Vertical Lift Modules (VLMs) for small items order picking: an economic evaluation

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Abstract

Small items order picking is a very labour-intensive activity which is often performed in a dedicated area equipped with carton racks and with the operators walking within the aisles to pick the required products. Recently, improvements in automated solutions have been introduced to ease the picking activities and to reduce the impact of human labor. In this paper, one of these automated systems, called dual-bay Vertical Lift Module (VLM) system, is compared to a manual warehouse with carton racks. Based on the lack reported from the literature analysis, the two order picking solutions are compared according to technical and cost factors, for the first time here. The total annual cost of both systems is modelled by analytical formulations, including fixed terms, related to space and equipment cost, and variable terms, linked to workforce cost. A multi-factorial analysis and an ANOVA permit to study the impact of each factor included in the models. The comparison of the systems leads to the definition of a VLM area of application, which can be used to understand the suitability of the VLM with respect to the warehouse with carton racks. The mathematical models are also applied to an industrial example. Finally, starting from the analysis of the models and from their application, some highlights are derived, and future researches are defined.

Keywords: Automated Warehouses, Vertical Lift Modules, Carton Racks Warehouses, Small Items Order Picking, Cost Model, Throughput Analysis.

1 Introduction

Order picking is the activity performed within a warehouse, usually by human operators, to fulfill the order of a customer through the retrieval of different items from their storage locations (De Koster, Le-Duc & Roodbergen, 2007). A picking warehouse can be picker-to-parts, with static storage locations visited by the pickers travelling within the warehouse aisles and characterized by a travel time which is usually about the 50% of the total order processing time (Tompkins et al., 2010). Consequently, the reduction of the travel time is one of the ways that is often suggested to improve the throughput of the system (Caron, Marchet & Perego, 2000; Battini et al., 2015a).

In case of small items order picking, this percentage of time spent in travelling could be higher, especially when the products are stocked in pallets (Battini et al., 2014). Moreover, previous studies have demonstrated that picking from full pallets heavily impacts also the ergonomic effort of the operators (Battini et al., 2014; Calzavara et al., 2017a). Therefore, small items are usually placed in a dedicated and very compact picking area, called forward area, where simple racks are used to stock a small amount of items, while the bulk storage is left in pallets in the reserve area (Choe & Sharp, 1991; Tompkins & Smith, 1998; Bartholdi & Hackman, 2017).

An alternative of the picker-to-parts system is the parts-to-picker one, in which the items are brought to the operator by an equipment, which is usually supported by automated systems, as well as computer software tools. Some examples are small AS/RS systems (Scheie et al., 2012) or robotic mobile fulfillment systems (Azadeh, De Koster & Roy, 2017; Lamballais, Roy & De Koster, 2017).

In the present paper, a parts-to-picker system (called VLM system) is analyzed and compared to the traditional picker-to-parts one, a warehouse with carton racks. It is composed by a dual-bay Vertical Lift Module (VLM), used for the high-density storage and the retrieval of the items, and an operator, for the picking activity. Recently, progresses in automation have driven the application of this kind of systems also to many different industrial sectors (Dukic, Opetuk & Lerher, 2015). Its installation is not so expensive, if compared to other automated solutions, but a global overview of the costs and technical characteristics of the VLM systems is strongly needed.

The contributions proposed so far, dealing with VLM systems, are not numerous. In fact, the literature analysis reported in the next section highlights the current lack in mathematical models for the economic evaluation of small items order picking systems.

Therefore, the main goal of the present paper is to study the applicability of a VLM system. Some analytical models are used to compare this one with a traditional warehouse with a cartons rack.

The used methodology refers to a mathematical model, which allows to describe the two order picking systems from an economic and from an operational perspective. A simple formula has been defined as the ratio between the total cost of the VLM solution and the total cost of the traditional warehouse with carton racks. This is relevant to understand the VLM suitable application, based on the fact that the implementation of this solution should consider an important trade-off, between the benefits that this system can carry (e.g. space and picking time reduction), and the related emerging costs (installed equipment). Subsequently, a multi-factorial analysis and an ANOVA (ANalysis Of VAriance) are carried out to highlight which parameters have a higher impact on the VLM implementation. Based on these most important factors, a VLM area of application is defined and represented in some decisional graphs, and then applied to a case study to show the validity of the proposed approach as well as its limitations.

The remainder of the paper is structured in six further sections. In the next one, the literature review about small items order picking systems and economic modelling, with emphasis on VLM systems, is presented. Section 3 explains the two studied order picking solutions: the warehouse with carton racks and the VLM system. Here, detailed mathematical equations to model the total cost of these systems are introduced and discussed. In Section 4, the multi-factorial analysis of the economic models and the comparison of the two systems are reported, introducing the decisional graphs with the VLM area of application. Then, in Section 5, a case study is used to explain step-by-step how to apply these graphs, while Section 6 proposes the discussion of the formulations and of the obtained results. Finally, Section 7 reports the conclusions and the future research.

2 Literature review

A Vertical Lift Module (VLM) is a parts-to-picker storage solution for small items composed of several trays, in which the items are stored, and of an automated storage and retrieval system, needed to retrieve, transport and deliver a tray at a time in front of the operator. Recently, the employ of VLMs has interestingly expanded, also to order picking contexts, thanks to their technological and performance development (Dukic, Opetuk & Lerher, 2015; Lenoble, Frein & Hammami, 2018). Among others, dual-bay VLMs turn out to be particularly suitable for picking activities, since they allow the picker to work in parallel to the system: while the picker has a certain tray in front of him, the crane can independently store the previous tray and retrieve the following one. Of course, this can lead to a higher productivity of the order picking system, since the picker does not have to walk to reach the items to pick, and also the search and the pick of the items are eased (Battini et al., 2016).

Although Vertical Lift Modules possible applications are promising, so far only a few contributions in literature are specifically related to this kind of systems. The first significant contribution, dealing with single-bay VLMs, is by Meller and Klote (2004). It is focused on the proposal of mathematical models to estimate the storage and retrieval cycle times of the system. Similarly, Rosi et al. (2016) explores the advantages of single-bay VLMs as an alternative to other traditional order picking systems, through a discrete event simulation technique. Another research is by Dukic, Opetuk & Lerher (2015), which derives some formulas for the estimation of the throughput of dual-bay VLMs, considering the interactions between the VLM and the picker, working in parallel. Battini et al. (2016) propose to employ dual-bay VLMs for a fast picking of small items' orders, by studying some possible solutions that can speed up the overall configuration, like class-based storage assignment of the items, grouped retrievals of the trays and order batching. Order batching applied to VLMs is investigated also by Lenoble, Frein & Hammami (2018), through the derivation of optimization models for minimizing the total time required to pick a given set of customers' orders.

The existing literature on warehousing suggests that the study of the economic contribution of a storage system should generally consider its most relevant costs items. For example, for Tompkins & Smith (1998) it should be taken into account the building, the equipment within it, the value of the material to be stored and the cost of the operation. On the other hand, Rosenblatt & Roll (1984) propose to focus on the initial investment, on the shortage costs and on the costs associated to the storage policy. In their review, Gu, Goetschalckx & McGinnis (2010) state that the warehouse layout and configuration can affect its construction and maintenance costs, the material handling costs, as well as the storage capacity, the space utilization and the equipment utilization.

In case of warehouse picking activities, the comparison of different picking approaches from an economic perspective has not received, for now, a proper attention. Some researches state that the most important costs of a picking warehouse are related to the time needed to process a picking order (De Koster, Le-Duc & Roodbergen, 2007; Gu, Goetschalckx & McGinnis 2010). Therefore, researches on this topic mainly propose to reduce costs by reducing the picking time (Daniels, Rummel & Schantz 1998). This can be achieved, for example, by reducing the travel time, through the reduction of the distances travelled by the operators, or by using paperless picking devices, that can decrease the search and pick time (Tompkins et al., 2010; Battini et al., 2015b).

On the other side, Thomas & Meller (2015) develop design guidelines for a case-picking warehouse, through a statistical-based methodology and considering the number of labour hours. Other contributions are more focused on the operational aspects of a picking warehouse, like the ones related to forward area dimensioning, items allocation, replenishment impact and related costs (Frazelle, Hackman & Platzman, 1989; Frazelle et al., 1994; Bartholdi & Hackman, 2008).

This literature analysis suggests a lack in the economic modelling of order picking systems with VLMs, which is the aim of the present paper. In fact, some of the reported contributions are mainly investigating the performance of a VLM, in terms of times estimation and throughput improvement. On the other side, for now there are no studies dealing with the economic impact that a VLM can have in a warehouse, especially compared to a traditional warehousing system.

3 Total cost functions for small items order picking systems

Here the total cost functions are introduced and discussed for the two analyzed order picking systems, the warehouse with carton racks and the Vertical Lift Module picking system (Figure 1).

