Local Voting Protocol Step-Size Choice for Consensus Achievement

Konstantin Amelin

Faculty of Mathematics and Mechanics,
Saint Petersburg State University,
198504, Universitetskii pr. 28, St. Petersburg, Russia
E-mail: konstantinamelin@gmail.com

Natalia Amelina

Faculty of Mathematics and Mechanics,
Saint Petersburg State University,
198504, Universitetskii pr. 28, St. Petersburg, Russia
E-mail: ngranichina@gmail.com

Yury Ivanskiy

Faculty of Mathematics and Mechanics,
Saint Petersburg State University,
198504, Universitetskii pr. 28, St. Petersburg, Russia
E-mail: ivanskiy.yuriy@gmail.com

Yuming Jiang

Department of Telematics,
Norwegian University of Science and Technology
NO-7491, Trondheim, Norway
Email: jiang@item.ntnu.no

Abstract: In the paper a multi-agent network system of different computing nodes is considered. A problem of load balancing in the network is addressed. The problem is formulated as consensus achievement problem and solved via local voting protocol. Agents exchange information about their states in presence of noise in communication channels. For the system operating in noised conditions analytically obtained estimation of control protocol optimal step size value is given. The dependence of the system behaviour on value of control protocol step-size is demonstrated in simulation examples.

Keywords: local voting protocol; step size choice; consensus achievement; load balancing; multi-agent networks.

Reference to this paper should be made as follows: Amelin, K., Amelina N., Ivanskiy, Y. and Jiang, Y. (201X) 'Local Voting Protocol Step-Size Choice for Consensus Achievement', *International Journal of Intelligent Engineering Informatics*, Vol. x, No. x, pp.xxx–xxx.

Biographical notes: Konstantin Amelin received his PhD in Software Engineering at St. Petersburg State University. He is working as the PostDoc at the Software Engineering Department of Saint Petersburg State University. His current research interests include Multi-Agent Technology, Embeded Systems and Data Mining. He published 3 books and about 10 papers in referred international journals.

Natalia Amelina received her PhD in mathematical cybernetics from the Department of Theoretical Cybernetics, St. Petersburg State University. She is a researcher at the Saint Petersburg State University, Faculty of Mathematics and Mechanics. Her research interests include distributed control of multiagent systems, consensus problem, load balancing, cooperative control of UAVs, sensor and wireless networks.

Yury Ivanskiy received his PhD in mathematical cybernetics at St. Petersburg State University. He is a researcher at Lab. Control of Complex Systems, Institute of Problems of Mechanical Engineering, Russian Academy of Sciences. His research interests include distributed control of multiagent systems, consensus problem, load balancing, randomized algorithms, stochastic optimization.

Yuming Jiang received his PhD in Electrical and Computer Engineering from National University of Singapore. He is a full Professor at the Department of Information Security and Communication Technology, Faculty of Information Technology and Electrical Engineering, Norwegian University of Science and Technology. He is first author of the book "Stochastic Network Calculus". His research interests are the provision, analysis and management of quality of service guarantees in communication networks. In the area of wireless networks, his focus has been on developing / applying analytical models and investigating their basic properties in performance analysis of wireless networks.

1 Introduction

Usage of multi-agent systems gains more popularity in solving optimization-involved problems. Multi-agent approach implies joining a number of autonomous agents (nodes, paricles) in a group, network or swarm for solving problems such as clustering, UAV soaring, adaptive scheduling of road transport, data mining, semantic processing, grid energy management and other Granichin et al. (2015); Elamine et al. (2016); Kumar et al. (2015); Rzevski and Skobelev (2014). Load balancing is an important practical problem in network systems. It may arise in such network systems as computer, production, transport, logistics, and other service networks. In computational networks load balancing can be applied to improve the system efficiency. A multi-agent approach is used to address this problem in network systems. A possible control goal in such systems is to improve the network speed of operation using communication among agents in the system. In Amelina at al. (2015b) it was shown that the problem of almost optimal task distribution among agents could be reformulated as a problem of the consensus achievement in the network.

The consensus approach was widely applied for solving various practical problems such as cooperative control of multivehicle networks Granichin et al. (2012); Ren et al. (2007), distributed control of robotic networks Bullo et al. (2009), flocking problem Virágh et al. (2014); Yu et al. (2010a), optimal control of sensor networks Kar et al. (2010), distributed node scheduling in multihop wireless networks Vergados et al. (2017) and others. Works Chebotarev and Agaev (2009); Lewis et al. (2014); Li and Zhang (2009); Proskurnikov (2013); Ren and Beard (2007); Yu et al. (2010b) considered formulating the conditions for achieving consensus in such systems.

In Amelina et al. (2015a) a choice of an optimal step-size of consensus-type protocol for task redistribution among agents in a stochastic network with randomized priorities is considered. It is shown that a trade-off is made between noise sensitivity and the rate of convergence of control protocol while choosing its step-size. The paper proposes a way of choosing step-size to maximize convergence precision. An optimal step-size of control protocol could be chosen for the network system under certain conditions such as parameters of noise during information exchange, system topology etc. In Amelin et al. (2016) we proposed stochastic approximation type algorithm for choosing step-size of control protocol which can be used for adjustment of the protocol step-size in the case of unknown system parameter values. In this paper we consider dependence of behaviour of the system operating by proposed protocol on the value of its step-size.

The paper is organized as follows. Notation used in the paper and the problem formulation are given in Section 2. The control protocol for achieving the consensus is introduced in Section 3. In Section 4 the main assumptions and main results are presented. Simulation results are given in Section 5. Section 6 contains conclusion remarks.

2 Problem Statement

Let's consider a dynamic network system of n agents, which exchange information among themselves during tasks processing. Tasks may come to different agents of the system in different discrete time instants $t = 0, 1, \ldots$. Agents process incoming tasks in parallel. Tasks can be redistributed among agents based on a feedback.

