Design Issues in Adaptive Control

RICHARD H. MIDDLETON, GRAHAM C. GOODWIN, FELLOW, IEEE, DAVID J. HILL, AND DAVID Q. MAYNE, FELLOW, IEEE

Abstract—**The contributions of this paper are in two main areas. The first is an *integrated* approach to the design of practical adaptive control algorithms. In particular, we bring many existing ideas together and explore the effect of various design parameters available to a user. Secondly, we extend the theory by showing how the problem of stabilizability of the estimated model can be overcome by running parallel estimators. We also show how asymptotic tracking of deterministic set points can be achieved in the presence of unmodeled dynamics.**

I. Introduction

A GENERAL problem in control theory is to design control laws which achieve good performance for any member of a specified class of systems. Various strategies have been proposed for achieving this result. For example, robust controllers [1]–[3] are aimed at achieving good performance for the class of systems which are close (in a well-defined sense) to some *fixed* nominal system. Adaptive controllers are a special class of nonlinear control laws which are intended to stabilize all members of a wider class of systems, namely those systems which are close to *any member* of a given *set* of nominal systems, e.g., those nominal systems having some fixed McMillan degree [4]. Adaptive controllers generally have the additional property that the detailed structure of the control law can be decomposed into an on-line parameter estimation module together with an on-line control law synthesis procedure [4]–[6].

The first stability results for adaptive controllers were obtained in the late 1970's. At this time several research groups (see, for example, [7]–[12]) published results which established global convergence for a class of model reference adaptive control laws applied to linear minimum-phase plants of known order. Subsequently, the results were extended to nonminimum-phase plants provided pole-zero cancellations were avoided in the estimated model. Several methods have been proposed for achieving this, e.g., persistent excitation [14], restricting the parameter space to a single convex region [13], [5], and searching the parameter space for suitable points [15]. These analyses were based on the assumption that the system order was equal to the assumed model order. More recently, the sensitivity of these analyses to the modeling assumptions has been questioned [16] and examples have been presented showing that the algorithms can fail if they are applied blindly to systems having unmodeled dynamics. On the other hand, there is some practical evidence [17], [18] which suggests that certain adaptive control algorithms perform well.

There has been considerable research recently aimed at gaining a better understanding of adaptive control algorithms under nonideal conditions [19]–[51]. One attractive idea that has emerged in this research is that of using either normalization or dead zones to handle disturbances and unmodeled dynamics [5], [44], [54]–[58]. This work is a development of earlier ideas due to Egardt [52], Samson [53], and Praly [31]. It has also been incorporated in many practical realizations of adaptive control laws [18], [59].

The present paper builds on this line of research and brings together a range of techniques relevant to the design of adaptive control algorithms. Some of the techniques that are drawn upon include normalization or dead zones [5], [44], [54]–[58], regression vector filtering [60], [71], pole-assignment [5], [13], [14], internal model principle adaptive control [5], [61], and least squares [5], [61]. A discussion of convergence issues is presented for the resulting algorithm and this is related to design considerations. In particular, we show how pole-zero cancellations in the estimated model can be avoided, e.g., by running parallel estimators; and we show how asymptotic tracking of a desired set point y^* is achievable for plants having unmodeled dynamics. Our analysis shall be presented in the continuous-time domain. The reasons for doing this are: first the analysis then applies, with minor modification, to the discrete case also and, second, this gives confidence in the performance when rapid sampling is used. In practice, one would invariably use a discrete-time implementation, in which case we suggest use of the delta operator [61], [63] which gives enhanced numerical properties.

II. The System Model

We consider a plant described by the following model:

$$A_a y = B_a u + C_a \nu \tag{2.1}$$

where the degrees of the polynomial operators A_a, B_a, and C_a satisfy $\partial_{A_a} > \partial_{B_a}$, $\partial_{A_a} \geq \partial_{C_a}$; and ν is a bounded term which may include noise and deterministic disturbances. We rewrite (2.1) by introducing a nominal transfer function, $H_0 = B/A$, as

$$Ay = Bu + \eta + d \tag{2.2}$$

where u, y denote the plant input and output, respectively, η denotes an unmodeled component, and d denotes a purely deterministic noise term. The polynomial operators A, B are coprime and of degree ∂_A, ∂_B, respectively, where $\partial_B < \partial_A$ and A is monic.

Remark 2.1: Models of the form (2.2) have been used extensively in the literature, e.g., in [31], [44], [50], [70], [73]. The term η includes bounded noise and unmodeled components in the system response. For example, if the plant is modeled in transfer function form as

$$y = [H_0(1+\bar{H}) + \bar{\bar{H}}]u \tag{2.3}$$

where $H_0 = B/A$, $\bar{H} = \bar{B}/\bar{A}$, and $\bar{\bar{H}} = \bar{\bar{B}}/\bar{\bar{A}}$, then

$$\eta = \left[\frac{B\bar{B}}{\bar{A}} + \frac{A\bar{\bar{B}}}{\bar{\bar{A}}}\right] u. \tag{2.4}$$

▽▽▽

The deterministic component of the noise d is assumed to

Manuscript received October 3, 1986; revised May 11, 1987. Paper recommended by Past Associate Editor, R. R. Bitmead.

R. H. Middleton, G. C. Goodwin, and D. J. Hill are with the Department of Electrical and Computer Engineering, University of Newcastle, New South Wales, Australia.

D. Q. Mayne is with the Department of Electrical Engineering, Imperial College of Science and Technology, London, England.

IEEE Log Number 8717740.

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satisfy a model of the form

$$Sd=0 \tag{2.5}$$

where S is a known monic polynomial of degree ∂_S having nonrepeated zeros on the stability boundary. For example, if d is a sinewave of angular frequency ω_0, then $S = D^2 + \omega_0^2$; $D = d/dt$.

Finally, to ensure that the final control system based on the internal model principle is well posed, we require the following.

