Novel $\mathcal{L}_1$ Neural Network Adaptive Control Architecture With Guaranteed Transient Performance

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***Abstract*—In this paper, we present a novel neural network (NN) adaptive control architecture with guaranteed transient performance. With this new architecture, both input and output signals of an uncertain nonlinear system follow a desired linear system during the transient phase, in addition to stable tracking. This new architecture uses a low-pass filter in the feedback loop, which consequently enables to enforce the desired transient performance by increasing the adaptation gain. For the guaranteed transient performance of both input and output signals of the uncertain nonlinear system, the $\mathcal{L}_1$ gain of a cascaded system, comprised of the low-pass filter and the closed-loop desired reference model, is required to be less than the inverse of the Lipschitz constant of the unknown nonlinearities in the system. The tools from this paper can be used to develop a theoretically justified verification and validation framework for NN adaptive controllers. Simulation results illustrate the theoretical findings.**

***Index Terms*—Adaptive control, approximation region, neural network (NN), radial basis function (RBF), transient.**

I. Introduction

Neural networks (NNs) were conventionally introduced to control systems in the presence of matched nonlinear uncertainties that could not be globally linearly parameterized in unknown parameters. Starting from the seminal paper of Narendra and Parthasarathy [1], where rigorous stability proofs were first introduced, the field has evolved significantly over the past one and half decades. Topics of interest included computation of the gradients needed for backpropagation-type tuning [2], [3], use of radial basis functions (RBFs) for feedback [4], dead-zone methods for NN parameters tuning [5], [6], projection-based adaptation [7]–[9], use of e-modification schemes in adaptive laws [10], use of NNs for discrete-time systems [11], use of dynamic NNs for feedback [12], use of NNs for general nonlinear systems in state feedback and output feedback [13], [14], and use of NNs in decentralized control [15], to name only a few. Parameter convergence in RBF networks has been addressed in [16]. NNs have also proved to be a useful tool in a wide range of applications from robotic manipulators [17] to aircraft control [18], and beyond.

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However, even though the stability proofs are available, the lack of systematic methods for selection of basis functions, distribution of their centers, and the choice of adaptive gains render design and application of NN adaptive controllers overly challenging. This is due to the fact that the system uncertainties during the transient can lead to unpredictable/undesirable situations, involving control signals of high-frequency or large amplitudes, large transient errors, or slow convergence rate of tracking errors, to cite a few. Extensive tuning of adaptive gains and Monte Carlo runs are the primary method today enabling the transition of NN adaptive control solutions to real-world applications.

It is worthwhile to mention that the transient performance characterization is challenging with any model reference adaptive control (MRAC) scheme, the NN-based adaptive controllers being one of those. Various approaches have been reported in literature over the past couple of decades that have addressed this issue in conventional MRAC scheme using in most of the cases high-gain feedback component in the feedback loop in addition to the adaptive signals [19]–[31]. However, all the bounds in these papers are computed for tracking errors only, and not for control signals. Although the latter can be deduced from the former, it is straightforward to verify that the ability to adjust the former may not extend to the latter in case of nonlinear control laws. Moreover, since the purpose of adaptive control is to ensure stable performance in the presence of modeling uncertainties, one needs to ensure that the changes in reference input and unknown parameters due to possible faults or unexpected uncertainties do not lead to unacceptable transient deviations or oscillatory control signals, implying that a retuning of adaptive parameters is required. Finally, it is highly desirable to ensure that whatever modifications or solutions are suggested for performance improvement of adaptive controllers, they are not achieved via high-gain feedback.

In this paper, we define a new type of model following NN adaptive controller, for which the transient performance can be characterized both for the system input and output signals. The methodology in this paper is an extension of the $\mathcal{L}_1$ adaptive controller, reported in [32]–[34], for general MRAC scheme. This new adaptive control architecture has a low-pass system in the feedback loop that enables to enforce the desired transient performance by increasing the adaptation gain. For the proof of ultimate boundedness, the $\mathcal{L}_1$ gain of a cascaded system, comprised of this filter and the closed-loop desired transfer function, is required to be less than the Lipschitz constant of the unknown nonlinearities. The ideal (nonadaptive) version of this $\mathcal{L}_1$ NN adaptive controller is used along with the main system dynamics to define a closed-loop reference system, which gives an opportunity to estimate performance bounds in terms of $\mathcal{L}_\infty$ norms for both system's input and output signals as compared to the same signals of this reference system. These bounds immediately imply that the transient performance of the control signal in conventional MRAC scheme cannot be characterized. Design guidelines for selection of the low-pass filter ensure that the closed-loop reference system approximates the desired linear system response, despite the fact that it depends upon the unknown nonlinearity. Thus, the desired tracking performance is achieved by systematic selection of the low-pass filter, which in its turn enables fast adaptation, as opposed to high-gain designs leading to increased control efforts. The last but not the least, the $\mathcal{L}_1$ NN adaptive control architecture gives the ability also to reduce the ultimate bound for the tracking error by increasing the adaptation gain.

The paper is organized as follows. Section II states some preliminary definitions, and Section III gives the problem formulation. In Section IV, the new $\mathcal{L}_1$ NN adaptive controller is presented. Stability and performance bounds for the $\mathcal{L}_1$ NN adaptive controller are presented in Section V. Design techniques for $\mathcal{L}_1$ neural adaptive controller are discussed in Section VI. In Section VII, simulation results are presented, while Section VIII concludes this paper. Throughout this paper, $\chi(s)$ will denote the Laplace transformation of signal $\chi(t)$.

II. Preliminaries

In this section, we recall some basic definitions and facts from linear systems theory, [24], [35], [36]. For any positive-definite matrix P, we let $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ be its minimum and maximum eigenvalues.

Definition 1: For a signal $\xi(t), t \geq 0, \xi \in \mathbb{R}^n$, its truncated $\mathcal{L}_\infty$ norm and $\mathcal{L}_\infty$ norm are defined as

$$\|\xi_t\|_{\mathcal{L}_\infty} = \max_{i=1,..,n}\left(\sup_{0\leq\tau\leq t}|\xi_i(\tau)|\right)$$
$$\|\xi\|_{\mathcal{L}_\infty} = \max_{i=1,..,n}\left(\sup_{\tau\geq 0}|\xi_i(\tau)|\right)$$

where ξ_i is the ith component of ξ.

Definition 2: The $\mathcal{L}_1$ gain of a stable proper single-input–single-output (SISO) system $H(s)$ is defined to be $\|H(s)\|_{\mathcal{L}_1} = \int_0^\infty |h(t)|dt$, where $h(t)$ is the impulse response of $H(s)$, computed via the inverse Laplace transform $h(t) = (1/2\pi i)\int_{\alpha-i\infty}^{\alpha+i\infty} H(s)e^{st}ds, t \geq 0$, in which the integration is done along the vertical line $x = \alpha > 0$ in the complex plane.

Proposition: A continuous-time linear time invariant (LTI) system (proper) with impulse response $h(t)$ is stable if and only if $\int_0^\infty |h(\tau)|d\tau < \infty$. A proof can be found in [24, Th. 3.2.2, p. 81].

Definition 3: For a stable proper m-input–n-output system $H(s)$, its $\mathcal{L}_1$ gain is defined as

$$\|H(s)\|_{\mathcal{L}_1} = \max_{i=1,..,n}\left(\sum_{j=1}^m \|H_{ij}(s)\|_{\mathcal{L}_1}\right) \tag{1}$$

where $H_{ij}(s)$ is the ith row jth column element of $H(s)$.

The next lemma extends the results of [35, Example 5.2, p. 199] to general multiple-input–multiple-output (MIMO) systems.

Lemma 1: For a stable proper MIMO system $H(s)$ with input $r(t) \in \mathbb{R}^m$ and output $x(t) \in \mathbb{R}^n$, we have

$$\|x_t\|_{\mathcal{L}_\infty} \leq \|H\|_{\mathcal{L}_1}\|r_t\|_{\mathcal{L}_\infty} \qquad \forall\, t > 0.$$

Proof: Let $x_i(t)$ be the ith element of $x(t)$, $r_j(t)$ be the jth element of $r(t)$, $H_{ij}(s)$ be the ith row jth element of $H(s)$, and $h_{ij}(t)$ be the impulse response of $H_{ij}(s)$. Then, for any $t' \in [0, t]$, we have

$$x_i(t') = \int_0^{t'}\left(\sum_{j=1}^m h_{ij}(t'-\tau)r_j(\tau)\right)d\tau. \tag{2}$$

From (2), it follows that:

$$\begin{aligned}|x_i(t')| &\leq \int_0^{t'}\left(\sum_{j=1}^m |h_{ij}(t'-\tau)|\,|r_j(\tau)|\right)d\tau\\ &\leq \int_0^{t'}\left(\sum_{j=1}^m |h_{ij}(t'-\tau)|\right)d\tau\left(\max_{j=1,..,m}\sup_{0\leq\tau\leq t'}|r_j(\tau)|\right)\\ &\leq \sum_{j=1}^m\left(\int_0^{t'}|h_{ij}(\tau)|\,d\tau\right)\left(\max_{j=1,..,m}\sup_{0\leq\tau\leq t'}|r_j(\tau)|\right)\end{aligned}$$

and hence, $\|x_{i_t}\|_{\mathcal{L}_\infty} \leq (\sum_{j=1}^m \|H_{ij}(s)\|_{\mathcal{L}_1})\|r_t\|_{\mathcal{L}_\infty}$. It follows from (1) that:

$$\begin{aligned}\|x_t\|_{\mathcal{L}_\infty} &= \max_{i=1,..,n}\|x_{i_t}\|_{\mathcal{L}_\infty}\\ &\leq \max_{i=1,..,n}\left(\sum_{j=1}^m \|H_{ij}(s)\|_{\mathcal{L}_1}\right)\|r_t\|_{\mathcal{L}_\infty}\\ &= \|H(s)\|_{\mathcal{L}_1}\|r_t\|_{\mathcal{L}_\infty}\end{aligned}$$

for any $t \geq 0$. The proof is complete. □

Corollary 1: For a stable proper MIMO system $H(s)$, if the input $r(t) \in \mathbb{R}^m$ is bounded, then the output $x(t) \in \mathbb{R}^n$ is also bounded as $\|x\|_{\mathcal{L}_\infty} \leq \|H(s)\|_{\mathcal{L}_1}\|r\|_{\mathcal{L}_\infty}$.

