Optimization of sample size for two-point diameter verification in coordinate measurements

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Abstract. This paper investigates the possibility for reduction of sample size for inspection of two-point diameters with a coordinate measuring machine, by use of statistical methods. The statistical methods implement the parametric and non-parametric statistic. As confirmed by the simulation results it is possible to keep the 95% confidence level with a relatively small data sample. A low sample size would be especially important for an operative online dimension inspection with CNC machine and immediate correction of a suspected part.

Keywords: two-point diameter, sample size, test statistic, model simulation, measuring strategy, CMM.

1 Introduction

The main goal of Geometrical Product Specifications (GPS) inspection is to verify that the geometry and the dimensions of a part are inside of the tolerance limits specified by the drawing requirements, with some given confidence level. One of the important parameters of a measuring strategy in a coordinate measuring machine (CMM) is the number of the measuring points [1]. On one hand, a large sample size provides a better accuracy. On the other hand, a large sample size increases costs and time consumption in CMM inspection. The necessary sample size depends on many factors such as tolerance type (form, dimension etc.), magnitude of deviation from the desired value, and its ratio to the tolerance interval. Thus, the proper choice of an optimal number of measuring points is a nontrivial task.

Different approaches were suggested to discover the sample size problem. Parametric statistic principles based on the normal distribution of the measured variable has been suggested [2, 3]. A fuzzy logic approach to reduce the number of measuring points with CMM was proposed by Cappetti et al. [4]. Other approach-

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es with genetic algorithms [5], adaptive sample strategy with Kriging models [6] and combinations of analytical methods with uncertainty simulations [7] has been applied to solve the measuring strategy problem.

In a previous paper, the authors estimated the optimal sample size for detecting 95% of the radius variation range (roundness form deviation of cylinder cross-sections) with 95% confidence level [8].

In this paper, we have investigated reduction of number of measurements in two-point size verification of a circular feature. According to ISO 14405-1 [9], "*the two point size is the distance between two opposite points on an extracted integral linear feature of size*". The two-point size of the cylinder is also called "*two-point diameter*". The two-point size of a circular feature is illustrated in Fig.1.

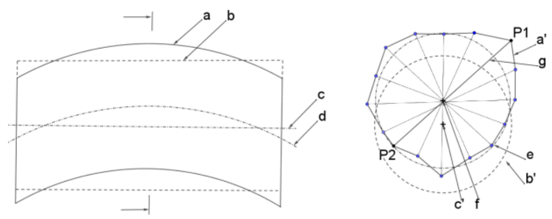

Fig. 1. Two-point diameter: a-actual cylinder; a'- section profile; b, b'-Gauss associated cylinder; c, c'- axis of Gauss associated cylinder; d-cylinder median line; e- Gauss associated circle (of section); f- Gauss associated circle center of e; g-actual local size (two-point diameter), the straight line between two opposite points P1 and P2, which goes through the center f

Research in the sample strategy field is generally focusing on the evaluation of geometrical form deviation (e.g. roundness, flatness) [5, 7, 8, 11]. In this paper: we consider the case when the diameter tolerance band is assumed to be larger than the variation of the two-point diameter in one single part. The parameters of interest in part inspection is the mean value and the variation of the two-point diameter.

2 Method and Material

We use the statistical hypotheses test approach to analyse the measurement data from CMM. The data sets need to be standardized and estimated before they are further applied for simulation. The more detail description is given bellow.

