

## Improved PI control for a surge tank satisfying level constraints<sup>\*</sup>

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**Abstract:** This paper considers the case of averaging level control, where the main objective is to reduce flow variations by using varying liquid levels. However, to avoid overflowing or emptying the tank, the liquid level needs to satisfy safety-related constraints. In the simplest case, a P-controller can be used, but may not give acceptable averaging of the flow, especially if the surge tank is relatively small. In addition, the P-controller does not allow the level setpoint to be adjusted. We propose a simple scheme with a PI-controller for normal operation and two high-gain P-controllers to avoid the liquid level constraints, which is compared with a benchmark MPC strategy. We demonstrate that the proposed method has similar performance, but with less modeling effort, less computational time and simpler tuning.

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### 1. INTRODUCTION

Liquid level control can have two purposes (Shinskey, 1988; Faanes and Skogestad, 2003): to tightly control the level (setpoint tracking) or to dampen flow disturbances. The latter, where the tank acts as a surge tank, is also known as averaging level control and is the focus in this paper. The controller tuning for the two cases are completely different, because for tight level control we need a high controller gain, whereas for averaging level control we want a low controller gain. For a surge tank, the actual value of the level may not be important as long as it is kept within its allowable safety limits (Shinskey, 1988; Åström and Hägglund, 1995), that is, to avoid overflowing or emptying the tank.

Fields of applications for setpoint tracking and safety control for levels in tanks are as diverse as drum boilers in power plants, where both, dry-running and complete filling should be avoided (Åström and Bell, 2000), gravity separators in the mining as well as the oil- and gas industry, where setpoint tracking and avoidance of complete filling are the most important control tasks (Backi and Skogestad, 2017), and waste-water sumps in the chemical industry and surge tanks (Åström and Hägglund, 2001). Especially for the latter, minimization of the change in the outflow is highly desired, since the incoming surge should be distributed further with reduced amplitude. In recent years, not only PI(D) controllers were designed for level control of tanks, but also fuzzy control approaches (Tani et al., 1996; Petrov et al., 2002), as well as optimal averaging strategies (McDonald et al., 1986; Campo and Morari, 1989; Rosander et al., 2012).

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In this work, we propose a PI-based control structure that efficiently allows for setpoint tracking with low usage of the manipulated variable (MV) and safety-related constraint satisfaction. Model Predictive Control (MPC) is well known for its capability of following a setpoint while following constraints and limiting rate of change of MVs. For this reason, we compare the performance of the proposed structure with model predictive control (MPC).

The rest of this paper is structured as follows: Section 3 introduces the problem, while the proposed control structure is presented in Section 4. Section 5 introduces the MPC formulation and simulation results are presented in Section 6. A performance comparison is shown Section 7, while the paper is concluded in Section 8.

### 2. PROBLEM FORMULATION

The control task is dampen flow disturbances in a simple tank system, modeled with the following differential equation

$$\frac{dh}{dt} = \frac{1}{a} (q_{in} - q_{out}), \quad (1)$$

where  $h$  is the level (controlled variable - CV),  $a$  denotes the cross-sectional area of the liquid (here  $a = 1 \text{ m}^2$ ),  $q_{in}$  denotes the volumetric inflow (disturbance variable - DV), and  $q_{out}$  is the volumetric outflow. The nominal residence of the tank is  $\tau = V/q = 1 \text{ m}^3/0.5 \text{ m}^3 \text{ min}^{-1} = 2 \text{ min}$ .

We assume that we have implemented a lower-layer flow controller so that  $q_{out}$  is the MV<sup>2</sup>. The inflow and outflow are assumed to be limited within  $q_{min} \leq q \leq q_{max}$ . With  $q_{min} = 0 \text{ m}^3 \text{ min}^{-1}$  and  $q_{max} = 1 \text{ m}^3 \text{ min}^{-1}$  The tank is

<sup>2</sup> Here, we assume that we have level control in the direction of flow, so that the inflow is the DV and the outflow is the MV, but in other cases it may be opposite. It will not affect the results in this paper.

at its maximum level when  $h = 1$  m, while an empty tank corresponds to  $h = 0$  m. Actually, to be on the safe side, the level should stay within  $0.1 \leq h \leq 0.9$  m. So  $h_{min} = 0.1$  m and  $h_{max} = 0.9$  m. These limits are shown by the yellow dotted lines in the figures. Two types of inflow disturbances are assumed to act upon the process (1); namely, step-changes and sinusoidal variations. The period of the sinusoids is 6.28 min which is quite long compared to the nominal residence time of 2 min, which means that it will be difficult to dampen large sinusoidal disturbances without violating the level constraints. Furthermore, the level measurement can be noisy.

