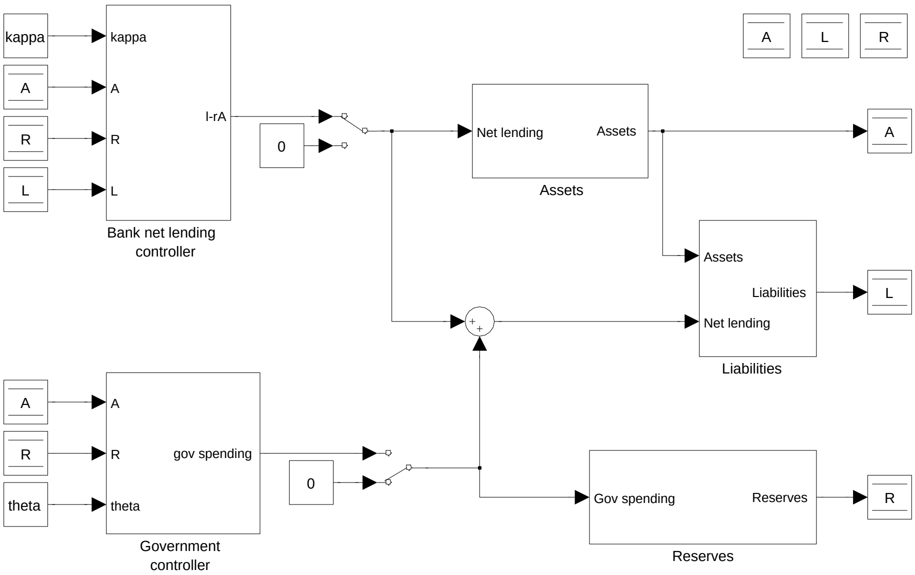

Figure 4.2: Macro economic Simulink model with government, banks, households and firms.

which is corrected by PID a controller (4.14).

$$\gamma(t) = K_p e_\theta(t) + K_i \int_0^t e_\theta(\tau) d\tau + K_d \frac{de_\theta}{dt}(t) \tag{4.14}$$

The output of the PID controller is government deficit spending to increase liabilities compared to assets, or fiscal policy or bond sales to reduce liabilities compared to assets. The PID output is added to (or subtracted from) liabilities and reserves to back government spending[3]. The equation of liability dynamics, after introducing government spending, now becomes

$$\dot{L}(t) = \gamma(t) + l - rA(t) - \beta(i_A A(t) - i_L L(t)) \tag{4.15}$$

and this is depicted in the Simulink subsystem in Figure 4.5.

4.4 Simulation

We will now simulate our macro economic model in Figure 4.2, to find out if our new government controller works as it should, namely to indirectly control the DS/GDP ratio by controlling the assets/liabilities ratio in equation (4.12), with the help of θ and κ. As we can see in Figure 4.2, some changes have been made from Figure 3.10. In addition to the change from the included government controller, the assets, Central Bank reserves, and liabilities are packed into subsystems, as showed in Figure 4.3, 4.4 and 4.5, respectively. Another big difference that can be seen between Figure 3.10 and 4.2 is merely a technical

[3]We assume Central Bank reserves to fluctuate with government spending and taxation. Interests on reserves however, are assumed to be zero throughout our analysis

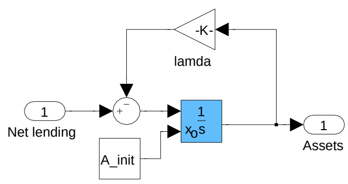

Figure 4.3: Simulink model of asset dynamics according to equation (3.3)

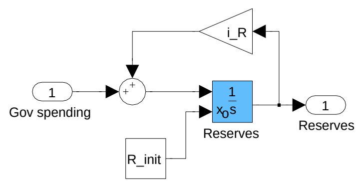

Figure 4.4: Simulink model of Central Bank reserve dynamics according to equation (4.10)

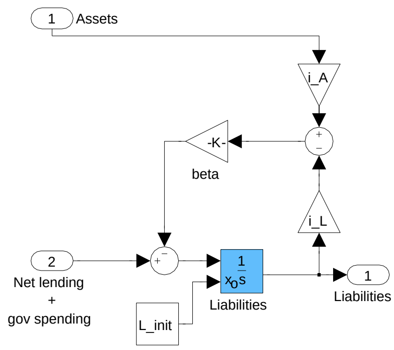

Figure 4.5: Simulink model of liability dynamics according to equation (4.15)

one. Instead of connecting measured assets, liabilities and reserves, to the inputs of the controllers by drawing lines, Signal Routing Blocks[20] from the Simulink Library Browser are used to simplify the model appearance. To make this clearer in an example, look at how the two subsystems in Figure 4.6 are connected with the usual line connection. This connection could also be done (as we have done it here) by the use of Signal Routing Blocks[20] *Read, Write* and *Storage* as depicted in Figure 4.7. The point made here is that the functionality in Figure 4.6 and 4.7 are essentially the same, they just look different, and the signal routing boxes are used to get a better overview of the model.

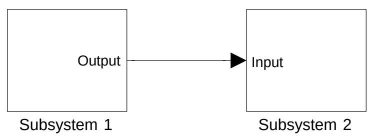

Figure 4.6: Simulink model of asset dynamics according to equation (3.3)

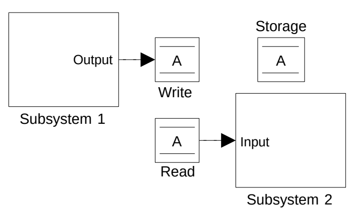

Figure 4.7: Simulink model of Central Bank reserve dynamics according to equation (4.10)

4.4.1 Debt service compared to GDP, and a sound value for θ

It is important to notice that as debt service is paid as a fraction of GDP, it is therefore impossible to have a debt service higher than GDP, because this would mean the borrowers financed their debt service (DS) with money that are not in circulation.

Before we start the simulation, a sound value for θ needs to be found. Together with κ, θ describes the recommended, or suggested relationship between total debt (assets of the Bank), and total deposits (liabilities of the Bank) according to equation (4.12). This relationship is considered to be "recommended" in terms of being the most stable $A(t)/L(t)$ ratio because it keeps confidence, and thereby spending and hoarding at a stable level, and economic fluctuation to a minimum. This suggested, or recommended, relationship will now be found from the average historical debt/GDP ratio from Australia and the U.S. From Figure 4.8 found from zfacts.com[30] we find average historic debt/GDP ratio in the U.S. between 1940 and 2009 to be about 60%. From Figure 4.9 found in Steve Keen's Debtwatch[16] (Published in November 30th, 2008), we get the average of the debt/GDP ratio in Australia between 1870 and 2007 to be around 60%. From these figures we choose suggested debt/GDP ratio to be about 60%, and use this further to find a reference for θ.[4] From Figure 4.8 and 4.9 we find

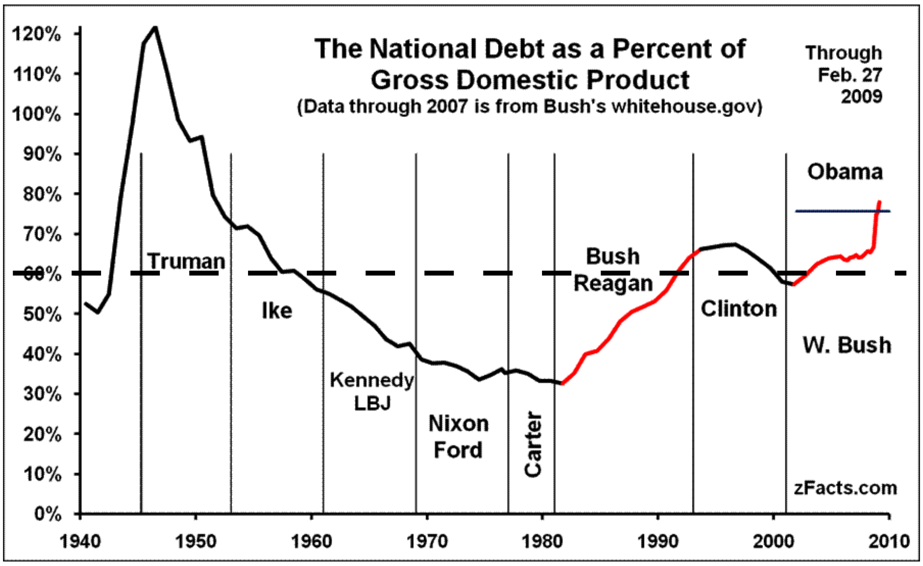

Figure 4.8: National Debt/GDP ratio in the U.S. between 1940 till 2009[30]

the historical average value of debt/GDP ratio to be

$$(\frac{Debt}{GDP})_{ref} = 0,6 \tag{4.16}$$

[4]The most important issue in this chapter is to show how a government spending controller can keep a certain DS/GDP ratio, and not exactly what value this ratio has. It is therefore acceptable to choose a reference value of this ratio the way we do it here.

Figure 4.9: Australian private Debt/GDP ratio from 1880 till 2007[16]

which corresponds to an assumed healthy DS/GDP ratio of

$$(\frac{DS}{GDP})_{ref} = (\frac{Debt}{GDP})_{ref}(\frac{1}{T_r} + i_A) = 0,122 \tag{4.17}$$

This historical value is assumed to be a healthy debt/GDP ratio that ensures a stable economy.

By the use of equations (4.12), (3.9) and (4.16) we calculate θ

$$\theta_{ref} = \frac{T}{(\frac{Debt}{GDP})_{ref}} + \kappa - 1 \tag{4.18}$$

which means that, for a Basel type bank regime with $\kappa = 0,08$, we need to have

$$\theta_{ref} = 0,246667 \tag{4.19}$$

to ensure a stable economy free from debt crisis and fluctuations from changes in confidence.

4.5 A Basel-type bank regime with government

We will now analyze the Basel-type bank regime with government controller to stabilize the economy by controlling the DS/GDP ratio. θ_{ref} for the Basel-type bank regime ($\kappa = 0.08$) is found from equation (4.19) to be $\theta = 0,24667$.

Parameter	Value	Denomination
A_{init}	0,00	[$]
L_{init}	0,00	[$]
R_{init}	10,00	[$]
i_A	0,07	[-/y]
i_L	0,03	[-/y]
i_R	0,00	[-/y]
κ	0,08	[-]
θ	0,24667	[-]
β	0,20	[-]
λ	0,005	[-/y]
T	0,70	[y]
T_r	7,50	[y]
$K_{p,bank}$	10,00	[-]
$T_{i,bank}$	1,00	[-]
$K_{p,gov}$	5,00	[-]
$K_{i,gov}$	0,05	[-]
$K_{d,gov}$	1,00	[-]

Table 4.1: Parameters for Simulink simulation of a Basel-type bank regime with government control

Parameters for our simulation are depicted in Table 4.1. With these parameters we first look at how the debt/GDP ratio develops without a government control action. We can see from Figure 4.10 that debt/GDP ratio stabilizes over a 150

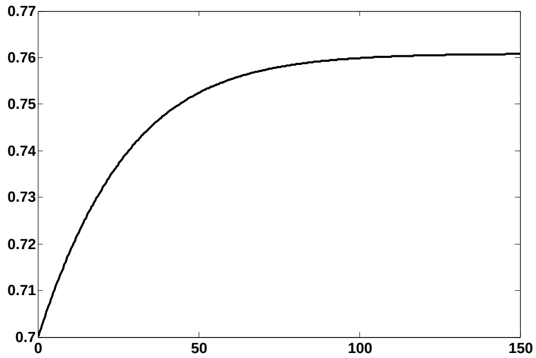

Figure 4.10: Debt/GDP ratio over a time period of 150 years in a Basel-type bank regime without a government controller.

years period, to a value of 0,76. This is because both Bank assets and Bank liabilities grows very big over time compared to government reserves, as can be seen in Figure 3.10, because we assume interest rate on government reserves to be zero without government control action included. Over time, the BIS rule from equation (3.1) now becomes:

$$\lim_{t\to\infty}\left(\frac{A(t)+R(t)-L(t)}{A(t)}\right)=\frac{A(t)-L(t)}{A(t)}=\tilde{\kappa}=0.08 \tag{4.20}$$