Without loss of generality, agents in the system are numbered. Assume $N = \{1, \ldots, n\}$ denotes the set of agents in the network system. Let $i \in N$ be the number of an agent. The network topology may switch over time. Let the dynamic network topology be modelled by a sequence of digraphs $\{(N, E_t)\}_{t \geq 0}$, where $E_t \subset E$ denotes the set of edges at time t of topology graph (N, E_t). The corresponding adjacency matrices are denoted as $A_t = [a_t^{i,j}]$, where $a_t^{i,j} > 0$ if agent j is connected with agent i and $a_t^{i,j} = 0$ otherwise. Here and below, an upper index of agent i is used as the corresponding number of an agent (not as an exponent). Denote $\mathcal{G}_{A_t}$ as the corresponding graph.

To introduce some properties of the network topology, the following definitions from the graph theory will be used. Define *the weighted in-degree* of node i as the sum of i-th row of matrix A: $indeg^i(A) = \sum_{j=1}^n a^{i,j}$; $\mathcal{D}(A) = \mathrm{diag}\{indeg^i(A)\}$ is the corresponding diagonal matrix; $indeg_{\max}(A)$ is the maximum in-degree of graph $\mathcal{G}_A$. Let $\mathcal{L}(A) = \mathcal{D}(A) - A$ denote the *Laplacian* of graph $\mathcal{G}_A$; $\cdot^{\mathrm{T}}$ is a vector or matrix transpose operation; $||A||$ is the Euclidean norm: $||A|| = \sqrt{\sum_i \sum_j (a^{i,j})^2}$; $Re(\lambda_2(A))$ is the real part of the second eigenvalue of matrix A ordered by the absolute magnitude; $\lambda_{\max}(A)$ is the maximum eigenvalue of matrix A.

It is said that digraph $\mathcal{G}_B$ is a subgraph of a digraph $\mathcal{G}_A$ if $b^{i,j} \leq a^{i,j}$ for all $i, j \in N$.

Digraph $\mathcal{G}_A$ is said to contain a *spanning tree* if there exists a directed tree $\mathcal{G}_{tr} = (N, E_{tr})$ as a subgraph of $\mathcal{G}_A$ which includes all vertices of $\mathcal{G}_A$.

The behaviour of agent $i \in N$ is described by characteristics of two types:

- lengths of queue of tasks at time instant t: q_t^i,
- productivity: p^i.

Let random variable η_j denote complexity (or a number of computational operations needed to execute the task) of a task which came to the system. Dynamics of the system can be written in the following way:

$$\sum_{q_{t+1}^i} \eta_j = \sum_{q_t^i} \eta_{j'} - p^i + \sum_{z_t^i} \eta_{j''} + \sum_{u_t^i} \eta_{j'''},$$

where $\sum_{q_{t+1}^i} \eta_j$ is number of computational operations needed to execute all tasks in the queue of agent i at time instant $t+1$, p^i is productivity of agent i or the number of computational operations it can perform during one tact of the system (assume it is constant), $\sum_{z_t^i} \eta_{j''}$ is the complexity of tasks which came to the system on agent i at time instant t and $\sum_{u_t^i} \eta_{j'''}$ is the complexity of tasks which already came to other agents at previous time instants and were redistributed to agent i according to control protocol.

Assume random variable η has mathematical expectation $\bar{\eta} < \infty$. Let's take expectation of left and right parts of the equation of system dynamics.

$$E\left(\sum_{q_{t+1}^i} \eta_j\right) = E\left(\sum_{q_t^i} \eta_{j'} - p^i + \sum_{z_t^i} \eta_{j''} + \sum_{u_t^i} \eta_{j'''}\right)$$

$$\sum_{q_{t+1}^i} \bar{\eta} = \sum_{q_t^i} \bar{\eta} - p^i + \sum_{z_t^i} \bar{\eta} + \sum_{u_t^i} \bar{\eta}$$

Left and right parts are now equal to number of tasks at agent i multiplied by their average complexity.

$$\bar{\eta} \sum_{q_{t+1}^i} 1 = \bar{\eta} \sum_{q_t^i} 1 - p^i + \bar{\eta} \sum_{z_t^i} 1 + \bar{\eta} \sum_{u_t^i} 1$$

$$\bar{\eta} q_{t+1}^i = \bar{\eta} q_t^i - p^i + \bar{\eta} z_t^i + \bar{\eta} u_t^i$$

Divide both parts of the equation by constant value $\bar{\eta}$. We get discrete model which allows as to analyse system dynamics without information about complexities of each task in the system (but with assumption their average value is bounded). For all $i \in N,\ t = 0, 1, \ldots$, the dynamics of the network system in a vector form is as follows

$$q_{t+1}^i = q_t^i - \tilde{p}^i + z_t^i + u_t^i, \tag{1}$$

where $\tilde{p}^i = p^i / \bar{\eta}$, z_t^i the amount of new tasks, which came to the system and were received by agent i at time instant t; u_t^i is control action (redistributed tasks to agent i at time instant t), which is chosen based on some information about queue lengths of neighbours $q_t^j,\ j \in N_t^i$, where N_t^i is the set $\{j \in N : a_t^{i,j} > 0\}$.

Denote

$$x_t^i = \frac{q_t^i}{\tilde{p}^i} \tag{2}$$

the *load* of agent $i \in N$. Assume, that $\tilde{p}^i \neq 0,\ \forall i \in N$. In Amelina at al. (2015b) it was proven that from all possible options for the redistribution of all tasks the minimum operation time of the system is achieved when loads x_t^i are equalized throughout the network. Hence, it is important to consider the achievement of the following goal.

It is required to maintain balanced (equal) loads across the network under conditions of changing network topology.

At this setting we can consider the consensus problem for states x_t^i of agents, where x_t^i is a state of agent $i \in N$. We use the following definitions.

Definition 2.1: n agents of a network are said to reach a *consensus* at time t if $x_t^i = x_t^j\ \ \forall i, j \in N, i \neq j$.

