Adaptive Control with Model Based Supervision and Non-convex Parameter Estimation

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Abstract

Model based supervision and constrained parameter estimation prevent parameter drift and ensure that the estimated models are admissible. The supervisor selects informative data using a second adaptive model and a switch. The new formulation of the constrained least squares estimation problem uses a one-step objective and McCormick over-estimators for the bilinear constraints. Stability results show that the supervised adaptive control algorithms are robust with respect to plant/model mismatch and unknown, bounded disturbances. Input and output signals are attracted to an invariant set independent of choice of initial conditions. The control parameters converge. Monte Carlo simulations compare the supervision algorithm's ability to handle bursting and drifting to unsupervised approaches. Simulation of a chemical reactor with nonlinear model and a pilot plant heat exchanger experiment show that parameter drift is eliminated.[1]

Keywords: adaptive control; model reference control; supervised control; stability; robustness; chaotic system; transient analysis; non-convex optimization; controllability; predictive control

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1 Introduction

Supervision: "Managing by overseeing the performance or operation of a person or system."

Certainty equivalence adaptive control needs supervision to decide if an event is informative before it is allowed into the data record. Without event detection the parameter estimates may drift and the model may become inaccurate since poor information overwhelms the identification process. The controller performance then suffers and closed loop instability results. The drift may also cause the models to become ill-conditioned. The control design process then breaks down and large spikes in the controller output may be created.

Detecting events which cause drift can be viewed as the opposite of large error detection. Small prediction errors cause drifting since they are associated with poor signal to noise ratio. One goal of the supervisor is therefore to detect small errors and delete these from the data record. This is easy to do if an upper bound for the current disturbance is known. The supervisor then simply compares the prediction error to the bound and rejects the data point if it is smaller than the bound. A review of the supervision problem can be found in the paper by Hägglund and Aström, (2000)[6]. The drift problem is described Golden (1992)[5] and more extensively in the textbook by Mareels and Polderman (1996) [11].

The supervisor must also ensure that the estimated model is well-conditioned so that a controller can be designed to meet nominal performance requirements. For example, suppose that the aim is to assign closed loop poles. The design procedure then breaks down if the estimated model has pole-zero cancelations. Such events must therefore be avoided. This second problem, referred to as the admissibility problem, is NP hard. The set of co-prime models, called the admissible set, is disconnected and the number of non-convex regions grows faster than exponentially as the model order increase. A description of this problem is given by Middleton *et al.*, (1988)[12], Staus *et al.* (1997) [15] and Mareels and Polderman (1996) [11].

Many methods have been developed to solve parameter drift and admissibility problems in adaptive control. Some methods rely on **excitation**. In this approach the estimator is presented with informative data all the time. The parameter estimates remain close to the "true parameters" and the estimated model is well conditioned, provided that the true model is well-conditioned. The problem with the excitation method is that the excitation must be strong enough to overcome the noise yet subtle enough so that performance does not suffer. It must also be band-limited so that it does not excite high frequency modes. Radenkovic and Ydstie (1995) [14] showed that hard bounds for the parameter estimation error can be established if these conditions are satisfied. The **deadzone** method works by switching the estimator off when the prediction error gets below a certain threshold [4, 13]. The parameters converge if the threshold is chosen large enough. The problem now is that the performance may suffer if the deadzone is too large. Parameter drift is re-introduced if the deadzone is too small. Finally, **parameter projection and/or leakage** constrain the parameters so that they do not wander out of the admissible set. Projection solves the problem using hard constraints [4]. Leakage [9] uses soft constraints and is similar to the barrier methods used in nonlinear programming. Neither parameter projection nor leakage solve the drift problem completely and poor closed loop performance may result if the parameter bounds and/or leakage parameters are not well-chosen [8]. The leakage and projection methods are difficult to implement if the admissible set is not convex.

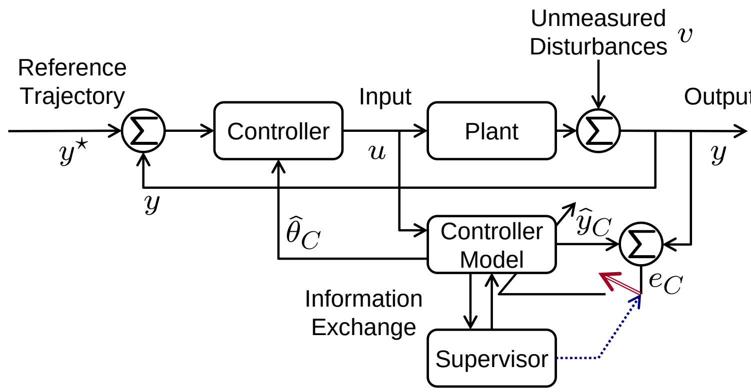

Figure 1: Supervised adaptive control.

Our paper addresses parameter drift by using a supervisor to detect informative events. The supervisor uses an adaptive model to estimate the disturbance and a switch to turn on adaptation if the prediction error of the control design model is larger than the disturbance estimate provided by the supervisor. We do not provide a bound for the magnitude of the external disturbances since they are estimated. The approach is advantageous in process control where the disturbances are difficult to characterize *apriori* and may vary considerably.

The proposed algorithm solves the admissibility problem using global optimization. McCormick over-estimators are used to estimate bilinear constraints and we solve a sequence of quadratic programs to global optimality using branch and bound as proposed by Staus *et al.* (1994) [15]. They reported that the calculations took no more than a small fraction of a second for a second order model and 1000 data points. Our method is considerably more efficient since we have reformulated the objective function so that problem complexity is independent of the estimation horizon. Significant advances have been made in algorithm design and computer speed since the work by Staus *et al.* It is now possible to apply such methods in process control applications where the sampling rate is in the order of a second and the model order is limited to have three poles or less, which is typically sufficient to model chemical process systems.

The stability theory is quite general and can be used to study model predictive, H_∞, as well as pole-assignment indirect adaptive control systems. We use a simple model predictive controller to illustrate the performance of the closed loop adaptive control system. The scope of the theory is quite broad and opens up for implementing considerably more complex noise estimation and control design methods than we have discussed here.

2 Supervised Adaptive Control

The schematic in Figure 1 shows an adaptive control system with a supervisor using a *switch* to determine when to update the estimated model. Data is considered informative when the model prediction error differs from a result calculated by the supervisor. The switch is then closed and the controller model is updated, otherwise, the switch is kept open and the model remains unchanged. Many different strategies can be implemented. We implement a switch which

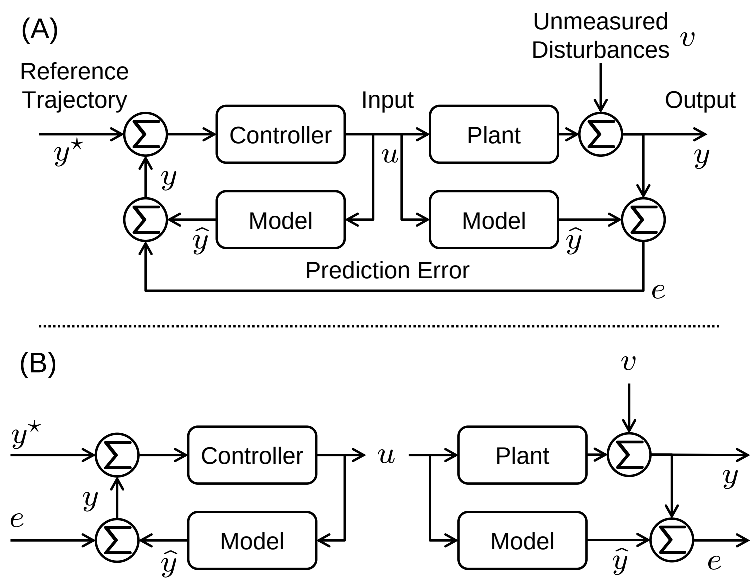

Figure 2: Framework for Certainty Equivalence Adaptive control.

executes "small error detection" to prevent over-fitting the data. By careful management of the switch we are able to select informative data so that the parameter estimates are not corrupted by small errors which accumulate over time. The method can be used for identification of linear and nonlinear models alike. The specific aim of our paper is to use the method to solve the problem of parameter drift in the context of adaptive linear control.

The supervisor in our paper is implemented so that

$$\Delta(t) = \begin{cases} 1 & \text{if } \bar{e}(t)^2 \geq T_0 s(t) + T_1 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where $\Delta(t)$ represents the switch, $T_i \geq 0, i = \{0, 1\}$ are tolerances set by the user and $\bar{e}(t)$ is the (*apriori*) prediction error. The trigger signal $s(t)$ depends on supervision strategy. For example, a fixed deadzone (simplest supervisor) is derived by setting $T_0 = 0$ and $T_1 > 0$ in equation 1. Hill and Ydstie (2004) [7] developed a trigger based on the Fisher Information Matrix which can be interpreted in the framework described here. More recently Dozal-Mejorada *et al.* (2007) and Dozal-Mejorada and Ydstie presented a method using parallel adaptive models to estimate and filter out the plant disturbances [2, 3]. Their supervision method uses $T_0 > 0$ and $T_1 = 0$ and selects data whenever the prediction error of the control design model exceeds the error provided by a second estimator.

To motivate the supervision method and technical results needed to establish stability and convergence, it is helpful to break the feedback control loop into two parts as shown in Figure 2. First insert the controller design model ("model" for short) as shown in Figure 2A. Doing so has no overall effect since the model output is added and then subtracted. Splitting the loop between the two models gives the systems in Figure 2B. The "nominal feedback system" on the left corresponds to the model and the controller in closed loop. The model error (e) and the setpoint ($y^\star$) generate the control signal (u). The "model error system" on the right measures how accurately the control model represents the plant. The control (u) and disturbance (v) signals are the inputs to the plant which generate the model error (e) and plant output (y).

It stands to reason that the control system described above performs to specifications if two conditions are met: first, the model error should be small; and second, the nominal feedback system should satisfy the control specifications.

3 Assumptions

Let M_θ denote the operator mapping the control signals to the predicted outputs. The estimated control model is indexed by the parameter vector θ. The mapping $\mathbb{C} : M_\theta \mapsto C_\theta$ represents a control design procedure. In order to carry out the control design it is necessary that the parameter vector θ belongs to a compact set Θ, called the *admissible set*. Otherwise the control design procedure breaks down. The problem of delineating the set Θ and keeping the estimated parameters in Θ is the aforementioned admissibility problem.

We now introduce all assumptions needed to delineate the admissible set and establish the existence of an invariant set for input and output signals and convergence of the parameter estimates.

Assumption 1. Bounded external disturbances: *Real constants k_v and k_{y^*} exist so that for all t*

$$v(t)^2 \leq k_v \tag{2}$$

$$y^\star(t)^2 \leq k_{y^\star} \tag{3}$$

where $v(t)$ and $y^\star(t)$ are the external disturbances and output setpoint respectively.

