Closest Security Boundary for Improving Oscillation Damping through Generation Redispatch using Eigenvalue Sensitivities

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Abstract — **This paper proposes an algorithm for determining a minimum generation redispatch capable of moving specific oscillation modes to the closest small-signal security boundary that guarantees them a desired damping factor. The algorithm combines nonlinear optimization techniques with numerical eigenvalue sensitivities obtained from multiple runs of a load flow solver and partial eigensolutions. The algorithm, namely Closest Security Boundary for Generation Redispatch using Eigenvalue Sensitivities (CSBGRES) relies on sparse matrices and is applicable to large-scale power systems. The work presents results on the Brazilian Interconnected Power System (BIPS) and the Nordic 44 Test System (N44S).**

Keywords — **Small-signal stability; Hopf bifurcations; Newton-Raphson method; Stability margins; Security boundaries; Nonlinear programming.**

1. Introduction

Bifurcation theory [1] applied to power system voltage stability problems had a large momentum in early 90's [2,3], particularly with saddle-node bifurcation analysis in load flow equations. In sequence, investigations considering the Hopf bifurcation analysis in differential-algebraic equations (DAE) [4-7] were performed under the voltage stability phenomenon.

The Hopf bifurcation theory was also applied to small-signal stability and control system evaluations [8-14]. The early works on Hopf bifurcation in small-signal stability have been concentrated on the analysis of eigenvalues with focus on controller parameters, such as power oscillation dampers (POD) and power system stabilizers (PSS). Some other works on Hopf bifurcations were related to subsynchronous stability analysis [15-18].

Some attempts to study Hopf bifurcation with respect to generation or demand parameters were made in [19-23], but the analytical determination of eigenvalue sensitivities with respect to the load flow parameters is complex and necessary to solve large scale problems [24].

This paper uses a numerical computation of eigenvalue sensitivities with respect to generation dispatch, obtained through multiple runs of a load flow solver combined with partial eigensolutions, namely Generation Sensitivities (GenSens). These numerically-computed sensitivities become the basis of the algorithm used to calculate a minimum redispatch for power systems, considering a damping factor criterion for oscillation modes, which is being proposed in this paper. Some previous works deal with this kind of sensitivities [14,24,25], but this is the first work where these sensitivities are used to solve the Hopf bifurcation problem considering minimum redispatch.

Using nonlinear programming, this algorithm determines the minimum generation redispatch of a selected number of power plants capable of moving a specific oscillation mode to the closest small-signal security boundary, defined by a desired damping ratio locus in the s-plane.

Moreover, if the system is unstable and a null damping ratio locus is chosen, corresponding to the geometric locus in the s-plane of Hopf bifurcations, the algorithm will find the minimum redispatch to stabilize the system. The algorithm can also be used, in the other way around, to calculate the small-signal security or stability margins with respect to generation redispatch.

The proposed algorithm, namely Closest Security Boundary for Generation Redispatch using Eigenvalue Sensitivities (CSBGRES), uses complete models of system components, numerical generation sensitivity calculation and optimization techniques, considering a damping factor criterion for system oscillation modes.

The CSBGRES method relies on sparse matrices and is applicable to large-scale power systems, as it was done in [8,13], where only controller parameter variations were considered, but, in the case of this paper, dispatch variations can also be considered. This work presents simulation results on the Brazilian Interconnected Power System (BIPS) and the Nordic 44 Test System (N44S). The proposed method is the first one to solve this kind of problem for large scale power system.

The remainder of the paper is organized as follows: Section II reviews the small-signal stability concept and its relation to the problem being solved; Section III presents the numerical eigenvalue sensitivity computation with respect to active power to be used in the development of the proposed algorithm, which is described in Section IV; Section V and VI present simulation results for the BIPS and the N44S, respectively; and Section VII presents the conclusions of this paper.

2. Small-signal Stability Review

The small-signal stability and modal analysis are very important to understand the power system dynamic behavior, which can be evaluated through calculating system eigenvalues or oscillation modes. The main concepts related to these subjects are the basis of the CSBGRES method definition, therefore, they will be reviewed in this section.

The set of nonlinear differential and algebraic equations that describes electric power systems for electromechanical transient analysis can be represented through (1) and (2), according to [8,13]:

$$\boldsymbol{T}\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{p}, u) \tag{1}$$

$$y = \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{p}, u) \tag{2}$$

where $\boldsymbol{x}$ is the vector of state and algebraic variables, $\boldsymbol{u}$ and y are, respectively, input and output variables of the system, $\boldsymbol{p}$ is a vector of a system parameter set and $\boldsymbol{T}$ is a diagonal matrix with ones and zeros, identifying the algebraic and differential equations of the system.

The differential and algebraic equations are initialized through using the results of a power flow solution, considering that the system is operating in a steady-state condition. The correct initialization guarantees that the relation between the differential equations in steady-state, the power flow equations and the other algebraic equations are consistent.

Small-signal assessment is obtained through the linearization of (1) and (2) around the steady-state condition, yielding (3) and (4):

$$\boldsymbol{T}\Delta\dot{\boldsymbol{x}} = \boldsymbol{J}\,(\boldsymbol{x_0}, \boldsymbol{p_0})\,\Delta\boldsymbol{x} + \boldsymbol{b}\,(\boldsymbol{x_0}, \boldsymbol{p_0})\,\Delta u \tag{3}$$

$$\Delta y = \boldsymbol{c}\,(\boldsymbol{x_0}, \boldsymbol{p_0})\,\Delta\boldsymbol{x} + d(\boldsymbol{x_0}, \boldsymbol{p_0})\,\Delta u \tag{4}$$

where $\boldsymbol{J}$ is the augmented jacobian matrix, $\boldsymbol{b}$ and $\boldsymbol{c}$ are, respectively, the input and output matrices, d is the direct transmission term and the subscript "$\boldsymbol{0}$" in $\boldsymbol{x_0}$ and $\boldsymbol{p_0}$ means the corresponding initial condition of vectors $\boldsymbol{x}$ and $\boldsymbol{p}$.

The generalized system eigenvalues, which describe the frequency and damping factor of natural oscillations of the system, can be computed for the linear matrix pencil $(\boldsymbol{J}, \boldsymbol{T})$, commonly performed in small-signal stability solvers [8,13]. These eigenvalues are the poles of the system.

The problem being solved in this paper consists in obtaining the vector $\boldsymbol{p}$, displaced from $\boldsymbol{p_0}$, according to a minimum norm, that moves a critical oscillation mode to lie on a specified security boundary locus in the s-plane (which is a constant damping ratio line). The solution of this problem was proposed in [8,13], where parameters in $\boldsymbol{p}$ were PSS or POD gains.

