Mixed-mode (I/II) rupture assessment of rubber-like materials weakened by cracks using the averaged strain energy density criterion

M.Heydari-Meybodi M.R.Ayatollahi F.Berto

Abstract

In the present study, the application of the averaged strain energy density (ASED) criterion is extended from pure mode-I to mixed-mode (I/II) loading for hyperelastic materials. Indeed, the use of a recently proposed method by the present authors for determination of the critical values of strain energy density and radius of control volume in mode-I loading has been generalized to the mixed-mode loading conditions. The key point in the generalization is the almost uniaxial state of stress fields near the crack tip in rubber-like materials. To validate the accuracy of the criterion in mixed-mode (I/II) loading, a new set of experiments on rubbers weakened by cracks has been carried out. The experimental results confirm the accuracy of the ASED criterion in the case of cracked rubbers under mixed-mode (I/II) loading.

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Keywords

Strain energy density (SED)Hyperelastic materialsFracture testMixed-mode loadingFracture criterion

1. Introduction

Experimental and theoretical analysis of failure in different materials is a primary concern for designers of engineering structures, especially those weakened by a geometrical discontinuity like a crack. A key issue in development of a criterion is that the criterion has a sufficiently simple formulation and high reliability.

One of the most frequently used theories for failure analysis of engineering structures is the strain energy density (SED) concept. In the case of cracked specimens, Sih [1], [2], [3], [4] was the first in introducing the minimum SED criterion according to the critical value of SED at the boundary of a core region (or damage zone) around the crack tip. The main idea in development of the core region concept is that the continuum mechanics is not valid very close to the crack tip [5]. According to the Sih's criterion, brittle fracture in a cracked component occurs when the SED factor, S, reaches its critical value at the boundary of core region. Moreover, the crack growth direction can be determined by imposing a minimum condition on the SED factor [3], [6].

On the other hand, averaged SED (ASED) criterion is a SED-based fracture model in which the SED value is averaged over a control volume. The ASED criterion benefits from Neuber's concept of "elementary control volume" [7], Gillemot's concept of "specific absorbed energy" [8], [9], and "the dominance of local mode-I concept" proposed by Erdogan and Sih [10]. According to the ASED criterion, the critical load in a cracked specimen is obtained when the mean value of SED over a control volume, , attains its critical value, [11]. Indeed, the ASED approach is based on a precise definition of the radius of control volume, , as well as the value of . In this regard, for a linear elastic material and under static loading conditions, some useful expressions for these two parameters have been provided in the past based on the ultimate tensile stress and the fracture toughness of the selected material. However, these relations cannot directly be used for materials with non-linear behavior like elastomers and rubbers.

Rubber, which is often used interchangeably with the term elastomer, exhibits a non-linear stress-strain relationship. The wide variety of applications of rubber-like materials, ranging from automobile tires and gaskets in jet aircraft to heart valves, necessitates a good understanding of their behaviors under different types of mechanical loading.

Mode-I fracture analysis of hyperelastic materials weakened by cracks has been the topic of relatively extensive research in the past. In these cases, researchers have attempted to assess the load bearing capacity of cracked rubbers subjected to pure mode-I loading by using different fracture criteria like tearing energy [12], local energy release rate [13], SED [14], ASED [15], [16], effective stretch [17], and J-integral [14], [18] criteria. A good review of these criteria can be found in Ref. [17]. However, fracture in most structures does not occur only under one type of loading [19] and very often takes place under a combination of tension and shear loading which is known as mixed-mode (I/II) loading. Since under the mixed-mode (I/II) conditions the problem is more complex, only a limited number of studies have dealt with mixed-mode (I/II) fracture in rubber-like components. To the best of the authors' knowledge, only two sets of experiments performed on rubbers under mixed-mode (I/II) loading are available in the literature [20], [21]. Based on these experiments, some limited criteria have been proposed. Hamdi et al. [21] investigated the maximum principal stretch, maximum principal stress, J-integral and SED criteria for rupture assessment of rubbers under mixed-mode loading. Their results revealed that the SED fracture criterion could not be a good candidate for estimating mixed-mode fracture loads in rubbers. Moreover, Pidaparti et al. [20] applied the tearing energy criterion to predict fracture loads for arbitrarily shaped rubbers containing mixed-mode cracks. In addition, the effective stretch criterion, as a non-energy based model, has recently been extended to the mixed-mode (I/II) loading of hyperelastic materials [22].

