Multi-directional Irregular Wave Modelling with CFD

Weizhi Wang[*], Hans Bihs[†], Arun Kamath[†],and Øivind Asgeir Arntsen[†]

Department of Civil and Environmental Engineering, Norwegian University of Science and Technology (NTNU), 7491 Trondheim, Norway

Presented at *4th International Conference in Ocean Engineering.*

Abstract

The design of coastal structures requires accurate simulations of the wave conditions. Computational Fluid Dynamics (CFD) captures most complexities of the wave physics with few assumptions and therefore is considered to be an ideal alternative for the offshore wave simulation. However, the conventional uni-directional regular wave CFD simulation does not represent the reality and tends to overestimate the wave conditions. The irregular bottom topography and varying water depth in the coastal area make the simulation more complicated. To give a more realistic simulation in a coastal area, a directional irregular wave model is to be implemented in a CFD code. This paper presents a multi-directional irregular wave implementation in the open-source CFD model REEF3D. The non-directional frequency spectra JONSWAP (Joint North Sea Wave Observation Project) together with a cos-squared-type directional spreading function is used for the simulation. REEF3D solves the incompressible Navier-Stokes equations with the finite difference method on a staggered Cartesian grid and uses the level-set method to capture the free surface under the two-phase flow approximation. The relaxation method is used for the wave generation and numerical beaches. The irregular waves are generated by the superimposition of a finite number of regular waves. The resulting significant wave heights are compared with another numerical model SWASH. The comparisons show good performance of CFD simulations in predicting irregular wave behaviours. The differences are also discussed for future references.

Keywords: Multi-directional Irregular Wave, CFD, REEF3D, SWASH

1 INTRODUCTION

Ocean waves are random by nature. An accurate prediction of the random seas requires a good knowledge of the energy distribution over both frequencies and directions. A conventional

---

[*] Corresponding author, weizhi.wang@ntnu.no

Postprint, published in 4th International Conference in Ocean Engineering,

uni-directional irregular wave study tends to over-estimate the wave condition. The study of the directionality in a random wave field is the key for an accurate prediction of the wave properties and the design of coastal structures.

Many spectra describing the energy density distribution have been developed in the past decades. Pierson and MoskowitzPierson and Moskowitz (1964), Hasselmann et.alHasselmann et al. (1973) and Bouws et. al.Bouws et al. (1985) developed different frequency spectra based on their observations to describe the fully developed sea, the non-fully developed sea and to include shallow water effects. To describe the directionality, Pierson et.alPierson et al. (1955) proposed the Pierson-Neumann-James (PNJ) directional spreading function as a function of only the directional angles. Longuet-Higgins et.alLonguet-Higgins et al. (1963), Mitsuyasu Mitsuyasu (1975) and Ochi Ochi (1998) further developed directional spreading functions of both angles and frequencies.

Many studies have been conducted using the empirical spectra. Even though most studies have been focusing on the uni-directional irregular wave, some experimental and numerical studies of the multi-directional irregular waves are also progressing. Li et.alLi et al. (2012)Li et al. (2014) and Ji et.alJi et al. (2017) experimentally studied the interaction between multi-directional irregular waves and a vertical cylinder. Numerical models HOS-NWT Ducrozet et al. (2012) and HOS-Ocean Ducrozet et al. (2016) were also developed in Ecole Centrale de Nantes for the multi-directional irregular wave simulation in wave basin and larger scale.

However, the coastal region presents more complicated scenarios compared to the idealised studies in experiments. The dramatic change of the bottom topography and water depth cause complicated wave transformation phenomena. Few studies have been made to numerically investigate the multi-directional random sea over irregular bottoms. The complexity, however, can be accounted for to a great extent by the Computational Fluid Dynamics (CFD) method, as CFD solves most of the kinematics and dynamics in the fluid domain with few assumptions. Wang et.al Wang et al. (2017*a*)Wang et al. (2017*b*) has explored the application of the CFD method on the regular wave and uni-directional irregular wave modelling over irregular bottom topography using the wave model REEF3D. REEF3D is an open-source CFD wave model developed in NTNU specialising in wave hydrodynamics and free-surface flow. It has been widely used to simulate various wave phenomena, such as 6 degree-of-freedom floating bodyBihs et al. (2016), the breaking wave kinematicsAlagan-Chella et al. (2017) and breaking wave-structure interactionsKamath et al. (2016).

