Controller and observer design for first order LTI systems with unknown dynamics

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ABSTRACT

The design of controllers and observers often relies on first order models of the system in question. These models are often obtained either through step-response tests, through on-line or off-line identification or through development of a mathematical model. When the system in question has unknown or uncertain parameters, the developed model also contains uncertainties and the controller/observer design may result in bad performance or even instability. In this paper we present a combined design of a controller and an observer for scalar linear time-invariant systems with unknown parameters. We combine a model reference adaptive controller, which does not require a model of the system, with a Luenberger observer which uses the desired closed-loop dynamics as its model. We show through Lyapunov theory and by application of Barbălat's lemma that all error states in the closed-loop system converge to zero and that all signals are bounded.

CCS CONCEPTS

• **Computing methodologies → Control methods**; Modeling and simulation; • **Applied computing** → *Engineering*;

KEYWORDS

Adaptive control, estimation, unknown systems

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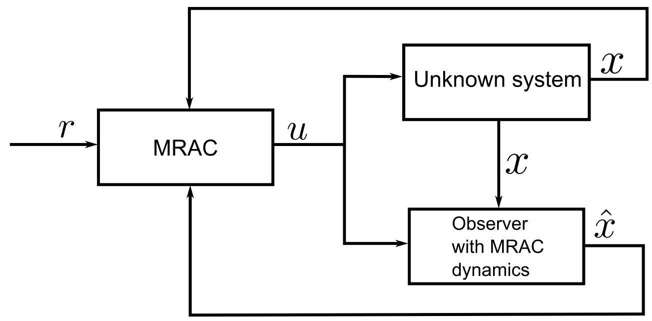

Figure 1: The proposed method consists of a MRAC and an observer with the same dynamics as the reference model.

1 INTRODUCTION

Controller and observer design for linear time-invariant (LTI) systems has for decades revolved around the assumption that the system dynamics, or at least an approximation of these, are known. A dynamic model of the system, or some knowledge about the open-loop behaviour, is often required in order to tune controllers and observers. The well-known, and widely used, Ziegler-Nichols [11] and Skogestad IMC (SIMC) [7] tuning methods cannot be applied without knowledge of the systems critical gain and oscillation period, and the gain and time constant, respectively.

The models are often obtained by constructing a mathematical model based on physical properties of the system (first-principle models) and linearizing this around the desired operating point. This model will only be valid in a region close to the linearization point. The model can also be obtained through step response analysis [6, Ch. 4], or through on-line or off-line identification/estimation of the system parameters [1, Ch. 4]. The last two approaches requires that the plant we wish to control is up and running and that the necessary measurements exist. This may not be feasible for all plants.

These methods of modelling, though quite different, have one thing in common: They all attempt to find out what the dynamics of the real system is and design an observer or a controller based on these dynamics. It is well known that any model of a system is an approximation at best. If the process itself changes, e.g. an actuator is worn down, or physical parameters, such as mass or pressure, changes, the linear controller may fail or perform worse than originally intended. Moreover, if there are uncertainties in the model parameters, it is impossible to guarantee that the designed linear controller or observer will work.

To handle the uncertainty often found in systems we can introduce measures to increase the robustness of our controller or observer. This can be done through non-linear control methods

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e.g., sliding-mode controllers (see for instance [2]) or adaptive controllers (see for instance [1]), and in the observer case through high-gain observers [3].

In this paper we introduce a different approach to the design of controllers and observers for unknown systems. This paper contains preliminary work and hence, only considers first-order LTI systems. Since the unknown system dynamics will converge to the dynamics of the reference model used in the MRAC scheme, we can design the observer based on these known dynamics, instead of on the unknown open-loop dynamics. A block diagram of our proposed method is shown in Figure. 1. In this paper we show that this approach also works during the transient period.

To the best of the authors knowledge the specific idea presented here is novel. Only two related works [9, 10] could be found in published literature.

This idea is touched upon in [9] where it is argued that an observer cannot be designed for a system with unknown parameters and [9] propose that the observer should contain the known closed-loop dynamics. A stability proof for the observer/controller solution is not presented in [9], but the scheme is demonstrated through simulations on a discrete-time system.