This means, without a government controller, assets evolve over time with this equation

$$\lim_{t\to\infty}(A(t)) = \frac{L(t)}{1-\kappa} \tag{4.21}$$

By inserting GDP as a function of liabilities (3.9), equation for how assets evolves over time without a government controller (4.21), and equation for debt service (4.7), we get debt/GDP ratio over time, without government control action to be

$$\lim_{t\to\infty}(\frac{A(t)}{GDP}) = \frac{(\frac{L(t)}{1-\kappa})}{(\frac{L(t)}{T})} = \frac{T}{1-\kappa} = 0,76087 \tag{4.22}$$

which is what we get close to after about 150 years, as can be seen in Figure 4.10. If we now connect the government controller we get a very good control over the debt/GDP ratio as seen in Figure 4.11. An equivalent measure of

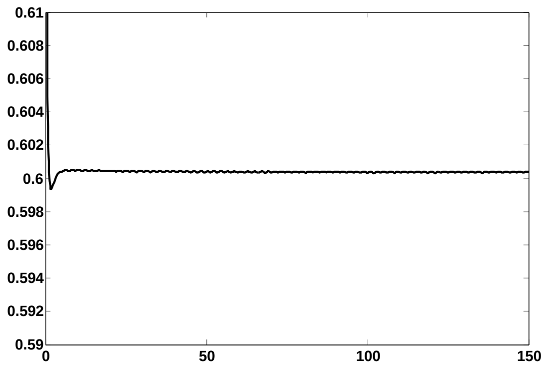

Figure 4.11: Government controlled debt/GDP ratio for a Basel-type bank regime with government controller $\theta = 0,24667$.

debt/GDP ratio is the debt service to GDP (DS/GDP) ratio over time. This is found by multiplying equation (4.22) with the relationship between assets and debt service, found in equation (4.7)

$$\lim_{t\to\infty}(\frac{DS}{GDP}) = \lim_{t\to\infty}(\frac{(\frac{1}{T_r}+i_A)A(t)}{GDP}) = 0,15471 \tag{4.23}$$

This result is also found by simulation, depicted in Figure 4.12. If we now look at how government control action can set a stable DS/GDP ratio, this controlled level of DS/GDP should be the same as

$$(\frac{DS}{GDP})_{ref} = (\frac{1}{T_r}+i_A)(\frac{Debt}{GDP})_{ref} = 0,6\cdot 0,203333 = 0,122 \tag{4.24}$$

found with recommended debt/GDP ratio from equation (4.16), multiplied with the relationship between assets and debt service found in (4.7). The simulation in Figure 4.13 shows how the government controller is able to set DS/GDP ratio to the recommended level of 0,122.

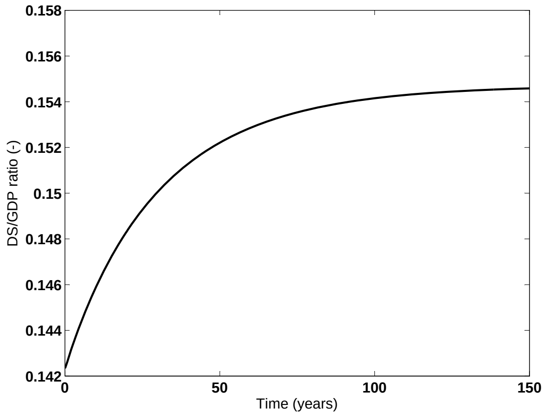

Figure 4.12: DS/GDP ratio for a Basel-type bank regime without controller.

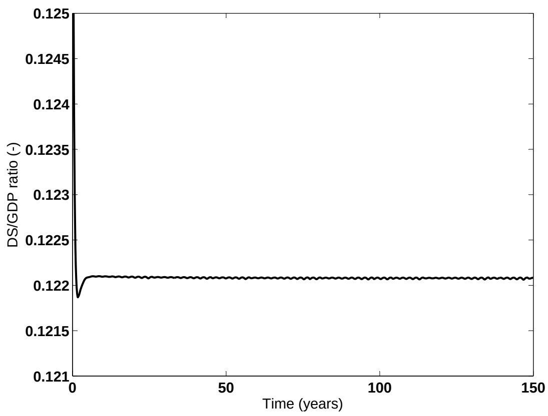

Figure 4.13: DS/GDP ratio for a Basel-type bank regime with government controller, $\theta = 0,246667$.

4.5.1 Simulations of a Basel-type bank regime with different values of θ

We will now look at how different values of θ affects the economy over a time span of 50 years. Values for θ is chosen to simulate the economy for different debt/GDP ratios around the reference debt/GDP ratio we found to be 60%. It is assumed that it is more important to look at how system reacts to a higher debt/GDP ratio of 60%, than a lower ratio. This is because the economies in Australia and the U.S have had a higher debt/GDP ratio than 60% during the recent years, which is believed to be the cause of todays financial crisis, as mentioned before. Values for θ from 0,01 to 0,30 are therefore chosen to simulate how these values thus debt/GDP ratios between 57% and 75%, affects the economy in terms of DS/GDP ratio (see Figure 4.16), GDP (in Figure 4.14), and to show how bank net lending to government net spending ratio is affected (Figure 4.17). The latter simulation is done to show how much the government needs to interfere in the economy to achieve the given relation between CB reserves and Bank assets, and thereby a given relation between debt and GDP as a result of the different values chosen for θ. The different values for θ, and the corresponding different debt/GDP and DS/GDP ratios can be found in Table 4.2.

θ	Debt/GDP	DS/GDP
0,01	0,753	0,153
0,05	0,723	0,147
0,10	0,686	0,140
0,15	0,654	0,133
0,20	0,625	0,127
0,246667	0,600	0,122
0,30	0,574	0,117

Table 4.2: Values for θ, debt/GDP and DS/GDP ratios for Simulink simulation of a Basel-type bank regime with government control

GDP

It can be seen from Figure 4.14 of GDP with different values of θ that a low value of θ creates more money into to the system through government deficit spending and bank lending, and thereby increasing GDP. To find this same result mathematically, we will now develop the GDP growth. To do this we start with the equation for the liability rate found in equation (4.15).

$$\dot{L}(t) = \gamma(t) + l - rA(t) - \beta(i_A A(t) - i_L L(t)) \tag{4.25}$$

The expression for $\gamma(t)$ is found from (4.8) and (4.10) to be

$$\gamma(t) = \dot{R}(t) = \theta \dot{A}(t) \tag{4.26}$$

and the expression for Bank net lending is found from (3.3) to be

$$l - rA = \dot{A}(t) + \lambda(t) \tag{4.27}$$

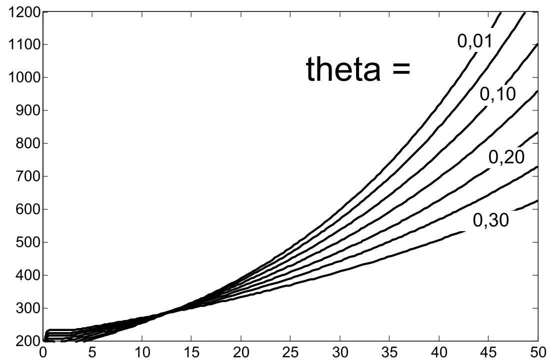

Figure 4.14: GDP as a function of different values for θ in a Basel-type bank regime with government controller, for different values of θ.

If expressions for Bank net lending (4.27) and Government net spending (4.26) is inserted in (4.25), the expression for liability rate becomes

$$\dot{L}(t) = \dot{A}(t)(\theta + 1) + \lambda A(t) - \beta(i_A A(t) - i_L L(t)) \tag{4.28}$$

To get an expression for asset rate, $\dot{L}(t)$ and $L(t)$ are changed with equation (4.12), and (4.28) is now

$$\dot{A}(t)(1 + \theta - \kappa) = \dot{A}(t)(\theta + 1) + \lambda A(t) - \beta(i_A A(t) - i_L A(t)(1 + \theta - \kappa)) \tag{4.29}$$

which is

$$-\kappa \dot{A}(t) = \lambda A(t) - \beta(i_A A(t) - i_L A(t)(1 + \theta - \kappa)) \tag{4.30}$$

By manipulation we find the growth rate in assets

$$\dot{A}(t) = gA(t) \tag{4.31}$$

with the expression for growth rate g

$$g = \frac{\beta(i_A - i_L(1 + \theta - \kappa)) - \lambda}{\kappa} \tag{4.32}$$

which is the same growth rate as for liabilities. To verify this equation, parameters from Table 4.1 are inserted into (4.32), which gives

$$g = 0,025 \tag{4.33}$$

and this can be verified by simulation, depicted in Figure 4.15. To continue develop GDP rate, growth rate g for liabilities is looked at

$$\dot{L}(t) = gL(t) \tag{4.34}$$

The relationship between GDP and $L(t)$ is found from (3.9) to be

$$L(t) = T \cdot GDP \tag{4.35}$$

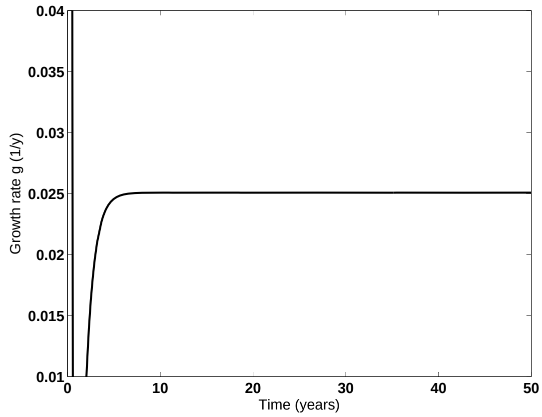

Figure 4.15: Growth rate g for a Basel-type bank regime with $\theta = 0,246667$.

and this is inserted into (4.34), and we get

$$\frac{d}{dt}(GDP) = g \cdot GDP \tag{4.36}$$

which means that growth rate g also holds for GDP growth rate. Eventually, Figure 4.14 is found to be mathematically correct with the use of g from (4.32). This is because θ is found in the numerator of g with a minus sign from i_L, meaning a higher value of θ will decrease GDP growth rate.

DS/GDP ratio

A low value of θ increases the DS/GDP ratio as can be seen in Figure 4.16. To verify this result mathematically, DS/GDP ratio is developed from equation

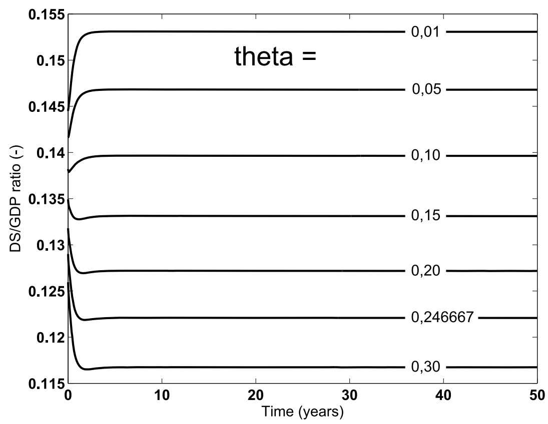

Figure 4.16: DS/GDP ratio as a function of θ in a Basel-type bank regime with government controller, for different values of θ, over a time period of 50 years.

(4.7) and (3.9) to be

$$\frac{DS}{GDP}(t) = \frac{(\frac{1}{T_r} + i_A)A(t)}{\frac{1}{T}L(t)} \tag{4.37}$$

With the help of (4.12), (4.37) becomes

$$\frac{DS}{GDP}(t) = \frac{T(\frac{1}{T_r} + i_A)}{1 + \theta - \kappa} \tag{4.38}$$

and we can now see that with θ in the denominator, DS/GDP will decrease with an increase in θ, like it does in Figure 4.16, hence the graph verifies our model.