### 3. SIMPLE CONTROLLER SCHEMES

For a surge tank, the actual value of the level may not be important as long as it is kept within its allowable safety limits (Shinskey, 1988). Therefore, Shinskey argued that integral action should not be used in some cases, and proposed to use a P-only controller in the form,

$$q_{out} = K_c \cdot h, \quad (2)$$

$$K_c = \frac{q_{max}}{h_{max}}. \quad (3)$$

This controller gives  $q_{out} = 0$  when  $h = 0$  and  $q_{out} = q_{max}$  when  $h = h_{max}$ . Note that there is no level setpoint. Rewritten in terms of deviation variables there will be a "setpoint", but it has no practical significance as it is not well tracked (Rosander et al., 2012). For averaging level control, where we want a low controller gain, this is the P-controller with the lowest controller gain that satisfies the safety constraints. However, one problem is that the gain  $K_c = q_{max}/h_{max}$  may still be too large when the process is operating at normal conditions, resulting in too large variations in  $q_{out}$  (MV) when there are smaller inflow disturbances.

This has led many authors to consider nonlinear controllers and MPC. The simplest nonlinear controller is a P-controller with a varying gain, that is, the gain is larger when the level approaches its safety limits. A simple implementation is to use three gain values as shown in Figure 1. The low gain works as an averaging controller when the flow changes are small (normal operation), and the two high gains track each boundary (Åström and Hägglund, 2006). During normal operation, inflow disturbances are dampened by the low gain P-controller. Then, when the level approaches the upper limit, the P-controller with high gain takes over, avoiding overflow with a fast response. Similarly, the other high-gain P-controller takes over when the level approaches the lower limit. The scheme may be implemented with three P-controllers and a mid-selector which selects the middle controller output as the MV.

The main drawback of the three P-controller scheme is that the normal range (with low controller gain) can be quite narrow in terms of flow rates, as illustrated in Figure 1. Once we get out of the normal range and one of the high-gain P-controllers takes over, it remains controlling the level tightly at the high or low limit and dampening of inflow disturbances is lost, as shown in Fig. 2.

Another problem with P-only control is that there is no level setpoint which the operator or a higher-level master

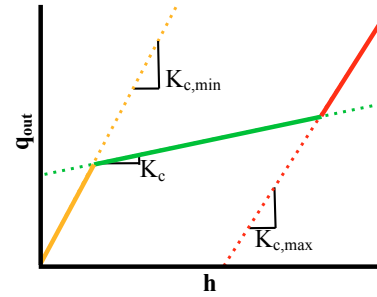


Fig. 1. Nonlinear relationship (solid lines) between level and outflow for case with three P-controllers.

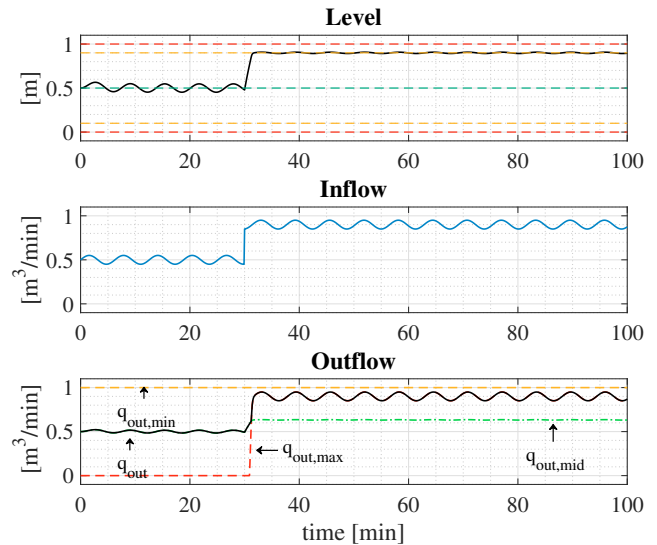


Fig. 2. Level control with three P-controllers.  $K_c = 0.33$ ,  $K_{c,max} = K_{c,min} = 6.67$ .

controller can manipulate. For example, the operator or master controller may want to set the level temporarily to a low value to prepare the systems for an expected large increase in the inflow. We therefore propose to use a modified three-controller scheme with the slow (normal) P-controller being replaced by a PI-controller, as discussed in the next section. However, before looking at this, let us consider the response with a single linear PI-controller.