We do not assume that the values of the constants k_v and $k_{y^\star}$ are known. Nominal stability of the feedback system illustrated on the left in Figure 2B and the model mismatch condition for the system illustrated on the right in Figure 2B are characterized using the comparison signal[2]

$$x(t+1) = \sigma^2 x(t) + (1-\sigma^2)e(t)^2,\ 0 \leq \sigma < 1 \tag{4}$$

where $e(t)$ is the (*a posteriori*) model error.

Assumption 2. Stability of the nominal feedback system: *Real constants K_{yu} and k_{yu} exist so that for all t*

$$y(t)^2 + u(t)^2 \leq K_{yu}x(t) + k_{yu} \tag{5}$$

The constant K_{yu} measures the gain of the nominal closed loop operator and k_{yu} is related to the magnitude of the setpoint so that $k_{yu} \leq c_0 k_{y^\star}$ for some constant c_0.

Assumption 3. Small errors: *There exists a parameter $\theta^\star \in \Theta$ so that the corresponding model error $\gamma(t)$ is bounded by the normalization signal $x(t)$. Namely,*

$$\gamma(t)^2 \leq K_\gamma x(t) + k_\gamma \tag{6}$$

[2]Throughout this paper we deal with sampled data systems and discrete models using the Swedish backward-shift operator q^{-1}. Similar results can be developed for continuous time systems and systems modeled using the δ-operator

The constant K_γ measures the gain of the unmodeled dynamics whereas k_γ is related to the magnitude of the external disturbances. K_γ is small in a way to be determined and $k_\gamma \leq c_0 k_v$ for some constant c_0.

Ydstie (1992) derived conditions A2 and A3 using minimum variance control and a nearly stably invertible plant [17]. Kelly and Ydstie (1993) showed how the conditions can be derived for H_∞ control and Ydstie and Wahlberg (1997) derived the condition for Model Predictive Control [10, 18].

Assumption 4. The admissible set: *The admissible set, Θ, consists of $l < \infty$ subsets Θ_i so that if $\theta_1 \in \Theta_j$ and $\theta_2 \in \Theta_k$ with $j \neq k$ then $||\theta_1 - \theta_2|| \geq \epsilon > 0$. Moreover, for any θ_1 and $\theta_2 \in \Theta$ we have $||\theta_1 - \theta_2||^2 \leq K_\Theta$.*

4 Constrained Parameter Estimation

The assumptions in the previous section can be applied to linear and nonlinear problems alike. We are interested in adaptive linear control. With this in mind we define the equation error for the model M_θ so that

$$e(t) = A(q^{-1})y(t) - B(q^{-1})u(t) \tag{7}$$

where

$$A(q^{-1}) = 1 + a_1 q^{-1} + \cdots + a_n q^{-n} \tag{8}$$

$$B(q^{-1}) = b_1 q^{-1} + \cdots + b_m q^{-m} \tag{9}$$

The linear model can be adapted to data and used to design a controller to meet the nominal performance requirements. The objective may be to minimize the tracking error, place closed loop poles, solve a robust or model predictive control problem. The calculation can be be carried out as long as the parameters belong to the admissible set. Assuming that this is the case we obtain the linear feedback law

$$R(q^{-1})u(t) = T(q^{-1})y^\star(t) - S(q^{-1})y(t) \tag{10}$$

where $R(q^{-1}), S(q^{-1}), T(q^{-1})$ are controller polynomials.

For example, in the case of pole-assignment it is necessary to solve Bezout's equation

$$P(q^{-1}) = A(q^{-1})R(q^{-1}) + B(q^{-1})S(q^{-1})$$

for R and S. The polynomial $P(q^{-1})$, with roots strictly inside a circle with radius $\sigma < 1$ defined by equation (4) in conjunction with Assumptions A2 and A3, represent the closed loop characteristic polynomial.

The Bezout equation can be re-written as

$$\begin{pmatrix} 1 & 0 & \dots & \dots & \dots & 0 \\ a_1 & \ddots & & b_1 & & \vdots \\ \vdots & \ddots & 1 & \vdots & \ddots & 0 \\ a_n & & a_1 & b_n & & b_1 \\ & \ddots & \vdots & & \ddots & b_1 \\ 0 & & a_n & 0 & & b_n \end{pmatrix} \begin{pmatrix} 1 \\ r_1 \\ \vdots \\ r_{n-1} \\ s_0 \\ \vdots \\ s_{n-1} \end{pmatrix} = \begin{pmatrix} 1 \\ p_1 \\ \vdots \\ p_{n-1} \\ p_n \\ \vdots \\ p_{2n-1} \end{pmatrix} \tag{11}$$

with

$$\max_i |a_i| \leq c_a, \quad \max_i |b_i| \leq c_b, \quad \max_i |r_i| \leq c_r, \quad \max_i |s_i| \leq c_s \tag{12}$$

The matrix on the left is called the Sylvester eliminant matrix. It is invertible if and only if the polynomials A and B are co-prime. The eliminant matrix exposes the bilinear nature of the estimation problem since the a_i, r_i, b_i, s_i are variables to be optimized using adaptation.

Other design procedures lead to different definitions of the admissible set. For example, if we use minimum variance control to control a stably invertible plant then the admissible set simply consists of two disconnected convex regions. One region with models with positive high frequency gain and another with negative high frequency gain.

We now introduce the parameter and regression vectors

$$\begin{aligned} \theta(t) &= (a_1, \ldots, a_n, b_1, \ldots, b_m)^T & (13) \\ \varphi(t-1)^T &= (-y(t-1), \ldots, -y(t-n), u(t-1), \ldots, u(t-m)) & (14) \end{aligned}$$

The prediction error of the supervisor (1) and the equation error are then defined so that

$$\begin{aligned} \bar{e}(t) &= y(t) - \varphi(t-1)^T\theta(t-1) & (15) \\ e(t) &= y(t) - \varphi(t-1)^T\theta(t) & (16) \end{aligned}$$

The prediction error is used to assess the accuracy of the model, whereas the model error is used to drive the nominal feedback system. This is because the most current parameters are used to design the feedback control law.

Constrained least squares solves the problem

$$\theta(t) = \arg\min_{\theta\in\Theta} J(\theta, t) \tag{17}$$

where Θ is the admissible set and the objective function given by

$$J(\theta, t) = \sum_{i=1}^{t} \Delta(i)\frac{q(i)}{r(i)}(y(i) - \varphi(i-1)^T\theta)^2 + (\theta - \theta(0))^T Q(t)(\theta - \theta(0)) \tag{18}$$

The supervisor switch (1) determines if the current data point is selected for estimation or not. Weights defined by the backwards recursion

$$q(i-1) = \lambda(i)q(i),\ i = t, t-1, ..., 1$$

with $q(t) = 1$ and forgetting factor $0 < \lambda_{\min} < \lambda(t) \leq \lambda_{\max} < 1$, allow for fading past data. The signal

$$r(t+1) = \max\left\{\sigma^2 r(t) + (1-\sigma^2)e(t)^2,\ \sigma^t r(0)\right\} \tag{19}$$

represents a moving average estimate of the variance of the model error and $r(0)$ represents its initial condition. The matrix $Q(t) \geq 0$ is introduced to bias the estimated parameters towards some pre-set values $\theta(0)$. This approach is called leakage. We assume that the following is satisfied

$$Q(t) \leq Q(t-1)$$

with $Q(0) > 0$. Sometimes we set

$$Q(t) = q(0)Q(0)$$

The effect of the leakage then dies away exponentially fast as data accumulates. The estimation problem posed above is not computationally tractable since its complexity grows as the number of data points grows.

Staus *et al.* (1994) [15] solved the problem of tractability using moving horizon estimation. In this approach only the most recent data points are kept and old ones are deleted from the data record. Convex (McCormick) overestimates were used to represent the bilinear constraints and branch and bound was used to find the global optimum. The algorithm is explained in [15].

One innovation in our current paper is that we convert the objective function (18) to a one-step objective. This reduces computational cost significantly while retaining the possibility of using growing data records. The solution time is now independent of the number of data-points as it is in the recursive least squares method frequently used in adaptive control[3].

Constrained Least Squares Estimation:

Initialization: Define the admissible set Θ, choose $0 \leq \sigma < 1, T_0, T_1, Q(0) > 0$ and $\theta(0) \in \Theta$.

Set $t = 1$ and execute the following algorithm:

Step 1: Update the signal $r(t)$ in equation (19) and the switch $\Delta(t)$ using supervisor (1) with prediction error (15).

Step 2: If $\Delta(t) = 1$ update $\lambda(t)$ and set

$$\begin{aligned} F(t) &= \lambda(t)F(t-1) + \frac{1}{r(t)}\varphi(t-1)\varphi(t-1)^T \\ W(t) &= \lambda(t)W(t-1) + \frac{1}{r(t)}\varphi(t-1)y(t) \end{aligned}$$

Choose $Q(t) \leq Q(t-1)$ so that

$$Q(0) \leq F(t) + Q(t) \tag{23}$$

[3]The estimation problem is often implemented using the familiar recursive method when the set Θ is convex or consists of multiple convex regions. Using the rank one update formula we write

$$\theta(t) = \theta(t-1) + \Delta(t)\mathbb{P}(t)\left\{P(t-1)\varphi(t-1)\frac{y(t) - \varphi(t-1)^T\theta(t-1)}{n(t)}\right\} \tag{20}$$

The projection operator $\mathbb{P}(\cdot)$ prevents the estimates from leaving the admissible set. The initial covariance is updated recursively so that

$$P(t) = \frac{1}{\lambda(t)}\left(P(t-1) - \Delta(t)\frac{1}{n(t)}P(t-1)\varphi(t-1)\varphi(t-1)^T P(t-1)\right) \tag{21}$$

where

$$n(t) = \lambda(t)r(t) + \varphi(t-1)^T P(t-1)\varphi(t-1) \tag{22}$$

The covariance update is typically implemented using Cholesky factorizations to improve numerical stability. The constrained least squares can be implemented using the recursive approach for solving the convex sub-problems in the branch an bound algorithm. A discussion of a similar approach can be found in Middleton *et al.* (1988)

Solve the following optimization problem using the algorithm of Staus et al.

$$\theta(t) = \arg\min_{\theta\in\Theta} \left\{-2(W(t) + Q(t)\theta(0))^T\theta + \theta^T(F(t) + Q(t))\theta\right\}$$

Redesign the controller (10) using the new estimates.

If $\Delta(t) = 0$ set

$$\begin{aligned} F(t) &= F(t-1) \\ W(t) &= W(t-1) \\ \theta(t) &= \theta(t-1) \end{aligned}$$

Leave the controller (10) unchanged.
Step 3: Implement the control action
Step 5: Set $t = t + 1$ and go to Step 1.