Bank net lending to government net spending

Figure 4.17 shows that Bank net lending to government net spending ratio increases with a decrease in θ. It should also be mentioned that the graph in Figure 4.17 does not show this ratio for $\theta = 0,01$ because this value is above 100 which means that if included in the graph, it would be difficult to see how the other ratios are affected by different values for θ. The mathematical expression

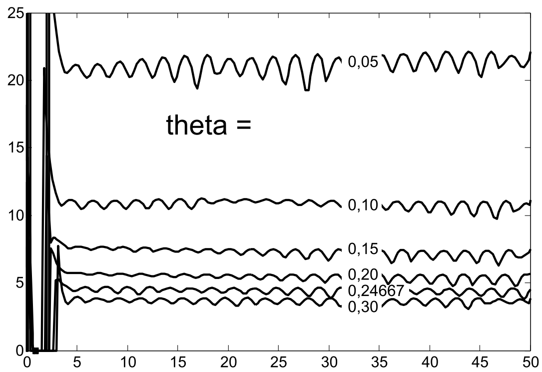

Figure 4.17: Bank net lending to government net spending in a Basel-type bank regime with government controller, for different values of θ, over a time period of 50 years.

for Bank net lending to government net spending is now found to conclude with why the Bank net lending to government net spending ratio increases with an increase in θ. Bank net lending to government net spending can be expressed by

$$\frac{l - rA}{\gamma} = \frac{\dot{A} + \lambda}{\theta \dot{A}} \tag{4.39}$$

found from (4.27), and (4.26), which is the same as

$$\frac{l - rA}{\gamma} = \frac{(g + \lambda)A}{\theta g A} = \frac{g + \lambda}{\theta g} \tag{4.40}$$

found from (4.31). With some manipulation and help from equation (4.32) describing the asset growth rate g, (4.40) becomes

$$\frac{l - rA}{\gamma} = \frac{1}{\theta}(1 + \frac{\lambda}{g}) \tag{4.41}$$

and eventually we have

$$\frac{l - rA}{\gamma} = \frac{1}{\theta}(1 + \frac{\kappa\lambda}{\beta(i_A - i_L(1 + \theta - \kappa)) - \lambda}) \tag{4.42}$$

Equation (4.42) now shows us the same relationship between θ and the Bank net lending to government net spending ratio as Figure 4.17 does, namely that the ratio increases with a decrease in θ because θ is found in the denominator of (4.42).

4.6 A 100% reserve system with government

In a 100% reserve system we have $\kappa = 1$, meaning total deposits of households and firms (=Bank liabilities $L(t)$) must mirror CB reserves 100% as discussed in the last chapter. The Simulink model look just as it does for the Basel-type bank regime (with $\kappa = 0,08$) in Figure 4.2. As we did for the Basel-type bank regime, we will first look at how the Debt/GDP and DS/GDP ratios develops without government control action over a 150 years period in Figure 4.18 and 4.19, respectively. Without government control we can see that, over a time

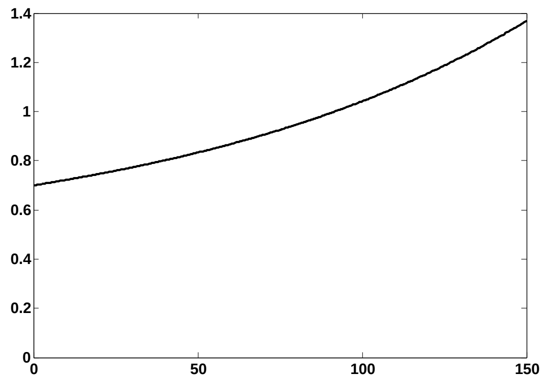

Figure 4.18: Debt/GDP ratio for a 100% reserve system without a government controller, over a time span of 150 years.

period of 150 years, debt/GDP and DS/GDP ratios grow exponentially, and are well above our preferred levels of 0,6 and 0,122, respectively. The initial value of debt/GDP ratio in Figure 4.10 is about 0,7, and reaches almost 1,4 after 150 years. A debt-crisis have most probably occurred before reaching a debt/GDP ratio of 1,4 which suggests the need for a government controller.

With government controller to set debt/GDP ratio to 0,6 and DS/GDP ratio to 0,122 which is assumed to be good values for a healthy economy as explained before, the confidence as discussed in Appendix A would be held stable. By simulation we now get a controlled ratio for debt/GDP in Figure 4.20, and controlled DS/GDP ratio in Figure 4.21.

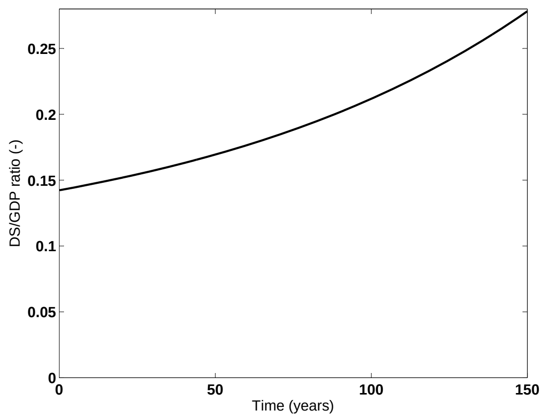

Figure 4.19: DS/GDP ratio for a 100% reserve system without a government controller, over a time span of 150 years.

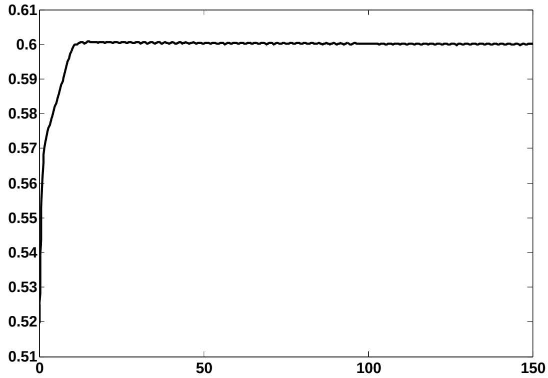

Figure 4.20: Debt/GDP ratio for a 100% reserve system with government control $\theta = 7/6 = 1,1667$, over a time span of 150 years.

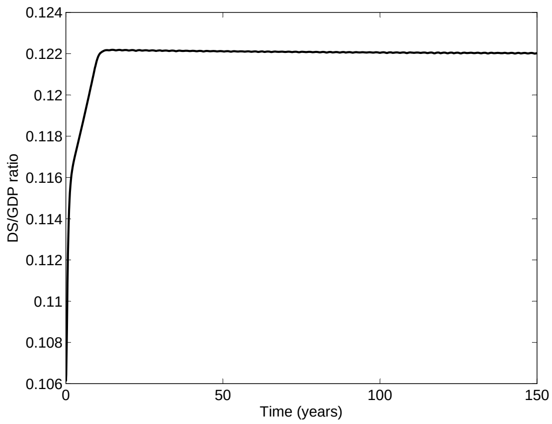

Figure 4.21: DS/GDP ratio for a 100% reserve system with government control $\theta = 7/6 = 1,1667$, over a time span of 150 years.

4.6.1 Simulations of a 100% reserve system for different values of θ

To make corresponding simulations for a 100% reserve system to the simulations done for the Basel-type bank regime, we need to find values for θ that corresponds to the same debt/GDP and DS/GDP ratios that was used for the Basel-type bank regime. These values for θ, and the corresponding controlled debt/GDP and DS/GDP ratios can be found in Table 4.3. The graphs from the

θ	Debt/GDP	DS/GDP
0,930	0,753	0,153
0,968	0,723	0,147
1,020	0,686	0,140
1,070	0,654	0,133
1,120	0,625	0,127
1,16667	0,600	0,122
1,220	0,574	0,117

Table 4.3: Values for θ, debt/GDP and DS/GDP ratios for Simulink simulation of a 100% reserve system with government control

simulations of this 100% reserve system, corresponding to the Basel-type bank regime, of GDP and DS/GDP ratios with government control with different values for θ can be found in Figure 4.22 and 4.24, respectively. It can be seen that these graphs bend after about 8 years. This is because $\kappa < 1$ until about 8 years have past thus Bank net lending have to be zero for this period according to the BIS rule, but the essential results can still be found. The result from simulation of Bank net lending to government net spending for different values of θ in this 100% reserve system can be found in Figure 4.27.

GDP

It is difficult to see from Figure 4.22, how GDP grows with different values of θ. The mathematical expression for this, from (4.36) and (4.32), with $\kappa = 1$, is

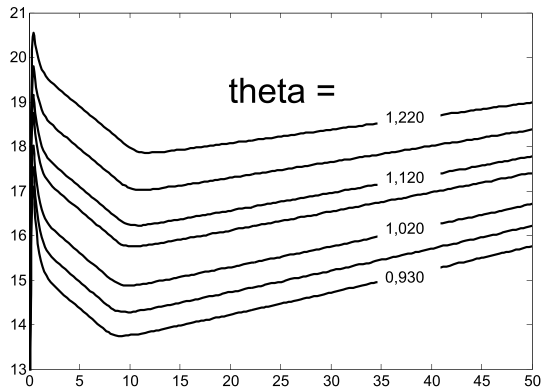

Figure 4.22: GDP in a 100% reserve system with government controller for different values of θ

found to be

$$\frac{d}{dt}(GDP) = g \cdot GDP = (\beta(i_A - i_L\theta) - \lambda) \cdot GDP \tag{4.43}$$

This means that GDP ([\$/y]) decreases with increased values for θ in a 100% reserve system. To see this better, initial values for assets A_{init}, and liabilities L_{init} are changed to fulfill the requirements of the Bank net lending controller with $\kappa = 1$, which is

$$R(t) = L(t) \tag{4.44}$$

and for the requirement from the government net spending controller

$$R(t) = \theta A(t) \tag{4.45}$$

With $R_{init} = 10$ [\$] the same as before, the new initial values for assets and liabilities are $A_{init} = R_{init}/\theta$ and $L_{init} = R_{init}$. These new initial values make the relationship between how GDP grow for differet values easier to find from the new graph in Figure 4.23. It is now easier to find the same relationship in

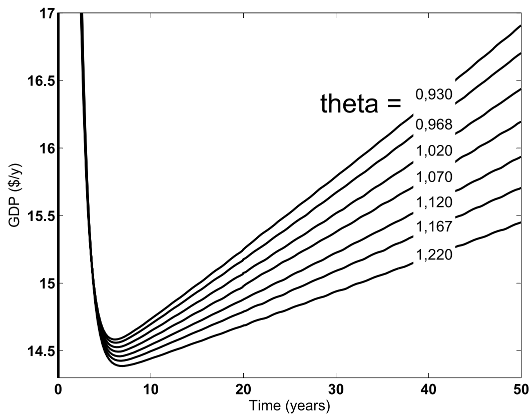

Figure 4.23: Relationship between θ and GDP for a 100% reserve system

Figure 4.23, as we find in the mathematical relationship from (4.43), namely that GDP decreases with an increase in θ.

DS/GDP ratio

The DS/GDP ratio increases with a decrease in θ as Figure 4.24 shows. The mathematical expression for this is found in equation (4.38), with $\kappa = 1$ to be

$$\frac{DS}{GDP}(t) = \frac{T(\frac{1}{T_r} + i_A)}{\theta} \tag{4.46}$$

This means DS/GDP ratio decreases with an increase in θ because θ is the denominator, which is essentially what Figure 4.24 shows.

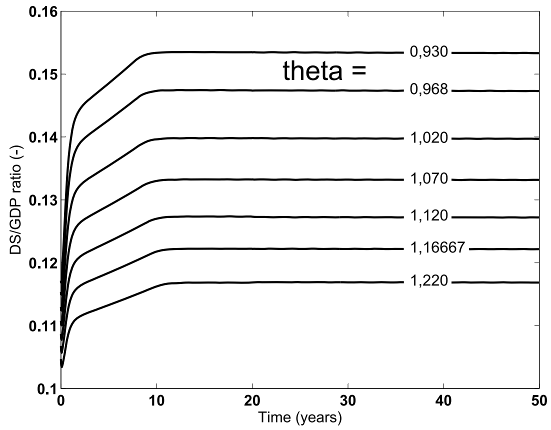

Figure 4.24: DS/GDP ratio for different values of θ in a 100% reserve system, over a time period of 50 years.