Some mathematical difficulty arises when elements of $\boldsymbol{p}$ are related to power flow parameters, such as generation or demand. In this case, using the same approach of [8,13], the whole set of power flow and initialization equations must be included in the equationing, which brings a large complexity to the proposed optimization problem and its modelling.

The main contribution of this paper lies on solving the aforementioned problem in a different way, using eigenvalue sensitivities obtained by a numerical differentiation procedure. This methodology effectively solves this problem considering an alternating solution, involving data communication between a small-signal stability solver and a power flow solver.

In this work, the small-signal stability software PacDyn [26] was used for the computational implementation of the proposed algorithm and the software ANAREDE [27] was used for the power flow computations. The communications between them were performed in memory, using a Windows Dynamic Link Library version of ANAREDE inside PacDyn's programming code.

3 Generation Sensitivity Calculation

This section presents the procedure to calculate eigenvalue sensitivities with respect to the active power dispatched by plants in power systems, namely generation sensitivities. The GenSens can be mathematically defined as the derivative of an eigenvalue λ related to the active power P dispatched by a power plant specified in the power flow data.

These sensitivities could be analytically computed through building a complete jacobian matrix, which includes the linearization of power flow equations, dynamic modeling equations and pole definition. However, it yields a very complex mathematical expression.

Additionally, some power flow controls (such as transformer tap changes, switching devices or intertie power flow controls), which are usually implemented as an alternating procedure in power flow solvers, should be fixed or described by approximate equations.

Due to the aforementioned difficulties, in this work, a numerical procedure was adopted to calculate the eigenvalue sensitivities with respect to each power plant dispatch. This numerical method is based on the central difference procedure.

In this procedure, a small positive variation $(P + \Delta P)$ and a small negative variation $(P - \Delta P)$ are applied in the active power dispatched by a given power plant and the corresponding oscillation modes are obtained for both situations ($\lambda_{+\Delta P}$ and $\lambda_{-\Delta P}$), through running the power flow solver, initializing the system dynamic model and using the DPSE method [28], where a transfer function with high residues for the mode under analysis should be used.

Note that, this GenSens calculation may consider any power flow controls of the system, since the power flow computation is solved stand-alone. Both power flow solution and model initialization are performed prior to the eigenvalue calculations, as commonly done in a conventional small-signal stability software, in an alternating procedure. Therefore, there is no need of simultaneous solution for determining these eigenvalue sensitivities.

After the calculation of these new oscillation modes, obtained for the mentioned conditions, the generation sensitivities of an oscillation mode λ, computed in the original steady-state condition of the power system, with respect to the dispatch P of a specific power plant can be approximately determined through (5) and (6):

$$\frac{\partial \lambda}{\partial P} \approx \frac{\Delta \lambda}{2\,\Delta P}, \quad \Delta P \to 0 \tag{5}$$

$$\frac{\partial \lambda}{\partial P} \approx \frac{\lambda_{+\Delta P} - \lambda_{-\Delta P}}{2\,\Delta P}, \quad \Delta P \to 0 \tag{6}$$

The choice of the value of ΔP is a very important task, which impacts directly on the reliability of the generation GenSens. If this magnitude is very small, problems related to convergence tolerance of power flow solver may occur. On the other hand, if this magnitude is very high, the sensitivity may not be precise. So, it is necessary to choose an intermediate value considering these two limit situations.

Some tests were made using small-scale power systems and the magnitude chosen to be used in the GenSens computations was 0.1 per unit in system power base.

Note that, in general, any change in power plant dispatches would be compensated by the slack bus in the power flow calculation, but, in this method, it does not occur. Instead of leaving the slack bus to supply this dispatch variation, in the proposed procedure, the dispatch of the machine of interest is varied in 0.1 per unit in system base power and the dispatches of the remaining power plants are varied in the opposite direction, weighted by their nominal capability.

In this way, a higher variation is applied to the machine of interest and small variations are applied to the remaining ones. This procedure seems to be a better approximation to represent an isolated variation in the dispatch of the power plant of interest, being a good way to perform the numerical calculation of the generation sensitivities.

The numerical method proposed to calculate the GenSens uses two directions of variation (positive and negative) for the power plant dispatches. If the power flow calculation does not converge to one of these directions, then only the converged one is used to determine the generation sensitivity. If both directions have convergence problems, then the generation sensitivity is assumed equal to zero for the specific power plant to which the convergence problem occurred.

The GenSens represents the sensitivity of an oscillation mode λ with respect to a certain power plant dispatch P and shows the displacement trend of this specific mode in the s-plane when increasing the dispatch of this specific power plant. These generation sensitivities are complex values and may be decomposed into their real and imaginary components.

The GenSens will be the basis of the CSBGRES method, since it will be used in the modelling of the proposed problem and in the ranking of the best power plants for redispatch, where the machines with highest sensitivities should be selected to be utilized in the optimization method, which will be presented in the next section of the paper.

4 Redispatch and Closest Security Boundary

This section describes the proposed CSBGRES method, which can be used to determine the minimum variation in the active power dispatches of system power plants capable of placing a complex eigenvalue λ at a security boundary defined in the s-plane.

This security boundary is defined as a geometric locus in the s-plane, given by a function that relates the real (σ) and the imaginary (ω) components of the eigenvalue, as given in (7):

$$B(\sigma,\omega) = 0 \tag{7}$$

In this work, the security boundary was considered as a line in the s-plane that represents the desired damping factor ξ_d, whose related functions are presented in (8) and (9), as it was done in [8,13]:

$$\xi_d = \frac{-\sigma}{\sqrt{\sigma^2+\omega^2}} \rightarrow \xi_d + \frac{\sigma}{\sqrt{\sigma^2+\omega^2}} = 0 \rightarrow \sigma + \frac{\xi_d}{\sqrt{1-{\xi_d}^2}}\omega = 0 \tag{8}$$

$$B(\sigma,\omega) = \sigma + \frac{\xi_d}{\sqrt{1-{\xi_d}^2}}\omega = 0 \tag{9}$$

The particular case where the desired damping factor is zero ($\xi_d = 0$) corresponds to the stability boundary or points of Hopf bifurcation occurrences, which are related to stability limits.

Optimization methods and techniques are needed to minimize an objective function capable of ensuring the minimum variation of the active power dispatches of the power plants selected to be used in the CSBGRES method. The sum of the square differences of these active powers will be used as the objective function to be minimized.