As mentioned earlier, the ASED criterion is a frequently used criteria during the last decade for the fracture assessment in a large number of materials. Indeed, various types of materials subjected to different loading modes, and under static and fatigue loading, have successfully been analyzed by the ASED criterion [23], [24]. The application of ASED criterion in the prevalent mode-I conditions of hyperelastic materials has also confirmed its efficiency for these types of materials as well [15]. However, there is still no ASED-based fracture criterion in the literature for rubbers under mixed-mode (I/II) loading.

Considering the previously mentioned points, the present contribution deals with the mixed-mode (I/II) fracture assessment of cracked components made of rubber. The main purpose of this study is twofold:

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First, to provide a new set of experimental results on rupture displacement in cracked rubber subjected to mixed-mode (I/II) loading. These experimental data enlarge the scarce number of results available in the literature for rubbers subjected to mixed-mode (I/II) loading conditions and can be useful for future studies.

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The second aim is to extend the application of the ASED criterion from pure mode-I to in-plane mixed-mode (I/II) loading in hyperelastic materials weakened by cracks. Indeed, according to a novel method proposed recently by the present authors for determination of the values of and in cracked rubbers under pure mode-I loading [16], an attempt is made in this study to generalize the use of the related procedure for mixed-mode loading conditions. To validate the accuracy of this generalization, the above-mentioned mixed-mode (I/II) experimental data are utilized.

2. Experimental procedure

The experiments were performed on a Styrene Butadiene Rubber (SBR) filled with 25 phr of calcium carbonate (purchased from a local factory in Iran). The rubber shows substantially non-linear behavior and large deformations. For characterization of the basic mechanical properties of SBR, monotonic uniaxial tensile tests were conducted, using a universal testing machine (Santam Inc., Iran). The tensile tests were carried out at room temperature (25 °C) under a crosshead speed of 1 mm/min on the dumbbell-shaped samples with the thickness of 1.5 mm. For accurate measurement of the strain during the test, a Canon CMOS camera equipped with a Macro USM Fixed Lens was used. Indeed, the camera records the distance between two fine marks placed on the surface of specimen at any time interval and afterwards, the engineering strain can be measured with respect to the marks motions.

In the next stage, a set of mixed-mode (I/II) fracture tests was performed using the single edge crack (SEC) specimens with different crack angles. The cracks were introduced into the specimens by a sharp razor blade and with crack inclinations of β = 15, 30 and 45 degrees (see Fig. 1). The SEC specimens were prepared with the dimensions of 40 mm (length), 20 mm (width) and 1.5 mm (thickness). The mode-I fracture test (i.e., β = 0 degrees) was also conducted in order to obtain the critical radius of control volume, , required for the ASED criterion as described in more details later. The crack length, , is not constant for the explored SEC samples and its value will be presented later. The geometry and the dimensions of the SEC specimens are shown in Fig. 1.

Fig. 1. The geometry and dimensions of the SEC specimens used in this study (a represents the crack length).

Moreover, for more accurate determination of the onset of rupture in the SEC specimens, a digital camera in conjunction with two white-light sources (LED) were utilized during the fracture tests. Some white speckle pattern was painted near the crack tip in each case for better determination of the onset of crack propagation. Fig. 2 shows an illustration of the test set up.

Fig. 2. The experimental set up.

3. Theoretical background and statement of criterion

The strain energy is defined as the energy stored in a member due to the deformation. The strain energy density (SED), , for a material is generally known as the strain energy per unit volume, i.e.:

(1)

Although the SED value in a finite volume near the crack tip theoretically becomes infinite, the energy in the control volume practically has a finite value [6], [25], [26]. According to this concept, the ASED criterion states that for a cracked specimen when the SED value averaged over a circular control volume of radius reaches its critical value, , fracture will start. As a result, and are two main parameters involved in the ASED criterion whose determination has a paramount importance.