Bank net lending to government net spending

To simulate how much the Bank interferes by Bank net lending compared to how much the government interferes with our economic system in terms of government net spending, there was a need to pass these measurements through a low-pass filter to be able to read the essential results in a graph. The reason that the graph made from raw data is hard to read is due to large variations in the signals, making even larger fluctuatuations in the ratio between them, as can be seen in Figure 4.25. Figure 4.25 shows Bank net lending to government net spending, without the low-pass filter in action, making the essential result hard to read. The low-pass filter made to fix this problem is made in Simulink,

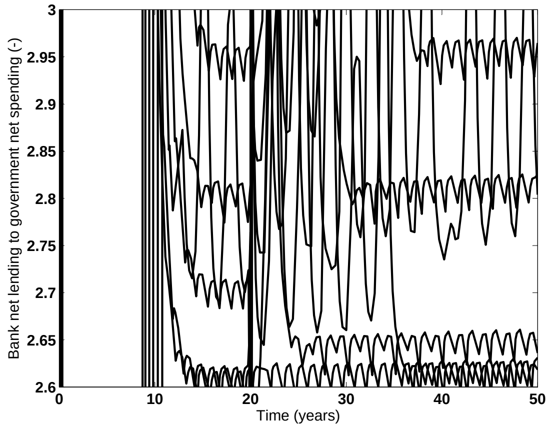

Figure 4.25: Unfiltered Bank net lending to government net spending for a 100% reserve system with different values for θ.

depicted in Figure 4.26. This filter is a simple one, and corresponds to the generic tank unit where the input to the filter, which is the raw measurements of either bank lending or government spending, is equivalent to an inflow to the

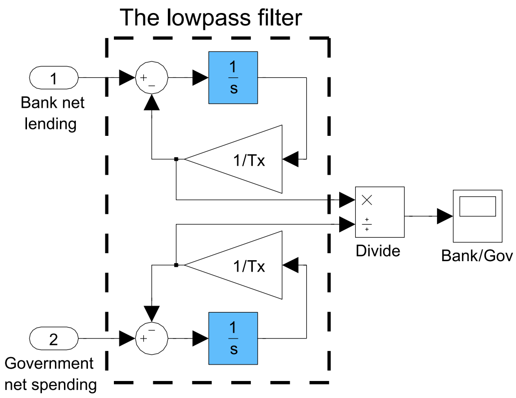

Figure 4.26: Simulink diagram for the low-pass filter made to make a better figure to interpret.

tank, and the output from the filter is equivalent to the outflow from the tank. The outflow of the tank, as described in the Foundation chapter is

$$F_o(t) = \frac{1}{T_x M(t)} \tag{4.47}$$

which has the transfer function

$$h_{io}(t) = \frac{1}{1 + T_x s} \tag{4.48}$$

where T_x is the time lag for our filter, chosen to be $T_x = 2$ years, which is quite large because it means it takes 2 years for the output to drop to 63% of the original value if the input is cut from some positive stable inflow to zero. However, it is assumed to be reasonable in terms of illuminating the essential relationship between Bank and government interference for different values of θ. The initial values for the integrators in Figure 4.26 are chosen to be 0. $M(t)$ in equation 4.47 for our filter is the intermediate volume of aggregate net lending from the bank, or the intermediate volume of aggregate net spending from the government. After measured Bank net lending and government net spending has passed through each low-pass filter, Bank net lending is divided by government net spending, as can be seen in the Simulink subsystem in Figure 4.26, to find how this ratio is affected by different values of θ. How the Bank net lending to government net spending reacts to different values of θ can be seen in Figure 4.27 with the filter in action to make the results more readable. To gain better understanding of our model from Figure 4.27, we need to keep in mind the quite large time lag introduced in the filter, namely $T_x = 2$ years. With this in mind, we can see that a higher value of θ gives a higher Bank net lending to government net spending, than a lower value of θ does.

The expression for Bank net lending to government net spending, found from (4.42), with $\kappa = 1$ is

$$\frac{l - rA}{\gamma} = \frac{1}{\theta}\left(1 + \frac{\lambda}{\beta(i_A - i_L\theta) - \lambda}\right) \tag{4.49}$$

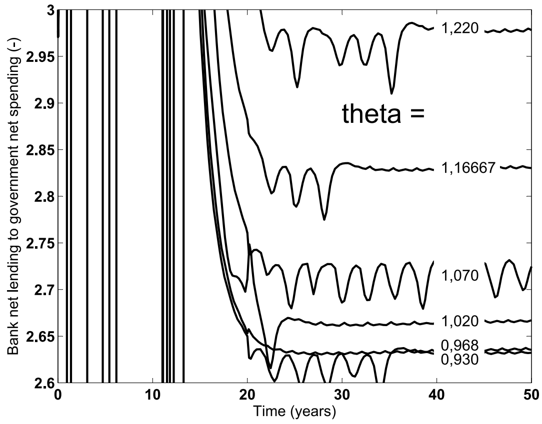

Figure 4.27: Bank net lending to government net spending ratio for a 100% reserve system.

This gives us the same relationship as Figure 4.27 does, namely that Bank net lending to government net spending increases with an increase in θ. It is not easy to see this from the equation, but if parameters from Table 4.1 are inserted into (4.49) with $\theta = 0.93$, we get the ratio to be 2,647, which is lower than for $\theta = 1,220$ which gives a ratio of 3,259.

Chapter 5

Model 4: Silvio Gesell's proposal to negative interest rates

In this chapter we will analyze Silvio Gesell's solution[25] to a good economic system in terms of his proposal of a negative interest rate on deposits to drive circulation of money by punishment on hoarding. This would drive the economy as a whole by increased trading and household and firm expenditure because hoarding money is punished by the negative deposit interest rate. To assess his proposal, a Simulink model, slightly different from the one we have before will be build. The model will then be simulated with Basel-regime bank system ($\kappa = 0,08$), and a 100% reserve system ($\kappa = 1,00$).

5.1 Brief, selected history of Silvio Gesell

Silvio Gesell was born in Belgium in 1862. After attending gymnasium, he started to work for the postal system of the German Empire, and then worked as a merchant under his brother in Berlin. After working for his brother some years in whereas places, he moved to Buenos Aires to open a branch of his brothers business. The depression in Argentina hurt the business, which caused him to reflect upon the structural problems caused by the money system. This lead to his first work on this topic, a book that came out in 1891, where the translated title from German to English is *The reformation of the monetary system as a bridge to a just state*, and later he published *Nervus Rerum* and *The nationalization of money*, and then his most familiar book, *The Natural Economic Order*[25].

Celebrated by his work

Although Silvio Gesells proposal of a better economic system could be said to be cheered mostly by the "fringe", his ideas are also celebrated by other well-known economists. Among others, economist John Maynard Keynes, the author of *General Theory of Employment, Interest and Money*[17], celebrated Gesell's thoughts by writing in his book: "I believe that the future will learn more from the spirit of Gesell than from that of Marx."(p. 355), and "The idea behind Gesell's stamped money is sound."(p. 357). In John Maynard Keynes' book[17], he describes the political and economic views in Gesells book[25]: "The purpose of the book may be described as the establishment of an anti-Marxian socialism, a reaction against laissez-faire built on theoretical foundations totally unlike those of Marx in being based on an unfettering of competition instead of its abolition"(p. 355)

5.2 Idea

Silvio Gesell sets up three criterions of "good money" in his famous book *The Natural Economic Order*[25]. This is how he puts it:

> "The criterion of good money, of an efficient instrument of exchange, is:
>
> 1. That it shall secure the exchange of goods - which we shall judge by the absence of trade depressions, crises and unemployment.
> 2. That it shall accelerate exchange - which we shall judge by the lessening stocks of wares, the decreasing number of merchants and shops, and the correspondingly fuller storerooms of the consumers.
> 3. That it shall cheapen exchange - which we shall judge by the small difference between the price obtained by the producer and the price paid by the consumer. (Among producers we here include all those engaged in the transport of goods)."

Gesell focuses on the strange property of money when it is seen as a commodity. Wereas other commodities gets devalued by time because of i.e. wear and tear of clothing, corrosive attacks on cars or bicycles, or the decomposing of almost anything we eat, money (when deposited in a bank, that is,) actually increases in quantity. Growing money deposited in banks makes the rich even richer, merely because they own a lot of money and not because they work to have them there. It is therefore thought to be better if every economic unit able to produce goods and services is encouraged to do so by inducing a negative interest rate on his or her deposits.

5.3 Is Gesells system possible to implement?

One can argue that if there would exist a negative interest rate on currency, people, or any economic unit, would i.e. store their money somewhere else than in a bank, to prevent the decreasing of their money stock induced by a negative interest rate. As explained by Willem H. Buiter in his paper "Overcoming the Zero Bound on Nominal Interest Rates: Gesell's Currency Carry Tax vd. Eisler's Parallel Virtual Currency":

> "The reason it is difficult to pay interest, positive or negative, on currency is that currency is a negotiable bearer bond. Its holder is anonymous: his identity is not known to the issuer - the central bank."

[9] Buiter [9] also argues that the authorities could announce an expiring date for currency, but in that case, there must exist a credible penalty for possession of such expired currency. One such penalty has been suggested by Mankiw:

> "At one of my recent Harvard seminars, a graduate student proposed a clever scheme to do exactly that. ... Imagine that the Fed were to announce that, a year from today, it would pick a digit from zero to 9 out of a hat. All currency with a serial number ending in that digit would no longer be legal tender. Suddenly, the expected return to holding currency would become negative 10 percent. That move would free the Fed to cut interest rates below zero. People would be delighted to lend money at negative 3 percent, since losing 3 percent is better than losing 10."[18]

Mankiw argues further, that this would make people spend money instead of hoarding, which is exactly the point of introducing a negative interest rate on currency and deposits. Other possibilities for people to hoard could be to buy precious metals i.e. gold and platinum, but it is assumed here that no economic unit can hide from the imposed negative interest rates. It is also in the author's opinion, possible that people could choose to have money in the bank for security reasons, and for convenience to have an account to collect income and spend from in term of i.e. a Visa debit card. In the future, one could also imagine a system that is totally digitalized; meaning tax on digital currency could easily be taxed by authorities as it would be easy to tax automatically with such technology, and because all money would have an owner. Physical hoarding of currency or cash to escape the negative interest rate would thus be impossible.

5.4 Building the model

Our new Gesell model in Figure 5.1 is similar to the model we have from before (Figure 4.2) in terms of Bank and government controllers, but because the interest rate on loans is now assumed to be zero, we have no longer a direct influence from the asset subsystem to the liability subsystem. The new equation for Bank liabilities rate, earlier described in equation (4.15), now becomes

$$\dot{L}(t) = l - rA(t) - \beta i_L L(t) + \gamma(t) \tag{5.1}$$

It is important to notice that the negative sign on Bank deposits rate, as proposed by Gesell and implemented here, is moved outside the parameter i_L to make this influence from a negative sign in our model clearer. This means that that $i_L > 0$ in our Matlab script and list of parameter values in Table 5.1 still describes a negative Bank deposit interest rate, because the negative sign is just moved outside the parameter in the Simulink model, and also in our equations. Net bank income surplus is still paid by the deposits from households and firms as it was for the systems with "normal" interest rates. I our system this is done with the term $-\beta i_L L(t)$ from liability change in equation (5.1). The liabilities subsystem in Simulink, with negative Bank deposit interest rates, corresponding to equation 5.1, becomes as depicted in Figure 5.2. The differential equation for asset change is still the same as before, namely

$$\dot{A}(t) = l - rA(t) - \lambda A(t) \tag{5.2}$$

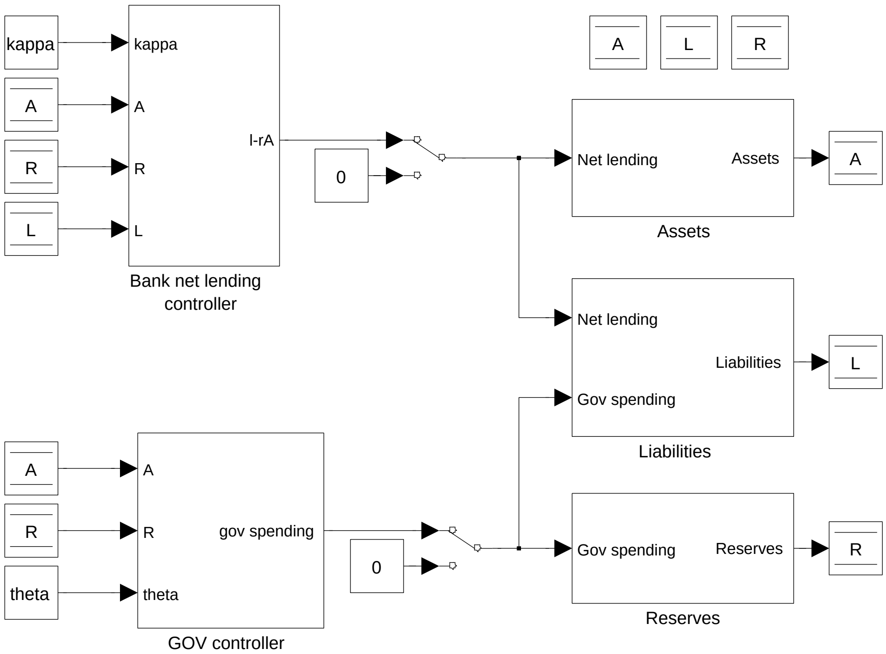

Figure 5.1: The new upper level Simulink macro economic Gesell model.

Parameter	Value	Denomination
A_{init}	0,000	[$]
L_{init}	0,000	[$]
R_{init}	10,000	[$]
i_A	0,000	[-/y]
i_L	0,040	[-/y]
i_R	0,000	[-/y]
κ	0,080	[-]
θ	0,24667	[-]
β	0,200	[-]
λ	0,005	[-/y]
T	0,700	[y]
T_r	7,500	[y]
$K_{p,bank}$	10,000	[-]
$T_{i,bank}$	1,000	[-]
$K_{p,gov}$	5,000	[-]
$K_{i,gov}$	0,050	[-]
$K_{d,gov}$	1,000	[-]

Table 5.1: Parameters for Simulink simulation of a Gesell system.