The algorithm does not allow the leakage to disappear completely unless the matrix $F(t)$ is positive definite, indicating that the regression vector is excited in all directions. Leakage comes back if excitation is lost.

The optimized objective function is given by

$$M(t) = J(\theta(t), t) \tag{24}$$

Lemma 1. *For all $t \geq 1$*

$$M(t) \geq \lambda(t)M(t-1) + \Delta(t)\frac{e(t)^2}{r(t)}$$

with $M(0) = 0$.

Proof. From the definition of $M(t)$ we have

$$M(t) = \lambda(t)J(\theta(t), t-1) + \frac{1}{r(t)}(y(t) - \phi(t-1)^T\theta(t))^2$$

However $M(t-1) \leq J(\theta(t), t-1)$ since $\theta(t-1)$ is the minimizer for $J(\theta, t-1)$ and $Q(t) \leq Q(t-1)$. The result follows. □

5 Stability of a Supervisor Class

Two signals, $z_1(t), z_2(t)$ are close of order $o(t)$ if

$$|z_1(t) - z_2(t)| \leq k_o\sigma^t \tag{25}$$

where k_o is constant. We write

$$z_1(t) = z_2(t) + o(t) \tag{26}$$

A switching function, not used in the algorithm but rather used as a tool in the stability analysis, is defined so that

$$A(t) = \begin{cases} 1 & \text{if } r(t+1) \geq \sigma r(t) \\ 0 & \text{otherwise} \end{cases} \tag{27}$$

We see that the switch $A(t)$ is equal to 1 if the normalization signal does not decrease faster than the given exponential and it is zero otherwise. Define the constants

$$K_\varphi = K_{yu} \frac{n_\theta^2}{\sigma^{\bar{n}}} \text{ and } k_\varphi = k_{yu} \frac{n_\theta^2}{\sigma^{\bar{n}}}$$

where

$$n_\theta = n + m \text{ and } \bar{n} = \max\{n, m\}$$

From Assumption 2 and definition (14) we deduce that the regressor is bounded in terms of the comparison signal so that

$$\|\phi(t-1)\|^2 \leq K_\varphi x(t) + k_\varphi \tag{28}$$

Assumption A5 below provides a sufficient condition for stability of a class of supervised adaptive control algorithms. This assumption can be used to check if a particular supervisor provides stable adaptive control.

Assumption 5. Supervisor Condition: *Suppose there exist constants ϵ_0 and R so that if $r(t) \geq R$ for all $t \in [t-N, t]$, then*

$$\frac{1}{N} \sum_{i=t-N}^{t} A(i)(1 - \Delta(i)) \frac{s(i)}{r(i)} \leq \epsilon_0 \tag{29}$$

and

$$K_\gamma + \epsilon_0 < \frac{\sigma}{1+\sigma} \frac{\ln \sigma}{\ln \sigma - \ln g_1}$$

where $g_1 = K_\varphi K_\theta + K_\gamma$. The constants K_φ, K_θ and K_γ derive from equation (28), Assumptions A3 and A4 respectively.

The supervisor condition implies that the estimator is turned on if the comparison signal $r(t)$ increases and at the same time $s(t)$ is large relative to $r(t)$. We can assert that the supervisor condition is satisfied for any supervisor with $s(t) \leq s_0 < \infty$.

Theorem 1. Supervised Adaptive System Stability: *If Assumptions A1-A5 are satisfied, then there exist constants R_0, R_1 and $\delta_1 > 0$ so that*

$$\|u(t)\|^2 + \|y(t)\|^2 \leq R_0(k_v + k_{y^*}) + e^{-\delta_1 t} R_1$$

The constant R_1 depends on the initial conditions whereas the other constants do not.

The result shows that there exists an attractive invariant set which does not depend on the choice of initial conditions. The size of the set scales with the size of the external perturbations. The result therefore shows that the steady state performance can be optimized by filtering and removing biases so that k_v and k_{y^*} are as small as possible. The result also shows that the nominal performance and robustness can be met if the model mismatch is sufficiently small. It implies that the controller must be designed so that it does not excite the unmodelled dynamics. It is easy to check that Assumption A5 is satisfied using a fixed deadzone. The problems with the fixed deadzone are that drift is introduced if the deadzone is too small and that performance suffers if it is too large.

To help the reader follow the proof we present a brief outline of the main ideas. First we establish the relationship between the signals $x(t)$ and $r(t)$. Then we introduce a new Lyapunov candidate function suitable for analyzing least squares estimation with non-convex admissibility constraints. Next we use the results obtained in the first two steps to establish properties of the indicator function $A(t)$. In the last step we tie everything together using the Switching Lemma introduced for analysis of hybrid systems by Ydstie (1989) [16]. The switching lemma is reviewed for completeness in the Appendix.

Step 1: The comparison sequence

Lemma 2. *Suppose* $r_0 > 0$. *Then*

$$\frac{x(t)}{r(t)} \leq 1 + o(t)$$

The Lemma shows that the signals $x(t)$ and $r(t)$ are identical apart from the choice of initial condition whose influence decays exponentially over time.

Proof. From the definitions of $r(t)$ and $x(t)$ in equations (4) and (19) we get

$$x(t) - r(t) = \left(\sigma^2 x(t-1) + (1-\sigma^2)e(t-1)^2\right) - \left(\sigma^2 r(t-1) + (1-\sigma^2)e(t-1)^2\right)$$

Hence

$$x(t) - r(t) = \sigma^{2t}(x_0 - r_0)$$

Therefore,

$$\frac{x(t)}{r(t)} = 1 + \sigma^{2t}\frac{x_0 - r_0}{r(t)}$$

Equation (19) gives $r(t) \geq \sigma^t r_0$. Hence

$$\frac{x(t)}{r(t)} \leq 1 - \sigma^t + \sigma^t \frac{x_0}{r_0}$$

The result follows. □

Step 2: The Estimator

In this section we define a Lyapunov function candidate $V(t)$ for the estimator and relate the size of the prediction error to the comparison sequence $x(t)$. This intermediate result aims to establish properties of the feedforward system in Figure 2B.

Define

$$M^\star(t) = J(\theta^\star, t) \tag{30}$$

where $\theta^\star \in \Theta^\star$ is any parameter satisfying Assumption 3. It follows that we can write the recursion

$$M^*(t) \leq \lambda(t)M^*(t-1) + \Delta(t)\frac{\gamma(t)^2}{r(t)} \tag{31}$$

with

$$M^*(0) = (\theta^* - \theta(0))^T Q(0)(\theta^* - \theta(0))$$

The equation error corresponding to $\theta^\star$ is defined so that

$$\gamma(t) = y(t) - \varphi(t-1)^T\theta^* \tag{32}$$

Lemma 3. *Let $V(t) = M^*(t) - M(t)$. Then there exists a constant M^* so that $0 \leq V(t) \leq M^*(t) \leq M^*$ and*

$$V(t) \leq \lambda(t)V(t-1) - \Delta(t)\left(\frac{e(t)^2}{r(t)} - \frac{\gamma(t)^2}{r(t)}\right)$$

with $V(0) = M^*(0)$.

Proof. The objective function, $J(\theta, t)$, in equation (17) is minimized for $\theta = \theta(t)$. Therefore by comparing (24) and equation (31) we see that

$$M(t) \leq M^*(t) \tag{33}$$

From Lemma 2 and (31) we have

$$M^*(t) - M(t) \leq \lambda(t)(M^*(t-1) - M(t-1)) - \Delta(t)\left(\frac{e(t)^2}{r(t)} - \frac{\gamma(t)^2}{r(t)}\right)$$

The result follows using the definition of $V(t)$, Assumption 3 and the fact that the forgetting factors are less than 1. □

Step 3: The Indicator Function $A(t)$

The following lemma bounds the indicator function using the ratio between the model error and the comparison signal.

Lemma 4. *We have*

$$\frac{\sigma}{(1+\sigma)}A(t) \leq A(t)\left(\frac{e(t)^2}{r(t)} + o(t)\right) \tag{34}$$

Proof. First multiply expression (4) through with $A(t)$. We get

$$A(t)r(t+1) = A(t)\left(\sigma^2 r(t) + (1-\sigma^2)e(t)^2 + o(t)\right) \tag{35}$$

By dividing through with $r(t)$ and re-arranging we get

$$A(t)\left(\frac{r(t+1)}{r(t)} - \sigma^2\right) = A(t)\left((1-\sigma^2)\frac{e(t)^2}{r(t)} + o(t)\right) \tag{36}$$

From (27) it follows that $r(t+1) \geq \sigma r(t)$ if $A(t) = 1$. The expression above therefore simplifies to

$$\frac{\sigma - \sigma^2}{1-\sigma^2}A(t) \leq A(t)\left(\frac{e(t)^2}{r(t)} + o(t)\right) \tag{37}$$

and the result follows by noting that $(1-\sigma^2) = (1-\sigma)(1+\sigma)$. □

Lemma 5. *Suppose that $r(i) \geq R > 0$ for all $i \in [t-N, t]$, then we have*

$$\frac{\sigma}{(1+\sigma)} \frac{1}{N} \sum_{t-N}^{t} A(i) \leq \bar{U}(R, N) + \epsilon_0 \tag{38}$$

with

$$\bar{U}(R, N) = \left(\frac{V(t-N)}{N} + K_\gamma + \frac{k_\gamma}{R} \right) \tag{39}$$

Proof. From Lemma 4

$$\frac{\sigma}{(1+\sigma)} A(t) \leq A(t) \left(\Delta(t) \frac{e(t)^2}{r(t)} + (1 - \Delta(t)) \frac{e(t)^2}{r(t)} \right) \tag{40}$$

From Lemma 3

$$\Delta(t) \frac{e(t)^2}{r(t)} \leq \lambda(t) V(t-1) - V(t) + \Delta(t) \frac{\gamma(t)^2}{r(t)} + o(t) \tag{41}$$

Supervisor (1) gives

$$(1 - \Delta(t)) \frac{e(t)^2}{r(t)} \leq (1 - \Delta(t)) \frac{T_0 s(t) + T_1}{r(t)} \tag{42}$$

Now we sum equation (40) from $i = t - N$ to $i = t$ and use inequalities (41) and (42) and Lemma 2 to give

$$\begin{aligned} \frac{1}{(1+\sigma)} \sum_{i=t-N}^{t} A(i) \leq \sum_{i=t-N}^{t} A(i) \bigg(& \lambda(t) V(i-1) - V(i) + \Delta(i) \frac{\gamma(i)^2}{r(i)} \\ & + (1 - \Delta(i)) \frac{T_0 s(i) + T_1}{r(i)} + o(t) \bigg) \end{aligned}$$

Now $\lambda(t)V(i-1) - V(i) = V(i-i) - V(i) - (1 - \lambda(i))V(i-1)$. With $(1 - \lambda(i))V(i-1) \geq 0$ and $V(t) \geq 0$ we get

$$\sum_{i=t-N}^{t} (V(i-1) - V(i)) = V(t-N) - V(t) \leq V(t-N)$$

The result follows by dividing with N and using the facts that $r(i) \geq R$, $\lambda(i) \leq 1$ and

$$\frac{1}{r(t)} = \frac{1}{x(t)} (1 + o(t))$$

□

Step 4: Proof of Theorem 1

In this section we establish boundedness of the comparison signal $x(t)$ using the Switching Lemma from the Appendix.