Differing from the previous researches, discussed in the literature analysis, an economic comparison is allowed thanks to the introduction of these two cost functions, with a deep analysis of the impact of the cost factors composing the two models.

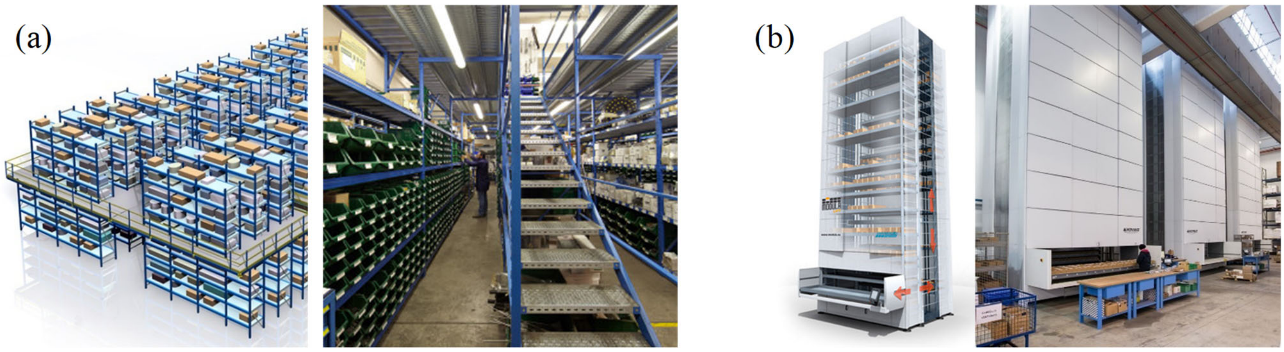

Figure 1: Analysed systems. (a) Warehouse with carton racks, (b) Dual bay Vertical Lift Module.

In the proposed models it is assumed that the items are stored with a random storage assignment, in both systems, and that the picking orders are processed individually, with a simple order picking strategy (and no batching). Moreover, both systems are considered to be always available, and the replenishment activity, needed for refilling the storage locations with items, is assumed to be performed in an additional time with a similar strategy for both systems (Bartholdi & Hackmann, 2017). All the indirect costs are assumed to be included in the total annual cost, and the throughput of the systems are described through a picking time, which could depend on the adoption of paperless picking technologies (Battini et al., 2015b).

Two different terms are considered into the general total cost function: the fixed cost component is related to the annual costs of facilities, equipment and devices, including the indirect costs; on the other side, the variable cost component is connected to the resources necessary yearly to perform the required items picking:

$$TC^s = C^s_{fix} + C^s_{var} \tag{1}$$

where $s = W$ for the warehouse with carton racks and $s = V$ for the dual bay VLM.

Table 1 reports all the input parameters and the variables used for the two cost models.

Symbol	**Description**
i	Stored item index, $i = 1 \dots n$
s	Storage system index, $s \in \{W, V\}$
Q [picks/h]	Total hourly required throughput, $Q = \sum_{i=1}^{n} Q_i$
N [lines/order]	Average number of lines per picking order
V [m^3]	Total storage volume, $V = \sum_{i=1}^{n} V_i$
H [m]	Plant height

C_{sp} [€/(m^{2}year)]	Annual space cost per square meter
C_{op} [€/h]	Hourly operator cost
$\bar{v}$ [m/s]	Average walking speed of the picker
h_y [€/year]	Number of working hours in a year
TC^s [€/year]	Annual order picking system total cost, $s \in \{W, V\}$
C_{fix}^s [€/year]	Annual order picking system fixed costs, $s \in \{W, V\}$
C_{var}^s [€/year]	Annual order picking system variable costs, $s \in \{W, V\}$
k^s	Order picking system space cost coefficient, $s \in \{W, V\}$
SL^s	Order picking system saturation level, $s \in \{W, V\}$
C^s [€/(year)]	Order picking system annual cost, $s \in \{W, V\}$
Q^s [picks/h]	Order picking system hourly throughput, $Q^s = 3600/t_l^s$, $s \in \{W, V\}$
t_l^s [s]	Average cycle time per line $s \in \{W, V\}$
t_{search}^s [s]	Search time, $s \in \{W, V\}$
t_{pick}^s [s]	Pick time, $s \in \{W, V\}$
t_{fix}^s [s]	Time for fixed activities, $s \in \{W, V\}$
t_{travel}^s [s]	Travel time, $s \in \{W, V\}$
j	Warehouse with carton racks aisles index, $j = 1 \dots a$
C^M [€/(year)]	Mezzanine system annual cost
A^W [m^2]	Area of the warehouse with carton racks
L^W [m]	Length of the warehouse with carton racks
L_{travel}^W [m]	Distance traveled by the picker in the warehouse with carton racks
L_I^t [m]	Distance traveled by the picker within the aisles in the warehouse with carton racks
L_E^t [m]	Distance traveled by the picker across the aisles in the warehouse with carton racks
v	Number of expected aisles that have to be visited during an order
a	Total number of aisles on one side of the warehouse with carton racks
f	Value of the farthest couple of aisles to visit in the warehouse with carton racks

N^V	Number of VLMs
V^V [m^3]	Storage volume of one VLM
A^V [m^2]	Operating area of one VLM, including the VLM area and the space for the operator
p_T [s]	Net operator's pick time while using a VLM
$E[DC]$ [s]	Time of the VLM crane to perform a dual command
$E[DC]_A$ [s]	Expected dual command time from the picking position A
$E[DC]_B$ [s]	Expected dual command time from the picking position B
$t_{p/d}$ [s]	Delay time to pick up and to deposit a tray
m	VLM tray index, $m = 1 \dots M$
t_{mj} [s]	Travel time of the crane for moving between m and j
p_m	Single probability of extracting each VLM tray
X	Same VLM tray extraction probability factor
R_{TC}	Total costs ratio
R_Q	Systems throughput ratio

Table 1. Notations.

3.1 Warehouse with carton racks cost model

The traditional warehouse with carton racks is the base solution of the research, since it is widely used for the picking of small items (Bartholdi & Hackman, 2017). In order to use all the available space, it is often composed by the ground floor and a mezzanine system, where the racks are placed (Figure 1, a). This is a typical example of picker-to-parts system, where the operators completes the picking list walking within the picking area.

In this case, the total cost TC^W can be written as:

$$TC^W = C^W_{fix} + C^W_{var} = C_{sp} \cdot k^W \cdot \frac{V}{SL^W \cdot H} + C_{op} \cdot \frac{Q}{Q^W} \cdot h_y \quad (2)$$

where the floor space cost coefficient k^W includes the racks annual cost C^W and the mezzanine system annual cost C^M, and it is calculated with:

$$k^W = \frac{2 \cdot C^W/A^W + C^M/A^W}{C_{sp}} \tag{3}$$

Moreover,

$$Q^W = 3600/t_l^W \tag{4}$$

is the throughput of the picking system, and t_l^W is the average cycle time per line. This is the time averagely needed to pick an item reported on the picking list, and it can be defined as the sum of the times of the different activities performed during picking. These are, respectively, the time for searching the item reported on the picking list, the time to physically perform the pick of the item, the time of fixed activities (e.g. order set up) and the time for travelling among all locations to retrieve the needed items (Tompkins et al., 2010):

$$t_l^W = t_{search}^W + t_{pick}^W + \frac{t_{fix}^W + t_{travel}^W}{N} \tag{5}$$

Whilst t_{search}^W, t_{pick}^W and t_{fix}^W can here be considered as constants and not particularly depending on the warehouse area and layout, t_{travel}^W changes according to the distance traveled by the picker L_{travel}^W, and, hence, to the dimensions of the stocking area and on the number of aisles, as already demonstrated by Caron et al. (1998).

The values of the different times making t_l^W can be mainly estimated through on the field measurements of pickers' activities during a significative period; it has been observed that in a warehouse with carton racks usually t_l^W can be from 30 to 60 seconds per line, according to the storage area (Caron et al., 2000; Bartholdi & Hackman, 2008).

Otherwise, in case the warehouse with carton racks has not been installed yet, and, hence, there is not the possibility of taking direct time measures, it is then necessary to design it, following the well-known approaches that consider the required storage capacity and then the throughput of the system (Caron et al., 1998).