2.1 Experimental data

In this case study, we have inspected an internal cylinder of an aluminium workpiece with the internal diameter 60 mm and the length 130 mm. The measurements have been performed in a Leitz PMM-C-600 coordinate measuring machine with an analogue probe. The least square cylinder method has been used to establish the cylinder axis, which is the z-axis in the coordinate system. Three cross sections (A, B, C) were measured, the first on the top, the second in the middle and the third on the bottom respectively. There are 500 measuring points in each cross-section, and 250 diameter values D_i were calculated:

$$D_i = \sqrt{(x_i - x_{i+250})^2 + (y_i - y_{i+250})^2} \ , \qquad i = 1...250 \qquad (1)$$

For simplicity of presentation and further data processing, the array D_i has been transformed into $\xi_i = (\bar{D} - D_i) \cdot 1000$, where $\bar{D}$ is the mean value of the diameter in each cross-section. We make an assumption that all the lines, which connect the two opposite points agree with two-point diameter definition terms shown in Fig.1.

In order to derive the distribution shape of probability density function (pdf) $f(\xi)$ of standardized variable ξ_i, the kernel density estimator (KDE) has been used [12]:

$$\hat{f}(\xi) = \frac{1}{bn} \sum_{i=1}^{n} K\left(\frac{\xi - \xi_i}{b}\right) \ , \qquad -\infty < \xi < \infty \ . \qquad (2)$$

We have applied the Epanechnikov kernel K, the default MATLAB bandwidth b, and the sample size n with 250 variables ξ_i. The estimation results of pdf $f_A(\xi), f_B(\xi), f_C(\xi)$ for all three cross-sections are shown in Fig.2.

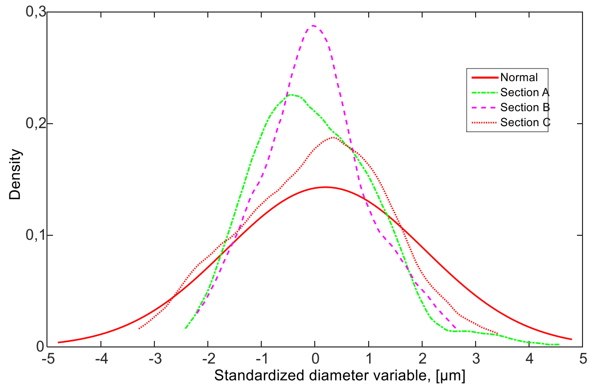

Fig. 2 The KDE $\hat{f}_A(\xi), \hat{f}_B(\xi), \hat{f}_C(\xi)$ based on 250 standardized diameter variables, and the adjusted normal distribution

At this point, for further purposes, we need to adjust the standard deviation σ_0 of the normal distribution in such way that the six-sigma interval could cover any of the cross-section variation ranges. In the same time, we want this value as small as possible to avoid an unnecessary reduction of the tolerance interval. In this particular case study, the standard deviation $\sigma_0 = 1.6$ μm has been chosen. The pdf objects with estimated parameters will be further used in simulation for generating of random measurements of workpiece to evaluate the influence of the sample size on the two-point diameter verification.

2.2 Statistical method and simulation

In order to estimate the influence of the sample size, we are apply the statistical hypotheses test with a test statistic criteria and pre-specified significance level [13]. First of all this method tests the sample strategy, which was exploited for the CMM inspection. By other words, we establish the statistical method to evaluate if the chosen sample size are sufficient or not. Next, we consider if such method can be applied for verification of the two-point diameter. We are applying the conventional two-sample hypotheses test for solving of a nonstandard problem with some small modifications. The principle of these modifications is illustrated in Fig. 3. An example of unknown non-normal distribution of the workpiece diameter variable are depicted as 5, 6, and 7. As long as we do not know in which direction from the nominal size the deviation of the workpiece size can occur in advanced, then the two independent hypothesis tests must be prepared. However only one of them will be carried out for each single case.