In [10], an observer-based solution for non-linear systems is presented with stability proofs. The observer is designed based on a reference model and the controller is an iterative learning controller based on a filtered fuzzy neural network. The general idea is the same as the one presented in this paper, but in this paper we use a simple model reference adaptive controller instead of a fuzzy neural network, and since this paper contains preliminary work, we do not consider non-linear systems.

In this paper we present a new adaptive control method for systems with unknown dynamics and we prove the stability of the method through Lyapunov theory. The stability proof also holds during the transient period.

2 PROBLEM FORMULATION

Consider the scalar linear time-invariant system

$$\dot{x} = ax + bu\,. \tag{1}$$

Designing a controller for this system if a and b were known would be rather trivial: We choose $u = -k^* x$ and choose k^* such that $a - bk^* < 0$.

2.1 Control design for unknown systems

If the parameters of the system in (1) are unknown or uncertain, one possible controller choice is to use model reference adaptive control (MRAC) [1, Ch. 6]. When designing an MRAC for the system in (1), we do not concern ourselves with the dynamics of the real system as we are only required to know $\operatorname{sign}(b)$.

The goal of MRAC is to force the output or state of the system to track the output or state of a reference model with dynamics

$$\dot{x}_m = -a_m x_m + b_m r \tag{2}$$

where $a_m > 0$ and $b_m \neq 0$ are chosen by the control designer. If we know a and b of (1) we can simply choose

$$u = -k^* x + l^* r \tag{3}$$

where $k^* = (a_m + a)/b$ and $l^* = b_m/b$.

Since we do not know a and b we must instead use estimates of the controller gains, $\hat{k}$ and $\hat{l}$. These estimates are updated by respective update laws that ensures convergence of the system state to the state of the reference model. Boundedness of the controller gains are also ensured.

2.2 Observer design for unknown systems

If a and b of (1) are known the Luenberger observer

$$\dot{\hat{x}} = a\hat{x} + bu + L(x - \hat{x}) \tag{4}$$

where $L > 0$, can be designed trivially. The observed state can be used in a feedback control law $u = -k^*\hat{x}$. By application of the well-known separation principle [8, Ch. 9.2], we can guarantee that the observer error converges to zero and that the controlled state converges to the desired value. Furthermore, the stability of the closed-loop system can be determined by choosing, independently, suitable controller and observer gains.

If a and b are unknown we cannot use the Luenberger observer in (4). We could instead use an adaptive observer [1, Ch. 5] and identify a and b, but this would require a persistently exciting signal to guarantee that the estimated a and b converge to the real values.

A high gain observer, [3], could be used for unknown systems, but our focus in this paper is to present an alternative to this method.

3 COMBINED CONTROLLER AND OBSERVER DESIGN

We consider the system given in (1) with unknown a and b, with the following assumptions:

Assumption 1. *In* (1), *the maximum value of a is known, i.e.,* $|a| \leq a_{\max}$.

Assumption 2. *In* (1), $b \neq 0$ *and* $\operatorname{sign}(b)$ *is known.*

Assumption 3. *In* (1), *a and b are constant.*

Theorem 1 (Main result). *Let the system be given by* (1) *with unknown a and b satisfying Assumptions 1–3. Let the controller be given by*

$$u = -\hat{k}\hat{x} + \hat{l}r \tag{5}$$

where $\hat{k}$ and $\hat{l}$ satisfy the update laws

$$\dot{\hat{k}} = \operatorname{sign}(b)\gamma_1 \hat{x}(e_1 m_1 + e_2 m_2) \tag{6}$$

$$\dot{\hat{l}} = -\operatorname{sign}(b)\gamma_2 r(e_1 m_1 + e_2 m_2) \tag{7}$$

with $\gamma_1, \gamma_2, m_2 > 0$, $m_1 = c a_m m_2$, $c > 0$ and the state estimate $\hat{x}$ is given by