The proposed optimization problem can be formulated according to the objective function (10), considering the constraints (11) and (12):

$$Min\, f_{obj}(P) = \sum_{i=1}^{n}\left(P_i - P_{i_0}\right)^2 \tag{10}$$

S.t.: $\sigma(P) + \frac{\xi_d}{\sqrt{1-\xi_d{}^2}}\omega(P) = 0$ (11)

$\sum_{i=1}^{m} P_i - \sum_{i=1}^{m} P_{i_0} = 0$ (12)

where P is the dispatch vector, P_i is the dispatch of power plant i, P_{i_0} is the initial dispatch of power plant i, σ is the real component of mode λ, ω is the imaginary component of mode λ, n is the number of selected power plants and m is the total number of power plants in service in the system, (11) represents the displacement of mode λ in the damping factor ξ_d line in the s-plane and (12) represents the power flow balance of the system, neglecting the variation of active losses.

This loss variation can be absorbed by the slack bus or can be divided proportionally through the selected generators during the iterative process of the method.

A special heuristic was used in the computational implementation of the CSBGRES method to deal with this specific issue. In each iteration, the method calculates the loss variation and redistributes it into the dispatches obtained for the selected machines. This redistribution of the active losses is made through a weighted way, which is based on the nominal capability of the selected power plants. In this way, the variation of the losses, which really happens when redispatching the system power plants, is indirectly considered in the method.

Note that, in the proposed problem, economic aspects are not being considering in the redispatch, which could be easily done through using weights for the elements of the vector P.

In [8,13], the components σ and ω were independent variables of the optimization problem, since they are directly calculated in each iteration of the algorithm, as well as the parameters of interest and the lagrangian multipliers. On the other hand, in the CSBGRES method, these components are considered dependent variables of the proposed optimization problem. It means that they are not directly calculated in each iteration of the method, but only the active power dispatches of interest and the lagrangian multipliers.

The oscillation modes are (indirectly) obtained in the beginning of the iterations, using the most recent values determined for the active power dispatches. Because of this characteristic, these variables can be considered dependent variables of the optimization problem defined in the development of the proposed method. This is an important contribution of this work.

Now, using the Lagrange method to solve the optimization problem, the lagrangian function (LF), which must be minimized, can be defined in (13):

$$Min\ LF = \sum_{i=1}^{n}\left(P_i - P_{i_0}\right)^2 + l_1\left(\sigma(P) + \frac{\xi_d}{\sqrt{1-{\xi_d}^2}}\omega(P)\right) + l_2\left(\sum_{i=1}^{m} P_i - \sum_{i=1}^{m} P_{i_0}\right) \qquad (13)$$

where l_1 and l_2 are, respectively, the first and second lagrangian multipliers.

The problem is solved when the lagrangian function gradient is null, in other words, $\nabla LF = 0$. Through this gradient, (14), (15) and (16) can be defined:

$$\frac{\partial LF}{\partial P} = 2\ (P - P_0) + l_1\left(\frac{\partial \sigma}{\partial P} + \frac{\xi_d}{\sqrt{1-{\xi_d}^2}}\frac{\partial \omega}{\partial P}\right) + l_2 = 0 \qquad (14)$$

$$\frac{\partial LF}{\partial l_1} = \sigma + \frac{\xi_d}{\sqrt{1-{\xi_d}^2}}\omega = 0 \qquad (15)$$

$$\frac{\partial LF}{\partial l_2} = \sum_{i=1}^{m} P_i - \sum_{i=1}^{m} P_{i_0} = 0 \qquad (16)$$

where the derivatives $\frac{\partial \sigma}{\partial P}$ and $\frac{\partial \omega}{\partial P}$ can be obtained through the calculation of generation sensitivities given in (5) and (6) for the mode λ with respect to dispatch vector P.

The derivatives $\frac{\partial \sigma}{\partial P}$ are the real components of the GenSens and the derivatives $\frac{\partial \omega}{\partial P}$ are the imaginary components of these sensitivities, as shown in (17) and (18):

$$\frac{\partial \sigma}{\partial P} = Re\left\{\frac{\partial \lambda}{\partial P}\right\} \qquad (17)$$

$$\frac{\partial \omega}{\partial P} = Im\left\{\frac{\partial \lambda}{\partial P}\right\} \qquad (18)$$

The nonlinear system defined by (14), (15) and (16) can be solved through using an alternating Newton-Raphson method, with a linear approximation given by (19), (20) and (21):

$$2\,\Delta P + \left(\frac{\partial\sigma}{\partial P} + \frac{\xi_d}{\sqrt{1-{\xi_d}^2}}\frac{\partial\omega}{\partial P}\right)\Delta l_1 + \Delta l_2 = \Delta\frac{\partial LF}{\partial P} \tag{19}$$

$$\left(\frac{\partial\sigma}{\partial P} + \frac{\xi_d}{\sqrt{1-{\xi_d}^2}}\frac{\partial\omega}{\partial P}\right)\Delta P = \Delta\frac{\partial LF}{\partial l_1} \tag{20}$$

$$\Delta P = \Delta\frac{\partial LF}{\partial l_2} \tag{21}$$

In (19), the second order derivatives of oscillation mode λ in relation to the dispatch vector P was not considered to simplify the optimization problem and improve the computational time performance, since it would require additional eigensolutions for their numerical computation.

Upper and lower active power limits should be modeled through using a variable replacement in the CSBGRES method. Considering minimum and maximum limits to the dispatches, the dispatch vector P may be replaced by an auxiliary variable vector a [8,13], according to (22) and (23):

$$P = \frac{P_{max}+P_{min}}{2} + \frac{P_{max}-P_{min}}{2}\sin a \tag{22}$$

$$a = \sin^{-1}\left(\frac{P-\frac{P_{max}+P_{min}}{2}}{\frac{P_{max}-P_{min}}{2}}\right) \tag{23}$$

where P_{max} is a vector that represents the active power upper limits and P_{min} is a vector that represents the active power lower limits of the system power plants.

