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Abstract

Recently, the authors in this paper proposed a correction function to determine material's equivalent stress-strain curve with axisymmetric-notched tensile specimens. In this study, tensile tests were performed at room temperature, -30°C and -60°C with axisymmetric notched tensile specimens to verify this method and to identify the equivalent stress-strain curves of a 420 MPa structural steel. A high-speed camera was used together with the so-called edge-tracing method to calculate average true strain. The material's equivalent stress-strain curve was also measured with extensometer and smooth round bar specimens. Experimental results show that equivalent stress-strain curve of this structural steel is sensitive to test temperature. Equivalent stress-stress curves obtained from axisymmetric notched tensile specimens by using the proposed correction function show good agreement with those from extensometer before diffuse necking and from Bridgman correction at large strain using smooth tensile specimens. Since fracture strain strongly depends on the notch geometry, it is recommended to use axisymmetric notched tensile specimens with smaller when applying the proposed correction function to measure material's equivalent stress-strain curve.

Keywords	equivalent stress-strain curve; low temperature; axisymmetric notched tensile specimen; Bridgman correction; large strain.
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Highlights

- A newly proposed correction function for deriving equivalent stress-strain curve with axisymmetric notched tensile specimens was verified experimentally.
- Significant temperature effect on the equivalent stress-strain curves was observed.
- Results obtained with the proposed correction method show good agreement with the well-known Bridgman correction at large strain.

Experimental measurement of temperature-dependent equivalent stress-strain curves of a 420 MPa structural steel with axisymmetric notched tensile specimens

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Nomenclature

a	current minimum cross-section radius
a_0	initial minimum cross-section radius
A	current minimum cross-section area
d_0	specimen outer diameter
E	Young's modulus
H	material zone height in the notch
P	tensile load
R	current notch curvature radius
R_0	initial notch curvature radius
a/R	current notch radius ratio
a_0/R_0	initial notch radius ratio
T	stress triaxiality
ε	average true strain
ε'	engineering strain
$\varepsilon_{P_{\max}}$	strain at the maximum load
ξ	correction factor for axisymmetric notched tensile specimen
ξ_B	Bridgman correction factor
σ_0	yield stress
σ'	engineering stress
σ_{eq}	von Mises equivalent stress
σ_T	true stress

1 Experimental measurement of temperature-dependent equivalent stress- 2 strain curves of a 420 MPa structural steel with axisymmetric notched 3 tensile specimens

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8 Abstract

9 Recently, we proposed a correction function to determine material's equivalent stress-strain curve with
10 axisymmetric notched tensile specimens. In this study, we performed tensile tests at room temperature,
11 -30°C and -60°C with axisymmetric notched tensile specimens to verify this method and to identify the
12 equivalent stress-strain curves of a 420 MPa structural steel. A high-speed camera was used together
13 with the so-called edge-tracing method to calculate average true strain. We also measured the material's
14 equivalent stress-strain curve with extensometer and smooth round bar specimens. Experimental results
15 show that equivalent stress-strain curve of this structural steel is sensitive to test temperature. Equivalent
16 stress-strain curves obtained from axisymmetric notched tensile specimens by using the proposed
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18 Bridgman correction at large strain using smooth tensile specimens. Since fracture strain strongly
19 depends on the notch geometry, it is recommended to use axisymmetric notched tensile specimens with
20 smaller a_0/R_0 when applying the proposed correction function to measure material's equivalent stress-
21 strain curve.

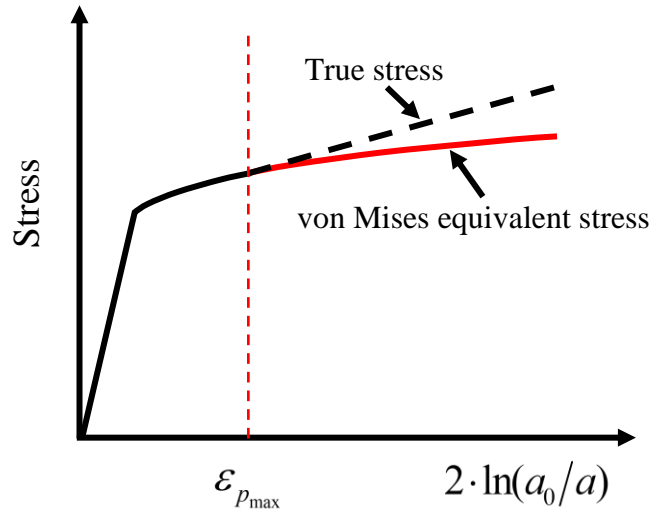
22 **Keywords:** *equivalent stress-strain curve; low temperature; axisymmetric notched tensile specimen;*
23 *Bridgman correction; large strain.*

24

25 1. Introduction

26 Identifying material's equivalent stress-strain curve in large strain is very important for large
27 deformation analysis, such as plastic forming [1, 2], ductile fracture analysis with finite element method
28 [3-8]. Usually, we use smooth round bar specimen [9, 10] or smooth specimen with rectangular cross-
29 section [11-13] to measure material's equivalent stress-strain curve with extensometer. The limitation
30 of such method is that only the data before diffuse necking can be used directly. There are several
31 methods to determine material's true stress-strain curve in large range of strain. For thick materials,
32 smooth round bar specimen can be used when the instantaneous minimum cross-section area is measured.
33 The strain ε is then characterized by the specimen minimum cross-section area reduction:

34 $\varepsilon = 2\ln(a_0/a)$. a_0 and a are the specimen initial and current minimum cross-section radius,
 35 respectively. The true stress or axial average stress σ_T is calculated by dividing the load P by the
 36 instantaneous minimum cross-section area. For very thin plate material, Zhang [14] proposed a method
 37 to calculate the post-necking minimum cross-section area of rectangular cross-section specimens, as a
 38 function of specimen thickness reduction. With Zhang's method, true stress-strain curves from flat
 39 tensile specimens can be obtained at large strain. It should be noted that after diffuse necking, tri-axial
 40 stress state occurs in the necked region. The true stress differs with von Mises equivalent stress σ_{eq} [9,
 41 15] and should be corrected. Fig. 1 schematically presents the difference of the true stress and von Mises
 42 equivalent stress after diffuse necking.



43
 44 *Fig. 1 Illustration of the difference between true stress and von Mises equivalent stress for tensile test*
 45 *with smooth round bar specimen after diffuse necking ($\varepsilon > \varepsilon_{p_{max}}$).*

46 Diffuse necking occurs after the maximum tensile load, hence the true stress should be corrected when
 47 the strain is larger than the strain corresponding to the maximum tensile load, $\varepsilon_{p_{max}}$. Bridgman [9]
 48 performed analytical analysis with necked round bar specimen and proposed a correction factor ξ_B :

$$49 \quad \xi_B = (1 + 2R/a) \cdot \ln(1 + a/2R) \quad (1)$$

$$\sigma_{eq} = \sigma_T / \xi_B$$

50 where R is the neck curvature radius. By dividing the true stress in Fig.1 by ξ_B , the material's equivalent
 51 stress can be calculated. Indeed, R is very difficult to measure accurately. Le Roy [16] proposed an
 52 empirical formula to calculate the notch curvature radius ratio a/R :

$$53 \quad a/R = 1.1 \cdot (\varepsilon - \varepsilon_{p_{max}}) \quad (2)$$

54 Combined with Eq. (1) – (2), true stress-strain curve from a smooth round bar specimen can be converted
 55 to material's equivalent stress-strain curve after diffuse necking. The Bridgman correction factor ξ_B

56 works well at strain slightly larger than $\varepsilon_{P_{\max}}$. As the strain further increases, errors between the
57 material's equivalent stress and the Bridgman corrected equivalent stress occurs and increases with the
58 increase of strain [15]. The errors range from several percentages to more than 10% [15, 17]. Recent
59 numerical analyses [18-20] show that the stress distribution at the necked specimen minimum cross-
60 section differs significantly with Bridgman's analytical solution. These errors are mainly attributed to
61 the assumption that the equivalent strain is uniform in the specimen minimum cross-section. Similar to
62 the Bridgman method, several other correction methods have been proposed [21]. The main difference
63 of these methods is the determination of the curvature radius of the longitudinal stress trajectories.
64 Though the Bridgman correction method is not very accurate when the strain is large, it still can be used
65 as reference. Ling [22] proposed a so-called weighted average method to measure the true stress-strain
66 curve from rectangular cross-section specimen, by setting the power law hardening as lower bound and
67 the linear hardening as the upper bound for the equivalent stress. The correction proposed by Ling is a
68 kind of hybrid experimental-numerical modeling method and the determination of the weight constant
69 is time consuming. Scheider [23] proposed a correction factor as a function of strain and $\varepsilon_{P_{\max}}$ to derive
70 equivalent stress-strain curve with flat tensile specimen. However, Scheider's method can only be used
71 for specimens with the aspect ratio of 1:4. Choung [24, 25] also proposed a method to measure equivalent
72 stress-strain curves with flat tensile specimens. The minimum cross-section area should be measured
73 manually with digital calipers and a micrometer. It is worth noting that both Shceider [23] and Choung's
74 [24, 25] method are based on inverse numerical analyses.

75
76 To measure the true stress-strain curve of each individual material zone in a weldment, Zhang [26]
77 proposed a correction function, with which the true stress-strain curve from an axisymmetric notched
78 tensile specimen can be converted to the corresponding one from a smooth round bar specimen. This
79 method is not accurate at large strain, but lay a foundation for our recent work [27, 28]. With further
80 numerical studies, we identified a 'magic' axisymmetric notched tensile specimen [28]. With only one
81 single correction factor, true stress-strain curve from the 'magic' notched specimen can be converted to
82 material's equivalent stress-strain curve in a large range of strain accurately, and no Bridgman correction
83 is needed. The limitation is that failure strain of this 'magic' notched specimen can be much smaller than
84 that from a smooth round bar specimen, sometimes.

85
86 Recently, we found a new correction function to determine material's equivalent stress-strain with 'any'
87 axisymmetric notched tensile specimens [27]. The correction function can be used to the perfectly plastic
88 material and hardening material, and also to weldments. In this study, we performed tensile tests at room
89 temperature, -30 °C and -60 °C with axisymmetric notched tensile specimens machined from a 420 MPa
90 structural steel plates to verify the proposed correction method. The correction function is introduced in

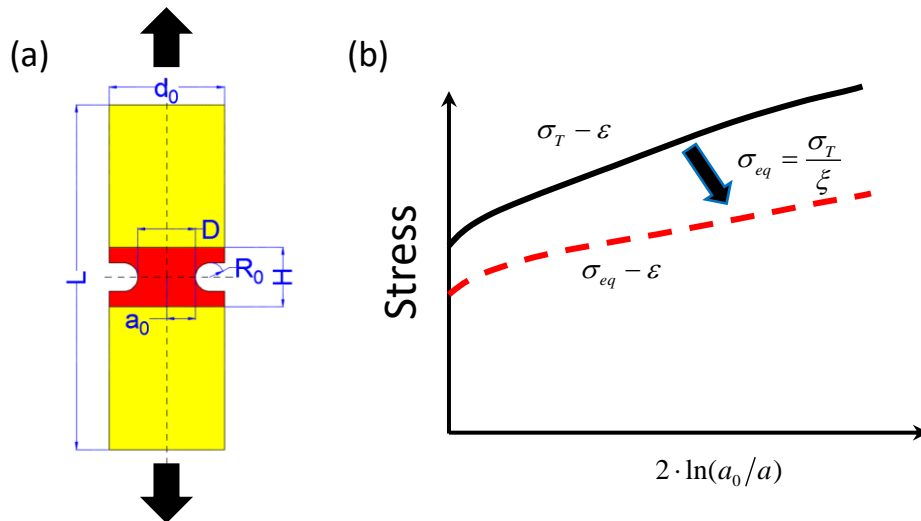
91 detail in section 2. The experimental procedure is presented in section 3. We also measured the material's
 92 equivalent stress-strain curve with extensometer and smooth round bar specimens. Before diffuse
 93 necking, the equivalent stress-strain curves from axisymmetric notched tensile specimens are compared
 94 with those from extensometer. With Eq. (1)-(2), we also performed Bridgman correction with smooth
 95 round bar specimen to obtain reference equivalent stress-strain curves after diffuse necking. Results and
 96 discussions are presented in section 4. The equivalent stress-strain curves are then verified by numerical
 97 analyses in section 5. Main conclusions are presented in section 6.

