

Experimental evaluation of adaptive three-tank level control

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Abstract

Liquid level control through regulation of mass flow rates is an important application in various areas of the power industry. Very often a PID controller is used for these applications. This paper compares a nonconventional PID controller and three different types of adaptive controller, a direct model reference adaptive controller (MRAC), an indirect MRAC with Lyapunov estimation, and an indirect MRAC with recursive least-squares (RLS) updating estimation, for liquid level control. By implementing all four controllers on a three-tank system, the performances of each are compared. All controllers track a sinusoidal input very well and overall exhibit somewhat varying performance. The direct MRAC and the indirect MRAC with RLS estimation give the best performance. With Lyapunov estimation and RLS estimation, all the system parameter estimates converge to the reference model values. However, RLS estimation has a much faster convergence. It is concluded that adaptive liquid level control is an improvement over traditional liquid level control when precise level control in three coupled tanks is desired. © 2005 ISA—The Instrumentation, Systems, and Automation Society.

Keywords: Adaptive control; Liquid level control; Direct MRAC; Indirect MRAC

1. Introduction

Liquid level control through regulation of mass flow rates is an important application in various engineering areas, such as steam generators in power generating processes, reactors in many chemical plants, and storage tanks in oil/gas production industry. Whether the inlet or outlet flow is controlled may vary depending on the particular application. Very often a PID controller is used for liquid level control in most applications [1,2]. However, tuning of PID controllers such that satisfactory performance is maintained over a wide operating range has not been thoroughly treated [3,4]. In many instances, there are continuously changing parameters in the plant. In the case of the steam generator, the steam pressure, as well as the total dissolved solids (TDS), is constantly changing which affects the liquid level. Failure to keep a

tight control level is a major cause of plant shut-downs in nuclear and conventional power plants [3,4]. This is particularly true on smaller steam generators with programmed water level control such as used in naval reactor plants. The authors' current experimental apparatus does not include boiling; instead, it drains water to affect similar mass balance relationships. Actual boiling is the likely next step for this current research. This paper presents the design and comparison of both nonconventional and adaptive control schemes for liquid level control applications.

2. Methods and apparatus

2.1. Nonconventional controller design

The three-tank plant used in this paper is very nearly a first-order system without delay. This can

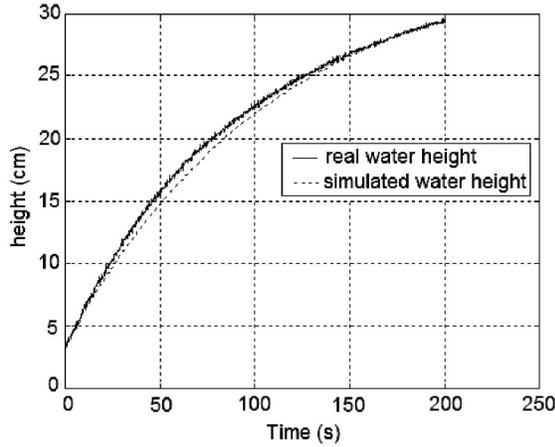


Fig. 1. Open-loop step response comparison of plant model.

be seen in the open-loop step response shown in Fig. 1. For this kind of system, a first-order model, having transfer function $G(s) = b/(s + a)$, is sufficient to capture the significant system behavior. Fig. 1 will be completely discussed later. In order to test the controller’s performance over a wide operating range, a time varying reference input such as a ramp or a sinusoidal input is required. Therefore a type-II open-loop process, including the controller, which has zero steady-state error to a ramp input, is required. Since the system used in this paper is a type-0 system, a conventional PID controller can only achieve zero steady-state error for step changes in reference level. However, by using a double integrating PID controller (PIID), one can design a controller which tracks ramp references with zero steady-state error [5,6]. In this paper, a nonconventional PIID of the form $D(s) = K(s + z_1)(s + z_2)/s^2$ is designed and implemented.

It should be noted that many liquid level plants are best modeled as first-order systems with time delay [7]. Incorporating a first-order delay approximation, the plant model is $G_d(s) = e^{-Ts} b/(s + a) \approx b/(Ts + 1)(s + a)$, where T is a small delay relative to the plant dynamics, $T \ll 5/a$. Though not presented, the techniques of this paper are appropriate for this second-order approximation of $G_d(s)$. One needs to consider the limitations on K and any feedback gains (discussed later in this paper) due to decreased phase margin caused by the maximum expected T [8]. Such considerations often lead to compromised

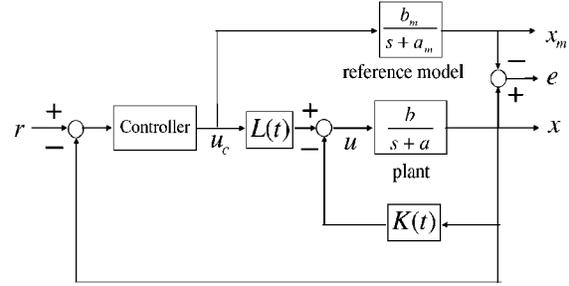


Fig. 2. Direct MRAC scheme.

performance of adaptive systems to ensure stability and robustness. Recently there has been much activity in developing adaptive control systems that minimize this compromise in performance for plants with significant delay [9–12].

2.2. Direct model reference adaptive controller (MRAC) design

The nonconventional controller design is based on the plant model with fixed parameters. When the system parameters vary with time, the nonconventional controller may not give desired performance. In this case, an adaptive controller, such as a direct or indirect MRAC, may serve better. Both the direct and indirect MRAC have been well established [13–17]. In this paper, we will follow the direct and indirect MRAC models discussed in Ref. [15].

The direct MRAC scheme is shown in Fig. 2. In Fig. 2, the controller is the PIID controller discussed in the previous section. The adaptive feedforward and feedback control laws $L(t)$ and $K(t)$ are designed to ensure that the plant follows the reference model in response to a reference input r .

