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Martin N. Hjelmeland

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In a Multi-Market Setting

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Norwegian University of Science and Technology

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Thesis for the Degree of Philosophiae Doctor
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Preface

This doctoral thesis was written at the Department of Electric Power Engineering at the Norwegian University of Science and Technology. My main supervisor was Professor Magnus Korpås, with Dr. Arild Helseth and Professor Gerard Doorman as co-supervisors. The work was carried out between 2015 and 2018. During this time a research stay was conducted at Georgia Institute of Technology with Dr. Shabbir Ahmed.

The PhD has been carried out in cooperation with SINTEF Energy Research's project "*Integrating Balancing Markets in Hydropower Scheduling Methods*". The project is supported by the Norwegian Research Council and the hydropower companies: Lyse, Statkraft, Vattenfall, Hydro, E-CO Energi and Agder Energi.

Martin N. Hjelmeland

Trondheim, April 2019

Acknowledgements

The three and a half years in which this thesis has been conducted has felt like a roller coaster, with a lot of ups and downs. I have been so fortunate in meeting numerous people that have accompanied me during this time and enabled me to fulfill this PhD. I would first and foremost like to thank my supervisors Professor Magnus Korpås, Dr. Arild Helseth and Professor Gerard Doorman. I have been lucky enough to work with a great group of supervisors, where Arild was always open for in-depth discussions about core subjects, Magnus with his exceptional trait of encouraging me with motivational speeches and insightful input for current and future work, and Gerard for providing an additional perspective to the discussion. Thank you all for your valuable guidance that was indispensable for this work.

During my first year as a PhD candidate I had the pleasure of co-supervising two exceptional master's students; Espen Flo Bødal and Jacob Koren Brekke. Working on a PhD requires a great deal of independent work, and thus working together with them was very gratifying. Following graduating, Espen started on his PhD and joined the astonishing environment at the department where I would like to thank my awesome office mate Martin Kristiansen, along with Sigurd Hofsmo Jackobsen, Salman Zaferanlouei, Camilla Thorrud Larsen, Markus Löschenbrand, Hans Kristian Meyer, Martin Håberg, Phillip Härtel, Emre Kantar, Erlend Engevik and Christian Øyn Naversen.

I was honored to be hosted by Professor Shabbir Ahmed at the Georgia Institute of Technology in my second year. Working with Shabbir and his PhD student Jikai Zou was truly an outstanding experience and I can surely say that this thesis has benefited greatly from the research stay. The memories of the places I have visited and the friends I have made that year will never be forgotten. I am also grateful for the team at Piedmont Atlanta Hospital for stitching me up so the stay was not cut short.

My new employer, Lyse Produksjon AS, provided me with data and valuable discussion with the view from a hydropower producer. Thank you for giving me the opportunity to apply what I have learned in this thesis on a regular basis and to continue to developing this knowledge.

I have also been blessed with the flatmates I had in Trondheim. Arne Øvrebø Lie, Per-Dimitri Sønderland and Erik Prytz certainly made life outside school formidable. However, I have to say that my last cohabitant, Tara Berg, has been the very best. She has helped me through tough times with her loving support and awesome humor. You are truly a wonderful life partner who never fails to put a smile on my face.

Lastly, thank you to my mom, dad and the rest of my family for cheering me on throughout these years. Especially my dad who will always be my greatest idol. I would never have been able to start or fulfill a PhD without the education you gave me. I know you regretted not taking a higher education yourself, I think my birth can partly be blamed.. With this thesis, I hope that I have balanced out the score for both of us. I want to dedicate this thesis to my dad and best friend who passed away in December 2017. I miss you.

Abstract

Regulated hydropower is a flexible resource that is well suited for the provision of ancillary services. Given the opportunity to participate in different markets, the difficult task is to find the optimal allocation between them. The scope of this thesis is to investigate the multi-market problem and provide models for the optimal utilization of regulated hydropower in the Medium-Term Hydropower Scheduling (MTHS) setting. This is conducted from the view of a hydropower producer, participating in the day-ahead electricity market and in reserve capacity markets for providing rotating reserves.

A major part of Norway's electricity generation comes from hydropower. Since the annual inflow displays considerable fluctuation, it is imperative that hydropower reservoirs are efficiently managed. The stored water should be exerted when it is required, i.e. when the market prices are high compared to the value of storing it for later use, at the same as spillage should be avoided and market revenues maximized. With better grid connection to the rest of Europe and tighter market coupling, flexible hydropower producers have an edge in providing their flexible resources to larger market shares. The tighter market coupling also ensures a higher security of supply in the Nordics and the ability to absorb more renewable energy at times when there is, alternately, a great deal of wind or sun in continental Europe. The objective of this thesis is thus to provide methods for decision support for hydropower producers in a changing power market.

The initial work in this thesis investigates how effectively a current Medium-Term Hydropower Scheduling (MTHS) model, based on Stochastic Dual Dynamic Programming (SDDP), performs in a multi-market setting. The SDDP algorithm is a state-of-the-art method for solving multistage stochastic programming problems with extensive adoption for Hydropower Scheduling (HS) problems. The work found that the SDDP model overestimated the hydropower system's ability to provide reserve capacity. For the given case studies it was around 30%, illustrating the importance of detailed modeling when considering reserve capacity.

To undertake the issue with modeling details the newly proposed Stochastic Dual Dynamic integer Programming (SDDiP) method was applied to the MTHS problem. It was shown that the method provided convergence of a nonconvex MTHS problem, but with a substantial computational burden. A new type of cut in the SDDiP framework, called a strengthened Benders cut, showed beneficial properties in terms of an improved optimality gap and manageable computation time. These results were further sustained by another study that included more detailed modeling of the hydropower system with a recently proposed method to include uncertainty in objective term coefficients for Dynamic Programming (DP) problems. The study showed that an approach based on Benders cuts was inferior when the problem formulation was so complex.

Furthermore, a study of a constructed power system with increasing shares of wind power was investigated, with an emphasis on the provision of both upwards and downwards reserve capacity from hydropower and wind power. The study was conducted with a SDDP model showing that wind power could effectively provide downwards reserve capacity, and in certain cases when the wind penetration was very high some upwards reserve could also be provided.

Based on the possibility of including nonconvexities in the MTHS problem with the SDDiP method, a study on the modeling of the generation function from a hydropower station was also conducted. It was found that the convex relaxation of this function leads to an overestimation of what is physically possible. This is particularly true when environmental constraints and a reserve capacity market was included.

In short, this thesis investigated both current and recently proposed algorithms used for solving the MTHS problem. Realistic case studies were applied with the aim of presenting state-of-the-art algorithms that are applicable in operational use. This is especially evident for an improved type of Benders cuts called Strengthened Benders cuts, used by the SDDiP method.

Abbreviations

ACER Agency for the Cooperation of Energy Regulators. 3, 4

ARMA Autoregressive-Moving-Average. 42

BSP Balance Service Provider. 19, 51

DOR Degree of Regulation. 23, 26, 49

DP Dynamic Programming. viii, 28, 31, 32, 37, 45, 59

EFP Expected Future Profit. xiv, 5, 32–34, 37–39, 42, 48, 57, 59, 61, 62, 66, 68

EFTA European Free Trade Association. 3

ENTSO-E European Network of Transmission System Operators for Electricity. 3

EU European Union. 3, 4, 12

FCR Frequency Containment Reserves. 15, 19

FCR-D Frequency Containment Reserves - Disturbance. 15, 16

FCR-N Frequency Containment Reserves - Normal. 15–17, 43

FRR-A Frequency Restoration Reserves - Automatic. 14, 17–19

FRR-M Frequency Restoration Reserves - Manual. 19, 21

HS Hydropower Scheduling. vii, 2, 4, 9, 24, 25, 28, 29, 55, 57, 67, 68

HVDC High-Voltage Direct Current. 3

IEA International Energy Agency. 14

IEM Internal Electricity Market. 3

LP Linear Programming. 5, 34, 37, 39–42, 48, 51, 59

LTHS Long-Term Hydropower Scheduling. 25, 26, 33, 45, 54

MILP Mixed Integer Linear Programming. 40, 41, 59

MIP Mixed Integer Programming. 28, 37, 39

MIQP Mixed Integer Quadratic Programming. 41

MSSP Multistage Stochastic Programming. 25, 27, 29, 31, 34, 37, 57, 61

MTHS Medium-Term Hydropower Scheduling. vii, viii, 2, 4, 5, 7, 26, 27, 42, 47, 52, 54, 55, 57, 58, 62, 65–68

OR Operations Research. 28

RPM Regulating Power Market. 19

RPO Regulating Power Options. 19, 21

SDDiP Stochastic Dual Dynamic integer Programming. viii, 5, 29, 37–40, 42, 44, 58, 59, 61, 62, 66–68

SDDP Stochastic Dual Dynamic Programming. vii, viii, 4, 5, 28, 29, 32–34, 37, 39, 43–45, 47, 48, 53, 57–59, 61, 62, 66–68

SDP Stochastic Dynamic Programming. 32, 62

SO System Operator. 1

SP Stochastic Programming. 28

STHS Short-Term Hydropower Scheduling. 27, 28, 52, 54, 57, 68

TSO Transmission System Operator. 9, 11, 12, 15, 18, 19, 21, 51, 52

UC Unit Commitment. 27

Definitions and Terms Used

Ancillary services
Necessary services provided to a system operator to ensure reliable operation of the power grid.

Central dispatch
The scheduling of production to cover net load is performed simultaneously in an integrated process. The entity performing the process thus defines which power stations should be operational or not, at a given time.

Energy system
Includes, for a region, all units generating, consuming and transporting energy.

Hydropower system
Includes all reservoirs, power plants and other connected components used for the purpose of generating hydropower in a river system.

Marginal pricing
The highest accepted bid in a market sets the price for the commodity.

Net load
Total demand of electricity subtracted by intermittent energy.

Power system
Includes, for a region, all units generating, consuming and transporting electric energy.

Reserve capacity
Spinning or rotating capacity that a power producer can provide to the system operator to quickly react to deviations in scheduled system operation.

Stability The power system's ability to return to an acceptable steady state after a disturbance.

Structural imbalances
Imbalance of generation and consumption caused by the market structure, e.g. rapid changes of net load around whole hours.

System operator
Entity responsible for the reliable transfer of electricity from producers to consumers.

Water value
The expected opportunity cost of water associated with storable hydro reservoirs.

Contents

Chapter 1

Introduction

The Regulation establishing a guideline on electricity balancing (EB) entered into force on 18 December 2017. The Balancing guideline will set down rules on the operation of balancing markets, i.e. those markets that Transmission System Operators (TSOs) use to procure energy and capacity to keep the system in balance in real time. The objectives of the guideline include increasing the opportunities for cross-border trading and the efficiency of balancing markets.

—European Commission, *on Electricity network codes and guidelines*

Hydropower scheduling is a complex problem that aims at utilizing the water resources in the most efficient manner. The electricity generated in Norway is predominantly derived from hydropower, in 2016 this share was 96.3% [1]. It is clear that the tools used for managing the country's hydropower resources should ensure as much value creation as possible. Storable hydropower with ample installed capacity provides a unique flexibility to shift generation to the hours throughout the year that provide the highest value, either the value is created in the wholesale energy market or by providing ancillary services for the System Operator (SO).

The Norwegian power system is not the only one which is predominately dependent on hydropower. The Brazilian system generates around three-quarters of its electric energy from hydropower, where the country's largest hydropower system[1] consists of 20 turbines with an installed capacity of 700 MW each. Other systems with large shares of hydropower include China, Canada, New Zealand and the USA. In fact, 16.4% of the electricity generated in the world came from hydropower in 2016 [2]. Given the large share of electricity generated by hydropower, it is thus imperative to have good tools for performing hydropower scheduling.

[1] Itaipu river system

This thesis aims at tackling the Medium-Term Hydropower Scheduling (MTHS) problem from a Nordic hydropower producer. Many of the concepts in this work are applicable for use in other Hydropower Scheduling (HS) problems, as the problems are similar except on the market side. Since the Nordic electricity market is a liberalized market, the producers have no commitment to supply electricity, they merely do so to maximize their profits in the free market. HS in Brazil, on the other hand, aims at minimizing the overall system costs, and does so by performing a central dispatch of the power system. Moreover, a HS problem in Canada aims at fulfilling the load requirements from metal smelters and any excess production can be sold to a market. The remainder of this thesis therefore take the Nordic hydropower perspective, unless stated otherwise.

In recent years, the majority of the income for hydropower producers has come from selling energy. A more extensive power market consisting of multiple commodities from which the hydropower producer can capitalize from requires tools that incorporate this added complexity. This can be seen in context with the expected need for more flexible power plants that can provide rapid changes in power output, resulting in greater volumes traded in these balancing reserve markets. In essence, this thesis provides models that are suited for the framework of a future power market where balancing services are highly valued and remunerated, and where higher levels of modeling details are required. The main objective has focused on making more comprehensive MTHS models that can supply reserve capacity in addition to selling energy.

1.1 Motivation

In order for the power system to transition into a sustainable system with high shares of renewable energy, some fundamental aspects have to change. Traditionally the flow of electricity has been from large power producers to end-users. The price of electricity has generally been stable and driven by the price of fuel by thermal units, such as nuclear and coal power plants [3]. Deviations in forecasted net load have been low and, subsequently, the grid stability has been good. With increasing shares of renewable power generation the deviations of the forecasted net load have significantly increased. The term "duck curve" has been repeatedly discussed and describes the rapid change in net load in the afternoon when the solar generation is low while the demand increases, resulting in a sharp increase of net load [4], an illustration is given in Figure 1.1. The marginal costs of renewable energies are also very low compared to thermal power plants. For wind turbines the marginal cost may even be negative due to feed-in tariffs. The generation is also often on the distribution grid, which can, in some cases, make the power flow in the opposite direction to that of the grid's original purpose. It is evident that there

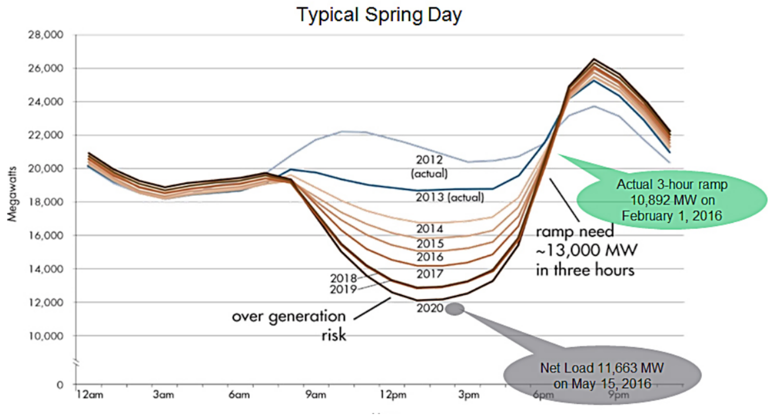


Figure 1.1: Illustration of the duck curve. It clearly illustrates the increasing ramping requirements a power system would need if the levels of solar generation increases [5].

are major changes happening within the power systems, that will require different market structures to better handle imbalances than the power system of the past. Therefore, it is imperative that hydropower producers adapt and develop tools that are suited for a future with higher uncertainty and variability.

In addition to the abovementioned challenges, a tighter grid connection between European countries is expected. This will enable the renewable energy to be transported from rural areas, which often have favorable conditions for renewable generation, to high load areas. Moreover, when the wind is not blowing and the sun is not shining, flexible power plants can balance out the net load. For Norway, this means more High-Voltage Direct Current (HVDC) cables to the European continent, which enable us to import cheap renewable energy and export from our flexible hydropower plants. This push towards a tighter coupled power system was outlined in 1996 when the European Union (EU) issued a directory (1996/92/EC) to gradually open the electricity markets for all member states. Following this, other legislation has appeared that aims to ensure a fully functional Internal Electricity Market (IEM) in the EU, such as the third energy package that came into force in 2009. The package aimed at liberalizing the European energy markets and obtaining a level playing field for all market participants. To oversee and ensure this vision two agencies were established; Agency for the Cooperation of Energy Regulators (ACER) and European Network of Transmission System Operators for Electricity (ENTSO-E). Even though Norway is not a member of the EU, it is a member of European Free Trade Association (EFTA) that gives access to the internal markets in the EU, and is therefore encompassed by the EU directives. Such regulations include the recently developed Network Codes that ENTSO-E and ACER are harmonizing [6]. It is evident that the EU has very ambitious targets for a renewable energy sector, with a vision of being climate-neutral by 2050 [7]. With common rules, guidelines, the merging of markets and tighter grid connection across Europe, one can expect that power markets will be more streamlined and products standardized [8].

In central Europe, the revenue of hydropower obtained from balancing markets is significantly higher than in the Nordic market. Given the current focus on integrating European electricity markets, one can expect this gap to close in the years to come. One can therefore expect that a larger share of the income from hydropower generation is likely to come from markets other than the day-ahead market. The impression Nordic producers have given implies that this development has already started and that their revenues from reserve and regulating markets are increasing.

The Norwegian hydropower producers are in a unique position to both facilitate a sustainable power system and capitalize on it. This requires, however, sufficient support tools that can be relied upon to make the best operational decisions in a multi-market setting. The support tool will also assist hydropower producers when constructing new or refurbishing old plants. Without these tools, a significant risk of lost future market opportunities may occur, as stated in [9].

1.2 PhD Project Scope

The overall goal is to analyze and develop methods for Medium-Term Hydropower Scheduling (MTHS), considering participation in multiple power markets. The PhD project is a part of a research project investigating how balancing markets affect Hydropower Scheduling (HS) and how methods for generation scheduling in multiple markets can be incorporated in existing operative models [10]. The methodological approaches used in this thesis have therefore been based on improvements of the renowned Stochastic Dual Dynamic Programming (SDDP) algorithm [11], a sampling-based nested Benders decomposition algorithm for solving multistage stochastic programming problems [12].

This thesis has investigated the different power markets in which a Norwegian hydropower producer can participate, with a discussion of potential future markets. It has also been motivated by new algorithms that have been published during the course of this PhD work, and a substantial amount of work has been conducted towards applying and developing these algorithms for MTHS problems.

1.3 Contributions

Hydropower producers rely on proficient decision support software to assist their generation scheduling. This thesis has investigated methods for benchmarking existing methods, new models that provide a better description of the underlying hydropower system and develop them to solve the MTHS problem. The main contributions can be summarized as having:

- Built a simulator used to benchmark the performance of a standard SDDP implementation used for solving multi-market problems and other problems where the underlying problem cannot be modeled in sufficient details as a Linear Programming (LP) problem.
- Applied SDDiP to an MTHS problem that included nonlinear constraints such as minimum generation limits and variable head. This work was further extended to include generation with variable head and to emphasize the promising properties strengthened Benders cut have for solving nonconvex problems.
- Implemented and tested a recent methodology for objective term uncertainty in SDDP, to model correlation between different stochastic processes, such as inflow, energy and reserve capacity price. The approach included variable head and combined methods used in SDDiP to provide a dominant solution, compared to an existing approach.
- Proposed a method to include correlation between inflow and price in the SDDiP framework.
- Developed methods to describe the generation function, representing the power stations power output, in an MTHS problem.
- Proposed a new method to visualize the Expected Future Profit (EFP) function, in order to determine the extent of nonconvexity of the underlying problem.

Furthermore, the main contributions deducted from the case studies have been to:

- Analyze how a current MTHS model based on the SDDP framework performs when including sales of reserve capacity. The model has proven to be state-of-the-art when only selling energy in a day-ahead market, but when a reserve capacity market is included the precision of the model was shown to be inadequate.
- Analyze the impacts of more low load generation in an MTHS problem. The results showed that when minimum discharge was added, the current model overestimated its available generation capacity.

- Analyze the provision of reserve capacity from wind power in a constructed power system. It was found that wind power can effectively provide downwards reserve capacity, and only in some extreme cases, with extensive wind penetration, provide upwards reserve capacity. The study also expressed how well hydropower can facilitate intermittent energy sources.

1.4 List of Publications

The following is a list of publications associated with this thesis.

I. M. N. Hjelmeland, M. Korpås, and A. Helseth, "Combined SDDP and simulator model for hydropower scheduling with sales of capacity," in *2016 13th International Conference on the European Energy Market (EEM)*, pp. 1–5, June 2016. © 2016 IEEE

II. M. N. Hjelmeland, A. Helseth, and M. Korpås, "A case study on medium-term hydropower scheduling with sales of capacity," *Energy Procedia*, vol. 87, pp. 124 – 131, 2016. 5th International Workshop on Hydro Scheduling in Competitive Electricity Markets

III. J. K. Brekke, M. N. Hjelmeland, and M. Korpås, "Medium-term hydropower scheduling with provision of capacity reserves and inertia," in *2016 51st International Universities Power Engineering Conference (UPEC)*, pp. 1–6, Sept 2016. © 2016 IEEE

IV. M. N. Hjelmeland, C. T. Larsen, M. Korpås, and A. Helseth, "Provision of rotating reserves from wind power in a hydro-dominated power system," in *2016 International Conference on Probabilistic Methods Applied to Power Systems (PMAPS)*, pp. 1–7, Oct 2016. © 2017 IEEE

V. M. N. Hjelmeland, A. Helseth, and M. Korpås, "Impact of modelling details on the generation function for a Norwegian hydropower producer," *Journal of Physics: Conference Series*, vol. 1042, no. 1, p. 012010, 2018

VI. M. N. Hjelmeland, J. Zou, A. Helseth, and S. Ahmed, "Nonconvex medium-term hydropower scheduling by stochastic dual dynamic integer programming," *IEEE Transactions on Sustainable Energy*, 2018. © 2018 IEEE

VII. M. N. Hjelmeland, A. Helseth, and M. Korpås, "Medium-term hydropower scheduling with variable head under inflow, energy and reserve capacity price uncertainty," *Energies*, vol. 12, no. 1, 2019

List of other publications related to the work carried out in this thesis:

- A. Helseth, B. Mo, M. Fodstad, and M. N. Hjelmeland, “Co-optimizing sales of energy and capacity in a hydropower scheduling model,” in *PowerTech, 2015 IEEE Eindhoven*, pp. 1–6, June 2015
- E. F. Bødal, M. N. Hjelmeland, C. T. Larsen, and M. Korpås, “Coordination of hydro and wind power in a transmission constrained area using SDDP,” in *2016 51st International Universities Power Engineering Conference (UPEC)*, pp. 1–6, Sept 2016

1.5 Thesis Outline

This thesis is a collection of seven publications carried out during the PhD. The publications are provided in Appendix A. The remainder of this thesis is carried out as follows.

The research context and background is outlined in Chapter 2. The chapter gives insights into the Norwegian power market, the fundamentals of hydropower scheduling with a main focus on MTHS and methods to solve these types of problems. The main findings and results of the thesis are presented in Chapter 3. Finally, Chapter 4 presents conclusions and recommendations for further work.

Chapter 2

Research Context and Methodology

The Nordic market solution for automatic secondary reserves will enable allocation of transmission capacity between bidding zones when this is beneficial. This market will, indirectly, also result in increased demand for Norwegian flexibility and thus greater value creation.

—Statnett, *System operations and market development plan 2017-2021*

The following chapter provides an insight into the markets in which a Norwegian hydropower producer can participate and their turnover. Following this, the HS problem is outlined. Lastly, some background on the methods used in this PhD is given.

Due to the physical properties of the power system, power markets with different timescales and usages are required to ensure stable grid operation. A list of the markets in the Nordic countries is provided in Table 2.1. For HS it is primarily the physical markets that are of interest, which are presented in the following.

2.1 Energy Markets

2.1.1 Elspot

Elspot is the day-ahead auction for power in Northern Europe. The auction consists of demand and supply bids for every hour for the following day. For each hour a system price is calculated under the assumption of unlimited grid capacity [22]. Due to congestion management by the Transmission System Operators

Table 2.1: List of power markets in the Nordic system.

Market place	Physical trade	Financial trade
Nord Pool Spot	Day-ahead (Elspot) Intraday (Elbas/XBID)	
NASDAQ OMX Commodities		Futures DS Futures options CfDs
TSO	Primary reserve market Secondary reserve market Tertiary reserve market	

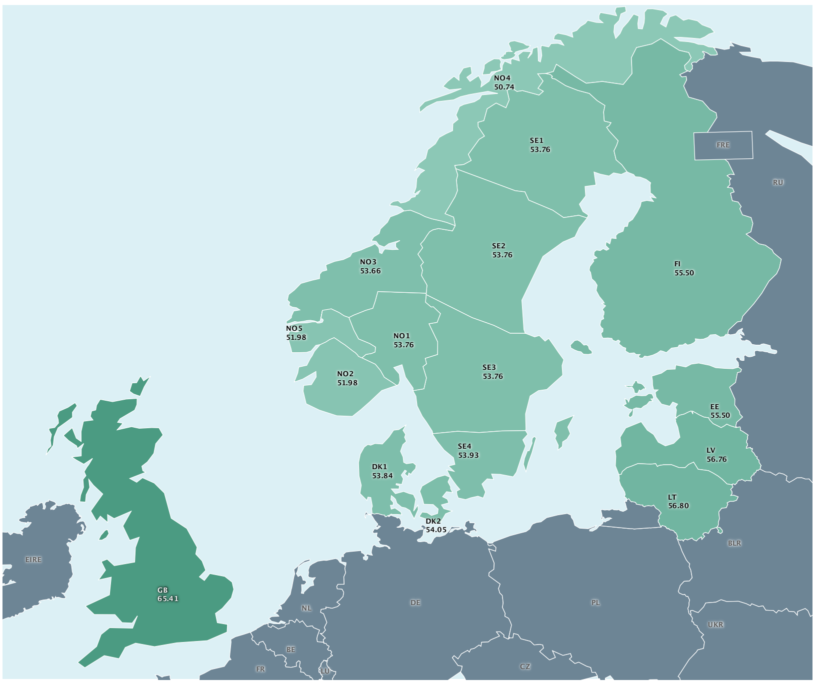


Figure 2.1: Overview of the Elspot areas.

(TSOs) different price areas are made, as seen in Figure 2.1. Grid congestion is thus implicitly taken into consideration between price areas, reducing the balancing management for the TSO. Currently, there are 15 areas in the Elspot area: five in Norway, four in Sweden, two in Denmark, and Finland, Estonia, Lithuania and Latvia each comprise of one price area. Producers and consumers have to place their bid in their respective price area. The price in one area is set such that the capacity to other areas is within limits.

The monthly historical system and NO2 price are shown in Figure 2.2, illustrating the large seasonal variations that can be observed. The NO2 area has a surplus of production and is thus a net exporter, that normally observes a lower energy price than the system price when the grid capacity is fully utilized. Since the bidding on Elspot is portfolio based, producers have the opportunity to move production or consumption in-house within each Elspot area, after clearing, rather than adjusting their Elspot commitment in the balancing market or paying the TSO for imbalance.

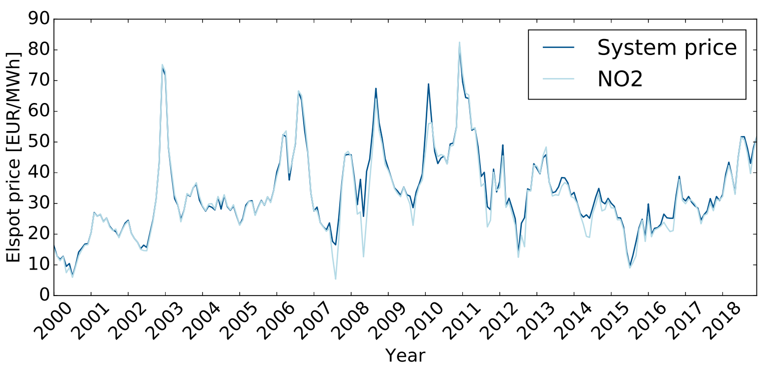


Figure 2.2: Historic monthly prices on Elspot for the system and the NO2 area.

Elspot is currently the largest day-ahead market in the world. A total of 512 TWh were traded on the entire Nord Pool platform in 2017. Moreover, 394 TWh were traded in Nord Pool Spots Nordic and Baltic day-ahead market, with a turnover of approx. 100 billion NOK [23]. Likewise, the turnover in NO2 alone was around 13 billion NOK. This high liquidity is one of the main reasons why the system price on Elspot is used to value the financial contracts traded on NASDAQ OMX Commodities.

2.1.2 Elbas

Elbas is an intraday market for power trading. Through the European Cross-Border Intraday Market (XBID) solutions 12 different intraday markets can be accessed,

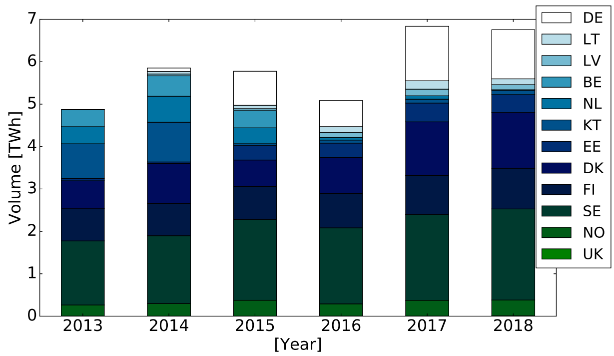


Figure 2.3: Sales volume traded on Elbas for the different countries. DE includes both the German and the Austrian market. It should be noted that the UK market was included in 2018.

which encompass the Nordic, Baltic, German, Luxembourg, French, Dutch, Belgian, Austrian and UK markets [24]. Elbas works as a balancing market to the Elspot market since it offers near real-time power trading, reducing the economic risk for participants and enhancing system stability as the market participants can balance any potential imbalance from the day-ahead schedule themselves. At 14:00 CET the TSO announces available transfer capacity and Elbas opens. It is open until an hour before delivery, the following day. Settlements are bilateral, anonymous and set on a first-come, first-served principle. Matching bids, lowest sell and highest buy price, will be settled independently of the time of bidding. The volume on Elbas was 6.7 TWh in 2017 [23]. The volume traded on Elbas in Norway was only 0.33 TWh the same year. Figure 2.3 shows the yearly traded volume on Elbas for different countries. Compared to previous years, the volume is increasing, but is still only a fraction of the volumes in Elspot.

A study from 2009 analyzed the optimal bidding strategy for a medium-sized hydropower producer. It was found that the value from considering Elbas was minor, never greater than 0.12% [25]. Since the study was performed new initiatives have come from the EU, aiming at an integrated intraday market across Europe, such as XBID. The volume on Elspot has also almost tripled since 2009. Another study, from 2016, investigated the trading behavior on Elbas over a year from February 2012 to March 2013 [26]. It was pointed out that a reason for the low Elbas trading in Norway could be due to the fact that there is no available transmission capacity towards Continental Europe, with the same holding true for limited export possibilities to Finland and Denmark that had higher Elbas prices. Another finding from the study was that the most trades occur in the last hours before closure, which is reasonable considering that renewable generation predictions are better closer to delivery. An in-depth study of trading in Elbas for a single hydropower plant was carried out in [27]. The thesis showed only a slight overall gain from participating in the Elbas market but argued that a producer with a larger plant portfolio, i.e. more flexibility, could have more to gain. However, an interesting remark is that the gain was considerably higher when it was assumed to be an Elbas market with greater more liquidity, that should represent a future power market with a large share of intermittent renewable energy.

It is clear that Elbas does and will play a role in the future for renewable energy producers that have to balance their generation commitments. Since 2012/2013, the period analyzed by the abovementioned study [26], the volumes on Elbas have increased by around 40% in 2017 [23]. With the launch of XBID in 2018 and more automated market solutions by the market participants, one can expect that the volumes will continue to increase.

2.2 Reserve Capacity Markets

The balancing markets refer to the three markets for providing primary, secondary and tertiary reserves. The main distinctions between them are on their responsiveness and time of activation. Figure 2.4 shows the principal reaction of the balancing reserves after a fault has occurred. The illustration shows how the markets complement each other to bring the frequency back to nominal value.

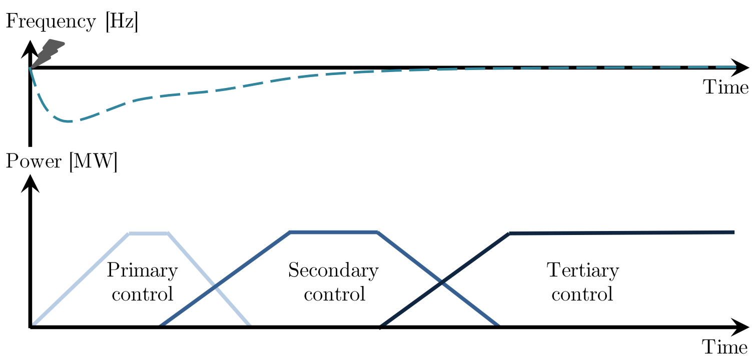


Figure 2.4: Principal activation of balancing reserves after imbalance.

The Elspot and Elbas markets are used for scheduling before the hourly physical delivery. Since the system's balance of generation and consumption will almost never be as predicted, the balancing markets are needed in order to ensure a stable operation of the grid in real time. Typical causes of imbalance include tripping of lines, transformers, generators or other components in the grid, forecasting errors and increasing amount of uncontrollable renewable power production. In recent years, structural imbalances have reduced the frequency quality, mainly by increased trade in the Nordic region with a more loaded grid, and interconnection to the continent while the amount of automatic reserves has been low [28]. In Norway, this increasing trend has, however, improved after the use of automatic secondary reserves from 2013, see Figure 2.5. The procurement of Frequency Restoration Reserves - Automatic (FRR-A) was halted in 2016, leading to the spike in the figure, more on this in Section 2.2.2.

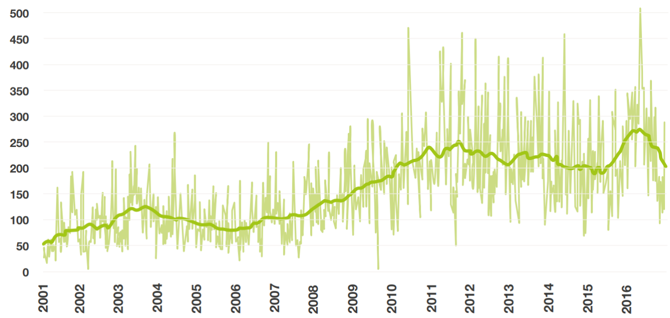


Figure 2.5: Frequency deviation outside 49.9 - 50.1 Hz given in minutes per week. The light green line portrays minutes per week, while the darker green shows moving average over 52 weeks. [28]

With increasing transmission capacity to continental Europe even more reserve capacity could be required, to back-up the expected increased changes of net load. [29] shows the positive effects of interconnections between countries and that a wind and solar production share above 30% would dramatically increase the need for flexible capacity. In 2017 this threshold was reached with a German renewable energy generation of 33.3% of the generation mix [30]. Comparably, in 2007, the share was 13.7%. A study from 2015 gave Germany a 76% chance of reaching the 30% level by 2020 [31]. The fact that this target has already been achieved proves how fast transitions can occur, and how difficult forecasting can be. The International Energy Agency (IEA) has recently been criticized for being overly conservative in their predictions [32]. In fact, they have always underpredicted the renewable generation.

In the following sections, the different reserve capacity markets will be outlined. Subsequently, a summary of the different markets is given in Section 2.4 to put them into context for the remainder of this thesis.

2.2.1 Primary Reserve Market

Figure 2.4 shows how the first reaction to an imbalance comes from the primary reserves, also called Frequency Containment Reserves (FCR). These energy reserves are automatic and fast acting, typically up to 30 seconds of activation and symmetric [33]. In a hydropower system FCR is assured by the droop settings in the turbine governors. With a negative imbalance[1] the turbine governor would increase the power output from the generator and vice versa for a positive imbalance. In a thermal system, the most common way to provide FCR is to throttle the inlet valves on the high-pressure turbine in conventional coal power plants. This process would reduce the overall efficiency by around 0.5% and additional investments and fuel costs are needed [22].

In Norway, a market for primary reserves was first established in 2008. Before this point, primary reserves were considered to be balanced out over time, so there was no market or any remuneration of the service. If the available primary reserves were too low, the Norwegian TSO Statnett would ask power producers to increase their droop settings [34].

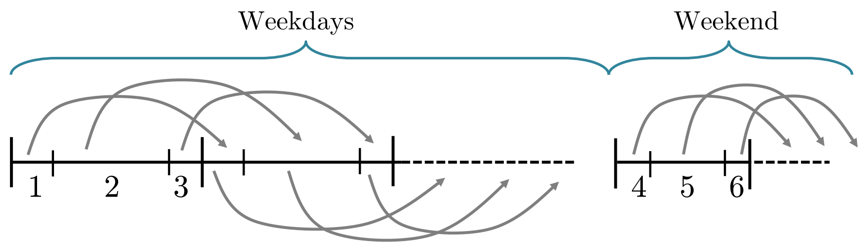


Figure 2.6: Visualization of the capacity allocation periods for FCR-N weekly reserves. There is a total of 6 different periods.

The primary reserve market consists of two products; Frequency Containment Reserves - Normal (FCR-N) and Frequency Containment Reserves - Disturbance (FCR-D), where FCR-N is divided into a weekly and daily market. FCR-N is automatically activated if the frequency deviates from 50 Hz by ± 0.1 Hz. FCR-D is partially activated when the frequency falls below 49.9 Hz and fully if the frequency reaches 49.5 Hz [33]. Statnett decides which offers that will be accepted based on marginal pricing. Local grid-structure may force Statnett to buy contracts over marginal price with a pay-as-bid principle. Pricing for the weekly market (only FCR-N) is divided into three periods for weekdays and weekends: night (00:00-08:00), day (08:00-20:00) and evening (20:00-00:00). A visualiza-

[1] Lower production or higher demand than planned will result in a reduced frequency.

tion of the allocation periods can be seen in Figure 2.6. The bids are given by 12:00 Thursday for the coming weekend and Friday 12:00 for coming weekdays. The daily market is open for both FCR-N and FCR-D. The bids are symmetric, i.e. an equal amount of positive and negative power should be provided, and should be sent in by 18:00 the day before delivery.

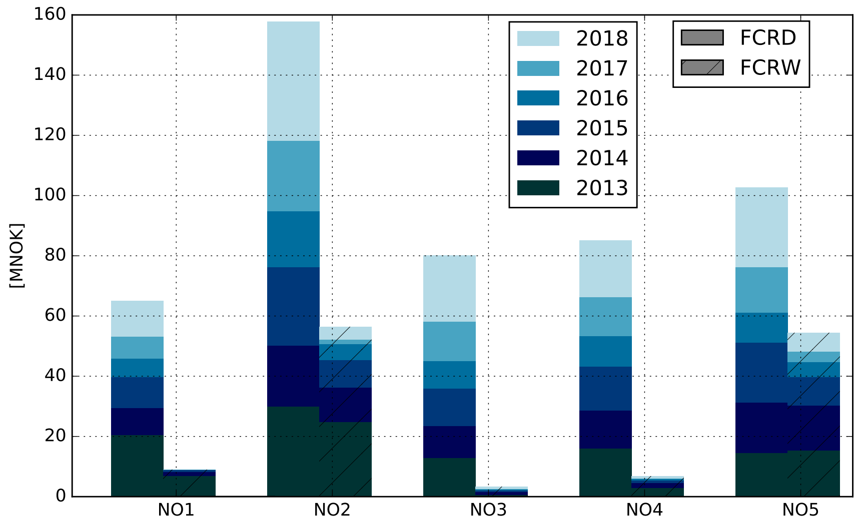


Figure 2.7: Turnover for the FCR-N daily and weekly market.

Figure 2.7 shows the turnover for the FCR-N weekly and daily market since 2013. It is clear that the turnover in the daily market is larger than the weekly market and that the turnover in 2018 was higher compared to previous years. This can be seen in context with the hydropower producers requirement for a higher premium to commit their generation before the day-ahead market is cleared. Thus, the TSO can acquire the primary reserves more cheaply the day before as the hydropower producers now know their commitments and can more easily price their primary reserves. Moreover, it should be noted that the turnover in the NO2 is significantly higher than in the other areas. This can be seen in context with the share of large and flexible hydropower stations located in this area. According to Statnett, 95% of the primary reserves in Norway come from hydropower [28].

Figure 2.8 and 2.9 depict the FCR-N price and volume in NO2 for the daily and weekly market, respectively. The values are averaged over 50 days to reduce some of the variability of the data, for purpose of visualization. It can still be observed that the prices can be correlated with the inflow; high in the spring and some peaks in the fall. The reason is that when the inflow is high the Elspot prices tend to be low as well and the large flexible hydropower producers will, therefore, shut down and store water for later use, leading to a scarce supply of rotating reserves. This aspect illustrates a core issue of this dissertation; how to optimally allocate the resource between the markets.

