Fracture analysis of V-notched rubbers: An experimental and theoretical study

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Abstract

In this study, the rupture load in rubbers weakened by sharp V-notch is investigated under mode I loading. To this end, first, mode I fracture tests are performed on V-notched samples made of styrene-butadiene rubbers and the corresponding rupture loads are obtained. Then, the effective stretch (ES) criterion, which was recently developed by the present authors for rupture assessment of cracked rubber parts, is extended and used for the V-notched rubbers. It is shown that similar to cracked rubbers, the state of stress near the notch tip is also nearly uniaxial. By employing the ES criterion, the critical displacements corresponding to the rupture in the tested samples are calculated. Finally, the predictions of the criterion are compared with the corresponding experimental values, and good consistency is shown to exist.

1 INTRODUCTION

Fracture mechanics seeks to predict the failure of structural components containing geometrical irregularities like cracks and notches. Among various types of these discontinuities, sharp V-notches are frequently found in engineering structures. A notch may be regarded as sharp when the notch tip radius is small enough compared with the overall sizes of the component. Due to high stress concentration, the notch tip is a potential zone for fracture initiation and eventually the catastrophic failure of structures. For this reason, presentation of a reliable criterion for fracture prediction in V-notched samples is of paramount importance in the optimal design of the structures.

Fracture assessment of sharp V-notches made of brittle or quasi-brittle materials has been widely investigated in the literature. Strain energy release rate,1 notch stress intensity factor-based criterion,2, 3 modified McClintock's stress,1 finite fracture mechanics concept,4, 5 maximum tangential stress,6 generalized maximum tangential stress,7 non-local stress fracture criterion,1 strain energy density (SED),1 and averaged SED8-12 are among the frequently employed fracture criteria in the past studies. It is important to notice that these criteria are developed based on the linear elastic fracture mechanics theory, and thus, they should be applicable only for linear elastic materials and under small deformation conditions. However, V-notched rubbers normally experience large deformations, and hence, the mentioned criteria are not applicable to them. Therefore, special attention should be paid to the development of new fracture criteria for rubbers.

Rubber, which is known for its high deformability and reversibility of deformation, has attracted great attention in recent years, especially for its damping capacity.13 Transmission belts, high-tech sealing elements, rubber bearings, and coatings for vessels are among the main applications of rubbers. Moreover, under external loads, rubber-like materials deform totally different from ordinary solids, like metals. To be more precise, when a metal is subjected to an external load, the distance between two adjacent atoms may not change by more than a fraction of an angstrom, and the resulting elasticity is mainly energy driven. However, a typical rubber which consists of cross-linked long polymer chains, deforms mainly via straightening its chains and can be stretched up to about 10 times of its initial length.14 This considerable feature of rubber (i.e., high stretchability) makes it possible that the rubber tolerates the existence of a flaw by rearranging its network and subsequently, reducing the local state of stress. Therefore, the rubber can be applicable in situations where severe deformation without substantial loss of load-bearing capacity is required.15 As a result, the rubber elasticity is mainly entropic rather than energetic.

Although analysis of rubber under fatigue loading has attracted much attention during the last two decades,16-23 the rupture assessment of elastomers under quasi-static loading is less documented.24 Moreover, due to the unique physics of rubber as stated before, presentation of a fracture criterion considering the rubber physics as the central part of criterion is of paramount importance. In this regard, the present authors recently developed a displacement-based criterion for rupture prediction of cracked rubbers which is called the effective stretch (ES) criterion.25, 26

A review of literature shows that in addition to the ES criterion, some other criteria have also been presented for fracture prediction of rubbers weakened by cracks (see, for example, literatures 27-34). However, to the best knowledge of the authors, there is no study concentrated on the rupture assessment of rubber-like materials containing V-notches.

The aim of the present contribution is to predict the rupture in rubber parts containing a sharp V-notch. To this end, several mode I fracture tests are performed on single edge V-notched (SEVN) samples made of styrene-butadiene rubber (SBR). Then, the ES criterion is extended and used for rupture prediction in V-notched hyperelastic materials. Here, the main hypothesis of the ES criterion, i.e., the nearly uniaxial state of stress around the stress concentrator, should be reexamined for sharp V-notches. Finally, the finite element method is employed to study the stress distribution around the notch tip where the stress state is shown to be almost uniaxial. Therefore, it is concluded that the ES criterion is applicable to V-notched rubbers as well.