In this case, defining the layout as reported in Figure 2 and assuming random storage assignment of the items and traversal routing, t_{travel}^W can be expressed as:

$$t_{travel}^W = \frac{L_{travel}^W}{\bar{v}} \tag{6}$$

and L_{travel}^W is the sum of the distance traveled within aisles L_I^t and across aisles L_E^t (Caron et al., 1998):

$$L_{travel}^W = L_I^t + L_E^t = v \cdot (l + e) + 2 \cdot f \cdot w + 2 \cdot D_{I/O} \tag{7}$$

with

$$v = a \cdot \left(1 - \left(1 - \frac{1}{a}\right)^N\right) \tag{8}$$

number of expected aisles that have to be visited, according to the total number of aisles on one side of the warehouse a and the number of picking lines per order N, and

$$f = \sum_{j=2}^{a/2} \left((j-1) \cdot \left(\left(\frac{2j}{a}\right)^N - \left(\frac{2j-2}{a}\right)^N \right) \right) \tag{9}$$

expected value of the farthest couple of aisles to visit.

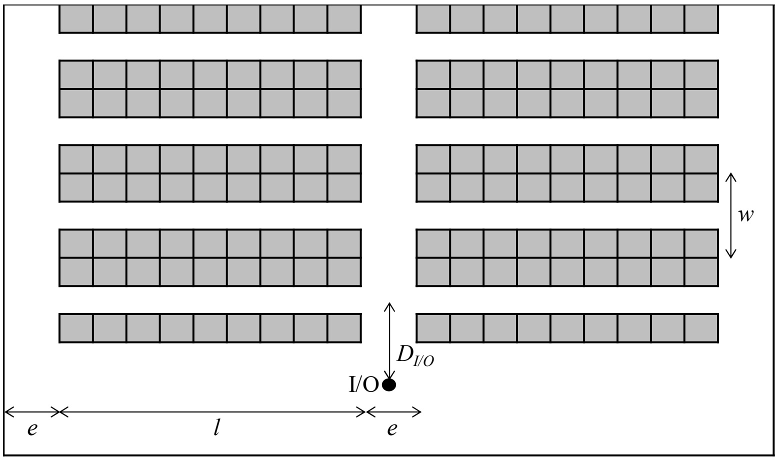

Figure 2. Warehouse with carton racks layout and considered parameters for traveled distance estimation.

Table 2 shows some examples of calculation of t_l^W, expressed in seconds, for different warehouse configurations, varying V (i.e. the number of aisles a, assuming an aisle width of 2.3 m and $a = 2 \dots 10$), N and t_{pick}^W. The warehouse width is considered to be fixed and equal to 38 m, while t_{fix}^W is high and equal to 60 s due to the setting of the warehouse and to the presence of the mezzanine system.

***N* [*lines/order*]**	**5**			**10**			**20**		
t_{pick}^W [s] / **V [m^3]**	**5.0**	**10.0**	**20.0**	**5.0**	**10.0**	**20.0**	**5.0**	**10.0**	**20.0**
113	33.2	38.2	48.2	21.7	26.7	36.7	15.9	20.9	30.9
225	39.3	44.3	54.3	26.3	31.3	41.3	18.4	23.4	33.4
338	42.8	47.8	57.8	29.7	34.7	44.7	20.8	25.8	35.8
451	45.2	50.2	60.2	32.2	37.2	47.2	22.9	27.9	37.9
563	47.0	52.0	62.0	34.1	39.1	49.1	24.7	29.7	39.7

Table 2. Values of average cycle time per line t_l^W [s] varying the total storage volume V, the average number of lines per order N and the pick time t_{pick}^W.

As it can be seen from Table 2, the average cycle time per line t_l^W varies according to the storage volume V, since it depends on the travel time (equation 6).

Such an analysis of t_l^W is useful to understand the throughput of this order picking system, in order also to compare it with the dual bay VLM introduced in the following section.

3.2 Dual-bay VLM system cost model

In this section, the total cost function for the dual-bay Vertical Lift Module system, composed by a set of VLMs and as many operators, is introduced and discussed. In the VLM, the items are stocked in trays placed in several racks in a closed space (Figure 1, b). An automated crane is responsible for the storage and retrieval of these trays, between racks storage levels and the picking bay. Here, the operator is standing, waiting to pick the required items from the delivered trays. Recently, dual-bay VLMs have been developed to improve the system throughput, since while the operator is picking from a certain tray, the crane can store the previous tray and then retrieve the following one (Lenoble, Frein & Hammami, 2018).

As done for the previous solution, the total cost function for the VLM system can be written as:

$$TC^V = C_{fix}^V + C_{var}^V = N^V \cdot \left[\left(C_{sp} \cdot A^V + C^V \right) + C_{op} \cdot \frac{Q}{N^V \cdot Q^V} \cdot h_y \right] \tag{10}$$

where the number of VLMs N^V, working in parallel, is calculated based on the required throughput of the system Q and the total storage volume V:

$$N^V = \max\left(\left\lceil \frac{Q}{Q^V} \right\rceil ; \left\lceil \frac{V}{V^V} \right\rceil \right) \tag{11}$$

Typically, N^V can be defined according to the following equality, to reduce the investment cost:

$$N^V = \left\lceil \frac{Q}{Q^V} \right\rceil = \left\lceil \frac{V}{V^V} \right\rceil \tag{12}$$

In fact, in case

$$\left\lceil \frac{Q}{Q^V} \right\rceil < \left\lceil \frac{V}{V^V} \right\rceil \tag{13}$$

it means that it is possible to install a slower VLM system, with a lower installation cost; on the other side, when

$$\left\lceil \frac{Q}{Q^V} \right\rceil > \left\lceil \frac{V}{V^V} \right\rceil \tag{14}$$

a smaller VLM system can be used, with a lower investment cost. In the model, it is assumed that, in case of multiple VLMs, all the different products are stocked in each VLM, to warrant a continuous availability of all products.

As done in the previous section, the total cost model is composed by the fixed term, referring the space occupied by the VLM and its annual cost, and by the variable terms, related to the required workforce.

Then, by introducing

$$k^V = \frac{C_{sp} + C^V / A^V}{C_{sp}} = 1 + \frac{C^V}{C_{sp} \cdot A^V} \tag{15}$$

as the floor space cost coefficient for the VLM, including the required operating space and the VLM, and considering

$$A^V = \frac{V^V}{SL^V \cdot H} \tag{16}$$

equation (10) can be rewritten in a similar way to equation (2) as:

$$TC^V = N^V \cdot \left[\left(C_{sp} \cdot k^V \cdot \frac{V^V}{SL^V \cdot H} \right) + C_{op} \cdot \frac{Q}{N^V \cdot Q^V} \cdot h_y \right] \tag{17}$$

Similarly to equation (5), the average cycle time per line t_l^V can be estimated as follows:

$$t_l^V = t_{search}^V + t_{pick}^V + \frac{t_{fix}^V + t_{travel}^V}{N} \tag{18}$$

Here, t_{search}^V, t_{fix}^V, and t_{travel}^V can be considered constant values and not depending on the warehouse layout. Moreover, t_{search}^V and t_{travel}^V are typically lower than the same factors related to the warehouse with carton racks t_{search}^W and t_{travel}^W, since the search is related only to the items in a certain tray in front of the picker (and usually supported by a signaling device) and the VLM is a parts-to-picker system.

On the other side, the picking time t_{pick}^V is the cycle time of the whole system and, since the picker and the VLM are working in parallel, it depends on some characteristics of the storage system (i.e. the dual command time of the VLM crane) and on the picker's performance (Battini et al., 2016). In fact, the resulting cycle time derives from the comparison between the time spent by the crane to perform a dual command $E[DC]$ and the time spent by the picker to perform his/her activities, such as picking items from the trays and other tasks like counting, weighing or stocking items to new locations. These activities are the same the operator performs in the warehouse with carton racks during t_{pick}^{W}; this net picking time needed by the picker while using a VLM is here called p_T, following the definition of some previous contributions (Dukic, Opetuk & Lerher, 2015; Battini et al., 2016).

Usually, since the VLM system is evaluated as an alternative solution of the warehouse with carton racks, and it is not already physically available in the storage area, the average cycle time per line t_l^V cannot be estimated through direct measurements. It is rather derived through mathematical or simulation models.

Based on the models proposed by Bozer & White (1990), Dukic, Opetuk & Lerher (2015) and Battini et al. (2016), the actual time to pick a line of the order t_{pick}^{V} using the VLM system can be estimated mathematically with:

$$t_{pick}^{V} = \begin{cases} E[DC] & 0 < p_T < t_1 \\ \frac{{p_T}^2 - 2p_T t_1 + {t_2}^2}{2(t_2 - t_1)} & t_1 \leq p_T < t_2 \\ p_T & t_2 \leq p_T < \infty \end{cases} \quad (19)$$

where t_1 and t_2 are the times defined by Bozer & White (1990), depending on $E[DC]$ and its standard deviation. It is considered a constant maximum velocity of the crane, reached with a constant acceleration (Dukic, Opetuk & Lerher, 2015).

