A Distributed Optimization Strategy for Large Scale Oil and Gas Production Systems*

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***Abstract*— In this paper, we consider the problem of real-time optimization of an oil and gas production network. The production network is often made of several wells from different reservoir sections producing to a common processing facility. In this work, we exploit this spatial structure of the system and cast the production optimization problem into a distributed optimization problem and solve it using Lagrangian decomposition. We extend previous work on decomposition for static real-time production optimization problem by considering a dynamic real-time optimization problem. We demonstrate the applicability of the proposed method using a gas-lifted well network with shared resources and common capacity constraints.**

I. INTRODUCTION

The optimal operation of an oil and gas production network is a challenging task that has received considerable attention recently. In order to meet goals and objectives ranging from long-term decisions to small corrective actions, the decision making process is typically decomposed in time as described in [1]. Real-time production optimization (also known as daily production optimization) is a crucial step in maximizing the revenue of day-to-day operations of the production network. This typically involves making decisions such as allocating production target from different wells, allocation of shared resources, etc. Hence this is equivalent to real-time optimization from a process systems perspective.

The optimal decision variables can be computed by employing mathematical optimization. The authors in [2] reported production increase in the range of 1-4% by using mathematical optimization tools. Although the more traditional approach is to formulate the production optimization problem as a static problem, it was noted in [3] that many production networks may benefit from dynamic formulations of production optimization problem. By employing a dynamic optimization problem, the transients are also optimized, leading to more economical operation during the transients. In the face of volatile oil prices and higher recovery costs, optimizing during the transients may be desirable.

In this work, we consider such a dynamic production optimization problem. Recent works such as [4], [5] and [6] presented a dynamic optimization framework for oil and gas production networks. However, optimization of a large scale production network may be complex with several hundred decision variables. The computational cost of dynamic optimization problems has been a prohibitive factor for implementation in many oil and gas applications [7]. An oil and gas production network typically consists of several wells producing to a common production manifold, which takes the produced fluids to the processing facility. For example, the Troll oil field in the Norwegian continental shelf has more than one hundred subsea wells producing to a common offshore facility [8].

To this end, the wells are coupled by the processing facility. Due to the structure of the production network, it can be easily decomposed into several smaller subsystems, for example, into each well or cluster of wells. The authors in [9] presented Lagrangian decomposition approaches to solve the static production optimization problem for such a large scale production network by decomposing it into several smaller subsystems. The nonlinear steady-state model was described using a piecewise affine (PWA) model and was solved using mixed-integer framework. In this paper, we extend the work in [9] to dynamic production optimization problems. The system is modelled as a nonlinear dynamic model and is solved using as a nonlinear programming (NLP) problem.

The Lagrangian decomposition splits the problem into several smaller subproblems such that the subproblems can be solved in parallel, significantly reducing the computation time of the dynamic optimization problem. Besides reducing computational time and enabling parallelization, decomposing the optimization problem into several smaller systems also addresses other drawbacks of using a single centralized optimizer to control a large scale system [10].

- *Modelling Effort*- With a decentralized approach, it may be easier to maintain the models for smaller subsystems than for a large centralized optimizer. Modelling effort is often complimented with plant experimentation and system identification. As such, it is also easier to evaluate, identify and update fewer models for smaller subsystems. A decentralized approach fits well with the existing workflow of modelling in the upstream oil and gas industry.
- *Monitoring* - A centralized optimizer may be difficult to understand and monitor the performance due to

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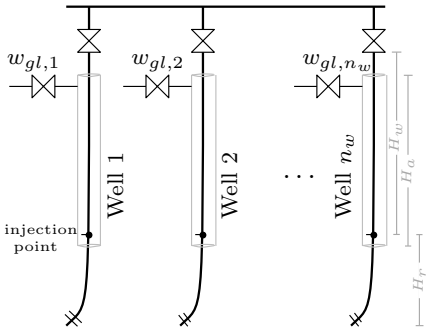

Fig. 1: Production network with gas lifted wells.

the complex interactions. Breaking it up into smaller subsystems makes the control structure more transparent and easier for the operators to understand. It also enables some subsystems to be turned off independently, instead of turning off the optimizer for the entire production network.