The reference model is given as

$$\dot{x}_m = -a_m x_m + b_m u_c, \tag{1}$$

where x_m is the state of the reference model, a_m and b_m are the system parameters of the reference model, and u_c is the input to the reference model.

The real plant is given as

$$\dot{x} = -ax + bu, \tag{2}$$

where x is the plant state, a and b are the plant system parameters, and u is the control input to the plant,

$$u = -K(t)x + L(t)u_c. \tag{3}$$

Choose adaptive control laws

$$\begin{aligned} K &= \gamma_1 \int \operatorname{sgn}(b)xe \, dt \text{ and} \\ L &= -\gamma_2 \int \operatorname{sgn}(b)u_c e \, dt. \end{aligned} \quad (4)$$

Then the Lyapunov function and its derivative are

$$\begin{aligned} V(e, \tilde{K}, \tilde{L}) &= \frac{1}{2}e^2 + \frac{|b|}{2\gamma_1}\tilde{K}^2 + \frac{|b|}{2\gamma_2}\tilde{L}^2 \geq 0, \quad (5) \\ \dot{V} &= \dot{e}e + \frac{|b|}{\gamma_1}\dot{\tilde{K}}\tilde{K} + \frac{|b|}{\gamma_2}\dot{\tilde{L}}\tilde{L} \\ &= -a_m e^2 - |b|\tilde{K}\left(\frac{1}{\gamma_1}\dot{\tilde{K}} - \operatorname{sgn}(b)xe\right) \\ &\quad + |b|\tilde{L}\left(\frac{1}{\gamma_2}\dot{\tilde{L}} + \operatorname{sgn}(b)u_c e\right) \\ &= -a_m e^2 \leq 0, \end{aligned} \quad (6)$$

where $e = x - x_m$, $\tilde{K} = K - (a_m - a)/b$, $\tilde{L} = L - b_m/b$, and $\gamma_i > 0$.

It can be seen that if $u(t) \in L_\infty$ and $x(t) \in L_\infty$ are allowed then reference model following is guaranteed by Lyapunov Stability Theory. This is not always true for the typical PID (or PIID) control based on the linearization of nonlinear systems such as tank liquid level. Analytically, there are several ways to determine the appropriate value range for the tuning parameters γ_1 and γ_2 [13–15,18]. In practice, and due to model uncertainty and unknown delay, γ_1 and γ_2 are first given some very small numbers and tuned up gradually to reach a best performance. Generally speaking, the value of γ_i is a tradeoff between the convergence speed and the implementation difficulty, in that high adaptation rates must be avoided to prevent saturation of physical devices. Large γ_i leads to a faster convergence of the plant states to the reference model states. However, too large γ_i will lead to control saturation and the divergence of the adaptation laws, and subsequently the states, as the controller tries to minimize the error function.

Following a similar derivation, with capital letters indicating the vector and matrix versions of the scalar quantities above, this direct MRAC algorithm can also be applied to higher-order systems [14]. For an n th order plant with q inputs,

$$\dot{X} = AX + BU, \quad (7)$$

where $X \in \mathfrak{R}^n$, $A \in \mathfrak{R}^{n \times n}$, $B \in \mathfrak{R}^{n \times q}$ are unknown constant matrices and (A, B) is controllable. The reference model is given as

$$\dot{X}_m = A_m X_m + B_m U_c \quad (8)$$

where $A_m \in \mathfrak{R}^{n \times n}$, $B_m \in \mathfrak{R}^{n \times q}$, and $U_c \in \mathfrak{R}^q$ is a bounded input vector to the reference model. The control law is

$$U = -K(t)X + L(t)U_c \quad (9)$$

and the Lyapunov function and its derivative are

$$V(E, \tilde{K}, \tilde{L}) = E^T P E + \operatorname{tr}[\tilde{K}^T \Gamma \tilde{K} + \tilde{L}^T \Gamma \tilde{L}], \quad (10)$$

$$\begin{aligned} \dot{V} &= -E^T Q E + 2E^T P B_m L^{*-1}(-\tilde{K}L + \tilde{L}U_c) \\ &\quad + 2\operatorname{tr}(\tilde{K}^T \Gamma \dot{\tilde{K}} + \tilde{L}^T \Gamma \dot{\tilde{L}}), \end{aligned} \quad (11)$$

where $E = X - X_m$, $K^* = B^-(A - A_m)$, $L^* = B^{-1}B_m$, $\tilde{K} \equiv K - K^*$, $\tilde{L} \equiv L - L^*$, $\Gamma = L^* \operatorname{sgn}(l)$, and $P = P^T > 0$ satisfies the Lyapunov equation $PA_m + A_m^T P = -Q$ for some arbitrary $Q = Q^T > 0$. This leads to the control laws

$$\begin{aligned} \dot{K} &= B_m^T P E X^T \operatorname{sgn}(l) \text{ and} \\ \dot{L} &= -B_m^T P E U_c^T \operatorname{sgn}(l), \end{aligned} \quad (12)$$

where $l = 1$ if L^* is positive definite and $l = -1$ if L^* is negative definite. Note that $B_m^T P$ acts as an adaptive gain, thus Q is chosen similar to γ_1 and γ_2 for the scalar case. Again, the Lyapunov stability theory guarantees reference model following for this n th-order plant.

2.3. Indirect model reference adaptive controller (MRAC) with parallel Lyapunov estimation design

From Eqs. (1)–(3), it can be seen that if the system parameters a and b are known, then control law $u = -K^*x + L^*u_c$ can be used, where $K^* = (a_m - a)/b$ and $L^* = b_m/b$ yield perfect reference model following Refs. [13–15]. This is the principle of indirect MRAC. A parallel Lyapunov estimator estimates the system parameters a and b . The estimated parameters are noted by \hat{a} and \hat{b} . After replacing unknown a and b by their esti-

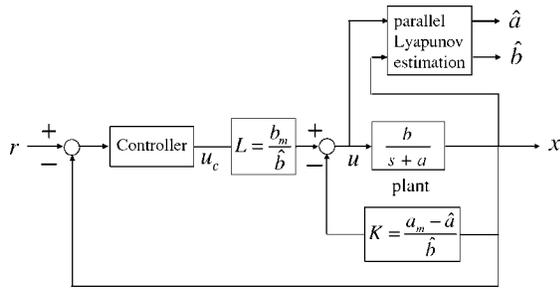


Fig. 3. Indirect MRAC with Lyapunov estimation.

mates \hat{a} and \hat{b} , the control laws become $K = (a_m - \hat{a})/\hat{b}$ and $L = b_m/\hat{b}$. The indirect MRAC scheme is shown in Fig. 3.