For a linear elastic and homogeneous material subjected to small strain loading, some closed-form relations for determination of the values of and have been presented in the past, as follows [27], [28], [29]:

(2)

where and are the fracture toughness and the ultimate tensile stress, respectively. Moreover, and denote the elastic modulus and Poisson's ratio of the material, respectively.

However, as describer earlier, the above-mentioned relations cannot be directly used for materials with non-linear behavior like rubbers. In this regard, and in order to facilitate the use of the ASED criterion in rubber-like materials, the present authors have recently developed a novel procedure for determination of the critical value of SED, , and the radius of control volume, , for these types of materials [16]. According to this method, the value of for a cracked rubber subjected to pure mode-I loading is equal to the critical value of SED obtained in the uniaxial tensile test, i.e.:

(3)

As noted in Ref. [16], the fundamental point in developing the novel method is the nearly uniaxial state of the stress fields next to the crack tip in hyperelastic materials.

The aim of the following subsections is to extend the use of the previously obtained procedure for determination of the values of and to the mixed-mode loading conditions. Therefore, its prerequisite is to reexamine the key point of the presented concept (i.e., the almost uniaxial state of stress field near the crack tip) in mixed-mode (I/II) loading as well.

3.1. Stress field near the crack tip in mixed-mode loading

In ordinary linear elastic materials and under small strain conditions a complex state of stress exists near the crack tip. However, for hyperelastic materials, a nearly uniaxial state of stress field occurs in the proximity of the crack tip not only in pure mode-I but also in mixed-mode (I/II) loading (Fig. 3). This crucial feature of rubbers in mixed-mode loading has been proven in the past via the numerical [22], [30] and analytical [30] studies.

Fig. 3. Nearly uniaxial state of stress field in proximity of the crack tip in mixed-mode loading of a cracked rubber.

3.2. Determination of in mixed-mode loading

As described above, since the crack tip stress field is still in the state of uniaxial tension, it is expected that the same conditions exist between the rupture of a cracked rubber subjected to mixed-mode loading and its uniaxial tensile test. Therefore, the method presented for determination of the value of in mode-I loading can now be extended to the mixed-mode conditions as follows:

“The value of critical SED for a cracked hyperelastic material, subjected to either mode-I loading or mixed-mode one, is equal to the critical value of SED obtained in the uniaxial tensile test”, i.e.:

(4)

From Eq. (4), it can be further deduced that the value of does not depend on the mode of loading and in fact, it is merely a material-dependent parameter.

More details about the determination of according to the experimental data is discussed in the next subsection.

3.3. ASED criterion in mixed-mode loading

The general concepts of ASED criterion are the same for the linear elastic materials and for the hyperelastic ones. According to this criterion, fracture occurs when the value of averaged over the circular control volume of radius , reaches the critical value of . The only difference between the linear elastic and hyperelastic materials is that in the case of rubber-like materials, a non-linear analysis should be performed to evaluate the values of main parameters involved in the ASED criterion (i.e., and).

Before describing the procedure needed for determination of the values of and , and because of the importance of SED functions in hyperelastic materials, it might be helpful to discuss first about SED functions. The SED function plays an important role in mechanical characterization of hyperelastic materials. Indeed, the SED functions are hyperelastic constitutive laws which are used to model materials subjected to very large strain. Moreover, in a hyperelastic material the stress-strain relation is calculated via the SED function. To this end, various types of constitutive models have been presented in the past for characterization of hyperelastic materials. These models, in general, can be categorized into phenomenological models (e.g. Neo-Hookean, Mooney-Rivlin [31], Ogden [32]) and physically-based ones (e.g. 3-chain network [33], [34], 4-chain network [35], [36] and 8-chain network [37] models). Refs. [38], [39] provide a good review for these types of models.