Assumption A.1: S has no zeros in common with the numerator B_a of the true plant transfer function B_a/A_a. ▽▽▽

III. Parameter Estimation

We now introduce a bandpass filter to eliminate the deterministic disturbance and also to mitigate the effect of unmodeled dynamics on the estimator. This is a combination of the ideas in, for instance [60], [61], and [71]. We define the following filtered variables:

$$\bar{y}=\frac{GS}{JQ}y;\ \bar{u}=\frac{GS}{JQ}u \tag{3.1}$$

where G, J, Q are monic Hurwitz polynomials $\partial_G = \partial_J$ and $\partial_Q = \partial_S$.

Using (3.1) and (2.5), the model (2.2) can be rewritten as

$$A\bar{y}=B\bar{u}+\bar{\eta} \tag{3.2}$$

where

$$\bar{\eta}=\frac{GS}{JQ}\eta. \tag{3.3}$$

Equation (3.2) will, in general, be unsuitable for parameter estimation since the *equation error* term $\bar{\eta}$ involves *differentiation* of the unmodeled error which may include noise. We, therefore, introduce an additional filter, $1/E$, where E is monic, Hurwitz, and $\partial_E = \partial_A$. We then define the following:

$$y_f=\frac{1}{E}\bar{y};\ u_f=\frac{1}{E}\bar{u};\ \eta_f=\frac{1}{E}\bar{\eta}. \tag{3.4}$$

Substituting (3.4) into (3.2) gives

$$\bar{y}=(E-A)y_f+Bu_f+\eta_f. \tag{3.5}$$

This equation can be expressed in regression form as

$$\bar{y}=\phi^T\theta_*+\eta_f. \tag{3.6}$$

Note that in (3.6), we have ignored *initial condition* terms which decay exponentially fast to zero due to the stability of $F = EJ$. They can be treated without further difficulty as in [10] and [6].

In (3.6) (with $n \triangleq \partial_A$, $m \triangleq \partial_B$, $s \triangleq \partial_S$, $g \triangleq \partial_G$) the various terms are

$$\phi^T=[y_f, \cdots, D^{n-1}y_f, u_f \cdots D^m u_f] \tag{3.7}$$

$$\theta_*^T=[e_0-a_0, \cdots, e_{n-1}-a_{n-1}, b_0, \cdots, b_m] \tag{3.8}$$

where

$$E(D) \triangleq D^n+e_{n-1}D^{n-1}+\cdots+e_0 \tag{3.9}$$

$$A(D) \triangleq D^n+a_{n-1}D^{n-1}+\cdots+a_0 \tag{3.10}$$

$$B(D) \triangleq b_mD^m+b_{m-1}D^{m-1}+\cdots+b_0. \tag{3.11}$$

In some earlier papers, e.g., [29] θ_* has been called the *tuned parameters*. Note that θ_* is not required to be unique and, in general, will not be.

The filtering operation outlined above is crucial to the success of adaptive controllers since it focuses the parameter estimator on the relevant frequency band thereby significantly reducing the deleterious effects of unmodeled dynamics and disturbances. In some cases this may be sufficient to obtain satisfactory performance. For example, robust stability has been established under these conditions provided persistently exciting signals are added [34], [36], [40], [46], [47], [60], and [71]. However, in many cases, it is impractical to add these signals and thus a separate line of development has evolved to treat this case. A key feature of this development has been the incorporation of additional normalization or dead zones into the parameter estimator to reduce the estimator gain when the unmodeled response is large. This line of development can be traced to early work on bounded disturbances of Egardt [11], Samson [53], and later to Praly [74] who treated the case of unmodeled dynamics. This idea has also been investigated in [31], [44], [50], [53], [57], [58], [61], [64], [70], [72], and [76].

A key feature of the above development has been the introduction of overbounding functions for the unmodeled response. This is possible provided the unmodeled response is stable in the sense made precise below.

Assumption A.2: The filtered unmodeled error η_f is the sum of a bounded term, plus a term related to $\bar{u}$ by a strictly proper exponentially stable transfer function. ▽▽▽

The following result then establishes the fact that $\{|\eta_f(t)|\}$ is overbounded by a function $\{\rho(t)\}$ which depends on *past* values of $|\bar{u}(t)|$ and $|y(t)|$.

Lemma 3.1: For all members of the class of systems satisfying Assumption A.2 there exist constants $\sigma_0 \in (0, 1)$, $\epsilon_0 \geq 0$, $\epsilon \geq 0$, and a constant vector v such that

$$|\eta_f(t)| \leq \epsilon\rho(t)+\epsilon_0 \qquad \text{for all } t \tag{3.12}$$

where

$$\rho(t)=\sup_{0\leq\tau\leq t}\{|v^Tx(\tau)|e^{-\sigma_0(t-\tau)}\} \tag{3.13}$$

x is the following state vector:

$$x \triangleq [D^{n+s+g-1}z_f', \cdots, z_f', D^{n-1}u_f, \cdots, u_f]^T \tag{3.14}$$

and

$$z_f' \triangleq \frac{1}{FQ}z; \qquad z \triangleq y-y^*. \tag{3.15}$$

Proof (Outline): We introduce an arbitrary Hurwitz polynomial $V = D^{n-1} + v_{n-2}D^{n-2} + \cdots v_o$ of degree $n - 1$. In view of (2.4), η_f can then be expressed as

$$\eta_f=\left[\frac{B\bar{B}}{V\bar{A}}+\frac{A\bar{\bar{B}}}{V\bar{\bar{A}}}\right]Vu_f+\text{(bounded terms)}. \tag{3.16}$$

Then, if we define

$$v=[0, \cdots 0, 1, v_{n-2}, \cdots, v_o]^T$$

it follows that $Vu_f = v^Tx$. The remainder of the proof is straightforward as in [5, Appendix B]. ▽▽▽

Remark 3.1: Other bounding functions can be used [31], [44], [50], [57] in place of $\rho(t)$, e.g., based on weighted integrals of $\bar{u}(t)$, etc. ▽▽▽

In our subsequent analysis we will replace knowledge of $\{\eta_f(t)\}$ by *any* suitable overbounding function $\rho(t)$. We therefore introduce the following assumption.