For the Lyapunov based estimation, the real plant is described by

$$\dot{x} = -ax + bu. \tag{13}$$

The estimated plant is

$$\dot{\hat{x}} = -\hat{a}\hat{x} + \hat{b}u, \tag{14}$$

where \hat{x} is the estimated plant state, \hat{a} and \hat{b} are estimated system parameters of the plant.

Choosing adaptive laws

$$\dot{\hat{a}} = -\gamma_1 e_1 x \text{ and } \dot{\hat{b}} = \gamma_2 e_1 u \tag{15}$$

then the Lyapunov function and its derivative are

$$V(e_1, \tilde{a}, \tilde{b}) = \frac{1}{2} e_1^2 + \frac{1}{2\gamma_1} \tilde{a}^2 + \frac{1}{2\gamma_2} \tilde{b}^2, \tag{16}$$

$$\begin{aligned} \dot{V} &= -\hat{a}e_1^2 + \tilde{a}\left(\frac{1}{\gamma_1}\dot{\tilde{a}} + e_1x\right) + \tilde{b}\left(\frac{1}{\gamma_2}\dot{\tilde{b}} - e_1u\right) \\ &= -\hat{a}e_1^2 \leq 0, \end{aligned} \tag{17}$$

where $e_1 = x - \hat{x}$, $\tilde{a} = \hat{a} - a$, $\tilde{b} = \hat{b} - b$, and $\gamma_i > 0$.

Applying Barbalat's Lemma it can also be shown that

$$\lim_{t \rightarrow \infty} e_1(t) = 0, \lim_{t \rightarrow \infty} \dot{\tilde{a}} = 0, \text{ and } \lim_{t \rightarrow \infty} \dot{\tilde{b}} = 0.$$

Note that, though it is guaranteed that the tracking error e_1 will converge to zero, it is not guaranteed that the estimates \hat{a} , \hat{b} will converge to the actual system parameters a , b .

This method can also be applied to high-order systems [14]. For an n th order plant with q inputs

$$\dot{X} = AX + BU, \tag{18}$$

where the states $X \in \mathfrak{R}^n$, $A \in \mathfrak{R}^{n \times n}$, and $B \in \mathfrak{R}^{n \times q}$ are unknown matrices. As in the first-order case shown above, the parallel estimated model is

$$\dot{\hat{X}} = \hat{A}\hat{X} + \hat{B}U. \tag{19}$$

Choosing adaptive laws

$$\dot{\hat{A}} = \gamma_1 E_1 \hat{X}^T \text{ and } \dot{\hat{B}} = \gamma_2 E_1 U^T \tag{20}$$

then the Lyapunov function and its derivative are

$$V(E_1, \tilde{A}, \tilde{B}) = E_1^T P E_1 + \text{tr}\left(\frac{\tilde{A}^T P \tilde{A}}{\gamma_1}\right) + \text{tr}\left(\frac{\tilde{B}^T P \tilde{B}}{\gamma_2}\right), \tag{21}$$

$$\begin{aligned} \dot{V} &= -E_1^T E_1 + 2\text{tr}\left(\frac{\tilde{A}^T P \dot{\tilde{A}}}{\gamma_1} - \tilde{A}^T P E_1 \hat{X}^T + \frac{\tilde{B}^T P \dot{\tilde{B}}}{\gamma_2} \right. \\ &\quad \left. - \tilde{B}^T P E_1 U^T\right) = -E_1^T E_1 \leq 0, \end{aligned} \tag{22}$$

where $E_1 = X - \hat{X}$, $\tilde{A} = \hat{A} - A$, $\tilde{B} = \hat{B} - B$, $\gamma_1, \gamma_2 > 0$ are constant scalars, and $P = P^T > 0$ is chosen as the solution of the Lyapunov equation $PA + A^T P = -I$. This implies that the equilibrium, $\hat{A} = A$, $\hat{B} = B$, and $E_1 = 0$, is uniformly stable.

2.4. Indirect model reference adaptive controller (MRAC) with updating recursive least-squares (RLS) estimation design

In the indirect MRAC scheme shown in Fig. 3, replacing the Lyapunov parallel estimation with a RLS updating estimation, the scheme becomes an indirect MRAC with RLS estimation.

RLS updating method uses the current estimate parameters θ_k to find the next parameter estimate θ_{k+1} ,

$$\hat{\theta}_{k+1} = \hat{\theta}_k + K_{k+1}(y_{k+1} - \varphi_{k+1}^T \hat{\theta}_k), \tag{23}$$