(5.2) is now manipulated to find an expression for Bank net lending

$$l - rA(t) = \dot{A}(t) + \lambda A(t) \tag{5.3}$$

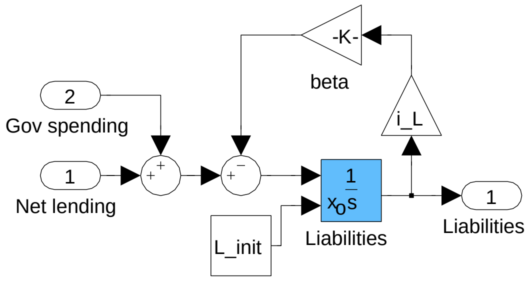

Figure 5.2: Simulink subsystem of liabilities for a Gesell system with zero borrowing rate, and negative interests on deposits.

(5.3) is inserted into (5.1) and we get

$$\dot{L}(t) = \dot{A}(t) + \lambda A(t) - \beta i_L L(t) + \gamma(t) \tag{5.4}$$

To derive an expression for asset growth rate, expressions for liabilities (5.5), and liabilities differentiated (5.6),

$$L(t) = R(t) + (1 - \kappa)A(t) \tag{5.5}$$

$$\dot{L}(t) = \dot{R}(t) + (1 - \kappa)\dot{A}(t) \tag{5.6}$$

are found from the Basel rule (3.1), and inserted into equation (5.4)

$$\dot{R}(t) + (1 - \kappa)\dot{A}(t)\dot{A(t)} + \lambda A(t) - \beta i_L(R(t) + (1 - \kappa)A(t)) + \gamma(t) \tag{5.7}$$

With the general expression for Central Bank reserves (4.9) inserted into (5.7), $\gamma(t)$ cancels out on both sides, and we finally get a general expression for asset growth rate

$$\dot{A}(t) = \frac{\beta i_L(1 - \kappa) - \lambda}{\kappa} A(t) + \frac{\beta i_L + i_R}{\kappa} R(t) \tag{5.8}$$

If we simulate our model without government interaction and $i_R = 0$, over a long time (=150 years), then reseves are not increased, but assets are, so we get $R(t) << A(t)$. Mathematically, this means

$$\lim_{t\to\infty} \dot{A}(t) = gA(t), \text{ where } g = \frac{\dot{A}(t)}{A(t)} = \frac{\beta i_L(1 - \kappa) - \lambda}{\kappa} \tag{5.9}$$

If we insert parameter values in (5.9) from Table 5.1 we get

$$g = 0,0295 \tag{5.10}$$

This result can also be found by simulation over 150 years, as can be seen from the graph in Figure 5.3. This graph veryfies our assumptions which lead to (5.9), and finally (5.10).

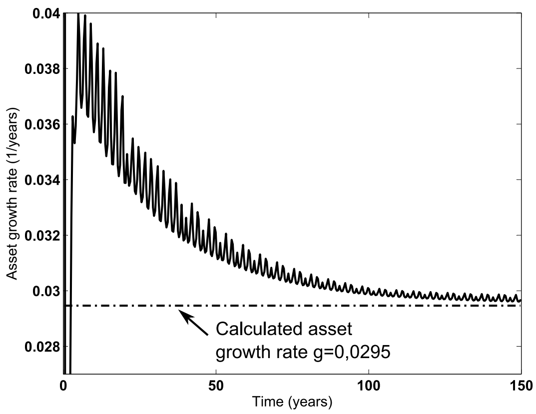

Figure 5.3: Asset growth rate develops accordingly with equation (5.9), and approaches $g = 0,0295$, as foreseen in equation (5.10).

Government enforces $R(t) = \theta A(t)$

With a government enforcing equation (4.8), namely $R(t) = \theta A(t)$, the new equation for asset growth rate can now be derived by substituting $R(t)$ in (5.8) with equation (4.8). If we do this, we get

$$\dot{A}(t) = \frac{\beta i_L(1+\theta-\kappa) + i_R\theta - \lambda}{\kappa} A(t) \tag{5.11}$$

If we use parameter values from Table 5.1, the parameter value for θ can now be found by assuming a sound value of debt/GDP ratio should be 60%, as suggested before. From equation (4.18) θ is calculated

$$\theta = \frac{T}{\frac{Debt}{GDP}_{ref}} + \kappa - 1 = 0,246667 \tag{5.12}$$

From equation (5.11) we have an asset growth rate g

$$g = \frac{\beta i_L(1+\theta-\kappa) + i_R\theta - \lambda}{\kappa} \tag{5.13}$$

With parameter values from Table 5.1, g is calculated and found to be

$$g = 0,0541667 \tag{5.14}$$

When we simulate our model with these settings, we find the same result as depicted in Figure 5.4, and our assumptions and analysis is thereby verified. With or without government control, the internal structure of Bank assets in our Simulink model however, is still the same as in Figure 4.3 and equation (3.3).

5.5 Simulation

As done for the former macro economic systems, we will now see how different values for θ affects our model. There will be two kinds of assessments of the

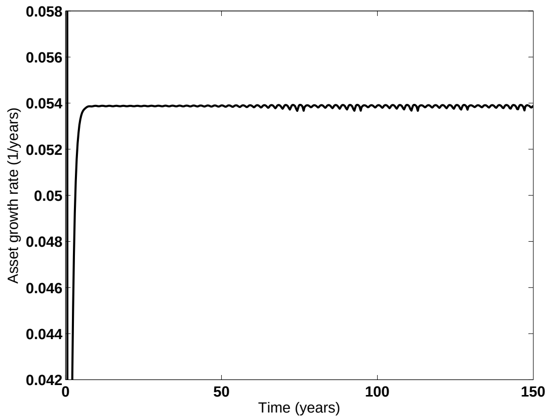

Figure 5.4: Asset growth rate g, according to equation (5.13), in a Gesell system with government control action to ensure (4.8), and with Bank capital/assets ratio $\kappa = 0,08$.

Gesell system, with all parameters held the same as in the former chapter, except from the interest rates $i_A = 0$, and $i_L = 0,04$ negative, as explained before. The first simulation will be of the Gesell interest rates induced on a Basel-type bank regime, which we call the Gesell-Basel-type bank regime, with values for θ the same as in the last chapter, from Table 4.2. The second simulation will be done with the Gesell interest rates imposed on a 100% reserve system, from now on referred to as the Gesell-100% reserve system, with values for θ the same as in the last chapter, from Table 4.3.

5.5.1 A Gesell-Basel-type bank regime

The simulations done for the Gesell-Basel-type bank system are for GDP in Figure 5.5, the DS/GDP ratio in Figure 5.6, and for the bank net lending to government net spending ratio in Figure 5.7.

GDP

We can see from Figure 5.5 that the higher the value of θ is, the more GDP increases, and it does so exponentially by the growth rate g as for assets in equation (5.13), divided by T.

$$\frac{d}{dt}(GDP) = \frac{\beta i_L(1+\theta-\kappa) + i_R\theta - \lambda}{\kappa} \cdot GDP \tag{5.15}$$

This is as expected since θ appears in the numerator in (5.15) as a positive value, and with only positive parameters in which could change its sign and effect, such as κ, β and i_A. It should be stressed once again, that i_A contains a positive value in this Gesell system, because the sign is moved outside to make the effect of a negative interest rate clearer in our equations and Simulink model.

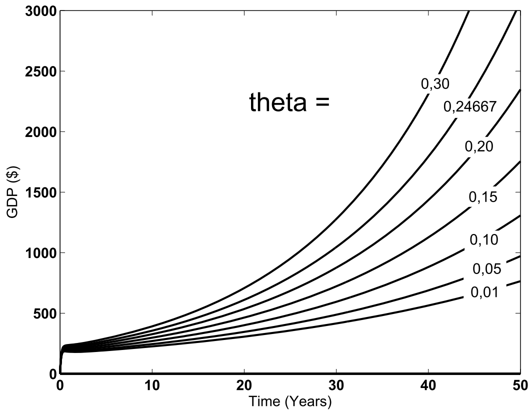

Figure 5.5: GDP for different values of θ in a Gesell-Basel-type bank regime, over a time period of 50 years.

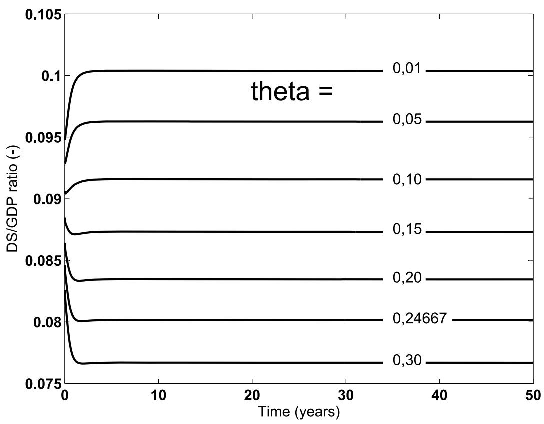

Figure 5.6: DS/GDP ratio for different values if θ in a Gesell-Basel-type bank regime, over a time period of 50 years.

DS/GDP ratio

The graph for DS/GDP ratio in Figure 5.6 shows a higher value of θ gives a lower DS/GDP ratio, meaning a lower debt burden on households and firms. The equation for DS/GDP ratio found from equation (4.7) and (3.9), when Bank net lending controller and government net spending controllers are assumed to be perfect, is

$$\frac{DS}{GDP} = \frac{T(\frac{1}{T_r} + i_A)}{1 + \theta - \kappa} \tag{5.16}$$

With Gesell interest rate on deposits $i_A = 0$, (5.16) becomes

$$\frac{DS}{GDP} = \frac{T/T_r}{1 + \theta - \kappa} \tag{5.17}$$

The essential result from Figure 5.6, that DS/GDP ratio decreases with an increase in θ is now easier to understand with θ found in the denominator of equation (5.17). It should be noticed for later discussion that this DS/GDP ratio is less than the DS/GDP ratio in a normal Basel-type regime system for the same values of θ, because of the Gesell interest rate $i_A = 0$ different from the normal Basel-type system with a positive interest rate of 7%.

Bank net lending to government net spending ratio

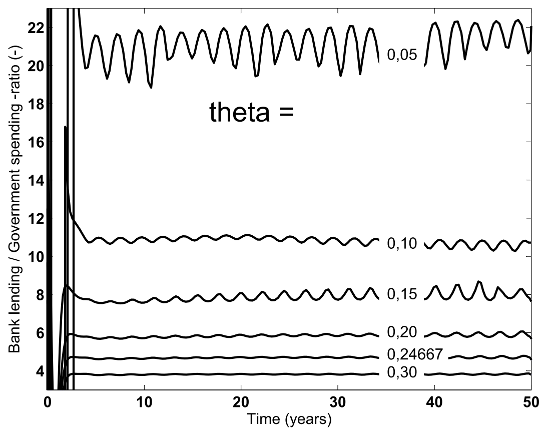

Figure 5.7: Bank net lending to government net spending ratio for a Gesell-Basel-type bank regime - one missing!!

For simulation of Bank net lending to government net spending ratio, we can see from Figure 5.7 that a lower value of θ corresponds to a hihger Bank net lending to government net spending, than a lower value of θ gives. The equation for this ratio is developed in Appendix B, and found to be

$$\frac{l - rA}{\gamma} = \frac{1}{\theta}(1 + \frac{\lambda}{g}) \tag{5.18}$$

which also suggests the same as Figure 5.7 shows, since θ appears in the denominator, namely that Bank net lending to government net spending increases with a decreased value of θ. It must be recognised that the growth rate g in equation (5.19) also varies with θ, but the effect from this is actually a further increase in the ratio due to a decreased θ. The full effect from θ in the Bank net lending to government net spending can be seen by inserting the expression for g found in (5.13), with $i_R = 0$, into (5.18)

$$\frac{l - rA}{\gamma} = \frac{1}{\theta}(1 + \frac{\kappa\lambda}{\beta i_L(1 + \theta - \kappa) - \lambda}) \tag{5.19}$$

It must be recognised that the simulation with $\theta = 0,01$ is not depicted in Figure 5.7. This is because it resulted in a Bank net lending to Government net spending ratio of right above 100 (109,23), and this would make the other simulation results in Figure 5.7 difficult to compare.