According to these models, the SED function, W, can be expressed in terms of the principal invariants (i.e.,) of the right Cauchy–Green deformation tensor, C, or principal stretches of , and [40]. It is worth mentioning here that the invariants of C can be written in terms of the principal stretches as follows:

(5)

On the other hand, because of the incompressible nature of rubber-like materials, W is not dependent on the third invariant of C. Therefore, W can be described as a function of two other invariants of C as follows:

(6)

For determination of the values of and , a coupled experimental-numerical procedure is suggested as follows:

For determining the value of , only a simple uniaxial tensile test is needed. In fact, after obtaining the stress-strain curve of the selected material from a uniaxial tensile test, the rupture stretch of rubber, , should be collected. It is helpful to note that the stretch ratio is defined as the ratio of the final length of a material line to its initial length.

After that, based on the hyperelastic material model chosen for the target rubber, the value of should be inserted into the selected SED function.

Considering the incompressible nature of rubbers, the following relations are established from the rupture condition of a uniaxial tensile test:

(7)

Furthermore, upon substitution of Eq. (7) into Eq. (5), the critical values of the first two invariants of C can be expressed in terms of as follows:

(8)

Therefore, the critical value of SED, depending on the type of chosen SED function (i.e., stretch-based or invariant-based one), can be computed as follows:

(9)

Determination of the value of : it should be pointed out that providing a closed-form relation for the value of in hyperelastic materials is far from easy [15], but the shape of control volume centered at the crack tip in these types of materials is supposed to be the same as that used for linear elastic materials (see Fig. 4).

Fig. 4. The shape of control volume centered at the crack tip based on the ASED criterion.

To obtain the value of , first, a mode-I fracture test should be conducted on a rubber weakened by a crack in order to determine its fracture load (or stretch). Afterwards, the mode-I fracture test should be simulated by finite element modeling (FEM). Finally, the graph of the SED values averaged over the control volume, , versus different radii of the control volume, should be drawn. The radius at which meets gives the value of .

4. Finite element analyses

For evaluation of the values of SED averaged over the control volume and the value of , finite element analysis should be employed, in particular, because of the non-linear nature of the problem for hyperelastic materials. Therefore, in the present study, the SEC samples were numerically modeled using the commercial finite element code Abaqus®. In addition, the Mooney-Rivlin strain energy potential was employed in order to apply the non-linear material behavior in FEM. It should be further noted that because of the large deformation nature of the problem in rubber-like materials, especially near the stress-concentrator regions, the crack tip is usually modeled as a very narrow blunted notch [17], [22]. However, in the computation of the value of SED averaged over the control volume, the region near the crack tip should be included in the control volume. Thus, a very small crack tip radius of 0.005 mm was considered in our finite element analyses. In addition, a very fine mesh was used in the vicinity of the crack tip to preserve the convergence of numerical analyses. Eight-node plane-stress elements were utilized in the finite element simulations. Fig. 5 shows a typical mesh pattern employed near the crack tip as well as the SED contour lines inside the control volume for the case of .

Fig. 5. (a) A typical mesh pattern near the crack tip, (b) The SED contour lines (in MJ/m3) used in a mixed-mode SEC sample .

5. Results and discussions

5.1. Determination of the constitutive law

Selection of a hyperelastic material model for the rubber is the primary step towards employing the ASED criterion. To this end, a curve fitting analysis was performed with the uniaxial tensile test results obtained for the selected SBR sample (shown in Fig. 6) and then, the Mooney-Rivlin SED function, which had a good correlation with the experimental data, was employed. According to the Mooney-Rivlin hyperelastic material model, W is a combination of the two first invariants of C as follows [41]:

Fig. 6. Comparison of the fitted Mooney-Rivlin material model and the uniaxial tensile test results.

In this study, and were obtained for the SBR. The comparative stress-strain curves obtained from the fitted hyperelastic material model and the corresponding experimental data are depicted in Fig. 6. As can be seen from Fig. 6, very good agreement exists between the fitted Mooney-Rivlin model and the experiments.

5.2. Fracture tests

The mixed-mode fracture tests for the SEC samples containing the crack with the length of a = 10 mm were performed until the final rupture of the specimens. Moreover, as stated earlier, the onset of crack growth was determined by using a digital camera. The pure mode-I fracture test (i.e.,) was also carried out for the SEC sample with the crack length of a = 8 mm in order to compute the radius of control volume in the next subsection. Table 1 presents the vertical displacement at the onset of crack propagation, , for the SEC samples.