where

$$\theta \equiv \begin{bmatrix} -a_1 \\ b_0 \end{bmatrix}$$

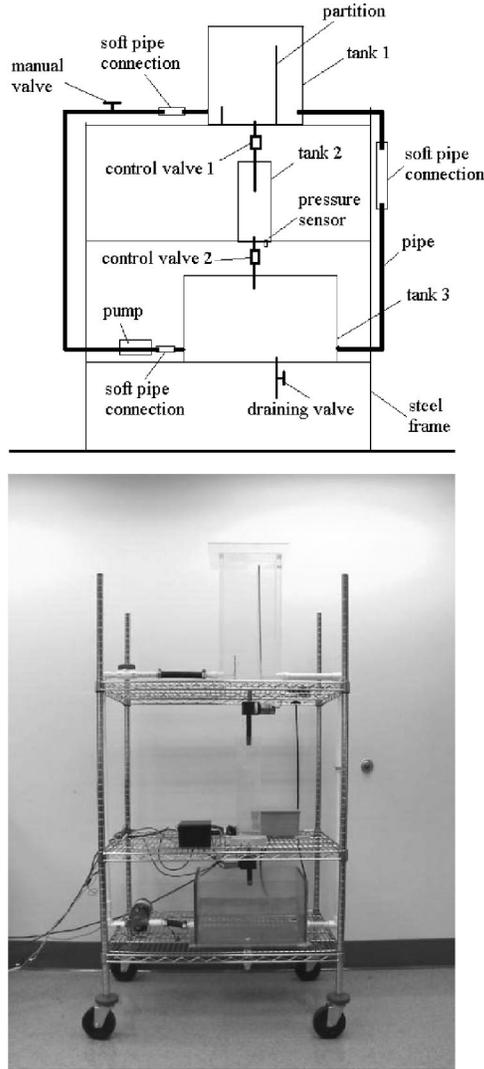


Fig. 4. Schematic drawing and picture of the three-tank system.

is the parameter vector of the discrete system model $M(\theta) = b_0 / (1 + a_1 q^{-1})$ to be estimated, $\varphi_k = [y_{k-n} \cdots y_{k-1} : u_{k-m} \cdots u_k]^T$. The Kalman gain K is given as

$$K_{k+1} = \frac{(X_k^T X_k)^{-1} \varphi_{k+1}}{1 + \varphi_{k+1}^T (X_k^T X_k)^{-1} \varphi_{k+1}}. \quad (24)$$

2.5. Apparatus description

The three-tank system shown in Fig. 4 consists of three rectangular tanks made of Plexiglas. They are named tank 1, tank 2, and tank 3, respectively,

from top to bottom. Tank 3 serves merely as a reservoir. Two proportional control valves are used to regulate the flow between each tank. Valve 1 is located between tank 1 and tank 2, and valve 2 is located between tank 2 and tank 3. There is a pump between tank 1 and tank 3, continuously pumping water from tank 3 to tank 1. A pressure sensor is mounted at the bottom of tank 2 to measure the water height inside the tank.

The objective is to control the water height in tank 2 to follow some reference input. By keeping the water height in tank 1 constant at 43.18 cm through physical design and the voltage to valve 1 constant at 0.32 V, the inlet mass flow rate of tank 2 is also constant at 24.87 ml/s. The control input is the voltage to valve 2, which in turn controls the outlet mass flow of tank 2. The ability exists to vary the pressure in each tank. However, this is left for future work. Additionally, this apparatus is developed as part of a web based instruction project in liquid level control.

Typically, a flow system's dynamic model can be derived from the mass conservation equation and Bernoulli's equation [19]. As shown in Fig. 4, after we account for the water head loss caused by control valves, pipes, and cross sectional area changes and apply the extended Bernoulli's equation and mass conservation equation, the system dynamic model becomes

$$\rho A_2 \dot{h} = -c_2 \sqrt{g(h + h_{p2})} + C_1, \quad (25)$$

where h is the water height in tank 2, A_2 is the cross-sectional area of tank 2, c_2 is the valve constant for control valve 2 and C_1 is the constant inlet mass flow rate of tank 2 (see Fig. 5).

It can be seen that the system is nonlinear. Using a Taylor's series to linearize the model about the middle height of tank 2 h_0 [20], where $h = h_0 + \Delta h$, the rate of change of the water level is

$$\Delta \dot{h} = -\frac{c_2}{2\rho A_2} \sqrt{\frac{g}{h_0 + h_{p2}}} \Delta h. \quad (26)$$

Therefore the system can be modeled as a first-order system with varying parameters due to the variation of the water height in tank 2. From the open-loop step system response shown in Fig. 1, a first-order reference model, $G(s) = b_m / (s + a_m) = 0.4 / (s + 0.009)$, is derived for the system. Fig. 1 also shows the comparison of the simulated water height response of the model and the real open-

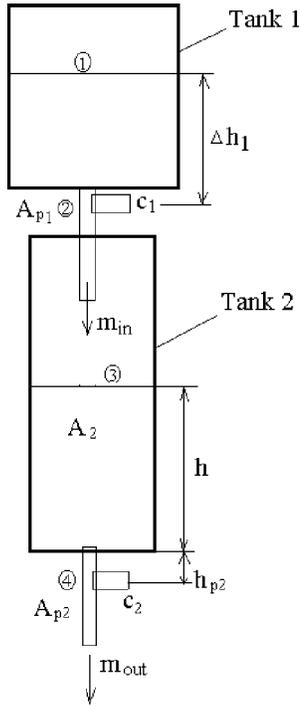


Fig. 5. Dynamics of the system.

loop water height response. It can be seen that the first-order reference model depicts the real open-loop system very well.

3. Results and discussion

The real-time operation of the entire system is conducted using digital control hardware and soft-

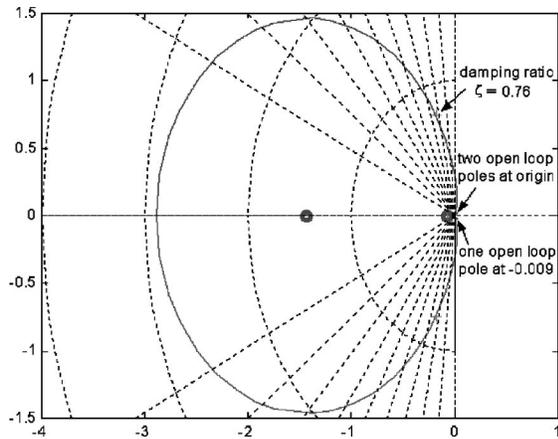


Fig. 6. Root locus design diagram w/pole placement for the PIID controller.