2 ANALYTICAL FORMULATIONS

In this section, the basic concepts of the ES criterion are reviewed. Then, some additional points required to extend the ES criterion to notched samples are presented.

2.1 A brief review of ES criterion in cracked rubbers

The main concepts of the ES criterion are briefly reviewed in the following lines.

The first hypothesis of the ES criterion is based on the physics of rubber. Rubber is a polymer-based material with long and entangled chains randomly distributed in a cross-linked network. Because of the existence of cross-links, the chains can return to their initial positions if the external loading is removed. In addition, since the elasticity of rubber-like materials is mainly entropic, a statistical mechanics approach should be utilized to model the rubber behavior.35 In fact, the behavior of individual chains should be related to the rubber constitutive framework considering appropriate representative volume elements.36

Based on the employed representative volume element, four well-known models, namely, three-chain, four-chain, eight-chain, and full network models are proposed, among which the eight-chain model has shown the best agreement with experiments.37 Hence, the eight-chain model has been used for development of the ES criterion. A schematic of chains arrangement in the eight-chain model is shown in Figure 1.

where $\lambda 1$, $\lambda 2$, and $\lambda 3$ are the macroscopic principal stretches. Accordingly, the ES criterion considers this ES as a fracture controlling parameter and postulates that the rupture in a rubber sample occurs when the stretch of the chains, λch, reaches a critical value called the critical ES, λces. The way to obtain this critical value is elaborated hereafter.

The second hypothesis of the ES criterion is the existence of a damage zone near the stress concentrator under external loads which has been experimentally confirmed for cracked rubbers.39 This zone no longer sustains any further stresses, and thus, the rupture will occur from its boundary. The radius of the damage zone is usually denoted by rc.

The final hypothesis is the nearly uniaxial state of the stress fields near the stress concentration zone in hyperelastic materials. Indeed, unlike cracked ordinary brittle materials (i.e., glass and rock) which possess a triaxial stress state at the crack tip, stress fields around the crack tip in rubbers are almost uniaxial due to their large deformations.40, 41

2.2 Extension of the criterion to notched rubber samples

Among the three bases of the ES criterion, explained above for cracked samples, the last one should be reexamined for V-notched samples. In fact, the first base is independent of the discontinuity geometry as it is related to the rubber microstructure. For the second one, although there is no experimental evidence available yet, considering the observations on cracked rubbers, it is expected to have the damage zone around V-notched rubbers as well.

Regarding the third base, as is shown later in Section 4.3, the stress state is indeed nearly uniaxial close to the notch tip. Consequently, one can extend the ES criterion to make it applicable for V-notched rubbers as well. The rupture predictions of this extended criterion for notched rubbers are evaluated later in Section 4.4.

2.3 Implementation and statement of the criterion

According to the ES criterion, rupture in a rubber-like material weakened by a sharp V-notch initiates when λch evaluated at the distance rc (from the notch tip) reaches a critical value, λces. Therefore, as can be realized, determination of rc and λces is the primary step to apply the ES criterion.

With the aim of obtaining λces for the V-notched rubbers, we remember the uniaxial state of stress fields near the notch tip in hyperelastic materials. According to this feature, the rupture conditions for a material point on the damage zone boundary of the notch are expected to be almost the same as the corresponding conditions for material points in the uniaxial tensile test. Therefore, the critical value of the ES, λces, can be computed based on the results of the uniaxial tensile test. In this regard, if the final stretch of a rubber in the uniaxial tensile test is equal to λten, the value of λces for an incompressible rubber can be obtained as follows25, 26:

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As it can be seen from Equation 2, the critical parameter λces depends only on the final stretch of the uniaxial tensile test (i.e., λten). Therefore, λces is independent of the notch depth and can be used properly as a fracture controlling parameter.