5.5.2 A 100% reserve Gesell system

As done for the Gsell-Basel-type bank regime, we wil simulate our model for different values of θ found in Table 4.3, and briefly discuss the results depicted in Figure 5.8 for GDP, Figure 5.9 for DS/GDP ratio, and Figure 5.10 for Bank net lending to government net spending ratio.

It can be seen in these graphs that there is a bend after around 8 years as it was for the normal interest rate 100% reserve system. This is because of the same reason, namely that during this period between zero to eight years, the Bank is not allowed to lend out money according to the BIS rule for $\kappa = 1$. This does not have any effect our essential results derived from these simulations however.

GDP

GDP for different values of θ is depicted in Figure 5.8. The essential information found here is the same as for the Gesell-Basel-type bank regime, namely that GDP increases with higher values of θ. This can also be explained by the growth rate g from (5.13)which holds for assets and liabilities, and is therefore also proportional with GDP with the relation

$$\frac{d}{dt}(GDP) = g \cdot GDP) \tag{5.20}$$

With $i_R = 0$ and $\kappa = 1$ in (5.13), we get

$$g = \beta i_L \theta - \lambda \tag{5.21}$$

From (5.21) it can be seen that GDP grows more with higher θ, which corresponds to the Figure 5.8.

DS/GDP ratio

Figure 5.9 shows how DS/GDP ratio settles over time for the Gsesell-100% reserve system, and it can be seen that lower values of θ leads to higher DS/GDP ratio. This is as expected from (5.17), which also holds for this Gesell-100% reserve system, because θ appears in the denominator, meaning an increase in θ

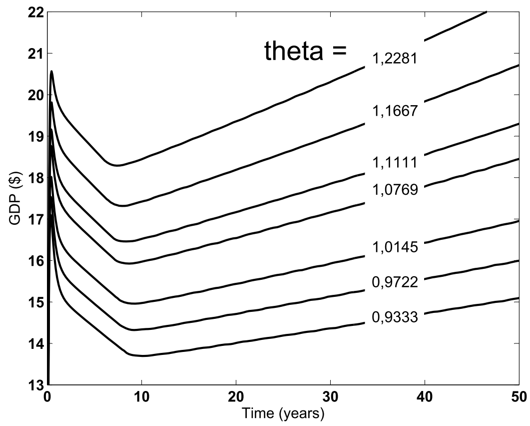

Figure 5.8: GDP for different values of θ in a Gesell-100% reserve system, over a time period of 50 years.

leads to a decrease in DS/GDP ratio. With $\kappa = 1$, as it is for this Gesell-100% reserve system simulation, (5.16) becomes

$$\frac{DS}{GDP} = \frac{T}{\theta T_r} \tag{5.22}$$

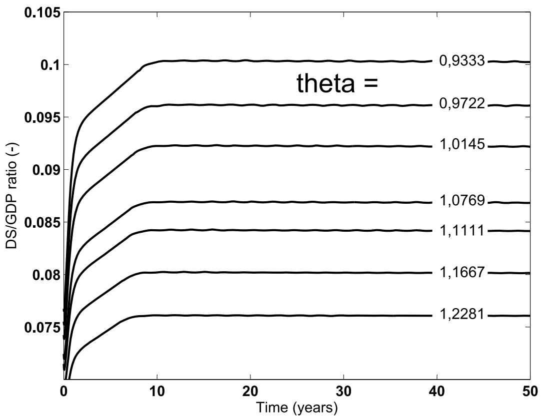

Figure 5.9: DS/GDP ratio for differnet values of θ in a Gesell-100% reserve system, over a time period of 50 years.

Bank net lending to government net spending ratio

Bank net lending to government net spending ratio can be calculated from equation

$$\frac{l - rA}{\gamma} = \frac{1}{\theta}(1 + \frac{\lambda}{g}) \tag{5.23}$$

from Appendix B. With the inclusion of g from equation (5.13), $i_R = 0$, and $\kappa = 1$, equation (5.23) becomes

$$\frac{l - rA}{\gamma} = \frac{1}{\theta}(1 + \frac{\lambda}{\beta i_A \theta - \lambda}) \tag{5.24}$$

which can be used to verify our results from simulation of Bank net lending to

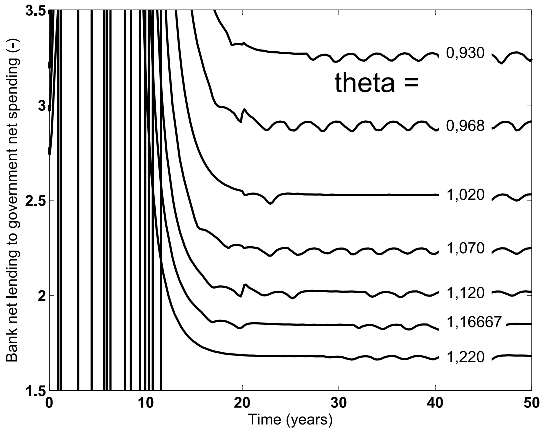

Figure 5.10: Bank net lending to government net spending 100 ges

government net spending for different values of θ, as depicted in Figure 5.10. To achieve these results, the measurements of government net spending and Bank net lending are sent through a filter, as can be seen in Figure 4.26, with initial integrator conditions of zero, and $T_x = 2$, before the ratio is found and presented in Figure 5.10. This is done, as explained before, to make it easier to see the essential relationship between different values of θ, and the Bank net lending to government net spending ratio. Without filtered measurements, Bank net lending to government net spending ratio looks like in Figure 5.11.

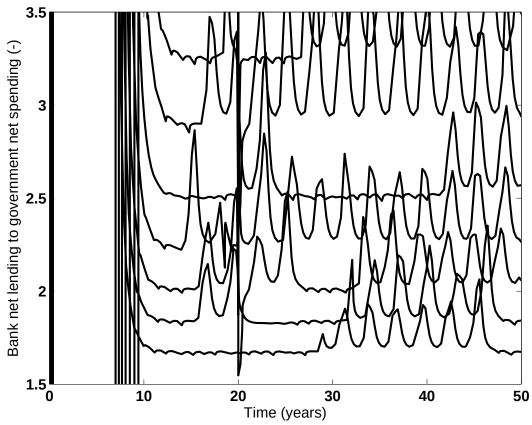

Figure 5.11: Bank net lending to government net spending 100 ges

Chapter 6

Discussion

How GDP, DS/GDP ratio, and Bank net lending to government net spending evolves with time with government control action for different macro economic systems have been found from simulations and equation development. The four macroeconomic systems these reseults are produced from are the Basel-type bank regime, the 100% reserve system, and these same two same systems, but with negative interest rate on deposits introduced, as proposed by Silvio Gesell in his book “The Natural Economic Order”(1920)[25].

Different values for θ found in Table 4.2 with government control action have been used for simulation of the Basel-type bank regime, and the 100% reserve system is simulated with different values of θ found in Table 4.3. The different values for θ was chosed to find how all the four macroeconomic systems evolved with the same debt/GDP ratios, thus the values for θ needed to be different to achieve this. Dynamics for how confidence affects the economy is not included in our model because to develop this is thought by the author to be highly speculative, and it would only serve to show how the economy would go into crisis from time to time with poor, or no, government control. As done by Fitje(2008)[13], confidence is included and it is assumed there exists a neutral confidence with a certain level of debt/GDP. If a government can enforce this certain debt/GDP ratio which gives neutral confidence, the effect from confidence could be ignored and the resulting system would be like the one made in our model. Our government controller is assumed to be able to keep such a neutral confidence at all times by keeoing a certain debt/GDP ratio. This means the effect from confidence can be ignored for our model. To read more about confidence dynamics, and the effect from a varying confidence, see Appendix A.

6.1 Results produced with “normal” interest rates

In chapter 4, the Basel-type bank regime and the 100% reserve system are simulated and equations for the dynamics in these systems are developed with what is assumed to be “normal” interest rates, namely i_A=7% and i_L=3%. Expressions and simulations for GDP, DS/GDP ratio and Bank net lending to government net spending ratio are made, which will now be discussed.

6.1.1 A normal Basel-type bank regime

This Basel-type bank regime is thought to normal in terms of normal interest rates as explained above. The essential results will now be presented very short, before a discussion of the results are made. The expression for GDP growth from equation (4.36) and (4.32) is

$$\frac{d}{dt}(GDP) = \frac{\beta(i_A - i_L(1 + \theta - \kappa)) - \lambda}{\kappa} \cdot GDP \tag{6.1}$$

which means GDP growth decreases with an increase in θ as can be seen in Figure 4.14. The equation for DS/GDP ratio is found in (4.38) to be

$$\frac{DS}{GDP}(t) = \frac{T(\frac{1}{T_r + i_A})}{1 + \theta - \kappa} \tag{6.2}$$

This means DS/GDP ratio is in inverse ratio with θ, thus a higher θ results in a lower DS/GDP ratio as depicted in Figure 4.16. The last result from this model is Bank net lending to government net spending ratio, where the expression for this, found form (4.42), is

$$\frac{l - rA}{\gamma} = \frac{1}{\theta}(1 + \frac{\kappa\lambda}{\beta(i_A - i_L(1 + \theta - \kappa)) - \lambda}) \tag{6.3}$$

which leads to an increase in the ratio with a decrease in θ as depicted in Figure 4.17, because θ is found in the denominator in (6.3).

These results means essentially that in order to increase economic output GDP, as authorities might want to do, yhe DS/GDP ratio must be increased, which could lead to a debt-crisis, and money creation would mostly be done through Bank net lending, with only a small fraction of government nest spending.

6.1.2 A normal 100% reserve system

This 100% reserve system is thought to be normal in terms of normal interest rates as explained above. A short presentation of the most essential results from the simulation and expressions of this model is presented here. GDP is the first of these results for a normal 100% reserve system, and the expression for this, found from (4.43), is

$$\frac{d}{dt}(GDP) = (\beta(i_A - i_L\theta) - \lambda) \cdot GDP \tag{6.4}$$

which means GDP growth increases with a decrease in θ, as can be seen in Figure 4.23, and from (4.43) because θ is found in the numerator with a negative sign. Secondly, the DS/GDP ratio, found from (4.46) is

$$\frac{DS}{GDP}(t) = \frac{T(\frac{1}{T_r + i_A})}{\theta} \tag{6.5}$$

which gives us the same result as Figure 4.24, namely that DS/GDP ratio decreases with a higher level of θ because θ is found as the denominator. At last, Bank net lending to government net spending can be seen in Figure 4.27 to be high for high values of θ. This result can also be found from equation (4.49),

$$\frac{l - rA}{\gamma}(t) = \frac{1}{\theta}(1 + \frac{\lambda}{\beta(i_A - i_L\theta) - lambda}) \tag{6.6}$$

with tests from different values of θ but it is not so easy to see.

These results show that if there is a desire to increase GDP (thus decreasing θ), this also means that DS/GDP ratio needs to be increased which probably would lead to a debt crisis if the debt/GDP ratio grows well above 60%. Further more, and increase in GDP would also lead to a decreased Bank net lending to government net spending ratio, thus the fraction of new money created would come from government net deficit spending would increase compared to money created from Bank net lending.

6.2 Results produced with Gesell's proposal of negative interest rates

In chapter 5 results for GDP, DS/GDP ratio and Bank net lending to government net spending are produced in terms of equations and graphs. These results are achieved from Gesell negative interest rates introduced in both the Basel-type bank regime and the 100% reserve system. The new interest rates introduced in these systems are zero interest rate on debt $i_A = 0$, and minus 4% interest rate on deposits $i_L = 0,04$. We will first look at the results produced by a Basel-type bank regime with Gesell interest rates, from no on called the Gesell-Basel-type bank regime, and then look at results found from the 100% reserve system, from now on called the Gesell-100% reserve system.

6.2.1 A Gesell-Basel-type bank regime

GDP for a Gesell-Basel-type bank regime grows as depicted in Figure 5.5 with increased GDP growth for increased values og θ. The same result is found from (5.15)

$$\frac{d}{dt}(GDP) = \frac{\beta i_L(1 + \theta - \kappa) - \lambda}{\kappa} \cdot GDP \tag{6.7}$$

because of the positive effect of θ in the numerator. The DS/GDP ratio for this system for different values of θ can be expressed by

$$\frac{DS}{GDP} = \frac{T(\frac{1}{T_r} + i_A)}{1 + \theta - \kappa} \tag{6.8}$$

Fond from (5.17). Equation (6.8), explains how the DS/GDP ratio decreases with an increase in θ, because θ is found with a positive effect in the denominator. This result is confirmed with the graph in Figure 5.6. The last result for this system is the Bank net lending to government net spending ratio, which can be expressed by equation (5.19) to be

$$\frac{l - rA}{\gamma} = \frac{1}{\theta}(1 + \frac{\kappa\lambda}{\beta i_L(1 + \theta - \kappa) - \lambda}) \tag{6.9}$$

From this equation, with corresponding graph in Figure 5.7, we can conclude that Bank net lending to government net spending ratio decreases with an increase in θ.