In this procedure, the derivatives calculated with respect to vector P must be corrected through using the factors f_1 and f_2, in order to obtain the derivatives with respect to vector a. These corrective factors can be calculated according to (24) and (25):

$$f_1 = \frac{\partial P}{\partial a} = \frac{P_{max}-P_{min}}{2}\cos(a) \tag{24}$$

$$f_2 = \frac{\partial^2 P}{\partial a^2} = -\frac{P_{max}-P_{min}}{2}\sin(a) \tag{25}$$

Using the corrective factors f_1 and f_2, the linearized system shown in (19), (20) and (21) can be rewritten by (26), (27) and (28), where vector P is replaced by vector a:

$$\left(2f_1{}^2+\left(2\,(P-P_0)+l_1\left(\frac{\partial\sigma}{\partial P}+\frac{\xi_d}{\sqrt{1-\xi_d{}^2}}\frac{\partial\omega}{\partial P}\right)+l_2\right)f_2\right)\Delta a+f_1\left(\frac{\partial\sigma}{\partial P}+\frac{\xi_d}{\sqrt{1-\xi_d{}^2}}\frac{\partial\omega}{\partial P}\right)\Delta l_1+f_1\Delta l_2=f_1\Delta\frac{\partial LF}{\partial P} \tag{26}$$

$$f_1\left(\frac{\partial\sigma}{\partial P}+\frac{\xi_d}{\sqrt{1-\xi_d{}^2}}\frac{\partial\omega}{\partial P}\right)\Delta a=\Delta\frac{\partial LF}{\partial l_1} \tag{27}$$

$$f_1\Delta a=\Delta\frac{\partial LF}{\partial l_2} \tag{28}$$

Thus, the model represented by (26), (27) and (28) should be used in an alternating Newton-Raphson algorithm to solve the proposed problem, which works in two steps. First, the power flow execution, model initialization and the eigensolution are made. After that, a Newton-Raphson iteration must be done, using the relation defined in (22), to obtain new estimates for the redispatch.

During the iterative process, the power flow data is updated, through running a load flow solver, and the pole of interest is recalculated, through using the DPSE method, at the beginning of each iteration. Then, new estimates for the minimum redispatches of the selected power plants are determined. This procedure is very similar to a predictor-corrector method. The CSBGRES method should execute these iterations until the desired damping factor is reached for the oscillation mode of interest.

Note that, if a convergence problem occurs in the power flow calculation during the execution of an iteration of the CSBGRES method, then the proposed algorithm aborts its execution and does not calculate the optimum redispatch for the power system. A step control should be used in order to increase the robustness of the proposed method, avoiding power flow calculation problems and improving the convergence of the method.

The algorithm of the CSBGRES method can be summarized in the flow chart presented in Figure 1, where the proposed alternating procedure can be easily understood.

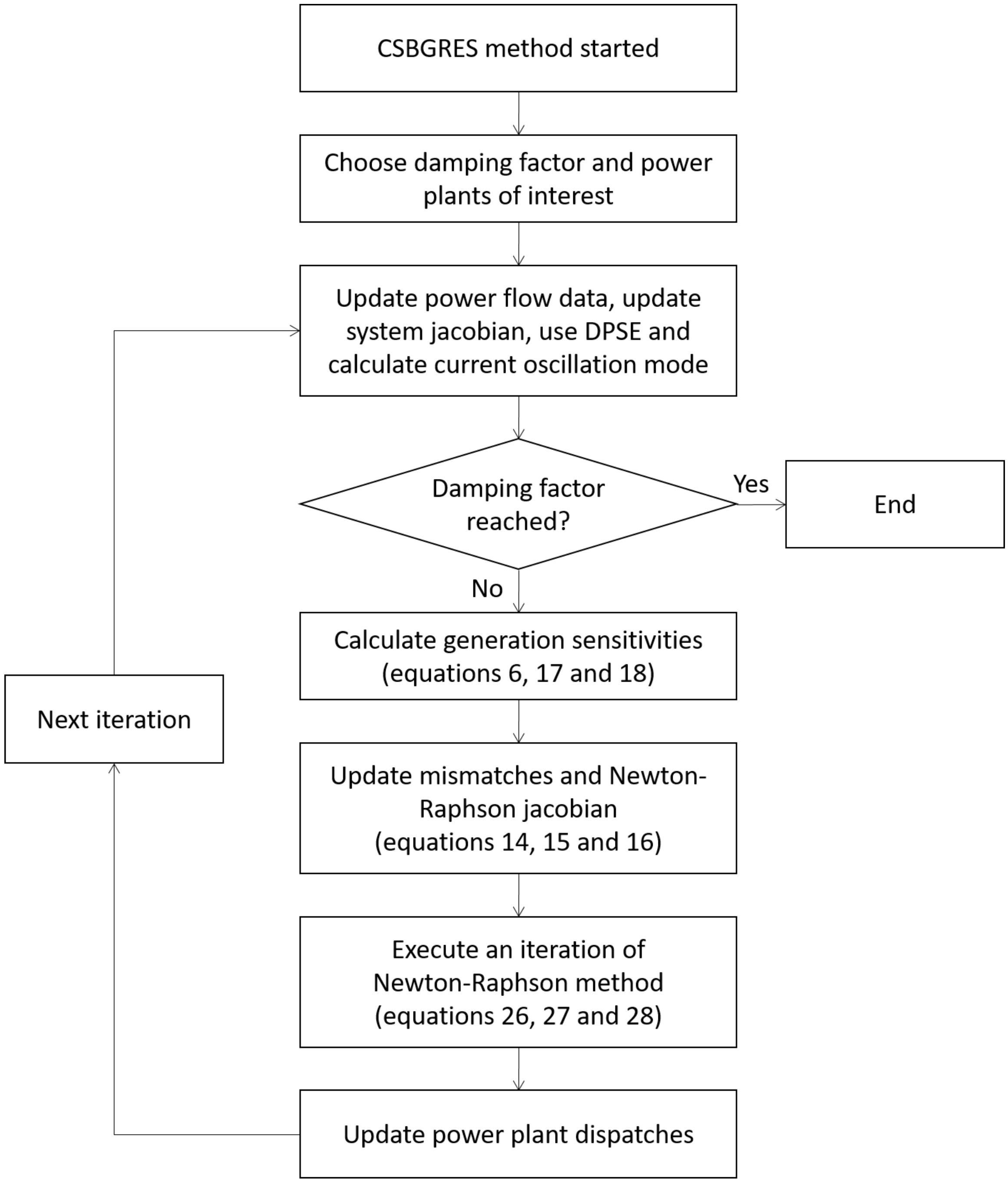

Figure 1 – CSBGRES method algorithm for computational implementation.

The lagrangian multipliers should be initialized with a small value in this optimization problem. In the CSBGRES method, both lagrangian multipliers have been initialized with value "one" ($l_1 = l_2 = 1$) and this choice worked very well for the proposed algorithm.