On the other hand, for determining the value of rc in notched rubbers, a fracture test on a specimen weakened by the sharp V-notch should be firstly performed, and its critical load (i.e., the load at the rupture initiation) should be recorded. Then, this fracture test should be simulated via the FE method. The obtained ES values should be plotted in a horizontal path ahead of the notch tip against the distance from the notch apex. Finally, the distance at which the calculated ES equals λces is recorded as the critical distance rc.

3 EXPERIMENTS

An SBR, which was purchased from Bandare-Imam Co in Iran, is used in this study. The recipe of this rubber is given in Table 1. For synthesizing the rubber, the compound was mixed on a laboratory two roll mill (Brabender PM300, Germany) with the friction ratio of 1.3. The time of mixing was about 10 minutes. Afterwards, according to the curing data obtained from oscillating disc rheometer, the optimum cure time (t90) was found to be equal to 380 seconds. It is useful to note that oscillating disc rheometer is an equipment used to characterize the curing behavior of rubbers. Moreover, t90 is defined as the required time for the torque to reach 90% of the maximum achievable torque.42 Finally, the sample was laid in a 1.5-mm-thickness steel mold and cured in a laboratory hydraulic hot press at 155°C for 380 seconds.

After preparation of rubber samples, the well-known dog-bone shaped specimens and the SEVN samples were fabricated for the uniaxial and fracture tests, respectively. The geometry and sizes of the specimens are shown in Figure 2. It may be useful to note that the tensile test specimens were prepared according to ASTM D 412, a standard test method for measuring tensile properties of rubbers. However, for the fracture tests of V-notched rubbers and to the best knowledge of the authors, there is still no standard test method.

Geometry and sizes of samples prepared for mechanical testing. A, Dog-bone shaped sample for the tensile test; B, SEVN sample for the fracture tests [Colour figure can be viewed at wileyonlinelibrary.com]

As shown in Figure 2B, four different values of notch depth to width ratio (a/w = 0.2, 0.4, 0.5, and 0.6) have been selected for fracture tests of the SEVN specimens. Two specimens were tested for each SEVN sample.

Moreover, all the mechanical tests were conducted at room temperature and under the tensile rate of 1 mm/min by using a Santam STM 150 universal testing machine (Iran). In addition, during the tests, a Canon CMOS camera equipped with a Macro USM Fixed Lens was used. In the tensile tests, the camera was utilized to accurately obtain the specimen stretches while the rupture initiation (i.e., crack nucleation) ahead of the notch tip in the fracture tests was detected by tracing the captured pictures. It should also be noted that, due to large deformation and incompressibility of SBR, a significant reduction in the thickness of the specimens is expected during the test, especially in the tensile test. Therefore, in order to prevent any slipping during the tests, a special fixture with roller grips, which can compensate the thickness reduction, was used in the experiments. The test setup and employed fixture are shown in Figure 3.

4 RESULTS AND DISCUSSION

4.1 Uniaxial tensile test: Selection of a hyperelastic model

In the tensile loading, for obtaining accurate strains, two marks were firstly painted on the dog-bone-shaped sample, and, by using the camera, the distance between these two marks was recorded during the test. Then, the tensile strain was evaluated via image processing and motion of the marks. The obtained nominal stress-stretch curve of the tested SBR is plotted in Figure 4. Subsequently, the Mooney-Rivlin hyperelastic model with the following SED function was fitted to the experimental stress-stretch curve, and the values of C10 = 0.228 MPa and C01 = 0.553 MPa were obtained for the SBR:

Figure 4

The nominal stress-stretch curve of the SBR [Colour figure can be viewed at wileyonlinelibrary.com]

Here, it should be noted that I1 and I2 are the first two principal invariants of the right Cauchy-Green deformation tensor. Moreover, it might be useful to note that the curve fitting procedure in the present study was performed using the Abaqus software. Indeed, the material coefficients of the hyperelastic models can be calibrated by Abaqus from experimental stress-strain data. The material constants are determined through a least-squares fitting procedure, which minimizes the relative error in stress.

Finally, from Figure 4, it is obvious that there is a very good agreement between the predictions of the Mooney-Rivlin model and the corresponding experimental data.

4.2 Fracture tests: The tearing behavior

Prior to the fracture tests, some white speckles were painted near the notch tip. These speckles were used as guide points for better detection of the rupture initiation in the SEVN samples. Hence, the critical displacements corresponding to the rupture initiation of the notched rubbers were obtained, and the average data for SEVN samples are given in Table 2.

