Available online at www.sciencedirect.com

ScienceDirect

IFAC PapersOnLine 51-8 (2018) 19–24

Gas-lift Optimization by Controlling Marginal Gas-Oil Ratio using Transient Measurements ⋆

Dinesh Krishnamoorthy Esmaeil Jahanshahi Sigurd Skogestad

Dept. of Chemical Engineering, Norwegian Univ. of Science & Technology , NO-7491 Trondheim , (e-mail: dinesh.krishnamoorthy@ntnu.no, esmaeil.jahanshahi@hotmail.com, skoge@ntnu.no).

Abstract: This paper presents the application of a steady-state gradient control using transient measurements to a gas-lift optimization problem. Optimal operation of a gas-lifted field involves controlling the marginal gas-oil ratio (mGOR), which is the steady-state gradient of the oil rate with respect to the gas lift rate. In this paper, we apply a novel method to estimate the marginal GOR online using a dynamic model and transient measurements, without the need for additional perturbation. The proposed method is based on linearizing the dynamic model around the current operating point to estimate the marginal GOR, which is then controlled using simple feedback controllers to achieve optimal operation. In case of disturbances, the proposed method is able to adjust fast to the new optimal point, without the need to solve computationally expensive optimization problems. By using transient measurements, it does not need to wait for the process to reach steady-state to update the model. The proposed method was tested in simulations and was shown to provide similar performance as an economic MPC.

Keywords: Production optimization, Measurement-based optimization, real-time optimization, plant-wide control

1. INTRODUCTION

In many mature oil and gas production fields, when the reservoir pressure is not sufficient to lift the fluids economically to the surface, artificial lift methods are used to boost the production. Gas-lift is one such commonly used artificial lift method, where compressed gases are injected into the well tubing to reduce the hydrostatic pressure drop and hence increase production. Injecting too much gas also has a detrimental effect on the oil production due to increased frictional pressure losses in the well tubing. Each well then has an optimal gas-lift injection rate that optimizes the production. This paper deals with the problem of finding the optimal gas lift injection rates for the different wells.

Daily production optimization is an important aspect of operating an oil and gas production network. Many different approaches are available in literature to optimize the production from a gas-lifted well network. Traditionally, production engineers use commercially available steady-state multiphase simulators to generate the so-called *gas-lift performance curves*, which gives the static mapping between the oil production rate and the gas lift injection rate (Rashid, 2010). Nonlinear steady-state optimization tools may then be used to compute the optimal gas lift injection rates. However, a common approach to optimizing production from a gas-lifted well network is to use the gas-lift performance curves directly. The optimal allocation of gas-lift rate is known to occur when the marginal gas-oil ratio is equal for all the wells. Marginal gas-oil ratio or simply known as marginal GOR, is a quantity that describes the increase in oil rate per unit change in the gas-lift injection rate. In other words, marginal GOR is given by the slope of the gas-lift performance curves (Bieker et al., 2007).

The principle of marginal GOR has been proven to be the optimal solution for any parallel unit such as the gas lift production network (Downs and Skogestad, 2011), and was also used by Urbanczyk et al. (1994) and Kanu et al. (1981). This was also later shown to fulfill the necessary condition of optimality (Sharma and Glemmestad, 2013). Therefore, the simplest approach to optimal gas lift allocation is by controlling the marginal GOR to be equal for all the wells.

Recently, the use of centralized dynamic optimization solutions such as economic NMPC has been gaining popularity in the process control literature. The use of economic NMPC for the gas lift optimization was considered by Codas et al. (2016) and Krishnamoorthy et al. (2016a). There is no doubt that theoretically optimal performance can be achieved by using such centralized optimizing controllers, however, solving a numerical optimization problem may be computationally intensive and can potentially lead to computational delays. Moreover controller tuning and pro-

⋆ The authors gratefully acknowledge the financial support from SFI SUBPRO, which is financed by the Research Council of Norway, major industry partners and NTNU.

2405-8963 © 2018, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.
Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2018.06.349

longed maintenance over time is crucial to ensure good performance (Skogestad and Postlethwaite, 2007).

There have also been developments in other optimization methods, where instead of solving a numerical optimization problem, optimal operation is achieved via feedback control. Such methods are classified as *direct input adaptation* methods (Chachuat et al., 2009), where the optimization problem is converted into a feedback control problem.