$$\dot{\hat{x}} = -a_m \hat{x} + b_m r + L(x - \hat{x}) \tag{8}$$

where $a_m \geq \epsilon > 0$, $b_m \neq 0$ and L satisfies $L \geq \bar{\epsilon}_2 + \frac{c}{4}(a_{\max} + a_m)^2 + a_{\max}$ with $\bar{\epsilon}_2 > 0$. Furthermore, let the reference model be given by

$$\dot{x}_m = -a_m x_m + b_m r\,, \tag{9}$$

the error variables by

$$e_1 = x - x_m \tag{10}$$

$$e_2 = x - \hat{x}\,, \tag{11}$$

and let r be a bounded signal. Then e_1, e_2, x, x_m, $\hat{x}$, $\hat{k}$ and $\hat{l}$ are bounded and e_1, e_2 converge to zero.

Proof. Consider the observer system

$$\dot{\hat{x}} = -a_m \hat{x} + b_m r + L(x - \hat{x}) \tag{12}$$

where $L > 0$, r is a bounded reference and $a_m > 0$ and $b_m \neq 0$ are the desired time constant and gain of the reference system

$$\dot{x}_m = -a_m x_m + b_m r \ . \tag{13}$$

Trivially, x_m is bounded.

If a and b of (1) had been known, we could have chosen u as in (3) and guaranteed that the transfer function of the closed-loop plant $x(s)/r(s)$ is equal to that of the reference model $x_m(s)/r(s)$. By adding and subtracting $-k^* x + l^* r$ to (1) and inserting (5) we get

$$\begin{aligned}\dot{x} &= ax + b\left(k^* x - k^* x + l^* r - l^* r + \hat{k}\hat{x} + \hat{l}r\right) \\ &= (a - bk^*)x + bl^* r + b\left(k^* x - l^* r + \hat{k}\hat{x} + \hat{l}r\right) \\ &= -a_m x + b_m r + bk^* x - b\hat{k}\hat{x} + b\tilde{l}r \ .\end{aligned} \tag{14}$$

where $\tilde{l} = \hat{l} - l^*$. Now we define the error variable $e_1 = x - x_m$ with time derivative

$$\begin{aligned}\dot{e}_1 &= -a_m x + b_m r + bk^* x - b\hat{k}\hat{x} + b\tilde{l}r + a_m x_m - b_m r \\ &= -a_m e_1 + b\tilde{l}r + bk^* x - b\hat{k}\hat{x} \ .\end{aligned} \tag{15}$$

To this we add and subtract the term $bk^*\hat{x}$, which gives

$$\begin{aligned}\dot{e}_1 &= -a_m e_1 + b\tilde{l}r + bk^*(x - \hat{x}) + b\hat{x}\left(k^* - \hat{k}\right) \\ &= -a_m e_1 + b\tilde{l}r + bk^* e_2 - b\tilde{k}\hat{x}\end{aligned} \tag{16}$$

where $e_2 = x - \hat{x}$ and $\tilde{k} = \hat{k} - k^*$.

Using the observer in (12), the error variable $e_2 = x - \hat{x}$ has time derivative

$$\begin{aligned}\dot{e}_2 &= -a_m x + b_m r + bk^* x + b\tilde{l}r - b\hat{k}\hat{x} + a_m \hat{x} - b_m r - Le_2 \\ &= -(a_m + L)e_2 + b\tilde{l}r + bk^* x - b\hat{k}\hat{x} \ .\end{aligned} \tag{17}$$

We again add and subtract the term $bk^*\hat{x}$ and get

$$\begin{aligned}\dot{e}_2 &= -(a_m + L)e_2 + b\tilde{l}r + bk^* e_2 - b\tilde{k}\hat{x} \\ &= -(a_m + L - bk^*)e_2 + b\tilde{l}r - b\tilde{k}\hat{x} \\ &= -(L - a)e_2 + b\tilde{l}r - b\tilde{k}\hat{x}\end{aligned} \tag{18}$$

where we use $bk^* = a + a_m$.