98 2. Axisymmetric notched tensile specimen method

99 Axisymmetric notched tensile specimen has been widely applied in characterizing material's mechanical
 100 properties [29-31]. For inhomogeneous material, such as weldment, it is impossible to measure the
 101 equivalent stress-strain curve in a targeted material zone with cross-weld smooth round bar specimen or
 102 flat tensile specimen, due to the nature of unpredictable fracture position. By introducing an
 103 axisymmetric notch on the smooth round bar specimen, the deformation is restrained mainly in the
 104 notched region under uniaxial tensile loading [26-28]. Fig. 2 (a) schematically shows the geometry
 105 information of the axisymmetric notched tensile specimen. Similar with smooth round bar specimen, the
 106 strain is defined by the minimum cross-section area reduction and the true stress is calculated by dividing
 107 load by the current minimum cross-section area:

$$108 \quad \varepsilon = 2 \cdot \ln(a_0/a) \quad (3)$$

$$109 \quad \sigma_T = P/\pi a^2 \quad (4)$$



110
 111 *Fig. 2 (a) Geometry of axisymmetric notched tensile specimen. The yellow part can be overmatch,*
 112 *under match or even match with the remain part of the specimen. (b) Conversion of true stress-strain*
 113 *curve from notched specimen to equivalent stress-strain curve by the proposed correction function.*

114

115 Stress concentration occurs due to the existence of the notch. True stress-strain curve from an
 116 axisymmetric notched tensile specimen differs significantly with the material's equivalent stress-strain
 117 curve and should be corrected. Our previous study [28] shows that when the specimen geometry
 118 requirement $d_0 \geq 3.5a_0$ is fulfilled, true stress-stress curves from axisymmetric notched tensile
 119 specimens with the same initial notch radius ratio a_0/R_0 are identical for homogeneous materials. This
 120 is true for inhomogeneous material when a_0 is smaller than the material zone length: $a_0 \leq H$.

121

122 Recently, we proposed a correction function to convert the true stress-strain curve from any
 123 axisymmetric notched tensile specimens to the material's equivalent stress-strain curve [27]. The
 124 correction function is written in a general form:

$$125 \quad \xi = g_{a_0/R_0, n=0}(\varepsilon) \cdot f(\varepsilon_{P_{\max}}) \quad (5)$$

126 Eq. (5) consists of two parts: the first part describes the notch effect on the true stress-strain curves of
 127 the perfectly-plastic material, and displays as a linear function of the true strain ε , Eq. (6). The slope,
 128 $b_{1, n=0}$, in Eq. (6) depicts the initial notch geometry effect on the evolution of true stress-strain curve from
 129 axisymmetric notched tensile specimen. While the intersection, $b_{2, n=0}$, can be explained as the notch
 130 induced stress concentration, sharper notch yields higher stress concentration. The slope and intersection
 131 are given in Eq. (7) and Eq. (8) as a function of the initial notch radius ratio, respectively. The second
 132 part, as shown in Eq. (9), is a function of $\varepsilon_{P_{\max}}$, describing the effect of strain hardening on the true
 133 stress-strain curve of a notched specimen.

$$134 \quad g_{a_0/R_0, n=0}(\varepsilon) = (b_{1, n=0} \cdot \varepsilon + b_{2, n=0})_{a_0/R_0} \quad (6)$$

$$135 \quad b_{1, n=0} = 0.03232 \cdot (a_0/R_0)^2 - 0.27 \cdot (a_0/R_0) + 0.3866 \quad (7)$$

$$136 \quad b_{2, n=0} = -0.04084 \cdot (a_0/R_0)^2 + 0.3557 \cdot (a_0/R_0) + 1.0577 \quad (8)$$

$$137 \quad f(\varepsilon_{P_{\max}}) = -0.22942 \cdot \varepsilon_{P_{\max}}^2 - 0.36902 \cdot \varepsilon_{P_{\max}} + 1 \quad (9)$$

138 When ε and $\varepsilon_{P_{\max}}$ are known, the $\sigma_T - \varepsilon$ curve from an axisymmetric notched tensile specimen can be
 139 converted to the material's equivalent stress-strain curve by Eq. (10), as demonstrated in Fig. 2 (b).
 140 Details about the derivation of this correction function can be referred to ref. [27].

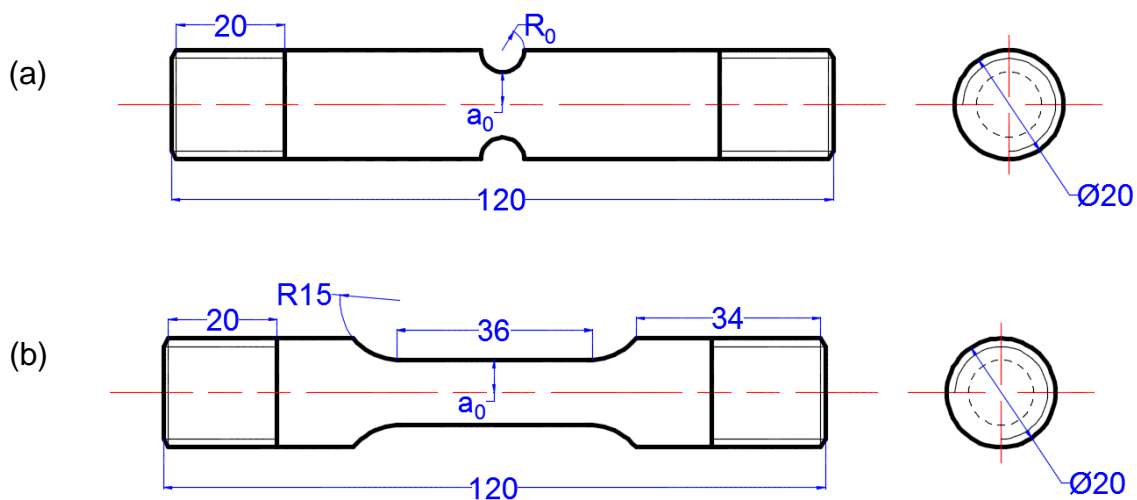
$$141 \quad \sigma_{eq} = \sigma_T / \xi \quad (10)$$

142 3. Experiment procedure

143 To experimentally verify the correction function, we conducted tensile tests with smooth round bar
144 specimens and axisymmetric notched tensile specimens with initial notch radius ratio, a_0/R_0 , ranging
145 from 0.5 to 3. The specimens were machined from 50 mm thick plates of a 420 MPa steel, along the
146 rolling direction. The specimen configurations are shown in Fig. 3. Our previous numerical studies
147 provide a conservative geometry requirement for axisymmetric notched tensile specimens: $d_0 \geq 3.5a_0$.
148 d_0 is the specimen outer diameter, as seen in Fig. 2. In this study, $a_0 = 6$ mm and $d_0 = 20$ mm. The
149 specimen outer diameter is 1 mm smaller than the geometry requirement ($d_0 \geq 3.5a_0$). In order to
150 guarantee that the specimen geometry can be used, we simply performed numerical analysis with power-
151 law hardening material and found that the correction function was still valid.

152
153 The tests were carried out at room temperature, -30°C, and -60°C using a universal test machine Instron
154 5985, with the loading cell of 250 KN. A liquid nitrogen-cooled temperature chamber was used to create
155 low temperature environment. We divided the tests into two packages: in the first package we tested
156 smooth round bar specimens with extensometer at each test temperature, to provide reference equivalent
157 stress-strain curves; in the second package, we used a digital high speed camera to record the specimen
158 deformation for axisymmetric notched tensile specimens, as well as for smooth round bar specimens.
159 The specimen minimum cross-section diameter in the second package was identified with a so-called
160 ‘edge-tracing’ or ‘edge-detection’ method [32]. For all the tests, the specimen was loaded in
161 displacement control with the crosshead speed of 0.3 mm/minute.

162



163

164

165 *Fig. 3 Sketches of the tensile specimens: (a) axisymmetric notched tensile specimen; (b) smooth*
166 *round bar specimen.*

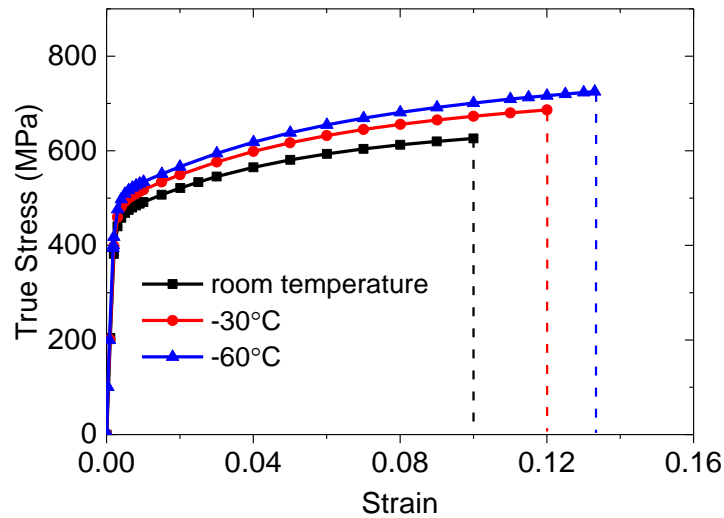
167 **4. Results and discussion**

168 For the smooth round bar tensile tests with extensometer, the engineering stress σ' is calculated by
 169 dividing load by the initial cross-section area ($\sigma' = P/\pi a_0^2$). Engineering strain ε' directly from
 170 extensometer and corresponding engineering stress are converted to true strain and true stress by Eq.
 171 (11) and Eq. (12):

172
$$\sigma_T = \sigma'(1 + \varepsilon') \quad (11)$$

173
$$\varepsilon = \ln(1 + \varepsilon') \quad (12)$$

174 Fig. 4 presents the true stress-strain curves at room temperature, -30°C and -60°C . Obvious temperature
 175 effect can be found: true stress-strain curve obtained at lower test temperature presents to be higher. It
 176 can also be found that the strain corresponding to the onset of diffuse necking ($\varepsilon_{P_{\max}}$, intersections of the
 177 dash lines and the horizontal axis) also increases slightly with decreasing testing temperature. Before
 178 diffuse necking, the smooth round bar specimen deforms uniformly, true stress-strain curve also
 179 represents material's equivalent stress-strain curve. Therefore, true stress-strain curves in Fig. 4 will be
 180 used as reference before diffuse necking in the following discussion.