Definition 2.2: n agents are said to achieve *asymptotic mean square ϵ-consensus* for $\epsilon > 0$ when

$$\overline{\lim}_{t\to\infty} \mathrm{E}\|x_t^i - x_t^j\|^2 \leq \epsilon.$$

To ensure balanced loads across the network (e.g., in order to increase the overall throughput of the system and to reduce the execution time), it is naturally to use a redistribution protocol over time. We assume that to form the control (redistribution) strategy each agent $i \in N$ has noisy observations about its neighbours' states

$$y_t^{i,j} = x_t^j + w_t^{i,j},\ j \in N_t^i, \tag{3}$$

where $w_t^{i,j}$ is a noise occurring during transmission from node j to node i.

3 Control Protocol

In Amelina at al. (2015b), properties of a control algorithm, called local voting protocol, were studied for stochastic networks in the context of load balancing problem. For each agent the control (amount of redistributed tasks) was determined by the weighted sum of differences between the information about the state of the agent and the information about its neighbours' states. Let's consider a protocol as follows. We define

$$u_t^i = \gamma \tilde{p}_t^i \sum_{j \in \bar{N}_t^i} b_t^{i,j}(y_t^{i,j} - x_t^i), \tag{4}$$

where $\gamma > 0$ is a step-size of the control protocol and $\bar{N}_t^i \subset N_t^i$ is the neighbour set of agent i (note, that we could use not all the available connections, but some subset of them), $b_t^{i,j}$ are protocol coefficients.

Let $B_t = [b_t^{i,j}]$ be the matrices of task redistribution protocol for every time instant t. (We set $b_t^{i,j} = 0$ when $a_t^{i,j} = 0$ or $j \notin \bar{N}_t^i$.) The corresponding graph $\mathcal{G}_{B_t}$ may have the same topology as graph $\mathcal{G}_{A_t}$ of matrix A_t or more poor.

The dynamics of the closed loop system with protocol (4) will be as follows:

$$x_{t+1}^i = x_t^i - 1 + \tilde{z}_t^i + \gamma \sum_{j \in \bar{N}_t^i} b_t^{i,j}(y_t^{i,j} - x_t^i) =$$

$$x_t^i - 1 + \tilde{z}_t^i + \gamma \left(\sum_{j \in \bar{N}_t^i} b_t^{i,j} x_t^j \right) - \gamma \; indeg^i(B_t) x_t^i + \gamma \tilde{w}_t^i, \; i \in N, \tag{5}$$

where $\tilde{w}_t^i = \sum_{j \in \bar{N}_t^i} b_t^{i,j} w_t^{i,j}$ and $\tilde{z}_t^i = z_t^i / \tilde{p}^i$.

Let us rewrite Eq. (5) in a more compact form. Define the $\mathbb{R}^n$-valued vectors $X_t = [x_t^i]$, $\mathbf{1}_n$ - vector with all elements equal to 1, $Z_t = [\tilde{z}_t^i]$ and $W_t = [\sum_{j \in \bar{N}_t^i} b_t^{i,j} w_t^{i,j}]$. The dynamics of the closed loop system with protocol (4) may be represented as

$$X_{t+1} = X_t + \gamma(B_t - \mathcal{D}(B_t))X_t - \mathbf{1}_n + Z_t + \gamma W_t. \tag{6}$$

Due to the view of Laplacian matrices $\mathcal{L}(B_t)$ we can rewrite the dynamics of the system in the following vector-matrix form:

$$X_{t+1} = X_t - \gamma \mathcal{L}(B_t) X_t - \mathbf{1}_n + Z_t + \gamma W_t. \tag{7}$$

4 Main Results

4.1 Assumptions

Let $(\Omega, \mathcal{F}, P)$ be the underlying probability space corresponding to the sample space, the collection of all events, and the probability measure respectively, and E be a mathematical expectation symbol.

Assume that the following conditions are satisfied:

- **A1**. **a)** For all $i \in N, \; j \in N_t^i$, observation noise vectors $w_t^{i,j}$ are zero-mean, independent identically distributed (i.i.d.) random vectors with bounded variances: $\mathrm{E}(w_t^{i,j})^2 \le \sigma_w^2$.

  b) Graphs $\mathcal{G}_{B_t}, t = 1, \ldots$ are i.i.d. (independent identically distributed), i.e. the random events of appearance of of "time-varying" edge (j, i) in graph $\mathcal{G}_{B_t}$ are independent and identically distributed for the fixed pair (j, i), $i \in N, j \in N_{\max}^i = \cup_t \bar{N}_t^i$. For all $i \in N, \; j \in N_t^i$ weights $b_t^{i,j}$ in the control protocol are independent random variables with mean values (mathematical expectations): $\mathrm{E}b_t^{i,j} = b_{av}^{i,j}$, and bounded variances: $\mathrm{E}(b_t^{i,j} - b_{av}^{i,j})^2 \le \sigma_b^2$. Let B_{av} be the corresponding adjacency matrix.

  c) For all $i \in N, \; t = 0, 1, \ldots$ random values z_t^i are independent with expectations: $\mathrm{E}z_t^i = \bar{z}$ which do not depend on i, and variances: $\mathrm{E}(z_t^i - \bar{z})^2 \le \sigma_z^2$.

  Additionally, all mentioned in Assumption **A1** independent random variables and vectors are mutually independent.

- **A2**. Graph $\mathcal{G}_{B_{av}}$ has a spanning tree (for the consensuses to be achievable throughout the system Chebotarev and Agaev (2009)).

- **A3.** For step-size γ of control protocol (4) the following conditions are satisfied:

$$0 < \gamma < \frac{1}{indeg_{\max}(B_{av})}, \; |\delta(\gamma)| < 1, \tag{8}$$

  where $\delta(\gamma) = 1 - \gamma Re(\lambda_{max}(\mathcal{L}(B_{av}))) - \gamma^2 \lambda_{\max}(\mathcal{L}(B_{av})^T \mathcal{L}(B_{av}))$.