Operator understanding, ease of monitoring and maintenance and flexibility are important aspects in the success of such advanced control solutions [11], [10]. These are some of the important reasons to adopt a distributed controller approach rather than a centralized approach.

The main contribution of this paper is the extension of the Lagrangian decomposition framework in [9] to dynamic production optimization problems. The remainder of the paper is organized as follows. Section II describes the process and formalizes the problem statement. The Lagrangian decomposition framework is presented in Section III. Section IV presents the simulation results. Discussions are provided in Section V before concluding the paper in Section VI.

II. Problem Formulation

In this work we consider a network of gas lifted oil wells where n_w gas lifted wells are connected to a common production manifold as shown in Fig.1. In gas lifted wells, compressed gases are injected into the well tubing via the annulus to boost production. Injecting gas reduces the fluid mixture density in the well tubing resulting in reduced hydrostatic pressure drop and hence increased production. However injecting too much gas increases the frictional pressure drop which has a counteracting effect. Therefore, there is an optimum gas lift injection rate that increases the oil production. Additionally, the total gas processing capacity for the production network may be constrained or the total available gas for gas lift may be limited. The optimization problem thus involves computing the optimal allocation of gas lift among the different wells such that the total production is maximized while satisfying the process constraints. The gas-lifted wells are typically controlled using the gas lift injection rate which are the manipulated variables.

Each gas lifted well model is modelled with two control volumes, one for the annulus and the other for the well tubing. The models are made of four different parts, namely, (i) mass balance of the different phases, (ii) density models, (iii) pressure models and (iv) flow models [12]. The mass balances in each control volume for each phase is given by,

$$\dot{m}^i_{ga} = w^i_{gl} - w^i_{iv} \tag{1a}$$
$$\dot{m}^i_{gt} = w^i_{iv} - w^i_{pg} + w^i_{rg} \tag{1b}$$
$$\dot{m}^i_{ot} = w^i_{ro} - w^i_{po} \tag{1c}$$
$$\forall i \in \{i, \dots, n_w\}$$

where, m^i_{ga} is the mass of gas in the annulus. Note that the annulus is only filled with the lift gas. The mass of the gas phase and the oil phase inside the well tubing is denoted by m^i_{gt} and m^i_{ot} respectively. w^i_{gl} is the gas lift injection rate at the top of the annulus and the gas rate that actually flows from the annulus into the well tubing down hole is denoted by w^i_{iv}. The oil and gas production rates that comes out of the well is denoted by w^i_{pg} and w^i_{po} respectively and the oil and gas flow rate that enters the well tubing from the reservoir are denoted by w^i_{rg} and w^i_{ro} respectively. The superscript i refers to the i^{th} well.

We assume an isothermal system and the average temperature of the fluid in the annulus and the well tubing are assumed to be known. Ideal gas laws are used to compute the density of gas in the annulus and the fluid mixture density inside the well tubing. The density of gas in the annulus ρ^i_a and the fluid mixture density in the tubing ρ^i_m for each well i are given by:

$$\rho^i_a = \frac{M_w p^i_a}{T^i_a R}, \tag{2a}$$
$$\rho^i_m = \frac{m^i_{gt} + m^i_{ot} - \rho^i_o L^i_r A^i_r}{L^i_w A^i_w} \tag{2b}$$
$$\forall i \in \{i, \dots, n_w\}$$

where M_w is the molecular weight of the gas, R is the gas constant, T^i_a is the annulus temperature and the density of oil in the reservoir is denoted by ρ^i_o. L^i_r and L^i_w are the length of the well sections above and below the injection point respectively, and the corresponding cross-sectional areas are denoted by A_r and A_w respectively.