181
 182 *Fig. 4 True stress-strain curves from smooth round bar specimens with extensometer.*

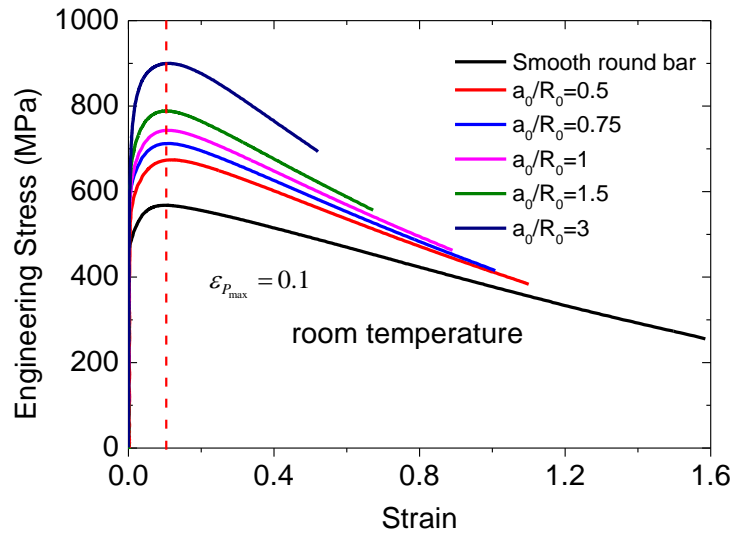
183 For the tensile tests in the second package, the specimen deformation was recorded with a digital high
 184 speed camera. The strain for smooth round bar specimens is calculated by Eq. (3), the same for the
 185 axisymmetric notched tensile specimens. Engineering stress-strain curves for all the tests in the second
 186 package are presented in Fig. 5. As expected, the engineering stress increases with strain firstly; after
 187 reaching the maximum value it decreases. Axisymmetric notched tensile specimen with a higher initial
 188 notch radius ratio corresponds to a larger peak engineering stress. For example, for the tests performed
 189 at room temperature, the maximum engineering stress for specimen with $a_0/R_0 = 0.5$ is 673.55 MPa;

190 while for specimen with $a_0/R_0 = 3$, the maximum engineering stress is 903.11 MPa. $\varepsilon_{P_{\max}}$ is shown with
191 red dash lines in Fig. 5. It can be seen that $\varepsilon_{P_{\max}}$ for smooth round bar specimen and axisymmetric
192 notched tensile specimens is approximately the same at same testing temperature. This result indicates
193 that for this 420 MPa structural steel, $\varepsilon_{P_{\max}}$ is independent of the specimen notch geometry. It can also
194 be observed that $\varepsilon_{P_{\max}}$ for this material is sensitive to temperature, and it increases slightly with
195 decreasing testing temperatures.

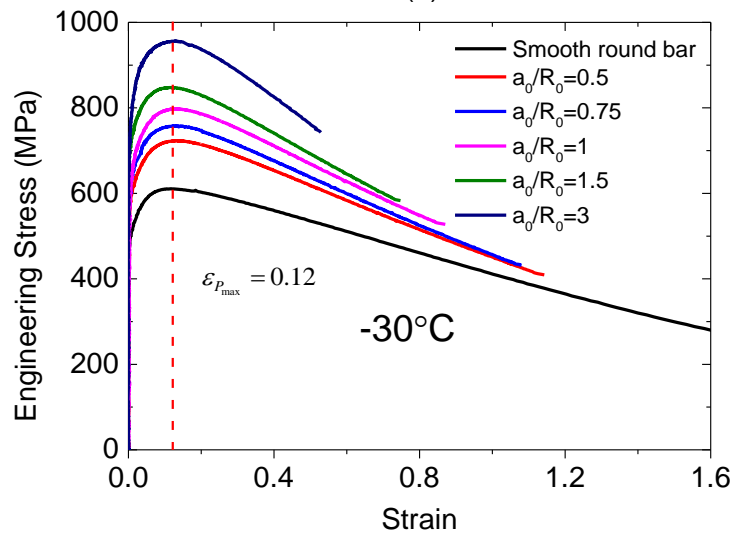
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197 True stress for all the tests in the second package are calculated with Eq. (4). Corresponding true stress-
198 strain curves are presented in Fig. 6. For the smooth round bar specimens in the second package, true
199 stress-strain curve before diffuse necking is exactly the material's equivalent stress-strain curve. After
200 diffuse necking, true stress-strain curves of smooth round bar specimens in Fig. 6 are corrected by
201 Bridgman correction: Eq. (1) and Eq. (2). True stress-strain curves for axisymmetric notched tensile
202 specimens in Fig. 6 are then corrected with Eq. (10). Corresponding equivalent stress-strain curves are
203 presented in Fig. 7, together with the true stress-strain curves with extensometer and equivalent stress-
204 strain curves after performing Bridgman correction with smooth round bar specimens in the second
205 package. Very good agreements can be seen in Fig. 7 between the true stress-strain curves from
206 extensometer and equivalent stress-strain curves corrected by Eq. (10) with axisymmetric notched tensile
207 specimens, at each test temperature. After diffuse necking, equivalent stress-strain curves corrected by
208 Eq. (10) with the axisymmetric notched tensile specimens agree well with the Bridgman corrected
209 equivalent stress-strain curve from smooth round bar specimen, when the strain is smaller than 0.528,
210 0.699, 0.742 for the tests performed at room temperature, -30°C , and -60°C , respectively. After then,
211 slight difference can be found. The equivalent stress corrected by Eq. (10) is slightly lower than those
212 from the Bridgman correction.

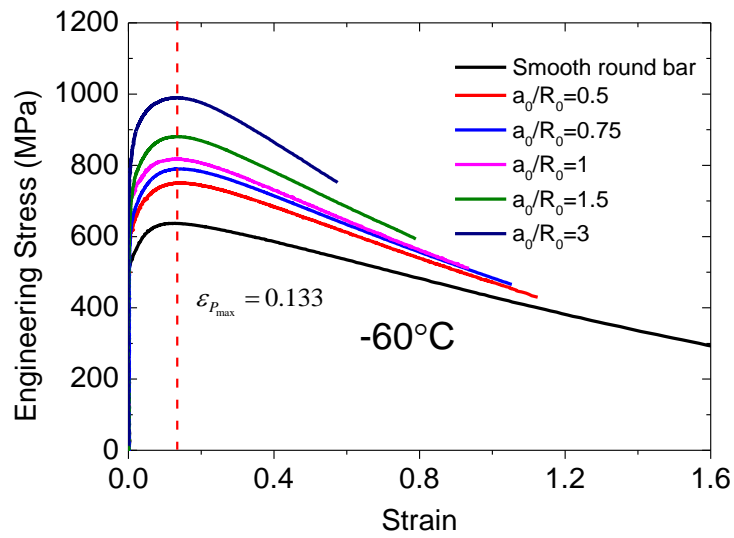
213 For axisymmetric notched tensile specimen with sharper initial notch (larger a_0/R_0), the specimen failed
214 at smaller strain than that with smaller initial notch radius ratio. For example, for the tests conducted at
215 -30°C , the specimen with $a_0/R_0 = 3$ failed when $\varepsilon = 0.525$; while for the specimen with $a_0/R_0 = 0.5$, it
216 failed at the strain $\varepsilon = 1.14$. This can be explained that the strain at fracture is strongly dependent of
217 stress triaxiality T , which is defined by the ratio of hydrostatic stress and von Mises equivalent stress
218 [33-36]. Fracture strain decreases with the increase of stress triaxiality in the range $T \geq 1/3$. For
219 axisymmetric notched tensile specimen, the stress triaxiality is a function of notch radius ratio and larger
220 than $1/3$. Larger a_0/R_0 corresponds to a higher stress triaxiality, therefore, resulting in a smaller failure
221 strain. On the purpose of measuring equivalent stress-strain curve with our correction function in large
222 strain, it is therefore not recommended to use specimens with very larger initial notch radius ratio.



(a)



(b)



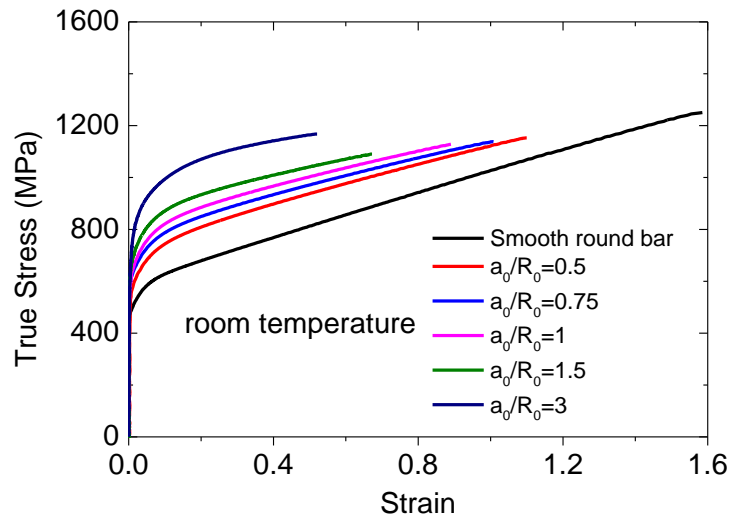
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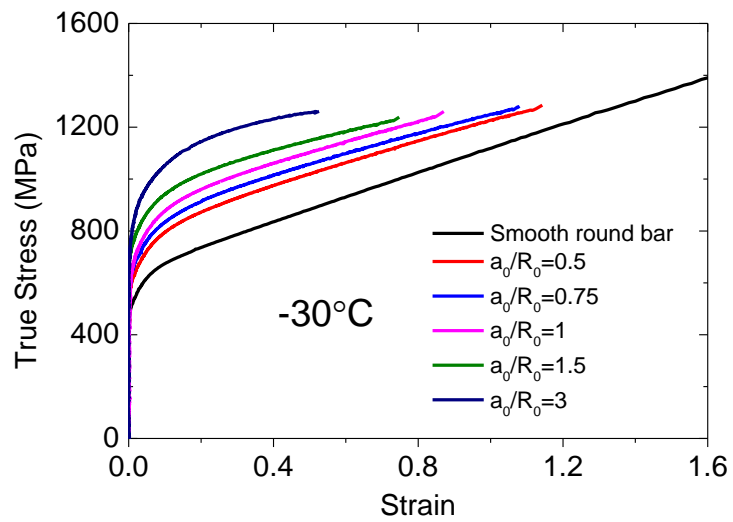
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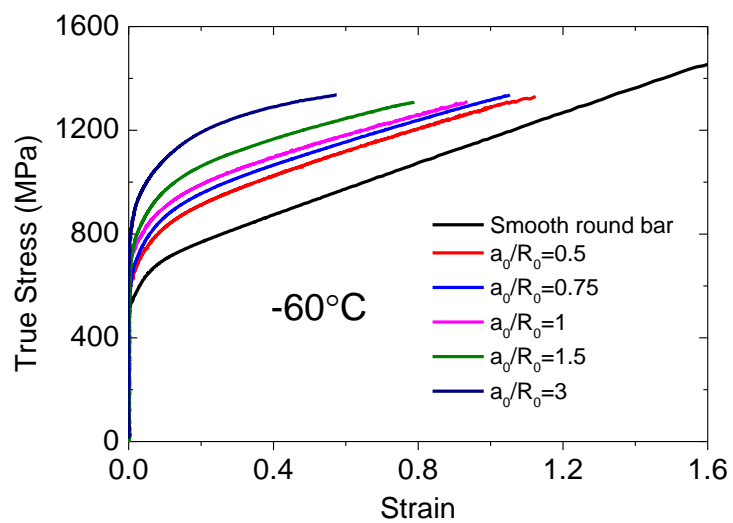
Fig. 5 Engineering stress-strain curves of smooth round bar and axisymmetric notched tensile specimens: (a) room temperature; (b) -30°C ; (c) -60°C . $\varepsilon_{p,\max}$ is also shown with red dash lines.