4.2 Averaged Models

Let $x_0^\star$, be the weighted average of the initial states

$$x_0^\star = \frac{\sum_i g_i x_0^i}{\sum_i g_i}$$

where g^T is the left eigenvector of matrix B_{av} Lewis et al. (2014) ($x_0^\star = \frac{1}{n}\sum_{i=1}^n x_0^i$ in the case of balanced topology graph $\mathcal{G}_{B_{av}}$) and $\{x_t^\star\}$ is the trajectory of averaged systems

$$x_{t+1}^\star = x_t^\star + \bar{z} - 1. \tag{9}$$

where $\bar{z}$ is expectation defined by Assumption **A1.c**.

4.3 Theoretical result

Consider vector $X_t^\star \in \mathbb{R}^n,\ t = 0, 1, \ldots$ which consists of $x_t^\star$ at all places.

Theorem 1: If *Assumptions* **A1–A3** *hold* **then** *for averaged squared difference* $\nu_t = \mathrm{E}||X_t - X_t^\star||^2$ *of trajectories of closed-loop systems* (5) *and* (9) *following inequalities are satisfied:*

$$\nu_t \leq \frac{\gamma^2 H + S}{1 - \delta(\gamma)} + (\delta(\gamma))^t \left(\nu_0 - \frac{\gamma^2 H + S}{1 - \delta(\gamma)}\right), \tag{10}$$

$H = \sigma_w^2 ||B_{av}||^2$, $S = n\sigma_z^2$, *i.e.* **if** *additionally* $\nu_0 < \infty$, **then** *the asymptotic mean square* ϵ*-consensus in* (5) *is achieved with* $\epsilon = \frac{\gamma^2 H + S}{1 - \delta(\gamma)}$.

The proof is a particular case of the proof in Amelina et al. (2013).

Theorem 2: If *Assumptions* **A1–A3** *hold* **then** *optimal step-size* $\gamma^\star$ *of control protocol* (4) *can be calculated by formula:*

$$\gamma^\star = -\frac{S}{H}\Delta + \sqrt{\frac{S^2}{H^2}\Delta^2 + \frac{S}{H}} \tag{11}$$

where $\Delta = \frac{\lambda_{\max}(\mathcal{L}(B_{av})^T \mathcal{L}(B_{av}))}{Re(\lambda_{\max}(\mathcal{L}(B_{av})))}$.

The proof is similar to proof given in Amelina et al. (2015a).

5 Simulation Results

Let's consider network on $n = 10$ agents connected as a directed circle. Amount of tasks coming to the system at time instant t is a Poisson random variable distributed with parameter $\sigma_z = 10$. Complexity of incoming tasks equals 1.Agent productivities $p^i,\ i = 1 \cdots n$ are constant and have values distributed uniformly in interval $[0.9, 1.1]$. Noise occurring during information exchange between agents $w_t^{i,j}$ is a random variable with uniform distribution on interval $[-15, 15]$. The agents have initial queue lengths uniformly distributed in interval $[0, 1000]$.

Consider behaviour of the given system. Fig. 1 shows how the average value of squared norm of difference between system state vector X and vector $X^\star$ depends on step-size value of the control protocol. Here, $X^\star$ is $[1 \times n]$-vector with all elements equal to the consensus value $x^\star$. The value $\|X - X^\star\|^2$ was calculated at each of time instants of system operation with chosen step-size. Averaged value $\|X - X^\star\|^2_{av}$ was computed by taking average of $\|X - X^\star\|^2$ at all T time instants, $T = 100$. The faster and more precisely the consensus is achieved, the smaller is the value of $\|X - X^\star\|^2_{av}$. Actual values depend on initial conditions but the form of graph differs little for the same system topology. Figures 2–7 show behaviour of the system operating by proposed protocol with different step-sizes in stationary and non-stationary case (in absence and presence of incoming task flow). Figures 2, 3, 4 describe the system behavior in stationary case. If the step-size is less than optimal value (Fig. 2) the system becomes less noise-sensitive but the agents reach consensus considerably slower. Step-size larger than optimal (Fig. 3) (but when the agents still converge to consensus value) gives better speed of convergence, but worse convergence precision since the influence of noise in communication channels grows when step-size increases. The choice of step-size close to optimal value (Fig. 4) provides a balance between noise sensitivity of the system and speed of agents' states convergence to the consensus value (which corresponds to averaged system load). Figures 5, 6, 7 describe system in presence of incoming task flow. The choice of step-size also influences the ability of the system to cope with incoming task flow (which unbalances agents' loads). If the step size is smaller than optimal (Fig. 5) the agents are not able to balance their states since tasks exchange rate is too slow because of small step-size. When step-size value is larger than optimal, (Fig. 7) agents don't maintain small deviation from the consensus value. Though the consensus is reached rather quickly, the increased noise sensitivity prevents the agents to achieve (and maintain) equal load. The agents in the system operating by protocol with the optimal step-size (Fig. 6) are able to achieve and maintain consensus of their state with the smallest deviation in given conditions i.e. the intensity of incoming task flow, statistical characteristics of noise, system topology.

6 Conclusion

In this paper we considered dependence of network system operating by proposed algorithm on its step-size value. The network model was assumed to have noise in measurements. A proposed control strategy is based on of a local voting protocol for load balancing of network system. A simulation of the system operating by introduced control strategy is provided. Both stationary and non-stationary cases are considered. Analytically obtained estimated optimal value of step-size is given.

Acknowledgements

Paper was supported by RFBR (projects 15-08-02640 and 16-07-00890) and by SPbSU (grant 6.38.230.2015).

References

Amelin, K. and Amelina, N. and Ivanskiy, Y. and Jiang, Y. (2016) ‘Choice of step-size for consensus protocol in changing conditions via stochastic approximation type algorithm’, *Proc. of 2016 International Conference on Control, Decision and Information Technologies (CoDIT)*, pp. 007–011.