The annulus pressure p^i_a, wellhead pressure p^i_{wh}, well injection point pressure w^i_{iv} and the bottom hole pressure p^i_{bh} are given by:

$$p^i_a = \left(\frac{T^i_a R}{V^i_a M_w} + \frac{g L^i_a}{L^i_a A^i_a}\right) m^i_{ga}, \tag{3a}$$
$$p^i_{wh} = \frac{T^i_w R}{M_w}\left(\frac{m^i_{gt}}{L^i_w A^i_w + L^i_r A^i_r - \frac{m^i_{ot}}{\rho^i_o}}\right), \tag{3b}$$
$$p^i_{wi} = p^i_{wh} + \frac{g}{A^i_w L^i_w}(m^i_{ot} + m^i_{gt} - \rho^i_o L^i_r A^i_r) H^i_w, \tag{3c}$$
$$p^i_{bh} = p^i_{wi} + \rho^i_m g H^i_r \tag{3d}$$
$$\forall i \in \{i, \dots, n_w\}$$

where L^i_a and A^i_a are the length and cross sectional area of the annulus respectively. The average well temperature inside the well tubing is given by T^i_w. H^i_r and H^i_w are the vertical height of the well tubing below and above the injection point

respectively, and g is the acceleration of gravity constant. The cross-sectional area of the annulus and the tubing are computed using their respective diameters, D_a^i and D_w^i.

The flow through the downhole gas lift injection point w_{iv}^i is modelled as a flow through an orifice with no back-flow. The mixture flow through the wellhead production choke, w_{pc}^i is modelled using a choke model with no back-flow. Using the mass fractions, the individual produced gas and oil phase flow rates, w_{pg}^i and w_{po}^i are given. A simple linear inflow model is used to model the reservoir oil and gas flow rates, w_{ro}^i and w_{rg}^i are given by [13]:

$$w_{iv}^i = C_{iv}^i \sqrt{\rho_a^i \max(0, p_{ai}^i - p_{wi}^i)}, \tag{4a}$$

$$w_{pc}^i = C_{pc}^i \sqrt{\rho_w^i \max(0, p_{wh}^i - p_m^i)}, \tag{4b}$$

$$w_{pg}^i = \frac{m_{gt}^i}{m_{gt}^i + m_{ot}^i} w_{pc}^i, \tag{4c}$$

$$w_{po}^i = \frac{m_{ot}^i}{m_{gt}^i + m_{ot}^i} w_{pc}^i, \tag{4d}$$

$$w_{ro}^i = PI^i (p_r^i - p_{bh}^i), \tag{4e}$$

$$w_{rg}^i = GOR^i \cdot w_{ro}^i, \tag{4f}$$

$$\forall i \in \{i, \ldots, n_w\}$$

where C_{iv}^i and C_{pc}^i are the valve flow coefficients for the downhole injection orifice and the production choke, respectively. The reservoir productivity index PI^i and the reservoir pressure p_r^i describes the drawdown from the reservoir. GOR^i is the gas–oil ratio in the reservoir. Although the reservoir conditions change slowly over time, it is fair to assume that these parameters are constant in the timescale of short-term production optimization. The manifold pressure p_m^i is assumed to be tightly regulated to a constant pressure.

Each gas lifted well is modelled as an index-1 semi-explicit DAE system of the form,

$$\dot{\mathbf{x}}^i = f^i(\mathbf{x}^i, \mathbf{z}^i, \mathbf{u}^i) \tag{5a}$$

$$0 = g^i(\mathbf{x}^i, \mathbf{z}^i, \mathbf{u}^i) \tag{5b}$$

$$\forall i \in \{i, \ldots, n_w\}$$

where $\mathbf{x}^i \in \mathbb{R}^{n_x}$ and $\mathbf{z}^i \in \mathbb{R}^{n_z}$ are the differential and algebraic states respectively for each well $i \in \{1, \ldots, n_w\}$ and $\mathbf{u}^i \in \mathbb{R}^{n_u}$ is the control input. $f^i(\mathbf{x}^i, \mathbf{z}^i, \mathbf{u}^i)$ and $g^i(\mathbf{x}^i, \mathbf{z}^i, \mathbf{u}^i)$ are the set of differential and algebraic equations for each well i. For each well, the variables on the left hand side of equations (1a) form the differential states $\mathbf{x}^i$ and (2a)-(4a) form the algebraic states $\mathbf{z}^i$. As such there are $n_x = 3$ differential states, $n_z = 12$ algebraic states and $n_u = 1$ control input for each gas lifted well.