Self-optimizing control is one example of such a method. It involves finding the right controlled variable which when kept constant leads to near optimal operation (i.e. minimum loss). Alstad (2005) demonstrated the application of self-optimizing control using nullspace method for gas lift optimization. This method however is based on local linearization around the nominal operating point and may lead to steady-state losses if the disturbances moves the operation of the process far away from the nominal operating point. The ideal self-optimizing variable for the gas-lift problem would indeed be the marginal GOR. However, the major challenge is that the marginal GOR is not a readily available measurement for control.

Model-free methods such as extremum seeking control has been applied for gas lift wells by Peixoto et al. (2015) and Krishnamoorthy et al. (2016b). Extremum seeking control involves estimating the steady-state gradient (the marginal GOR) directly using the measurements. The estimated marginal GOR is then controlled using simple integral action. The main advantage of these methods is that it does not require a model. However, to estimate the marginal GOR accurately, constant perturbations of the manipulated inputs are required, which may be undesirable in many oil production wells. It also requires direct measurement of the cost function.

More importantly, the use of transient measurements in such model-free methods leads to erroneous gradient estimation. Therefore, such methods often require clear time scale separation between the plant dynamics , excitation signal and the convergence to the optimum, such that the plant can be approximated as a static map (Krstić and Wang, 2000). This results in very slow convergence to the optimum. Gas-lift wells typically have long settling times due to compressibility of the gas in the annulus and transport time inside the well tubing. Therefore, the time scale separation can make such model-free methods prohibitively slow for gas lift optimization. Additionally, abrupt disturbances may cause undesired responses during the transients, which was motivated in Krishnamoorthy et al. (2016b) using a gas-lift optimization problem.

In this paper, we propose to use a new model-based steady-state gradient estimation method to drive the process to optimal operation (Krishnamoorthy et al., 2018). It uses available transient measurements along with a dynamic model online to estimate the exact steady-state gradient around the current operating point without any additional perturbation. Consequently, it converges to the new optimum point in the fast time scale following a disturbance. The proposed method also does not require the need to measure the cost directly. Furthermore, the proposed method is computationally cheap, since the optimization is done via feedback.

The main contribution of this paper is the application of the new steady-state gradient estimation method using transient measurements to the gas-lift optimization problem, which is demonstrated using a simple case example with two gas lifted wells connected to a common manifold.

The reminder of the paper is organized as follows. Section 2 introduces the proposed method and its application for both, unconstrained and constrained gas-lift optimization cases. Simulation results for both the cases are provided in section 3 and compared with economic NMPC. Some discussions are provided before concluding the paper in section 4.

2. PROPOSED METHOD

In this paper we consider a gas lifted well network with n_w wells. The gas lifted well network can be modelled as a dynamic model,

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), \mathbf{d}(t)) \tag{1a}$$

$$\mathbf{y}(t) = \mathbf{h}(\mathbf{x}(t), \mathbf{u}(t)) \tag{1b}$$

where $\mathbf{x} \in \mathbb{R}^{n_x}$ is the vector of differential variables, $\mathbf{u} \in \mathbb{R}^{n_u}$ is the vector of manipulated variables, $\mathbf{d} \in \mathbb{R}^{n_d}$ is the vector of process disturbances and $\mathbf{y} \in \mathbb{R}^{n_y}$ is the vector of measurements. $\mathbf{f} : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_d} \to \mathbb{R}^{n_x}$ is the set of differential equations and $\mathbf{h} : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \to \mathbb{R}^{n_y}$ is the measurement model. The reader is referred to Krishnamoorthy et al. (2016a) for detailed description of the model.

In a gas lifted well, the flow from the wells are predominantly controlled by the gas lift injection rates under normal operating conditions. Typically, well head chokes are used only under unusual conditions such as to dampen slug flow or to control casing-heading etc., which are not considered in this work for the sake of simplicity. Therefore, in this work, the manipulated inputs are set to be the gas lift injection rates $\mathbf{u} = [w_{gl,1}, \ldots, w_{gl,n_w}]^{\mathsf{T}}$.

