Comparison of Cost Allocation Strategies among Prosumers and Consumers in a Cooperative Game

Abstract

As the higher penetration of distributed generation (DG) and electrical energy storage (EES) is emerging, end-users are taking a more active role in the power grid. With an increased amount of DG and EES available, opportunities for cooperation in the operation of power exchange arises. In cooperative game theory, for all players in a game cooperate under joint benefits. Preliminary studies show such cooperation among prosumers and consumers provides reduced annual electricity cost compared to independent operation. Focusing on cost allocation among end-users equipped with rooftop PV and batteries, we want to evaluate two solution concepts from game theory; the nucleolus and the Shapley value. By changing parameters that increase the value of the battery system in terms of reduced cost, this paper aims to examine whether the deviation between the cost allocations proposed by the methods increases as the value of the battery system is changed. The simulated case is based on data from private residences in Norway. Results from our case show that both nucleolus and Shapley provide stable cost allocations under minor deviations. However, results also imply that the deviation between the methods increases with an increased battery system value.

Nomenclature

$\eta_{bat,z}$	Charg./discharg. efficiency of battery z [%]
λ_{EES}	Relative reduced cost provided by the battery system [%]
$\theta_{min}(\boldsymbol{x})$	Lexicographic smallest excess vector for payoff vector $\boldsymbol{x}$ [NOK]
$v(N)$, $v(S)$	Worth of coalition set N and S [NOK]
$C_{el}(t)$	Total electricity cost in time step t [NOK]
$E_{bat,z}(t)$	Energy capacity of battery z in time step t [kWh]
$E_{bat,z}^{max}$	Max. energy capacity of battery z [kWh]

$P_{bat,z}(t)$	Charg./discharg. power of battery z in time step t [kW]
$P_{bat,z}^{max}$, $P_{bat,z}^{min}$	Max. and min. charge rate of battery z [kW]
$P_{grid}(t)$	Power supplied or delivered to the grid in time step t [kWh/h]
$P_{load,n}(t)$	Load demand for all n players in time step t [kWh/h]
$P_{PV}(t)$	Total photovoltaic power production in time step t [kWh/h]
C, $C_{\varepsilon}(N,\upsilon)$	The core and the ε-core of a cooperative game
$e(S,\boldsymbol{x})$	Excess experienced by players in S from payoff vector $\boldsymbol{x}$ [NOK]
F_k	Union of previously binding coalitions in k
k	Number of iterations in *least cores*
N, n, i	Set of all players, total number of players, and their index
$SOC_z(t)$	State of charge of battery z in time step t [%]
SOC_z^{max}	Max. battery state of charge of battery z [%]
SOC_z^{min}	Min. battery state of charge of battery z [%]
S	Subset of N
T, t, Δt	Set of all discrete time steps, total number of discrete time steps, and time interval
Z, z	Set of all batteries, total number of batteries
$\phi(\upsilon)$	Shapley value [NOK]
$\phi_i(\upsilon)$	Payoff assigned to player i by Shapley [NOK]
ε_k	Max. excess vector in k [NOK]

1 Introduction

By 2050, solar photovoltaic (PV) and wind power might account for 52 % of the world's total electricity generation [1]. To support the increase of renewables, electrical energy storage (EES) will play a crucial role.

As the costs of rooftop PV and batteries become more competitive economical, these applications are becoming more attractive for private end-users in the distribution grid. With an increased amount of DG and EES available among the end-users, possibilities for cooperating operation arise. In cooperative game theory, joint benefits for all players in a game to cooperate is assumed. Results from preliminary studies show that cooperating in bidding at a power exchange provides reduced electricity cost compared to independent operation. However, as a prerequisite for rational players to cooperate, the profitability after cost allocation for all players must exist. This, in turn, depends on the cost allocation among them.