Lemma 2: For a cascaded system $H(s) = H_2(s)H_1(s)$, where $H_1(s)$ is a stable proper system with m inputs and l outputs and $H_2(s)$ is a stable proper system with l inputs and n outputs, we have $\|H(s)\|_{\mathcal{L}_1} \leq \|H_2(s)\|_{\mathcal{L}_1}\|H_1(s)\|_{\mathcal{L}_1}$

Proof: Let $y(t) \in \mathbb{R}^n$ be the output of $H(s) = H_1(s)H_2(s)$ in response to input $r(t) \in \mathbb{R}^m$. It follows from Lemma 1 that for any $t \geq 0$

$$\begin{aligned}\|y(t)\|_\infty \leq \|y_t\|_{\mathcal{L}_\infty} &\leq \|H_2(s)\|_{\mathcal{L}_1}\|H_1(s)\|_{\mathcal{L}_1}\|r_t\|_{\mathcal{L}_\infty}\\ &\leq \|H_2(s)\|_{\mathcal{L}_1}\|H_1(s)\|_{\mathcal{L}_1}\|r\|_{\mathcal{L}_\infty}.\end{aligned} \tag{3}$$

Let $H_i(s)$, $i = 1, \ldots, n$, be the ith row of the system $H(s)$. It follows from (1) that there exists i such that

$$\|H(s)\|_{\mathcal{L}_1} = \|H_i(s)\|_{\mathcal{L}_1}. \tag{4}$$

Let $h_{ij}(t)$ be the jth element of the impulse response of the system $H_i(s)$. For any T, let

$$r_j(t) = \mathrm{sgn} h_{ij}(T-t), \qquad t \in [0,T] \qquad \forall j = 1,\ldots,m. \tag{5}$$

It follows from Definition 1 that $\|r\|_{\mathcal{L}_\infty} = 1$, and hence, $\|y(t)\|_\infty \le \|H_2(s)\|_{\mathcal{L}_1}\|H_1(s)\|_{\mathcal{L}_1}, \forall t \ge 0$. For $r(t)$ satisfying (5), we have

$$\begin{aligned} y(T) &= \int_{t=0}^{T} \sum_{j=1}^{m} h_{ij}(T-t) r_j(t) dt \\ &= \int_{t=0}^{T} \sum_{j=1}^{m} |h_{ij}(T-t)|\, dt = \sum_{j=1}^{m} \left(\int_{t=0}^{T} |h_{ij}(t)|\, dt \right). \end{aligned}$$

Therefore, it follows from (3) that for any T, $\sum_{j=1}^{m}(\int_{t=0}^{T}|h_{ij}(t)|dt) \le \|H_2(s)\|_{\mathcal{L}_1}\|H_1\|_{\mathcal{L}_1}$. As $T \to \infty$, it follows from (4) that:

$$\begin{aligned} \|H(s)\|_{\mathcal{L}_1} = \|H_i(s)\|_{\mathcal{L}_1} &= \lim_{T\to\infty} \sum_{j=1}^{m} \left(\int_{t=0}^{T} |h_{ij}(t)|\, dt \right) \\ &\le \|H_2(s)\|_{\mathcal{L}_1} \|H_1(s)\|_{\mathcal{L}_1} \end{aligned}$$

and this completes the proof. □

Consider a linear time invariant system

$$\dot{x}(t) = A_m x(t) + bu(t) \tag{6}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}$, $b \in \mathbb{R}^n$, and $A_m \in \mathbb{R}^{n\times n}$ is Hurwitz, and assume that the transfer function $(sI - A_m)^{-1}b$ is strictly proper and stable. Notice that it can be expressed as

$$(sI - A_m)^{-1} b = n(s)/d(s) \tag{7}$$

where $d(s) = \det(sI - A_m)$ is a nth-order stable polynomial, and $n(s)$ is a $n \times 1$ vector with its ith element being a polynomial function

$$n_i(s) = \sum_{j=1}^{n} n_{ij} s^{j-1}. \tag{8}$$

Lemma 3: If $(A_m \in \mathbb{R}^{n\times n}, b \in \mathbb{R}^n)$ is controllable, the matrix N with its ith row jth column entry n_{ij} is full rank.

Proof: Controllability of (A_m, b) for the LTI system in (6) implies that given an initial condition $x(t_0) = 0$, arbitrary $x_{t_1} \in \mathbb{R}^n$, and arbitrary t_1, there exists $u(\tau)$, $\tau \in [t_0, t_1]$ such that $x(t_1) = x_{t_1}$. If N is not full rank, then there exists a nonzero vector $\mu \in \mathbb{R}^n$, such that $\mu^\top n(s) = 0$. Then, it follows that for $x(t_0) = 0$ one has $\mu^\top x(\tau) = 0, \forall \tau > t_0$. This contradicts $x(t_1) = x_{t_1}$, in which $x_{t_1} \in \mathbb{R}^n$ is assumed to be an arbitrary point. Therefore, N must be full rank, and the proof is complete. □

Lemma 4: If (A_m, b) is controllable and $(sI - A_m)^{-1}b$ is strictly proper and stable, there exists $c_o \in \mathbb{R}^n$ such that the transfer function $c_o^\top(sI - A_m)^{-1}b$ is minimum phase with relative degree one, i.e., all its zeros are located in the left half-plane, and its denominator is one order larger than its numerator.

Proof: It follows from (7) that

$$c_o^\top (sI - A_m)^{-1} b = \frac{c_o^\top N [s^{n-1} \cdots 1]^\top}{d(s)} \tag{9}$$

where $N \in \mathbb{R}^{n\times n}$ is matrix with its ith row jth column entry n_{ij} introduced in (8). We choose $\bar{c} \in \mathbb{R}^n$ such that $\bar{c}^\top[s^{n-1}\cdots 1]^\top$ is a stable $(n-1)$-order polynomial. Since (A_m, b) is controllable, it follows from Lemma 3 that N is full rank. Let $c_o = (N^{-1})^\top \bar{c}$. Then, it follows from (9) that $c_o^\top(sI - A_m)^{-1}b = \bar{c}^\top[s^{n-1}\cdots 1]^\top/d(s)$ has relative degree 1 with all its zeros in the left half-plane. □

III. Problem Formulation

Consider the following SISO system dynamics:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + b\left(u(t) - f(x(t)) + \zeta(t)\right) \\ y(t) &= c^\top x(t) \quad x(0) = x_0 = 0 \end{aligned} \tag{10}$$

where $x \in \mathbb{R}^n$ is the system state vector (measurable), $u \in \mathbb{R}$ is the control signal, $b, c \in \mathbb{R}^n$ are known constant vectors, A is a known $n \times n$ matrix with (A, b) controllable, $y \in \mathbb{R}$ is the regulated output, $\zeta(t) \in \mathbb{R}$ is time-varying disturbance, and $f : \mathbb{R}^n \to \mathbb{R}$ is an unknown Lipschitz continuous map, i.e., there exists L such that

$$|f(x_1) - f(x_2)| \le L\|x_1 - x_2\|_\infty. \tag{11}$$

We notice that the Lipschitz continuity is expressed using the ∞ norm without loss of generality. We further assume that the upper bound for $f(0)$ is known

$$|f(0)| \le B. \tag{12}$$

The unknown bounded disturbance $\zeta(t)$ is continuously differentiable with uniformly bounded derivative

$$|\zeta(t)| \le \Delta \quad \left|\dot{\zeta}(t)\right| \le d_\Delta. \tag{13}$$

The control objective is to design a neural adaptive controller to ensure that $y(t)$ tracks a given bounded continuous reference signal $r(t)$, with known upper bound of $\|r\|_{\mathcal{L}_\infty}$, both in transient and steady state, while all other error signals remain bounded. More rigorously, the control objective can be stated as

$$y(s) \approx D(s) r(s) \tag{14}$$

where $D(s)$ is a strictly proper stable LTI system that specifies the desired transient and steady-state performance.

Following [4], consider an RBF NN approximation of $f(x)$ over a compact set

$$f(x) = W^\top \phi(x) + \epsilon(x) \quad |\epsilon(x)| \le \epsilon^*, \qquad x \in \mathcal{D}_x \tag{15}$$

where $\phi(x)$ is a vector of Gaussian RBFs with its ith element

$$\phi_i(x) = \exp\left(-\frac{(x - z_i)^\top (x - z_i)}{\delta_i^2}\right)$$

the parameters z_i and δ_i are the prefixed centers and widths, W is a vector of unknown weights, ϵ^* is a uniform bound for the approximation error, and the set $\mathcal{D}_x$ will be specified shortly. We further assume that a compact convex set Ω is known *a priori* such that

$$W \in \Omega.$$

Remark 1: To streamline the subsequent derivation, we have set the initial condition $x_0 = 0$. Extension to nonzero x_0 is straightforward, and is discussed in Remark 9.