Another interesting point observed in the rupture tests of rubbers under mixed-mode loading is the nearly horizontal crack growth path. In other words, as it can be observed in Fig. 7, independent of the crack angle, the inclined cracks propagate almost perpendicular to the loading direction. Similar to this behavior has been already reported in other studies [20], [21] as well.

Fig. 7. The crack growth path for the SEC specimens subjected to mixed-mode (I/II) loading.

5.3. Verification of ASED criterion in mixed-mode loading

The ASED criterion described in the previous section is utilized here to assess the critical displacement of the SEC rubber specimens under mixed-mode loading. In the first step, the value of for the selected SBR should be computed. According to the uniaxial tensile test data presented in Fig. 5, the critical rupture stretch, , is equal to 2.76. Substitution of this value into Eq. (8) yields and . Therefore, by substitution of the obtained critical values of the first two invariants of C into the Mooney-Rivlin SED function (Eq. (10)), the value of Wc = 0.487 MJ/m3 is obtained for the employed rubber.

In the next step, to calculate the value of , the results obtained from the mode-I SEC specimen are taken into account and then, a finite element simulation for the specimen with its critical displacement reported in Table 1 is performed. During the simulation, various values of are examined and their related values of are obtained. Afterwards, the graph of versus is plotted and the point where meets is determined. As shown in Fig. 8, the critical radius is determined to be equal to mm for the target SBR.

Fig. 8. Determination of the critical radius of control volume from the mode-I rupture experiment for the tested SBR.

After determining the material properties of the employed SBR (i.e., and), the ASED criterion proposed for mixed-mode loading of hyperelastic materials should be employed to predict the critical displacements in the SEC specimens of different crack angles. To this end, finite element simulation is performed for each of the mixed-mode test specimens. The magnitude of the applied displacement in the finite element model is increased up to a point where the value of inside the control volume reaches the critical value of MJ/m3. The corresponding displacement applied in the finite element model is then considered as the critical displacement predicted using the ASED criterion.

The theoretical estimates based on the ASED criterion are shown in Fig. 9 in comparison with the experimental data obtained for the SEC specimens of different crack angles. From the good agreement that exists between the experimental results and theoretical estimates, it can be deduced that in addition to mode-I loading, the ASED criterion can also be utilized successfully to predict the critical displacement of the rubbers under the mixed-mode (I/II) loading conditions. However, it would be very useful to perform further experiments on other rubber-like materials and mixed-mode test specimens of different shapes to provide additional verifications for the extended criterion proposed in this study.

Fig. 9. Predicted results of the ASED criterion for critical displacements in comparison with the experimentally obtained rupture results.

6. Conclusions

The averaged strain energy density (ASED) criterion is mainly based on the precise definitions of two independent parameters (namely the radius of control volume, , and the critical strain energy density value,). However, the available conventional relationships for these two parameters are restricted to linear elastic hypotheses and they cannot be utilized directly in materials with large deformation and non-linear behavior. Therefore, having a reliable method for determination of these two parameters in hyperelastic materials makes it possible to use the ASED criterion in rubber-like materials as well. Recently, a novel method was proposed by the authors for determination of the value of in cracked rubber specimens under pure mode-I loading [16].

In the present study, the strain energy density criterion averaged over the control volume (i.e., ASED criterion) was used to investigate the static strength of cracked rubber components subjected to in-plane mixed-mode (I/II) loading. The key point in the generalization of the ASED criterion was mainly based on the almost uniaxial state of stress field next to the crack tip in hyperelastic materials. Moreover, a new set of experiments on rubber subjected to mixed-mode loading was carried out. The results of experiments highlighted that, despite different initial crack inclinations, the crack growth path was almost perpendicular to the loading direction. Finally, in order to evaluate the accuracy of the extended criterion in mixed-mode loading, the ASED criterion was used through a non-linear finite element analysis to assess the critical displacements in the mixed-mode experiments. Very good agreement was shown to exist between the predictions of the extended criterion and the corresponding experimental data.