(a)



(b)



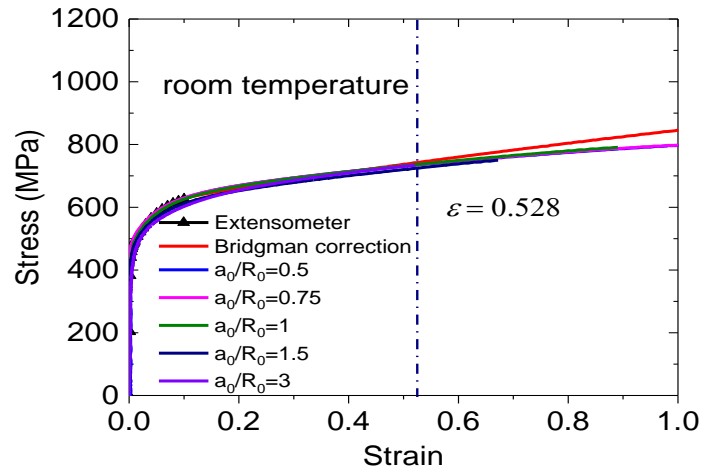
(c)

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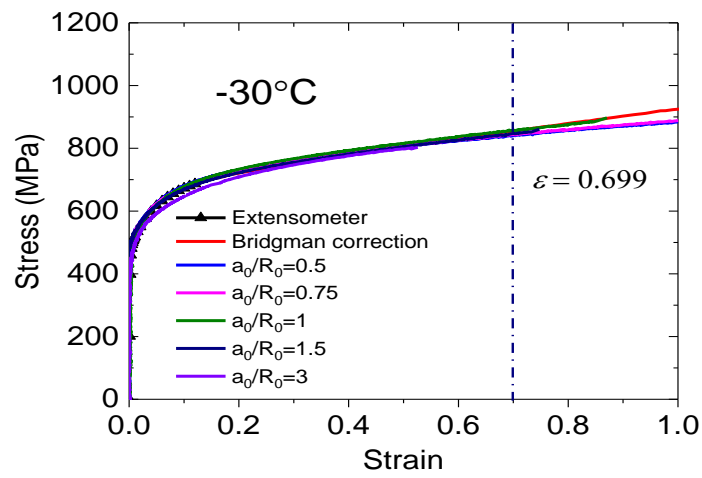
227 Fig. 6 True stress-strain curves of smooth round bar and axisymmetric notched tensile specimens: (a)

228

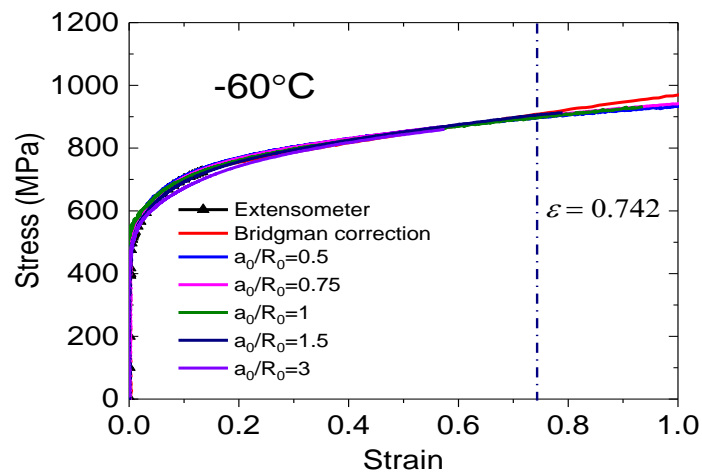
room temperature; (b) -30°C; (c) -60°C.



(a)



(b)



(c)

229

230

231

232

233

Fig. 7 Equivalent stress-strain curves obtained from axisymmetric notched tensile specimens with the correction function: (a) room temperature; (b) -30°C ; (c) -60°C . Equivalent stress-strain curve from smooth round bar specimen from extensometer (before diffuse necking) and from Bridgman correction are also shown for reference.

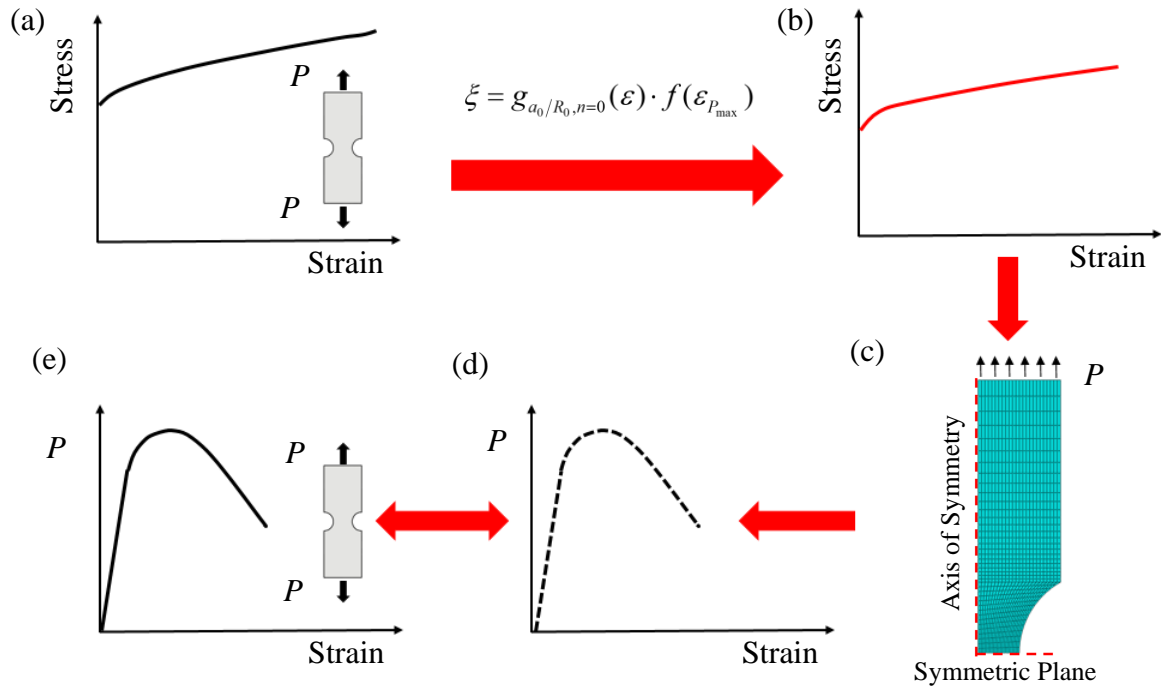
234 **5. Validation of the equivalent stress-strain curve**

235 The correction method is derived with power-law hardening materials in an inverse manner. Attention
236 should be paid for the application of this correction method, since materials can follow different
237 hardening rules. To guarantee the accuracy of the equivalent stress-strain curve obtained with the
238 correction method, a fast and efficient way is to compare load-strain curves from tests and from
239 numerical analysis, assuming the derived equivalent stress-strain curve as material's equivalent stress-
240 strain curve and used for numerical modeling. Fig. 8 schematically presents the validation procedure.
241 True stress-strain curve from axisymmetric notched tensile specimen in Fig. 8 (a) are corrected with Eq.
242 (10) to obtain the equivalent stress-strain curve in Fig. 8 (b). The equivalent stress-strain curve in Fig. 8
243 (b) is then used as input stress-strain curve for numerical analysis. Load-strain curves from numerical
244 simulation (see in Fig. 8 (d)) are then compared with those from test, as shown in Fig. 8 (e). When the
245 load-strain curves from test and from numerical simulation show very good agreement, it indicates that
246 the equivalent stress-strain derived with the proposed correction method is accurate.

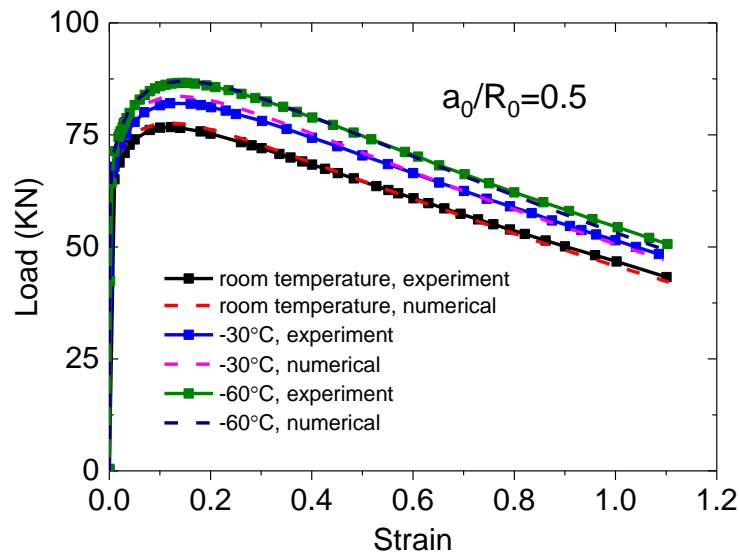
247
248 As an example, equivalent stress-strain curves derived with the axisymmetric notched tensile specimen
249 with $a_0/R_0 = 0.5$ at each test temperature are used for numerical analyses. The geometry used for
250 numerical analyses is the same as in experiments. Numerical analyses were performed with
251 Abaqus/Standard 6.14. Axisymmetric model is used with the 4-noded axisymmetric reduced integration
252 element (CAX4R). The element size is approximately 0.4*0.4 mm in the notch region. Larger
253 deformation is accounted. Symmetric boundary condition is applied in the symmetric plane. The
254 specimen is modelled in displacement control, the same as in the experiment. Load-strain curves from the
255 experiments and from numerical analyses are presented in Fig. 9.

256
257 It can be seen that the load-strain curves from numerical analyses present very good agreement with
258 those from experiments, at each test temperature. It indicates that the deformation on the specimen
259 during loading process can be well captured. It also indicates that the equivalent stress-strain curves
260 derived with the correction function are accurate for this 420 MPa structural steel.

261



262
 263 Fig. 8 Procedure for the validation of the equivalent stress-strain curves from axisymmetric notched
 264 specimens. (a) true stress-strain curve from axisymmetric notched specimens tensile tests; (b)
 265 equivalent stress-strain curve obtained with the proposed correction method; (c) Numerical
 266 simulation of tensile tests. (d) Load-strain curves from numerical simulation; (e) Load-strain curves
 267 from test.
 268



269
 270 Fig. 9 Comparison of load-strain curves from experiments and from numerical analyses for
 271 axisymmetric notched specimen with $a_0/R_0 = 0.5$ at each test temperature.

272 **6. Concluding remarks**

273 In this paper, we performed tensile tests with axisymmetric notched tensile specimens with a_0/R_0
274 ranging from 0.5 to 3 to experimentally verify the recent proposed correction function, by measuring
275 equivalent stress-strain curve of a 420 MPa structural steel at room temperature, -30°C and -60°C,
276 respectively. Equivalent stress-strain curves by converting true-strain curves from axisymmetric notched
277 tensile specimens with the proposed correction function agree very well with true stress-strain curves
278 from smooth round bar specimen with extensometer together with Bridgman correction. **Comparing**
279 **load-strain curves from the experiments and numerical simulations, it indicates that our correction**
280 **method works well to explore the material's stress-strain behavior.** It is worth noting that the proposed
281 correction function can also be used to measure the equivalent stress-strain curve of each individual
282 material zone in a weldment, by locating the notch in the targeted material zone, once the specimen
283 geometry requirements ($d_0 \geq 3.5a_0$, $a_0 \leq H$) are fulfilled. Due to the stress triaxiality dependence of
284 fracture strain, it is not suggested to use specimens with very sharp notch (large a_0/R_0) to measure
285 material's equivalent stress-strain curve. We recommend to run numerical analysis to verify the
286 equivalent stress-strain curve derived with the correction function to guarantee the validity of test results.