Therefore, it is an important task to implement the multi-directional irregular wave module in the REEF3D framework. In order to fulfil the task, the first step is to verify the model using a simple numerical tank. In this paper, a flat bottom numerical wave tank without any structures is used for the verification of the newly developed module. The results are also compared to another numerical model SWASHZijlema et al. (2011).

2 NUMERICAL MODEL

2.1 The Governing Equations

The CFD model REEF3D solves the incompressible Navier-Stokes equation together with the continuity equation as shown in Eqn. (1) and Eqn. (2).

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\nu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right] + g_i \tag{2}$$

where u is the velocity, ρ stands for the fluid density, p represents the pressure, ν is the kinematic viscosity and g is the acceleration due to gravity.

REEF3D uses a finite difference method to solve the governing equations on a structured Cartesian grid. High-order numerical schemes such as a conservative fifth-order weighted essentially non-oscillatory (WENO) scheme Guang-Shan and Chi-Wang (1996) and a third-order Total-Variation-Diminishing (TVD) Runge-Kutta scheme Shu and Osher (1988) are used in REEF3D. The Poisson pressure equation is solved using the PFMG preconditioned BiCGStab algorithm provided by HYPRE library Falgout et al. (2006).

SWASH solves the non-linear shallow water equations using a finite element framework on a staggered grid. Different from a Boussinesq-type model, SWASH compensates the dispersion relation by increasing the number of vertical layers rather than increasing the order of derivatives Zijlema et al. (2011).

2.2 Multi-directional Irregular Wave Generation in a Numerical Tank

The 3D irregular wave can be regarded to be a superposition of a finite number of regular wave components travelling in different directions at different frequencies and phases. The properties of each wave components are ramped up to the theoretical values in the wave generation zone using a relaxation method. The theoretical values for the wave components are calculated from the directional energy spectrum, which is formed by multiplying a frequency spectrum with a directional spreading function. For the frequency spectrum, the JONSWAP spectrum recommended by DNV DNV (2011) is used in this paper, as shown in Eqn. (3).

$$S = \frac{5}{16}\frac{H_s^2\omega_p^4}{\omega^5}exp(-\frac{5}{4}\left(\frac{\omega}{\omega_p}\right)^{-4})(1-0.287ln(\gamma))\gamma^{exp(-0.5(\frac{\omega-\omega_p}{\sigma\omega_p})^2)} \tag{3}$$

where ω_p is the peak frequency in the unit of rad, γ is a constant, $\gamma = 3.3$ in this paper.

The PNJ Pierson et al. (1955) directional spreading function is used in this paper due to its simplicity and wide application, as shown in Eqn. (4). In the formula, $\overline{\beta}$ is the principal direction representing the main energy propagation direction and β_i is the incident wave direction of each wave components, as illustrated in Fig. 1.

$$G(\beta) = \begin{cases} \frac{2}{\pi}\cos^2(\beta - \overline{\beta}) & , if\ |\beta| < \frac{\pi}{2} \\ 0 & , else \end{cases} \tag{4}$$

By multiplying Eqn. (3)and Eqn. (4) one gets the directional spectrum JONSWAP-PNJ. The velocities and surface elevations can be derived based on the directional spectrum using a single-array method. As an example, the formulation of the surface elevation is shown in Eqn. (5). k_n can be calculated based wave theory and the amplitude is calculated in Eqn. (6) using the spectrum. N denotes the total number of the wave components, which is a multiplication of the number of frequency components and directional components. ε_n is the phase generated randomly between 0 and 2π.

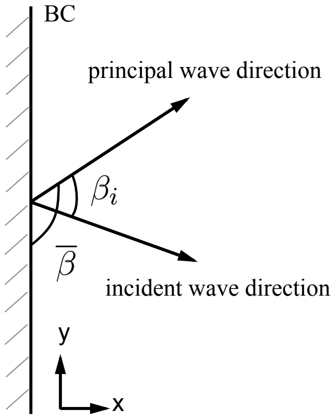

Figure 1: The definition of the principal angle and the incident wave angle

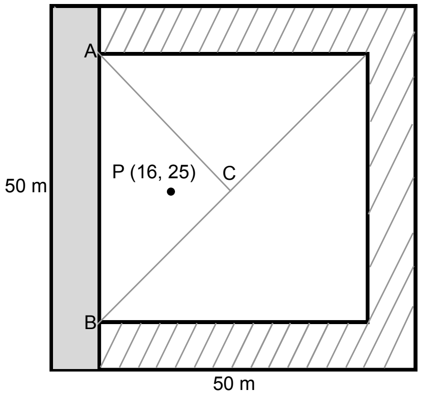

Figure 2: The configuration of the numerical wave tank. The shaded zone to the left-hand side is the wave generation zone. Numerical beaches are shown in the zones with stripes.