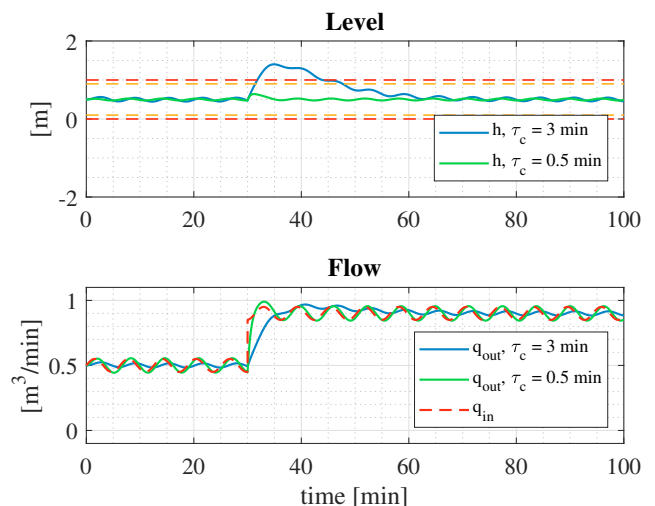


Fig. 3. Level control with PI-controller.

Figure 3 depicts the response with a slow and a fast PI-controller to step and sinusoidal inflow changes. Both are tuned using the SIMC rules (Skogestad, 2003), in which the tuning parameter,  $\tau_c$ , corresponds to the closed loop time constant. Anti-windup with back-calculation is also implemented. The fast PI-controller (green lines), with a short closed loop time constant,  $\tau_c = 0.5$  min ( $K_c = 2, \tau_I = 2$  min), keeps  $h$  within the safety constraints, but fails to dampen the sinusoidal input during normal operation ( $q_{out} \approx q_{in}$ ). On the other hand, the slow PI-controller (blue lines), with  $\tau_c = 3$  min ( $K_c = 0.33, \tau_I = 12$  min), performs well during normal operation, dampening the sinusoidal signal on  $q_{out}$ . However, the response is too slow when  $q_{in}$  has sudden changes close to its limits, and safety and physical constraints on  $h$  are clearly violated.

#### 4. PROPOSED CONTROL STRUCTURE FOR IMPROVED LIQUID LEVEL CONTROL

The purpose of this study is to develop a simple, yet efficient control structure for averaging level control based on easy-to-tune P and PI algorithms. We propose a non-linear control scheme in which the selection is done based on the output of three controllers. The overall structure of the proposed controller is demonstrated in Fig. 4.

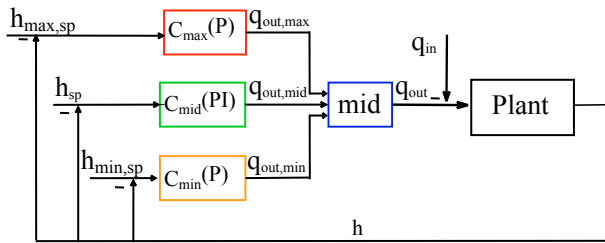


Fig. 4. Proposed PIPP control structure with one PI-controller and two P-controllers to track safety limits

Three different controllers calculate  $q_{out}$  (PIPP control strategy):

- $c_{mid}$ : PI-controller that tracks the actual desired value for the level,  $h_{sp}$ . This is a "slow" controller with a low gain  $K_c$ , designed to dampen the response for disturbances in  $q_{in}$  during normal operation.
- $c_{max}$ : P-controller with a large gain,  $|K_{c,max}| \gg |K_c|$ , which avoids violation of the maximum liquid level.
- $c_{min}$ : P-controller with a large gain,  $|K_{c,min}| \gg |K_c|$ , which avoids violation of the minimum liquid level.