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291

292 **Acknowledgement**

293

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Highlights

- A newly proposed correction function for deriving equivalent stress-strain curve with axisymmetric notched tensile specimens was verified experimentally.
- Significant temperature effect on the equivalent stress-strain curves was observed.
- Results obtained with the proposed correction method show good agreement with the well-known Bridgman correction at large strain.

Experimental measurement of temperature-dependent equivalent stress-strain curves of a 420 MPa structural steel with axisymmetric notched tensile specimens

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Nomenclature

a	current minimum cross-section radius
a_0	initial minimum cross-section radius
A	current minimum cross-section area
d_0	specimen outer diameter
E	Young's modulus
H	material zone height in the notch
P	tensile load
R	current notch curvature radius
R_0	initial notch curvature radius
a/R	current notch radius ratio
a_0/R_0	initial notch radius ratio
T	stress triaxiality
ε	average true strain
ε'	engineering strain
$\varepsilon_{P_{\max}}$	strain at the maximum load
ξ	correction factor for axisymmetric notched tensile specimen
ξ_B	Bridgman correction factor
σ_0	yield stress
σ'	engineering stress
σ_{eq}	von Mises equivalent stress
σ_T	true stress

1 **Experimental measurement of temperature-dependent equivalent stress-** 2 **strain curves of a 420 MPa structural steel with axisymmetric notched** 3 **tensile specimens**

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7

8 **Abstract**

9 Recently, the authors in this paper proposed a correction function to determine material's equivalent
10 stress-strain curve with axisymmetric-notched tensile specimens. In this study, tensile tests were
11 performed at room temperature, -30°C and -60°C with axisymmetric notched tensile specimens to verify
12 this method and to identify the equivalent stress-strain curves of a 420 MPa structural steel. A high-
13 speed camera was used together with the so-called edge-tracing method to calculate average true strain.
14 The material's equivalent stress-strain curve was also measured with extensometer and smooth round
15 bar specimens. Experimental results show that equivalent stress-strain curve of this structural steel is
16 sensitive to test temperature. Equivalent stress-stress curves obtained from axisymmetric notched tensile
17 specimens by using the proposed correction function show good agreement with those from
18 extensometer before diffuse necking and from Bridgman correction at large strain using smooth tensile
19 specimens. Since fracture strain strongly depends on the notch geometry, it is recommended to use
20 axisymmetric notched tensile specimens with smaller a_0/R_0 when applying the proposed correction
21 function to measure material's equivalent stress-strain curve.

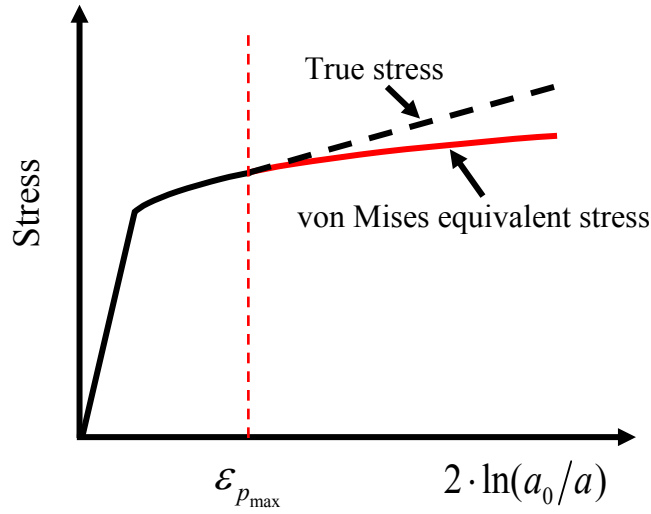
22 **Keywords:** *equivalent stress-strain curve; low temperature; axisymmetric notched tensile specimen;*
23 *Bridgman correction; large strain.*

24

25 **1. Introduction**

26 Identifying material's equivalent stress-strain curve in large strain is very important for large
27 deformation analysis, such as plastic forming [1, 2] and ductile fracture analysis with finite element
28 method [3-8]. Usually, smooth round bar specimens [9, 10] or smooth specimens with rectangular cross-
29 section [11-13] are used to measure material's equivalent stress-strain curves with extensometer. The
30 limitation of such method is that only the data before diffuse necking (different to localized necking)
31 can be used directly. There are several methods to determine material's true stress-strain curve in large
32 range of strain. For thick materials, smooth round bar specimen can be used when the instantaneous
33 minimum cross-section area is measured. The strain ε is then characterized by the specimen minimum

34 cross-section area reduction: $\varepsilon = 2 \ln(a_0/a)$, where a_0 and a are the specimen initial and current
 35 minimum cross-section radius, respectively. The true stress or axial average stress σ_T is calculated by
 36 dividing the load P by the instantaneous minimum cross-section area. For very thin plate material, Zhang
 37 [14] proposed a method to calculate the post-necking minimum cross-section area of rectangular cross-
 38 section specimens, as a function of specimen thickness reduction. With Zhang's method, true stress-
 39 strain curves from flat tensile specimens can be obtained at large strain. It should be noted that after
 40 diffuse necking, tri-axial stress state occurs in the necked region. The true stress differs with von Mises
 41 equivalent stress σ_{eq} [9, 15], as shown in Fig. 1.



42
 43 Fig. 1 Illustration of the difference between true stress and von Mises equivalent stress for tensile test
 44 with smooth round bar specimen after diffuse necking ($\varepsilon > \varepsilon_{P_{max}}$).

45 Diffuse necking occurs after the maximum tensile load, hence the true stress should be corrected when
 46 the strain is larger than the strain corresponding to the maximum tensile load, $\varepsilon_{P_{max}}$. Bridgman [9]
 47 performed analytical analysis with necked round bar specimen and proposed a correction factor ξ_B :

$$48 \quad \xi_B = (1 + 2R/a) \cdot \ln(1 + a/2R) \quad (2)$$

$$\sigma_{eq} = \sigma_T / \xi_B$$

49 where R is the neck curvature radius. By dividing the true stress in Fig. 1 by ξ_B , the material's equivalent
 50 stress can be calculated. Indeed, R is very difficult to measure accurately. Le Roy [16] proposed an
 51 empirical formula to calculate the notch curvature radius ratio a/R :

$$52 \quad a/R = 1.1 \cdot (\varepsilon - \varepsilon_{P_{max}}) \quad (3)$$

53 Combined with Eq. (1) – (2), true stress-strain curve from a smooth round bar specimen can be converted
 54 to material's equivalent stress-strain curve after diffuse necking. The Bridgman correction factor ξ_B
 55 works well at strain slightly larger than $\varepsilon_{P_{max}}$. As the strain further increases, errors between the
 56 material's equivalent stress and the Bridgman corrected equivalent stress occurs and increases with the

57 increase of strain [15]. The errors range from several percentages to more than 10% [15, 17]. Recent
58 numerical analyses [18-20] show that the stress distribution at the necked specimen minimum cross-
59 section differs significantly with Bridgman's analytical solution. These errors are mainly attributed to
60 the assumption that the equivalent strain is uniform in the specimen minimum cross-section. Similar to
61 the Bridgman method, several other correction methods have been proposed [21]. The main difference
62 of these methods is the determination of the curvature radius of the longitudinal stress trajectories.
63 Though the Bridgman correction method is not very accurate when the strain is large, it still can be used
64 as reference. Ling [22] proposed a so-called weighted average method to measure the true stress-strain
65 curve from rectangular cross-section specimen, by setting the power law hardening as lower bound and
66 the linear hardening as the upper bound for the equivalent stress. The correction proposed by Ling is a
67 kind of hybrid experimental-numerical modeling method and the determination of the weight constant
68 is time consuming. Scheider [23] proposed a correction factor as a function of strain and $\varepsilon_{p_{max}}$ to derive
69 equivalent stress-strain curve with flat tensile specimen. However, Scheider's method can only be used
70 for specimens with the aspect ratio of 1:4. Choung [24, 25] also proposed a method to measure equivalent
71 stress-strain curves with flat tensile specimens. The minimum cross-section area should be measured
72 manually with digital calipers and a micrometer. It is worth noting that both Scheider [23] and Choung's
73 [24, 25] method are based on inverse numerical analyses.

74
75 To measure the true stress-strain curve of each individual material zone in a weldment, Zhang [26]
76 proposed a correction function, with which the true stress-strain curve from an axisymmetric notched
77 tensile specimen can be converted to the corresponding one from a smooth round bar specimen. This
78 method is not accurate at large strain, but lay a foundation for our recent work [27, 28]. With further
79 numerical studies, Tu et al. identified a 'magic' axisymmetric notched tensile specimen [28]. With only
80 one single correction factor, true stress-strain curve from the 'magic' notched specimen can be converted
81 to material's equivalent stress-strain curve in a large range of strain accurately, and no Bridgman
82 correction is needed. The limitation is that failure strain of this 'magic' notched specimen can be much
83 smaller than that from a smooth round bar specimen, sometimes.

84
85 Recently, Tu et al. found a new correction function to determine material's equivalent stress-strain curve
86 with 'any' axisymmetric notched tensile specimens [27]. The correction function can be used to the
87 perfectly plastic material and hardening material, and also to weldments. In this study, tensile tests were
88 performed at room temperature, -30 °C and -60 °C with axisymmetric notched tensile specimens
89 machined from a 420 MPa structural steel plates to verify the proposed correction method. The
90 correction function is introduced in detail in section 2. The experimental procedure is presented in
91 section 3. The material's equivalent stress-strain curve were also measured with extensometer and

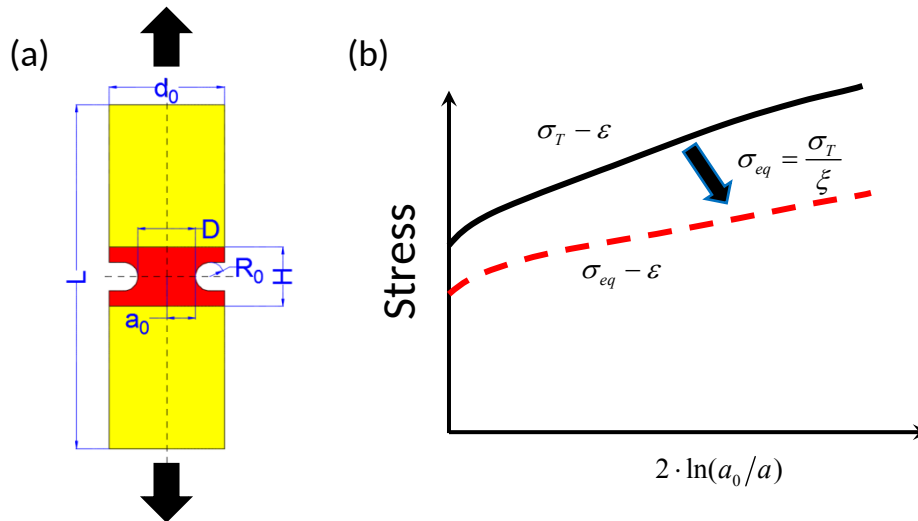
92 **smooth round bar specimens.** Before diffuse necking, the equivalent stress-strain curves from
 93 axisymmetric notched tensile specimens are compared with those from extensometer. With Eq. (1)-(2),
 94 we also performed Bridgman correction with smooth round bar specimen to obtain reference equivalent
 95 stress-strain curves after diffuse necking. Results and discussions are presented in section 4. The
 96 equivalent stress-strain curves are then verified by numerical analyses in section 5. Main conclusions
 97 are presented in section 6.