Lemma 6. *Suppose that there exist positive numbers R and N so that*

$$(1+\frac{1}{\sigma})(\bar{U}(R,N)+\epsilon_0) \le \frac{\ln\sigma}{\ln\sigma - \ln g_1}$$

where

$$g_1 = 2(K_\varphi K_\theta + K_\gamma)$$

Then we have

$$x(t) \le \sigma^{-N}\max\left\{R, e^{-\delta(N+t)}x(0)\right\}$$

Proof. We have from definition (4) and the definition of $A(t)$ in equation (27)

$$x(t) \le A(t)\left(\sigma^2 x(t-1) + (1-\sigma^2)e(t)^2\right) + (1-A(t))\sigma x(t-1) \tag{43}$$

Now

$$e(t) = \varphi(t-1)^T\tilde{\theta}(t) + \gamma(t)$$

Hence, using the fact that $||\theta(t)||^2 \le K_\theta$ from A4 and applying A3 and inequality (28) we get

$$\begin{aligned} e(t)^2 &\le 2((\varphi(t-1)^T\tilde{\theta}(t))^2 + \gamma(t)^2) \\ &\le 2\left(\|\varphi(t-1)\|^2 K_\theta + K_\gamma x(t-1) + k_\gamma\right) \\ &\le 2\left(\left(K_\varphi x(t-1) + k_\varphi\right)K_\theta + K_\gamma x(t-1) + k_\gamma\right) \\ &\le 2\left(\left(K_\varphi K_\theta + K_\varphi\right)x(t-1) + k_\varphi + k_\gamma\right) \end{aligned}$$

It follows from inequality (43) that we can write

$$x(t) = A(t)\left(g_1 x(t-1) + k_1\right) + (1-A(t))\sigma x(t-1) \tag{44}$$

where g_1 was defined above and

$$k_1 = 2(k_\phi K_\theta + k_\gamma)$$

The Switching Lemma applies with the following assignments

$$\begin{aligned} g_1 &= G_1 \\ \sigma &= G_2 \\ k_1 &= K_1 \\ 0 &= K_2 \end{aligned}$$

The result as stated above follows. □

Proof of Theorem 1: From the definition of $\bar{U}(R,N)$ in (39) (lemma 5) we have

$$\bar{U}(R,N) \le \frac{M^*}{N} + K_\gamma + \frac{k_\gamma}{R}$$

It follows that for any $\epsilon > 0$, by choosing R and N sufficiently large we have

$$\bar{U}(R,N) \le K_\gamma + \epsilon$$

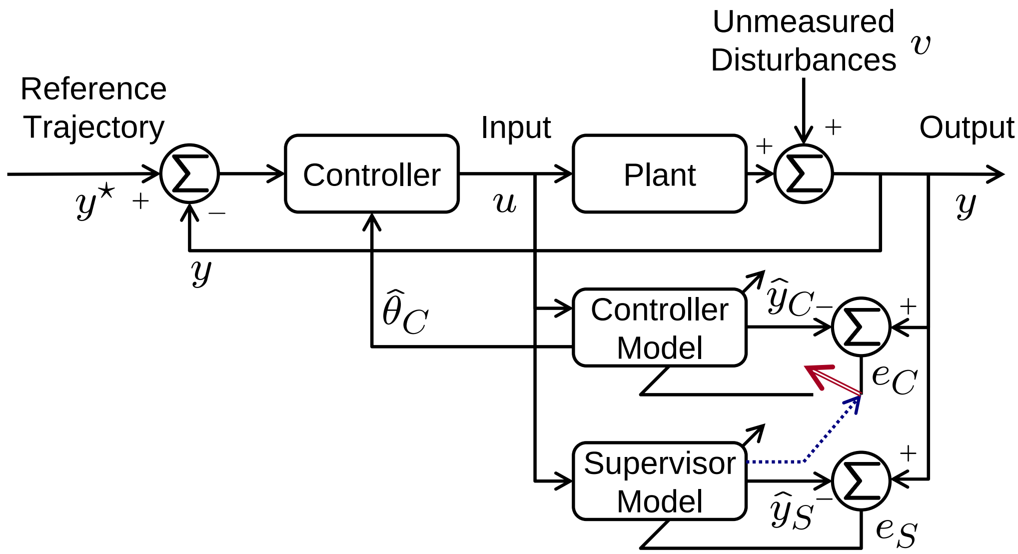

Figure 3: Block diagram of the supervised dual-model adaptive control approach.

hence

$$\bar{U}(R,N) + \epsilon_0 \leq K_\gamma + \epsilon + \epsilon_0$$

From Assumption A5 we know that there exists $\epsilon > 0$

$$\bar{U}(R,N) - \epsilon + \epsilon_0 \leq \frac{\sigma}{1+\sigma} \frac{\ln \sigma}{\ln \sigma - \ln g_1}$$

Hence for large enough R and N

$$\bar{U}(R,N) + \epsilon_0 \leq \frac{\sigma}{1+\sigma} \frac{\ln \sigma}{\ln \sigma - \ln g_1}$$

Lemma 6 applies and Theorem 1 follows by appropriate choice of constants.

6 Model Based Supervision with Parameter Convergence

Lemma 3 shows that the Lyapunov function decreases if $s(t)$ approximates the prediction error $\gamma(t)^2$. This observation leads to the idea of model based supervision. In this approach a second estimator generates the noise model. We then simply compare the prediction errors of the two models and update the control model when these are sufficiently different. Figure 3 shows the architecture used to adapt the control parameters and the noise model on-line. We call the approach model based supervision.

Definition 1. Model based supervision. *Let $\theta_S(t)$ be a vector of supervisor parameters and define the supervisor error*

$$\bar{e}_S(t) = y(t) - \varphi(t-1)^T \theta_S(t-1)$$

The trigger signal used in model based supervision defined by equation (1) is given by

$$s(t) = e_S(t)^2 \tag{45}$$

The parameters are set so that $T_1 \geq 1$ and $T_1 > 0$.

We have the following convergence result concerning model based supervision.

Lemma 7. *Suppose that Assumptions A1-A5 are satisfied and that*

$$\lim_{t\to\infty} \|\theta_S(t) - \theta_S(t-1)\| = 0$$

Theorem 1 is then satisfied and the parameter estimates used in the control design do not drift, i.e.

$$\lim_{t\to\infty} \theta(t) = \theta_0$$

where $\theta_0 \in \Theta$ is constant.

Proof. (The development is similar to the one used to prove Lemma 3). From equation (18) we define

$$M_S(t) = J(\theta_S(t), t)$$

It follows that we can write

$$\begin{aligned} M_S(t) =& \Delta(t)\frac{e_S(t)^2}{r(t)} \\ &+ \sum_{i=1}^{t-1} \Delta(i)\frac{q(i)}{r(i)}(y(i) - \varphi(t-i)^T\theta_S(t))^2 + (\theta_S(t) - \theta_S(0))^T Q(t)(\theta_S(t) - \theta_S(0)) \end{aligned}$$

where $e_S(t) = y(t) - \varphi(t-1)^T\theta_S(t)$ is the model error. By using the fact that the forgetting factors are not equal to one we can conclude that

$$M_S(t) = \lambda(t)M_S(t-1) + \Delta(t)\frac{\bar{e}_S(t)^2}{r(t)} + \epsilon_1(t)$$

where $\lim \epsilon_1(t) = 0$ since $\lim \|\theta_S(t) - \theta_S(t-1)\| = 0$. It follows that for every $\epsilon_S > 0$ there exists a time t_S so that for all $t > t_S$ we have

$$M_S(t) \leq \lambda(t)M_S(t-1) + \Delta(t)\frac{e_S(t)^2}{r(t)} + \epsilon_S$$

We now define the Lyapunov function $V_S(t) = M_S(t) - M(t) \geq 0$ where the inequality follows due to the fact that $\theta(t)$ is the minimizer. For large t we obtain the recursion

$$V_S(t) \leq \lambda(t)V_S(t-1) + \Delta(t)\left(-\frac{e(t)^2}{r(t)} + \epsilon_S + \frac{e_S(t)^2}{r(t)}\right)$$

The supervisor (1) with Switch (1) gives

$$\bar{e}(t)^2 \geq s(t) + T_1 \text{ for } \Delta(t) = 1$$

We now note that

$$\frac{e_S(t)}{\sqrt{r(t)}} = \frac{\bar{e}_S(t)}{\sqrt{r(t)}} + \frac{\varphi(t-1)^T(\theta(t-1)-\theta(t)}{\sqrt{r(t)}}$$

For large t we can therefore interchange the prediction and model errors for the supervisor since $\frac{\varphi(t-1)}{\sqrt{r(t)}}$ is bounded according to inequality (28) and $\theta(t-1)-\theta(t)$ converges to zero. For $\Delta(t)=1$ we can therefore write

$$-\frac{e(t)^2}{r(t)} + \epsilon'_S + \frac{e_S(t)^2}{r(t)} \le -T_1 + \epsilon'_S$$

Hence

$$V_S(t) \le V_S(0) - \sum_{i=1}^{t} \Delta(i)\left(T_1 - \epsilon'_S\right)$$

But for every $T_1 > 0$ there exists t_S so that $\epsilon'_S < T_1$ for all $t > t_S$. We must therefore conclude that $\Delta(t)$ converges to zero. The parameter estimator stops and the parameter estimates converge to a finite limit since the admissible set Θ is compact.

□

We now get to the issue of how to implement the supervisor. Many different methods can be proposed. One simple supervisor results using constrained least squares so that

$$\theta_S(t) = \arg\min_{\theta\in\Theta}\left\{-2(W_S(t)+Q(0)\theta(0))^T\theta + \theta^T(F_S+Q(0))\theta\right\} \tag{46}$$

The update equations are given by

$$W_S(t) = W_S(t-1) + y(t)\frac{\varphi(t-1)}{r(t)}$$

$$F_S(t) = F_S(t-1) + \frac{\varphi(t-1)^T\varphi(t-1)}{r(t)}$$

The minimization solves problem (17) with $\Delta(t)=1$ and $\lambda(t)=1$.