IV. $\mathcal{L}_1$ Neural Adaptive Controller

For the system in (10), consider the following control structure:

$$u(t) = u_1(t) + u_2(t) \quad u_1(t) = -K^\top x(t) \tag{16}$$

where $u_2(t)$ is the adaptive controller to be determined later, while K is a nominal design gain needed to ensure that $A_m = A - bK^\top$ is Hurwitz or, equivalently, that

$$H_o(s) = (sI - A_m)^{-1} b$$

is stable. Notice that if A is Hurwitz, then one can set $K = 0$. The control signal in (16) leads to the following partially closed-loop dynamics:

$$\begin{aligned} \dot{x}(t) &= A_m x(t) + b\left(u_2(t) - f(x(t)) + \zeta(t)\right) \\ y(t) &= c^\top x(t) \quad x(0) = x_0 = 0. \end{aligned} \tag{17}$$

For the linearly parameterized system in (17), we consider the following state predictor:

$$\begin{aligned} \dot{\hat{x}}(t) &= A_m \hat{x}(t) + b\left(u_2(t) - \hat{W}^\top(t)\phi(x(t)) + \hat{\zeta}(t)\right) \\ \hat{y}(t) &= c^\top \hat{x}(t) \quad \hat{x}(0) = x_0 \end{aligned} \tag{18}$$

along with the adaptive law for $\hat{W}(t)$ and $\hat{\zeta}(t)$

$$\begin{aligned} \dot{\hat{W}}(t) &= \Gamma_c \mathrm{Proj}\left(\hat{W}(t), \phi(x(t))\,\tilde{x}^\top(t) P b\right), \qquad \hat{W}(0) = \hat{W}_0 \\ \dot{\hat{\zeta}}(t) &= \Gamma_c \mathrm{Proj}\left(\hat{\zeta}(t), -\tilde{x}^\top(t) P b\right), \qquad \hat{\zeta}(0) = 0 \end{aligned} \tag{19}$$

where $\tilde{x}(t) = \hat{x}(t) - x(t)$ is the tracking error, $\Gamma_c \in \mathbb{R}^+$ is adaptation gain, $\mathrm{Proj}(\cdot,\cdot)$ denotes the projection operator [37], and $P = P^\top > 0$ is the solution of the algebraic Lyapunov equation $A_m^\top P + P A_m = -Q$, $Q > 0$. Letting

$$\bar{r}(t) = \hat{W}^\top(t)\phi(x(t)) - \hat{\zeta}(t)$$

the state predictor in (18) can be viewed as a low-pass system with $u_2(t)$ being the control signal and $\bar{r}(t)$ being a time-varying disturbance, which is not prevented from having high-frequency oscillations. We consider the following control design for (18):

$$u_2(s) = C(s)\bar{r}(s) + k_g r(s) \tag{20}$$

where $C(s)$ is a stable and strictly proper system with low-pass gain $C(0) = 1$, and

$$k_g = \lim_{s\to 0} \frac{1}{c^\top H_o(s) b} = \frac{1}{c^\top H_o(0) b}.$$

Consider the closed-loop state predictor in (18) with the control signal defined in (20). It can be viewed as an LTI system with two inputs $r(t)$ and $\bar{r}(t)$

$$\hat{x}(s) = \bar{G}(s)\bar{r}(s) + G(s) r(s) \tag{21}$$
$$\bar{G}(s) = H_o(s)\left(C(s) - 1\right) \tag{22}$$
$$G(s) = k_g H_o(s). \tag{23}$$

We note that $\bar{r}(t)$ is related to $\hat{x}(t)$, $u(t)$, and $r(t)$ via nonlinear relationships.

Remark 2: Since both $H_o(s)$ and $C(s)$ are strictly proper stable systems, one can check straightforwardly that $\bar{G}(s)$ and $G(s)$ are strictly proper stable systems, even though $1 - C(s)$ is proper.

Now, we give the performance requirement that ensures boundedness of the entire system and desired transient performance, as discussed in Section V.

$\mathcal{L}_1$-gain requirement: Design K and $C(s)$ to satisfy

$$\left\|\bar{G}(s)\right\|_{\mathcal{L}_1} < \frac{1}{L} \tag{24}$$

where L is introduced in (11).

Since with RBF approximation the performance results are always local, one needs to characterize the set $\mathcal{D}_x$, on which the RBFs are distributed. Let

$$\mathcal{D}_x = \{x \,|\, \|x\|_\infty \le \gamma_r + \gamma_1 + \gamma_0 + \sigma\} \tag{25}$$

where $\sigma > 0$ is an arbitrary positive constant, while

$$\begin{aligned} \gamma_r = &\left(\|G(s)\|_{\mathcal{L}_1} \|r\|_{\mathcal{L}_\infty} + \left\|\bar{G}(s)\right\|_{\mathcal{L}_1}(B + \epsilon^* + \Delta)\right. \\ &\left. + \|H_o(s)\|_{\mathcal{L}_1}\epsilon^*\right) / \left(1 - \left\|\bar{G}(s)\right\|_{\mathcal{L}_1} L\right) \end{aligned} \tag{26}$$

$$\gamma_0 = \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}\left(\frac{2\|Pb\|}{\lambda_{\min}(Q)}\left(\epsilon^* + \frac{d_\Delta}{\Gamma_c}\right)\right)^2 + \frac{W_{\max}}{\lambda_{\min}(P)\Gamma_c}} \tag{27}$$

$$\gamma_1 = \frac{5\epsilon^* \left\|\bar{G}(s)\right\|_{\mathcal{L}_1} + \|H_o(s)\|_{\mathcal{L}_1}\epsilon^* + \left(1 + \|C(s) - 1\|_{\mathcal{L}_1}\right)\gamma_0}{1 - \left\|\bar{G}(s)\right\|_{\mathcal{L}_1} L} \tag{28}$$

where $W_{\max} \triangleq \max_{W\in\Omega} 4\|W\|^2 + 4\Delta^2$. The complete $\mathcal{L}_1$ neural adaptive controller consists of (16) and (18)–(20), subject to (24) with $\mathcal{D}_x$ defined in (25). The closed-loop system with it is illustrated in Fig. 1.

Remark 3: Since the $\mathcal{L}_1$ neural controller will lead only to local results and guarantee $x(t) \in \mathcal{D}_x$ for any $t \ge 0$, as proved in the subsequent sections, the relationships in (12) and (11) can be assumed only for the compact set $\mathcal{D}_x$.

V. Analysis of $\mathcal{L}_1$ Adaptive Controller

A. Reference System

In this section, we characterize the reference system that the $\mathcal{L}_1$ neural controller in (16) and (18)–(20) tracks both in transient and steady state, and this tracking is valid for system's

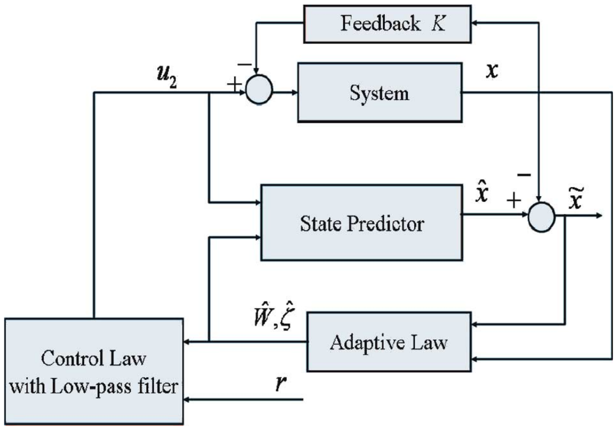


Fig. 1. Closed-loop system with $\mathcal{L}_1$ neural adaptive controller.

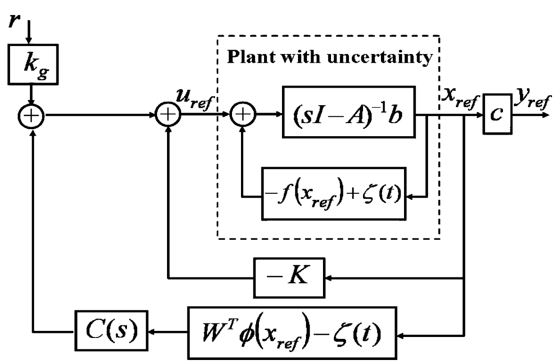


Fig. 2. Closed-loop reference system.

both input and output signals. Towards that end, consider the following ideal version of the adaptive controller in (16) and (20):

$$u_{\rm ref}(s) = k_g r(s) + \eta(s) - K^\top x_{\rm ref}(s) \tag{29}$$

where $\eta(s)$ is the filtered output of $W^\top \phi(x_{\rm ref}(t)) - \zeta(t)$ by $C(s)$ and $x_{\rm ref}(s)$ is introduced to denote the Laplace transformation of the closed-loop system state with the controller (29). If $x_{\rm ref}(t) \in \mathcal{D}_x$, the control law in (29) leads to the following closed-loop dynamics:

$$\begin{aligned} x_{\rm ref}(s) &= G(s)r(s) + \bar{G}(s)\eta_1(s) - H_o(s)\epsilon_{\rm ref}(s) \\ y_{\rm ref}(s) &= c^\top x_{\rm ref}(s) \end{aligned} \tag{30}$$

where $\epsilon_{\rm ref}(s)$ and $\eta_1(s)$ are the Laplace transformations of $\epsilon(x_{\rm ref}(t))$ and $W^\top\phi(x_{\rm ref}(t)) - \zeta(t)$, and $x_{\rm ref}(0) = x_0$. The closed-loop system with the controller (29) is given in Fig. 2. The following lemma states that the closed-loop system with the controller (29) is stable, and its state $x_{\rm ref}(t)$ remains inside $\mathcal{D}_x$ for all $t \geq 0$.

Lemma 5: The control signal given by (29), subject to the condition in (24), ensures that the state of the closed-loop system in (30) remains inside $\mathcal{D}_x$ for all $t \geq 0$

$$\|x_{\rm ref}\|_{\mathcal{L}_\infty} \leq \gamma_r \tag{31}$$

where γ_r is defined in (26).