In order to examine the tearing process and as an example, Figure 5 illustrates a sequence of the captured frames during the fracture test of the SEVN specimen with the notch depth of a = 10 mm. In the first frame of this figure, the specimen is undeformed. Then, as the deformation proceeds, the notch opens and notch blunting continuously increases. As is obvious in this figure, this blunting remains symmetric with respect to the notch bisector line during the test. The deformation continues until a crack initiates from the notch border, and then this initiated crack propagates through the whole specimen ligament.

From the last frame presented in Figure 5, it is also clear that the fracture trajectory is nearly perpendicular to the direction of the applied load. Similar fracture paths are also observed for SEVN samples of other notch depths (see Figure 6).

It may also be useful to study the failure surface of the tested samples. Indeed, it is interesting to compare the morphology of a ruptured surface with that cut with the razor blade. In this regards, some images taken by using an optical microscope from the ruptured and notch-border surfaces of a typical SEVN sample are shown in Figure 7.

Fractographic analysis of tested SBR: A, notch border surface cut with razor blade; B,C, ruptured surfaces

As it is clear from Figure 7, in the region located at the notch border, the surface has a very low roughness. This is because the notch border was cut via the razor blade. However, in the ruptured surface, higher roughness exists when compared with the cut surface. The higher roughness of the ruptured surface can be attributed to the resistance of rubber chains against the crack growth.

4.3 State of stress and orientation of principal stresses

The state of stress fields around the notch tip is investigated in this subsection via the FE method. It is useful to note that the FE analyses in the present study are performed using the commercial finite element code Abaqus. Two-dimensional models with eight-node plane-stress quadrilateral elements are used for the entire FE simulations. Furthermore, the reduced integration technique has been selected to remove the likely over stiffening effects due to volumetric locking which may occur considering the almost incompressible behavior of our rubber.43

Moreover, very fine mesh is employed around the notch tip to accurately capture the high stress/strain gradients in this region. Furthermore, since practically the tip of V-notch is not perfectly sharp and in order to avoid potential divergence problems due to large deformation, a very small notch tip radius is examined in the FE models (see Figure 8). This radius has been selected here to be 0.001 mm which is also consistent with the recommendations in Gomez and Elices.3 Figure 8 illustrates a typical mesh pattern employed in the FE simulation of the SEVN specimen. Also shown in this figure is the von Mises stress distribution around the notch apex for the SEVN sample with the notch depth of 4 mm in the undeformed configuration.

image

Figure 8

Open in figure viewerPowerPoint

A, A typical mesh pattern utilized in the FE analyses; B, von Mises stress distribution for the SEVN sample with the notch depth of 4 mm in the undeformed configuration [Colour figure can be viewed at wileyonlinelibrary.com]

On the other hand, and in order to study the state of stress near the notch tip at fracture initiation, the normal nominal stress components in the vertical (i.e., loading) and horizontal directions, namely σyy and σxx, are calculated through the notch bisector line by utilizing the FE data. The obtained results are shown in Figure 9 for a typical SEVN sample (a = 12 mm). According to this figure, the stress ratio urn:x-wiley:8756758X:media:ffe12947:ffe12947-math-0004 in the regions near the notch tip (i.e., r ~ 0.5 mm) is always larger than 10, and, thus, it can be inferred that the stress state in the proximity of the notch tip is almost uniaxial. It is important to emphasize that the obtained predominant uniaxial state of the stress fields close to the notch tip is also in agreement with the results of the previous work published by Arfaoui et al44 who theoretically analyzed the asymptotic finite plane deformation fields at the notch vertex of a hyperelastic material obeying the incompressible Mooney-Rivlin model.