98 2. Axisymmetric notched tensile specimen method

99 Axisymmetric notched tensile specimen has been widely applied in characterizing material's mechanical
 100 properties [29-31]. For inhomogeneous material, such as weldment, it is **practically** impossible to
 101 measure the equivalent stress-strain curve in a targeted material zone with cross-weld smooth round bar
 102 specimen or flat tensile specimen, due to the nature of **practically** unpredictable fracture position. By
 103 introducing an axisymmetric notch on the smooth round bar specimen, the deformation is restrained
 104 mainly in the notched region under uniaxial tensile loading [26-28]. Fig. 2 (a) schematically shows the
 105 geometry information of the axisymmetric notched tensile specimen. Similar with smooth round bar
 106 specimen, the strain is defined by the minimum cross-section area reduction and the true stress is
 107 calculated by dividing load by the current minimum cross-section area:

$$108 \quad \varepsilon = 2 \cdot \ln(a_0/a) \quad (4)$$

$$109 \quad \sigma_T = P/\pi a^2 \quad (5)$$



110
 111 *Fig. 2 (a) Geometry of axisymmetric notched tensile specimen. The yellow part can be overmatch,*
 112 *under match or even match with the remain part of the specimen. (b) Conversion of true stress-strain*
 113 *curve from notched specimen to equivalent stress-strain curve by the proposed correction function.*

114
 115 Stress concentration occurs due to the existence of the notch. True stress-strain curve from an
 116 axisymmetric notched tensile specimen differs significantly with the material's equivalent stress-strain

117 curve and should be corrected. Our previous study [28] shows that when the specimen geometry
 118 requirement $d_0 \geq 3.5a_0$ is fulfilled, true stress-strain curves from axisymmetric notched tensile
 119 specimens with the same initial notch radius ratio a_0/R_0 are identical for homogeneous materials. This
 120 is true for inhomogeneous material when a_0 is smaller than the material zone length: $a_0 \leq H$.

121

122 Recently, we proposed a correction function to convert the true stress-strain curve from any
 123 axisymmetric notched tensile specimens to the material's equivalent stress-strain curve [27]. The
 124 correction function is written in a general form:

$$125 \quad \xi = g_{a_0/R_0, n=0}(\varepsilon) \cdot f(\varepsilon_{P_{\max}}) \quad (6)$$

126 Eq. (5) consists of two parts: the first part describes the notch effect on the true stress-strain curves of
 127 the perfectly-plastic material, and displays as a linear function of the true strain ε , Eq. (6). The slope,
 128 $b_{1, n=0}$, in Eq. (6) depicts the initial notch geometry effect on the evolution of true stress-strain curve from
 129 axisymmetric notched tensile specimen. While the intersection, $b_{2, n=0}$, can be explained as the notch
 130 induced stress concentration, sharper notch yields higher stress concentration. The slope and intersection
 131 are given in Eq. (7) and Eq. (8) as a function of the initial notch radius ratio, respectively. The second
 132 part, as shown in Eq. (9), is a function of $\varepsilon_{P_{\max}}$, describing the effect of strain hardening on the true
 133 stress-strain curve of a notched specimen.

$$134 \quad g_{a_0/R_0, n=0}(\varepsilon) = (b_{1, n=0} \cdot \varepsilon + b_{2, n=0})_{a_0/R_0} \quad (7)$$

$$135 \quad b_{1, n=0} = 0.03232 \cdot (a_0/R_0)^2 - 0.27 \cdot (a_0/R_0) + 0.3866 \quad (8)$$

$$136 \quad b_{2, n=0} = -0.04084 \cdot (a_0/R_0)^2 + 0.3557 \cdot (a_0/R_0) + 1.0577 \quad (9)$$

$$137 \quad f(\varepsilon_{P_{\max}}) = -0.22942 \cdot \varepsilon_{P_{\max}}^2 - 0.36902 \cdot \varepsilon_{P_{\max}} + 1 \quad (10)$$

138 When ε and $\varepsilon_{P_{\max}}$ are known, the $\sigma_T - \varepsilon$ curve from an axisymmetric notched tensile specimen can be
 139 converted to the material's equivalent stress-strain curve by Eq. (10), as demonstrated in Fig. 2 (b).
 140 Details about the derivation of this correction function can be referred to ref. [27].

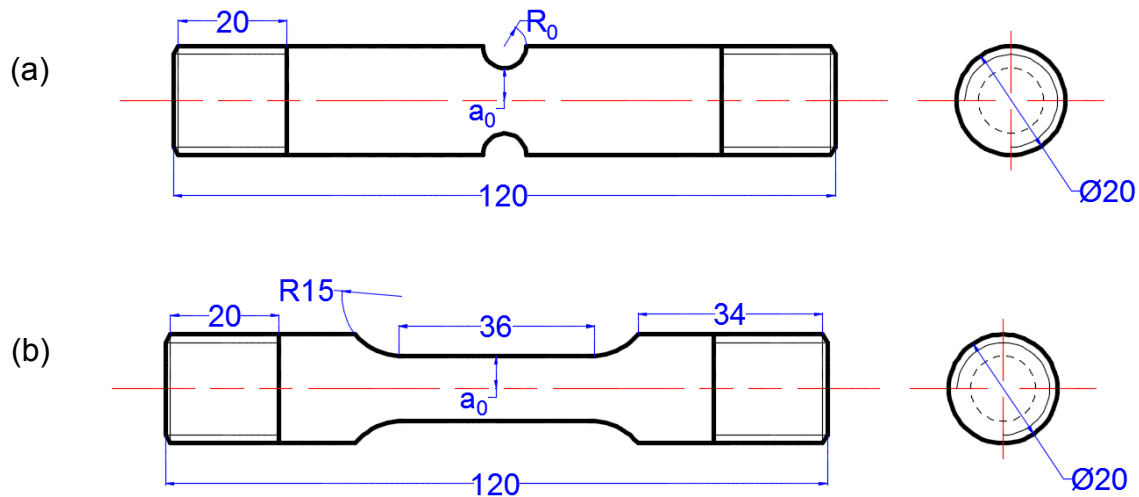
$$141 \quad \sigma_{eq} = \sigma_T / \xi \quad (11)$$

142 3. Experiment procedure

143 To experimentally verify the correction function, we conducted tensile tests with smooth round bar
 144 specimens and axisymmetric notched tensile specimens with initial notch radius ratio, a_0/R_0 , ranging
 145 from 0.5 to 3. The specimens were machined from 50 mm thick plates of a 420 MPa steel, along the

146 rolling direction. The specimen configurations are shown in Fig. 3. Our previous numerical studies
 147 provide a conservative geometry requirement for axisymmetric notched tensile specimens: $d_0 \geq 3.5a_0$.
 148 d_0 is the specimen outer diameter, as seen in Fig. 2. In this study, $a_0 = 6 \text{ mm}$ and $d_0 = 20 \text{ mm}$. The
 149 specimen outer diameter is 1 mm smaller than the geometry requirement ($d_0 \geq 3.5a_0$). In order to
 150 guarantee that the specimen geometry can be used, we simply performed numerical analysis with power-
 151 law hardening material and found that the correction function was still valid.

152
 153 The tests were carried out at room temperature, -30°C , and -60°C using a universal test machine Instron
 154 5985, with the loading cell of 250 kN. A liquid nitrogen-cooled temperature chamber was used to create
 155 low temperature environment. The tests were divided into two packages: in the first package we tested
 156 smooth round bar specimens with extensometer at each test temperature, to provide reference equivalent
 157 stress-strain curves; in the second package, we used a digital high speed camera to record the specimen
 158 deformation for axisymmetric notched tensile specimens, as well as for smooth round bar specimens.
 159 The specimen minimum cross-section diameter in the second package was identified with a so-called
 160 ‘edge-tracing’ or ‘edge-detection’ method [32]. For all the tests, the specimen was loaded in
 161 displacement control with the crosshead speed of 0.3 mm/minute.



163
 164
 165 Fig. 3 Sketches of the tensile specimens: (a) axisymmetric notched tensile specimen; (b) smooth
 166 round bar specimen.

167 4. Results and discussion

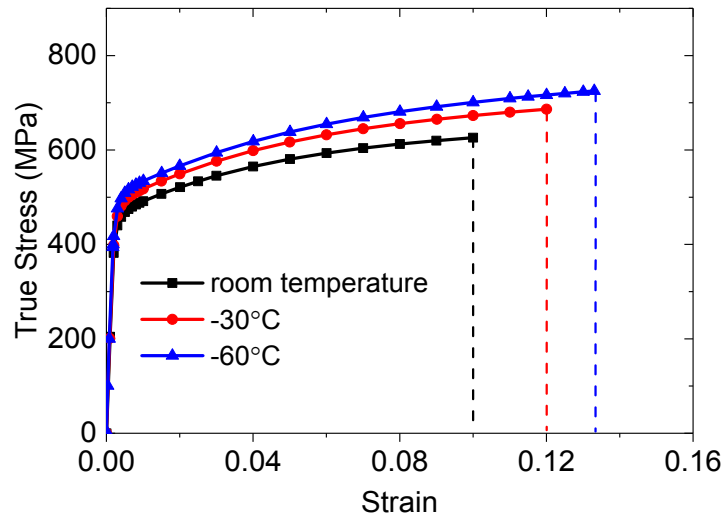
168 For the smooth round bar tensile tests with extensometer, the engineering stress σ' is calculated by
 169 dividing load by the initial cross-section area ($\sigma' = P/\pi a_0^2$). Engineering strain ε' directly from

170 extensometer and corresponding engineering stress are converted to true strain and true stress by Eq.
 171 (11) and Eq. (12):

$$172 \quad \sigma_T = \sigma'(1 + \varepsilon') \quad (12)$$

$$173 \quad \varepsilon = \ln(1 + \varepsilon') \quad (13)$$

174 Fig. 4 presents the true stress-strain curves at room temperature, -30°C and -60°C. Obvious temperature
 175 effect can be found: true stress-strain curve obtained at lower test temperature presents to be higher. It
 176 can also be found that the strain corresponding to the onset of diffuse necking ($\varepsilon_{P_{max}}$, intersections of the
 177 dash lines and the horizontal axis) also increases slightly with decreasing testing temperature. Before
 178 diffuse necking, the smooth round bar specimen deforms uniformly, true stress-strain curve also
 179 represents material's equivalent stress-strain curve. Therefore, true stress-strain curves in Fig. 4 will be
 180 used as reference before diffuse necking in the following discussion.