Proof: We will prove the lemma by contradiction. Assume that (31) is not true. Since $\|x_{\rm ref}(0)\|_\infty = 0 \leq \gamma_r$ and $x_{\rm ref}(t)$ is continuous, there exists t' such that $\|x_{\rm ref}(t')\|_\infty > \gamma_r$ and $\|x_{\rm ref}(\tau)\|_\infty \leq \gamma_r + \gamma_1$ for any $\tau \in [0, t']$, which consequently leads to the following inequalities for $\|x_{ref_{t'}}\|_{\mathcal{L}_\infty}$:

$$\gamma_r < \left\|x_{ref_{t'}}\right\|_{\mathcal{L}_\infty} \leq \gamma_r + \gamma_1. \tag{32}$$

It follows from (25) that:

$$x_{\rm ref}(\tau) \in \mathcal{D}_x \qquad \forall \tau \in [0, t']$$

and hence, (15) implies that

$$\left\|\epsilon_{ref_{t'}}\right\|_{\mathcal{L}_\infty} \leq \epsilon^*.$$

For the closed-loop system state in (30), application of Lemma 1 leads to the following result:

$$\begin{aligned} \left\|x_{ref_{t'}}\right\|_{\mathcal{L}_\infty} \leq \|G(s)\|_{\mathcal{L}_1} \|r_{t'}\|_\infty + \left\|\bar{G}(s)\right\|_{\mathcal{L}_1} \left\|\eta_{1_{t'}}\right\|_{\mathcal{L}_\infty} \\ + \|H_o(s)\|_{\mathcal{L}_1} \epsilon^*. \end{aligned} \tag{33}$$

Notice that $\eta_1(\tau)$ can be upper bounded as

$$\begin{aligned} \eta_1(\tau) &\triangleq W^\top \phi\left(x_{\rm ref}(\tau)\right) - \zeta(t) \\ &= f\left(x_{\rm ref}(\tau)\right) - \epsilon\left(x_{\rm ref}(\tau)\right) - \zeta(t) \\ &\leq |f(0)| + L\,\|x(\tau)\|_\infty + \epsilon^* + \Delta \end{aligned}$$

for all $\tau \in [0, t']$ and, hence

$$\left\|\eta_{1_{t'}}\right\|_{\mathcal{L}_\infty} \leq |f(0)| + L\left\|x_{ref_{t'}}\right\|_{\mathcal{L}_\infty} + \epsilon^* + \Delta.$$

This consequently leads to the next upper bound for the expression in (33)

$$\begin{aligned} \left\|x_{ref_{t'}}\right\|_{\mathcal{L}_\infty} &\leq \|G(s)\|_{\mathcal{L}_1} \|r_{t'}\|_\infty + \left\|\bar{G}(s)\right\|_{\mathcal{L}_1} \\ &\times \left(|f(0)| + L\left\|x_{ref_{t'}}\right\|_{\mathcal{L}_\infty} + \epsilon^* + \Delta\right) + \|H_o(s)\|_{\mathcal{L}_1} \epsilon^*. \end{aligned}$$

Using the condition in (24) and the upper bound in (12), one gets

$$\begin{aligned} \left\|x_{ref_{t'}}\right\|_{\mathcal{L}_\infty} &\leq \left(\|G(s)\|_{\mathcal{L}_1} \|r_{t'}\|_{\mathcal{L}_\infty} + \|H_o(s)\|_{\mathcal{L}_1} \epsilon^* \right. \\ &\left. + \left\|\bar{G}(s)\right\|_{\mathcal{L}_1} (B + \epsilon^* + \Delta)\right) / \left(1 - \left\|\bar{G}(s)\right\|_{\mathcal{L}_1} L\right). \end{aligned} \tag{34}$$

Since $\|r_{t'}\|_{\mathcal{L}_\infty} \leq \|r\|_{\mathcal{L}_\infty}$, then it is straightforward to see that

$$\left\|x_{ref_{t'}}\right\|_{\mathcal{L}_\infty} \leq \gamma_r$$

which contradicts $\gamma_r < \|x_{ref_{t'}}\|_{\mathcal{L}_\infty}$ in (32). Hence, the relationship in (31) holds, and this completes the proof. □

Thus, the control signal ensures that for any $t \geq 0$ the state $x_{\rm ref}(t) \in \mathcal{D}_x$, on which the RBF approximation has been defined, and therefore, $\|\epsilon_{\rm ref}\|_{\mathcal{L}_\infty} \leq \epsilon^*$.

B. Uniform Boundedness and Guaranteed Transient Performance of $\mathcal{L}_1$ NN Adaptive Controller

We note that $(A - bK^\top, b)$ is the state–space realization of $H_o(s)$. Since (A, b) is controllable, it can be proved straightforwardly that $(A-bK^\top, b)$ is also controllable. It follows from Lemma 4 that there exists $c_o \in \mathbb{R}^n$ such that

$$c_o^\top H_o(s) = \frac{N_n(s)}{N_d(s)} \tag{35}$$

where the order of $N_d(s)$ is one more than the order of $N_n(s)$, and both $N_n(s)$ and $N_d(s)$ are stable polynomials.

Theorem 1: Given the system in (10), the reference system in (29) and (30), and the $\mathcal{L}_1$ NN adaptive controller defined via (16) and (18)–(20), subject to (24), we have

$$\|x-x_{\rm ref}\|_{\mathcal{L}_\infty} \le \gamma_1 \tag{36}$$
$$\|y-y_{\rm ref}\|_{\mathcal{L}_\infty} \le \|c^\top\|_{\mathcal{L}_1}\gamma_1 \tag{37}$$
$$\|u-u_{\rm ref}\|_{\mathcal{L}_\infty} \le \gamma_2 \tag{38}$$

where γ_1 is defined in (28)

$$\gamma_2 = \left\|C(s)\frac{1}{c_o^\top H_o(s)}c_o^\top\right\|_{\mathcal{L}_1}\gamma_0 + 3\left\|C(s)\right\|_{\mathcal{L}_1}\epsilon^* + \left(\|C(s)\|_{\mathcal{L}_1}L + \|K^\top\|_{\mathcal{L}_1}\right)\gamma_1 \tag{39}$$

and $\|c^\top\|_{\mathcal{L}_1}$ and $\|K^\top\|_{\mathcal{L}_1}$ are the $\mathcal{L}_1$ gains of $c^\top$ and $K^\top$, respectively.

Proof: The proof will be done by contradiction. Assume that (36) is not true. Then, since $\|x(0)-x_{\rm ref}(0)\|_\infty = 0 \le \gamma_1$, and both $x(t)$ and $x_{\rm ref}(t)$ are continuous, there exists t' such that $\|x(t')-x_{\rm ref}(t')\|_\infty > \gamma_1$ and $\|x(\tau)-x_{\rm ref}(\tau)\|_\infty \le \gamma_1+\sigma$ for any $\tau \in [0,t']$. This leads to the following inequalities:

$$\gamma_1 < \|(x-x_{\rm ref})_{t'}\|_{\mathcal{L}_\infty} \le \gamma_1 + \sigma \tag{40}$$

which, consequently, implies that

$$x(\tau) \in \mathcal{D}_x \qquad \forall\, \tau \in [0,t'].$$

Since $x(\tau) \in \mathcal{D}_x$, and the regressor in the state predictor in (18) operates over x, one can use the relationships in (15) and (17) to derive the following error dynamics:

$$\dot{\tilde{x}}(\tau) = A_m\tilde{x}(\tau) - b\tilde{W}^\top(\tau)\Phi\left(x(\tau)\right) + b\tilde{\zeta}(\tau) + b\epsilon\left(x(\tau)\right)$$
$$\tilde{x}(0) = 0 \qquad \forall\, \tau \in [0,t'] \tag{41}$$

where $\tilde{W}(\tau) = \hat{W}(\tau) - W$ and $\tilde{\zeta}(\tau) = \hat{\zeta}(\tau) - \zeta(\tau)$. Consider the following candidate Lyapunov function:

$$V\left(\tilde{x}(\tau), \tilde{W}(\tau)\right) = \tilde{x}^\top(\tau)P\tilde{x}(\tau) + \frac{\tilde{W}^\top(\tau)\tilde{W}(\tau)}{\Gamma_c} + \frac{\tilde{\zeta}^2(\tau)}{\Gamma_c}.$$

The following is straightforward to derive:

$$\dot{V}(\tau) \le -\tilde{x}^\top(\tau)Q\tilde{x}(\tau) + \frac{2\tilde{x}^\top(\tau)Pb\dot{\zeta}(\tau)}{\Gamma_c} + 2\tilde{x}^\top(\tau)Pb\epsilon\left(x(\tau)\right)$$

for any $\tau \in [0,t']$. Therefore, $\dot{V}(\tau) \le 0$ for any $\tau \in [0,t']$ if

$$\|\tilde{x}(\tau)\| \ge \frac{2(\epsilon^* + d_\Delta/\Gamma_c)\|Pb\|}{\lambda_{\min}(Q)}.$$

The projection algorithm ensures that $\hat{W}(\tau) \in \Omega \forall \tau \in [0,t']$, and therefore

$$\max_{\tau\in[0,t']}\left(\Gamma_c^{-1}\tilde{W}^\top(\tau)\tilde{W}(\tau) + \Gamma_c^{-1}\tilde{\zeta}^2(\tau)\right) \le \frac{W_{\max}}{\Gamma_c}$$

where $W_{\max}$ is defined in (27). Thus, $\dot{V}(\tau) \le 0$, $\tau \in [0,t']$, outside the compact set

$$\left\{\|\tilde{x}\| \le \frac{2(\epsilon^* + d_\Delta/\Gamma_c)\|Pb\|}{\lambda_{\min}(Q)}\right\} \bigcap \left\{\left\|[\tilde{W}^\top \tilde{\zeta}]^\top\right\| \le \sqrt{W_{\max}}\right\}. \tag{42}$$

This consequently implies that for any $\tau \in [0,t']$

$$\|\tilde{x}(\tau)\| \le \gamma_0$$

and, hence,

$$\|\tilde{x}_{t'}\|_{\mathcal{L}_\infty} \le \gamma_0. \tag{43}$$

Moreover, using the upper bounds in (40) and (31) along with $\|\tilde{x}(\tau)\| \le \gamma_0$, implies that

$$\hat{x}(\tau) \in \mathcal{D}_x \qquad \forall \tau \in [0,t'].$$

From (21) and (30), one can write

$$\begin{aligned}\hat{x}(s) - x_{\rm ref}(s) &= \bar{G}(s)\left(\bar{r}(s) - \eta_1(s)\right) + H_o(s)\epsilon_{\rm ref}(s)\\ &= \bar{G}(s)\left(\eta_2(s) - \eta_3(s) + \eta_4(s)\right)\\ &\quad + H_o(s)\epsilon_{\rm ref}(s)\end{aligned} \tag{44}$$

where $\eta_2(s)$, $\eta_3(s)$, and $\eta_4(s)$ are the Laplace transformations of signals $\tilde{W}^\top(t)\phi(x(t)) - \tilde{\zeta}(t)$, $W^\top(\phi(\hat{x}(t)) - \phi(x(t)))$, and $W^\top(\phi(\hat{x}(t)) - \phi(x_{\rm ref}(t)))$, respectively. The relationship in (41) leads to

$$\tilde{x}(s) = H_o(s)\left(-\eta_2(s) + \epsilon_{\rm ref}(s)\right). \tag{45}$$