Assumption A.3: Constants ϵ_0, ϵ, v, σ_0 are assumed known such that (3.12) is satisfied. ▽▽▽

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We next propose the following parameter estimator:

$$D\hat{\theta}=bP\phi e \tag{3.17}$$

$$DP=-bP\phi\phi^TP+R;\ P(0)=P(0)^T>0 \tag{3.18}$$

where

$$e=\bar{y}-\phi^T\hat{\theta} \tag{3.19}$$

$$R=R^T\geq 0 \tag{3.20}$$

and where b represents a time-varying gain. The basic idea in both normalization and dead zones is to reduce b when the unmodeled dynamics are large.

In the case of normalization one may use

$$b=\frac{\alpha_1}{1+\alpha_2\rho^2+\phi^TP\phi} \tag{3.21}$$

where α_1, $\alpha_2>0$ are constants, chosen such that $\alpha_1\epsilon/\alpha_2$ and $\alpha_1\epsilon_0$ are small. Note that in this case the algorithm does not depend explicitly on knowledge of ϵ or ϵ_0.

In the case of a dead zone, we select $\beta>1$, and define

$$s(t)\triangleq\begin{Bmatrix} 0 \text{ if } |e(t)|\leq\beta(\epsilon\rho(t)+\epsilon_0) \\ f(\beta[\epsilon\rho(t)+\epsilon_0],\ e(t))/e(t) \text{ otherwise}\end{Bmatrix} \tag{3.22}$$

where

$$f(g,e)\triangleq\begin{Bmatrix} e-g & \text{if } e>g \\ 0 & \text{if } |e|\leq g \\ e+g & \text{if } e<-g\end{Bmatrix}. \tag{3.23}$$

We then use

$$b=\frac{\alpha_1 s}{1+\phi^TP\phi}. \tag{3.24}$$

Remark 3.2: The term $R(t)$ in (3.18) has been introduced to ensure that the algorithm gain does not *turn off*. Any choice of $R(t)=R(t)^T\geq 0$ that ensures $P(t)$ and $P(t)^{-1}$ are bounded may be used: indeed it can be shown that many of the usual least-squares modifications, including covariance resetting, variable forgetting factors, gradient algorithms, and constant trace algorithms, are described by a suitable choice of R.

It is also possible to add additional refinements to the algorithm. For example, it is possible to constrain the parameter estimates to lie in prespecified convex regions, and/or to add parameter searches to satisfy additional constraints. It is particularly important to constrain the estimates in the case of normalization since, unlike the dead zone algorithm, this algorithm does not guarantee boundedness automatically.

There are basically two ways of constraining the estimates; to include a hard constraint implemented using projection [5], [6], or to use a soft constraint implemented via a σ-modification [44], [50]. $\nabla\nabla\nabla$

The convergence properties of the least-squares parameter estimator (possibly modified as above) are summarized in the following.

Lemma 3.2 (Dead Zones): Consider the parameter estimator (3.17), (3.18), (3.24) applied to any system of the form (2.2), subject to Assumption A.2. Then the following properties hold irrespective of the control law:

i) $$\bar{f}^2\triangleq\frac{f(\beta[\epsilon\rho(t)+\epsilon_0],\ e(t))^2}{1+\phi(t)^T\phi(t)}\in L_1 \tag{3.25}$$

ii) $$D\hat{\theta}\in L_2 \tag{3.26}$$

iii) for all t, $$\|\hat{\theta}(t)-\theta_*\|\leq\nu(P_0)\|\hat{\theta}(0)-\theta_*\| \tag{3.27}$$

where ν (P_0) denotes the condition number of the matrix P_0.

Proof (Outline): Consider the "Lyapunov" function $V=\tilde{\theta}^TP^{-1}\tilde{\theta}$ with $\tilde{\theta}=\hat{\theta}-\theta_*$. Then

$$\dot{V}\leq b(\eta_f^2-e^2) \tag{3.28}$$

$$\leq b((\epsilon\rho+\epsilon_0)^2-e^2) \tag{3.29}$$

where we have used (3.12). Using the expression for b in the case of dead zones (3.24), we then have

$$\dot{V}\leq\frac{\alpha_1 s\left(\dfrac{e^2}{\beta^2}-e^2\right)}{1+\phi^TP\phi} \tag{3.30}$$

since $s=0$ when $e^2\leq\beta^2(\epsilon\rho+\epsilon_0)^2$ [see (3.22)].

Now in view of (3.22), (3.23); $se^2=fe\geq f^2$ and thus

$$\dot{V}\leq-\alpha_1\left(\frac{\beta^2-1}{\beta^2}\right)\frac{f^2}{1+\phi^TP\phi}. \tag{3.31}$$

The remainder of the proof follows as in [5], [6]. $\nabla\nabla\nabla$

Remark 3.3: This lemma is an extension to the continuous-time least-squares case of results in [43], [50], and [58]. $\nabla\nabla\nabla$

Lemma 3.3 (Normalization): Consider the parameter estimator (3.17), (3.18), (3.21) applied to any system of the form (2.1), subject to Assumption A.3. Then the following properties hold irrespective of the control law:

i) $$\int_0^t\bar{e}^2(\tau)\,d\tau\leq k_0t+k_1 \tag{3.32}$$

where

$$\bar{e}\triangleq\frac{e}{(1+\alpha_2\rho^2+\phi^TP\phi)^{1/2}} \tag{3.33}$$

$$k_0=\frac{(\epsilon\rho+\epsilon_0)^2}{1+\alpha_2\rho^2} \tag{3.34}$$

and

$$k_1=\frac{1}{\alpha_1}\tilde{\theta}_0^TP_0^{-1}\tilde{\theta}_0 \tag{3.35}$$

ii) $$\int_0^t\dot{\hat{\theta}}^T(\tau)\dot{\hat{\theta}}(\tau)\,d\tau\leq\alpha_1^2(\lambda_{\max}P)[k_0t+k_1]. \tag{3.36}$$

Proof (Outline): Equation (3.29) also holds in this case. Then using (3.21) we have

$$\dot{V}\leq\frac{\alpha_1(\epsilon\rho+\epsilon_0)^2}{1+\alpha_2\rho^2}-\frac{\alpha_1e^2}{1+\alpha_2\rho^2+\phi^TP\phi}. \tag{3.37}$$