Battini et al. (2016) demonstrated that the expected dual command time of a dual-bay VLM $E[DC]$ is defined as the average of the two expected dual command times from the two picking positions A and B (corresponding to the two picking bays), as follows:

$$E[DC] = \frac{E[DC]_A + E[DC]_B}{2} \quad (20)$$

with

$$E[DC]_A = X \cdot \left[\sum_{\substack{m=1 \\ m \neq j}}^{M} \sum_{\substack{j=1 \\ j \neq i}}^{M} (t_{Am} + t_{mj} + t_{jA}) \cdot p_m p_j + 4t_{p/d} \right] \quad (21)$$

$$E[DC]_B = X \cdot \left[\sum_{\substack{m=1 \\ m \neq j}}^{M} \sum_{\substack{j=1 \\ j \neq i}}^{M} (t_{Bm} + t_{mj} + t_{jB}) \cdot p_m p_j + 4t_{p/d} \right] \quad (22)$$

which both take into account that some items can be picked from the same tray, according to the single probability of extracting each tray p_m, thanks to

$$X = [1 - \sum_{m=1}^{M} p_m{}^2 \cdot (1 - p_m)^3] \tag{23}$$

In equations (21) and (22), $t_{p/d}$ is the delay time to pick up and to deposit a tray, while t_{mj} (t_{Am} or t_{Bm}) is the travel time of the crane for moving between two different positions or tray locations m and j (A and m or B and m). Moreover, these show that the dual command times depend on the total number of trays M and, consequently, on the VLM height.

Table 3 shows some examples of calculation of t_l^V varying the number of picking lines per order N and the operator's picking time p_T, considering a VLM 10 meters high storing V^V= 55 m^3. The times t_{fix}^V and t_{travel}^V are equal to 60 s and 5 s, respectively. The search time t_{search}^V is assumed to be equal to 0, since it is considered that the VLM is equipped with a signaling system.

***N* [*lines/order*]**	**5**			**10**			**20**		
p_T *[s]*	**5.0**	**10.0**	**20.0**	**5.0**	**10.0**	**20.0**	**5.0**	**10.0**	**20.0**
t_{pick}^V *[s]*	28.1	28.2	28.6	28.1	28.2	28.6	28.1	28.2	28.6
t_l^V *[s]*	41.1	41.2	41.6	34.6	34.7	35.1	31.3	31.4	31.8

Table 3. Values of average cycle time per line t_l^V [s] varying the average number of lines per order N and the net operator's picking time p_T.

First of all, the times are all quite similar and mainly related to the performance of the VLM, since the VLM is dual-bay and the operator's picking time p_T is always lower than the time of the dual command cycle (condition $0 < p_T < t_1$ of equation 19). On the other side, the number of picking lines per order N is influencing t_l^V, as also seen for t_l^W.

The values reported on Table 3 allow to compare the throughput of a dual-bay VLM system with the throughput of a warehouse with carton racks (Table 2). Of course, for increasing the storage volume V in case of a dual-bay VLM system it would be necessary to add more VLMs, working in parallel.

From the comparison of Table 2 and Table 3 it can be derived that a VLM system would outperform the warehouse with carton racks and, then, would be better from a variable costs C_{var}^{s} perspective, when p_T= 10 or 20 s, like in case of complex kitting activities or small items counting. On the other side, if p_T is lower, the number of lines per order N has to be considered as well. Again, if N is low (e.g. 5, like in case of urgent or e-commerce orders, picking of spare-parts or slow-moving items, preparation of assembly kits with few parts), the VLM turns out to be the best solution.

4 Systems economic and throughput evaluation and comparison

In this section, the ratio between the two total cost functions is defined in order to compare the related small items order picking solutions. A deep multi-factorial analysis of this ratio allows to understand the impact of each parameters in the functions and the applicability of the VLM is depicted thanks to some decisional graphs.

4.1 Economic comparison and analysis

First, the definition of the following ratio permits to compare the two solutions from an economic perspective:

$$R_{TC} = \frac{TC^V}{TC^W} = \frac{C_{fix}^V + C_{var}^V}{C_{fix}^W + C_{var}^W} = \frac{N^V \cdot \left[\left(C_{sp} \cdot k^V \cdot \frac{V^V}{SL^V \cdot H} \right) + C_{op} \cdot \frac{Q}{N^V \cdot Q^V} \cdot h_y \right]}{C_{sp} \cdot k^W \cdot \frac{V}{SL^W \cdot H} + C_{op} \cdot \frac{Q}{Q^W} \cdot h_y} \quad (24)$$

So, when R_{TC} is lower than 1, the VLM system is preferable than the traditional solution, while R_{TC} greater than 1 means that the VLM solution is more expensive than the basic one.

In the following, a multi-factorial analysis on R_{TC} and related ANOVA are reported in order to understand which factors have greater impact on the applicability of the VLM system.

The input varying factors are:

- V=50; 100; 250; 500 m^3
- Q=100; 200; 500; 1000 lines/h
- C_{sp}=80; 120 m^2
- Q^W=60; 80; 90 lines/h
- C_{op}=15; 25; 35 €/h
- h_y=1,800; 3,600 h/year

- C^V= 18,000; 24,000 €/year
- Q^V=90; 100; 120; 150 lines/h

While the input fixed parameters are H=10 m, k^W=2, SL^W=10%, A^V=20 m^2 and V^V=60 m^3. All the other factors are derived from these ones.

Figures 3, 4 and 5 show the results of the ANOVA performed with Minitab software v. 17. Figure 3 shows the interaction plot for R_{TC}, highlighting how the relationship between one categorical factor and a continuous response depends on the value of the second categorical factor, with the categorical factors shown on the diagonal of the plot. The interactions are represented through lines: a parallel line means no interaction, while a non-parallel line indicates interaction. The more non-parallel the lines are, the greater the strength of the interaction is. Looking at the plot, it can be seen that in some cases (e.g. for Q and V or V and Q) the lines are not parallel.

In particular, it is interesting to see how R_{TC} is influenced by V and Q: the ratio has a similar average value (about 0.9) for the smallest storage volume (V=50 m^3) and for all throughputs Q; on the other side, by increasing V it significantly changes, with different trends according to Q.

Figure 4 reports the Pareto chart of the standardized effects related to R_{TC} calculation, useful to quickly understand which are the factors that have a greater impact on it. The chart shows that some factors are predominant, i.e. Q^W, Q^V, Q and the combination of Q and V.

Figure 5 refers to the same factors but showing the normal plot of the standardized effects to the resulting R_{TC}. If the effect of a factor (or of a combination of factors) lays far from the straight line, it means that this is significant for the calculation of the ratio. Moreover, its position with respect to the straight line (left or right) indicates, respectively, the inverse or direct relation.

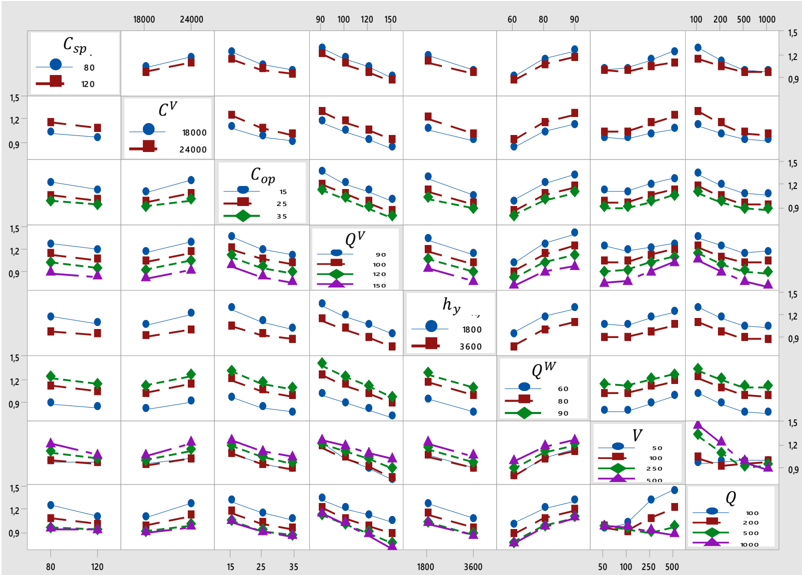

Figure 3. Interaction plot for R_{TC}.