The core of the proposed scheme is a mid-selector, based on the output of the three controllers. The proportional parts of the controllers behaves in a similar fashion as the non-linear three P-controllers described in Section 3 (Fig. 1). During normal operation, the output of the PI-controller,  $q_{out,mid}$ , will be the mid-value. When the level approaches the upper limit, the controller  $c_{max}$  will give an output signal  $q_{out,max} > 0$ , which becomes the middle value, increasing the outflow to avoid overflow. Accordingly, when the level decreases close to the lower limit,  $c_{min}$  will take over, preventing the tank from emptying.

Contrary to the scheme with three P-controllers presented in Section 3 (Fig. 2), the "slow" PI-controller will always take over after some time due to integral action, which

should not be limited by anti windup. It will bring the level back to normal operation and dampen oscillations.

#### 4.1 Tuning

For tuning of the  $c_{mid}$  PI-controller for normal operation,

$$K(s) = K_c \left( 1 + \frac{1}{\tau_I s} \right),$$

we recommend to use the SIMC tuning rules (Skogestad, 2003), with the following parameters for integral processes:

$$K_c = \frac{1}{k'(\tau_c + \theta)} \text{ and } \tau_I = 4(\tau_c + \theta), \quad (4)$$

where  $\theta$  is the process time delay, and  $k'$  is the slope of the integral process ( $\Delta y / (\Delta t \cdot \Delta u)$ ). In our case study  $\theta = 0$  and  $k' = 1$ . The only tuning variable for the PI-controller is the desired closed-loop time constant,  $\tau_c$ , which should be selected long enough to dampen the response for inflow disturbances. Instead of selecting  $\tau_c$ , one can select the controller gain and from this get  $\tau_c$ . As a starting point for the controller gain one may use the value  $K_c = q_{max}/h_{max} \approx 1/1 = 1$  for the slowest single P-controller, see (3). Here, we reduce it by a factor 3, because we want to have smaller MV (outflow) variations. Thus, we select  $\tau_c = 3$  min which gives  $K_c = 0.33, \tau_I = 12$  min.

For the two P-controllers,

$$q_{out,max} = K_{c,max}(h - h_{max,sp}) + q_{out,bias} \quad (5a)$$

$$q_{out,min} = K_{c,min}(h - h_{min,sp}) + q_{out,bias} \quad (5b)$$

In order to have a wide operation range for the PI-controller (dampening effect), we select a large controller gain for the P-controllers,  $K_{c,max} = K_{c,min} = 20 K_c \approx 6.7$ . We use (5) to find  $h_{max,sp}$  and  $h_{min,sp}$ , such that we have a fully open valve ( $q_{out} = q_{out,max} = 1 \text{ m}^3 \text{ min}^{-1}$ ) when the level is at the upper limit ( $h = h_{max} = 0.9 \text{ m}$ ), and a fully closed valve ( $q_{out} = q_{out,min} = 0 \text{ m}^3 \text{ min}^{-1}$ ) when the level is at the lower limit ( $h = h_{min} = 0.1 \text{ m}$ ). We use the nominal value for  $q_{out}$  as the bias,  $q_{out,bias} = 0.5 \text{ m}^3 \text{ min}^{-1}$ . For example,

$$h_{max,sp} = h_{max} - (q_{out,max} - q_{out,bias})/K_{c,max},$$

$$h_{max,sp} = 0.9 \text{ m} - \frac{(1 - 0.5) \text{ m}^3 \text{ min}^{-1}}{6.7 \text{ m}^2 \text{ min}^{-1}} = 0.825 \text{ m}$$

#### 4.2 Simulation

Fig. 5 shows the response of the proposed PIPP control structure when the process is subject to a sinusoidal disturbance and a large step change. The process starts at steady state, with  $h = h_{sp} = 0.5 \text{ m}$ , which represents normal operation. Hence, the selected MV-signal is  $q_{out,mid}$ , the output of  $c_{mid}$ . The output of the P-controller  $c_{max}$  is a closed valve,  $q_{out,max} = 0 \text{ m}^3 \text{ min}^{-1}$ , while the output of  $c_{min}$  is a fully open valve, corresponding to  $q_{out,min} = 1 \text{ m}^3 \text{ min}^{-1}$ . At  $t = 30$  min,  $q_{in}$  increases to an average of  $0.9 \text{ m}^3 \text{ min}^{-1}$ . Then, P-controller  $c_{max}$  takes over as  $q_{out,max}$  increases and becomes the middle value. Eventually, at  $t \approx 55$  min, the output from the PI-controller,  $q_{out,mid}$ , again becomes the middle value and brings the level back to its nominal setpoint. When this happens the variations in the outflow again become much reduced.