From these results, we find that if authorities want to increase economic output GDP, then θ would need to be increased, thus leading to a lower DS/GDP ratio, and thereby moving away from the threat of debt crisis. Furthermore, for an increase in GDP, the fraction of total money creation created by government would increase.

6.2.2 A Gesell-100% reserve system

The expression of GDP for a 100% reserve system can be found in (5.21) and (5.20) to be

$$\frac{d}{dt}(GDP) = (\beta i_L \theta - \lambda) \cdot GDP \tag{6.10}$$

This shows that the higher the value of θ is, the higher the GDP growth is, which can be confirmed by Figure 5.8. The second result of DS/GDP ratio is expressed as

$$\frac{DS}{GDP} = \frac{T}{\theta T_r} \tag{6.11}$$

found from equation (5.22). This equation shows that the DS/GDP ratio is decreased with an increase in θ, as θ can be found in the denominator, which also can be seen in Figure 5.8. At last, Bank net lending to government net spending ratio is found to decrease with an increase in θ, which can be explained by equation

$$\frac{l - rA}{\gamma} = \frac{1}{\theta}(1 + \frac{\lambda}{\beta i_A \theta - \lambda}) \tag{6.12}$$

found from (5.24), because θ can be found in the denominator. This result can also be confirmed by Figure 5.10.

If authorities should want to increase GDP, that is an increase in θ, this would lead to a lower DS/GPP ratio, meaning the system would be more robust against a debt-crisis, and to enforce such an increase in GDP, government contribution to new money creation, would be higher than for a lower GDP, thus a lower θ.

6.3 Discussion

To prepare for the conclusion in the next chapter, the main results are presented in this chapter, followed by this discussion.

With normal interest rates, we have the same relationship between θ, GDP and the DS/GDP ratio for the Basel-type bank regime and the 100% reserve system, namely that to increase GDP, θ needs to be decreased, which leads to a higher DS/GDP ratio that could cause debt-crisis if the debt/GDP ratio is increased well above our "recommended" ratio of 60%, as discussed before. The Bank net lending to government net spending ratio however, reacts the opposite way to θ in the Basel-type bank regime compared to the 100% reserve system. This is because for an increase in θ, the Bank net lending to government net spending ratio also increases for the Basel type bank regime, but decreases with increse in θ for the 100% reserve system.

For the Gesell interest rates, the same essential relationship between GDP, θ, DS/GDP, and the Bank net lending to government net spending ratio exists for both the Gesell-Basel-type bank regime and the 100% reserve system. This means that, for both Gesell systems, an increase in θ leads to an increase in GDP, a decrease in the DS/GDP ratio, and a decrease in the Bank net lending to government net spending ratio. This means an increase in GDP for both systems leads to a decrease in the DS/GDP ratio which makes the Gesell systems secure against debt-crisis for firms and households.

The Essential dynamics for the four systems is shown in Table 6.1. The arrows mean an increase if pointing upwards, and decrease if pointing downwards. Each cell with an arrow in one row, meaning an increase or decrease, corresponds to the other cells in the same row. If one arrow is turned around to point the other way, every other arrow in the same row needs to be turned, and this would correspond to the opposite scenario, i.e. if one wants to see what happens when GDP is set to decrease instead of increase in a normal Basel-type bank regime, one can turn every arrow in this row, which will correspond to the opposite scenario, namely that with a decrease in GDP, this leads to an increased θ, decreased DS/GDP ratio, and a decreased Bank net lending to government net spending ratio. From Appendix A explaining confidence dynamics, the effect from DS/GDP ratio is thought to be in inverse ratio with confidence, thus a growth in DS/GDP ratio leads to decrease in confidence as argued by Fitje(2008)[13]. Furtermore it is argued that a rise in confidence will decrease circulation inertia T thus higher liquidity and GDP because people would hoard less and spend more. If we add the essential change in confidence from the change in DS/GDP ratio in Table 6.1, and also add the effect on liquidity, Table 6.1 becomes: A table of stationary values for GDP growth rate g, θ, DS/GDP ratio, and Bank net lending to government net spending is also made for each system with respective values of θ corresponding to a neutral con-

System	GDP	θ	$\frac{DS}{GDP}$	$\frac{Bank}{Gov}$
Normal Basel:	↑	↓	↑	↑
Normal 100%:	↑	↓	↑	↓
Gesell Basel:	↑	↑	↓	↓
Gesell 100%:	↑	↑	↓	↓

Table 6.1: The essential dynamics for the four different systems.

System	GDP	θ	$\frac{DS}{GDP}$	$\frac{Bank}{Gov}$	Confidence	liquidity
Normal Basel:	↑	↓	↑	↑	↓	↓
Normal 100%:	↑	↓	↑	↓	↓	↓
Gesell Basel:	↑	↑	↓	↓	↑	↑
Gesell 100%:	↑	↑	↓	↓	↑	↑

Table 6.2: The essential dynamics for the four different systems.

fidence, and with other parameters as simulated for in each chapter or section. This can be seen in Table 6.3.

System	g	θ	$\frac{DS}{GDP}$	$\frac{Bank}{Gov}$
Normal Basel:	0,025	0,247	0,122	4,865
Normal 100%:	0,002	1,167	0,122	3,000
Gesell Basel:	0,054	0,247	0,080	4,430
Gesell 100%:	0,004	1,167	0,080	1,846

Table 6.3: The essential stationary values describing the four different systems for a debt/GDP ratio corresponding to neutral confidence.

Here is a discussion of the different macro economic systems and how they could be compared.
real growth
inflation
debt burden -¿ confidence: see appendix

Chapter 7

Conclusion

7.1 This was the motivation

The main motivation for this report has been to find another conseptually different macroeconomic system to solve the debt crisis dynamics we can see today in the late spring of 2009, or to prevent such debt crisis to happen. Such a system should be able to avoid recession, explained by the Penguin Reference Dictionary of Economics to be:

> "... Two successive declines in seasonally adjusted quarterly, real gross domestic product would constitute a recession"[11]

A system should be able to avoid such a recession without finding itself in the liquidity trap described by the Penguin Reference Dictionary of Economics to be:

> "A situation in which the rate of interest is so low that no one wants to hold interest-bearing assets, and people only wants to hold cash. ... In this situation the liquidity preferance is absolute."[11]

The Penguin Reference Dictionary of Economics also describes the term liquidity preference:

> "The liquidity preference is the desire to hold money rather than other forms of wealth, e.g. stocks and bonds. ... A high degree of liquidity preference implies that a given supply of money flows relatively slowly through the economy, resulting in a low velocity of circulation."[11]

This liquidity preference is what is refered to in this report at confidence; a high liquidity preference corresponds in our terminology to low confidence, which leads to low liquidity. One objective has been to find a system that can create high GDP growth with a low DS/GSP ratio to prevent debt-crisis mechanisms. The prefered system should therefore have high liquidity, by having high confidence from a low debt/GDP ratio, because this means a higher economic growth in GDP can be achieved.

7.2 This has been done

The essential results from the discussion in the last chapter was extracted into Table 6.2 and 6.3, showing how the dynamics for the four different macro economic systems are for certain values of θ. The four systems were simulated with the same debt/GDP ratios between 57% and 75% by the use of different values for θ. This intervall in debt/GDP ratios was chosen to be around what is regarded as a healthy debt/GDP ratio. This healthy ratio of 60% was found from historical data from Australia and the U.S. and it is assumed to be healthy in terms of leading to a neutral confidence. With neutral confidence kept fixed, the confidence can be assumed to have no effect on the economic system thus it can be, and has been, excluded from our simulations. A healthy economy in this report means a macroeconomic system robust against debt-crisis and the effect of flutuations due to change in confidence. A healthy economy should also be able to avoid the liquidity trap.

7.3 The specifications for a prefered macroeconomic system

Specifications for a new and better macroeconomic system should therefore be a system that can increase its economic growth in terms of increased GDP, without the side-effect of increased DS/GDP-ratio. This is because an increase in DS/GDP-ratio to a certain point eventually leads to a debt-crisis mechanism with decreased confidence and spending, possibly leading to a liquidity trap. The system chosen to be the better one should also be able to avoid such a liquidity trap.

7.4 The solution

From the specifications of a better macroeconomic system and the different essential system dynamics descriptions in Table 6.2 and 6.3, one of the Gesell systems must be favoured. This is because the DS/GDP ratio decreases with an increase in economic growth measured by GDP, thus increasing confidence and liquidity. Furthermore, the liquidity trap can not occur in a Gesell system induced with negative interest rate on deposits because the negative interest rates could always be lowered more in order to keep people from hoarding money.

From Table 6.3 we can see that the highest GDP growth, $g = 5,4\%$, comes from the Gesell-Basel-type regime, while both the 100% reserve systems has a relatively low GDP growth rate. The Gesell-Basel-type regime also has a higher bank net lending to government net spending than the 100% reserve systems, and is not very different from today's Basel-type regime. It is therefore thought to be relatively easy to implement compared to the 100% reserve systems, because in the 100% reserve systems the financial banking structure would have to be radically changed. A challenge with the Gesell-Basel-type bank regime however, is that it needs to make sure to prevent people from hoarding money in terms of cash when negative interest rates on deposits are introduced.

7.5 However

Challenges in how to implement such systems are merely looked at in this report. Whether the 100% reserve system or a Gesell system is hard to implement is only discussed briefly. The prefered system in this report can therefore not be chosed on the basis of whether or not it is easy to introduce.

Assessment of the different models in this report has only been done with parameter values that are regarded as reasonable for today's Basel-type regime. Whether these parameters are reasonable for the other systems we have assessed is not accounted for.

Price level is not accounted for, and by this, neither is inflation. Whether e.g. a growth in GDP corresponds to a real or nominal gross domestic growth is thereby hard to say. The same issue of inflation also holds for debt burden on households and firms. We can see the DS/GDP ratio is lower for the Gesell than for the normal interest rate systems. This is of course because there are no interest rates on loans in the Gesell system. The real debt burdon however is difficult to assess because this means inflation rate would have to be accounted for, and this is not included in our model.

Endogenous money growth is thought by Andresen[7] to be a crucial fact that needs to be recognised for today's Basel-type regime. The Gesell-Basel-type regime also has this property, but challenges induced by endogenous money growth for any of the systems in this report is neither taken into account or discussed.

The saturation introduced in subsection 3.4.3 that Bank net lending must be zero or positive was found to be a minor flaw in the last days of writing this report and has therefore not been changed. It is wrong because Bank net lending $l - rA$ could be negative if Bank stops issuing new loans, and borrowers continue to pay interest and principal. However, some simulations have been done with the correct saturation on lending, as it should be instead of net lending, and the error was found to induce small, if any, impact. The simulations that have been done to find the impact from this error is not documented here because there was no time, and there was no time to correct it and do simulations over again either.

7.6 Future work

Future work could be interesting to do on the basis of this report, and here are my suggestions of what I think could be interesting tasks:

- John Maynard Keynes' zero employment solution[17], with the wage of the Employer of Last Resort as a tool for regulation could be simulated, (I am not sure how), and compared to the gold standard, or today's standard. This, he argues, should be used as a new "gold standard" because the wage of the unemployed, which from definition by Keyens will always be lower than for the employed, reflects the state of economy and demand for money, and therefore offers an effective tool to control and stabilize the economy. His argument that wages to the unemployed is a great effective tool for stabilization of the economy could be tested in comparison with other alternative systems for stbilization.

- An expansion of the model presented here could be interesting to do to incorporate more advanced and interconnected relationships which surely exists in terms of e.g. influence from international business and exchange rates. which are not included here due to probably a more potentially complex system hard to analyze.

- Only $\kappa_{ref} = 0,08$ and $\kappa_{ref} = 1$ are used for our system assessments, and these are used as fixed parameters. Control through a change in κ could therefore be interesting to study.