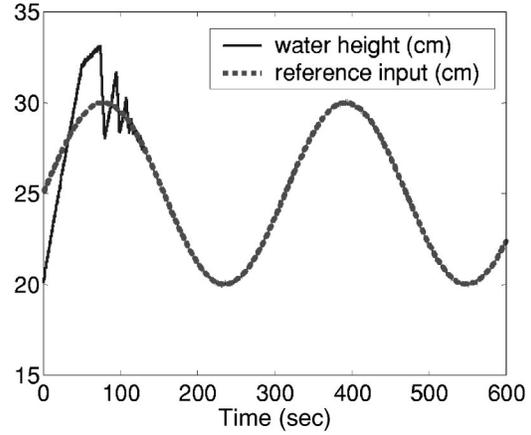


Fig. 7. Performance of PIID controller.

ware. Using pole placement and root locus design methods, the PIID is designed as $D(s) = (s^2 + 1.5s + 0.1)/s^2$. For this, a root locus design procedure was used to place the closed-loop poles in a suitable region of the root locus diagram for this controlled system, as shown in Fig. 6.

For implementation, the designed controller is discretized at a sampling rate of 1/100 sec. Both and adaptive controllers are implemented in the actual system to force the water height to follow a sinusoidal wave input $u = \sin(0.02t)$. During operation of the system, high-frequency noise corrupts measurements from the pressure sensor. To alleviate this problem, a discrete, fourth-order, low-pass Butterworth filter is used to obtain better measurements of the water height.

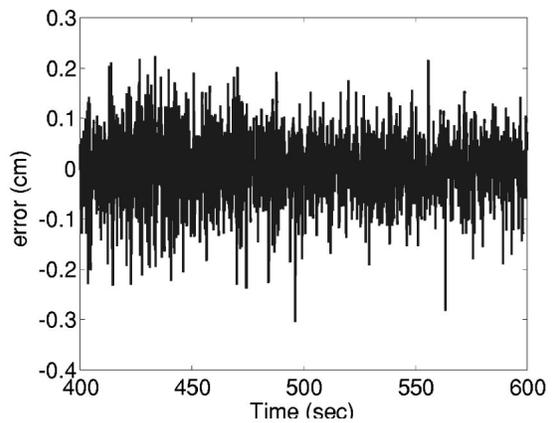


Fig. 8. Steady-state tracking error of PIID controller.

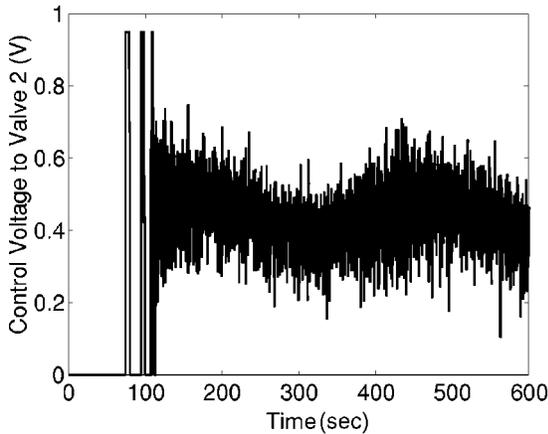


Fig. 9. Control voltage of PIID controller.

3.1. Nonconventional PIID controller

The closed-loop performance of the PIID is shown in Fig. 7. The steady-state tracking error is shown in Fig. 8. It can be seen that the controller exhibits very effective tracking performance to the reference input.

Over the given steady-state time window, (400–600 sec), the RMS value of the PIID tracking error is 0.0704 cm. The corresponding control voltage to valve 2 is shown in Fig. 9.

Initially, the control voltage to valve 2 saturates because of the initial difference between the water height and the reference input. After about 120 sec, control voltage is within the operating range, and the system response reaches steady state. During steady state, the control voltage varies between

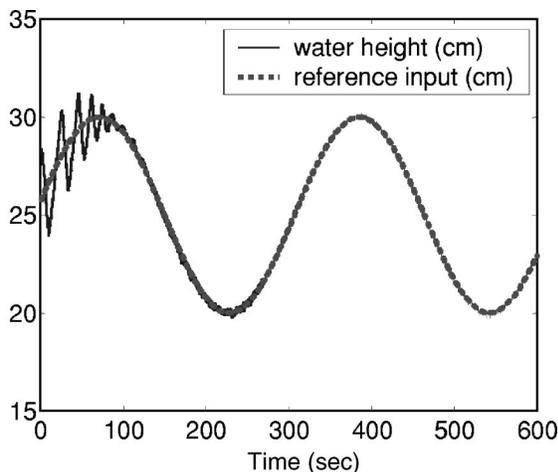


Fig. 10. Performance of direct MRAC.

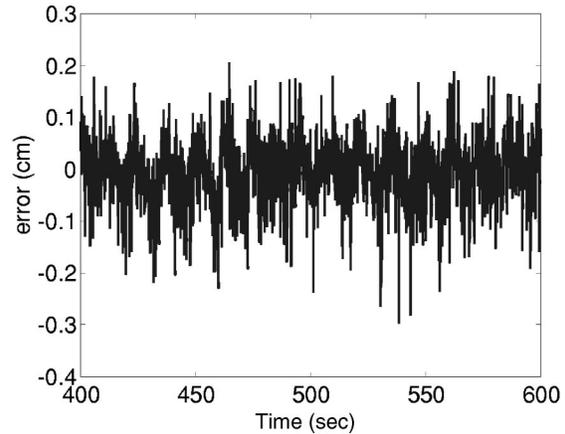


Fig. 11. Steady-state tracking error of direct MRAC.

about 0.25 and 0.65 V. From the results, it can be seen that the PIID design is successful.

3.2. Direct MRAC

The direct MRAC is implemented with constants $\gamma_1 = 10^{-6}$ and $\gamma_2 = 10^{-6}$, which are determined by tuning during implementation. The performance of the direct MRAC is shown in Fig. 10 and the steady-state tracking error is shown in Fig. 11.

It can be seen that the direct MRAC also tracks the reference input very well. During steady state (400–600 sec), the RMS value of the tracking error is 0.0628 cm. The control voltage is shown in Fig. 12. After about 60 sec, the control voltage is within the operating range of the valve, varying between about 0.45 and 0.6 V.