Recalling the definition of $\bar{G}(s)$ from (22), one can further write

$$\begin{aligned}\hat{x}(s) - x_{\rm ref}(s) = {}& \bar{G}(s)\left(-\eta_3(s) + \eta_4(s) + \epsilon(s)\right)\\ & - \left(C(s) - 1\right)\tilde{x}(s) + H_o(s)\epsilon_{\rm ref}(s).\end{aligned}$$

Recalling the RBF approximation in (15), one gets the following upper bound:

$$\left|W^\top\left(\phi\left(\hat{x}(\tau)\right) - \phi\left(x(\tau)\right)\right) - \left(f\left(\hat{x}(\tau)\right) - f\left(x(\tau)\right)\right|\right. \le 2\epsilon^*$$

for $\tau \in [0,t']$, and hence, the relationship in (11) implies that

$$\begin{aligned}\left\|\eta_{3_{t'}}\right\|_{\mathcal{L}_\infty} &= \left\|\left(W^\top\left(\phi(\hat{x}) - \phi(x)\right)\right)_{t'}\right\|_{\mathcal{L}_\infty}\\ &\le \|(f(\hat{x}) - f(x))_{t'}\|_{\mathcal{L}_\infty} + 2\epsilon^*\\ &\le L\|\tilde{x}_{t'}\|_{\mathcal{L}_\infty} + 2\epsilon^*.\end{aligned} \tag{46}$$

Similarly, we have

$$\left\|\eta_{4_{t'}}\right\|_{\mathcal{L}_\infty} \le L\left\|(\hat{x} - x_{\rm ref})_{t'}\right\|_{\mathcal{L}_\infty} + 2\epsilon^*.$$

Lemma 1 leads to the following upper bound:

$$\|(\hat{x}-x_{\rm ref})_{t'}\|_{\mathcal{L}_\infty} \leq \left\|\bar{G}(s)\right\|_{\mathcal{L}_1}\left(L\|(\hat{x}-x_{\rm ref})_{t'}\|_{\mathcal{L}_\infty}+5\epsilon^* \right. \\ \left. +L\|\tilde{x}_{t'}\|_{\mathcal{L}_\infty}\right)+\|C(s)-1\|_{\mathcal{L}_1}\|\tilde{x}_{t'}\|_{\mathcal{L}_\infty}+\|H_o(s)\|_{\mathcal{L}_1}\epsilon^*$$

which along with the condition in (24) reduces to

$$\|(\hat{x}-x_{\rm ref})_{t'}\|_{\mathcal{L}_\infty} \leq \Big(\left\|\bar{G}(s)\right\|_{\mathcal{L}_1}(L\|\tilde{x}_{t'}\|_{\mathcal{L}_\infty}+5\epsilon^*) \\ +\|C(s)-1\|_{\mathcal{L}_1}\|\tilde{x}_{t'}\|_{\mathcal{L}_\infty}+\|H_o(s)\|_{\mathcal{L}_1}\epsilon^*\Big)\Big/\Big(1-\left\|\bar{G}(s)\right\|_{\mathcal{L}_1}L\Big).$$

Since

$$\|(x-x_{\rm ref})_{t'}\|_{\mathcal{L}_\infty} \leq \|(\hat{x}-x_{\rm ref})_{t'}\|_{\mathcal{L}_\infty}+\|\tilde{x}_{t'}\|_{\mathcal{L}_\infty}$$

it follows from (43) that:

$$\|(x-x_{\rm ref})_{t'}\|_{\mathcal{L}_\infty} \leq \gamma_1$$

which contradicts the condition in (40). Hence, the relationship in (36) holds. The upper bound in (37) follows from (36) and Lemma 2 directly.

It follows from (16), (20), and (29) that

$$u(s)-u_{\rm ref}(s)=\eta_6(s)+C(s)\eta_5(s)-K^\top(x(s)-x_{\rm ref}(s))$$

where $\eta_5(s)$ is the Laplace transformation of $W^\top(\phi(x(t))-\phi(x_{\rm ref}(t)))$ and $\eta_6(s)=C(s)\eta_2(s)$. Using the same steps as in (46), we obtain

$$\|\eta_5\|_{\mathcal{L}_\infty} \leq L\|x-x_{\rm ref}\|_{\mathcal{L}_\infty}+2\epsilon^*.$$

From (45), it follows that $\eta_6(s)$ can be presented as

$$\begin{aligned}\eta_6(s) &= C(s)\frac{1}{c_o^\top H_o(s)}c_o^\top H_o(s)\eta_2(s)\\ &= C(s)\frac{1}{c_o^\top H_o(s)}c_o^\top(-\tilde{x}(s)+H_o(s)\epsilon_{\rm ref}(s))\\ &= -C(s)\frac{1}{c_o^\top H_o(s)}c_o^\top\tilde{x}(s)+C(s)\epsilon_{\rm ref}(s)\end{aligned}$$

where c_o is introduced in (35). Using the expression in (35), one can further write

$$C(s)\frac{1}{c_o^\top H_o(s)}=C(s)\frac{N_d(s)}{N_n(s)}$$

where $N_d(s)$ and $N_n(s)$ are stable polynomials and the order of $N_n(s)$ is one less than the order of $N_d(s)$. Since $C(s)$ is stable and strictly proper, the complete system $C(s)(1/c_o^\top H_o(s))$ is proper and stable, which implies that its $\mathcal{L}_1$ gain exists and is finite. Hence, we have

$$\|\eta_6\|_{\mathcal{L}_\infty} \leq \left\|C(s)\frac{1}{c_o^\top H_o(s)}c_o^\top\right\|_{\mathcal{L}_1}\|\tilde{x}\|_{\mathcal{L}_\infty}+\|C(s)\|_{\mathcal{L}_1}\epsilon^*$$

which leads to the following upper bound:

$$\|u-u_{\rm ref}\|_{\mathcal{L}_\infty} \leq \left\|\frac{C(s)}{c_o^\top H_o(s)}c_o^\top\right\|_{\mathcal{L}_1}\|\tilde{x}\|_{\mathcal{L}_\infty} \\ +\left(\|C(s)\|_{\mathcal{L}_1}L+\|K^\top\|_{\mathcal{L}_1}\right)\|x-x_{\rm ref}\|_{\mathcal{L}_\infty}+3\|C(s)\|_{\mathcal{L}_1}\epsilon^*.$$

□

Remark 4: From the relationships in (31) and (36), it is straightforward to verify that

$$\|x\|_{L_\infty} \leq \gamma_r+\gamma_1$$

and hence, $x(t)$ belongs to $\mathcal{D}_x$ for any $t \geq 0$.

Corollary 2: Given the system in (10) and the $\mathcal{L}_1$ NN adaptive controller defined via (16) and (18)– (20), we have

$$\begin{aligned}\lim_{\Gamma_c\to\infty,\epsilon^*\to 0}(x(t)-x_{\rm ref}(t)) &= 0 \qquad \forall t\geq 0\\ \lim_{\Gamma_c\to\infty,\epsilon^*\to 0}(y(t)-y_{\rm ref}(t)) &= 0 \qquad \forall t\geq 0\\ \lim_{\Gamma_c\to\infty,\epsilon^*\to 0}(u(t)-u_{\rm ref}(t)) &= 0 \qquad \forall t\geq 0.\end{aligned}$$

Corollary 2 states that $x(t)$, $y(t)$, and $u(t)$ follow $x_{\rm ref}(t)$, $y_{\rm ref}(t)$, and $u_{\rm ref}(t)$ not only asymptotically but also during the transient, provided that the adaptive gain is selected sufficiently large and the NN approximation is accurate enough. Thus, the control objective is reduced to designing K and $C(s)$ to ensure that the reference system with unknown parameters has the desired response $D(s)$. Before then, the following remarks are in order.

Remark 5: Notice that if we set $C(s)=1$, then the $\mathcal{L}_1$ neural controller degenerates into an MRAC type. In that case, $\|C(s)(1/c_o^\top H_o(s))c_o^\top\|_{\mathcal{L}_1}$ cannot be finite, since $H_o(s)$ is strictly proper. Therefore, from (39) it follows that $\gamma_2 \to \infty$, and hence, for the control signal in conventional MRAC-type NN adaptive controller, one cannot reduce the bound in (38) by increasing the adaptive gain.

Remark 6: Recall that in conventional MRAC scheme the ultimate bound is given by γ_0 defined in (27). Notice that γ_0 depends upon ϵ^*, $W_{\max}$, and Γ_c. While ϵ^* and $W_{\max}$ are interconnected via the choice of RBFs, Γ_c is a design parameter of the adaptive process that can be used to reduce the ultimate bound. However, increasing Γ_c in the conventional MRAC scheme leads the control signal into high-frequency oscillations. With the $\mathcal{L}_1$ adaptive control architecture, the ultimate bound of the tracking error is given by γ_1 in (36). From the definition of it in (28), it follows that $\gamma_1 > \gamma_0$. Nevertheless, the ability of the $\mathcal{L}_1$ control architecture to tolerate large adaptive gain implies that γ_0 can be reduced leading to overall a smaller value for γ_1. This is enabled via the low-pass system $C(s)$ in the feedback path that filters out the high-frequencies in $\eta_1(t)$ excited by large Γ_c. The $\mathcal{L}_1$ adaptive control architecture gives a scheme for fast adaptation without generating high-frequency oscillations in the control signal.