Historically, cooperative game theory has been a tool for investment analysis, mainly focusing on power generation and transmission facilities. In [2], the authors propose a generic framework for flexibility analysis in transmission expansion planning using the concept of Shapley value. In recent year, the interest of applying cooperative game theory in the distribution system is increased, as it can serve as a well-performing tool for optimizing electricity costs and available resources. In [3], the authors analyze the value of sharing storage among consumers in a cooperative manner, and conclude that all players in a community would benefit from such cooperation. Ref. [4] studies cooperation among energy communities using cooperative game theory, and proves that each grid increases their individual profit by cooperating with the other energy communities. Furthermore, [5] studies how cooperative game theory can be applied for cost minimization within an energy community. Here, the authors propose the Shapley value for cost allocation, and conclude that both prosumers and consumers will obtain reduced cost when participating in the cooperative game.

In contrast to previous studies, this paper analyzes the deviation between the cost allocations proposed by nucleolus and the Shapley value among end-users within a single energy community. Furthermore, we want to examine whether the deviation between the methods is related to the value of the battery system within the energy community.

We analyze a case study consisting of four end-users as a cooperative game. Among the end-users, there are two prosumers, whereas the two remaining end-users are consumers. We consider both the prosumers and consumers as players in the cooperative game. By changing the parameters: 1) demand, and 2) electricity spot price, four different scenarios are obtained. The parameters are changed to obtain the deviation between the nucleolus and Shapley value with varying battery system values. The simulations are conducted by applying a dynamic programming (DP) optimization algorithm which calculates the annual electricity cost for all sub coalitions in each scenario. As the objective of the paper is to evaluate the game theoretical methods on a conceptual level, perfect foresight and deterministic input data are used for simplicity, but not applicable to real case studies.

This paper is structured as follows. Section II presents the DP algorithm. Section III presents the game theoretical concepts used for cost allocation. Section IV presents the scenarios and the input data used. Results are presented in section V, followed by a discussion in section VI. Conclusion and further work are presented in chapter VII.

2 Dynamic programming algorithm

To obtain the annual electricity cost for all possible coalitions $S \subseteq N$, a dynamic programming (DP) optimization algorithm is utilized. The objective function aims to minimize the annual cost of a number of cooperating end-users, by minimizing the cost of grid imported energy. The algorithm optimizes operation of two batteries in parallel over a year, thus minimizing the annual cost for the end-users. By calculating the cost of every possible charge and discharge decision in every time step within the optimization time horizon, a set of nodes is derived which results in lowest possible costs. In this paper, the time horizon is one year, thus $t = 8\ 760$ hours. The objective function is shown in Eq. (1). Eq. (2a) shows the energy balance, which includes all players in the coalition and their respective PV and batteries. For coalitions $S \subseteq N$ where PV and batteries are unavailable, the demand is met by buying power from the grid. $P_{grid}(t)$ operates as a slack variable, as expressed in Eq. (2a). Eq. (2b) and (2c) show the maximum and minimum battery state of charge (SOC), whereas Eq. (2d) and (2e) reflect the maximum charge and discharge power[1]. Eq. (2f) shows the stored energy in a given time step. Finally, Eq. (2g) shows the battery SOC equation.

$$\min f(P_{bat}) = \sum_{t \in T} C_{el}(t) P_{grid}(t) \tag{1}$$

s.t.

$$P_{grid}(t) = \sum_{n \in N} P_{load,n}(t) + \sum_{z \in Z} P_{bat,z}(t) - P_{PV}(t) \tag{2a}$$

$$SOC_z(t+1) \leq SOC_z^{max} \quad z \in Z \tag{2b}$$

$$SOC_z(t+1) \geq SOC_z^{min} \quad z \in Z \tag{2c}$$

$$P_{bat,z}(t) \leq P_{bat,z}^{max} \quad z \in Z \tag{2d}$$

$$P_{bat,z}(t) \geq -P_{bat,z}^{max} \quad z \in Z \tag{2e}$$

$$E_{bat,z}(t+1) \geq E_{bat,z}(t) + \eta_{bat,z} P_{bat,z}(t) \Delta t \quad z \in Z \tag{2f}$$

$$SOC_z(t+1) = \frac{E_{bat,z}(t+1)}{E_{bat,z}^{max}} \quad z \in Z \tag{2g}$$

Note that

$$\eta_{bat,z} = \eta_{ch,z}, \quad P_{bat,z}(t) \geq 0 \quad z \in Z$$
$$\eta_{bat,z} = \eta_{dis,z}, \quad P_{bat,z}(t) < 0 \quad z \in Z$$

[1]The rated power is considered to be the continuous rated power.