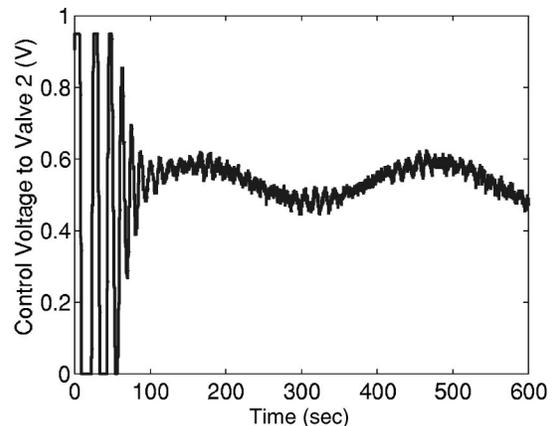


Fig. 12. Control voltage of direct MRAC.

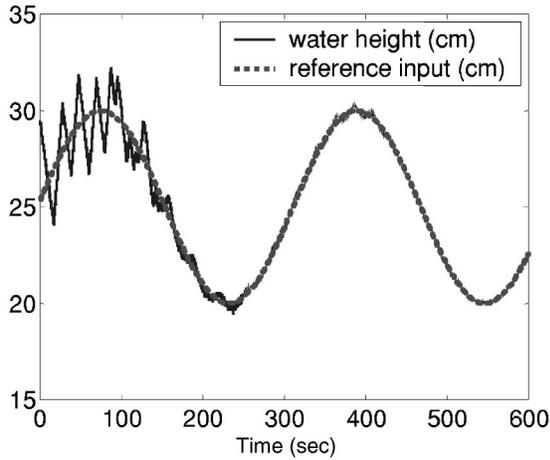


Fig. 13. Performance of indirect MRAC with Lyapunov estimation.

3.3. Indirect MRAC with Lyapunov estimation

The indirect MRAC is implemented with Eq. (4) constants $\gamma_1=10^{-4}$ and $\gamma_2=10^{-4}$, which are determined by tuning during implementation. The performance is shown in Fig. 13 and the steady-state tracking error is shown in Fig. 14.

It can be seen that the performance is not as good as the previous two controllers. It takes longer to reach steady state. During steady state (500–600 sec), the RMS value of the tracking error is 0.0719 cm. The control voltage is shown in Fig. 15. After about 160 sec the control voltage reaches the operating range of the valve, as it varies between about 0.3 and 0.75 V. The parameter

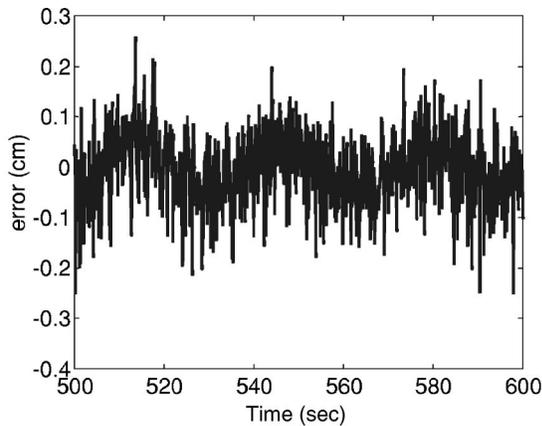


Fig. 14. Steady-state tracking error of indirect MRAC with Lyapunov estimation.

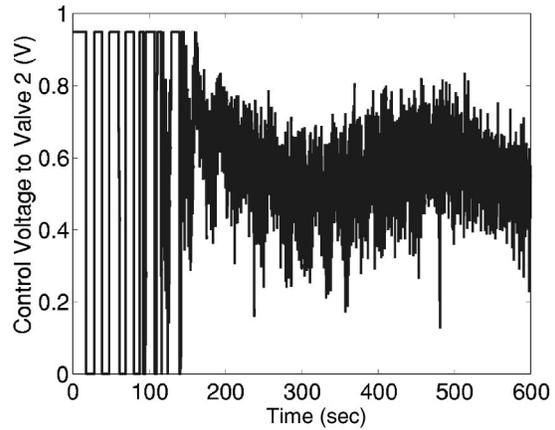


Fig. 15. Control voltage of indirect MRAC with Lyapunov estimation.

estimates \hat{a} and \hat{b} are shown in Figs. 16 and 17, respectively. It can be seen that both converge approximately to the parameter values in the reference model $G(s)$ with $a=0.009$ and $b=0.4$, respectively.

3.4. Indirect MRAC with RLS estimation

The performance of the indirect MRAC with RLS estimator is shown in Fig. 18. The steady-state tracking error is shown in Fig. 19. It can be seen that the indirect MRAC with RLS estimation tracks the input very well and has the fastest response among all four controllers. During steady state (400–600 sec), the RMS value of the tracking error is 0.1356 cm. Although the RMS value is

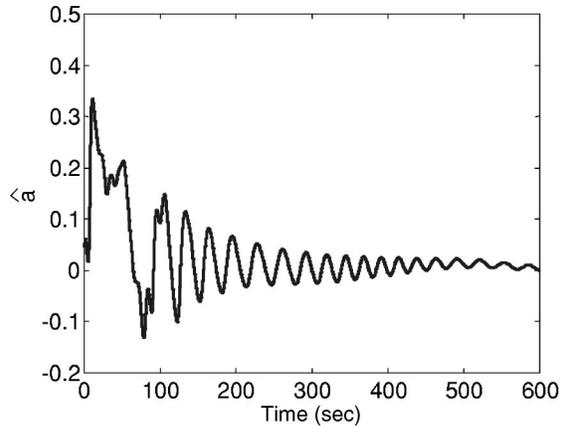


Fig. 16. Parameter estimate \hat{a} of indirect MRAC with Lyapunov estimation.