The objective of the CSBGRES method is the determination of the minimum dispatch variation of selected power plants to move a specific mode to a desired damping factor line in the s-plane. However, special heuristics were made in the computational implementation of this method, so it could consider a set of oscillation modes. In this case, the method will consider this set of modes and will choose the worst damped one to be used in each iteration.

Then, the CSBGRES method will present the results of redispatch obtained to reach the desired damping factor for the worst damped mode of the oscillation mode set. If the desired damping factor could not be reached, considering this set of oscillation modes, then the proposed method will present the redispatch calculated for the best situation obtained during the execution of its iterations, where the damping factor was as close as possible to the desired value.

It should be emphasized that a step-length control in the desired damping factor and in the active power dispatches should be used to avoid convergence issues in the power flow solver execution and in the eigensolver execution. This mathematical trick helps the convergence to the global minimum solution (solution of minimum redispatch) for the oscillation mode of interest, since it limits the amplitude of redispatch, avoiding convergence to local minimum solutions.

5 Brazilian Interconnected Power System

An example case of the BIPS, obtained from the data base of the Brazilian Transmission System Planning Company (EPE) [29], was used to test the CSBGRES method.

This example case was presenting, 8,065 buses, 11,547 branches, 19 HVDC links, 46 FACTS devices, 329 synchronous machines and 911 control systems.

After running the QR method [30,31], which computes the all system eigenvalues, the North-South electromechanical oscillation mode -0.0527 + j 2.5482, with 2% of damping factor, was obtained and selected to have its damping enhanced by the CSBGRES method.

First, the generation sensitivities were calculated for the mode of interest. The results obtained can be visualized in Figure 2 and Table I, where the power plants TUC70, TUC71, BMT1, BMT2, CNV, MCD, GBM1, GMB2 and PPM can be highlighted, due to their sensitivity results and their importance to the Brazilian Interconnected Power System.

The power plants TUC70, TUC71, BMT1 and BMT2 are located in the North region of Brazil, whereas the power plants CNV, MCD, GBM1, GMB2 and PPM are located in the South.

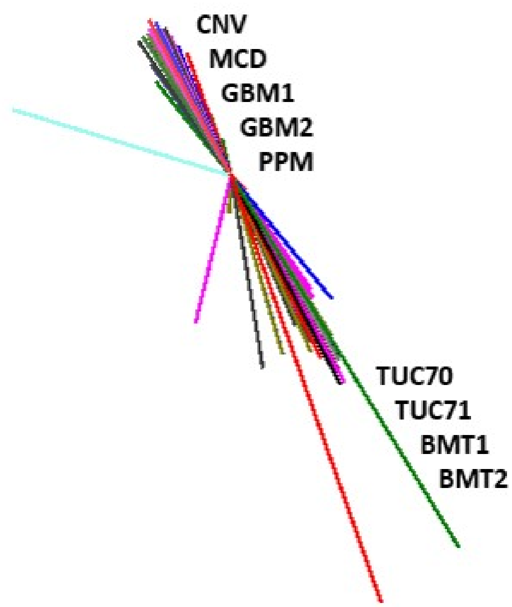

Figure 2 – Normalized GenSens phasors for the BIPS.

Table I – Normalized GenSens list for the BIPS (main results).

Power Plant	Module	Phase
TUC71	0.4577	-60.615
TUC70	0.4516	-60.697
BMT1	0.4490	-62.937
BMT2	0.4249	-62.908
CNV	0.3593	116.490
MCD	0.3464	118.750
GBM1	0.3455	122.110
PPM	0.3438	119.380
GBM2	0.3384	121.860

The GenSens results show the displacement trends of the oscillation mode position in the s-plane when trying to increase the power plant dispatches. These sensitivities are very important, since their amplitudes and these displacement trends of the desired oscillation mode will define the most adequate generators to be redispatched in the CSBGRES method. In other words, the generation sensitivities may be used to select the set of power plants that should be used for the redispatch.

An excessive number of redispatched generators, besides being not practical from an operation point of view, increases unnecessarily the computational time of the proposed method, since each sensitivity involves the execution of two complete power flow calculations and eigensolutions, and the results are quite similar when neglecting the power plants with small sensitivity from the redispatch.

The example system used in the GenSens test had 7868 state variables. Using a processor Intel (R) Core (TM) i7-3537U CPU @ 2.00 GHz, the processing time for the generation sensitivity calculation, considering all 329 generators of the system, was around 18 minutes. This processing time could be significantly reduced if using parallel computing.

After that, the CSBGRES method was used for the selected generators in Table I to calculate the active power dispatch variations capable of moving the oscillation mode -0.0527 + j 2.5482 to a 5% damping factor line in the s-plane.

The objective function defined in the optimization problem, proposed in this paper, is of particular high interest for Brazil. Being a hydroelectricity powerhouse, with many river basins running across a continent-size country and having a sole transmission system operator (ONS) [32] for the whole country, the centralized dispatch takes heavily into account the hydrological conditions. Therefore, the solution based on generation redispatch, like the one proposed in this paper, should target a minimum deviation from a previously optimized energetic solution.

The redispatches determined through the CSBGRES method for the previously selected power plants can be observed in Figure 3 and Table II.