Amelina, N. and Granichin, O. and Kornivetc, A. (2013) ‘Local Voting Protocol in Decentralized Load Balancing Problem with Switched Topology, Noise, and Delays’, *Proc. of 52nd IEEE Conference on Decision and Control (CDC 2013)*, pp. 4613–4618.

Amelina, N. and Granichin, O. and Granichina, O. and Ivanskiy, Y. and Jiang, Y. (2015a) ‘Optimal Step-Size of a Local Voting Protocol for Differentiated Consensuses Achievement in a Stochastic Network with Priorities’, *Proc. of 2015 European Control Conference (ECC-2015)*, pp. 628–633.

Amelina, N. and Fradkov, A. and Jiang, Y. and Vergados, D.J. (2015b) ‘Approximate consensus in stochastic networks with application to load balancing’, *IEEE Transactions on Information Theory*, Vol. 61, No. 4, pp. 1739–1752.

Bullo, F. and Cortés, J. and Martinez, S. (2009) *Distributed Control of Robotic Networks: a Mathematical Approach to Motion Coordination Algorithms*, Princeton University Press.

Chebotarev P.Yu. and Agaev R.P. (2009) ‘Coordination in multiagent systems and Laplacian spectra of digraphs’, *Automation and Remote Control*, Vol. 7, Is. 3, pp. 469–483.

Granichin, O. and Skobelev, P. and Lada, A. and Mayorov, I. and Tsarev, A. (2012) ‘Comparing adaptive and non-adaptive models of cargo transportation in multi-agent system for real time truck scheduling’, *Proc. of the 4th International Joint Conference on Computational Intelligence*, pp. 282–285.

Granichin, O. and Volkovich, Z. and Toledano-Kitai, D. (2015) *Randomized algorithms in automatic control and data mining*, Intelligence Systems Reference Library, Vol. 67, Springer-Verlag: Heidelberg New York Dordrecht London.

Elamine, D.O. and Nfaoui, E.H. and Boumhidi, J. (2016) ‘Intelligent multi-agent system for smart microgrid energy management’, *International Journal of Intelligent Engineering Informatics*, Vol. 4, No. 3–4, pp. 245–266.

Kar, S. and Moura, J. MF (2010) ‘Distributed consensus algorithms in sensor networks: Quantized data and random link failures’, *IEEE Transactions on Signal Processing*, Vol. 58, No. 3, pp. 1383–1400.

Kumar, S.S. and Inbarani, H.H. and Azar, A.T. and Hassanien, A.A. (2015) ‘Rough set-based meta-heuristic clustering approach for social e-learning systems’, *International Journal of Intelligent Engineering Informatics*, Vol. 3, No. 1, pp. 23–41.

Lewis, F.L. and Zhang, H. and Hengster-Movric, K. and Das, A. (2014) *Cooperative Control of Multi-Agent Systems: Optimal and Adaptive Design Approaches (Communications and Control Engineering)*, Springer.

Li, T. and Zhang, J.F. (2009) 'Mean square average-consensus under measurement noises and fixed topologies: Necessary and sufficient conditions', *Automatica*, Vol. 45, No. 8, pp. 1929–1936.

Proskurnikov, A. (2013) 'Average consensus in networks with nonlinearly delayed couplings and switching topology', *Automatica*, Vol. 49, No. 9, pp. 2928–2932.

Ren, W. and Beard, R. (2007) *Distributed Consensus in Multi-vehicle Cooperative Control: Theory and Applications*, Springer.

Ren, W. and Beard, R.W. and Atkins, E.M. (2007) 'Information consensus in multivehicle cooperative control', *Control Systems, IEEE*, Vol. 27, No. 2, pp. 71–82.

Rzevski, G. and Skobelev, P. (2014) *Managing complexity*, Wit Press.

Vergados, D.J. and Amelina, N. and Jiang, Y. and Kralevska, K. and Granichin, O. (2017) 'Towards optimal distributed node scheduling in a multihop wireless network through local voting', to appear in *IEEE Transactions on Wireless Communications* arXiv:1701.09010.

Virágh, C. and Vásárhelyi, G. and Tarcai, N. and Szörényi, T. and Somorjai, G. and Nepusz, T. and Vicsek, T. (2014) 'Flocking algorithm for autonomous flying robots', *Bioinspiration & biomimetics*, Vol. 9, No. 2, pp. 25012–25022.

Yu, W. and Chen, G. and Cao, M. (2010a) 'Distributed leader–follower flocking control for multi-agent dynamical systems with time-varying velocities', *Systems & Control Letters*, Vol. 59, No. 9, pp. 543–552.

Yu, W. and Chen, G. and Cao, M. (2010b) 'Some necessary and sufficient conditions for second-order consensus in multi-agent dynamical systems', *Automatica*, Vol. 46, No. 6, pp. 1089–1095.