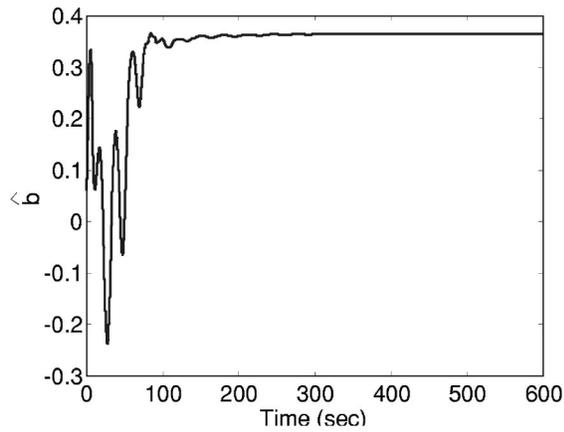


Fig. 17. Parameter estimate \hat{b} of indirect MRAC with Lyapunov estimation.

higher than the other controllers, it can be seen from Fig. 19 that the RLS steady-state tracking error has a tighter band than other controllers. There is some variation of the steady-state tracking error that leads to a higher RMS value.

The control voltage is shown in Fig. 20. After about 40 sec the control voltage reaches within the operation range of the valve, and it varies between about 0.45 and 0.6 V. The parameter estimates \hat{a} and \hat{b} are shown in Figs. 21 and 22, respectively. We can see that they converge quickly to the discrete parameter values in the reference model $a = -0.999$ and $b = 0.003931$, respectively.

Table 1 gives the comparison of the performance of all four controllers. It can be seen that

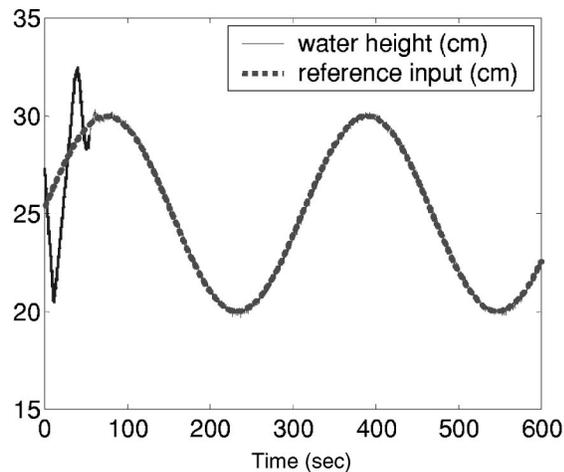


Fig. 18. Performance of indirect MRAC with RLS estimation.

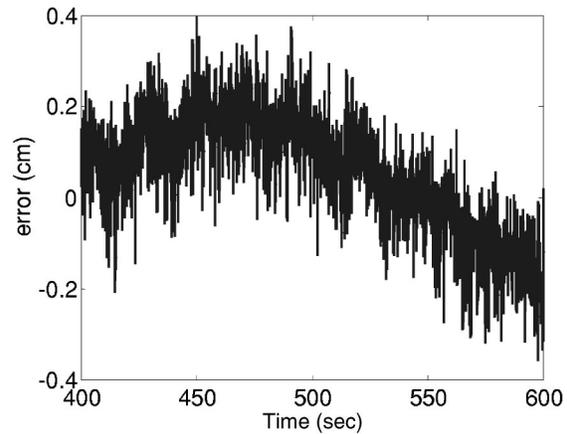


Fig. 19. Steady-state tracking error of indirect MRAC with RLS estimation.

direct MRAC and indirect MRAC with RLS estimation have the best performance. Adaptive control is an improvement over PIID control in the three-tank system.

4. Conclusion

This paper presents a comparative examination of PIID control and three adaptive control algorithms for liquid level control applications in a simple three-tank system. By implementing these adaptive control algorithms and a nonconventional controller, PIID, in the three-tank system, the performance improvement of the adaptive controllers has been demonstrated. Four controllers for three-tank liquid level control are designed and imple-

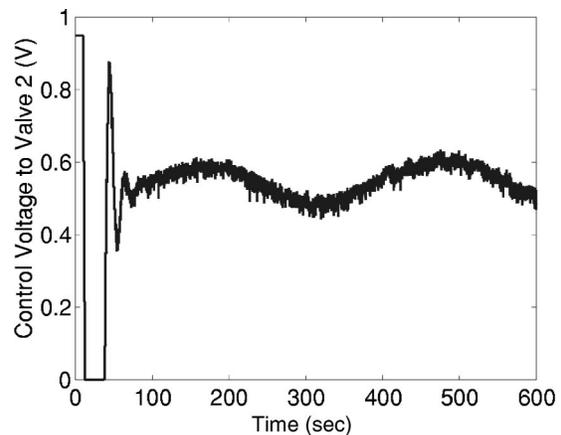


Fig. 20. Control voltage of indirect MRAC with RLS estimation.

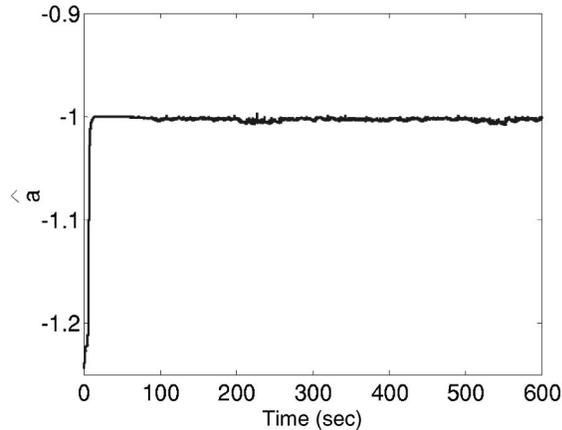


Fig. 21. Parameter estimate \hat{a} of indirect MRAC with RLS estimation.