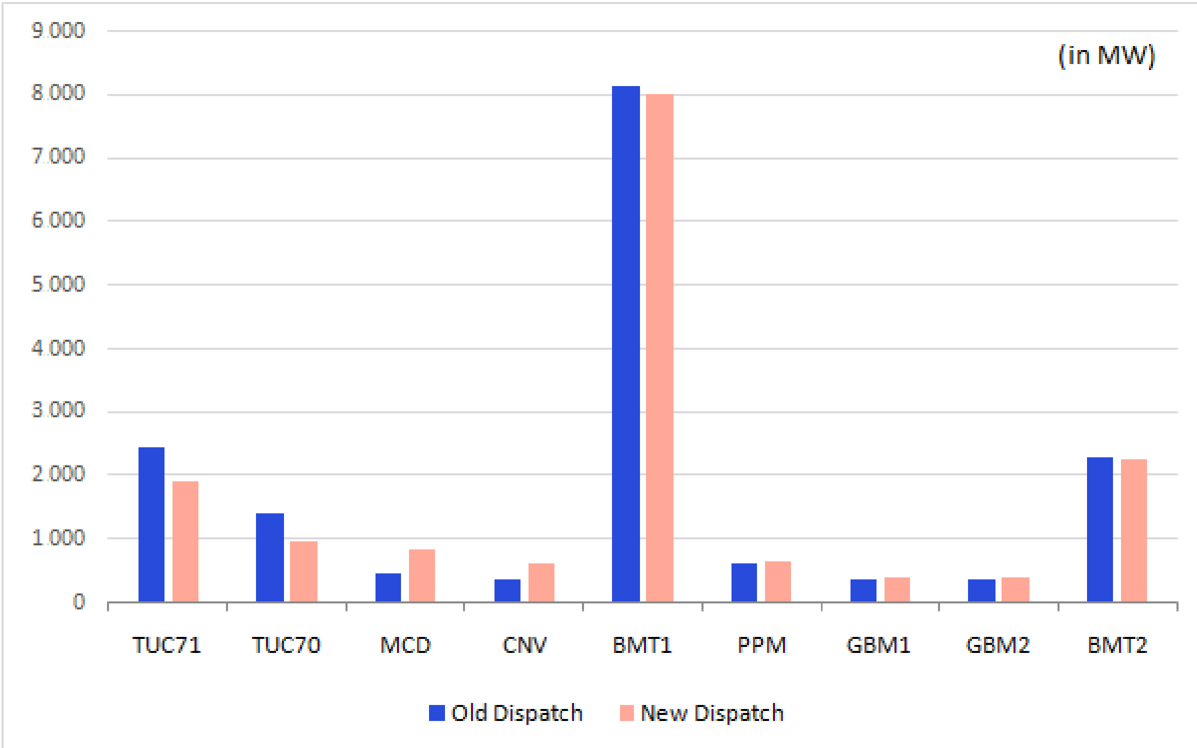

Figure 3 – CSBGRES histogram for the BIPS, in MW.

Table II – CSBGRES list for the BIPS (main results), in MW.

Power Plant	**Old Dispatch (MW)**	**New Dispatch (MW)**	**Variation (MW)**
TUC71	2460.00	1930.70	-529.30
TUC70	1406.00	987.14	-418.86
MCD	468.50	837.00	368.50
CNV	364.20	622.00	257.80
BMT1	8151.00	8014.50	-136.50
PPM	616.00	672.00	56.00
GBM1	376.65	419.00	42.35
GBM2	376.65	419.00	42.35
BMT2	2299.00	2260.50	-38.50

The CSBGRES method has two possible applications for small-signal security assessment in power systems. The first application consists of determining security margins, where the damping factor of the mode of interest should be decreased to the value that represents this security limit for the system, and the second application consists of determining corrective measures, where the damping factor of the mode of interest should be increased to respect the security limits.

In the BIPS example, the proposed method was used to determine a corrective measure, increasing the damping factor of the chosen mode to 5%, which could be understood as a security limit.

Considering the dispatches calculated through the CSBGRES method, the mode -0.1351 + j 2.6886 can be obtained, with a damping factor of 5.0186%, in 16 iterations.

The damping factor obtained by the proposed method is not exactly 5%, previously established value, because of the tolerance of 0.1% used in CSBGRES algorithm, which is directly associated to the damping factor of the oscillation mode of interest.

The processing time for the CSBGRES method calculation, using a processor Intel (R) Core (TM) i7-3537U CPU @ 2.00 GHz, was around 1 minute, to reach the damping factor of 5% for the oscillation mode of interest, without using parallel computing.

6 Nordic 44 Test System

An example case of the N44S, which is a Nordic Power System equivalent presented in [33], was used to test the CSBGRES method and show its on-line application.

The Nordic 44 Test System represents interconnections between Norway, Sweden and Finland. The example case used in this section was presenting, 44 buses, 79 branches, 18 synchronous machines, 40 control systems and no HVDC link.

In the performed tests, an on-line monitoring of oscillations was simulated, where the software PacDyn was used to monitor the frequency and the damping factor of two inter-areas electromechanical oscillation modes of major importance for the system.

The modes -0.3750 + j 3.7519, with 10% of damping factor, and -0.1021 + j 2.0400, with 5% of damping factor, obtained in the initial operating point, were monitored in this simulation.

An actual historical power flow database of the Nordic Power System was used to create the variation of operating points for the N44S. Then, PacDyn was used to calculate the oscillation modes for all the operating points created, in sequence, building a timeline of frequency and of damping factor for these oscillation modes. The CSBGRES method was used to calculate the dispatch needed to keep the power system with, at least, 5% of damping factor.

The operating points of the N44S were being sent to PacDyn every 6 seconds, updating the system power flow data. Each operating point was representing one hour of system operation, such that a sequence of 22 days (or 527 hours) of continuous operating conditions could be simulated.

Figure 4 presents the damping factors and the frequencies of the monitored oscillation modes, considering the operating point modifications along the time.

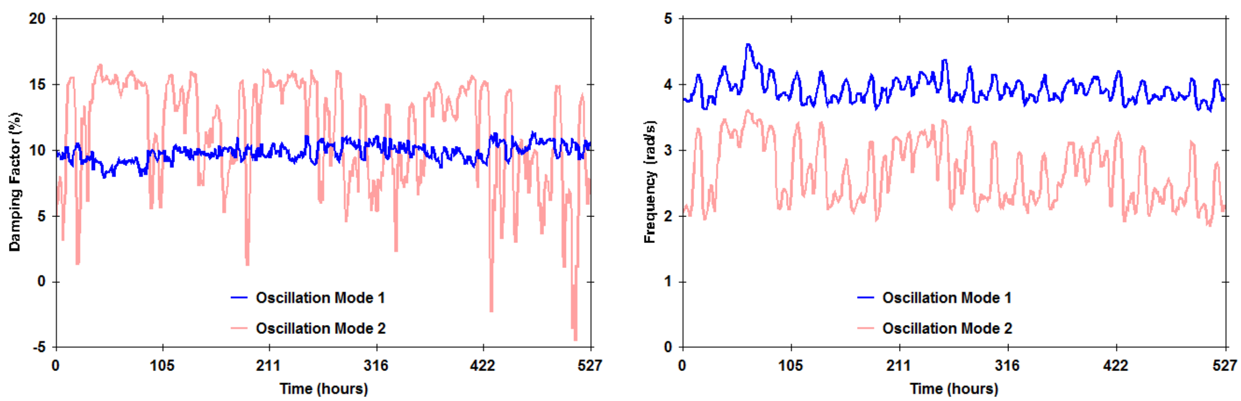

Figure 4 – Damping factor and frequency timelines for the N44S.

The damping factor of oscillation mode 2 presents undesired values in some operating conditions, lower than 5%, which is the minimum damping factor allowed for the system.

The CSBGRES method can be utilized to determine an optimum redispatch, which is the minimum variation in the power plant dispatches, that should be used in the system to improve the damping factor of this oscillation mode, ensuring the minimum damping factor of 5% for all the monitored poles, considering the whole set of generators for redispatch.