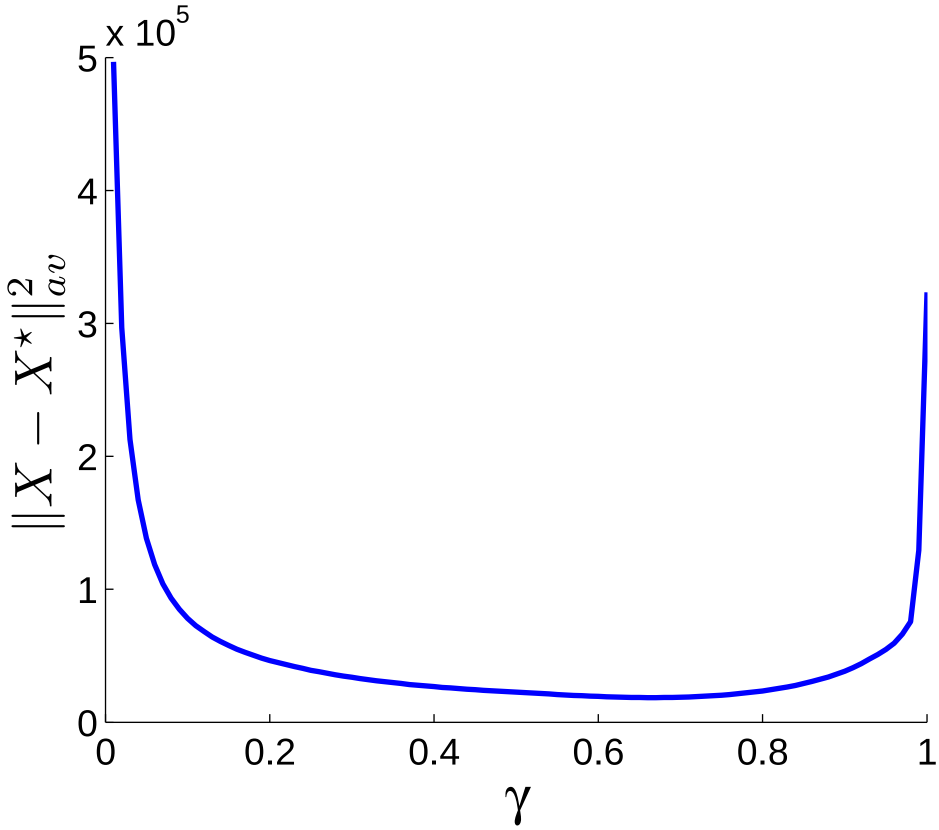

Figure 1 Dependence of average deviation of agent states from consensus value on γ for ring-type topology.

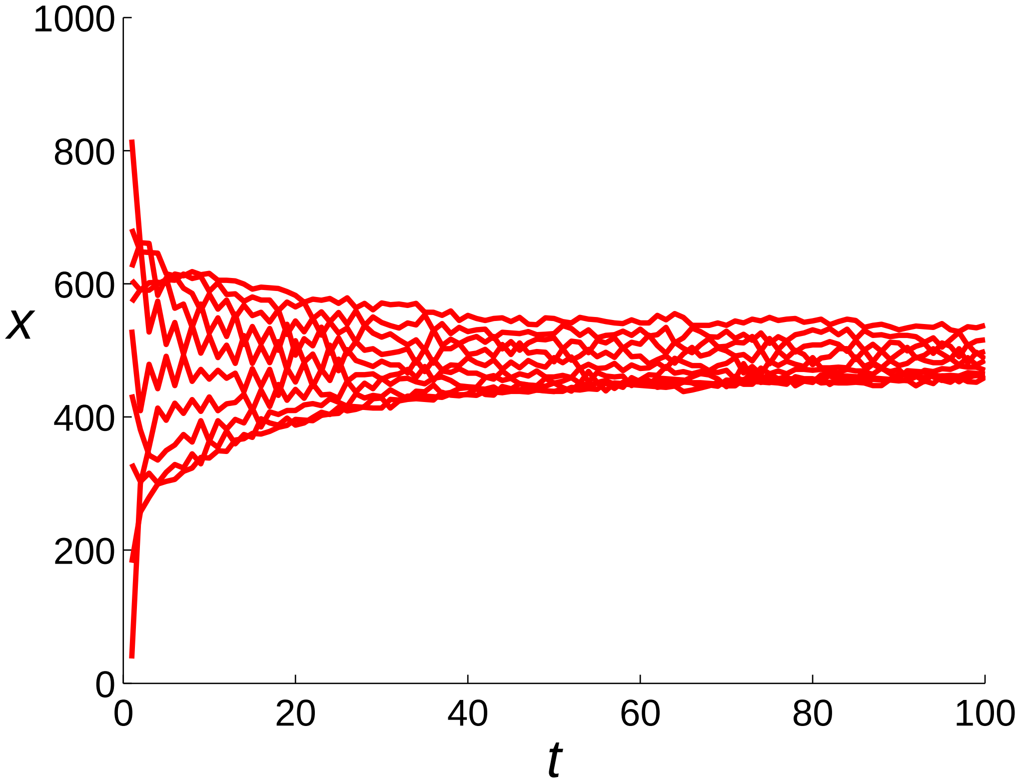

Figure 2 Consensus achievement in the system with step-size $\gamma = 0.5$ without incoming tasks.