In the gas lift optimization problem, the stage cost $J_i : \mathbb{R}^{n_x} \times \mathbb{R}^{n_z} \times \mathbb{R}^{n_u} \to \mathbb{R}$ is expressed as

$$J_i(\mathbf{x}^i, \mathbf{z}^i, \mathbf{u}^i) := -c_o w_{po}^i + c_{gl} w_{gl}^i \tag{6}$$

where c_o and c_{gl} are the oil price and cost of gas lift compression respectively. w_{po}^i represents the oil production rate from well i and w_{gl}^i represents the gas lift injection rate for well i. The gas lifted well system also has some common resource constraints that couple the different subsystems together, which are enforced as nonlinear constraints.

$$\sum_{i=1}^{n_w} \left(\mathbf{G}^i(\mathbf{x}^i, \mathbf{z}^i, \mathbf{u}^i) \right) - r \leq 0 \tag{7}$$

This could be either in the form of gas capacity constraint where $r := w_g^{max}$ and (7) is expressed as

$$\sum_{i=1}^{n_w} \left(w_{pg}^i \right) - w_g^{max} \leq 0 \tag{8}$$

Alternatively the coupling constraint could also be due to the limited gas available for lifting that has to be optimally allocated among the wells. In this case, $r := w_{gl}^{max}$ and (7) is expressed as

$$\sum_{i=1}^{n_w} \left(w_{gl}^i \right) - w_{gl}^{max} \leq 0 \tag{9}$$

Before solving the infinite dimensional optimal control problem, the system dynamics are first discretized into a finite dimensional optimal control problem using a third order direct collocation, which gives a third order approximation of the system (5). The reader is referred to [12] for more detailed description. The resulting discretized system for each well i at any time step k can then be expressed as,

$$\mathbf{F}^i(\boldsymbol{\chi}_{k+1}^i, \boldsymbol{\chi}_k^i, \boldsymbol{\nu}_k^i, \boldsymbol{\zeta}_k^i) = 0 \tag{10}$$

where $\boldsymbol{\chi}$, $\boldsymbol{\nu}$ and $\boldsymbol{\zeta}$ are the discretized counterparts of the differential states $\mathbf{x}$, algebraic states $\mathbf{z}$ and the control inputs $\mathbf{u}$ respectively.

The centralized dynamic optimization problem can then be formalized as,

$$\boldsymbol{\Theta} := \min_{\boldsymbol{\chi}_k, \boldsymbol{\nu}_k, \boldsymbol{\zeta}_k} \quad \sum_{k=0}^{N} \sum_{i=1}^{n_w} J_i(\boldsymbol{\chi}_k^i, \boldsymbol{\nu}_k^i, \boldsymbol{\zeta}_k^i) \tag{11a}$$

s.t.: for all $i \in \{1, \ldots, n_w\}$, for all $k \in \{1, \ldots, N\}$:

$$\mathbf{F}^i(\boldsymbol{\chi}_{k+1}^i, \boldsymbol{\chi}_k^i, \boldsymbol{\nu}_k^i, \boldsymbol{\zeta}_k^i) = 0 \tag{11b}$$

$$\left(\boldsymbol{\chi}_k^i, \boldsymbol{\nu}_k^i, \boldsymbol{\zeta}_k^i\right) \in \mathcal{Z} \tag{11c}$$

$$\boldsymbol{\chi}_0^i = \mathbf{x}^i(t) \tag{11d}$$

$$\sum_{i=1}^{n_w} \left(\mathbf{G}^i(\boldsymbol{\chi}_k^i, \boldsymbol{\nu}_k^i, \boldsymbol{\zeta}_k^i) \right) - r \leq 0, \qquad \forall k \in \{1, \ldots, N\} \tag{11e}$$