Result i) follows upon integration of both sides of (3.37). Part ii) follows immediately since

$$\dot{\hat{\theta}}^T\dot{\hat{\theta}}=\frac{\alpha_1^2e^2\phi^TP^2\phi}{(1+\alpha_2\rho^2+\phi^TP\phi)^2}\leq\alpha_1^2(\lambda_{\max}P)\bar{e}^2. \tag{3.38}$$

$\nabla\nabla\nabla$

Remark 3.4: This lemma is a simple extension to the results in [43] and [44]. $\nabla\nabla\nabla$

IV. Control System Design

When the nominal model is known, there exist many possible choices for the control system design procedure including frequency domain methods, model reference control (for minimum-phase plants), pole assignment, linear quadratic optimal

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control, etc. Our analysis will be based on pole assignment since this includes several other design procedures as special cases. We will also use the internal model principle [68], [5], [77], so that the classical three term control law will be a special case of our design procedure.

The input u is generated using

$$LGSu = P(y^* - y) = -Pz; \qquad z = y - y^* \tag{4.1}$$

or equivalently

$$Lu_f = -Pz_f'; \qquad z_f' = \frac{1}{FQ} z \tag{4.2}$$

where L, P are polynomial in D determined from the following pole-assignment equation:

$$ALGS + BP = A^* \tag{4.3}$$

where A^* is a monic Hurwitz polynomial of degree $\partial_{A^*} = 2\partial_A + \partial_G + \partial_S$. The degree of L and P are, respectively, $\partial_L = \partial_A$ and $\partial_P = \partial_A + \partial_G + \partial_S - 1$. When substituted into (4.2) this yields a strictly proper control law which can be implemented as

$$\bar{u} = (E - L)u_f - Pz_f' \tag{4.4}$$

$$u = \frac{JQ}{GS} \bar{u}. \tag{4.5}$$

V. Adaptive Control Algorithm

The essential idea in obtaining the adaptive control law is to combine the least-squares algorithm of Section III with the certainty equivalence form of the feedback control law (4.2). i.e., given $\hat{\theta}(t)$ we form $\hat{A}$, $\hat{B}$, and $\hat{A}^*$ and solve

$$\hat{A}\hat{L}GS + \hat{B}\hat{P} = \hat{A}^* \tag{5.1}$$

for $\hat{L}$ and $\hat{P}$. Then implement the control law as

$$\hat{L}u_f = -\hat{P}z_f' \tag{5.2}$$

which may be rewritten as

$$\bar{u} = (E - \hat{L})u_f - \hat{P}z_f' \tag{5.3}$$

$$u = \frac{JQ}{GS} \bar{u}. \tag{5.4}$$

In (5.1) we have allowed the desired closed-loop polynomial to depend upon the parameter estimates. (Hence, the notation $\hat{A}^*$.) We require the following condition.

Assumption A.4: The partial derivative $\partial \hat{A}^* / \partial \hat{\theta}$ is bounded and $\hat{A}^*$ is uniformly strictly Hurwitz. ▽▽▽

A key technical point is that we must ensure that all limit points of the algorithm correspond to a stabilizable model. Several techniques have been proposed in the literature for achieving this. One method is the use of parameter space search techniques as suggested in, for example, [15], [72], and [76]. Another method involves constraining the parameter estimates to a single convex region in which the tuned parameters lie and such that all models in the region are stabilizable [5], [6], [13], [50].

Here we extend the latter strategy to include any finite union of convex sets which satisfies Assumption A.4.

Assumption A.5: There exists a finite set $\{D_1, \cdots, D_p\}$ of convex sets (not necessarily disjoint) such that

i) $\theta_* \in \bigcup_{i=1}^{p} D_i$

ii) for all $\theta \in \cup_{i=1}^{p} D_i$, the corresponding model is uniformly stabilizable. ▽▽▽

The basic idea of our proposed adaptive control algorithm is to run a separate parameter estimator in each of the p convex regions. A suitable performance index then allows one of these regions to be selected. The corresponding parameter estimates are then used to implement a certainty equivalence form of the control law.

An alternative to running parallel estimators is to start with any particular convex region and to monitor the parameter estimation performance criteria. If these criteria exceed a preset threshold, one recommences estimation in another of the regions.

The parameter estimator performance index needs to be selected so as to ensure that swapping between different regions does not occur infinitely often and to guarantee that the key properties of the parameter estimator are retained. Note that this need not imply that θ^* belongs to the region ultimately chosen by the algorithm.

In the case of the dead zone algorithm, one suitable form for the parameter estimator index is

$$M_i(t) = \int_0^t \frac{f(\beta(\epsilon\rho(\tau) + \epsilon_0), e_i(\tau))^2}{1 + \phi(\tau)^T P_i(\tau)\phi(\tau)} \, d\tau \tag{5.5}$$

where $e_i(\tau)$, $P_i(\tau)$ refer to the estimator operating in the ith region D_i. The selection procedure is then as follows.

Let $j(t)$ denote the region selected at time t ($j(0)$ being arbitrary). If $M_{j(t)}(t) < M_i(t) + \gamma$ for $i \neq j(t)$, then $j(t)$ is unaltered. Otherwise, $j(t) = \text{Arg}\min_{i=1\cdots p} [M_i(t)]$.

V. Analysis of the Algorithm

The key equations that we shall use are: the parameter estimator (3.17), (3.18), (3.22)–(3.24), the prediction error (3.19), the design identity (5.1), and the feedback control law (5.3), (5.4).