Theorem 2. ***Performance of model based supervision:*** *Suppose that Assumptions A1-A4 are satisfied with*

$$K_\gamma < \frac{\sigma}{2(1+\sigma)}\frac{\ln\sigma}{\ln\sigma - \ln g_1}$$

and that the constrained least squares algorithm is implemented using Switch 1 and trigger signal (45), with $T_0 = 1$ and $T_1 > 0$ and supervisor model (46). The following holds

1. *There exist constants R_0, R_1 and $\delta_1 > 0$ so that*

$$\|u(t)\|^2 + \|y(t)\|^2 \le R_0(k_v + k_{y^*}) + e^{-\delta_1 t}R_1$$

The constant R_1 depends on initial conditions, the other constants do not.

2. *The parameter estimates do not drift since the limit*

$$\lim_{t\to\infty} \theta(t) = \theta_0$$

exists and is finite.

Proof. We first note that least squares with no forgetting forces the difference between consecutive parameter estimates to converge. This result generalizes to constrained least squares using Assumption A4. We get

$$\lim_{t\to\infty} ||\theta_S(t) - \theta_S(t-1)|| = 0 \tag{47}$$

We now define a Lyapunov function for the supervisor and derive the inequality

$$V_S(t) \leq V_S(t-1) - \frac{e_S(t)^2}{r(t)} + \frac{\gamma(t)^2}{r(t)}$$

We can then write

$$\frac{1}{N}\sum_{i=t-N}^{t} \frac{e_S(i)^2}{r(i)} \leq \frac{1}{N}\sum_{i=t-N}^{t} \frac{\gamma(i)^2}{r(i)} + \frac{1}{N}V(t-N)$$

It follows from Assumption 3 that

$$\frac{1}{N}\sum_{i=t-N}^{t} \frac{e_S(i)^2}{r(i)} \leq K_\gamma + \frac{k_\gamma}{R} + \frac{1}{N}V(t-N)$$

Equation (47) holds and we can interchange the model and prediction errors so that we can write

$$\frac{1}{N}\sum_{i=t-N}^{t} \frac{\bar{e}_S(i)^2}{r(i)} \leq K_\gamma + \frac{k_\gamma}{R} + \frac{1}{N}V(t-N) + \epsilon_2(t)$$

where $\lim_{t\to\infty} \epsilon_2(t) = 0$. Assumption 5 is now satisfied with trigger signal $s(t) = \bar{e}_S(t)^2$ for large t. We get stability according to Theorem 1. Lemma 7 also applies since the limit (47) holds and the result follows.

□

The algorithm can be modified in many ways. It can be restarted by including a second switch which resets the supervisor information matrix so that $F_S(t) \leq F_S(t-1)$. This can be done periodically or by using a special supervisor which determines that re-adaptation is needed. The stability follows in the sense of Theorem 1. However the parameters do not converge as long as the resetting continues. Many other modifications can be made and more robust supervisors with large and small error detection can be implemented. The old parameters can be stored to build a bank of models which are matched to different operating conditions.

The results of the paper can be developed without using the normalization signal $r(t)$ in the constrained least squares algorithm. The analysis, which follows the format developed by Ydstie (1992), is considerable more cumbersome and the bounds obtained on unmodelled dynamics are more conservative.

7 Adaptive Predictive Control with Supervision

The adaptive supervision algorithm is now combined with a Model Predictive Controller (MPC) minimizing the T-step ahead prediction

$$\min_{\delta u(t)} J = [y^\star(t+T) - y(t+T)]^2 + r\delta u(t)^2 \tag{48}$$

subject to

$$\delta u(t+i) = \delta u(t), \qquad i \geq 1$$

where $\delta = 1 - q^{-1}$ is the increment operator.

The controller is developed by defining $F(q^{-1})$ and $G(q^{-1})$ satisfying the Diophantine equation

$$F(q^{-1})A(q^{-1})(1-q^{-1}) + q^{-T}G(q^{-1})(1-q^{-1}) = (1-q^{-T}) \tag{49}$$

Using the method of Lagrange multipliers we find that

$$\begin{aligned} u(t) = &u(t-1) + \frac{\sum_{i=1}^{T}\beta_i}{r + \sum_{i=1}^{T}\beta_i}(y(t+T)^\star - y(t) \\ &- \alpha_1\delta y(t) - \cdots - \alpha_n \delta y(t-n) - \beta_1 \delta u(t-1) - \cdots - \beta_m \delta u(t-m)) \end{aligned} \tag{50}$$

where α_i are the coefficients of the polynomial $G(q^{-1})$ and β_i are the coefficients of the polynomial $F(q^{-1})B(q^{-1})$. In direct adaptive control we use the equation error

$$e(t) = y(t) - y(t-T) - \varphi(t-1)^T\theta \tag{51}$$

with

$$\varphi(t-1)^T = (\delta y(t), \delta y(t-1), ..., \delta y(t-n_a)\delta u(t-1), \delta u(t-1), ..., \delta u(t-n_b-T)) \tag{52}$$

$$\theta = (\alpha_1, ..., \alpha_{n_a}, \beta_1, ..., \beta_{n_b+T})^T \tag{53}$$

Indirect control estimates a_i and b_i in equation (13) using the equation error (16). The Diophantine equation (49) is then solved to get the control parameters (50).

In the CSTR simulation and heat-exchanger experiments we use the direct approach. We also make the assumptions that the plant is stable and that the sign and lower bound for the high frequency gain is known. The admissible set is then convex. The forgetting factor is chosen so that

$$\lambda = 1 - \frac{1}{M_0}$$

where M_0 is called the memory length. Setting $M_0 = \infty$ gives $\lambda = 1$ which corresponds to regular least squares estimation.

Monte Carlo Simulations

The plant is given by the stochastic process

$$d(t) = d(t-1) + \mu v(t) \tag{54}$$

$$y(t) = ay(t-1) + u(t-1) + d(t) \tag{55}$$

Here $v(t)$ is an i.i.d. noise signal with mean = 0 and standard deviation = 1 generated by the Matlab® `rand` command. The disturbance and plant parameters are $\mu = 0.05$ and $a = 0.5$ respectively. The initial conditions for the simulations are $y(0) = d(0) = u(0) = 0$. The equation error is given by

$$e(t) = y(t) - \varphi(t-1)^T \theta(t) \tag{56}$$

where the regression vector is $\varphi(t-1) = y(t-1)$. The model initial conditions are $\theta(0) = \theta_S(0) = 1\text{E-}4$ and the leakage is set so that $Q(0) = Q_S(0) = 1\text{E-}9$.

The control law update is given by the following 1-step ahead stochastic predictive controller

$$u(t) = y^\star - \theta(t)y(t) - e(t) \tag{57}$$

where $y^\star = 0$ is the output setpoint. Each simulation run is based on 6,000 samples. We use 1000 realizations to calculate the averages. The initial tuning period (1000 sampling points) is not reported since we are interested in the long term behavior of the adaptive control system.

The results are presented in three figures and one table. Figures 4, 5 and 6 show representative simulations for the recursive least squares, fixed deadzone and adaptive model based supervision algorithms. Each figure is composed of four plots. The topmost plot shows the control input as well as the signal-to-noise ratio (snr) for a representative run. The signal to noise ratio is defined as the absolute value of the ratio of the plant output without noise to the noise, i.e.

$$\text{snr} = |\frac{y}{d} - 1|$$

Next, we show the process output along with its setpoint. The estimated control model parameter θ_C along with the switching condition (for the supervised case) are shown on the next graph. In figure 6 the third plot also shows the supervisor estimated parameter θ_S for the same run. The histogram shows the distribution of the estimated parameter at the conclusion of each run. The results are summarized in Table 7.

The following points are worth highlighting.

1. The table shows that all algorithms have similar average performance as measured by the standard deviation of the control signal and the tracking error

2. The algorithm with model based supervision has similar performance to the optimal controller using the true parameter. The other methods investigated show drift and burst. This shows up as occasional spikes and large tracking errors, resulting in poor l_∞ performance. The bursts do dot happen frequently enough to impact the average performance significantly

3. The RLS algorithm and the deadzone approach show output bursting.The instability is seen with and without exponential forgetting. The supervision approach does not show signs of the bursting behavior. Figure 6 shows that the supervisor model acts as an exploratory controller always estimating parameters

4. The histograms show the distribution of the final value of the parameter estimate. The RLS approach has a wide distribution centered around the true parameter. The fixed deadzone approach gives bias towards 1. The supervisor allows the parameter to be centered closer to the true parameter. The RLS and deadzone algorithms occasionally

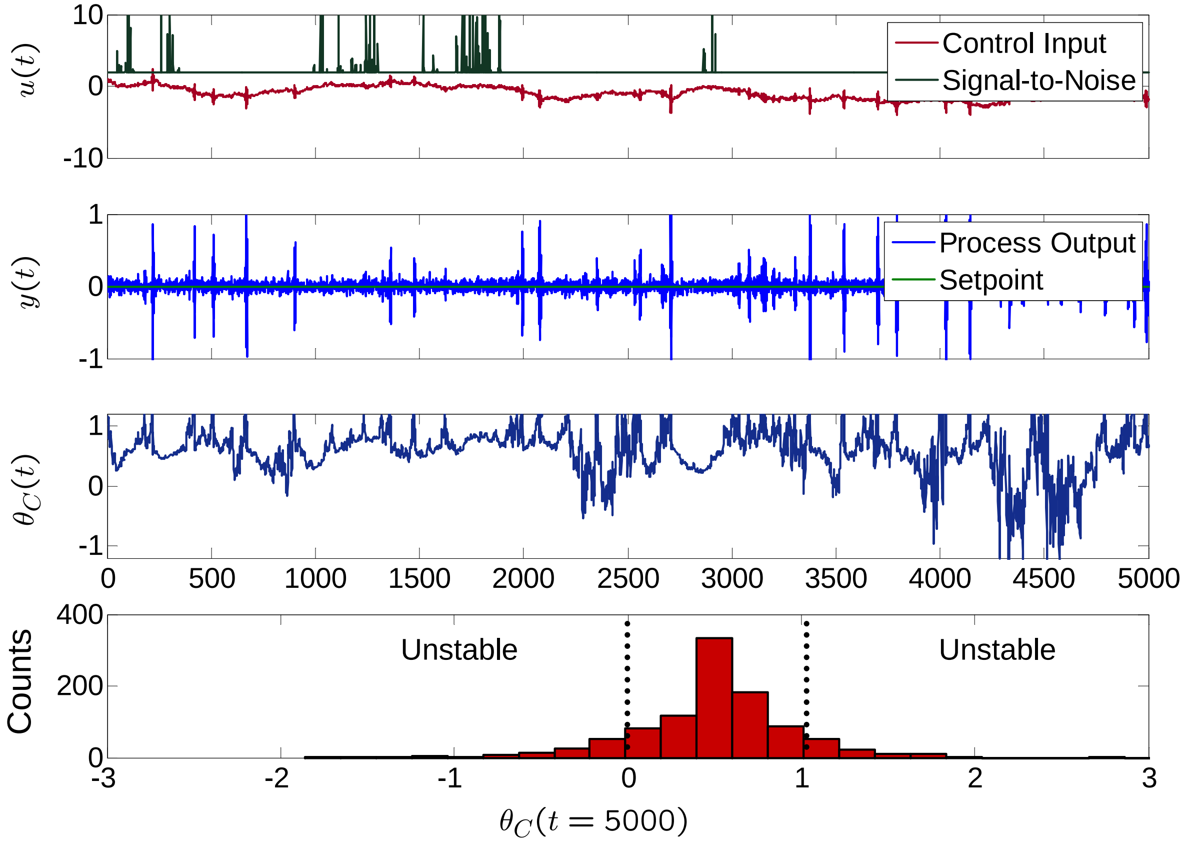

Figure 4: Recursive least squares estimation with exponential forgetting.