Let the total oil production rate w_{to} be given by,

$$w_{to} = \sum_{i=1}^{n_w} w_{po,i} = \mathbf{g}(\mathbf{x}(t), \mathbf{u}(t)) \tag{2}$$

where, $\mathbf{g} : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \to \mathbb{R}$ and $w_{po,i}$ is the oil production rate from the i^{th} well..

Marginal GOR is defined as the change in oil rate per unit change in the gas lift rate which is equivalent to the steady-state gradient of (2) with respect to the control inputs. Let the marginal GOR be represented by the symbol ν

$$\nu_i = \frac{\partial w_{to}}{\partial w_{gl,i}} \quad \forall i \in \{1, \ldots, n_w\} \tag{3}$$

In order to control the marginal GOR, we propose a new approach to estimating the marginal GOR using transient measurements. The proposed method uses a dynamic model (1) online to estimate the states and the disturbances using any state estimator such as the extended Kalman filter (EKF) (Simon, 2006). The dynamic model from the total oil production rate (2) to the manipulated variables $\mathbf{u}$ is then linearized around the current operating point to get a local linear dynamic model approximation of the form,

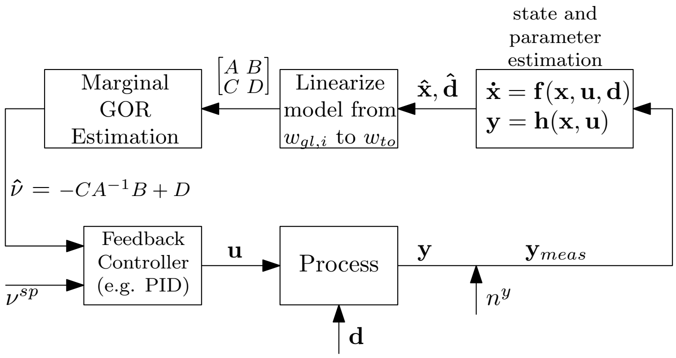

Fig. 1. The proposed method to estimate and control the steady-state gradient (marginal GOR) using transient measurements.

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{4a}$$
$$w_{to} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} \tag{4b}$$

where,

$$\mathbf{A} = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right|_{x=\hat{x}} \quad \mathbf{B} = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{u}}\right|_{x=\hat{x}}$$
$$\mathbf{C} = \left.\frac{\partial \mathbf{g}}{\partial \mathbf{x}}\right|_{x=\hat{x}} \quad \mathbf{D} = \left.\frac{\partial \mathbf{g}}{\partial \mathbf{u}}\right|_{x=\hat{x}}$$

The corresponding local linear steady-state model is given by setting $\dot{\mathbf{x}} = 0$. Consequently, the estimated steady-state marginal GOR is given by,

$$\hat{\boldsymbol{\nu}} = -\mathbf{C}\mathbf{A}^{-1}\mathbf{B} + \mathbf{D} \tag{5}$$

where, $\hat{\boldsymbol{\nu}} = [\nu_1 \ \dots \ \nu_{n_w}]^\mathsf{T}$ is the vector of estimated marginal GOR values.

The estimated marginal GOR can then be controlled using any feedback controller to a constant setpoint to achieve optimal operation. This is schematically represented in Fig.1. In the following subsections, we consider simple control structures to control the marginal GOR for different cases.

2.1 Unconstrained Optimization problem

In the unconstrained optimization case, we assume that there is an unlimited supply of gas available for gas lift, and the objective is to maximize the total oil production by computing the optimal gas lift injection rates $w_{gl,i}$ for all the wells i.

$$\min_{\mathbf{u}} \quad J = -\sum_{i=1}^{n_w} w_{po,i} \tag{6a}$$
s.t.
$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), \mathbf{d}(t)) \tag{6b}$$

$$\hat{\mathbf{J}}_{\mathbf{u}} = \frac{\partial J}{\partial \mathbf{u}} = \begin{bmatrix} \frac{\partial w_{to}}{\partial w_{gl,1}} \\ \vdots \\ \frac{\partial w_{to}}{\partial w_{gl,n_w}} \end{bmatrix} = \begin{bmatrix} \hat{\nu}_1 \\ \vdots \\ \hat{\nu}_{n_w} \end{bmatrix} \tag{7}$$

The estimated marginal GOR can then be controlled to drive the system to its optimum using any feedback controller such as a PI controller. To maximize the total oil production in the unconstrained case, the marginal GOR of each well is driven to a constant setpoint of $\nu_i^{sp} = 0$ (thus satisfying the necessary conditions of optimality).