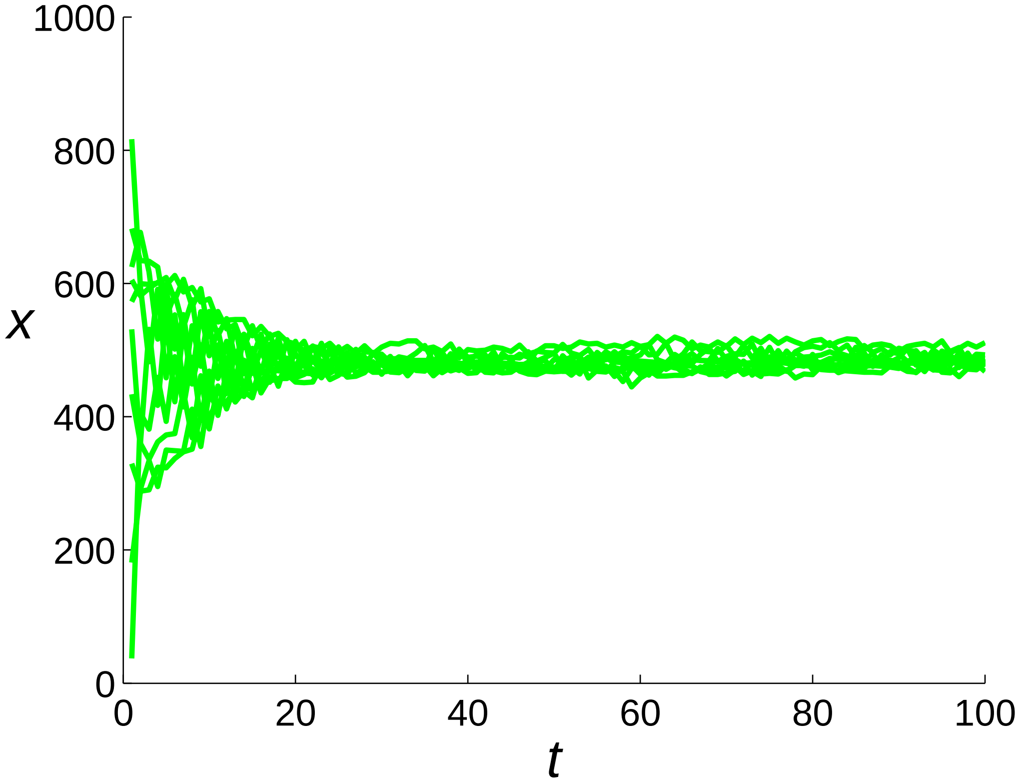

Figure 3 Consensus achievement in the system with step-size $\gamma = 0.7$ without incoming tasks.

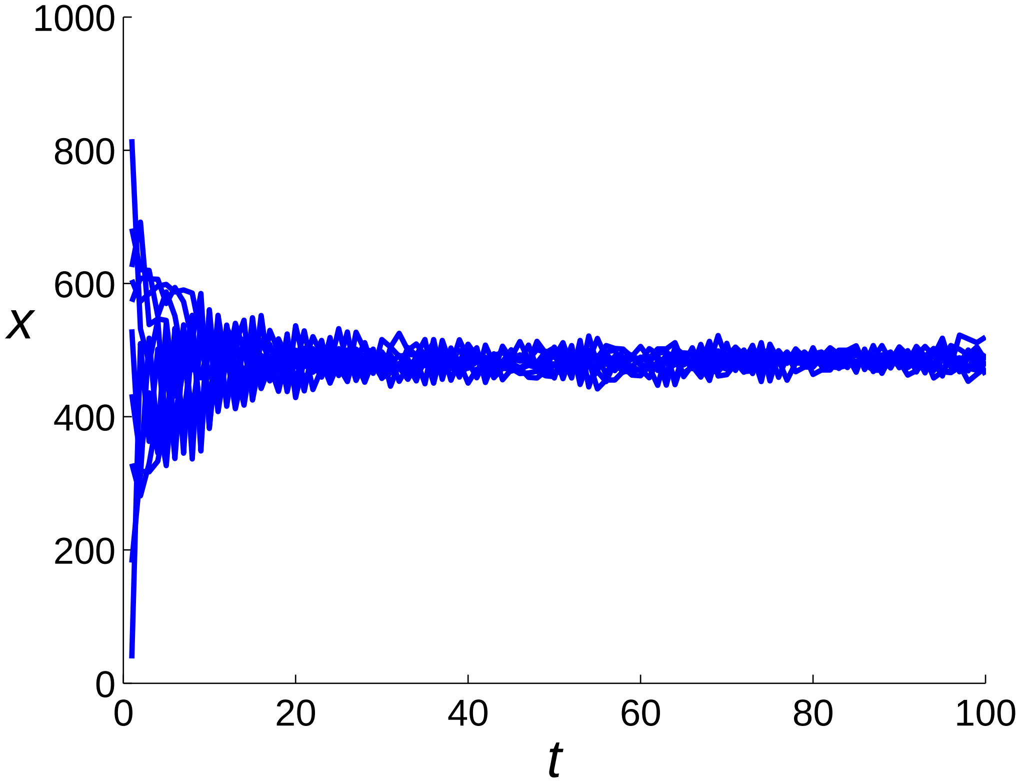

Figure 4 Consensus achievement in the system with step-size $\gamma = 0.9$ without incoming tasks.

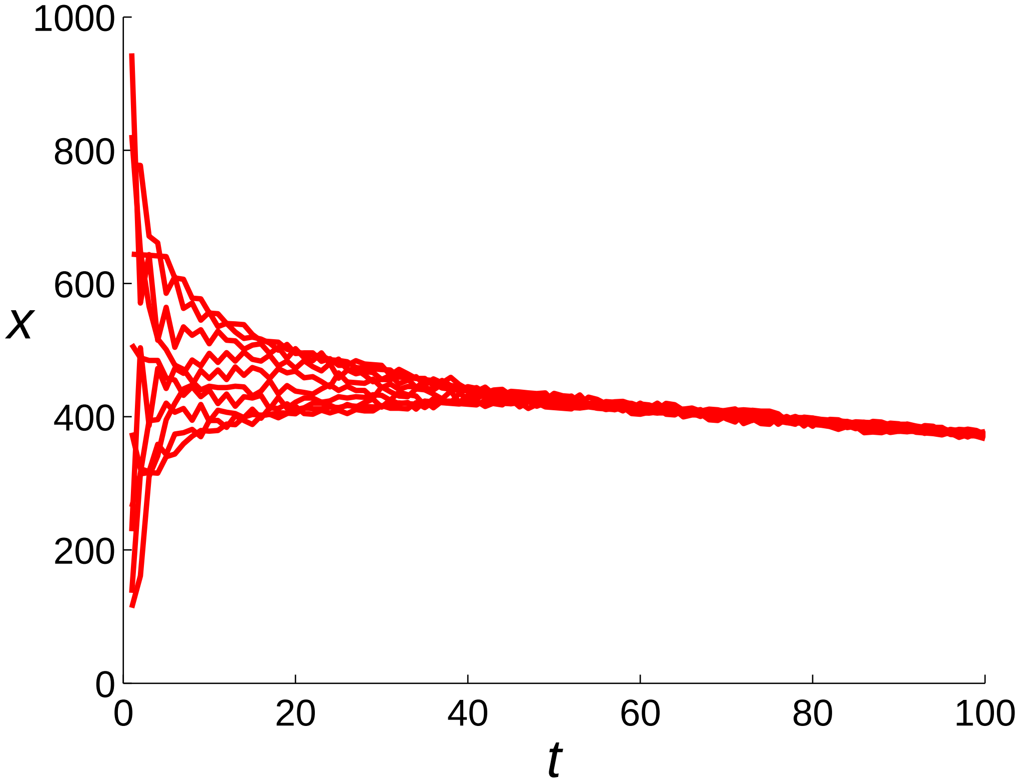

Figure 5 Consensus achievement in the system with step-size $\gamma = 0.5$ with incoming tasks.