$$\eta(\overrightarrow{r},\ t) = \sum_{i=1}^{N} a_n \cos(\overrightarrow{k_n} \cdot \overrightarrow{r} - \omega_n t + \varepsilon_n) \tag{5}$$

$$a_n = \sqrt{2S(\omega_n, \beta_n)\triangle\omega\triangle\beta} \tag{6}$$

2.3 Numerical Tank Configurations

A numerical wave tank 50 m long and 50 m wide is used with a water depth of 0.5 m. As shown in Fig. 2, a wave generation zone of one wavelength is provided on the left-hand side of the tank and numerical beaches are located both at the end of the tank and also along the side walls. The wave probes are located near the geometric centre of the triangle $\triangle ABC$ formed by the diagonals of the numerical tank in Fig. 2. This arrangement helps to minimise the influence of reflected waves from the boundaries.

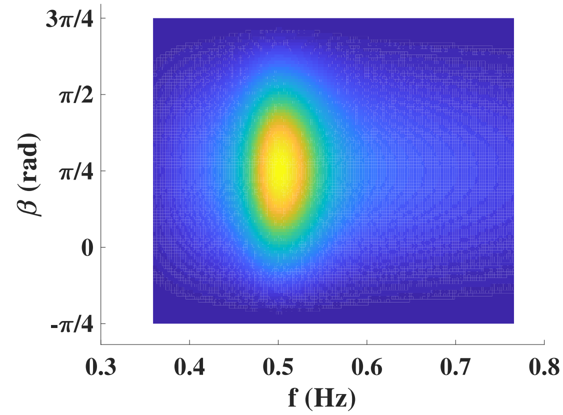

Figure 3: The plot of the spectrum density of the directional spectrum combining the JONSWAP frequency spectrum and the PNJ directional spreading function.

3 Numerical Simulation Results

The input wave has a H_s of 0.1 m and a T_p of 2 s. The principal direction is 45° counterclockwise from the x-axis. The corresponding wave directional spectrum based on the wave input is plotted in Fig. 3. The mesh convergence study is conducted with three mesh sizes: dx=0.08 m, dx=0.10 m and dx=0.20 m. The frequency spectra obtained at the probes at different mesh sizes are compared with the theoretical spectrum in Fig. 4. Concluding from the comparison, a mesh size of dx=0.08 m is considered to be sufficient to obtain accurate information about the irregular wave field. The following CFD results are all based on this mesh size. With the mesh size of 0.20 m, 0.25 m and 0.50 m, SWASH gives very similar predictions of the significant wave height. A mesh size of 0.20 m is considered to be sufficient for the SWASH model. Two vertical layers are used in the SWASH simulation. The simulation time is 200 s for both REEF3D and SWASH simulations. As the first verification study, a small number of wave components is used in the CFD simulation. 20 frequencies are used with the range of [2.26, 4.82] rad/s. Six directional angles are used within the range of $[\frac{-\pi}{2}, \frac{\pi}{2}]$ rad.

Fig. 5 and Fig. 6 show the surface elevation results from the REEF3D and SWASH simulations respectively. The principal direction of the irregular wave field is clearly visible in both simulations. The H_s at the probe from both simulations are compared to the input wave data as shown in Table. 1. The H_s in the numerical tank from the REEF3D simulation gives a very satisfactory agreement with the input data. SWASH gives a lower value in this simulation.

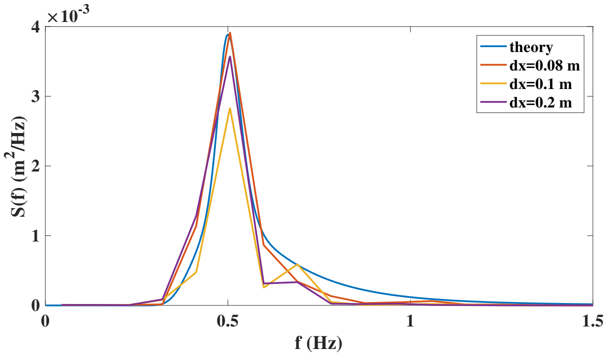

Figure 4: The comparison of the frequency spectrum in the tank with different mesh sizes