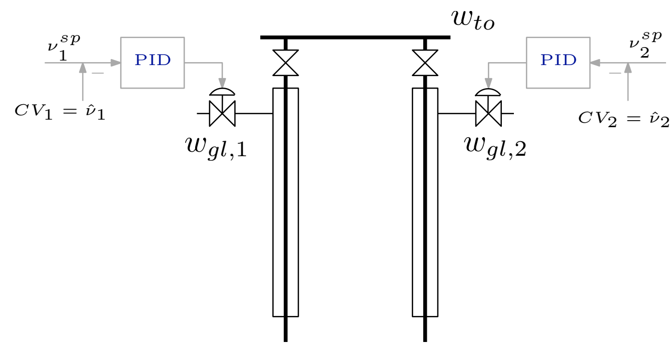

Fig. 2. A gas lifted field with $n_w = 2$ wells and the proposed controller design in the case of unlimited gas lift supply.

In many gas lifted fields, the objective is not only to maximize the oil production, but also minimize the usage of gas lift, due to the costs associated with compressing the gas. The modified cost function J' then has additional terms that penalizes input usage.

$$\min_{\mathbf{u}} \quad J = -c_o \sum_{i=1}^{n_w} w_{po} + c_{gl} \sum_{i=1}^{n_w} w_{gl,i} \tag{8a}$$
s.t.
$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), \mathbf{d}(t)) \tag{8b}$$

where c_o and c_{gl} are the value of oil and cost of gas compression respectively. In this case, the steady-state gradient of the modified cost function is given by

$$\hat{\mathbf{J}}_{\mathbf{u}} = \frac{\partial J}{\partial \mathbf{u}} = \begin{bmatrix} -c_o\hat{\nu}_1 + c_{gl} \\ \vdots \\ -c_o\hat{\nu}_{n_w} + c_{gl} \end{bmatrix} \tag{9}$$

At the optimum $\hat{\mathbf{J}}_{\mathbf{u}} = 0$. Therefore the estimated marginal GOR $\hat{\nu}_i$ in (5) is now controlled to a constant setpoint of $\nu_i^{sp} = c_{gl}/c_o$ to achieve optimal operation.

For a gas lifted field with n_w wells, we can use n_w decentralized PI controllers to control the marginal GOR as shown in Fig.2.

$$w_{gl,i} = \left(Kp_i + \frac{K_{I_i}}{s}\right)(\nu_i^{sp} - \hat{\nu}_i) \ , \forall i = \{1, \dots, n_w\} \tag{10}$$

2.2 Constrained Optimization problem

In many cases, the total gas available for gas lift is limited due to the limited compression capacity. In such cases, the total available gas lift must be optimally allocated among the different wells. The optimization problem can be written as,

$$\min_{\mathbf{u}} \quad J = -c_o \sum_{i=1}^{n_w} w_{po} + c_{gl} \sum_{i=1}^{n_w} w_{gl,i} \tag{11a}$$
s.t.
$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), \mathbf{d}(t)) \tag{11b}$$
$$\sum_{i=1}^{n_w} w_{gl,i} \leq w_{gl}^{max} \tag{11c}$$

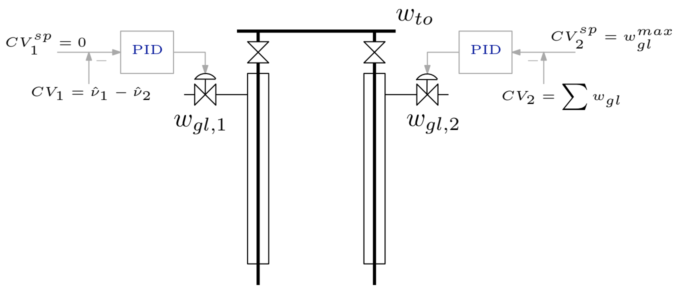

Fig. 3. A gas lifted field with $n_w = 2$ wells and the proposed controller design in the case of limited gas supply.

where w_{gl}^{max} is the maximum gas lift rate.