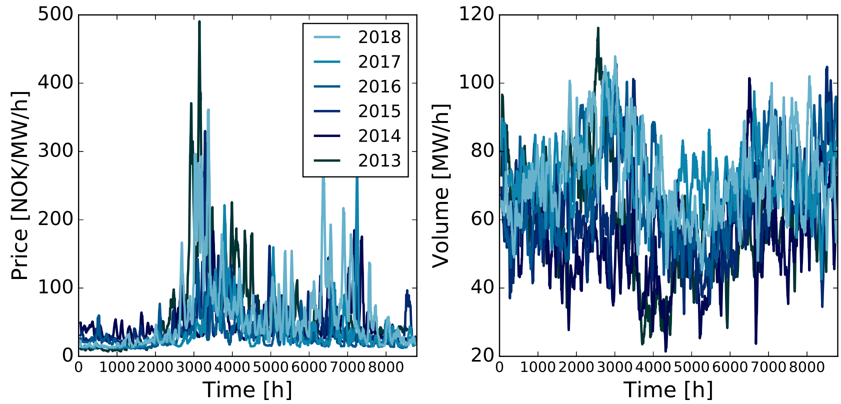


Figure 2.8: Price and volume for the daily FCR-N market in NO2. Values are averaged over 50 days to reduce noise.

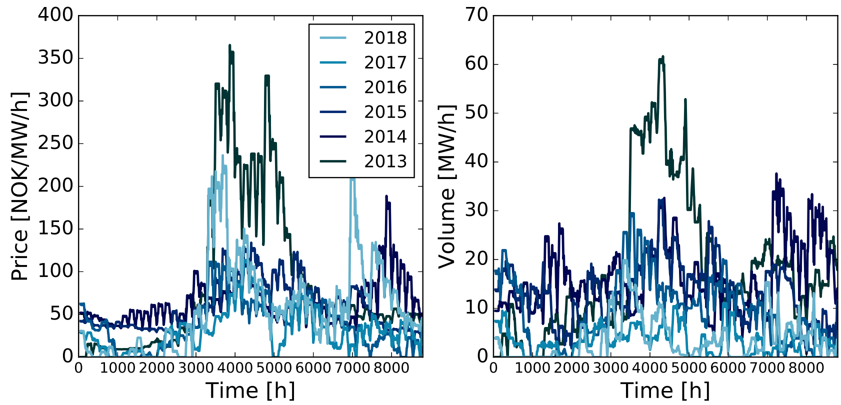


Figure 2.9: Price and volume for the weekly FCR-N market in NO2. Values are averaged over 50 days to reduce noise.

2.2.2 Secondary Reserve Market

The objective of FRR-A is to relieve the primary reserves, which may be activated up to 2 minutes while the activation period for FRR-A is from 2-15 minutes [28]. FRR-A is also a spinning reserve, but activation is performed directly by a signal from Statnett to either increase or reduce the set-point of the production unit. This requires some physical infrastructure installed in the hydropower station, therefore only certified power stations are granted access to the market [35]. The volume of FRR-A is set by Statnett on a basis of system requirements.

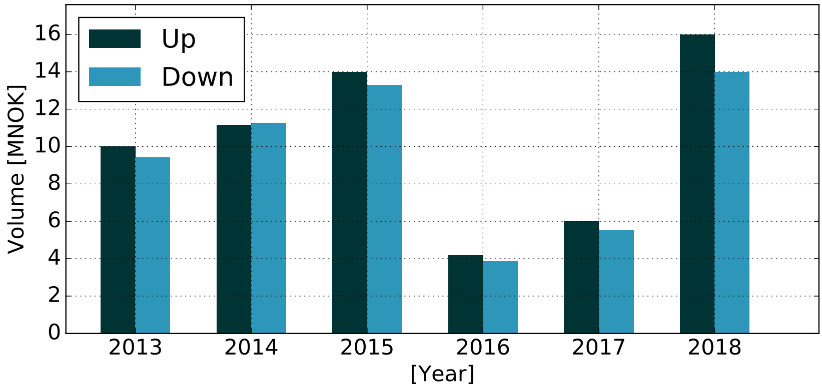


Figure 2.10: Turnover in the FRR-A market.

Figure 2.10 shows the turnover in the secondary reserve market since 2013. In 2016 the procurement of FRR-A was halted as a result of an effort to harmonize a Nordic platform for FRR-A. In 2018 the turnover has recovered and could be expected to continue to increase as the common Nordic market becomes operative from mid-2019, giving flexible hydropower producers access to a larger market share.

The price is based on marginal bidding, as with the primary reserve market, but this is dependent on potential bottlenecks in the system, which are treated separately (outside the market) by the TSO. Therefore the resulting prices might sometimes be higher than the marginal price. The time periods that active bids should operate in are set 2 weeks in advance of bidding. Some details about the specifications for bidding in the secondary reserve market can be seen in Table 2.2.

Table 2.2: FRR-A specifications.

What	Specifications
Allocation period	One week
Product	Up-regulating or down-regulating
Volume	Min 5 MW, max 35 MW and dividable by 5
Price	[NOK/MW/h]

According to Statnett's concession to build the two new HVDC interconnectors to Germany and UK, there will be requirements for more FRR-A to handle rapid variations of net load [36]. It is also planned to have a dynamic allocation of up to 300 MW on each cable for FRR-A. It is dynamic as the TSOs will analyze the social benefits of allocating to FRR-A compared to the spot market. Their prediction is that this would result in 100 MNOK/yr added value. This follows from the conclusion from a technical report by Statnett stating that the potential to export highly valued FRR-A to other Nordic countries and Europe is substantial [28].

2.3 Additional Ancillary Services

In addition to the abovementioned reserves in Section 2.2, there are other ancillary services that the TSOs rely on to ensure reliable operation of the power grid. In the following section, the tertiary reserve market is outlined followed by a discussion on other ancillary services.

2.3.1 Tertiary Reserve Market

The tertiary reserve market, called Regulating Power Market (RPM), in the Nordic system, is used as a third mechanism by Statnett to stabilize the grid. Capacity traded on the tertiary market, called Frequency Restoration Reserves - Manual (FRR-M), is used to release the primary and secondary control, so that these reserves may be ready to act on other imbalances. The reaction time for FRR-M is considerably longer than FCR and FRR-A. The activation is performed by a telephone call from Statnett to the load dispatch center of the activated producer, called Balance Service Provider (BSP). The activation time is up to 15 minutes after the call is received. In Norway, Statnett requires a total volume of 1200 MW and additional 800 MW to handle local bottlenecks in the grid [28]. In periods with scarce volume in the RPM an additional market is used to stimulate the volume. This is achieved by introducing the Regulating Power Options (RPO) market, where power producers can sell options to Statnett on their balancing capacity.

The tertiary reserve market is somewhat different for each Nordic country, depending on the specific needs for the TSO. A pilot study is currently carried out by Statnett to assess the possibility for exchanging tertiary control out of the Nordic countries [28]. In the future, this might open up new market shares for Norwegian power producers.

Figure 2.11 shows how the different balancing markets restrict the production set point for a single turbine. As FCR and FRR-A are both automatically handled, the sum of these allocations subtracted from the maximum power output restricts the new power set point upwards, and vice versa with the minimum generation limit downwards. This, in turn, limits how much FRR-M reserves the hydropower

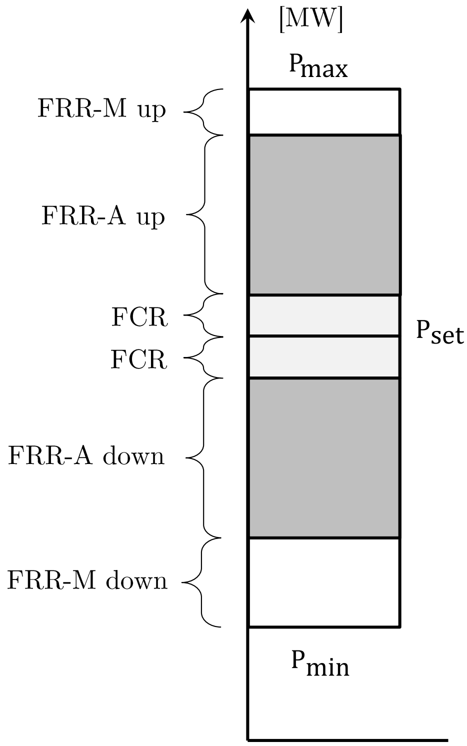


Figure 2.11: Illustration of allocation in the primary, secondary and tertiary reserve markets. P_{set} represents the set-point of the hydropower unit, given available capacity between the maximum and minimum generation limits, the unit may participate in the different markets with the remaining capacity.

station can deliver.

Predictions with the two HVDC interconnectors show that during the summer months, with low load and high generation from nonstorable hydro, there might be a scarcity of flexible power stations to deliver downward FRR-M. This is similarly to the issue during winter when there is a scarcity of upwards FRR-M, as the prices are good and flexible producers generate close to maximum. Statnett is, therefore, considering including an RPO market during summer for selling downwards FRR-M, providing additional reveneue potential for flexible hydropower producers. An alternative would be to reduce the transmission capacity on the HVDC interconnectors [36].

2.3.2 Discussion

Other ancillary services are often restricted spatially and are not suitable for incorporation in a market structure, e.g. reactive power control for voltage stability. They are thus more suitable for inclusion on a case-to-case basis, rather than in a generic hydropower scheduling approach.

An intriguing grid stability issue is the amount of rotating inertia in the system. More inertia gives a sturdy system that can handle contingencies better than a system with low inertia. With many HVDC cables and a high share of electricity generation from intermittent energy sources the inertia may become too low for the TSOs to ensure reliable grid operation. In the Nordic power system the TSOs have set a lower limit of 100 GWs of inertia.

According to [28] the amount of inertia in the Norwegian power system is now reaching critical limits in certain hours throughout the year. This is especially true during the night and some hours during summer, when energy prices are low. Projections show that this falling trend will continue, in such a way that measures to combat this should be implemented. This issue is discussed in [15] from a hydropower producers' perspective. Even though the impacts of an inertia market were moderate in the study, such a future market will contribute to another complicating layer for hydropower producers in their scheduling.

2.4 Market Overview

In order to systematize the different markets, Table 2.3 shows which time periods the markets are active, the resolution of the time periods, what the commodity is and the time-sequence for participating in the markets. The table is deducted from [37].

Table 2.3: Time-sequence for the different power markets in Norway. The table shows the time period for the market, the resolution and what the commodity is. N/D/E refers to the periods Night/Day/Evening. The right-hand side of the table indicates the gate closure of the different markets.

Market	Period	Res.	Comm.	October	Thursday - 10:00	Thursday - 12:00	Friday - 12:00	Day-1 - 12:00	Day-1 - 18:00	Hour-1	Hour - 0:45
RKOM	Winter	Season	Capacity	✓							
FFR-M	Week	N/D/E	Capacity		✓						
FCR-N	Weekend	N/D/E	Capacity			✓					
RKOM	Week	N/D	Capacity				✓				
FCR-N	Weekday	N/D/E	Capacity				✓				
Elspot	Day	Hour	Energy					✓			
FCR-N/D	Day	Hour	Capacity						✓		
Elbas	Cont.	Hour	Energy							✓	
FRR-M	Hour	Hour	Energy								✓

After the clearing of Elspot at 12:42[2], it is crucial for the producer to perform an appropriate planning on which market to participate in, maximizing profits and ensuring sufficient capacity for every market.

To sum up the turnover in the different markets in Norway, Figure 2.12 illustrates the relative turnover in 2017. It is undeniable that turnover in the balancing markets and Elbas constitutes only a small share compared to that of the Elspot market. According to the Ministry of Petroleum and Energy, 75% of the installed capacity in Norway is hydropower generators with upstream reservoir capacity [38]. These are the stations with the flexibility that can participate in the balancing markets and will take most of the turnover in them. A flexible hydropower producer could, therefore, have a larger share of its turnover from Elbas and the balancing markets, than which is illustrated in Figure 2.12.

[2]The system and area price are published at this time given that the market clearing was successful. In case of the contrary, a new clearing is needed and 4 minutes' notice is sent to the participants.

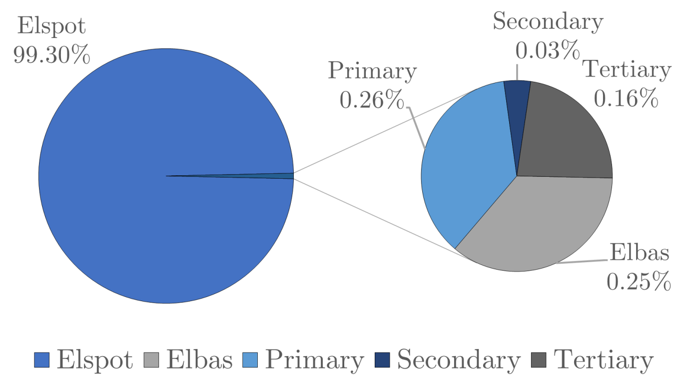


Figure 2.12: Pie chart of the relative turnover for the different power markets in 2017 in Norway. The turnover is calculated as price times the volume in the different markets.

2.5 Expectations for the Future Power Market

This section has, until now, shown that hydropower producers have many different markets to consider in their scheduling. Nonetheless, the volumes in the Elspot market are significantly more substantial for the overall profits than the other markets. Some already highlighted studies show, however, that flexible hydropower producers will acquire added profits by taking multiple-markets into consideration, e.g. [39, 40, 41, 9, 42, 43].

Studies taking into consideration a potential future power market, such as [44, 45], show that the overall costs for balancing, provided by secondary reserves, were significantly lower with an integrated power market. Compared to the case when the power market was not integrated, Norwegian hydropower also delivered considerably more of the balancing reserves. With this in mind and the ongoing integration of European power markets and increasing cross-border capacity, one can expect in the future that a larger share of Norwegian hydropower producers' profit will come from the balancing markets.

2.6 Hydropower Scheduling

The following chapter aims to clarify how storable hydropower scheduling is generally performed. An important matter for storable hydropower is the reservoir's Degree of Regulation (DOR), i.e. the ratio of the reservoir's storage capacity to the yearly expected inflow. An illustration of the DOR from an extensive case study is shown in Figure 2.13 [46]. The figure provides an insight into the size and DOR of different Norwegian hydropower reservoirs. Note that some reservoirs have been aggregated in order to reduce the computation time in the study.

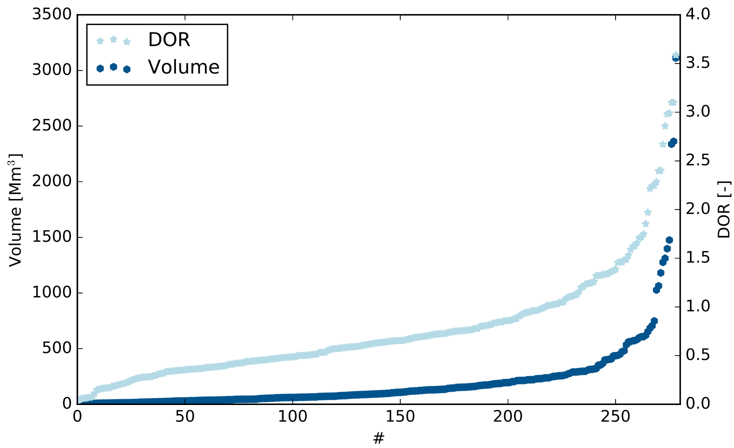


Figure 2.13: DOR overview over Norwegian reservoirs.

Nonetheless, the key question for storable hydropower producers in a liberalized market is to decide when best to generate power in order to maximize the expected profits. A highly simplified illustration of the HS process is given in Figure 2.14. If the water value is known, one can better decide whether to store the water for later use or to participate in the power market. The difficult part however is how to set the water value, as it should take into consideration all future profit opportunities for which the water may be used. In order to compute this, the different markets in which to participate in should be modelled, and both their, and the inflow's uncertainty should be accounted for. This shows how important, and applicable, stochastic programming is for hydropower scheduling, as it is necessary to consider the different outcomes and find a scheduling plan that hedges against them. The HS problem has been applied to numerous optimization models and an extensive review of some of them can be found in [47].

A literature review on HS in multiple markets is given by [42]. The review states that eventhough the profit potential is limited, hydropower producers can provide gains by providing flexibility and ancillary services. Services that are expected to be remunerated more beneficially in the future. Another literature review on the same subject can be found in [40].

For a power system with a central dispatch, such as the Brazilian, the HS process shown in Figure 2.14 is somewhat different. The aim is to perform a total system cost minimization rather than maximizing the individual power plant's profit. The HS decision must, therefore, store water until the hours with the highest system cost, avoiding spillage and rationing of energy. It is clear that both methods solve the same problem, they simply approach it from two different angles.

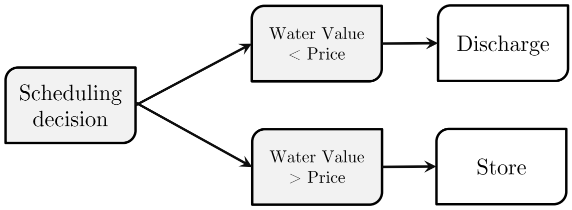


Figure 2.14: Illustration of the decision problem for a hydropower producer in a liberalized electricity market, such as the Nordic. Price refers to what the hydropower producer can obtain for their water in the power market.

The hydropower scheduling problem is ultimately a Multistage Stochastic Programming (MSSP) problem. It is multistage due to the long time periods that must be considered, and stochastic, as mentioned, due to the uncertainty of inflow, and also future energy and reserve capacity prices. The MSSP problem is a class of problems that are particularly difficult to solve and a distinct classification is used for hydropower scheduling to divide and conquer the problem. The problem is typically divided into a long-term, medium-term and short-term hydropower scheduling problem [48, 49, 50], depending on the analyzed time period and system details. A study on the contrary, with an integrated long- and short-term HS model can be found in [39]. Still, the fundamental principle of the segregation of different HS models can be seen in Figure 2.15. Since the water is storable it has some opportunity costs associated with it, in contrast to thermal power plants, where a marginal cost is given by the cost of the fuel. Since the inflow ("fuel") to the hydropower station is free, it's the opportunity cost, defined as the water value, that provides insight to whether or not the water should be dispatched, as illustrated in Figure 2.14. Thus, the water value is an important term within the field of hydropower scheduling. One can, very briefly, state that if the energy price is lower than the water value, then the hydropower should not dispatch, as there is an expectancy for higher income in the future. The water value must, therefore, be able to incorporate all future income opportunities for the stored water. This is the essential idea behind the different HS models, described with further detail in the following.

2.6.1 Long-Term Hydropower Scheduling (LTHS)

In an effort to describe the entire power system, the Long-Term Hydropower Scheduling (LTHS) problem is defined as a fundamental market model describing the power market with producers and consumers. Power producers and consumers are typically aggregated and the time resolution is normally on a daily, weekly or

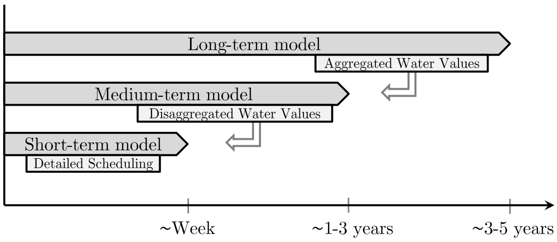


Figure 2.15: Illustration of a Nordic hydropower scheduling problem with different layers of models. The overall problem is unbundled into different tractable models with different objectives. Aggregated water values refer to an aggregated reservoir, whereas the disaggregated water values are given for individual reservoirs.

monthly basis to make the problem tractable. A main concern of the LTHS problem is to tackle the long-term uncertainty of inflow. It is often used for expansion planning, price forecasting, system studies and finding aggregated water values [51]. The aggregated water values typically describe the value of the water in an aggregated reservoir, describing an area in the power system.

Since some reservoirs have storage capacity over several years[3], the LTHS problem should have a sufficiently long time horizon to ensure that the initial stages (weeks) in the MTHS are not affected by the prices in the final stages. Thus, using the LTHS model for price forecasting the time horizon is typically set from 5-10 years. For expansion planning and system studies the time horizon may be even longer, spanning up to decades ahead.

An example of a LTHS model is the NEWAVE model in Brazil [52], used with monthly decision stages and four aggregated reservoirs. Another model, more widely used in the Nordics, is the EMPS model [51], that uses a heuristic approach for aggregating and disaggregating the hydropower reservoirs in the system. It is not only used for LTHS but also for general power system studies, such as what-if analysis where one can study how certain changes in the power system affect the system as a whole, e.g. [53] and [54].

2.6.2 Medium-Term Hydropower Scheduling (MTHS)

The MTHS problem can be seen as a refinement of the LTHS problem, where the level of detail is increased with a shorter time horizon. For example from the LTHS model one can obtain aggregated water values that the MTHS problem uses as an end-value statement and prices of the market clearing can be used for input. In the Nordics the MTHS problem typically describes a hydropower river system

[3] DOR is greater than one.

with individual reservoirs, instead of an aggregated representation. In contrast, in Brazil the MTHS problem can describe the same region, but in greater detail and a shorter time horizon.

Again, the different market structures decide how the MTHS problem is defined. For example in the Nordics it is assumed that the prices in the MTHS problem are exogenous, the hydropower station is thus a risk-neutral price taker. This is an assumption that is often debated, as it requires the market players to withstand any opportunity to exploit market power and behave rationally. Due to the market structure in Brazil the central dispatch has gone from being a risk-neutral to a risk-averse problem. This is due to the fact that the worst outcomes (scarce inflow) have a significantly worse societal impact than the best outcomes.

The MTHS problem is a well studied, and often the go-to problem when validating new MSSP methods. An extensive exploration of MTHS can be found in the PhD dissertation [41], which also investigates the MTHS problem in a multi-market setting.

2.6.3 Short-Term Hydropower Scheduling (STHS)

After individual water values are computed a more complex problem can be solved in order to obtain as detailed results as possible. The Short-Term Hydropower Scheduling (STHS) problem normally has a time horizon of 1-2 weeks, where the water values from the MTHS problem indicate the opportunity cost of storing the water after this period. The STHS problem is generally divided into two problems. The first problem is often referred to as the bidding problem, where the aim is to generate bids to the power market. After market clearing the hydropower producer, if the bids were activated, has some obligations to fulfill in the respective markets. Following this, the second problem of the STHS is to be solved, often described as the Unit Commitment (UC) problem. The UC problem is a complex combinatorial optimization problem, where the aim is to decide the optimal combination of units that should be operating [55, chp. 14].

Due to the high degree of modeling details in the STHS problem, most commercial models solve a deterministic problem, where the assumption is that the time periods are so small that uncertainty can be neglected. With more computational force and faster mathematical solvers stochastic STHS models have been more prevalent, such as the work in [56], where a slight gain of stochastic optimization is observed.

[43] is another recent study on hydropower bidding in multiple markets. The study tackles the STHS problem with participation in the primary and tertiary reserve markets and finds a maximum gain of coordinating the bidding process of 1%,

compared to a sequential one. The effect decreased to around 0.5% for larger hydropower systems with more flexibility.

2.6.4 Discussion

The proposed approach is predominantly focused on the Norwegian HS case. In other systems, such as the Brazilian, a central dispatch is performed so that even though the methodology of dividing the HS problem is used, the main objective is to minimize the entire system cost [57]. Moreover, in the Canadian power system the hydropower may be owned by large energy-intensive industries for self-use, sold over long-term contracts or sold to a day-ahead market. Regardless of the objective for the hydropower, it all essentially relates to maximizing the utilization of the water.

Some work, however, has been conducted on how to integrate the STHS model with a more long-term model [39]. The fundamental principle was to use a fine time resolution early in the model and a more coarse resolution going forward. The case study consisted of one reservoir, deterministic inflow and a time horizon of 6 months, which resulted in a Mixed Integer Programming (MIP) with approximately 70 thousand constraints and 1 million variables. The procedure might therefore not be adequate for scaling larger problems, which is the essential idea behind dividing the HS problem into different models.

2.7 Stochastic Optimization

The models used for solving HS problems are confined by the discipline of Operations Research (OR). The discipline originated from World War I where the task was to schedule convoys. It was later advanced in World War II where around 200 OR scientists worked in the British Army [58, p. 117]. Today, the field has matured and its applications are widely used around the world. Stochastic Programming (SP) is a sub-field of OR and consists of decision making under uncertainty. For example, the hydropower producer's task of scheduling the water discharge over a time period considering uncertain inflow and market price(s). The first known publication where SP was applied to HS comes from [59]. An interesting discussion on the early history of HS and how it matured until recent years can be found in [60].

Even though [59] described a DP approach, i.e. the problem is divided into subproblems for each time stage, DP was formally proposed by R. Bellman. The framework is described in [61], where the original version came out in 1957. The concepts of DP form the basis for the models adopted in this thesis, where the focus has been on the SDDP algorithm [62] with extensions.

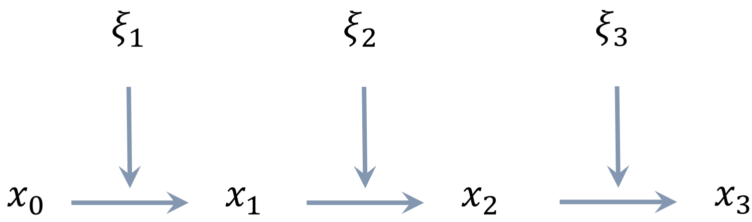


Figure 2.16: Illustration of the decision process for multistage stochastic programming problems.

In the following section, some concepts of multistage stochastic programming will be outlined. The concepts are then used to show how SDDP and the extension SDDiP are solved.

2.8 Multistage Stochastic Programming (MSSP)

MSSP is used when the underlying problem has a structure as illustrated in Figure 2.16. The problem consists of four time stages where for each time stage t, some decision x_t has to be made. After a decision has been made some new information, ξ_t, is revealed described by a random or stochastic process. It is clear that the complicating part of this problem is to perform an action in the early stages without knowing what will happen in the later stages. Stochastic programming therefore aims at finding an action that is robust for the potential outcomes of the stochastic process [63]. For completeness, the following problem describes a canonical *deterministic* programming problem

$$\max_{(x_1,y_1),\ldots,(x_T,y_T)} \left\{ \sum_{t=1}^{T} f_t(x_t, y_t) : (x_{t-1}, x_t, y_t) \in X_t, \forall t = 1, \ldots, T \right\}, \quad (2.1)$$

where x_t and y_t describe state and stage variables, respectively. The state variable carries information about the state/condition of the underlying problem between time stages and x_0 is the known initial state of the problem. For HS the reservoir level is a state variable. The stage variables, y_t, are decision variables for the given time stage, e.g. how much water should be discharged from a hydro reservoir. The objective value, i.e. what function should be maximized/minimized, is described by f_t. For a Norwegian hydropower producer the objective would be to maximize the profit from its hydropower system. The set X_t describes the solution space or feasible region of the problem, thus consisting of constraints and bounds that describe the underlying problem. These would include the efficiency of a power

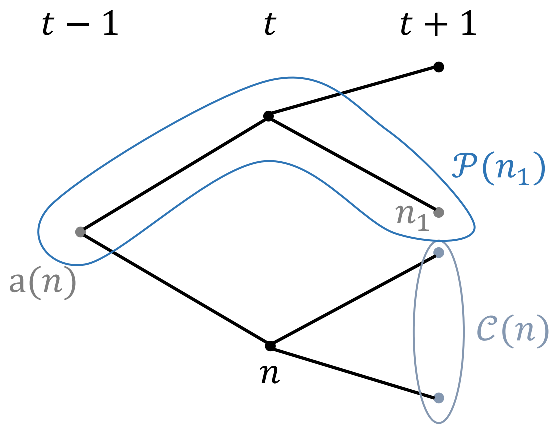


Figure 2.17: Illustration of a scenario tree and relationships between the nodes.

station, reservoir limits, etc. The reservoir constraint, also called water balance constraint, is a time-coupling constraint as it is used to carry information between the time stages, written on the form

$$W_t x_t + T_{t-1} x_{t-1} + G_t y_t = h_t. \tag{2.2}$$

G_t, T_{t-1} and W_t are matrices of suitable dimensions, the latter two are often referred to as the technology and recourse matrix, respectively. The term *recourse* comes from the fact that given an action in the former stage and with new revealed information it is possible to perform another action in the next stage. The action in the latter stage would thus be a recourse action. The flow of how new information is disclosed can be given by a scenario tree, as depicted in Figure 2.17.

The scenario tree, $\mathcal{T}$, describes the underlying stochastic process. For each node, n, in the tree, new information is disclosed. The set $\mathcal{C}(n)$ describes the children nodes, whereas $a(n)$ is a node's parent or ancestor node. The cardinality of $\mathcal{C}(n)$ also refers to the number of branches for node n. The set $\mathcal{P}(n_1)$ contains all parent nodes of node n_1 and the node itself and is thus describing a unique path in the scenario tree. For the nodes in the final stage T, the unique paths of each of them describe a scenario. For a scenario tree with $T > 1$ stages and 2 branches for each stage, the amount of nodes will equal $1 + 2^{T-1}$. For problems over larger time periods, it is therefore evident that the stochastic process has to be described in a reasonable manner for the problem to be tractable. The modeling of stochastic processes is a topic in itself, and is therefore in the broader sense not a core topic for this work. Some important resources for the topic should, however, be mentioned. The following book on time series analysis gives a comprehensive introduction to different stochastic processes [64]. The work by [65] and [66] describes some methods for constructing scenario trees. A key concept of generating scenario trees is how to optimally describe the underlying stochastic process, thus the concept of distance measurements is important. The problem can be described by minimizing the difference in shape (distance) between two probability distributions, i.e. how to optimally design a distribution that resembles an original distribution. Given a scenario tree, the following *extensive* formulation of a MSSP is given as

$$\max_{(x_n,y_n)} \left\{ \sum_{n\in\mathcal{T}} p_n f_n(x_n, y_n) : (x_{a(n)}, x_n, y_n) \in X_n, \forall n \in \mathcal{T} \right\}. \tag{2.3}$$

Here the objective function is multiplied with the total probability, p_n, associated with node n. As discussed above, this formulation is not very convenient for solving MSSPs with many time stages. In an effort to combat this issue a stagewise decomposition is applied to divide the problem into smaller sub-problems.

2.8.1 Dynamic Programming (DP)

DP aims at solving sequential decision problems by dividing a large problem into smaller stagewise problems. This was the key idea behind the methods proposed by [59] and [61]. The well known *Bellman equation* is thus given as the stagewise problem that can be solved recursively to solve (2.1). It is defined as

$$Q_t(x_{t-1}) := \max_{(x_t,y_t)} \left\{ f_t(x_t, y_t) + Q_{t+1}(x_t) : (x_{t-1}, x_t, y_t) \in X_t \right\}. \tag{2.4}$$

$Q_{t+1}(x_t)$ is referred to as the value function, profit-to-go function or future profit function, since it describes the future value for the state x_t. The backward DP approach discretizes the state variables and starts from the last time stage with a known end value $Q_{T+1}(x_T)$ and solves (2.4) for all discrete state variables. The process moves backward in time since $Q_T(x_{T-1})$ is already computed it is possible to solve $Q_{T-1}(x_{T-2})$, and thus continue the process until the first time stage is reached. Thereby, the DP approach evaluates all possible states so that for a given initial state one can compute the optimal (shortest) path between the stages. There are numerous ways to implement a DP problem, the common concept is to discretize the state space and use a recursive approach, either forward or backward.

There is, however, an obvious downside with the DP approach; the requirement to discretize the state space. For a problem with T time stages and D discretizations for the state variables with dimension d, it would be necessary to solve T times D^d

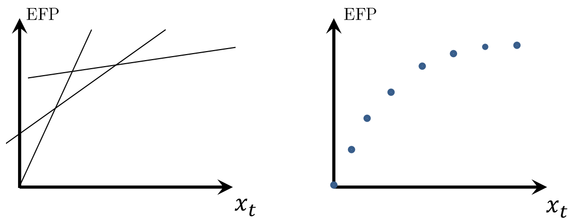


Figure 2.18: Illustration the EFP in a SDDP problem (left) and DP problem (right).

problems, which clearly becomes untractable for problems with a high dimension of state variables.

To handle the drawback of exponentially growing subproblems[4] of DP and nodes in a scenario tree an approximate Stochastic Dynamic Programming (SDP) approach was proposed by [12], followed by the Stochastic Dual Dynamic Programming (SDDP) alogorithm in [62, 11].

2.8.2 Stochastic Dual Dynamic Programming (SDDP)

In essence, the SDDP method circumvents the curse of dimensionality of DP problems by approximating the Expected Future Profit (EFP) function by a piecewise linear function and by sampling a subset of scenarios from the scenario tree. Then statistical bounds can be computed for controlling convergence of the method. The piecewise linear EFP function does not require discretization of the state variables, as illustrated in Figure 2.18. The SDDP method is an iterative method that gradually builds the EFP function by using Benders cuts [67]. Consider the following DP formulation of (2.3)

$$\mathcal{Q}_1(x_0) := \max_{(x_1,y_1)} \{f_1(x_1, y_1) + \mathbb{E}_{\tilde{\xi}_1}[Q_2(x_1, \tilde{\xi}_1)]\} \tag{2.5a}$$

$$\text{s.t.} \quad (x_1, y_1) \in X_1(x_0), \tag{2.5b}$$

and

$$\mathcal{Q}_t(x_{t-1}, \xi_{t-1}) := \max_{(x_t,y_t)} \{f_t(x_t, y_t) + \mathbb{E}_{\tilde{\xi}_t|\tilde{\xi}_{t-1}}[Q_{t+1}(x_t, \tilde{\xi}_t)]\} \tag{2.6a}$$

$$\text{s.t.} \quad (x_t, y_t) \in X_t(x_{t-1}, \xi_{t-1}), \tag{2.6b}$$

where (2.5) is the problem in the first stage, referred to as the overall or master problem. (2.6) describes the recursive problems for the rest of the time stages,

[4]often referred to as the *curse of dimensionality*

where Q_{T+1} is a known function. We assume that for all feasible x_{t-1} solutions and realizations of the random data vector $\tilde{\xi}_t$ the problem (2.6) is feasible, i.e. *relative complete recourse*. Outcomes of the random data vector are denoted as ξ_t.

For a Nested Benders Decomposition approach the expectation is taken over $\xi_{t+1}|\xi_t$. The outcome of the stochastic process is therefore dependent on the previous nodes. Consequently, the problem must be solved for all nodes in the scenario tree. A key aspect for the success of SDDP is that the stochastic process is assumed to be *stage-wise independent*, so it does not matter which node you are in at stage t for the outcome of ξ_{t+1}. The expectation is thus taken over ξ_{t+1} instead of $\xi_{t+1}|\xi_t$. The number of nodes in the scenario tree effectively collapses to $\sum_{t=1}^{T-1} |\xi_t|$ nodes instead of $\prod_{t=1}^{T-1} |\xi_t|$. This means, in practice, that the EFP function for a node n in time stage t is valid for all other nodes in the same time stage. Even though the stage-wise independent sampling might seem like a significant drawback it is possible to use normalization and affine functions to better describe the stochastic process, such as autoregressive processes [68].

The SDDP method consists of two main procedures, the forward and backward iteration, given in Algorithm 1. The stagewise decision problem in iteration i is given by (2.7) and (2.8), where (2.7) is the master problem given as

$$\mathcal{S}_1^i: \ \mathcal{Q}_1^i(x_0^i) := \max_{(x_1^i, y_1^i)} \{f_1(x_1^i, y_1^i) + \phi_1^i(x_1^i)\} \tag{2.7a}$$

$$\text{s.t.} \quad (x_1^i, y_1^i) \in X_1(x_0^i), \tag{2.7b}$$

and for the rest of the stages t and stochastic outcomes ξ_t^{ik} the problem is given as

$$\mathcal{S}_t^i: \ \mathcal{Q}_t^i(x_{t-1}^{ik}, \xi_{t-1}^{ik}) := \max_{(x_t^{ik}, y_t^{ik})} \{f_t(x_t^{ik}, y_t^{ik}) + \phi_t^i(x_t^{ik})\} \tag{2.8a}$$

$$\text{s.t.} \quad (x_t^{ik}, y_t^{ik}) \in X_t(x_{t-1}^{ik}, \xi_{t-1}^{ik}). \tag{2.8b}$$

The piecewise linear function ϕ_t^i is given as

$$\phi_t^i(x_t) := \{\theta_t \le U_t, \tag{2.9a}$$

$$\theta_t \le \pi_t^j x_t + b_t^j, \forall j \in \mathcal{C}_t^i\}, \tag{2.9b}$$

representing an upper bound of the EFP function, where ϕ_T^i is known[5]. The forward iteration is used to sample a set of scenarios, Ω^i, from the scenario tree and

[5] If not taken from a LTHS model or estimated by future market prices it is typically set to zero, given sufficient long time horizon.

generate candidate solutions x_t^{ik} that will be used in the backward iteration to improve the piecewise linear EFP function. More information on the cut generation cab be found in Section 2.9.2. By computing the expected objective values from the candidate solutions a lower statistical convergence bound can be computed. The deterministic upper bound comes from the solution of the master problem (2.7), that is solved after the backward iteration[6]. It is in the backward iteration that most of the computational work is performed, with the complicating part being how to generate the EFP function. Given that the problem is a LP problem and uncertainties are in the right-hand-side parameters of the constraints, the EFP functions are concave functions that can be upper approximated by Benders cuts. The backward iteration starts at the final stage with a given candidate solution to compute a cut for the earlier stage. Thus, by iterating between the forward and backward iteration the description of the EFP improves and convergence can eventually be observed. The method is described in Algorithm 1.

2.8.3 Solution Approach

The SDDP algorithm has benefited from the fact that the stagewise decision problems are LP problems and thus effectively utilize efficient resolving methods such as the dual simplex method [69]. The dual simplex is especially efficient when the number of constraints is large. The traditional approach for efficiently solving SDDPs has therefore been to solve the stagewise decision problem without any cuts and after a solution is obtained, perform a check over the set of cuts which violate the current solution the most. The cut is added to the problem, dual simplex is applied, and a new solution is obtained. This continues until there are no cuts that violate the given solution. The approach is explained in Algorithm 2. The algorithm has proven to be very effective in practice. For a majority of stagewise decision problems one only one or two cuts need to be added to find the optimal solution. The list of cuts can, therefore, be extremely large without affecting the computational burden considerably.

2.9 Stochastic Dual Dynamic Integer Programming (SDDiP)

The SDDP has become a widely used method for solving MSSP problems. A drawback has, however, been that it does not easily support nonconvex problems. By including binary or integer variables in the problem the EFP function is no longer convex with respect to the state variables, and may not be accurately de-

[6]Note that this is a maximization problem. For a minimization problem, the master problem would provide the lower bound and a statistical upper bound is computed from in the forward iteration.

Algorithm 1 The SDDP Method

Set x_0^i, ξ_0^i, $i \leftarrow 1$, $\mathrm{UB} = +\infty$ and $\mathrm{LB} = -\infty$,
while $i < i^{\max}$ **or** some other stopping criteria **do**
    Sample N scenarios $\Omega^i = \xi_1^{ik}, \ldots, \xi_T^{ik}{}_{k=1,\ldots,N}$
    /* Forward iteration */
    for $k = 1, \ldots, N$ **do**
        for $t = 1, \ldots, T$ **do**
            Solve $\mathcal{Q}_t^i(x_{t-1}^{ik}, \xi_{t-1}^{ik})$ given by (2.8)
            Collect solution $f_t(x_t^{ik}, y_t^{ik}, \xi_t^{ik}), x_t^{ik}, y_t^{ik}$
        $\mathrm{lb}^k \leftarrow \Sigma_{t=1,\ldots,T} f_t(x_t^{ik}, y_t^{ik}, \xi_t^{ik})$
    /* Compute lower bound */
    $\mu \leftarrow \frac{1}{N}\Sigma_{k=1}^N \mathrm{lb}^k$ and $\sigma^2 \leftarrow \frac{1}{N-1}\Sigma_{k=1}^N(\mathrm{lb}^k - \mu)^2$
    $\mathrm{LB} \leftarrow \mu + \mathrm{z}_\alpha \frac{\sigma}{\sqrt{\mathrm{N}}}$
    /* Backward iteration */
    for $t = T, \ldots, 2$ **do**
        for $k = 1, \ldots, N$ **do**
            for $m \in \mathcal{C}(t)$ **do**
                Solve $\mathcal{Q}_m^i(x_t^{ik}, \xi_t^{ik})$ given by (2.8)
                Collect cut coefficients and parameters
            Add Benders cut(s)
    /* Compute upper bound */
    Compute $\mathrm{UB} \leftarrow \mathcal{Q}_1^i(x_0^{ik})$ from (2.7)
    $i \leftarrow i + 1$

Algorithm 2 Cut Relaxation

Stagewise decision problem $\mathcal{S}_t^i$
Set of cuts $\mathcal{C}_t^i$ from (2.9b), where $c_t^j(x) = \pi_t^j x + b_t^j$
Choose cut violation parameter ϵ
Set $v = -\infty$
/* Remove all cuts from the stagewise decision problem */
for all $c_t^j \in \mathcal{C}_t^i$ **do**
  if $c_t^j \in \mathcal{S}_t^i$ **then**
    $\mathcal{S}_t^i = \mathcal{S}_t^i \setminus c_t^j$
while $v < \epsilon$ **do**
  Compute

$$z = \max\{\mathcal{S}_t^i\}$$
$$x = \arg\max_{x_t}\{\mathcal{S}_t^i\}$$

  /*Find the most violating cut*/
  for all $c_t^j \in \mathcal{C}_t^i$ **do**
    if $z - c_t^j(x) > v$ **then**
      $v = z - c_t^j(x)$
      $c^{\max} = c_j^t(x)$
  /* Add the most violating cut to the stagewise decision problem */
  $\mathcal{S}_t^i = \mathcal{S}_t^i \cup c^{\max}$

scribed by piecewise linear functions. For problems that are not very nonconvex the piecewise linear functions could provide sufficient solutions. One could, therefore, use the nonconvex problem formulation in the forward iteration and solve the LP relaxation in the backward iteration to generate the EFP function. If the convergence gap is within reasonable limits, this could be considered satisfactory, more on this in Article VII. This might work for some problems, but not in general.