To investigate the stability of the error system $\mathbf{e} = \begin{bmatrix} e_1 & e_2 \end{bmatrix}^T$ we propose the Lyapunov function candidate

$$V(\mathbf{e}, \tilde{k}, \tilde{l}) = \frac{1}{2}\mathbf{e}^T \mathbf{M}\mathbf{e} + \frac{|b|}{2\gamma_1}\tilde{k}^2 + \frac{|b|}{2\gamma_2}\tilde{l}^2 \tag{19}$$

where $\gamma_1, \gamma_2 > 0$ are constants and

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \tag{20}$$

with $m_1, m_2 > 0$.

The time derivative of (19) along the system trajectories is

$$\begin{aligned}\dot{V} &= -a_m m_1 e_1^2 + m_1(a + a_m)e_1 e_2 + b\tilde{l}rm_1 e_1 \\ &\quad - b\tilde{k}\hat{x}m_1 e_1 - (L - a)m_2 e_2^2 + b\tilde{l}rm_2 e_2 \\ &\quad - b\tilde{k}\hat{x}m_2 e_2 + \frac{|b|}{\gamma_1}\tilde{k}\dot{\tilde{k}} + \frac{|b|}{\gamma_2}\tilde{l}\dot{\tilde{l}} \ .\end{aligned} \tag{21}$$

Since k^* and l^* are constants, $\dot{\tilde{k}} = \dot{\hat{k}}$ and $\dot{\tilde{l}} = \dot{\hat{l}}$. We gather some terms and express (21) as

$$\begin{aligned}\dot{V} &= -\mathbf{e}^T \mathbf{Q}\mathbf{e} + \tilde{k}\left(\frac{|b|}{\gamma_1}\dot{\hat{k}} - b\hat{x}(m_1 e_1 + m_2 e_2)\right) \\ &\quad + \tilde{l}\left(\frac{|b|}{\gamma_2}\dot{\hat{l}} + br(m_1 e_1 + m_2 e_2)\right)\end{aligned} \tag{22}$$

where

$$\mathbf{Q} = \begin{bmatrix} a_m m_1 & -\frac{1}{2}m_1(a + a_m) \\ -\frac{1}{2}m_1(a + a_m) & m_2(L - a) \end{bmatrix} \ . \tag{23}$$

Inserting the update laws defined in (6) and (7) reduces (22) to

$$\dot{V} = -\mathbf{e}^T \mathbf{Q}\mathbf{e} \tag{24}$$

To ensure that $\mathbf{Q}$ is positive definite we require $a_m m_1 \geq \epsilon_1 > 0$ and $\det(\mathbf{Q}) \geq \epsilon_2 > 0$ where ϵ_1, ϵ_2 are constants. Thus,

$$\det(\mathbf{Q}) = a_m m_1 m_2 (L - a) - \frac{1}{4}(a + a_m)^2 m_1^2 \geq \epsilon_2 > 0 \tag{25}$$

We choose $\epsilon_2 = \bar{\epsilon}_2 a_m m_1 m_2$ and $m_1 = ca_m m_2$, where $\bar{\epsilon}_2, c > 0$ are constants, reducing (25) to

$$L \geq \bar{\epsilon}_2 + \frac{c}{4}(a + a_m)^2 + a \ . \tag{26}$$

The above is satisfied if

$$L \geq \bar{\epsilon}_2 + \frac{c}{4}(a_{\max} + a_m)^2 + a_{\max} \ . \tag{27}$$

Thus

$$\dot{V} = -\mathbf{e}^T \mathbf{Q}\mathbf{e} \leq 0 \tag{28}$$

and the origin of the $e, \tilde{k}, \tilde{l}$ system is stable [2, Th. 4.1]. This also implies that $x, \hat{x}, \hat{k}$ and $\hat{l}$ are bounded.