When the total available gas lift rate is insufficient to operate all the wells at their local optimum, then the optimum occurs when all the available gas is optimally allocated among the different wells, i.e. the maximum gas lift rate becomes active at the optimal operation. Therefore, according to good plant-wide control practice, one of the wells is used to control this active constraint tightly (Skogestad, 2000). The remaining $(n_w - 1)$ unconstrained degrees of freedom are used to maintain the marginal GOR of the wells to be equal. This is because the optimal operation of any parallel unit happens when the marginal cost of the different units (wells) are equal (Downs and Skogestad, 2011). The $(n_w - 1)$ self-optimizing controlled variables would then be

$$\nu_i - \nu_{i+1} \quad \forall i \in \{1, \dots, n_w - 1\} \tag{12}$$

In such a case, $(n_w - 1)$ feedback controllers are used to maintain equal marginal GOR and 1 controller is used to control the active constraint tightly. For example, in a production network with two wells, if well 2 is used to control the active constraint tightly, then well 1 maintains the marginal GOR of all the wells to be equal, i.e. the controlled variable is set to $\nu_1 - \nu_2$ and controlled to a constant setpoint of zero. This is schematically represented in Fig.3.

3. SIMULATION RESULTS

In this section, we demonstrate the proposed method for gas lift optimization using a case study with two gas lifted wells. For the state and parameter estimation, we use a discrete time extended Kalman filter with a sampling time of 1s as used by Krishnamoorthy et al. (2017). Decentralized PID controllers were used to control the estimated marginal GOR. The PI controllers were tuned using the SIMC tuning rules (Skogestad, 2003). The plant simulator was implemented using `IDAS` integrator.

The proposed method was compared with an centralized optimizing control structure. An economical nonlinear model predictive control was implemented with a sampling time of 5 min and a prediction horizon of 60 samples. The continuous time model was discretized using a third order collocation. The reader is referred to Krishnamoorthy et al. (2016a) for more detailed description on this.

3.1 Unconstrained case

In this subsection, we consider that the total gas available for gas lift is unlimited. In the first simulation, we want to maximize the oil production from the two wells using the cost function (6). The estimated marginal GOR for each well is then controlled to a constant setpoint of zero. The PI controller gains were tuned using the SIMC tuning rules and the resulting PI tuning values used for the two controllers are shown in Table 1. The disturbance enters in the form of step changes in gas-oil ratio (GOR) from the reservoir. The GOR for well 1 increases from 0.1 to 0.12 at time $t = 2h$ and the GOR of well 2 decreases from 0.12 to 0.1 at $t = 3h$. The simulation results using the proposed method compared with economic NMPC is shown in Fig.4.

Table 1. PI controller tunings.

		K_p	K_I
Unconstrained case	Well 1	7.0934	0.0149
	Well 2	11.1111	0.0214
Constrained case	Well 1	7.0934	0.0149
	Well 2	0	1

It can be clearly seen that the computed optimal gas lift rates by the proposed method converges to the same solution as the economic NMPC.

We then simulate the same problem, but now the objective is to maximize the oil production and at the same time minimize the costs associated with gas compression as shown in (8). The value of oil $c_o = 1\$$ and the cost of compression $c_{gl} = 0.5\$$. Therefore the marginal GOR of both wells are now controlled to a constant setpoint of $c_{gl}/c_o = 0.5$ instead of 0. The PI controller tunings were the same as shown in Table 1. We consider the same disturbances in GOR as in the previous simulation case. The simulation results are shown in Fig.5 and are compared with the solution provided by the economic NMPC. It can be clearly seen that the proposed method is able to provide similar performance as that of an economic MPC.

3.2 Limited Gas lift case

In this simulation case, we now consider that the total available gas for gas lift is limited to $w_{gl}^{max} = 4kg/s$. Well 1 is used to control the marginal GOR of the two wells to be equal, whereas well 2 tightly controls the active constraint. The proposed method estimates the marginal GOR of both the wells and a PI controller is used to control $\hat{\nu}_2 - \hat{\nu}_1$ to a constant setpoint of zero. By doing so, we ensure that the marginal GOR of the two wells would be equal. The PI controller tuning is shown in Table 1. We consider the same disturbances in GOR as in the previous simulation case. The simulation results are shown in Fig.6 and compared with the optimal solution provided by economic NMPC. It can be clearly seen that the proposed method is able to provide a similar performance as that of an economic MPC.