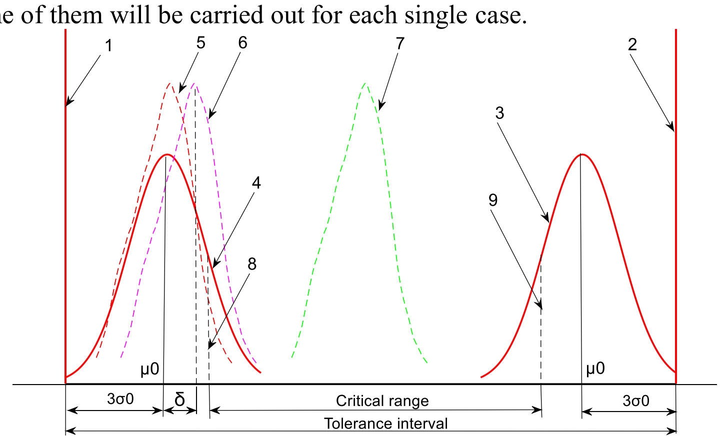

Fig. 3 Statistical hypothesis tests for verification of two-point diameter: 1-lower tolerance limit; 2-upper tolerance limit; 3-upper Gauss (null hypothesis); 4-lower Gauss (null hypothesis); 5-KDE object with large deviation; 6-KDE object with medium deviation; 7-KDE object with small deviation; 8-lower boundary of the statistical test; 9-upper boundary of the statistical test

The mean values of both normal distributions are located in such way that the distance between one of the means and one of the tolerance limits (1 – lower, 2 – upper) equals to $3 \cdot \sigma_0$ on each side as shown in Fig.3. The cases when the sample mean is below or above the mean of the Gauss curves (Fig. 3) correspond to the null hypotheses H_0^L *or* H_0^U. When one of the null hypotheses is accepted then the special procedure with large number of points will be suggested. Thereby, the method does not reject a part, but it recommends larger sample size when it is necessary.

As it was recently noticed, we need to test only one of the two hypotheses at the same time either for the lower tolerance (LTL) or for the upper tolerance limit (UTL). We consider the tolerance H7 for 60 mm diameter hole as an example, according to ISO 286-1 ($EI = 0, ES = +30$ μm) [14]. As long as the diameter variable D_i has been transformed, it has simplified a calculation of the tolerance limits: $LTL = EI$ and $UTL = ES$. Then the theoretical mean values are: $\mu_0^L = LTL + 3\sigma_0$ for the null hypothesis H_0^L and $\mu_0^U = UTL - 3\sigma_0$ for H_0^U (the lower and the upper tolerance limits respectively). Thereby the tested hypotheses H_0^L or H_0^U are formulated in this way: the sample mean of measurements is equal to the one of theoretical means of the normal distributions $N(\mu_0^L, \sqrt{\sigma_0 / n})$, $N(\mu_0^U, \sqrt{\sigma_0 / n})$ either $\mu_S = \mu_0^L$ (for H_0^L) or $\mu_S = \mu_0^U$ (for H_0^U), hence the alternative hypotheses H_1^L or H_1^U are $\mu_S > \mu_0^L$ or $\mu_S < \mu_0^U$ respectively. We use the sample mean $\bar{\xi} = \frac{1}{n}\sum_{i=1}^{n} \xi_i$ (to estimate μ_S) as the test statistic. The sample mean is computed from the sample data generated by KDE, which has unknown non-normal distribution $K(\mu_S, \sigma_S)$. The alternative hypotheses H_1^L, H_1^U assume that the variation range of the sample data is inside of the critical range V_k. Then the critical range is defined with the lower bound $\bar{\xi}_k^L$ for the LTL and the upper bound $\bar{\xi}_k^U$ for UTL respectively by:

$$\bar{\xi}_k^L = u_{1-\alpha} \cdot \sqrt{\sigma_0^2 / n} + \mu_0^L, \tag{3}$$

$$\bar{\xi}_k^U = u_\alpha \cdot \sqrt{\sigma_0^2 / n} + \mu_0^U, \tag{4}$$

where u_α and $u_{1-\alpha}$ are the quantiles of levels α , $1-\alpha$ respectively for $N(0,1)$, and n is the number of observations. We have used the level of significance $\alpha = 0.05$ in the statistical simulation (i.e. $u_{0.05} = -1.645$ and $u_{0.95} = 1.645$).