We now look at the time derivative of (28), i.e.,

$$\begin{aligned}\ddot{V} &= -2\mathbf{e}^T \mathbf{Q}\dot{\mathbf{e}} - \mathbf{e}^T \dot{\mathbf{Q}}\mathbf{e} \\ &= -2\mathbf{e}^T \mathbf{Q}\dot{\mathbf{e}} \ .\end{aligned} \tag{29, 30}$$

From (16) and (18) and the above, $\dot{\mathbf{e}}$ is bounded, and thus so is $\ddot{V}$. Hence, Barbălat's lemma, as described in [2, Ch. 8.3] can be applied and both boundedness of V and convergence of V to zero is ensured, i.e., asymptotic stability of e_1 and e_2 and boundedness of $\tilde{k}$ and $\tilde{l}$ is proven. □

4 SIMULATION RESULTS

4.1 Applied to an example system

The proposed solution is applied to a first order example system on the form given in (1). The parameters a and b are chosen randomly from a set $a, b \in [-10, 10]$ where $b \neq 0$ and $a_{\max} = 10$. The sign of b is given to the controller.

Note that the system can be either open-loop stable or unstable depending on the sign of a. The parameters used in the simulations are given in Table 1.

In the first simulation we choose $a = 10$, to demonstrate that the method works when the open-loop system is at its most unstable, and $b = 6.6$ was chosen randomly. For comparison, we also simulate

Table 1: Simulation parameters

Parameter:	a_m	b_m	$\bar{\epsilon}_2$	c	m_2	m_1	L	γ_1, γ_2
Value:	1	1	1	0.1	10	1	14	10

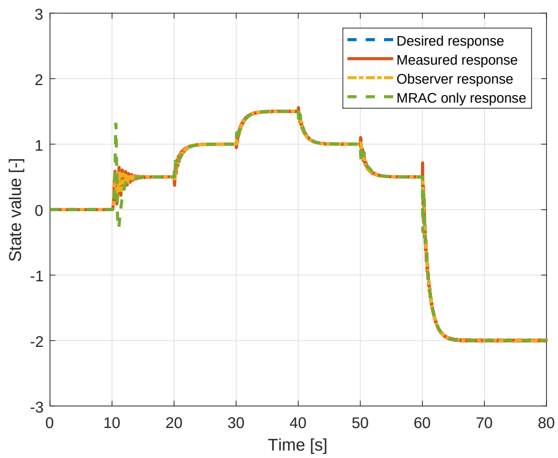

Figure 2: The observer state and the measured state tracks the state of the reference system.

using a standard MRAC without the observer, i.e., the update laws are

$$\dot{\hat{k}} = \text{sign}(b)\gamma_1 x e_1$$
$$\dot{\hat{l}} = -\text{sign}(b)\gamma_2 r e_1 \tag{31}$$

As can be seen from Figure 2, the observer state response and the measured state response converges to the desired state response for a variety of setpoint changes. When we use the MRAC alone, however, the convergence during the first step is significantly worse than for the case when we use the proposed method. The presence of the extra terms in the update laws $\dot{\hat{k}}$ and $\dot{\hat{l}}$ when we use the proposed method is the cause of this difference. In Figure 3 we see the controller parameters. The parameters are oscillating much more when we use the proposed method because of the extra term present in the update laws.

We perform another simulation where we choose a random, but stable, $a = -3.7$ and a smaller $b = 0.57$. The controller tuning used is the same as in the first simulation. As can be seen in Figure 4 the states are not oscillatory, as was the case in the first simulation. We see the same behaviour for the case where we only use the MRAC as in the first simulation. The convergence during the first step is slower. The controller parameters, shown in Figure 5, are quite similar for both cases, but we see that the extra term present in the update laws for the proposed solution causes a slightly faster convergence during the first step.