A trade-off with improved modeling details associated with MIP problems over LP problems is the computational burden affiliated with them. Moreover, it is possible to utilize the dual simplex method used in the cut relaxation technique in Algorithm 2. Nonetheless, recent years' developments in mathematical solvers, like Gurobi and CPLEX, along with the utilization of multiple cores makes MIP increasingly more tractable.

The recent work by [70], and as a part of the PhD thesis [71], proposed the SDDiP method for solving nonconvex MSSP problems that is general with finite convergence, under some assumptions. The main assumption is that the state variables have to be binary. There are three pillars that separate the method from SDDP:

Binary state variables
: All state variables have to be binary.

Copy variables and constraints
: Some auxiliary variables and constraints have to be added.

A set of cut families
: Different types of cuts that can be used.

In order to transform all state variables into binary variables, the binary expansion approach is used [72]. The method is based on the fact that if λ_t is an integer variable, $\lambda_t \in \{0, \dots, L\}$, it can be represented by κ binary variables, $\kappa = \lfloor \log_2(\mathrm{L}) \rfloor + 1$, such that $\lambda_t = \Sigma_{j=1}^{\kappa} 2^{j-1} \lambda_{tj}$. Similarly, for the continuous case, $\lambda_t \in [0, L]$, where λ_t is given with ε accuracy; $\kappa = \lfloor \log_2(\mathrm{L}/\varepsilon) \rfloor + 1$, hence $\lambda_t = \Sigma_{j=1}^{\kappa} 2^{j-1} \varepsilon \lambda_{tj}$. The binarization does in essence mean that the state space is discretized to finite values. It is, however, not similar to the discretization performed in the DP method. An illustrative example of why this method can approximate a nonconvex EFP function is given in the following sections.

2.9.1 Visualization of the Approximation of a Nonconvex EFP Function

Figure 2.19 illustrates a function $\phi_t(x_t)$ that is to be upper approximated by some cuts. The cut C3 illustrates the tightest cut that can be used to approximate the

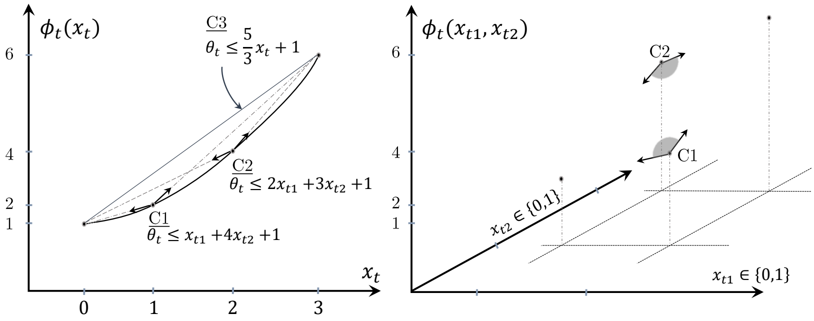


Figure 2.19: (Left) The EFP function in the continuous state space. (Right) The EFP function in the discrete state space via the transformation $x_t = x_{t1} + 2x_{t2}$, where x_{t1} and x_{t2} are binary variables. C3 represents the tightest cut one can generate with continuous state variables. The cuts C1 and C2 illustrate how discrete state variables can be used to provide cuts to approximate a nonconvex function. © IEEE.

function for continuous state variables. However, if the state variables are transformed to binary variables, as seen in the right figure, cuts that are tight for the discretized points can be computed. Binary expansion is used, and the continuous state variable x_t is transformed to $x_t = x_{t1} + 2x_{t2}$. Observe that the state space is reduced from $x_t \in [0, 3]$ to $x_t \in \{0, 1, 2, 3\}$. Nevertheless, this transformation enables us to generate cuts that are tight for the discrete state space. For instance, cut C1 can then be enumerated in the following way; along the x_{t1} dimension the cut will have a coefficient of $(2-1)/(1-0) = 1$, in x_{t2} dimension the coefficient will be $(6-2)/(1-0) = 4$. Moreover, it can easily be observed that the right-hand-side parameter will be 1. This yields the cut $\theta_t \leq x_{t1} + 4x_{t2} + 1$. The same approach is used for computing cut C2. Hence, we have found two cuts that are tight at the discrete state space values. The cuts C1 and C2 are given in the left figure, in an effort to visualize them in the continuous state space. In the given example it was fairly straightforward to enumerate the cuts. When the dimension of the state space becomes larger, this is, however, not so straightforward. The complicating problem is thus how to find these cut coefficients and right-hand-side parameters that provide tight cuts. [70] proposed a family of cuts that can be used to solve the overall problem, outlined in the following section.

2.9.2 Cut Families

The second pillar of SDDiP tackles the issue of computing cut coefficients and right-hand-side parameters. By introducing the copy variable $z_t \in [0, 1]$ and copy constraint $z_t = x_{t-1}$ it was proved in [70] that it is possible to compute cuts that are guaranteed to be tight. Cuts that might not be tight can also be used, as long as they provide an upper approximation of the EFP function. The stagewise problem solved in SDDiP is given by

$$\bar{\mathcal{Q}}_1(x_0) := \max_{(x_1,y_1,z_1)} \{f_1(x_1,y_1) + \phi_1(x_1)\} \quad (2.10a)$$
$$\text{s.t.} \quad (x_1,y_1,z_1) \in X_t \quad (2.10b)$$
$$z_1 = x_0 \quad (2.10c)$$
$$x_1 \in \{0,1\}, y_1 \in \mathbb{R} \times \mathbb{Z}, z_1 \in [0,1], \quad (2.10d)$$

and

$$\bar{\mathcal{Q}}_t(x_{t-1},\xi_{t-1}) := \max_{(x_t,y_t,z_t)} \{f_t(x_t,y_t) + \phi_t(x_t)\} \quad (2.11a)$$
$$\text{s.t.} \quad (x_t,y_t,z_t) \in X_t(\xi_{t-1}) \quad (2.11b)$$
$$z_t = x_{t-1} \quad (2.11c)$$
$$x_t \in \{0,1\}, y_t \in \mathbb{R} \times \mathbb{Z}, z_t \in [0,1], \quad (2.11d)$$

where the EFP function is given as

$$\phi_t(x_t) := \{\theta_t \le U_t, \quad (2.12a)$$
$$\theta_t \le \pi_t^j x_t + b_t^j, \forall j \in \mathcal{C}_t\}. \quad (2.12b)$$

The iteration and scenario indices, i and k, are neglected for simplicity. The different types of cut families that can be used are accordingly

Benders Cuts (B)

The well-known Benders cut [67] was initially proposed as a method for solving MIP problems and is used by the standard SDDP approach. The cut is generated by solving an LP relaxation of 2.11, $\bar{\mathcal{Q}}_t^{LP}(x_{t-1},\xi_{t-1})$,

$$\theta_t \le \sum_{m\in\mathcal{C}(t)} q_{tm} \bar{\mathcal{Q}}_m^{\text{LP}}(x_t^*,\xi_t) + \sum_{m\in\mathcal{C}(t)} q_{tm} (\pi_m^{\text{LP}})^\top (x_t - x_t^*), \quad (2.13)$$

where $(*)$ indicates that it is the candidate solution from the forward iteration and π^{LP} is the dual value for the copy constraint of the LP relaxation. The cut is not guaranteed to be tight, but on the upside it is very quick to generate as it involves solving LP problems.

Lagrangian Cuts (L)

This cut family is based on Lagrangian relaxation and provides cuts that are tight. The copy constraint (2.11c) is relaxed and the Lagrangian relaxation problem is given as

$$\mathcal{R}_t(\pi_t) := \min_{\pi_t} \left\{ \mathcal{L}_t(\pi_t) + \pi^\top x_{t-1} \right\}, \tag{2.14}$$

where $\mathcal{L}_t$ is defined as

$$\begin{aligned} \mathcal{L}_t(\pi_t) := \max_{x_t, y_t, z_t} \; & f_t(x_t, y_t) + \phi_t(x_t) - \pi_t^\top z_t && \text{(2.15a)} \\ \text{s.t.} \quad & (z_t, x_t, y_t) \in X_t(\xi_{t-1}) && \text{(2.15b)} \\ & z_t \in [0, 1] && \text{(2.15c)} \\ & x_t \in \{0, 1\}, y_t \in \mathbb{R} \cdot \mathbb{Z}. && \text{(2.15d)} \end{aligned}$$

Lagrangian relaxation often aims to relax complicating constraints and divide the problem into smaller subproblems that can more easily be computed. Our aim, however, is to find good multipliers that make the Lagrangian cuts as tight as possible. It is essential for the convergence of SDDiP that the Lagrangian cuts can be generated in a sufficient manner, in regard to both computation time and tightness. There are several methods to solve the Lagrangian problem, such as sub-gradient and bundle methods. Regardless of which method is used to get the Lagrangian multiplier, $\bar{\pi}_t^i$, the cuts we construct are on the following form

$$\theta_t \leq \sum_{m \in \mathcal{C}(t)} q_{tm} [\mathcal{L}_m(\bar{\pi}_m) + (\bar{\pi}_m)^\top x_t]. \tag{2.16}$$

Since solving the problem, (2.11), is not too time consuming it is possible to compute the optimal value of the stagewise decision problems, denoted Q_{t-1}^{MIP}, allowing for a more rapid solution of the Lagrangian problem, since we then know the lower bound of the problem. Figure 2.20 illustrates the concave function $\mathcal{R}_t^i(\pi_t)$ with an optimal value of $\mathcal{R}_t^i(\bar{\pi}_t) = Q_t^{MIP}$.

A common method to solve the Lagrangian problem is by a subgradient method [73]. This initilly involves solving the relaxed LP problem to obtain an initial solution for the Lagrangian multipliers, π_t^i. Then, by setting the step size parameters at desired values, iteratively compute $\bar{\pi}_t$. The process is computationally consuming as it requires solving a MILP problem for each iteration of the subgradient method until a convergence criterion is reached.

Another promising method for solving the Lagrangian problem is the level method [74]. One of the benefits from this approach is that when the optimal value solution is known, it is possible to cleverly tune a step size parameter for describing

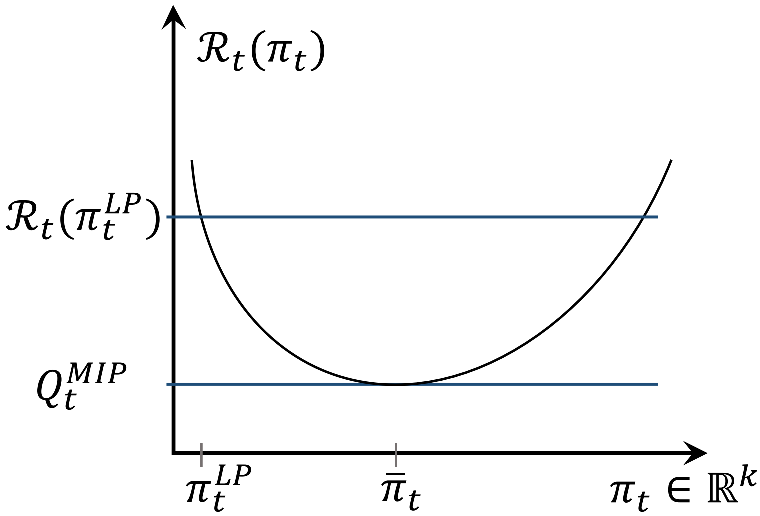


Figure 2.20: Illustration of the convex outer Lagrangian problem.

the level set, yielding a rapid solution. The drawback with the method is that it is necessary to compute the l_2-norm and hence a Mixed Integer Quadratic Programming (MIQP) problem for each iteration of the method. Practical experience has, nevertheless, shown that it is less time consuming to compute the MIQP than the entire Mixed Integer Linear Programming (MILP) problem (2.14), as in the subgradient method.

Strengthened Benders Cuts (SB)

The Lagrangian cut was made by solving a Lagrangian problem and obtaining both cut coefficients and right-hand-side parameters. The Lagrangian formulation does, however, also provide a valid cut for all Lagrangian multipliers that solves (2.15). Thus, by using the cut coefficients from the LP relaxation and solving (2.15) once, one can obtain a new cut with a right-hand-side parameter that is at least as tight as the Benders cut, called the Strengthened Benders cut, given as

$$\theta_t \leq \sum_{m \in \mathcal{C}(t)} q_{tm} [\mathcal{L}_m(\pi_m^{\text{LP}}) + (\pi_m^{\text{LP}})^\top (x_t)]. \tag{2.17}$$

Figure 2.21 depicts a Benders and Strengthened Benders cut. It illustrates how the Strengthened Benders cut may improve the tightness of the cut from the LP relaxation to the original function.

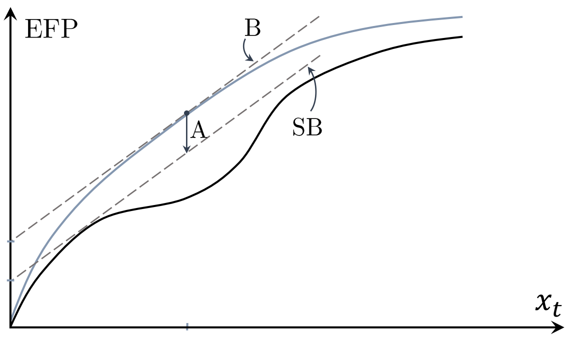


Figure 2.21: Illustration of a Benders (B) and Strengthened Benders (SB) cut. The blue EFP function refers to the LP relaxation of the true function (in black).

Integer Optimality Cuts (I)

Another type of cut that can be used for solving the SDDiP problem is the integer optimality cut [75, 76]. The integer optimality cuts are valid, tight and finite and will, therefore, guarantee convergence. They are also very fast to generate, as they only rely on the candidate solution obtained in the forward iteration.

$$\theta_t \leq (v_t - V_t)\left(\sum_{j \in \mathcal{S}(x_t^*)} x_{tj} - \sum_{j \notin \mathcal{S}(x_t^*)} x_{tj} - |\mathcal{S}(x_t^*)| \right) + v_t, \qquad (2.18)$$

where $v_t = \sum_{m \in \mathcal{C}(t)} q_{tm} \bar{\mathcal{Q}}_m^i(x_t^i, \xi_t^i)$, V_t is an upper boundary to the EFP function and $\mathcal{S}(x_t^i) := \{j \ : \ x_{tj}^i = 1\}$, given that x_t^i is a candidate solution. The integer optimality cut is tight for the candidate solution from which it was made, but very loose at other solutions. In practice, these cuts have been proven to contribute moderately to improving the convergence properties, while the computation time has increased as a result of large coefficients in the constraint matrix. The integer optimality cuts have therefore limited usefulness for practical purposes.

2.9.3 Stochastic Processes in SDDP

In the following, the core aspects of the uncertainty modeling in this thesis are presented.

For the MTHS problem, the key stochastic processes are energy price and inflow to the reservoirs. There are numerous approaches to model these stochastic processes. A common approach for modeling the inflow is by an Autoregressive-Moving-Average (ARMA) process [64] and a Markov process for the energy price [77]. This is done in order to circumvent the complex task of solving the bilinear

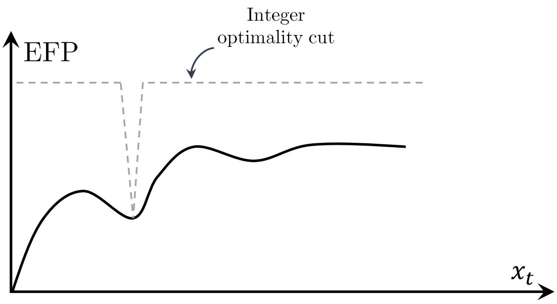


Figure 2.22: Illustration of integer optimality cut.

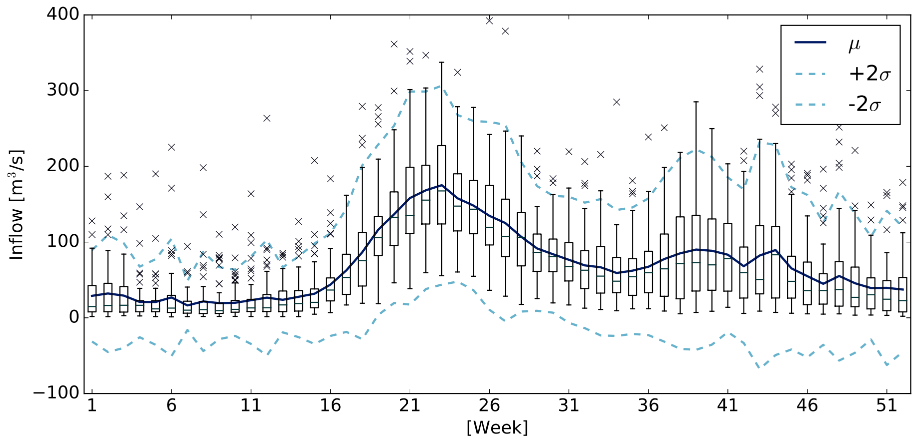


Figure 2.23: Standard box plot with outliers of the inflow measured in Bulken, a hydro system in south-western Norway. A line depicting the mean and a 95% confidence interval is added on top. The figure illustrates an important aspect when considering inflow modeling; inflow may not be sufficiently well described by a normal distribution, e.g. negative inflows may occur. It is, nonetheless, a requirement in the SDDP framework that the stochastic variables are independent and identically distributed to ensure convergence. Notice the correlation between the inflow and FCR-N prices, in Figure 2.8 and 2.9, as mentioned in Section 2.2.1.

term of energy price and generation. First, we elaborate on the stochastic process affiliated with the inflow.

The inflow is typically given on a weekly basis, due to a strong weekly correlation. The inflow is normalized in an effort to remove the seasonal variations of the inflow, as observed in Figure 2.23. This is primarily derived from the accumulation of snow in the winter that is followed by the melting in the spring. The inflow can

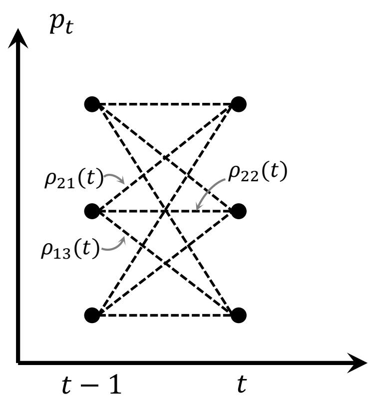


Figure 2.24: Illustration of a Markov chain, describing the uncertainty of the energy price. The transition probabilities, ρ_{ij}, is given as probability of going from a node i to a node j.

then be described by a VAR1 model, given by (2.19) in the following

$$\delta_t = \Phi\delta_{t-1} + \tilde{a}_t \tag{2.19}$$

$$I_t = \mu_t + \sigma_t\delta_t., \tag{2.20}$$

where δ_t is the vector of the normalized stochastic process, Φ is a time-invariant correlation matrix and $\tilde{a}_t$ is a vector of white noise. The realized inflows I_t are given by Eq. (2.20), where μ_t and σ_t are the expected value and standard deviations for time stage t.

The approach can be seen as a compromise between accuracy and computational feasibility [77]. For this reason, the same approach of using a VAR1 model for the inflow was adapted in this thesis. The SDDP algorithm also requires a convex stochastic model. This is not the case for the SDDiP model but is deemed to be outside of the scope for this thesis.

The discrete-time Markov process describing the underlying energy price process, p_t, can be given as

$$P(p_t = \zeta_t^j | p_{t-1} = \zeta_{t-1}^i) = \rho_{ij}(t), \forall i, j \in \mathcal{M}(t). \tag{2.21}$$

For all $t \in \{2, \ldots, T\}$, where T is the time horizon, $\mathcal{M}(t)$ is the set of nodes in the Markov process for stage t, ζ_t^j represents the realized energy price for a node j and $\rho_{ij}(t)$ is the transition probability of going from node i to j. An illustration of the Markov process can be seen in Figure 2.24. IT shows how the Markov chain is constructed as given by [78].

Observe that incorporating correlations between the two different stochastic processes is not straightforward and is considered a drawback of the approach. However, the energy price process is generated by an LTHS that takes into consideration the inflow uncertainty, thus the realized inflow and energy prices exhibit some correlation, but not in the noise term.

A recent publication by [79], and followingly [80], proposes a method to include objective term uncertainty for DP problems. The key concept in [80] comes from the fact that the outcomes of the stochastic process are sampled a-priori in the forward iteration of SDDP and that the problem is convex with respect to the uncertain objective coefficient. The objective term therefore becomes a parameter in the stagewise decision problem, and an additional term in the Benders cut is added. The method proposes a very elegant approach for including correlations between the different stochastic processes, such as the inflow and energy price. It is also possible to avoid the exponential growing set of nodes in the discrete Markov process when adding additional price uncertain parameters.

Chapter 3

Results and Discussion

The following chapter gives a brief presentation of the results from the articles that form the main contribution of this thesis.

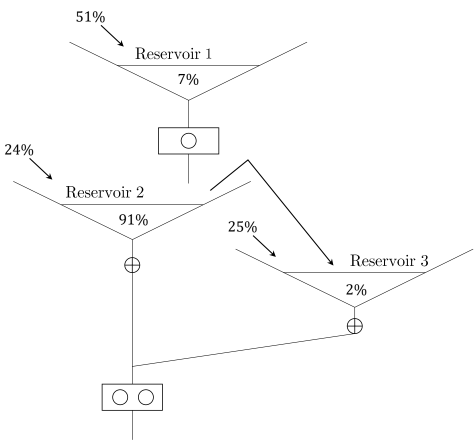


Figure 3.1: Illustration of the hydropower system used for case studies in this thesis. There are three reservoirs, each represented by their relative storage capacity and inflow compared to the system as a whole. As an example, Reservoir 3 has 2% of the system's reservoir capacity and 25% of the system's inflow. Note that Reservoir 2 and 3 are connected to the same power station.

3.1 Convex Medium-Term Hydropower Scheduling with Sales of Reserve Capacity

In the first two articles, a SDDP model that included sales of reserve capacity was developed and tested. The model was a prototype of an operative MTHS

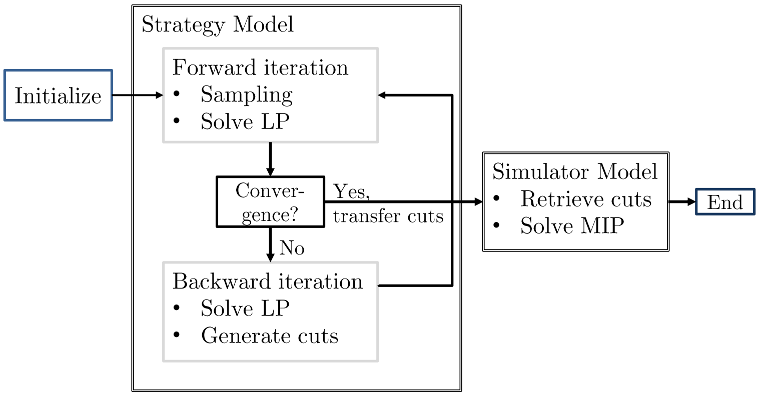


Figure 3.2: Overview of the model structure. The strategy model refers to the SDDP model that was developed. The results from the strategy model were then validated by the simulator model.

model that is currently used by many hydropower producers in the Nordics [81]. The overall aim of the research project, of which this dissertation forms a part of, was to investigate methods to include reserve capacity markets in that model [10]. Since the model is based on LP it was interesting to investigate how one could approximate minimum generation limits, start-up costs and other nonconvexities. In order to benchmark the results a simulator model was developed.

3.1.1 Article I: SDDP and Simulator Methodology

In the first article, a more detailed description of the SDDP and Simulator model was given. An illustration of the information flow between the models is given in Figure 3.2. The EFP function that was generated by the SDDP model, referred to as the strategy model, was used in the simulator model that had minimum generation limits, binary start-up costs, binary hydraulic coupling and a nonconcave generation function, dependent only on discharge.

The stagewise problem for the simulator model had approximately 2 and 4 times as many variables and constraints, respectively, indicating the rate at which the problem grows in size for more complex modeling. Both models had 21 time periods within each weekly stage, i.e. three periods per day. This appears as a natural choice since the included reserve capacity market described the weekly primary reserve market.

In order to ensure an even comparison between the results the simulator model used the same scenarios as the final iteration in the strategy model. The results showed good convergence properties of the strategy model after only a couple of iterations. Moreover, the computed lower bound from the simulator model was also close to that of the strategy model. This indicated that the nonconvexities included did not alter the policy to any undue extent. On the other hand, the scheduled generation achieved more realistic results and a significant drop in income from providing reserve capacity was observed, at around 30%. As there was no firm lower generation limit on the hydropower stations in the strategy model it operated at low power outputs during periods where the energy price was low but the reserve capacity price was good.

3.1.2 Article II: Validation of Results

A more in-depth investigation of the results from the same model was performed in the second article. The two models had one inconsistency, and that was a constraint in the strategy model that aimed at limiting the amount of reserve capacity the model could provide for operation at low capacity, as described in [37]. The constraint, which was not included in Article II, was given as

$$c_t \leq \frac{c^{\max}}{P^{\min} + c^{\max}} p_t, \tag{3.1}$$

where c_t and p_t was the reserve capacity and generation capacity, respectively. $P^{\min}$ was the minimum generation of the power station and $c^{\max}$ was the maximum reserve capacity the station could provide. It should be observed that for the station to deliver the maximum available reserve capacity the generation capacity had to be greater or equal than $P^{\min} + c^{\max}$, thus limiting the available reserve capacity for lower power outputs. This led to an even larger gap in revenue from reserve capacity from the strategy to the simulator model of approximately 40%.

Another compelling differentiation between the Strategy and Simulator models was the generation schedule they provided. The power output of one of the power stations, for a period of 10 weeks, is shown in Figure 3.3. The power station is connected to two different reservoirs with a DOR of 0.8 and 0.05, and with an almost equal amount of inflow in both. A good dispatch strategy is therefore needed to avoid spilling of the smallest reservoir. Due to the different heads at the two reservoirs, the power station cannot generate from both of them at the same time. As seen from the figure, the strategy model is not able to include this, as it dispatches some from both reservoirs at the same time. In the simulator model, however, this is included and one can clearly see that the model switches its generation between the reservoirs.

The study of the marginal costs for providing reserve capacity was also investigated, see Figure 3.4. One could observe a clear differentiation of the marginal

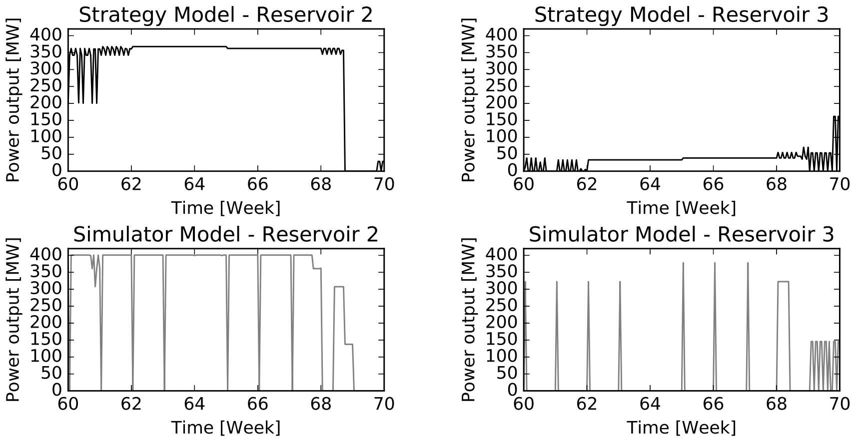


Figure 3.3: Generation capacity/power output for a power station that is connected to two reservoirs. The columns refer to the two different reservoirs. The top row is the result for a scenario in the strategy model and the bottom row is for the same scenario in the simulator model.

costs due to the different sizes of the reservoirs and installed capacity. For example, Reservoir 1, with small volume and low installed capacity of the connected power station, had a high marginal cost for providing more capacity when the spring flood occurred, around week 20. Since the station was generating at maximum capacity to avoid spillage there was no capacity left to provide reserve capacity. On the other hand, if the reservoir size and installed capacity is large, as with Reservoir 2, it can be observed that for the same period the marginal cost was very negative, indicating that the profits would be lower if more power was needed to be generated to deliver reserve capacity. This comes as a result of the low energy price in these periods and that the model wanted to store water for use in the winter when the energy price was higher.

Lastly, it was shown that the overall profits were increased by 2.56% for the Simulator model when comparing a simulation with both sales of energy and reserve capacity to sales of energy only. This number could be seen as a theoretical upper bound for the given price series as it can assumed that the reserve capacity market was modeled as an exogenous price, thus not taking into account the current low liquidity in the reserve market. Nevertheless, the study shows that there is potential for additional profits when participating in different power markets.

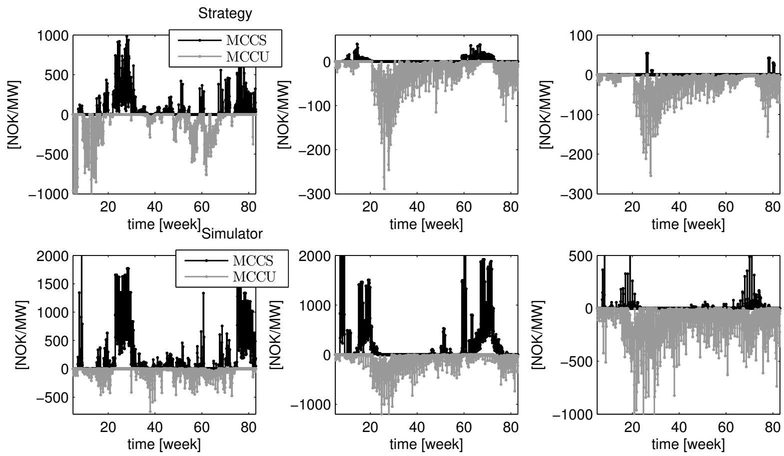


Figure 3.4: Marginal cost and simulated marginal cost for reserve capacity for the Strategy (top) and Simulator (bottom) model, respectively. The simulated marginal costs are computed first by solving the Simulator model, then fixing the binary variables and solving the resulting LP problem. Reservoir 1, 2 and 3 are given from left to right. The Marginal Cost of Capacity Spinning (MCCS) and Marginal Cost of Capacity Up (MCCU) refer to the dual values from the two constraints limiting the provision of reserve capacity.

3.2 Provision of inertia

As discussed in Section 2.3, another commodity that a hydropower producer can deliver to the power system is inertia. When the hydropower station is operating it provides inertia to the system. Given the proper equipment, the hydropower station might also operate as a synchronous condenser without having to discharge water to provide inertia. Given large-scale integration of renewable energies and HVDC cables, the TSO might be forced to keep more synchronous generators running to ensure sufficient inertia in the system.

3.2.1 Article III

How a market for inertia would influence a hydropower producer was studied in the thesis's third article [15]. A thorough discussion about the subject was presented. Three measures for ensuring sufficient system inertia were discussed:

Reducing dimensioning fault. This would, in practice, mean to split up the largest active power unit into smaller units to reduce the risk of losing a large unit in the system. Not very applicable in practice.

Existing market solutions. Since the primary and secondary reserve market require the BSPs to be rotating, they indirectly motivate inertia. This would, however, give both small and large units the same incentive for operating. Tertiary reserves, on the other hand, could be used to regulate up large units, while regulating down smaller units. If current market solutions were to be used, this could be an effective way to ensure sufficient inertia for certain periods.

New market designs. If required, the TSO could set up an additional market for providing inertia, i.e. after the day-ahead market is cleared an analysis could be conducted to determine any potential inertia requirements. A discussion of whether all units or just additional units providing inertia should be remunerated is also provided.

Regardless of the approach to ensuring sufficient inertia by the TSO, it is imperative that the exploitation of market power is mitigated. This could involve holding back generation in the day-ahead market due to expected higher profits by being active in the inertia market. By limiting the available technologies which participate, i.e. only flywheels or hydropower plants operating in synchronous condenser mode, the issue of exploiting market power could be diminished, as the profits from participating in the day-ahead market are assumed to be dominating.

It was further studied how a market for providing inertia would affect the MTHS problem, while considering participation in the day-ahead and a primary reserve market. The background for the study was that it could be expected that a shorter period in the summer that would require additional inertia in the system. In order to estimate a price for supplying the inertia the price was set so that if the hydropower producer was active in the period when the inertia market was open they would recover the investment cost. A case where the hydropower producer could co-optimize/speculate all markets (run 1) and where they could not co-optimize/speculate in the inertia market (run 2) was performed. Run 2 thus considered that the hydropower producer scheduled in the two markets up-front and whatever capacity was available for the inertia market could be sold.

The results showed that run 1 achieved sufficient income from the inertia market to recover its investment costs, whereas run 2 did not. This indicates that a price for inertia of approx. 10.5 NOK/MWs/h is was not sufficient for providing investment incentives for the hydropower producer. The results should be seen in context with the modeling framework, since a STHS model could provide more operational details and thus more robust conclusions could be drawn. Nevertheless, the study highlighted some of the important aspects that should be considered in the multi-market setting for a hydropower producer.

3.3 Future Power System with High Shares of Intermittent Wind Energy

As the introduction of this thesis and that others have expressed, it is expected that the amount of variable renewable energy in the generation mix will only continue to grow. Given such a system, would it be economically reasonable for wind turbines to provide reserve capacity[1], or would flexible hydropower provide this service? Another interesting question is, what would be the expected price for providing this service?

3.3.1 Article IV: Provision of Rotating Reserves from Wind Power in a Hydro-Dominated Power System

The abovementioned research questions were investigated in the fourth article of the thesis. A power system that resembled the Nordic was constructed and modeled with no internal grid limitations. A connection to the continental European power system was given by an exogenous power price. The reserve capacity market resembled the market for secondary reserves, consisting of two products; upwards and downwards reserve capacity. The case study consisted of two main cases, one where only hydropower could provide reserve capacity and the other where wind turbines could also provide it. For a wind turbine to provide upwards reserve capacity the possible generation has to be reduced in such a way that if the reserve is activated the wind turbine could increase power output. The problem was modeled with SDDP to solve the problem with the uncertainty of inflow and winds, on a weekly basis.

The work from [82] was used to estimate the required amount of reserve capacity for different wind penetrations. It was assumed that the Nordic system would require less reserve capacity than the UK system but somewhat more than what was found for the Swedish system. It is trivial that the amount of reserve requirements is not linearly dependent on the wind penetration, due to grid integration between areas, geographical distribution of wind turbines and improved forecasting techniques. Further discussion on large-scale wind integration was thoroughly performed in [83].

The results showed that after a wind penetration of 20% the average price for providing upwards reserve capacity significantly increased, as seen in Figure 3.5. This was seen in context with increased energy and reserve capacity requirements in the system, in such a way that the hydropower units required a higher price for providing upwards regulation as it would limit discharge and subsequently increase the risk of spillage during the spring flood, and that it also limited the amount of

[1] referred to as rotating reserves in the article.

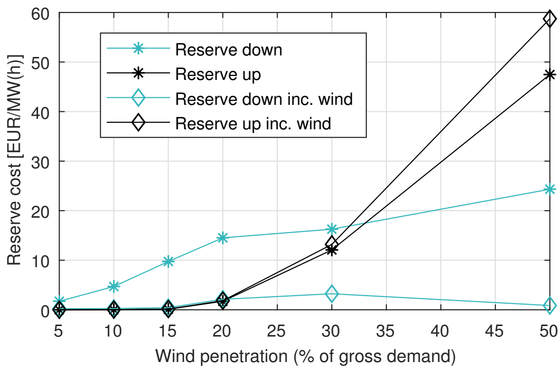


Figure 3.5: The black and blue lines indicate the expected cost of providing downwards and upwards reserve capacity, respectively. The results are from two case studies, where wind turbines also could contribute.

energy that could be sold during the highest paying hours in the winter. Another interesting result occurred when wind turbines could provide reserve capacity, where the price for upwards reserve capacity increases for wind penetrations of 30% and above, as the average price is higher than when only hydropower could deliver this service. Even though this seems counter-intuitive since the system is more flexible, the overall gain is higher as hydropower can generate more during high price hours and wind power can alleviate the reserve capacity requirements. Moreover, it was found that wind turbines could deliver downward reserve capacity at a cheap cost, freeing up some of the hydropower units so they are not required to be generating at periods with low energy price just to deliver reserve capacity.

It should be stated that the study has several limitations and may require potential improvements for future work. The time resolution is coarse, in such a way that the short-term uncertainty of wind power is not sufficiently included. Therefore, the inclusion of a short-term model for better describing the problem within the week and conducting a more detailed study could provide more rigorous results. Internal grid congestion is also likely to have an impact on the results. The problem is so complex and compelling that a research project has been started with the title *Pricing Balancing Services in the Future Nordic Power Market* [84].

3.4 Generation Function Modeling

The generation function describes the efficiency of the outputted power from a hydropower station. In MTHS and LTHS it is often assumed that it is a concave function, only dependent on the water discharge in the station. For more detailed STHS models the generation function can be given for each unit in the hydropower station and with head dependency. A research question for the following work was to find a method for how to describe a nonconcave generation function, given some measurement data. Due to advancements in modeling details, as discussed in the next section, the results from the work are further used for modeling a nonconvex MTHS problem.

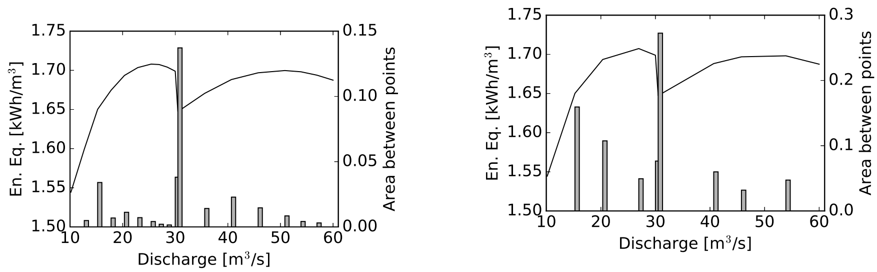
At the beginning of this work a hydropower producer in Norway that had an ongoing project of online measurements of the power station was contacted [85]. The goal was to obtain data from online measurements on the power output, water head and discharge for a power station and subsequently generate a generation function that could further be used in a HS model. The complicating aspect of performing these measurements is how to accurately measure the water discharge, which is one of the main topics in [85]. Such an online measurement system of the power station could provide an up-to-date generation function, as the efficiency tends to decrease over time, real-time surveillance with fault detection and as an initial control that the turbine and generator efficiencies are in line with the manufacturer's specifications. Currently, hydropower producers perform an initial measurement of the power station to determine the station's overall efficiency and use this data as input for the generation function. The work in this section comes from these measurements from two different hydropower stations where head dependencies have been neglected.

3.4.1 Article V: Impact of Modelling Details on the Generation Function for a Norwegian Hydropower Producer

As discussed earlier, more profit opportunities in additional markets than the day-ahead energy market may lead to different generation scheduling. In recent years, there has also been an increased focus on environmental issues associated with hydropower. This has led to more environmental constraints for hydropower systems, e.g. ramping restrictions and minimum discharge in rivers to facilitate a strong stock of fish in the rivers. This work therefore also considered a case with and without a minimum discharge limit in the summer months to comply with these restrictions. The environmental aspect of hydropower scheduling has been studied in detail in the research project CEDREN [86].

The main contribution from the paper comes from finding a method to reduce the dimensionality of the generation function, with the trade-off between the representation of the generation function and reduced computational time. Figure 3.6(a) shows the generation function with all data points. After removing eight data points the generation function is shown in Figure 3.6(b).

The work proposed two procedures for reducing the dimension of the data points



(a) Original data points.

(b) Removed 8 data points using the area removal algorithm.