3 Simulation Results

By using two-sample tests of hypotheses, we must be aware of the significant tolerance interval reduction especially for the small sample sizes. The computed results for the boundaries and the critical range (Eq. 3 and 4) are shown in the Table 1.

Table 1. Reduction of 60mm H7 tolerance interval ($\alpha = 0.05, \sigma_0 = 1.6$)

Sample size	Lower bound, µm	Upper bound, µm	Critical range, µm	Tolerance reduction,%
5	6.0	24.0	18.0	40.00
10	5.6	24.4	18.8	37.33
15	5.5	24.5	19.0	36.67
20	5.4	24.6	19.2	36.00
30	5.3	24.7	19.4	35.33
40	5.2	24.8	19.6	34.67
50	5.2	24.8	19.6	34.67
60	5.1	24.9	19.8	34.00

After the boundaries of the critical range are established, we can proceed with simulation. A number of $N = 10^5$ iterations have been simulated for each sample size n_i. We consider the deviation of the mean μ_S to the *LTL* as denoted with 5 and 6 in Fig.3. Then three different kernel distributions have been simulated for the mean difference δ_j^L with the following values $\delta_1^L = \sigma_0$, $\delta_2^L = 0.5 \cdot \sigma_0$, and $\delta_3^L = 0$ such that $\mu_S = \mu_0^L + \delta_j^L$. The sample mean $\bar{\xi}$ was comparing with either the low bound $\bar{\xi}_k^L$ or the upper bound $\bar{\xi}_k^U$ Table 1. When the conditions of $\bar{\xi} > \bar{\xi}_k^L$ or $\bar{\xi} < \bar{\xi}_k^U$ are fulfilled, the iteration assigned as 1 (0 otherwise) and summed up as the counters C_0^L or C_0^U , then the rejecting rates η_0^L or η_0^U were calculated as C_0^L / N or C_0^U / N respectively. The simulation results for each cross-section A, B and C are presented in Table 2, 3 and 4 (simulation results for the opposite side *UTL* are similar and not presented in the paper).

Table 2 Rejecting rates η_0^L of the sample mean location for the *Section A*

Mean difference, δ_j^L	**Sample size, n**						
	5	**10**	**15**	**20**	**30**	**40**	**50**
$\delta_1^L = \sigma_0$	0.78	0.98	1	1	1	1	1
$\delta_2^L = 0.5 \cdot \sigma_0$	0.22	0.46	0.65	0.79	0.93	0.98	1
$\delta_3^L = 0$	0.02	0.02	0.02	0.02	0.02	0.02	0.02

Table 3 Rejecting rates η_0^L of the sample mean location for the *Section B*

Mean difference, δ_j^L	**Sample size,** n						
	5	**10**	**15**	**20**	**30**	**40**	**50**
$\delta_1^L = \sigma_0$	0.82	0.99	1	1	1	1	1
$\delta_2^L = 0.5 \cdot \sigma_0$	0.21	0.46	0.66	0.82	0.95	0.99	1
$\delta_3^L = 0$	0.01	0.01	0.01	0.01	0.01	0.01	0.01

Table 4 Rejecting rates η_0^L of the sample mean location for the *Section C*

Mean difference, δ_j^L	**Sample size,** n								
	5	**10**	**15**	**20**	**30**	**40**	**50**	**60**	**65**
$\delta_1^L = \sigma_0$	0.75	0.95	0.99	1.00	1.00	1.00	1.00	1.00	1.00
$\delta_2^L = 0.5 \cdot \sigma_0$	0.29	0.47	0.63	0.75	0.88	0.95	0.98	0.99	1.00
$\delta_3^L = 0$	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03