The proposed method was, then, applied inside the on-line monitoring execution loop, so, in every operating point where the system presents a minimum damping factor lower than 5%, an optimum redispatch could be obtained to solve the small-signal security problem.

Figure 5 presents the damping factors and the frequencies of the monitored oscillation modes, with the corrective solution determined by the CSBGRES method.

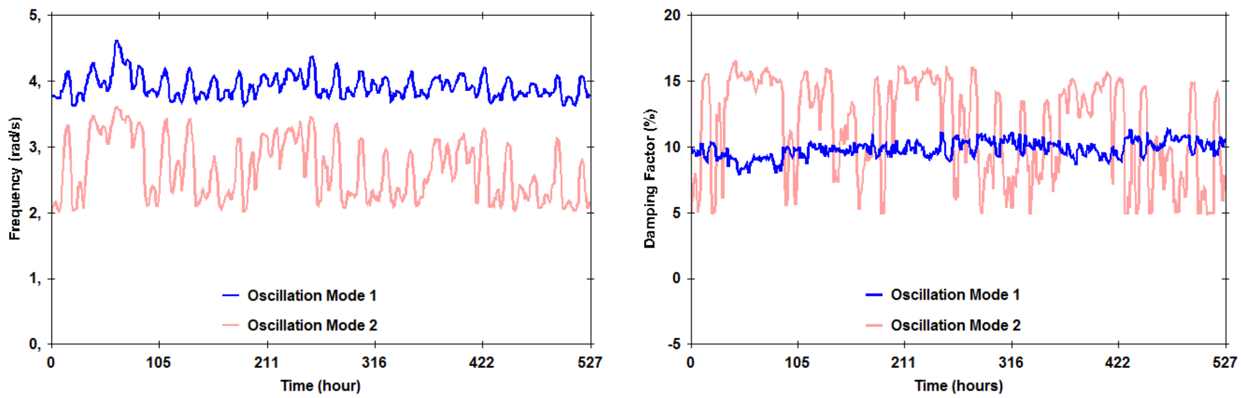

Figure 5–Damping factor and frequency timelines for the N44S, with CSBGRES.

Figure 6 presents a comparison between the results with and without the application of the CSBGRES during the monitoring simulation for the damping factor of mode 2.

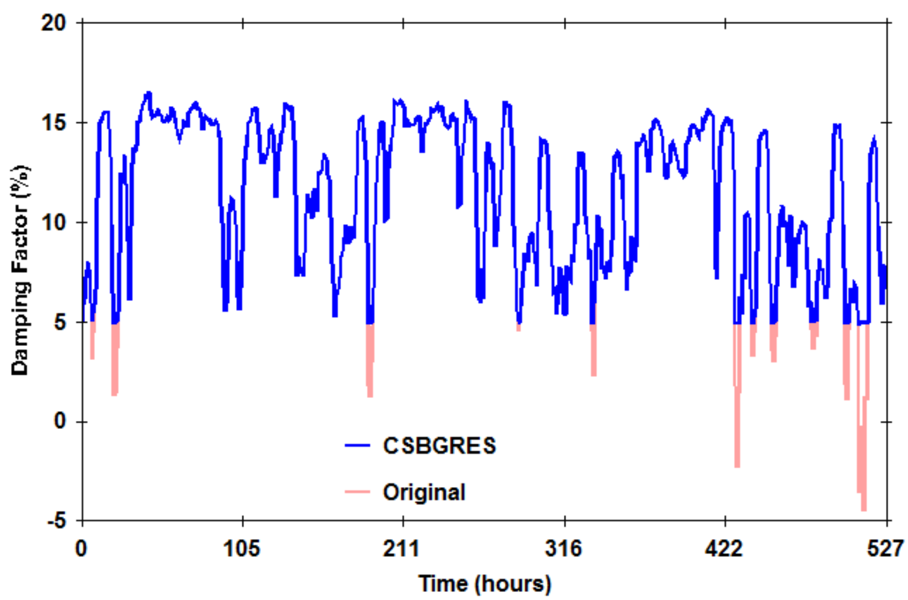

Figure 6 – Comparison between the results with and without the use of CSBGRES.

The results show that the CSBGRES method was able to calculate new dispatches for the system, which are minimum variations in relation to the original dispatches that solved the undesired damping factor problem of the oscillation mode 2. Whenever it was needed, the algorithm converged to an adequate solution, ensuring the desired damping factor for the system.

The worst scenario of the N44S observed in this on-line monitoring simulation was presenting the critical mode 0.0830 + j 1.8273, with damping factor of -4.5%.The results obtained in the generation sensitivity calculation for this mode can be visualized in Figure 7 and Table III.

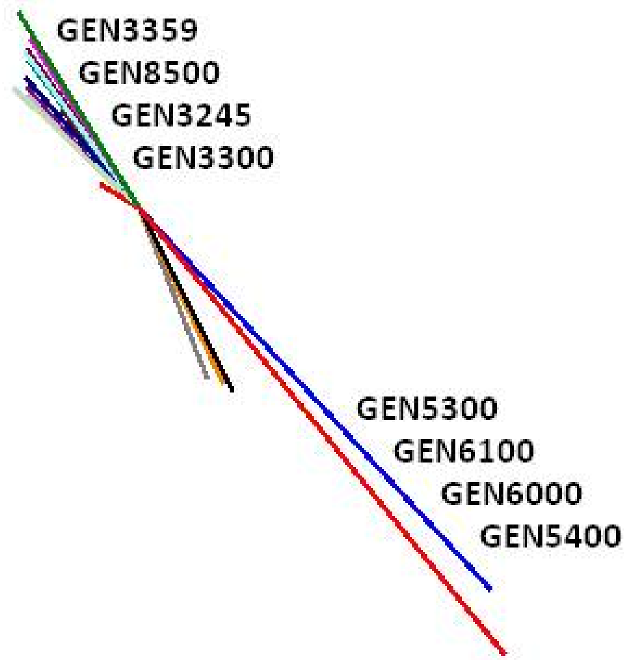

Figure 7 – Normalized GenSens phasors for the worst scenario of the N44S.

Table III – Normalized GenSens list for the worst scenario of the N44S (main results).