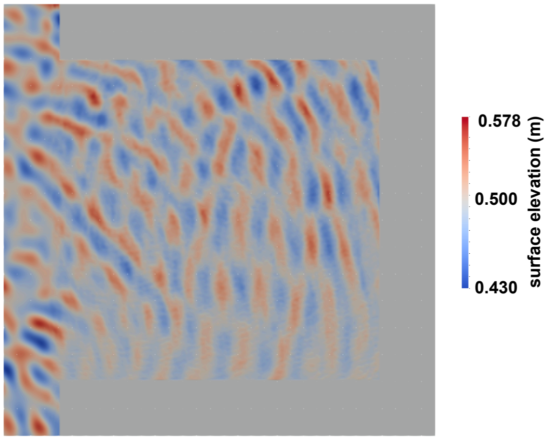

Figure 5: The surface elevation in the numerical tank of the REEF3D simulation at t=200s

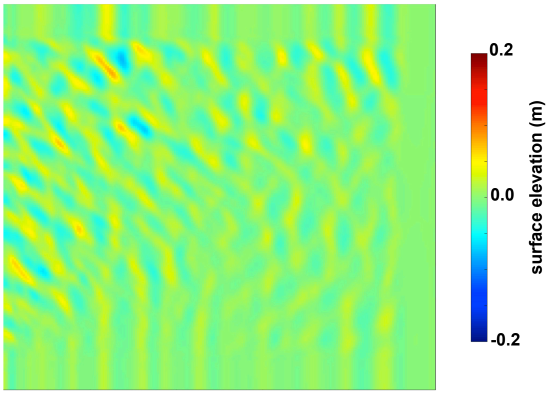

Figure 6: The surface elevation in the numerical tank of the SWASH simulation at t=200s

Table 1: The comparison of the H_s in the REEF3D and SWASH simulations

	H_s (m)
Input	0.1000 m
REEF3D	0.1001 m
SWASH	0.07103 m

4 CONCLUSIONS

In order to develop a CFD model for directional random sea over irregular bottom near the coastal area, a study of the newly developed multi-directional irregular wave module in REEF3D is described in this paper. A flat bottom tank is used for verification. The CFD simulation successfully produced a directional random sea in the numerical wave tank and gives a good prediction of the frequency spectrum and H_s. SWASH performs well in producing a random sea, but the prediction of H_s is not very accurate based on the current set-up. Due to the complexity in the random wave field, more thorough studies are needed for further conclusions. The following works are to be completed in the future: 1) A sensitivity study of the number of the frequency and direction components; 2) A sensitivity study of the simulation time; 3) The reconstruction of the directional spectrum using an array of probes in the numerical wave tank; 4) A validation study to compare the numerical results with the data from physical experiments.

Acknowledgements. This study has been carried out under the E39 fjord crossing project (No. 304624) and the authors are grateful to the grants provided by the Norwegian Public Roads Administration. This study was supported in part with computational resources at the Norwegian University of Science and Technology (NTNU) provided by NOTUR project (No. NN2620K), http://www.notur.no.

References

Alagan-Chella, M., Bihs, H., Myrhaug, D. and Muskulus, M. (2017). Breaking solitary waves and breaking wave forces on a vertically mounted slender cylinder over an impermeable sloping seabed. *Journal of Ocean Engineering and Marine Energy*, **3**(1), 1–19. ISSN 2198-6452. doi:10.1007/s40722-016-0055-5.

Bihs, H., Kamath, A., Alagan-Chella, M., Aggarwal, A. and Arntsen, O. (2016). A new level set numerical wave tank with improved density interpolation for complex wave hydrodynamics. *Computers Fluids*, **140**(Supplement C), 191 – 208. ISSN 0045-7930. doi: https://doi.org/10.1016/j.compfluid.2016.09.012.

Bouws, E., Gnther, H., Rosenthal, W. and Vincent, C.L. (1985). Similarity of the wind wave spectrum in finite depth water: 1. spectral form. *Journal of Geophysical Research: Oceans*, **90**(C1), 975–986. ISSN 2156-2202. doi:10.1029/JC090iC01p00975.

DNV (2011). Modelling and analysis of marine operations. Standard DNV-RP-H103, Det Norske Veritas, Veritasveien 1, Hvik, Norway.

Ducrozet, G., BONNEFOY, F., Le Touzé, D. and Ferrant, P. (2012). A modified High-Order Spectral method for wavemaker modeling in a numerical wave tank. *European Journal of Mechanics - B/Fluids*, **34**, http://dx.doi.org/10.1016/j.euromechflu.2012.01.017. doi: 10.1016/j.euromechflu.2012.01.017.