VI. Design of the $\mathcal{L}_1$ Neural Controller

We proved that the error between the state and the control signal of the closed-loop system with $\mathcal{L}_1$ neural controller in (10), (16), and (18)–(20) (Fig. 1) and the state and the control signal of the closed-loop reference system in (29) and (30) (Fig. 2) can be rendered arbitrarily small by increasing the adaptive gain. Therefore, the control objective is reduced to determining K and $C(s)$ to ensure that the reference system in (29) and (30) (Fig. 2) has the desired response $D(s)$ from $r(t)$ to $y_{\rm ref}(t)$. Notice that the reference system in Fig. 2 is nonlinear and depends upon the unknown nonlinearity $f(x)$.

Consider the following signals:

$$y_{\rm des}(s) \triangleq c^\top x_{\rm des}(s) \triangleq c^\top G(s)r(s) = k_g c^\top H_o(s) r(s) \quad (47)$$

where $x_{\rm des}(0) = x_0$ and

$$u_{\rm des}(s) \triangleq k_g r(s) - K^\top x_{\rm des}(s) + C(s)\eta_7(s) \quad (48)$$

where $\eta_7(s)$ is the Laplace transformation of $f(x_{\rm des}(t)) - \zeta(t)$. We note that $u_{\rm des}(t)$ depends on the system uncertainty $f(x(t))$, while $y_{\rm des}(t)$ does not. Since $r(t)$ is bounded and $G(s)$ is stable, $x_{\rm des}(t)$ is also bounded and we can straightforwardly derive its upperbound: $\|x_{\rm des}\|_{\mathcal{L}_\infty} \leq \|G(s)\|_{\mathcal{L}_1}\|r\|_{\mathcal{L}_\infty}$. It is straightforward to verify that $x_{\rm des}(t) \in \mathcal{D}_x$ for any $t \geq 0$.

Lemma 6: For the system in (10) and the reference system in (29) and (30), subject to (24), the following upper bounds hold:

$$\|x_{\rm ref} - x_{\rm des}\|_{\mathcal{L}_\infty} \leq \gamma_3 \quad (49)$$

$$\|y_{\rm ref} - y_{\rm des}\|_{\mathcal{L}_\infty} \leq \|c^\top\|_{\mathcal{L}_1}\gamma_3 \quad (50)$$

$$\|u_{\rm ref} - u_{\rm des}\|_{\mathcal{L}_\infty} \leq \left(\|K^\top\|_{\mathcal{L}_1} + \|C(s)\|_{\mathcal{L}_1} L\right)\gamma_3 + \|C(s)\|_{\mathcal{L}_1}\epsilon^* \quad (51)$$

where $\gamma_3 = \|\bar{G}(s)\|_{\mathcal{L}_1}(\|f(0)\|_\infty + L\gamma_r + \epsilon^* + \Delta) + \|H_o(s)\|_{\mathcal{L}_1}\epsilon^*$.

Proof: It follows from (30) and (47) that:

$$x_{\rm ref}(s) - x_{\rm des}(s) = \bar{G}(s)\eta_1(s) - H_o(s)\epsilon_{\rm ref}(s)$$

and, hence

$$\|x_{\rm ref} - x_{\rm des}\|_{\mathcal{L}_\infty} \leq \left\|\bar{G}(s)\right\|_{\mathcal{L}_1}\|\eta_1\|_{\mathcal{L}_\infty} + \|H_o(s)\|_{\mathcal{L}_1}\epsilon^*. \quad (52)$$

Since

$$\eta_1(t) = f\left(x_{\rm ref}(t)\right) - \epsilon\left(x_{\rm ref}(t)\right) - \zeta(t)$$

and

$$\|f\left(x_{\rm ref}(t)\right)\|_\infty \leq \|f(0)\|_\infty + L\left\|x_{\rm ref}(t)\right\|_\infty$$

we have

$$\|\eta_1\|_{\mathcal{L}_\infty} \leq \|f(0)\|_\infty + L\|x_{\rm ref}\|_{\mathcal{L}_\infty} + \epsilon^* + \Delta.$$

From Lemma 5, we have $\|x_{\rm ref}\|_{\mathcal{L}_\infty} \leq \gamma_r$, which leads to

$$\|x_{\rm ref} - x_{\rm des}\|_{\mathcal{L}_\infty} \leq \left\|\bar{G}(s)\right\|_{\mathcal{L}_1}(\|f(0)\|_\infty + L\gamma_r + \epsilon^* + \Delta) + \|H_o(s)\|_{\mathcal{L}_1}\epsilon^*. \quad (53)$$

The relationship in (50) follows directly.

Using the relationships in (29) and (48), we get

$$\|u_{\rm ref} - u_{\rm des}\|_{\mathcal{L}_\infty} \leq \|K^\top\|_{\mathcal{L}_1}\|x_{\rm ref} - x_{\rm des}\|_{\mathcal{L}_\infty} + \|C(s)\|_{\mathcal{L}_1}\|\eta_8 - \eta_7\|_{\mathcal{L}_\infty} + \|C(s)\|_{\mathcal{L}_1}\epsilon^* \quad (54)$$

where

$$\eta_8(t) = f\left(x_{\rm ref}(t)\right).$$

Using the Lipschitz condition for $f(\cdot)$ from (11), it follows that:

$$\|\eta_7 - \eta_8\|_{\mathcal{L}_\infty} \leq L\|x_{\rm ref} - x_{\rm des}\|_{\mathcal{L}_\infty}$$

which, when substituted back into (54), leads to the upper bound in (51). □

We notice that the condition in (24) is crucial for characterization of the transient performance, as stated by the bounds in Lemma 6. Thus, the problem is reduced to finding a strictly proper stable $C(s)$ and a gain K to satisfy the performance requirement in (24). It follows from (49) that for achieving $y_{\rm ref}(s) \approx y_{\rm des}(s)$ it is desirable to ensure that $\|\bar{G}(s)\|_{\mathcal{L}_1}$ is sufficiently small. Minimization of $\|\bar{G}(s)\|_{\mathcal{L}_1}$ can be achieved from the following two different perspectives: 1) fix $C(s)$ and minimize $\|H_o(s)\|_{\mathcal{L}_1}$ and 2) fix $H_o(s)$ and minimize the $\mathcal{L}_1$-gain of the cascaded systems $\|H_o(s)(C(s)-1)\|_{\mathcal{L}_1}$ via the choice of $C(s)$.

1) High-gain design. Set $C(s) = D(s)$, where $D(s)$ is introduced in (14). Then, minimization of $\|H_o(s)\|_{\mathcal{L}_1}$ can be achieved via high-gain feedback by choosing K sufficiently large. Minimized $\|H_o(s)\|_{\mathcal{L}_1}$ via large K leads to large poles of $H_o(s)$. Since $C(s)$ is a strictly proper system containing the dominant poles of the closed-loop system in $k_g c^\top H_o(s)C(s)$ and $k_g c^\top H_o(0) = 1$, we have $k_g c^\top H_o(s)C(s) \approx C(s) = D(s)$. Hence, the system response will be $y_{\rm ref}(s) \approx D(s)r(s)$. We note that with large feedback K, the performance of $\mathcal{L}_1$ neural controller degenerates into a high-gain type. The shortcoming of this design is that the high-gain feedback K leads to a reduced phase margin and, consequently, affects robustness.
2) Design without linear feedback. As in MRAC, assume that we can select A to ensure

$$k_g c^\top H_o(s) \approx D(s).$$

Let

$$C(s) = \frac{\omega}{s+\omega}. \quad (55)$$

Lemma 7: For any single-input–n-output strictly proper stable system $H_o(s)$, the following is true:

$$\lim_{\omega\to\infty}\|(C(s)-1)\,H_o(s)\|_{\mathcal{L}_1} = 0.$$

Proof: It follows from (55) that:

$$(C(s)-1)\,H_o(s) = \frac{-s}{s+\omega}H_o(s) = \frac{-1}{s+\omega}sH_o(s).$$

Since $H_o(s)$ is strictly proper and stable, $sH_o(s)$ is stable and has relative degree ≥ 0, and hence, $\|sH_o(s)\|_{\mathcal{L}_1}$ is finite. Since $\|(-1)/(s+\omega)\|_{\mathcal{L}_1} = 1/\omega$, it follows from (2) that:

$$\|(C(s)-1)\,H_o(s)\|_{\mathcal{L}_1} \leq \frac{1}{\omega}\|sH_o(s)\|_{\mathcal{L}_1}$$

□

Lemma 7 states that if one chooses $k_g c^\top H_o(s)r(s) \approx D(s)$, then by increasing the bandwidth of the low-pass system $C(s)$, it is possible to render $\|\bar{G}(s)\|_{\mathcal{L}_1}$ arbitrarily small, and, hence

$$y_{\rm ref}(s) \approx k_g c^\top H_o(s)r(s) \approx D(s)r(s).$$

We note that $k_g c^\top H_o(s)$ is exactly the reference model of the MRAC design. Therefore, this approach is equivalent to

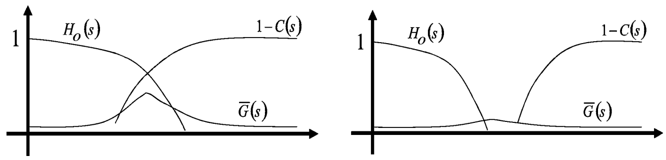


Fig. 3. Cascaded systems.

mimicking MRAC, and, hence, high-gain feedback can be completely avoided.

However, increasing the bandwidth of $C(s)$ is not the only choice for minimizing $\|\bar{G}(s)\|_{\mathcal{L}_1}$. Since $C(s)$ is a low-pass filter, its complementary $1 - C(s)$ is a high-pass filter with its cutoff frequency approximating the bandwidth of $C(s)$. Since both $H_o(s)$ and $C(s)$ are strictly proper systems, $\bar{G}(s) = H_o(s)(C(s) - 1)$ is equivalent to cascading a low-pass system $H_o(s)$ with a high-pass system $C(s) - 1$. If one chooses the cutoff frequency of $C(s) - 1$ larger than the bandwidth of $H_o(s)$, it ensures that $\bar{G}(s)$ is a "no-pass" system, and hence, its $\mathcal{L}_1$ gain can be rendered arbitrarily small. This can be achieved via higher order filter design methods. The illustration is given in Fig. 3.