3 Game theoretical modelling

Cooperative game theory constitutes a mathematical framework for evaluating cooperation among a group of players. A cooperative game with *transferable utility* (TU) is represented as a pair (N,v) [6]. With n players, 2^n possible coalitions can be obtained. Further, v denotes the characteristic function, representing the value of a coalition. For every coalition $S \subseteq N$, there exists a value $v(S)$. The value of the empty set, $v(\emptyset) = 0$. The coalition consisting of all players is termed the *grand coalition.* Due to the concept of *superadditivity*[2], the $v(N)$ provides the highest payoff.

In this paper, each coalition formed by the players will lead to an outcome in form annual electricity cost. The outcome of each coalition depends on the interaction among the players. Thus, each simulated scenario satisfies the definition of a cooperative game. Secondly, the players are assumed rational and to act in their self-interest. Due to the concept of superadditivity, the grand coalition provides the lowest electricity cost for the players. Despite this, the players will only join the grand coalition if the proposed allocation provides the players the highest payoff.

To satisfy the equilibrium state, it must be ensured that none of the players want to leave the grand coalition N in order to join another sub coalition S. Due to rationality, the players seek to form the coalition where they expect to obtain the highest payoff. Let $\boldsymbol{x} = [x_1,x_2,...,x_n]$ be a proposed cost allocation of the total payoff $v(N)$. If $\boldsymbol{x}$ fulfills the requirements of both individual and group rationality[3], it is denoted and *imputation.* Furthermore, an imputation $\boldsymbol{x}$ is stable if no alternative coalition will provide a higher payoff for any of its players. Hence, a stable imputation is said to be in the *core* of the game.

In order to fairly allocate the total payoff $v(N)$ for the players, the concepts of nucleolus and the Shapley value are introduced. These value concepts propose a unique allocation $\boldsymbol{x}$ based on some fairness principles. Nucleolus and Shapley differ in their interpretation of fairness, thus they do not necessary provide equal cost allocations. Before presenting these methods, we introduce the concept of the core.

[2] *Superadditivity*: The value of a union of two disjoint coalitions is equal to, or greater than the sum of the coalitions' separate values.

[3] *Individual rationality*: A player will only join a coalition if this leads to at least the utility obtained by operating individually.
Group rationality: The total utility from a coalition should be divided among all the players within the coalition.

3.1 The Core

The core C of a TU game (N,υ) is the set consisting of all stable imputations, mathematically expressed through Eq. (3) [7].

$$C = \Big\{ x_i \mid x_i \in \{x_1, ..., x_n\}, \sum_{i \in N} x_i = \upsilon(N), \sum_{i \in S} x_i \geq \upsilon(S),\ \forall S \subseteq N \Big\} \tag{3}$$

To ensure that all end-users in a cooperative game want to form the grand coalition, the proposed value allocation needs to be in the core of the game.

3.2 The Shapley Value

Lloyd Shapley proposed a solution concept whose interpretation of fairness is in terms of each player's individual contribution to a coalition [8]. Shapley provides a simple method for cost allocation for all the players in the game based on four axioms. These axioms are as follows:

1. *Efficiency*: All utility obtained by any player should be allocated. The total value of the players is the value of the grand coalition, hence $\upsilon(N) = \sum_{i \in N} \upsilon(i)$.
2. *Symmetry*: Two players i and j that contribute the same to each coalition are substitutes, hence they should be treated equally. Player i and j are symmetric if $\upsilon(S \cup i) = \upsilon(S \cup j)$.
3. *Null player*: A player i that contributes nothing, should receive nothing. Such a player is referred to as a null or a zero player. A player is a null if $\upsilon(S) = \upsilon(S \cup i)$.
4. *Additivity*: The sum of two independent TU games, u and v must be the sum of the value of each game, hence $\phi(u + v) = \phi(u) + \phi(v)$.