Ducrozet, G., Bonnefoy, F., Le Touzé, D. and Ferrant, P. (2016). HOS-ocean: Open-source solver for nonlinear waves in open ocean based on High-Order Spectral method. *Computer Physics Communications*, **203**, 245–254. doi:10.1016/j.cpc.2016.02.017.

Falgout, R.D., Jones, J.E. and Yang, U.M. (2006). Conceptual interfaces in hypre. *Future Gener. Comput. Syst.*, **22**(1-2), 239–251. ISSN 0167-739X. doi:10.1016/j.future.2003.09.006.

Guang-Shan, J. and Chi-Wang, S. (1996). Efficient implementation of weighted eno schemes. *Journal of Computational Physics*, **126**(1), 202–228.

Hasselmann, K. et al. (1973). Measurements of wind-wave growth and swell decay during the joint north sea wave project (jonswap). *Ergänzung zur Deut. Hydrogr. Z., Reihe A (8)*, **12**, 1–95.

Ji, X., Liu, S., Li, J. and Jia, W. (2017). Numerical investigation of multidirectional random wave interaction with a large cylinder. *Proceedings of the Institution of Mechanical Engineers, Part M: Journal of Engineering for the Maritime Environment*, **231**(1), 271–283. doi:10.1177/1475090216642464.

Kamath, A., Alagan-Chella, M., Bihs, H. and Arntsen, O. (2016). Breaking wave interaction with a vertical cylinder and the effect of breaker location. *Ocean Engineering*, **128**, 105 – 115. ISSN 0029-8018. doi:http://dx.doi.org/10.1016/j.oceaneng.2016.10.025.

Li, J., Wang, Z. and Liu, S. (2012). Experimental study of interactions between multi-directional focused wave and vertical circular cylinder, part i: Wave run-up. *Coastal Engineering*, **64**(Supplement C), 151 – 160. ISSN 0378-3839. doi: https://doi.org/10.1016/j.coastaleng.2012.02.003.

Li, J., Wang, Z. and Liu, S. (2014). Experimental study of interactions between multi-directional focused wave and vertical circular cylinder, part ii: Wave force. *Coastal Engineering*, **83**(Supplement C), 233 – 242. ISSN 0378-3839. doi: https://doi.org/10.1016/j.coastaleng.2013.06.004.

Longuet-Higgins, M.S., Cartwright, D.E. and Smith, N.D. (1963). Observations of the directional spectrum of sea waves using the motions of a floating buoy. *Proceedings of the Conference of Ocean Wave Spectra*, 111–132.

Mitsuyasu, H. (1975). Observations of the directional spectrum of ocean waves using a cloverleaf buoy. *Journal of Physical Oceangraphy*, 750–760.

Ochi, M.K. (1998). *Ocean Waves: The Stochastic Approach.* Cambridge University Press. ISBN 9780521017671.

Pierson, W.J. and Moskowitz, L. (1964). A proposed spectral form for fully developed wind seas based on the similarity theory of s. a. kitaigorodskii. *Journal of Geophysical Research*, **69**(24), 5181–5190. ISSN 2156-2202. doi:10.1029/JZ069i024p05181.

Pierson, W.J., Neumann, G. and James, R.W. (1955). Practical methods for observing and forecasting ocean waves by means of wave spectra and statistics.

Shu, C. and Osher, S. (1988). Efficient implementation of essentially non-oscillatory shock-capturing schemes. *Journal of Computational Physics*, **77**(2), 439–471.

Wang, W., Bihs, H., Kamath, A. and Arntsen, Ø. (2017*a*). Large scale cfd modelling of wave propagation in sulafjord for the e39 project. *Proceedings of MekIT17 - 9th national conference on Computational Mechanics*.

Wang, W., Bihs, H., Kamath, A. and Arntsen, Ø. (2017*b*). Large scale cfd modelling of wave propagation into mehamn harbour. *Proceedings of MARINE 2017 Computational Methods in Marine Engineering VII*.

Zijlema, M., Stelling, G. and Smit, P. (2011). Swash: An operational public domain code for simulating wave fields and rapidly varied flows in coastal waters. *Coastal Engineering*, **58**(10), 992 – 1012. ISSN 0378-3839. doi:https://doi.org/10.1016/j.coastaleng.2011.05.015.