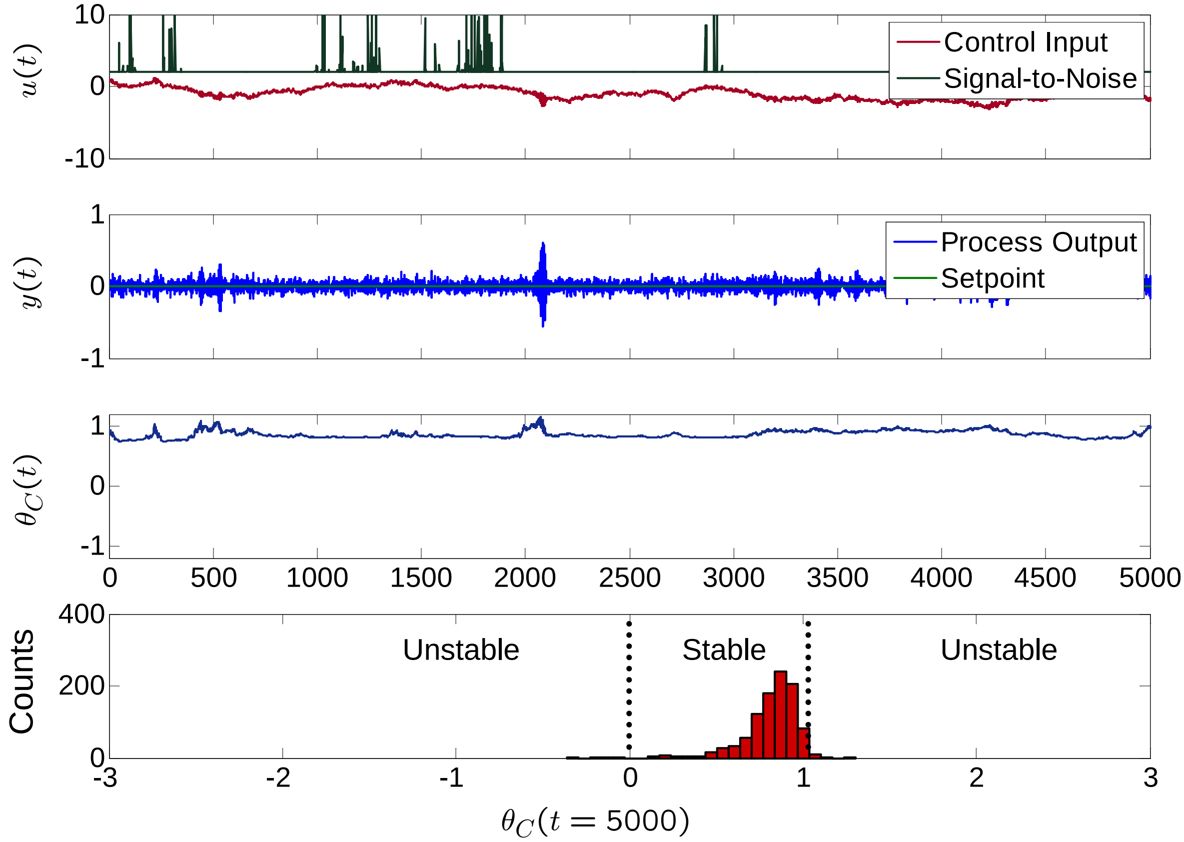

Figure 5: Fixed deadzone.

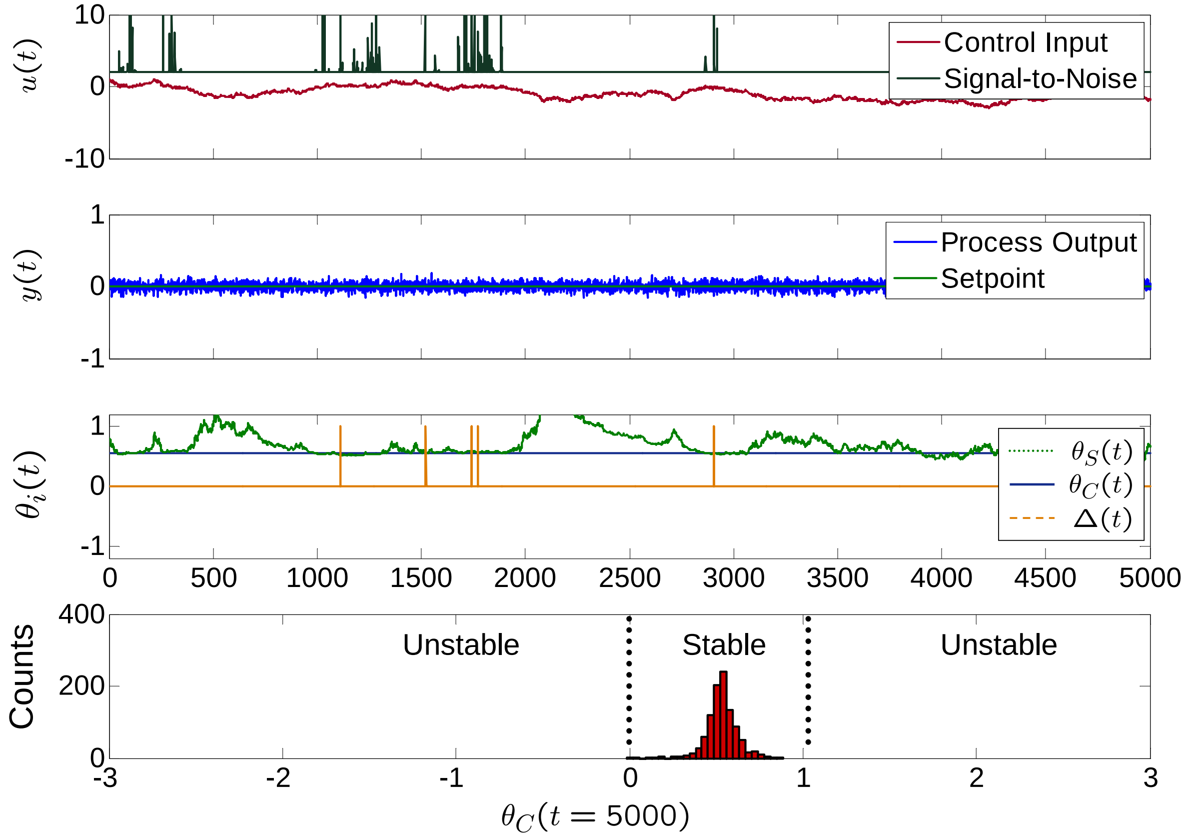

Figure 6: Supervised adaptive control.

	Std e	Std $y - y^*$	Max $\lvert y - y* \rvert$
Optimal Model ($a = 0.5$)	0.0500	1.9668	0.2276
Least squares $M_0 = \infty$	0.0502	1.8308	6.4977
Forgetting Factor $M_0 = 1000$	0.0970	1.8707	4.3228
Deadzone $= 0.1, M_0 = 1000$	0.0774	1.8177	1.5905
Deadzone $= 1, M_0 = 1000$	0.0749	1.6831	1.8199
Deadzone $= 5, M_0 = 1000$	0.0624	1.8529	1.6311
Supervised $M_0 = \infty$, $T_0 = 3$, $T_1 = 3$	0.0500	2.0305	0.2321
Supervised $M_0 = 1000$, $T_0 = 3$, $T_1 = 3$	0.0500	1.8946	0.2230

Figure 7: Summary of MC simulations with various adaptive control algorithms. Supervised adaptive control results in italics.

produce parameters yielding an unstable closed-loop. The histogram for the supervision approach shows that the estimated control model parameter stabilize the plant

5. The supervised algorithm updates the parameter very rarely and only during periods when the signal to noise ratio is favorable. The parameter estimate stays close to its optimal value

Chemical Reactor Simulation

Consider an isothermal CSTR with reactions

$$A + R \rightarrow B$$
$$B + R \rightarrow C$$
$$C + R \leftrightarrow D$$
$$D + R \rightarrow E$$

We assume that component R is present in excess so that we get pseudo-first order kinetics. Thus the material balances can be written

$$V\frac{dA}{dt} = QA_f - qA - k_1VA \tag{58}$$
$$V\frac{dB}{dt} = -qB + k_1VA - k_2VB \tag{59}$$
$$V\frac{dC}{dt} = -qC + k_2VB - k_3VC + k_3'VD \tag{60}$$
$$V\frac{dD}{dt} = -qD + k_3VC - k_3'VD - k_4VD \tag{61}$$

where V is the vessel volume, Q and q are the inlet and outlet flow rates, $\{A_f, A, B, C, D, E\}$ are component compositions. The feed compositions of B, C, D, and E are equal to zero. All system parameters are specified in [1].

The objective is to maintain the composition of component C close to its setpoint despite variations in the feed composition R. The composition of C is measured and the inlet flow rate, Q is manipulated, as seen in figure 8.

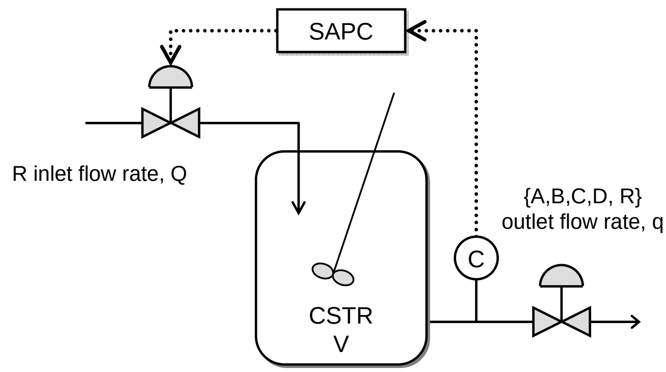

Figure 8: CSTR schematic with control objective.