$\mathcal{Z}$ represents the path constraints which are enforced in the form of bounds on the optimization variables (differential states, algebraic states and the control inputs),

$$\begin{pmatrix} \underline{\boldsymbol{\chi}}^i \\ \underline{\boldsymbol{\nu}}^i \\ \underline{\boldsymbol{\zeta}}^i \end{pmatrix} \leq \begin{pmatrix} \boldsymbol{\chi}_k^i \\ \boldsymbol{\nu}_k^i \\ \boldsymbol{\zeta}_k^i \end{pmatrix} \leq \begin{pmatrix} \overline{\boldsymbol{\chi}}^i \\ \overline{\boldsymbol{\nu}}^i \\ \overline{\boldsymbol{\zeta}}^i \end{pmatrix} \tag{12}$$

where the $\overline{(\cdot)}$ and $\underline{(\cdot)}$ represents the upper and lower bounds, respectively. (11d) enforces the initial states $\boldsymbol{\chi}_0^i$ to be equal to the current measured/estimated state $\mathbf{x}^i(t)$ at time t.

The centralized problem can be solved by repeatedly solving (11) at each sampling time k to compute the optimal

input trajectory $\boldsymbol{\zeta}^*_{[k,k+N]}$ over the prediction horizon. The first step of the optimal control sequence is implemented on the plant, i.e. $\mathbf{u}^*_k = \boldsymbol{\zeta}^*_k$.

III. Decomposition method

It can be seen clearly from the optimization problem (11) that it is easily separable for each well i except for the coupling constraint (11e). There are many different approaches to decomposing a system with coupling constraints which are broadly categorized into primal and dual approaches [14]. In this work, we consider a dual decomposition approach, also known as Lagrangian decomposition [15].

The basic idea behind Lagrangian decomposition is to solve the dual problem obtained by relaxing the coupling constraint (11e). A master problem then manipulates the dual variables to co-ordinate the subproblems. To illustrate this, consider the centralized optimization problem (11) and let the optimization variables be represented as

$$\boldsymbol{X}_k = \begin{bmatrix} \boldsymbol{\chi}_k^{i^\top} & \boldsymbol{\nu}_k^{i^\top} & \boldsymbol{\zeta}_k^{i^\top} \end{bmatrix}^\top \tag{13}$$

The Lagrangian function for (11) can be written as,

$$\begin{aligned} \mathcal{L}(\boldsymbol{X}, \boldsymbol{\mu}, \boldsymbol{\lambda}) = \sum_{k=1}^{N} \Bigg[\sum_{i=1}^{n_w} J_i(\boldsymbol{X}_k^i) + \sum_{i=1}^{n_w} \left(\boldsymbol{\mu}_k^i c^i(\boldsymbol{X}_k^i) \right) \\ + \boldsymbol{\lambda}_k \left(\sum_{i=1}^{n_w} \left(\mathbf{G}^i(\boldsymbol{X}^i) \right) - r \right) \Bigg] \\ \forall i \in \{1, \dots, n_w\}, k \in \{1, \dots, N\} \end{aligned} \tag{14}$$

where $\boldsymbol{\mu}_k^i$ represents the Lagrange multiplier for the constraints (11b),(11c) and (11d) collectively represented as $c^i(\boldsymbol{X}_k^i)$ which are unique to each subsystem i and do not have any impact on the other subsystems. $\boldsymbol{\lambda}_k$ represents the Lagrange multiplier for the coupling constraint (11e) which are common for all the subproblems. Notice that since we consider dynamic optimization over a prediction horizon of $[k, k+N]$, the Lagrange multipliers are also a function of time and hence are given a subscript k.

It is evident that the Lagrange function (14) is now additively separable for each subsystem. Hence the problem may be easily decomposed by assigning prices of the shared resources, and adding the cost of the resources to the objective function of each subproblem. In other words, $\boldsymbol{\lambda}_k$ is the price variable and the n_w subproblems are solved by fixing $\boldsymbol{\lambda}_k$. The resulting optimal solution from each subproblem is then used to assess the constraint in the master problem and iteratively update $\boldsymbol{\lambda}_k$.