Figure 3.6: Generation function for a power station with two units. Note that the y-axis is not given by power but the energy equivalent to normalize the plot for better visualization.

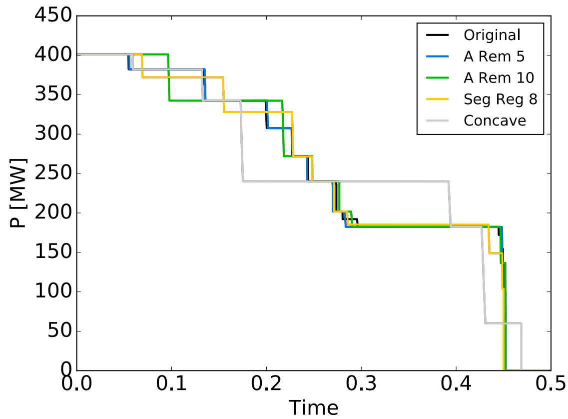


Figure 3.7: Duration curve for a selection of the results. (A Rem) refers to the area removal algorithm and how many points on the generation function were removed from the original one. (Seg Reg 8) refers to the segmented regression approach where eight points were removed. The grey line refers to the concave generation function.

in the generation function; the *area removal algorithm* and a segmented regression method. The goal was to find a method that kept the inherent shape of the generation function while reducing the data dimension. The first method computed the area the different points had with two adjacent points and removed the point that had the least area associated with it. The procedure persisted until a given amount of points was removed. The segmented regression approach used a library implemented in R to find linear regression lines that most accurately fit the data.

The results showed, first of all, that a significant amount of data points could be removed by the area removal algorithm with little impact on the generation function, compared to the original data. It also had the benefit of notably reduced computation time. A comparison to a concave representation of the generation function was performed, where the computational time was considerately lower, ranging from about half to 10 times lower. As expected, the objective value was also just over 2% better, for both stations, compared to the original generation function. This difference increased even more when a limit on minimum discharge was included in the summer months, where the concave generation function had a 14.1% higher expected generation compared to the original.

Given the nature of the data that described the generation function, it was also found that the segmented regression method did not perform as well. The operating point that had the highest efficiency was slightly shifted compared to the original data and different generation schedules were observed. It is believed that the method would perform considerably more effectively had there been more data available.

The given work consisted of a compelling topic for hydropower producers and much work could be done in order to more accurately describe the generation function and how it should be incorporated in HS models. This article has only scratched the surface of methods that could be used. The usage of a nonconcave generation function becomes more clear in the next section.

3.5 Nonconvex Medium-Term Hydropower Scheduling

A significant contribution from this thesis is the work on nonconvex[2] MTHS, where the SDDP algorithm has been the main focus due to its applicability for solving Multistage Stochastic Programming (MSSP) problems. In the early work of this thesis, models to test how well SDDP performed in a multi-market setting was created. It was found that the models overestimated the profits from selling reserve capacity, even though they contributed to a fraction of the overall profit. With increasing volume and, potentially, price in these markets, it can be expected that their shares of the profits will increase. In such a future market setting, it is therefore imperative that good decision-support tools are available such that the hydropower producer can allocate its resources in an optimal way. Improved modeling of the hydropower system would also contribute to more realistic results that are currently left for the STHS problem to handle.

The complicating aspect for nonconvex SDDP problems is how to model the EFP function. There have been several publications focusing on solving nonconvex MTHS problems, and until recently none of them could guarantee that the method

[2]The terms nonconvex and nonconcave are used interchangeably, both referring to a problem that cannot directly be described by a piecewise linear function, as a concave or convex problem can.

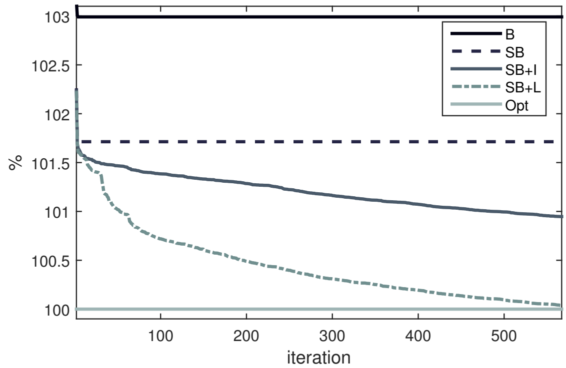


Figure 3.8: An illustration of the upper bound for a validation study of the SDDiP method. The optimal value is given as a base.

actually converged to an optimal solution [87, 88, 89]. In the following section, the results from Article VI and VII are presented. The work is based on the recently published SDDiP method, discussed in Section 2.9.

3.5.1 Article VI: Nonconvex Medium-Term Hydropower Scheduling by Stochastic Dual Dynamic Integer Programming

This article was written during the author's research visit with Dr. Shabbir Ahmed at the Georgia Institute of Technology. The visit was conducted in the wake of the SDDiP publication, where the aim was to assess the method for an MTHS problem with sales of reserve capacity.

A drawback with the SDDP method is that it did not easily incorporate stochastic objective term coefficients, e.g. energy price. In an effort to test whether this could be implemented in a satisfactory manner, the case study that was tested included some cases with an uncertain energy price. In order to circumvent the bilinear/nonconvex issue of a stochastic variable of price times generation in the objective term, McCormick envelopes were used [90]. Moreover, other nonconvexities including a nonconcave generation function, minimum generation limit and other hydraulic constraints were modeled.

To validate the convergence of the different cut families a validation study was performed. The convergence for the different cuts can be seen in Figure 3.8. The study consisted of a simplified MTHS problem that enabled us to compute the optimal value prior to using SDDiP. The illustration shows the gap provided by Benders cuts and the improvement provided by strengthened Benders cuts. To obtain convergence of the SDDiP algorithm Lagrangian cuts had to be used.

The study showed that SDDiP is more computationally demanding than SDDP. The Lagrangian cuts that are required for guaranteeing convergence require significant computational force to generate a cut. For comparison it could be required to solve around 50 MILP problems, compared to a single LP problem, to generate a Benders cut. The strengthened Benders cuts did, however, provide promising results in terms of computation time and improved convergence.

The case with uncertain energy price led to very large upper bounds, but by using Jensen's inequality it was found that the provided policy was adequate. The proposed method to include uncertain energy price could not be deemed very practical in practice as it requires solving an additional problem to verify the solution.

3.5.2 Article VII: Medium-Term Hydropower Scheduling with Variable Head under Inflow, Spot and Reserve Capacity Price Uncertainty

In the thesis's final article an aim was to assess how the strengthened Benders cuts from the SDDiP framework would perform with even more complexity than what was included in Article VI. This included modeling head dependencies in a nonconvex generation function. The modeling of the energy price in Article V was considered a drawback, that used the McCormick envelopes to relax the bilinear objective terms. The final article therefore included a recent method to incorporate uncertain objective term coefficients for DP problems [79, 80]. The SDDP model used in the earlier articles was extended with strengthened Benders cuts and used to perform the case studies.

Another contribution from the article was to propose a method to visualize the EFP function. This enables an inspection of the shape of the EFP function and insight to the modeler as to whether a piecewise-linear approximation is sufficient or more complex methods has to be performed for adequate results.

The case study was performed on the system given in Figure 3.1. The uncertain parameters were inflow, energy and reserve capacity price. Studies with only uncertainty of inflow were also conducted to compare how well the approach from [80] performed. The inflow to the system was described by historical data, the energy price from a fundamental market model and the reserve capacity price was described by historical data from the daily primary reserve market.

To benchmark the performance of the strengthened Benders cuts, some studies that made use of Benders cuts were also performed. Convergence of the problem for the two different types of cuts and with uncertainty of inflow, energy and reserve capacity price can be seen in Figure 3.9. A total of 50 iterations were performed with a final simulation to evaluate the policy from both approaches. It can

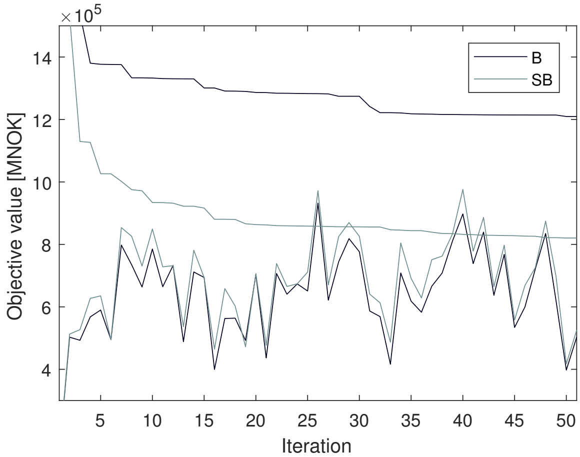


Figure 3.9: Convergence when using either strengthened Benders (SB) or Benders (B) cuts.

observed that even though the strengthened Benders cuts provided a significantly tighter upper bound, the gap between the upper and lower bound is still significant; approximately 36%. The approach would benefit from increasing the number of iterations, with the cost of added computational time. Verification studies that were performed for the implementation of the uncertainty modeling did provide convergence, but at a significantly slower rate than without the uncertain objective term coefficients.

One of the findings from the article was that when more complex nonconvexities were modeled, such as a nonconvex generation function, Benders cuts were inferior to the strengthened Benders cuts. This is expressed in the way the reservoir handling is performed, where the approach with Benders cuts resulted in very high reservoir levels as the model overestimated the value of the water in the future, leading to a higher risk of spillage. This phenomenon can be observed in Figure 3.10, where the water values for week 17[3] are shown. The water values are given by the coefficients of the binding cuts, thus representing the marginal costs of water for different reservoir volumes. From Figure 3.10 it can can be observed that the Benders cuts generally provide higher water values, but a significant drop is observed for high fillings as high inflow may lead to floods and subsequently the water value would be zero. The strengthened Benders cuts provide lower water values and consequently lower reservoir volumes.

To summarize, the article showed how superior strengthened Benders cuts are

[3] Around the time when the spring flood occurs.

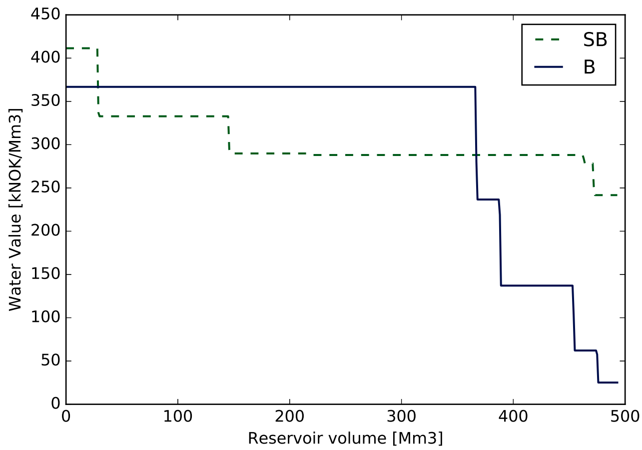


Figure 3.10: Water values for week 17.

compared to Benders when solving nonconvex problems. A drawback is the increased computational time required to solve the problem. With appropriate parallelization of the model this drawback can be diminished. Furthermore, the uncertainty modeling provided a unified stochastic model that can handle correlations and time lags in a more tractable way for SDDP problems, with the downside of reduced convergence rate.

3.6 Putting the Research in Context

There is a great deal of ongoing research on MSSP, both theoretical work and applied, such as the work carried out in this thesis. Alongside the proposed SDDiP algorithm the research in [87, 88, 89, 91, 92] all investigates nonconvex MSSP problems.

[87], which used McCormick envelopes [90] to approximate the nonconvex relations between head, discharge and power output. In Article VII of this thesis, another method to model this function was proposed where the function was divided into concave regions, similar to using McCormick envelopes, and a set of hyperplanes was used to describe the function. The idea of using hyperplanes to describe the generation function is also proposed in [93], where a concave function of hyperplanes was fitted to the generation function.

[88] investigated both convexification of the hydropower systems' properties and a method with Lagrangian relaxation on the EFP function, for approximation. The work concluded that the selection of the convexification approach is highly de-

pendent on the given system and that choosing the correct approach can prove challenging task. It is with this in mind that the approach for visualization of the EFP function in Article VII was presented, yielding an insight into how it is affected by different nonconvexities.

Another model using Lagrangian relaxation was proposed in [89]. The work also proposed a method to use locally valid cuts, i.e. dividing the EFP function into convex regions, where the model performed well compared to a SDP model used as a benchmark. The most recent work, built on the SDDP framework, is the SDDiP method proposed in [70], more on this in the next section.

A method that uses step-functions instead of hyperplanes to model the EFP function is given in [91]. This is a novel approach that would be very interesting to pursue, but has not been undertaken by this thesis.

As already specified, there is also a great deal of ongoing research on multi-market scheduling. The previously mentioned work in [41] is one example of extensive research on the topic. A key contribution was to apply the method of multi-horizon modeling for a MTHS problem. The underlying concept of the method is to divide the problem into two stages with different temporal composition, e.g. investment and operational stages. This thesis has primarily based its uncertainty modeling on the conventional approach of using a VAR-1 process to model the inflow and a Markov chain for the energy price. Extensions were, however, included in Article VI and VII with two different methods for modeling the uncertainty of energy price with use of McCormick envelopes and with use of the recent work by [80]. Both methods are able to include correlations between the stochastic processes and are unambiguous in extending its dimension to higher orders. The complicating task is to construct these stochastic processes, which has not been investigated in greater detail but rather left for future work.

Looking at more work conducted on MTHS in the multi-market setting the research by [37, 9, 94] requires acknowledgment. [37] extended the Markov chain to include a primary reserve market in addition to the energy market. Moreover, the primary market was cleared the week prior to delivery, resembling how the weekly market for primary reserves is cleared today. The work showed how computationally demanding the problem becomes when extending the dimension of the Markov chain. Which is why other methods were pursued in Article VI and VII.

In [9] an SDP model was used to model the small nonconvex MTHS problem. SDP requires a small state dimension to be tractable with the benefit of being able to solve nonconvex problems. As with the work in [94], the work aimed at understanding how the water values change in a multi-market setting. This work shows how the water values become more sensitive to changing reservoir levels and states the importance of taking reserve capacity markets into consideration when refurbishing old power plants. The work gives a great deal of insight into how the multi-market setting affects the hydropower scheduling, which is what this thesis also aims to achieve.

Chapter 4

Conclusions

> *We still expect increased imbalances that the system operator must handle close to and in real time through the use of balancing services for up- and down-regulation of generation or consumption.*
>
> —Statnett, *System operations and market development plan 2017-2021*

The following chapter summarizes the conclusions drawn from the work in this thesis.

4.1 Main Results

Article I and II outlined how existing MTHS models could be used to include sales of reserve capacity and how viable the results were. It was found that the existing models significantly overestimated the hydropower system's flexibility to provide reserve capacity and that more detailed modeling is desired. Moreover, it was found that even though the provision of reserve capacity was overestimated it still amounted to a fraction of the overall profit compared to selling energy. The expectation is, however, that there will be an increased demand for flexibility in the future in such a way that models that include more complexity will have a more prominent value.

A commodity that can provide additional profits for hydropower producers in the future is the provision of inertia, which was discussed in Article III. Much is still uncertain on how scarcity of inertia in the power system should be handled to ensure a reliable power grid. It is especially during periods with low demand, high intermittent renewable generation and import over HVDC cables that can come below a certain security threshold. Since the electricity prices are typically low in these periods, flexible hydropower producers with the proper equipment can provide inertia without using any water. The article discussed these issues and concluded that such a market would have a small impact on the MTHS with a short period of critically low inertia.

In Article IV the provision of reserve capacity from wind and hydropower was investigated. It was found that wind power can provide downward reserve capacity at a low cost. The activation of the energy would, however, be expensive as the opportunity cost of wind power is zero. It was only for the most extreme wind penetration scenarios, 30% and 50%, that there was observed some provision of upwards reserve capacity from wind power, the rest was provided by the hydropower. This can be seen as an indicator of how effectively wind power can be facilitated in a power system with a larger share of hydropower.

Extending on the work carried out in the previous articles the method of SDDiP was tested on a MTHS problem in Article VI. It was concluded that the method was significantly more computationally demanding than the standard SDDP method, but it was able to better handle nonconvexities such as minimum generation limits and start-up costs. A validation study of the MTHS problem showed that the method provided convergence when the Lagrangian cuts were used, but proved too computationally demanding for the full problem formulation. Nonetheless, the strengthened Benders cuts provided a significantly tighter optimality gap compared to the classical Benders cuts.

In order to improve the representation of the generation function in each power station Article VI proposed two methods. It was concluded that representation of the function could be reduced without compromising the accuracy of the results and with favorable reduction of computational time. Moreover, the results from the two methods were dependent on the underlying data used to describe the generation function, and if more data had been available the accuracy of the results would be more reliable. For instance, it was found that slightly shifting the operating point with the best efficiency had great impact on how the station was operated.

The favorable aspects with the strengthened Benders cuts used for MTHS were further supported in the seventh and final article, where the results were superior compared to that which the Benders cuts could provide. The method to visualize the EFP function showed that the nonconvexities of the problem were not too profound, in such a way that a piecewise linear approximation would be sufficient. Furthermore, the modeling of the price uncertainty gave a slower convergence and more iterations would be required to reduce the optimality gap even further.

4.1.1 Concluding Remarks

This thesis has investigated the impacts of additional markets and more detailed modeling has been performed on the MTHS problem. Current approaches based on a convex problem formulation, such as SDDP, are not able to incorporate the level of detail required to describe the hydropower system with sales of reserve capacity and environmental constraints. The proposed methods based on SDDiP are able to do this, but with the cost of significantly increasing the computational burden. The strengthened Benders cuts from the SDDiP framework should, however, be pointed out as a very promising tool for solving nonconvex MTHS problems. The main conclusions from this thesis can thus be summarized by the following.

- A MTHS model based on SDDP will overestimate the hydropower system's opportunity to sell reserve capacity. For the case studies performed in this thesis the profit was overestimated with around 30%.
- A market for providing inertia to the power system may be a solution for ensuring reliability of the power grid, especially during low load and price hours. The provided case study showed that a inertia market had little impact on the HS, mainly due to the few hours it was open and that the price that was used was too low. A higher price could lead to the hydropower producer to switch operation from generating energy to provide inertia in synchronous condenser mode, making it a very inefficient market. This could be mitigated by choosing wisely the operating hours for the inertia market or apply other mechanisms, such as a fixed premium and/or investment support.
- The cost of providing reserve capacity, for the case study representing the Nordic system, significantly increases for wind penetration levels above 20%.
- Wind power is able to provide downwards regulating reserve capacity at a cheap cost. The activation of the reserves will, however, come at a cost that can be represented by the value of the curtailed energy.
- How the generation function for a hydropower station is constructed has high impact on how the station is operated. It is especially evident to model the generation function accurately when environmental constraints and markets for reserve capacity are included, such that the generation capacity is not overestimated.
- The SDDiP algorithm enables a nonconvex problem formulation, enabling high levels of details for MTHS problems. Eventhough the SDDiP algorithm

is computationally demanding, the strengthened Benders cut is superior when solving nonconvex problems, compared to the Benders cut.

- Bilinear objective terms, as imposed by an uncertain energy price, are hard to solve by McCormick envelopes in the SDDiP framework.

- The shape of a hydropower reservoir's head function highly determines the shape of the EFP function. By visually inspecting the EFP function it is easier to decide which solution approach to use.

Furthermore, the methodology proposed by the SDDiP algorithm is also relevant to use as a benchmark when new convexification methods are developed in the SDDP framework. More research is, however, required for generalizing SDDiP for a variety of hydropower systems, reducing the computational time and adapting methods to the different problems, such as when is binaryzation needed, how accurate should it be, and so on.

As a final remark to put some perspective on the multi-market HS problem, the turnover from selling energy on Elspot, only in NO2 area, was approx 1.46 billion NOK in 2017. Comparing this to 100 million NOK in both the primary and secondary reserve market it is obvious that there is some way to go until the market for reserve capacity plays a greater role in the power market. Nonetheless, finding methods that better describe the underlying problem is imperative for generating good and useful results. For a hydropower producer with a yearly turnover of 1 000 MNOK, hydropower scheduling models that provide just a 1% increased utilization of the water would make a significant impact on the bottom line. For a nation where over 96% of the electricity comes from hydropower it is therefore imperative with efficient scheduling models.

4.2 Recommendations for Future Research

In this thesis, the MTHS problem has been investigated. It is therefore hard to make concise conclusions on the impacts additional markets have on the HS problem. It would, therefore, be beneficial to combine the MTHS with a detailed STHS model to better verify the impacts of multi-market modeling.

The time resolution of the problem in Article IV is also too coarse to capture the short-term variability of the wind power. The results would, therefore, be more viable if a stochastic short-term model had been included to describe this. Furthermore, for the results to be applicable and potentially provide price-forecasts for energy and reserve capacity, the power grid should be included. This could, in turn, be used as price input to a MTHS problem.

All the studies rely on data that should describe the underlying hydropower system in the best manner. Obtaining these data is not straightforward and, as seen in Article VI, the format of the data highly influences the results. It would, therefore, be beneficial to perform an in-depth data collection, such as online measurements used to generate the generation function.

The results have shown that it is possible to include a higher level of modeling details at the cost of a higher computational burden. For the models proposed in this thesis to be used for operational use, it would therefore almost certainly be required to include parallel processing to utilize computers with multiple cores and reduce the computation time.

In addition to the above-mentioned, further testing on other hydropower systems could be done to further strengthen the findings of this work. Case studies on revision planning could also be an interesting problem for investigation using the models in this thesis.

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Appendix A

Papers

A.1 EEM 2016

M. N. Hjelmeland, M. Korpås, and A. Helseth, "Combined SDDP and simulator model for hydropower scheduling with sales of capacity," in *2016 13th International Conference on the European Energy Market (EEM)*, pp. 1–5, June 2016. © 2016 IEEE

Combined SDDP and simulator model for hydropower scheduling with sales of capacity

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***Abstract*—In following work, we investigate the importance of detailed hydropower scheduling modelling when including sales of capacity, which adds complexity that is not easily incorporated in a Linear Programming (LP) problem. In the proposed approach, we use the profit-to-go function obtained from a Stochastic Dual Dynamic Programming (SDDP) scheduling-model in a Simulator Model, based on Mixed Integer Programming (MIP), and perform detailed simulations. The Simulator Model allows a more complex problem description, than by the LP formulation in the SDDP model. The Simulator Model may therefore be used to give an estimate of the LP approximation, which is used for providing the opportunity cost in short-term hydropower scheduling models or conceivably for making investment decisions.**

For the given case study, the expected profit from selling capacity was 29.2% higher than the linear SDDP Model to the Simulator Model. The overall profit loss was reduced by 0.93%, quantifying the overestimation of profit in the SDDP Model. This illustrates the importance of detailed modelling when considering sales of capacity.

***Index Terms*—Stochastic processes, Power system analysis computing, Optimal scheduling, Hydroelectric power generation**

I. Introduction

With the introduction of a liberalized power market, power producers have been more eager for robust and efficient optimization models for their power scheduling. In Norway the liberalization started in 1991 after the Energy act, stating that power producers were no longer obliged to supply electricity but instead could maximize their profits. In the following year a paper on the EMPS model (EFI's Multi-area Power-market Simulator), which had been developed and used in the Nordic regions since the 1970's, was published [1]. The model mimics the market clearing process and is frequently used by market players to provide price forecasts. A single producer's hydropower strategy need to consider relevant uncertainties, such as inflows and energy prices. The decision process consist of the choice to either store water today and produce tomorrow or vice-versa, i.e. a multi-stage stochastic problem.

In 1996, the European Union issued an internal electricity market directive (1996/92/EC) to gradually open the electricity markets for all member states. Alongside the political agenda to assemble a common European energy market there has been an even greater push towards a sustainable energy development. With the large-scale entry of non-dispatchable generation, strengthened and possible new market mechanisms are required to attain secure operation of the electricity grid. Due to the flexibility of hydropower, volumes in the balancing markets in Norway have been low, and therefore low incentives for hydropower producers to assess models with multiple market clearing. With an increasing share of renewables and stronger grid connection to other countries, this perception may alter.

Due to the complexity and large computation time detailed model imposes, a clear differentiation has been done for hydropower scheduling. Typically the scheduling comprises a long-, medium- and short-term model depending on the modelling objective [2]. The long-term model may represent an aggregated system model and is e.g. used for expansion planning, price forecasting, and system studies, preferably with large system boundaries [3]. The medium-term model is linked with the long-term model by price and individual water values are calculated for a more detailed part of the larger system. Detailed water values for a river-system may then be used as the opportunity cost as the end-of-horizon valuation of water when running the short-term scheduling, a detailed model often defined as a Mixed Integer Programming (MIP) problem [4–6].

Around 1990, two papers on the Stochastic Dual Dynamic Programming (SDDP) algorithm for multi-stage stochastic optimizations problems was presented in [7, 8]. In the years to come numerous alternations and improvements on the algorithm has been presented. Such as grid modelling in [9], convergence properties in [10] and bidding strategy for a large hydropower portfolio in [11]. Latter years developments include the relations of non-convexities. [12] was treated with use of McCormick envelopes to model the non-convex relationship between power, discharge and water head. Later, [13] used Lagrangian relaxation to assess two procedures on convexification of the cost-to-go function and the non-convex

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components. In [14] a SDDP algorithm with local cuts was proposed in order to capture the non-convexities resulting from a detailed system description and supply of spinning reserves.

The primary frequency reserve (PFR) market in Norway for providing weekly Frequency Containment Reserves - Normal (FCR-N) has especially shown an income potential for hydropower producers during the summer months. With high reservoir fillings, low prices and high inflow, hydropower producers are set to either produce at maximum to avoid spillage or not produce due to allegedly low power prices. The market for PFR in Norway displays many of the same characteristics to other markets and minor changes are need to represent these differences. Comprehensive market details can be found in [15].

In this paper a combined Stochastic Dynamic Programming (SDP)/SDDP method presented in [16] and extended to incorporate capacity reservation in [17], is applied together with a simulator. First, a strategy (represented by the profit-to-go function) is found from a updated version of the SDP/SDDP model described [17], hereby referred to as the Strategy Model. This model allows both delivery of energy to the day-ahead market and sales of capacity to the PFR market. Subsequently the obtained strategy is applied in the MIP-based Simulator Model over the two year period of analysis to find detailed schedules for hydropower stations considering operation in both markets. The Simulator Model has the capability to represent more accurately a non-convex generation-discharge relation, minimum power requirement and head dependencies, if desirable. These details are of particular importance when considering operation in a capacity reserve market, which requires the committed capacity to be spinning. The Simulator Model can therefore act as benchmark to quantify the approximation errors that the Strategy Model impose.

The purpose of the presented approach is to quantify the influence a capacity reserve market has on the medium-term hydropower scheduling, considering accurate modelling of the physical system.

In Section II the Strategy Model and the Simulator Model will be presented. Section III describes a case study on a Norwegian hydropower system followed by results and discussions. Conclusions from the research and case study are finally given in Section IV.

II. Methodology

An overview of the two models can be seen in Figure 1. Inflow series and power prices used in this model has weekly stochastic time stages. In order to scale the model further down, a weekly profile with resolution similar to the PFR market is added, represented by 21 time steps within the week.

It is assumed that the reader has previous knowledge about SDDP, such that details regarding the method are neglected. For interested readers the following articles are suggested [7, 8, 16, 17].

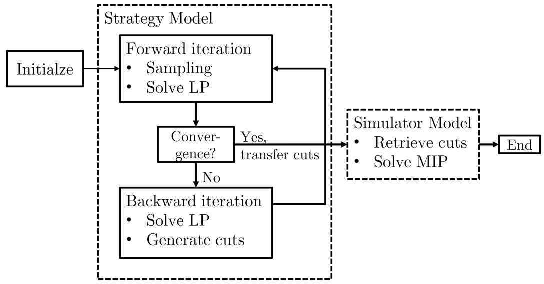

Figure 1. Simplified overview over the models. The Simulator Model is run after convergence is reached in the Strategy Model. Cuts refer to the hyperplanes that the profit-to-go function consists of.

A. Strategy Model

For each time stage, t, and realizations of stochastic variables, ω_t, the hydropower scheduling problem is solved, called the one-stage dispatch problem. The forward and backward iterations of the Strategy Model are carried out iteratively to generate an improved representation of the profit-to-go function until convergence. The LP problem representing the one-stage dispatch problem in the Strategy Model will be discussed in this section. Section II-B will cover the one-stage dispatch problem for the Simulator Model.

$$\alpha_t(\mathbf{x}_{t-1}, \omega_t) = \max \left\{ L_t(\mathbf{x}_t, \mathbf{u}_t, \mathbf{c}_t) + \alpha_{t+1}(\mathbf{x}_t, \omega_{t+1}) \right\} \quad (1)$$

subject to

$$\mathbf{A}\mathbf{x}_t + \mathbf{B}\mathbf{u}_t = \mathbf{C}\mathbf{x}_{t-1} + \mathbf{D}\mathbf{a}_t(\omega_t) \quad (2)$$
$$\mathbf{E}\mathbf{u}_t = \mathbf{d}_t \quad (3)$$
$$\mathbf{F}\mathbf{u}_t + \mathbf{c}_t \leq \mathbf{b}_t \quad (4)$$
$$\mathbf{u}^{\mathrm{lb}} \leq \mathbf{u}_t \leq \mathbf{u}^{\mathrm{ub}} \quad (5)$$
$$\mathbf{x}^{\mathrm{lb}} \leq \mathbf{x}_t \leq \mathbf{x}^{\mathrm{ub}} \quad (6)$$
$$\mathbf{c}^{\mathrm{lb}} \leq \mathbf{c}_t \leq \mathbf{c}^{\mathrm{ub}} \quad (7)$$

Problem (1)-(7) exhibit the main structure in the one-stage dispatch problem with state variables, $\mathbf{x}_t$, decision variables, $\mathbf{u}_t$, and realization of the stochastic parameter ω_t. $\mathbf{x}_t$ represents, in this model, the amount of water in a reservoir and the state of the power station, while $\mathbf{u}_t$ includes the water discharge, spillage and bypass. The function L_t in the objective function is made up by three major terms; income from selling energy and capacity, and a penalty function. The penalties includes spillage, tank water and start-up costs. Inflow to the reservoir is modelled with a first order autoregressive model, elaborated in [18], such that for extreme cases when the reservoir is empty you might also receive negative inflow. The model can therefore use the very costly tank water to fill up the reservoir. This ensures that relatively complete recourse of the stochastic model is obtained.

One of the characteristic with hydropower scheduling models is the relation between state variables from one time stage to another. This is expressed in constraint (2), where the reservoir balance is included. Matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ represent coupling between reservoirs and $\mathbf{E}$ consists of the efficiencies regarding production and discharge in the energy balance constraint (3). Linearized start-up costs, included in (2), were implemented in a similar manner as in [19,20], with a linear approximation of the power stations state (on/off).

The delivery of capacity is incorporated in (4), where $\mathbf{c}_t$ is the amount of capacity available for sale, hence also included in the objective function. The constraints includes a symmetrical delivery of capacity, such that for delivering a certain amount of capacity the power station must be able to regulate both up and down an equal amount. The matrix $\mathbf{F}$ represents the relations between power and discharge and $\mathbf{b}_t$ the power limits.

The delivery of PFR is limited by the turbine governor, such that both the droop setting and size of the power station limits the amount for providing PFR, included in (7). The other variables are bounded by (5) and (6).

Impacts of head variations impose non-convex properties that hydropower scheduling models should consider. Due to its complexity and minor significant impact for the proposed case study, this was currently not treated.

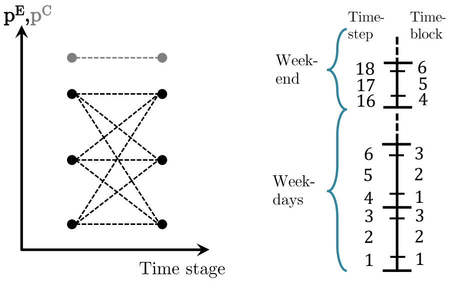

Figure 2. (Left) Illustration of the dynamic recursion between time stages for energy and capacity prices. For the energy price, different price nodes are used with probabilities from going to one price node to another in next time stage. The figure illustrates the underlying modelling of the prices and not the magnitudes. (Right) Illustration of the six different time-blocks for the weekly PFR market, i.e. the capacity for time-block 1 must be available for certain hours for all weekdays, i.e. time step 1,4,7,10 and 13.

1) Price models: In order to ensure a convex function in SDDP, the power price will have to be described in other terms than in the continuous state space. This is achieved by discretizing the power price and using a Markov chain to describe the transition for the power price between time stages. In short, the SDDP model is solved for each of the discretized power prices and transitions between them are used to calculate the expected profit-to-go function, illustrated by the transition between different price nodes from one time stage to another in Figure 2. The PFR price is also described by a deterministic price series in this work, but could be described stochastically as in [21], this would however significantly increase the problem size.

As seen in Figure 2 there are six different periods for providing PFR. If you commit a certain amount in period 1 you must provide the same amount every weekday.

TABLE I. Problem characteristics for case study.

	Strategy	Simulator
Variables	1 036	2 233 (585 binary)
Constraints	567	2 121
Dispatch problems, per iteration	293 600	5 200

For both markets it is assumed that the hydropower producer is a risk-neutral price-taker. With many actors and the large energy volumes traded on the day-ahead market this is a good assumption. However, the volumes for the PFR market are low, such that an improved representation could be beneficial for this market.

B. Simulator Model

The core distinctions of the Simulator Model is the MIP formulation of the one-stage dispatch problem. This enables a more complex modelling and subsequently more realistic results. The hydropower system for both models are the same, however, the Simulator Model is able to include binary state variables, non-convex power-discharge function and distinct operation from different reservoirs, such that a power station could not generate while discharging from multiple reservoirs.

During the research it was found that one of the main issues with a linear model when considering provision of PFR is that in time periods with low power prices and high PFR prices, the model tended to operate the power station at a minimum level. E.g. the power station was operated at 10 MW in order to provide 10 MW of PFR, even though the technical lower limit of the station was 50 MW. This weakness was significantly improved by reducing the solution space, such that when selling maximum amount of PFR, the power station had to operate above minimum generation limit. This issue was omitted by introducing binary variables in the Simulator Model. Together with the restriction to only generate from one reservoir at the time, this significantly improved the validity of the results given by the Simulator Model.

As Figure 1 illustrates, after the Strategy Model has converged and a solution is obtained, the profit-to-go function, represented by cuts, is utilized in the Simulator Model. Since the profit-to-go function was made for a different optimization problem, a distinct convergence gap is expected. This may be seen as the main drawback of the model, but it ensures a quick and efficient evaluation of the results obtained by the Strategy Model.

III. Case Study

The case study is based on a hydropower system under construction in south-west of Norway, shown in Figure 5. Power station 1 and 2 has installed capacity of 13.8 MW and 400 MW, respectively. The price series for capacity reserves are given by the actual prices from 2013 and 2014. A simulation with the EMPS model from 2013 gave the price series for energy, such that the capacity and energy prices were coupled in time.

In order to get a honest comparison between the two models, all Simulator Model runs utilized the scenarios for inflow and

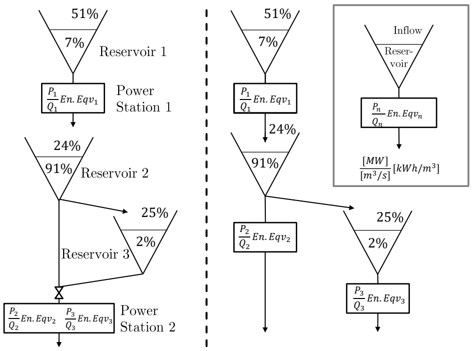

Figure 5. System used in the Simulator Model (left). Simplified system used in the Strategy Model (right). Size of reservoir and inflow given as percentage of the whole system. Power station 2 has two operating points in regard to which reservoir it is operating from. P_n, Q_n and $En.eqv_n$ refer to the power-discharge relations for the given power station and reservoir n.

spot price that were sampled in the last forward iteration by the Strategy Model.

The simulation was carried out with 50 scenarios in the forward iteration of the Strategy Model and hence the same amount in the Simulator Model. The sampling of 50 scenarios could favourably have been increased, but due to long computation time and promising in-sample stability this was not done. The power prices were discretized into 7 price nodes while the number of inflow samples in the backward iteration were 8. There were carried out 15 iterations of the Strategy Model and a final simulation with the Simulator Model.

The number of variables and constraints in the one-stage dispatch problem for the two models is displayed in Table I, together with the total number of problems. It can be seen that the Simulator Model has a significant increase in both variables and constraints for the problem described in the case study.

A. Results and Discussion

All simulations were carried out on a Dell Latitude E7240 with an Intel Core i7-4600U processor with a 2.7 GHz clock

TABLE II. Expected values for the Simulator Model, given in percentage with the Strategy Model as benchmark.

Generation	-1.0	Profit	-1.48
Energy Profit	-0.99	End Value of Water	3.26
Capacity Profit	-29.18	Total Profit	-0.93

rate and 16 GB RAM. The CPU time for the Strategy Model was 11 h and 4 min and 41 min for the Simulator Model.

Some selected results, in percentage with the Strategy Model as benchmark, are given in Table II. It is shown that the expected total profit, referring to the sum of profits and the end value of the water, was reduced by 0.93%. The profit from sales of capacity was reduced by 29.2%. This significant reduction from sales of capacity comes as a result of the tightened system description with minimum generation and operating reservoir constraints, and it shows how vital the system description is when considering provision of PFR.

Figure 3 shows the convergence of both models. It can be seen that there is some uncertainty concerning the gap in the Strategy Model, as a new sampling of inflow and power price is carried out for each iteration. As expected an convergence gap is observed in the Simulator Model, in percentage calculated to 0.68%.

The duration curve depicted in Figure 4 shows how generation, for the entire hydropower system, is shifted from less high power output to increased output at around 150 MW. This is first seen because of the binary constraints, but also as a result of the detailed power-discharge modelling, where the power station 2 has beneficial properties around this point. Also, due to the minimum generation limit in the Simulator Model, no generation beneath this limit is obtained, as opposed to the Strategy Model where this is observed with some power outputs of around 50 MW.

The proposed method does not improve the profit-to-go strategy used by short-term models, as in [12–14], but instead gives a rapid benchmark of how far away the linear approximation is from the true value. This can be very efficient for making investment decisions, where additional electricity

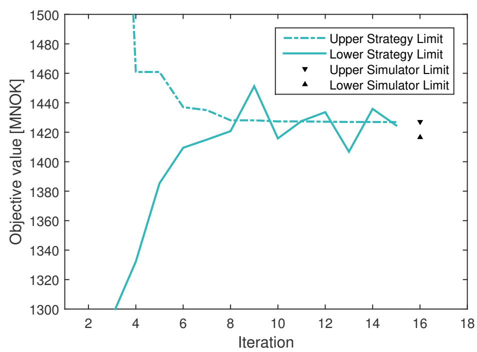

Figure 3. Convergence of both the Strategy and Simulator Model.

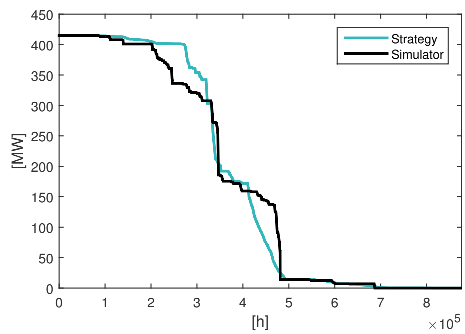

Figure 4. Duration curve from the Strategy and Simulator Model.

markets are included. An example could be to either invest in a Pelton turbine that provides good generation efficiency over a large amount of different discharge values, such that delivery of reserves are fairly cheap, or to invest in a Francis turbine with a higher maximum efficiency but with a higher cost of providing capacity reserves.

IV. Conclusion

A realistic approach for medium-term hydropower scheduling operating in both day-ahead and capacity reserve markets has been formulated and tested. The Simulator Model proposed has a detailed description of the power-discharge function and minimum generation for hydropower. When modelling the accurate power-discharge function, generation will give more realistic results regarding the unit's operating point. The minimum generation restriction also provides realistic results concerning how the hydropower units are operated. Following by the reduced flexibility in the system a profit reduction was observed, adjusted with the end value of water, by 0.93%.

By generating the profit-to-go function from the Strategy Model and applying it in the same system with above-mentioned modifications, a prompt solution is obtained, dependent on the system size and simulation period. It can be observed that the Strategy Model overestimated the value of selling capacity by around 29%. Even though this value is case-specific, it demonstrates the importance of detailed modelling when considering sales of capacity.

Further work should consist of generating an improved representation of the profit-to-go function that is to be used by the Simulator Model and hence reducing its convergence gap. Nevertheless, the finding from the case study with 0.68% gap demonstrate encouraging properties in terms of finding the optimal solution. Improvements of the representation of the capacity market should also be empathized in regard to validity of prices and conceivable volumes.