Power Plant	Module	Phase
GEN5300	1.0000	-48.187
GEN6100	0.9045	-48.815
GEN3359	0.3926	123.960
GEN6000	0.3468	-60.783
GEN8500	0.3434	125.020
GEN3245	0.3343	128.650
GEN3300	0.3337	127.660
GEN5400	0.3287	-62.383
GEN6500	0.3166	129.310
GEN3000	0.3111	128.460

The redispatch obtained through the CSBGRES method to solve the problem observed in the worst scenario of the N44S can be seen in Figure 8 and Table IV.

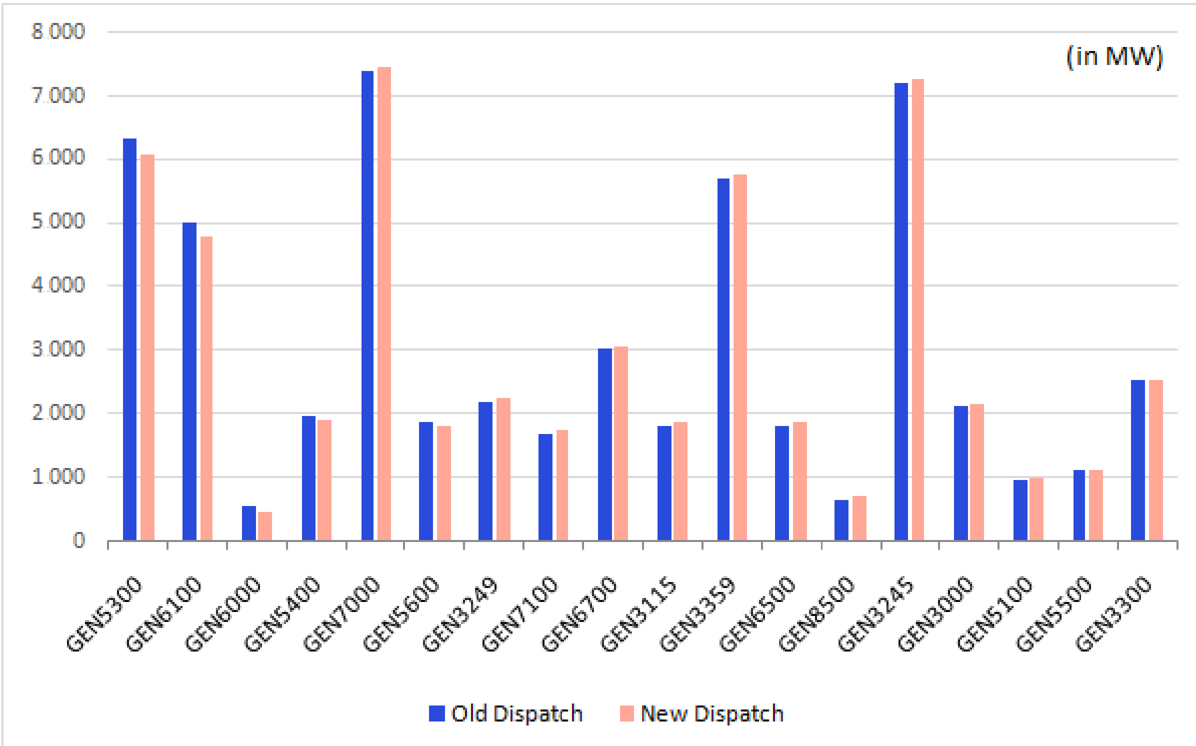

Figure 8 – CSBGRES histogram for the worst scenario of the N44S, in MW.

Table IV – CSBGRES list for the worst scenario of the N44S (main results), in MW.

Power Plant	Old Dispatch (MW)	New Dispatch (MW)	Variation (MW)
GEN5300	6326.00	6081.20	-244.80
GEN6100	5022.30	4785.40	-236.90
GEN6000	555.32	486.19	-69.13
GEN5400	1972.80	1907.90	-64.90
GEN7000	7416.80	7472.70	55.90
GEN5600	1883.60	1828.10	-55.50
GEN3249	2196.60	2249.90	53.30
GEN7100	1707.20	1760.20	53.00
GEN6700	3034.00	3086.70	52.70
GEN3115	1823.40	1875.90	52.50
GEN3300	2537.70	2537.70	0.00

Using the dispatches calculated through the CSBGRES method in this worst scenario, the oscillation mode -0.1005 + j 2.0300 can be obtained, with a damping factor of 4.9436%, in 19 iterations.

The example system used in these tests had 224 state variables. Using a processor Intel (R) Core (TM) i7-3537U CPU @ 2.00 GHz, the processing time for the GenSens determination was around 3 seconds and for the CSBGRES method calculation was around 7 seconds to reach the desired damping factor for the oscillation mode of interest.

7 Conclusions

To achieve the objective of moving specific eigenvalues by active power injection, the paper proposed an optimization method, namely CSBGRES, based on a numerical computation of eigenvalue sensitivities with respect to generation dispatches, which are calculated through multiple runs of a load flow solver combined with a partial eigensolver. A computational framework was developed for an effective and reliable communication between both solvers.

By knowing the numerically-computed generation sensitivities, the mentioned optimization problem was defined. The sum of the square differences of selected power plant dispatches was used as the objective function that should be minimized, so an optimum redispatch of the system, needed to meet a damping factor criterion for oscillation modes, could be determined.

The proposed method uses a numerical calculation of generation sensitivities, which may be inaccurate and not robust if inadequate choices of power variations are used. However, with adequate deviations, this alternative solved a very difficult problem, which is the analytical determination of these generation sensitivities.

The CSBGRES method relies on sparse matrices, being applicable to small and large-scale power system models. Besides, the method may be used on-line to determine corrective measures to improve the dynamic behavior of these systems through power plant redispatches.

The proposed algorithm was applied in the Brazilian Interconnected Power System (BIPS, which has a large-scale model) and in the Nordic 44 Test System (N44S, which has a small-scale model).

In the former, the CSBGRES method was applied to determine an optimum redispatch of major power plants to enhance the damping factor of the North-South inter-area oscillation mode.

In the latter, the proposed method was applied in an on-line monitoring simulation. Whenever poorly-damped operating conditions were detected, the CSBGRES method was used to determine the redispatch needed to improve the dynamic behavior of the power system.

In essence, this paper presented a method for improving damping factor of natural oscillations through the minimum active power redispatch of selected power plants, showing important results obtained with the CSBGRES method application in power system analysis.

8 Acknowledgement

The authors would like to thank the former student Knut Bjørsvik, from Norwegian University of Science and Technology, for the collaboration in obtaining the Nordic equivalent system results, in the real-time monitoring of oscillations. Prof. Taranto would like to thank the financial support provided by CNPq, CAPES and FAPERJ.

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