Remark 7: From Corollary 2 and Lemma 6 it follows that the $\mathcal{L}_1$ adaptive controller can generate a system response to track (47) and (48) both in transient and steady state if we set the adaptive gain large and minimize $\|\bar{G}(s)\|_{\mathcal{L}_1}$. Notice that $u_{\rm des}(t)$ in (48) depends upon the unknown nonlinearity $f(x(t))$, while $y_{\rm des}(t)$ in (47) does not. This implies that for different nonlinearities $f(x)$, the $\mathcal{L}_1$ neural controller will generate different control signals [dependent on $f(x(t))$] to ensure uniform system response [independent of $f(x(t))$]. This is natural, since different nonlinearities imply different systems, and to have similar response for different systems the control signals have to be different. Here is the obvious advantage of the $\mathcal{L}_1$ neural controller in a sense that it controls a partially known system as a nonadaptive feedback controller would have done if the unknown nonlinearities were known. Finally, we note that if the term $C(s)\eta_7(s)$ is dominated by $-K^\top x(s)$, then the controller in (48) turns into a high-gain type, and, consequently, the $\mathcal{L}_1$ adaptive controller degenerates into a high-gain design.

Remark 8: We notice that the following limiting relationships:

$$\lim_{\|\bar{G}(s)\|_{\mathcal{L}_1} \to 0, \epsilon^* \to 0} \gamma_r \to \|G(s)\|_{\mathcal{L}_1} \|r\|_{\mathcal{L}_\infty}$$
$$\lim_{\Gamma_c \to \infty, \epsilon^* \to 0} \gamma_0 \to 0$$
$$\lim_{\Gamma_c \to \infty, \epsilon^* \to 0} \gamma_1 \to 0$$
$$\lim_{\|\bar{G}(s)\|_{\mathcal{L}_1} \to 0, \epsilon^* \to 0} \gamma_3 \to 0$$

imply that the set $\mathcal{D}_x$ of the RBF distribution converges to the set

$$\mathcal{D}_d = \left\{x \in \mathbb{R}^n \middle| \|x\|_\infty \le \|G(s)\|_{\mathcal{L}_1} \|r\|_{\mathcal{L}_\infty}\right\}$$

while $\mathcal{D}_d$ contains all possible trajectories $x_{\rm des}(t)$ for a bounded input $r(t)$. In the presence of the limiting relationships $\|\bar{G}(s)\|_{\mathcal{L}_1} \to 0, \Gamma_c \to \infty$, and $\epsilon^* \to 0$, Theorem 1 and Lemma 6 imply that both system response $x(t)$ and control signal $u(t)$ follow the corresponding desired signals of $x_{\rm des}(t)$ and $u_{\rm des}(t)$, respectively. Without $\|\bar{G}(s)\|_{\mathcal{L}_1} \to 0, \Gamma_c \to \infty$, and $\epsilon^* \to 0$, the corresponding $\mathcal{L}_\infty$ bounds are explicitly given for both system response and control signals in Theorem 1 and Lemma 6.

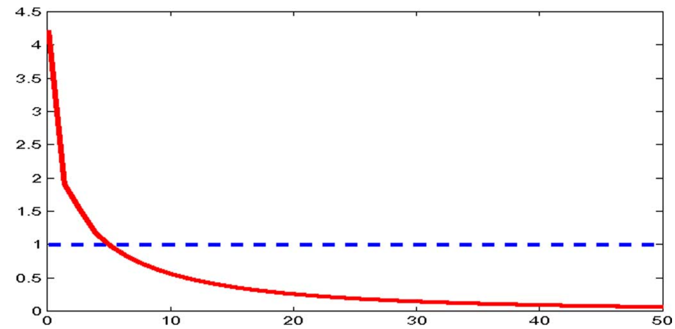


Fig. 4 $\|\bar{G}(s)\|_{\mathcal{L}_1} L$ (solid) with respect to ω and constant 1 (dashed). $\|\bar{G}(s)\|_{\mathcal{L}_1} L$ (solid) defined in (58).

Remark 9: To accommodate nonzero initial conditions, one needs to define the compact set $\mathcal{D}_x$ for RBF approximation larger to ensure that all the trajectories of $x(t)$, $x_{\rm ref}(t)$, and $x_{\rm des}(t)$ remain inside $\mathcal{D}_x$ for all $t \ge 0$. It is straightforward to verify that the bounds given in Theorem 1 and Lemma 6 hold for nonzero initial values as well, as long as $x(t)$, $x_{\rm ref}(t)$, and $x_{\rm des}(t)$ do not leave $\mathcal{D}_x$. Towards this end, notice that

$$\|x\|_{\mathcal{L}_\infty} \le \|x_{\rm des}\|_{\mathcal{L}_\infty} + \|x_{\rm ref} - x_{\rm des}\|_{\mathcal{L}_\infty} + \|x - x_{\rm ref}\|_{\mathcal{L}_\infty}$$

leading to

$$\|x\|_{\mathcal{L}_\infty} \le \|G(s)\|_{\mathcal{L}_1} \|r\|_{\mathcal{L}_\infty} + \gamma_1 + \gamma_3$$

where $\|G(s)\|_{\mathcal{L}_1}\|r\|_{\mathcal{L}_\infty}$ is the bound for $x_{\rm des}(t)$. Since the closed-loop desired system has linear response, the bound for $x_{\rm des}(t)$ can be computed for arbitrary initial condition, and the definition of the set $\mathcal{D}_x$ can be modified accordingly.

Remark 10: We notice that the performance bounds in (36)–(38) were computed assuming zero trajectory initialization error in (41), i.e., $\tilde{x}_0 = 0$, which is in the spirit of the methods for transient performance improvement in [38]. In [39], we have proved that nonzero trajectory initialization error leads only to an exponentially decaying term in both system state and control signal, without affecting the performance throughout.

VII. Simulations

Consider the following nonlinear system:

$$\dot{x}(t) = Ax(t) - bf(x(t)) + b\zeta(t)) + bu(t), \qquad x(0) = x_0 = 0 \tag{56}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

and $x = [x_1\ x_2]^\top \in \mathbb{R}^2$ is the measurable state vector, $u \in \mathbb{R}$ is the control signal, $f(x)$ is an unknown nonlinear function of system states, and $\zeta(t)$ is an additional time-varying bounded

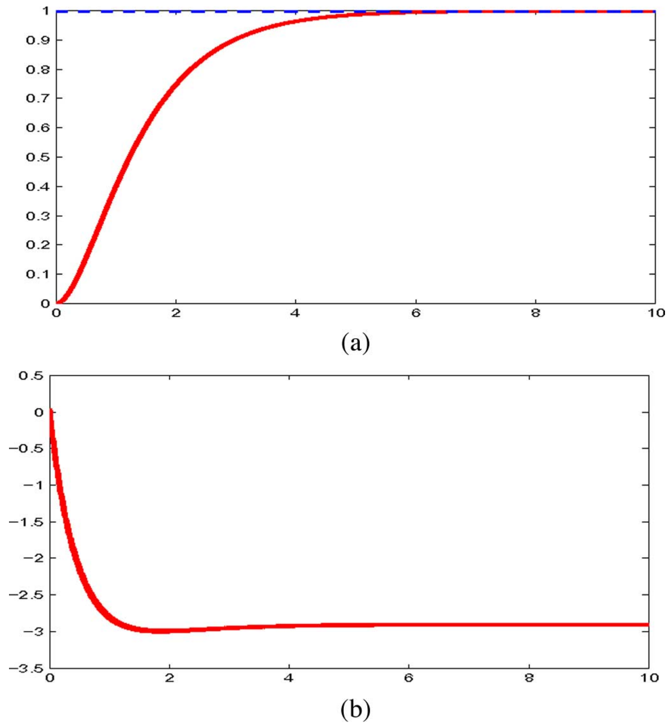


Fig. 5. Performance of $\mathcal{L}_1$ neural controller for $r = 1$ and $\zeta(t) = 0$. (a) $y(t)$ (solid) and $r(t)$ (dashed); (b) time history of $u(t)$.

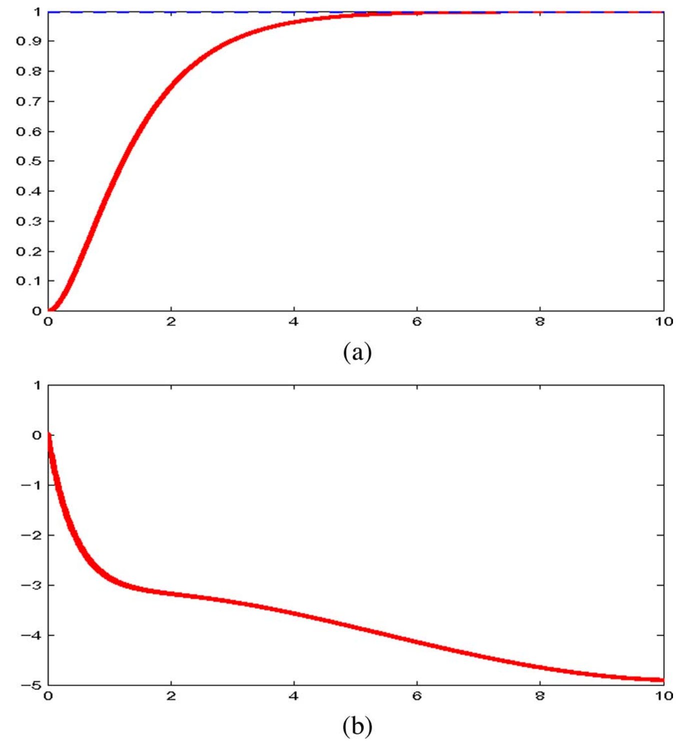


Fig. 6. Performance of $\mathcal{L}_1$ neural controller for $r = 1$ and $\zeta(t) = 1 - \cos(0.3t)$. (a) $y(t)$ (solid) and $r(t)$ (dashed); (b) time history of $u(t)$.

disturbance. The control objective is to design an NN adaptive controller to ensure that $x_1(t)$ tracks any continuous $r(t)$, subject to $|r(t)| \leq 1$, both in transient and steady state. Let

$$L = 4 \quad B = 3. \tag{57}$$

In the following simulations, we consider the uncertainty

$$f(x) = -x_1 - \left(0.5 - 0.1x_1^2\right)x_2 - \frac{2}{1 + 0.05x_1^2}$$

for which the value of $L = 4$ is a sufficiently conservative estimate for the bound in (11), and $B = 3$ is a similarly conservative for the upper bound in (12) on the compact set of RBF distribution.