We then apply a sinusoidal reference and use the same controller parameters and open-loop system from the first simulation. In Figure 6 we see that the states are able to track the desired response given by the reference model. The convergence is better when we use the proposed method compared to an MRAC alone. In this case, the controller parameters converge to the actual values as seen in

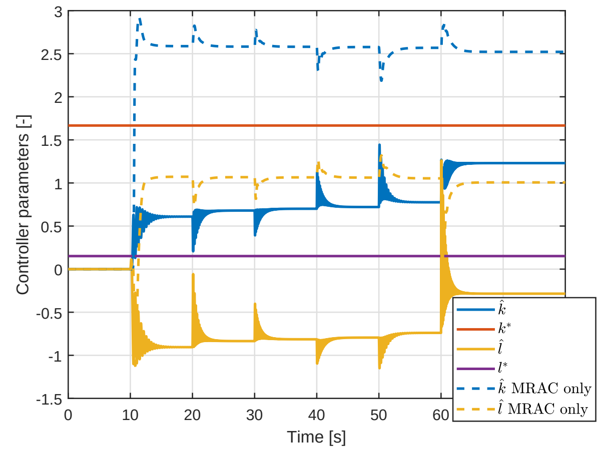

Figure 3: The controller parameters during the first simulation.

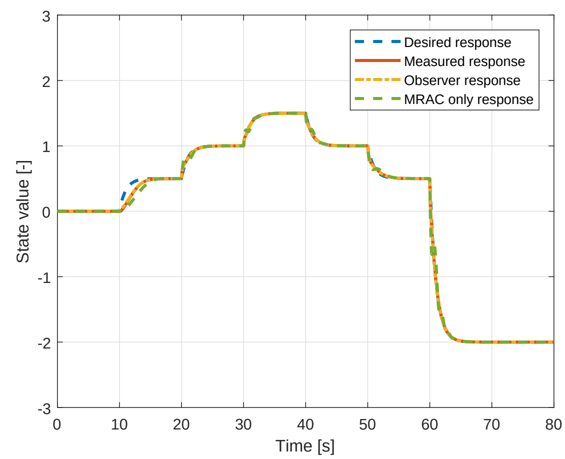

Figure 4: When the open loop system is stable, but has a low gain, the performance during the first transient is less oscillatory.

Figure 7. The convergence is faster for the proposed method than for a conventional MRAC due to the additional terms present in the update laws.

4.2 Applied to a complex system

At this stage, the method has only been proven for a scalar, linear case. In this section, we use the method on a complex, nonlinear vector case in simulation. We do not present a proof, and include it here only to illustrate the method's potential broader applicability.

We consider a model of a gas liquid cylindrical cyclone (GLCC), presented in [4]. The GLCC is a compact gas/liquid separation device used in the oil and gas industry.

To ensure effective separation of gas and liquid it is important to keep the liquid level at a certain setpoint. The gas pressure in the GLCC must also be controlled in order to ensure safe operation. The GLCC model contains nonlinearities in the valve-openings controlling the liquid level and gas pressure as well as highly complex

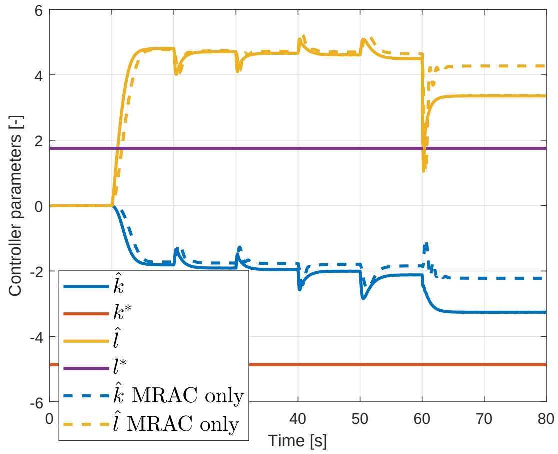

Figure 5: The controller parameters are not subject to the same oscillations as in the first simulation since the open loop system is stable.