4. DISCUSSION AND CONCLUSIONS

In this paper, we presented some simple plant-wide control structure design for optimal operation of gas lifted wells.

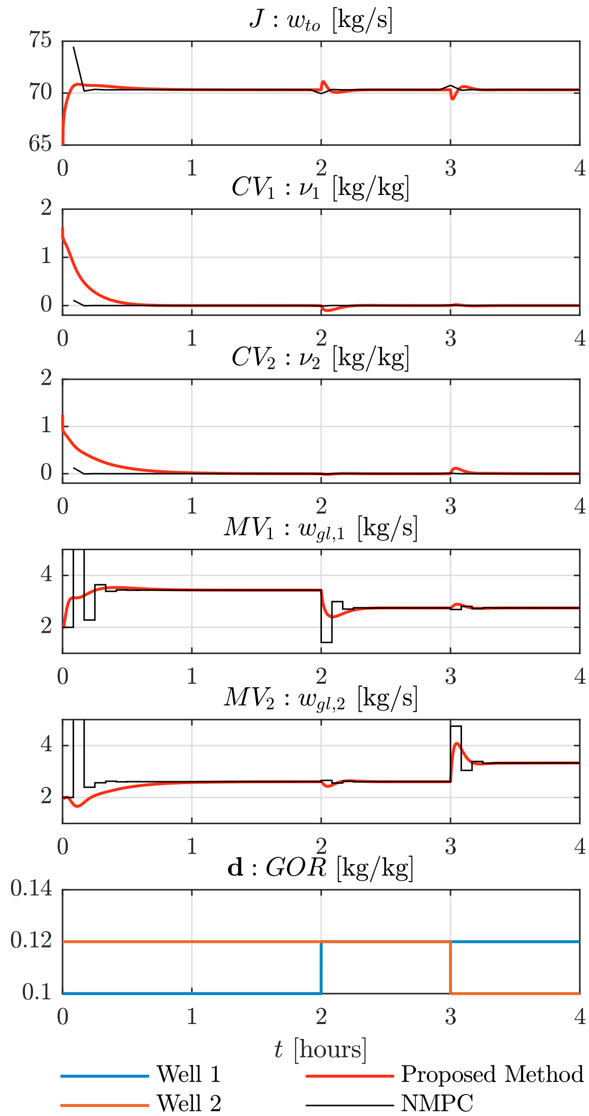

Fig. 4. Simulation results for the unconstrained case when the cost function is given by (6). The proposed method is shown in red lines and the economic NMPC is shown in thin black lines.