Nomenclature

ω_t Stochastic parameter for time stage t.

$\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{E}, \mathbf{F}$ System specific matrices. Representing coupling between reservoirs, relations between water discharge, energy and capacity.

$\mathbf{a}_t(\omega_t)$ Inflow as a function of a realization of the stochastic parameter ω_t.

$\mathbf{c}_t$ Capacity for sale.

$\mathbf{u}_t$ Decision variable. Representing discharge of water.

$\mathbf{x}^{\mathrm{lb}}, \mathbf{u}^{\mathrm{lb}}, \mathbf{c}^{\mathrm{lb}}$ Lower bounds for the variables.

$\mathbf{x}^{\mathrm{ub}}, \mathbf{u}^{\mathrm{ub}}, \mathbf{c}^{\mathrm{ub}}$ Upper bounds for the variables.

$\mathbf{x}_t$ State variable. Representing reservoir volume and state of a power station.

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A.2 Energy Procedia 2016

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A case study on medium-term hydropower scheduling with sales of capacity

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Abstract

This paper conducts a case study on hydropower scheduling considering sales of capacity reserves and impacts of detailed modelling. In latter years a growing demand for reserve capacity has been needed to ensure stable operation in the power grid which has given the power producers incentives to commence methods for co-optimization of energy and capacity. In this paper we will assess the value of providing primary reserves and how decisive accurate modelling is for sale of capacity. The results are based on a model consisting of a Strategy and a Simulator part. The Strategy Model is based on a combined Stochastic Dynamic Programming (SDP)/Stochastic Dual Dynamic Programming (SDDP) model where variables and functional relationships are linear. Subsequently the obtained profit-to-go function is used in the Simulator Model; a Mixed Integer Program (MIP) based simulator allowing a more detailed system description. The case study represents a Norwegian water course comprising of two minor and a large regulating reservoir. The Simulator Model manages to incorporate a binary unit commitment and to represent the non-convex relationship between power and discharge, giving more viable results and identifying dependencies between the reservoirs. As a result it was found that the expected profit from sales of capacity was reduced by 40 % when comparing results from the Simulator with the Strategy Model. Correspondingly, sales of capacity gave a clear shift of power outputs between the models as unrealistic results were eliminated in the Simulator Model.

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Keywords: Hydropower scheduling, capacity reserves, Primary Frequency Reserve (PFR) market, Stochastic Dual Dynamic Programming (SDDP), simulator

1. Introduction

For many years the Nordic power market has been isolated from the European continental power grid. The large share of flexible hydropower has secured a stable grid operation and incentives for providing reserve capacity has been scarce. Recent years grid connection to both Denmark and Netherlands has tightened the coupling between the

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systems, belittling the boundary between the hydro and thermal power system. Recent licence to build two new cables to UK and Germany will enhance this effect even further, utilizing synergies between the systems where flexibility and back-up capacity will play an even greater role as the share of intermittent energy resources continue to increase. For a hydropower producer this transition towards the new energy system will require improved scheduling models that can incorporate additional markets.

There has been conducted a large amount on studies regarding the amount of optimal capacity and energy reserves in order to fulfil the system operators requirements to obtain stable grid operation. The authors of [1] investigates the topic with regard to optimal allocation of spinning and non-spinning reserves in a system with a high share of wind on a short-term basis. Co-optimization of both energy, spinning and non spinning reserves was performed through a detailed market representation in [2] where a demand curve for the operating reserves was constructed. The proposed work outlined in this paper will focus on the optimal amount of capacity reserves in regard of a hydropower producer, rather than an optimal overall system optimization, where energy and reserve prices are given as an exogenous parameter.

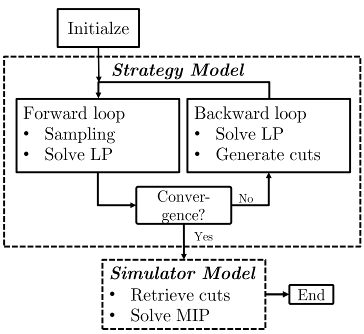
A SDDP model for long-term hydropower scheduling in a hydrothermal system considering sales of energy was proposed by the authors in [3]. Furthermore, a hybrid SDP/SDDP scheduling model was presented in [4] to include spot price uncertainty that has been further developed and in [5] to incorporate scheduling of both energy and Primary Frequency Reserves (PFR). The outlined method described a model that performed a simultaneous optimization, which is a simplification of the original market with sequential allocation. The common factor the above-mentioned papers shares is the requirement of a convex system description with linear state and decision variables.

The novel contribution from this work is to analyze and quantify the impacts detailed modelling imposes when including capacity sales. Long- and medium-term hydropower operational strategies are traditionally found using linear models, and drawbacks with the linear models should be estimated. A Simulator Model is used to evaluate the validity of a linear and convex Strategy Model. The model is performed on the hydro system Lysebotn in the south-western part of Norway. Firstly the weekly generation dispatch is analysed, followed by the simulated marginal cost for providing capacity and the amount of capacity reserves. Lastly, impacts on the duration curve over the period of analysis.

Section 2 contains a brief overview of the applied model and description of the analysed hydropower system. An extensive outline of graphical and quantitative simulation results are given in Section 3 followed by an conclusions in Section 4.

2. Methodology

A thorough outline of the mathematical description and model limitation can be found in [6]. The model has been slightly improved as a tighter bound on the minimum production level when supplying spinning reserve has been



Strategy Model (SDP/SDDP)	*Simulator Model (MIP)*
Objective: Profit maximization	
s.t.	s.t.
Reservoir balance	Reservoir balance
Energy balance	Energy balance
Spinning reserve	Spinning Reserve
Linear start-up cost	Binary start-up cost
Linear hydraulic coupling	Binary hydraulic coupling
Convex power-discharge function	Non-convex power-discharge function Minimim production level

Fig. 1: Overview of the Strategy and Simulator Model presented in this paper.

introduced in the Strategy Model. A narrative representation of the model and case system description is given in the following section.

2.1. Short Model Description

The Strategy consist of a hybrid SDP/SDDP model with incorporation of start-up cost and restrictions on available capacity reserves for sale. The Simulator Model is representing the same physical system and with the same time resolution as the Strategy Model, however with a MIP structure, enabling much higher details in regard to start-up cost, hydraulic connection, representation of the power-discharge function and minimum production level. The forward loop in the Strategy Model provides sampling of spot prices and inflow while the backward loop generates cuts, representing the future profit-to-go function, that is added to the one-stage dispatch problem. The one-stage dispatch problem represents the weekly scheduling problem as the uncertainty in inflow and power price is given with weekly resolution, the week is then divided into 21 time-blocks, all representing a time-block in the 6 different weekly PFR bidding intervals, c.f. [7]. To obtain an equal comparison as possible the exact same sampling of inflow and price scenarios performed by the Strategy Model is applied to the Simulator Model, such that the only difference in modelling is the one-stage dispatch problem. A flowchart of the fundamental model description and how the one-stage dispatch problem is built up can be seen in Figure 1.

The inflow model was based on a first-order autoregressive model, where the statistical properties were extracted from 70 years of weekly inflow data. Day-ahead prices came from a simulation with the fundamental market model Efi's Multi area Power market Simulator (EMPS) [8], while the PFR market price came from historical price data extracted from [9]. PFR market were represented by the historical prices from the years 2013 and 2014. In order to achieve a good comparison in prices as possible the day-ahead prices used came from a EMPS simulation in the start of 2013.

2.2. Case Study Description

A new power station, Lysebotn-2 c.f. Figure 2, is under construction in the hydro system replacing an old one. The input values to the model are hence based on expectations. A slight simplification is made by aggregating the reservoir

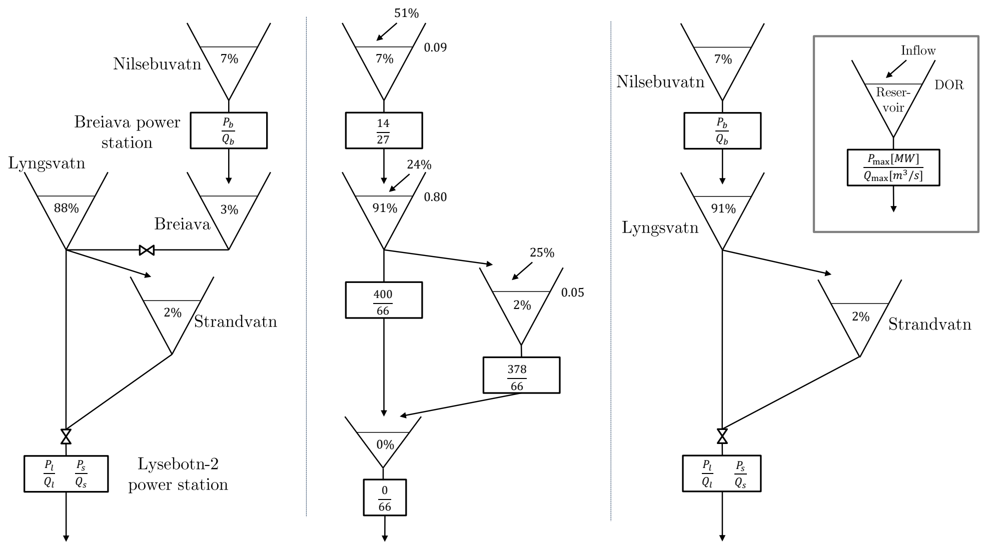


Fig. 2: Physical system (left), Strategy Model system (middle), Simulator Model system with figure explanations (right). Reservoir capacity and expected inflow is given as percentage of the total.

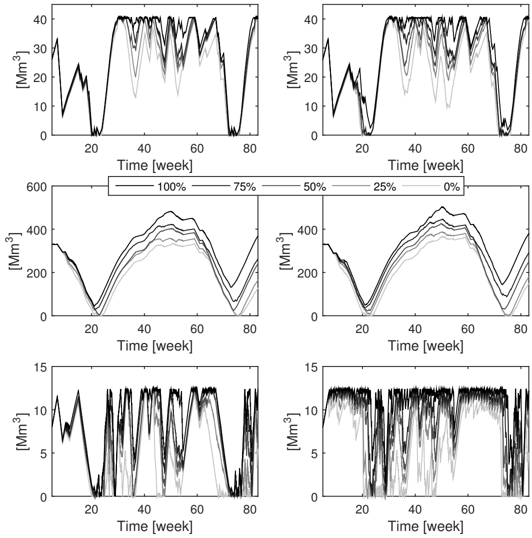


Fig. 3: (Left) Strategy Model. (Right) Simulator Model. Percentile plot of reservoir volume in all reservoirs. (Top) Nilsebutvatn, (middle) Lyngsvatn and (bottom) Strandvatn.

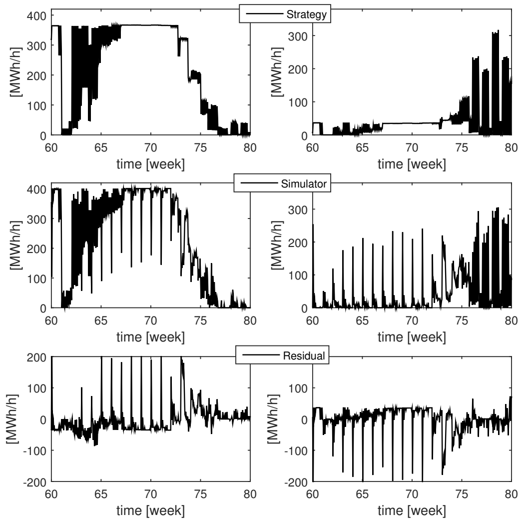


Fig. 4: Production from (left) Lyngsvatn and (right) Strandvatn around from week 60 to 80 for both Strategy Model (top), Simulator Model (middle) and the residual (bottom).

Lyngsvatn and Breiava, while for the Strategy Model a constraint on outflow for Lyngsvatn and Strandvatn is added as the power station, Lyngsvatn-2, can only operate from one reservoir at the time. In the Simulator Model, binary variables were added and hence a rigid unit commitment is obtained. In Figure 2 both expected inflow and reservoir values are given as percentage of the whole system. The Degree of Regulation (DOR) represents how the reservoir size is compared to its annual expected inflow. It can be seen that the upper and lower reservoir act as short-term storages, while Lyngsvatn acts as a regulating reservoir with long planning horizon.

Over the simulation period of two years 50 scenarios were sampled, in the backward iteration 8 inflow samples were applied and there were 7 different price nodes for the day-ahead price. For comparison there were done two simulations of the system; the single scenario where the model had only access to the day-ahead market and a dual scenario where the model had access to both day-ahead and the PFR market. The Strategy Model was not connected to a long-term model, resulting in some complications when calibration the end-of-horizon statement. The following results is therefore given for the first 78 weeks of the simulation, starting from week 5.

3. Results and Discussion

3.1. Dual Case Scenario

A percentile plot of the hydro reservoir handling for the dual scenario is given in Figure 3. It is clear that both Nilsebuvatn and Strandvatn has a much higher utilization of the reservoirs with rapid filling an emptying, whereas Lyngsvatn follows the characteristics of a regulating reservoir; storing water during low price periods and depletion during high price periods.

As seen from the illustration of percentile reservoir filling in Figure 3, the are hardly any noticeable differentiable patterns between the two models reservoir handling. However, there are two observable changes in regard to the production; firstly, due to the rigid production schedule from Lyngsvatn and Strandvatn a much more jagged reservoir filling pattern is observed in the Simulator Model for Strandvatn. Secondly, lower production in the Simulator Model

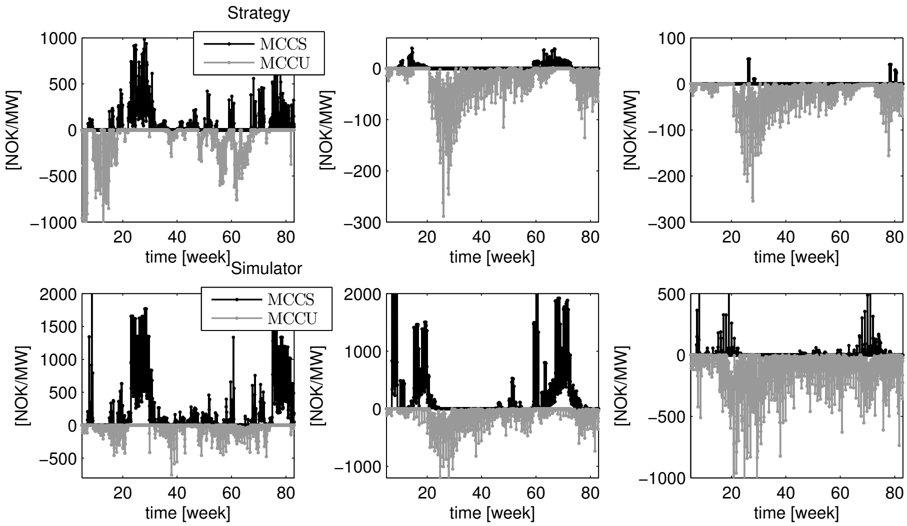


Fig. 5: Simulated marginal cost for capacity reserve for both Strategy Model (top) and Simulator Model (bottom). Nilsebuvatn (left), Lyngsvatn (middle) and Strandvatn (right).

results in an overall higher reservoir fillings as the gain from providing capacity reserves has been reduced due to the tightened system description.

A clear evidence for the above mentioned matter is illustrated in Figure 4, that shows the production from week 60 to 80. Since the Strategy Model could operate from both Lyngsvatn and Strandvatn it tended to operate at the best point of Lyngsvatn and with the rest of the available water capacity from Strandvatn. It could then sell energy from both reservoirs and still provide PFR. Similar the start-up cost is also held at a modest level. The production pattern for the Simulator Model differs significantly. As it is no longer possible to operate the power station from both reservoirs the model will now have to switch between them. In addition a constraint on minimum production level is added, ensuring proper utilization of the power aggregates. The Simulator Model therefore chooses to produce at higher power outputs for short time-periods. This will subsequently result in less capacity reserves available and increased start-up costs. This effect of increasing costs of providing capacity reserves can be seen in Figure 5, where the simulated marginal cost of providing capacity are significantly increased. Both Marginal Cost of Capacity Spinning (MCCS) and Marginal

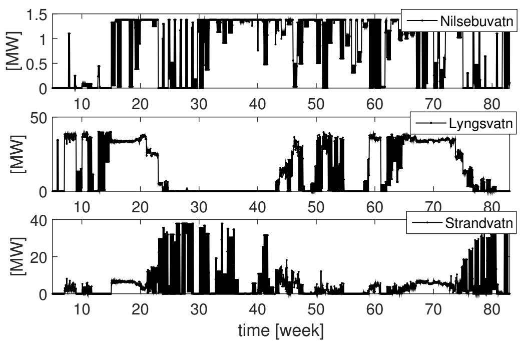


Fig. 6: Sales of capacity reserves for the Strategy Model.

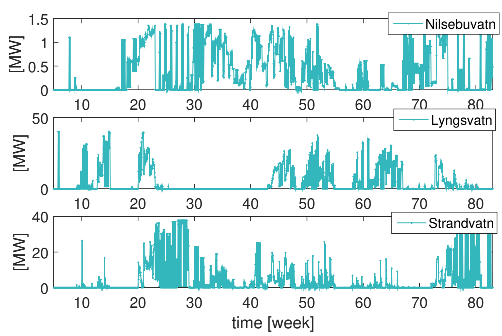


Fig. 7: Sales of capacity reserves for the Simulator Model.

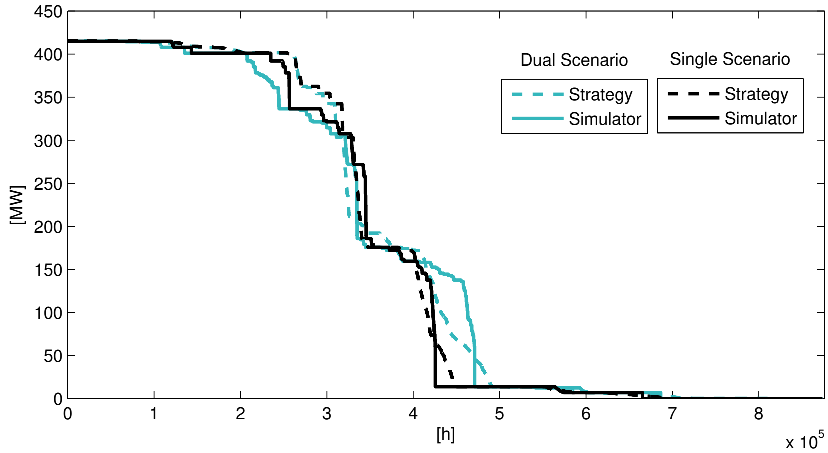


Fig. 8: Duration curve for all market scenarios and models. The duration curve displays values for all power stations in the power system.

Cost of Capacity Up (MCCU) respectively the simulated marginal cost of moving the power station set-point up and down. This comes as a result of the restriction that the power station has to supply symmetrical amounts of capacity. The positive values of MCCS and negative value of MCCU indicates the change in profit obtained if the set-point was increased by one unit. In Nilsebuvatn a considerable increase of profits would be obtained if additional capacity was available in the Strategy Model, which can be seen as a result of the high degree of inflow and minor installed capacity in the power station. As Strandvatn and Lyngsvatn were no longer able to deliver a sufficient amount of capacity, added capacity from Nilsebuvatn would have been highly favourable and hence and the magnified spike of MCCS in the Simulator Model.

The scheduling change in the Simulator Model between Lyngsvatn and Strandvatn from week 60 to 80 is distinctively represented by the high MCCS in both reservoirs. How this affected the actual sales of capacity reserves is shown in Figure 6 and 7, respectively from the Strategy and the Simulator Model. It is evident from Figure 6 that the period around week 20 and 70 served as high income periods from both the day-ahead and PFR market. Nevertheless, in the Simulator Model operation from Lyngsvatn and Strandvatn was not allowed, such that in order to avoid spillage production was set to maximum and almost no sale of capacity reserves were observed.

3.2. Case Scenario Comparisons

Figure 8 depicts the different duration curves of sum hydropower generation for the different models and scenarios, indicating the impact both a dual scenario and detailed modelling has on the hydropower scheduling. From the single to the dual scenario the Strategy Model shows a small shift from production around 350 MW, to a larger amount around 170 MW and down. A similar effect is seen for the Simulator Model, then however the shift in production comes from high power outputs around 400 MW to increased outputs around 150 MW, which has a favourable operating point in regard to efficiency. Both indicating the influence a capacity reserve market constitute, with higher amount of low power outputs.

For the dual scenario the low power outputs around 40 MW in the Strategy Model has almost completely dispersed in the Simulator Model to outputs around 150 MW. This follows from the previous discussed results from the production scheduling in Figure 4. The amount is however modest as the tightened constraint on the spinning reserve at minimum production level has performed well.

In Table 1 some of the important performance and economic factors are displayed. With a total amount of 3.00 TWh generated electricity over the two year period and profit of 1 213 MNOK the average day-ahead price obtained for the Strategy Model would be around 400 NOK/MWh.In the dual market scenario the model tended to store more

Table 1: Overview over significant results for scenario with and without sales of capacity. For comparison reasons the objective value is given by the lower objective value. Values in [MNOK] and [GWh].

	Single market		Dual market	
	Strategy	Simulator	Strategy	Simulator
Objective Value	1 418.90	1 410.74	1 447.64	1 436.01
Total Generation	3 002.78	2 960.31	3 060.01	3 014.06
Energy Profit	1 212.52	1 195.55	1 251.97	1 232.87
Capacity Profit	-	-	25.09	15.06
Terminal Value	207.75	217.09	190.22	200.83
Total Profit	1 420.27	1 412.64	1 467.28	1 448.76
CPU Time	7 h 37 min	33 min	10 h 37 min	39 min

Table 2: Percentage differentiation between the different models and different market scenarios.

Comparing	Strategy $\mapsto$ Simulator		Single $\mapsto$ Dual	
	Single	Dual	Strategy	Simulator
Objective Value	-0.58	-0.80	2.03	1.79
Total Generation	-1.41	-1.50	1.91	1.82
Energy Profit	-1.40	-1.53	3.25	3.12
Capacity Profit	-	-39.98	-	-
Terminal Value	4.50	5.58	-8.44	-7.49
Total Profit	-0.54	-1.26	3.31	2.56

terminal water, which is the value of the water left in the reservoirs after the simulation period, and thereby reducing the profits compared to the single market scenario. In order to compare both scenarios the terminal values of the water were calculated, and hence the model calculated overall total profit gain of 3.31% and 2.56% for respectively the Strategy and Simulator Model. Compared to the Strategy Model obtained the Simulator Model a reduced total expected profit of 0.54% and 1.26%, respectively for the single and dual scenario. It should be pointed out that the objective value includes a number of penalty function and the terminal value of water and is hence somewhat lower than the total profit.

It is clear that there is a profit potential for co-optimizing energy and sales, though limited and with added complexity. The detailed modelling also shows that the actual profit potential is significantly lower than the Strategy Model produces, with a total capacity profit reduction of 52%, c.f. Table 2.

As a result of added options by the PFR market the CPU time was somewhat increased. All simulations were carried out on a Intel Core i7-4600U processor with 2.7 GHz clock rate and 16 GB RAM. It would be reasonable to assume that an parallel processing implementation of the model would significantly reduce computation time [10].

4. Conclusion

The case study presented in this paper has assessed the differentiation in hydropower scheduling between a energy only and energy with sales of capacity scenario. The analysis was performed with the SDP/SDDP based Strategy Model and the Simulator Model, which is a more accurate model of the physical system. The study has achieved to give a quantitative valuation of including sales of capacity and the impact a detailed system description impose. It should be noted that the findings from the paper may be case specific, considering that coupling between reservoirs strongly influenced the results.

Firstly, a short description of the applied model and case study was revised. Following results on reservoir handling and production scheduling were outlined with a discussion on validity, the value of providing capacity was discussed where findings showed a strong link between two of the reservoirs and how this was encapsulated in the detailed modelling. It showed that the Strategy Model overestimated the amount of available capacity reserves. This was especially evident in periods where prices where beneficial but risk of spillage resulted in maximum production and sales of energy only. Lastly, a thorough comparison of the two case scenarios were carried out. It were deducted that the total profit from the dual scenario were 3.31% and 2.56% higher than the single scenario for respectively the Strategy and the Simulator Model. The profit from capacity sales alone were reduced by 40% in the Simulator Model, hence quantifying the impacts of detailed modelling.

The evidence from this case study indicates that The Simulator model proposed promising results in regard to validity of the scheduling and numerical values. The robustness and comprehension of the results could however be improved by drastically increase the number of sampled scenarios. This would however come at the expense of even longer CPU time, but could i.e. simulate with a modest sampling amount and in the last iteration increase it significantly.

It should be addressed that the profit-to-go function is not the true one for the Simulator Model. The importance of generating viable cuts should hence be emphasized . Other important considerations are the amount of volume allocated in the PFR market and the validity of using historical PFR prices, further work should therefore consider these limitations and investigate improvements.

Acknowledgements

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A.3 UPEC 2016

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Provision of rotating reserves from wind power in a hydro-dominated power system

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***Abstract*—This paper investigates how wind power can contribute to the provision of rotating reserves in a hydro-dominated power system with limited transmission capacity to an exogenous power market. We emphasize on the impacts different schemes for providing rotating reserves has on the generation dispatch and rotating reserve (RR) cost. Due to the flexibility provided by hydropower, the system is well suited for facilitating a large share of intermittent energy.**

We approached this by building a model based on Stochastic Dual Dynamic Programming (SDDP), which efficiently handles multistage stochastic problems.

A case study is presented based on the properties from the Nordic power system. Results shows that for wind penetration levels above 20%, some wind power is used for the provision of upwards RR at higher costs than the hydropower could provide, but freeing up more flexibility for the hydropower units and subsequently higher overall gain. The use of wind power to provide downwards RR proved to be very cost efficient, as there is no opportunity cost associated with the use of wind power.

NOMENCLATURE

A. Indices

c	Capacity reserve period
i, j	Power station unit
k	Time-step within week
l	Cut
s	Discharge segment
t	Weekly time-stage

B. Sets

$\mathcal{C}$	Periods within week for capacity reserves
$\mathcal{K}$	Time-steps within the week
$\mathcal{K}(c)$	Time-steps for capacity reserve period c
$\mathcal{L}(t)$	Cuts for time-stage t
$\mathcal{S}(i)$	Discharge segments for unit i
$\mathcal{T}$	Weekly time-stages
$\mathcal{U}$	Hydropower units
$\mathcal{U}(i)$	Upstream hydropower units of i
$\mathcal{U}^T$	Thermal power units
$\mathcal{U}^W$	Wind farm

C. Parameters

α_t	Future expected value of water [€]
β_l	Right-hand-side of cut l [€]
ϵ	Purchase fee [1.01]
η_{kis}	Discharge segment energy equivalent [MWh/Mm^3]
γ_i	Scalar limmiting the use of downward capacity reserves ≥ 1
λ^D	Curtailment price [€/MWh]
λ^R	Load curtailment price [€/MWh]
λ_k^S	Energy price for time-step k [€/MWh]
λ_i^T	Marginal energy cost of thermal unit i [€/MWh]
τ_k	Duration of time-step k [h]
$\widetilde{\tau}_k$	Relative duration of time-step k
D_t	Demand [MWh]
K^{Dn}	Downward RR requirement [MW]
K^{Up}	Upward RR requirement [MW]
$P_i^{\max}$	Maximum installed capacity for unit i [MW]

D. Stochastic Parameters

E^W	Weekly wind energy [MWh]
I_i	Weekly inflow to hydropower unit i [Mm^3]

E. State and Decision Variables

κ_c^{Dn}	Total downward RR in period c [MW]
κ_{ki}^{Dn}	Downward RR from unit i in time-step k [MW]
κ_c^{PenDn}	Artificial capacity for downward RR in period c [MW]
κ_c^{PenUp}	Artificial capacity for upward RR in period c [MW]
κ_c^{Up}	Total upward RR in period c [MW]
κ_{ki}^{Up}	Upward RR from unit i in time-step k [MW]
e_k^D	Curtailed energy [MWh]
e_k^P	Purchased energy [MWh]
e_k^R	Curtailed load [MWh]
e_k^S	Sold energy [MWh]
$e_k^T i$	Thermal energy for unit i [MWh]
e_k^W	Generated wind energy in time-step k [MWh]
q_{kis}^D	Discharge in time-step k for unit i and segment s [Mm^3]
q_{ki}^S	Spillage in time-step k for unit i [Mm^3]
v_{ki}	Reservoir volume in time-step k for unit i [Mm^3]
w_{ki}	Artificial resource (wind/water) supply for unit i [Mm^3/MWh]

I. INTRODUCTION

To deal with the impact of climate change, the need for a more sustainable power industry has never been more prominent[1]. This shift towards renewable electricity generation does, however, require different mechanisms to facilitate high penetration levels of intermittent energy sources. In recent years, the European power system has experienced a revolutionary growth of reneweables, where renewables accounted for 77% of new installations in 2015, the amount from wind power was 44% [2]. Independent of large-scale integration of renewables or not; it is crucial to ensure a sufficient high level of power quality, such that the stability of the power grid is not diminished.

Reduced net load caused by wind and PV generators results in a reduced amount of dispatchable generators, mainly displaced by non-synchronous generators. Subsequently, the system's ability to stay balanced in terms of frequency and sufficient amount of inertia is reduced, and serious reliability issues might emerge [3]. Issues with power system reliability and security are foremost observed for larger shares of renewable generation, as the net demand fluctuations are comparable to the gross demand for low levels [4]. Thus, additional rotating reserves (RR) are foremost required when the renewable share is high. A collection of studies regarding the systems rotating reserve requirements can be found in [5], where the authors have summarized experiences regarding the topic of wind integration into power systems.

It is perceived that in the future, generation companies will profit less from selling energy and more from capacity markets and ancillary services [6]. The reduced income from energy markets is a result from the merit order effect [7], as renewables with low marginal costs are replacing more expensive conventional power generation. From a power system perspective it is therefore necessary to have enough flexible power generation that can provide ancillary services, e.g. the authors in [8] pointed out that for a renewable share of more than 30% this flexibility requirement was evident from a system perspective. With a large share of non-dispatchable generation this might require expensive reserve capacity. Nevertheless, an aspect that should be examined further is how wind power can contribute as a provider of capacity reserves, and thereby not only being a sustainable solution for electricity generation but also to provide balancing services to the grid.

Automatic generation control of variable speed turbines has given wind power plants the possibility to provide Primary Control Reserves (PCR) [9]. The feasibility of the approach was confirmed in [10]. The approach does, however, require that the wind turbines are operated below their maximum wind power curve, i.e. some energy is curtailed in order to provide reserve capacity. It is evident that the benefit from providing such a service should exceed the benefit of selling the energy.

A central issue with wind power is its uncertain and variable nature. There are, however, several factors that contribute to improve this, such as; increased grid integration, technical and geographical distribution of wind turbines, improved forecasting and, as investigated further in this work, flexible generation which can provide capacity reserves and energy storage. All these issues are thoroughly discussed in [11]. Another mechanism mentioned by the authors to facilitate large shares of wind generation is power curtailment. This would unfortunately lead to loss of energy and lost income to the generation company. An interesting aspect, and a contribution from this work, would be to investigate if the wind power could operate below maximum output and then provide rotating reserves for upward regulation, in a power system with large share of wind power. Consider the following scenario; if the wind share is high, the requirement of reserve capacity could be so high that there is a limited amount of conventional generators available to deliver this service, and potentially at a very high price. This is especially evident during summer period in the Scandinavian power system, when the marginal cost of hydro is high, compared to the energy price. It could therefore become more efficient to curtail some wind power that can provide balancing reserves than using the water.

A stochastic time resolution of one week is considered suitable to describe inflow uncertainty [12], but is probably too coarse to describe wind uncertainty properly. Given the fluctuating nature of wind, it can seem unreasonable to assume that the wind generation is known for a whole week in advance. One can argue that such a representation is not able to capture the actual need for balancing wind power. However, the model aims to mange the hydro reservoirs on a long-term basis, a problem in itself that is difficult enough to solve. Such that the short-term variations and uncertainty of wind, and more complex modelling of the power stations and river systems, should be handled by the short-term planning.

From a system perspective the possibility to co-optimize hydro- and wind-generation has proven beneficial as the study in [13] has pointed out, and the impact storage has for integration of wind and solar power is investigated in [14]. Therefore, in a hydro dominated system, added wind power could efficiently be absorbed. The hydropower producer will receive a price signal in hours were there is substantial wind power, such that the producer would then store the water until sufficient energy prices are obtained.

The novel contribution of this paper is to investigate whether it is economical viable to use wind power as source for providing capacity reserves and demonstrate how a power system could react to such a scheme. It is important to clarify that this is not a feasibility study on the technical side. We have demonstrated this by building a Stochastic Dual Dynamic Programming (SDDP) model used for solving multistage stochastic problems. The model is based on weekly stochastic time-stages, which overestimates the predictability of wind power, nonetheless, it enables us to very neatly incorporate correlations between the stochastic parameters and thus analyze how hydropower and wind power operate together to fulfil the load and capacity reserves by the system.

The paper is outlined as follows. First, the generic market for providing rotating reserve, used in the study, is discussed and outlined. Secondly, a brief methodology of the SDDP algorithm is given with details on how the weekly dispatch problem is modelled. Then the case study is described, followed by results and discussions. Lastly, a conclusion of the study is drawn.

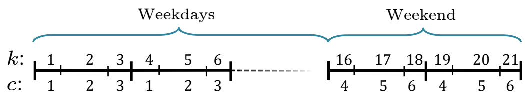

Fig. 1. Illustration of the time and rotating reserve periods within the week. The days are divided into 3 time periods, from 00:00-08:00, 08:00-20:00 and 20:00-00:00. As illustrated in the figure the there are 21 time-steps during a week with 6 different time-periods for providing rotating reserves (RR).

II. Market for Rotating Reserves

One of the mandates for a Transmission System Operator (TSO) in Europe is to handle real-time balancing of the electricity grid [15]. A set of mechanisms are set up by the TSO for this purpose, such as markets for providing primary, secondary and tertiary control reserves, congestion management and other measures to uphold both reliability and security of the power system. The exact specifications for the different mechanisms may vary between different TSOs, depending on their system characteristics, but the overall objective is still the same.

The objective for secondary control reserves (SCR) is initially to alleviate the primary control reserves (PCR) such that these reserves are available to react if a new disturbance should occur. Both SCR and PCR requires that the power station is rotating, whereas PCR requires a turbine governor with droop settings to react immediately if a deviation in normal frequency is observed. SCR requires that the TSO have a direct access to the power station and can thus alter the stations power output. Should the disturbance persist for longer time periods the tertiary control reserves (TCR) should be activated.

The design of control reserve markets varies between countries, and in this work we consider what can be seen as a generic weekly market for providing rotating reserves (RR), i.e. what is typically provided by PCR and SCR. A weekly market implies that the reserves have to be available throughout the entire week. The market consists of three different periods for weekdays and three for weekends, indicated in Figure 1. To deliver RR in time period 1 (weekday morning) the same amount has also to be delivered in time period 4, 7, 10 and 13, and so on for the other periods. Furthermore, we assume that the generic RR market allows asymmetric provision of up-and downward reserve capacity.

III. Methodology

The purpose of this work is to evaluate the long-term economic benefit of using wind power, in addition to hydropower, for providing rotating reserves to the power system. We consider a hydro-dominated region which also comprise wind and thermal generation. There is no internal congestion within the region, which is connected to the rest of the power system through a transmission line with limited capacity. The region operation is centrally planned, where the objective is to maximize expected revenue from trades in the day-ahead market while satisfying a given requirement for RR, as well as other system constraints.

The problem is a multistage stochastic programming problem which in compact notation can be formulated as

$$\max \left\{ \sum_{t=1}^{T} f_t(\mathbf{x}_t) + F(\mathbf{x}_T) : \mathbf{A}\mathbf{x}_t \leq \mathbf{b}, \right. \qquad (1)$$
$$\left. \mathbf{T}\mathbf{x}_{t-1} + \mathbf{W}\mathbf{x}_t = \mathbf{h}, \mathbf{x}_t \in \mathbf{X}_t, \forall t \right\}.$$

The function, $f_t(\mathbf{x}_t)$, in the objective includes sales of energy and penalty functions, i.e. for ensuring relative complete recourse. State and decisions variables are represented by the vector $\mathbf{x}_t$, which is subject to a set of constraints. It is assumed that the initial state $\mathbf{x}_0$ is known. The second equation reflects the dynamic structure of the problem. Due to the reservoirs ability to store water from one period to another, the decisions made today will impact future decisions. $F(\mathbf{x}_T)$ denotes the end-value function, which estimates the value of water left in the reservoirs by the end of the planning horizon and is included to ensure reservoirs are not emptied completely in the last stage.

The problem in (1) constitute a large-scale multistage stochastic programming problem, which is generally hard to solve. The problem is therefore stage-wise decomposed into a sequence of one-stage problems and solved using Stochastic Dual Dynamic Programming (SDDP) [16]. SDDP is an iterative algorithm that employs a combination of sampling and Benders decomposition, and is considered the state-of-the-art approach to solve such problems.

In SDDP each one-stage problem consists of immediate profit, plus the 'profit-to-go' function which represents the expected future profit. The technique of Benders decomposition is used to find a set of cuts that constructs a linear approximation to the profit-to-go function in each stage. One main iteration of the algorithm consist of a forward pass and a backward pass. Each forward pass samples outcomes of the stochastic parameters and generates trial solutions which are used in the next backward pass. At the end of each forward pass a lower bound to the objective function is calculated. The backward pass updates the solutions by adding Benders cuts to the profit-to-go function and provides an upper bound to the objective function. By iterating over this procedure an enhanced representation of the profit-to-go function is achieved and lower and upper bounds to the optimal objective function value can be calculated. The algorithm is stopped when a predefined convergence criterion is met. For more details on SDDP we refer to [16], [12].

A. Weekly Dispatch Problem

For conciseness only the weekly one-stage problem is presented. The problem where only hydropower has the opportunity to provide rotating reserves is described by (2) - (12). The problem when also wind power can contribute in providing rotating reserves is defined in Section III-A2.

$$\alpha_t = \max \sum_{k\in\mathcal{K}} [\lambda^S_k(e^S_k - \epsilon e^P_k) + \lambda^D e^D_k - \lambda^R e^R_k \quad (2)$$

$$- \sum_{i\in\mathcal{U}^T} \lambda^T_i e^T_{ki}] - \Phi(q^S_{ki}, w_{ki}, \kappa^{\text{PenDn}}_c, \kappa^{\text{PenUp}}_c) + \alpha_{t+1}$$

s.t.

$$\sum_{i\in\mathcal{U}} \sum_{s\in\mathcal{S}(i)} \eta_{kis} q^D_{kis} + \sum_{i\in\mathcal{U}^T} e^T_{ki} + e^W_k - e^S_k \quad (3)$$

$$+e^P_k - e^D_k + e^R_k = D_k \quad \forall k$$

$$v_{ki} + \sum_{s\in\mathcal{S}(i)} q^D_{kis} + q^S_{ki} - w_{ki} - \sum_{j\in\mathcal{U}(i)} \Big[\sum_{s\in\mathcal{S}(j)} q^D_{kjs} \quad (4)$$

$$+q^S_{kj}\Big] = v_{k-1,i} + \widetilde{\tau}_k I_i \quad \forall k, \forall i$$

$$e^W_k - w_{ki} \leq \widetilde{\tau}_k E^W \qquad \forall k, \forall i \in \mathcal{U}^W \quad (5)$$

$$\sum_{s\in\mathcal{S}(i)} \frac{1}{\tau_k} \eta_{kis} q^D_{kis} - \gamma_m \kappa^{\text{Dn}}_{ki} \geq 0 \qquad \forall k, \forall i \in \mathcal{U} \quad (6)$$

$$\sum_{s\in\mathcal{S}(i)} \frac{1}{\tau_k} \eta_{kis} q^D_{kis} + \kappa^{\text{Up}}_{ki} \leq P^{\max}_i \qquad \forall k, \forall i \in \mathcal{U} \quad (7)$$

$$\sum_{k\in\mathcal{K}(c)} \sum_{i\in\mathcal{U}} \kappa^{\text{Dn}}_{ki} - \kappa^{\text{Dn}}_c = 0 \qquad \forall c \in \mathcal{C} \quad (8)$$

$$\sum_{k\in\mathcal{K}(c)} \sum_{i\in\mathcal{U}} \kappa^{\text{Up}}_{ki} - \kappa^{\text{Up}}_c = 0 \qquad \forall c \in \mathcal{C} \quad (9)$$

$$\kappa^{\text{Dn}}_c + \kappa^{\text{PenDn}}_c \geq K^{\text{Dn}} \qquad \forall c \in \mathcal{C} \quad (10)$$

$$\kappa^{\text{Up}}_c + \kappa^{\text{PenUp}}_c \geq K^{\text{Up}} \qquad \forall c \in \mathcal{C} \quad (11)$$

$$\alpha_{t+1} - \sum_{i\in\mathcal{U}} \pi_{kil} v_{ki} \leq \beta_l \qquad k = |\mathcal{K}|, l \in \mathcal{L}(t) \quad (12)$$

The objective function (2) comprise income from selling energy e^S_k, the possibility to curtail energy and load, a penalty function $\Phi()$ for ensuring relatively complete recourse, and the profit-to-go function α_t. Energy and hydro reservoir balances are respectively given by (3) and (4). The relationship between generation and discharge, is linearized by different discharge segments, q^D_{kis}. (5) represents the wind energy balance, where the system is not required to use all the wind. An artificial variable w_{ki}, associated with a high cost, is included in the reservoir- and wind-balance, subsequently (4) and (5), to ensure that the stochastic model has complete recourse. Values for the stochastic variables, inflow to reservoirs I_i and wind energy E^W, are distributed over the time-steps in the week, by the weighting factor $\widetilde{\tau}_k$. Representing the number of hours for time-step k divided by the hours in a week.