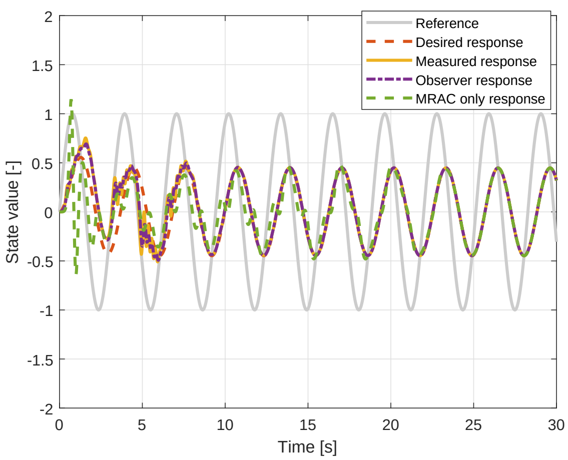

Figure 6: When we apply a sinusoidal reference the measured state and the observer state tracks the output of the reference model.

nonlinearities describing the separation factors. The entire model of the GLCC is too extensive to present here, but the liquid level and gas pressure can be described by [5]

$$\dot{h} = \frac{1}{\alpha}\left(f_1 + f_{1,s} - w_l\right) \tag{32a}$$

$$\dot{p} = \frac{\beta}{\alpha H - m_l}\left(f_3 + f_{3,s} - \sigma w_g + \frac{m_g(f_1 + f_{1,s} - w_l)}{aH - m_l}\right) \tag{32b}$$

where α and β are model equations, H is the height of the GLCC separator, the functions f_x describes inlet flows and separation flows between the gas and liquid phases (these functions are highly nonlinear), σ is the gas mass fraction in the gas outlet (also highly nonlinear), m_g and m_l is the total amount of gas and liquid in the GLCC, respectively, and w_g, w_l are the outlet mass flows described

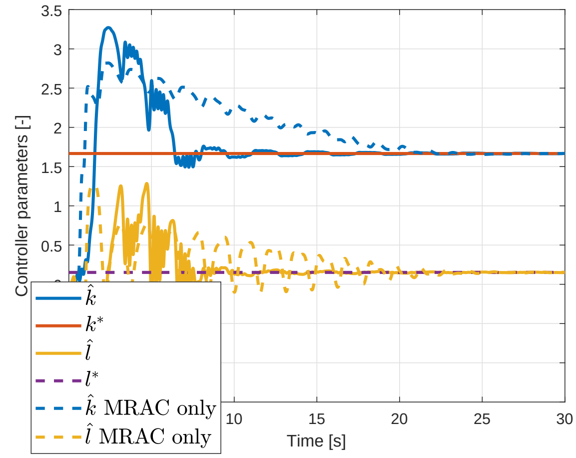

Figure 7: The controller parameters converges to the true values for k^* and l^* when the reference is persistently exciting.

Table 2: Simulation parameters, GLCC system

Param.:	a_m	b_m	$\bar{\epsilon}_2$	c	m_2	m_1	L	γ_1, γ_2	$a_{\max}$
w_l:	0.1	0.1	1	1	10	1	36.5	1	10
w_g:	0.1	0.1	1	1	10	1	36.5	0.01	10

using a valve equation on the form

$$w_x = C_{d,x} A_x z_x \sqrt{\rho_x \Delta P_x} \tag{33}$$

where $C_{d,x}$ is the valve coefficient, A_x is the cross-sectional area of the valve, z_x is the valve opening, ρ_x is the density of the fluid and ΔP_x is the pressure differential over the valve.

We used the proposed control structure to control the GLCC, using two decoupled controllers, w_l and w_g, controlling liquid level h and pressure p, respectively. The sign of b is easily deduced from (32), (33) as an open valve reduces liquid level and pressure. Trial and error was used to determine an appropriate L.

In use, GLCCs are usually given constant set-points. The parameters used for the GLCC controllers is listed in Table 2. The model parameters are as in [4].

As can be seen from Figure 8, the controller/observer method also appears to work on more advanced models containing nonlinearities. In the simulation, we also introduce disturbances in the form of changes in the mass flow and gas mass fraction going into the GLCC. The controller is able to handle these disturbances very well. A conventional MRAC is also applied to this system and we see a similar behaviour as we did in the simulations on the example system. The conventional MRAC has the same adaptation gain as the proposed method. Looking at the controller parameters in Figure 9, we see that $\hat{k}$ and $\hat{l}$ are, for both controllers, quite small. This could indicate that the system is open-loop stable or that the open-loop gain is high such that little control action is required to bring the states to the desired values.