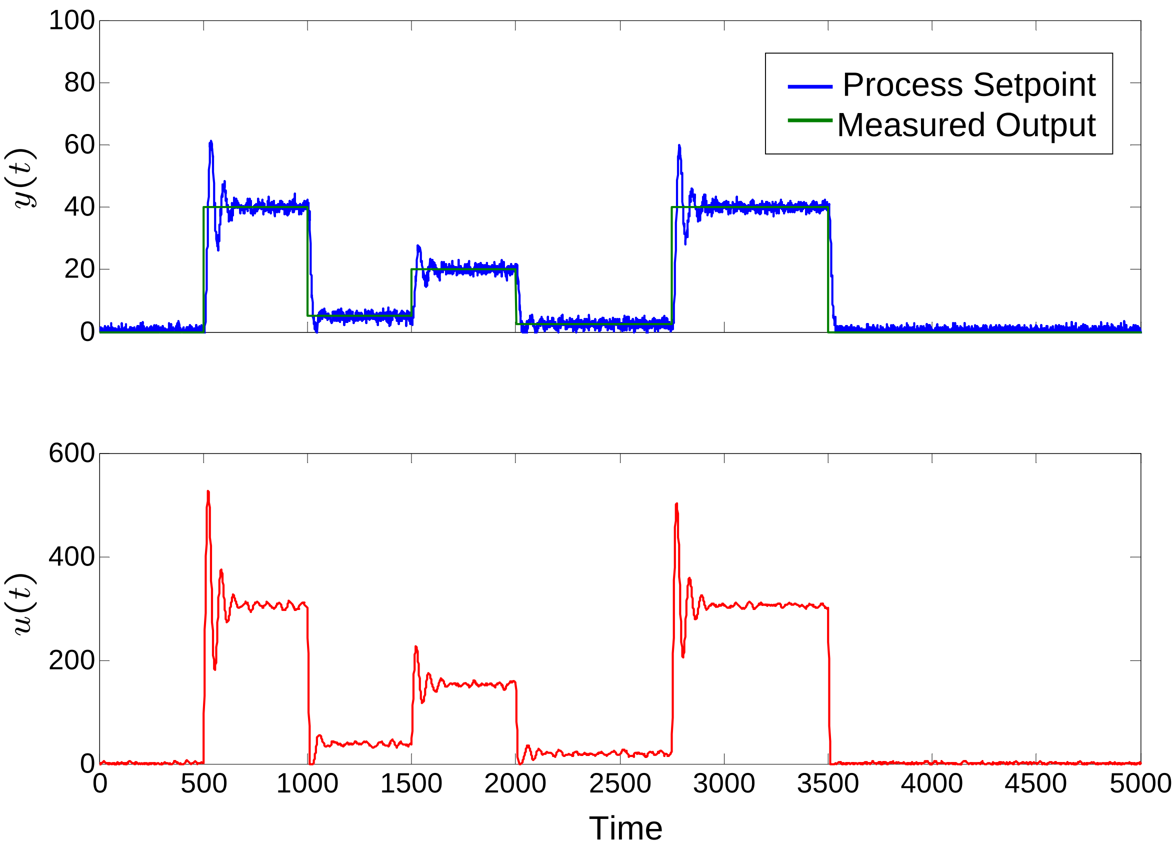

Figure 9: CSTR simulation showing the reactor signals while undergoing set point changes.

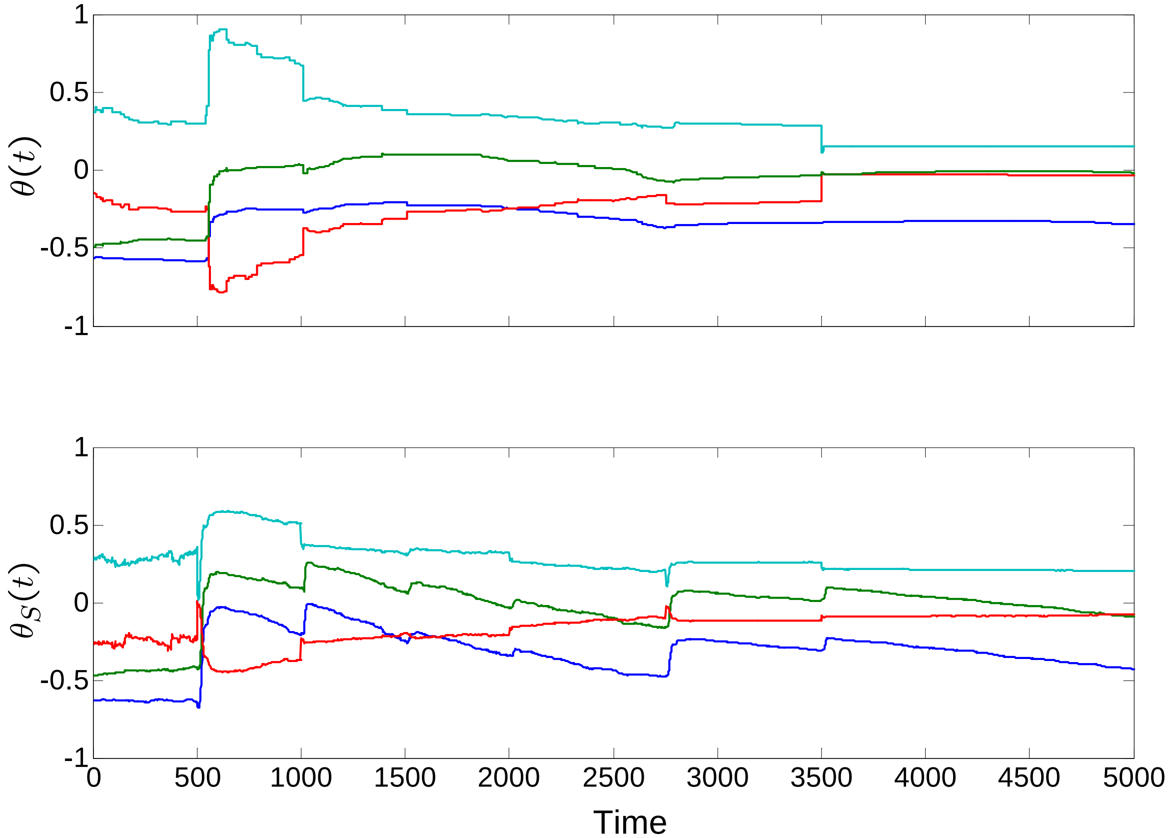

Figure 10: Controller and supervisor estimated parameters for the CSTR simulation during set point changes.

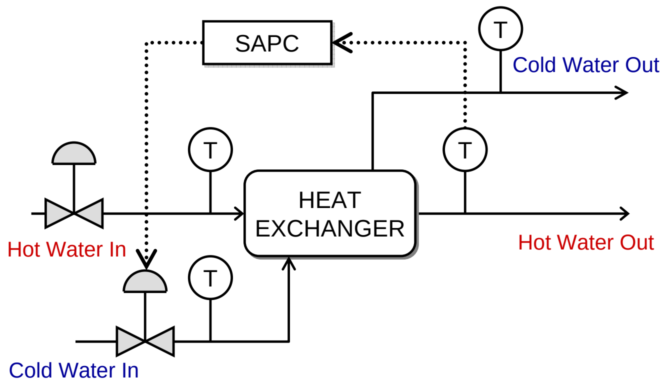

Figure 11: CSTR schematic with control objective.

Figure 9 shows the output set point, measured output and manipulated input signals for the system. The adaptive controller successfully achieves the desired set points with little overshoot. Figure 10 shows the parameter estimates for both the control (top) and supervisor (bottom) models. Note that the control model parameters are stable while a noticeable drift is evident in the supervisor parameters.

Heat Exchanger Experiments

Figure 11 shows the schematic for the pilot plant scale shell and tube heat exchanger used in the experiments. The hot water flows through the shell side and cold water flows through the tube side. The supervised adaptive predictive control algorithm was implemented in LabVIEW® with adjacent field point boxes holding the A/D and D/A converters. Thermocouples were used to obtain temperature measurements. The sampling time used was $T_s = 2$ seconds. The control objective was to regulate the hot water outlet temperature to its set point. The cold water flow rate was used as the manipulated input. Disturbances enter the system due to variations in the hot water flow rate and temperature.

The results are shown in the Figures below. Figure 12 shows the hot water outlet temperature controlled by an adaptive regulator *without* supervision. The typical output bursting reviewed in [7] is observed. This behavior is due to slow parameter drift. In this case, the estimated parameters cross the linear stability boundary and bursts are seen first around $t = 1100$ and then again around $t = 2600$. Figure 13 shows that bursting is eliminated with the supervisor algorithm as. Furthermore, better steady state control is achieved as compared to the RLS adaptive control algorithm.

8 Conclusions and Discussions

We have presented the theory and application of adaptive control using model based supervision. The method uses a pair of adaptive models. The supervisor model is used to estimate the noise. The control model is used to design controller. The supervisor uses a switch to decide when to update the control model. The admissibility problem is solved using non-convex parameter

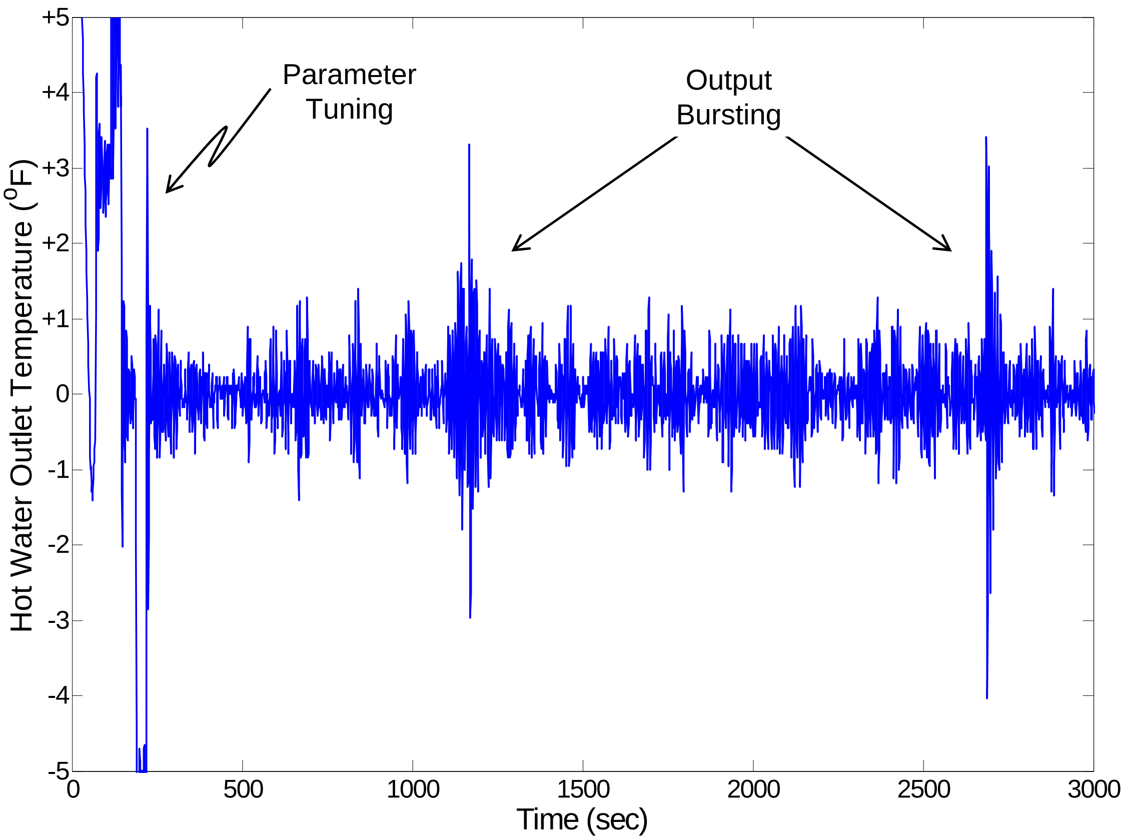

Figure 12: Heat exchanger experiment showing the bursting output behavior observed for regular adaptive control scheme *without* supervision.