Each hydropower unit has the possibility to provide both upwards and downward RR, given by (6) and (7). The amount of available upwards and downwards RR for each hydropower unit is illustrated in Figure 2. [17] showed that the potential income for selling PCR was overestimated with almost 30% by SDDP, for the given case study. This indicates the importance of detailed modelling when considering provision of rotating reserves. In order to improve the representation of available

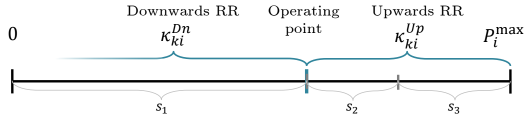

Fig. 2. Illustration of a hydropower unit discharge segments, operating point and the upwards and downwards RR. The amount of downwards RR is dependent on the factor γ_i, such that the available amount is dependent on this linear approximation of the lower limit.

downwards RR, the factor γ_m is added to the problem, as in [18].

Equations 8 and 9 allocates the available capacities to the different capacity reserves periods over the week, as illustrated in Figure 1. The amount of RR required is then set in (10) and (11), where a penalty variable is included to ensure feasibility. The required amount of RR, depicted in Figure 3 and represented by K^{Dn}_t and K^{Up}_t, does also depend on the average weekly load, such that weeks with load above average requires more RR, and vice versa for low demand weeks.

1) Wind and Inflow Model: Reservoir inflow and available wind generation are the stochastic parameters in this problem. The stochastic time resolution in the optimization model is one week, and the wind and inflow model is based on a stochastic time series model fitted to weekly data. A finer stochastic time resolution could be used, but would increase computational complexity substantially.

We use a vector autoregressive model of order one (VAR-1), as proposed in [19]. Both processes exhibit a seasonal pattern, and are as such non-stationary processes. Seasonality is first extracted by subtracting the sample seasonal mean and dividing by the seasonal standard deviation. The seasonally adjusted series are then assumed to be (weekly) stationary and modelled jointly by a VAR-1 model. This model accounts for seasonal effects, serial correlation (within and between series) and covariation in the stochastic error term.

The optimization model requires the stochastic component to be discretized into a moderate number of wind-inflow outcomes with corresponding probabilities. This is achieved through a combination of scenario generation and reduction. First, a large discrete sample is generated from the stochastic model by random sampling from the multivariate error distribution. Each outcome in this sample has the same probability of occurrence. Subsequently, this distribution is reduced to a manageable number of scenarios with corresponding probabilities using the 'Fast forward selection' algorithm of Heitsch and Römisch [20].

2) Rotating Reserves from Wind:

$$e^W_k - \tau_k \kappa^{\text{Dn}}_{ki} \geq 0 \qquad \forall k, i \in \mathcal{U}^W \quad (13)$$

$$e^W_k + \tau_k \kappa^{\text{Up}}_{ki} \leq \widetilde{\tau}_k E^W \qquad \forall k, i \in \mathcal{U}^W \quad (14)$$

In order to model the possibility of using wind for providing rotating reserves (13) and (14) is added to the problem, subsequently (8) and (9) are updated to:

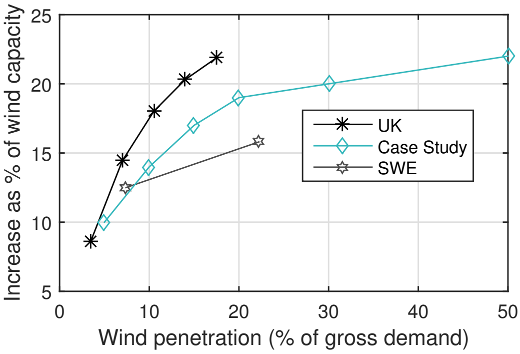

Fig. 3. Increase in reserve requirements. Results from previous studies in UK [21] and Sweden [22] are illustrated together with the values used in present case study.

$$\sum_{k\in\mathcal{K}(c)}\sum_{i\in\mathcal{U}\cup\mathcal{U}^W}\kappa_{ki}^{\mathrm{Dn}}-\kappa_c^{\mathrm{Dn}}=0 \qquad \forall c\in\mathcal{C} \tag{8a}$$

$$\sum_{k\in\mathcal{K}(c)}\sum_{i\in\mathcal{U}\cup\mathcal{U}^W}\kappa_{ki}^{\mathrm{Up}}-\kappa_c^{\mathrm{Up}}=0 \qquad \forall c\in\mathcal{C} \tag{9a}$$

IV. Case Study

The system in the case study comprise a multi-reservoir hydropower system, a wind farm and two types of thermal generators - representing base- and peak load units, illustrated in Figure 4. The hydropower system consists of five interconnected reservoirs and power units, their properties are given in Table I. The system is connected to a exogenous power market through a single transmission line with limited capacity. There is a local load which can be fulfilled either by own generation or by purchase from the power market. In such a way the system can be seen as a simplification of the Nordic power system, with limited connection to the European continent, hydro-dominated with some thermal generation. Table II shows the characteristics of the different system components, except for the wind farm for which we let installed capacity vary.

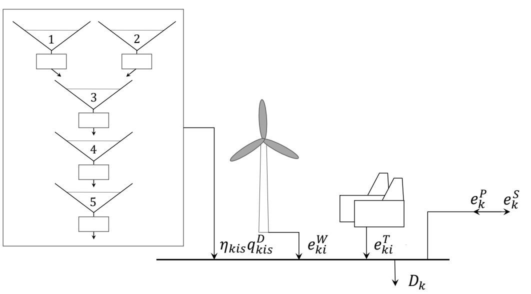

Fig. 4. Power system overview with watercourse topology and associated variables and parameters.

TABLE I
OVERVIEW OVER HYDROPOWER UNITS PROPERTIES.

Unit	Size [Mm^3]	Exp. Inflow [Mm^3]	Inst. Cap. [MW]
1	145.0	423.3	90
2	896.6	1289.4	140
3	26.2	50.5	350
4	86.9	240.8	20
5	11.2	357.3	400

TABLE II
GENERATION AND DEMAND CAPACITY [MW].

Hydro	1000
Base thermal	300
Peak thermal	200
Exchange	300
Peak Demand	1200
Average Demand	777

We evaluate 12 cases, differing in the amount of RR for up- and downward regulation the system as a whole must provide, the share of wind power installed, and whether both wind and hydro, or only hydro, can provide RR. The amounts chosen for RR was based on the work by [5], where studies on wind integration from 15 countries was summarized and presented. We considered six different levels for wind penetration, amounting to 5, 10, 15, 20, 30 and 50 percent of gross energy demand. Figure 3 displays the values chosen for RR requirement for the various wind power penetrations (blue line) together with values estimated in two previous studies described in [21] and [22].

We consider a planning horizon of one year, starting January 1. The model is, however, run for a period of two years in order to reduce the impact of the end value function which is generally difficult to estimate. To evaluate and compare the results from the different cases a final forward simulation was performed using 500 sampled scenarios of wind and inflow. The scenarios were kept fixed across cases.

For the purpose of this study, a deterministic representation of energy price and demand is used.

V. Results and Discussion

To illustrate the operation of the power system, the energy generation for a given scenario with a 30% wind power penetration is illustrated in Figure 5. The figure also depicts the load and the exogenous energy price. It can be seen that the system has a typical dispatch profile for a Nordic country, with low load and energy prices during summer and the opposite during winter. The system therefore chooses to export energy in high-price periods and import during summer when the prices are low.

An assessment over different metrics for flexibility in a power system was conducted in [23]. As the paper states there are numerous metrics for assessing the flexibility of a power system as a whole, each fitted for the purpose of the study. For the analysis of a hydro dominated system a valuable metric is the amount of spillage, i.e. the hydro system's ability to

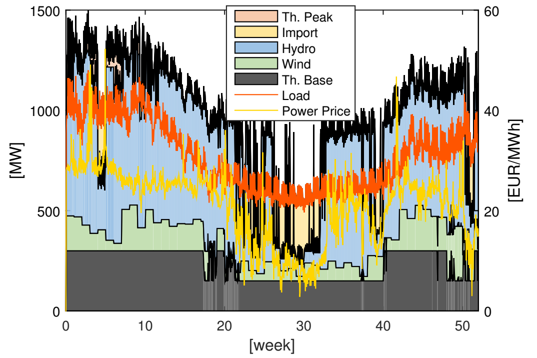

Fig. 5. Energy load balance for a given scenario with 30% wind penetration. Generation above the load levels (red) are exported to an external system, represented by the exogenous energy price (yellow).

move water between time-periods for maximum utilization of available energy (water). As expected, we observed that as the wind penetration increases the amount of spillage also increases, due to the transmission constraint.

A. RR by hydropower

For analyzing the cost aspects of providing capacity reserves the shadow price of the reserve requirements, given by (10) and (11), is used. The average expected values for both upward and downward RR are illustrated in Figure 6. The optimal operation point for the hydropower stations is around 80% of installed capacity, such that the provision of upward RR is fairly cheap for low levels of wind penetration. As the provision of downward RR requires the machines to be rotating, a moderate cost is observed for this service, even for low levels of wind penetration. For wind penetrations above 20% more striking results are observed, as provision cost of upward RR increases significantly. This is seen as a result of the increase of available energy in the system and the increased requirement for upwards RR, limiting the maximum discharge, such that in order to avoid spillage in the hydro reservoirs a large marginal cost is associated with providing upward RR. Consequently, the hydropower stations have to discharge more at hours with low energy prices.

B. RR by hydro- and wind power

For the case when wind can also provide RR, a considerable change is observed for the downward RR. As expected the cost is reduced as the system does no longer have to use water for this service, that instead can be stored and used at a later stage with more beneficial income conditions. This can be seen in Figure 7, where a large amount of the available wind power offers downwards RR. The impact of actually activating this reserve could, however, impose a higher cost than for hydropower, as the energy that has to be curtailed cannot be stored for later purpose. For upwards RR an higher cost is observed, compared to the hydropower case. This result seems counter-intuitive at first sight, as the system now is more

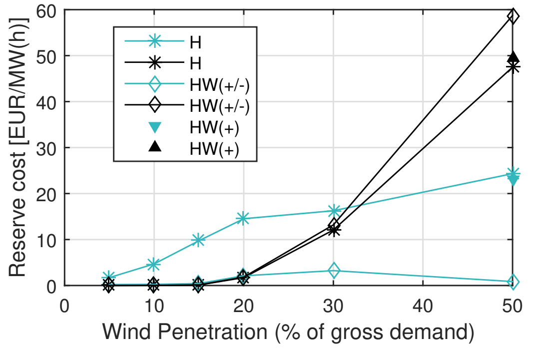

Fig. 6. Average cost for providing capacity reserves given as a function of wind penetration. The lines and marker indicates upwards RR (black) and downwards RR (blue). ”H” refers to the case when only wind can provide RR, ”HW(+/-)” and ”HW(+)” respectively refers to the case when wind power can provide upwards and downwards reserves and when wind only can provide upwards RR.

flexible, but the fact that wind power now can alleviate some of the upward RR, which is observed for wind penetration levels above 30% (ref. Figure 7), the hydropower units can now generate more during periods with high prices. This therefore results in a higher cost of upwards RR, but also an increase in objective function value, as seen in Figure 8.

It is evident that even-though the costs of providing upwards RR increase for the hydro and wind case, the corresponding objective value also increase, as seen in Figure 8. To investigate this results further, we constructed another case study where wind power only could provide upwards RR. The results is displayed by the dark bar in Figure 8 and the triangles in Figure 6. It is seen that the objective value is somewhat similar to the hydro-only case, but the costs for upwards and downwards RR increased and decreased, correspondingly. Even though the amount of wind power for provision of upwards RR is only minor (see Figure 7), it comes

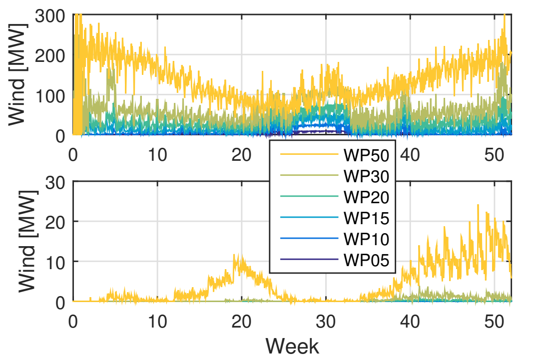

Fig. 7. Expected RR provided by wind for different levels of wind penetration (WP). Downwards RR (top) and upwards RR (bottom).

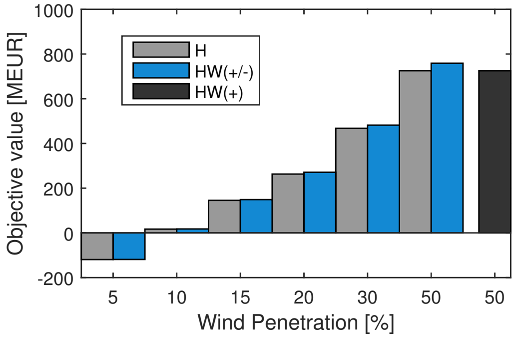

Fig. 8. Lower bound objective values for the different case studies. The cases labeled "HW(+/-)" refers to when both hydro and wind can provide upwards and downwards RR, while the case "HW(+)" wind power could only deliver upwards RR.

at a higher cost than hydropower since some energy has to be curtailed. Nevertheless, hydropower stations now have a higher generation capacity and can use this to generate more in periods with higher prices, and then provide more downwards RR. Consequently, the results are similar as for the case when wind could provide both upwards and downwards RR.

VI. Conclusion

In this work we have presented a long-term model for weekly generation scheduling of a power system which considers uncertainty in available wind generation and reservoir inflow. The model was used to assess the value of using wind power to contribute in providing rotating reserves. The model was applied to a power system with limited transmission capacity to an exogenous market.

In order to determine the amount RR required by the system, this work built on previous studies that analyzed the impact of large-scale integration of wind power. A novel contribution of this work was then to investigate how provision of RR by wind power affected the generation dispatch and the costs associated with providing RR. We found that the system operation and costs changed significantly when allowing wind power to deliver RR. The provision of downwards RR are cheap as there is no opportunity cost associated with wind power, and since activation was neglected in this study. For upwards regulation the cost of RR increased slightly for the case when wind could provide RR, then again this freed up capacity for the hydropower such that it could be utilized more efficiently.

For future studies a higher resolution of the stochastic time-stages could be evaluated, as the weekly resolution for wind power overestimates its capabilities to provide RR. Another interesting extension is to include activation of the different reserves, as this could potentially impact which power station the system will procure for RR.

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A.5 IOP Conference Series

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Impact of Modelling Details on the Generation Function for a Norwegian Hydropower Producer

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Abstract. In the following work the generation function relating water discharge to power output for a hydropower station is considered. The conventional approach to modelling the generation function has been to simplify it as a concave function. In the future it is expected that the stations will operate at more varied power outputs, motivating the need for a more detailed modelling of the generation function. Therefore, an investigation on how the nonconcave generation function may be simplified, without having to lose the function's inherent geometric shape, is performed. A greedy algorithm and a Minimum Least Square Error (MLSE) approach is used. It was found that the function can be reduced until a point where the changes become too prominent. A benchmark test against the conventional modelling approach found that it is important with a detailed modelling of the generation function when environmental constraints, such as minimum discharge, are included.

1. Introduction

The following work is founded on recent advancements in hydropower scheduling methods that enables a higher level of modelling details. There has been a fundamental change in the electricity sector with large shares of renewables entering the market and increased cross-border exchange. It is expected that this will lead to increased demand of flexible power plants that can provide ancillary services [1]. Given a market environment where these services can be traded across the European countries it is estimated that there is a income potential for Norwegian hydropower producers [2]. Due to the flexibility storable hydropower possesses they have an opportunity to provide many of the ancillary services at a competitive cost. However, in order for them to adapt to this changing market environment and provide these services it is evident that adequate decision support tools are needed to optimally allocate generation between different markets. Earlier studies have shown that conventional scheduling algorithms based on linear programming (LP) tend to overestimate the amount of capacity reserves that the hydropower producer can deliver [3]. There is also a tendency to increase the environmental restrictions on new concessions for hydropower systems in Norway, leading to more complex constraints in the optimization problem. Developing more advance methods should there be of priority to include these constraints and capture the added income potential by participating in multiple-markets.

2. Hydropower Scheduling

Due to the uncertainty of inflow, energy prices and the possibility of storing water over longer time periods, the hydropower scheduling problem is generally handled as multistage stochastic programming problem. An important element of hydropower scheduling is the water value; even though inflow to the reservoirs are free, there is an opportunity cost associated with the water stored in the reservoirs. The producer can choose to use the water for selling energy today and earn an immediate income or store it for future use with a potential higher profit. In terms of hydropower scheduling, this opportunity cost is referred to as the water value.

Given the complex nature of the hydropower scheduling problem, it can be divided into three problems with different time horizon and modelling details [4]. The Long-Term Hydropower Scheduling (LTHS) problem aims at solving a fundamental market problem that provides energy price forecasts and aggregated water values [5] [6]. The Medium-Term Hydropower Scheduling (MTHS) problem can then use the aggregated water values as end statement and price forecasts to provide individual water values for each reservoir in a local hydropower system [7]. Further, these individual water values can be used as an end statement for the Short-Term Hydropower Scheduling (STHS) problem that has high modelling details, e.g. modelling of individual generators and water courses. The time resolution is fine, typically hourly for a period of one or two weeks. For liberalized electricity markets this problem aims at first generating bids to the power market and after the market clearing perform a re-optimization that incorporates any commitments that occurred in the clearing [8].

2.1. Multistage Stochastic Optimization

This paper depicts an application that will be used in further studies on the MTHS problem. A time horizon of 1-2 years is typical for this problem type, using weekly decision stages as the inflow is highly correlated on a weekly basis. The problem is cast as a Stochastic Dynamic Programming (SDP) problem, with weekly decision stages. To circumvent the "curse of dimensionality" associated with conventional (S)DP problems, a method called Stochastic Dual Dynamic Programming (SDDP) is widely used [9]. The method approximates the expected profit function (EPF), also referred to as cost-to-go function for minimization problems. One of the drawbacks with the method is that it requires the decision stages to be modelled as LP problems, ruling out the possibility of modelling nonlinearities without simplifications. A recent extension of the method has been proposed called the Stochastic Dual Dynamic integer Programming (SDDiP) method [10]. The method can solve nonlinear problems with finite convergence. However, it comes at a cost of added computation time and some adjustments on the structure of the problem, i.e. it requires all the state variables to be binary. Previous work has tested the method on a MTHS problem with promising results, compared to the SDDP method [11]. With this in mind, the following work investigates how the generation function of a hydropower plant can be modelled in the SDDiP framework and how it can be simplified to reduce the computational burden.

2.2. Contributions

The following paper applies a greedy algorithm and a Minimum Least Square Error (MLSE) approach to simplify the generation function of a hydropower station. The greedy algorithm is based on the idea of reducing the number of points on a bid curve in [12]. The approach is tested on two real power station equivalents and the importance of detailed modelling of the generation function is shown. The following section outlines the hydropower scheduling problem, followed by case studies and results.

3. Medium-Term Hydropower Scheduling

The problem aims at allocating resources simultaneously in an energy market and a capacity market. For the sake of simplification, only uncertainty of inflow is considered. The weekly decision problem can be written on dense form as

$$\max_{(x_t,y_t)} \; f_t(x_t,y_t) + \alpha_t(x_t) \tag{1}$$

$$\text{s.t.} \quad Wx_t + Gy_t = h_t(\xi_t) - Hx_{t-1} \tag{2}$$

$$By_t = 0 \tag{3}$$

$$Cy_t - Dx_t \geq 0 \tag{4}$$

$$Cy_t + Dx_t \leq Cy^{\max} \tag{5}$$

$$x_t, y_t \in Y_t \tag{6}$$

$$x_t \in \mathbb{R}^{k_1} \cdot \mathbb{Z}^{k_2}, y_t \in \mathbb{R}^{l_1} \cdot \mathbb{Z}^{l_2}, \tag{7}$$

where the objective function (1) consists of a present profit function, $f_t(x_t, y_t)$, and the expected future profit function, $\alpha_t(x_t)$. x_t and y_t are respectively the state and stage variables. State variables carry information between stages, e.g. reservoir levels, whereas the stage variables only represent variables within the stages. The inflow is given by the function $h_t(\xi_t)$, where ξ_t is the normalized inflow. The matrices W, G, H, B, C, D are of suitable dimensions, representing the given hydropower system. The time-linking constraint (2) includes all reservoir balances and generator state time-couplings. The energy balance is given by (3) and the constraints describing the system's ability to provide capacity reserves is given in (4) and (5). The generation function and other miscellaneous constraints, such as bypass limits, spillage limits and other system dependent constraints are included in (6).

3.1. Generation function

The generation function describes how the hydropower station's discharge relates to the output power, usually modelled as a piecewise linear function. The function is typically computed when the hydropower station is built. This function provides data input to the scheduling models and as a control that the station meets the efficiency as promised by the contractors. As the efficiency may deteriorate over time, the more recent the measurements, the better. Advantageously, a system with continuous measurement could give the best result in terms of describing the generation function most accurately. The methods proposed in this work will work irrespective of the measurement approach, it would just require some prepossessing of the data.

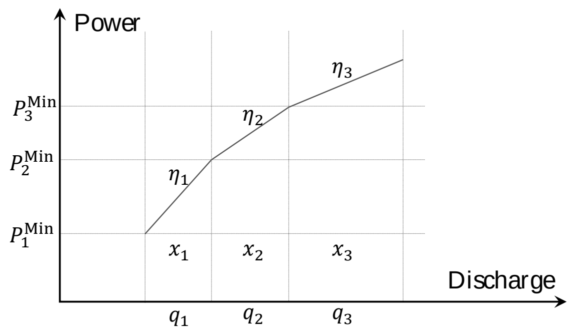

Figure 1: Illustration showing the mathematical notation of the generation function.

The line between two points in the generation function is referred to as a segment. Extending on the notation from previous section, the generation function is modelled as follows

$$0 \leq q_i\left(Q_i^{\text{Max}} - Q_i^{\text{Min}}\right)x_i \qquad \forall\, i\, \in S \tag{8}$$

$$p_i = P_i^{\text{min}} x_i + \eta_i q_i \qquad \forall\, i\, \in S \tag{9}$$

$$\sum_{i \in S} x_i \leq 1 \tag{10}$$

$$p = \textstyle\sum_{i \in S} p_i, \tag{11}$$

where q_i is the discharge (in Mm^3) for segment i, Q_i^{Min} and Q_i^{Max} are the minimum and maximum discharge for each segment, p_i is the generation (in MW) for each segment, P_i^{Min} is the minimum generation for each segment, x_{i} is a binary variable indicating whether the segment i is active or not, p and p_i is, respectively, the overall generation and generation for the individual segments in the set S of segments. An illustration of the generation function is given in Figure 1. Observe that even though the illustration depicts a concave function the method also applies to the nonconcave case, which is an important aspect in this paper. A concave generation function would imply that the first segment has the best efficiency, i.e. best power output for a given discharge, while the other segments are decreasingly

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worse. This case would need no binary variables to ensure that the efficiency of each segment is used correctly, since the model will always use the segment with the best efficiency first and the rest followingly.

3.1.1. Area removal algorithm
The following section describes the greedy algorithm used to reduce the number of segments in the generation function. The idea is taken from [12], where the authors apply the method on bid curves for a hydropower producer. The underlying idea is that you should remove points in the function that alter its geometric shape as little as possible. Illustration given in Figure 2.

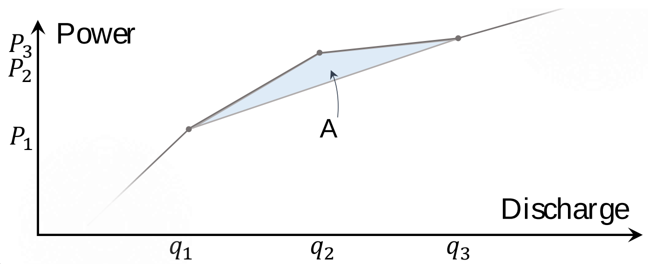

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Figure 2: The figure shows a part of the generation function and how the area (A) is computed between three adjacent points.

The algorithm computes the area between two adjacent point for all the points on the curve. The point that has the lowest computed area is removed. This procedure is repeated until a desired amount of points are removed.

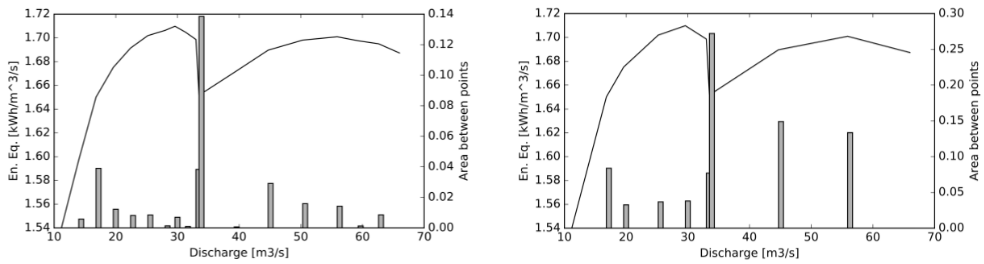

Figure 3: Illustration of the generation function. Right axis indicates the area of the triangle computed with the two adjacent points. Note that we are illustrating the energy equivalent [$\mathbf{kWh/m^3}$]. This is done to get a better visualization of the curve. Left: The original generation function. Right: The generation function after eight points have been removed by the area removal technique.

Figure 3 shows the generation function and the area between the adjacent points for the original function and for the function when eight points have been removed. As expected the geometric shape of the function is very much kept, even though half of the points on the curve has been removed. Below, in Figure 4, the accumulated area that is removed from the function is plotted. It is evident that there are many points on the curve that can be removed with almost no impact. After a certain threshold the accumulated area increases significantly, giving an initial indication for how much the function can be reduced.

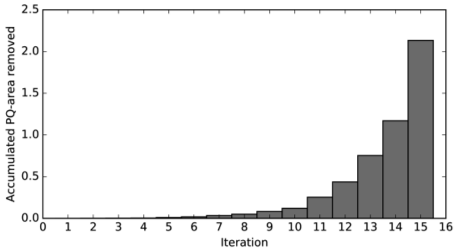

Figure 4: Bar graph of the accumulated area between adjacent points in the generation function that are removed. As illustrated, there are many points that can be removed with little impact on the shape of the curve, followingly the more points removed, the more significant this impact becomes.

3.1.2. Segmented regression method

Segmented regression is applied to reduce the number of points in the generation function. This is done with the R library *segmented* [13]. It is an iterative approach where the user suggests starting breakpoints. Being a statistical approach, it would be beneficial with a lot of data to fit the curve. In such sense, the approach would be well suited for a system that does continuous measurements. Nonetheless, the approach provides results well fitted to the original data, as seen in Figure 5.

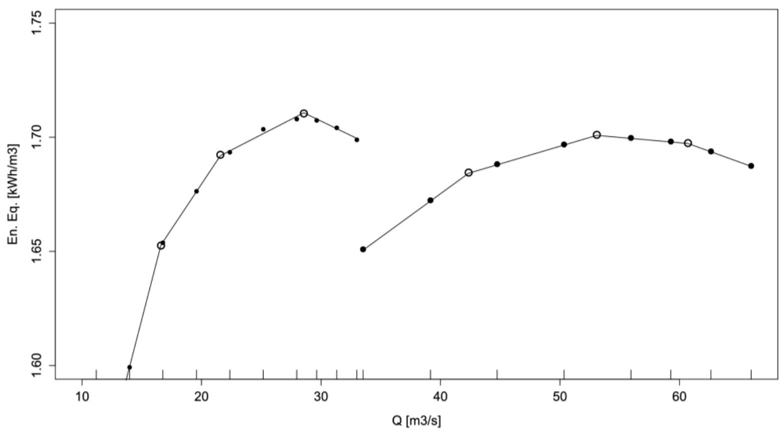

Figure 5: The generation function is given by the filled dots. The line going through the circles are the results from the segmented regression. Observe that there are two generators in the power station, each fitted individually.

4. Case Study

Two power stations are investigated in the study. The first, Station A, consists of a single generator unit with 45 MW installed capacity. The second, Station B, consists of two generator units, each with 210 MW installed capacity. Both stations only have Francis turbines installed.

The two systems are solved with a time horizon of two years with weekly uncertainty of inflow. For simplicity, the energy prices and capacity reserve prices are modelled as deterministic. The week is then separated into 21 time periods, three for each day indicating morning, mid-day and evening.

The SDDiP approach is applied, described in [10] [11], to solve the hydropower scheduling problem. The problem is solved with the original generation function and an EFP function is obtained, which in turn is used to simulate with the different reductions of the generation function. For each simulation 100 different scenarios are generated that are used to evaluate the expected results, for all simulations. To compare the results to what is normal approach for this problem type in operative models today, a case where the generation function is concave is performed. Lastly, some tests are performed on how Station A would be operated given that there is a time period in the summer months that requires a minimum discharge, due to environmental constraints.

5. Results and Discussion

The following section outlines some results from the case studies and discusses its implications.

Table 1 and Table 2 shows some of the results obtained from the different simulations. The area removal algorithm (A. Rem.), segmented regression method (Seg. Reg.) and Concave case (Conc.) was run. The tables depict the computation time of each simulation, the expected objective value, deviation in shape of the duration curve and total generation from the original problem, given by the first column. The second row of the table indicates how many points on the curve has been removed from an original total amount of 18 and 11 in System A and B, respectively.

Table 1: Results from System A. Simulation time is given in seconds, whereas the other values are given as a percentage. Val. represents the expected objective value, Dur. indicates the percentage deviation of the duration curve compared to the original problem. Gen. is the expected generation for the two years.

	A. Rem						Seg. Reg.				Conc.
	0	2	4	6	8	10	8	9A	9B	10	
Time	920	895	763	668	614	523	555	522	521	475	59
Val.	0.00	0.00	0.01	0.00	0.00	-0.03	-0.01	-0.02	-0.01	-0.01	2.22
Dur.	0.00	0.67	0.69	0.67	1.33	2.62	3.10	4.39	4.77	4.28	13.02
Gen.	0.00	-0.01	0.02	-0.01	-0.30	-0.15	-0.14	0.46	-0.24	-0.11	0.34

As expected the computation time drops significantly with the number of points being removed. This is especially evident for the concave case where there are less constraints and binary variables in the problem. Moreover, one can observe that the expected objective is not changed much between the cases. This is due to the fact that points on the generation curve are being removed regardless whether it results in an increase or decrease in overall efficiency. The changes in shape of the duration function increases the more points are removed from the generation function. This is especially clear for the segmented regression case, but not unexpected as seen from Figure 5, where the best operating points shift slightly. Lastly, another expected result occurs where there is a positive change in generation for the concave case.

Table 2: Results from System B. Simulation time is given in seconds, whereas the other values are given as a percentage. Val. represents the expected objective value, Dur. Indicates the percentage deviation of the duration curve compared to the original problem. Gen. is the expected generation for the two years.

	A. Rem.					Seg. Reg.		Conc.
	0	2	4	6	8	6	7	
Time	335	284	238	218	180	197	178	77
Val.	0.00	-0.03	-0.01	-0.05	-0.58	0.01	0.09	2.05
Dur.	0.00	0.16	0.50	0.52	9.99	1.04	2.95	15.09
Gen.	0.00	0.00	-0.04	0.06	-0.76	0.09	0.01	1.02

The duration curve for selected simulations are shown in Figure 6 illustrates some duration curves for System A. The degree of regulation, i.e. the system's ability to store water for usage at a later stage, is very high for the system. Thus, the model will shift as much of the generation as possible to the hours in the winter where the prices are highest. Also, considering that the efficiency of the station is low at low power outputs the model will tend to avoid such operation. The best efficiency of the station is obtained around 180 MW, where a large portion of the generation comes from. Note that the concave

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generation function is not able to represent the low efficiency at low power outputs, some generation is therefore observed around 60 MW for this simulation. The yellow line shows the duration curve for the simulations with the segmented regression method applied. As expected the generation points are somewhat shifted from the original problem. This illustrates the importance of the generation function modelling on the operation of the power station. This method would, as above-mentioned, get a much more robust representation of the generation function had there been more measurements to fit.

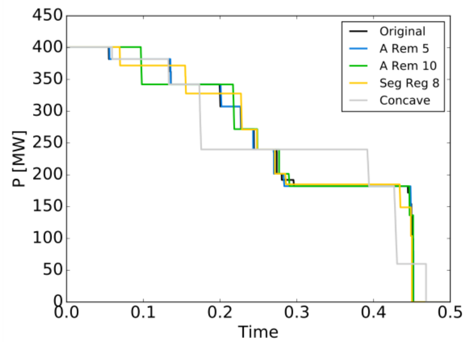

Figure 6: Duration curve for a selection of the results from System A. A. Rem. refers to the Area removal method and how many points are remove. Seg Reg 8 is for the segmented regression and the grey line is for the concave generation function.

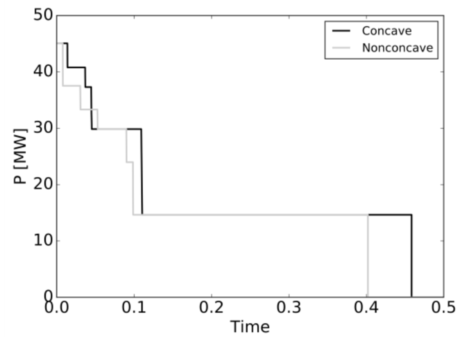

Figure 7: Duration curve for a selection of the results from System B. Minimum discharge limit during summer months.

Figure 7 shows the duration curve when Station B ran with a minimum generation requirement during the summer months. This would result in much less flexibility in terms of storing the water for the hours of the year with highest prices. The simulations were performed as a test to investigate how the system reacts to generation on lower power outputs. The minimum power output for the station is 15 MW, as seen in the figure where most of the power station operation is performed. The tests were performed with the original generation function and the concave one. It can be seen that the shape of the duration curves is very much similar, but with much less generation for the original, nonconcave, problem. This shows a significant drawback with the concave generation function, it overestimates the efficiency of the station and therefore assume that it can generate more electricity than what is possible. For the given case the simulation with concave generation function had an expected generation of 14.1% more than the original one, indicating how much the generation can be overestimated for simplifications of the generation function. For the other simulations this number was lower (1.02% and 0.34%), showing that how the station is operated has an impact on how much detail should be included in the generation function. Since it is expected that the market for ancillary services will grow and increased environmental constraints will occur, more operation of power stations at low power output can be more common in the future.

6. Conclusion and Further Work

In the given paper a segmented regression approach and a greedy algorithm have been proposed and tested to simplify the piecewise linear generation function used to model hydropower stations. Impacts from environmental constraints and how they alter the operation has been examined, demonstrating the importance of a more detailed modelling of the generation function.

The assumption that the generation function is only dependent on discharge is reasonable for stations with high head and less regulation of the reservoirs. On the contrary, if this is not the case further work will investigate methods that can include the head dimension in the generation function.

Acknowledgments
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Nonconvex Medium-Term Hydropower Scheduling by Stochastic Dual Dynamic Integer Programming

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***Abstract*—Hydropower producers rely on stochastic optimization when scheduling their resources over long periods of time. Due to its computational complexity, the optimization problem is normally cast as a stochastic linear program. In a future power market with more volatile power prices, it becomes increasingly important to capture parts of the hydropower operational characteristics that are not easily linearized, e.g. unit commitment and nonconvex generation curves.**

Stochastic dual dynamic programming (SDDP) is a state-of-the-art algorithm for long- and medium-term hydropower scheduling with a linear problem formulation. A recently proposed extension of the SDDP method known as stochastic dual dynamic integer programming (SDDiP) has proven convergence also in the nonconvex case. We apply the SDDiP algorithm to the medium-term hydropower scheduling (MTHS) problem and elaborate on how to incorporate stagewise dependent stochastic variables on the right-hand sides and the objective of the optimization problem. Finally, we demonstrate the capability of the SDDiP algorithm on a case study for a Norwegian hydropower producer.

The case study demonstrates that it is possible but time-consuming to solve the MTHS problem to optimality. However, the case study shows that a new type of cut, known as strengthened Benders cut, significantly contributes to closing the optimality gap compared to classical Benders cuts.

***Index Terms*—Stochastic processes, Dynamic programming, Hydroelectric power generation, Integer programming.**

I. Introduction

THE hydropower scheduling problem is difficult given its stochastic and multistage nature, and a variety of different solution techniques have been applied to it, see e.g. [1], [2]. In a liberalized power market, one typically decomposes the overall problem into three hierarchies (long-, medium-, and short-term) according to the system boundary, the level of required details in the hydro system and the representation of different stochastic variables [3], [4], [5]. In this work we consider the medium-term hydropower scheduling (MTHS) problem, aiming at maximizing a single producer's profit. The MTHS problem links the long-term fundamental market model problem with the short-term (ST) operational scheduling problem, as discussed in [3]. The MTHS problem can be formulated as a multistage stochastic programming (MSSP) problem, with many decision stages and uncertainty of inflow and market prices.

Due to the complexity of the MTHS problem, it is typically approximated as a multistage stochastic linear programming (MSSLP) problem. To further exploit the flexibility of hydropower by contributing in ancillary service markets, it becomes increasingly important to capture parts of the operational characteristics that are inherently nonconvex. Nonconvexities arise from binary variables used to model minimum generation limits and unit commitments, and are vital for capturing the generation units' capability to provide reserve capacity. There are also other types of nonconvexities that occur in the MTHS problem, such as the generation-discharge function that is dependent on the water head, discharge from multiple reservoirs to one power station and other topological constraints in the hydropower system. These nonconvexities should not only be represented in the ST operative models but also in the MTHS models that provide the *expected* opportunity cost of water to them.

A. MSSP Problems

The MTHS problem can be formulated as a multistage stochastic program, of the following general extensive form

$$\max_{(x_n,y_n)} \left\{ \sum_{n\in\mathcal{T}} p_n f_n(x_n,y_n) : (x_{a(n)},x_n,y_n) \in X_n, \forall n \in \mathcal{T} \right\}. \quad (1)$$

In the above formulation, $\mathcal{T}$ is a scenario tree given by a set of nodes, n, that describes the underlying stochastic process $\{\tilde{\xi}_t : \forall t = 1,\dots,T\}$. Each node is assigned a probability p_n, and has a unique ancestor node $a(n)$ and set of children nodes $\mathcal{C}(n)$. For each node we define x_n and y_n as vectors comprising the *state* and *stage* variables, respectively. The state variables are those used to carry information from one period to the next. The initial state of the system is x_0, f_n is the objective function and X_n is the set of constraints. Note that the constraint set comprises time-linking constraints, connecting the state variables in $x_{a(n)}$ and x_n.

The size of the MSSP in (1) grows dramatically with the number of decision stages and number of children nodes considered in the scenario tree. Thus, MSSP problems are often solved by decomposition [6]. Stagewise decomposition [7] has become a popular technique for efficiently solving

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the hydropower scheduling problems [8], [9]. Such a decomposition scheme involves dividing (1) into subproblems for each decision stage. Each subproblem comprises a part of the objective function corresponding to that stage and an expected future profit (EFP)[1] function.

B. Stochastic Dual Dynamic Programming

The stochastic dual dynamic programming algorithm (SDDP) presented in [10] is a stagewise decomposition method for solving the long- and medium-term hydropower scheduling problem. For a scenario tree corresponding to a stagewise independent stochastic process, the SDDP method proceeds by sampling in the scenario tree and sharing Benders cuts between nodes belonging to the same stage. Proof of almost sure convergence for the SDDP algorithm has been given by [11], and for risk averse problems in [12], [13]. The SDDP algorithm has been frequently addressed in the recent literature. Some of the work involves time-inconsistent policies [14], co-optimization of hydro- and wind power [15], joint treatment of energy and ancillary services [16], [17], uncertainty in price [18] and uncertainty in inflow [19].