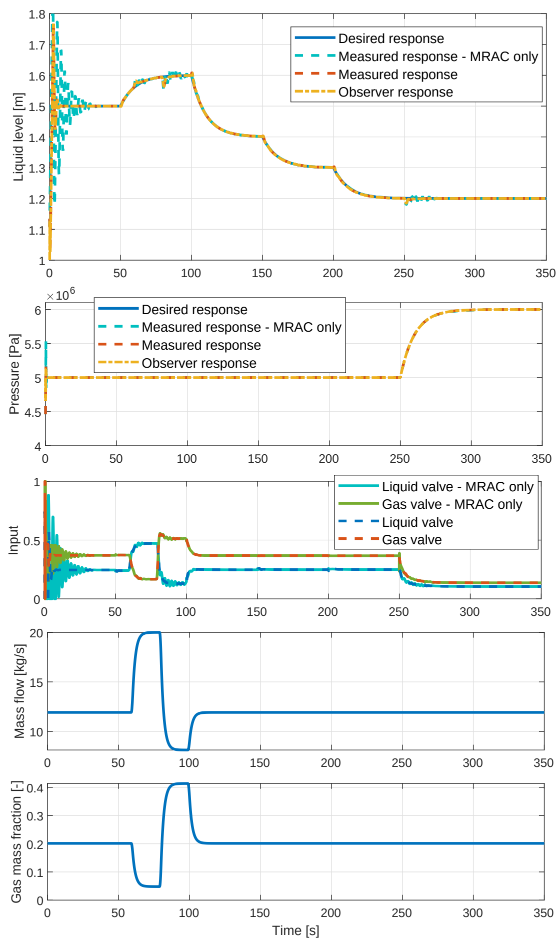

Figure 8: Simulation of the proposed method on a GLCC system. The observer state is used as input to the controller and the internal state tracks the state of the reference model.

5 CONCLUSION

In this paper we have presented an approach for designing a controller and an observer for first-order linear time-invariant systems with unknown dynamics. The controller is similar to a conventional MRAC, but the update laws are different. The observer is a Luenberger observer which uses the reference model dynamics of the MRAC, rather than the (unknown) plant dynamics, as its model. The main result proves, by the use of Lyapunonv theory, that the system state converges to the reference model state, and that the observer state converges to the system state. By utilizing this method we have shown that we can, without any identification of the open-loop model, design a controller and an observer for a first-order LTI system.

We show trough simulations that the method can handle a variety of unknown systems as long as the observer gain is chosen appropriately, and that the transient performance is slightly better than that of a classical MRAC working alone. We also apply the method to a more complex system to illustrate that the same basic method may work on more complex systems. However, proving that this is the case remains future work.

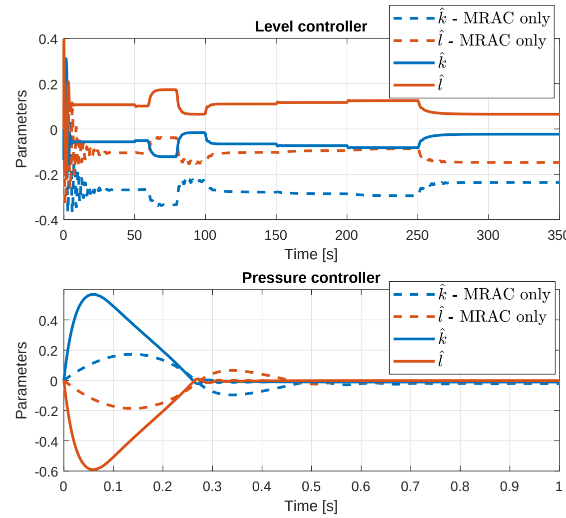

Figure 9: The controller parameters during the GLCC simulation. The parameters for the pressure controller is only shown for the first second to better display the difference.

The results presented here is only the first step. As future work we would like to extend the stability proof to systems with nonlinear dynamics and to vectorial systems. Another interesting application of the idea is to replace the Luenberger observer with a Kalman filter which should give the system increased robustness against noise.

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