We choose $K = [2, 3]^\top$ for (16), and implement the following state predictor as:

$$\dot{\hat{x}}(t) = A_m\hat{x}(t) - b\hat{W}^\top(t)\phi(x(t)) + bu_2(t) + b\hat{\zeta}(t), \qquad \hat{x} = x_0$$

where $A_m = A - bK^\top$ and b are the same as in (56), $\hat{W}^\top(t)$ is updated following (19) with

$$Q = 50I_{2\times 2}$$
$$P = \begin{bmatrix} 62.5 & 12.5 \\ 12.5 & 12.5 \end{bmatrix}$$
$$\Gamma_c = 5000.$$

Since we have no knowledge of the uncertainty $f(x)$ except for some conservative bounds, we set $\hat{W}(0) = \hat{W}_0 = 0$. The adaptive increment $u_2(t)$ is implemented following (20) with

$$C(s) = \frac{3\omega^2 s + \omega^3}{(s + \omega)^3}$$

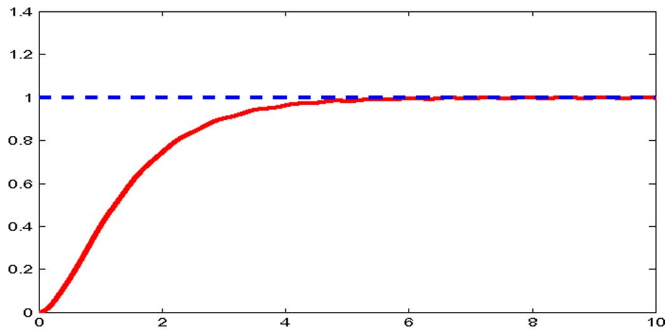
and $\omega = 30$. The state predictor is designed with the use of nine identical Gaussian RBFs. They are distributed over the grids $x_1 \in [-3, 3]$ and $x_2 \in [-3, 3]$ with the step size equal to two in both dimensions and width $\delta_i = 2$. We set the norm of upper bound for the projection operator to $W^* = 15$, i.e., the weight parameter of every RBF is restricted to the set $[-15, 15]$. In Fig. 4(a), we plot

$$\left\|\bar{G}(s)\right\|_{\mathcal{L}_\infty} L = \left\| \begin{bmatrix} \dfrac{1}{s^2 + 3s + 2} & \dfrac{3\omega^2 s + \omega^3}{(s+\omega)^3} \\ \dfrac{s}{s^2 + 3s + 2} & \dfrac{3\omega^2 s + \omega^3}{(s+\omega)^3} \end{bmatrix} \right\|_{\mathcal{L}_1} L \tag{58}$$

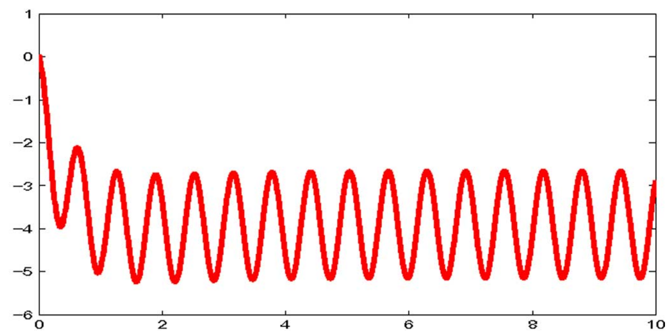
with respect to ω and compare it to 1. We notice that when $\omega > 5$, we have $\|\bar{G}(s)\|_{\mathcal{L}_1} L < 1$ and the $\mathcal{L}_1$-gain requirement in (24) is satisfied. By setting $\omega = 30$, we have $\|\bar{G}(s)\|_{\mathcal{L}_1} = 0.036 < (1/L) = 0.25$, which leads to smaller bounds for $\|x - x_{\text{ref}}\|_{\mathcal{L}_1}$ and $\|x_{\text{ref}} - x_{\text{des}}\|_{\mathcal{L}_1}$.

For a constant reference input $r = 1$ and $\zeta(t) = 0$, the system response and the control signal are plotted in Fig. 5(a) and (b). We now consider the performance of the $\mathcal{L}_1$ adaptive controller in the presence of additional disturbance signal. The plots are given in Figs. 6(a) and 7(b) for disturbances $\zeta(t) = 1 - \cos(0.3t)$ and $\zeta(t) = 1 - \cos(10t)$, respectively. We note that the control signal compensates for the unknown disturbance leading to the desired system response.

The performance of the $\mathcal{L}_1$ NN adaptive controller for the reference input $r(t) = \cos(0.2t)$ in the presence of $\zeta(t) = 1 - \cos(0.3t)$ is plotted in Fig. 8(a) and (b). Fig. 9(a) and (b) plots the simulation results for a different disturbance signal $\zeta(t) = 1 - \cos(10t)$. If these frequencies in the control signal are undesirable, then one can reduce the bandwidth of $C(s)$,

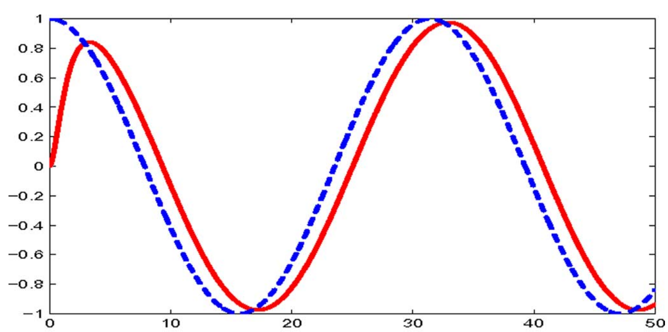


(a)

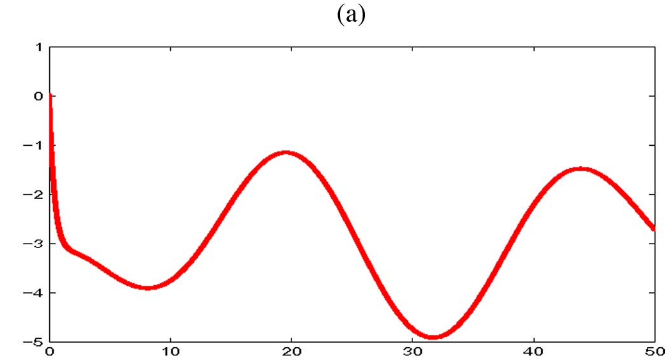


(b)

Fig. 7. Performance of $\mathcal{L}_1$ neural controller for $r = 1$ and $\zeta(t) = 1 - \cos(10t)$. (a) $y(t)$ (solid) and $r(t)$ (dashed); (b) time history of $u(t)$.



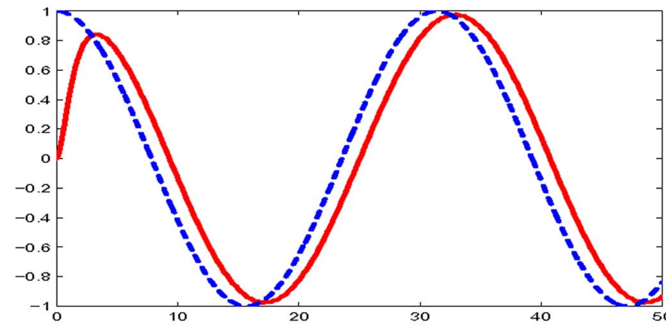
(a)



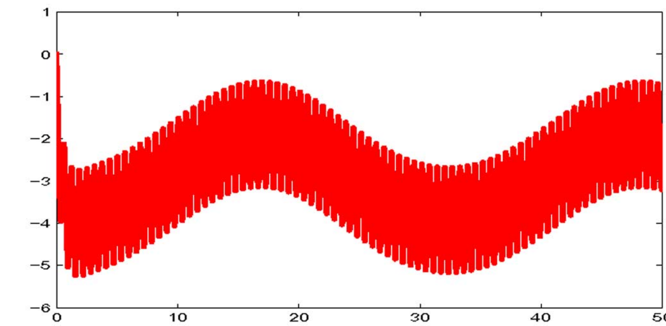
(b)

Fig. 8 Performance of $\mathcal{L}_1$ neural controller for $r = \cos(0.2t)$ and $\zeta(t) = 1 - \cos(0.3t)$. (a) $y(t)$ (solid) and $r(t)$ (dashed); (b) time history of $u(t)$.

and the corresponding bounds for the degradation in the performance can be computed from the analysis of the $\mathcal{L}_1$ adaptive controller.



(a)



(b)

Fig. 9. Performance of $\mathcal{L}_1$ neural controller for $r = \cos(0.2t)$ and $\zeta(t) = 1 - \cos(10t)$. (a) $y(t)$ (solid) and $r(t)$ (dashed); (b) time history of $u(t)$.

VIII. Conclusion

A novel NN adaptive control architecture is presented that has guaranteed transient response in addition to stable tracking. The new low-pass control architecture adapts fast without generating high-frequency oscillations in the control signal, which is otherwise not possible to achieve using conventional MRAC-based NN adaptive controllers. The compact set on which NN approximation is performed is given explicitly. Extension of the methodology to MIMO systems with unknown control effectiveness is reported in [40].

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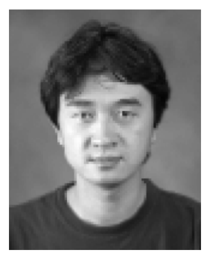

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