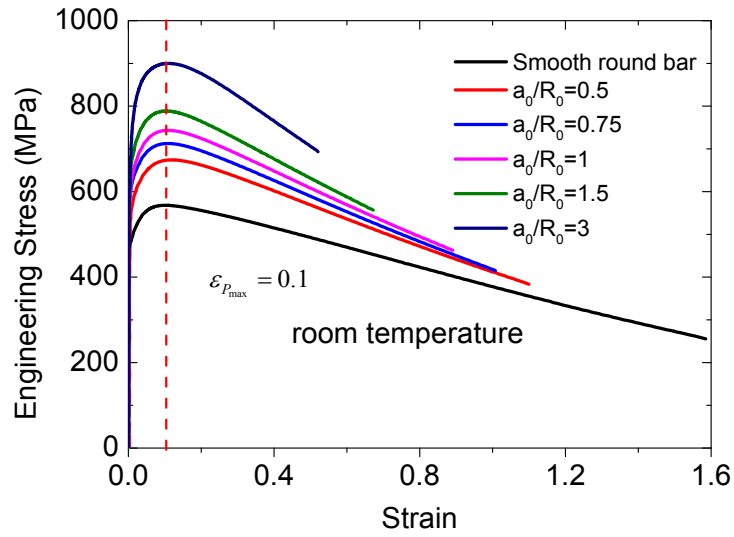
181
 182 *Fig. 4 True stress-strain curves from smooth round bar specimens with extensometer.*

183 For the tensile tests in the second package, the specimen deformation was recorded with a digital high
 184 speed camera. The originally circular cross sections almost remained circular just before fracture, as
 185 indicated by the minor difference in diameter measurement in the minimum cross-section. The strain for
 186 smooth round bar specimens is calculated by Eq. (3), the same for the axisymmetric notched tensile
 187 specimens. Engineering stress-strain curves for all the tests in the second package are presented in Fig.
 188 5. As expected, the engineering stress increases with strain firstly; after reaching the maximum value it
 189 decreases. Axisymmetric notched tensile specimen with a higher initial notch radius ratio corresponds
 190 to a larger peak engineering stress. For example, for the tests performed at room temperature, the
 191 maximum engineering stress for specimen with $a_0/R_0 = 0.5$ is 673.55 MPa; while for specimen with
 192 $a_0/R_0 = 3$, the maximum engineering stress is 903.11 MPa, $\varepsilon_{P_{max}}$ is shown with red dash lines in Fig. 5.
 193 It can be seen that $\varepsilon_{P_{max}}$ for smooth round bar specimen and axisymmetric notched tensile specimens is

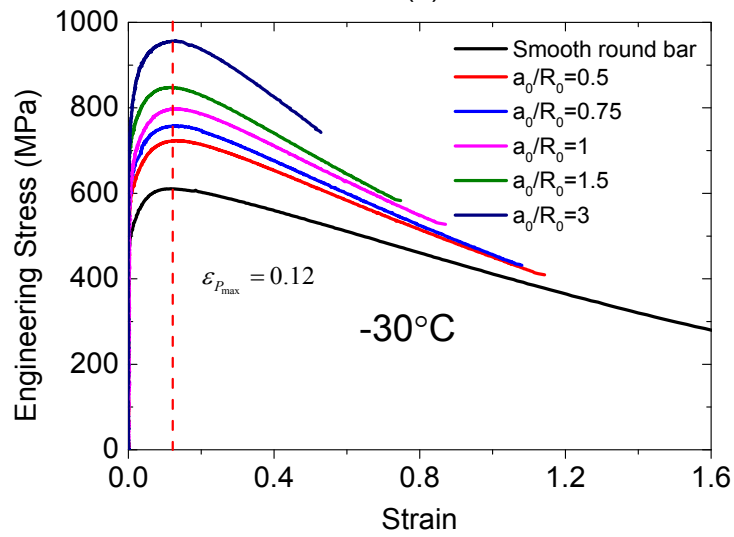
194 approximately the same at same testing temperature. This result indicates that for this 420 MPa structural
195 steel, $\varepsilon_{P_{\max}}$ is independent of the specimen notch geometry. It can also be observed that $\varepsilon_{P_{\max}}$ for this
196 material is sensitive to temperature, and it increases slightly with decreasing testing temperatures.

197
198 True stress for all the tests in the second package are calculated with Eq. (4). Corresponding true stress-
199 strain curves are presented in Fig. 6. For the smooth round bar specimens in the second package, true
200 stress-strain curve before diffuse necking is exactly the material's equivalent stress-strain curve. After
201 diffuse necking, true stress-strain curves of smooth round bar specimens in Fig. 6 are corrected by
202 Bridgman correction: Eq. (1) and Eq. (2). True stress-strain curves for axisymmetric notched tensile
203 specimens in Fig. 6 are then corrected with Eq. (10). Corresponding equivalent stress-strain curves are
204 presented in Fig. 7, together with the true stress-strain curves with extensometer and equivalent stress-
205 strain curves after performing Bridgman correction with smooth round bar specimens in the second
206 package. Very good agreements can be seen in Fig. 7 between the true stress-strain curves from
207 extensometer and equivalent stress-strain curves corrected by Eq. (10) with axisymmetric notched tensile
208 specimens, at each test temperature. After diffuse necking, equivalent stress-strain curves corrected by
209 Eq. (10) with the axisymmetric notched tensile specimens agree well with the Bridgman corrected
210 equivalent stress-strain curve from smooth round bar specimen, when the strain is smaller than 0.528,
211 0.699, 0.742 for the tests performed at room temperature, -30°C, and -60°C, respectively. After then,
212 slight difference can be found. The equivalent stress corrected by Eq. (10) is slightly lower than those
213 from the Bridgman correction.

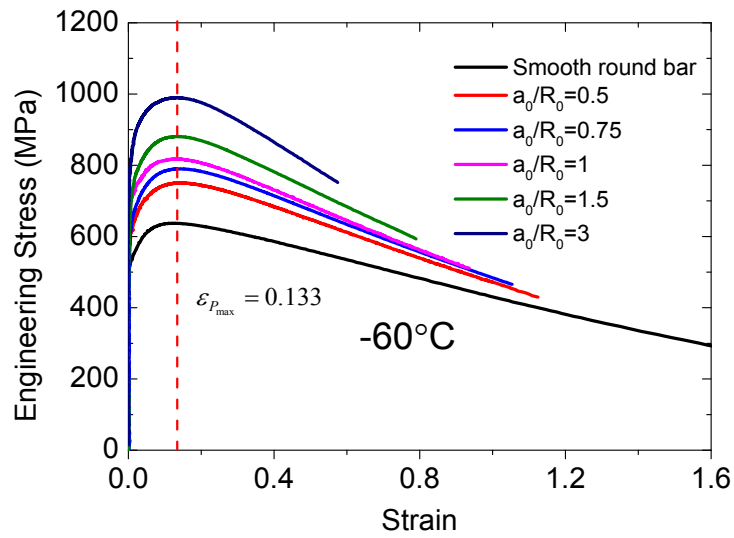
214
215 For axisymmetric notched tensile specimen with sharper initial notch (larger a_0/R_0), the specimen failed
216 at smaller strain than that with smaller initial notch radius ratio. For example, for the tests conducted at
217 -30°C, the specimen with $a_0/R_0 = 3$ failed when $\varepsilon = 0.525$; while for the specimen with $a_0/R_0 = 0.5$, it
218 failed at the strain $\varepsilon = 1.14$. This can be explained that the strain at fracture is strongly dependent of
219 stress triaxiality T , which is defined by the ratio of hydrostatic stress and von Mises equivalent stress
220 [33-36]. Fracture strain decreases with the increase of stress triaxiality in the range $T \geq 1/3$. For
221 axisymmetric notched tensile specimen, the stress triaxiality is a function of notch radius ratio and larger
222 than 1/3. Larger a_0/R_0 corresponds to a higher stress triaxiality, therefore, resulting in a smaller failure
223 strain. On the purpose of measuring equivalent stress-strain curve with our correction function in large
224 strain, it is therefore not recommended to use specimens with very larger initial notch radius ratio.



(a)



(b)



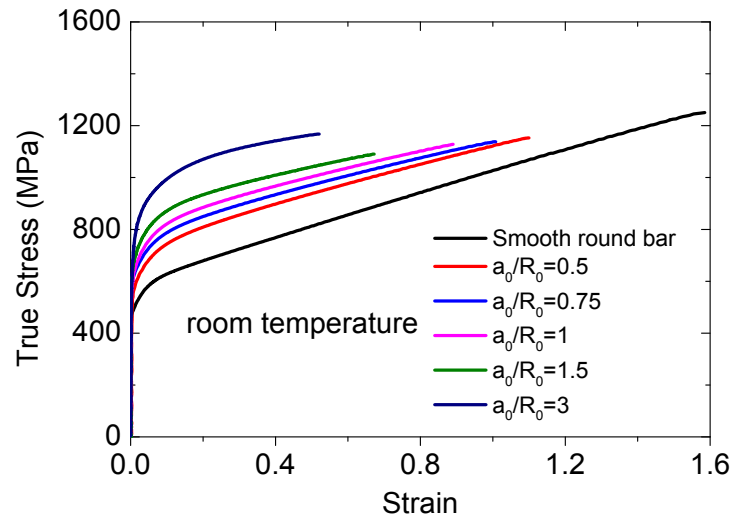
(c)

225

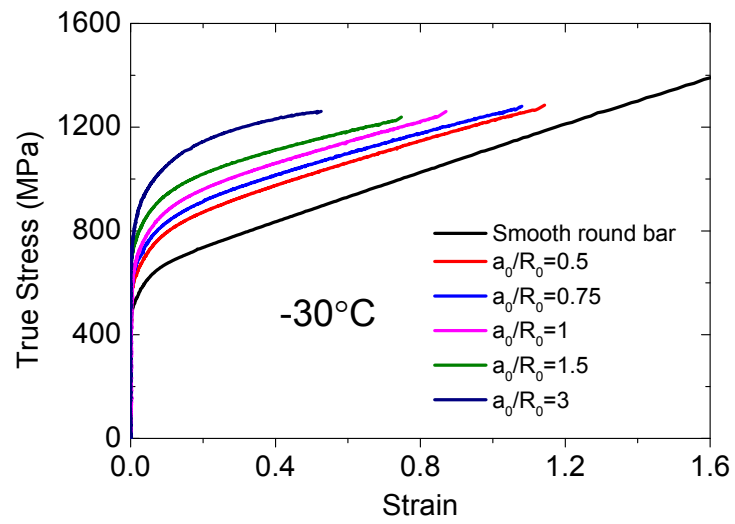
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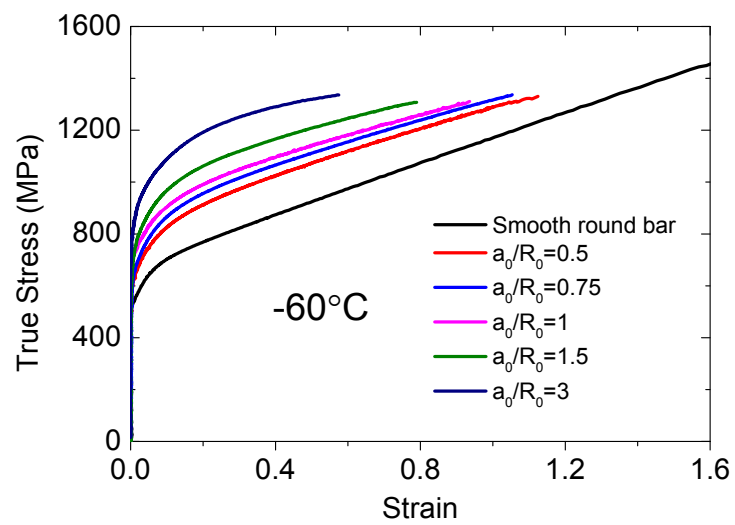
Fig. 5 Engineering stress-strain curves of smooth round bar and axisymmetric notched tensile specimens: (a) room temperature; (b) -30°C ; (c) -60°C . $\varepsilon_{p,\max}$ is also shown with red dash lines.