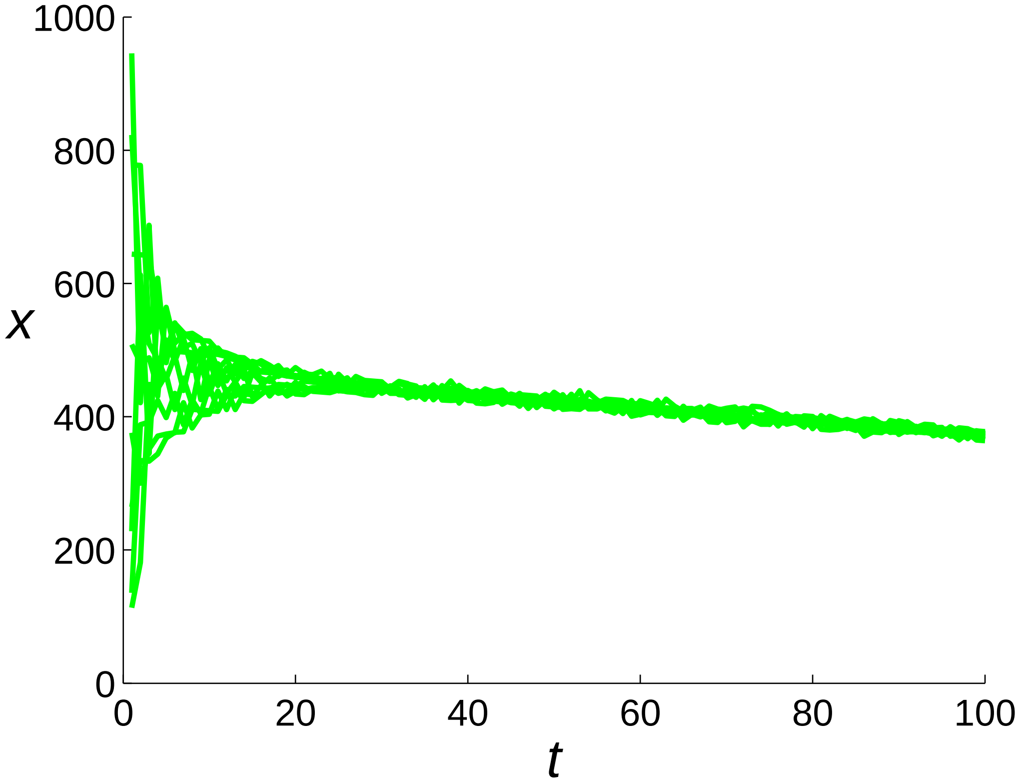

Figure 6 Consensus achievement in the system with step-size $\gamma = 0.7$ with incoming tasks.

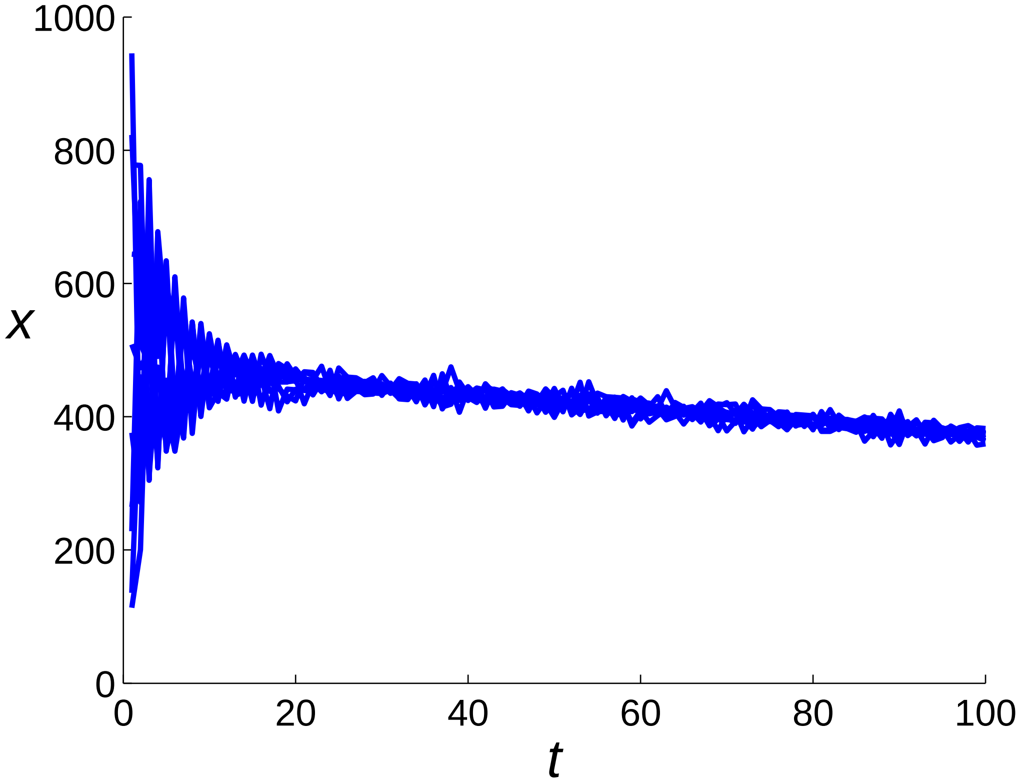

Figure 7 Consensus achievement in the system with step-size $\gamma = 0.9$ with incoming tasks.