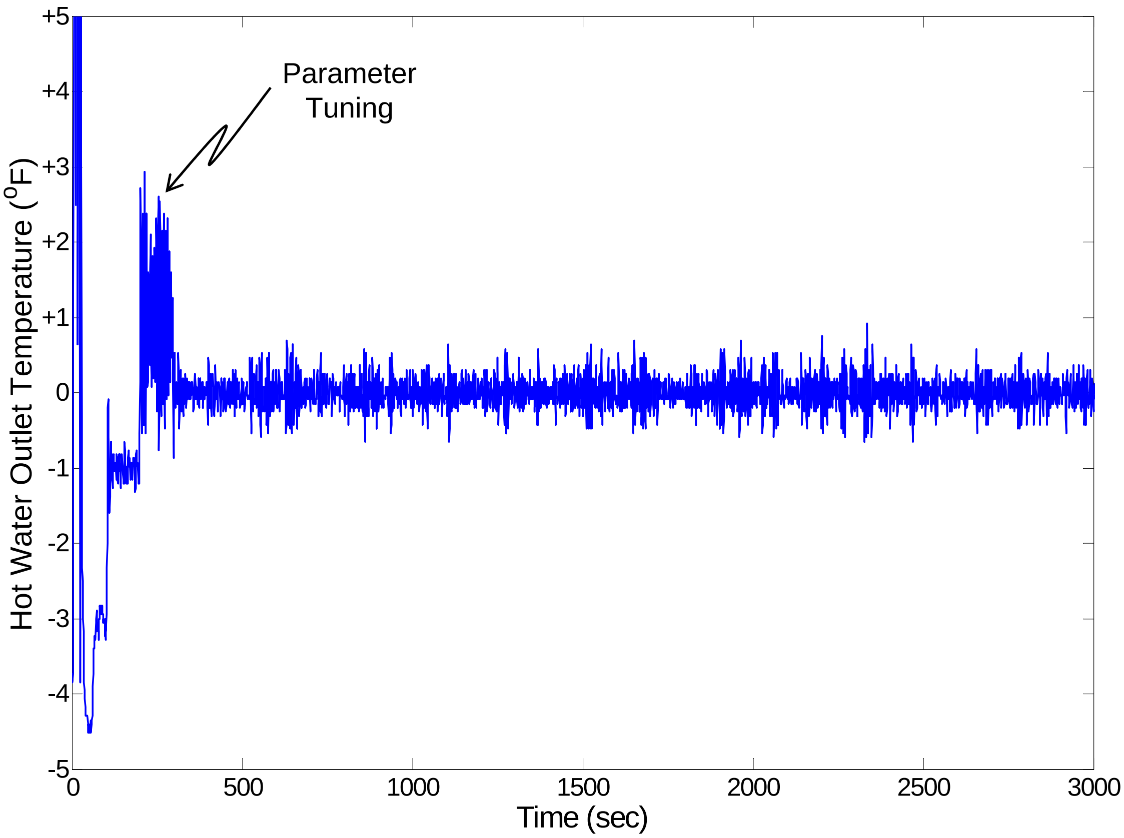

Figure 13: Heat exchanger experiment under supervised adaptive control.

estimation. The latter method is computationally expensive and cannot at present be applied in applications where the sampling rate has to be high. We expect that this limitation will become less important as advances in algorithm design are made and computer speed increases. At present it takes a small fraction of a second to solve each problem to global optimality. The method can therefore be applied in process control where the sampling rate is typically one second.

The stability theory shows that supervised adaptive control with non-convex parameter estimation provides a robust solution to the adaptive control problem. We show that the input and output signals converge to an invariant set even in the presence of unmodeled dynamics and unknown but bounded external noise. We do not require that the frequency distribution and upper bounds for the disturbances are known *apriori*. We do not rely on persistent excitation. We furthermore show that the parameters estimates converge.

The analysis highlights the importance of filtering signals so that the magnitude of external perturbations are reduced as much as possible. this does not effect stability, but it improves performance. It is also important to use controllers that do not excite high frequency unmodelled dynamics.

Applicability is demonstrated in simulation and experiments. Monte Carlo simulations show convergence properties when there are large (moving average) disturbances present. A simple simulation of a chemical reactor and a pilot plant heat exchanger experiment show that parameter drift and burst do not happen.

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References

[1] J. M. Douglas. *Process Dynamics and Control, Volume 2.* Prentice Hall, Inc, Englewood Cliffs, NJ, 1972.

[2] E.J. Dozal-Mejorada, P. Thakker, and B.E. Ydstie. Supervised adaptive predictive control using dual models. *In Proceedings of DYCOPS 2006, Cancún, México*, 2006.

[3] E.J. Dozal-Mejorada and B.E. Ydstie. Stability and robustness of supervised adaptive control. *In Proceedings of DYCOPS 2006, Cancún, México*, 2006.

[4] B. Egardt. *Stability of Adaptive Controllers.* Springer-Verlag, New York, NY, 1979.

[5] M.P. Golden and B.E. Ydstie. Ergodicity and small amplitude chaos in adaptive control. *Automatica*, 28:11–25, 1992.

[6] T. Hägglund and K.J. Aström. Supervision of adaptive control algorithms. *Automatica*, 36:1171–1180, 2000.

[7] J.H. Hill and B.E. Ydstie. Adaptive control with selective memory. *International Journal of Adaptive Control and Signal Processing*, 18:571–587, 2004.

[8] M. Hovd and R.R. Bitmead. Directional leakage and parameter drift. *International Journal of Adaptive Control and Signal Processing*, 20:27–39, 2006.

[9] P.A. Ioannou and P.V. Kokotovic. *Adaptive Systems with Reduced Models*. Springer, Berlin, Germany, 1983.

[10] J.H. Kelly and B.E. Ydstie. Adaptive h_∞ control with application to systems with structural flexibility. *IEEE Transactions on Automatic Control*, 42(10):1358–1369, 1997.

[11] I. Mareels and J.W. Polderman. *Adaptive Systems: An Introduction*. Birkhauser, Boston, 1996.

[12] R.H. Middleton, G.C. Goodwin, D.J. Hill, and D.Q. Mayne. Design issues in adaptive control. *IEEE Transactions on Automatic Control*, 33(1):50–58, 1988.

[13] B.B. Peterson and K.S. Narendra. Bounded error adaptive control. *IEEE Transactions on Automatic Control*, 27(6):1161–1168, 1982.

[14] M.S. Radenkovic and B.E. Ydstie. Using persistent excitation with fixed energy to stabilize adaptive controllers and obtain hard bounds for the parameter estimation error. *SIAM Journal of Control and Optimization*, 33(4):1224–1246, 1995.

[15] G.H. Staus, L.T. Biegler, and B.E. Ydstie. Adaptive control via non-convex optimization. In C.A. Floudas and P.M. Pardoulos, editors, *Nonconvex Optimization and its Applications*, volume 9, pages 119–138. Kluwer Academic Publishers, Boston, MA, 1996.

[16] B.E. Ydstie. Stability of discrete model reference adaptive control-revisited. *Systems and Control Letters*, pages 429–438, 1989.

[17] B.E. Ydstie. Transient performance and robustnes of direct adaptive control. *IEEE Transactions on Automatic Control*, 37(8):1091–1105, 1992.

[18] B.E. Ydstie and B. Wahlberg. Iterative adaptive control. *In Proceedings of IFAC Triennial World Congress 1996, San Francisco, CA*, 1996.

Appendix: The Switching Lemma

The premise of the Switching lemma [16] is that the dynamics of an adaptive system may switch between stable and unstable (exploratory) modes in a quasi-periodic manner over finite intervals so that chaotic bursting may be captured. This property is constructed by utilizing an indicator function $A(t) \in [0, 1]$ which can switch between the unstable and stable systems. The switching Lemma shows that there exists a positively invariant set if the switching conditions are satisfied.

Consider the nonnegative comparison signal $a(t)$

$$a(t+1) = A(t)[g_1 a(t) + K_1] + [1 - A(t)][g_2 a(t) + K_2], \; a(0) < \infty \tag{62}$$

where

$$0 < g_2 < 1 < g_1 < \infty \qquad \text{and} \qquad K_1, K_2 \geq 0 \tag{63}$$

Now assume that there exist constants R and N so that

$$a(t-i) \geq R \qquad \forall i = 0, 1, \ldots, N \tag{64}$$

implying that

$$\frac{1}{N} \sum_{i=t-N}^{t} A(i) = U < \frac{u_2}{u_1 + u_2} \tag{65}$$

where

$$u_1 = \ln \left\{ g_1 + K_1 R^{-1} \right\} \tag{66}$$
$$u_2 = -\ln \left\{ g_2 + K_2 R^{-1} \right\} \tag{67}$$

We then get

Lemma 8. *Assuming that expressions (64) through (66) hold, and applying them to (62) then*

1. $a(t+1) \leq \max \left\{ R, e^{-\delta(N+t)} a(0) \right\}$
   where $\delta = -\ln \left(g_2 + K_2 R^{-1} \right) - \left(\frac{1}{N} \sum_{i=t-N}^{t} A(i) \right) \ln \left(\frac{g_1 + K_1 R^{-1}}{g_2 + K_2 R^{-1}} \right) > 0$

2. $\frac{k_2}{1-g_2} \liminf a(t) \leq R$ *and* $\limsup a(t) \leq g_2^{-N} R$

Proof. Given in [17]. □

The sequence of events corresponding to $A(t) = 1$ belong to an unstable system where all signals increase and output bursting is observed and a Lyapunov-like candidate function decreases. These events inherently result in excited signals and the parameters tune while stabilizing the closed-loop. The second case involves events of the form of $A(t) = 0$ corresponding to exponentially stable paths and divergence of the parameter estimates. It is along these paths that the Lyapunov-like candidate grows.