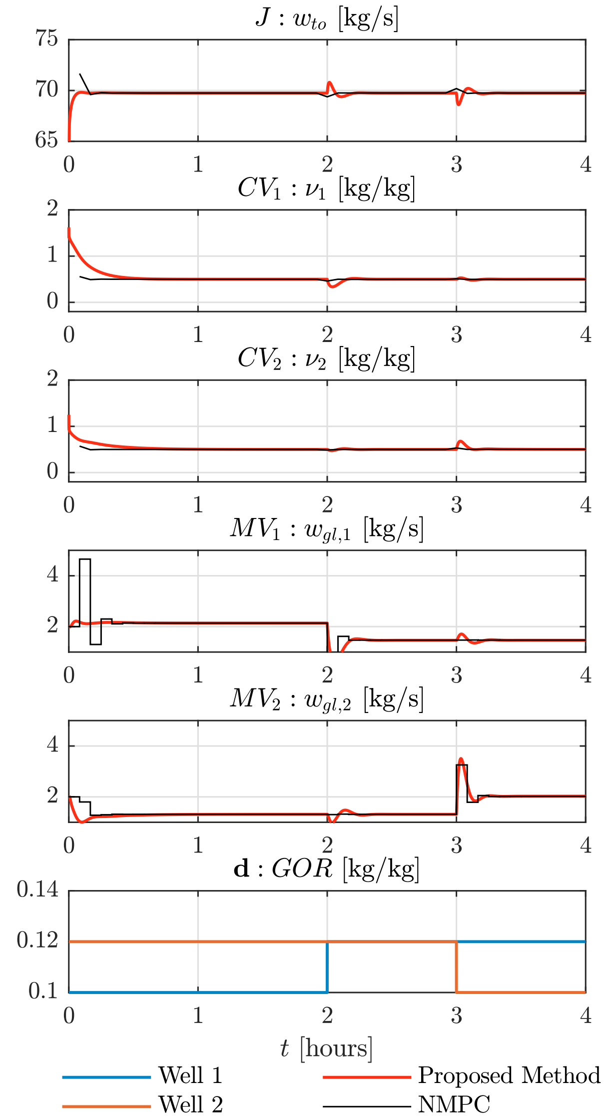

Fig. 5. Simulation results for the unconstrained case when the cost function is given by (8). The proposed method is shown in red lines and the economic NMPC is shown in thin black lines.

We showed that optimal operation can be achieved by using a dynamic model online to estimate the marginal GOR using transient measurements and control the marginal GOR to constant setpoints. The performance of the proposed control strategy was compared to economic NMPC and was shown to provide similar response as the economic NMPC. The proposed method is based on feedback control and hence is computationally cheap to implement. The average computation times for the case study used here were $0.005s$ for the proposed method as opposed to $0.924s$ for the economic NMPC. The proposed control structure is easier for the operators to understand. Additionally, the PI controllers are also easier to tune and maintain than the economic NMPC solution.

It is however important to note that when the active constraint set changes, then the control structure design is different as shown in Fig.2 and Fig.3. This may require redesign and re-tuning of the PI controllers. The economic NMPC can however easily handle changes in the active constraint set.

Rashid (2010) noted that, in practice, the wells are often considered independently neglecting the back-pressure effects imposed by interconnected wells. Optimization based on marginal GOR from individual gas lift performance curves may only lead to pseudo-steady-state solutions. The proposed method can include the interaction terms as well, thereby overcoming this limitation.

In terms of plant model mismatch, since the model used in the proposed method and the economic NMPC are the same, both the methods are equally affected by the plant-model mismatch. Model-free slow optimizing controllers such as extremum seeking control or NCO-tracking control

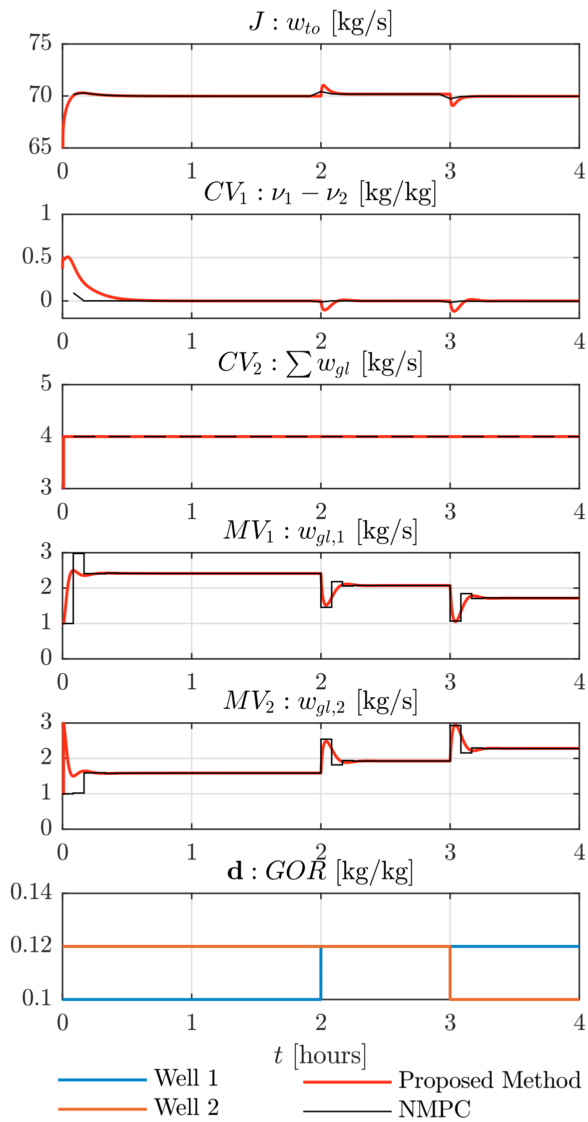

Fig. 6. Simulation results when the total available gas for injection is limited to $w_{gl}^{max} = 4kg/s$. The proposed method is shown in red lines and the economic NMPC is shown in thin black lines.

can be employed on top of the proposed method to account for any plant-model mismatch (Jäschke and Skogestad, 2011; Straus et al., 2017). In the simulation case study shown here, the same model structure was used to estimate the marginal GOR and in the plant simulator. A more realistic case would be to test with the plant modelled in advanced multiphase simulators such as OLGA which is an ongoing work.