As suggested in [30], [50], [53], and [76], the control law equation (5.3), (5.4), and the prediction error equation (3.19), may be combined into the following closed-loop equation:

$$Dx(t) = \bar{A}(t)x(t) + B_1(e + r) \tag{6.1}$$

where

$$\bar{A}(t) \triangleq \begin{bmatrix} -\hat{c}_{n+g+s-1} & & & -\hat{c}_0 & \hat{b}_{n-1} & & & \hat{b}_0 \\ 1 & 0 & & 0 & 0 & & & 0 \\ 0 & & & & & & & \\ 0 & 0 & 1 & 0 & 0 & & & 0 \\ -\hat{p}_{n+g+s-1} & & & -\hat{p}_0 & -\hat{l}_{n-1} & & & -\hat{l}_0 \\ 0 & & & 0 & 1 & 0 & & 0 \\ & & & & 0 & & & \\ 0 & & & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{6.2}$$

$$\begin{aligned} \hat{C}(D) &\triangleq D^{n+g+s} + \hat{c}_{n+g+s-1}D^{n+g+s-1} + \cdots + \hat{c}_0 \\ &= \hat{A}(D)S(D)G(D) \end{aligned} \tag{6.3}$$

$$B_1^T \triangleq [1, 0 \cdots 0] \tag{6.4}$$

$$r \triangleq \hat{A}\left(\frac{GS}{FQ} y^*\right). \tag{6.5}$$

The analysis of the closed-loop system described above will depend on two key facts relating first to the parameter estimator and second to the homogeneous part of (6.1). We therefore begin by establishing these preliminary facts.

Properties of the multiple convex region parameter estimator are described in the following lemma for the dead zone algorithm.

Lemma 6.1:

i) There exists a $t_0 > 0$ such that

$$j(t) = j(t_0) \qquad \text{for all } t \geq t_0.$$

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ii) With the time origin moved to t_0, then

$$\bar{f} \in L_2 \text{ and } D\hat{\theta} \in L_2.$$

Proof: In view of Assumption A.5 it is clear that there exists a $\bar{j}$ such that $\theta^* \in D_{\bar{j}}$. From Lemma 3.2 it follows that $M_{\bar{j}}$ is bounded. The result then follows since the algorithm ensures $M_{j(t)} \le M_{\bar{j}} + \gamma$. $\nabla\nabla\nabla$

Properties of the homogeneous part of (6.1) are given in the following.

Lemma 6.2: Consider the following homogeneous linear time-varying system:

$$D\bar{x}(t) = \bar{A}(t)\bar{x}(t). \tag{6.6}$$

Provided

i) $\bar{A}(t)$ is bounded.

ii) $\int_T^{T+t} \|\dot{\bar{A}}(\tau)\|^2\, d\tau \le k_0 t + k_1;\ \forall t,\ T$ where k_0 is sufficiently small.

iii) The eigenvalues of $\bar{A}$ are strictly inside the stability boundary for all t.

Then (6.6) is exponentially stable.

Proof (Outline): (See also [43], [44], [57], [65], [74], and [76] for closely related results.)

Choose $\Gamma = \Gamma^T \ge 0$ and let $\Omega(t)$ denote the positive definite symmetric solution to

$$\bar{A}^T(t)\Omega(t) + \Omega(t)\bar{A}(t) = -\Gamma \tag{6.7}$$

then using i)–iii) we can show that $\Omega(t)$ is uniformly positive definite, bounded, and $\exists\ k_2$ s.t.

$$\|\dot{\Omega}(t)\|_2 \le k_2 \|\dot{\bar{A}}(t)\|_2. \tag{6.8}$$

Now consider the system

$$D(w(t)) = (\bar{A}(t) - \frac{1}{2}\Omega^{-1}(t)\dot{\Omega}(t))w(t) \tag{6.9}$$

and define

$$V(t) = w^T(t)\Omega(t)w(t) \tag{6.10}$$

we then have

$$\dot{V}(t) = -w^T(t)\Gamma w(t) \tag{6.11}$$

which establishes exponential stability of (6.9).

Rewriting (6.6) gives

$$D\bar{x}(t) = (\bar{A}(t) - \frac{1}{2}\Omega^{-1}(t)\dot{\Omega}(t))\bar{x}(t) + \frac{1}{2}\Omega^{-1}(t)\dot{\Omega}(t)\bar{x}(t). \tag{6.12}$$

The solution to (6.12) can be written

$$\bar{x}(t) = \bar{\phi}(t, T)\bar{x}(T) + \int_T^t \bar{\phi}(t, \tau)\frac{1}{2}\Omega^{-1}(\tau)\dot{\Omega}(\tau)\bar{x}(\tau)\, d\tau \tag{6.13}$$

where $\bar{\phi}(t, \tau)$ is the state transition matrix of the system in (6.9).

In view of the exponential stability of $\bar{\phi}$ established in (6.10), (6.11), and using Schwarz's inequality, we have

$$\|\bar{x}(t)\|^2 \le c_0 e^{-2\sigma(t-T)}\|\bar{x}(T)\|_2^2 + \int_T^t c_1 e^{-\sigma(t-\tau)}\|\dot{\Omega}(\tau)\|_2^2 \|\bar{x}(\tau)\|_2^2\, d\tau \tag{6.14}$$

for some c_0, c_1, $\sigma > 0$, and thus

$$\|\bar{x}(t)\|_2^2 e^{\sigma t} \le c_0\|\bar{x}(T)\|_2^2 + \int_T^t c_1\|\dot{\Omega}(\tau)\|_2^2 e^{\sigma t}\|\bar{x}(\tau)\|_2^2\, d\tau. \tag{6.15}$$

Using Gronwall's lemma we then have

$$\|\bar{x}(t)\|_2^2 e^{\sigma t} \le c_0\|\bar{x}(T)\|_2^2 \exp\left\{\int_T^t c_1 k_2^2 \|\dot{\bar{A}}(\tau)\|_2^2\, d\tau\right\}$$
$$\le c_0\|\bar{x}(T)\|_2^2 \exp(c_1 k_2^2 k_1)\exp(k_0 c_1 k_2^2 (t-T)). \tag{6.16}$$

From (6.16), it is clear that provided $k_0 < \sigma/c_1 k_2^2$, (6.6) is exponentially stable. $\nabla\nabla\nabla$

Remark 6.1: Lemma 6.2 is useful in the analysis of adaptive controllers employing either relative dead zones or normalization. In the case of relative dead zones, property ii) of Lemma 3.2 implies that k_0 in Lemma 6.2 is zero. In the case of normalization, property ii) of Lemma 3.3 implies that k_0 is nonzero in general. $\nabla\nabla\nabla$

We will now illustrate the stability analysis of the adaptive algorithm for the case of dead zones.