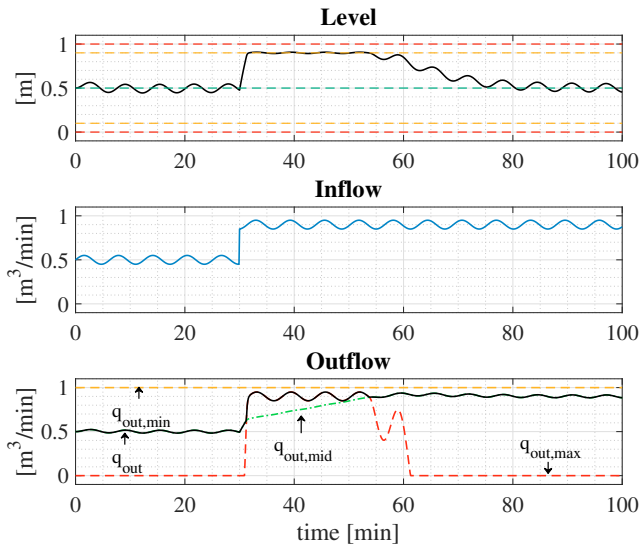


Fig. 5. Simulation of proposed PIPP control structure.

## 5. MPC IMPLEMENTATION

In order to have a benchmark to compare our simple PIPP scheme, we design a standard MPC controller. The optimal control problem is first discretized into a finite dimensional optimization problem divided into  $N$  elements, which represents the length of the prediction horizon. Hence, each interval is in  $[t_k, t_{k+1}]$  for all  $k \in \{1, \dots, N\}$ , where we use a third order direct collocation Radau scheme for the polynomial approximation of the system trajectories for each time interval  $[t_k, t_{k+1}]$ . The resulting discretized system model is represented as:

$$h_{k+1} = f(h_k, q_{in,k}, q_{out,k}), \quad (6)$$

where  $h_k$  represents the differential state from (1),  $q_{in,k}$  is the DV (inflow) and  $q_{out,k}$  denotes the MV (outflow), all at time step  $k$ . Once the system is discretized, the MPC problem can be formulated as

$$\min \sum_{k=1}^N \omega_1 \|(h_k - h_{sp})\|^2 + \sum_{k=1}^N \omega_2 \|(q_{out,k} - q_{out,k-1})\|^2$$

$$\text{s.t.} \quad (6) \quad (7a)$$

$$h_{min} \leq h_k \leq h_{max} \quad (7b)$$

$$q_{out,min} \leq q_{out,k} \leq q_{out,max} \quad (7c)$$

$$h_0 = h_{init} \quad (7d)$$

$$q_{out,0} = q_{out,init} \quad (7e)$$

with  $h_{min} = 0.1$  m,  $h_{max} = 0.9$  m,  $q_{out,min} = 0$  m<sup>3</sup> min<sup>-1</sup> and  $q_{out,max} = 1$  m<sup>3</sup> min<sup>-1</sup>. The objective function comprises of a term for level setpoint tracking as well as a term penalizing changes in the manipulated variable  $q_{out}$  between time steps  $k - 1$  and  $k$ . Constraint (7a) defines the model dynamics, whereas constraint (7b) enforces the level to remain between the bounds,  $h_{min}$  and  $h_{max}$ , respectively. Upper and lower bounds are also enforced for the manipulated variable as  $q_{out,min}$  and  $q_{out,max}$  in (7c). We assume that the level is measured. At each iteration, the initial conditions for the states are enforced in (7d) and (7e).

The dynamic optimization problem is setup as a QP problem in CasADi v3.1.0 (Andersson, 2013), which is then

solved using qpOASES (Ferreau et al., 2014). The plant simulator is solved with an ode15s solver. We simulate 2000 MPC iterations with a sample time of  $\Delta t = 0.1$  min. The prediction horizon of the MPC controller is set to 5 min resulting in  $N = 50$  prediction steps.