In spite of considerable research effort, see e.g. [20], [21], [22], [23], [24], the SDDP method does not easily facilitate nonconvexities. As mentioned previously, nonconvexities arise, for example, when representing the detailed relationship between power output and water discharge [25], and the exact unit commitment of generators [26]. The core issue is how the nonconvex EFP function can be modeled. In [21] McCormick envelopes were applied to regions of the nonconvex function between power, discharge and water head. In [22] Lagrangian relaxation was used to convexify the EFP function, whereas [24] applied locally valid cuts to represent the nonconvex EFP function. All the above methods produce solutions to different forms of relaxations rather than the original problem.

Recently [27] proposed a method referred to as stochastic dual dynamic integer programming (SDDiP) allowing integer variables within the SDDP method, i.e. a solution to the multistage stochastic integer programming (MSSIP) problem. By approximating all state variables with binary variables, the authors were able to prove finite convergence as long as the cuts satisfy some sufficient conditions.

Another new and promising method that can be used to solve the nonconvex MTHS problem is reported in [28]. As SDDiP it is an extension of SDDP for handling nonconvexities. The significant difference is that instead of using cutting planes to describe the EFP function, step functions are used and that a monotonic increasing EFP function is required.

C. Contributions

In this work we apply the SDDiP method presented in [27] to the MTHS problem with a nonconvex function of power and discharge, incorporate stagewise dependencies in stochastic variables and correlations between them (inflow and energy price). We particularly extend the modeling from [27] with stagewise dependency in energy price, since this dependency enters the objective function and thus introduces a nonconvex term that is challenging to handle.

[1] or expected future cost when considering cost minimization problems.

The SDDiP method is tested and verified on a hydropower system in Norway. We evaluate the performance of the SDDiP algorithm on the MTHS problem using different types of cuts and provide recommendations for which types of cuts are most efficient in solving the MTHS problem.

The paper is organized as follows. In the next section, we will describe the basic modeling of the MTHS problem followed by the fundamentals of the SDDiP method. The case study is presented in Section VI and computational results in Section VII followed by concluding remarks in Section VIII.

II. The MTHS Problem

In a liberalized electricity market the hydropower producer's primary objective is to find operational strategies for each decision stage and maximize the total profit. For the Nordic electricity market, it is assumed that there is a sufficient number of players such that a price-taker assumption is reasonable.

The state variables, x_t, in the MTHS problem considered here are reservoir levels, generator status, inflow and energy price. The stage variables are represented by y_t and include the operational decisions at that stage. First, let us assume that the set of state variables consists of both continuous and binary variables. For the MTHS problem, given by (2), we want to find an operating strategy that maximizes profit in (2a), comprising the revenues from selling energy and capacity reserves, start-up cost and penalty functions ensuring relatively complete recourse. The expectation in (2a) is taken over the stochastic parameter $\tilde{\xi}_t$, representing energy price and inflow.

$$\max_{(x_1,y_1),\dots,(x_T,y_T)} \mathbb{E}\left\{ \sum_{t=1}^{T} f_t(x_t, y_t) \right\} \tag{2a}$$

$$\text{s.t.} \quad W x_t + H x_{t-1} + G y_t = h(\tilde{\xi}_t) \tag{2b}$$

$$B y_t = 0 \tag{2c}$$

$$C y_t - D x_t \geq 0 \tag{2d}$$

$$C y_t + D x_t \leq C y^{\max} \tag{2e}$$

$$x_t, y_t \in Y_t \tag{2f}$$

$$x_t \in \mathbb{R}^{k_1} \cdot \mathbb{Z}^{k_2}, y_t \in \mathbb{R}^{l_1} \cdot \mathbb{Z}^{l_2} \tag{2g}$$

$$\forall t \in \{1, 2, \dots, T\}, \tag{2h}$$

where the initial state vector x_0 is given and W, H, G, B, C and D are matrices of suitable dimensions. The right-hand-side parameter vector, $h(\tilde{\xi}_t)$, is dependent on the random data vector $\tilde{\xi}_t$ whose distribution is known, and where ξ_t are the realizations. Due to strong autocorrelation, both inflow and the energy price should be considered as an affine function of state variables. Thus the profit function for stage t is

$$f_t(x_t, y_t) = c_t(x_t) y_t^G + g_t(y_t), \tag{3}$$

comprising the energy price, $c_t(x_t)$, multiplied with generation, y_t^G, and the remaining linear relationships in $g_t(y_t)$. Treatment of the bilinear expression in (3) will be addressed in Section III-B.

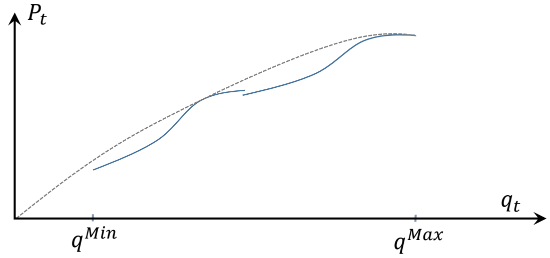

Fig. 1. Illustration of the nonconvex function of power, P_t, and discharge, q_t in a power station with two units in blue. It is conditioned that the first unit is started before the second. The dotted line illustrates the convex relaxation of the function.

The time-linking constraint, (2b), contains all reservoir balances, generator state time-couplings and the Vector Autoregressive model with lag-1 (VAR-1) constraints, outlined in Section III-A. The energy balance is given by (2c), where we assume that all power is sold to the market. Generation and reserve capacity limits are included in (2d) and (2e). We assume that the hydropower producer can offer spinning reserve capacity, meaning that the generator has to be operating to provide this service. The reserve capacity is also assumed symmetric, i.e. an equal amount of both upwards and downwards reserve capacity is sold. For conciseness, we also include constraints that limit spillage, bypass and generation from different reservoirs in (2f). In practice, spillage may only occur when the water head is greater than the bottom of the spillway. If, however, the optimization model finds that the water has better utilization in a lower reservoir, it will spill or bypass water to that reservoir, even-though this might not be feasible in practice. By including binary variables and additional constraints in (2f), we can prohibit this behavior of the model. Similar to when a power station can operate from two different reservoirs with uneven head, we can restrict the model to only operate from one reservoir at the time.

As an extension to the MTHS problem we include a nonconvex relationship between power and discharge for each hydropower station, referred to as the generation function. Much work has been done recently, see [21], [22], to include a nonconvex generation function, also with water head dependencies. Considering the modest water head variations of most Norwegian hydropower stations and to limit the complexity of the modelling, with added computational time, we omit this in our approach. Our experience has shown that it is important to include the nonconvex generation function when selling both energy and reserve capacity in order not to overestimate the sales of capacity reserves [15]. This is especially imperative for periods with low energy price and high capacity price. Then the model wants to operate the units at low power outputs in order to participate in the capacity reserve market. An illustration of a generation function is given in Figure 1.

For presentation simplicity, the reserve capacity price is assumed to be deterministic, but as the work done in [16] shows, it could be extended to a stochastic variable.

III. The SDDiP Formulation

In this section, we reformulate the problem (2) to a form suitable for SDDiP. First, we describe how the stochastic processes are handled.

A. Stochastic Processes

We consider two types of uncertainties in the problem; inflow to the hydro reservoirs and energy price. For the sake of simplicity both processes are assumed to have a normal distribution. Since both stochastic processes exhibit a seasonal pattern and are non-stationary processes, these are normalized by subtracting the seasonal mean and dividing by the seasonal standard deviation. Moreover, we shift the mean of the normalized value to ensure non-negative values. The normalized and shifted series are then assumed to be stationary and fitted to a Vector Autoregressive model with lag-1 (VAR-1), which enables us to account for seasonal effects and capture correlations between the time series within the SDDiP framework. In order to get a manageable number of stochastic outcomes of $\tilde{\xi}_t$, we applied the “Fast forward selection” scenario reduction algorithm in [29], resulting in a confined amount of scenarios with a corresponding probability. Another known method to handle the uncertainty of energy price in SDDP is by an outer Markov chain as proposed in [30]. The drawback is then that there are two independent stochastic processes in the model.

The VAR-1 model includes a state variable, δ_t. Since it is a state variable, binary expansion has to be applied to it. After the scenario reduction and sampling of the stochastic parameters are completed, a bound can be computed so that a finite support is guaranteed. Subsequently, we need to shift the mean of the process by U such that δ_t is non-negative. The bound is then given by (4b) and δ_t is the normalized and shifted values. We can derive the following VAR-1 model, which is included in (2b)

$$\delta_t = \Phi\delta_{t-1} + \tilde{\xi}_t \tag{4a}$$
$$\delta_t \in [0, 2U]^2, \tag{4b}$$

where Φ is the correlation matrix. Note that $\mathbb{E}[\delta_t] = U$ and $\mathbb{E}[\tilde{\xi}_t] = U$. The stochastic parameters for inflow and energy price are respectively given by

$$\begin{bmatrix} I_t(\delta_t) \\ c_t(\delta_t) \end{bmatrix} = \sigma_t\big(\delta_t - U\big) + \mu_t. \tag{5}$$

where the inflow, $I_t(\delta_t)$, is the right-hand side of the reservoir constraints and the energy price, $c_t(\delta_t)$, is expressed in the objective function (3). Note that δ_t in the stochastic model is identical with x_t in (3) as the stochastic model was not yet introduced. In order to solve the bilinear term in (3) an approximation is applied.

B. Approximation of the Energy Price

To circumvent the computational complexity introduced by the bilinear term in (3), we apply an approximation by linear relaxation of the objective function term, $c_t(\delta_t)y_t^G$. The relaxation is applied by using the binary expansion method to

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the energy price state variable, δ_t. The method is based on the fact that if λ_t is an integer variable, $\lambda_t \in \{0, \dots, L\}$, it can be represented by κ binary variables, $\kappa = \lfloor \log_2(\mathrm{L}) \rfloor + 1$, such that $\lambda_t = \Sigma_{j=1}^{\kappa} 2^{j-1} \lambda_{tj}$. Similarly, for the continuous case, $\lambda_t \in [0, L]$, where λ_t is given with ε accuracy; $\kappa = \lfloor \log_2(\mathrm{L}/\varepsilon) \rfloor + 1$, hence $\lambda_t = \Sigma_{j=1}^{\kappa} 2^{j-1} \varepsilon \lambda_{tj}$. See e.g. [31] for further details about the binary expansion. In the following we use the notation $\lambda_t = \sum_{j=1}^{\kappa} A_{tj} \lambda_{tj}$. Thus, we redefine $\delta_t = \sum_{j=1}^{r} A_{tj} x_{tj}$, with $r = \lfloor \log_2(2\mathrm{U}/\varepsilon) \rfloor + 1$ and substitue δ_t in (5). Moreover, we then replace the bilinear terms $x_{tj} y_t^G$ with auxiliary variables w_{tj}, and constrain these by (6b)-(6e), for all $j \in \{1, \dots, r\}$. The objective function becomes,

$$f_t(x_t, y_t) = \sigma_t \sum_{j=1}^{r} A_{tj} w_{tj} + (\mu_t - \sigma_t U) y_t^G + g(y_t) \quad (6a)$$

$$\text{s.t.} \quad w_{tj} - y_t^G \le 0 \quad (6b)$$

$$w_{tj} - M_{tj} x_{tj} \le 0 \quad (6c)$$

$$w_{tj} - y_t^G + M_{tj}(1 - x_{tj}) \ge 0 \quad (6d)$$

$$w_{tj} \in \mathbb{R}^r, \ x_{tj} \in \{0, 1\}^r \quad (6e)$$

$$\forall j \in \{1, \dots, r\}. \quad (6f)$$

We fix M_{tj} to $y^{G,\max}$ for all j, in order to get an accurate relaxation.

Note that after the binary expansion is applied to the state variable, δ_t, (4a) becomes a constraint with only binary variables and a stochastic parameter. Thus, in order to ensure feasibility of the model with all possible outcomes of $\tilde{\xi}_t$, the constraint is altered to a less than or equal constraint. Implications of this are discussed in Section VII.

C. Dynamic Programming (DP) Formulation

From the MTHS problem given by (2) and the energy price approximation in (6), we have the following DP equation

$$\mathcal{Q}_t(x_{t-1}, \xi_{t-1}) := \max_{x_t, y_t} f_t(x_t, y_t) + \phi_t(x_t) \quad (7a)$$

$$\text{s.t.} \quad (x_t, y_t) \in X_t(x_{t-1}, \tilde{\xi}_t) \quad (7b)$$

$$x_t \in \mathbb{R}^{k_1} \cdot \mathbb{Z}^{k_2}, y_t \in \mathbb{R}^{l_1} \cdot \mathbb{Z}^{l_2}. \quad (7c)$$

Problem (7) consists of the present objective function $f(x_t, y_t)$, the *true* EFP function $\phi_t(x_t)$ and constraint set X_t, with parameters x_{t-1} and $\tilde{\xi}_t$, described by (2b)-(2f) and (6b)-(6f).

As the MTHS problem contains integer stage variables the EFP function is nonconvex with respect to the state variables. Existing nested decomposition methods rely on convex relaxation to approximate the EFP function, and convergence can therefore not be guaranteed.

The traditional approach to solving the nonconvex problem in (7) has been to relax the problem formulation to an LP. We are then able to define a convex relaxation of the EFP function given by,

$$Q_t^i(x_{t-1}, \xi_{t-1}) := \max_{x_t, y_t} f_t(x_t, y_t) + \varphi_t^i(x_t) \quad (8a)$$

$$\text{s.t.} \quad (x_t, y_t) \in X_t^{'}(x_{t-1}, \tilde{\xi}_t) \quad (8b)$$

$$x_t \in \mathbb{R}^{k_3}, y_t \in \mathbb{R}^{l_3}. \quad (8c)$$

The EFP function is then $\varphi_t^i(x_t)$, for a given iteration i of the SDDP method, and constraint set $X_t^{'}$ that describes a LP relaxation of X_t. We see that this DP formulation is similar to the SDDP method, where the EFP function is described by hyperplanes, defined as

$$\varphi_t^i(x_t) := \max\{\theta_t \le V_t \quad (9a)$$

$$\theta_t \le v_{t+1}^l + \pi_{t+1}^l x_t, \ \forall l \in \mathcal{H}(i), \quad (9b)$$

$$x_t \in \mathbb{R}^{k_3}\}. \quad (9c)$$

Where the set $\mathcal{H}(i)$ contains the cuts that are used to approximate the EFP function up to iteration i. The cuts are represented with a right hand side parameter v_{t+1}^l, the coefficient π_{t+1}^l and θ_t is a scalar variable representing the value of the EFP function. The different type of cuts that we have used in our implementation will be outlined in Section IV-A.

To solve the nonconvex MTHS problem we apply the SDDiP method proposed in [27]. The key concept of the method is that any function of binary variables can be represented as a convex polyhedral function. This can, therefore, be achieved by applying the above-mentioned binary expansion to all integer and continuous state variables of the original problem.

After the binary expansion is applied to the state variables in (2g), $x_t \in \mathbb{R}^{k_1} \times \mathbb{Z}^{k_2}$, it is reformulated to $x_t \in \{0, 1\}^k$, such that all state variables in the MTHS problem are binary and representing the original continuous state variables to ε-accuracy. Note that $k \neq k_1 + k_2$, as it depends on the binary expansion.

Since the possible state variables solutions are given by a finite set of binary variables, we can generate a finite number of cuts to approximate it, as the convex hull of the binary state space ensures that we can compute tight bounds. See Figure 2 and its explanation below.

Another reformulation used in the SDDiP method is to generate local copies of the state variables. This reformulation is crucial to ensure that one is able to generate cuts that accurately approximate the EFP function. The DP formulation of the problem with binary state variables becomes

$$\underline{Q}_t^i(x_{t-1}, \xi_{t-1}) := \max_{x_t, y_t, z_t} f_t(x_t, y_t) + \psi_t^i(x_t) \quad (10a)$$

$$\text{s.t.} \quad (x_t, z_t, y_t) \in X_t^{''}(\tilde{\xi}_t) \quad (10b)$$

$$z_t = x_{t-1} \quad (10c)$$

$$z_t \in [0, 1]^k \quad (10d)$$

$$x_t \in \{0, 1\}^k, y_t \in \mathbb{R}^{l_1} \cdot \mathbb{Z}^{l_2}. \quad (10e)$$

Where (10c) is the essential copy constraint, connecting the previous state solution, x_{t-1}, and the copy variable z_t. The set $X_t^{''}$ is obtained by transforming all state variables in X_t into binaries. The EFP function, $\psi_t^i(x_t)$, is defined as (9), with the differentiation that all the state variables has to be binary.

IV. Review of the SDDiP Approach

The SDDiP approach builds on the same principles as SDDP, where each main iteration comprises a forward and a backward iteration, as shown in Algorithm 1. We can summarize the main differences between SDDiP and SDDP into

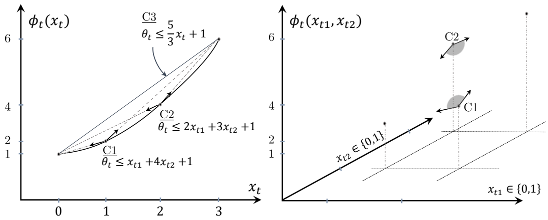

Fig. 2. Illustration of the nonconvex EFP function (left) and the transformation into the binary state (right). Note that we are solving a maximization problem and thus the function is nonconcave, but in line with standard terminology, we refer to it as a nonconvex function. Assume that the continuous variable x_t is transformed by two binary variables x_{t1} and x_{t2}, such that $x_t = x_{t1} + 2x_{t2}$. By constructing two hyperplanes (C1 and C2) in the binary state space, we can illustrate how they are projected to the continuous space. This illustrates how by using binary state variables we are able to construct cuts that approximate a nonconvex function. The hyperplane C3 is added to show the tightest cut we are able to generate from the convex relaxation of the nonconvex function.

Algorithm 1 The SDDiP Method

1: Set x_0^i, ξ_0^i, $i \leftarrow 1$, UB $= +\infty$ and LB $= -\infty$,
2: Apply binary expansion on continuous and integer state variables
3: **while** $i < i^{\max}$ **or** some other stopping criteria **do**
4: Sample N scenarios $\Omega^i = \xi_1^k, \ldots, \xi_T^k{}_{k=1,\ldots,N}$
5: /* Forward Iteration */
6: **for** k=1,. . . ,N **do**
7: **for** t=1,. . . ,T **do**
8: Solve $\underline{Q}_t^{ik}(x_{t-1}^{ik}, \xi_{t-1}^{ik})$
9: Collect solution $f_t(x_t^{ik}, y_t^{ik}, \xi_t^{ik}), x_t^{ik}, y_t^{ik}, z_t^{ik}$
10: $\mathrm{lb}^k \leftarrow \Sigma_{t=1,\ldots,T} f_t(x_t^{ik}, y_t^{ik}, \xi_t^{ik})$
11: /* Compute lower bound */
12: $\mu \leftarrow \frac{1}{N}\Sigma_{k=1}^N \mathrm{lb}^{\mathrm{k}}$ and $\sigma^2 \leftarrow \frac{1}{N-1}\Sigma_{k=1}^N(\mathrm{lb}^{\mathrm{k}} - \mu)^2$
13: $\mathrm{LB} \leftarrow \mu + \mathrm{z}_\alpha \frac{\sigma}{\sqrt{\mathrm{N}}}$
14: /* Backward Iteration */
15: **for** t=T,. . . , 2 **do**
16: **for** k=1,. . . , N **do**
17: **for** m$\in \mathcal{C}(t)$ **do**
18: Solve a suitable relaxation of
19: $\underline{Q}_m^{ik}(x_t^{ik}, z_t^{ik}, \xi_t^{ik})$
20: Collect cut coefficients and parameters
21: Add desired cut(s) as described in Sec. IV-A
22: /* Compute upper bound */
23: $\mathrm{UB} \leftarrow \underline{\mathrm{Q}}_1^{\mathrm{i}}(\mathrm{x}_0^{\mathrm{i}}, \xi_0^{\mathrm{i}})$
24: $i \leftarrow i + 1$

three pillars; the requirement of discretization of state variables into binary variables, introduction of the copy variable and the different cut families it facilitates.

Scenarios are sampled from the underlying stochastic process in line 4 in Algorithm 1. The forward iteration solves the problem given in (10) for each stage and scenario, providing candidate solutions to the problem in line 9 and an α confidence lower bound on the optimal value.

The backward iteration operates along the N state trajectories obtained in the previous forward iteration to compute cuts that are passed backward in time. The different types of cuts considered are described in the next section. The value of the objective function in the first stage will then represent an upper bound of the true optimal solution in line 23. Convergence is achieved if the lower bound is within a statistical confidence interval of the upper bound.

A. Cut Families

SDDiP is defined on a notion of *valid* and *tight* cuts. A cut is *valid* and *tight* if it supports the EFP function. For maximization problems, it is only *valid* if it provides an upper approximation of the EFP function. An illustration is given in Figure 2, where the cuts C1, C2 and C3 are all valid, but only C1 is tight when the state variable has the value 1.

The different cut families that could be used within SDDiP are briefly described below.

1) Benders Cuts (B): Benders cuts (see [32]) are constructed by solving the LP relaxation of problem (10). We obtain the following Benders cut for stage t in iteration i

$$\theta_t \leq \sum_{m \in \mathcal{C}(t)} q_{tm} \left[\underline{Q}_m^{\mathrm{LP}} + (\pi_m^i)^\top (x_t - x_t^i)\right]. \tag{11}$$

Where π_m^i is a vector of dual values corresponding to (10c), q_{tm} is the probability of going to a child node m, in decision stage t and $\underline{Q}_m^{\mathrm{LP}}$ is the optimal LP relaxation value. Convergence is not guaranteed with only Benders cuts for SDDiP. In some practical scheduling cases, the Benders cuts may be far from tight, as we will see in the Section VI-B.

2) Lagrangian Cuts (L): This cut family is based on Lagrangian relaxation, where we relax the copy constraint (10c). The Lagrangian multiplier is obtained by

$$\bar{\pi}_t^i := \underset{\bar{\pi}_t}{\operatorname{argmin}} \left\{ \mathcal{L}_t^i(\bar{\pi}_t) + \bar{\pi}_t^\top x_{t-1} \right\}, \tag{12}$$

and $\mathcal{L}_t^i$ is defined as

$$\mathcal{L}_t^i(\bar{\pi}_t) := \max_{x_t, y_t, z_t} f_t(x_t, y_t, \xi_t) + \psi_t^i(x_t) - \bar{\pi}_t^\top z_t \tag{13a}$$

$$\text{s.t.} \quad (z_t, x_t, y_t) \in X_t^{''}(\tilde{\xi}_t) \tag{13b}$$

$$z_t \in [0,1]^k \tag{13c}$$

$$x_t \in \{0,1\}^k, y_t \in \mathbb{R}^{l_1} \cdot \mathbb{Z}^{l_2}. \tag{13d}$$

Lagrangian relaxation often aims to relax complicating constraints and divide the problem into smaller subproblems that can more easily be computed. Our aim, however, is to find good multipliers that make the Lagrangian cuts as tight as possible. It is essential for the convergence of SDDiP that the Lagrangian cuts can be generated in a sufficient manner, in regard to both computation time and tightness. By repeatedly solving (12) and updating the Lagrange multipliers $\bar{\pi}_t$, e.g. using the subgradient [33] or level [34] methods, one can construct tight and valid Lagrangian cuts, as described in [27]. Nevertheless whichever method we use to get the Lagrangian multiplier, $\bar{\pi}_t^i$, the cut we construct is of the following form,

$$\theta_t \leq \sum_{m \in \mathcal{C}(t)} q_{tm} [\mathcal{L}_m^i(\bar{\pi}_m^i) + (\bar{\pi}_m^i)^\top x_t]. \tag{14}$$

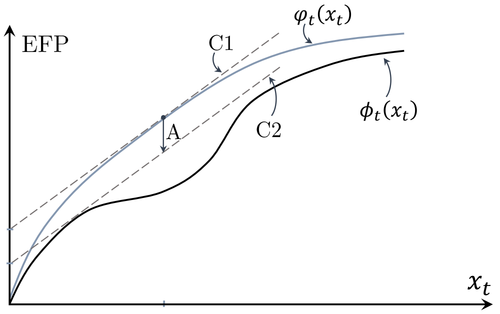

Fig. 3. Illustration of improvement for the strengthened Benders cut (C2) compared to Benders cut (C1) by generating a tighter right-hand-side parameter, equaling the distance of A. $\varphi_t(x_t)$ is the LP relaxation of the original nonconvex function $\phi_t(x_t)$.

3) Strengthened Benders Cuts (SB): The *strengthened Benders cuts* presented in [27] are parallel to the regular Benders cut and are at least as tight. They are not guaranteed to be tight, but they may improve the solution quality significantly compared to the Benders cuts, as illustrated in Figure 3. The increased precision comes at a modest increase in computation time.

Strengthened Benders cuts are constructed as follows. First, we solve the LP relaxation of problem (10). Then problem (13) is solved with $\bar{\pi}_t$ equals to the optimal LP dual solution with respect to constraint (10c), π_t^i. From the solution of the latter problem we obtain the right-hand side $\mathcal{L}_t^i(\pi_t^i)$ and then construct the following strengthened Benders cut

$$\theta_t \leq \sum_{m \in \mathcal{C}(t)} q_{tm}[\mathcal{L}_m^i(\pi_m^i) + (\pi_m^i)^\top x_t]. \tag{15}$$

The strengthened Benders cuts do not require the state variables to be binary and can such be used to achieve good solutions to problems that do not possess highly nonconvex properties.

4) Integer Optimality Cuts (I): The last family of cuts we have used in our method is the *integer optimality cuts* [35], [36]. The integer optimality cuts are valid, tight and finite and will, therefore, guarantee convergence. They are also very fast to generate, as they only rely on the solution obtained in the forward iteration.

B. Cut Discussion

As mentioned in Section I-B, the Lagrangian cuts proposed by [27], and used in this paper, differentiate from the Lagrangian cuts proposed by [22], [21]. Instead of introducing copy constraints they dualize the time-linking constraints and are therefore not able to guarantee tight cuts, even-though they in many cases are stronger than the Benders cuts. This can be shown by a small example. Consider,

$$Q(x) = \min_{y_1, y_2}\{y_1 + y_2 : 2y_1 + y_2 \geq 3x, \quad y_1 \in \{0,1,2\}, y_2 \in [0,3]\} \tag{16}$$

Where $Q(x)$ is a value function dependent on the binary state x, and y_1 and y_2 are local variables. First, by dualizing the time-linking constraint and providing $x = 1$ as a candidate solution one gets the following Lagrangian cut $\theta \geq 1.5x$, this is essentially the method proposed by [22], [21] and as the small example shows it does not necessarily provide tight cuts. Furthermore, let's add a copy constraint $z = x, z \in [0,1]$ and swap x with z in the time-linking constraint. By dualizing the copy constraint we can compute the Lagrangian cut $\theta \geq -1 + 3x$, with $x = 1$ as a candidate solution. For $x = 0$ we get that $\theta \geq 0$, such that we are able to provide a tight approximation of $Q(x)$.

As proven in [27] and partially illustrated above, the SDDiP algorithm will converge if Lagrangian or integer optimality cuts are added. Benders and strengthened Benders cuts will not guarantee convergence, but will often serve to improve convergence rate in concert with Lagrangian or integer optimality cuts. Benders cuts are by far the least computational demanding type of cuts since their construction only involves solving LP problems. For this reason, we only add Benders cuts until a stable upper bound is reached. Subsequently, we add the other cut families to further tighten the EFP functions.

C. The Approximate SDDiP Problem

The SDDiP method depends on the aforementioned three pillars to ensure finite convergence. However, we are still able to use some of its features if we do not restrict the state variables to be binary. Finite convergence can then no longer be guaranteed, with the trade-off of reduced computational complexity. We reformulate (7) by applying the same concepts of copy variables and copy constraints. For this problem we can only compute Benders and Strengthened Benders cuts. This is done in the same manner as for the (10) in Section IV-A1 and IV-A3. Results from this approach is also reported in Section VI-B

V. Bounding the EFP Function

Due to the bilinear objective term introduced when considering uncertain energy price, we relaxed the problem by artificial variables and constraints containing the big-M notation in (6). As we will elaborate, it is difficult to obtain tight bounds for that problem formulation. Consider the case where a given energy price state variable is $x_{tj} = 1$, we then have

$$\pi_{t,j}^E = \frac{\partial \underline{Q}_t^i}{\partial x_{tj}} = \frac{\partial \underline{Q}_t^i}{\partial w_{tj}} \cdot \frac{\partial w_{tj}}{\partial x_{tj}} = \frac{\partial \underline{Q}_t^i}{\partial w_{tj}} \cdot M_{tj}, \tag{17}$$

where $\pi_{t,j}^E$ is the dual value for the copy constraints for that energy price state variable in (10c).

From a practical standpoint, it is obvious that there are time periods and scenarios where y_t^G is far from $y^{G,\max}$. In this sense the proposed relaxation method has the same drawback as the integer optimality cut, but now the big-M notation cascades into all types of cuts, making it hard to compute cuts that are tight for not only the given candidate solution but nearby solutions as well. A modest upside is that the big-Ms depend on a physical value and not the expected future profit, contributing to a tighter formulation than for the integer optimality cuts.

Given the nature of the problem at hand, different techniques could be applied in order to make the big-Ms tighter by adding some heuristics on the bounds of y_t. Nevertheless, for an MTHS problem with a price taker assumption and no demand, this has proven difficult, as the only limiting constraints are generation limits and available water in the reservoirs.

Observe that the big-M formulation is only required for our implementation of the uncertainty of the energy price. The following theorem establishes a method for constructing an upper bound to this problem so as to be able to evaluate the policy quality.

Theorem 1. *Let v^* be the optimal value of the MTHS problem in* (2) *with price uncertainty, and v^{LP} be the optimal value of its LP relaxation. Let $\bar{v}^{\mathrm{LP}}$ be the optimal value of the LP relaxation where the uncertain energy price is replaced by its expected value. Then*

$$v^* \leq v^{\mathrm{LP}} \leq \bar{v}^{\mathrm{LP}}.$$

For brevity, we omit a detailed proof of the above result and provide a sketch. Note that the first inequality is simply owing to the LP relaxation. Since the price uncertainties are only on the right-hand side (recall the modeling approach discussed in Section III-B), we know that the optimal value of LP relaxation is a concave function of the uncertain parameters [6]. It follows by Jensen's inequality that $v^{\mathrm{LP}} \leq \bar{v}^{\mathrm{LP}}$, hence an upper bound to the original problem can be constructed by using the upper bound computed with only Benders cuts applied to the problem where energy price is modeled by its expected value.

VI. Case Study

For the case study, we considered a Norwegian hydropower reservoir system that consists of reservoirs with both short- and long-term storage capacity. The case study is documented in previous publications [15], [26]. The system contains three hydropower reservoirs, with two power stations and a total installed capacity of 414 MW. One of the modeling issues with this system is that the two lower reservoirs, with uneven water head, are connected to the same power station, such that the station can only generate from one of them at a time. Considering that one of the reservoir's size is 2% of the other with an equal amount of inflow, linear SDDP tended to overestimate its ability to avoid spillage. This is handled more precisely in the SDDiP framework.

The mean and standard deviation parameters for the stochastic processes were extracted from price forecasts obtained from a fundamental market model using inflow from 70 years of historical data as input. For simplicity, we only use a single inflow and energy price series in this work, as extensions to multiple are straightforward.

The stagewise decision problem of the SDDiP formulation consists of 137 constraints, 182 variables (120 continuous and 62 binary), 52 decision stages and 9 branches for each decision stage. In each forward iteration we sampled 2 scenarios and restricted the number of forward and backward iterations to

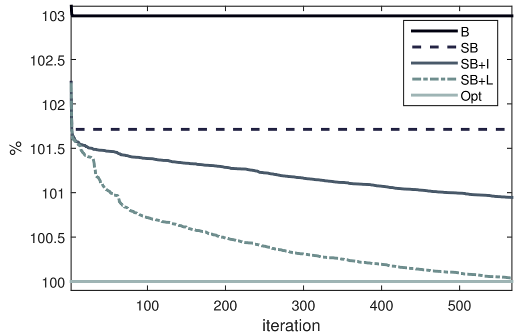

Fig. 4. Illustration of the upper bound for SDDiP of the MTHS problem with 6 stages, convex generation function and expected values for the stochastic parameters. Values are given with the optimal value as base.

50. Lastly, a final simulation of 300 forward scenarios was performed to evaluate the policy by calculating a 95% confidence interval of the lower bound. For an even comparison between the different methods we use the same scenarios in the forward iteration. Computations were performed on a Dell Latitude E7240 with an Intel Core i7-4600U processor with 2.7 GHz clock rate and 16 GB RAM. The problem was formulated in Python 3.5 with Gurobi 7.0 as the mathematical solver. We did not implement any parallel processing except from default settings in the MIP solver.

A. Validation

In order to validate the convergence properties of our implementation of the SDDiP method for the MTHS problem, a case was constructed with only 6 decision stages, convex generation function and the stochastic parameters modeled by their expected value. The upper bounds for different combinations of cut families are illustrated in Figure 4. Due to the reduced scale of the problem it is possible to compute the true optimal value, illustrated by the lower line in the figure. The figure shows the results from iteration 3 to 567, where convergence is obtained using strengthened Benders and Lagrangian cuts (SB+L). The reason to use Lagrangian and integer optimality cuts combined with strengthened Benders is that strengthened Benders cuts rapidly find a good stable solution, that the other cuts can start from. From the figure, it is clear that Benders (B) and strengthened Benders (SB) does improve only up to a certain point, from there either Lagrangian or integer optimality cuts are required to obtain convergence. This does, however, come at a cost of significant increase of computational time. The benefit of using strengthened Benders cuts compared to Benders cuts is also evident, as we see the gap is almost reduced by half.

B. Results

Now we turn to the 52-stage MTHS problem defined in Section II where two different case studies were performed. First, in Case I we modeled only uncertainty of inflow, then in Case II we also included uncertainty of the energy price.

TABLE I
CASE I, INFLOW UNCERTAINTY. SDDIP (TOP) AND APPROXIMATE PROBLEM (BOTTOM).

	UB [kEUR]	stat. LB [kEUR]	gap [%]	time [s]
B	47 353.72	40 776.87	13.89	16 792
B-SB	42 907.58	41 135.43	4.13	17 716
B-SB-I	42 854.80	41 197.25	3.87	30 277
B	47 030.08	40 973.28	12.88	742
B-SB	42 906.90	41 350.85	3.63	1 176

TABLE II
CASE II, INFLOW AND ENERGY PRICE UNCERTAINTY. SDDIP (TOP) AND APPROXIMATE PROBLEM (BOTTOM).

	UB [kEUR]	stat. LB [kEUR]	gap [%]	$\underline{\text{gap}}$ [%]	time [s]
B	107 469.94	42 276.74	60.66	11.24	11 788
B-SB	84 380.16	43 802.85	48.09	7.37	13 896
B-SB-I	84 291.78	43 748.69	48.10	7.50	18 966
B	107 449.36	42 444.20	60.50	10.80	2 279
B-SB	84 367.26	43 891.43	47.98	7.15	2 869

Subsequently, we tested the SDDiP method using Benders, strengthened Benders and integer optimality cuts. Lagrangian cuts were omitted due to computational requirement. We also computed the approximate problem, as defined in Section IV-C, with the Benders and strengthened Benders cut families. Recall that for the approximate problem the state variables are not required to be binary. The solution of the approximate problem, when only using Benders cuts, will also give an indication on how well the generic SDDP method performs for the MTHS problem.

The expected profit for the given case study with a final simulation of 300 forward scenarios is shown in Table I and Table II for Case I and II, respectively. The reported computation time includes final simulation and the gap is given as $\frac{UB-LB}{UB}$.

We obtained good solutions after 50 iterations in both the SDDiP and for the approximate problem. The approximate problem is solved faster as there are less binary variables, and the convergence gap is better for the given amount of iterations. When comparing the solutions from the Benders to the strengthened Benders cut families, for both SDDiP and the approximate problem, a significant improvement of the convergence gap is observed. This can also be seen in the convergence plot in Figure 5. We see that after iteration 20 when the strengthened Benders cuts were added, both the upper and lower bound for both cases improves.

The case with uncertain energy price results in a large gap. When using strengthened Benders cuts a significant improvement is observed compared to only Benders cuts. By using the upper bound from Theorem 1 we can use the approximate problem's value with Benders cuts and compute a tighter gap, referred to as $\underline{\text{gap}}$ in Table II. It shows that eventhough the upper bound initially was very high, the policy that it provides is quite good. Notably is the improvement with strengthened Benders cuts for the approximate problem with superior policy and computation time compared to the other results.

VII. DISCUSSION

We found that the integer cuts are not very computationally efficient compared to solution improvement when applied for the given case study. Lagrangian cuts were omitted from the large case due for the same reason, and are therefore not represented. On the other hand, the strengthened Benders cut significantly improves solution quality compared to ordinary Benders cuts with only a modest increase in computational

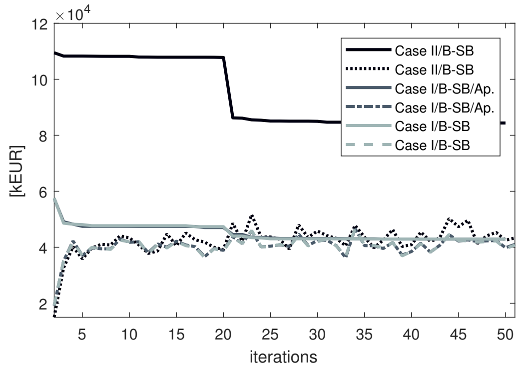

Fig. 5. Convergence plot for the different case studies with Benders and strengthened Benders cuts. The solid line indicates the upper bound and the other the expected lower bound. One can clearly observe how large the gap difference is between Case I and II. The approximate method also follows SDDiP close as seen for the dashed and dash-dot lines.

effort. We believe this is due to the characteristics of the presented MTHS case; it seems more important to adjust the right-hand side of the EFP function than to fine-tune its shape by adjusting the cut coefficients. Many Norwegian hydropower stations have high water head and their generation functions are not significantly affected by head variations. It may be that a nonconvex generation function with water head dependencies found in other systems, e.g. as reported in [22], will lead to a more pronounced shape of the EFP function, further justifying the need for Lagrangian and integer cuts.

Due to the introduction of very large numbers in the constraint matrix when adding integer optimality cuts, we have observed increased computation time for the subproblems. When comparing the computation time between the first stage subproblem for Case II after 50 iterations with SB and SB+I we observe an increase from 0.02 to 0.08 seconds. It is clear that the added constraints with large coefficients make it difficult for the solver to find a solution quickly. Subsequently, the integer optimality cuts do not provide good enough solution improvements to justify the added computational burden we observe, as seen in Table I and II.

Lastly, we observed that the lower bounds are higher for the approximate problem, when comparing computations with the same cut families, e.g. for Case I with Benders cuts the approximate problem has a lower bound of 40 973.28 and the SDDiP problem has 40 776.87. This follows from how the VAR-1 model is altered to a less than or equal constraint to ensure feasibility of the model, as discussed in Section III-B. In the final simulation, where we compute the lower bound, there are some extreme scenarios where the model are required to use an artificial penalty variable that ensures relative complete recourse. Since the inflow state in the SDDiP formulation is somewhat less than that of the approximate, due to the relaxation of the equality constraint, a subsequently higher penalty is observed in the order of the difference of the lower bounds that explains this observation.

VIII. Conclusion

With the earlier mentioned challenges flexible power producers encounter, we believe that binary state expansion and the SDDiP algorithm gives the producers a useful tool to address new challenges that involve nonconvexities in the modeling.

We observed that our method performs very well when omitting the uncertainty modeling of energy price, due to the drawback of the big-M formulation. Nonetheless, it allowed us to model the stochastic processes within a unified framework and we showed that the resulting policy from SDDiP is very good. Methods to improve the implementation of the uncertainty of the energy price will be investigated in future research, including methods on how to avoid the relaxation of the VAR-1 constraint.

In the case study, we found that the strengthened Benders cuts are superior to Benders cuts in finding good policies and bounds. We also found that the strengthened Benders cuts can be used in an approximate SDDiP method to provide satisfying results for nonconvex MTHS problems, that also includes a nonconvex generation function. To eventually close the optimality gap, however, one would have to use SDDiP and add either Lagrangian or Integer optimality cuts. Further, we observed that adding Integer optimality cuts did not improve the results enough to justify the increased computational time. We emphasize that these findings are case-specific, and expect that the efficiency of Lagrangian and Integer optimality cuts will be higher in scheduling problems with more pronounced nonconvexities.