Theorem 6.1 (Dead Zones):

1) Subject to Assumptions A.1–A.5 and provided ϵ [in (3.12)] is sufficiently small, then the adaptive control law, applied to the plant (2.2), is globally stable in the sense that y and u (and, hence, all states) are bounded for all finite initial states and any bounded y^*.

2) If, in addition, y^* is purely deterministic, satisfying $Sy^* = 0$, and $\epsilon_0 = 0$, it then follows that

$$\lim |y(t) - y^*(t)| = 0. \tag{6.17}$$

Proof: The proof is based on (6.1). The various terms in this equation are dealt with as follows: $\bar{A}(t)$ is exponentially stable; r is bounded and thus a BIBS argument can be used; e can be decomposed into a sum of terms in f and $e - f$; the term in f is dealt with by Gronwall's lemma since $\bar{f} \in L_2$; and the term in $e - f$ is handled by a small gain type argument since $|e - f| < \beta[\epsilon\rho + \epsilon_0]$.

The algorithm ensures there are no finite escapes. We then redefine the time origin as $t_0 = 0$ where t_0 is as in Lemma 5.1.

1) Noting that the eigenvalues of $\bar{A}(t)$ are the $2n + g + s$ zeros of $\hat{A}^*$ and in view of Assumption A.4 ii), it follows that the conditions of Lemma 6.2 are satisfied.

From (6.7)

$$x(t) = \phi(t, 0)x(0) + \int_0^t \phi(t, \tau)B_1(e(\tau) + r(\tau))\, d\tau \tag{6.18}$$

where $\phi(t, \tau)$ is the state transition matrix corresponding to $\bar{A}(t)$.

In view of Lemma 6.2 and since r and x_0 are bounded, we have

$$\|x(t)\| \le k_0 + \int_0^t k_1 e^{-\sigma(t-\tau)}|e(\tau)|\, d\tau \tag{6.19}$$

for some k_0, k_1, $\sigma > 0$.

From (3.23), and using Schwarz's inequality, it follows that

$$\|x(t)\| \le k_0 + \int_0^t k_1 e^{-\sigma(t-\tau)}(\beta\epsilon_0 + \beta\epsilon\rho(\tau))\, d\tau + \frac{k_1}{\sqrt{\sigma}}\left\{\int_0^t f^2(\tau)\, d\tau\right\}^{1/2} \tag{6.20}$$

$$\le k_0' + \frac{k_1\beta\epsilon\|\nu\|}{\sigma}\sup_{0\le\tau\le t}\|x(t)\| + \frac{k_1}{\sqrt{\sigma}} \cdot \left\{\int_0^t f^2(\tau)\, d\tau\right\}^{1/2} \tag{6.21}$$

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where we have used (3.13). Since the right-hand side of (6.21) is monotonic nondecreasing in t, it follows that

$$\left\{\sup_{0\le\tau\le t}\|x(t)\|\right\}\le k_0'+\frac{k_1\beta\epsilon}{\sigma}\|v\|\left\{\sup_{0\le\tau\le t}\|x(t)\|\right\}+\frac{k_1}{\sqrt{\sigma}}\left\{\int_0^t f^2(\tau)\,d\tau\right\}^{1/2}. \tag{6.22}$$

Then provided $\epsilon < \sigma/k_1\beta\|v\|$, we can show that

$$\|x(t)\|^2\le k_2+k_3\int_0^t f^2(\tau)\,d\tau \tag{6.23}$$

$$=k_2+k_3\int_0^t \bar{f}^2(\tau)\,d\tau+\int_0^t \bar{f}^2(\tau)\|\phi(\tau)\|^2\,d\tau. \tag{6.24}$$

Since $\bar{f}\in L_2$ (Lemma 3.2) and $\|\phi(\tau)\|\le\|x(\tau)\|$, it follows using Gronwall's lemma that $x(t)$ is bounded. x bounded implies ϕ, ρ, e bounded, and thus Dx is bounded in view of (6.1). Thus, $\bar{u}$ and y are bounded.

To show u is bounded we use Assumption A.1, i.e., S and B_a are coprime polynomials. If we take A_a^+ Hurwitz and of the same degree as A_a, then GS/JQ and B_a/A_a^+ are coprime in the ring of rational, stable, proper transfer functions [66]. That is, there exist stable, proper, transfer functions χ, Λ such that

$$\chi\frac{GS}{JQ}+\Lambda\frac{B_a}{A_a^+}=1 \tag{6.25}$$

$$\chi\frac{GS}{JQ}u+\Lambda\frac{B_a}{A_a^+}u=u \tag{6.26}$$

or

$$\chi\bar{u}+\Lambda\left(\frac{A_a}{A_a^+}y-\frac{C_a}{A_a^+}v\right)=u \tag{6.27}$$

and thus u is bounded, since $\bar{u}$, y, and v are bounded.

2) Equation (6.1) holds in this case also, however, since $Sy^*=0$, we have $r=0$ in view of (6.5); and since ϕ is bounded (as established in 1), $\bar{f}\in L_2$ implies $f\in L_2$. We then have

$$\|x(t)\|\le k_1e^{-\sigma t}\|x_0\|+\int_0^t k_1e^{-\sigma(t-\tau)}|e(\tau)|\,d\tau \tag{6.28}$$

$$\le k_1e^{-\sigma t}\|x_0\|+\int_0^t k_1e^{-\sigma(t-\tau)}|f(\tau)|\,d\tau+k_1\epsilon\beta\int_0^t e^{-\sigma(t-\tau)}\sup_{0\le T\le\tau}\{e^{-\sigma_0(\tau-T)}|v^Tx(T)|\}\,d\tau. \tag{6.29}$$