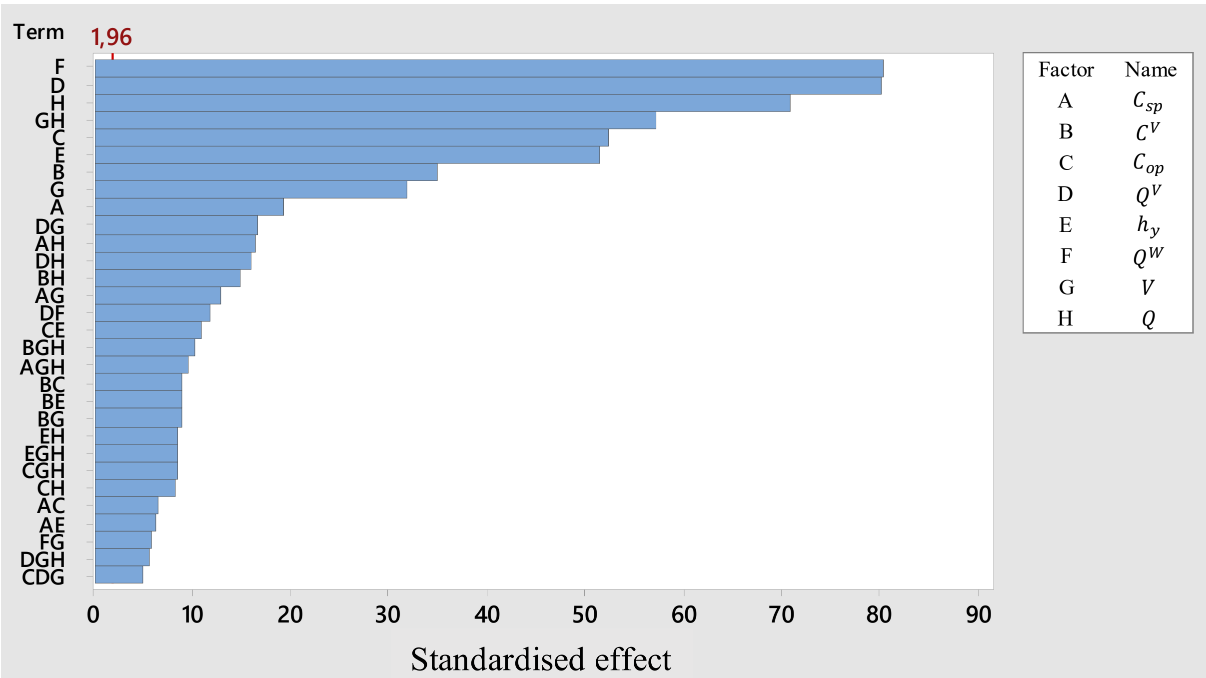

Figure 4. Pareto chart for the standardised effects (30 effects shown), response R_{TC}, α=0.05.

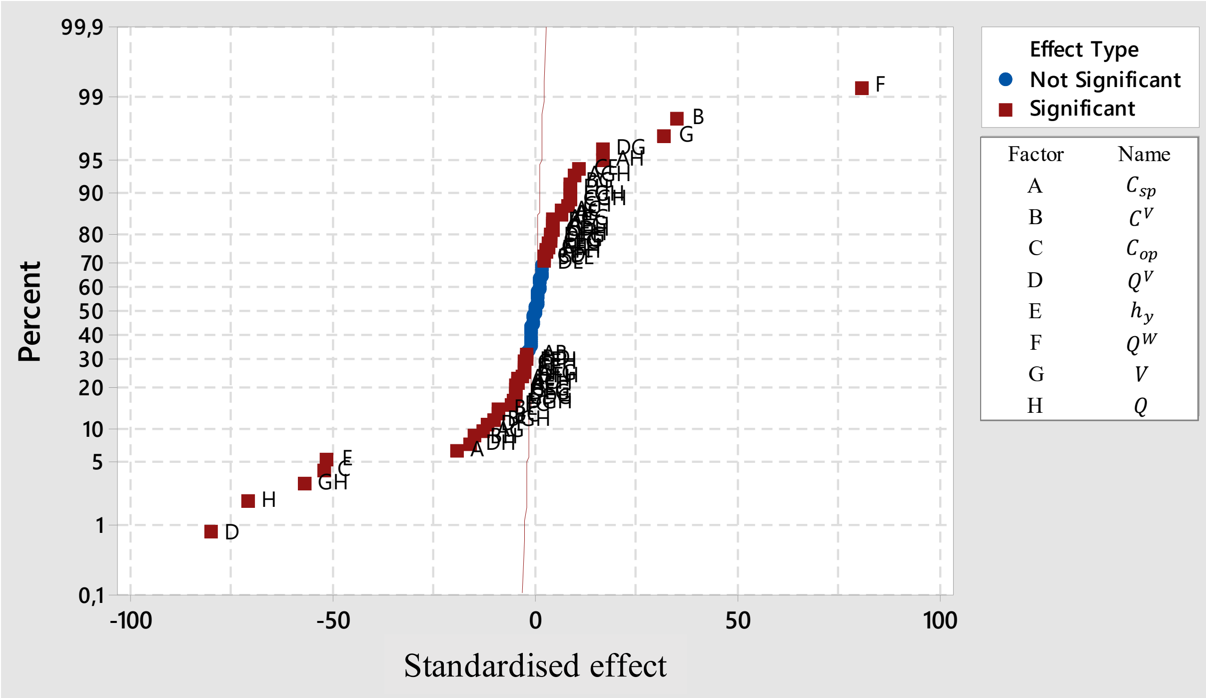

Figure 5. Normal plot of the standardised effects, response R_{TC}, α=0.05.

4.2 VLM area of application definition and analysis

Following the multi-factorial analysis outcomes, the *VLM area of application* is defined and calculated, as a set of conditions in which the VLM system is preferable than the warehouse with carton racks.

The *VLM area of application* is defined by this condition:

$$R_{TC}=\frac{TC^V}{TC^W}=\frac{C_{fix}^V+C_{var}^V}{C_{fix}^W+C_{var}^W}=\frac{N^V\cdot\left[\left(C_{sp}\cdot k^V\cdot\frac{V^V}{SL^V\cdot H}\right)+C_{op}\cdot\frac{Q}{N^V\cdot Q^V}\cdot h_y\right]}{C_{sp}\cdot k^W\cdot\frac{V}{SL^W\cdot H}+C_{op}\cdot\frac{Q}{Q^W}\cdot h_y}\leq 1 \quad (25)$$

Referring to equations (2) and (17), it is easy to notice that, for the fixed cost component (C_{fix}^W and C_{fix}^V), the warehouse with carton racks is always preferable to the VLM system, since for this latter system there is a further cost related to the VLM annual cost C^V, even if the required space is lower.

This is simply verified for typical values of the annual space cost per square meter C_{sp} (about 80-120 €/(m^2year) and k^W coefficient (about 1.5-2), VLM annual cost C^V (about 18,000-24,000 €/year), and related k^V coefficient (about 8-16) and saturation levels SL^W ≈ 10% and SL^V ≈ 30%.

Therefore, if there are not particular restrictions on space availability, the applicability of the VLM system mainly depends on its performance in terms of system throughput Q^V.

Introducing the term R_Q as a throughput ratio between the two solutions:

$$R_Q = \frac{t_l^W}{t_l^V} = \frac{Q^V}{Q^W} \tag{26}$$

and considering the condition of equation (12) and assuming

$$V^V = \frac{V}{N^V} \tag{27}$$

after some mathematical derivations (see Appendix A for the extensive procedure), the final formulation for the systems comparison turns out to be:

$$\frac{C_{sp} \cdot \frac{V^V}{H} \cdot \left(\frac{k^V}{SL^V} - \frac{k^W}{SL^W}\right)}{C_{op} \cdot h_y \cdot (R_Q - 1)} \leq \frac{Q}{N^V \cdot Q^V} \leq 1 \tag{28}$$

where

$$\frac{C_{sp} \cdot \frac{V^V}{H} \cdot \left(\frac{k^V}{SL^V} - \frac{k^W}{SL^W}\right)}{C_{op} \cdot h_y \cdot (R_Q - 1)} = \frac{Q^*}{Q^V} \tag{29}$$

is the threshold value and it is verified if the throughput ratio is as follows:

$$R_Q > 1 + \frac{C_{sp} \cdot \frac{V^V}{H} \cdot \left(\frac{k^V}{SL^V} - \frac{k^W}{SL^W}\right)}{C_{op} \cdot h_y} \tag{30}$$

Therefore, the adoption of a VLM system made of N^V machines is convenient from an economic point of view when

$$Q^* \leq Q/N^V \leq Q^V \tag{31}$$

This is applicable if the throughput ratio R_Q is higher than the threshold value R_Q^*, expressed by the following equation (see Appendix B for mathematical proof):

$$R_Q > R_Q^* = 1 + \frac{C_{sp} \cdot \frac{V^V}{H} \cdot \left(\frac{k^V}{SL^V} - \frac{k^W}{SL^W}\right)}{C_{op} \cdot h_y} \tag{32}$$

Based on these equations, several decisional graphs have been developed to easily understand in which set of conditions one solution is preferable than the other. These graphs represent the behaviour of equation (29) between 0% and 100%. Each line in the graph represents the lower bound of the VLM area of application according to a certain set of input parameters, which lays in the upper delimited part of the graph area up to 100%, which is 1 in equation (28).