Plot of the normal nominal stress components σyy and σxx through the notch bisector line near the notch tip [Colour figure can be viewed at wileyonlinelibrary.com]

Utilizing the FE results, the principal stress directions (θp in Figure 10) for some typical points around the notch tip in the SEVN sample with notch depth of a = 12 mm are calculated, and the results are presented in Table 3. Based on this table, the principal directions in the proximity of the notch tip are 90° ≤ θp ≤ 104°. This outcome also confirms the nearly uniaxial state of the stress field near the notch apex in the notched rubber with the Mooney-Rivlin constitutive model.

image

Figure 10

Table 3. The principal orientation θp(o) for several typical points near the notch tip in the SEVN specimen with a = 12 mm

Therefore, based on the results of this subsection, the key point of the ES criterion is upheld in the V-notched samples, and, hence, the extension of the criterion to the hyperelastic materials containing sharp V-notches is also feasible.

4.4 Experimental verification of the ES criterion

To employ the ES criterion in V-notched rubber parts, the first step is to obtain the critical values of λces and rc. It can be seen from Figure 4 that the rupture stretch of the employed SBR under tensile loading (i.e., λten) is 2.67. Hence, utilizing Equation 2, the critical value of the ES λces is obtained to be equal to 1.62. Then, the value of rc should be evaluated. Following the procedure explained in Section 2.3 and utilizing the experimental data obtained for the sample with the notch depth of 4 mm, the critical distance (rc) is found to be equal to 0.42 mm for the tested SBR. Figure 11 illustrates the details on how the critical distance is determined. It should also be noted that the reported distance is evaluated in the undeformed configuration of the notch.

It might be helpful to notice that the critical distance value rc should be obtainable from any of the tested samples with a good accuracy. Indeed, according to the concepts of the critical distance theory,45, 46 a notched sample fractures once its controlling parameter (here the chain stretch λch) at a critical distance from the notch tip (i.e., rc in the ES criterion) reaches the critical value of the criterion (i.e., λces).

In other words, if the ES value is plotted against the distance ahead of the notch tip, ideally, for all tested samples, the obtained curves should meet each other at a same point (rc, λces). Therefore, ideally, it is expected that samples with different notch depths should have the same rc. However, in reality and due to many unavoidable differences between the tested samples, a small reasonable scatter may exist between the obtained values of the critical distance for the notched specimens.

To shed more light on this point, the variations of the ES against the distance from the notch tip for the four tested SEVN samples have been calculated via the FE model and are illustrated in Figure 12.

ES value against the distance from notch tip for four different tested SEVN samples [Colour figure can be viewed at wileyonlinelibrary.com]

From Figure 12, it is clear that nearly all the curves pass through the same point at the critical ES. Thus, it can be concluded that, in our study, the critical distance can be considered as a constant value with a good accuracy.

After calibration of the main parameters of the ES criterion, the criterion can be utilized for rupture evaluation of other SEVN samples. To achieve this goal, a finite element analysis should be performed for each SEVN specimen following the points explained in Subsection 4.3. Then, the corresponding ES value at the critical distance of 0.42 mm should be monitored while the external load is increasing. When the ES at this point reaches the critical value of λces = 1.62, the corresponding applied load is the fracture load predicted based on the ES criterion.

The results of the ES criterion in comparison with their corresponding experimental values are shown in Figure 13. From this figure, good agreement is seen between the fracture displacement predictions and their corresponding experimental values. This consistency confirms the accuracy and applicability of the ES criterion for rupture assessment of V-notched rubbers under mode I loading. Moreover, since we have restricted our study to SBR under mode I loadings in the current study, the next logical step to assess the efficiency of the suggested ES criterion is to employ it for other rubbers and under mixed mode loading conditions as well.

5 CONCLUSIONS

In the present study, the ES criterion has been employed to investigate the fracture behavior of V-notched rubbers made of SBR under mode I loading. The ES criterion, which was originally suggested for cracked rubbers, is mainly based on the physics of rubbers and their large deformations. Therefore, first, we have explored the applicability of the ES criterion for the case of V-notched rubbers. Apart from the rubber microstructure which is the same for cracked and notched components, we have shown that, similar to cracks, the stress state near the notch tip is also almost uniaxial. Hence, the ES criterion can be applied in the case of V-notches as well. Next, we have calibrated the model via uniaxial and mode I fracture tests to obtain the values of critical ES and critical distance, respectively. The calibrated model is then employed to predict the fracture displacement of SEVN samples with different notch depths. The predicted displacements are finally compared with mode I fracture tests on SEVN samples, and good agreement is observed.