The result in part 1) relied on $\epsilon<\sigma/k_1\beta\|v\|$; and so we select λ such that $0<\epsilon k_1\beta\|v\|<\lambda<\sigma$. We now rearrange the third term on the right-hand side of (6.29) as:

$$k_1\epsilon\beta\int_0^t e^{-\sigma(t-\tau)}\sup_{0\le T\le\tau}\{e^{-\sigma_0(\tau-T)}|v^Tx(T)|\}\,d\tau$$

$$\le k_1\epsilon\beta\|v\|\int_0^t e^{-\lambda(t-\tau)}e^{-(\sigma-\lambda)(t-\tau)}\sup_{0\le T\le\tau}\{e^{-\sigma_0(\tau-T)}\cdot\|x(T)\|\}\,d\tau \tag{6.30}$$

$$\le\left(\frac{k_1\epsilon\beta\|v\|}{\lambda}\right)e^{-\lambda_0 t}\sup_{0\le\tau\le t}\{e^{\lambda_0\tau}\|x(\tau)\|\} \tag{6.31}$$

(where $\lambda_0=\min\{(\sigma-\lambda),\sigma_0\}$).

Using (6.31) in (6.29) we have

$$e^{\lambda_0 t}\|x(t)\|\le k_1\|x_0\|+\int_0^t k_1e^{\lambda_0\tau}|f(\tau)|\,d\tau+\frac{k_1\epsilon\beta\|v\|}{\lambda}\sup_{0\le\tau\le t}\{e^{\lambda_0\tau}\|x(\tau)\|\}. \tag{6.32}$$

Since the right-hand side of (6.32) is monotonic nondecreasing in t, and $k_1\epsilon\beta\|v\|<\lambda$, we then have

$$\sup_{0\le\tau\le t}\{e^{\lambda_0 t}\|x(t)\|\}\le k_4\|x_0\|+k_4\int_0^t e^{\lambda_0\tau}|f(\tau)|\,d\tau \tag{6.33}$$

and thus

$$\|x(t)\|\le k_4e^{-\lambda_0 t}\|x_0\|+k_4\int_0^t e^{-\lambda_0(t-\tau)}|f(\tau)|\,d\tau. \tag{6.34}$$

Since $\lambda_0>0$ and $f\in L_2$, it follows from (6.34), that

$$\lim_{t\to\infty}x(t)=0. \tag{6.35}$$

Thus, from (3.13), it follows that $\rho(t)\to 0$. Since $f\in L_2$ and Df is bounded, we have $f(t)\to 0$. So $e(t)\to 0$ and from (6.1) we have

$$\lim_{t\to\infty}(Dx(t))=0. \tag{6.36}$$

From (6.36) and (6.35) it follows that $z(t)=y(t)-y^*(t)\to 0$. $\nabla\nabla\nabla$

VII. Discussion of Design Parameters

The adaptive control law depends on the following user choices:

a) $\hat{A}^*$
b) the orders of A and B
c) the internal model polynomial S
d) the estimator filter
e) the sampling rate
f) the details of the least-squares algorithm
g) the choice of the overbounding functions
h) the method of dealing with model stabilizability.

Clearly, the choice of $\hat{A}^*$ is influenced by the desired closed-loop bandwidth. We have made $\hat{A}^*$ a function of $\hat{A}$, $\hat{B}$ since this is essential in practice. For example, the stable well damped part of $\hat{B}$ should be cancelled, i.e., included in $\hat{A}^*$. The unstable part of $\hat{B}$ places an upper bound on the achievable bandwidth; a sensible choice being to reflect the unstable zeros through the imaginary axis. The part of $\hat{A}$ which is not stable and well damped places a lower limit on the bandwidth. When these various requirements conflict, one is faced with a very difficult control problem and one ought to investigate alternative architectures including additional measurements if possible.

The choice of orders of A, B is a compromise between reducing the potential unmodeled dynamics, on the one hand, and increased complexity including additional burden on the parameter estimator and possible model stabilizability difficulties on the other hand. In many practical cases, a third-order model is adequate.

The internal model polynomial S will almost always include integral action. This is essential to eliminate load disturbances, input offsets, etc. One should only attempt to cancel sinewave type disturbances if they are well within the achievable closed-loop bandwidth. If an internal model polynomial is used, then it is desirable to place stable zeros in $\hat{A}^*$ near the zeros of S, this being the classical equivalent of having an integral bandwidth well below the system bandwidth. In fact, for the case of ordinary

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integral action, this separation in bandwidths implies that one can do the control system design without considering the integrator and then to retrofit the integrator with a suitably small gain. The stability analysis can be readily extended to cover this procedure—see [77] for further details.

The prime function of the estimator filter is to focus the parameter estimator on an appropriate bandwidth. There are two aspects, namely high-pass filtering to eliminate d.c. offsets, load disturbances, etc., and low-pass filtering to eliminate irrelevant high frequency components including noise and system response. The net effect is a bandpass filter. The rule of thumb governing the design of the filter is that the upper frequency should be about twice the desired system bandwidth and the lower frequency should be about one tenth the desired bandwidth.

The choice of sampling rate is uncontroversial, it should be, if at all possible, ten times the maximum system bandwidth.

The key requirement on the least-squares parameter estimator is that an upper and lower limit be placed on the algorithm *gain*. The exact choice of gain represents a compromise between rapid tracking of time-varying parameters and smoothing of the noise. Further discussion is given in [78]–[81].

In the choice between normalization and dead zones, one consideration is that dead zones require knowledge of the *size* (i.e., constants ϵ, ϵ_0) of the errors. On the other hand it has the advantages over normalization of directly bounding the estimates, of ensuring that the parameters do not drift leading to potential bursting phenomena, and of giving zero tracking errors in the presence of unmodeled dynamics. It would seem that, in practice, some combination of dead zones and normalization would give best performance.

The choice of constants (υ, σ_0, ϵ_0, ϵ) in the overbounding function ρ is problem dependent. In view of (3.16), V plays the role of an additional filter prior to the signal entering ρ. This filter should also be linked to the closed-loop bandwidth, and will normally have small d.c. gain. The choice of σ_0 depends on the damping of the unmodeled dynamics.