Each decomposed subproblem using Lagrangian relaxation can be expressed as,

$$\begin{aligned} \mathcal{P}_i(\boldsymbol{\lambda}_k) := \min_{\boldsymbol{X}^i, \boldsymbol{\mu}^i} \sum_{k=1}^{N} \Big[J_i(\boldsymbol{X}_k^i) + \boldsymbol{\mu}_k^i \left(c^i(\boldsymbol{X}_k^i) \right) \\ + \boldsymbol{\lambda}_k \left(\mathbf{G}^i(\boldsymbol{X}^i) - \frac{r}{n_w} \right) \Big] \\ \forall i \in \{1, \dots, n_w\} \end{aligned} \tag{15}$$

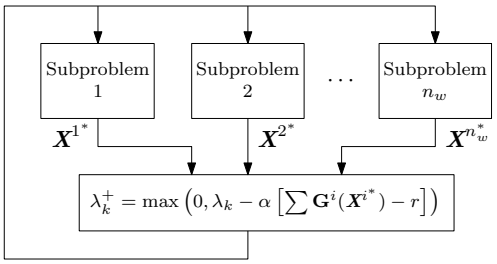

Fig. 2: Schematic representation of the dual decomposition approach.

The optimal solution $\boldsymbol{X}^{i^*}_{[k,k+N]}$ provided by each of the subproblem is then used in the master problem to update the dual variable $\boldsymbol{\lambda}_k$ for the next iteration.

Lemma 1: If $\boldsymbol{X}^{i^*}_{[k,k+N]}$ is the optimal solution computed by each subproblem by fixing $\boldsymbol{\lambda}_k$, the solution to the master problem is then given by the descent step,

$$\begin{aligned} \boldsymbol{\lambda}_k^+ = \max\left(0, \boldsymbol{\lambda}_k + \alpha \left[\sum_{i=1}^{n_w} \left(\mathbf{G}^i(\boldsymbol{X}_k^{i^*}) \right) - r \right] \right) \\ \forall k \in \{1, \dots, N\} \end{aligned} \tag{16}$$

where $\boldsymbol{\lambda}_k^+$ represents the updated $\boldsymbol{\lambda}_k$ for the next iteration and α is any suitable step length size.

Proof: The original problem (11) is equivalent to the dual master problem,

$$\boldsymbol{\Phi} := \max_{\boldsymbol{\lambda}} \sum_{i=1}^{n_w} \mathcal{P}_i(\boldsymbol{\lambda}_k) \tag{17}$$

Assuming that $\boldsymbol{\Phi}$ is differentiable, a steepest-ascent algorithm can be used to solve the dual problem. The descent direction $\mathbf{p}$ for the master problem is then given by, [16]

$$\begin{aligned} \mathbf{p} &= -\nabla_{\boldsymbol{\lambda}} \boldsymbol{\Phi} \\ &\Rightarrow -\sum_{i=1}^{n_w} \left(\mathbf{G}^i(\boldsymbol{X}^{i^*}) - r \right) \end{aligned} \tag{18}$$

The solution to the master problem is then along the descent direction with a suitable step size α.

$$\boldsymbol{\lambda}_k^+ = \max\left[0, (\boldsymbol{\lambda}_k + \alpha \mathbf{p})\right] \qquad \forall k \in \{1, \dots, N\} \tag{19}$$

Since the coupling constraint is an inequality constraint, the dual feasibility from Karush-Kuhn-Tucker(KKT) conditions dictate that the Lagrange multiplier must be non-negative at all times ($\boldsymbol{\lambda}_k \geq 0$). ■

The primal subproblems and the dual master problem is solved iteratively until the Lagrange multipliers converge from an initial set of multipliers. Upon convergence, the relaxed coupling constraints in the subproblems will become feasible. A sketch of the Lagrange decomposition method is summarised in Algorithm 1.