(a)



(b)



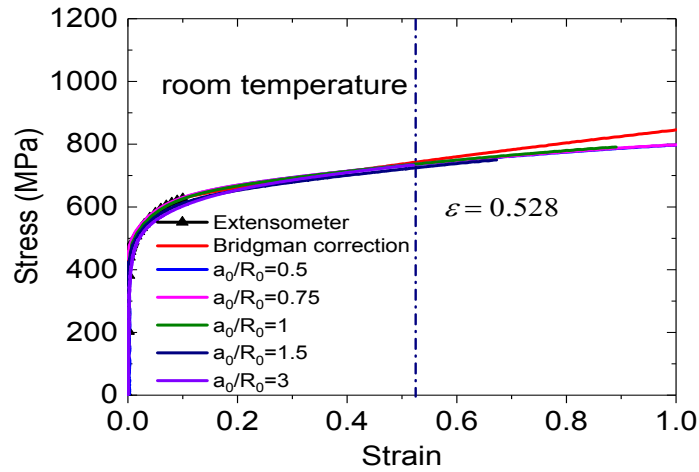
(c)

228

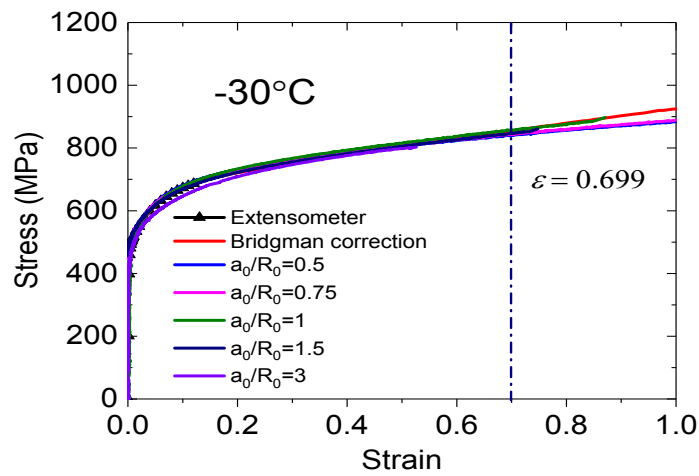
229 Fig. 6 True stress-strain curves of smooth round bar and axisymmetric notched tensile specimens: (a)

230

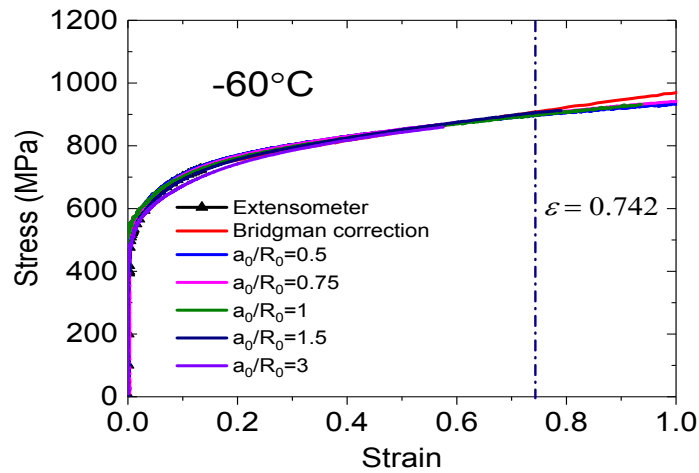
room temperature; (b) -30°C ; (c) -60°C .



(a)



(b)



(c)

231

232 Fig. 7 Equivalent stress-strain curves obtained from axisymmetric notched tensile specimens with

233 the correction function: (a) room temperature; (b) -30°C; (c) -60°C. Equivalent stress-strain curve from

234 smooth round bar specimen from extensometer (before diffuse necking) and from Bridgman

235 correction are also shown for reference.

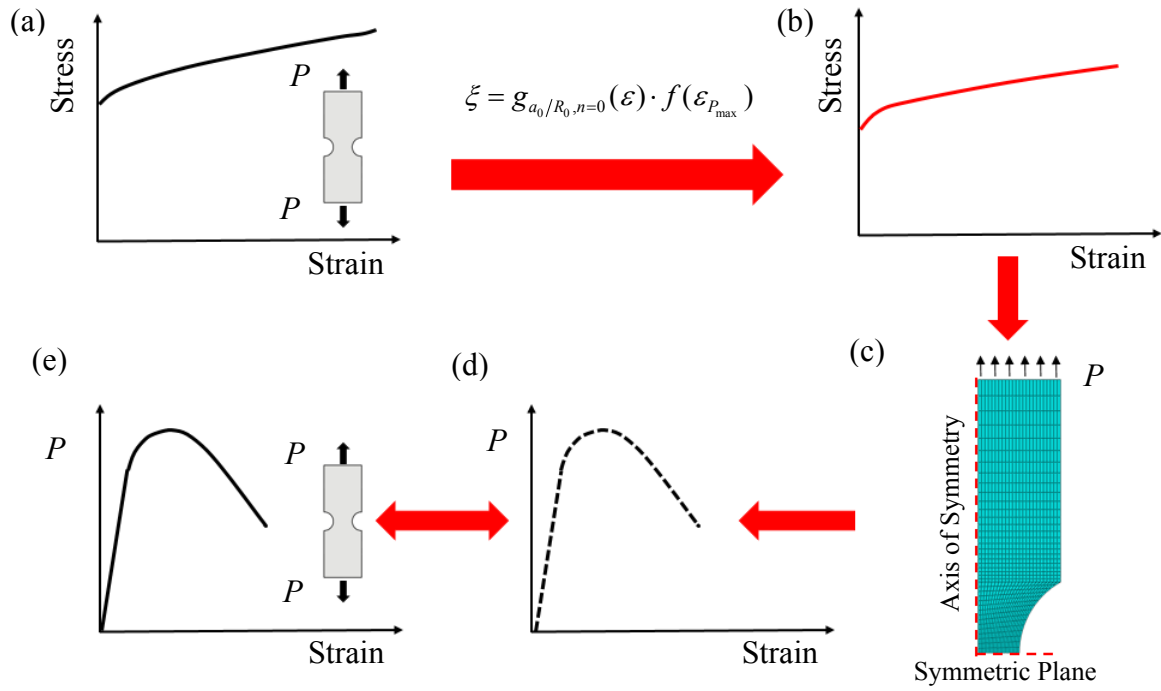
236 **5. Validation of the equivalent stress-strain curve**

237 The correction method is derived with power-law hardening materials in an inverse manner. Attention
238 should be paid for the application of this correction method, since materials can follow different
239 hardening rules. To guarantee the accuracy of the equivalent stress-strain curve obtained with the
240 correction method, a fast and efficient way is to compare load-strain curves from tests and from
241 numerical analysis, assuming the derived equivalent stress-strain curve as material's equivalent stress-
242 strain curve and used for numerical modeling. Fig. 8 schematically presents the validation procedure.
243 True stress-strain curve from axisymmetric notched tensile specimen in Fig. 8 (a) are corrected with Eq.
244 (10) to obtain the equivalent stress-strain curve in Fig. 8 (b). The equivalent stress-strain curve in Fig. 8
245 (b) is then used as input stress-strain curve for numerical analysis. Load-strain curves from numerical
246 simulation (see in Fig. 8 (d)) are then compared with those from test, as shown in Fig. 8 (e). When the
247 load-strain curves from test and from numerical simulation show very good agreement, it indicates that
248 the equivalent stress-strain derived with the proposed correction method is accurate.

249
250 As an example, equivalent stress-strain curves derived with the axisymmetric notched tensile specimen
251 with $a_0/R_0 = 0.5$ at each test temperature are used for numerical analyses. The geometry used for
252 numerical analyses is the same as in experiments. Numerical **analyses** were performed with
253 Abaqus/Standard 6.14. Axisymmetric model is used with the 4-noded axisymmetric reduced integration
254 element (CAX4R). The element size is approximately 0.4*0.4 mm in the notch region. Larger
255 deformation is accounted. Symmetric boundary condition is applied in the **symmetry plane**. The
256 specimen is modelled in **displacement** control, the same as in the experiment. Load-strain curves from
257 the experiments and from numerical analyses are presented in Fig. 9.

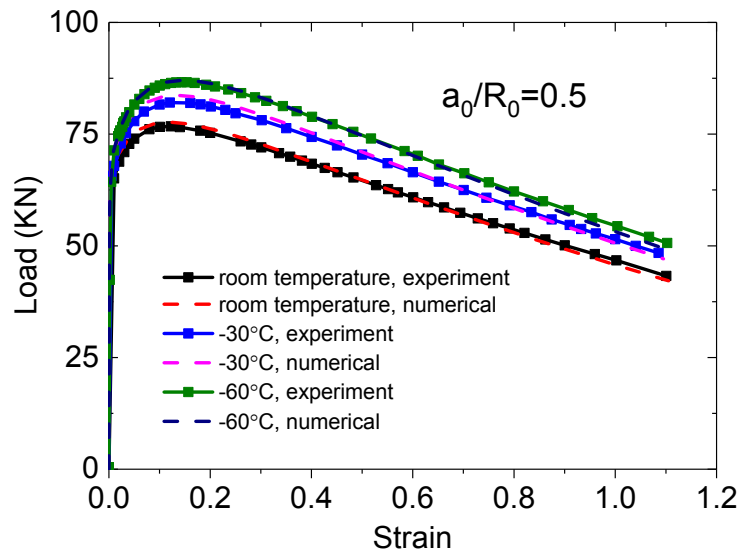
258
259 It can be seen that the load-strain curves from numerical analyses present very good agreement with
260 those from experiments, at each test temperature. It indicates that the deformation on the specimen
261 during loading process can be well captured. It also indicates that the equivalent stress-strain curves
262 derived with the correction function are accurate for this 420 MPa structural steel.

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Fig. 8 Procedure for the validation of the equivalent stress-strain curves from axisymmetric notched specimens. (a) true stress-strain curve from axisymmetric notched specimens tensile tests; (b) equivalent stress-strain curve obtained with the proposed correction method; (c) Numerical simulation of tensile tests. (d) Load-strain curves from numerical simulation; (e) Load-strain curves from test.



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Fig. 9 Comparison of load-strain curves from experiments and from numerical analyses for axisymmetric notched specimen with $a_0/R_0 = 0.5$ at each test temperature.

274 **6. Concluding remarks**

275 In this paper, we performed tensile tests with axisymmetric notched tensile specimens with a_0/R_0
276 ranging from 0.5 to 3 to experimentally verify the recent proposed correction function, by measuring
277 equivalent stress-strain curve of a 420 MPa structural steel at room temperature, -30°C and -60°C,
278 respectively. Equivalent stress-strain curves by converting true-strain curves from axisymmetric notched
279 tensile specimens with the proposed correction function agree very well with true stress-strain curves
280 from smooth round bar specimen with extensometer together with Bridgman correction. Comparing
281 load-strain curves from the experimental tensile tests and those from numerical modeling, it indicates
282 that the proposed correction method works well to explore the material's stress-strain curve. Considering
283 the derivation of this correction function, the authors suggest to use axisymmetric specimens with a_0/R_0
284 ranging from 0.5 to 3 and the hardening exponent n from 0 to 0.35. It is worth noting that the proposed
285 correction function can also be used to measure the equivalent stress-strain curve of each individual
286 material zone in a weldment, by locating the notch in the targeted material zone, once the specimen
287 geometry requirements ($d_0 \geq 3.5a_0$, $a_0 \leq H$) are fulfilled. Due to the stress triaxiality dependence of
288 fracture strain, it is not suggested to use specimens with very sharp notch (large a_0/R_0) to measure
289 material's equivalent stress-strain curve. For materials display highly anisotropy, this method may not
290 be suitable. We recommend to run numerical analysis to verify the equivalent stress-strain curve derived
291 with the correction function to guarantee the validity of test results.

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