The graphs reported in Figure 6 have been elaborated varying some factors: V^V, between 40 m^3 and 60 m^3 with a step of 2 m^3, C^V, equal to 18,000 €/year or 24,000 €/year, and C_{sp}, equal to 80 €/m^2 or 120 €/m^2. The throughput ratio R_Q is equal to 1.25 and 1.5, corresponding, for example, to t_l^W= 37.5 s and t_l^V= 30 s or t_l^W= 45 s and t_l^V= 30 s, respectively. Different hourly cost of the operator C_{op} has been analysed as well (15, 20, 25 and 30 €/h). It has been considered only the case where operators work in two work shifts per day, resulting in h_y=3,600 h/year. However, it is easily demonstrated that when there is only one shift per day (h_y=1,800 h/year), the threshold values are exactly twice the previous ones, since h_y is in the denominator of equation (29).

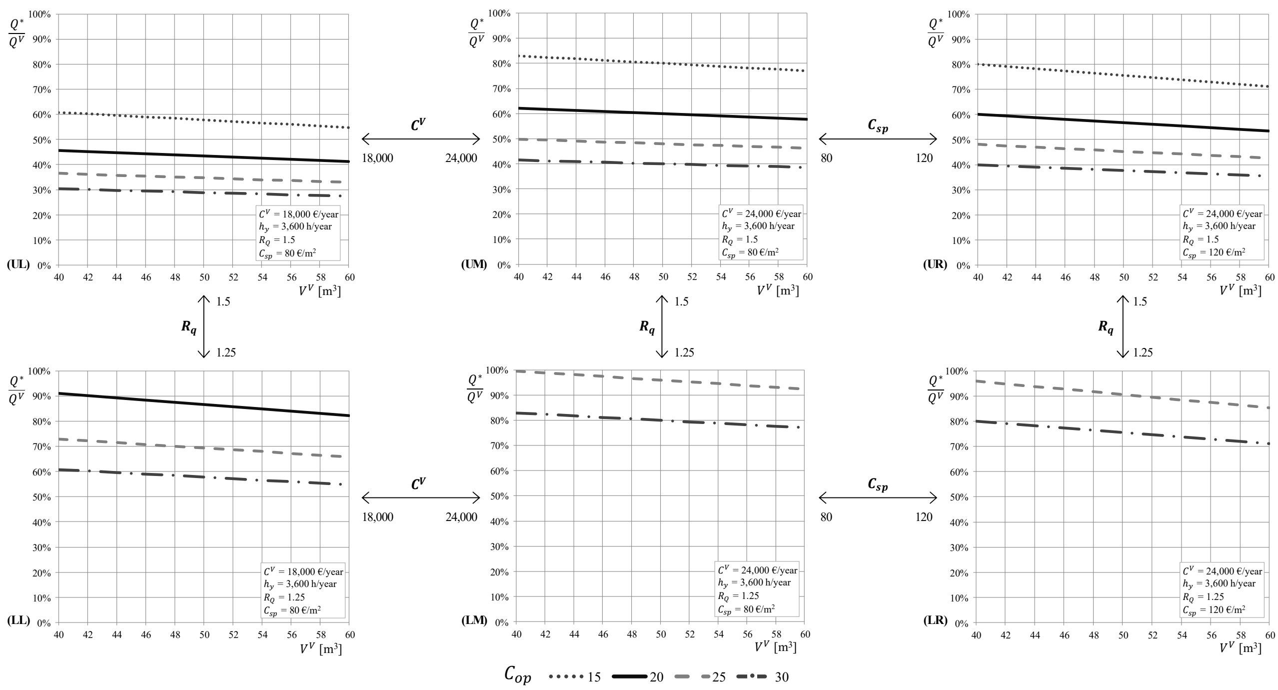

Figure 6. Scenarios for the VLM area of application with different input parameters values.

In Figure 6, the graph UM, in the middle of the upper part, is the base scenario of this analysis. It shows the trend of equation (29), representing the threshold value between the traditional warehouse with carton racks and the VLM, with C^V = 24,000 €/year, C_{sp} = 80 €/m^2, h_y=3,600 h/year and R_Q =1.5.

The other graphs of the figure are linked to the base scenario. They are obtained varying one or more parameters in the analysis in order to understand the impact of their changes.

On the right, the two graphs (UR, in the Upper Right part of the figure, and LR, in the Lower Right part of the figure) report the analysis with a higher space cost C_{sp} = 120 €/m^2. On the left, in the UL and LL graphs, the cost of VLM is C^V = 18,000 €/year, lower than the base scenario. Finally, the graphs in the lower part of the figure (LL, LM and LR) are obtained with a low value of the throughput ratio R_Q.

Analysing the figure, it can be noticed that the hourly cost of the operator has a relevant impact on the threshold values: lower hourly operator costs bring to higher threshold values. Contrarily, the VLM storage volume V^V leads to a decreasing trend of Q^*/Q^V: the decrease is then steeper for the lower values of C_{op}. Generally, when the VLM system is less used, then it is more probable that the warehouse with carton racks is the best option. Moreover, the value of Q^*/Q^V is less affected by the space cost, but, usually, the higher this cost is, the lower is the threshold value, especially for higher V^V. On the other side, in case of a lower VLM cost, of course, the threshold is lower, with a wider area of application compared with the base scenario.

Comparing the graphs in the upper part of the figure where $R_Q = 1.5$, with those depicted in the lower part where $R_Q = 1.25$, the line of the Q^*/Q^V ratio moves up, reducing the VLM area of application. The threshold could be higher than 100% especially when the operator cost is low; in this case, the VLM system could be not the best solution.

Generally, from this analysis some insights can be summarized: the factors V^V and C_{sp} have a low impact on the ratio Q^*/Q^V. This is because the total cost functions of the both solutions are linked to these two parameters in the same way, as described in equations (2) and (17).

On the other side, the factor C^V has a relevant influence on the behaviour of the threshold value. The reason is that the annual cost of the VLM C^V is included only in the total cost TC^V, so it has a direct impact on the threshold value.

Finally, the VLM system could be not applicable if it works just one shift per day, since in this case the VLM system would turn out to be too expensive and not appropriately used.

The analysis of the ratio R_Q is reported in Figure 7. As in Figure 6, the graph in the middle (M) represents the base scenario. On the right, the graph R reports the analysis with a higher space cost C_{sp} = 120 €/m^2, while on the left, in the graph L, the cost of VLM C^V = 18,000 €/year is lower than the base scenario.

The results are similar to the ones obtained in the analysis of the previous graphs: R_Q is not very affected by V^V and C_{sp} while it can change according to the hourly cost of the operator C_{op}. The factor C^V, instead, has a relevant impact, as demonstrated by the comparison of the graphs M and L: if the VLM has a lower cost, it can be applicable also for lower values of R_Q^*.

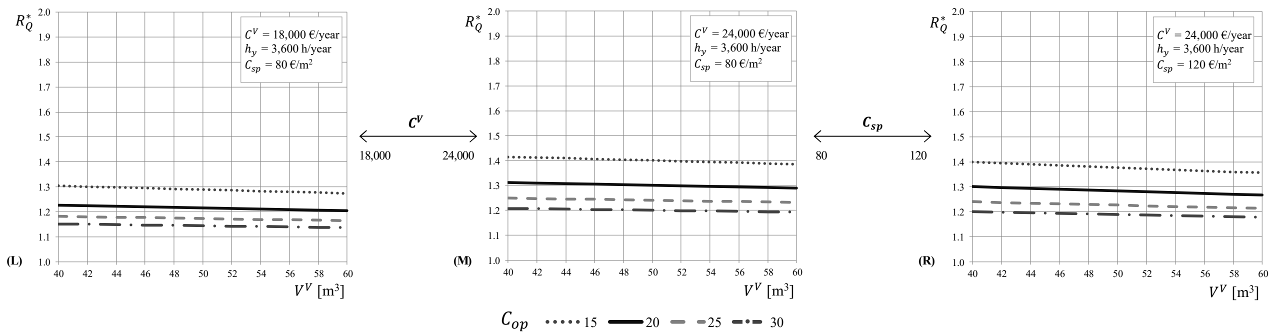

Figure 7. Trend of R_Q^* for different input parameters values.

5 Case Study: applicability of VLM system

The following case study shows the application of the economic model developed in the present paper. A step-by-step procedure is used to explain which data are necessary and how to compare the VLM system to the traditional warehouse with carton racks.

The application of the method can go through the following procedure.

1. Input data collection and estimation

The first step for the application of the method consists in retrieving the data that are needed for the formulations. General information and useful input data about the warehouse and the pickers are reported in Table 4.

The studied warehouse has an area A^W of about 26 x 10 m^2 with a mezzanine where two different categories of products are stocked in carton racks. These two groups are stocked in the two floors: group A on the mezzanine level (with a resulting k^W=2.59) and group B on the ground floor (then, k^W=1.47).