## 6. COMPARISON OF SIMPLE PIPP SCHEME WITH MPC

In this section we present simulation results for four different cases, in which the inflow,  $q_{in}$ , is the disturbance:

- (1) Step changes in  $q_{in}$
- (2) Step changes in  $q_{in}$  and measurement noise
- (3) Step changes in sinusoidal  $q_{in}$
- (4) Step changes in higher frequency sinusoidal  $q_{in}$

In all simulations, the level setpoint is  $h_{sp} = 0.5$  m, and the plant is subject to the same step changes of  $q_{in}$ : +0.2 m at  $t = 50$  min, +0.2 m at  $t = 100$  min, and +0.05 m at  $t = 150$  min, with an initial value of  $q_{in} = 0.5$  m<sup>3</sup> min<sup>-1</sup>. In case 3, the amplitude is 0.05 m<sup>3</sup> min<sup>-1</sup> and the frequency is 1 rad min<sup>-1</sup>. In case 4, the frequency is increased to 2 rad min<sup>-1</sup>.

The parameters for the plant model (1) are  $k' = 1$  and  $\theta = 0$  min. For every case, the SIMC tuning parameter,  $\tau_c$ , was set to 3 min. Then,  $K_c = \frac{1}{3}$ ,  $\tau_I = 12$  min, and  $K_{c,max} = K_{c,min} = 20 K_c$ . For every case, we compare the response of our proposed structure with the aforementioned MPC implementation. The MPC tunings were the same for all the cases with  $\omega_1 = 1$  and  $\omega_2 = 130$ .

### 6.1 Case 1: steps in the inflow

As seen in Fig. 6, the constraints on the level and the output are satisfied and overall tracking performance is satisfactory for both controllers in this simple tracking case without disturbances or added noise. Note that in the case of the proposed PIPP controller, the PI-controller effectively dampens the oscillations in the beginning, and  $q_{out} = q_{mid,out}$ . When the disturbance is large and the level approaches the upper limit at  $t \approx 50$  min,  $c_{max}$  takes over and  $q_{out} = q_{max,out}$ . This avoids overflow of the tank. Then, at  $t \approx 60$  min,  $c_{mid}$  takes over again. We observe a similar behavior at  $t \approx 110$  min.

### 6.2 Case 2: steps in the inflow plus noisy measurement

Fig. 7 shows the effect of the added measurement noise for the level in both controllers. The level can still be maintained around the nominal value  $h_{sp} = 0.5$  m and all constraints are satisfied. A drawback of using a high gain for the P-controllers is that measurement noise is magnified in  $q_{out}$  when the  $h$  is close to the limits.

### 6.3 Case 3: sinusoidal inflow

In this case we aim for the minimization of the change in  $q_{out}$ . Fig. 8 shows the effect of the different gains of the proposed controller on the dampening of the sinusoidal  $q_{in}$ . It can be seen that  $q_{out}$  is heavily reduced in amplitude compared to  $q_{in}$  and that the level constraints are satisfied. For the MPC, we penalize the difference in two subsequent values for  $q_{out}$  more heavily than deviations from the level setpoint  $h_{sp} = 0.5$  m.

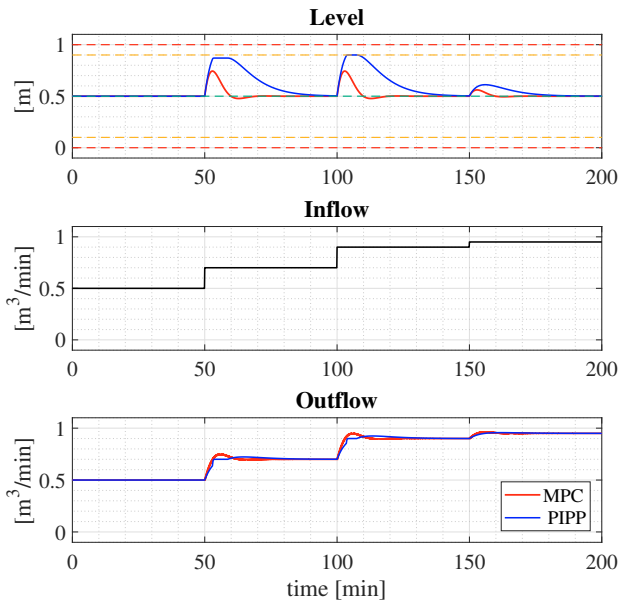


Fig. 6. Response of proposed control structure and MPC with step changes in  $q_{in}$  (case 1).