For each player, there exists a unique value satisfying these axioms. This unique value is the Shapley value, denoted $\phi(\upsilon)$. For a TU game (N,υ) the Shapley value for each player i is calculated by the following:

$$\phi_i(\upsilon) = \sum_{i \in S} \frac{(|S|-1)!(n-|S|)!}{n!} [\upsilon(S) - S \backslash \{i\})]. \tag{4}$$

Once the Shapley value for each player is calculated, the value allocation $\boldsymbol{\phi} = [\phi_i, ..., \phi_n]$ can be obtained. However, the method does not ensure that the allocation is stable. Once the Shapley values for the game are calculated, additional examination of the core is required to verify stability. Thus, $\boldsymbol{\phi}$ must fulfill the following condition:

$$\boldsymbol{\phi} \in C \tag{5}$$

3.3 The Nucleolus

The concept of the nucleolus was first introduced in [9]. While Shapley focuses on fairness in terms of individual contribution, the nucleolus is based on minimizing the players' dissatisfaction with their payoff. The idea behind the nucleolus is to minimize the maximum dissatisfaction the players in coalition S experience from a proposed imputation $\boldsymbol{x}$. Dissatisfaction is measured through an *excess function* $e(S,\boldsymbol{x})$, expressed in Eq. (6).

$$e(S, \boldsymbol{x}) = \upsilon(S) - \sum_{i \in S} x_i = \upsilon(S) - \boldsymbol{x}(S) \tag{6}$$

A negative $e(S,\boldsymbol{x})$ represents the additional payoff coalition S obtains from $\boldsymbol{x}$. Thus, an imputation $\boldsymbol{x}$ is in the core if and only if all excesses are negative or equal to zero:

$$C(N, v) = \{\boldsymbol{x} \in X \mid e(S, \boldsymbol{x}) \leq 0 \quad \forall S \subseteq N\} \tag{7}$$

A payoff vector $\boldsymbol{x}$ provides an excess vector $\theta(\boldsymbol{x})$ = [e($\boldsymbol{S}_1$,$\boldsymbol{x}$),..., e(S_{2^n-2},$\boldsymbol{x}$)] for $S \subseteq N \backslash S = \emptyset$. Different allocations provide different excess vectors. The excess vectors are ordered *lexicographically*[4], thus there exists an allocation which corresponds to the lexicographic smallest excess vector $\theta_{min}(\boldsymbol{x})$. This unique payoff vector is the nucleolus. The nucleolus of a cooperative game always exist. If the game is non-empty, then nucleolus is always in the core. In this paper, the nucleolus is obtained by finding $\theta_{min}(\boldsymbol{x})$ from *least cores*, a method based on [10], [11]. The least core concept leads to the introduction of the ε-core $C_\varepsilon(N,v)$, expressed through Eq. (8). By letting $\varepsilon < 0$, the ε-core becomes more restrictive than the core represented in Eq. (3).

$$C_\varepsilon(N, v) = \boldsymbol{x} \in X \mid e(S, \boldsymbol{x}) \leq \varepsilon \quad \forall S \subseteq N\} \tag{8}$$

By iteratively solving a linear programming (LP), the least core is obtained. The LP problem representing the least core is expressed in Eq. (9)-(9c). For each iteration *k*, constraints based on previous maximum excesses are added, thus reducing the feasible region. The feasible region is reduced until the nucleolus is obtained. Eq. (9a) ensures that ε_k does not exceed the maximum excess for imputation $\boldsymbol{x}$. Group rationality is fulfilled by Eq. (9b), while Eq. (9c) ensures that the previous minimized maximum excess is still maintained. F_j represents the set including all coalitions which the excess constraint Eq. (9a) was binding

[4]Lexicographic ordering means that the excesses are ordered in the same way as words are ordered in the dictionary.

at previous stages. Thus, F_k is the union of previously binding coalitions and $F_1 = \emptyset$.