- It should be studied how and if the negative Gesell rate on deposits i_L could be a usefull control variable. Gesell i_L is interesting when seen as a control variable because it has a very direct influence on decreasing real value of people's money accounts. In today's system with an inflation of lets say 3% and a interest rate on bank deposits of zero, it is in the authors opinion that people are less able to understand this real decrease on their deposits caused by inflation. Zero interest rates and high inflation is therefore regarded by the author as a bad control variable to use to encourage spending, and thereby increase liquidity. If spending should be encouraged by i_L in a Gesell system however, people would instantly realize their real change in value of their bank deposit accounts caused by this negative i_L. i_L as a control variable in a Gesell system is therefore regarded to have a more directly influence on economy than interest rates for a Basel-type regime does. i_L for a Gesell system is therefore thought to be a good control variable that would be interesting to study the effect of.

- Simulations for the liquidity trap with different economic systems, e.g. the four different systems that has been assessed in this report, could be interesting to study.

Appendix A

Confidence

Economic confidence could be included to show how the debt service to GDP ratio influence the economy. It will not be included however because to develop such a model is thought by the author to be highly speculative. If included with dynamics as Fitje(2008)[13] choosed to make it, confidence would flutuate with the DS/GDP ratio. This means that with low DS/GDP ratio confidence would be high, and the other way around for a low DS/GDP ratio. With low and high values of DS/GDP ratio it is ment in comparioson with the DS/GDP ratio that gives neutral confidence. Neutral confidence is confidence with no influence on the rest of the economy.

The word "confidence" is used here because if embedded in our model as a variable, a high or low confidence would make sense, whereas high or low "mood" could be confusing. The way confidence dynamics are included in Fitje[13] is found to be reasonable. His development of confidence dynamics will not be reproduced here, but merely the essentials will be explained, to find out how it would effect our systems for a change in DS/GDP ratio.

One essential aspect of this model is that it has a level of neutral confidence, at a certain level of DS/GDP ratio found to be corresponding with a debt/GDP ratio of 60%. If this confidence kan be kept by controlling DS/GDP ratio, as is done in this report, the effect on the economy from confidence is zero and confidence dynamics are therefore excluded from our model. When discussing the model and when a conclusion is drawn however, the essential effect from high and low confidence as described here is taken into consideration.

A.1 The dynamics of confidence

Input

The confidence state variable can be chosen to vary with differnt measurements in the economy, i.e. Debt/Deposits-, Debt/GDP-, Debt/Export-, or Debt service/GDP-ratio. Surely many other types of indicators could be measured to influence the economic confidence as well, but the DS/GDP ratio is thought by the author the be superior to other infuences on confidence. This is in accordance with what Fitje[13] chose, because it sounds reasonable. Therefore, the

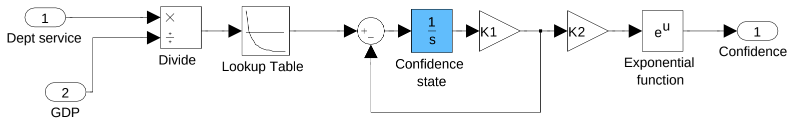

Figure A.1: All parts of the confidence subsystem.

bigger share of GDP used to service debt, the more confidence is decreased, and we have by this established the input to the confidence state variable.

Output and influence on the economy

In times of low confidence in future economy, people tend to hoard money if times get worse, thus time lag T, also called circulation inertia, representing the throughput of money should be increased to show a decrease in spending thus market liquidity. A higher DS/GDP ratio would also make the loss rate λ increase, because this means a higher percentage of income to firms and people is used to service their loans. The confidence could also have the effect that people and firms would decrease their eagerness to borrow because they would be affraid they could not service future repayments. Low confidence could also cause banks to be less eager to lend because they would be afraid of a higher future percentage of borrowers defaulting on their loans. The two latter cases of banks being afraid to lend and borrowers being afraid to borrow would only decrease liquidity more. The essential effect that will be in foucus here is that DS/GDP ratio and confidence are in inverse ratio with each other, thus e.g. higher DS/GDP ratio would lead to a lower confidence, which would lead to decreased liquidity.

A.1.1 Inner workings of confidence dynamics

After establishing what should be the input to change confidence, and how the output confidence, should influence the rest of the economy, the more detailed inner workings of the input output dynamics of confidence can be found in Fitje(2008)[13], but this will not be discussed more in detail here. Figure A.1 shows how the Simulink confidence subsystem for Fitje's inclusion of confidence looks like.

Appendix B

Development of Bank net lending to government net spending ratio for a Gesell system

This development will be long. The notation $-(t)$, meaning it is not a constant, but a variable varying with time will therefore be dropped, i.e. $L(t)$ becomes L, to save time and space.

We want to develop Bank net lending

$$l - rA = \dot{A} + \lambda A \tag{B.1}$$

to government net spending

$$\gamma = \dot{L} - (l - rA) + \beta i_L L \tag{B.2}$$

ratio, which can be written

$$\frac{l - rA}{\gamma} = \frac{\dot{A} + \lambda A}{\dot{L}} = \frac{(1)}{(2)} \tag{B.3}$$

The notation (1) and (2) in (B.3) is done so we can develop each statement by itself. We will start with Bank net lending (1), then look at government net spending (2), and finally find the ratio we are looking for.

B.1 Bank net lending (1)

Bank net lending (1) can be expressed as

$$(1) = \dot{A} + \lambda A = gA + \lambda A = (g + \lambda)A \tag{B.4}$$

with

$$g = \frac{\beta i_A (1 + \theta - \kappa) - \lambda}{\kappa} \tag{B.5}$$

found from (5.13), with interest rates on Central Bank reserves $i_R = 0$.

B.2 Government net spending (2)

Government net spending (2) can be expressed as

$$(2) = \dot{L} - (l - rA) + \beta i_L L \tag{B.6}$$

found from (5.1). By exchange of $l - rA$ with equation (B.1), found from (5.3), we get

$$(2) = \dot{L} - \dot{A} - \lambda A + \beta i_L L \tag{B.7}$$

If we now exchange L with

$$L = (1 + \theta - \kappa)A \tag{B.8}$$

and exchange $\dot{L}$ with

$$\dot{L} = (1 + \theta - \kappa)\dot{A} \tag{B.9}$$

(2) becomes

$$(2) = (1 + \theta - \kappa)\dot{A} - \dot{A} - \lambda A + \beta i_L(1 + \theta - \kappa)A \tag{B.10}$$

The equation for asset growth rate g for a Gesell system is found in (5.13) to be $\dot{A} = gA$. By inserting (5.13) for $\dot{A}$ in (2), (2) becomes

$$(2) = (1 + \theta - \kappa)gA - gA - \lambda A + \beta i_L(1 + \theta - \kappa)A \tag{B.11}$$

In (B.11) we find the second term gA cancels out, and we get

$$(2) = (\theta - \kappa)gA - \lambda A + \beta i_L(1 + \theta - \kappa)A \tag{B.12}$$

The two last terms are now put together and multiplied with κ/κ

$$(2) = (\theta - \kappa)gA - \frac{(\lambda + \beta i_L(1 + \theta - \kappa)}{\kappa}\kappa A \tag{B.13}$$

which is the same as

$$(2) = (\theta - \kappa)gA - g\kappa A \tag{B.14}$$

Both a negative and positive $g\kappa A$ term exists, thus cancels each other out and (2) becomes

$$(2) = \theta gA \tag{B.15}$$

B.3 (1)and (2)are finally merged into Bank net lending to government net spending ratio

The Bank net lending to government net spending ratio is now found by merging Bank net lending (1), and government net spending (2), together.

$$\frac{(1)}{(2)} = \frac{(g + \lambda)A}{\theta gA} \tag{B.16}$$

which is

$$\frac{l - rA}{\gamma} = \frac{1}{\theta}(1 + \frac{\lambda}{g}) \tag{B.17}$$

with g found from (5.13).

Bibliography

[1] Bjørn Skogstad Aamo. The financial crisis and norwegian financial institutions.
http://www.kredittilsynet.no/archive/stab_pdf/01/06/08052054.pdf, 05.08.2009.

[2] Trond Andresen. A block diagram approach to macroeconomic dynamics, and one reason is/lm is fatally flawed. Working paper.

[3] Trond Andresen. Gjeldskrise - ikke finanskrise. Adresseavisen
http://www.adressa.no/meninger/kronikker/article1194541.ece, 11.21.2009.

[4] Trond Andresen. The dynamics of long-range financial accumulation and crisis. Technical report, Department of Engineering Cybernetics, Norwegian University of Science and Technology, 1999.

[5] Trond Andresen. The macroeconomy as a network of money-flow transfer functions. Technical report, Department of Engineering Cybernetics, Norwegian University of Science and Technology, 1999.

[6] Trond Andresen. Fundamental financial accumulation dynamics. Technical report, Department of Engineering Cybernetics, Norwegian University of Science and Technology, 2008.

[7] Trond Andresen. Basel rules, endogenous money growth, financial accumulation and debt crisis. Technical report, Department of Engineering Cybernetics, Norwegian University of Science and Technology, 2009.

[8] Bank for International Settlements. About BIS.
http://www.bis.org/about/index.htm, 04.16.2009.

[9] Willem H. Buiter. Overcoming the zero bound on nominal interest rates: Gesell's currency carry tax vd. eisler's parallel virtual currency. Technical report, Institute of Economic Research, Hitotsubashi University, Kunitachi, Tokyo, Japan, June 2005.

[10] Cappelen Damm. Intro/mortages.
http://introengelsk.cappelendamm.no/c35043/artikkel/vis.html?tid=35541, 05.22.2009.

[11] Graham Bannock R. E. Baxter & Evan Davis. *Dictionary of Economics 7th edition*. Penguin Books, 2003.

[12] Karl E. Case & Ray C. Fair. Principles of economics, 7th edition, chapter 8. http://wps.prenhall.com/bp_casefair_econf_7e/31/7936/2031653.cw/index.html, 05.31.2009.

[13] Tryggve Fitje. Monetary macroeconomic modeling, simulation and control. Master's thesis, NTNU, 2008.

[14] Takis Fotopoulous. The myths about the economic crisis, the reformist left and economic democracy. http://www.inclusivedemocracy.org/journal/vol4/vol4_no4_takis_economic_crisis.htm, October 2008.

[15] Bjarne A. Foss Jens G. Balchen, Trond Andresen. *Reguleringsteknikk*. NTNU-trykk, 2004.

[16] Steve Keen. Steve keen's debtwatch. http://www.debtdeflation.com/blogs/2008/11/30/debtwatch-no-29-december-2008/, 06.01.2009.

[17] John Maynard Keynes. *General Theory of Employment, Interest and Money*. Macmillan Cambridge University Press, for Royal Economic Society, 1936.

[18] N. Gregory Mankiw. It may be time for the fed to go negative. The New York Times, Economic View, 04.19.2009, 2009.

[19] matematikk.norsknettskole.no. Ferdige regneark. http://matematikk.norsknettskole.no/excel/excel/serie_anu_lan.xls, 05.22.2009.

[20] The MathWorks. Matlab - documentation. http://www.mathworks.com/access/helpdesk/help/techdoc/index.html?/access/helpdesk/help/techdoc/, 05.24.2009.

[21] U.S. Department of State / Byreau of International Information Programs. The global financial system: Six experts look at the crisis. http://www.america.gov/media/pdf/ejs/0509.pdf#popup, 05.29.2009.

[22] Karl Johan Åstrøm. chapter 6, lecture notes for me155a, control system deesign. Technical report, Department of Mechanical and Environmental Engineering, University of California, 2002.

[23] Oak Road Systems. Loan or Investment Formulas. http://oakroadsystems.com/math/loan.htm, 22.05.2009.

[24] Norbert Janssen Charles Noland Ryland Thomas. Money, debt and prices in the uk, 1705–1996. Technical report, Henley Business School University of Reading, 1999/2000.

[25] Silvio Gesell translated by Philip Pye M.A. 1958. *The Natural Economic Order*. http://wikilivres.info/wiki/The_Natural_Economic_Order, 1920.

[26] Ricardo Caballero Bengt Holmstrom Andrew Lo James Poterba & William Wheaton. Webcast of an mit panel discussion. "the u.s. financial crisis: What happened? what's next?". http://web.mit.edu/smcs/mit/2008/econ-financial_crisis-08oct2008-350k.asx, 05.29.2009.

[27] Wikipedia. Financial crisis. http://en.wikipedia.org/wiki/Financia_crisis, 05.29.2009.

[28] Wikipedia.org. Bank. http://en.wikipedia.org/wiki/, 06.09.2009.

[29] L. Randall Wray. *Understanding Modern Money.* Edward Elgar Publishing Limited, 1998.

[30] zfacts.com. National debt graph: Bush goes for wwii stimulus. http://zfacts.com/p/318.html, 06.02.2009.