Direct measurement method has permitted to estimate the average picking time per order line t_l^W. The total amount of time spent by the operators in the overall picking operations has been divided by the total amount of order lines performed in a period of 20 weeks, resulting in t_l^W=40.9 s for group A and t_l^W=46.6 s for group B.

	Group A	**Group B**
Type of products	Merchandising products (t-shirts, cap, gloves etc.)	Small metal parts (sealings, small bearings, etc.) and kits for motorcycle engines
Number of stored items	About 2,500	About 3,500
V [m^3]	About 165 m^3 on the mezzanine floor	About 165 m^3 on the ground floor
Q [picks/h]	250	310
N [lines/order]	10	20
t_l^W [s]	40.9 s	46.6 s
h_y [h]	1,800 h	3,600 h
k^W	2.59	1.47
SL^W	0.128	
C_{sp} [€/(hm^2)]	120	
C_{op} [€/h]	20	

Table 4. Information about the case study.

From a first analysis of the available height of the warehouse area, it could be installed a VLM system 10 m high, with 55 m^3 as storage volume V^V. From the information provided by different technical sheets and also confirmed by previous researches, typical values of t_{pick}^{V}, t_{fix}^{V} and t_{travel}^{V} for this kind of system are about 28 s, 60 s and 5 s, respectively. Thus, considering the different average number of lines per order N, the average picking time t_{l}^{V} is 34.5 s for the items of group A and 31.3 s for the items of group B.

2. R_Q calculation and VLM applicability evaluation

Starting from the average cycle times per line t_{l}^{W} and t_{l}^{V} it can be estimated the throughput ratio R_Q between the traditional solution and the VLM system for both cases: 1.19 for the group A and 1.49 for group B.

Then, based on the input data available from the previous step, for the products of group A, R_Q^* (equal to 1.30) is higher than R_Q, and consequently, the traditional warehouse with carton racks is preferable.

For the other group of items (B) the throughput ratio is higher, R_Q=1.49. In this particular case, R_Q^*=1.23 because the warehouse works for h_y=3,600 h/year. Thus, the VLM system is preferable for this group of products, since $R_Q > R_Q^*$.

3. Area of application and system position representation

The VLM application areas for the two analysed cases, group A and group B, are reported in Figure 11. For the items of group A, it can be seen that there is no VLM area of application, and the position of the adopted solution, related to V^V= 55 m^3, lays on the area of the warehouse with carton racks ($\frac{Q}{N^V \cdot Q^V} = 0.799$). For group B, the VLM area of application is for values of Q^*/Q^V higher than about 65%, for the storage volume V^V of 55 m^3. Here, the condition $\frac{Q}{N^V \cdot Q^V} > \frac{Q^*}{Q^V}$ is verified, since $\frac{Q}{N^V \cdot Q^V} = 0.897$.

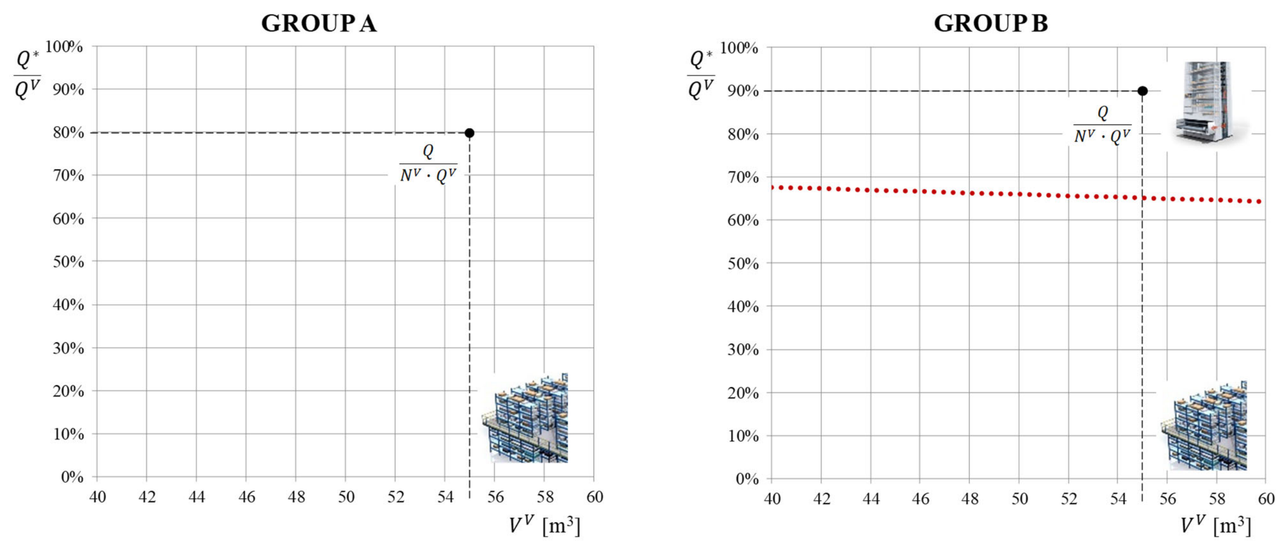

Figure 11. Trend of Q^*/Q^V and VLM area of application for the analysed cases.

4. Total costs and saving calculation

For group B, the applicability of the VLM system can be demonstrated also by calculating the total cost functions of the two alternatives. For the as-is scenario of the warehouse with carton racks, the total cost is:

$$TC^W = C_{sp} \cdot k^W \cdot \frac{V}{SL^W \cdot H} + C_{op} \cdot \frac{Q}{Q^W} \cdot h_y = 22{,}932.00 + 289{,}206.79 = 312{,}138.79 \text{ €}/year \quad (33)$$

Based on the input data, the required number of VLMs to be installed N^V is:

$$N^V = \max\left(\left\lceil\frac{Q}{Q^V}\right\rceil; \left\lceil\frac{V}{V^V}\right\rceil\right) = \max\left(\left\lceil\frac{310}{115}\right\rceil; \left\lceil\frac{165}{55}\right\rceil\right) = 3 \quad (34)$$

and the total cost can be estimated as:

$$TC^V = N^V \cdot \left[\left(C_{sp} \cdot k^V \cdot \frac{V^V}{SL^V \cdot H}\right) + C_{op} \cdot \frac{Q}{N^V \cdot Q^V} \cdot h_y\right] = 79{,}200.00 + 193{,}750.00 = 272{,}950.00 \text{ €}/year \quad (35)$$

The total saving obtained by installing the VLM system is about 39,000 €/year, therefore about 12.6%.

6 Model and results discussion

The multi-factorial analysis presented in Section 4 and the industrial study reported in Section 5 suggest some comments and insights about the proposed model.

Generally, it turned out that a warehouse with carton racks is preferable to a VLM system when the VLM has lower time performances, even if it occupies less space, since a VLM has higher fixed costs. For example, this could happen in case of fast manual order picking solutions, like in a forward picking area (Bartholdi & Hackman, 2017).

On the other side, in case the VLM system is faster than the traditional one, further analysis is needed to fully understand its applicability. In this case, a VLM could turn out to be suitable only for the slow moving (C-class) products, which are usually characterised by high picking times in traditional manual order picking systems, resulting in a throughput ratio R_Q higher than 1.5. Another aspect that influences the applicability of a VLM system is the number of hours per year in which the warehouse is used. If the warehouse is accessed only for one shift per day (h_y=1,800 hours/year), the VLM could turn out to be too expensive; then, it would be better to consider a traditional warehouse with carton racks.

The analysis showed that the smaller are the orders (in terms of number of items to pick) and the better is the VLM system as order picking solution. Moreover, the applicability of the VLM system is linked to the space cost: when the cost per square meter is high or when it is recommended to save the available space for other value-added activities, the VLM system is preferable to the warehouse with carton racks. However, it is important to point out that a VLM system leads to proper savings when it is adequately exploited, hence, when its saturation level is high. It is therefore fundamental to adopt the most suitable system for the considered case in terms of storage volume and performance, and at the most adequate cost.

Most of the results that have been derived from the previous sections can be explained looking deeply at the equations (28) and (29). Here, it can be noticed that the upper part of the threshold ratio is related to the fixed terms of the total cost, while the lower part is linked to the variable costs. Then, it is simple to understand that the VLM system turns out to be better than the warehouse with carton racks when the savings obtained by reducing the variable costs using a faster system (i.e. the VLM system) are higher than the additional fixed costs that are required for installing the new equipment.

A possible limitation of a VLM system compared to the warehouse with carton racks is related to its lower flexibility. In fact, usually a VLM system has to be designed considering the peak of the required throughput over time, while in the warehouse with carton racks a higher throughput can be satisfied by increasing the number of operators working within. Of course, a decrease in the required throughput would lead to a reduction of the number of pickers for both systems, but with a higher hardware investment cost for the VLM solution.