$$\min \varepsilon_k \tag{9}$$

$$\text{s.t.} \quad \upsilon(S) - \sum_{i \in S} x_i \leq \varepsilon_k \quad \forall S \subseteq N \, \text{and} \, S \notin F_k \tag{9a}$$

$$\sum_{i \in N} x_i = \upsilon(N) \tag{9b}$$

$$\begin{aligned} &\upsilon(S) - \sum_{i \in S} x_i = \varepsilon_j, \\ &\forall S \subseteq F_j, j \in \{1, ..., k-1\} \end{aligned} \tag{9c}$$

$$\varepsilon_k \in R, \, x_i \in R \quad \forall i \in N$$

4 The Case Study

4.1 The Scenarios

In this paper, we analyze four scenarios, each consisting of four players. Player 1 and 2 are prosumers with one rooftop PV system and one battery each, whereas player 3 and 4 are consumers. Using two different sets of load profiles and two different sets of electricity spot prices, nucleolus and the Shapley value are applied to all scenarios to study their performance under different conditions. The load profiles are denoted Load P1 and Load P2. The two sets of spot prices are taken from Norway and Germany, denoted NO3 and GER respectively. The simulated scenarios are presented in Tab. 1.

Table 1: The four simulated scenarios with different load profiles (Load P1/P2) and electricity spot prices (NO3/GER).

	Scenario			
Parameter	**#1**	**#2**	**#3**	**#4**
Load	Load P1	Load P1	Load P2	Load P2
Electricity spot price	NO3	GER	NO3	GER

4.2 Load Demand and PV Production

Load data are based on Norwegian end-users located in Trondheim, Norway. Key data for the total load in each load scenario are given in Tab. 2. The PV production for player 1 and 2 are calculated by using a PV production model based on [12]. Irradiation and temperature data are taken from the Norwegian weather service [13]. Fig. 1 shows the daily load in each load scenario along with the total PV production provided by player 1 and 2, during the year. As shown in Fig. 1, the total PV production does only exceed demand of load profile 2.

Table 2: Key data for the two load profiles.

Value	Load profile 1	Load profile 2
Annual consumption	114 270 kWh	28 868 kWh
Average consumption	13.04 kWh/h	3.30 kWh
Maximum consumption	35.40 kWh/h	10.00 kWh/h

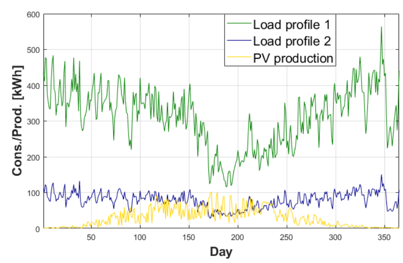

Figure 1: Daily consumption for the two load profiles, along with the daily PV production in 2015.

4.3 Battery Specifications

Two different batteries are used for the simulations. Battery specifications are presented in Tab. 3. Battery 1 is based on data from a Tesla Powerwall [14], while battery 2 is based on data from a LG house battery [15].

As we want to obtain the value of the battery system, the term λ_{EES} is introduced. The λ_{EES} represents the relative reduced cost for the grand coalition, provided by the battery system consisting of both batteries. Thus, there exists a λ_{EES} for each scenario.

Table 3: Battery specifications.

	$P_{bat,z}^{max}$	$E_{bat,z}^{max}$	SOC_z^{max}	SOC_z^{min}	$\eta_{bat,z}$
Battery 1	7 kW	13.5 kWh	100 %	0 %	0.95
Battery 2	7 kW	9.8 kWh	100 %	0 %	0.90

4.4 Electricity Spot Prices

The prices used for the case studies are taken from Nordpool [16] and EPEX spot markets [17] from 2015, both with hourly resolution. The prices are shown in Fig. 2. The spot price in the respective EPEX area[5] had an average of 0.1898 NOK/kWh with very low fluctuations, whereas the German spot price had an average price of 0.2831 NOK/kWh and much higher fluctuations. In addition to the electricity spot price, an obligatory green certificate cost, a monthly fixed cost and a retailers revenue margin are added to every purchased kWh.

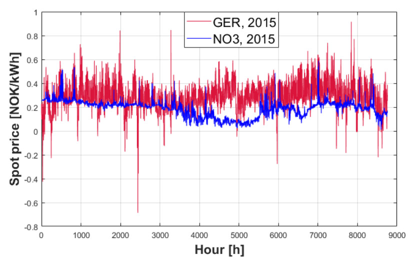

Figure 2: Electricity spot price from Norway (NO3) and Germany (GER) in 2015.