REFERENCES

Alstad, V. (2005). *Studies on selection of controlled variables*. Ph.D. thesis, Norwegian University of Science and Technology (NTNU).

Bieker, H.P., Slupphaug, O., Johansen, T.A., et al. (2007). Real-time production optimization of oil and gas production systems: A technology survey. *SPE Production & Operations*, 22(04), 382–391.

Chachuat, B., Srinivasan, B., and Bonvin, D. (2009). Adaptation strategies for real-time optimization. *Computers & Chemical Engineering*, 33(10), 1557–1567.

Codas, A., Jahanshahi, E., and Foss, B. (2016). A two-layer structure for stabilization and optimization of an oil gathering network. *IFAC-PapersOnLine*, 49(7), 931–936.

Downs, J.J. and Skogestad, S. (2011). An industrial and academic perspective on plantwide control. *Annual Reviews in Control*, 35(1), 99–110.

Jäschke, J. and Skogestad, S. (2011). NCO tracking and self-optimizing control in the context of real-time optimization. *Journal of Process Control*, 21(10), 1407–1416.

Kanu, E.P., Mach, J., Brown, K.E., et al. (1981). Economic approach to oil production and gas allocation in continuous gas lift (includes associated papers 10858 and 10865). *Journal of Petroleum Technology*, 33(10), 1–887.

Krishnamoorthy, D., Foss, B., and Skogestad, S. (2017). Gas lift optimization under uncertainty. *Computer Aided Chemical Engineering*, 40, 1753–1758.

Krishnamoorthy, D., Jahanshahi, E., and Skogestad, S. (2018). A feedback rto strategy using transient measurements. *Journal of Process Control*, In preparation.

Krishnamoorthy, D., Foss, B., and Skogestad, S. (2016a). Real time optimization under uncertainty - applied to gas lifted wells. *Processes*, 4(4). doi:10.3390/pr4040052.

Krishnamoorthy, D., Pavlov, A., and Li, Q. (2016b). Robust extremum seeking control with application to gas lifted oil wells. *IFAC-PapersOnLine*, 49(13), 205–210.

Krstić, M. and Wang, H.H. (2000). Stability of extremum seeking feedback for general nonlinear dynamic systems. *Automatica*, 36(4), 595–601.

Peixoto, A.J., Pereira-Dias, D., Xaud, A.F., and Secchi, A.R. (2015). Modelling and extremum seeking control of gas lifted oil wells. *IFAC-PapersOnLine*, 48(6), 21–26.

Rashid, K. (2010). Optimal allocation procedure for gas-lift optimization. *Industrial & Engineering Chemistry Research*, 49(5), 2286–2294.

Sharma, R. and Glemmestad, B. (2013). On generalized reduced gradient method with multi-start and self-optimizing control structure for gas lift allocation optimization. *Journal of Process Control*, 23(8), 1129–1140.

Simon, D. (2006). *Optimal state estimation: Kalman, H infinity, and nonlinear approaches*. John Wiley & Sons.

Skogestad, S. (2000). Plantwide control: the search for the self-optimizing control structure. *Journal of process control*, 10(5), 487–507.

Skogestad, S. (2003). Simple analytic rules for model reduction and PID controller tuning. *Journal of Process Control*, 13(4), 291–309.

Skogestad, S. and Postlethwaite, I. (2007). *Multivariable feedback control: analysis and design*. Wiley New York, 2 edition.

Straus, J., Krishnamoorthy, D., and Skogestad, S. (2017). Combining self-optimizing control and extremum seeking control- applied to ammonia reactor.

Urbanczyk, C.H., Wattenbarger, R.A., et al. (1994). Optimization of well rates under gas coning conditions. *SPE Advanced Technology Series*, 2(02), 61–68.