Finally, it has to be highlighted the importance of the measurement and of the estimation of the input data, which, of course, can influence the possible outcomes. However, the study of the area of application of the VLM system allows to be aware of how near (or far) is the result to the VLM application threshold, and, hence, how robust the decision is. Moreover, the ANOVA and the industrial case show the parameters that have a higher impact on the final result, which are the required throughput and the one warranted by the two storing systems, the total storage volume and the labour cost.

7 Conclusions and future researches

In this paper, a mathematical model to evaluate and compare a warehouse with carton racks and a dual-bay Vertical Lift Module system, has been proposed. The model starts with the proposal of two cost formulations, which consider, for both systems, the most common emerging fixed and variable costs, like the cost per square meter, also depending on its saturation, and the operators' cost. Then, a single formulation has been derived, which depends on the annual floor space cost per square meter, on the annual operator cost, on the volume saturation levels of the two systems and on the throughput ratio of the two systems. This allowed to define a VLM area of application, which can be also easily integrated in a decisional graph. The study of the behaviour of the formulation in the multi-factorial analysis and the ANOVA highlighted that the ratio Q^*/Q^V is not very influenced by the total storage volume V, while it is by the annual floor space cost per square meter and to the annual cost of the VLM. Moreover, the reported industrial case showed that differences in the required throughput Q, in the number of working hours per year h_y and/or in the number of lines per order N can affect the applicability of the VLM system.

Aside from for the comparison of the two systems and for the study of the applicability of a VLM system, the model introduced in this paper can be used also for the design and the sizing of the two storage solutions. Moreover, the model of the dual-bay VLM system could be applied also for other kinds of VLM, for example the single bay VLM, by changing the system picking times and the related throughput Q.

As already stated, this paper contributes to the economic modelling of small items order picking systems. In future researches, it would be interesting to model also the refilling activity of the items within the storage locations, which could have an impact on the storage allocation, on the travelled distances and, then, on the total picking time (Battini et al., 2014; Bartholdi & Hackman, 2017). Another interesting study could concern the modelling of batch picking instead of single order picking. Although the already introduced formulations can be easily adapted to the batch picking activity, since it would imply only an increase of the picking times and on the number of lines per order (Bartholdi & Hackman, 2017), these should then be integrated with some factors considering the subsequent sorting activity, which is not usually related to the used storage and order picking system.

The presented analysis could also be extended to other small items order picking systems, with the aim of deriving a complete tool for their evaluation and comparison. A further development could also integrate the adoption of various paperless picking solutions, having an impact on the searching and picking time and on the costs of the manual systems (Battini et al., 2015b). This could help the choice of their most proper field of application in real warehouse picking contexts.

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Appendix A: Mathematical derivation for equation (28)

Comparing equations (2) and (17), as defined in equation (25):

$$TC^W = C_{fix}^W + C_{var}^W = C_{sp} \cdot k^W \cdot \frac{V}{SL^W \cdot H} + C_{op} \cdot \frac{Q}{Q^W} \cdot h_y \tag{2}$$

$$TC^V = C_{fix}^V + C_{var}^V = N^V \cdot \left[\left(C_{sp} \cdot k^V \cdot \frac{V^V}{SL^V \cdot H}\right) + C_{op} \cdot \frac{Q}{N^V \cdot Q^V} \cdot h_y\right] \tag{17}$$

in the total costs ratio $R_{TC} = \frac{TC^V}{TC^W}$, it is possible to derivate the final equation (28) with the following mathematical steps:

$$R_{TC} = \frac{C_{fix}^V + C_{var}^V}{C_{fix}^W + C_{var}^W} = \frac{N^V \cdot \left[\left(C_{sp} \cdot k^V \cdot \frac{V^V}{SL^V \cdot H}\right) + C_{op} \cdot \frac{Q}{N^V \cdot Q^V} \cdot h_y\right]}{C_{sp} \cdot k^W \cdot \frac{V}{SL^W \cdot H} + C_{op} \cdot \frac{Q}{Q^W} \cdot h_y} \leq 1 \tag{25}$$

As explained in Section 3.2, assuming $V^V = \frac{V}{N^V}$, then $V^V \cdot N^V = V$, the previous equation becomes:

$$R_{TC} = \frac{C_{sp} \cdot k^V \cdot \frac{V}{SL^V \cdot H} + C_{op} \cdot \frac{Q}{Q^V} \cdot h_y}{C_{sp} \cdot k^W \cdot \frac{V}{SL^W \cdot H} + C_{op} \cdot \frac{Q}{Q^W} \cdot h_y} \leq 1 \tag{A.1}$$

Then:

$$C_{sp} \cdot k^V \cdot \frac{V}{SL^V \cdot H} + C_{op} \cdot \frac{Q}{Q^V} \cdot h_y \leq C_{sp} \cdot k^W \cdot \frac{V}{SL^W \cdot H} + C_{op} \cdot \frac{Q}{Q^W} \cdot h_y \tag{A.2}$$

$$C_{sp} \cdot k^V \cdot \frac{V}{SL^V \cdot H} - C_{sp} \cdot k^W \cdot \frac{V}{SL^W \cdot H} \leq C_{op} \cdot \frac{Q}{Q^W} \cdot h_y - C_{op} \cdot \frac{Q}{Q^V} \cdot h_y \tag{A.3}$$

$$C_{sp} \cdot \frac{V}{H} \cdot \left(\frac{k^V}{SL^V} - \frac{k^W}{SL^W}\right) \leq C_{op} \cdot h_y \cdot \left(\frac{Q}{Q^W} - \frac{Q}{Q^V}\right) \tag{A.4}$$

$$C_{sp} \cdot \frac{V}{H} \cdot \left(\frac{k^V}{SL^V} - \frac{k^W}{SL^W}\right) \leq C_{op} \cdot h_y \cdot \frac{Q}{Q^V} \cdot \left(\frac{Q^V}{Q^W} - 1\right) \tag{A.5}$$

Using the throughput ratio $R_Q = \frac{Q^V}{Q^W}$, and the assumption $V^V \cdot N^V = V$, equation (A.5) can be written as follows:

$$C_{sp} \cdot \frac{V^V \cdot N^V}{H} \cdot \left(\frac{k^V}{SL^V} - \frac{k^W}{SL^W}\right) \leq C_{op} \cdot h_y \cdot \frac{Q}{Q^V} \cdot \left(R_Q - 1\right) \tag{A.6}$$

$$\frac{C_{sp} \cdot \frac{V^V \cdot N^V}{H} \cdot \left(\frac{k^V}{SL^V} - \frac{k^W}{SL^W}\right)}{C_{op} \cdot h_y \cdot \frac{Q}{Q^V} \cdot (R_Q - 1)} \leq 1 \tag{A.7}$$

Finally, based on the assumption $N^V = \max\left(\left\lceil \frac{Q}{Q^V} \right\rceil ; \left\lceil \frac{V}{V^V} \right\rceil\right)$, the final equation (28) is demonstrated:

$$\frac{C_{sp} \cdot \frac{V^V}{H} \cdot \left(\frac{k^V}{SL^V} - \frac{k^W}{SL^W}\right)}{C_{op} \cdot h_y \cdot (R_Q - 1)} \leq \frac{Q}{Q^V \cdot N^V} \leq 1 \tag{A.8}$$

Appendix B: Mathematical derivation for equation (30)

Equation (28) is validated if and only if the following inequation is true:

$$\frac{C_{sp} \cdot \frac{V^V}{H} \cdot \left(\frac{k^V}{SL^V} - \frac{k^W}{SL^W}\right)}{C_{op} \cdot h_y \cdot (R_Q - 1)} \leq 1 \quad \text{(B.1)}$$

Then

$$C_{sp} \cdot \frac{V^V \cdot N^V}{H} \cdot \left(\frac{k^V}{SL^V} - \frac{k^W}{SL^W}\right) \leq C_{op} \cdot h_y \cdot \frac{Q}{Q^V} \cdot (R_Q - 1) \quad \text{(B.2)}$$

$$\frac{C_{sp} \cdot \frac{V^V}{H} \cdot \left(\frac{k^V}{SL^V} - \frac{k^W}{SL^W}\right)}{C_{op} \cdot h_y} \leq (R_Q - 1) \quad \text{(B.3)}$$

Since $\frac{k^V}{SL^V}$ is always greater than $\frac{k^W}{SL^W}$, in the domain of values of these parameters, equation (30) is demonstrated:

$$R_Q > 1 + \frac{C_{sp} \cdot \frac{V^V}{H} \cdot \left(\frac{k^V}{SL^V} - \frac{k^W}{SL^W}\right)}{C_{op} \cdot h_y} \quad \text{(30)}$$