4.5 Grid Tariffs

In order to capture the annual cost that the end-user actually pays, the grid tariff from the relevant DSO is used. The current grid utility tariff structure in Norway is energy based, and has a fixed annual cost plus an extra fee per kWh. All cost elements in addition to the spot price are shown in Tab. 4.

Table 4: Grid, energy, tax and VAT costs for end-users. Note that all costs shown in the table are not including VAT.

Cost element	**Cost**
Fixed monthly cost [NOK/Month]	37.6
Grid energy cost [NOK/kWh]	0.22
Fixed grid cost [NOK/year]	1 340
Energy tax [NOK/kWh]	0.124
Green certificate fee [NOK/kWh]	0.0369
Retailer margin [NOK/kWh]	0.025
VAT [%]	25

[5]Region NO3 in the Nord Pool spot market.

5 Results

The initial question is if the core is non-empty. For each scenario, there exists a non-empty core, thus the nucleolus is in the core. Furthermore, the Shapley value is shown to be in the core for each scenario. Consequently, both methods provide stable cost allocations and are suitable for comparison.

Tab. 5 shows the relative deviation in cost allocations provided by nucleolus and the Shapley value, along with the preferred method by each player. As the table content shows, the deviation between the methods is modest in all scenarios. Further, it can be seen that player 3 and 4 experience the same deviation. Due to equal demand, they prefer the same cost allocation method within each scenario. Cells marked with '≈' in Tab. 5, indicate that the deviation between the method is less than 0.1 %. Deviation less than this quantity can be considered negligible, thus both methods propose almost identical cost allocations. Cells marked with '(**Nu**)' indicate that nucleolus is the preferred method, whereas '(**Sh**)' correspond to the Shapley value.

Table 5: Relative deviation in [%] between nucleolus and Shapley for each player in each scenario, along with their preferred method.

	Scenario			
Player	**#1**	**#2**	**#3**	**#4**
1	0.02 (≈)	0.02 (≈)	0.01 (≈)	1.01 (**Sh**)
2	0.00 (≈)	0.04 (≈)	1.37 (**Nu**)	3.01 (**Nu**)
3	0.02 (≈)	0.03 (≈)	0.43 (**Sh**)	0.57 (**Sh**)
4	0.02 (≈)	0.03 (≈)	0.43 (**Sh**)	0.57 (**Sh**)

6 Discussion

Fig. 3 shows the deviation between the nucleolus and Shapley, for each player plotted along with the cost reduction provided by the battery system, λ_{EES}. As Fig. 3 shows, there is an increase in the value of the battery system for each scenario. By changing the parameters load and electricity spot price, the batteries' contribution to cost reduction varies for each scenario. In scenario 1, the total load demand is high (Load profile 1) and the fluctuations in electricity spot prices are modest. The batteries are only able to lower the cost with 0.39 %. The value of the battery system increases in scenario 2 and 3, whereas the highest cost reduction provided by the batteries is obtained in scenario 4. In this scenario, there are periods where PV production exceeds the demand (Load profile 2), as illustrated in Fig. 1. In addition, the electricity spot price in scenario 4 is the most fluctuating (GER), which implies that the batteries are utilized for both storing power produced by the PV system, and for buying power when prices are low in order to store the power for high peak-periods.

The high utilization of the batteries is reflected through a cost reduction of of 8.84 %. With a varying λ_{EES}, we aim to examine whether the deviation between nucleolus and Shapley is affected.

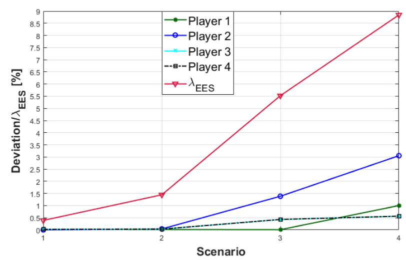

Figure 3: Relative deviation between nucleolus and Shapley for each player along with the value of batteries, in each scenario.