IV. Simulation Results

In this paper, we consider a production network with 2 gas lifted well, i.e. $n_w = 2$. The dynamic optimization problem was setup as a nonlinear programming problem in `CasADi`

Algorithm 1 Lagrangian decomposition

Define tolerance $\epsilon > 0$ and $\Delta\lambda > \epsilon$.
Input: At each time step t initial state $\mathbf{x}_t^i$, initial $\boldsymbol{\lambda}^0$.
  while $\Delta\lambda > \epsilon$ **do**
    for i = $1, 2, \ldots, n_w$ **do**
      $\left[\boldsymbol{X}^{i^*}, \boldsymbol{\mu}^{i^*}\right] \leftarrow$ solution $\mathcal{P}_i(\boldsymbol{\lambda})$.
    end for
    Compute gradient $\mathbf{p} \leftarrow -\sum_{i=1}^{n_w} \left(\mathbf{G}^i(\boldsymbol{X}^{i^*}) - r\right)$.
    Update Lagrange multiplier $\boldsymbol{\lambda}_k^+ \leftarrow \max\left[0, \boldsymbol{\lambda}_k + \alpha\mathbf{p}\right]$
    Update $\Delta\lambda \leftarrow \|\boldsymbol{\lambda}_k^+ - \boldsymbol{\lambda}_k\|$
  end while
  Reset $\Delta\lambda > \epsilon$
  $\boldsymbol{\lambda}^0 = \boldsymbol{\lambda}^+$
Output: $\boldsymbol{X}^{i^*}$

`v3.0.1-rc1` [17] using `MATLAB 2017a` programming environment. The resulting NLP problem is then solved using `IPOPT` version 3.12.2 [18] running with a mumps linear solver. The plant simulator was solved using `IDAS` integrator.

The dynamic optimization problem for the centralized approach was solved over a prediction horizon of $N = 60$ samples with a sample time of $T_s = 300$s.

For the distributed optimization, the two primal subproblems were posed as dynamic optimization problem and solved with the same sampling times and prediction horizon. The dual master problem was posed as a gradient step update with a fixed step length $\alpha = 10$. At each time step, the tolerance was reset to $\Delta\lambda = 1$. A tolerance limit of $\epsilon = 0.1$ was chosen as the stopping criteria for master and subproblem iterations. Upon convergence of the dual variable $\boldsymbol{\lambda}_k$, the coupling constraints will become feasible. The optimal solution is implemented on the plant using a receding horizon principle. The first sample of the computed optimal input trajectory is implemented on the plant. At the next time step, when new measurements are available, the optimization problem is solved again to compute a new optimal input trajectory. The closed-loop implementation incorporates the inherent feedback law $\mathbf{u} = \mathcal{K}(\mathbf{x})$.

A. Limited gas lift constraint

In the first part of the simulation, we assume that the coupling constraints arise from the limited gas supply available for gas lift injection. In this case, the limited gas lift injection rate of $w_{gl}^{max} = 3$kg/s must be optimally allocated among the wells to maximize oil production. This problem was first solved using the centralized approach. The problem was then solved using the Lagrangian decomposition framework described in Section III and compared with the centralized approach. The closed loop simulation results are shown in Fig.3. It can be clearly seen that the solution provided by the decomposed problem accurately tracks the solution provided by the centralized optimizer. In terms of the CPU times, the centralized approach on average required 0.51s as opposed to the distributed approach, where each subproblem on average was computed within 0.30s.

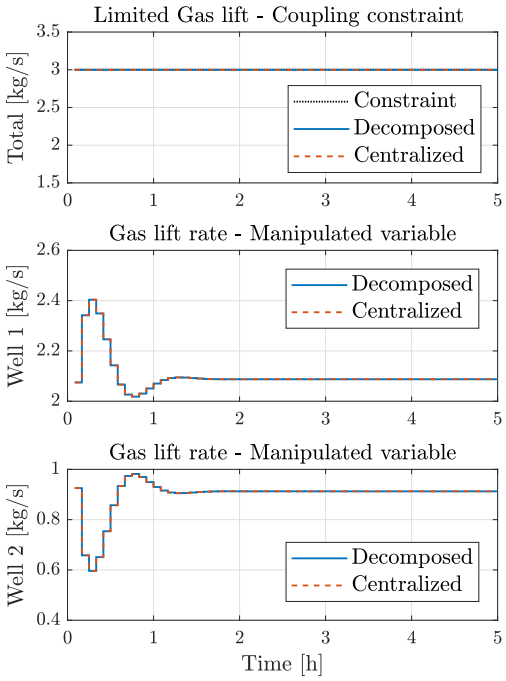

Fig. 3: Simulation results comparing the centralized and decomposed approach with limited gas lift as the coupling constraint.