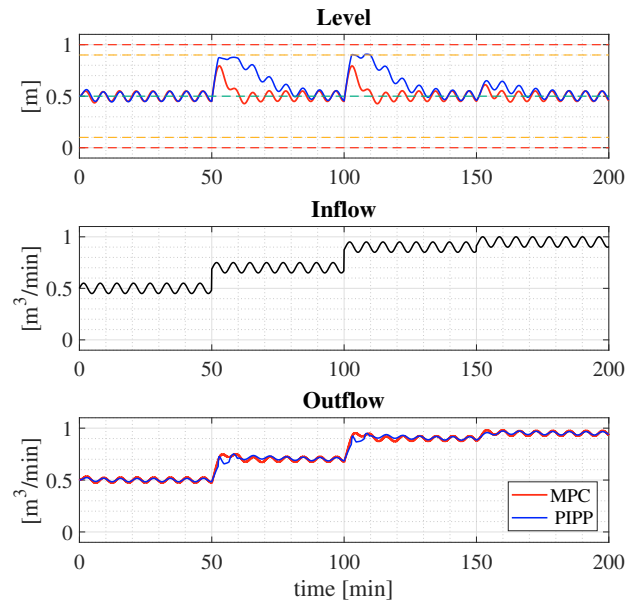


Fig. 8. Response of proposed control structure and MPC with sinusoidal  $q_{in}$  (case 3).

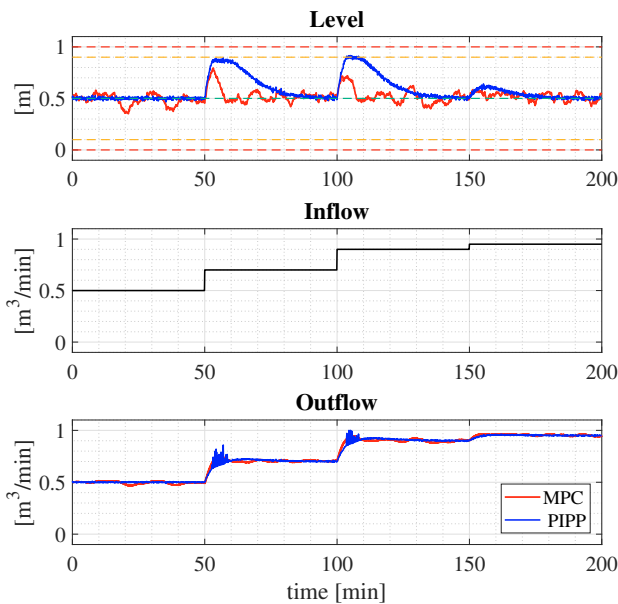


Fig. 7. Response of proposed control structure and MPC with step changes in  $q_{in}$  and measurement noise (case 2).

6.4 Case 4: Higher frequency sinusoidal inflow

Fig. 9 shows the results with a higher frequency sinusoidal disturbance ( $2 \text{ rad min}^{-1}$ ). The faster sinusoid is easier to handle and by comparing Fig. 9 with Fig. 8, we observe that  $q_{out}$  is smoother. Level constraints are also satisfied in this case. We note that the outflow variations are smaller with PIPP than with MPC in this case.

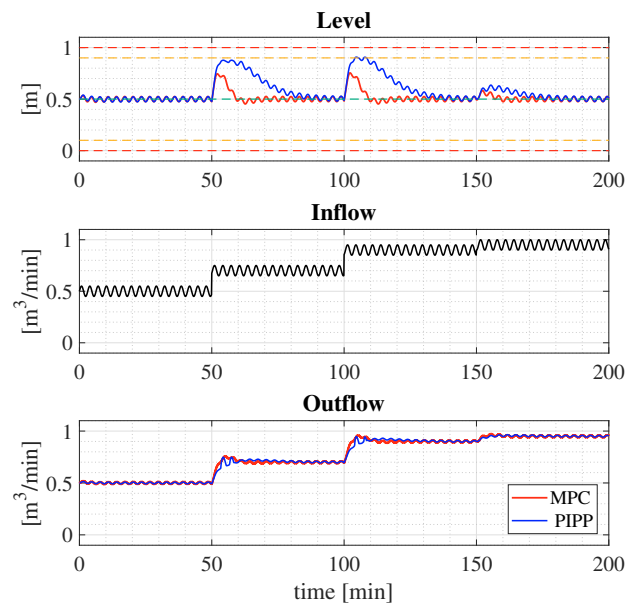


Fig. 9. Response of proposed control structure and MPC with higher frequency sinusoidal  $q_{in}$  (case 4).