As Fig. 3 shows, player 3 and 4 experience exactly the same deviation within each scenario. Although the overall tendency for these players is a slight increase in deviation from 0.02 % in scenario 1 to 0.57 % in scenario 4, nucleolus and Shapley provide approximately similar cost allocations irrespective of the value of the battery system.

In scenario 1, player 1 experiences a deviation of 0.02 %, similar to player 3 and 4. However, in scenario 4, the deviation between the methods increases to 1.01 %. In the same scenario, player 2 experiences a deviation of 3.01 %. Even though these values can be considered marginal, there is a weak tendency that the prosumers experience a higher deviation in allocation method as the value of the battery system increases. Thus, the results might imply that the prosumers are more concerned regarding their preferred method when the value of the battery system is high. A high λ_{EES} can be interpreted as a higher contribution from player 1 and 2 to the energy community.

Despite this, player 1 and 2 do not necessary prefer the same allocation method. The overall highest deviation in method is found in scenario 4. For player 2, the cost proposed by nucleolus is over 3 % lower than the Shapley value. In contrast, player 1 prefers the Shapley value within the same scenario. Player 1 is equipped with the most efficient battery with largest energy capacity, as shown in Tab. 3. Thus, battery 1 is utilized more than battery 2. In other words, player 1 contributes more to the overall cost reduction. This difference in individual contribution is reflected through the preferred methods. While Shapley is preferred by player 1, player 2 prefers nucleolus within the same scenario.

In this paper, nucleolus and the Shapley value propose approximately similar cost allocations. Despite this, there is a tendency that the deviation in cost allocation increases as the value of the battery system increases. The value of the batteries is dependent on parameters such as renewable power production, price fluctuations and load demand. Thus, it can be interpreted as if the deviation in cost allocation method depends on these parameters. For an energy community where the players' available resources lead to marginal cost reduction, nucleolus and the Shapley propose almost similar cost allocations. Hence, for the case study presented in this paper, the players will not be concerned regarding their preferred method. In contrast, the deviation between the methods increases in scenarios where the available resources play a greater role in cost reduction within the energy community. Although the deviation between the methods is shown to be small in the presented scenarios, it might increase in larger energy communities consisting of more diversity among the players.

In an energy community consisting of solely consumers without neither PV production nor battery systems, there is no incentive for cooperation, as there are no resources to operate in a cooperative manner. Thus, for an energy community to operate cooperative, it is essential to facilitate the prosumers to join the cooperating operation. Although the consumers prefer the Shapley method in all presented scenarios, they never experience a deviation higher than 0.57 %. In contrast, player 2 experiences a deviation of 3 % in the same scenario. We believe that both nucleolus and the Shapley value are well-suited for cost allocation for the set-up of our case. Further we believe that it is of high importance to evaluate what is the aim of the cooperation. In scenarios where there are large deviations between the proposed methods, we believe that it is of high interest to study which players that are attractive for the cooperative operation to be beneficial. Both methods show similar fitness to solve the problem setting, whereas several aspects worth considering prior to implementation. The study shows that both methods provide solid cost allocations for local energy communities.

7 Conclusion

In this paper we have evaluated nucleolus and the Shapley value for cost allocation among a set of cooperating end-users within an energy community. As the presented results show, both methods provide stable cost allocations. The deviation between the methods is small, and can be even considered negligible in some of the simulated scenarios. However, there is a tendency of a slight increase in deviation in scenarios where the battery system is able to contribute with a certain cost reduction. Thus, we interpret this as if the deviation in cost allocation method is affected by externalities that are able to increase the value of the battery system, such as renewable power production and fluctuation in electricity spot price.

Based on the presented results, both nucleolus and Shapley value serve the intended purpose. However, we believe that it is of importance to study how to encourage valuable players to join the cooperation. The results also imply that in larger systems or systems with price volatility or high use of flexibility, the methods deviate slightly and should be compared.

We believe that the definition of fairness in the context of game theoretical method in the considered case, has to be extended to consider other externalities such as renewable generation from other sources, different grid tariff structures and the size of the energy community. For future work, it is of interest to study how Shapley and nucleolus perform in larger energy communities with higher deviation between the players' individual resources. Another interesting aspect would be to include the grid operator as a player in the cooperative game.

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