B. Gas capacity constraint

In the second part of the simulation, we assume the coupling constraints arise from the total gas processing capacity downstream. In this case, the total produced gas from the different wells must be less than the maximum processing capacity of $w_g^{max} = 8$kg/s. This problem was first solved using the centralized approach. The problem was then solved using the Lagrangian decomposition framework described in Section III and compared with the centralized approach. The closed loop simulation results are shown in Fig.4. It can be clearly seen that the solution provided by the decomposed problem accurately tracks the solution provided by the centralized optimizer. In terms of the CPU times, the centralized approach on average required 0.92s as opposed to the distributed approach, where each subproblem on average was computed within 0.29s.

V. Discussion

The simulation results in the previous section showed that the Lagrangian decomposition method is able provide equivalent results compared to solving the problem as a centralized large-scale optimization problem. It may be noted that the accuracy of the decomposed method for the simulations presented in Fig.4 may be improved by further reducing the tolerance ϵ.

To speed up the convergence of the Lagrange multipliers "warm-starting" was used. At time $t = 0$, $\boldsymbol{\lambda}_k = 20$ was

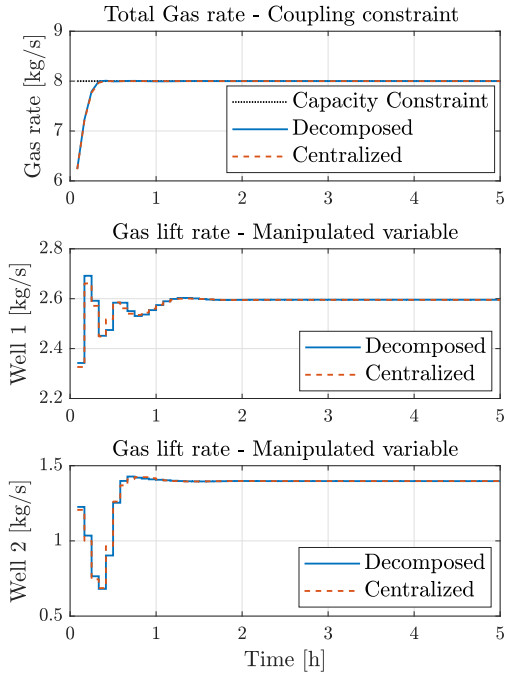

Fig. 4: Simulation results comparing the centralized and decomposed approach with total gas processing capacity as the coupling constraint.

chosen for all $k \in \{1, \ldots, N\}$. At each consecutive time $t > 0$, The $\boldsymbol{\lambda}_k$-update was initialized with the $\boldsymbol{\lambda}_k$ obtained in the previous time step. Warm-starting often results in far fewer iterations to converge [19].

Lagrangian decomposition methods can also be further improved by using an augmented Lagrangian decomposition algorithm, where an additional quadratic penalty term is added to the objective term. However, the improved convergence of the augmented Lagrangian method comes at a cost. The augmented Lagrangian function is no longer separable. The subproblems cannot be solved in parallel, but must be solved sequentially using the alternating directions method of multipliers [19].

Another future research direction to improve the performance could be to use parametric optimization, where the subproblems can be parameterized with respect to the dual variable λ. At each iteration, when the master problem updates λ, the subproblems can then be solved using a predictor-corrector QP instead of solving a nonlinear programming problem (NLP) [20],[21].

Although the simulation results were presented for a two well network, the decomposition framework can be easily extended to any number of wells. Different wells may also be clustered together in the subproblems, for example wells from each subsea template may be grouped together [9].

VI. Conclusion

In this paper, we showed that the structure of large scale oil and gas production networks can be exploited to decompose it into several smaller subproblems that can be parallelized. Lagrangian decomposition method was presented to solve the resulting distributed dynamic optimization problem. A gas lifted production model was used to demonstrate the proposed method. Using simulations we showed that the Lagrangian decomposition method is able to provide the same solution as the centralized optimization problem.

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