Finally, there are a number of ways of dealing with the problem of stabilizability of the estimated models. We have introduced multiple convex regions as one way of dealing with the problem. In some cases, the choice of these regions is straightforward, e.g., when the model is of low order. In other cases, some sort of prior knowledge will be crucial in overcoming the problem.

VIII. Experimental Results

The theory, as outlined above, has been implemented in a practical adaptive controller [59]. This adaptive controller has been found to give excellent performance under a wide range of experimental conditions. For example, on electromechanical servo systems, we have been able to make 2000 percent step changes in gain, time constant, and d.c. offset and yet retain excellent closed-loop performance.

Fig. 1 shows a typical set of results obtained on a Feedback Ltd. ES1B Servo kit. The desired output y^*, was a square wave of amplitude $\pm 30°$ and period 1 second; the sampling rate was 50 Hz, the desired closed-loop polynomial was $A^* = (0.07\delta + 1)^3$; the parameter estimator was a constant trace version of least-squares incorporating a relative dead zone; G/F was chosen as an all pole second-order low-pass filter of bandwidth 15 Hz; S/Q was chosen as $\delta/\delta + 1$ to eliminate d.c. offsets in u; and an integrator was retrofitted to the system. These choices are in accordance with the guidelines given in Section VII. For the results in Fig. 1, the gain of the servo was switched between 5 and 100 percent as indicated by the arrows. The lower trace shows the estimate of this gain. The upper trace shows the output response of the servo system.

IX. Conclusions

This paper has discussed design issues in adaptive control. The theoretical results have been supported by experimental evidence of the claimed robustness properties. We believe that the success of the algorithm is a result of the judicious amalgamation of various techniques including regression vector filtering, pole assignment, choice of desired closed-loop polynomial as a function of estimated model, estimator dead zone, least-squares, and internal model principle.

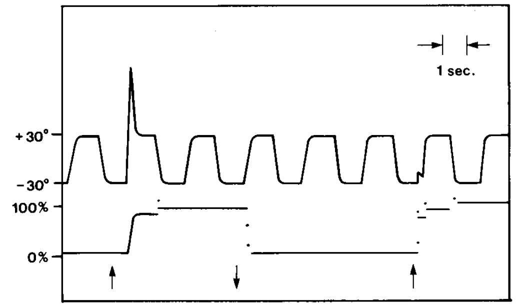

Fig. 1. Adaptive control of servo system. Upper trace: system response. Lower trace: estimate of system gain ($\hat{b}_o$). ↑ 20:1 gain increase. ↓ 20:1 gain decrease.

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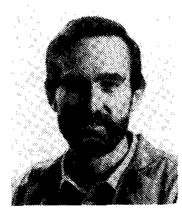
Richard H. Middleton was born in Waratah, Australia, on December 10, 1961. He received the B.Sc. and B.E. (Hons. I) degrees from the University of Newcastle, New South Wales, Australia, in 1984 and 1985, respectively, and the Ph.D. degree in 1987.

He is currently a Lecturer in the Department of Electrical and Computer Engineering, University of Newcastle, where his interests include adaptive control, computer control systems, and electronics.

Graham C. Goodwin (M'74–SM'84–F'86) was born in Broken Hill, Australia, in 1945. He received the B.Sc. degree in physics, the B.E. degree in electrical engineering, and the Ph.D. degree from the University of New South Wales, New South Wales, Australia.

From 1970 until 1974 he was a Lecturer in the Department of Computing and Control, Imperial College, London, England. Since 1974 he has been with the Department of Electrical and Computer Engineering, University of Newcastle, New South Wales, Australia. He is the coauthor of three books: *Control Theory* (London: Oliver and Boyd, 1970), *Dynamic System Identification* (New York: Academic, 1977), and *Adaptive Filtering, Prediction and Control* (Englewood Cliffs, NJ: Prentice-Hall, 1984).

Dr. Goodwin is a member of the Australian Research Grants Committee.



David J. Hill received the B.E. degree in electrical engineering and the B.Sc. degree in mathematics from the University of Queensland, Australia, in 1972 and 1974, respectively, and the Ph.D. degree in electrical engineering from the University of Newcastle, New South Wales, Australia, in 1976.

He is currently a Senior Lecturer in Electrical and Computer Engineering at the University of Newcastle. Previous appointments include research positions in the Electronics Research Laboratory, University of California, Berkeley, from 1978 to 1980 and a Queen Elizabeth II Research Fellowship at the University of Newcastle from 1980 to 1982. During 1986 he occupied a Guest Professorship in the Department of Automatic Control, Lund Institute of Technology, Lund, Sweden. His research interests are mainly in nonlinear systems and control, stability theory, and power system stability and security.

Dr. Hill is a member of I.E. Australia, the Society for Industrial and Applied Mathematics, and the International Conference on Large Electric High-Tension Systems.

David Q. Mayne (S'75–SM'77–F'81) was born in South Africa and received the B.Sc. and M.Sc. degrees in engineering from the University of Witwatersrand and the Ph.D. and D.Sc. degrees from the University of London.

During the period 1950 to 1959 he held the posts of Lecturer at the University of Witwatersrand and, for two years, Research and Development Engineer at the British Thomson Houston Company, Rugby, England. Since 1960 he has been with the Imperial College of Science and Technology, London, England. He became a Reader in 1967 and a Professor in 1971. He was a Visiting Research Fellow at Harvard University in 1971, and has spent several summers at the University of California, Berkeley, in collaborative research with Professor E. Polak. He is the joint author, with D. H. Jacobson, of *Differential Dynamic Programming* (New York: Elsevier) and joint Editor, with R. W. Brockett, of *Geometric Methods in System Theory* (Dordrecht: Reidel). He is an Associate Editor of the *Journal of Optimization Theory and Applications.* His major research interests are in optimization, computer-aided design, and adaptive estimation and control.

Dr. Mayne is a Fellow of the IEEE and the Institute of Electrical Engineers and a member of the Mathematical Programming Society. He was recently elected a Fellow of the Royal Society. He was also awarded a Senior Science Research Council Fellowship for the period 1979 to 1980.

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