presented cases, both for the proposed PIPP control structure and MPC.

Table 1. Deviation of the level from its set-point.

Case	Proposed PIPP structure	MPC
1	16.19	3.33
2	16.77	9.04
3	19.15	8.76
4	17.47	5.79

7. COMPARISON OF PERFORMANCE OF PROPOSED STRUCTURE AND MPC

Table 1 shows the Integral Absolute Error (IAE) for deviations from the level setpoint for each of the previously

Table 2 presents the IAE for deviations from the outflow to the steady inflow reference without added sinusoidal (compare Fig. 6 and 8). Furthermore, deviations from the steady inflow to the inflow that is used in the respective

cases (without and with added sinusoidal) is shown as 'inflow deviation'. A clear reduction in deviations from the outflows compared to the respective inflows can be seen for cases 3 and 4, which are the cases with added sinusoidal disturbances. We can also pinpoint that the proposed controller performs better with high frequency disturbances, as the deviation is lower in case 4 (high frequency) compared to case 3 (low frequency).

Table 2. Deviation of the outflow from the steady inflow setpoint.

Case	Proposed PIPP structure	MPC	Inflow deviation
1	1.75	1.19	0
2	2.24	2.46	0
3	3.24	4.06	6.37
4	2.56	2.42	6.35

Another performance index that could be used to quantify how the outflow is smoothed is the "total variation" or integrated absolute variation of the MV, corresponding to the sum of all "moves" of the MV:

$$\sum_{k=0}^{N-1} \left| \frac{q_{out,k} - q_{out,k-1}}{t_k - t_{k-1}} \right| \quad (8)$$

As we desire to smoothen  $q_{out}$ , this value should be as small as possible. Table 3 shows this performance index. It can be observed that the best performance of the proposed PIPP structure is when there is no measurement noise (cases 1, 3 and 4). In these cases, the PIPP performance is better than the presented MPC implementation. This can partly be explained because in the presence of noise (case 2), when the level (CV) is close to the limits, the high gain P-controllers take over and  $q_{out}$  (MV) is correspondingly moved aggressively, see Fig. 7.

Table 3. Total outflow variation

Case	Proposed PI structure	MPC
1	5.56	6.88
2	123.38	13.40
3	27.35	37.15
4	28.23	34.89

For all simulation cases, simulation times for the proposed structure were in the range  $0.9 \pm 0.04$  s, whereas the runtime for the MPC was in the range of 88.4 – 177.6 s, depending on the case. The long runtime for the MPC was mostly due to the relatively large horizon of  $N = 50$ .

## 8. CONCLUDING REMARKS

In this paper, we presented a simple, yet efficient level control structure for setpoint tracking and safety-related lower and upper constraint satisfaction in industrial tanks. The proposed PIPP control algorithm relies in simple and easy to tune P and PI controllers. The proposed method performs much better than standard PI controllers and has a performance comparable to standard MPC in the the exact same simulation cases. These cases include the investigation of sinusoidal and step disturbances for the inflow and white noise added to the level measurement, respectively.

The proposed controller is not only able to effectively smoothen the use of the controlled variable, it is furthermore able to avoid violation of the safety constraints on

upper and lower limits. Additionally, it gives the possibility to track the desired level setpoint in the presence of disturbances and noise. When compared to standard MPC, the proposed structure has the advantage that implementation of PI structures is simpler and computational times are consistently and substantially shorter. Additionally, tuning of PI controllers using the SIMC rule is fast and uncomplicated compared to tuning of MPC. The presented approach is particularly convenient for surge tanks with relatively small volumes, where it is difficult to get dampening of flow disturbances without violating liquid level constraints.

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