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A.7 Energies

M. N. Hjelmeland, A. Helseth, and M. Korpås, "Medium-term hydropower scheduling with variable head under inflow, energy and reserve capacity price uncertainty," *Energies*, vol. 12, no. 1, 2019

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Article

Medium-Term Hydropower Scheduling with Variable Head under Inflow, Energy and Reserve Capacity Price Uncertainty

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Abstract: We propose a model for medium-term hydropower scheduling (MTHS) with variable head and uncertainty in inflow, reserve capacity, and energy price. With an increase of intermittent energy sources in the generation mix, it is expected that a flexible hydropower producer can obtain added profits by participating in markets other than just the energy market. To capture this added potential, the hydropower system should be modeled with a higher level of detail. In this context, we apply an algorithm based on stochastic dual dynamic programming (SDDP) to solve the nonconvex MTHS problem and show that the use of strengthened Benders (SB) cuts to represent the expected future profit (EFP) function provides accurate scheduling results for slightly nonconvex problems. A method to visualize the EFP function in a dynamic programming setting is provided, serving as a useful tool for a priori inspection of the EFP shape and its nonconvexity.

Keywords: hydropower scheduling; stochastic programming; integer programming

1. Introduction

Increasing rates of renewable energy generation are resulting in a higher demand for flexible power units to balance the power system and to deliver ramping capacity. Regulated hydropower is a flexible renewable energy source that is well suited to provide such services. The increased demand for flexibility has led to the requirement of more detailed optimization models, such that the flexible power units can perform an optimal allocation of resources in the different power markets for energy and ancillary services. In this work, we focus on rotating reserve capacity, providing what is normally referred to as primary and secondary reserves.

The stochastic dual dynamic programming (SDDP) algorithm proposed in [1] is commonly used for hydropower scheduling and can be seen as a sampling-based approach of the nested Benders decomposition proposed in [2]. The sampling-based method for solving multistage stochastic programming problems consists of two main procedures: the forward and backward pass. Instead of visiting all nodes in the scenario tree, the forward pass samples a set of scenarios used to generate candidate solutions. The backward pass follows the trajectories of the candidate solutions computed in the forward pass, starting from the final stage, to approximate the expected future profit (EFP) function. Subsequently, a statistical confidence interval can be computed for controlling the convergence of the method. A more in-depth explanation of the method can be found in [3–5].

In this work, we investigate how improvements in the SDDP algorithm, derived from the stochastic dual dynamic integer programming (SDDiP) algorithm [6], can be used to improve the medium-term hydropower scheduling (MTHS) problem under uncertainty. The MTHS problem normally covers a planning horizon of one to three years, aiming at maximizing a single producer's expected profit. We have from ongoing research experienced that SDDiP requires considerably more computational force than SDDP [7]. Nevertheless, an improved type of the Benders (B) cuts, called strengthened Benders (SB) cuts, derived from the SDDiP method, show promising results by improving the convergence of the algorithm with a reasonable increase in computation time. The generation of SB cuts requires solving an additional mixed integer programming (MIP) problem to compute the right-hand-side parameter of the cut. This provides an at least as good a cut as the original B cut.

A considerable amount of research has been conducted for solving the nonconvex MTHS problem, such as [8–11]. Except [10], which proposed a novel approach that uses step functions to model a nonconvex EFP function, they all rely on solving some relaxation of the original problem. This is also the case for the SB cuts applied in this work. However, instead of solving the Lagrangian problem to convergence to obtain the cuts, one solves the Lagrangian problem only once, as elaborated in Section 3.2.

For the MTHS problem, a hybrid stochastic dynamic programming (SDP)-SDDP method is currently the state-of-the-art in the Nordic power system. This method was developed in the late 1990s and uses a discrete Markov chain to describe the price uncertainty and an autoregressive model to describe the inflow to the reservoirs [5,12]. The discrete Markov chain is used to circumvent the nonconvexity caused by the bilinear term where the energy price and the generation are multiplied. As the uncertainty is described by two different stochastic processes, it is challenging to model correlations between these processes. For an MTHS problem with weekly decision stages, it has been shown that the correlation between inflow and energy price has not been too significant on a weekly basis, yielding sufficient results by the hybrid SDP-SDDP method [12]. Nonetheless, adding additional markets would extend the dimension of the Markov chain. This comes at a significant increase of computational cost, as presented in [13], where a reserve capacity market was added to the Markov process for an MTHS problem. The recent works [14,15] proposed an elegant approach for including uncertainty in the objective function for dynamic programming (DP) problems, utilizing the fact that the EFP function is a saddle function that is convex w.r.t. to the objective coefficient and concave w.r.t. the state variables. In the following work, we build on the work in [15] to model the MTHS problem with uncertainty of inflow, energy, and reserve capacity price.

Contributions

The work carried out in this paper is based on earlier work on developing improved methods to solve the MTHS problem, as in [7,16]. The main contributions are:

- A procedure to visualize and evaluate the shape of the EFP function to give a better insight into the nonconvexities in dynamic programming problems.
- The application of SDDP with SB cuts on a realistic nonconvex MTHS case study. The SDDP model considers correlated stochastic processes of inflow, energy, and reserve capacity price.
- The representation of nonconcave generation functions that are dependent on discharge and water head by concave regions.

2. The Medium-Term Hydropower Scheduling Problem

A dense formulation of the MTHS problem is given as the following multistage stochastic programming problem:

$$\max_{(x_1,y_1),\ldots,(x_T,y_T)} \mathbb{E}_{\tilde{\xi}}\left\{ \sum_{t=1}^{T} f_t(x_t, y_t, \tilde{\xi}_t) \right\} \tag{1}$$

$$\text{s.t.} \quad Wx_t + Hx_{t-1} + Gy_t = h(\tilde{\xi}_t) \tag{2}$$

$$By_t = 0 \quad (3)$$
$$Cy_t - Dx_t \geq 0 \quad (4)$$
$$Cy_t + Dx_t \leq Cy^{\max} \quad (5)$$
$$(x_t, y_t) \in Y_t \quad (6)$$
$$x_t \in \mathbb{R}^{k_1} \cdot \mathbb{Z}^{k_2}, y_t \in \mathbb{R}^{l_1} \cdot \mathbb{Z}^{l_2} \quad (7)$$
$$\forall t \in \{1, 2, \ldots, T\}. \quad (8)$$

The state variables, x_t, carry information between stages with a known initial state, x_0. Stage variables are given by y_t. The objective is to maximize the expected value of some real value function f_t that describes the profit the system can obtain. The expectation is taken over $\tilde{\xi}_t$, which describes a stochastic process of the inflow to the reservoirs as well as energy and reserve capacity prices. The matrices W, H, G, B, C, D are of suitable dimensions and define the parameters for a given hydropower system. The time-linking constraints in Equation (2) constrain the unit commitment of hydro stations and provide reservoir balances, where the function $h(\tilde{\xi}_t)$ describes the inflows to the reservoirs. The energy balance is given by Equation (3). The system's ability to provide reserve capacity is included in Equations (4) and (5). The generation function defining the relationship between power output, discharge and net head for each station, and the head function, describing how the head is related to reservoir volume, are also described by these constraints. More details on the these functions are given in Section 2.1. Other system constraints, not imperative for this study, are given in Equation (6).

In contrast to earlier characterizations of the MTHS problem, such as [5], where the objective was to maximize income from selling only energy, we extend this to include sales of reserve capacity. In order to keep the MTHS tractable, we define the reserve capacity as a composition of the provision of primary and secondary reserves. Further operational details associated with participation in the different markets are left for the short-term hydropower scheduling (STHS) problem [17] to handle.

2.1. Generation Function

The generation function describes a power station's power output. An illustration is given in Figure 1, where the generation is a function of discharge and net head. Since the generation function describes the station's overall power output, one must assume that the units are started by a given sequel. The station consists of two units, as can be seen from the two concave regions along the water discharge in Figure 1.

Due to the computational complexity of the MTHS problem, the generation function is normally cast as a concave function, where the power output only depends on discharge. For a well-regulated and loosely-constrained hydropower system where the producer only considers sales of energy, this assumption is reasonable. Roughly speaking, the optimal solution is to discharge as much water as possible in the hours with the highest energy prices and produce nothing the rest of the year. However, as discussed in Section 1, we expect that hydropower plants will more frequently run at low-level power output in the future to provide ancillary services. Similar behavior might also be imposed by environmental constraints, such as minimum discharge limits for certain periods of the year. While operating at low power outputs, the linear optimization model will observe a higher power output than what is physically feasible and thus overestimate the system's potential profit, as discussed in [7,16]. This overestimation can be avoided by more accurate modeling of the generation function. However, such improvements bring about increased model complexity and computation time. In the following, we define the generation function as a nonconcave function of net head and discharge. The generation function of a power station, for a given stage t, is defined by the following:

$$p_c \leq \alpha_i q_c + b_i x_c + \beta_i h, \forall i \in \mathcal{K}(c), \forall c \in \mathcal{C} \tag{9}$$

$$Q_c^{\min} x_c \leq q_c \leq Q_c^{\max} x_c, \forall c \in \mathcal{C} \tag{10}$$

$$P_c^{\min} x_c \leq p_c \leq P_c^{\max} x_c, \forall c \in \mathcal{C} \tag{11}$$

$$\sum_{c \in \mathcal{C}} x_c \leq 1 \tag{12}$$

$$x_c \in \{0,1\}, (p_c, q_c) \in \mathbb{R}_+ \tag{13}$$

where h is the net head, i.e., height difference between the station's upstream reservoir and the tailwater level. The set $\mathcal{C}$ contains concave regions of the generation function, where the discharge, q_c, and generation, p_c, are constructed for each of these regions. For each concave region, the generation function is bounded above by a set of hyperplanes, $\mathcal{K}(c)$, with coefficients $\alpha_{i(c)}$ and $\beta_{i(c)}$, and a right-hand side parameter $b_{i(c)}$. The generation function describes the entire station's power output; therefore, one must make an assumption that the units are started in a certain sequence. This sequence of starting up new units leads to nonconcavities in the generation function, as seen in Figure 1. Furthermore, another source of nonconcavity comes from the fact that power output for a hydropower station is a nonconcave function w.r.t. head. Therefore, one must make a trade-off between accurate problem formulation and computation time. To tackle this, we base our implementation on the generation function presented in [16].

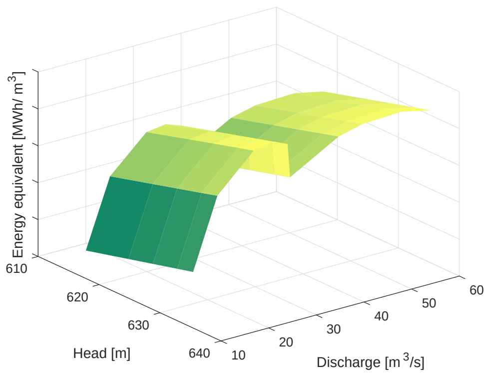


Figure 1. A generation function dependent on net head and discharge. For the purposes of illustration, the z-axis is given by the energy equivalent [MWh/m^3].

Several authors have investigated how to include variable head in the hydropower scheduling problem. Most of the publications are related to solving the STHS problem; see [17–20]. There has also been some work conducted on how to include variable head in MTHS problems [8,21]. A nonconcave generation function is described in [8], using a piecewise-linear approximation. The formulation in [8] requires one binary variable for each discrete point of the generation function, compared to one binary variable for each concave region in the generation function, as in Equations (9)–(13). In [21], the generation function is approximated by hyperplanes, and the authors propose a quadratic function to describe the relationship between head and reservoir volume. The bilinear terms are divided into a grid with different cells, where each cell is represented by McCormick envelopes [22]. Our approach avoids the bilinear term as the generation function is described by a set of hyperplanes for each of the concave regions. The method of using hyperplanes to describe the generation function is not novel, e.g., as proposed in [20], so our approach is thus an extension to an already established methodology.

The head function, relating head and reservoir volume, can for most Norwegian reservoirs be approximated by a concave function without significantly compromisingaccuracy. Most Norwegian hydropower plants have a relatively high head, and generation is typically less dependent on head variations than it is for hydropower systems in other parts of the world. An illustration of the head function is given in Figure 2, where the head function is illustrated with some constructed reservoirs. One can observe that the head function is concave for a reservoir with a monotonically-increasing cross-section, but it may be nonconcave for a reservoir that inhabits a subsurface cave.

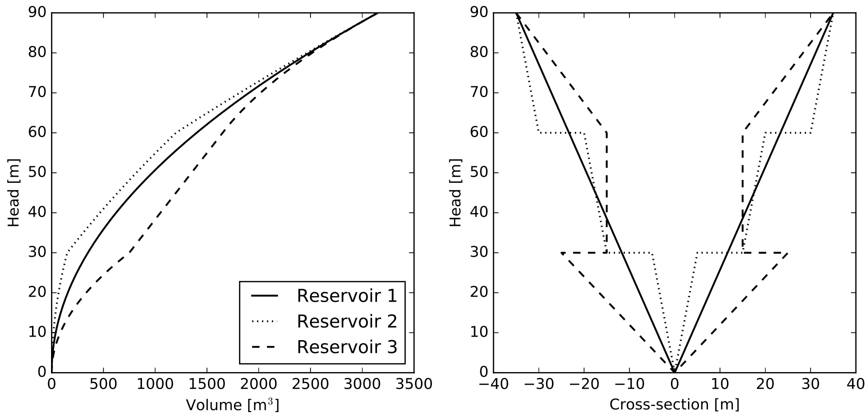


Figure 2. (**Right**) Cross-section of some artificial hydropower reservoirs. (**Left**) Head as a function of reservoir volume for the same reservoirs (corresponding line style). It is assumed that, from the view of the cross-section, the reservoirs has a depth of one unit.

2.2. Stochastic Processes

To solve the MTHS problem with uncertain inflow, energy, and reserve capacity price, we apply a vector autoregressive model of order one (VAR-1); see Equation (14).

$$n_t = \Phi n_{t-1} + \tilde{\xi}_t \tag{14}$$

$$r_t = \mu_t + \sigma_t n_t. \tag{15}$$

where n_t is the vector of the normalized stochastic processes, Φ is a time-invariant correlation matrix, and $\tilde{\xi}_t$ is a vector of white noise with realizations denoted as ξ_t. The physical realizations r_t of the normalized variables are given by Equation (15), where μ_t and σ_t are the expected value and standard deviation of the processes. p_t is defined as the subset of r_t that describes the objective term coefficients. As described in [15], these coefficients must be computed a priori to solving the stage-wise decision problem. Thus, the energy and reserve capacity prices are found beforehand and provided as parameters to the stage-wise decision problem, while the constraints on the normalized inflows are included in the weekly decision problem. Note that the normalized inflows can be calculated a priori and provided to the optimization problem as a parameter, but for modeling convenience, they are added as constraints. The treatment of objective term uncertainty in the SDDP method was first described in [15] and is further discussed in Section 3.3.

3. Methodology

The following section first describes how one can visualize and inspect the EFP function. Following that, it gives some insight into how the uncertain objective function is included and how the SB cuts introduced in [6] are used to solve the MTHS problem with variable head and price uncertainty.

3.1. The EFP Visualization Approach

In order to gain insight into the shape of EFP function, we solve the extensive form of the MTHS problem given by Equations (1)–(8). By visually inspecting the EFP function, one can get a first-hand impression whether it can be approximated using SB cuts with sufficient accuracy or not.

To solve the extensive MTHS problem, we rely on a tractable scenario tree representing some of the underlying uncertainty with a one-year planning horizon. Then, by looping over different initial reservoir levels, re-solving the MTHS problem, and storing the objective value, one can obtain an estimate of the EFP function for the first stage. This procedure is performed for some cases of the MTHS problem with and without capacity reserves, as seen in Figure 3.

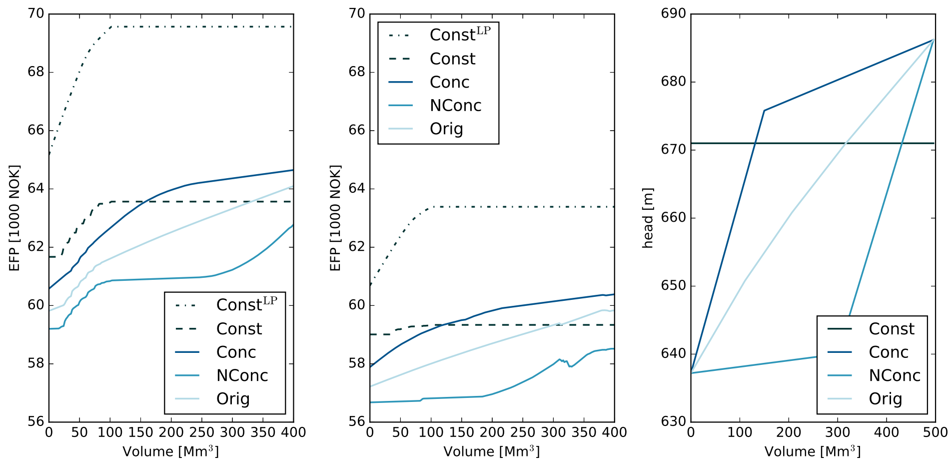


Figure 3. (**Left**) EFP with sales of reserve capacity and energy. (**Middle**) EFP with sales of energy. (**Right**) Different head functions. The colors of the lines in the two leftmost figures refer to the head function, $h(V)$, in the right figure. The different head functions are constant (Const), concave (Conc), original (Orig) and nonconcave (NConc), respectively. The Const$^{\mathrm{LP}}$ function refers to the linear programming relaxation of the MTHS problem. Both dashed lines refer to the constant head function; thus, the power output only depends on discharge.

The approach is limited in the sense that it only provides an approximation of the EFP function for a given time stage. Moreover, it can only provide meaningful visualizations a few state variables at a time. The results should therefore only assist the modeler in the choice of what approach to use. Note that this visual approach can be combined with numerical indices, such as those presented in [23], to indicate the degree of nonconcavity of the MTHS problem.

Inspecting the EFP for the MTHS at Hand

The MTHS problem, given by Equations (1)–(8), was defined with different head functions; a highly concave, highly nonconcave, constant and the actual function for a given reservoir. They are illustrated in the right plot in Figure 3. The resulting EFP function is given in the two leftmost plots, where the plot to the left includes sales of reserve capacity. It is clear that the shape of the head function has a significant impact on the corresponding shape of the EFP function, yielding information that can be used for deciding which solution strategy, and possible required simplifications, one should use.

The two EFP functions corresponding to a constant head are plotted (in black, dashed lines). One of them is the linear programming (LP) relaxation, denoted as PQH^{LP}. As expected, the LP relaxation yielded a concave EFP function. Moreover, the LP relaxation significantly overestimates the EFP function. Since the EFP function with the actual head function is not a highly nonconcave function, we expect that representing the generation function by piecewise-linear functions (cuts) provides acceptable results.

3.2. DP Formulation

In SDDP, the forward pass is used to generate valid candidate solutions that are used for computing the EFP function in the backward pass. All possible candidate solutions generated in the forward pass must, therefore, be present in the solution space in the problem used in the backward pass. The EFP function is described by an upper approximated piecewise-linear concave function, generated in the backward pass. For the forward pass, we define the following DP problem for iteration i:

$$\mathrm{FP}_t^i: \; Q_t^i(x_{t-1}, u_{t-1}, \xi_t) := \max_{x_t, y_t, z_t, u_t} f_t(x_t, y_t, u_t, \xi_t) + \phi_t^i(x_t, u_t, \tilde{\xi}_{t+1}) \tag{16}$$

$$\text{s.t.} \quad (z_t, x_t, y_t) \in X_t(\xi_t) \tag{17}$$

$$z_t = x_{t-1} \tag{18}$$

$$(z_t, u_t) \in \mathbb{R}, (x_t, y_t) \in \mathbb{R} \cdot \mathbb{Z}. \tag{19}$$

The objective function (16) consists of the present profit function, f_t, and the EFP function ϕ_t^i. The problem is constrained by the set X_t and the copy constraint Equation (18) as described in [6] together with the copy variable z_t. In addition to the state and stage variables (x_t and y_t), the additional state variable u_t is included in the formulation to include the uncertain objective coefficients, as discussed in [15].

In order to compute the EFP function, we define a backward pass problem, BP_t^i, where integrality has been relaxed.

$$\mathrm{BP}_t^i: \; \mathcal{Q}_t^i(x_{t-1}, u_{t-1}, \xi_t) := \max_{x_t, y_t, z_t, u_t} f_t(x_t, y_t, u_t, \xi_t) + \phi_t^i(x_t, u_t, \tilde{\xi}_{t+1}) \tag{20}$$

$$\text{s.t.} \quad (z_t, x_t, y_t) \in X_t(\xi_t) \tag{21}$$

$$z_t = x_{t-1} \tag{22}$$

$$(z_t, u_t, x_t, y_t) \in \mathbb{R}. \tag{23}$$

By solving BP_t^i, we obtain the cut coefficients π_t^i, from the dual values of the copy constraint in Equation (22). The cut coefficients aligned with the objective-term uncertainty are purely given by the sampled value, as described in [15]. Further, the cut used to describe the EFP function is enhanced by solving the following Lagrangian problem based on a Lagrangian relaxation of problem FP_t^i.

$$\mathrm{LG}_t^i: \; \mathcal{L}_t^i(\pi_t) := \max_{x_t, y_t, z_t, u_t} f_t(x_t, y_t, u_t, \xi_t) + \phi_t^i(x_t, u_t, \tilde{\xi}_{t+1}) - \pi_t^\top z_t \tag{24}$$

$$\text{s.t.} \quad (z_t, x_t, y_t) \in X_t(\xi_t) \tag{25}$$

$$(z_t, u_t) \in \mathbb{R}, (x_t, y_t) \in \mathbb{R} \cdot \mathbb{Z}, \tag{26}$$

By solving the Lagrangian problem, one can obtain the SB cut, as proposed in [6]. Note that the constant term $\pi_t^\top x_{t-1}$ is neglected in the Lagrangian problem as it would be subtracted in the SB cut, which is given as;

$$\theta_t \leq \sum_{m \in \mathcal{C}(t)} q_{tm} \mathcal{L}_m(\pi_m^i) + \sum_{m \in \mathcal{C}(t)} q_{tm} (\pi_m^i)^\top x_t - (p_t^i)^\top \mu_t. \tag{27}$$

Similarly, the B cut is given as:

$$\theta_t \leq \sum_{m \in \mathcal{C}(t)} q_{tm} \mathcal{Q}_m^{i*} + \sum_{m \in \mathcal{C}(t)} q_{tm} (\pi_m^i)^\top (x_t - x_t^*) - (p_t^i)^\top \mu_t. \tag{28}$$

where $\mathcal{Q}_m^{i*}$ is the objective value of BP_m^i and x_t^* is the candidate solution. The problem is solved for $m \in \mathcal{C}(t)$, where $\mathcal{C}(t)$ is the set of children nodes from a node in stage t and q_{tm} is the conditional probability. Recall that SDDP requires the stochastic variables to be stage-wise independent; therefore, the set $\mathcal{C}(t)$ is the same for all nodes in time stage t. The function $\phi_t^i(x_t, u_t, \tilde{\xi}_{t+1})$ is thus confined by some upper bound and the acquired set of B or SB cuts.

3.3. Uncertainty Modeling

In the following, we provide some insight into how the objective term uncertainty modeling is done. For the purposes of illustration, we assume that all state variables are fixed and that we only look at the terms where the auxiliary term u_t is present. Assume that two samples of the objective term coefficient p_t^1 and p_t^2 are available and that two cuts were constructed around these. Subsequently, a third sampling is done, and the problem to be solved can be given as:

$$\max_{u_t} \left\{ p_t^3 u_t + \theta_t : \theta_t \leq C_t^i - p_t^i u_t, (\theta_t, u_t) \in \mathbb{R}_+, \forall i \in \{1,2\} \right\}. \tag{29}$$

Since the state variables are assumed fixed, they are embedded in the constant term C_t^i. One can see that the problem consists of maximizing the present profit, $p_t^3 u_t$, and future profit, described by the two cuts. The problem given by Equation (29) is, therefore, able to assert whether there is an expectancy for greater profits in the future or not, depending on the current realization of the objective term coefficient. An illustration of this is given in Figure 4.

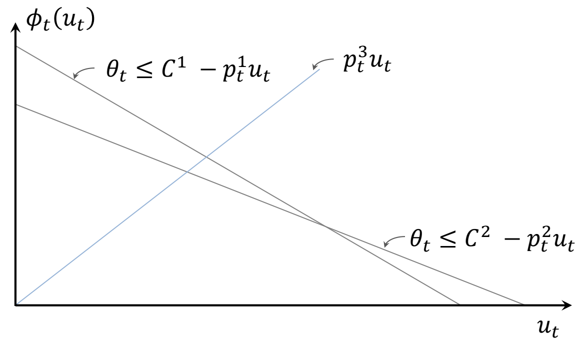


Figure 4. Illustration of the representation of the uncertain objective term price. The EFP function w.r.t. to the auxiliary variable u_t is given, and x_t is assumed fixed and added to the parameter C. Observe that the cuts represent the potential for future profit, whereas the sampled objective term coefficient p_t^3 describes the present profit potential. Thus, the model can compute the trade-off between them.

The algorithm for how the MTHS problem is solved is given in the next section.

3.4. Solution Approach

We define Algorithm 1 based on the SDDP framework. As seen from Lines 14 and 16 in the algorithm, one can choose which type of cut (B or SB) to use. After a certain amount of iterations, a final simulation is carried out on a larger set of scenarios.

Note that convergence cannot be guaranteed as FP_t^i is a nonconcave function w.r.t. the state variables. However, the approach will give an approximate solution that can yield good results depending on how nonconcave the true EFP function is. The nonconvexity of the EFP function can be visualized by the approach proposed in Section 3.1. Other measures to characterize how prominent the nonconcavities are could also be performed, as in Chp. 7.2. of [23].

Algorithm 1: Solving the MTHS problem.

Set x_0^i, $i \leftarrow 1$, $\text{UB} = +\infty$, and $\text{LB} = -\infty$
while $i < i^{\max}$ **or some other stopping criteria do**
  Sample N scenarios $\Omega^i = \xi_1^k, \ldots, \xi_{T k=1,\ldots,N}^k$
  `/* Forward iteration */`
  for $k = 1, \ldots, N$ **do**
    for $t = 1, \ldots, T$ **do**
      Solve FP_t^i, and collect solution f_t from Equation (16)
    $\text{lb}^k \leftarrow \Sigma_{t=1,\ldots,T} f_t$
  `/* Compute lower bound */`
  $\mu \leftarrow \frac{1}{N}\Sigma_{k=1}^N \text{lb}^k$ and $\sigma^2 \leftarrow \frac{1}{N-1}\Sigma_{k=1}^N(\text{lb}^k - \mu)^2$
  $\text{LB} \leftarrow \mu + z_\alpha \frac{\sigma}{\sqrt{N}}$
  `/* Backward iteration */`
  for $t = T, \ldots, 2$ **do**
    for $k = 1, \ldots, N$ **do**
      for $m \in \mathcal{C}(t)$ **do**
        Solve BP_t^i, and collect π_m^i from Equation (22)
        if *B cuts* **then**
          Collect $\mathcal{Q}_t^i$ from Equation (20)
        else if *SB cuts* **then**
          Solve LG_t^i, and collect $\mathcal{L}_t^i$ from Equation (24)
      Collect $p_t \subset r_t$ from Equation (15)
      Add desired cut to ϕ_t^i
  `/* Compute upper bound */`
  $\text{UB} \leftarrow \mathcal{Q}_1^i(x_0^i, u_0, \xi_0^i)$
  $i \leftarrow i + 1$
`/* Final simulation */`
Sample M scenarios $\Omega^i = \xi_1^k, \ldots, \xi_{T k=1,\ldots,M}^k$
for $k = 1, \ldots, M$ **do**
  for $t = 1, \ldots, T$ **do**
    Solve FP_t^i, and collect solution f_t, x_t^{ik}, y_t^{ik} from Equation (16)-(19)
  $\text{lb}^k \leftarrow \Sigma_{t=1,\ldots,T} f_t$
`/* Compute lower bound */`
$\mu \leftarrow \frac{1}{M}\Sigma_{k=1}^M \text{lb}^k$ and $\sigma^2 \leftarrow \frac{1}{M-1}\Sigma_{k=1}^M(\text{lb}^k - \mu)^2$
$\text{LB} \leftarrow \mu + z_\alpha \frac{\sigma}{\sqrt{M}}$

4. Case Study

The case study is a representation of a Norwegian hydropower system, comprised of three reservoirs and two power stations. The power stations have 13.8 MW and 365 MW of installed capacity. There is a short-term and long-term reservoir connected to the largest power station, as seen in Figure 5. Thus, the aim for the long-term reservoir is to store as much water for usage in the most remunerated hours during the year, while the short-term reservoirs need to be properly managed in order to avoid spillage. This system was also used as a study case in [7,24].

The VAR-1 model representing the stochastic processes was fitted to 70 historical years of inflow and energy prices obtained from a fundamental market model generating energy prices for those 70 years [25]. Historical prices for the primary reserve market are used to describe the reserve capacity prices.

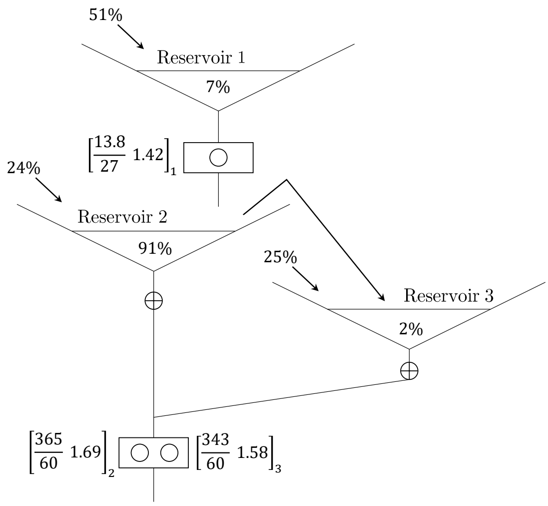


Figure 5. Illustration of the hydropower system. There are three reservoirs, each represented by its relative storage capacity and inflow compared to the system as a whole. As an example, Reservoir 1 has 7% of the system's storage capacity and 51% of the inflow. Reservoirs 2 and 3 have a hatch downstream that controls which reservoir is depleted. Only one of the reservoirs can be depleted at a time, due to their different head. Reservoir 2 can also bypass water, indicated by the arrow between the reservoirs. The hydropower stations are represented by their maximum power at nominal head, discharge and the energy equivalent ($\frac{\text{MW}}{\text{m}^3/\text{s}}$ MWh/m^3). Since the lower power station is connected to two reservoirs, it has different efficiencies, depending on which reservoir is depleted.

We use weekly decision stages. Each weekly decision problem consists of 1858 constraints (not considering the cuts) and 1152 variables (837 continuous and 315 binary). There are 104 weeks in the scheduling horizon, and we consider 15 branches in the backward pass of the SDDP algorithm. Each week has 21 time-steps representing three time blocks of the day. Three scenarios are sampled for each forward SDDP iteration, and the final simulation is carried out with 300 scenarios. We use the same sampled scenarios for all cases. From the final simulation, a confidence interval is computed. The problem was formulated in C++ with Gurobi 7.5 as the optimization solver. The computations were carried out on a computer cluster with two Intel Xeon E5-2690 v4 processors, 2.6 GHz, and 384 GB RAM. No parallelization was carried out except the one from the optimization solver. Parallelization in the SDDP framework is well studied, as in [26], and thus neglected in this work. It would, however, contribute to significantly reducing the CPU time.

Results and Discussion

In the following, the results from Algorithm 1 are outlined with the use of both B and SB cuts. The MTHS problem was first solved with uncertainty of inflow, reserve capacity, and energy price. A case with only uncertainty in inflow was performed for comparison.

The convergence plot for Algorithm 1 is shown in Figure 6, when B and SB cuts were used. It is clear that the SB cuts provided significantly tighter cuts and a better policy. One can also observe that the upper bound converged slowly and would most likely continue to improve with more iterations. The computation time did, however, become more prominent for SB cuts, as seen in Table 1. Algorithm 1 required approximately five-times more time with SB cuts than with B cuts. The use of parallel processing could easily drive the computation time down, making the use of SB cuts better suited for daily operational use.

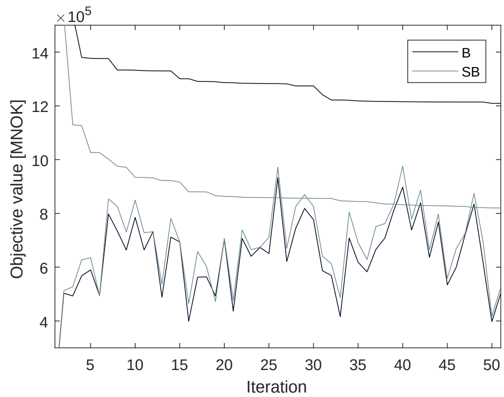


Figure 6. Convergence of the approach with Benders and strengthened Benders cuts.

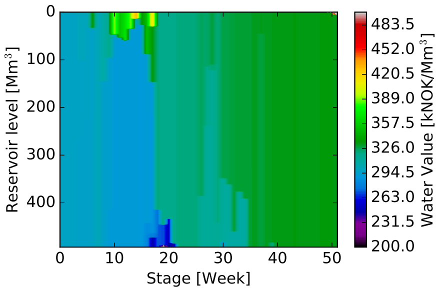
Table 1. Economic and computational performance of the two case studies with use of either B or SB cuts. Uncertainty of inflow, reserve capacity, and energy price (top) and uncertainty of inflow (bottom).

	UB (kNOK)	stat. LB (kNOK)	Gap (%)	Time (h min)
B	1,209,510	502,838	58.4	5 17
SB	820,466	525,493	36.0	23 59
B	1,017,580	537,808	47.1	5 52
SB	614,616	563,280	8.4	34 31

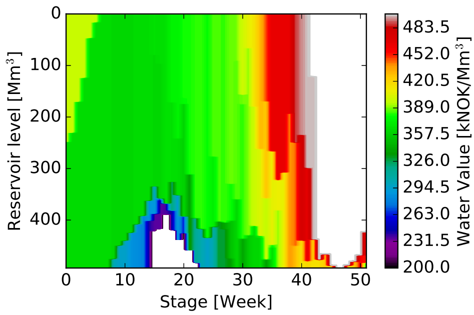
The expected value of water, or water values (WV), for the largest reservoir in the system are shown in Figure 7. The WVs were computed by fixing all the other state variables in the EFP function and finding the coefficients of the binding cuts. One can observe that even though the cut coefficients from both B and SB cuts came from the problem BP_t^i, the water values for the SB cuts were generally lower than the B cuts. This can be seen as a result of the right-hand-side in the SB cuts being lower, therefore lowering the cuts, which resulted in a lower water value for the same state, as the EFP function is concave. An illustration of this is given in Figure 8, where two cuts are generated in the first iteration of the SDDP algorithm. The coefficients of the cuts were the same for both SB and B cuts, but as seen from the top left and right plot, the SB cuts had a tighter right-hand side. This results in the water values given in the lower plot. After consecutive iterations, the water values with SB cuts tended to stabilize on lower values, which is a reasonable observation as the SB cuts made the model see less expected profit in the future.

A percentile plot of the reservoir trajectories for the largest reservoir is shown in Figure 9, for both B and SB cuts. The figure clearly shows how the algorithm was able to utilize the reservoir better when SB cuts were used. When B cuts were used, the algorithm saw a higher expected future profit than what was achievable, resulting in a simulated operation at very high reservoir volumes. Implications of this can be seen in Figure 7b,d, where the change in water values is substantial around Week 18, due to the spring flood, giving a high risk of spillage for large reservoir volumes.

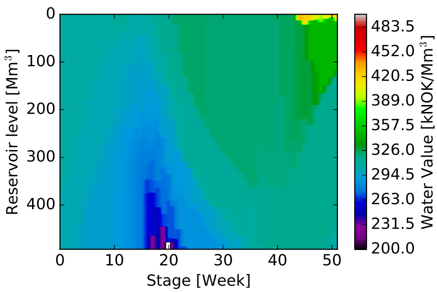
In Table 1, the bounds of the algorithm are shown, computed from the final simulation, together with the percentage-wise gap and computation time for the 50 forward and backward iterations. For comparison, the problem was solved with only uncertainty of inflow, which is given in the bottom half of the table. Observe how the convergence properties improve, indicating that the approach of including objective term uncertainty in SDDP by [15] requires more iterations. In validation studies using a smaller system with fewer decision stages, it was observed that the approach does slowly converge. This illustrates the difficulty of solving multi-stage stochastic problems with high dimensions of uncertainty.



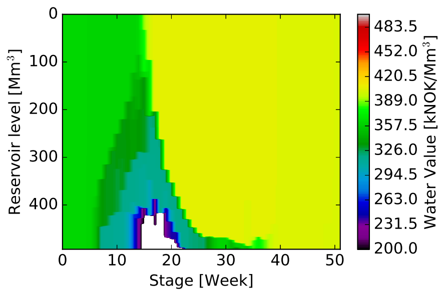
(**a**) With SB cuts.



(**b**) With B cuts.



(**c**) With SB cuts and only uncertainty of inflow.



(**d**) With B cuts and only uncertainty of inflow.

Figure 7. Water values for Reservoir 2 for the different case studies.

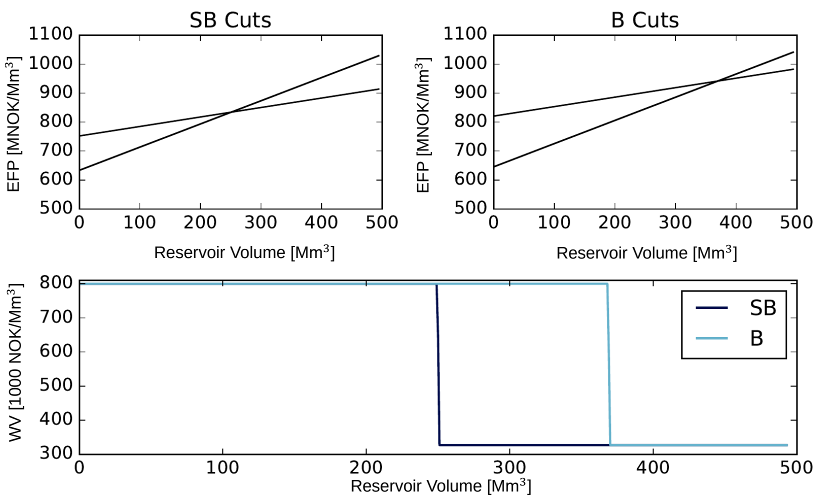


Figure 8. Illustration of the B and SB cuts and how this affects the water value.

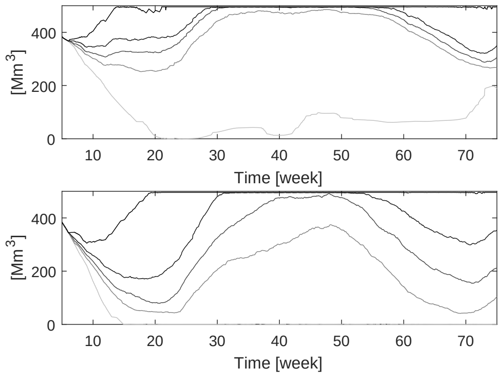


Figure 9. Percentile plot of reservoir trajectories for Reservoir 2. (**Top**) With Benders cuts and (**bottom**) with strengthened Benders cuts.

5. Conclusions

A medium-term hydropower scheduling (MTHS) problem with variable head and uncertainty in inflow, reserve capacity, and energy price was investigated. The proposed model based on the stochastic dual dynamic (SDDP) method included correlations between the different stochastic processes and allowed for representation of a detailed hydropower system.

By means of visualization, we found that the expected future profit (EFP) function for the MTHS case study was not highly nonconcave, and we argue that the approximation of the EFP as a concave function within the SDDP method is a fair compromise between accuracy and computation time. We compared two types of Benders cuts to approximate the EFP function, namely the Benders (B) and the strengthened Benders (SB) cuts.

In the presented case study, it was found that the use of SB cuts provided a significantly better policy than with the use of B cuts. The policy improvement comes at an increased computational time, around five-times higher for SB than B in the case study. Moreover, we found that the inclusion of objective term uncertainty led to significantly slower convergence.

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Abbreviations

The following abbreviations are used in this manuscript:

B	Benders
DP	Dynamic programming
EFP	Expected future profit
LP	Linear programming
MIP	Mixed integer programming
MTHS	Medium-term hydropower scheduling
SB	Strengthened Benders
SDDiP	Stochastic dual dynamic integer programming
SDDP	Stochastic dual dynamic programming
SDP	Stochastic dynamic programming
WV	Water values

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