mented, a nonconventional PIID, a direct model reference adaptive controller (MRAC), an indirect MRAC with Lyapunov estimation, and an indirect MRAC with RLS estimation. By implementing all four different types of controllers on an actual three-tank system, the performances of all controllers are compared. All controllers track a sinusoidal input very well and overall exhibit similar performance. However, the direct MRAC and the indirect MRAC with RLS estimation do deliver significantly better performance than the others. They both require the least control effort, while the indirect MRAC with RLS estimation has the fastest response and the direct MRAC has the smallest band of the steady-state tracking error. Additionally, RLS estimation has a much faster

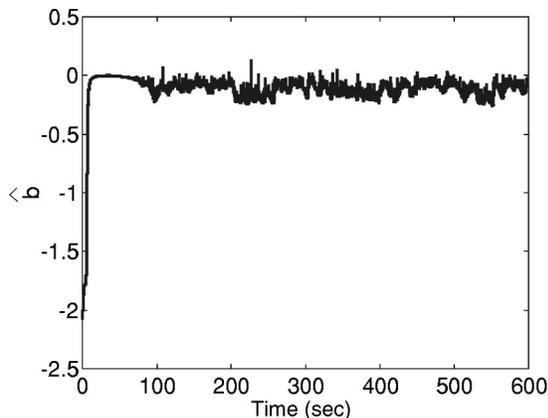


Fig. 22. Parameter estimate \hat{b} of indirect MRAC with RLS estimation.

Table 1
Controllers performance comparison.

	Settling time to steady state (sec)	Steady-state error RMS (cm)	Steady-state control effort range (V)	Parameter estimation
PIID	120	0.0704	0.25–0.65	
Direct	100	0.0628	0.45–0.6	
Indirect with Lyapunov	240	0.0719	0.3–0.75	converge after 100 sec
Indirect with RLS	60	0.1356	0.45–0.6	converge after 10 sec

convergence. The performance of the indirect MRAC with Lyapunov estimation is not an improvement to the PIID controller. With Lyapunov estimation and RLS estimation, the system parameter estimates all converge to the reference model values. By comparing the performance of the PIID and the adaptive controllers, it can be concluded that adaptive liquid level control is an improvement over PIID liquid level control when precise level control in three tanks is desired. However, the performance improvement is demonstrably limited to the three-tank apparatus presented. It is believed, but not implied here, that the presented adaptive control algorithms will deliver similar performance improvement when applied to complex and higher-order systems.

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References

- [1] Dukelow, S. G., *The Control of Boilers*, 2nd ed. Instrument Society of America, 1991.
- [2] Shinskey, F. G., Averaging level control. *Chem. Eng. Process.* **60** (9), 58 (1997).
- [3] Eborn, J., Panagopoulos, H., and Åström, K. J., Ro-

- bust PID control of steam generator water level. IFAC'99 14th World Congress of IFAC. Beijing, P. R. China, 1999.
- [4] Panagopoulos, H., Åström, K. J., and Hägglund, T., Design of PID controllers based on constrained optimization. 1999 American Control Conference. San Diego, California, 1999.
- [5] Kim, J. and Shin, D., Disturbance accommodating attitude control with reaction wheel. *Adv. Astronaut. Sci.* **97** (1), 929–948 (1997).
- [6] Kim, I. and Kim, J., Software-in-the-loop for attitude determination and control system using Simulink and Real-Time Workshop. *Adv. Astronaut. Sci.* **102** (1), 523–536 (1999).
- [7] Cohen, G. H. and Coon, G. A., Theoretical considerations of retarded control. *Trans. ASME* **75**, 827 (1953).
- [8] Fernandez, G., Ortega, R., and Begovich, O., Conditions for preservation of stability of adaptive controllers for systems with unmodeled delay. *IEEE Trans. Autom. Control* **33** (6), 601–603 (1988).
- [9] Wang, D., Zhou, D. H., Jin, Y. H., and Qin, S. J., Adaptive generic model control for a class of nonlinear time-varying processes with input time delay. *J. Process Control* **14**, 517–531 (2004).
- [10] Huzmezan, M., Gough, W. A., Dumont, G. A., and Kovac, S., Time delay integrating systems: A challenge for process control industries a practical solution. *Control Eng. Pract.* **10**, 1154–1161 (2002).
- [11] Diop, S., Kolmanovsky, I., Moraal, P. E., and van Nieuwstadt, M., Preserving stability/performance when facing an unknown time-delay. *Control Eng. Pract.* **9**, 1319–1325 (2001).
- [12] Shah, S., Iwai, Z., Mizumoto, I., and Deng, M., Simple adaptive control of processes. *J. Process Control* **7** (6), 439–449 (1997).
- [13] Haykin, S., *Adaptive Filter Theory*, 4th ed. Prentice-Hall, Englewood Cliffs, NJ, 2002.
- [14] Ioannou, P. A. and Sun, J., *Robust Adaptive Control*. Prentice-Hall, Englewood Cliffs, NJ, 1996.
- [15] Astrom, K. J. and Wittenmark, B., *Adaptive Control*, 2nd ed. Addison-Wesley, Reading, MA, 1995.
- [16] Nam, K. and Arapostathis, A., A model reference adaptive control scheme for pure feedback nonlinear systems. *IEEE Trans. Autom. Control* **33** (9), 738–752 (1988).
- [17] Krstic, M., Kanellakopoulos, I., and Kokotovic, P. V., Nonlinear design of adaptive controllers for linear systems. *IEEE Trans. Autom. Control* **39**, 738–752 (1994).
- [18] Kuo, S. M. and Morgan, D. R., *Active Noise Control Systems: Algorithms and DSP Implementations*. John Wiley & Sons, New York, 1996.
- [19] Ko, C. C., Chen, B. M., Chen, J., Zhuang, Y., and Tan, K. C., Development of a web-based laboratory for control experiments on a coupled tank apparatus. *IEEE Trans. Educ.* **44** (1), 76–86 (2001).
- [20] Holzenthall, L. L., Jr. and Masada, G. Y., Utilization of a linearized model for redesign of the feedwater control system of a drum boiler power plant. *Conference Proceedings—IEEE Southeastcon.* **3**, 914–918 (1990).