4 Discussion of results

There are four important categories of the measurements regarding to the simulation results of the data mean μ_S location. The first category G (good parts) corresponds to the intersection of two subsets $\{\mu_S \geq \mu_0^L + \sigma_0\} \cap \{\mu_S \leq \mu_0^U - \sigma_0\}$. The second category T (transitional parts) includes the subsets $\{\mu_0^L < \mu_S < \mu_0^L + \sigma_0\}$ and $\{\mu_0^U - \sigma_0 < \mu_S < \mu_0^U\}$, and the third category S (suspected parts) belongs to $\{\mu_S \leq \mu_0^L\}$ and $\{\mu_S \geq \mu_0^U\}$, which equivalents (according to the terms in section 2.2) with the fourth category F (fail parts) of subsets $\{\mu_S < LTL\}$ and $\{\mu_S > UTL\}$. Obviously, all the boundaries are fuzzy but they help to clarify the simulation results. For illustrational purposes, we presume a uniform distribution $U(0,30)$ of the manufacturing process over a long time period. The tolerance interval can be illustrated as follows in Fig.4. Thus, the content of each region (S,T,G respectively) can be easily evaluated:

$$\int_0^{3\sigma_0} \frac{1}{30} du = 0.16, \quad \int_{3\sigma_0}^{4\sigma_0} \frac{1}{30} du = 0.05, \quad \int_{\mu_0^L+\sigma_0}^{\mu_0^U-\sigma_0} \frac{1}{30} du = 0.57 \qquad (5, 6, 7)$$

A correct decision about the data mean location μ_S for the S category can be defined at least with 95% probability (the accepting rate $1-\eta_0^L$, $\delta_3^L = 0$) even with the five-observation sample size. Moreover, the solution is independent on the sample size.

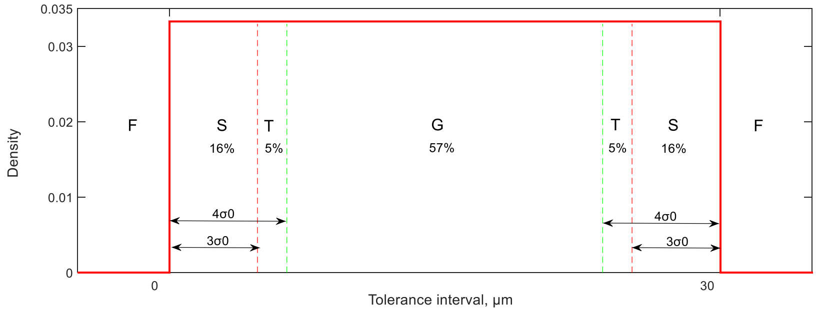

Fig. 4 The dimension tolerance interval based on the uniform distribution assumption

Similar for the G category, the correct conclusion about a low deviation of the mean value can be done at least with 95% probability (the rejecting rate η_0^L, $\delta_1^L = \sigma_0$) for the ten-observation sample size.

According to Table 2, 3, 4 ($\delta_2^L = 0.5 \cdot \sigma_0$), for the transitional category T, the large sample over 40 observations might be required to confirm the compliance of the size with the tolerance limits with 95% CL. Nevertheless, the T regions are only 10% regarding to (6) relative to whole the uniform distribution (Fig. 4). Even by using the 10 observations sample (corresponds to 20 measuring points for the two-point diameter), we are still able to make the right decision in about 45% of the cases. Namely, the total percentage will be below 10% of all possible issues.

5 Conclusion

In this paper, we have investigated the number of required points in measurement of two-point diameter, and performed a case study on a part with 60 mm diameter and H7 tolerance. The reliability of the method strongly depends on the proper choice of the six-sigma range. The method reduces to some degree the tolerance interval size (acceptance interval), but it is compensated by the significant minimization of the sample size, more than in 4 times relative to [8] (for approximately 89% cases estimated with (5) and (7) with the assumption of the uniform distribution of the process, Fig. 4). Such reduction of the sample strategy will be in high demand for online inspection with CNC machine, where the suspected part can be corrected immediately.

6 References

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